Supplement to 'Simulation of Steady State Energy Metabolism in Cyling and Running'

We take the equations for the activation of the oxygen uptake (Eq. 1) and of the lactate production (Eq. 2).

$$\dot{V}O_2 = \frac{\dot{V}O_{2,max}}{1 + (\frac{K_{ox}}{[ADP]})^{n_{ox}}}$$

$$\dot{c}La_{pr} = \frac{\dot{c}La_{max}}{1 + (\frac{K_{La}}{[ADP]})^{n_{La}}}$$

We normalize the rate of oxygen uptake and the rate of lactate production in relation to its natural reference values VO_2,max and cLa_max:

$$\%\dot{V}O_2 = \frac{\dot{V}O_2}{\dot{V}O_{2,max}} = \frac{1}{1 + (\frac{K_{ox}}{[ADP]})^{n_{ox}}}$$

$$\%\dot{c}La_{pr} = \frac{\dot{c}La_{pr}}{\dot{c}La_{max}} = \frac{1}{1 + (\frac{K_{La}}{[ADP]})^{n_{La}}}$$

Rewriting the normalized oxygen uptake equation for ADP:

$$
\begin{aligned}
\%\dot{V}O_2 &= \frac{1}{1+(\frac{K_{ox}}{[ADP]})^{n_{ox}}} \\
\iff \frac{1}{\%\dot{V}O_2} - 1 &= (\frac{K_{ox}}{[ADP]})^{n_{ox}} \\
\iff (\frac{1-\%\dot{V}O_2}{\%\dot{V}O_2})^{\frac{1}{n_{ox}}} &= \frac{K_{ox}}{[ADP]} \\
\iff [ADP] &= K_{ox}(\frac{1-\%\dot{V}O_2}{\%\dot{V}O_2})^{-\frac{1}{n_{ox}}}
\end{aligned}
$$

Plug in the result into the normalized lactate production equation:

$$
\%\dot{c}La_{pr} = \frac{1}{1+(\frac{K_{La}}{(K_{ox}(\frac{1-\%\dot{V}O_2}{\%\dot{V}O_2})^{-\frac{1}{n_{ox}}})})^{n_{La}}}
$$

$$
\iff \%\dot{c}La_{pr} = (1+\frac{K_{La}}{K_{ox}}(\frac{1-\%\dot{V}O_2}{\%\dot{V}O_2})^{\frac{1}{n_{ox}}})^{n_{La}})^{-1}
$$

The equation for the rate of lactate removal (Eq. 3) is:

$$
\dot{c}La_{re} = \dot{V}O_2 * k_{re}
$$

normalized to:

$$
\%\dot{c}La_{re} = \frac{\dot{c}La_{re}}{\dot{c}La_{max}} = \dot{V}O_2 * k_{re} * \dot{c}La_{max}{}^{-1}
$$

We replace VO2 with (%VO2*VO2max)

$$
\%\dot{c}La_{re} = \%\dot{V}O_2 * k_{re} * \frac{\dot{V}O_{2,max}}{\dot{c}La_{max}}
$$

We define the net lactate accumulation rate (Eq. 4):

$$
\dot{c}La_{net} = \dot{c}La_{pr} - \dot{c}La_{re}
$$

normalized to:

$$\%\dot{c}La_{net} = \frac{\dot{c}La_{net}}{\dot{c}La_{max}} = \%\dot{c}La_{pr} - \%\dot{c}La_{re}$$

Plug in the equations for %cLa_pr and %cLa_re into this yields Equation 6:

$$\%\dot{c}La_{net} = (1 + \frac{K_{La}}{K_{ox}}(\frac{1 - \%\dot{V}O_2}{\%\dot{V}O_2})^{\frac{1}{n_{ox}}})^{n_{La}})^{-1} - \%\dot{V}O_2 * k_{re} * \frac{\dot{V}O_{2,max}}{\dot{c}La_{max}}$$