Question #1

(א) y(x1, x2, x3, x4) = sigmoid(x1*w1 + x2*w2 + x3*w3 + x4*w4)

(ב)
Let z=x1*w1+x2*w2+x3*w3+x4*w4, so that y=σ(z)=1/(1 + e^-z)

The derivative of σ(z) with respect to z is: σ′(z)=σ(z)(1−σ(z))

Now, we compute ∂y/∂wi for each i=1,2,3,4Using the chain rule: ∂y/∂wi= dσ(z)/dz * ∂z/∂wi

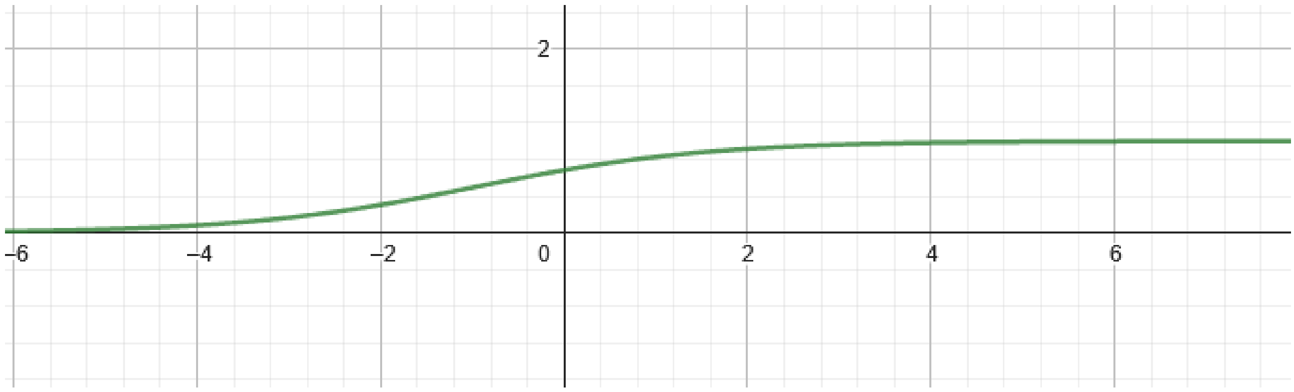
From the definition of z, we have: ∂z/∂wi=xi

∂y/∂wi=σ(z) *(1−σ(z))*xi

Result: ∂y/∂wi=σ(x1w1+x2w2+x3w3+x4w4)*(1−σ(x1w1+x2w2+x3w3+x4w4))*xi

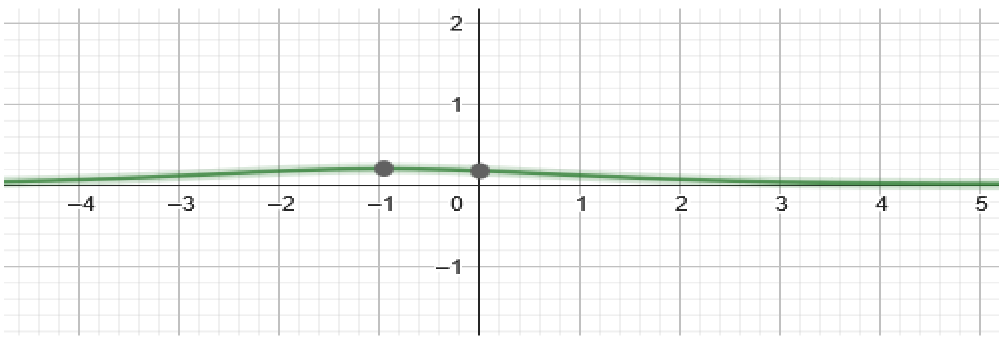
(ג) Random numbers:
x2 = 0.210851, x3 = 0.7781711, x4 = 0.3505987
w1 = 0.7944766, w2 = 0.6926461, w3 = 0.4358078, w4 = 0.7908785
x*w1 + x2*w2 + x3*w3 + x4*w4 = 0.7944766x + 0.7624591

y(x) = Sigmoid(0.7944766x + 0.7624591)



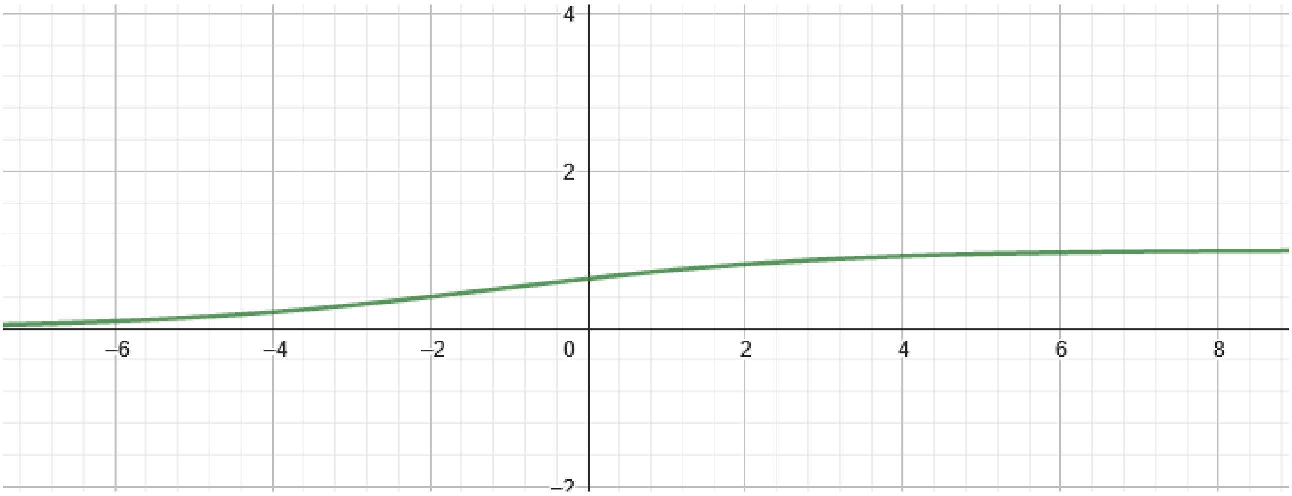
(ד)

$$\frac{0.7944766e^{-0.7944766x-0.7624591}}{\left(1+e^{-0.7944766x-0.7624591}\right)^2}$$



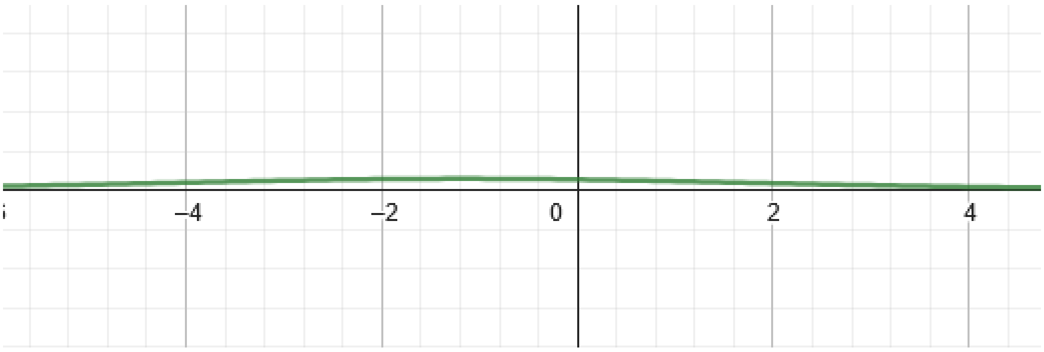
(ה)
set inputs and weights to random numbers:
x1 = 0.469499, x2 = 0.681902, x3 = 0.460511, x4 = 0.584383
w2 = 0.126323, w3 = 0.732136, w4 = 0.19543

z(w) = w*x1 + w2*x2 + w3*x3+w4*x4 = 0.554998 + 0.469499w
y(w) = Sigmoid(z(w))



y'(w) =

$$\frac{0.469499e^{-0.469499w-0.554998}}{\left(1+e^{-0.554998-0.469499w}\right)^2}$$



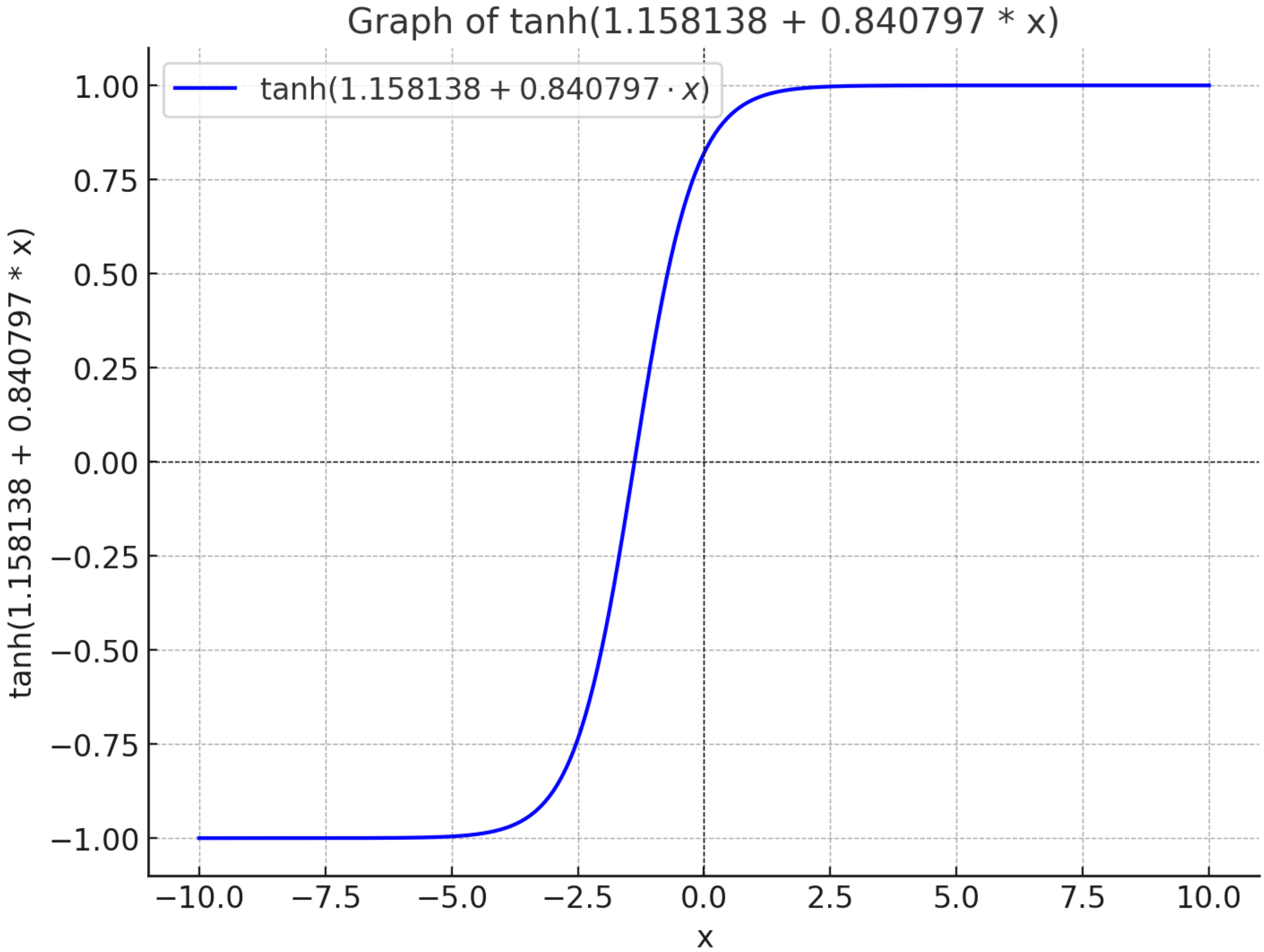
(ו)
y(x1, x2, x3, x4) = tanh(x1*w1 + x2*w2 + x3*w3 + x4*w4)

$$\frac{\partial y}{\partial w_i} = \frac{4e^{-2(x_1w_1+x_2w_2+x_3w_3+x_4w_4)}}{\left(1+e^{-2(x_1w_1+x_2w_2+x_3w_3+x_4w_4)}\right)^2} \cdot x_i$$

setting random values:

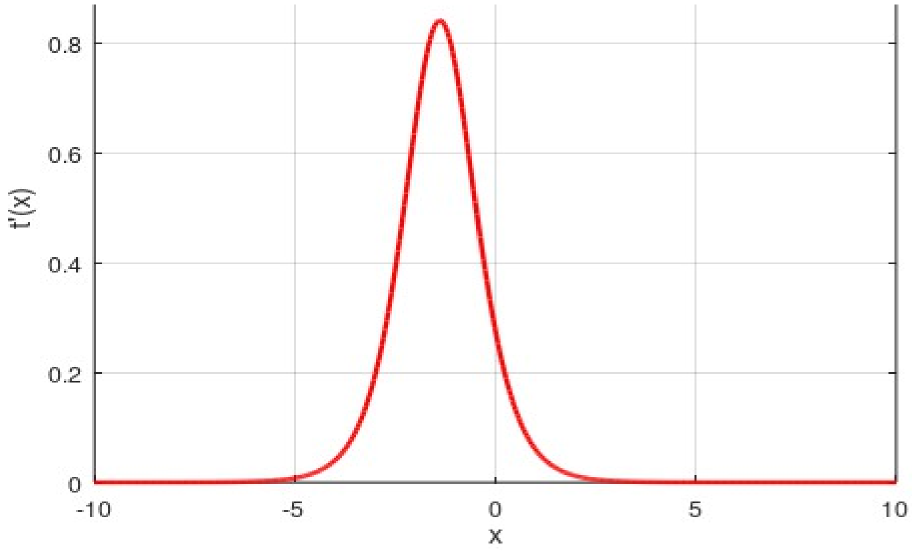
x2 = 0.625869, x3 = 0.109058, x4 = 0.681813
w1 = 0.840797, w2 = 0.955199, w3 = 0.100481, w4 = 0.805718

y(x) = tanh(w1*x + w2*x2 + w3*x3 + w4*x4) = tanh(1.158138 + 0.840797*x)



$$t'(x) = 3.363188 \cdot \frac{e^{-z}}{(1+e^{-z})^2}$$

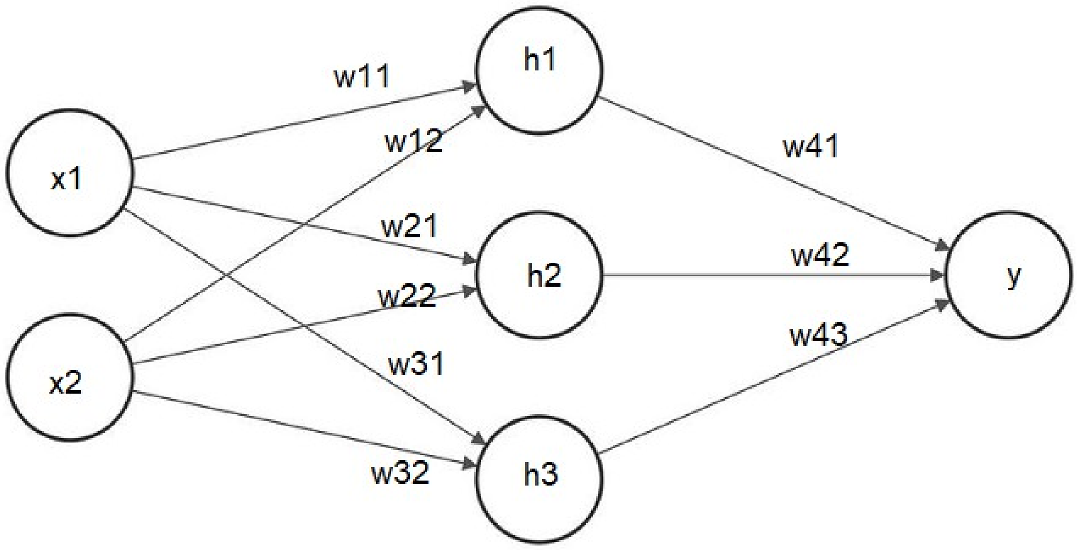
where $z = 2(1.158138 + 0.840797x)$.



(ז) tanh function is a shifted and stretched version of the sigmoid.

Question #2

(א)



h1=sigmoid(w11*x1+w12*x2)
h2=sigmoid(w21*x1+w22*x2)
h3=sigmoid(w31*x1+w32*x2)

(s = sigmoid)
y=s(w41*h1+w42*h2+w43*h3) =>
y=s(w41*s(w11*x1+w12*x2)+w42*s(w21*x1+w22*x2)+w43*s(w31*x1+w32*x2))

(ב)
Selected function is h1, then

dy/dh1 = s(w41*h1+w42*h2+w43*h3)*(1 – s(w41*h1+w42*h2+w43*h3))*w41
s = sigmoid

(ג)
Selected function is x1, then

$$\frac{\partial y}{\partial x_1} = f'(w_{41}h_1 + w_{42}h_2 + w_{43}h_3) \cdot \sum_{i=1}^{3} [w_{4i} \cdot f'(w_{i1}x_1 + w_{i2}x_2) \cdot w_{i1}]$$

f = sigmoid

(ד)
Set randoms:

w11=0.259893
w12=0.839863
w21=0.199095

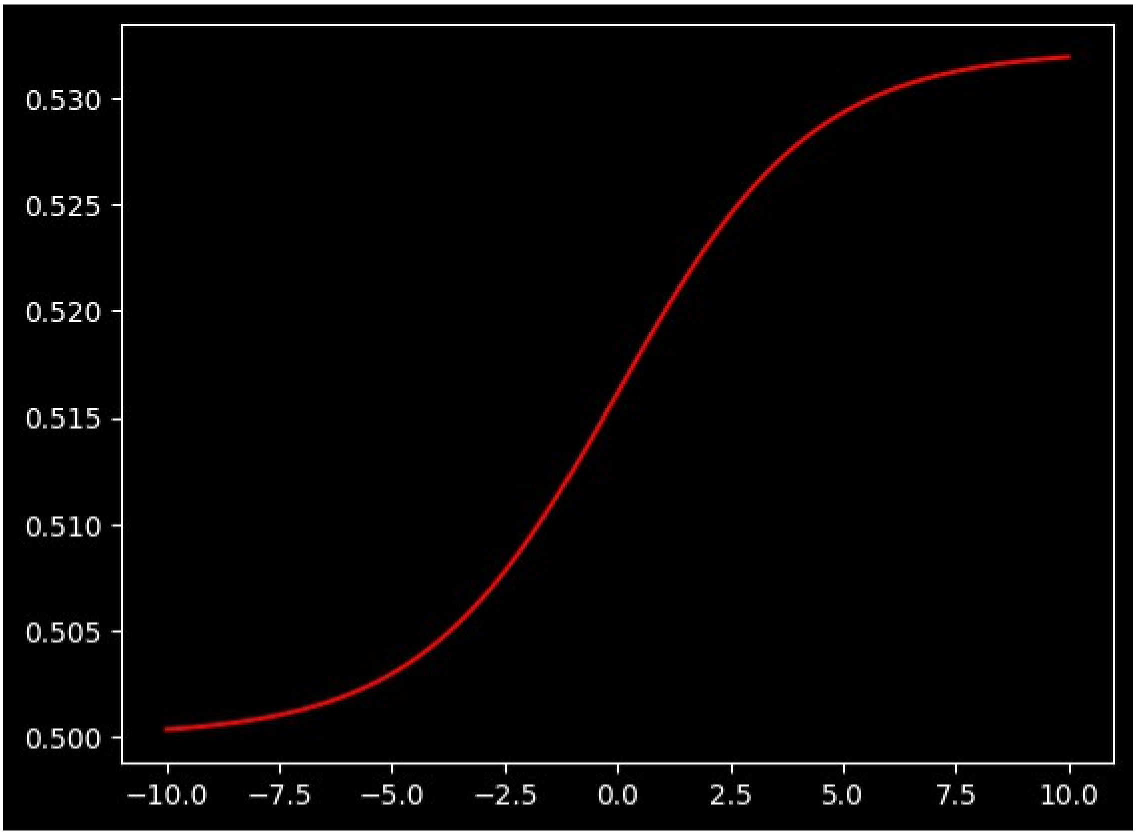
w22=0.750530
w31=0.450002
w32=0.122721
w42=0.063974
w43=0.018475
w41=0.046841
x2=0.295953

re-calculate functions:

h1=s(w11*x1+w12*x2)=s(0.5085*x1)
h2=s(w21*x1+w22*x2)=s(0.4212*x1)
h3=s(w31*x1+w32*x2)=s(0.4863*x1)

y=s(w41*h1+w42*h2+w43*h3)
 = s(0.046841*s(0.5085*x1) + 0.063974*s(0.4212*x1) + 0.018475*s(0.4863*x1))

plot:



code used to generate the plot:

```
import math
import numpy as np
import matplotlib.pyplot as plt

def s(x):
    return 1 / (1 + math.e ** (-x))
```

```
def y(x1):
    return s(0.046841*s(0.5085*x1) + 0.063974*s(0.4212*x1) + 0.018475*s(0.4863*x1))

x = np.linspace(-10, 10, 100)

plt.plot(x, y(x), color='red')
```