Chapter 6: Convective Heat Transfer

For convective heat transfer we use fluid as the medium to transport fluid. We have far more variables now so non dimensionalizing gets rid of too many things to track

Nusselt Number

$$Nu = \frac{h L_c}{k}$$

Physical Meaning of Number

$$\frac{\dot{q}_{conv}}{\dot{q}_{cond}} = \frac{h \Delta T}{k \Delta T / L} = \frac{hL}{k} = Nu$$

This number represents the enhancement of heat transfer due to the convection.

$$Nu \equiv 1 \quad \text{(means just conduction)}$$

Important Equations.

$$\dot{Q}_{conv} = h A_s (T_s - T_\infty) \qquad \tau_s = \mu \frac{\partial u}{\partial y}\bigg|_{y=0}$$

$$h = \frac{-k_f (\partial T / \partial y)_{y=0}}{T_s - T_\infty} \qquad \tau_s = C_f \frac{\rho V^2}{2}$$

Different types of fluid flow.

We classify flow by ~~its~~ properties.

Viscous or ~~Stat~~ Inviscid

Internal ~~vs~~ External

Compressible vs Incompressible

Prandtl Number

$$Pr = \frac{\nu}{\alpha} = \frac{\mu C_p}{k}$$

This is the number that tells you the ratio of diffusivity of momentum or heat.

Convective Analysis

For a circular pipe:

$$V_{avg} = \frac{2}{R^2}\int_0^R u(r)\cdot r\,dr$$

$$T_{avg} = \frac{2}{V_{avg}R^2}\int_0^R T(r)\cdot u(r)\cdot r\,dr$$

$$\boxed{\begin{aligned} Re &= \frac{4\cdot \dot{m}}{\mu \pi D} \\ &= \frac{\rho D}{\mu}\left(\frac{\dot{m}}{\rho \pi D^2/4}\right) \\ &= \frac{V_{avg}\cdot D}{\nu} \end{aligned}}$$

For pipe flow

Smooth surface → likely laminar

Rough surface → likely turbulent.

Circular Pipes vs. Rectangular Ducts

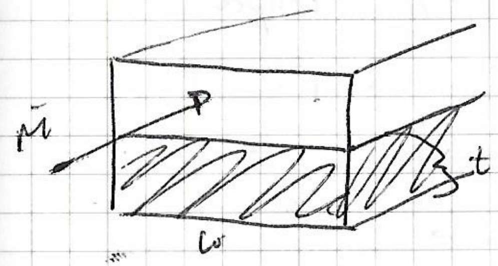
with steal pressure

bal at with standing pressure.

However we use the hydraulic diameter

$$D_h = \frac{4A_c}{P}$$

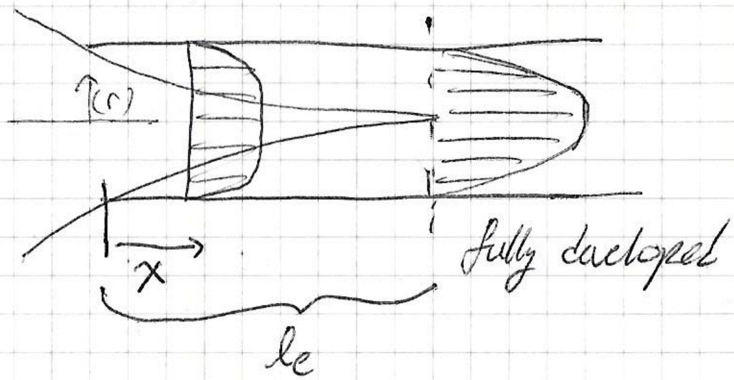
Where A_c are P are the dimensions of the wetted dimensions



$$D_h = \frac{4wt}{2w+2t}$$

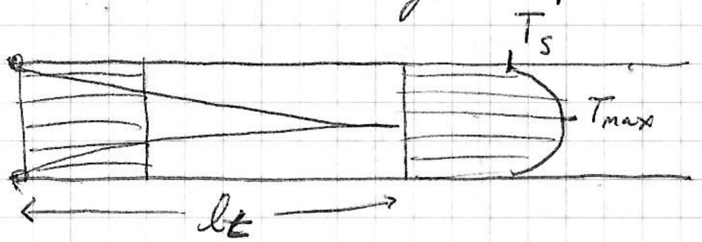
Circular are far more popular due to their symmetric conditions.

Entrance Regions:



The entrance region is the section where viscouse effects are important.

— Thermal Fully Developed —



$$T_s = \frac{T_{m,i} + T_{m,e}}{2}$$

l_t : thermal entrance region.

Property of Fully Developed

Velocity:

$$\frac{\partial u(r,x)}{\partial x} = 0$$

Temp:

$$\frac{\partial}{\partial x}\left[\frac{(T_s(x) - T(r,x))}{T_s(x) - T_m(x)}\right] = 0$$

The slope change of the dimensionless temp remains zero at fully developed regions.

Shear Stress

$$\tau_w = \mu \frac{\partial u}{\partial y}\bigg|_{y=0}$$

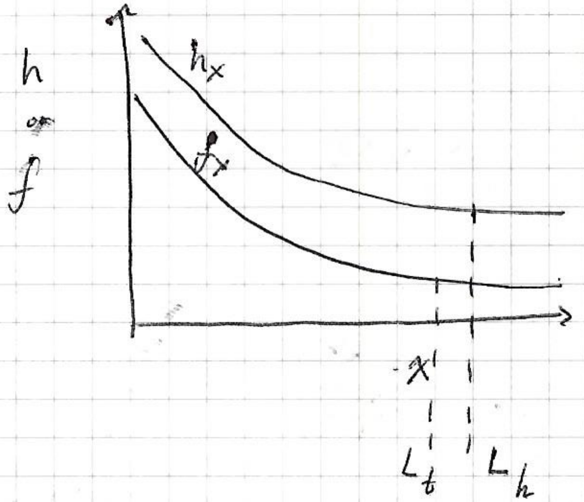
Surface Heat Flux

$$q_s = h_x(T_s - T_m)$$

$$= k \frac{\partial T}{\partial r}\bigg|_{r=R}$$

$$\boxed{h_x = \frac{k(\partial T \backslash \partial r)|_{r=R}}{T_s - T_m}}$$

Heat Transfer Coefficient and Friction Coeff



There are 2 different entrance effects

$$\boxed{L_f < L_h}$$

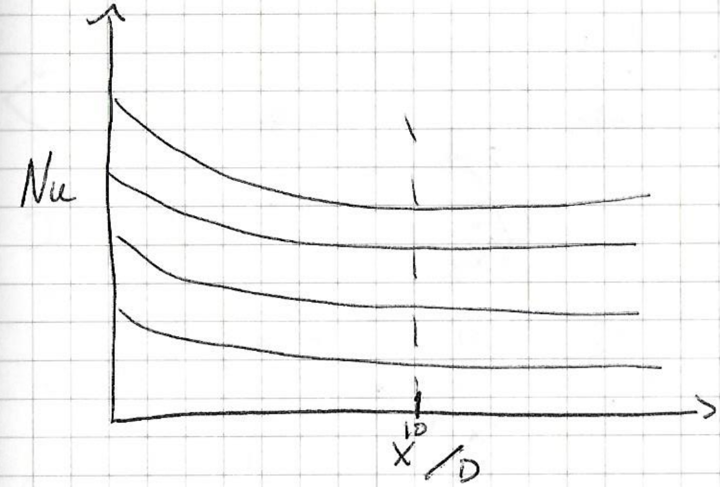
The thermal entrance length is always larger than the friction length coefficient.

$$L_{h,\,lam} = 0.05\,Re \cdot D$$

$$L_{t,\,lam} = 0.05\,Re\,Pr\,D$$

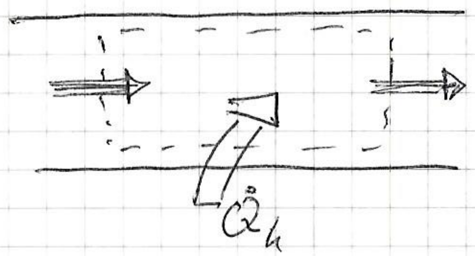
$$= L_{h,\,lam} \cdot Pr$$

Nusselt # and X/D



After 10D the Nu can be assumed to be constant. That is after significant difference we can make thing simpler.

Energy Balance



$$\dot{Q} = -\dot{m} c_p T_i + m c_p T_e$$

$$\dot{Q}_h = h_x A_s (T_s - T_M)$$

Temp at Exit

$$T_e = T_i + \frac{\dot{q}_s A_s}{m c_p}$$

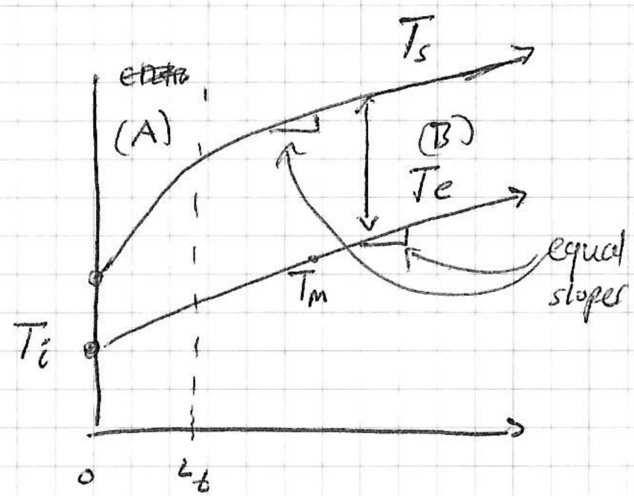
Surface Temp

$$T_s = T_M + \frac{\dot{q}_s}{h}$$

Graph

(A): Entrance Region

(B): Fully Developed



You will never find a case where you have

<Impossible> $\boxed{\text{constant surface and constant max temp}}$

It is impossible to have a costant surface temp and constant T_m. Or else there would be NO Heat Transfer!

Graphically

$$\frac{\partial T_s}{\partial x} = \frac{\partial T_M}{\partial x}$$

$$\frac{\partial T}{\partial x} = \frac{dT_s}{dx} = \frac{dT_M}{dx} = \frac{\dot{q}_s P}{\dot{m} c_p} \quad \text{(fully developed)}$$

Where p is the perimeter of a pipe.

T_s = Constant

$dA_s = p\,dx$

$$\dot{m} c_p \, dT_m = h(T_s - T_m)\, dA_s$$

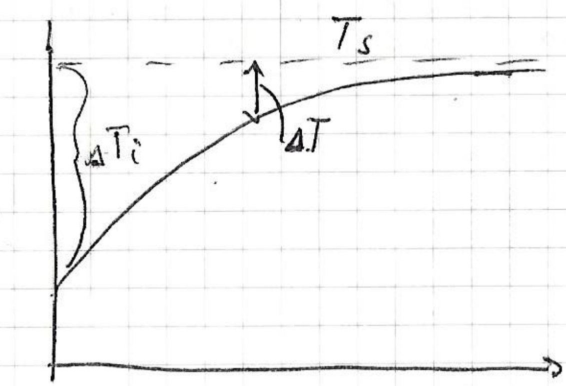
$$dT_m = -d(T_s - T_m)$$

$$\frac{d(T_s - T_m)}{T_s - T_m} = -\frac{hp}{\dot{m} c_p}\, dx$$

$$\ln\left(\frac{T_s - T_e}{T_s - T_i}\right) = -\frac{hA_s}{\dot{m} c_p}$$

$$\boxed{T_e = T_s - (T_s - T_i)\, e^{-\frac{hA_s}{\dot{m} c_p}}}$$

Graph



NTU : Number of Transfer Units

$$NTU = \frac{hA_s}{\dot{m} c_p}$$

A measure of how much heat can be transfered during flow.

If $NTU = 5$; $T_e = T_s$ at the end regardless of pipe length.

$$\ln \frac{T_s - T_e}{T_s - T_i} = -\frac{hA_s}{\dot{m} c_p}$$

$$\dot{m} c_p = \frac{hA_s}{\ln(\Theta_e / \Theta_i)}$$

$$\dot{Q} = hA\,\Delta T_{ln}$$

Where

$$\Delta T_{ln} = \frac{T_i - T_e}{\ln(\Theta_e / \Theta_i)}$$

$$= \frac{\Delta T_e - \Delta T_i}{\ln(\Theta_e / \Theta_i)}$$

We want to find "h" through in the study of Convective heat transfer.

$$\frac{dP}{dx} = K \quad \text{(fully developed)}$$

$$u(r) = 2V_{avg}\left(1 - \frac{r^2}{R^2}\right)$$

$$\Delta P = \frac{32 \mu L V_{avg}}{D^2}$$

Pump Power

$$\dot{W}_{pump} = \dot{V} \Delta P_L \qquad \dot{W}_{pump} = 8\pi\mu L (V_{avg})^2$$

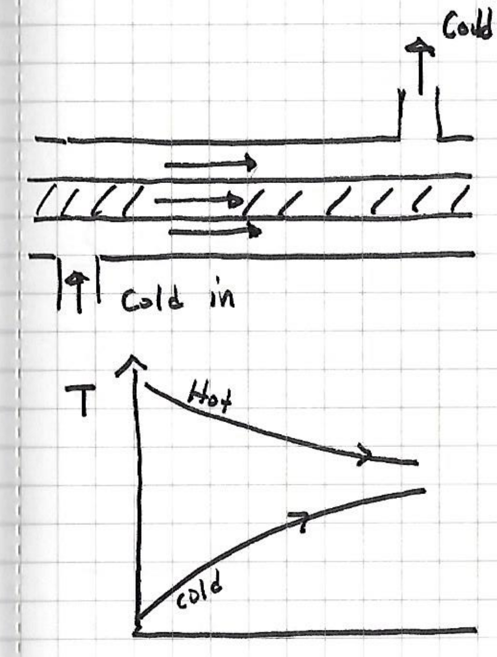
$$\dot{W}_{pump} = \dot{V} \rho g h_L \qquad = 8\pi\mu L \left(\frac{\dot{m}}{\frac{1}{4}\pi D^2}\right)^2$$

$$\dot{W}_{pump} = \dot{m} g h_L$$

Poiseuille's Law

$$Q = \frac{\Delta P \pi D^4}{128 \mu L}$$

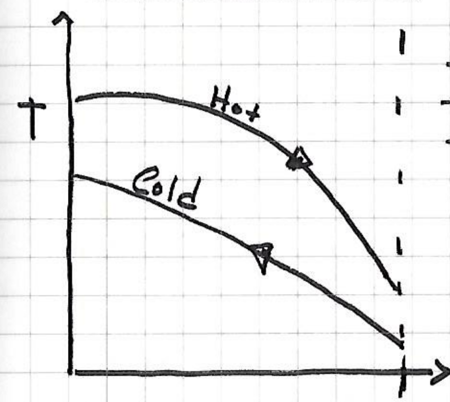
Heat Exchangers

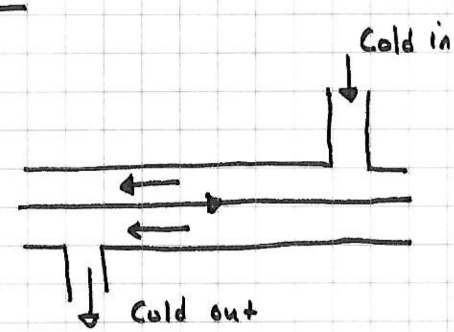


Parallel Flow

- doesn't wick away hot fluid
- Slower

Counter Flow





- Keeps an almost constant ΔT between fluids

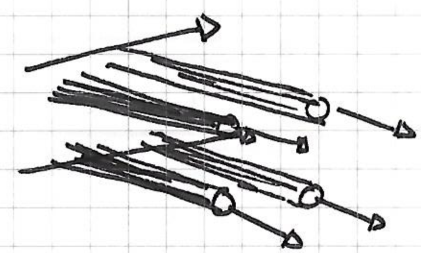
Compact Heat EXN

Increases surface area per volume to transfer heat in SMALL places

$$\alpha = \frac{S_A}{V} \cdot \frac{(\text{surface area})}{(\text{volume})}$$

Cross flow

The ΔT fluids cross perpindicular to each other.



Case 1

Mixed

No sepperation or ducts to guide fluid

Unmixed

fluid through tubes

CASE 2

Unmixed

- fluid passes through "guide plates"

Unmixed

- fluid through tubes

Tube and Shell

We send fluids through tubes encased by shells. Each will have its own $\dot{m}$ and easier flow passage.

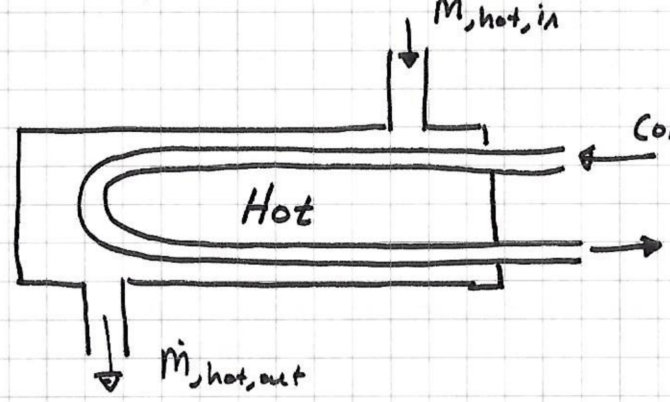
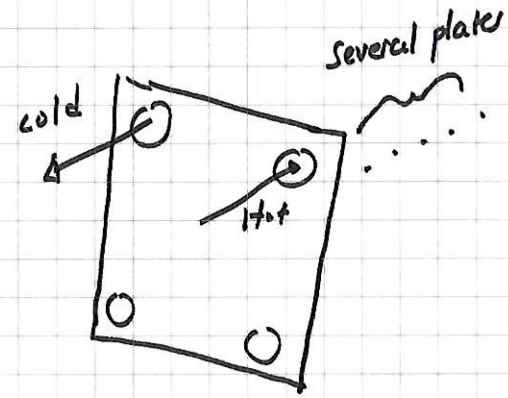


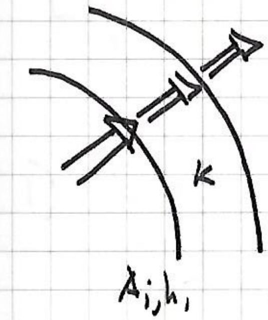
Plate and Frame



Good for fluid temp changes.

"Get my cold fluid hot"

Tube Analysis



If $R_{wall} \approx 0$

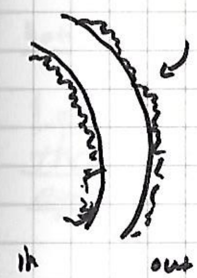
and $A_i \approx A_o \approx A_s$

$$\frac{1}{U} \approx \frac{1}{h_i} + \frac{1}{h_o}$$

Normally we seek to minimize the temperature effects of the tube material this means we what large K and a small ΔX.

$$R_{wall} = \frac{\Delta X}{K A_s}$$

Deposits and Effects



deposits add A_s more surface area and ΔX can change the R_{wall}.

Fouling Factor : R_f

We must ADD these terms to an inside and outside of a pipe.

$$\frac{1}{UA_s} = \frac{1}{h_i A_i} + \frac{R_{f,i}}{A_i} + \frac{1}{h_o A_o} + \frac{R_{f,o}}{A_o} + \frac{\ln(r_o/r_i)}{hA_i}$$

Log Temp Difference
& NTU method.

1) Select a heat exchanger that achieves a desired output temperature T_e. Must know $\dot{m}$

2) Use a NTU to determine a T_e.

Log Method

We have 2 fluids T_h and T_c these are the hot and cold fluids

cold $$\dot{Q} = \dot{m}_c C_{pc}(T_{c,out} - T_{c,in})$$

hot $$\dot{Q} = -\dot{m}_h C_{ph}(T_{h,out} - T_{h,in})$$

Let $C_c = \dot{m}_c C_{pc}$ and $C_h = \dot{m}_h C_{ph}$. Notice the reverse signs of the Hot and Cold eqns

$$Q = UA_s \Delta T_m$$

$$\ln \frac{T_{h,out} - T_{c,in}}{T_{h,in} - T_{c,out}} = -UA_s\left(\frac{1}{\dot{m}_h C_{ph}} + \frac{1}{\dot{m} C_{pc}}\right)$$

$$\Delta T_{ln} = \frac{\Delta T_1 - \Delta T_2}{\ln(\Delta T_1 / \Delta T_2)}$$

$\Delta T_1 :=$ inlet temp differences

$\Delta T_2 :=$ outlet temp differences

It doesn't even matter what side you choose because it will just result in a sign change.

Log Temp Difference $\Longrightarrow$ Finds A_s

NTU Method $\Longrightarrow$ Find $T_{h,s,exit}$'s

Effectiveness ϵ

$$\epsilon = \frac{\dot{Q}_a}{\dot{Q}_{max}} = \quad @\ \Delta T_{max} = (T_{h,in} - T_{c,in})$$

$$@\ C_{min} = \{C_h, C_c\}_{\underline{min}}.$$

ϵ parameters

i) ΔT_{max}

ii) $C_{min} = \min\{C_h, C_c\}$

Case 1: $C_{min} = C_c$

$$\epsilon = \frac{T_{c,out} - T_{c,in}}{T_{h,in} - T_{c,in}}$$

Case 2: $C_{min} = C_h$

$$\epsilon = \frac{T_{h,in} - T_{h,out}}{T_{h,in} - T_{c,out}}$$

Case 3: Parallel Flow

$$\epsilon = \frac{1 - e^{\left[-\frac{UA_s}{C_{min}}\left(1 + \frac{C_{min}}{C_{max}}\right)\right]}}{1 + \frac{C_{min}}{C_{max}}}$$

Remark: ϵ for parallel is a function of fluid properties, and heat transf properties.

$$NTU = \frac{UA_s}{(\dot{m}\, c_p)_{min}}$$

$$c = \frac{C_{min}}{C_{max}}$$

$$\epsilon = f(NTU, c)$$