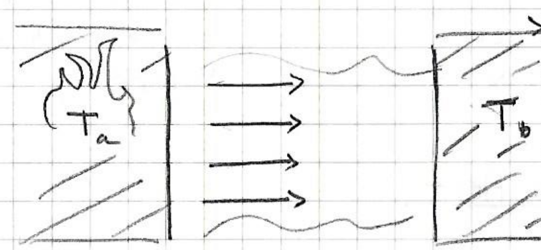
Thermodynamics	vs	Heat Transfer
State Driven Study		Rate Driven Study
total amounts		how fast, how long?

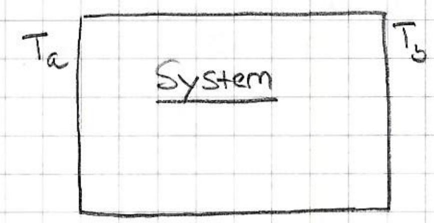
Voltage, Pressure, or Temperature.



If $T_b < T_a$
- 2nd law tells us the direction
- 1rst law tells us the amount of energy

Gradients drive process. These differences is where the engineering occurs.

Rating Task



Q: Given T_a, T_b what is the rate of heat transfere $\dot{Q}$

A: $\boxed{\text{Rating Task}}$

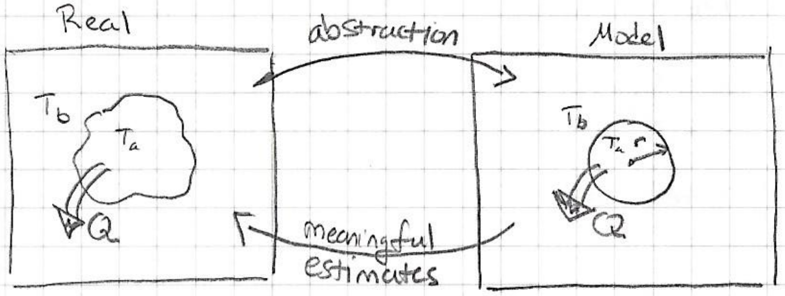
Sizing Problem

$\dot{Q}_a = A$

Q: What system is needed to get $\dot{Q}_a = A$

$\boxed{\text{A: Sizing Task}}$

Model Concerns: Simpe or Complex?



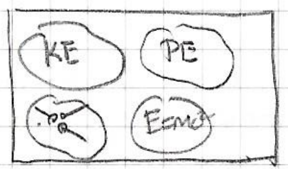
- oblong shape
- need CAS to get accurate readings.

- symmetric shape
- Estimates with pen and paper

Energy and Heat

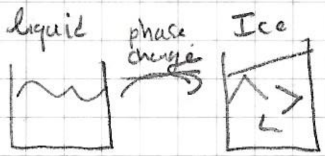
Total Energy

internals, Kinetic, potential.



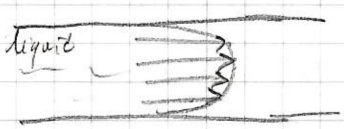
Latent Heat

* The needed energy to change phase



Flow Energy

$$h = u + \underbrace{PV}_{\text{flow energy}}$$



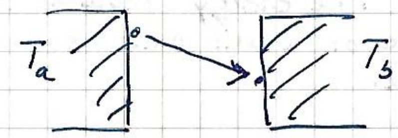
Lecture 1

This class is concerned with 2 major fundamentals.

Part 1: Conduction

Sections $\boxed{1,2,3,4,\sim 5}$

- equilibrium sols
- $\dot{Q} \approx 0$.



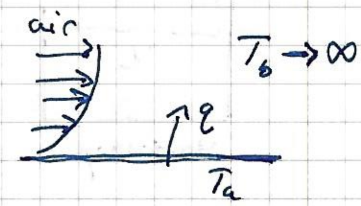
Equations have Dirichlet Boundary Conditions.

- How fast does energy transfer.
- Conduction applys to a heat sink (take Heat Out)!

Part 2: Convection

$\boxed{6,7,8,9,11,12,13}$

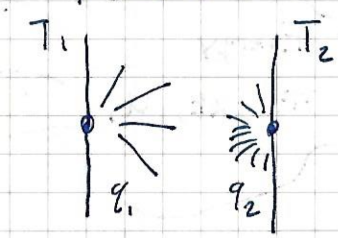
- We use the fluid mechanics to see how heat is wicked away.



- New more complex material.

Part 3: Radiation

Radiation is analogous to electromagnetic charge.



- Only heat transfer that requires no contact
- Method of Images?
- EM Techniques.

Design Bit

{ Making coats and cups shiney lowers this rate of transfer. }

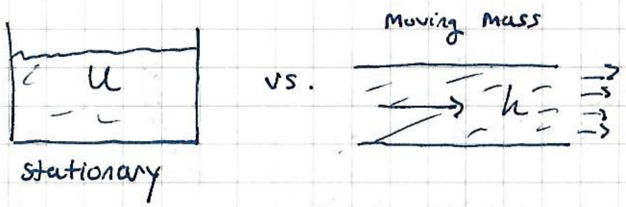
Heat Transfer is everywhere! It is not about the amount but about the Rates.

Lecture 2 : Quiz has been posted.

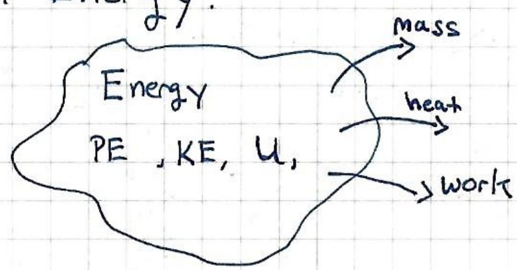
Specific Heat: The energy required to raise the temperature of a unit mass of a substance by one degree.

$$du = c_v dT \qquad dh = c_p dT$$

u, h are mainly used to describe 2 forms of total energy.



Change in Energy.



u: Internal energy ; PE = potential energy; KE = Kinetic.

More than likely the potential will remain constant.

$$E_{in} - E_{out} = \Delta E_{total}$$

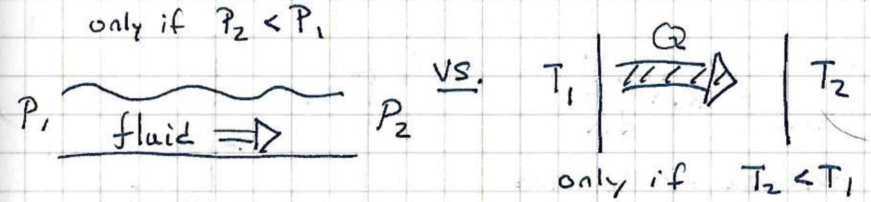
Since we have 3 methods to transfer energy Heat Transfer deals with one

Q : Heat.

The time rate is denoted

$$\dot{Q} = \frac{dQ}{dt}$$

How do we transfer heat? Temperature Gradients. Much similar to pressure differences in fluids



All 3 modes convection, conduction, radiation REQUIRE A TEMPERATURE GRADIENT.

Q : Heat transfered $\qquad \dot{Q}$: transfer rate

$\dot{q}$: flux

The flux is related to $\dot{Q}$ by

$$\dot{q} = \frac{\dot{Q}}{A}$$

$$Q = \int_0^{\Delta t} \dot{Q}\, dt$$

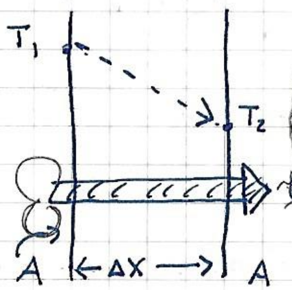
Heat Conduction.

Occurs by motion of molecules exchanging energy

$$\underbrace{\dot{Q}_{cond}}_{\text{time rate}} = \underbrace{-KA\frac{dT}{dx}}_{\text{spatial gradient}}$$

Conduction needs a gradient, medium to pass thru, contact. For the derivation of this look to Sobelev as he assumes the least and even accepts the randomness of the molecules.

$$\dot{Q}_{cond} = -kA\frac{dT}{dx}$$



pets firs...

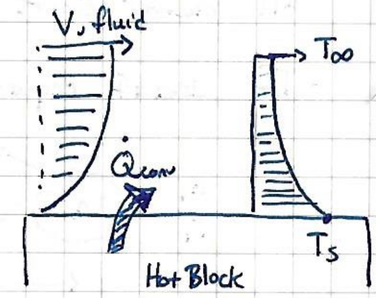
e don' t leave th

A: the area of the flux crossing.

K: conductivity constant.
↑ means heat move thru easy
↓ insulators

Convection Transfer

Convection occurs by fluid passing a surface and wicking away heat

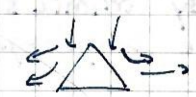


Convection is actually a combination of

- conduction
- motion.

$$\dot{Q}_{conv} = hA_s(T_s - T_\infty) \quad \text{(Newton's Cooling Law)}$$

Forced	Natural
use a fan or airjet	natural air currents



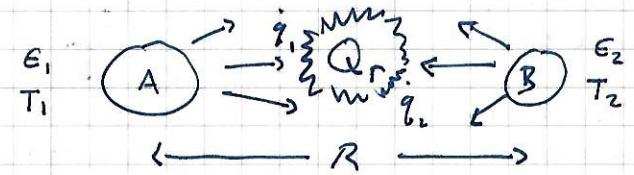
h is a heat transfer coefficient. Not Enthalpy!

h is NOT a fluid property. It is experimental.

Radiation

All matter emits electromagnetic waves, or photons.

- It doesn't require a medium. (only one like it)



Stefan-Boltzman Law

$$\dot{Q}_{emit} = \epsilon \sigma A_s T_s^4$$

$$\sigma = 5.67 \times 10^{-8} \frac{W}{m^2 \cdot K^4} \quad \text{(Boltzman constant)}$$

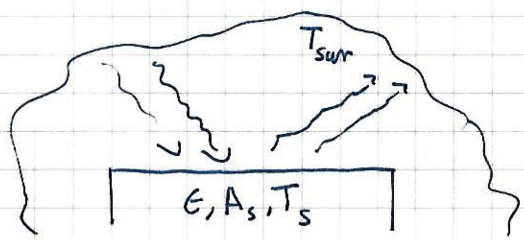
Emissity ϵ: $0 \leq \epsilon \leq 1$ Describes how closely the body mimics a blackbody. It is a surface property.

When $\epsilon = 1$ we have a "BlackBody" The perfect absorber and emitter.

$$\dot{Q}_{Max} = \sigma A_s T_s^4$$

For all practical purposes though...

$$\dot{Q}_{rad} = \epsilon \sigma A_s (T_s^4 - T_{surr}^4)$$



The 3 Differential Heat Equations.

conduction: $\dot{Q}_{cond} = -kA \frac{dT}{dx}$

convection: $\dot{Q}_{conv} = hA(T_s - T_\infty)$

radiation: $\dot{Q}_{rad} = \epsilon \sigma T_s^4 A_s$

The Heat Eqn

$$\frac{\partial^2 T}{\partial x^2} + \frac{\dot{e}_{gen}}{k} = \alpha \frac{\partial T}{\partial t}, \quad \alpha = \frac{k}{\rho c}$$

thermal diffusivity "α". The ratio of heat storage and heat conduction.

$$\frac{\partial^2 T}{\partial x^2} + \frac{\dot{e}_{gen}}{k} = \frac{1}{\alpha} \frac{\partial T}{\partial t}$$

If steady state $\partial T / \partial t \to 0$, $\dot{e}_{gen} \to 0$ (1)

$$\frac{\partial^2 T}{\partial x^2} = 0$$

No heat generation. (2)

$$\frac{\partial^2 T}{\partial x^2} = \frac{1}{\alpha} \frac{\partial T}{\partial t}$$

We want to find $T(x,t)$, $\dot{Q}$, or $\dot{q}$ at a given location.

Boundary Conditions.

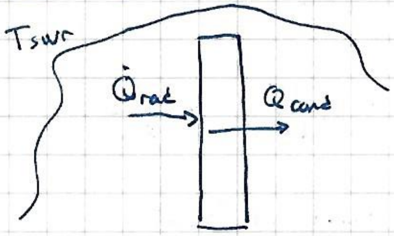
- Dirchlet, Neumann, and Robin

Dirichlet: Assigned boundary temp

Neumann: assigned heat flux.

Robin: Convection? Radiation.

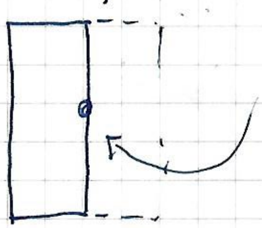
What ever goes in must come out. Surfaces can't store heat.



$\dot{Q}_{rad} = \dot{Q}_{cond}$

$$-\varepsilon\sigma \cancel{A}_s (T - T_{sur}) = -k\cancel{A}\frac{dT}{dx}$$

Continuity Boundaries.

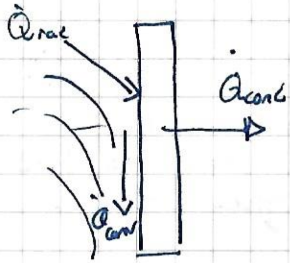


At this interface we must have a smooth transition rate.

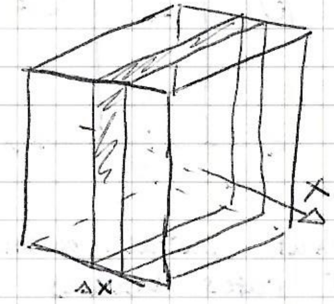
The derivatives must match.

General Boundary

$$\underbrace{\dot{Q}_{rad} + \dot{Q}_{conv}}_{\text{Coming in from outside}} = \underbrace{\dot{Q}_{cond}}_{\text{Inside}}$$



In 1D Heat Conduction we applied conservation of energy to a control volume



$$\frac{\partial^2 T}{\partial x^2} + \frac{\dot{e}_{gen}}{k} = \frac{1}{\alpha}\frac{\partial T}{\partial t}$$

A little more on $\dot{e}_{gen}$... This is a bulk chemical work or energy that is create within the material.

$$\dot{e}_{gen} = \left[\frac{W}{m^3}\right]$$ per unit volume

Other parameters (k), Thermal conductivity.

$$\alpha = \frac{k}{\rho c}$$ ← heat conduction / ← heat "storage"

If we want to lower the time derivative or rate of temperature evolution we would need to increase α.

Ways to decrease α

$$\alpha = \frac{k}{\rho c}$$

- Select ~~high~~ low k material
- Very dense material (high ρ)
- Very large c material.

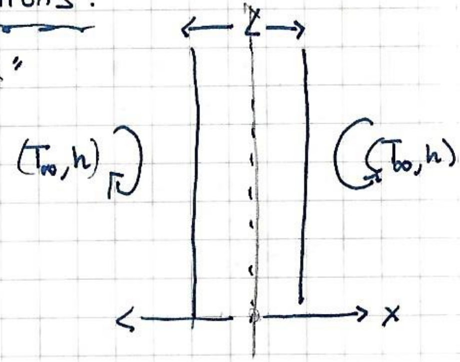
Ways to Increase α:

① Larger k ② small ρ ③ small c

Some Boundary Conditions.

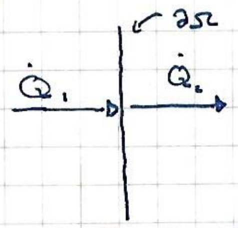
We can be ~~clever~~ "heroes" and select that

$$\left.\frac{\partial T}{\partial x}\right|_{x=0} = 0$$



- We want no heat crossing the center line

One prime example is that surface balance allows us to prescribe other rates becaus



$$\dot{Q}_{cond} = \dot{Q}_{conv}$$

A SURFACE CANNOT STORE ENERGY.
it is a priority to always surface balance when you are at a lose for Boundary conditions.

Steady-State Heat Transfer

We have seen that heat transfer rate is bulkly proportional to the ΔT

$$\Delta T = \dot{Q}\, R_{therm}$$
$$\updownarrow \quad \updownarrow \quad \updownarrow$$
$$\Delta V = I\, R$$

The transfers add just as resistence does

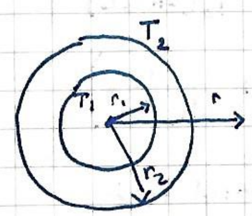
$$R_{cond} = \frac{L}{kA_c} \qquad (1)$$

$$R_{conv} = \frac{1}{hA} \qquad (2)$$

$$R_{rad} = \frac{1}{h_{rad}A} \qquad (3)$$

All the rules that apply to circuits apply now to heat transfer.

Cylindrical Coordinates <steady-state transfer>



Assume: i) Steady-state
ii) $\dot{e}_{gen} = 0$

$$\dot{Q} = 2\pi k L \frac{T_1 - T_2}{\ln(r_2/r_1)}$$

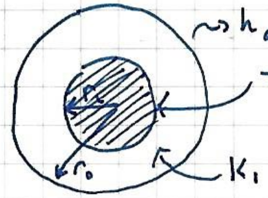
How do we define thermal resistence for cylindrical.

$$\dot{Q} = \frac{\Delta T}{\frac{\ln(r_2/r_1)}{2\pi k L}} = \frac{\Delta T}{R_{therm.}}$$

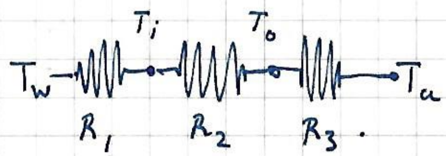
So we define then cylindrical resistence.

$$R_{cond} = \frac{\ln(r_2/r_1)}{2\pi K L}$$

The Rules to Find Total Resistence.



• The circuit



Where

$$R_1 = \frac{1}{h_i A_i}$$

$$R_2 = \frac{\ln(r_2/r_1)}{2\pi K l}$$ [only difference between methods]

$$R_3 = \frac{1}{h_o A_o}$$

Then

$$R_{tot} = \frac{1}{h_i(2\pi r_i L)} + \frac{\ln(r_2/r_1)}{2\pi K L} + \frac{1}{h_o(2\pi r_o L)}$$

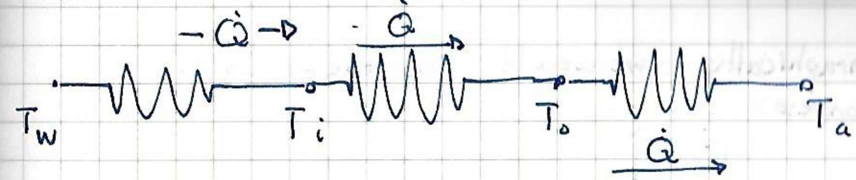
$$= \frac{1}{2\pi L}\left(\frac{1}{h_i r_i} + \frac{\ln(r_2/r_1)}{K} + \frac{1}{h_o r_o}\right)$$

Now we can equate to the total heat transfer.

$$\dot{Q} = \frac{\Delta T}{R_{tot}}$$

$$\dot{Q} = \frac{T_w - T_a}{R_{tot}}$$

But just as current is constant thru series resistors

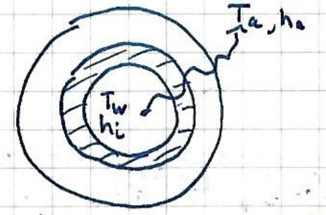


$$\frac{d\dot{Q}}{dt} = 0.$$

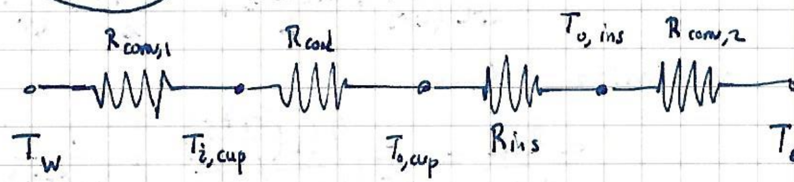
Then. to find T_o

$$\dot{Q} = \frac{\Delta T}{R_{tot}} = \frac{T_o - T_a}{R_{conv,2}}$$

Parallel Analogy Heat Circuit



This circuit is still a series resistor.



Sum resistence.

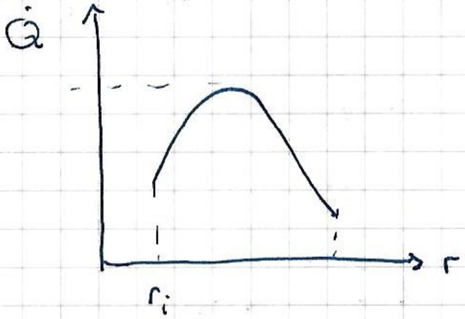
$$R_{tot} = \frac{1}{2\pi L}\left(\frac{1}{h_i r_i} + \frac{\ln(r_2/r_i)}{K} + \frac{\ln\left(\frac{r_o + r}{r_o}\right)}{K_i} + \frac{1}{h_o(r_o + r)}\right)$$

How much insulation should I put to lower the $\dot{Q}$

$$\dot{Q} = \frac{\Delta T}{R_{tot}}$$

Notice that as r increases the last term will decay.

Graphically we would see that $\dot{Q}$ is convex



Then we could find a maximum r using the derivative.

$$\dot{Q} = \frac{1}{R_{tot}} \Delta T$$

$$\dot{Q}(r) = \frac{\Delta T}{\frac{1}{2\pi L}\left(\frac{1}{h_i r_i} + \frac{\ln(r_2/r_1)}{k} + \frac{\ln\left(\frac{r_o+r}{r_o}\right)}{k} + \frac{1}{h_o(r+r_o)}\right)}$$

Or minimize R_{tot}

$$\frac{dR_{tot}}{dr} = \cancel{\frac{d}{dr}\left(\frac{\ln\left(\frac{r_o+r}{r_o}\right)}{K} + \frac{1}{h_o(r+r_o)}\right)}$$

$$\cancel{= \frac{d}{dr}\left(\frac{1}{h_o(r+r_o)}\right) + \frac{d}{dr}\left(\frac{\ln\left(\frac{r_o+r}{r_o}\right)}{K}\right)}$$

$$\cancel{= \frac{1}{h_o} \cdot \left(\frac{1}{h_o(r+r_o)^2}\right)}$$

$$\cancel{= \frac{1}{h_o} \cdot \ln(h_o(r+r_o)) + \frac{1}{K\left(\frac{r_o+r}{r_o}\right)}}$$

Then it turns out

$$\boxed{r_{cr} = \frac{K_o}{h_o}}$$

By optimization we have discovered an optimal range of insulation. Since

$$R_{tot} \sim \frac{1}{r}$$

We found critical value of cylindrical radius full of, The plan is ALWAYS to determine the design factors that effect R and then take derivatives.

$$\cancel{\dot{Q}(r)} = R_{tot}(\delta) = \Sigma$$

Then

$$\frac{dR_{tot}(\delta)}{d\delta} = \frac{d}{d\delta}\left(\frac{\ln\left(\frac{r_o+\delta}{r_o}\right)}{K}\right) + \frac{d}{d\delta}\left(\frac{1}{h_o(r+r_o)}\right)$$

- Effecting Heat Transfer:-

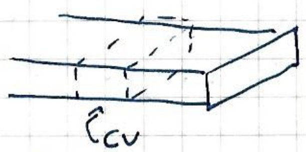
We use fins to create more convective heat transfer rate. Since

$$\dot{Q}_{conv} = hA_s(T_s - T_\infty)$$

Then $\dot{Q}_{conv} \sim A_s \sim h \sim \Delta T$. The easiest one is simply increasing A_s.

How effective is a fin design though?

An estimate is usually a control volume question.



$$\frac{d^2T}{dx^2} + \frac{\dot{e}_{gen}}{k} = 0$$

$\dot{Q}_{conv}$: heat loss ~ negative heat gain.

$$\dot{e}_{conv} = \frac{2h(\Delta x \cdot w + \Delta x \cdot 2b)(T - T_\infty)}{\Delta x \cdot 2b \cdot w} = \frac{h \cdot P}{k \cdot A_c}(T - T_\infty)$$

Then.

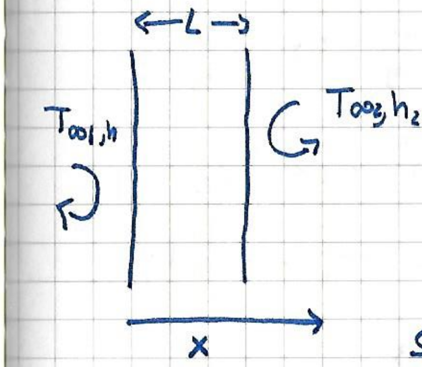
Knowing what sign to pick?

$T_\infty \longrightarrow T_0$ Means $h(T_\infty - T_0)$

We can remember this because

$$h(T_\infty \text{"}\longrightarrow\text{"} T_0)$$

Quiz Question (3)



- Write the convective boundary conditions

@ x = 0, energy balance.

$\dot{Q}_{conv,1} \longrightarrow \dot{Q}_{cond} \longrightarrow \dot{Q}_{conv,2} \longrightarrow$

So.

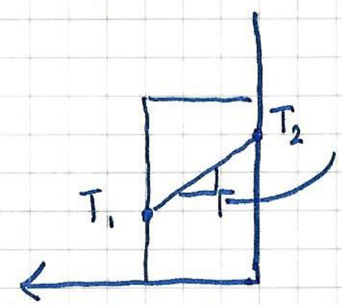
$$\dot{Q}_{conv,1} = \dot{Q}_{cond}$$

Using Newton's Law of Cooling.

$$\dot{Q}_{conv} = h(T_{\infty,1} - T(0))$$

$$\dot{Q}_{cond} = -k\frac{dT}{dx}\Big|_{x=0}$$

Coordinate Systems Matter



The gradient must be negative.

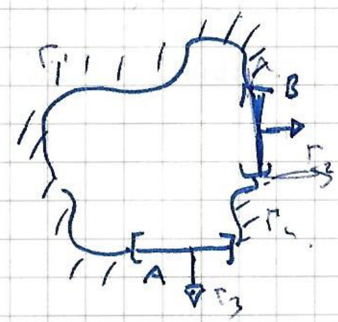
$$\frac{dT}{dx} = -\frac{(T_2 - T_1)}{L}$$

Area of A & B are given.
T_B, T_A are given as well.

$\frac{\partial T}{\partial y}\Big| = 30.$

$$\nabla^2 T = 0$$

What is $\frac{\partial T}{\partial x}$ $\quad \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} = 0$



$$\nabla^2 T = 0$$

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} = 0 \; ; \; \frac{\partial T}{\partial y} = 30 \Rightarrow \frac{\partial^2 T}{\partial y^2} = 0$$

$$\frac{\partial^2 T}{\partial x^2} = 0 \Rightarrow T(x) = C_1 x + C_2$$

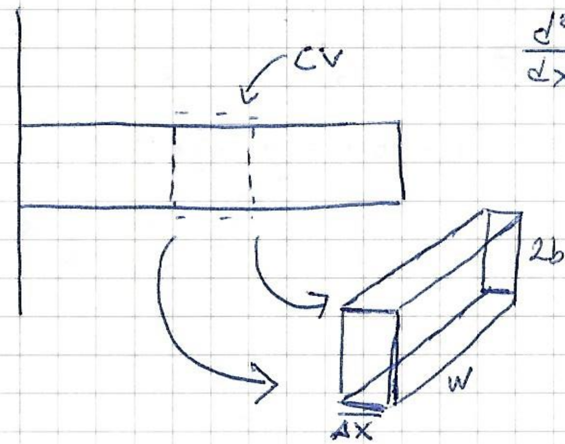
B.C.'s: $T_B(L) = 100°C$ $\quad T_B'(0) = -kA \frac{dT}{dx}\Big|_0$

$$T_A(y) = 30y + C_3$$

$$T_B(x) = C_1 x + C_2$$

The Fin Eqn

We want to maximize surface area because it is a direct design choice that proportionally grows with transfer rate.



$$\frac{d^2 T}{dx^2} = 0$$

Where is heat being transferred.

4 : outside surfaces

2 : faces conducting.

Convective Terms

$$\dot{Q}_{conv} = h\,(2\Delta x \cdot w + 2\Delta x \cdot 2b)(T - T_\infty)$$

$$\dot{q}_{conv} = \frac{\dot{Q}_{conv}}{\text{Volume}}$$

Conduction Equations

It has to equal the energy outside.

We make a ficticious $\dot{e}_{gen}$ to create the Power Solution

$$\frac{\partial^2 \Theta}{\partial x^2} - m^2 \Theta = 0$$

$$\Theta(x) = C_1 e^{mx} + C_2 e^{-mx}$$

$$\Theta(0) = \Theta_b = T_b - T_\infty$$

$$T(x,t) = h(x) \cdot \phi(t)$$

$$= \sum_{n=1}^{\infty} (A \sin(\sqrt{\lambda} x) + B \cos(\sqrt{\lambda} x)) \times e^{-\lambda t}$$

Transient Heat Transfer

$$\frac{1}{\alpha}\frac{\partial T}{\partial t} = \frac{\partial^2 T}{\partial x^2} + \frac{\dot{e}_{gen}}{k}$$

Solving for $T(x,t)$ the solution becomes for more complex.

$$\boxed{\frac{\partial T}{\partial t} \neq 0}$$

If though the temperature variation is assumed to be constant this is to say

$$\nabla^2 T = 0$$

Then the transient problem becomes

$$\frac{1}{\alpha}\frac{\partial T}{\partial t} = \frac{\dot{e}_{gen}}{k}, \quad \alpha = \frac{\rho C_p}{k}$$

The solution should then just be

$$\frac{T - T_\infty}{T_0 - T_\infty} = e^{(-t/\tau)}$$

Where τ is the time constant

$$\tau = \frac{\rho C_p L_c}{h}$$

As well L_c : is the characteristic length.

$$L_c = \frac{V}{A_s} = [m]$$

This derives the temperature varies along to an exponential decay.

$$T^* = \frac{T - T_\infty}{T_o - T_\infty} \quad \text{[no units]}$$

We also have a heat transfer rate solution.

$$\dot{Q}(t) = hA_s \Delta T$$
$$= hA_s(T(t) - T_\infty)$$

We have solved for a $(T(t) - T_\infty)$ for a transient problem.

But alternatively we can solve for the total amount of heat transfer between time 0 and $t = t_o$.

$$Q_{0 \to t} = \int_0^t hA_s(T(t) - T_\infty)\, dt$$

or

energy balance

$$Q_{0 \to t} = MC_p \Delta T = MC_p(T(t) - T_\infty)$$

If we want to find the maximum amount of heat transfer.

$$\frac{d}{dt}\dot{Q} = \frac{d}{dt}\int_0^t hA_s(T(\tau) - T_o)\, d\tau$$

$$Q_{max} = mC_p(T_\infty - T_o)$$

When when can we use the lumped system approach. $\nabla^2 T \equiv 0$.

- High Conductivity Term.
  - temperature almost immediately transfers
- $Bi = \frac{hL_c}{k}$, This Numerical Measure.

$$Bi = \frac{hA_c \Delta T}{\frac{k}{L_c} A_c \Delta T} \quad \begin{aligned} &= \text{convective out of body} \\ &= \text{conductive in body} \end{aligned}$$

$$Bi = \frac{\frac{L_c}{kA_c}}{\frac{1}{hA_c}} \quad \begin{aligned} &\longrightarrow \text{conductive Resistance} \\ &\longrightarrow \text{convective resistance} \end{aligned}$$

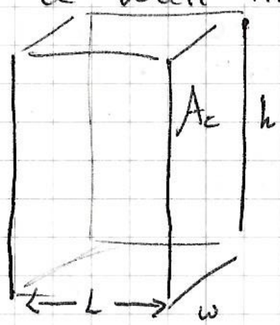
$$\boxed{\text{When } \underline{Bi \leq 0.1} \text{ we can use the lumped system approach!}}$$

The Bi number is a ratio for how much temperature can leave vs how much heat is going through

Recall that

$$L_c = \frac{V}{A_s} = \frac{\text{Volume of Body}}{\text{Boundary}}$$

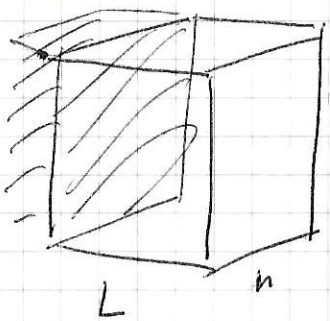
For a wall then.



$$L_c = \frac{V}{A_s} = \frac{L\cancel{wh}}{2\cancel{wh}} = L/2$$

• 2 A_c because total flux boundary.

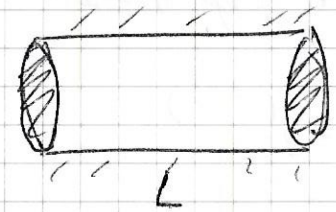
Insulated Boundary



$$L_c = \frac{V}{A_s} = \frac{L\cancel{wh}}{\cancel{wh}} = L$$

$$\boxed{A_s \text{ in } L_c \text{ calculations is using the total flux surface area}}$$

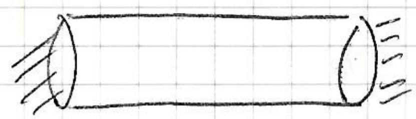
Cylinder Cases



$$L_c = \frac{V}{A_c} = \frac{L\cancel{\pi r^2}}{2\cancel{\pi r^2}} = \frac{L}{2}$$

So some parameter of these 2 devices are similar

⟨Insulated Ends for a Cylinder⟩



$$L_c = \frac{V}{A_c} = \frac{\cancel{L\pi}\left(\frac{D}{2}\right)^2}{\cancel{\pi}D\cancel{L}} = \boxed{\frac{D}{4}}$$

The Thermocouple Device

We must assume that the temperature will remain constant.

$D = 1\,mm$, $K = 33$, $\rho = 8500$, $c_p = 320$

$h = 210$

Determine the time (t) such that the measurement will be considered "accurate"

99% of original ΔT.

or

$$\frac{T - T_\infty}{T_i - T_\infty} = .99$$

Using a Bi calculation we should see if we can use the Lump system properties.

$$\frac{T(t)-T_\infty}{T_i - T_\infty} = 0.99 = e^{(-t/\tau)}$$

$$\ln(0.99) = -t/\tau$$

$$\tau \ln(0.99) = -t$$

$$-\tau \ln(0.99) = t$$

This is a general form then to see if it is the case that

$$Bi = \frac{hL_c}{K} \leq 0.1$$

Then for the temp ratio to reach a percentage β, the time it will take will be

$$\boxed{\begin{aligned} t &= -\tau \ln(\beta) \\ t &= -\frac{\rho L_c c_p \ln(\beta)}{h} \end{aligned}}$$

This number is telling you how long to wait before taking readings.

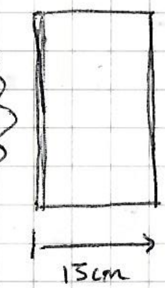
Quiz 3 Q's

$\dot{q} = 275 \frac{W}{m^2}$

$K = 1.1$

$\Delta T = ?$

$\dot{Q}_{cont} = -kA_c \frac{dT}{dx}$

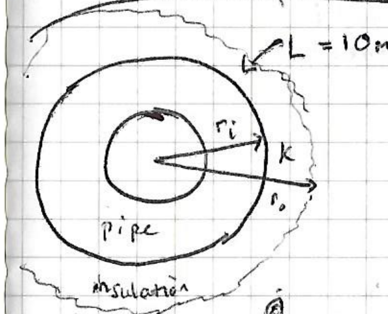


Then we have the following:

$$\dot{Q}_{cond} = -kA_c \frac{dT}{dx}$$

$$\frac{\dot{Q}_{cond}}{A_c} = \dot{q} = -k\frac{\Delta T}{\Delta x}$$

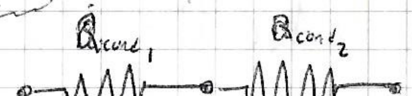
$\boxed{\text{Ans.}} \implies \Delta T = \frac{\dot{q}\,\Delta x}{K} = \frac{(275 \frac{W}{m^2})(0.15 m) m (K)}{1.1 W}$ ✓



Δr of ins $= 3cm$

$K = 0.05$

$\dot{Q} = 1000 W$



Here we have to solve for the potential.

$$\Delta T = \dot{Q}/R_{tot}$$

$$= (1000 W) \frac{\ln\left(\frac{r_2}{r_1}\right)}{2\pi r_o L}$$

Test 1 : 1 page of Notes.

Rates

$$\dot{Q}_{conv} = -k A_c \frac{dT}{dx} \quad [W]$$

$$\dot{Q}_{conv} = h A_c (T_s - T_\infty) \quad [W]$$

Flux can also be solved for because

$$q = \frac{Q}{A_c} = -k \frac{dT}{dx} \quad \text{or} \quad q_{conv} = h(T_s - T_\infty)$$

$$\dot{Q}_{rad} = \sigma \epsilon A T_s^4 \quad \left[\text{must use K scale}\right]$$

The Boltzman Eqn only tells you how much radiation a body emitts but it doesn't tell you net effects

$\boxed{\text{Special Radiation Case}}$

$$\dot{Q}_{rad} = \sigma \epsilon A_s (T_s^4 - T_{surr}^4)$$

Radiation: the only method of transfer that doesn't require any medium.

- They just need a view factor

Chapter 2: Steady-State Heat Transfer.

$$\frac{1}{\alpha} \cdot \frac{\partial T}{\partial t} = \nabla^2 T + \frac{\dot{e}_{gen}}{k}$$

$\alpha = \frac{k}{\rho c}$ and $\dot{e}_{gen}$ is generated heat.

1D Steady State;

$$T(x) = -\frac{\dot{e}_{gen}}{k} x^2 + C_1 x + C_2$$

Boundary Conditions can be

Dirichlet : $T(x_o) = A$

Neumann : $T'(x_o) = f(x)$.

Robin : $\alpha T(x_o) + \beta T'(x_o) = 0$

Can you solve the general $T(x,t)$ with Robin B.C's? If you could that would make the test fairly quick?

1D Demension Transient Heat Transfer

Lumped System: $Bi \le 0.1$

$$Bi = \frac{hL_c}{K} \; ; \; L_c = \frac{V}{S_A}$$

Graphic Solution : $Bi > 0.1$

$T(x,t)$.

if $t^* > 0.2$, just use the first eigen value λ_1

$$\left.\begin{matrix} x^*, X \\ T^*, \Theta \\ t^*, \tau \end{matrix}\right\} \text{non Demensional terms}$$

Product Solution: $Bi > 0.1$

$$T(\bar{x},t) = T(x,t) \cdot T(y,t)$$

If $t^* \le 0.2$

Use the solution chart

$$\frac{\Theta}{\Theta_{center}} = f(x) \qquad \frac{Q}{Q_{max}} = f(\Theta_{center})$$

Where,

$$\left[Q_{max} = M C_p (T_\infty - T_0) \right]$$

Fin Design

① Efficiency $\eta_{th} = \dfrac{Q_{fin}}{Q_{max}}$

Where the fin's temperature approaches T_b in zero seconds.

$$\dot{Q}_{max} = h \, SA_{,fin} (T_b - T_\infty)$$

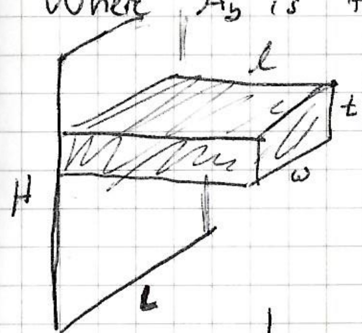
So

$$\dot{Q}_{fin} = \eta \cdot h \cdot A_{fin} (T_b - T_\infty)$$

② Effectiveness:

$$\epsilon = \frac{Q_{fin}}{Q_{no,fin}} = \frac{\eta \cancel{h} \cdot A_{fin} \cdot \cancel{\theta_b}}{\cancel{h} \cdot A_b \cdot \cancel{\theta_b}} = \eta \left(\frac{A_{fin}}{A_b} \right)$$

Where A_b is the base area.



$A_b = H \cdot L$

A_{fin} = surface area of fin

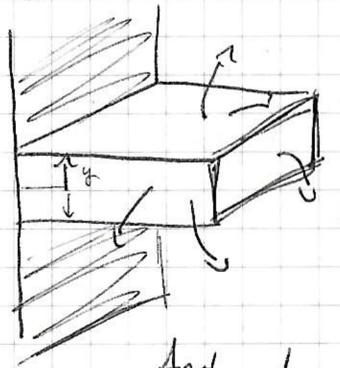
$= 2lw + 2tl + w,l$

Long Fin: $\epsilon = \sqrt{\dfrac{kP}{hA_c}}$

$$\epsilon \propto K \quad \underline{\text{and}} \quad \epsilon \propto \frac{1}{h}, \frac{P}{A_c}$$

- choose high K material
- choose smaller h
- choose largest $\dfrac{P}{A_c}$ ~ test note sheet these ratios for different geometry.

Bi # and Fin Design.



- How can we assume that the ∂y zero.

The Bi number

$$Bi = \frac{hL_c}{K}$$

And $L_c = \frac{V}{S_A}$ where

S_A is the flux area we don't care about the dimension in and out of the page

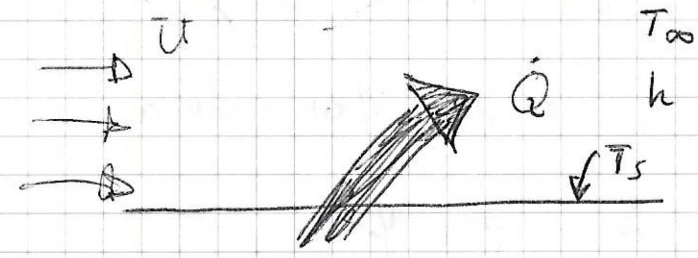
$$\frac{h\delta}{K} \leq 0.2$$

Let's use the lump system method.

Convection Heat Transfer

8 multiple choice } test
2 work out }

- For convective heat transfer fluid motion is the mechanism.



What is this h parameter?

$$h = \frac{-K_{fluid}\left(\frac{\partial T}{\partial y}\right)_{y=0}}{T_s - T_\infty}$$

That is the conduction heat transfer rate ratio to the temperature difference.

$$h = \frac{1}{A_s}\int_{A_s} h_{loc}\, dA_s$$

$$h = \frac{1}{L}\int_0^L h_x\, dx$$

For the most part we are concerned with finding the convective heat transfer coefficient.

$$\frac{\dot{q}_{conv}}{\dot{q}_{cond}} = \frac{h \Delta T}{-k \Delta T / L} = \frac{hL}{k} = Nu$$

Here Nu is not the Biot number.

h = fluid property

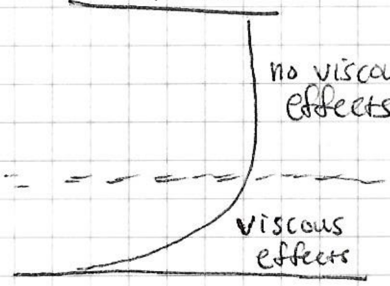
L_c = not Biot formula

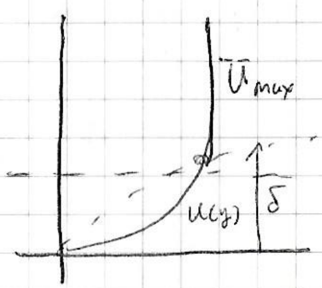
k = property.

What happens if Nu := 1.

- Then there is no heat transfer by convection.

Viscid vs Inviscid





Prandtl Number

$$Pr = \frac{\nu}{\alpha} = \frac{\mu C_p}{k}$$

The diffusion of momentum to diffusion of conductivity. Metals have very low prandtl number while oils have a very high Pr number.