Ion Trapped Quantum Computing

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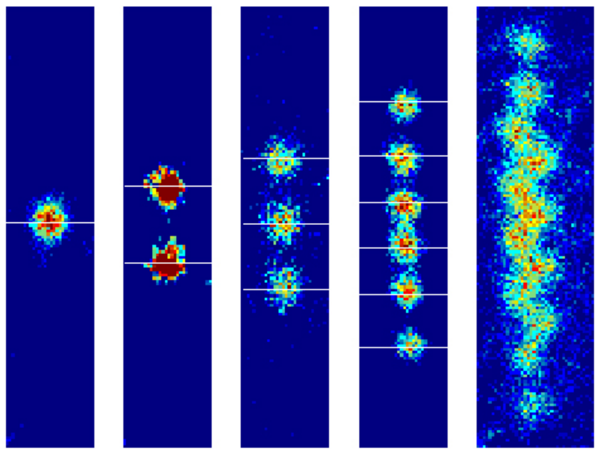

PHYS 468 - Introduction to Quantum Information Processing

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Contents

Abstract

An implementation of quantum computing must fulfill the DiVincenzo criteria in order to be considered a realistic model. The following shows that a quantum computer can be implemented with cold Ytterbium ions as qubits, confined in a linear Paul trap interacting with laser beams. Qubits are connected and entangled with single laser operations, creating high accuracy. State initialization is accomplished with very high fidelity (99.985 %) using optical pumping, and very accurate state-dependant fluorescence methods are used for detection. The decoherence time of Ytterbium qubits were found to be 2.5s with the demonstrated experiment, but are expected to be significantly larger by evidence through similar experiments. Current challenges and breakthroughs in ion-trapped computing are discussed to conclude.

1 Introduction

The conventional computers used today obey the laws of classical physics. The smallest unit of information is stored in bits, which are binary numbers represented by a microscopic dot on a chip. These dots hold a charge that determines whether the bit is equal to 0 or 1. Similarly, designing a quantum computer relies heavily on one basic element - finding a physical implementation of *quantum* bits. Quantum bits, or qubits, obey the laws of quantum mechanics, with the advantageous feature of being in a superposition of two states - $|0\rangle$, and $|1\rangle$ at the same time. This allows a computer to follow multiple computational paths simultaneously, giving it inherent parallelism not achievable through classical computation.

J. I. Cirac and P. Zoller showed that confined ions, interacting with laser light in a linear trap, is a promising model for a quantum computer [1]. Ions are created from an atom either gaining or loosing an electron, giving it an electric charge. This is useful, since charged particles may be manipulated by magnetic and electric fields. The two possible states $|0\rangle, |1\rangle$ of the system, are the two hyperfine components of the electronic ground state.

A model of quantum computation is not considered realistic until it has fulfilled all the Divincenzo criteria [8]. In summary, the Divencenzo criteria require that qubit states can be properly initialized, measured, and have a universal set of gates implemented between them. Additionally, the decoherence time of the qubit must be very long compared to the time of gate operations. This report is primarily based on both Cirac and Zollers *Quantum Computations with Cold Trapped Ions* [1] and a recent Ytterbium ion qubit experiment [4]. It is shown that a set of cold ions interacting with laser light, trapped in a linear chip, is an effective physical model of a quantum computer. The distinctive features of the implementation are (i) it can be measured and initialized with high efficiency using laser methods, (ii) decoherence can be made negligible during computation using precision spectroscopy, and (iii) n-bit qubit gates can be implemented between ions without them necessarily being side-by-side [1].

2 Ion trapping

2.1 Preparing Ions

The atoms chosen for ion trapping are not random - experiments are conducted on atoms with simple electronic structures (two valence electrons) and small mass. Since mass is proportional to inertia, a smaller mass will make it easier to apply forces on ions needed for interactions. To ionise an atom, a laser is applied to it which displaces an outer electron, and one outer charge remains for convenient manipulation. Thus, alkaline earth metals - or elements of similar electron configuration are best suited for ion traps. The experimental content of this report will be focused on a recent experiment [4] using Ytterbium-171. Ytterbium ions (Yb^+) are particularly attractive due to their strong $S^2_{\frac{1}{2}} \rightarrow \mathrm{P}^2_{\frac{1}{2}}$ electronic transition near 369nm, making laser schemes for qubit manipulation more feasible. The spin-$\frac{1}{2}$ nature of the Ytterbium nucleus allows for efficient and fast preparation and detection of the ground state hyperfine levels (i.e., the two qubit states). This is due to the usefully high hyperfine transition frequency of 12.6GHz, making the distinction of two qubit states with high efficiency possible [5]. Ytterbium ions are loaded into the trap by photoionization of an atomic beam of neutral Yb metal [4]. A stainless steel tube known as an *atom oven*, is filled with Yb metal and heated up. The vapourized Yb is directed toward the ion trap placed in an ultra-high vaccuum chamber. Once they hit the trap, the Ytterbium ions will be confined.

2.2 Ion confinement

An ion feels a force from an electric field due to its electric charge. In ion trap architecture, an electric field is generated by applying voltage to two opposing pairs of metal hyperbolic electrodes in a quadrupole geometry (see figure 1). These two pairs of electrodes are positioned along the radial and axial planes, with opposite charges applied to them. Tip electrodes of static electric fields are positioned in the z axis. This is known as a linear Paul trap, and is most commonly used in ion trapping experiments.

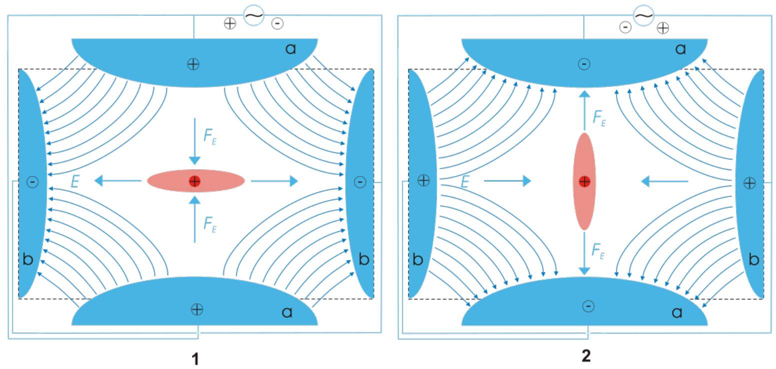

Figure 1: (1) electric fields flowing from + to - electrodes in the quadrupole configuration. the ion experiences an inward force from the axis with + charge, while expelling along the axis of - charge. (2) charges on different axis [2]

2.2.1 Earnshaw's theorem

Earnshaw's theorem states that a static electric field is not sufficient to produce a stable stationary equilibrium configuration for the ions. For a particle to remain confined, small perturbations of the particles should not break equilibrium - there should always be field lines pointing towards the center, bringing the particle back to it. In order to achieve this, the electric field must be switched rapidly at the appropriate RF frequency. The radio-frequency (RF) oscillations of the fields are used to radially confine the ions within the trap. Earnshaw's theorem can be explained mathematically - a potential U(r) with zero divergence will have no local minima or maxima in the field, i.e., nothing to pull the particle back in. Therefore, the the divergence must always be greater than 0.

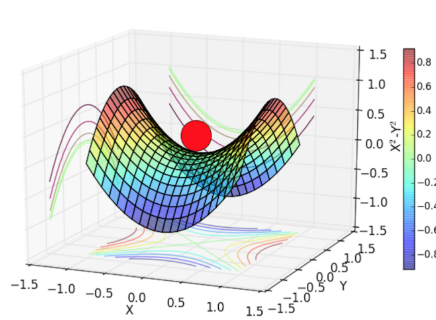

Figure 2: the electric field of the ion, where the center averages to 0 potential, and the more the ion drifts away from the center, the higher the field gets, pulling in back in [3]

$$\nabla^2 U = \frac{d^2}{dx^2}U + \frac{d^2}{dy^2}U + \frac{d^2}{dz^2}U > 0 \tag{1}$$

One drawback from the oscillating fields is that it causes a micromotion in the ions. To achieve better confinement, the micromotion must be minimized, which is achieved through laser cooling methods.

2.3 Laser Cooling

Lasers cool ions via the transfer of momentum in the appropriate direction. Ions absorb photons of the appropriate resonant frequency, reducing their momentum in the direction opposing the propagation of photons in the beam. The lasers are detuned so that the frequency of light is only at the resonant frequency of the transition when the ions are moving *counter* to the beam direction. This is achieved by cunning use of the Doppler effect to shift the wavelength of light as seen by the moving ions. Thus, ions at the centre of the trap which are (almost) stationary do not absorb photons and remain unaffected by the lasers. However, ions with high kinetic energy may absorb the Doppler-shifted photons and slow down. Once the trapped ions are sufficiently cool (1 μK) [9], a working ion-trapped qubit can be created. The Ytterbium ion is Doppler-cooled by red-detuned ($\approx$ 10 MHz) laser light of 369.53nm [4].

3 DiVencenzo Criteria

The requirements for the physical implementation of quantum computation include proper state initialization and detection of the qubits, a set of universal quantum gates, and long qubit coherence time compared to gate operation times. It is demonstrated that the ${}^2S_{\frac{1}{2}}$ hyperfine levels of ${}^{171}Yb^+$ serve as excellent qubit states for quantum information processing [4,7]. By applying additional laser frequency sources according to the hyperfine splittings of the relevant energy levels, efficient state initialization and detection is achieved, reliable single qubit operations are realized, and a qubit coherence time exists that far exceeds the duration of gate processes. The relevant energy levels of the Yb$^+$ ion are shown in Figure 3. The $S_{\frac{1}{2}}\ |F = 1, m_F = 0\rangle$ state is defined as the logical state $|1\rangle$; and the $S_{\frac{1}{2}}\ |F = 0, m_F = 0\rangle$ as the logical state $|0\rangle$. The hyperfine ground state levels are separated by a large frequency of $\approx$ 12.6 GHz.

3.1 State Initialization

State initialization is accomplished by optically pumping the ${}^{171}Yb^+$ ion to the $|0\rangle$ hyperfine state. Optical pumping is a laser scheme used to create population inversion in two energy states.

It works by shining a laser beam resonant with a specific energy transition onto the ground energy levels $(|0\rangle, |1\rangle)$. The particles are then pumped up to the excited state, and decay down, with equal probability, to the lower energy ground states. This is a cyclic process - the particles are pumped back up again with the same laser that only communicates with a particular ground state. Eventually, there will be no population left in the resonant state, and the ion will be fully pumped into the uncoupled ground state. In the case of Yb^+, a 369.53nm laser beam resonant with the $S_{\frac{1}{2}}$ $|F=1\rangle \rightarrow \mathrm{P}_{\frac{1}{2}}$ $|F=1\rangle$ transition is shined at the ground state. The population in the excited P state has a $\frac{1}{3}$ chance of decaying to each of $|1\rangle$, $|0\rangle, D_{\frac{3}{2}}$ $|F=2\rangle$. The hyperfine splittings are the only states of interest, so a tunable fiber electro-optic modulator is driven at 3.07 GHz, adding a frequency component to a 935.2nm beam resonant with the $D_{\frac{3}{2}}$ $|F=2\rangle$ $\rightarrow \mathrm{D}[3/2]_{\frac{1}{2}}$ $|F=1\rangle$ transition. This will depopulate the $D_{\frac{3}{2}}$ $|F=2\rangle$ state, as demonstrated in Figure 3. Cyclic optical pumping of this nature will result in the ion being fully pumped to the $|0\rangle$ in less than 500ns, with up to 99.985% fidelity [4].

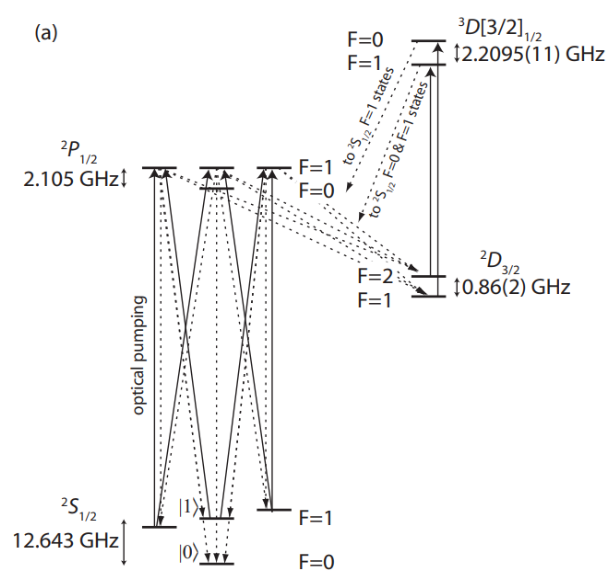

Figure 3: relevant energy levels of the Ytterbium ion, optical pumping applied to create population inversion between the hyperfine ground state $|1\rangle$ to $|0\rangle$ using a precise laser that only couples to the $S_{\frac{1}{2}}$ $|F=1\rangle \rightarrow \mathrm{P}_{\frac{1}{2}}$ $|F=1\rangle$ transition

3.2 State measurement

Reliable quantum computation and communication requires very high detection accuracy. In the Yb^+ model, detection of states is accomplished with state-dependant fluorescence techniques. When detecting a state, the 369.53nm light will once again be shined on the states, and detuned to be in resonance with the $|1\rangle$ $\rightarrow \mathrm{P}_{\frac{1}{2}}$ $|F=0\rangle$ transition.

This will only excite particles in the hyperfine state $|1\rangle$, as the transition from $|0\rangle$ to $P_{\frac{1}{2}}\,|F=0\rangle$ is forbidden due to selection rules. Thus, if the state was prepared in $|1\rangle$, the impinging light is nearly on resonance, photons will be scattered and light will be detected. If the state was prepared in $|0\rangle$, the state will stay dark because the laser used is very far from the resonance of $|0\rangle$. The hyperfine splitting frequency of 12.6GHz is too drastic to allow for any particles originating from the $|0\rangle$ to release a significant amount of photons. This explains the near-perfect efficiency of detection - the collection of even a small fraction of fluorescence concludes that the qubit state is in $|1\rangle$.

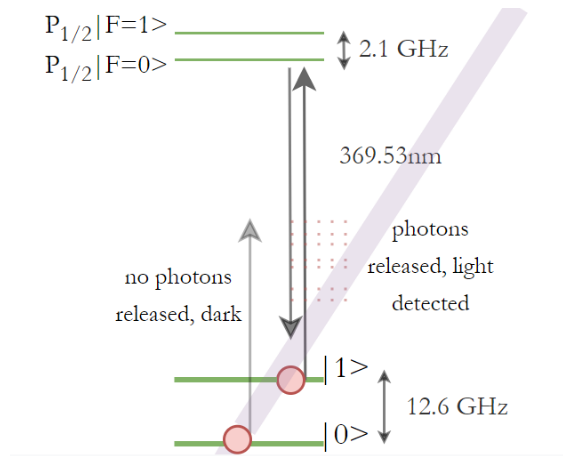

Figure 4: detection of the qubit state - if the qubit was prepared in state $|1\rangle$, photons will scatter and light will be detected.

One limiting source of error encountered in this state detection technique originates from the off-resonant coupling to the $P_{\frac{1}{2}}\,|F=1\rangle$ manifold. When the state is prepared in $|1\rangle$, some off-resonant excitation may occur into the $P_{\frac{1}{2}}\,|F=1\rangle$ state, considering the hyperfine splitting of the $P_{\frac{1}{2}}$ level has a low frequency of 2.1 GHz. An additional source of error may be the reduced scattering rate of the Yb+ ion due to coherent population trapping in the $|1\rangle$ manifold. The intensity and duration of the light must be experimented with to produce optimal results. It was concluded that 0.8 W of light focused at a radius of ≈ 30 m at the trap, incident on the ion for 1000 micro seconds, produced the most optimal results [4]. The results in the form of a histogram of state detection of $|0\rangle$ vs. $|1\rangle$ for the Ytterbium ion is shown in figure 5 [4].

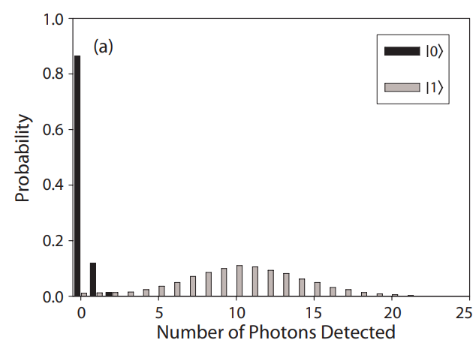

Figure 5: average number of 30 photons detected if ion is in the $|1\rangle$ state, almost no photons detected if prepared in the $|0\rangle$ state

3.3 Gate implementation

3.3.1 Motion control

A key consequence of the Earnshaw theorem is that the trapped ions possess an oscillating micromotion. This proves to be an advantage, since the motions of the ions can now be described in terms of normal modes [1]. The Lamb-Dicke limit (LDL) is assumed so that all the modes are in their corresponding quantum state [5]. This is possible because the frequency of the modes are much greater than the photon recoil frequency that corresponds to the transition used for laser cooling [1]. Thus, the change in the motional quantum number by sideband cooling is strongly suppressed [1]. As explained above, when a laser beam is applied on an ion, it will induce a transition between internal ground states and excited states. Assuming the LDL exists and low intensities are applied, the laser can be prepared so that it will only induce transitions that will modify the state of *one* of the modes, for any number of ions. Each mode has a different frequency, with the minimum frequency being the center of mass mode. For a mode x with frequency v_x, the laser can be detuned at $\delta = -v_x$, so that it exclusively excites mode x. Note that a laser is applied to each individual ion, so this technique is used to control the motion and interactions of multiple ions by simply adjusting the ions corresponding laser. The interaction-picture Hamiltonian of this situation is defined by the below operator [1].

$$\hat{H} = \frac{n}{\sqrt{N}}\frac{\Omega}{2}[|e_q\rangle_n \langle g|\, ae^{-i\theta} + |g\rangle_n \langle e_q|\, a^t e^{i\theta}] \tag{2}$$

In the above equation, a and a^t are the creation and annihilation operators of the mode, N is the number of ions, Ω is the Rabi frequency, θ is the phase of the applied laser, and n is the LDL parameter, which is equal to $n = [\hbar k^2 cos^2(\theta)/(2Mv_x)]^{\frac{1}{2}} \ll 1$. The $\frac{1}{\sqrt{N}}$ term appears as a result of the Mossbauer effect [1]: the mass of the collective mode motion is NM, and the amplitude of the mode scales by $1/\sqrt{NM}$. The $k^2cos^2(\theta)$ term comes from the oscillating motion of the qubit, as described in section 2.2 - the field is oscillating *and* the ion is oscillating, which is why the term is squared. Equation 1 is the general single ion Hamiltonian for the case of a linear Paul trap [1].

Gates can be applied by adjusting the time a laser is turned on for an ion, using π pulses. For instance, if a $k\pi$ pulse was applied to the system, the laser beam would be on the ion for a time duration of $t = k\pi\sqrt{N}/(\Omega n)$. The evolution of the Hamiltonian would be described using the unitary operator:

$$\hat{U}_n^{k,q}(\theta) = exp[-ik\frac{\pi}{2} \left|e_q\right\rangle_n \left\langle g\right| ae^{-i\theta} + H.C.] \tag{3}$$

where the k represents the π multiple, n is the mode number, and q is the state that will undergo excitement ($q = 1, 0$). This unitary would transform the states of interest accordingly:

$$\left|g\right\rangle_n \left|1\right\rangle \rightarrow cos(k\pi/2) \left|g\right\rangle_n \left|1\right\rangle - ie^{i\theta} sin(k\pi/2) \left|e_q\right\rangle_n \left|0\right\rangle$$
$$\left|e\right\rangle_n \left|0\right\rangle \rightarrow cos(k\pi/2) \left|g\right\rangle_n \left|0\right\rangle - ie^{-i\theta} sin(k\pi/2) \left|g\right\rangle_n \left|1\right\rangle$$

Using these types of interactions, gates can be performed by applying them in steps. A controlled-NOT gate would be equivalent to applying the unitary operators $\hat{U}_m^{1,0}\hat{U}_n^{2,1}\hat{U}_m^{1,0}$. The sign of the state changes if the two modes are both excited. The resulting states after applying these gates are in accordance with the expected behaviour of a CNOT:

$$\left|g_m\right\rangle \left|g_n\right\rangle \left|0\right\rangle : \hat{U}_m^{1,0} \left|g_m\right\rangle \left|g_n\right\rangle \left|0\right\rangle \rightarrow \hat{U}_n^{2,1} \left|g_m\right\rangle \left|g_n\right\rangle \left|0\right\rangle \rightarrow \hat{U}_m^{1,0} \left|g_m\right\rangle \left|g_n\right\rangle \left|0\right\rangle \rightarrow \left|g_m\right\rangle \left|g_n\right\rangle \left|0\right\rangle$$
$$\left|g_m\right\rangle \left|e_{0n}\right\rangle \left|0\right\rangle : \hat{U}_m^{1,0} \left|g_m\right\rangle \left|e_{0n}\right\rangle \left|0\right\rangle \rightarrow \hat{U}_n^{2,1} \left|g_m\right\rangle \left|e_{0n}\right\rangle \left|0\right\rangle \rightarrow \hat{U}_m^{1,0} \left|g_m\right\rangle \left|e_{0n}\right\rangle \left|0\right\rangle \rightarrow \left|g_m\right\rangle \left|e_{0n}\right\rangle \left|0\right\rangle$$
$$\left|e_{0m}\right\rangle \left|g_n\right\rangle \left|0\right\rangle : \hat{U}_m^{1,0} \left|e_{0m}\right\rangle \left|g_n\right\rangle \left|0\right\rangle \rightarrow \hat{U}_n^{2,1} - i \left|g_m\right\rangle \left|g_n\right\rangle \left|0\right\rangle \rightarrow \hat{U}_m^{1,0} i \left|g_m\right\rangle \left|g_n\right\rangle \left|0\right\rangle \rightarrow \left|e_{0m}\right\rangle \left|g_n\right\rangle \left|0\right\rangle$$
$$\left|e_{0m}\right\rangle \left|e_{0n}\right\rangle \left|0\right\rangle : \hat{U}_m^{1,0} \left|e_{0m}\right\rangle \left|e_{0n}\right\rangle \left|0\right\rangle \rightarrow \hat{U}_n^{2,1} - i \left|g_m\right\rangle \left|e_{0n}\right\rangle \left|0\right\rangle \rightarrow \hat{U}_m^{1,0} - i \left|g_m\right\rangle \left|e_{0n}\right\rangle \left|0\right\rangle \rightarrow - \left|e_{0m}\right\rangle \left|e_{0n}\right\rangle \left|0\right\rangle$$

3.4 Entanglement

State-dependant fluorescence is once again applied to achieve entanglement between ions. A laser light of 355nm [10] is shined at each ion that will be entangled. The laser beam works in a way that, dependant on the state that the ion is in, the ion will move according to that state. This is once again achieved through precise transition energies and selection rules. The lasers work as optical tweezers - they will transfer momentum and move the ion in the direction of the spin state that it is in. For instance, the $\left|\uparrow\right\rangle$ state will be pushed upwards, and the $\left|\downarrow\right\rangle$ will be pushed down. This creates a dipole-dipole coupling between the ions necessary for entanglement. The ions are now coupled with a quantizied motion achieved through the Coloumb repulsion force, and are entangled [10]. An extra phase term is included for entangled ions of opposite spin states, which originates from the difference in energy. This is visualized in the figure below.

$$|\uparrow\uparrow\rangle = |\uparrow\uparrow\rangle$$
$$|\downarrow\downarrow\rangle = |\downarrow\downarrow\rangle$$

$$|\uparrow\downarrow\rangle = e^{-i\theta}\,|\uparrow\downarrow\rangle$$
$$|\downarrow\uparrow\rangle = e^{-i\theta}\,|\downarrow\uparrow\rangle$$

$$\Delta E = \frac{(e\delta)^2}{2r^3}$$

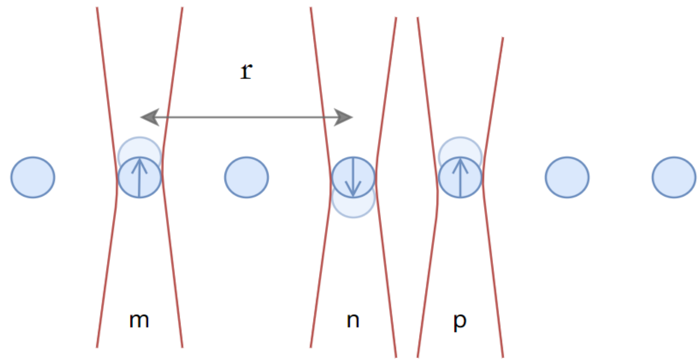

Figure 6: entanglement by optical tweezing. Laser light shined on both ends of ions m,n, and p. m,p demonstrate entanglement between same-state ions and m,n between opposing state ions.

3.5 Decoherence

A Ramsey-type experiment is performed to measure the coherence time of the qubit states [4]. In the Ytterbium experiment, the coherence of one ion with respect to another is measured. A single ion coherence time cannot be measured directly because the coherence time of the qubit exceeds the microwave stability time [4]. It is worth noting that, in previous experiments using the same qubit energy levels, a coherence time of up to 15 minutes has been achieved [6].

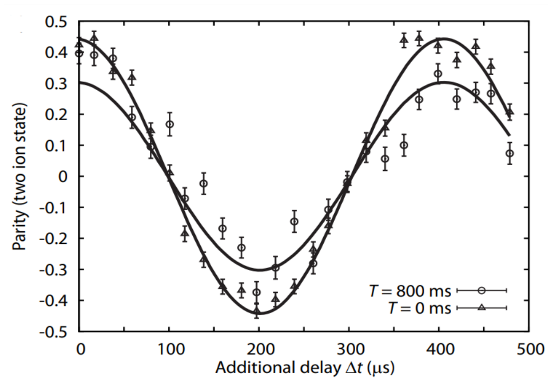

Figure 7: observed Ramsey fringes in the parity of the states of the two ions for two delay times: 0ms, and 800ms. The amplitude evidently decays with time [4]

The ions are first both initialized to $|0\rangle$. A microwave pulse of $\pi/2$ is applied to the two ions, and after waiting for a time of $T/2$, a microwave pulse of π is applied. After a time of $T/2 + \Delta t$, a pulse of $\pi/2$ is applied, and finally the state of each ion is measured. The decay of the Ramsey amplitudes with respect to Δt are plotted. The amplitude is then fit into a

Gaussian decay, and a coherence time of approximately 3 seconds is achieved. This is much less than what is normally achieved with Ytterbium ions [6]. The measured coherence time is likely limited by fluctuations of the differential magnetic field between the two ion positions through the second-order Zeeman shift [4]. The coherence time would be significantly longer when lower static magnetic fields are used [4]. However, in comparison to qubit gate times, which are around microseconds [10], this is still an impressive coherence time.

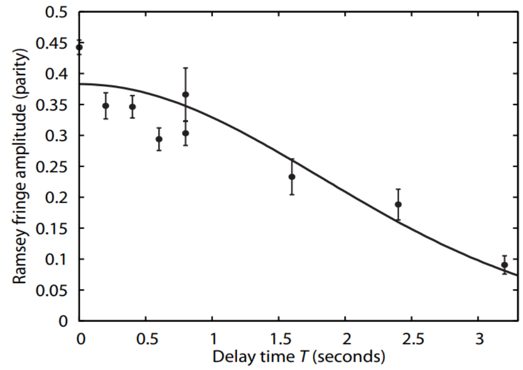

Figure 8: decay of ramsey fringe amplitude vs. the delay time T. A Gaussian fit to the data, where the 1/e decay time is achieved at 2.5s. [4]

4 Challenges and Progress

The major challenge with the ion trap implementation proposed by Cirac and Zoller in 1995, is the ability to manipulate a large number of ions in a single trap [11]. Full-scale computation requires the ability to manipulate 106 ions which causes a lot of technical difficulties in the current linear Paul trap scheme [11]. A linear trap containing 106 ions would be limited to a center of mass mode frequency of $\ll$ 100Hz [11], meaning the computational power required for the Shor algorithm would take 30 years to execute [11]. Some solutions to escape this limitation are to use arrays of micro-traps that act as multiple small ion-trap registers, and to apply quantum communication between them [11]. Other proposals use spin-dependant Coloumb interactions, as demonstrated in the entanglement section of this report.

Today, there are quite a few research initiatives dedicated to developing commercial trapped ion quantum computers. IonQ, a company working on bringing the trapped ion computer out of the lab and into the market, have achieved great success with the power, precision, and connection of the computer. IonQ's current computer is able to apply single qubit gates on a **79** ion chain. However, for more complex, multi-qubit gates, algorithms were run on chains of up to **11** ions [7]. IonQ's current computer has a program length of $>$ 60 two-qubit gates, and a two-qubit gate error of $< 1.0\%$. A main advantage with the ion trap architecture is the complete connectivity of the system - there are no wires the connect the qubit, only precise individual laser beams. Thus, any ion can interact with the other without any intermediate steps [7]. This creates a huge reduction in computational noise, and is why trapped ions provide such an accurate and efficient approach to quantum computing.

5 Conclusion

The use of ytterbium ions as quantum bits for quantum information processing is demonstrated to be successful. State initialization is accomplished with near perfect efficiency using optical pumping schemes, and final states are efficiently detected by monitoring the fluorescence of ions. Quantum logic gates of a universal set were demonstrated. The high efficiency and fidelity of these quantum operations is possible due to the stabilization, precision, and frequency modulation of ion-specific laser sources.

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