# Discrete Mathematics

Exercises & Solutions

# Truth Table

Construct a truth table for each of these compound propositions.

**a)** $p \wedge \neg p$

**b)** $p \vee \neg p$

**c)** $(p \vee \neg q) \to q$

**d)** $(p \vee q) \to (p \wedge q)$

**e)** $(p \to q) \leftrightarrow (\neg q \to \neg p)$

**f)** $(p \to q) \to (q \to p)$

# Truth Table

a)

| $p$ | $\neg p$ | $p \wedge \neg p$ |
|---|---|---|
| T | F | F |
| F | T | F |

b)

| $p$ | $\neg p$ | $p \wedge \neg p$ |
|---|---|---|
| T | F | T |
| F | T | T |

c)

| $p$ | $q$ | $\neg q$ | $p \vee \neg q$ | $(p \vee \neg q) \rightarrow q$ |
|---|---|---|---|---|
| T | T | F | T | T |
| T | F | T | T | F |
| F | T | F | F | T |
| F | F | T | T | F |

# Truth Table

e)

| $p$ | $q$ | $p \rightarrow q$ | $\neg q$ | $\neg p$ | $\neg q \rightarrow \neg p$ | $(p \rightarrow q) \leftrightarrow (\neg q \rightarrow \neg p)$ |
|---|---|---|---|---|---|---|
| T | T | T | F | F | T | T |
| T | F | F | T | F | F | T |
| F | T | T | F | T | T | T |
| F | F | T | T | T | T | T |

f)

| $p$ | $q$ | $p \rightarrow q$ | $q \rightarrow p$ | $(p \rightarrow q) \rightarrow (q \rightarrow p)$ |
|---|---|---|---|---|
| T | T | T | T | T |
| T | F | F | T | T |
| F | T | T | F | F |
| F | F | T | T | T |

# Logical Equivalence

Use a truth table to verify the distributive law $p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$.

| $p$ | $q$ | $r$ | $q \vee r$ | $p \wedge (q \vee r)$ | $p \wedge q$ | $p \wedge r$ | $(p \wedge q) \vee (p \wedge r)$ |
|---|---|---|---|---|---|---|---|
| T | T | T | T | T | T | T | T |
| T | T | F | T | T | T | F | T |
| T | F | T | T | T | F | T | T |
| T | F | F | F | F | F | F | F |
| F | T | T | T | F | F | F | F |
| F | T | F | T | F | F | F | F |
| F | F | T | T | F | F | F | F |
| F | F | F | F | F | F | F | F |

# Tautology

Show that each of these conditional statements is a tautology by using truth tables.

**a)** $(p \wedge q) \rightarrow p$

**b)** $p \rightarrow (p \vee q)$

**c)** $\neg p \rightarrow (p \rightarrow q)$

**d)** $(p \wedge q) \rightarrow (p \rightarrow q)$

**e)** $\neg(p \rightarrow q) \rightarrow p$

**f)** $\neg(p \rightarrow q) \rightarrow \neg q$

# Tautology

a)

| $p$ | $q$ | $p \wedge q$ | $(p \wedge q) \rightarrow p$ |
|---|---|---|---|
| T | T | T | T |
| T | F | F | T |
| F | T | F | T |
| F | F | F | T |

b)

| $p$ | $q$ | $p \vee q$ | $p \rightarrow (p \vee q)$ |
|---|---|---|---|
| T | T | T | T |
| T | F | T | T |
| F | T | T | T |
| F | F | F | T |

d)

| $p$ | $q$ | $p \wedge q$ | $p \rightarrow q$ | $(p \wedge q) \rightarrow (p \rightarrow q)$ |
|---|---|---|---|---|
| T | T | T | T | T |
| T | F | F | F | T |
| F | T | F | T | T |
| F | F | F | T | T |

# Predicate & Quantifier

Let $P(x)$ be the statement "$x = x^2$." If the domain consists of the integers, what are the truth values?

**a)** $P(0)$ **b)** $P(1)$ **c)** $P(2)$
**d)** $P(-1)$ **e)** $\exists x P(x)$ **f)** $\forall x P(x)$

Determine the truth value of each of these statements if the domain for all variables consists of all integers.

**a)** $\forall n(n^2 \geq 0)$ **b)** $\exists n(n^2 = 2)$
**c)** $\forall n(n^2 \geq n)$ **d)** $\exists n(n^2 < 0)$

# Predicate & Quantifier

Suppose that the domain of the propositional function $P(x)$ consists of the integers 0, 1, 2, 3, and 4. Write out each of these propositions using disjunctions, conjunctions, and negations.

**a)** $\exists x P(x)$ **b)** $\forall x P(x)$ **c)** $\exists x \neg P(x)$

**d)** $\forall x \neg P(x)$ **e)** $\neg \exists x P(x)$ **f)** $\neg \forall x P(x)$

# Predicate & Quantifier

Determine the truth value of each of these statements if the domain for all variables consists of all integers.

**a)** $\forall n \exists m(n^2 < m)$

**b)** $\exists n \forall m(n < m^2)$

**c)** $\forall n \exists m(n + m = 0)$

**d)** $\exists n \forall m(nm = m)$

**e)** $\exists n \exists m(n^2 + m^2 = 5)$

**f)** $\exists n \exists m(n^2 + m^2 = 6)$

**g)** $\exists n \exists m(n + m = 4 \wedge n - m = 1)$

**h)** $\exists n \exists m(n + m = 4 \wedge n - m = 2)$

**i)** $\forall n \forall m \exists p(p = (m + n)/2)$

# Negating Nested Quantifiers

$$\begin{aligned}
&\neg\forall\epsilon > 0\ \exists\delta > 0\ \forall x(0 < |x - a| < \delta \rightarrow |f(x) - L| < \epsilon) \\
&\qquad \equiv \exists\epsilon > 0\ \neg\exists\delta > 0\ \forall x(0 < |x - a| < \delta \rightarrow |f(x) - L| < \epsilon) \\
&\qquad \equiv \exists\epsilon > 0\ \forall\delta > 0\ \neg\forall x(0 < |x - a| < \delta \rightarrow |f(x) - L| < \epsilon) \\
&\qquad \equiv \exists\epsilon > 0\ \forall\delta > 0\ \exists x\ \neg(0 < |x - a| < \delta \rightarrow |f(x) - L| < \epsilon) \\
&\qquad \equiv \exists\epsilon > 0\ \forall\delta > 0\ \exists x(0 < |x - a| < \delta \wedge |f(x) - L| \geq \epsilon).
\end{aligned}$$

# Negating Nested Quantifiers

Express the negations of each of these statements so that all negation symbols immediately precede predicates.

**a)** $\forall x \exists y \forall z T(x, y, z)$

**b)** $\forall x \exists y P(x, y) \vee \forall x \exists y Q(x, y)$

**c)** $\forall x \exists y (P(x, y) \wedge \exists z R(x, y, z))$

**d)** $\forall x \exists y (P(x, y) \rightarrow Q(x, y))$