Based on National Curriculum of Pakistan 2022-23

Textbook of
Physics
12

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A Textbook of Physic for Grade 12
based on National Curriculum of Pakistan (NCP) 2022-23

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Note
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Preface

This Textbook for Physics Grade 12 has been developed by NBF according to the National Curriculum of Pakistan 2022-2023. The aim of this textbook is to enhance learning abilities through inculcation of logical thinking in learners, and to develop higher order thinking processes by systematically building the foundation of learning from the previous grades. A key emphasis of the present textbook is creating real life linkage of the concepts and methods introduced. This approach was devised with the intent of enabling students to solve daily life problems as they grow up in the learning curve and also to fully grasp the conceptual basis that will be built in subsequent grades.

After amalgamation of the efforts of experts and experienced authors, this book was reviewed and finalized after extensive reviews by professional educationists. Efforts were made to make the contents student friendly and to develop the concepts in interesting ways.

The National Book Foundation is always striving for improvement in the quality of its textbooks. The present textbook features an improved design, better illustration and interesting activities relating to real life to make it attractive for young learners. However, there is always room for improvement, the suggestions and feedback of students, teachers and the community are most welcome for further enriching the subsequent editions of this textbook.

May Allah guide and help us (Ameen).

Dr. Kamran Jahangir
Managing Director

Practical Applications of Physics-XII in Everyday Life

Welcome to the fascinating world of physics! This book is designed to take you on a journey through the fundamental principles and concepts that govern our universe. From the intricacies of gravitational potential to the mysteries of quantum physics, each unit in this book will reveal the beauty and relevance of physics in our daily lives.

As you explore the world of physics, you'll discover the numerous career paths available to you. Whether you're interested in pursuing a career in research, engineering, medicine, or technology, physics will provide you with a solid foundation for success.

Let's take a glimpse into the exciting world of physics that awaits you:

In **Unit 15, Gravitation,** you'll discover how the concept of gravitational potential energy is crucial in designing roller coasters, understanding the motion of planets, and even predicting the trajectory of spacecraft. Careers in aerospace engineering, astrophysics, and geophysics rely heavily on this concept.

Unit 16, Statistical Mechanics and Thermodynamics, will show you how the principles of thermodynamics govern and impacting our daily lives in countless ways. Careers in mechanical engineering, chemical engineering, and materials science rely on a deep understanding of thermodynamics.

In **Unit 17, Simple Harmonic Motion,** you'll explore how this fundamental concept is used in designing medical equipment, such as MRI machines, and musical instruments, like guitars and violins. Careers in biomedical engineering, mechanical engineering, sound and music technology rely on an understanding of simple harmonic motion.

Unit 18, Diffraction and Interference, will reveal the secrets and the behavior of waves in different phenomenon. Careers in photonics, optics, and acoustics rely heavily on this concept.

Unit 19, Electric Potential and Capacitor, will introduce you to the world of energy storage and transmission, crucial for understanding how batteries, capacitors, and electrical grids work. Careers in electrical engineering, renewable energy, and energy storage rely on a deep understanding of electric potential and capacitors.

In **Unit 20, Alternating Current,** you'll discover how AC circuits power our homes, industries, and technologies, and learn about the innovative applications of AC in medical equipment and transportation systems. Careers in electrical engineering, power engineering, and telecommunications rely heavily on this concept.

Unit 21, Quantum Physics, will take you on a journey into the fascinating realm of the tiny, where the principles of quantum mechanics govern the behavior of atoms, molecules, and subatomic particles. Careers in materials science, nanotechnology, and quantum computing rely on a deep understanding of quantum physics.

Unit 22, Nuclear Physics, will explore the mysteries of nuclear reactions, radioactivity, and the applications of nuclear energy in medicine, industry, and power generation. Careers in nuclear engineering, medical physics, and radiation therapy rely heavily on this concept.

In **Unit 23, Cosmology,** you'll embark on a cosmic journey to explore the origins, evolution, and fate of our universe, delving into the mysteries of dark matter, dark energy, and the expansion of the cosmos. Careers in astrophysics, cosmology, and space exploration rely on a deep understanding of cosmological principles.

Unit 24, Earth's Climate, will examine the complex relationships between our planet's atmosphere, oceans, and land surfaces, highlighting the impact of human activities on climate change and the importance of sustainable practices. Careers in environmental science, climate modeling, and sustainability rely heavily on this concept.

In **Unit 25, Medical Imaging,** you'll discover how physics principles, such as X-ray computed tomography (CT) scans, magnetic resonance imaging (MRI), and positron emission tomography (PET) scans, have revolutionized medical diagnosis and treatment. Careers in medical physics, biomedical engineering, and radiology rely on a deep understanding the principles of medical imaging.

Finally, **Unit 26, Nature of Science: A Debate,** will challenge you to think critically about the scientific method, the role of experimentation and observation, and the ethics of scientific inquiry. Careers in science policy, science communication, and science education rely heavily on a deep understanding of the nature of science.

As you embark on this physics journey, remember that the concepts and principles you'll learn are not just abstract ideas - they have a profound impact on our daily lives, from the technology we use to the environment we inhabit. By learning these fundamental physics concepts, you'll gain a deeper understanding of the world around you and develop problem-solving skills essential for innovative careers in science, technology, engineering, and mathematics (STEM).

Some potential career paths in physics include:

- Research scientist
- Medical physicist
- Biomedical engineer
- Materials scientist
- Nanotechnologist
- Environmental scientist
- Climate modeler
- Sustainability specialist
- Science policy analyst
- Quantum computing specialist
- Science communicator
- Science educator
- Engineer (mechanical, electrical, aerospace, etc.)

These are just a few examples of the many exciting career paths available to physics students. Get ready to explore, discover, and be amazed by the wonders of physics!

Managing Author
Physics-XII

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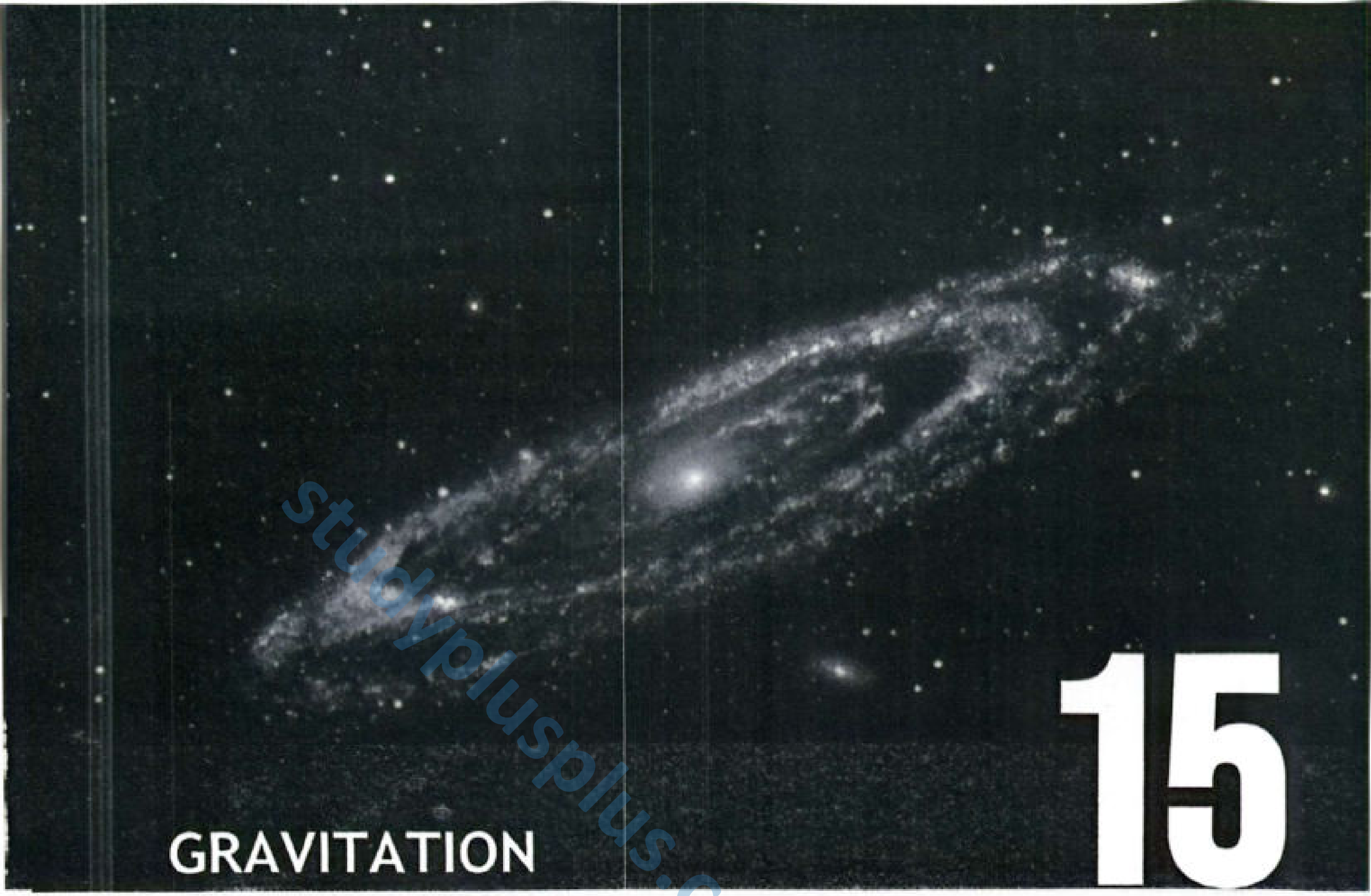


15 GRAVITATION

Student Learning Outcomes (SLOs)

The student will

- Define and calculate gravitational field strength [this will include more challenging problems than in Grade-9. It will involve use of $g = \frac{GM}{r^2}$]
- Analyse gravitational fields by means of field lines. This includes knowing that for a point outside a uniform sphere, the mass of a sphere may be considered to be a point mass at its center.]
- Apply Newton's law of gravitation to solve problems [$F = G\frac{m_1 m_2}{r^2}$ for the force between two-point masses to solve problems].
- Analyze circular orbits in gravitational fields [By relating the gravitational force to the centripetal acceleration it causes]
- Analyze the motion of geostationary satellites [This includes knowing that a geostationary orbit remains at the same point above the Earth's surface, with an orbital period of 24 hours, orbiting from west to east, directly above the Equator].
- Derive the equation for gravitational field strength [From Newton's law of gravitation and the definition of gravitational field, the equation $g = \frac{GM}{r^2}$ for the gravitational field strength due to a point mass].
- Analyse why g is approximately constant for small changes in height near the Earth's surface.
- Define and calculate gravitational potential [Use $\phi = \frac{-GM}{r}$ for the gravitational potential in the field due to a point mass] [At a point as the work done per unit mass in bringing a small test mass from infinity to the point]
- Justify how the concept of gravitational potential leads to the gravitational potential energy of two-point masses [Use $E_p = -G\frac{Mm}{r}$ in problems is expected]

Gravity, a force that has fascinated scientists for centuries, is essential in shaping the universe. It keeps us firmly held on Earth and controls the movements of stars in galaxies. Newton's law of gravitation has been crucial in helping us comprehend this force.

Our galaxy, the Milky-Way, is a prime example of gravity's strength. It is a vast disk of stars, dust, and gas held together by the powerful gravitational force of its center. Even though we are located near the edge of the galaxy, about 26,000 light-years from the center, we are not alone. The Milky-Way is part of the Local Group, a cluster of galaxies that includes the Andromeda Galaxy, just 2.5 million light-years away. Gravity is the master of this cosmic dance. It controls the movements of stars within our galaxy and influences how galaxies interact with each other. As we explore gravity further, we develop a deeper understanding of the force that shapes the universe on large scale.

15.1 NEWTON'S LAW OF UNIVERSAL GRAVITATION

Newton formulated a law called the law of universal gravitation to describe the force of attraction between various objects in the universe; which is stated as follows:

Every object in the universe attracts every other object with a force which is directly proportional to the product of their masses and inversely proportional to the square of the distance between their centres.

Newton's law of universal gravitation revolutionized our understanding of the physical world and provided a mathematical framework for explaining the motion of celestial bodies such as planets, moons, and stars. It laid the foundation for the development of classical mechanics and helped to explain many phenomena in the natural world, from the tides caused by the gravitational pull of the moon to the orbiting of planets around the sun.

Today, Newton's law of universal gravitation is still used as a fundamental principle in physics and astronomy, although it has been refined and expanded upon by later theories such as Einstein's theory of general relativity. Nevertheless, Newton's law of universal gravitation remains a cornerstone of our understanding of the force that governs the motion of objects in the universe.

Consider two spherical bodies of masses 'm_1' and 'm_2' separated by distance 'r', as shown in Fig. 15.1. By definition of Newton's law of universal gravitation, the force of gravity 'F_g' is:

$$F_g \propto m_1 \times m_2$$

$$F_g \propto \frac{1}{r^2}$$

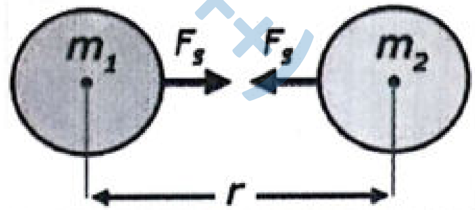


Figure 15.1: Illustration of gravitational force between two objects.

Combining both relations, we get:

$$F_g \propto \frac{m_1 \times m_2}{r^2}$$

Replacing proportionality with constant, we get:

$$F_g = G\frac{m_1 \times m_2}{r^2}$$

Where 'G' is constant of proportionality and is known as gravitational constant. Its value is 6.67×10^{-11} N m^2 kg^{-2}.

Gravitational force is always attractive and does not depend upon the medium between the masses. The gravitational force is an inverse-square force: it decreases by a factor of 4 when the distance increases by a factor of 2, it decreases by a factor of 9 when the distance increases by a factor of 3, and so on. Graph in Fig. 15.2 is a plot of the magnitude of the gravitational force as a function of the distance, between two objects.

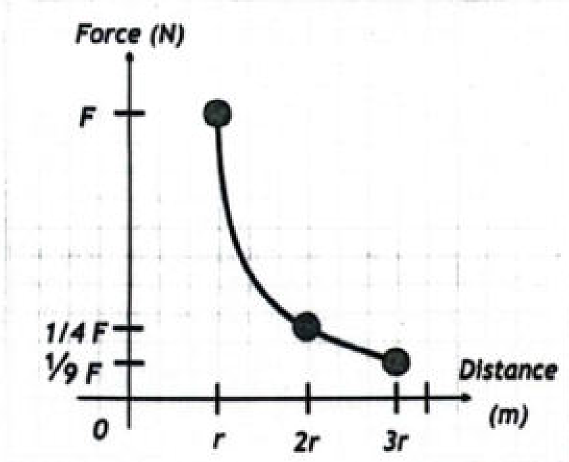


Figure 15.2: Variation of the gravitational force as a function of the distance.

According to Newton's law of universal gravitation, all objects attract each other. So, if the force on mass 'm_2' caused by mass 'm_1' is 'F_{21}', there is also a force on mass 'm_1' caused by mass 'm_2', which is 'F_{12}'. These forces are equal in magnitude but opposite in direction. Therefore, we can conclude that the forces acting on two objects due to gravitational force demonstrate action and reaction making a pair, as shown in the Fig. 15.3.

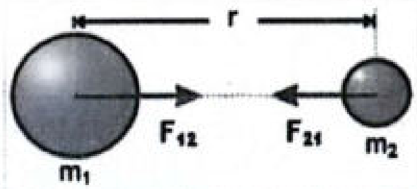


Figure 15.3: Illustration of gravitational action reaction pair.

The forces 'F_{12}' and 'F_{21}' can be mathematically equated as:

$$F_{12} = -F_{21}$$

These two forces act like action and reaction which are equal to each other in magnitude but opposite in direction. It means that Newton's universal law of gravitation is consistent with Newton's 3rd law motion.

Example 15.1: Two bodies A and B are placed at a distance of 1 m from each other. What will be the force of attraction between them if their masses are 45 kg and 50 kg respectively?

Given: $r = 1$ m $\quad m_1 = 45$ kg $\quad m_1 = 50$ kg

To Find: $F_g = ?$

Solution: Using Newton's law of universal gravitation: $F_g = G\dfrac{m_1 m_2}{r^2}$

Putting values: $$F_g = \frac{6.673\times10^{-11}\times45\times50}{(1)^2}$$

Hence $$F_g = 1.5\times10^{-7}\text{ N}$$

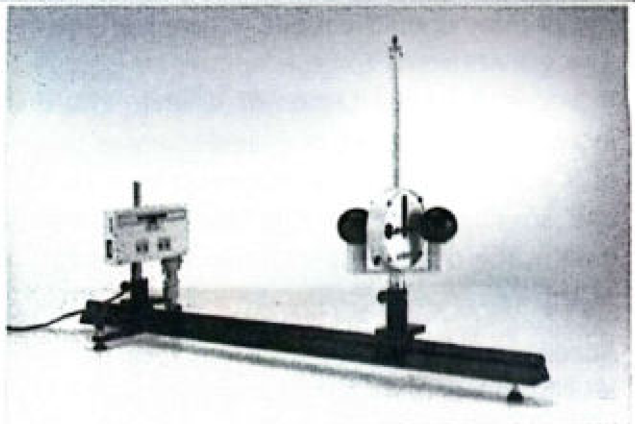
This implies that we are drawn towards each other, but the force is so minuscule, around 10^{-7} N, that it goes unnoticed unless we employ highly sensitive instruments. Even massive objects such as ships and buildings experience a very slight gravitational pull.

Assignment 15.1

1) The mass of Earth is 6×10^{24} kg and that of the moon is 7.4×10^{22} kg, with a distance of 3.84×10^5 km between them. Calculate the force exerted by the Earth on the moon.
2) Why do we say that law of gravitation is a universal law?

POINT TO PONDER

The gravitational constant 'G' is crucial for calculating gravitational forces accurately. However, measuring such a small number was a challenge. It took scientists 100 years after Newton's work to create a device capable of measuring G. In 1798, Henry Cavendish, an Englishman, used a torsion balance apparatus (shown in figure) to measure the gravitational force between two objects. This experiment not only helped determine G but also raised questions about whether gravitational force varies in different mediums, such as air and water. Knowing G also led to obtaining precise values for Earth's mass and other astronomical masses.



15.2 GRAVITATIONAL FIELD STRENGTH

The area around a massive object (like Earth, sun, etc.) where its gravitational force acts is known as gravitational field.

Gravitational field is represented by field lines that indicate the direction and strength of gravity, as shown in Fig. 15.4. Closer lines represent a stronger field, while farther apart lines indicate a weaker field.

The gravitational field lines point radially inward uniformly in all directions for a point mass (a point mass is treated as if all its mass is concentrated at a single point in space), forming a symmetric pattern. Since a gravitational force is an interaction between a test mass and the gravitational field created by the source mass, therefore from Newton's second law of motion:

$$a = \frac{F}{m}$$

Similarly, $$a_g = \frac{F_g}{m} = g$$

Gravitational field can be defined by gravitational field strength, denoted by 'g'. The gravitational field strength is a vector with a magnitude of 'g' pointing in the direction of the gravitational force. The value of 'g' is the gravitational force on a unit mass at that point, or '$g = F_g/m$'. Therefore, gravitational field strength and acceleration due to gravity are equivalent.

Value of 'g' on Surface of the Earth

Newton's law of universal gravitation shows that the value of 'g' depends on mass and distance.

For example, consider an object (stone) of mass 'm_O' placed on surface of Earth. Let 'M_E' be the mass of the Earth and radius of Earth 'R_E' is the distance between their centres (as radius of stone is very small compared to radius of Earth, therefore it is ignored). The gravitational force between the stone and Earth is:

$$F_g = G\frac{m_O M_E}{R_E^2} \qquad \text{\_\_\_\_\_ (15.1)}$$

The gravitational force F_g by Newton's second law is:

$$F_g = W = m_o g \qquad \text{\_\_\_\_\_\_ (15.2)}$$

Comparing Eq. (15.1) and Eq. (15.2), we get: $m_o g = G\dfrac{m_o M_E}{R_E^2}$

or $$g = G\frac{M_E}{R_E^2} \qquad \text{\_\_\_\_\_\_ (15.3)}$$

For Earth we know that mass of Earth $M_E = 6 \times 10^{24}$ kg and radius of Earth $R_E = 6.4 \times 10^6$ m and $G = 6.67 \times 10^{-11}$ N m^2 kg^{-2}, putting these values in Eq. (15.4), we get:

$$g = 6.67 \times 10^{-11} \times \frac{6 \times 10^{24}}{(6.4 \times 10^6)^2} \quad \text{or} \quad g = 9.77 \text{ m s}^{-2} = 9.8 \text{ m s}^{-2}$$

Eq. (15.3) shows that the value of 'g' does not depend upon the mass 'm_o' of the body. This means that light and heavy bodies should fall at same rate. This equation also shows that gravitational field strength depends only on mass of Earth 'M_E' and radius of Earth 'R_E'. Therefore, on any other planet's surface, both the value of 'g' and our weight will be influenced by the planet's mass 'm' and radius 'r', therefore the Eq. (15.3) in more general form can be expressed as:

$$g = G\frac{m}{r^2} \qquad \text{\_\_\_\_\_\_ (15.4)}$$

Variation of 'g' with Altitude

Moving away from surface of Earth may change 'g' and therefore our weight. The value of 'g' at a given place depends upon the distance from the centre of Earth, as shown in Fig. 15.4.

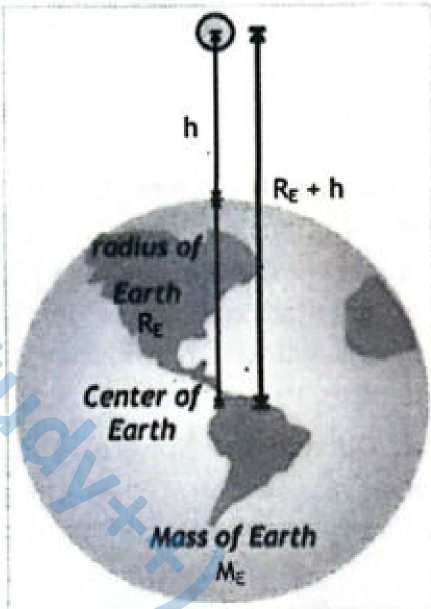


Figure 15.4: Variation of 'g' with altitude.

Let 'g_h' be the value of acceleration due to gravity at a height 'h' from the surface of the Earth. We can rewrite gravitational field strength from Eq. (15.3) as:

$$g_h = G\frac{M_E}{(R_E + h)^2} \qquad \text{\_\_\_\_\_\_ (15.5)}$$

As, $$g = G\frac{M_E}{R_E^2}$$

or $$GM_E = gR_E^2 \qquad \text{\_\_\_\_\_\_ (15.6)}$$

Putting Eq. (15.6) in Eq. (15.5), we get:

$$g_h = \frac{gR_E^2}{(R_E + h)^2} \qquad \text{\_\_\_\_\_\_ (15.7)}$$

This equation shows that:

As we go further away from the centre or surface of the Earth, the value of 'g' decreases.

Table 15.1: Variation in Gravitational Acceleration with Altitude

Altitude (km)	g_h (m s^{-2})	Example
0	9.8	Average Earth radius
8.8	9.8	Mount Everest
36.6	9.7	Highest manned balloon
400	8.7	Space shuttle orbit
35,700	0.2	Communication satellites

The change in 'g' is only noticeable at very long distances, as indicated in the Table 15.1.

Example 15.2: Shooting stars are meteors that burn up in Earth's atmosphere. Meteors become visible between about 75 to 120 km above Earth. What is the gravitational field strength at 120 km above earth surface?

Solution: The gravitational field strength of Earth is given by:

$$g_h = \frac{gR_E^2}{(R_E + h)^2}$$

Putting values, we get: $$g_h = \frac{9.8 \times (6.4 \times 10^6)^2}{(6.4 \times 10^6 + 0.12 \times 10^6)^2} \quad \text{or} \quad g_h = 9.4\ \text{m s}^{-1}$$

The value of 'g' at 120 km above Earth surface is slightly less than its value at surface of Earth.

Assignment 15.2

What will be the value of gravitational filed strength 'g' at 35,700 km, where geostationary satellites orbit around the Earth?

Gravitational Fields Lines

To better understand the gravitational field surrounding the Earth, we can visualize the planet as a perfectly uniform sphere. This simplification allows us to analyse the gravitational effects more easily. A uniform sphere has a consistent density throughout. The concept of a uniform sphere simplifies the understanding of complex mass distributions, which is crucial in astrophysics and planetary science. It allows for easier calculations of gravitational forces for stars, planets, and moons without considering differences in their density. Its gravitational field behaves as follows:

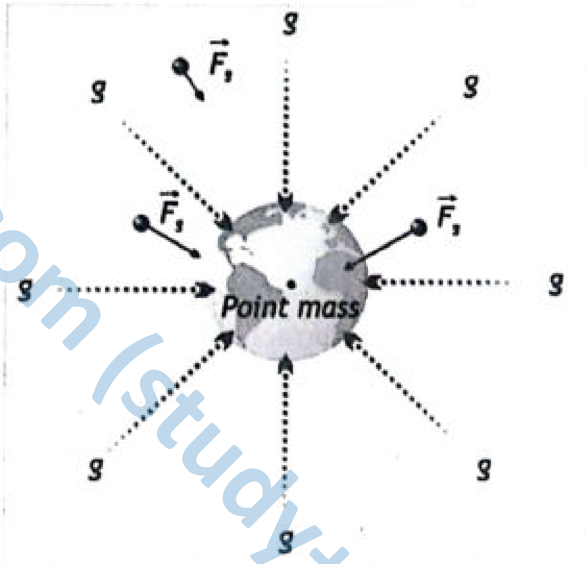


Figure 15.5: Gravitational Field Lines.

A. Outside the Sphere: The field acts as if all mass is concentrated at the centre, resembling that of a point mass with the same total mass.

For locations that are situated outside the Earth's surface, we can conceptualize the Earth's mass as being concentrated at a single point located at its centre. This model is useful because it simplifies the calculations involved in understanding gravitational forces.

As shown in Fig. 15.5, this representation illustrates how the strength of the gravitational field reduces rapidly as one moves away from the Earth. This behaviour aligns with our expectations as one would expect from the inverse-square nature of Newton's law of universal gravitation.

B. Inside the Sphere: The gravitational field strength increases steadily toward the surface, as only the mass within the radius affects the field at any point inside.

Example 15.3: The largest planet in our solar system is Jupiter, it has a mass of 1.898×10^{27} kg and radius as 7.15×10^{7} m. Determine the free-fall acceleration at the surface of Jupiter? Also calculate the weight of 60 kg astronaut on it.

Given: $m_J = 1.898 \times 10^{27}$ kg $\quad r_J = 7.15 \times 10^{7}$ m $\quad$ Mass of astronaut = 60 kg

To Find: $g_J = ?$ $\quad W_A = ?$

Solution: The gravitational field strength of Jupitar is given by:

$$g_J = G\frac{m_J}{r_J^2}$$

Putting values, we get: $\quad g_J = 6.67 \times 10^{-11} \times \frac{1.898 \times 10^{27}}{(7.15 \times 10^{7})^2}$ or $\quad g_J = 24.77\,\mathrm{m\,s^{-2}}$

So, value of 'g' on Jupiter's surface if 24.77 which is about 2.528 times of the acceleration due to gravity on Earth's surface ($g_J = 2.528\ g_E$). Weight of astronaut can be found by:

$$W_A = mg_J$$

Putting values, we get: $\quad W_A = 60 \times 24.77\,\mathrm{N}$ $\quad$ or $\quad W_A = 1486.2\,\mathrm{N}$

While on Earth, he will only weigh 588.6 N.

Assignment 15.3

The gravitational field strength on surface of moon is $1.6\ \mathrm{N\,kg^{-1}}$. The mass of moon is 7.3×10^{22} kg, what is its radius?

15.3 SATELLITES AND ORBITS

A satellite is any object that orbits the planet due to the force of gravity, maintaining a stable path around it.

A natural body orbiting a planet, dwarf planet, or minor planet, where the larger body's gravity dominates the system, is called a natural satellite. Six of the major planets possess natural satellites often termed as moons.

For Your Information

Space station is a space craft capable of supporting crew which is designed to remain in space for an extended period of time and to which other space-crafts can dock. International Space Station (ISS) is the largest satellite in the orbit; it can even be seen with the naked eye.

Artificial Satellites

Artificial satellites are objects intentionally placed into orbit around the Earth or other celestial bodies. Different orbits for artificial satellites are shown in Fig. 15.6. The first artificial satellite was launched in 1957, and since then, thousands have been sent for various purposes such as communication, military operations, and scientific research.

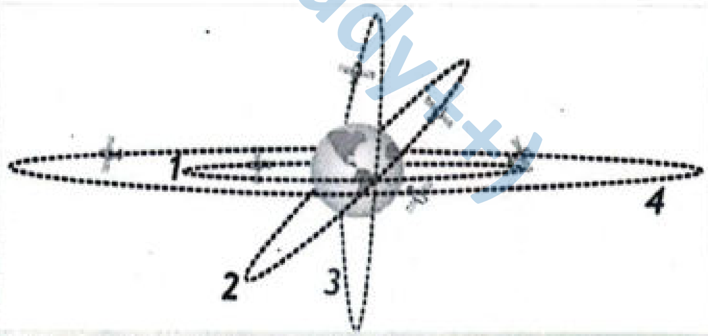


Figure 15.6: Different orbits for artificial satellites are shown with numbers. 1 shows plane of the equator, 2 shows inclined orbit, 3 shows the polar orbit, 4 shows the geostationary orbit.

These satellites orbit the Earth without the need for an engine, as they are held in place by the gravitational pull of the Earth. Engineers have developed different types of satellites, each serving a specific purpose or mission, such as: Communication satellites, Weather satellites, Navigation satellites etc.

Launching a satellite into orbit involves a complex and carefully planned process to ensure mission success. A satellite is put into orbit by moving it to high altitude and then accelerating it to a sufficiently high tangential speed with the help of space craft (e.g. rockets), as shown in Fig. 15.7. If the speed is too low the satellite will fall back to Earth. If the speed is too high, the satellite will either move in elliptical orbit or will escape out of the Earth's gravity, never to return (escape speed). However, if the speed is adjusted it will move in a circular orbit forever. Satellites are typically put into circular (or nearly circular) orbits, because such orbits require the least energy.

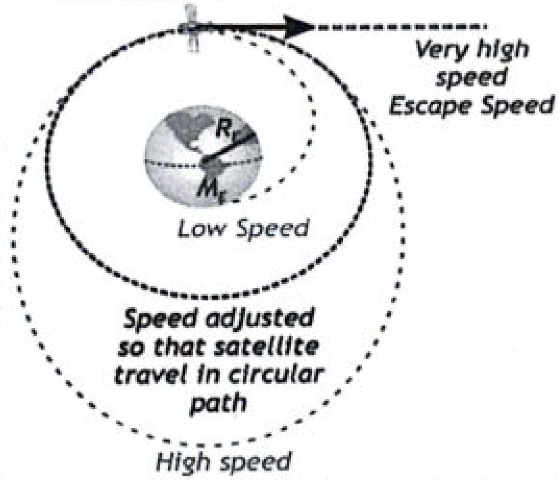


Figure 15.7: Artificial satellites launched at different speed.

Orbital Velocity

The orbital velocity of a satellite refers to the minimum velocity required for a satellite to orbit the Earth or another celestial body at a specific altitude.
When a satellite is moving with velocity 'v_o' in a circle of radius 'r' from the centre of earth, it has centripetal acceleration given by:

$$a_c = \frac{v_o^2}{r}$$

As this centripetal acceleration is supplied by gravity, i.e., $a_c = g$,

So, $$g = \frac{v_o^2}{r}$$

$$v_o^2 = g\,r$$

or $$v_o = \sqrt{g\,r}$$

By using $g = G\frac{M_E}{r^2}$, we get:

$$v_o = \sqrt{\frac{GM_E}{r}} \qquad \text{———} \quad (15.8)$$

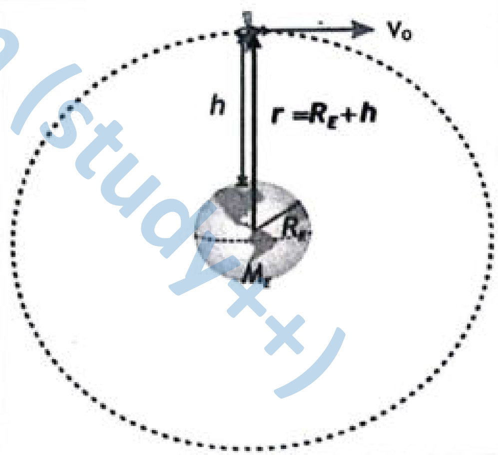


Figure 15.8: Satellite launched at orbital speed.

Hence orbital velocity also depends upon the mass of the larger body and the distance from the centre of the larger body to the centre of mass of the satellite.

Eq. (15.8) tells us that the satellite in the orbit moves faster when it is close to the gravitating body (i.e., Earth) and slower when it is further away.

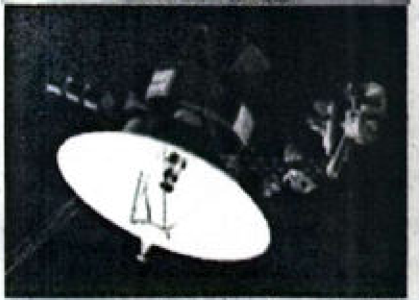
As the size of satellite is small as compared to the size of Earth and the distance in between therefore 'r' is taken as the sum of the radius of Earth 'R_E' and height 'h'.

Therefore, Eq. (15.9), can also be written as:

$$v_o = \sqrt{\frac{GM_E}{R_E + h}} \quad \text{———} \quad (15.9)$$

As the speed decreases due to height, we may have certain orbit such that the satellite covers a complete round trip in twenty-four hours and it appears stationary above certain fixed point.

Point to Ponder



Voyager 1, which was launched in 1977 alongside Voyager 2, holds the title of being the farthest manmade object. It has recently ventured into interstellar space, surpassing the distance between Earth and Pluto. These spacecrafts are equipped with a golden record containing various messages from Earth, such as music and speeches, intended for potential extraterrestrial beings to appreciate.

Example 15.4: The International Space Station (ISS) is the biggest structure ever placed in space (even visible in night sky with unaided eye) and serves as a space laboratory. It orbits at an average altitude of about 400 km above the Earth's surface in Low Earth Orbit (LEO). What is its orbital velocity?

Given: h = 400 km = 400,000 m

To Find: v_o = ?

Solution: The equation for orbital velocity is:

$$v_o = \sqrt{\frac{GM_E}{R_E + h}}$$

Putting values, we get: $v_o = \sqrt{\frac{6.67 \times 10^{-11} \times 6 \times 10^{24}}{6.4 \times 10^6 + 400{,}000}}$ or $v_o = 7.66 \text{ km s}^{-1}$

At this speed in a span of 24 hours, the space station completes 16 orbits around Earth, experiencing 16 sunrises and sunsets along the way.

Example 15.5: The mass of the Sun is 1.99×10^{30} kg, and the radius of the Earth's orbit around the Sun is 1.5×10^{11} m. What is the orbital velocity of the Earth?

Given: $M_s = 1.99 \times 10^{30}$ kg r = 1.5×10^{11} m

To Find: v_o = ?

Solution: The equation for orbital velocity is:

$$v_o = \sqrt{G\frac{M_s}{r}}$$

Putting values, we get: $v_o = \sqrt{6.67 \times 10^{-11} \times \frac{1.99 \times 10^{30}}{1.5 \times 10^{11}}}$ or $v_o = 30 \text{ km s}^{-1}$

This is indeed very high speed; it means that we all sitting stationary on surface of Earth are actually moving at 30 km s^{-1} around the sun.

Assignment 15.4

Calculate the orbital speed of satellite orbiting the Earth at an altitude equal to Earth's radius.

15.4 GEOSTATIONARY SATELLITES

At certain distance from the centre of Earth a satellite would take exactly 24 hours to circle the Earth. Such satellite would remain stationary above some point on Earth. These satellites are called geostationary (or geosynchronous satellites) and the orbit of these satellites is called geostationary orbit, as shown in Fig. 15.9.

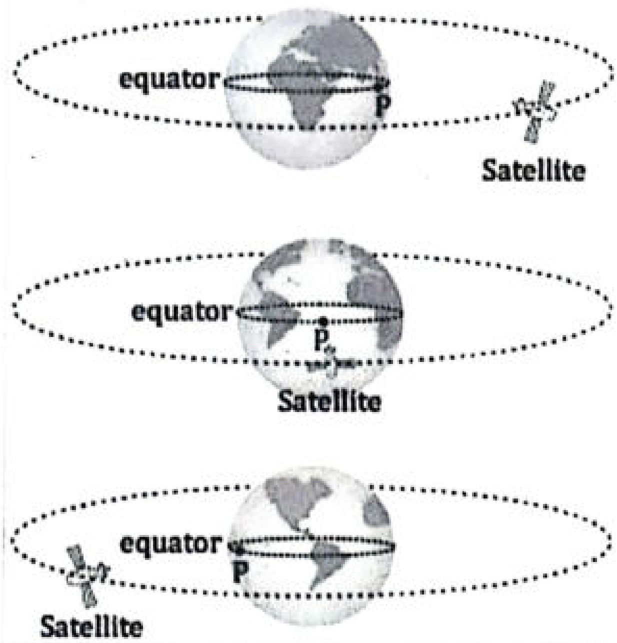


Figure 15.9: Geostationary satellite remains stationary above point 'P' on the surface of Earth.

Consider a satellite of mass m revolving in a geostationary orbit with velocity 'v_o' from Earth of mass 'M_E'. Let 'r' be the distance between the centre of Earth and the centre of satellite. Then the orbital velocity of satellite is given by:

$$v_o = \sqrt{\frac{GM_E}{r}}$$

For a satellite revolving in a circular geostationary orbit of radius 'r' the distance covered by satellite is the circumference of circle ($2\pi r$) and time taken is the time period 'T' of satellite, then velocity 'v_o' is given by:

$$v_o = \frac{S}{t} = \frac{2\pi r}{T}$$

Comparing the above two equations for velocity 'v_o', we get: $\frac{2\pi r}{T} = \sqrt{\frac{GM_E}{r}}$

Squaring both sides and rearranging for 'r', we get: $r^3 = \frac{GM_E T^2}{4\pi^2}$

Taking cube root on both sides, we get:

$$r = \left(\frac{GM_E T^2}{4\pi^2}\right)^{1/3} \qquad \text{——— (15.10)}$$

Eq. (15.10) gives the orbital radius of geostationary satellite.

Calculation of orbital radius for geostationary satellite

Since $G = 6.67 \times 10^{-11}\ \text{N m}^2\ \text{kg}^{-2}$, $M_E = 6 \times 10^{24}$ kg, T = 86400 s and $\pi = 3.14$ by substituting these values in Eq. (15.10), we get:

$$r = \left(\frac{6.67 \times 10^{-11} \times 6 \times 10^{24} \times (86400)^2}{4 \times (3.14)^2} \right)^{1/3}$$

$r = 4.23 \times 10^7$ m or $r = 4.23 \times 10^4$ km

This is the orbital radius as measured from the centre of Earth.

Calculation of orbital speed of geostationary satellite

The equation for orbital speed is: $v_o = \sqrt{\dfrac{GM_E}{r}}$

Since $G = 6.67 \times 10^{-11}$ N m^2 kg^{-2}, $M_E = 5.98 \times 10^{24}$ kg and $r = 4.23 \times 10^7$ m, by substituting values, we get:

$$v_o = \sqrt{6.67 \times 10^{-11} \times \frac{6 \times 10^{24}}{4.23 \times 10^7}} = 3.07 \times 10^3 \text{ m s}^{-1}$$

Point to Ponder

A satellite that stays fixed relative to the Moon's surface, known as a lunar-stationary satellite, cannot maintain a stable orbit due to the Moon's Hill Sphere. The Hill Sphere is a region around a celestial body where its gravitational pull is stronger than that of a larger body, in this case, Earth. To have a satellite stationary relative to the Moon, its orbital period would have to match the Moon's rotation period of about 27.3 days. The required orbital radius for this 27.3-day period around the Moon is approximately 88,417 km. However, the Moon's Hill Sphere extends only up to around 60,000 km. This means that the calculated radius of 88,417 km falls outside the Moon's Hill Sphere, causing Earth's gravitational force to interfere and destabilize the satellite's orbit. As a result, a lunar-stationary satellite would not be able to sustain a stable orbit around the Moon.

Example 15.6: Venus has a mass of 4.867×10^{24} kg and has a period of 243 days. What would be the radius of a synchronous satellite for this planet?

Given: $M_V = 4.867 \times 10^{24}$ kg $T = 243$ days $= 2.093 \times 10^7$ s

To Find: $r = ?$

Solution: The equation for radius of synchronous satellite for Venus is:

$$r = \left(\frac{GM_V T^2}{4\pi^2} \right)^{1/3}$$

Putting values, we get:

$$r = \left(\frac{6.67 \times 10^{-11} \times 4.867 \times 10^{24} \times (2.093 \times 10^7)^2}{4 \times (3.14)^2} \right)^{1/3}$$

or

$$r = 1.53 \times 10^9 \text{ m} = 1.53 \times 10^6 \text{ km}$$

Venus, due to its incredibly slow rotation, cannot have a stable geostationary orbit at 1,530,000 km (or even close to that distance). Due to Venus's lower mass compared to Earth, its Hill sphere is significantly smaller. At a distance of 1,530,000 km, the satellite would likely be outside Venus's Hill sphere and more influenced by the Sun's gravity. This would cause the satellite's orbit to become unstable, and it wouldn't remain stationary over a fixed point on Venus.

Therefore, while the mathematical formula provides a solution, it doesn't represent a feasible scenario for a synchronous satellite around Venus.

Assignment 15.5

Calculate the height (from surface of Earth) of a satellite in geostationary orbit.

15.5 GRAVITATIONAL POTENTIAL

The formula $\Delta P.E = mgh$ is accurate for potential energy near the Earth's surface where gravity is constant. However, as we move away from the surface, gravity weakens, making the equation invalid. Instead, we must use an expression based on Newton's law of universal gravitation.

Secondly, to calculate gravitational potential energy, we need to establish a zero-reference point, often the Earth's surface. This choice results in negative potential energy for any finite distance 'r' because potential energy decreases as objects move closer to the zero-reference point and increases as they move apart.

Consider the Fig. 15.10 in which a body of mass 'm' is placed at surface of Earth having distance from the centre of the Earth 'R_E' (equal to the radius of Earth). As mass of Earth is 'M_E' and to displace the body from point '1' to 'n', we divide the whole distance into number of small distances each of magnitude 'Δr', such that the force during each interval remains constant.

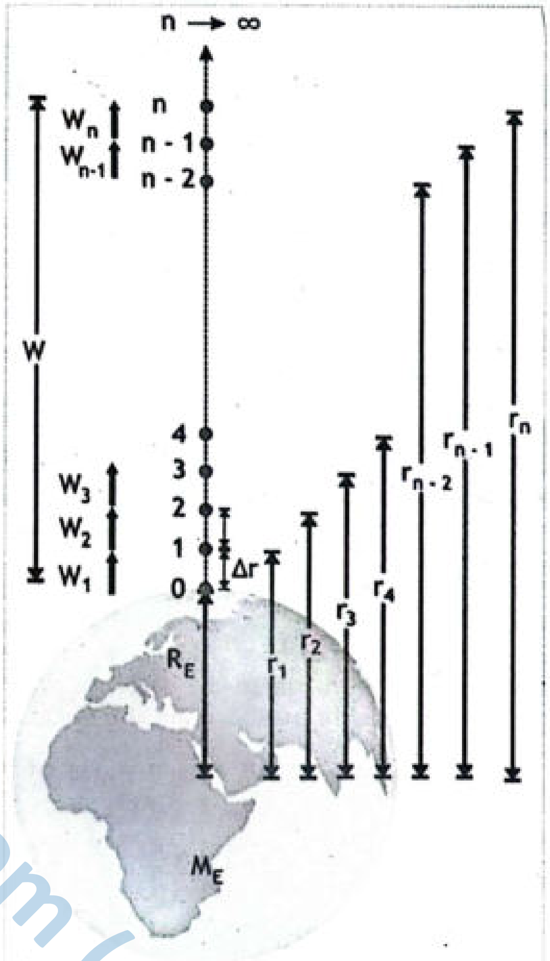


Figure 15.10: Gravitational Potential.

By work and potential energy principle, we get:

$$\Delta P.E = W_{net} \qquad \text{\_\_\_\_\_\_ (1)}$$

As we go up from the surface of Earth, the gravitational force decrease. So, for variable force, we can get the total work done by summing up all the individual work done, i.e.,

$$W_{net} = W_1 + W_2 + W_3 + \ldots\ldots + W_{n-1} + W_n \qquad \text{\_\_\_\_\_\_ (2)}$$

The work done from point '0' to point '1' is 'W_1', which can be written as:

$$W_1 = \mathbf{F_{av}} \cdot \mathbf{\Delta r} \qquad \text{or} \qquad W_1 = F_{av}\Delta r \cos\theta$$

Here $\theta = 180°$ and $\cos 180° = -1$

Therefore, $\quad W_1 = -F_{av}\,\Delta r \qquad$ ______ (3)

The average force from point '0' to point '1' is F_{av}, which can be written from Newton's law of universal gravitation as:

$$F_{av} = \frac{G M_E m}{r_{av}^2} \qquad \text{\_\_\_\_\_\_ (4)}$$

Now the average distance 'r_{av}' is neither 'R_E' nor 'r_1', it can be calculated as:

Calculation of r_{av}:

The average distance r_{av} is $r_{av} = \frac{R_E + r_1}{2}$ _____ (5)

From the Fig. 15.10: $\Delta r = r_1 - R_E$ or $r_1 = R_E + \Delta r$ _____ (6)

Putting Eq. (6) in Eq. (5), we get:

$$r_{av} = \frac{R_E + R_E + \Delta r}{2} \quad \text{or} \quad r_{av} = \frac{2R_E + \Delta r}{2}$$

Therefore. $r_{av} = R_E + \frac{\Delta r}{2}$ _____ (7)

Calculating $(r_{av})^2$:

Squaring both sides of Eq. (7), we get: $r_{av}^2 = \left(R_1 + \frac{\Delta r}{2}\right)^2$

Therefore, $r_{av}^2 = \left(R_E^2 + \frac{(\Delta r)^2}{4} + \frac{2R_E \Delta r}{2}\right)$

Since 'Δr' is very very small, the square of the term will be so small that it approaches to ZERO, compared to 'R_E' and 'r_1'. Therefore, we can neglect the term which include $(\Delta r)^2$, as it will have extremely minor effect on overall calculation, hence:

$$r_{av}^2 = R_E^2 + R_E \Delta r \quad \text{_____ (8)}$$

Putting value of Δr from equation (6) in equation (8), we get:

$$r_{av}^2 = R_E^2 + R_E(r_1 - R_E) \quad \text{or} \quad r_{av}^2 = R_E^2 + R_E r_1 - R_E^2$$

So $r_{av}^2 = R_E r_1$ _____ (9)

Putting Eq. (9) in Eq. (4), we get: $F_{av} = \frac{GM_E m}{R_E r_1}$ _____ (10)

From the Fig. 15.10, $\Delta r = r_1 - R_E$ _____ (11)

Putting values from Eq. (10) and Eq. (11) in Eq. (3), we get:

$$W_1 = -\frac{GM_E m}{R_E r_1}(r_1 - R_E)$$

Hence $W_1 = -GM_E m\left(\frac{1}{R_E} - \frac{1}{r_1}\right)$ _____ (12)

This is the value of work in which a force is supposed to remain constant during separation 'Δr', similarly,

$$W_2 = -GM_E m\left(\frac{1}{r_1} - \frac{1}{r_2}\right) \quad \text{_____ (13)}$$

and $$W_3 = -GM_E m\left(\frac{1}{r_2}-\frac{1}{r_3}\right) \quad \text{_____ (14)}$$

Similarly, the work done in last two steps is given by:

$$W_{n-1} = -GM_E m\left(\frac{1}{r_{n-2}}-\frac{1}{r_{n-1}}\right) \quad \text{_____ (15)}$$

and $$W_n = -GM_E m\left(\frac{1}{r_{n-1}}-\frac{1}{r_n}\right) \quad \text{_____ (16)}$$

The total work done in moving a body from point '0' to point 'n' can be obtained by adding all the individual work done, that is, by putting Eq. (12) to Eq. (16) in Eq. (2), we get:

$$W_{net} = -GM_E m\left(\frac{1}{R_E}-\frac{1}{r_1}\right)-GM_E m\left(\frac{1}{r_1}-\frac{1}{r_2}\right)-GM_E m\left(\frac{1}{r_2}-\frac{1}{r_3}\right)+\ldots\ldots\ldots$$

$$\ldots\ldots-GM_E m\left(\frac{1}{r_{n-2}}-\frac{1}{r_{n-1}}\right)-GM_E m\left(\frac{1}{r_{n-1}}-\frac{1}{r_n}\right)$$

or $$W_{net} = -GM_E m\left(\frac{1}{R_E}-\frac{1}{r_1}+\frac{1}{r_1}-\frac{1}{r_2}+\frac{1}{r_2}-\frac{1}{r_3}+\ldots\ldots\ldots\ldots+\frac{1}{r_{n-2}}-\frac{1}{r_{n-1}}+\frac{1}{r_{n-1}}-\frac{1}{r_n}\right)$$

Therefore; $$W_{net} = -GM_E m\left(\frac{1}{R_E}-\frac{1}{r_n}\right) \quad \text{_____ (17)}$$

Putting Eq. (17) in Eq. (1), we get:

$$\Delta P.E = -GM_E m\left(\frac{1}{R_E}-\frac{1}{r_n}\right) \quad \text{_____ (18)}$$

Now to exclude the choice of setting a reference point for calculation, we set point 'n' at infinity, such that we define the gravitational potential energy 'U' as the work done in bringing a mass 'm' from infinity to the surface of Earth.

When point 'n' is taken at infinity, then '$r_n = \infty$', and Eq. (18) becomes:

$$U = -GM_E m\left(\frac{1}{R_E}-\frac{1}{\infty}\right) \qquad \text{as} \qquad \frac{1}{\infty}=0$$

Therefore, the general expression for the gravitational potential energy 'U' of a body situated on the surface of Earth at distance 'r' from the centre of Earth is given by:

$$U = -\frac{GM_E m}{r} \quad \text{_____ (15.9)}$$

Eq. (15.9) is used when gravitational force is not constant. The variation of gravitational potential energy 'U' as a function of distance 'r' is shown in the Fig. 15.11. The absolute potential energy is defined as:

The amount of work done in moving a body from Earth's surface to a point at infinite distance where the value of g is negligible.

The gravitational potential (represented by ϕ or V) at a point is the work done per unit mass in bringing a small test mass from infinity to that point. The gravitational potential 'V' is therefore the gravitational potential energy 'U' per unit mass 'm', mathematically:

$$V = \frac{U}{m} \quad \text{——— (15.10)}$$

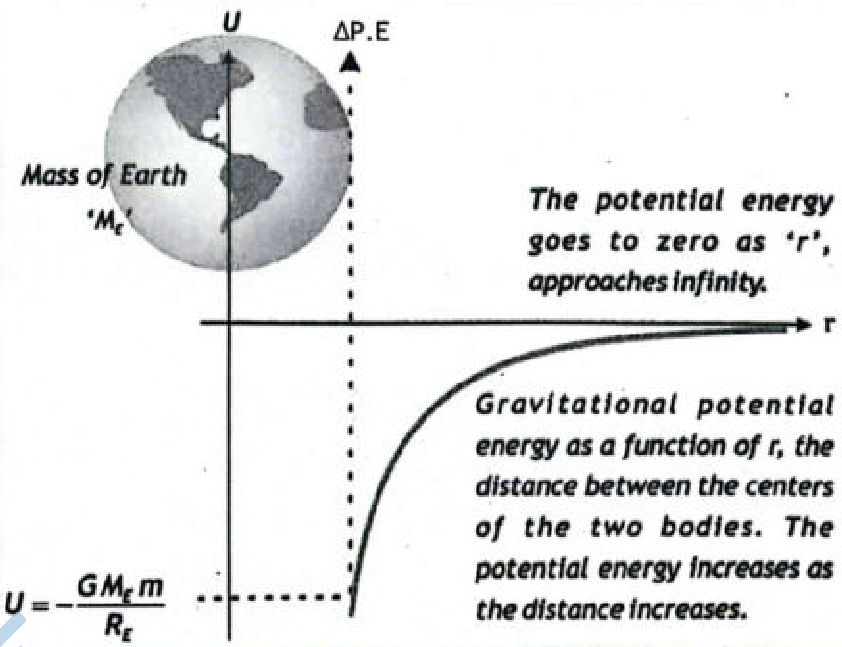


Figure 15.11: Variation of the gravitational field and potential, as a function of the distance from the center.

The expression for gravitational potential 'V' at a point 'P' at distance 'r' from the centre of Earth, can therefore be obtained by putting Eq. (15.9) in Eq. (15.10), such that:

$$V = -\frac{GM_E}{r} \quad \text{——— (15.11)}$$

Calculation of Gravitational Potential at the Surface of Earth:

Eq. (15.11) can be used to determine the gravitational potential at the surface of Earth as it does not depend upon the mass of the object. As gravitational constant G is 6.67×10^{-11} N m^2 kg^{-2}, mass of Earth M_E is 6×10^{24} kg and radius of Earth R_E is 6.4×10^6 m, therefore:

$$V = -\frac{6.67 \times 10^{-11} \times 6 \times 10^{24}}{6.4 \times 10^6}$$

or $$V = -62.3 \times 10^6 \text{ N m kg}^{-1} = -62.3 \text{ MJ kg}^{-1}$$

Although the equations are derived for Earth, but these are equally valid for other gravitating objects.

Example 15.7: What is the gravitational potential energy and gravitational potential with respect to the Sun at the position of the Earth? The mass of the Sun is 1.99×10^{30} kg and the mass of the Earth is 6×10^{24} kg. The mean Earth-to-Sun distance is 1.5×10^{11} m.

Given: $M_S = 1.99 \times 10^{30}$ kg $\quad M_E = 6 \times 10^{24}$ kg $\quad r = 1.5 \times 10^{11}$ m

To Find: U = ? $\quad$ V = ?

Solution: Gravitational potential energy formula for sun and Earth can be written as:

$$U = -\frac{G M_S M_E}{r}$$

Putting values from given data, we get:

$$U = -\frac{6.67 \times 10^{-11} \times 1.99 \times 10^{30} \times 6 \times 10^{24}}{1.5 \times 10^{11}}$$

Therefore, $U = -5.29 \times 10^{33}\ \text{N m} = -5.29 \times 10^{33}\ \text{J}$

Gravitational potential formula for sun and Earth can be written as:

$$V = -\frac{G M_S}{r}$$

Putting values from given data, we get:

$$V = -\frac{6.67 \times 10^{-11} \times 1.99 \times 10^{30}}{1.5 \times 10^{11}}$$

or $V = -8.85 \times 10^{8}\ \text{N m kg}^{-1} = -885\ \text{MJ kg}^{-1}$

Assignment 15.6

Calculate the value of gravitational potential at 1000 km, 50,000 km, and 100,000 km from the surface of Earth. Compare the values obtained with potential energy formula '$E_P = mgh$'.

SUMMARY

- **Newton's Law of Universal Gravitation:** Every object in the universe attracts every other object with a force which is directly proportional to the product of their masses and inversely proportional to the square of the distance between their centres.
- **Satellite:** A satellite is any object that orbits another object due to the force of gravity, maintaining a stable path around it.
- **Orbital velocity:** In circular orbit a satellite has a constant tangential speed called orbital velocity.
- **Geostationary satellite:** A geostationary satellite, also known as a geosynchronous equatorial orbit (GEO) satellite, is a type of satellite that orbits the Earth directly above the equator at an altitude where its orbital period matches the Earth's rotation period. This results in the satellite appearing stationary relative to a fixed point on the Earth's surface.
- **Absolute gravitational potential energy:** The potential energy possessed by a body at a certain height in a gravitational field with respect to reference point of zero potential is known as absolute potential energy.
- **Gravitational potential:** The gravitational potential is the gravitational potential energy per unit mass.

Formula Sheet

$$g = G\frac{m}{r^2} \qquad g_h = \frac{gR_E^2}{(R_E + h)^2} \qquad v_o = \sqrt{\frac{G M_E}{R_E + h}}$$

$$r = \left(\frac{GM_E T^2}{4\pi^2}\right)^{1/3} \qquad U = -\frac{GM_E m}{R_E} \qquad V = -\frac{GM_E}{R_E}$$

$$F_g = G\frac{m_1 \times m_2}{r^2}$$

EXERCISE

Multiple Choice Questions

Encircle the Correct option.

1) Two identical balls of masses 1 kg each having distance of 1 m between their centres then gravitational force between them is:

   A. 667×10^{-9} N B. 6.67×10^{-11} N

   C. 667×10^{11} N D. 6.67×10^{-13} N

2) Gravitational force between two objects is 'F'. If masses of bodies are doubled and distance between their centres is reduced to half, then gravitational force is:

   A. F B. 4 F C. F/4 D. 16 F

3) The value of 'g' at height of 1500 km above the surface of Earth in m s^{-2} is:

   A. 0 B. 6.4 C. 9.8 D. 12.2

4) If we consider Earth as perfect sphere and ignore the presence of air resistance. What will be the orbital velocity to launch a satellite in circular orbit just above the surface at $r = R_E = 6.4 \times 10^6$ m:

   A. 0 km s^{-1} B. 1.19 km s^{-1} C. 3.07 km s^{-1} D. 7.9 km s^{-1}

5) When a satellite is put into a higher circular orbit, its kinetic energy:

   A. increases B. decreases C. is zero D. remains the same.

6) The orbital velocity of geostationary satellite is

   A. 0 km s^{-1} B. 1.19 km s^{-1} C. 3.07 km s^{-1} D. 7.9 km s^{-1}

7) The minimum number of geostationary satellites for the complete global coverage is

   A. 1 B. 2 C. 3 D. 4

8) When an object moves away from a massive body, its gravitational potential energy:

   A. increases B. decreases C. remains constant D. becomes zero

9) The value of gravitational potential at an altitude of 35700 km above the Earth's surface, where communication satellites orbit the Earth is approximately:

   A. – 94.7 M J kg^{-1} B. – 62.3 M J kg^{-1}

   C. + 947.11 J kg^{-1} D. – 947.11 J kg^{-1}

Short Questions

1) Why can gravitation not account for the formation of molecules?

2) Why *don't* two books on your desk attract each other gravitationally, despite Newton's law of gravitation?

3) Why does an apple fall towards the Earth due to gravity, while the Earth doesn't move towards the apple?

4) Can gravitational field strength be negative? Explain.

5) What factors determine the strength of the gravitational field around a planet?

6) If two planets have the same mass but different radii. How would their gravitational field strengths compare?

7) Why satellites in higher orbits have lower orbital velocities?

8) How does the mass of the Earth affect the orbital velocity required for a satellite to stay in orbit?

9) Is it possible for an object's gravitational potential energy to become negative? If so, what does this mean for the object's motion?

10) How the gravitational potential energy of two point masses is related to concept of gravitational potential?

Comprehensive Questions

1) What is force of gravity? State and explain Newton's law of gravitation. Also show that Newton's law of gravitation is consistent with Newton's 3rd law of motion.

2) What are gravitational field and gravitational field strength? Explain. Derive the formula for gravitational acceleration 'g' on the surface of Earth and discuss the variation of 'g' with altitude.

3) How do engineers calculate the required velocity for a satellite to be placed into a specific orbit? Can you describe the steps involved in determining the orbital parameters necessary for a successful satellite launch and deployment?

4) What are geostationary satellites? Calculate the orbital radius 'r', orbital speed 'v_o' and height 'h' above surface of Earth for the geostationary orbit.

5) If 'M_E' is the mass of Earth and 'r' is the distance of unit mass from the centre of Earth and 'G' is the universal gravitational constant. Prove that gravitational potential 'V' can be written as: $V = -\frac{GM_E}{r}$

Numerical Problems

1) Find the mass of the Earth by considering a scenario where a small object is placed on the Earth's surface such that distance between their centres is equal to the Earth's radius, i.e., 6.4×10^6 m. **(Ans: 6.4×10^{24} kg)**

2) Alpha Centauri is a binary star system with two stars: Alpha Centauri A and Alpha Centauri B. The mass of Alpha Centauri A is 2.19×10^{30} kg and Alpha Centauri B is 1.80×10^{30} kg, they attract each other with a force of 2.24×10^{25} N. What is the distance between the two stars?

(Ans: 3.43×10^{12} m)

3) What will be value of 'g' on an exoplanet (a planet outside the Solar System) whose mass is five times the mass of Earth and its radius is twice the radius of Earth? **(Ans: 12.25 m s^{-2})**

4) The Hubble Space Telescope is in a circular orbit 613 km above Earth's surface. The average radius of the Earth is 6.4×10^6 m and the mass of Earth is 6×10^{24} kg. (a) What is the speed of the telescope in its orbit? (b) What is the period of the telescope's orbit?

[Ans: (a) 7550 m s^{-1}, (b) 5807 s]

5) To launch a satellite around planet Mars at an altitude of 300 km above its surface. Mars has a mass of 6.42×10^{23} kg and has a radius of 3.39×10^6 m, calculate its orbital velocity.

(Ans: 3.4 km s^{-1})

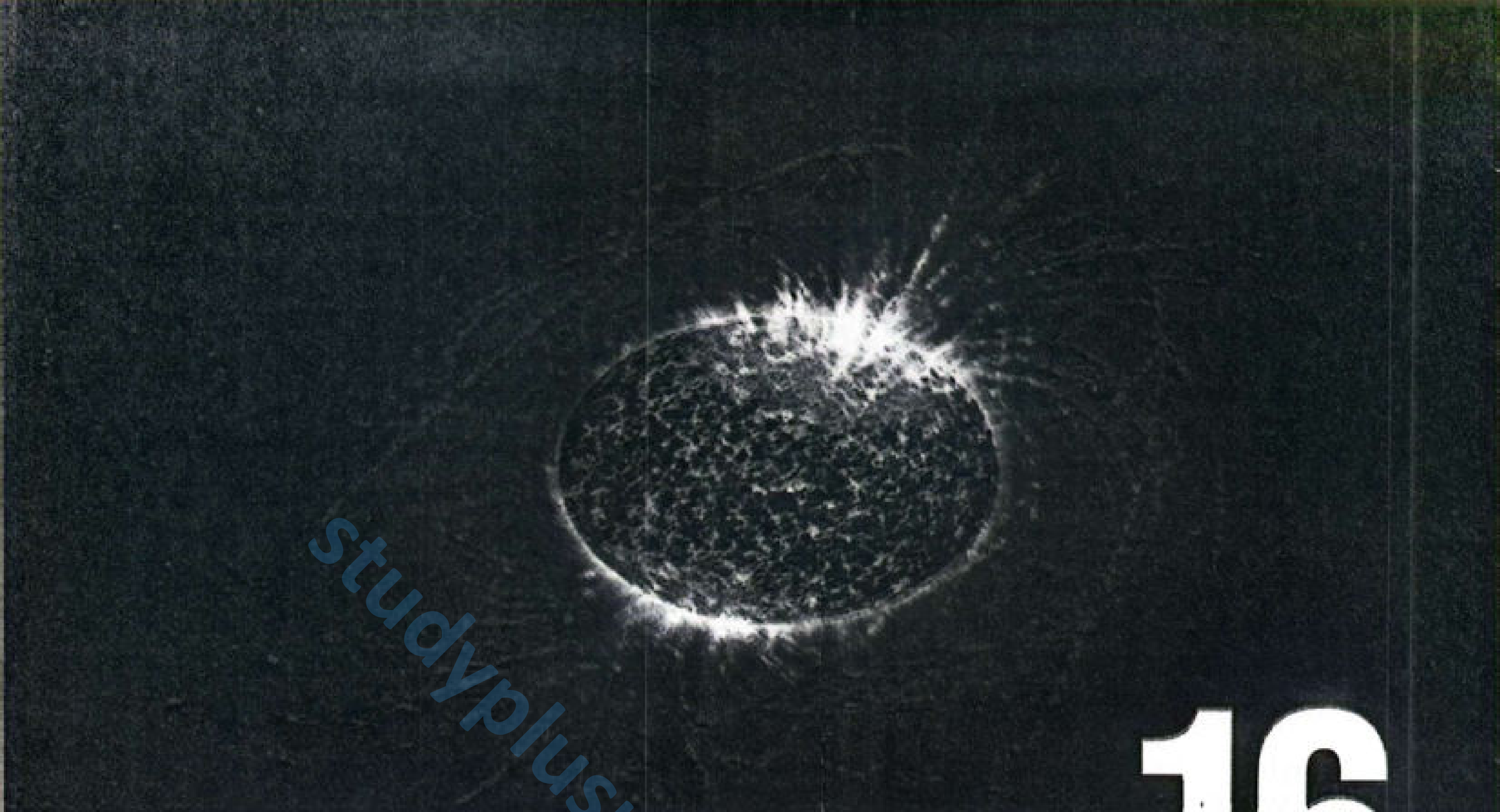
6) An asteroid orbits the sun 8.35×10^{11} m from it. (a) How fast must the asteroid travel to maintain its circular orbit around the sun? (b) How long will it take the asteroid to orbit the sun? **(Ans: 1.26×10^4 m s^{-1}, 13.2 years)**

7) Mars has a mass of 6.42×10^{23} kg and has a period of 88,642 s. What would be the radius of a stationary satellite for this planet? **(Ans: 20,428 km)**

8) Calculate the potential energy of the Moon having mass 7.35×10^{22} kg, relative to Earth with mass 6×10^{24} kg if the distance between their centres is 3.94×10^5 km. **(Ans: -7.4×10^{28} J)**

9) A 50 kg weather satellites move in circular orbits about the Earth at an altitude of 1000 km. A similar 50 kg communication satellite is at an altitude of 37,000 km in circular orbits about the Earth. Calculate the difference in the gravitational potential energies of the two satellites in their respective orbits? **(Ans: 2.2 GJ)**

10) What is the change in gravitational potential energy of a 64.5 kg astronaut, lifted from Earth's surface into a circular orbit of altitude 4.40×10^2 km? **(Ans: 2.6×10^8 J)**



16

STATISTICAL MECHANICS AND THERMODYNAMICS

Student Learning Outcomes (SLOs)

The student will

- explain how molecular movement causes the pressure exerted by a gas.
- Derive and use the relationship $PV=1/3Nm<c^2>$ [where $< c^2>$ is the mean-square speed (a simple model considering one-dimensional collisions and then extending to three dimensions using $1/3<c^2>=<c_x^2>$ is sufficient)].
- Calculate the root-mean-square speed of an ideal gas.
- Derive and use the formula for the average translational kinetic energy of a gas.
- Illustrate that the model of ideal gasses is used a base from which the field of statistical mechanics emerged [and has helped explain the behavior of 'non-ideal' gasses through modifications to the model e.g. the behavior of stars].
- State that under extreme physical conditions, atoms can break down into sub-atomic particles that can form unusual states of matter [Such as degenerate matter. Usually made of any one kind of subatomic particle such as neutron degenerate matter in neutron stars under strong gravity and heat) and Bose-Einstein condensates (created when certain materials are taken to very low temperatures and then exhibit remarkable properties like superconductivity and superfluidity)].

Statistical mechanics provides a foundation for understanding thermodynamic properties like energy, entropy, and temperature by using the principles of thermodynamics.

Statistical mechanics is a branch of physics that applies statistical methods to understand the behavior of physical systems composed of a large number of particles. It offers a framework for understanding the behavior of complex systems, from the microscopic to the macroscopic level. In the present chapter, we shall relate volume, pressure, and temperature to the microscopic properties like speed and kinetic energy of gas molecules. Statistical mechanics is applied to various complex systems, from materials science to biological systems.

16.1 PRESSURE EXERTED BY GAS MOLECULES

The molecules of a gas are in a state of continuous random motion in a container. They collide with one another and also with the walls of the container. Due to collisions of gas molecules with the walls, their momentum changes. Due to their elastic collisions, they transfer an equal amount of momentum to the walls of the container. According to Newton's second law of motion, the rate of transfer of momentum ($\Delta p/\Delta t$) by the gas molecules to the walls is equal to the force (F) exerted on the wall. This force exerted by the gas molecules per unit area (A) of the walls is equal to the pressure exerted by the gas.

$$\text{Pressure} = \frac{F}{A} = \left(\frac{\Delta p}{\Delta t}\right)\frac{1}{A}$$

Hence, the pressure that the gas exerts on the box depends upon the number of molecules that hit each side of the box in one second and the force with which a molecule collides with the wall.

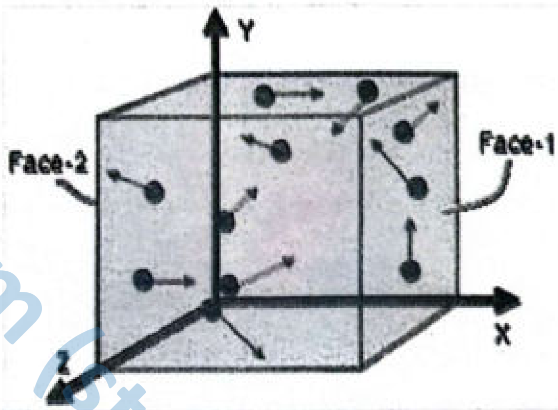


Figure 16.1: Collision of gas molecules with the wall of container.

Let us consider an ideal gas consisting of N number of identical molecules in a rapid, random motion contained in a cubic box of side l, as shown in Fig. 16.1. The gas molecules are moving with velocity (v) collide with the walls of the container and exert force on it. The force exerted per unit area of the wall is the pressure of the gas.

Consider a single gas molecule of mass "m" moving towards face-1 of the container with the x-component of its velocity v_{x1}. So, initial momentum of the molecule along the x-direction is mv_{x1}. At face-1 of the container, the molecule has perfectly elastic collision and bounces back, (its momentum is reversed). So, after the collision final momentum in the negative x-direction is $-mv_{x1}$. Therefore, change in momentum of a gas molecule along the x-direction is:

$$\Delta p = -mv_{x1} - (mv_{x1})$$

$$\Delta p = -2mv_{x1} \quad \text{\_\_\_\_\_\_} \quad (16.1)$$

According to the law of conservation of momentum, the momentum imparted to the wall by the molecule will be $2mv_{x1}$.

After colliding with face-1 of the container, the molecule travels to opposite face-2 and collide with it, rebounds, and travels back to face-1. So, the gas molecule cover distance '2l' before it again collides with face-1. The time Δt taken by gas molecule to covers this distance '2l' is given by:

$$\Delta t = \frac{2l}{v_{x1}} \quad \text{\_\_\_\_\_\_ (16.2)}$$

So, the rate of change of momentum of a gas molecule along x-axis is:

$$\frac{\Delta p}{\Delta t} = \frac{-2mv_{x1}}{\frac{2l}{v_{x1}}} = \frac{-mv_{x1}^2}{l} \quad \text{\_\_\_\_\_\_ (16.3)}$$

As the rate of change of momentum is equal to force. Therefore, force exerted by wall of cubical container on gas molecule is $\frac{-mv_{x1}^2}{l}$. According to Newton's third law of motion, the force exerted by the gas molecule on wall of cubical container is equal in magnitude but opposite in direction to the force exerted by wall on gas molecule. Force F_{x1} exerted by gas molecule on the wall of cubical container is given by

$$F_{x1} = -\left(\frac{-mv_{x1}^2}{l}\right) = \frac{mv_{x1}^2}{l}$$

Similarly, forces exerted by all other molecules along x-axis are:

$$F_{x2} = \frac{mv_{x2}^2}{l},\ F_{x3} = \frac{mv_{x3}^2}{l},\ \dots,\ F_{xN} = \frac{mv_{xN}^2}{l}$$

Total force exerted F_x due to all molecules along x-axis is

$$F_x = F_{x1} + F_{x2} + F_{x3} + \dots + F_{xN}$$

or

$$F_x = \frac{mv_{x1}^2}{l} + \frac{mv_{x2}^2}{l} + \frac{mv_{x3}^2}{l} + \dots + \frac{mv_{xN}^2}{l}$$

$$F_x = \frac{m}{l}(v_{x1}^2 + v_{x2}^2 + v_{x3}^2 + \dots + v_{xN}^2)$$

Multiplying and dividing R.H.S by N number of gas molecules.

$$F_x = \frac{mN}{l}\left(\frac{v_{x1}^2 + v_{x2}^2 + v_{x3}^2 + \dots + v_{xN}^2}{N}\right) \quad \text{\_\_\_\_\_\_ (16.4)}$$

Where 'mN' is total mass of N gas molecules. Putting $\left(\frac{v_{x1}^2 + v_{x2}^2 + v_{x3}^2 + \dots + v_{xN}^2}{N}\right) = \langle v_x^2 \rangle$ in Eq. (16.4), we get:

$$F_x = \frac{mN}{l}\langle v_x^2 \rangle \quad \text{\_\_\_\_\_\_ (16.5)}$$

Where $\langle v_x^2 \rangle$ is the mean square velocity of all the gas molecules traveling along the x-direction. The mean square velocity $\langle v^2 \rangle$ of the gas molecule is equal to the sum of the mean square velocities of the x, y, and z-components of velocities.

$$\langle v^2 \rangle = \langle v_x^2 \rangle + \langle v_y^2 \rangle + \langle v_z^2 \rangle \quad \text{\_\_\_\_\_\_ (16.6)}$$

Due to the random motion of a large number of gas molecules in the container, the components of the mean square velocities of the gas molecules are the same along three axes, i.e., $\langle v_x^2\rangle = \langle v_y^2\rangle = \langle v_z^2\rangle$. So, Eq. (16.6) becomes as:

$$\langle v^2\rangle = \langle v_x^2\rangle + \langle v_x^2\rangle + \langle v_x^2\rangle \quad \text{or} \quad \langle v^2\rangle = 3\langle v_x^2\rangle$$

or $$\langle v_x^2\rangle = \frac{\langle v^2\rangle}{3} \qquad \text{\_\_\_\_\_\_ (16.7)}$$

Putting value of $\langle v_x^2\rangle$ from Eq. (16.7) in Eq. (16.5), we get:

$$F_x = \frac{mN}{l}\frac{\langle v^2\rangle}{3}$$

As the force per unit area in unit time on all faces of a cubical container is equal, therefore generalizing the above equation, we get:

$$F = \frac{mN}{l}\frac{\langle v^2\rangle}{3} \qquad \text{\_\_\_\_\_\_ (16.8)}$$

Also, according to Pascal's law, the pressure of gas is equally transmitted on all faces of a cubical container. Since pressure is the force per unit area, therefore,

$$P = \frac{\text{Force}}{\text{Area}} = \frac{F}{l^2} \qquad \text{\_\_\_\_\_\_ (16.9)}$$

Putting value of F from Eq. (16.8) in Eq. (16.9), we get:

$$P = \frac{mN}{l}\frac{\langle v^2\rangle}{3}\left(\frac{1}{l^2}\right)$$

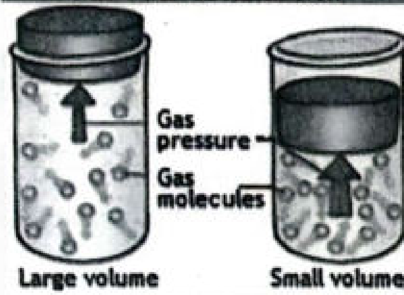
or $$P = \frac{mN}{l^3}\frac{\langle v^2\rangle}{3}$$

As l^3=V (Volume of the gas), so above equation becomes:

$$P = \frac{mN}{3V}\langle v^2\rangle \qquad \text{\_\_\_\_\_\_ (16.10)}$$

This is the expression for the pressures of ideal gas.

For Your Information



Pressure is affected by volume. As the volume of the container decreases molecules contained within a smaller volume. As a result, the particle collisions occur more frequently with the sides of the container, exerting a higher pressure.

When the volume of the container increases, there is more space, so less frequent collisions with the walls of the container, exerting a lower pressure.

Pressure in terms of Average Translational Kinetic Energy of Gas Molecules

To find the relation between pressure of gas and average translational kinetic energy of gas molecules, we multiply and divide the right-hand side of Eq. (16.10) by 2, i.e.,

$$P = \frac{2}{3}\frac{N}{V}\langle\frac{1}{2}mv^2\rangle \qquad \text{\_\_\_\_\_\_ (16.11)}$$

putting $\frac{N}{V} = N_o$ (number of molecules per unit volume), we get:

$$P = \frac{2}{3}N_o\langle\frac{1}{2}mv^2\rangle$$

As, $\frac{2}{3}N_o$ is constant, so;

$$P = \text{constant } \langle \frac{1}{2}mv^2 \rangle \qquad \text{or} \qquad P \propto \langle \frac{1}{2}mv^2 \rangle$$

or $$P \propto \langle K.E \rangle$$

This relation shows that:

The pressure exerted by the gas molecules on the walls of container is directly proportional to the average translational kinetic energy of the gas molecules.

Pressure in terms of Density

From Eq. (16.10) we can derive another formula for the pressure of gas in terms of density and mean square velocity of gas molecules.

$$P = \frac{mN}{3V}\langle v^2 \rangle$$

As, $\frac{mN}{V} = \rho$, so;

$$P = \frac{1}{3}\rho\langle v^2 \rangle \quad \text{———} (16.12)$$

This is another expression for the pressures of gas in terms of density of the gas.

Temperature in terms of Average Translational Kinetic Energy of Gas Molecules

According to ideal gas law:

$$PV = n\,RT \quad \text{———} (16.13)$$

Where 'n' is the number of moles of the gas, 'V' is the volume, 'T' is the absolute temperature and '$R = 8.314\ \text{J mol}^{-1}\text{K}^{-1}$' is the universal gas constant. As, the number of moles 'n' can be expressed as:

$$n = \frac{N}{N_A}$$

Where $N_A = 6.022 \times 10^{23}$ (molecules or atoms per mole) is the Avogadro number. So, Eq. (16.13) becomes as:

$$PV = N\left(\frac{R}{N_A}\right)T$$

Here $\frac{R}{N_A} = k$ is Boltzmann constant and its value is $1.38 \times 10^{-23}\,\text{JK}^{-1}$. Hence;

$$PV = NkT \quad \text{———} (16.14)$$

As, Eq. (16.11) can be written as:

$$PV = \frac{2N}{3}\langle \frac{1}{2}mv^2 \rangle \quad \text{———} (16.15)$$

Comparing the Eqs. (16.14) and (16.15), we get:

$$N\,k\,T = \frac{2N}{3}\langle\frac{1}{2}mv^2\rangle$$

or $$T = \frac{2}{3k}\langle\frac{1}{2}mv^2\rangle \qquad \text{\_\_\_\_\_ (16.16)}$$

As $\frac{2}{3k}$ = constant, so Eq. (16.16) can be written as:

$$T = \text{constant}\ \langle\frac{1}{2}mv^2\rangle$$

or $$T \propto \langle\frac{1}{2}mv^2\rangle \qquad \text{or} \qquad T \propto \langle K.E\rangle$$

The absolute temperature of an ideal gas is directly proportional to the average translational kinetic energy of the gas molecules.

Example 16.1: The density of the air is ρ = 1.296 kg m^{-3} at temperature 0 °C. The root mean square velocity of air molecules is 484 m s^{-1}. Determine the pressure of air.

Given: Density = ρ = 1.29 kg m^{-3} $\quad v_{r.m.s} = 484\ \text{m s}^{-1}$ $\quad T = 0\ °C = 273\ K$

To Find: Pressure = P = ?

Solution: By using, $$P = \frac{1}{3}\rho v_{r.m.s}^2$$

Putting values, we get: $$P = \frac{1}{3}\times 1.296\times(484)^2 = 1.012\times 10^5\ \text{Pa}$$

Assignment 16.1

Calculate the average translational kinetic energy of a gas molecule at a temperature 320 K.

16.2 ROOT MEAN SQUARE SPEED OF AN IDEAL GAS

According to the kinetic theory of gases, every molecule moves at a different velocity. If v_1, v_2, v_3, ..., v_N are the speeds of N gas molecules, then the mean square speed $\langle v^2\rangle$ can be determined by adding the square of the speeds of all the molecules and dividing it by the total number of gas molecules.

$$\langle v^2\rangle = \frac{v_1^2 + v_2^2 + v_3^2 + \ldots + v_N^2}{N}$$

Taking the square root of both sides, we get:

$$\sqrt{\langle v^2\rangle} = \sqrt{\frac{v_1^2 + v_2^2 + v_3^2 + \ldots + v_N^2}{N}}$$

Putting $\sqrt{\langle v^2\rangle} = v_{r.m.s}$ on the left side of above equation, we get root mean square speed

$$v_{r.m.s} = \sqrt{\frac{v_1^2 + v_2^2 + v_3^2 + \ldots + v_N^2}{N}} \qquad \text{\_\_\_\_\_ (16.17)}$$

For Your Information

The value of Boltzmann constant per molecule in different units are:

$k = 1.38\times10^{-16}\ erg\ K^{-1}$

$k = 1.38\times10^{-23}\ J\ K^{-1}$

$k = 3.3\times10^{-24}\ cal\ K^{-1}$

The square root of the mean square speed of the gas molecules is called the root mean square (rms) speed of the molecules.

Average speed: It is defined as the arithmetic mean of the speeds of the molecules of a gas at a given temperature.

$$v_{av} = \frac{v_1 + v_2 + v_3 + \dots + v_N}{N} \qquad \text{———— (16.18)}$$

In order to derive another expression for the root mean square speed of gas molecules, let us start from the Eq. (16.10) of the pressure of gas.

$$P = \frac{1}{3}\frac{mN}{V}\langle v^2 \rangle$$

or

$$PV = \frac{mN}{3}\langle v^2 \rangle \qquad \text{———— (16.19)}$$

As we know that: $PV = NkT$ ———— (16.20)

Comparing Eqs. (16.19) and (16.20), we get:

$$\frac{mN}{3}\langle v^2 \rangle = NkT$$

$$\langle v^2 \rangle = \frac{3kT}{m} \qquad \text{———— (16.21)}$$

Taking a square root on both sides, we get:

$$v_{r.m.s} = \sqrt{\frac{3kT}{m}} \qquad \text{———— (16.22)}$$

Here $\sqrt{\langle v^2 \rangle} = v_{r.m.s}$. From Eq. (16.22), we can also write as:

$$v_{r.m.s} \propto \sqrt{T}$$

Thus, the root mean square speed of the given gas molecule is directly proportional to the square root of the absolute temperature of the gas molecules.

Putting $k = \frac{R}{N_A}$ in Eq. (16.22), we get:

$$v_{r.m.s} = \sqrt{\frac{3RT}{mN_A}}$$

Putting $mN_A = M$ (molar mass of the gas), we get:

$$v_{r.m.s} = \sqrt{\frac{3RT}{M}} \qquad \text{———— (16.23)}$$

At constant temperature; $v_{r.m.s} \propto \frac{1}{\sqrt{M}}$

Thus, at constant temperature, the root mean square speed of the gas molecules is inversely proportional to the square root of the molar mass of the gas. A gas of smaller molar mass has comparatively high speed as compared to a gas of greater molar mass. Therefore, the root mean square speed of hydrogen molecules is four times greater than that of oxygen molecules at the same temperature.

Also, from the Eq. (16.12), the pressure of gas is $P = \frac{1}{3}\rho\langle v^2 \rangle$

or $$\langle v^2 \rangle = \frac{3P}{\rho}$$

Taking a square root on both sides of the above equation:

$$\sqrt{\langle v^2 \rangle} = \sqrt{\frac{3P}{\rho}}$$

Putting $\sqrt{\langle v^2 \rangle} = v_{r.m.s}$ on the left side of the above equation:

$$v_{r.m.s} = \sqrt{\frac{3P}{\rho}} \quad \text{------} \quad (16.24)$$

From Eqs. (16.22), (16.23) and (16.24) we get:

$$v_{r.m.s} = \sqrt{\frac{3kT}{m}} = \sqrt{\frac{3RT}{M}} = \sqrt{\frac{3P}{\rho}} \quad \text{------} \quad (16.25)$$

Example 16.2:
Calculate the root mean square speed of oxygen molecules in the air at standard temperature and pressure.

Given: At ST.P., we have:

Temperature = T = 0 °C = 273 K   Pressure = P = 1.01×10^5 Pa

To Find: Root mean square speed of oxygen molecules = $v_{r.m.s}$ = ?

Solution: To find mass m, of one molecule of oxygen, we use the formula

$$m = \frac{\text{Molecular mass of oxygen}}{\text{Avogadro's number}} \quad \text{or} \quad m = \frac{M}{N_A}$$ (For oxygen = M = 32 g)

Thus, $$m = \frac{32 \times 10^{-3} kg}{6.022 \times 10^{23}} = 5.31 \times 10^{-26} \text{ kg}$$

Now for root mean square speed, we use $v_{r.m.s} = \sqrt{\frac{3kT}{m}}$

Putting values, we get: $$v_{r.m.s} = \sqrt{\frac{3 \times 1.38 \times 10^{-23} \times 273}{5.31 \times 10^{-26}}} = 461.4 \text{ ms}^{-1}$$

Example 16.3:
Four molecules of a gas have speeds of 2 km s^{-1}, 4 km s^{-1}, 6 km s^{-1}, and 8 km s^{-1}, respectively. Calculate their average speed and root mean square speed.

Given: v_1 = 2 km s^{-1} v_2 = 4 km s^{-1} v_3 = 6 km s^{-1} v_4 = 8 km s^{-1}

To Find: v_{av} = ? $v_{r.m.s}$ = ?

Solution: For average speed, we use the formula: $$v_{av} = \frac{v_1 + v_2 + v_3 + v_4}{N}$$

Putting values, we get: $$v_{av} = \frac{2+4+6+8}{4} = 5 \text{ km s}^{-1}$$

For root mean square speed, we use the formula:

$$v_{r.m.s} = \sqrt{\frac{v_1^2 + v_2^2 + v_3^2 + v_4^2}{N}}$$

$$v_{r.m.s} = \sqrt{\frac{(2)^2 + (4)^2 + (6)^2 + (8)^2}{4}} = 5.48\ kms^{-1}$$

Assignment 16.2

Calculate the root mean square speed of nitrogen gas molecules in air under standard conditions of pressure and temperature.

16.3 MODIFICATION OF THE IDEAL GAS MODEL TO DISCUSS BEHAVIOUR OF NON-IDEAL GASES

Behaviour of Ideal Gases

As we know that in the ideal gas model, the size of gas molecules is negligible, there are no interactions between them, and the collisions between each other and with the walls of the container are perfectly elastic.

Gas that obeys the ideal gas law (PV = n RT) is called an ideal gas. The ideal gas law governs macroscopic properties, e.g., temperature, pressure, volume, quantity of gas, and entropy. Although there is no such thing in nature as a truly ideal gas, gases can approach the ideal state at low pressure and high temperatures. We have seen that the temperature of an ideal gas can be determined as $T = \frac{m}{3k}\langle v^2 \rangle$. This equation implies that temperature (a macroscopic property of a gas) is defined by the motion of an individual particle (a microscopic quantity). Hence, different results for ideal gas provide connections between its microscopic and macroscopic quantities. This gives rise to a branch of physics called **statistical mechanics.**

Statistical mechanics is a branch of physics that connects the microscopic details of a system, such as motion, energy, and the interaction of individual particles, with the macroscopic observables we measure, such as temperature, pressure, volume, and entropy.

Statistical mechanics provides a mathematical foundation for thermodynamics, which is otherwise a phenomenological theory.

Behaviour of Non-Ideal Gases

The ideal gas law works well for large volumes, high temperatures, and low pressure, but it fails to explain the behavior of gases under high pressure, high density, and low temperature (where molecules move slowly and interact with each other). It also fails when strong gravitational forces act like those in stars. To account for these deviations from ideal behavior, modifications were made to the ideal gas law.

Van Der Waals Equation

The ideal gas equation PV = n R T can be used for real gases at high temperatures as well as low pressures at which intermolecular forces are negligibly small. Van der Waals modified this equation so that it can be used for real gases at wide ranges of temperature and pressure. Van der Waals assumed that the gas molecules are hard spheres with definite volume and that two gas molecules somehow interact with each other, especially at low temperatures when molecules move slowly. He made the following corrections:

(i) Volume Correction: The size of gas molecules is not negligible, but it has a finite size, and some part of the volume of gas is occupied by the gas molecules. So, the space available for the motion of molecules of gas will be slightly less than the volume of gas 'V'. Roughly 'b' is the volume of one mole of gas molecules. Hence, effective volume becomes (V − b), as shown in Fig. 16.2.

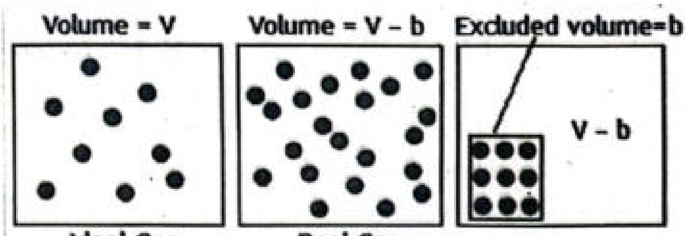


Figure 16.2: Illustration of volume correction.

(ii) Pressure Correction: At low temperature, molecules interact somehow. Inside the container, interaction between gas molecules cancel each other, but particles near the surface and walls of the container have net inward force, as shown in Fig. 16.3. The effective pressure of the real gas is calculated by $\left(P+\frac{a}{V^2}\right)$. Van der Waal's equation for 1 mole of a gas is:

$$\left(P+\frac{a}{V^2}\right)(V-b)=RT \quad \_\_\_\_\_ (16.26)$$

Here 'a' and 'b' are empirical constants, their values are different for different gases. For 'n' moles of the gas effective volume becomes (V -n b) and effective pressure becomes $\left(P+\frac{an^2}{V^2}\right)$, therefore Vander Waal's equation for 'n' mole of a gas is;

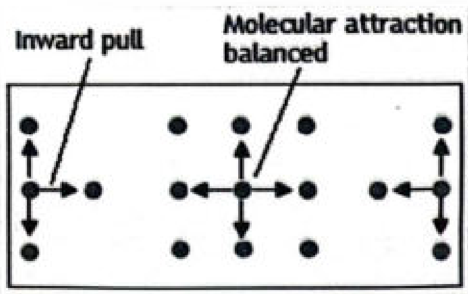


Figure 16.3: Illustration of pressure correction.

$$\left(P+\frac{an^2}{V^2}\right)(V-nb)=nRT \quad \_\_\_\_\_ (16.27)$$

This equation is designed to describe the behavior of real gases, but it can still be used for ideal gases as well. Real gases or non-ideal gases approach ideal behavior at a high temperature and low pressure. At high temperatures or at very low pressure, volume 'V' is very large. So, the terms '$\frac{a}{V^2}$' and 'b' can then be neglected. In this situation, Van der Waals' equation reduces to the ideal gas equation (P V = n RT).

16.4 Gravitational Effects on Ideal Gas Model

In the ideal gas model (i.e., the ideal gas law) at the laboratory scale, gravity is much weaker as compared to other factors such as the kinetic energy of the gas molecules and the pressure of the gas. That's why, it makes sense to ignore gravity for ideal gas. But on a large scale, e.g., in a star, gravity plays an important role in their stability. Accounting for gravity requires much more complex models as compared to the ideal gas model to understand the behaviour of gas molecules.

In the core of stars, heat produced by fusion, radiation pressure, and degeneracy pressure by quantum effects, generate net outward pressure that is counterbalanced by gravity, a state known as hydrostatic equilibrium, as shown in Fig. 16.4. This ensures stability in the stars.

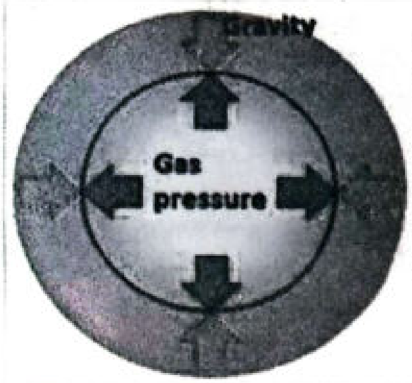


Figure 16.4: Hydrostatic equilibrium.

Example16.4: Calculate the pressure exerted by one mole of a gas at room temperature 300 K with fixed volume 0.022 m^3, using Van der Waals equation. Where a = 3.59 (liter)^2 atm (mole)$^{-2}$ and b = 0.0427 liter/mole.

Given: R = 0.0821 liter-atm-mol^{-1} K^{-1}

Temperature = T = 300 K   n =1 mol

Volume = V = 0.022 m^3 = 0.022×1000 liter = 22 liter

(as 1 m^3 = 1000 liter)

To Find: P = ?

Solution: By using Van der Waals equation: $\left(P+\frac{an^2}{V^2}\right)(V-nb)=nRT$

$$P=\frac{nRT}{V-nb}-\frac{an^2}{V^2}$$

Putting values, we get: $$P=\frac{1\times 0.0821\times 300}{22-(1)0.0427}-\frac{3.59\times(1)^2}{(22)^2}=1.1146 \text{ atm}$$

Assignment 16.3

(a) Using the Van der Waals equation, calculate the temperature of 20.0 moles of helium in a 10.0 *litre* cylinder at 120 atmosphere pressure. Van der Waals constants for helium: a = 0.0341 litre^2 atm mol^{-2}; b = 0.0237 litre mol^{-1}.

(b) Compare this value with the temperature calculated from the ideal gas equation.

16.5 BEHAVIOUR OF MATTER UNDER EXTREME PHYSICAL CONDITIONS

In the universe, there are many cases in which matter has evolved in to extreme physical conditions. Extreme physical conditions can alter the behavior of matter, leading to unique and unusual states of matter such as degenerate matter and phenomena like Bose-Einstein condensation, superconductivity states, superfluidity, and other quantum phenomena. These conditions are observable at extremely high pressure or at extremely low pressure, by materials of high density or by materials experiencing high gravitational force. We will learn how these conditions lead to unusual states of matter.

16.5.1 Degenerate Matter

Extremely high pressure can cause particles to interact closely where quantum mechanical effects, particularly the Pauli Exclusion Principle become significant. The Pauli Exclusion Principle states that:

Two electrons cannot occupy the same quantum state.

Degenerate matter happens when gravity squeezes atoms so tightly under extremely high pressure in a star that they change from their normal state. Normally, atoms are made up of a nucleus and electrons revolving around the nucleus. But when the pressure is extremely high, electrons are strongly pressed tightly against the nucleus in high energy states (according to Pauli Exclusion principle) and they cannot move freely. At this stage, electrons show resistance against gravity and other forces for further compression. This resistance is called degeneracy pressure, which counteract the effect of gravity and other compressive forces, as shown in Fig. 16.5. So, degenerated matter is matter that is packed so tightly that its electrons are forced into a tightly packed configuration.

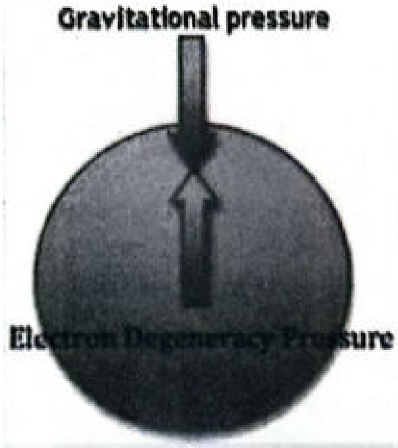


Figure 16.5: Degeneracy pressure.

Degenerate matter is typically formed in the cores of white dwarf and neutron stars, where gravity compress the material to incredibly high densities.

White Dwarfs

White dwarfs are formed due to remnants of low- to intermediate mass- stars. When a star has used up all of its nuclear fusion fuel, it removes its outer layers and contracts to form a white dwarf.

In white dwarfs, the high gravitational force is balanced by electrons degenerate pressure, which arises from the Pauli Exclusion principle, as shown in Fig. 16.6. Gravity pulls material together while electrons are forced into high-energy states, and it forms an equilibrium state.

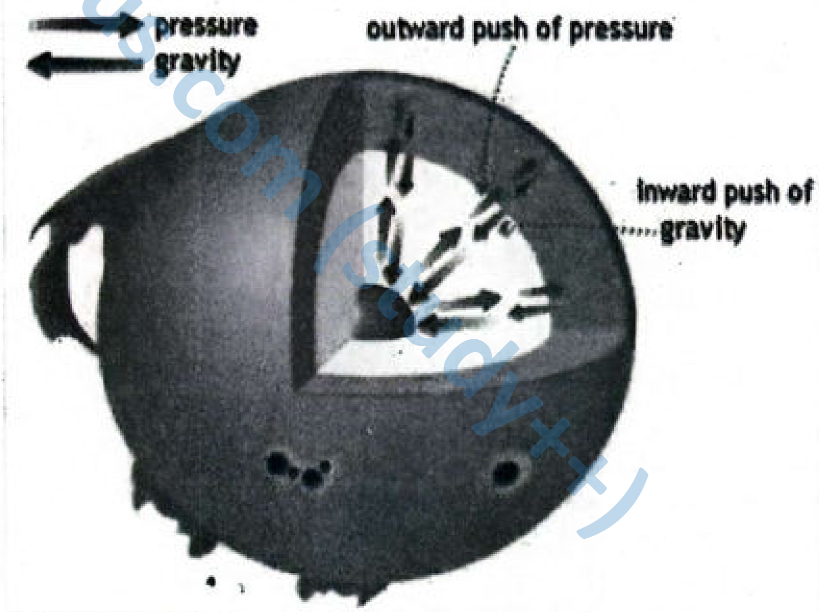


Figure 16.6: High gravitational force is balanced by electrons degenerate pressure.

16.5.2 Neutron Stars

Neutron stars are formed from the remnants of more massive stars. In neutron stars, matter is further compressed by the gravity, due to which electrons and protons start *to combine* to form neutrons. Therefore, neutrons degenerate pressure arises due to the Pauli Exclusion Principle, which balances gravity, preventing further compression, as shown in Fig. ,16.7. This stable state of matter is known as neutron Stars.

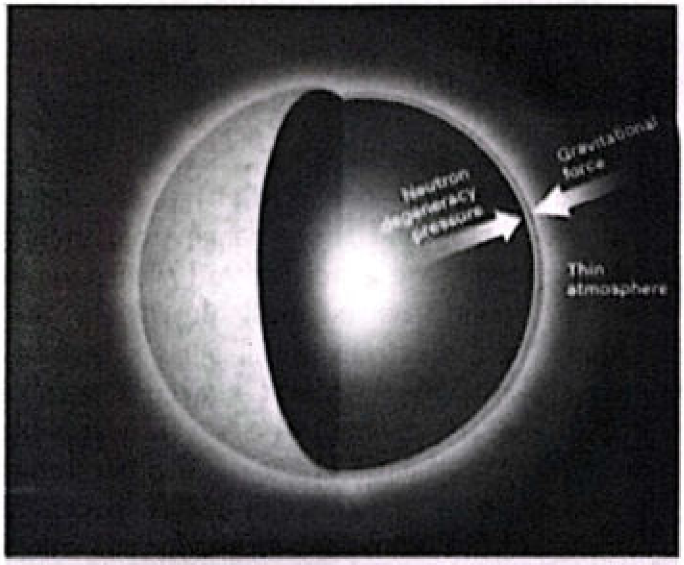


Figure 16.7: Neutron star.

Despite the fact, that a neutron star is only about 20 km in diameter, it is about 1.5 times more massive than the sun. Neutron stars have overall densities of the order of 10^{17} kg m^{-3}. Newly formed neutron stars, known as "hot neutron stars," can have a surface temperature of a several million degrees Celsius.

16.5.3 Bose-Einstein Condensation

Bose-Einstein Condensation (BEC) is a state of matter that forms at extremely low temperatures, such as close to absolute zero, causing particles kinetic energies to decrease significantly. At this low temperature, bosons (e.g., photons, W bosons, Z bosons, etc.) can condense in the same state known as Bose-Einstein condensation. This is different from degenerate matter, where two fermions, e.g., electrons, cannot exist in the same state. The Bose-Einstein condensation state of matter exhibits unique properties like superfluidity and superconductivity.

For Your Information

The temperature inside a newly formed neutron star is from around 10^{11} to 10^{12} K. However, the huge number of neutrinos it emits carries away so much energy that the temperature of an isolated neutron star falls within a few years to around 10^6 K.

16.5.4 Super Fluidity

A notable property of Bose-Einstein Condensation is superfluidity, where condensate shows zero viscosity, allowing it to flow without resistance. Superfluidity is observed when certain fluids are cooled to extremely low temperatures, close to absolute zero. This results in special qualities, like a fluid's capacity to pass through incredibly small spaces at a steady speed without the aid of outside forces.

It can also lead to remarkable behaviour, such as the ability to form persistent vortices. In fluid dynamics, a vortex is a region in a fluid in which the flow revolves around an axis line, which may be straight or curved.

16.5.5 Super Conductivity

In certain conditions, Bose-Einstein condensation leads to superconductivity, where electrical resistance drops to zero, allowing current to flow without resistance. This occurs when two electrons form boson-like entities called cooper pairs, which can then occupy the same state and move through the material without scattering.

Below the superconducting transition temperature, paired electrons form a condensate (a macroscopically occupied single quantum state that flows without resistance).

Superconductivity is the property of certain materials to conduct current without energy loss when they are cooled below a critical temperature or transition temperature.

Superconductors are those substances whose resistivity become zero at very low specific temperatures.

Critical temperature (T_c):

The low temperature at which and below which the resistivity ρ of substance become zero is called the critical temperature or superconducting transition temperature, as shown in Fig. 16.8.

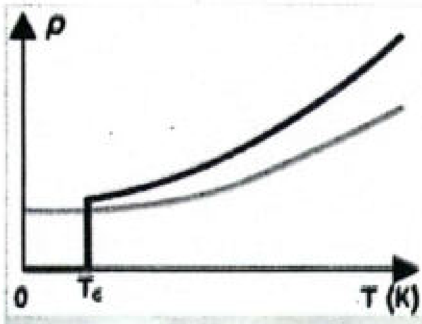


Figure 16.8: Graph of resistivity against temperature.

Superconductivity was first discovered in 1911 by Dutch physicist Heike Kamerlingh Onnes when he observed the sudden disappearance of electrical resistance in mercury at extremely low temperatures (4.2 K). In 1988, some new ceramic (thallium, calcium, barium, coper and oxygen) showed super-conductivity even at critical temperatures of 125 K.

A lanthanum superhydride (LaH_{10}) compound at a pressure of around 170 gigapascals is found to exhibit superconductivity with a critical temperature of 250 kelvin, the highest critical temperature that has been confirmed so far in a superconducting material.

Table: 16.1

Substance	T_c
Mercury (Sb)	4.2 K
Aluminum (Al)	1.18 K
Tin (Sn)	3.72 K
Lead (Pb)	7.2 K

Applications of superconductors:

Superconductors are used in powerful electromagnets (32 Tesla), particle accelerators, magnetic levitation (Maglev) trains, small but powerful electric motors, fast computer chips, and potentially more efficient power transmission lines. Researchers continue to explore ways to achieve superconductivity at higher temperatures, which would make it more practical for everyday use.

SUMMARY

- Pressure of gas: The pressure exerted by a gas molecule is a measure of the force exerted by gas molecules per unit area as they collide with the walls of their container.
- The pressure exerted by the gas molecules on the walls of the container is directly proportional to the average translational kinetic energy of the gas molecules.

- The temperature of gas molecules is directly proportional to the average translational kinetic energy of gas molecules.
- Mean kinetic energy of a moving gas molecule of an ideal gas is directly proportional to the absolute temperature "T".
- Pauli Exclusion Principle states that "two electrons cannot occupy same quantum state".
- Bose-Einstein condensate (BEC) Predicted by Bose and Einstein in the 1920s, this is a unique state of matter at extremely low temperatures, where a group of boson particles occupy the same quantum state.
- Degenerate matter: It's a state of matter where particles are so densely packed that quantum mechanical effects dominate over classical mechanics. This typically occurs in extremely high-pressure environments, such as the cores of massive stars like white dwarfs, neutron stars.
- White dwarfs are formed due to remnants of low to intermediate mass stars. When a star has used up all of its fusion fuel, it removes its outer layers and contracts to form a white dwarf.
- Neutron stars are formed from the remnants of more massive stars. In neutron stars, matter is further compressed by gravity. Due to this, electrons and protons start to combine to form neutrons.
- Super fluidity: A notable property of Bose-Einstein Condensation BEC is superfluidity, where condensate shows zero viscosity, allowing it to flow without resistance.
- Super conductivity: In certain conditions, Bose-Einstein Condensation BEC leads to super-conductivity, where electrical resistance drops to zero, allowing current to flow without resistance.

Formula Sheet

$$P=\frac{mN}{3V}\langle v^2\rangle \qquad P=\frac{2}{3}N_0\langle\frac{1}{2}mv^2\rangle \qquad P=\frac{1}{3}\rho\langle v^2\rangle \qquad T=\frac{2}{3k}\langle\frac{1}{2}mv^2\rangle$$

$$PV = nRT \qquad PV = NkT \qquad v_{r.m.s}=\sqrt{\frac{v_1^2+v_2^2+v_3^2+\dots+v_N^2}{N}}$$

$$v_{r.m.s}=\sqrt{\frac{3kT}{m}}=\sqrt{\frac{3RT}{M}}=\sqrt{\frac{3P}{\rho}} \qquad \left(P+\frac{an^2}{V^2}\right)(V-nb)=nRT$$

EXERCISE

Multiple Choice Questions

Encircle the Correct option.

1) Which of the following is the correct relation between pressure 'P' and density 'ρ' of the gas, at constant temperature?

A. $P \propto \frac{1}{\rho}$ B. $P \propto \frac{1}{\rho^2}$ C. $P \propto \rho$ D. $P \propto \rho^2$

2) Which of the following is correct formula for the root mean square speed of a molecule in a gas at the absolute temperature T?

A. $v_{r.m.s} = 1.5\sqrt{\frac{kT}{m}}$ B. $v_{r.m.s} = 1.5\sqrt{\frac{mk}{T}}$ C. $v_{r.m.s} = 1.73\sqrt{\frac{m}{kT}}$ D. $v_{r.m.s} = 1.73\sqrt{\frac{kT}{m}}$

3) Which of the following motions of ideal gas molecules determines the temperature?

A. Translational motion B. Rotational motion

C. Vibrational motion D. All types of motion.

4) Which of the following gases possesses the maximum root mean square velocity at the same temperature?

A. Oxygen B. Carbon dioxide C. Hydrogen D. Helium

5) Real gases deviate from ideal behavior because gas molecules:

A. are colorless B. are spherical

C. have intermolecular forces of attraction D. have high speeds

6) The root mean square speed of gas at 27 °C is $v_{r.m.s}$. If the temperature of the gas is raised to 327 °C , then which of the following will be the root mean square speed of gas?

A. $(1/\sqrt{2})\ v_{r.m.s}$ B. $v_{r.m.s}$ C. $\sqrt{2}\ v_{r.m.s}$ D. $2\ v_{r.m.s}$

7) Which of the following is the ratio of root mean square speed of oxygen O_2 and hydrogen H_2 at same temperature?

A. 1: 2 B. 2:1 C. 1:4 D. 4:1

8) Density of a gas is 5 kg m^{-3} at a pressure of 6×10^5 Pa. Which of the following is the root mean square velocity of the gas molecules?

A. 600 m s^{-1} B. 400 m s^{-1} C. 300 m s^{-1} D. 180 m s^{-1}

9) The average kinetic energy of gas molecules is:

A. directly proportional to pressure of gas B. inversely proportional to volume of gas

C. inversely proportional to the absolute temperature of gas

D. directly proportional to the absolute temperature of gas

10) The pressure is exerted by the gas on the walls of the container because its molecules:

A. lose kinetic energy B. stick to the walls

C. on collision with the walls, there is a change in momentum of gas molecules

D. on collision with the walls, gas molecules exert no force on wall

11) The rms speed of the molecules of enclosed gas is $v_{r.m.s}$. Which of the following will be the rms speed of gas molecule, if pressure is doubled keeping the temperature same?

A. 1/2 $v_{r.m.s}$ B. $v_{r.m.s}$ C. 2 $v_{r.m.s}$ D. 4 $v_{r.m.s}$

12) A neutron star has a super high _______________ .

A. density B. output of light C. input of neutrons D. output of x-rays

13) Which of the following is the electrical resistance of the superconductor below critical temperature?

A. Finite B. Large C. Zero D. Infinity

14) What is the process by which a Bose-Einstein Condensate is formed?

A. Fusion B. Fission

C. Decomposition D. Cooling to extremely low temperature

Short Questions

1) Mention the different ways of increasing the number of molecular collisions per unit time in a gas.
2) By reducing the volume of a gas at a constant temperature, the pressure of the gas increases. Explain it on the basis of kinetic theory.
3) What do you mean by the root mean square speed of the molecules of a gas? Is the root mean square speed the same as the average speed?
4) Why is the temperature below absolute zero not possible?
5) Estimate the average kinetic energy of a helium atom at the temperature on the surface of the sun (6000 K).
6) Show that the ratio of the root mean square speeds of molecules of two different gases at a certain temperature is equal to the square root of the inverse ratio of their masses.
7) Differentiate between the formations of white dwarfs and neutron stars.
8) Why do the gases at low temperatures and high pressure show large deviations from ideal behaviour?
9) What distinguishes degenerate matter from regular matter?
10) Show that the temperature of ideal gas is directly proportional to the average translational kinetic energy of gas molecules.
11) What happens to the electrical resistance of a superconductor when it is cooled below its critical temperature?
12) What are some potential applications of superconductors in transportation?
13) Provide an example of a high-temperature superconductor.

Comprehensive Questions

1) What is the pressure of gas? How do gas molecules exert pressure on the walls of a container?
2) Derive an expression for the pressure of an ideal gas and show that pressure is directly proportional to the average translational kinetic energy of gas molecules.
3) Describe the root mean square speed of the ideal gas molecule and drive its expressions.
4) Derive an expression for the average translational kinetic energy of the ideal gas molecule.
5) Describe the modification of the ideal gas model to discuss the behaviour of non-ideal gases using the Wan der Waals equation.
6) Describe the behaviour of matter under extreme physical conditions. What is degenerate matter, and how are white dwarf and neutron stars formed?
7) Discuss superfluidity and superconductivity on the basis of Bose-Einstein condensation.

Numerical Problems

1) The mass of a helium atom is 6.64×10^{-27} kg. Calculate the root mean square speed of helium atom in a gas at a temperature of 15 °C. **(Ans: 1.34×10^{3} m s^{-1})**

2) At which temperature will the root mean square velocity of the oxygen molecules become equal to the escape velocity of the earth (11.2 km s^{-1})? (Mass of one molecule of oxygen is 5.3×10^{-26} kg and Boltzmann constant $k = 1.38\times10^{-23}$ J K^{-1}). **(Ans: 1.6×10^{5} K)**

3) The mass of a molecule of a gas is 6.4×10^{-27} kg. Calculate the root mean square speed of gas molecule and kinetic energy per molecule at temperature 400 K.

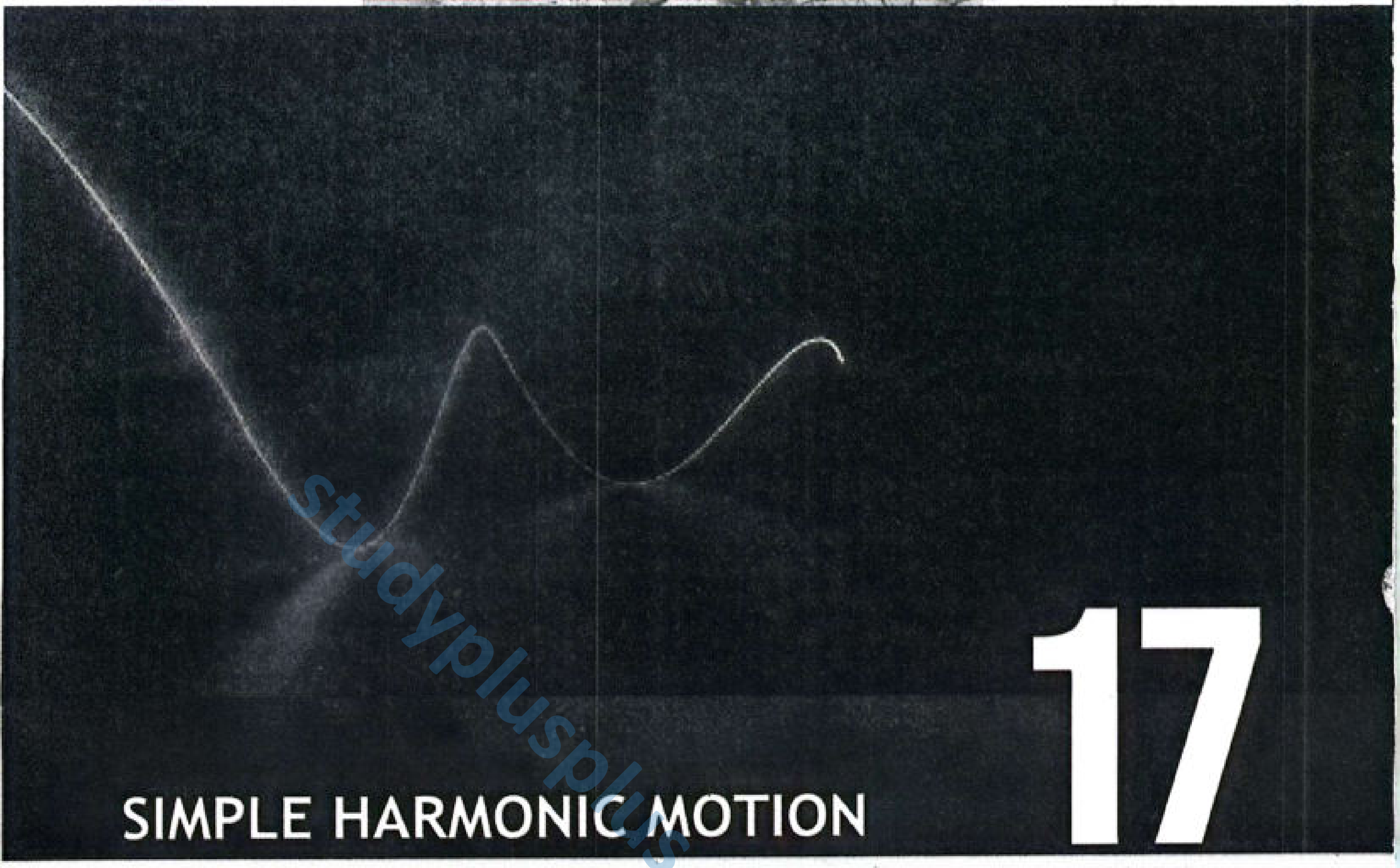
(Ans: 1.6×10^{3} m s^{-1}, 8.3×10^{-21} J)

4) At which temperature the root mean square speed of gas molecules becomes double than its speed at 27 °C, (pressure of gas is kept constant)? **(Ans: 927 °C)**

5) Determine the root mean square speed of argon atoms at temperature 40 °C. The molar mass of argon is 39.95 g mol^{-1}. **(Ans: 442 m s^{-1})**

6) Calculate the number of gas molecules in a cubic meter of gas at standard temperature and pressure (STP). **(Ans: 2.68×10^{25} molecules)**

7) A vessel A contains hydrogen and another vessel B whose volume is twice of vessel A contains same mass of oxygen at the same temperature. Find (i) the ratio of the root mean square speeds of hydrogen and oxygen gases, (ii) the ratio of pressures of gases in vessels A and B. Molecular mass of hydrogen and oxygen are 2 and 32 respectively. **(Ans: 4:1 and 32:1)**



17 SIMPLE HARMONIC MOTION

Student Learning Outcomes (SLOs)

The student will

- describe simple examples of free oscillations.
- use the terms displacement, amplitude, period, frequency, angular frequency and phase difference in the context of oscillations.
- Express the period of simple harmonic motion in terms of both frequency and angular frequency.
- Explain that simple harmonic motion occurs when acceleration is proportional to displacement from a fixed point and in the opposite direction.
- use $a = -\omega^2 x$ to solve problems.
- use the equations $v = v_o \cos(\omega t)$ and $v = \pm\omega\sqrt{x_o^2 - x^2}$ to solve problems.
- Analyze graphical representations of the variations of displacement, velocity and acceleration for simple harmonic motion.
- Analyse the interchange between kinetic and potential energy during simple harmonic motion.
- Apply $\frac{1}{2}m\omega^2 x_o^2$ for the total energy of a system undergoing simple harmonic motion.
- describe that a resistive force acting on an oscillating system causes damping.
- use the terms light, critical and heavy damping.
- sketch displacement-time graphs to illustrate light, critical and heavy damping.
- State that resonance involves a maximum amplitude of oscillations and that this occurs when an oscillating system is forced to oscillate at its natural frequency.
- Describe practical examples of free and forced oscillations.
- Describe practical examples of damped oscillations [with particular reference to the efforts of the degree of damping and the importance of critical damping in cases such as a car suspension system.]
- Justify qualitatively the factors which determine the frequency response and sharpness of the resonance.
- identify the use of standing waves and resonance in applications [such as rubens tubes, chladni plates and acoustic levitation (knowledge of wave harmonic modes is not required)]
- Justify the importance of critical damping in a car suspension system.
- Justify that there are some circumstances in which resonance is useful [such as tuning a radio, microwave oven and other circumstances in which resonance should be avoided such as airplane's wing or a suspension bridge].

Have you ever wondered why a pendulum swings back and forth, or how a guitar string vibrates to produce sound? Perhaps you've noticed the smooth motion of a child on a swing or the rhythmic movement of a spring-based toy. These phenomena are all connected by a fundamental concept in physics: Simple Harmonic Motion (SHM). SHM can be observed in various natural and man-made systems. It is in the rhythms of nature, how the universe moves, vibration of atoms, mechanical systems that engineers create, and countless other systems from everyday life.

In this chapter, we'll delve into the world of SHM, exploring topics such as oscillations, uniform circular motion, phase of motion, and energy conservation. We'll also examine the differences between free and forced oscillations, and discover the fascinating phenomenon of resonance.

Sometimes harmonic motion can cause problems. One famous example is the collapse of Tacoma Narrows Bridge. The bridge was designed by structural engineers who did not adequately take into account the role of harmonic motion and it led into its collapse. So, let's go to uncover the rhythm of such a motion!

17.1 OSCILLATIONS

What does a child on a swing, the pendulum of a clock and bouncing of children on the trampoline (as shown in Fig.17.1), all have in common? They all oscillate, i.e., they move back and forth between two points.

Figure 17.1: Examples of oscillation in our surrounding.

When a body moves to and fro about its mean position, then such motion is called vibratory or oscillatory motion.

In our surrounding, there are many other systems which oscillate. For examples,

- The motion of the Earth during earthquake.
- The wings of birds during flying.
- A string of a guitar producing music.
- Vibration of atoms in a crystal.
- The beating of heart.

For Your Information

Ocean waves or ripples on a pond exhibit oscillatory motion. Sound waves propagate through a medium, causing particles to oscillate. Light waves and other electromagnetic radiation exhibit oscillatory behavior.

- Mass attached to a compressed or stretched spring (horizontally or vertically). Once it is released, starts oscillating.

The complete round trip of an oscillating or vibrating body about its *mean position* is called oscillation or vibration.

One oscillation occurs when a particle moves from its mean position to extreme position (A) in one direction, moving back to extreme position (B) in the opposite direction through mean position and back once more to mean position, as shown in Fig. 17.2.

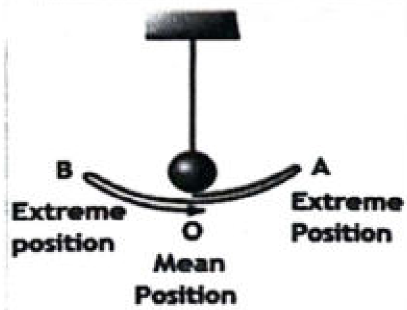


Figure 17.2: One oscillation.

Terms Related to Oscillations

In this section, we shall know about some important terms related to oscillatory motion.

(i) Displacement (x): The distance of vibrating body from the equilibrium position on either side at any instant of time is known as displacement. Its SI unit is meter (m).

(ii) Amplitude (x_o or A): The maximum displacement of a vibrating body from mean position is called amplitude. Its SI unit is meter (m).

(iii) Time Period (T): The time taken to complete one oscillation or vibration is called time period of the oscillation. It is denoted by T. Its SI unit is second (s).

(iv) Frequency (f): The number of vibrations or oscillations (n) per unit time (t) is called frequency. It is denoted by f, and $f = \frac{n}{t}$. Its SI unit is hertz (Hz).

One hertz is defined as: the one oscillation per second. The relationship between frequency and time period is:

$$f = \frac{1}{T}$$

(v) Angular Frequency (ω): Angular displacement per unit time is called angular frequency. It is related to the frequency 'f' and time period 'T' of the oscillation by the expressions as:

$$\omega = 2\pi f = \frac{2\pi}{T}$$

17.2 SIMPLE HARMONIC MOTION

Simple Harmonic Motion is a type of motion in which a system oscillates back and forth around a mean (equilibrium) position. It is characterized by a restoring force. Simple harmonic motion (SHM) is defined as:

Such oscillatory motion in which acceleration of a particle is directly proportional to its displacement from the mean position and is always directed towards the mean position.

Mathematically, simple harmonic motion is expressed as:

$$a \propto -x$$

SHM can be observed in many natural phenomena. A good example of SHM is motion of mass (m) attached to an elastic spring. The other end of spring is connected to a fixed support, as shown in Fig. 17.3.

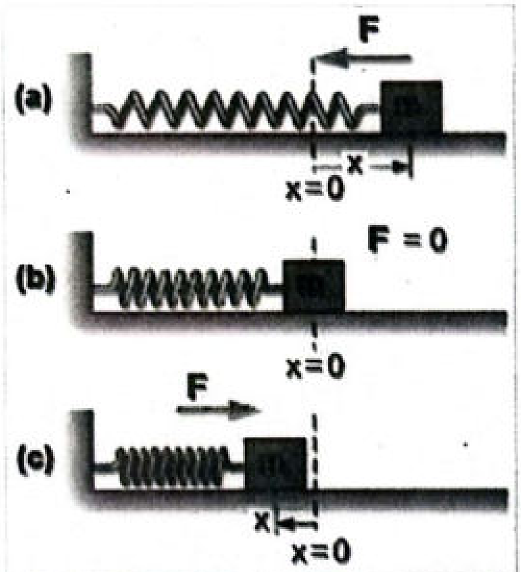


Figure 17.3: Mass 'm' oscillates on horizontal surface.

Let us assume that the mass of spring is ignored and the mass is free to move on a frictionless, horizontal surface. In the absence of an external force, the spring is neither stretched nor compressed. Hence, the mass stays at its equilibrium or mean position, which we identify as x = 0, as shown in Fig. 17.3 (b).

Let the mass is stretched towards right through a displacement 'x' from its equilibrium position by applying a force F, as shown in Fig. 17.3 (a). Due to elasticity, spring exerts an opposite force on the mass which is proportional to the displacement 'x' from mean position. This force is called restoring force. This restoring force obeys **Hooke's law**, i.e.,

$$F = -k\,x \qquad \text{\_\_\_\_\_\_ (17.1)}$$

Here 'k' is called the spring constant. The SI unit of k is N m^{-1}.

The negative sign in Eq. (17.1) shows that the force exerted by spring is always directed opposite to the displacement of the mass from mean position.

When the mass is displaced to the right of mean position, the displacement x is positive but the restoring force is directed to the left (mean position).

When the mass is released, it moves towards the mean position. Due to inertia, it cannot stop at mean position and goes ahead. Then the mass begins to compress the spring and slows down, coming to rest at the left side of the mean position equal to its initial distance on right side, as shown in Fig. 17.3 (c).

The compressed spring then pushes the mass back toward the mean position. Again, the mass cannot stop at the mean position and goes ahead. The result is that the mass oscillates back and forth about the mean position. Since the mass is continuously changing its direction, so it accelerates. According to **Newton's second law of motion**, the force acting on the mass is given by:

$$F = m\,a \qquad \text{\_\_\_\_\_\_ (17.2)}$$

By comparing Eq. (17.1) and Eq. (17.2), we get:

$$m\,a = -\,k\,x$$

or $$a = -\frac{k}{m}x \qquad \text{———— (17.3)}$$

In Eq. (17.3), $\frac{k}{m} = \omega^2$ is constant. So, angular frequency of 'mass attached to the spring' is:

$$\omega = \sqrt{\frac{k}{m}} \qquad \text{———— (17.4)}$$

Now Eq. (17.3) can be written as:

$$a \propto -x$$

This relation shows that the oscillating mass exhibits SHM; its acceleration (a) is proportional to its displacement (x) and is directed towards mean position.

For Your Information

The atoms in a material oscillates as if the atoms were connected by a tiny spring, hence exhibit SHM. The spring constant of this spring depends upon the type of bonding between the atoms. This model can be extended to solids, where atoms are often thought of as being connected to their neighbours by springs. This leads to an experimental way of obtaining information about interatomic forces in the solids.

Frequency and Time Period of an Oscillating Mass-Spring System

As angular frequency 'ω' is related to the frequency and time period by the following expressions:

$$\omega = 2\pi f = \frac{2\pi}{T}$$

So, $$T = \frac{2\pi}{\omega}$$

By putting $\omega = \sqrt{\frac{k}{m}}$, we get:

$$T = 2\pi\sqrt{\frac{m}{k}}$$

Similarly, for the frequency, we can get the following expression:

$$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$$

Example 17.1: A Fish is hung on a spring scale.
(a) What is the constant of the spring in such a scale if the spring stretches 8.0 cm for a 10.0 kg load? (b) What is the mass of the fish that stretches the spring 5.5 cm?

(a) Given: x = 8.0 cm = 0.08 m $\qquad$ m = 10.0 kg
To Find: k = ?

Solution: According to the Hooke's law, $F = k\,x$
Here, $F = W = mg$.

Therefore, $$k = \frac{mg}{x}$$

Putting values, we get: $$k = \frac{10 \times 9.8}{0.08} = 1225\ \text{N m}^{-1}$$

(b) Given: $x = 5.5\ \text{cm} = 0.055\ \text{m}$
To Find: $m = ?$

Solution: As $$m = \frac{kx}{g}$$

Putting values, we get: $$m = \frac{1225 \times 0.055}{9.8} = 6.875\ \text{kg}$$

Assignment 17.1

An object with mass 500 g is suspended from a spring. The spring is stretched by 9.8 cm. Calculate the spring constant.

17.3 UNIFORM CIRCULAR MOTION AND SHM

There is a close connection between circular motion and simple harmonic motion. Therefore, many aspects, such as displacement, velocity and acceleration of simple harmonic motion (SHM) can be understood by relating it with uniform circular motion.

Consider a point P is moving on a circular path of radius x_o in xy-plane at constant angular velocity ω, as shown in Fig. 17.4 (a). At the same time, Q be the projection of P on the x-axis undergoes oscillation along the diameter of circular path between $-x_o$ and $+x_o$.

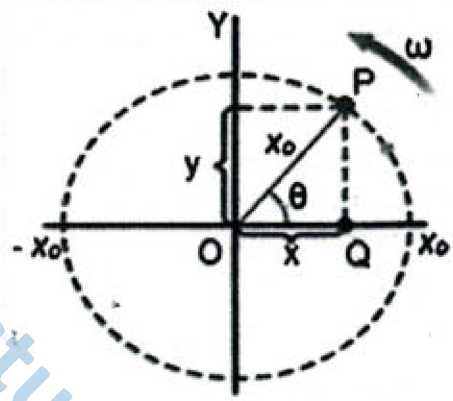


Figure 17.4 (a): A point P is moving on a circular path at constant angular velocity ω.

It is seen that the time period of one revolution of point P on the circular path is equal to the time period of one oscillation of point Q on the diameter. Therefore, the angular speed of P is the same as the linear speed of Q. Thus, the expressions for displacement, velocity and acceleration of P also hold for the Q.

Expressions for Displacement

As the point P moves on the circle, at some instant t, the angle made by the line OP with the x-axis is $\theta = \omega t$. From the Fig. 17.4 (a), the instantaneous displacement x of Q at that instant can be calculated as:

From ΔOPQ, $$\cos\theta = \frac{x}{x_o}$$

or $\qquad x = x_o \cos\theta = x_o \cos\omega t \qquad$ ______ (17.5)

Eq. (17.5) can be used for calculating displacement of a body executing simple harmonic motion.

Expressions for Velocity

Velocity of the point P at instant t is $v_P = x_o\,\omega$, which is directed along the tangent. Instantaneous velocity v of the point Q is the projection of v_P on the x-axis, which is the horizontal component of v_P, as shown in Fig. 17.4 (b).

$$v = v_P \cos(90^\circ - \theta)$$

or $\qquad v = v_P \sin\theta \qquad$ ______ (17.6)

as, $\sin^2\theta + \cos^2\theta = 1$ or $\quad \sin^2\theta = 1 - \cos^2\theta$

so, Eq. (17.6) becomes:

$$v = v_P \sqrt{1-\cos^2\theta}$$

or

$$v = x_o\,\omega \sqrt{1-\cos^2\theta}$$

Using Eq. (17.5), we get:

$$v = \omega \sqrt{x_o^2 - x^2} \qquad \text{______ (17.7)}$$

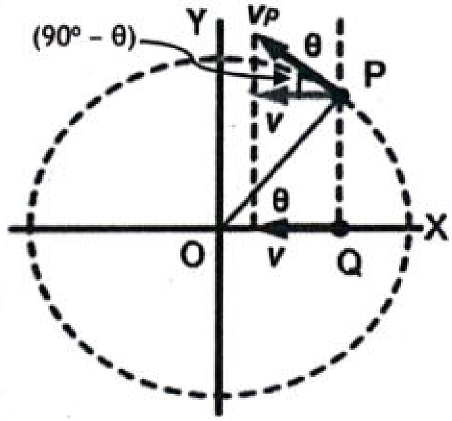


Figure 17.4 (b): Velocity of the point P and its horizontal component.

Eq. (17.7) can be used for calculating velocity of a body executing simple harmonic motion.

Expressions for Acceleration

Since ω is constant, so the acceleration of the point P is centripetal i.e., $a_P = x_o\,\omega^2$, which is directed inward towards O. The acceleration 'a' of Q is the horizontal component of a_P, as, shown in Fig. 7.4 (c).

$$a = -\,a_P \cos\theta = -\,x_o\,\omega^2 \cos\theta$$

Since the velocity is decreasing, so the negative sign shows that acceleration of Q is directed towards O. Using $\cos\theta = \dfrac{x}{x_o}$, we get:

$$a = -\,\omega^2 x \qquad \text{______ (17.8)}$$

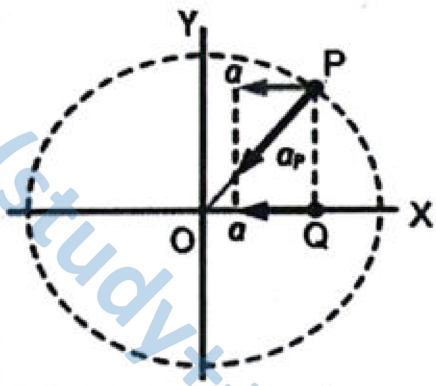


Figure 17.4 (c): Acceleration of the point P and its horizontal component.

This equation describes the SHM. Hence, the oscillatory motion of point Q is SHM; its acceleration is proportional to its displacement and is directed opposite to the displacement from mean position. Hence, we can conclude that:

When a body moves in a circle, its projection undergoes simple harmonic motion on the diameter of the circle.

So, acceleration of a body executing simple harmonic motion can be calculated by using Eq. (17.8).

Example 17.2: A body with mass 800 g attached to a spring, vibrates with amplitude 30 cm. The restoring force is 60 N when the displacement is 0.30 m. (a) Find out its angular frequency (b) Also calculate magnitude of its velocity and acceleration at x = 12 cm.

Given: $m = 800\ g = 0.8\ kg$ $\qquad$ $x_o = 30\ cm = 0.30\ m$

$F = 60\ N$ at $x_o = 30\ cm = 0.30\ m$ $\qquad$ $x = 12\ cm = 0.12\ m$

To Find: (a) angular frequency = ω = ?

(b) velocity = ? $\qquad$ Acceleration = a = ?

Solution: (a) As $\quad k = \dfrac{F}{x_o} = \dfrac{60}{0.30} = 200\ N\ m^{-1}$

Angular frequency is given by the relation:

$$\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{200}{0.8}} = 15.81\ rad\ s^{-1}$$

(b) For velocity, we use:

$$v = \omega\sqrt{x_o^2 - x^2}$$

Putting values, we get:

$$v = 15.81\sqrt{0.3^2 - 0.12^2} = 1.4\ m\ s^{-1}$$

For acceleration, we use:

$$a = \omega^2 x$$

Putting values, we get:

$$a = (15.81)^2\ (0.12) = 3\ m\ s^{-2}$$

Assignment 17.2

Time period of a mass attached at the end of a spring is 0.40 s. Find out the magnitude of acceleration when the displacement is 4 cm.

17.4 PHASE

We have studied that the instantaneous displacement of the point executing SHM is given by the Eq. (17.5), as:

$$x = x_o \cos\theta \qquad \text{or} \qquad x = x_o \cos\omega t$$

Here, $\theta = \omega t$ is the phase of motion and can be defined as:

The angle $\theta = \omega t$ which specifies the displacement as well as the direction of a point executing SHM is called phase of the motion.

In general, the Eq. (17.5) can be written as:

$$x = x_o \cos(\omega t + \varphi) \qquad \text{\_\_\_\_\_\_ (17.9)}$$

The term φ is called initial phase. The inclusion of φ gives the information regarding the starting or initial phase of oscillation.

Physical Significance of Phase

To give the physical significance of phase, we use displacement-time graph. Since x is periodic, so T is the time period of the oscillation.

If $\varphi = 0^\circ$, then Eq. (17.9) becomes:

$$x = x_o \cos\omega t$$

Putting t = 0, T/4, T/2, 3T/4, T ...., we get a graph, as shown in Fig. 17.5 (a). This graph shows that:

At t = 0, T/2 and T (corresponding to θ = 0, π and 2π), the point is at the extreme positions.

At t = T/4 and 3T/4 (corresponding to $\theta = \pi/2$ and $3\pi/2$), the point is at mean position.

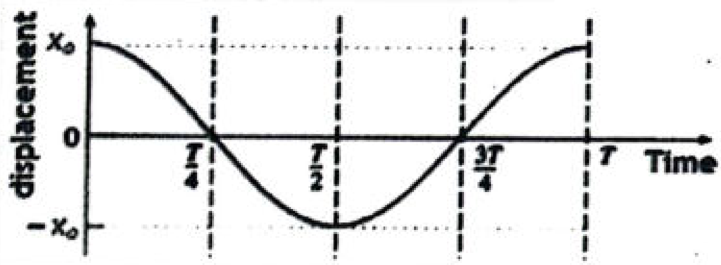


Figure 17.5 (a): Graph of $x = x_o \cos(\omega t + \varphi)$, for $\varphi=0^\circ$.

If $\varphi = 90^\circ$, then Eq. (17.9) becomes:

$$x = x_o \cos(\omega t + 90^\circ)$$

Putting t = 0, T/4, T/2, 3T/4, T ...., we get a graph, as shown in Fig. 17.5 (b). This graph shows that:

At t = 0, T/2 and T (corresponding to θ = 0, π and 2π), the point is at mean positions.

At t = T/4 and 3T/4 (corresponding to $\theta = \pi/2$ and $3\pi/2$), the point is at extreme positions.

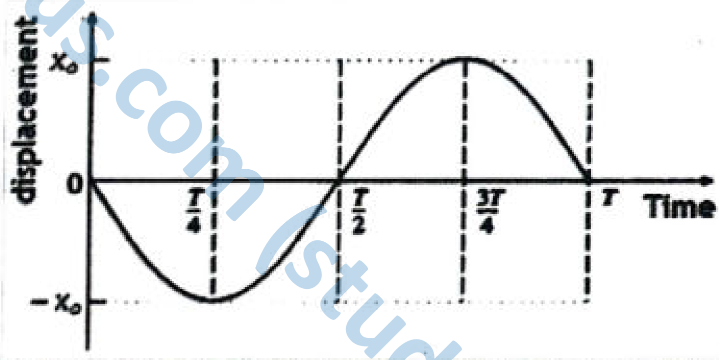


Figure 17.5 (b): Graph of $x = x_o \cos(\omega t + \varphi)$, for $\varphi=90^\circ$.

If $\varphi = 180^\circ$, then Eq. (17.9) becomes:

$$x = x_o \cos(\omega t + 180^\circ)$$

Putting t = 0, T/4, T/2, 3T/4, T ...., we get a graph, as shown in Fig. 17.5 (c).

It can be noted that:

- The curve in Fig. 17.5 (b) leads the curve in Fig. 17.5 (a) by 90°, because their phase difference is 90°.

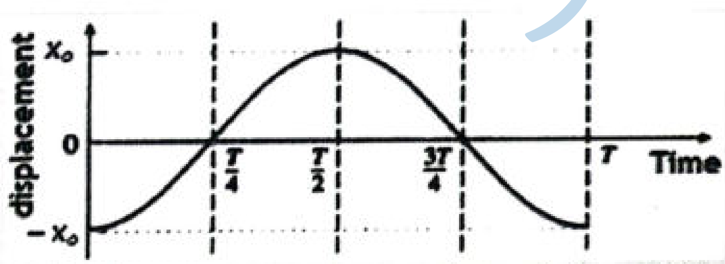


Figure 17.5 (c): Graph of $x = x_o \cos(\omega t + \varphi)$, for $\varphi=180^\circ$.

- Similarly, the curve in Fig. 17.5 (c) leads the curve in Fig. 17.5 (a) by 180°, because their phase difference is 180°.

When the phase difference between two oscillating systems is 180°, they are said to be oscillating **out of phase.**

When the phase difference between two oscillating systems is 0° or 360°, they are said to be oscillating **in phase.**

17.5 GRAPHICAL REPRESENTATIONS OF DISPLACEMENT, VELOCITY AND ACCELERATION FOR SHM

The graphical representations of displacement, velocity and acceleration of a body executing SHM is given in Fig. 17.5 (d).

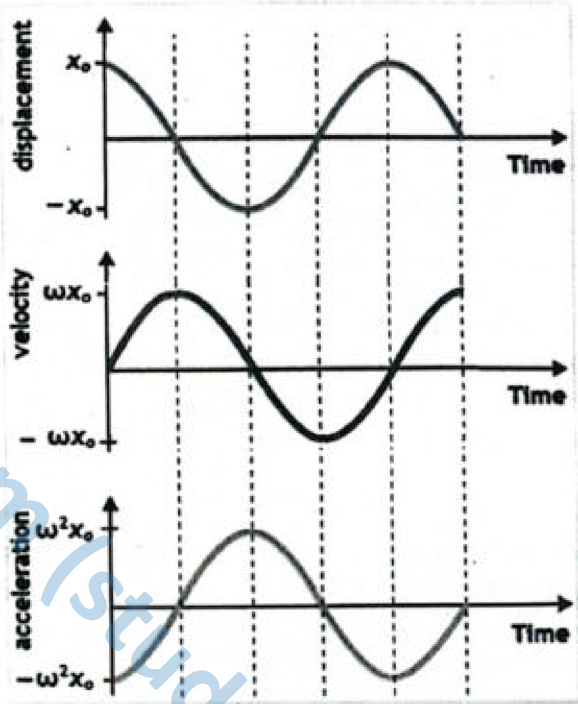


Figure 17.5 (d): Graphical representations of 'x', 'v' and 'a' of a body executing SHM.

(i) The displacement of the particle executing SHM is given by the expression:

$$x = x_o \cos\omega t \quad \text{\_\_\_\_\_\_ (i)}$$

As maximum value of displacement of the particle is x_o.

(ii) The velocity of the particle executing SHM is given by the expression:

$$v = x_o\,\omega\,\sqrt{1-\cos^2\theta}$$

or $$v = x_o\,\omega \sin(\omega t) \quad \text{\_\_\_\_\_\_ (ii)}$$

The velocity of the particle is maximum (i.e., $v = \pm x_o\,\omega$) at the mean position and zero at the extreme positions.

(iii) The acceleration of the particle executing SHM is given by the expression:

$$a = -\,\omega^2 x$$

Putting $x = x_o \cos\omega t$, we get:

$$a = -\,x_o\,\omega^2 \cos(\omega t) \quad \text{\_\_\_\_\_\_ (iii)}$$

The acceleration will be maximum (i.e., $x_o\,\omega^2$) at the extreme positions, and zero at the mean position.

The graph of the Eqs. (i), (ii) and (iii) is shown in Fig. 17.5 (c). It can be seen that:

- The phase difference between velocity and displacement is $\pi/2$.
- The phase difference between acceleration and displacement is π.

17.6 SIMPLE PENDULUM

An ideal simple pendulum consists of a small but heavy bob of mass m which is suspended by a light and inextensible string of length l. The other end of the string is attached to a fixed frictionless support at point P, as *shown in* Fig. 17.6 (a).

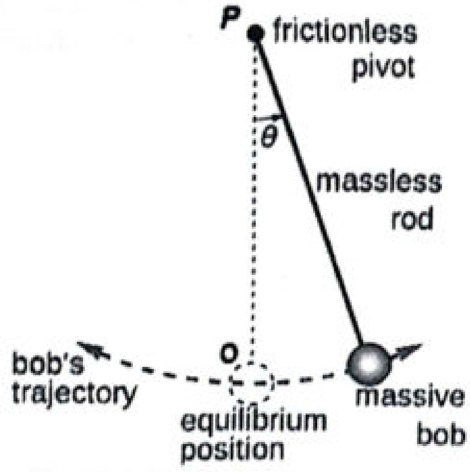


Figure 17.6 (a): Simple pendulum.

When the bob is displaced slightly from its mean position O, after releasing it oscillates to-and-fro along the arc of a circle with centre at P. Suppose that the oscillating bob is at point A at some instant, where the displacement is x and the angle OPA = θ. The forces acting on the bob are the tension T exerted by the string and its weight mg acting vertically downward.

The weight mg of the bob can be resolved into its two rectangular components, i.e.

i) $mg\cos\theta$ along the string.

ii) $mg\sin\theta$ along the tangent to the arc.

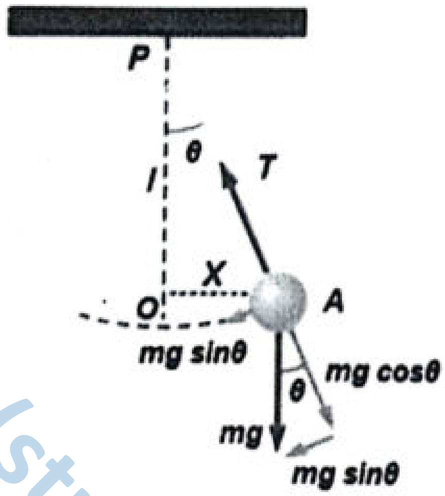


Figure 17.6 (b): Components of weight.

Tension in the string exactly cancels the component $mg\cos\theta$. The net force $mg\sin\theta$ on the bob at A is the restoring force which makes it to accelerate towards equilibrium position, i.e.,

$$F = -\,m\,g\sin\theta \quad \text{\_\_\_\_\_\_ (i)}$$

Negative sign indicates that the force is acting towards mean position O. When θ is small (less than 10° or 0.2 rad), as shown in Table (7.1), we make the approximation sin θ ≈ θ and Eq. (i) becomes:

$$F = -\,m\,g\,\theta \quad \text{\_\_\_\_\_\_ (ii)}$$

From Fig. 7.6 (b), arc length = OA, then,

$$\theta = \frac{OA}{l}$$

For small angle θ, the arc OA ≈ x, then Eq. (iii) becomes,

$$F = -\,m\,g\,\frac{x}{l} \quad \text{\_\_\_\_\_\_ (iii)}$$

According to Newton's second law of motion, the force acting on the bob is given by:

$$F = m\,a \quad \text{\_\_\_\_\_\_ (iv)}$$

Table 7.1: Values of θ and sin θ for small angle.

θ (degree)	θ (radian)	sin θ
0.00	0.0000	0.0000
1.00	0.0175	0.0175
2.00	0.0349	0.0349
3.00	0.0524	0.0524
4.00	0.0698	0.0698
5.00	0.0873	0.0872
6.00	0.1048	0.1046
7.00	0.1222	0.1219
8.00	0.1397	0.1392
9.00	0.1571	0.1565
10.00	0.1746	0.1737

By comparing Eq. (iii) and Eq. (iv), we get:

$$m\,a = -\,m\,g\,\frac{x}{l}$$

$$a = -\,\frac{g}{l}\,x \qquad \_\_\_\_\_\_ \text{(v)}$$

or

$$a \propto -x \qquad \_\_\_\_\_\_ \text{(vi)}$$

The Eq. (vi) describes the SHM. Hence, an oscillating simple pendulum exhibits SHM. As equation for acceleration of a body executing SHM is:

$$a = -\,\omega^2 x \qquad \_\_\_\_\_\_ \text{(vii)}$$

By comparing Eq. (v) and Eq. (vii), we get:

$$\omega = \sqrt{\frac{g}{l}} \qquad \_\_\_\_\_\_ \text{(viii)}$$

So, the expression for time period of the simple pendulum can be obtained by putting Eq. (viii) in $T = 2\pi/\omega$, thus we get:

$$T = 2\pi\sqrt{\frac{l}{g}} \qquad \_\_\_\_\_\_ \text{(17.10)}$$

Above equation shows that:

The time period of a simple pendulum depends on the length of the pendulum and the acceleration due to gravity. It is independent of the mass.

Example 17.3: A simple pendulum completes one vibration in one second. Calculate its length when $g = 9.8\ \text{m s}^{-2}$.

Given: Time period of the simple pendulum = $T = 1$ s
Gravitational acceleration = $g = 9.8\ \text{m s}^{-2}$
To Find: Length of the simple pendulum = $l = ?$

Solution: As $$T = 2\pi\sqrt{\frac{l}{g}}$$

Putting values, we get: $$1 = 2 \times 3.14\sqrt{\frac{l}{9.8}}$$

$$l = 0.248\ \text{m} = 24.8\ \text{cm}$$

Assignment 17.3

Find the time periods of a simple pendulum with length 1 m, placed on Earth and on Moon. The value of g on the surface of Moon is 1/6th of its value on Earth.

17.7 ENERGY CONSERVATION IN SHM

Energy of a body executing SHM remains conserved. To examine this fact, we again consider a vibrating mass-spring system, as shown in Fig. 17.7 (a). When the spring is stretched by the applied force 'F' through a displacement 'x', work must be done. This work W is given by:

$$W = F_{av}\ x \qquad \text{\_\_\_\_\_\_ (i)}$$

According to Hooke's law, the applied force is given by:

$$F = k\,x \qquad \text{\_\_\_\_\_\_ (ii)}$$

Since the force increases linearly from 0 to kx, the average force F_{av} is given by:

$$F_{av} = \frac{0+k\,x}{2} = \frac{1}{2}k\,x \qquad \text{\_\_\_\_\_\_ (iii)}$$

Substituting the value of F_{av} in Eq. (i) from Eq. (iii), we get:

$$W = (\frac{1}{2}k\,x)(x) = \frac{1}{2}k\,x^2$$

As, this work is stored in spring as elastic potential energy. Hence, the potential energy at any instant 'x' is given by:

$$P.E = \frac{1}{2}k\,x^2 \qquad \text{\_\_\_\_\_\_ (17.11)}$$

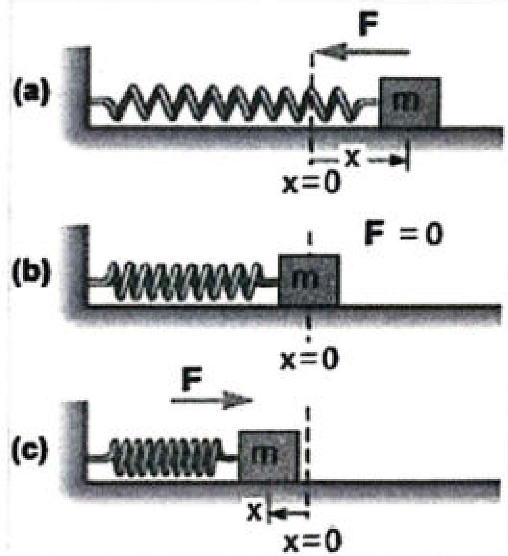


Figure 17.7: A vibrating mass-spring system.

At extreme position, where the displacement is maximum, i.e. $x = x_o$, the block is at rest. So, its kinetic energy is zero and total energy is entirely elastic potential energy, i.e.

$$T.E = P.E_{max} = \frac{1}{2}k\,x_o^2 \qquad \text{\_\_\_\_\_\_ (17.12)}$$

When the block is released, it moves toward mean position. As a result, its velocity increases and also its kinetic energy. However, the displacement of the block decreases and also its elastic potential energy.

As, the instantaneous velocity v of the block executing SHM is given by,

$$v = \omega\sqrt{x_o^2 - x^2} \qquad \text{\_\_\_\_\_\_ (iv)}$$

As, $\omega = \sqrt{\frac{k}{m}}$, so Eq. (iv) becomes:

$$v = \sqrt{\frac{k}{m}(x_o^2 - x^2)} \qquad \text{\_\_\_\_\_\_ (v)}$$

At mean position x = 0, the block gets maximum velocity, i.e.,

$$v_o = \sqrt{\frac{k}{m}}\ x_o \qquad \text{\_\_\_\_\_\_ (vi)}$$

Hence, at any instant where the displacement is 'x', the kinetic energy is given by:

$$K.E = \frac{1}{2}mv^2 = \frac{1}{2}k(x_o^2 - x^2) \qquad \text{\_\_\_\_\_\_ (17.13)}$$

At mean position (x = 0), the elastic potential energy is zero but the velocity is maximum, hence the total energy is entirely kinetic energy.

$$T.E = K.E_{max} = \frac{1}{2}mv_{max}^2 = \frac{1}{2}kx_o^2 \quad \text{———} \quad (17.14)$$

Let us now examine the total energy of the systems at displacement x. As, the total energy is the sum of kinetic and potential energy, i.e.

$$T.E = K.E + P.E \quad \text{———} \quad (vii)$$

After substituting the values of K.E and P.E from Eq. (17.11) and Eq. (17.13) in Eq. (vii), we get:

$$T.E = \frac{1}{2}k(x_o^2 - x^2) + \frac{1}{2}kx^2 = \frac{1}{2}kx_o^2 \quad \text{———} \quad (17.15)$$

As, $\omega = \sqrt{\frac{k}{m}}$, or $k = m\,\omega^2$, so Eq. (17.15) can also be written as:

$$T.E = \frac{1}{2}m\omega^2 x_o^2 \quad \text{———} \quad (17.16)$$

From Eqs. (17.12), (17.14) and (17.15), it is proved that:

Total energy of the mass-spring system is constant and is proportional to square of the amplitude of oscillation.

This statement of conservation of energy is equally valid for all bodies executing SHM.

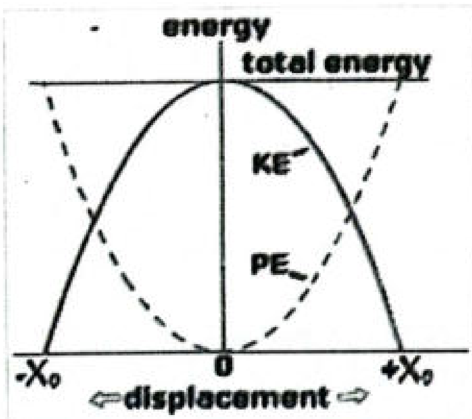


Figure 17.7 (d): Graph of energy (K.E, P.E and T.E) against displacement, for SHM.

The energy oscillates between K.E and P.E, but their sum remains constant. This can be illustrated by plotting the graph of K.E, P.E and T.E verses displacement, as shown in Fig. 17.7 (d).

Example 17.4: A 0.025 kg mass is attached to a spring which is displaced through 0.10 m to right of its mean position and then released. Time period of its oscillation is 1.57 s. Calculate its:

(a) Angular speed (b) The total energy (c) The maximum acceleration.

Given: m = 0.025 kg $\quad x_o = 0.10$ m $\quad$ T = 1.57 s

To Find: (a) Angular Speed = ω =? $\quad$ (b)Total Energy = $T.E$ = ?
(c) Maximum Acceleration = a = ?

Solution: (a) To find angular speed, we use the relation:

$$\omega = \frac{2\pi}{T}$$

Putting values, we get: $\quad \omega = \frac{2\,(3.14)}{1.57} = 4 \text{ rad s}^{-1}$

(b) The total energy can be found by using the relation:

$$T.E = \frac{1}{2} m\omega^2 x_o^2$$

Putting values, we get: $$T.E = \frac{1}{2}(0.025)(4)^2(0.1)^2 = 2 \times 10^{-3}\ J$$

(c) For the maximum acceleration, we use the relation:

$$a = x_o\,\omega^2$$

Putting values, we get: $$a = 0.1 \times 4^2 = 1.6\ m\ s^{-2}$$

Assignment 17.4

Find the amplitude, frequency and time period of an object oscillating at the end of a spring, if the equation for its position at any instant t is given by $x = 0.25\cos(\pi/8)t$. Also find the displacement of the object after 2.0 s

17.8 FREE, FORCED AND DAMPED OSCILLATIONS

Depending upon the situation, oscillations may be damped, free and forced oscillations. Here we discuss these three types of oscillations in detail.

Free Oscillations

Every oscillator has a natural frequency of vibration with which it vibrates freely after an initial disturbance. For example, an ideal simple pendulum oscillates freely with its natural frequency, when slightly displaced from its mean position, as shown in Fig.17.8 (a). The natural frequency of pendulum depends on its length. If you change the length of string, you may change its natural frequency.

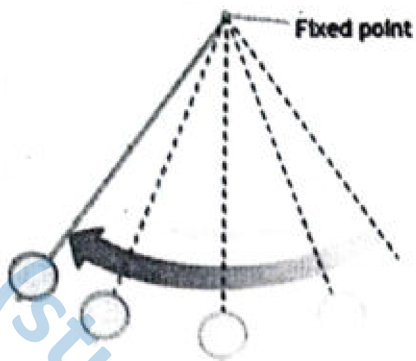


Figure 17.8 (a): Illustration of free oscillation.

If you pluck a guitar string, it continues to vibrate for some time after you have released it. It vibrates with its natural frequency and it gives rise to the particular note that you hear. The natural frequency of guitar string depends on its length. If you change the length of string, certainly you change its natural frequency.

A body is said to be executing free oscillations when it oscillates under the influence of a restoring force without any external force acting on it.

The free oscillations possess constant amplitude and period without any external force acts on it. Ideally, free oscillations do not undergo damping.

Forced Oscillations

If an external force acts on an oscillator, it can change its amplitude of oscillations. The external force shifts the energy to the oscillator at a certain frequency, not necessarily the same as the natural frequency of the oscillator. This frequency is called driven frequency.

For example, when you push a swing, you have to keep periodically pushing it so that the it doesn't reduce its amplitude and continue oscillating. These oscillations are called forced oscillations.

If the external forces make the object to oscillate at the frequency of applied force rather than its natural frequency, then such oscillations are called forced oscillations.

Movement of the pendulum of a clock (as shown in Fig.17.8-b) is also an example of a forced oscillations, because it is driven by a small motor.

Figure 17.8 (b): Illustration of force oscillation.

Damped Oscillations

Damping is the effect of resistive forces which dissipate energy from a vibrating object. When a simple pendulum is being set into oscillatory motion, then after some time, it stops oscillating due to air resistance. All oscillating systems experience such type of resistive force which are known as damping forces. Due to the damping force, amplitude of oscillation decreases over time from one oscillation to the next and eventually stops oscillating.

Examples of damping forces can include frictional forces between moving parts, air resistance or internal forces such as those in springs that tend to dissipate energy as heat.

We know that in reality, a spring won't oscillate forever. Frictional forces will diminish the amplitude of oscillations until eventually the system comes to rest.

If the amplitude of oscillations decreases under damping forces, then such oscillations are called damped oscillations.

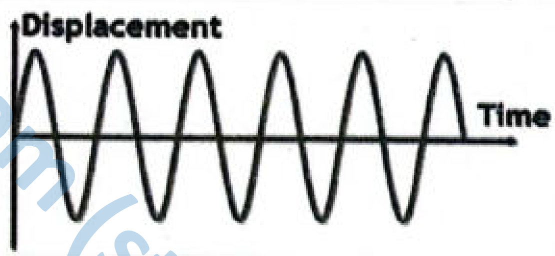


Figure 17.9 (a): Displacement-time graphs to illustrate undamped oscillations.

It can also be defined as: when an oscillator undergoes oscillations before coming to rest under the action of damping force, then such oscillations are called damped oscillations. As an oscillator vibrates, it performs work against force of friction, which result in the gradually decrease of oscillator's energy.

The damping is said to be light when the amplitude of oscillations decreases gradually with time.

Light damping gradually reduces the energy and amplitude of the vibrating object. An example of 'light damping' is a swing in playground, which gradually comes to rest when oscillates freely. Displacement-time graph of light damping is shown in Fig. 17.9 (a) by black curve.

Heavy damping takes long time before the object comes to rest; it is shown in Fig. 17.9 (b) by green curve.

In heavy damping, the oscillator merely moves back to its equilibrium position gradually without completing a single oscillation. It is the result of a very large resistive force.

Oscillations of a mass attached to a spring that is placed in a thick, viscous liquid, can be considered as heavy damping.

Critical damping returns the object to the equilibrium position in the shortest time possible. An example of critical damping is shock absorbers in a car; they increase the resistive force so that after being displaced when going over a bump, the vehicle returns to its original position without oscillating. Critical damping is shown in Fig. 17.9 (b) by red curve.

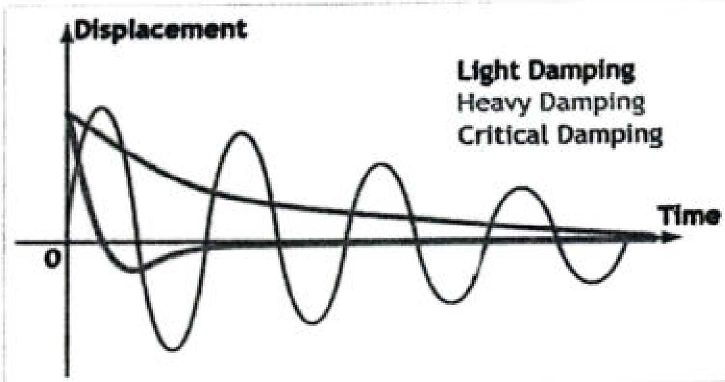


Figure 17.9 (b): Displacement-time graphs to illustrate light, critical and heavy damping.

When an oscillator comes to rest without any oscillation in the shortest time under damping force, then such damping is called critical damping.

Critical damping is often useful in an oscillating system because such systems return to equilibrium position rapidly after facing external deriving force. For example, when a car is passing over a bump on road, it would move up and down violently for sometimes which may cause injury to passengers. To overcome this problem, energy-absorbing devices (damping) known as shock absorbers are positioned parallel to the spring in automobiles. Vehicles have springs between the wheels and the frame, as shown in Fig. 7.9 (c), provide a smoother and comfortable ride.

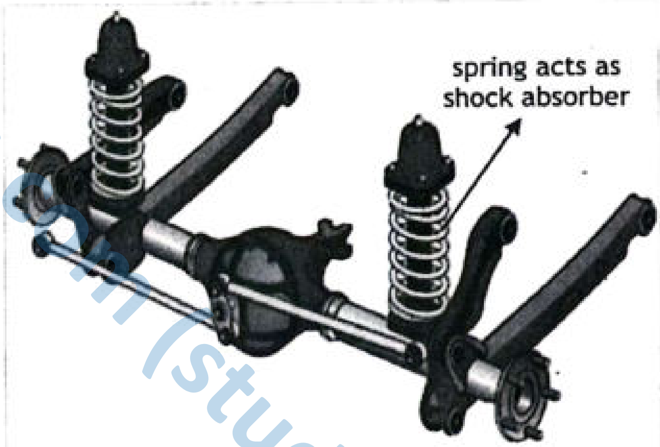


Figure 17.9 (c): Shock absorbers are fluid-filled tubes that turn the simple harmonic motion of the springs into damped harmonic motion.

Modern auto suspensions are set up so that all of a spring's energy is absorbed by the shock absorbers, eliminating vibrations in single oscillation. This prevents the car from continually bouncing.

17.9 RESONANCE

A driving force is always required to initiate the oscillations of any oscillator, damped or un-damped, by supplying the initial energy for the motion. The driving forces have their own frequencies, which cause the oscillator to vibrate at the driving frequency rather than at the natural frequency.

When the frequency of driving force gradually increases from zero, the oscillator begins to vibrate with small amplitude. As we increase the frequency, the amplitude of vibration also increases. The closer the deriving frequency to the natural frequency, the more efficiently the driving force transfers energy to the oscillator and the greater the resulting amplitude. When the deriving frequency equals the natural frequency of oscillator, the amplitude of vibrations reaches to its maximum value, this situation is called resonance. Hence resonance is defined as:

Resonance is the phenomena in which the amplitude of vibration of an oscillator attains maximum value when deriving frequency becomes equal to the natural frequency of oscillator. The natural frequency of the oscillator is called resonance frequency.

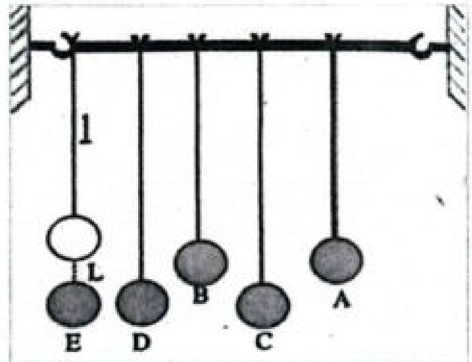


Figure 17.10: Setup to demonstrate resonance.

At resonance frequency, the efficiency of energy transfer from the driving force to the oscillator is maximum. This can be demonstrated with a simple experiment, as shown in Fig. 17.10. Here the five pendulums A, B, C, D and E are suspended vertically from the same horizontal rod. The lengths of A and B are same and equal to l, and the length of C and D are same and equal to L.

If length of pendulum E is made equal to the lengths of pendulum A and B, and E is set into vibrations in direction perpendicular to the plane of the paper. Then, after some time A and B start vibrations automatically but C and D will remain at rest. This is due to the reason that E has same length and frequency as A and B.

If length of pendulum E is made equal to the lengths of pendulum C and D, and pendulum E is set into vibrations in a direction perpendicular to the plane of the paper. Then after some time pendulum C and D start vibrating automatically but pendulum A and B will remain at rest. This is due to the reason that pendulum E has same length and hence frequency as pendulum C and D.

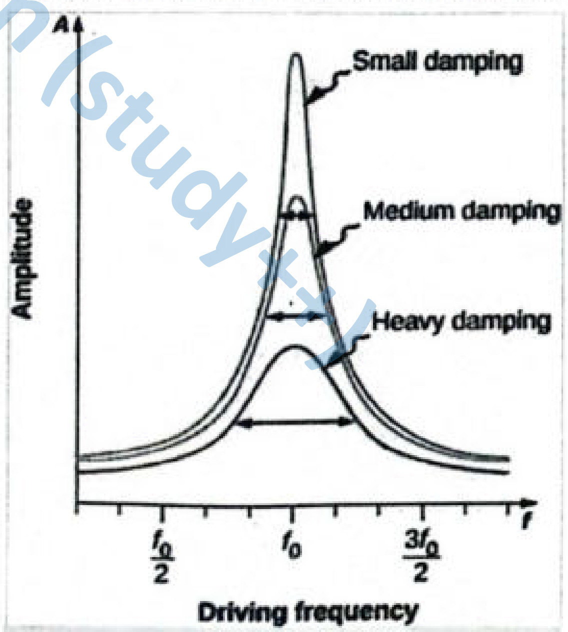


Figure 17.11: Graph of amplitude against frequency of driving force for small, medium and heavy damping.

Effect of Damping on Resonance

Damping reduces the maximum amplitude of an oscillator at its resonance frequency and broadens the resonance curve. This can be illustrated by plotting a graph of the amplitude of a damped harmonic oscillator against the frequency of driving force, as shown in Fig. 17.11. The graph shows that smaller damping results in a sharper resonance peak and larger amplitude. While heavy damping, decreases the amplitude.

In practical applications, such as car's suspension system, heavy damping is employed to minimize oscillations and ensure a smooth ride when traveling over bumps and jumps.

Applications of Resonance in Daily Life

Resonance plays very important role in various phenomena of daily life. Here we shall discuss some circumstances in which resonance is useful:

- Tuning of Radio and Electrical Resonance: When we turn the knob of a radio to tune a station. We are actually changing the natural frequency of the electrical circuit of receiver to make it equal to the transmission frequency of the radio station. When the two frequencies match with each other, then resonance occurs. In this way, energy absorption from the station is maximum and this is the only station we hear.
- Magnetic Resonance Imaging: Magnetic resonance imaging (MRI) is a widely used medical diagnostic tool in which atomic nuclei (mostly hydrogen nuclei) are made to oscillate by incoming strong radio waves (on the order of 100 MHz). When resonance occurs, maximum energy is absorbed by the nuclei. The pattern of energy absorbed can be used to produce computer enhanced photography.
- Heating/Cooking of Food in Microwave Ovens and Resonance: In a microwave oven, the microwave with a frequency similar to the natural frequency of vibration of water or fat molecules are used. When the food is placed in the oven, the water molecules in the food oscillate by absorbing maximum energy from the microwaves. Hence, it causes the food to heat up for cooking. The plastic or glass containers do not heat up in ovens, since they do not contain water molecules.
- Resonance in Guitar: When the guitarist strikes the guitar strings, a vibration is produced. The vibration transmits to the hollow wooden box. Thus, creating resonance, and the sound gets amplified.

Following are some circumstances in which resonance should be avoided:

- Resonance in Bridge: Soldiers while marching on the bridge are ordered to break their steps. This is because that the vibrations created by the rhythmic march on the bridge cause the bridge to oscillate with its natural frequency. Thus, amplitude of vibrations increases and resonance occurs that causes the bridge to collapse. One of the most studied examples of this is the collapse of Tacoma Narrows Bridge, as shown in Fig. 17.12. The strong continuous wind drove oscillations of the bridge deck that increased in amplitude until it broke apart.

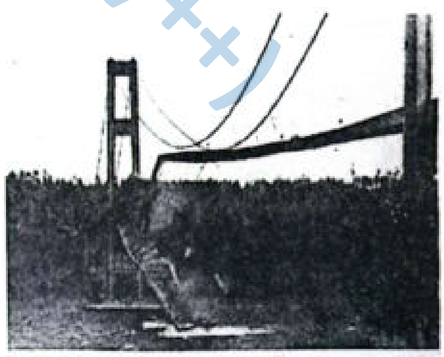


Figure 17.12: Collapse of Tacoma Narrows Bridge in 1940 during a windstorm.

- **Resonance in Airplane's Wing:** The wing is a very flexible part of the airplane. If the periodic vibrations of the wind gust have a frequency equal to the natural structural frequency of the wing, resonance occurs. At resonance, the amplified vibrations of the wing become too large. It eventually leads to its destruction. To avoid resonance, it is important to design the wing such that the natural frequency of the wing does not match the external frequencies of vibrations.

Science Tidbit

A singer can shatter a glass by loudly singing a note that matches the natural frequency of the glass. When the sound gets too loud for the glass to vibrate with large enough amplitude, it shatters the glass.

Applications of Resonance and Standing Waves

There are many applications of resonance in different devices that generate and use standing waves. Some of them are discussed here:

Rubens Tubes: A Rubens tube (also known as a standing wave flame tube) is used to demonstrate acoustic standing waves. It consists of a metal pipe with holes drilled along the top and sealed at both ends. One sealed end is attached to a small speaker or frequency generator while the other end is connecter to a supply of a flammable gas, as shown in Fig. 17.13. The pipe is filled with the gas, and the gas leaking from the perforations produces a resonant flame.

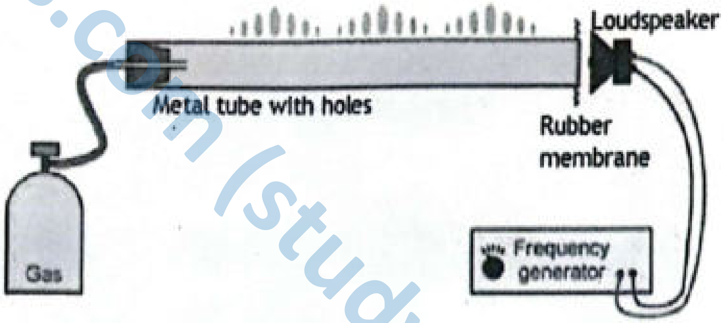
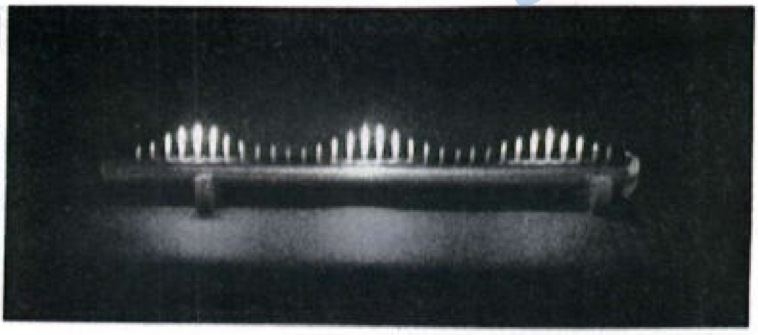


Figure 17.13: Rubens tube setup.

Science Tidbit

Rubens tube is invented by German Physicist Heinrich Rubens in 1905. It graphically shows the relationship between sound waves and sound pressure, acting as a primitive oscilloscope.



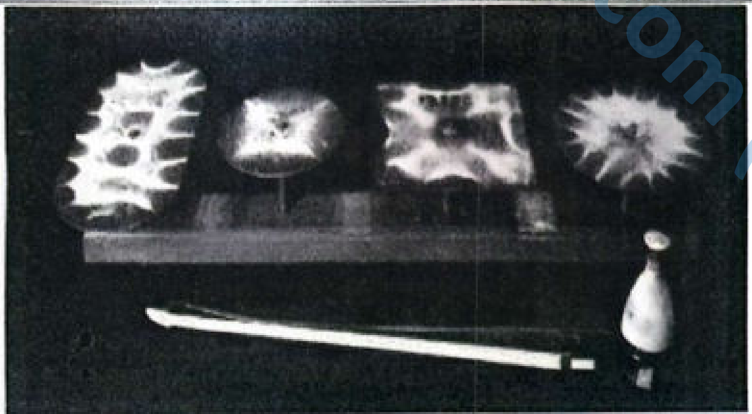
When sound waves of resonance frequency (based on the tube dimensions) is produced by frequency generator, a standing wave formed inside the tube. The standing wave creates points with oscillating (higher and lower) pressure within the tube. Less gas will escape from the perforations in the tube where pressure is low, hence the flames will be lower at these points. Large quantity of gas will escape from the perforations in the tube where pressure is high, hence the flames will be high at these points. The wavelength of the standing wave can be determined with a ruler by measuring the distance between low and high flame.

Figure 17.14: Acoustic levitation.

Acoustic Levitation: Acoustic levitation is a method for suspending matter in air against gravity by using acoustic pressure from high intensity sound waves, as shown in Fig 17.14. Acoustic levitation provides a container-less environment for some experiments. Acoustic levitation occurs when sound waves interact and create a standing wave with nodes that can trap a particle.

Chladni Plates: A chladni plate consists of a flat metal sheet (usually circular or square) mounted on a central stalk attached to a strong base. When the plate oscillates in a particular mode of vibration, the nodes and antinodes that formed produce a complex but symmetrical patterns on its surface. The positions of these nodes and antinodes can be seen by sprinkling sand upon the plates. The sand will vibrate away from the antinodes and gather at the nodes.

Science Tidbit



Scan the above QR code then search the appeared link to see the video of an interesting experiment. It is a good demonstration to show how complicated modes of vibration can be formed on the chladni plates of different shapes and dimensions.

SUMMARY

- When a body moves to and fro about its mean position, then such motion is called vibratory or oscillatory motion.
- The complete round trip of an oscillating or vibrating body about its mean position is called oscillation or vibration.

- The distance of vibrating body from the equilibrium position on either side at any instant of time is known as displacement. Its SI unit is meter.
- The maximum displacement of a vibrating body from mean position is called amplitude. Its SI unit is meter.
- The time taken to complete one oscillation or vibration is called time period of the oscillation. It is denoted by T. Its SI unit is second.
- The number of vibrations or oscillations per unit time is called frequency.
- SHM is an oscillatory motion in which acceleration of a particle is directly proportional to its displacement from the mean position and is always directed towards the mean position.
- When a body moves in a circle, its projection undergoes simple harmonic motion on the diameter of the circle.
- The angle which specifies the displacement as well as the direction of a point executing SHM is called phase of the motion.
- Energy of a body executing SHM remains conserved. The energy oscillates between K.E and P.E, but their sum remains constant.
- A body is said to be executing free oscillation when it oscillates under the influence of restoring force without any external force acting on it.
- If the external forces make the object oscillating at the frequency of applied force rather than its natural frequency, then such oscillations are called forced oscillations.
- If the amplitude of oscillations decreases under damping forces, then such oscillations are called damped oscillations.
- The damping is said to be light when the amplitude of oscillations decreases gradually with time.
- When an oscillator comes to rest without any oscillation in the shortest time under a damping force, then such damping is called critical damping.
- Resonance is the phenomena in which the amplitude of vibration of an oscillator attains maximum value when deriving frequency equals the natural frequency of oscillator.
- A Rubens tube is used for demonstrating acoustic standing waves.
- Acoustic levitation is a method for suspending matter in air against gravity by using acoustic pressure from high intensity sound waves.

FORMULA SHEET

$$f = \frac{1}{T} \qquad \omega = 2\pi f = \frac{2\pi}{T} \qquad F = -kx$$

$$\omega = \sqrt{\frac{k}{m}} \qquad T = 2\pi\sqrt{\frac{m}{k}} \qquad f = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$$

$v = \omega\sqrt{x_o^2 - x^2}$ $\quad a = -\omega^2 x$ $\quad x = x_o\cos(\omega t + \varphi)$

$T = 2\pi\sqrt{\frac{l}{g}}$ $\quad P.E = \frac{1}{2}kx^2$ $\quad K.E = \frac{1}{2}mv^2 = \frac{1}{2}k(x_o^2 - x^2)$

EXERCISE

Multiple Choice Questions

Encircle the Correct option.

1) Which of the following statements is true regarding the acceleration of the particle executing simple harmonic motion?
   A. Acceleration is zero at the mean position
   B. Acceleration is maximum at the mean position
   C. Acceleration is zero at the extreme position
   D. Acceleration is a constant
2) If your heart rate is 150 beats per minute during strenuous exercise, what is the time per beat in seconds?
   A. 150 B. 2.5 C. 0.4 D. 0.0067
3) Which of the following relationship between the acceleration 'a' and the displacement 'x' of a particle involve SHM?
   A. $a = 0.7\,x$ B. $a = -200\,x^2$ C. $a = -10\,x$ D. $a = 100\,x^3$
4) A mass-spring system undergoes simple harmonic motion has amplitude A. When the kinetic energy of the object equals twice the potential energy stored in the spring, what is the position x of the object?
   A. A B. $\frac{A}{3}$ C. $\frac{A}{\sqrt{3}}$ D. 0
5) Suppose we replace the spring in a simple harmonic oscillator with a stronger spring, having twice the spring constant. What is the ratio of the new period of oscillation to the original period?
   A. $\frac{1}{2}$ B. $\frac{1}{\sqrt{2}}$ C. 1 D. $\sqrt{2}$
6) A pinball machine uses a spring that is compressed 4.0 cm to launch a ball. If the spring constant is 13 N/m, what is the force on the ball at the moment the spring is released?
   A. 52 N B. 0.52 N C. 0.52 N m D. 5.2 N m
7) A particle is executing simple harmonic motion with period T. At time t = 0 it is at the equilibrium point. At which time is it furthest from the equilibrium point?
   A. 0.50 T B. 0.70 T C. 0.25 T D. 1.4 T
8) The acceleration of a body executing simple harmonic motion leads the velocity by what phase?
   A. 0 rad B $\pi/8$ rad C. $\pi/4$ rad D. $\pi/2$ rad

9) A simple pendulum has a period of 2.5 s. What is its period if its length is made four times larger?
A. 0.625 s B. 1.25 s C. 2.5 s D. 5 s

10) What is the frequency of a pendulum that swings at the rate of 45 cycles per minute?
A. 0.75 Hz B. 1.3 Hz C. 2700 Hz D. 60 Hz

11) A block at the end of a horizontal spring is pulled from equilibrium at x = 0 to x = A, and then released. Through what total distance does it travel in one full cycle of its motion?
A. $\frac{A}{2}$ B. A C. 2 A D. 4 A

12) A particle is vibrating simple harmonically with an acceleration of 16 cm s^{-2} when it is at a distance of 4 cm from the mean position. Its time period is:
A. 1 s B. 2.572 s C. 3.142 s D. 6.028 s

Short Questions

1) If we halve the length of a simple pendulum to its original length, what is the alteration in the period of this pendulum? What is its new frequency?
2) If the amplitude of vibration of a body executing SHM is doubled, what will happen to the maximum kinetic energy?
3) When marching soldiers are about to cross a bridge, they break steps. Why?
4) Suppose that a driving force has half frequency as compared to the frequency of an oscillator. Will it produce resonance? Similarly, if the driving frequency is twice the frequency of the oscillator, will it produce resonance?
5) Pendulum clocks are made to run at the correct rate by adjusting the pendulum's length. Suppose you move from one city to another where the acceleration due to gravity is slightly greater, taking your pendulum clock with you. Will you have to lengthen or shorten the pendulum to keep the correct time, other factors are remaining constant? Explain your answer.
6) Two mass-spring systems vibrate with simple harmonic motion. If the spring constants are equal and the mass of one system is twice that of the other, which system has a greater period?
7) Give some applications in which resonance plays an important role.
8) A simple pendulum is set into vibrations and left untouched, eventually stops, why?
9) Under what condition(s) the motion of a simple pendulum be simple harmonic motion?
10) At what position is the velocity of a particle executing simple harmonic motion a) maximum b) minimum?
11) Show that the motion of projection of a body revolving in a circle describes S.H.M.
12) sketch displacement-time graphs to illustrate light, critical and heavy damping.
13) Justify the importance of critical damping in a car suspension system.
14) Differentiate free and forced oscillations.
15) How the time period of a simple pendulum changes if mass of its bob is doubled? What change arises in time period of mass attached to a spring if same is done here?

Comprehensive Questions

1) Define and explain phase in simple harmonic motion.
2) Derive the expressions for instantaneous displacement, instantaneous velocity and acceleration of the projection of a particle moving in a circle.
3) Show that motion of a mass attached to a spring executes S.H.M.
4) Analyse the interchange between kinetic and potential energy during simple harmonic motion.
5) What is resonance? Give three of its applications in our daily life?
6) Justify some circumstances in which resonance should be avoided.
7) Derive equations for kinetic and potential energy of a body of mass m executing S.H.M.
8) Explain what is meant by damped oscillations?
9) Explain the relation between damping and sharpness of resonance.
10) Discuss the use of standing waves and resonance in (a) rubens tubes (b) chladni plates (c) acoustic levitation.
11) Analyze graphical representations of the variations of displacement, velocity and acceleration for simple harmonic motion.
12) Explain the terms light, critical and heavy damping, with the help of examples.

Numerical Problems

1) The amplitude of the motion of a mass attached to a spring is 2.48 m, while maximum speed of its mass is 4.36 m s^{-1}. What is the period of the motion? **(Ans: 3.57 s)**

2) A particle moves, whose displacement as a function of time is: $x = 3.0\cos(2t)$, where distance is measured in meter and time in second.

a) Calculate the amplitude, the frequency, the angular frequency and the period of this particle?

b) Calculate the time at which the particle reaches the midpoint (i.e., $x = 0$) and the turning point?

(Ans: (a) 3.0 m, 0.318 Hz, 2.0 rad s^{-1}, 3.14 s (b) 0.785 s, 1.57 s)

3) A particle executes simple harmonic motion, that moves back and forth along x-axis between points −0.20 m and +0.20 m. The period of the motion is 1.2 s. At the time t = 0, the particle is at +0.20 m and its velocity is zero.

a) What is the frequency of the motion and the angular frequency?

b) What is the amplitude of the motion?

c) At what time will the particle reach the point $x = 0$? At what time will the particle reach the point $x = 0.10$ m?

d) What is the speed of the particle when it is at $x = 0$? What is the speed of the particle when it reaches the point $x = 0.10$ m?

(Ans: (a) 0.83 Hz, 5.2 rad s^{-1} (b) 0.20 m (c) 0.30 s, 0.20 s (d) 1.05 m s^{-1}, 0.91 m s^{-1})

4) A mass of 8.0 kg is attached to a spring and oscillates with amplitude of 0.25 m and has a frequency of 0.60 Hz. What is the energy of the motion? (Ans: 3.6 J)

5) Calculate the period and frequency of a 3.5 m long pendulum at the following locations:

a) at Karachi, where g = 9.832 m s^{-2}.

b) at K-2, where g = 9.782 m s^{-2}.

(Ans: (a) 3.747 s, 0.2669 Hz (b) 3.757 s, 0.2662 Hz)

6) In a car engine, a piston executes SHM with amplitude of 0.41 m. The engine is running at angular frequency of 2400 rpm (251 rad s^{-1}). What is the maximum speed of the piston?

(Ans: 103 m s^{-1})

7) A ball connected to a spring executes SHM. At t = 0, its displacement is 0.50 m and its acceleration is -0.72 m s^{-2}. The phase constant for its motion is 0.84 rad. What is the ball's displacement at t = 3.4 s? (Ans: 0.15 m)

18 DIFFRACTION AND INTERFERENCE

Student Learning Outcomes (SLOs)

The student will

- Explain experiments that demonstrate two-source interference using water waves in a ripple tank, sound, light and microwaves.
- describe the conditions required if two-source interference fringes are to be observed.
- use $\Delta y = \lambda L/d$ for double-slit interference using light to solve problems.
- use $d \sin(\theta) = n\lambda$ to solve problems.
- describe the use of a diffraction grating to determine the wavelength of light [the structure and use of the spectrometer are not included].
- with the context of the electron diffraction double slit experiment, explain the below two of the many interpretations of quantum mechanics: (i) copenhagen interpretation (ii) many worlds interpretation.

Have you ever marveled at the vibrant colors of a rainbow, or wondered why sound waves can sometimes cancel each other out, creating silence? These phenomena are a testament to two fascinating wave properties: interference and diffraction.

Interference occurs when two or more waves traveling through the same medium interact and superimpose on each other. This interaction can lead to surprising results. Depending on how the waves are aligned, they can either strengthen each other (constructive interference), creating brighter light or louder sound, or cancel each other out (destructive interference), resulting in darkness or quieter sound. Understanding interference allows us to explain everyday occurrences like the beating sound between slightly off-tune instruments and the formation of colorful bands in soap bubbles. Interference isn't limited to light or sound waves. It's a fundamental property observed in all types of waves, from water ripples in a tank to the intricate quantum waves that make up matter. So, the next time you witness a dazzling rainbow or hear a mesmerizing musical chord, remember, it's all a captivating play of interference.

Diffraction, on the other hand, describes the bending of a wave around the edges of an obstacle or through a narrow slit. Unlike reflection, where a wave bounces off a surface, diffraction causes the wave to spread out and travel into regions beyond the obstacle's shadow. This phenomenon is responsible for the dazzling colors we see in rainbows, where sunlight diffracts through water droplets. Diffraction also plays a crucial role in various technologies like x-ray crystallography, which helps us understand the structure of materials, and optical instruments like diffraction gratings, which separate light into its constituent colors.

Interference and diffraction are not isolated concepts; they are intimately connected. Both phenomena arise from the wave nature of light, sound, and other waves. While interference describes how waves interact and superimpose, diffraction showcases how waves can bend and spread out when encountering obstacles. Understanding these principles unveils a deeper understanding of the behavior of waves and their remarkable effects in the world around us.

18.1 INTERFERENCE

Two-source interference, also known as double-slit interference, is a phenomenon in which waves from two coherent sources overlap and interfere with each other, creating a pattern of alternating constructive and destructive interference fringes. **Interference of two waves may lead to a resultant wave of either a larger or a smaller displacement.** This phenomenon can be demonstrated using various types of waves, including water waves in a ripple tank (see Fig. 18.1), sound waves, light waves, and microwaves.

Figure 18.1: Interference of water waves.

Conditions for Observing Interference Fringes:

To observe interference fringes in a two-source setup, the following conditions must be met:

1. **Coherent Sources:** The two sources emitting the waves must be coherent, meaning they have a constant phase relationship and emit waves with the same frequency and wavelength.
2. **Narrow Slits:** The slits must be narrow and close together compared to the wavelength of the waves. This ensures that the waves from each slit overlap and interfere with each other.
3. **Monochromatic Waves:** The waves emitted by the sources should ideally have a single wavelength (monochromatic) to produce a clear interference pattern.
4. **Stable Environment:** The experimental setup must be free from disturbances such as vibrations or air currents, which could disrupt the wave patterns and interfere with the observation of fringes.

By satisfying these conditions, two-source interference fringes can be observed across various wave types, demonstrating the wave nature of light, sound, and other phenomena.

18.1.1 Interference of Water Waves

Interference can be demonstrated in a ripple tank by using two-point sources. In a ripple tank experiment (see Fig. 18.2), two coherent sources are created by generating ripples in the water from two separate point sources. The tank is illuminated from above to enhance visibility. When the waves from both sources meet, they interfere with each other. This interference creates regions of constructive interference, where waves reinforce each other and produce larger waves, and regions of destructive interference, where waves cancel each other out. This results in a pattern of alternating crest and trough on the surface of the water.

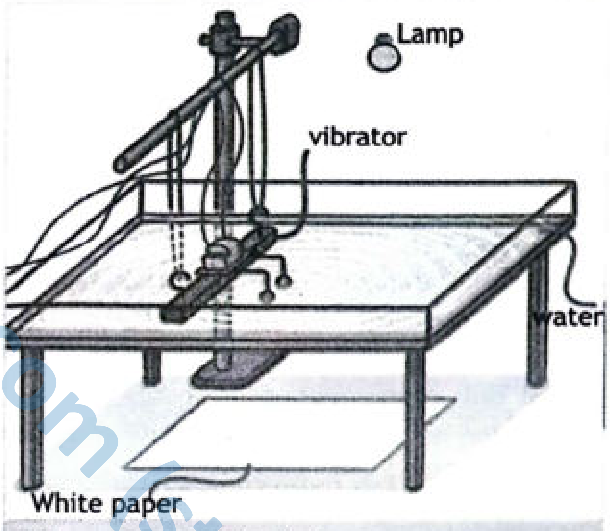


Figure 18.2: Ripple tank.

Interference of two circular waves is shown in Fig. 18.3. If two waves arrive in **phase** (their crests or trough arrive at exactly the same time), they will interfere **constructively**. A resultant wave will be produced, which has crests much higher than the two individual waves, and troughs much deeper.

If the two waves arrive in **anti-phase** (with a phase difference of π radians or 180°), the peaks of one wave arrive at the same time as the troughs from the other, and they will interfere **destructively**. The resultant wave will have smaller amplitude. This phase difference may be produced by allowing the two sets of waves to travel different distances. This difference in distance of travel is called the **path difference**.

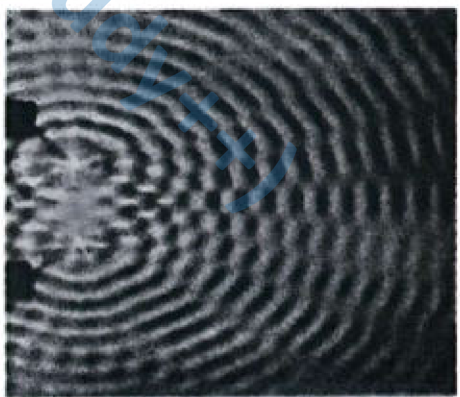


Figure 18.3: Shadow of Interference pattern of two circular waves obtained on white paper under the ripple tank.

18.1.2 Interference of Sound Waves

Similar to water waves, sound waves can also exhibit interference. Two speakers emitting coherent sound waves are placed facing a screen, as shown in Fig 18.4. The sound waves from both sources overlap, creating regions of constructive and destructive interference. This interference pattern can be visualized by using a microphone or a detector to measure the intensity of sound at various points on the screen.

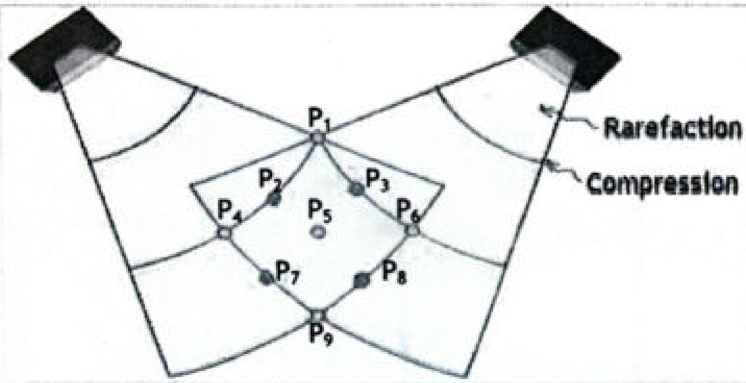


Figure 18.4: Interference of sound wave from two speakers.

Consider sound waves from two loudspeakers connected to the same signal generator and amplifier, producing notes of the same frequency. Sound waves being longitudinal waves, consists of **compressions and rarefactions.** At points such as P_1, P_4, P_5, P_6 and P_9 constructive interference occurs because here compressions or rarefactions align and the sound appears louder. Destructive interference occurs at points such as P_2, P_3, P_7 and P_8, when compression align with a rarefaction and vice versa, resulting in a quieter sound. This principle is used in noise-cancelling headphones (you have studied in grade XI).

18.1.3 Interference of Light Waves

In a double-slit experiment with light waves, a laser beam is typically used as the coherent light source. The light beam is directed through a barrier with two narrow slits (Figure 18.5), creating two coherent sources of light waves. A screen placed behind the slits displays the interference pattern. The pattern consists of alternating bright and dark fringes, which can be observed directly or captured using a camera. Here, light of a single wavelength passes through a pair of vertical slits and produces a diffraction pattern on the screen—numerous vertical bright and dark lines that are spread out horizontally. Without diffraction and interference, the light would simply make two lines on the screen.

Figure 18.5: Set up of Young's double-slit experiment.

For Your Information

In the early 1800s, the nature of light was still a topic of debate, with Christian Huygens proposing wave-like behavior, Isaac Newton offering an alternative explanation for color and observable effects.

Thomas Young's Double Slit Experiment

In 1801, the English physicist and physician Thomas Young conducted an experiment to demonstrate the wave nature of light. Young's experiment involves a pair of closely spaced vertical slits (the double slit) through which light passes. Initially, Young allowed sunlight (which contains multiple wavelengths) to pass through a single slit, making the light partially coherent (waves with a definite phase relationship). He then directed this partially coherent light through the double slit. The light passes through two narrow slits, producing semicircular waves that overlap and interfere on a screen placed behind the slits.

As the two slits (S_1 and S_2) are narrow, so the light spreads out (diffracts) from each slit, as shown in Fig. 18.6 (a). Two slits provide **two coherent light sources** that interfere and interference fringes will be obtained in the form of bright (constructive interference) and dark (destructive interference) pattern on the screen, as shown in the Fig. 18.6 (b).

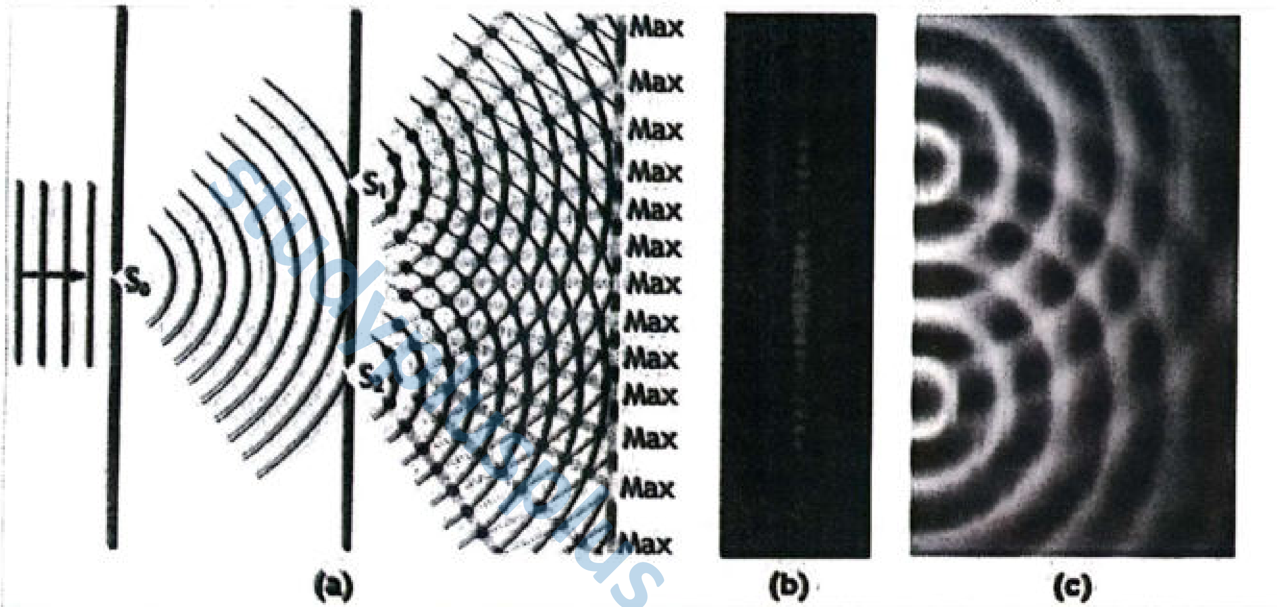


Figure 18.6: Young's double-slit experiment.

Figure 18.6 (c) shows that the double-slit interference pattern for water waves is nearly identical to that for light. Young's use of sunlight in his double-slit experiment made the effect easier to observe. This is because sunlight is a mixture of different wavelengths (colors), and each wavelength formed its own interference pattern on the screen. To observe clear pattern, monochromatic (single-wavelength) light is often used.

Young's experiment confirmed that light exhibits **wave-like behavior**, producing an interference pattern.

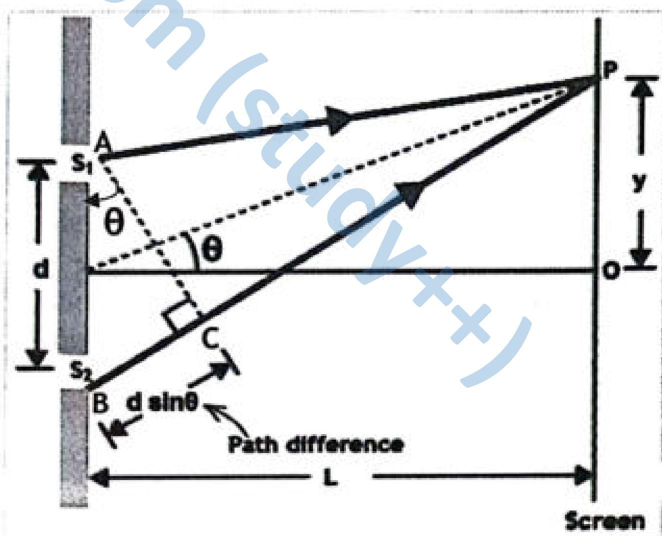


Figure 18.7: Schematic diagram of Young's double slit experiment.

Path Difference: The wave from slit S_2 has to travel slightly larger than that from S_1 to reach the point P on the screen, as shown in Fig. 18.7. The difference in this distance is the path difference. When two waves interfere, the resultant wave depends on the phase difference between the two waves, which is proportional to the path difference between the waves which can be expressed in terms of the wavelength 'λ' of the wave.

If 'd' is the separation distance between the slits, 'L' is distance between the slits and the screen, and 'y' is the distance from the central maximum to the point 'P' on the screen. Then

from the ΔABC: $\sin\theta = \frac{BC}{AB}$ or $BC = AB\sin\theta$

Hence, the path difference BC between the waves reaching a point on the screen from the two slits is:

$$\text{Path difference} = d\sin\theta \qquad \text{—— (18.1)}$$

Constructive Interference: Light waves show constructive interference when their crests (peaks) overlap, leading to a bright maximum on the screen. This occurs when the path difference is an integer multiple of the wavelength, i.e.,

$$\text{Path difference} = m\lambda$$

or

$$d\sin\theta = m\lambda \qquad \text{—— (18.2)}$$

Where m is an integer ($0, \pm1, \pm2, \pm3, \ldots$) representing the order of the maxima (central maxima is 0^{th} order). When $m = \pm1$, then two bright fringes are obtained one above and one below the central point O, which are called 1^{st} order bright fringes, and so on.

Destructive Interference: Destructive interference happens when the crests of one wave cancel out the troughs of another, resulting in a dark fringe called minima. This occurs when the path difference is an odd multiple of half the wavelength, i.e.,

$$\text{Path difference} = \left(m + \frac{1}{2}\right)\lambda$$

or

$$d\sin\theta = \left(m + \frac{1}{2}\right)\lambda \qquad \text{—— (18.3)}$$

Where 'm' is still an integer ($0, \pm1, \pm2, \pm3, \ldots$) representing the order of the minima (first minima occurs at $m = 0$, and is called 1st order minima).

Positions of Maxima and Minima: Let P is the position of m^{th} order bright fringe on screen

then $d\sin\theta = m\lambda$

If 'θ' is very small then $\sin\theta = \tan\theta$

so, $d\tan\theta = m\lambda$

As, $\tan\theta = \frac{y}{L}$, so $d\frac{y}{L} = m\lambda$

or

$$y = m\frac{\lambda L}{d} \qquad \text{—— (18.4)}$$

Similarly, for the position of m^{th} order dark fringe on screen, we can get:

or

$$y = \left(m + \frac{1}{2}\right)\frac{\lambda L}{d} \qquad \text{—— (18.5)}$$

These equations allow you to calculate the positions of maxima and minima on the screen based on the known values of d, L, and λ. Note that the central maxima ($m = 0$) will always be at $y = 0$.

Fringe Spacing: The distance between two consecutive bright or dark fringes is called fringe spacing. For bright fringes, the fringe spacing is:

$$\Delta y = y_{m+1} - y_m$$

or
$$\Delta y = (m+1)\frac{\lambda L}{d} - m\frac{\lambda L}{d}$$

or
$$\Delta y = \frac{\lambda L}{d} \qquad \text{\_\_\_\_\_\_ (18.6)}$$

Eq. 18.6 can be used to find fringe spacing if values of d, L, and λ are known. Similarly, for dark fringes, the fringe spacing is:

$$\Delta y = y_{m+1} - y_m$$

or
$$\Delta y = \left(m+1+\frac{1}{2}\right)\frac{\lambda L}{d} - \left(m+\frac{1}{2}\right)\frac{\lambda L}{d} = \frac{\lambda L}{d}$$

So, fringe spacing between the bright and dark fringe is same.

18.1.4 Interference of Microwaves

Microwaves can also be used to demonstrate two-source interference. Two-source interference experiments using microwaves are a fascinating demonstration of wave physics. These experiments typically involve two coherent microwave sources, i.e. microwaves of the same frequency and phase. When these microwaves overlap, they create an interference pattern characterized by alternating regions of constructive and destructive interference.

To observe this pattern, a detector is moved through the various points of interference, registering the intensity of the microwaves. At points of constructive interference, the detector records a higher intensity, while at points of destructive interference, the intensity is significantly lower or even zero. This pattern can be visualized on a screen or through a recording device connected to the detector.

Experimental setup: One common setup for such an experiment includes a microwave transmitter and a receiver, as shown in Fig 18.8. The transmitter emits microwaves towards

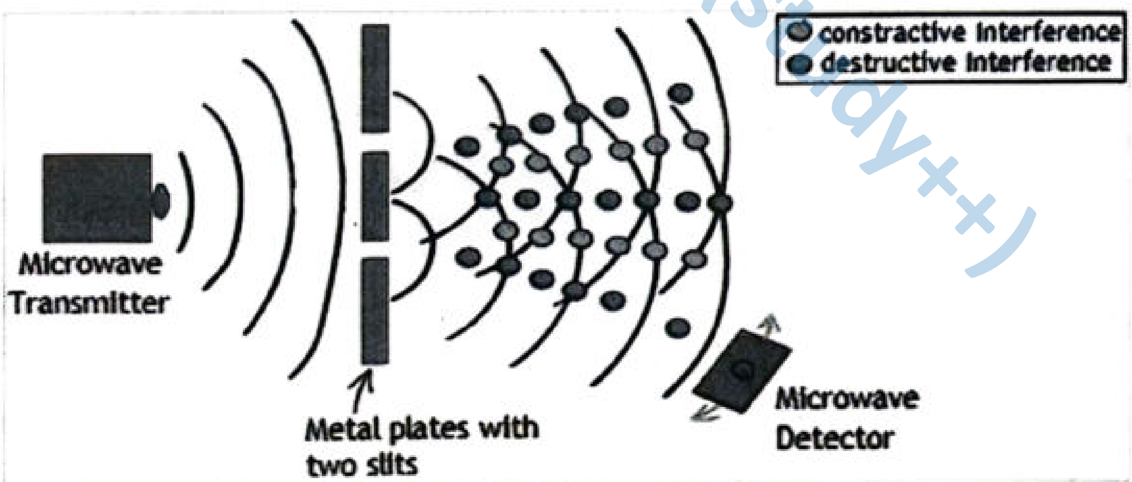


Figure 18.8: Microwave interference experiment.

two slits or openings, which then act as the new sources of waves. As the waves emanate from these slits, they overlap and interfere with each other. The receiver, which is placed at variable distances and angles from the slits, measures the intensity of the resulting microwaves.

The data collected from these experiments can be used to calculate the wavelength of the microwaves, as the distance between the points of maximum or minimum intensity is related to the wavelength and the geometry of the setup. This is a practical application of the principles of wave superposition and interference.

These experiments not only demonstrate the wave nature of microwaves but also have practical implications. For instance, understanding microwave interference is crucial in designing microwave communication systems to avoid signal loss due to destructive interference. Moreover, the principles observed in microwave interference are analogous to those in other wave phenomena, including sound waves, water waves, and even quantum mechanics, where particles exhibit wave-like behavior. Thus, two-source interference experiments with microwaves provide a valuable insight into the broader wave phenomena that govern various aspects of the physical world.

Example 18.1: The fringe spacing between the central maxima and 1st minima is 2 mm. If a light of 500 mm is used then find the separation between the slits. The distance between the slit and screen is 1 m.

Given: Distance between central maximum and 1st minimum $y_1 = 2.0$ mm $= 0.002$ m

Wavelength of light $\lambda = 500$ nm $= 5.00 \times 10^{-7}$ m

Distance between the slits and screen $L = 1.0$ m

To Find: Slit separation $= d = ?$

Solution: We can use the relationship between fringe spacing and slit separation, as:

$$y = m\,\frac{\lambda L}{d}$$

For 1st minima, put m = 1, so we get: $$y = \frac{\lambda L}{d}$$

Rearranging the equation to solve for d, we get: $$d = \frac{\lambda L}{y}$$

or $$d = \frac{(5.00 \times 10^{-7})(1.0)}{0.002} = 2.50 \times 10^{-4}\ \text{m}$$

Therefore, the separation between the slits is about 250 micrometers.

Assignment 18.1

Light from a He-Ne laser pass through two slits separated by 0.0100 mm. The third bright line on a screen is formed at an angle of 10.95° relative to the incident beam. What is the wavelength of the light?

18.2 DIFFRACTION GRATING

A diffraction grating is an optical device consisting of a transparent material, such as glass or plastic, having a large number of equally spaced parallel slits or grooves etched or ruled over it, as shown in Fig. 18.9 (a). These slits are closely spaced together and act as individual sources of secondary waves when illuminated by incident light.

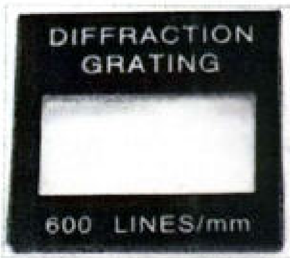


Figure 18.9 (a): Diffraction Grating

The distance between two adjacent slits is called grating element.

Grating element is represented by d, and can be calculated by using the following formula:

$$d = \frac{\text{unit length of grating}}{\text{total number of lines ruled on it}}$$

Grating element is typically of the order of the wavelength of light or smaller. The number of slits per unit length, denoted by "N," determines the resolving power of the diffraction grating.

18.2.1 Diffraction of Light through Diffraction Grating

The principle behind using a diffraction grating, to determine the wavelength of light, is based on the interference pattern produced when light passes through the grating. When monochromatic light (light of a single wavelength) illuminates a diffraction grating, the light waves passing through the slits interfere constructively and destructively, creating a pattern of bright and dark fringes, as shown in Fig. 18.9 (b).

The angle at which the bright fringes (maxima) occur depends on the wavelength of light and the spacing between the slits in the grating. This relationship is described by the equation:

$$d \sin\theta = m\lambda$$

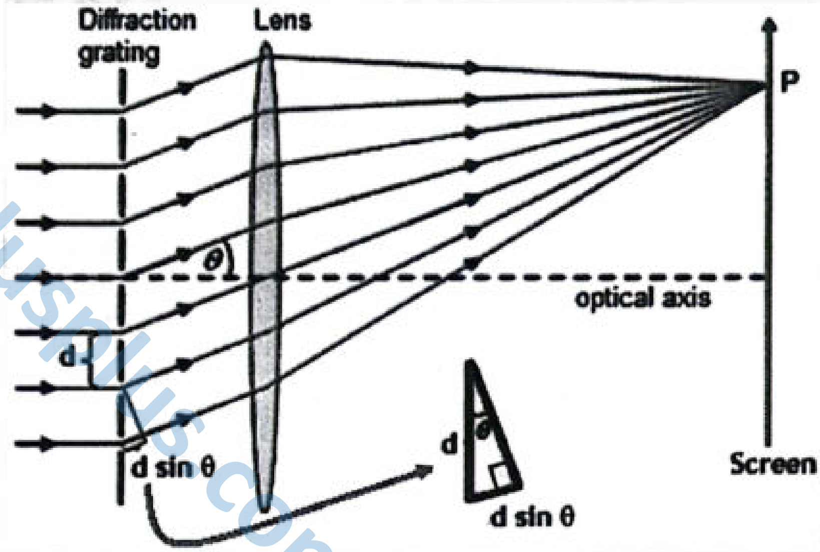


Figure 18.9 (b): Diffraction of light through diffraction grating.

By measuring the angle of diffraction (θ) for a specific order of the bright fringe (m) and knowing the spacing between the slits (d), one can determine the wavelength of light (λ).

By following this procedure, the wavelength of light can be accurately determined using a diffraction grating, making it a valuable tool in spectroscopy and various scientific applications.

An idealized graphs of the intensity of light passing through a double slit and a diffraction grating for monochromatic light is shown in Fig.18.10. Maxima can be produced at the same angles, but those for the diffraction grating are narrower, and hence sharper. The maxima become narrower and the regions between become darker as the number of slits is increased.

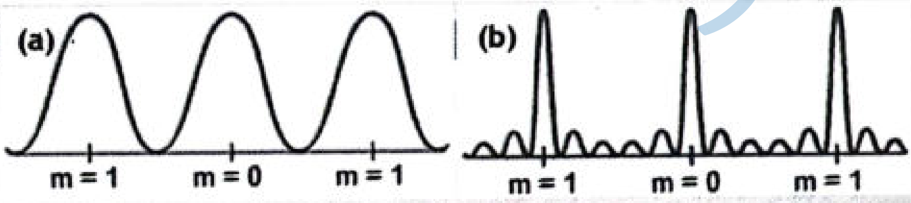


Figure 18.10: Graphs of the intensity of light passing through (a) double slit (b) diffraction grating.

18.2.2 Applications

1. **Wavelength Measurement:** Diffraction grating is used to determine the wavelength of light.

2. **Atomic Spectra Analysis:** Diffraction gratings help analyzing the wavelengths emitted by atoms and molecules.

3. **Biomedical Imaging:** Gratings can be used to selectively analyze specific wavelengths for disease detection in biopsy samples.

4. **Spectroscopy:** Gratings disperse light into its constituent wavelengths for detailed analysis.

Australian opal and the butterfly wings have rows of reflectors that act like diffraction gratings, reflecting different colors at different angles, as shown in Fig. 18.11.

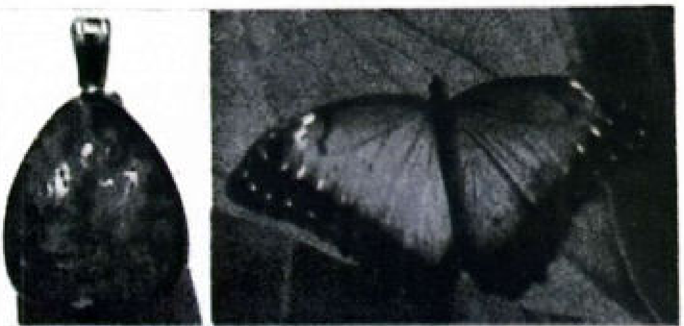


Figure 18.11: Australian opal and the butterfly wings.

Example 18.2: Diffracted light observed at an angle of 30° in the 2nd order ($m = 2$) with grating constant 2.0 μm. Find the wavelength of the diffracted light.

Given: $\theta = 30°$ $m = 2$

$d = 2.0\ \mu m$

To Find: $\lambda = ?$

Solution: We can use the grating equation, as:

$$m\lambda = d\sin\theta$$

Rearranging the equation to solve for λ, as:

$$\lambda = \frac{d\sin\theta}{m}$$

Putting values, we get:

$$\lambda = \frac{(2\times10^{-6})(\sin 30^\circ)}{2} = 0.50\ \mu m$$

Therefore, the wavelength of the diffracted light is about 0.50 micrometers.

Assignment 18.2

Visible light of wavelength 550 nm falls on a single slit and produces its second diffraction minimum at an angle of 45° relative to the incident direction of the light. What is the width of the slit?

For Your Information

Have you ever seen the grooves on a CD or DVD? Grooves are there, but they are extremely narrow—1,600 in a millimeter. Because the width of the grooves is similar to wavelengths of visible light, they form a diffraction grating. That is why you see rainbows on a CD. The colors are attractive, but they are incidental to the functions of storing and retrieving audio and other data.

18.3 Electron Double Slit Experiment and Interpretations of Quantum Mechanics

The Electron Double Slit Experiment is a classic physics experiment that demonstrates the principles of wave-particle duality and the nature of reality at the quantum level.

Experimental setup of electron double slit experiment is shown in Fig. 18.12. It consists of:

- Electron gun: Electrons are emitted from an electron gun.
- Double slits: The electrons pass through two parallel slits, creating a pattern on a screen behind the slits.
- Screen: The electrons hit the screen, creating a visible pattern.

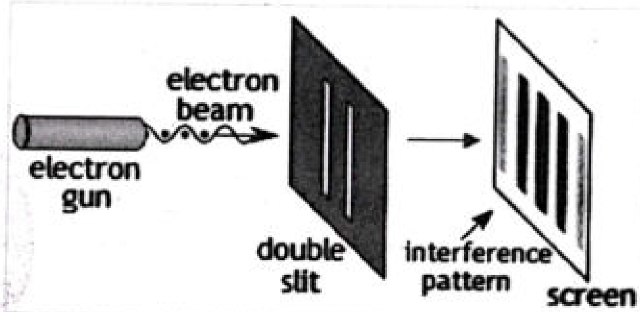


Figure 18.12: Experimental setup of electron double slit experiment.

If electrons were considered classical particles, we would expect two distinct patterns on the screen, corresponding to the two slits. However, the actual outcome is an interference pattern, similar to what we would expect from waves. This suggests that electrons exhibit wave-like behavior.

The Electron Double Slit Experiment has far-reaching implications for our understanding of reality, including:

- Quantum mechanics: It laid the foundation for quantum mechanics, which describes the behavior of particles at the atomic and subatomic level.
- Limits of classical physics: It showed that classical physics is insufficient to describe the behavior of particles at the quantum level.
- Philosophical implications: It raises questions about the nature of reality, observation, and the role of the observer in shaping reality.

The electron double-slit experiment has led to several interpretations of quantum mechanics. The two prominent interpretations in this context are Copenhagen interpretation and many-worlds interpretation.

i) Copenhagen Interpretation

The Copenhagen Interpretation (CI) is a widely-held interpretation of quantum mechanics that was formulated by Niels Bohr and Werner Heisenberg. It states that a quantum system, like an electron before measurement, doesn't possess a definite position or momentum. Instead, it exists in a superposition of all possible states simultaneously. Here's how CI explains the Electron Double Slit Experiment:

- Before detection, the electron is considered to be in a superposition of states, passing through both slits simultaneously (wave-like behavior).

- The act of measurement (detecting the electron on the screen) forces the wave function to collapse into a definite position, explaining why we see a single electron hit at a specific point on the screen (particle-like behavior).
- The interference pattern observed on the screen arises from the probability distribution of where the electron might be detected. Regions with higher probability density correspond to bright fringes, while regions with lower probability density correspond to dark fringes.

Criticisms:

- The "collapse of the wave function" due to measurement seems like an arbitrary break from the deterministic nature of physics.
- It doesn't explain what happens to the "other possibilities" in the superposition before measurement.

The Copenhagen Interpretation remains a widely-held and influential interpretation of quantum mechanics, but its implications and limitations continue to be debated among physicists and philosophers.

ii) Many-Worlds Interpretation

The many-worlds interpretation, proposed by Hugh Everett III in 1957, presents a radically different view of quantum mechanics. It proposes that every interaction in the quantum realm splits the universe into multiple realities (worlds), one for each possible outcome. Each branch of reality carries forward one possibility from the superposition. Here's how many-worlds interpretation explains the Electron Double Slit Experiment:

- The electron initially exists in a superposition, going through both slits in all the newly created worlds.
- In each world, the electron interacts with the detector screen and leaves its mark (particle-like behavior) at a specific location.
- The interference pattern observed on the screen is a reflection of the combined effect of electrons hitting the screen in all the parallel universes. We only experience one world (the one where the electron interacted with our detector), but the interference pattern signifies the existence of the other possibilities.

Criticisms:

- The concept of multiple universes is difficult to test experimentally.
- It doesn't provide a clear explanation of the transition from superposition to a definite state upon measurement.

Both the Copenhagen and Many-Worlds interpretations are attempts to explain the puzzling behavior of quantum systems at the microscopic level. Neither interpretation is universally accepted, and physicists continue to debate their implications. While the experiment demonstrates the wave-particle duality, the underlying reality behind it remains an ongoing exploration in quantum mechanics.

SUMMARY

- **Interference:** The interaction of two or more waves propagating through the same medium, resulting in a superposition effect that can lead to constructive or destructive interference.
- **Constructive Interference:** When the crests (peaks) of two or more waves overlap, they reinforce each other, producing a resultant wave with a higher intensity (brighter light, louder sound) compared to the individual waves.
- **Destructive Interference:** When the crest of one wave coincides with the trough (valley) of another wave, they cancel each other out partially or completely. This results in a weaker resultant wave with a lower intensity (darkness, quieter sound) compared to the individual waves.
- **Superposition Principle:** A fundamental principle in wave physics stating that the resultant wave at any point in space is the sum of the individual waves present at that point. The individual waves can add up constructively or destructively depending on their relative phases (alignment).
- **Coherent Sources:** Waves that have the same frequency and a constant phase relationship are considered coherent. This means the crests and troughs of the waves occur at the same time, allowing for predictable interference patterns.
- **Wavelength (λ):** The distance between two consecutive crests (or troughs) of a wave.
- **Frequency (f):** The number of wave cycles that pass a point in a given unit of time (usually measured in Hertz or Hz).
- **Diffraction:** The bending of a wave around the edges of an obstacle or through a narrow slit. Unlike reflection, where the wave bounces off a surface, diffraction causes the wave to spread out and travel into regions beyond the geometrical shadow of the obstacle.
- **Wave front:** A surface connecting points in a wave that are in the same phase (at the same point in their cycle). In diffraction, the wave front is altered as it interacts with the diffracting object.
- **Slit Width:** The width of the opening through which the wave passes in diffraction. The smaller the slit width compared to the wavelength, the more pronounced the diffraction effect will be.
- **Diffraction Grating:** A periodic structure with many closely spaced parallel slits or grooves. When light or other waves pass through a diffraction grating, they diffract and create a specific pattern of diffracted beams due to constructive and destructive interference.
- **Diffraction Pattern:** The spatial distribution of intensity observed after a wave diffracts around an obstacle or through a slit. This pattern typically consists of alternating bright and dark bands due to constructive and destructive interference of the diffracted waves.
- **Grating Constant (d):** The distance between the centers of two adjacent slits (or grooves) in a diffraction grating. The grating constant plays a crucial role in determining the angles at which the diffracted beams emerge.
- **Copenhagen interpretation** emphasizes indeterminism, complementarity, and the role of measurement, while the **many-worlds interpretation** posits a multitude of parallel universes where all quantum outcomes coexist.

Formula Sheet

$d \sin\theta = m\lambda$ $\quad$ $d \sin\theta = \left(m + \frac{1}{2}\right)\lambda$ $\quad$ $y = m\frac{\lambda L}{d}$

$y = \left(m + \frac{1}{2}\right)\frac{\lambda L}{d}$ $\quad$ $\Delta y = \frac{\lambda L}{d}$ $\quad$ $d = \frac{\text{unit length of grating}}{\text{total number of lines ruled on it}}$

EXERCISE

Multiple Choice Questions

Encircle the Correct option.

1) Rainbows are formed due to:
   A. Reflection of sunlight by water droplets B. Refraction of sunlight by water droplets
   C. Interference of sunlight by water droplets C. Diffraction of sunlight by water droplets
2) In the double-slit experiment, a pattern of bright and dark bands is observed on the screen. This pattern is evidence of:
   A. Reflection of light B. Refraction of light
   C. Wave nature of light D. All of the above
3) Monochromatic light from a laser passes through two slits separated by 0.00500mm. The third bright line on a screen is formed at an angle of 18.0° relative to the incident beam. What is the wavelength of the light?
   A. 51.5 nm B. 77.3 nm C. 515 nm D. 773 nm
4) What is the width of a single slit through which 610-nm orange light passes to form a first diffraction minimum at an angle of 30°?
   A. 0.863 µm B. 0.704 µm C. 0.610 µm D. 1.22 µm
5) Two slits are separated by a distance of 3500 nm. If light with a wavelength of 500 nm passes through the slits and produces an interference pattern, the m = ________ order minimum appears at an angle of 30°.
   A. 0 B. 1 C. 2 D. 3
6) What is a diffraction grating?
   A. A single slit that produces a diffraction pattern.
   B. A double slit that produces an interference pattern.
   C. A periodic arrangement of slits or lines that produces a diffraction pattern.
   D. A random arrangement of slits or lines that produces a diffraction pattern.
7) When light travels through a narrow slit, it bends slightly around the edges. This phenomenon is called:
   A. Reflection B. Refraction C. Diffraction D. Dispersion
8) A diffraction grating separates white light into its constituent colors because:
   A. It absorbs certain colors. B. It reflects different colors at different angles.
   C. It diffracts different colors at different angles.
   D. It refracts different colors at different angles.

9) Light passing through double slits creates a diffraction pattern. How would the spacing of the bands in the pattern change if the slits were closer together?
A. The bands would be closer together. B. The bands would spread farther apart.
C. The bands would remain stationary. D. The bands would fade and eventually disappear.

10) What is diffraction of light?
A. The bending of light around an obstacle. B. The reflection of light from a surface.
C. The refraction of light through a medium. D. The interference of light waves.

11) The tip of a needle does not give a sharp image. It is due to
A. Polarization B. Interference C. Diffraction D. Refraction

12) What is the pattern formed on a screen when light passes through two parallel slits?
A. A bright central maximum with alternating dark and bright fringes.
B. A dark central minimum with alternating bright and dark fringes.
C. A uniform intensity pattern.
D. A random pattern.

13) What is the wavelength of light falling on double slits separated by 2.0 μm if the third-order maximum is at an angle of 60°?
A. 667nm B. 471nm C. 333nm D. 577nm

14) What is the longest wavelength of light passing through a single slit of width 1.20 μm for which there is a first-order minimum?
A. 1.04 μm B. 0.849 μm C. 0.600 μm D. 2.40 μm

15) What is the distance between lines on a diffraction grating that produces a second-order maximum for 760-nm red light at an angle of 60°?
A. 2.28×10^4 nm B. 3.29×10^2 nm C. 2.53×10^1 nm D. 1.76×10^3 nm

Short Questions

1) How is an interference pattern formed by a diffraction grating different from the pattern formed by a double slit?
2) A beam of light always spreads out. Why can a beam not be produced with parallel rays to prevent spreading?
3) In the sunlight, the shadow of a building has fuzzy edges even if the building does not. Is this a refraction effect? Explain.
4) A laser pointer emits a coherent beam of parallel light rays. Does the light from such a source spread out at all? Explain.
5) A beam of light passes through a single slit to create a diffraction pattern. How will the spacing of the bands in the pattern change if the width of the slit is increased?
6) Describe a diffraction grating and the interference pattern it produces.
7) Suppose a monochromatic light falls on a diffraction grating. What happens to the interference pattern if the same light falls on a grating that has more lines per centimeter.
8) What is the significance of the equation $d\sin(\theta) = m\lambda$ in the context of (a) diffraction gratings (b) Young's double-slit experiment?
9) What is the effect of following aspects of diffraction grating on the resulting interference pattern: (a) number of slits (b) width of the slits?

10) Describe the conditions necessary for sustained interference patterns to be observed in Young's double-slit experiment.

Comprehensive Questions

1) Design an experiment using a ripple tank to demonstrate the principles of interference with water waves. Describe what you expect to observe when two wave sources are introduced.

2) Briefly explain why we see a spectrum of colors (red, orange, yellow, green, blue, indigo, violet) in a rainbow. Is it due to reflection, refraction, or interference of light?

3) Describe two examples of how interference of light waves plays a role in natural phenomena you can observe in everyday life (excluding rainbows).

4) Sometimes, when two tuning forks with slightly different frequencies are struck close together, you hear a wavering sound instead of a clear tone. Explain what causes this "beat" phenomenon, and how is it related to the interference of sound waves?

5) Both CDs and DVDs use lasers to store information, but DVDs hold much more data. How might the principles of diffraction be used in DVD design to achieve this higher storage capacity?

6) Describe a diffraction grating and its key features like grating constant and slit width. How does a diffraction grating separate white light into its constituent colors?

7) Scientists use X-ray diffraction to determine the structure of crystals. Briefly explain how the diffraction pattern of X-rays helps reveal the arrangement of atoms within a crystal lattice.

Numerical Problems

1) The light with wavelength (λ) 633 nm is used at a distance of 2 m from the screen. The separation between the slits is 10 μm. Find the distance between adjacent maxima (fringe width) for the first ($m = 1$) and second ($m = 2$) order of interference?

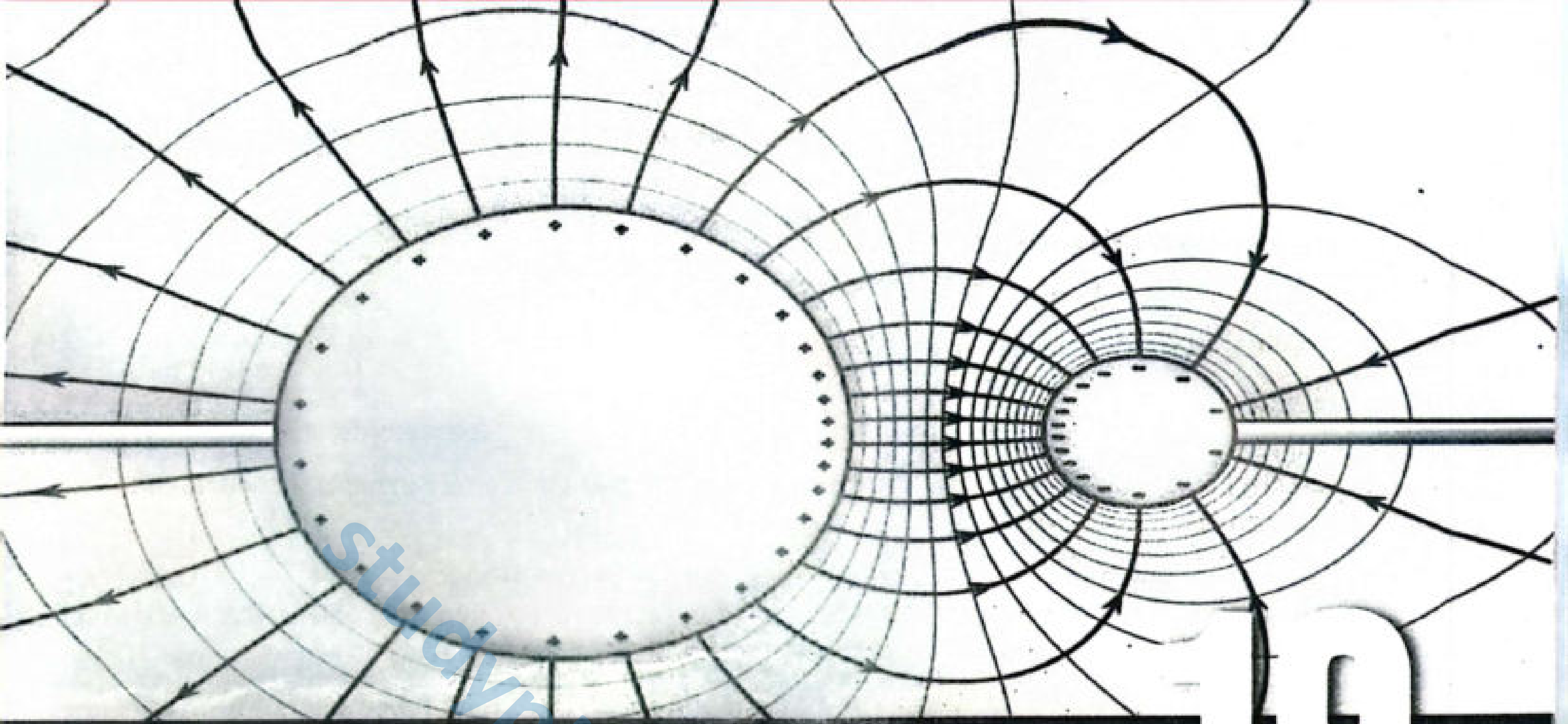
(Ans: 0.127 mm, fringe width for first order, 0.063 mm, fringe width for second order**)**

2) Find the distance from the central maximum to the third minimum on the screen, if the light of wavelength 400 nm is used. The distance between the slits and screen is 0.5 m and slit separation is 5 μm? **(Ans:** 0.024 mm**)**

3) The distance between the slits and screen is 1m, fringe spacing on screen is 1 mm. Find the slit separation when light wavelength is 500 nm. **(Ans: 0.05 mm)**

4) Find the minimum angle for non-zero diffracted light, having slit spacing 2.0 μm with 400 nm light wavelength. (Ans: $\theta_{min} = 11.54^\circ$ (minimum angle for first minimum))

5) The deviation of second order diffracted image formed by an optical grating having 5000 lines per centimeter is 32^0. Calculate the wavelength of light used. **(Ans: 5.3×10^{-5} cm)**



19 ELECTRIC POTENTIAL AND CAPACITOR

Student Learning Outcomes (SLOs)

The student will

- define and calculate electric potential [At a point as the work done per unit positive charge in bringing a small test charge from infinity to the point. Use $V = \frac{q}{4\pi\epsilon_o r}$ for the electric potential in the field due to a point charge].
- use the fact that the electric field at a point is equal to the negative of potential gradient at that point.
- state how the concept of electric potential leads to the electric potential energy of two point charges and use $E_p = \frac{qQ}{4\pi\epsilon_o r}$
- define and calculate capacitance [as applied to both isolated spherical conductors and to parallel plate capacitors].
- Derive and apply formulae for the combined capacitance of capacitors in series and in parallel.
- use the capacitance formula for capacitors in series and in parallel.
- determine the electric potential energy stored in a capacitor from the area under the potential charge graph [Use $W = 1/2QV = 1/2CV^2$ to solve physics related problems].
- analyze graphs of the variation with time of potential difference, charge and current for a capacitor discharging through a resistor [use $\tau = RC$ for the time constant for a capacitor discharging through a resistor]
- Use equations of the form $x = x_o \exp(-t/RC)$ [where x could represent current, charge or potential difference for a capacitor discharging through a resistor]
- List the use of capacitors in various household appliances [such as in flash guns, refrigerators, electric fans, rectification circuits, etc.]
- Illustrate how bioelectricity is generated in animals. [- cells control the flow of specific charged elements across the membrane with proteins that sit on the cell surface and create an opening for certain ions to pass through. These proteins are called ion channels.
  - When a cell is stimulated; it allows positive charges to enter the cell through open ion channels. The inside of the cell then becomes more positively charged, which triggers further electrical currents that can turn into electrical pulses, called action potentials.
  - The bodies of many organisms use certain patterns of action potentials to initiate the correct movements, thoughts and behaviors.]
- State that there are several species of aquatic life. such as Electrophorus Electricus, that can naturally generate external electric shocks through internal biological mechanisms that act as batteries.
- Explain, with examples of animals with this ability, that electroreception is the ability to detect weak naturally occurring electrostatic fields in the environment.

Imagine a world where energy is rare and devices can't function efficiently. Understanding how energy is stored and released is crucial for harnessing its power. In this chapter, we'll explore the fundamental concepts of energy storage, starting with electric potential energy and its relationship to energy storage. We'll then explore the working of capacitors, devices that store energy and regulate its flow.

Through interactive examples, diagrams, and real-life applications, you'll gain a deeper understanding of: electric potential energy, capacitors and its combination, energy stored in capacitors, charging and discharging of capacitors. Presence of electric potential in living organism (i.e., bioelectricity) will also be discussed at the end of this unit.

By mastering these concepts, you'll unlock the secrets of energy storage and its role in shaping our modern world. Get ready to explore the exciting world of energy storage and its applications!

19.1 ELECTRIC POTENTIAL ENERGY AND ELECTRIC POTENTIAL

Electricity is a form of energy that powers our devices and gadgets. Electric circuits usually use electric energy and transfer it to other forms like heat, light, or motion. The stored energy of an electric circuit is called electric potential energy. For example, we know that great amounts of electrical energy can be stored in batteries, are transmitted cross-country through power lines. Batteries are typically a few volts, the outlets in your home produce 220 volts, and power lines can be as high as hundreds of thousands of volts. Energy and voltage are two different quantities. A motorcycle battery, for example, is small and would not be very successful in replacing the much larger car battery, yet each has the same voltage. Here, we shall examine the relationship between voltage and electrical energy and begin to explore some of the many applications of electricity.

Consider a point charge q_o is placed in between two oppositely charged parallel plates, as shown in Fig. 19.1. Force F experienced by the charge q_o in the electric field E is:

$$F = q_o E$$

If the charge q_o is allowed to move freely in an electric field, it will move from positive plate to negative plate and acquire kinetic energy. If the charge is moved against the electric field, energy is required.

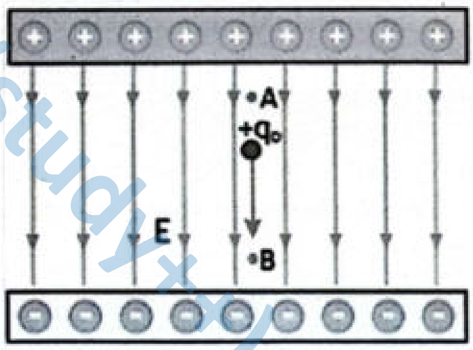


Figure 19.1: A charge is placed between two oppositely charged parallel plates.

Electric potential energy is the energy that is needed to move a charge against an electric field.

One needs more energy to move a charge further in the electric field, but also more energy to move it through a stronger electric field. So, there are two factors on which the electric potential energy of an object depends: Its own electric charge and its relative position with other electrically charged objects.

Thus, the electric potential energy can also be defined as:

Electric potential energy is the energy possessed by a unit charge if located at any point in the space, around a field charge.

The change in potential energy ΔU of charge q_o is equal to the "work done by the force in carrying the charge q_o from point B to the point A against the electrical field" i.e.,

$$\Delta U = W_{BA}$$

or $$U_A - U_B = W_{BA} \qquad \text{\_\_\_\_\_\_ (19.1)}$$

Where U_A and U_B represent the potential energies at point A and B respectively. W_{BA} is the work done by the force in carrying a positive charge q_o from point B to point A without disturbing the equilibrium state of the charge. Unit of electric potential energy is joule (J).

The concept of electric potential is used to express the effect of an electric field of a source in terms of the location within the electric field. A test charge with twice the quantity of charge would possess twice the potential energy at a given location; yet its electric potential at that location would be the same as any other test charge. A positive test charge would be at a high electric potential when held close to a positive source charge and at a lower electric potential when held further away. In this sense, electric potential becomes simply a property of the location within an electric field. Suppose that the electric potential at a given location is 12 Joules per coulomb, then that is the electric potential of a 1 coulomb or a 2 coulomb charged object. Stating that the electric potential at a given location is 12 joules per coulomb, would mean that a 2 coulomb object would possess 24 joules of potential energy at that location and a 0.5 coulomb object would experience 6 joules of potential energy at the location. Thus, Electric potential is purely location dependent. Electric potential is the potential energy per unit charge.

The potential difference between two points is defined as:

Potential difference is the work done in moving a unit positive charge from one point to another keeping the charge in electrostatic equilibrium.

If V_A and V_B are the electric potentials at points A and B, respectively, then the potential difference between these two points is

$$\Delta V = V_A - V_B = \frac{W_{AB}}{q_o} = \frac{\Delta U}{q_o} \qquad \text{\_\_\_\_\_\_ (19.2)}$$

It can also be written as:

$$\Delta U = q_o \Delta V \qquad \text{\_\_\_\_\_\_ (19.3)}$$

Unit of Electric Potential

The unit of electric potential is joule per coulomb (J/C) or volt (V).

The potential difference between two points in an electric field is one volt if one joule of work is done in moving one coulomb of charge from one point to the other.

So, $$1 \text{ volt} = \frac{\text{joule}}{\text{coulomb}}$$

The following multiples and sub-multiples of volt are commonly used;

$$1\text{ mV} = 10^{-3}\text{ V},\ 1\ \mu\text{V} = 10^{-6}\text{ V},\ 1\text{ kV} = 10^{3}\text{ V},\quad 1\text{ MV} = 10^{6}\text{ V},\ 1\text{ GV} = 10^{9}\text{ V}$$

Example 19.1: You have a 12.0 V motorcycle battery that can move 5000 C of charge, and a 12.0 V car battery that can move 60,000 C of charge. How much energy does each deliver?

Given: For the motorcycle battery, $q_m = 5000$ C

For the car battery, $q_c = 60{,}000$ C

$\Delta V = 12.0$ V

To Find: a) $\Delta U_m = ?$ **b)** $\Delta U_c = ?$

Solution: a) The total energy delivered by the motorcycle battery is

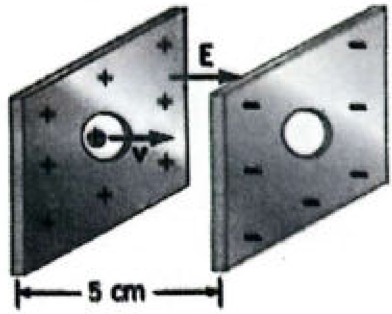
$$\Delta U_m = q\,\Delta V = (5000\text{ C})\ (12.0\text{ V}) = 6.00 \times 10^4\text{ J.}$$

b) The total energy delivered by the car battery is

$$\Delta U_c = q\,\Delta V = (60{,}000\text{ C})(12.0\text{ V}) = 7.20 \times 10^5\text{ J}$$

Assignment 19.1

A proton is injected at a speed of 1.0×10^6 m s^{-1} between two plates 5.0 cm apart, as shown in Figure. The proton accelerates across the gap and exits through the opening. (a) What must the electric potential difference be if the exit speed is to be 3.0×10^6 m s^{-1}? (b) What is the magnitude of the electric field intensity between the plates, if it is assumed constant?



19.2 ELECTRIC POTENTIAL DUE TO A POINT CHARGE

The equation of electric potential ΔV in constant electric field E is given by:

$$\Delta V = -E\,\Delta r$$

This equation is true only for constant electric field E. For most of the cases electric field is not uniform. Here we derive an expression of electric potential for such non uniform field. Let us suppose an isolated charge +Q is placed at origin of coordinate system, as shown in Fig. 19.2.

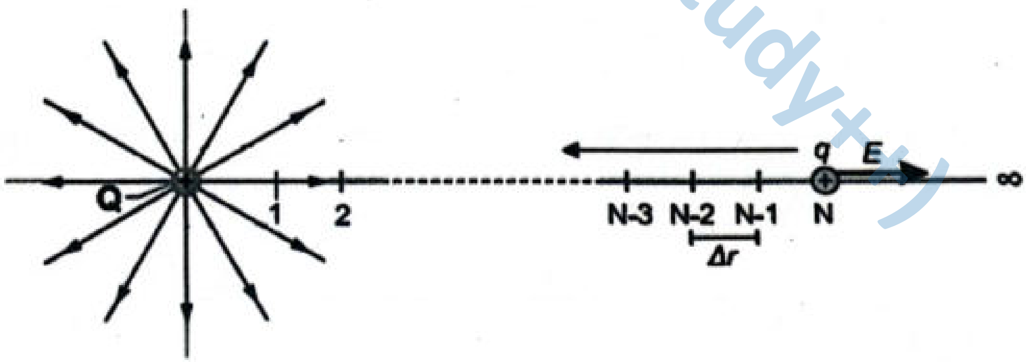


Figure 19.2: A test charge q is moved from a far point N to a point 1 against electric field E.

Let a test charge q is moved from a far point N to a point 1 against electric field E by applying force F. As the electric field does not remain constant but varies with the square of the distance from the charge +Q. So, we divide this large distance into small steps of length Δr such that E remains constant over each step. Here $r_1, r_2, r_3, \ldots, r_N$ are the distances of point 1, 2, 3, ... , N from charge Q, respectively. Let us find the electric potential from N to N−1.

The work done during the 1st step from N to N−1 is;

$$\Delta W_{N\to N-1} = F\,\Delta r\cos\theta$$

As $F = qE$, so we get;

$$\Delta W_{N\to N-1} = q\,E\,\Delta r\cos\theta$$

As E and Δr are in opposite direction, so we putting $\theta = 180^\circ$, hence;

$$\Delta W_{N\to N-1} = q\,E\,\Delta r\cos 180^\circ$$

$$= q\,E\,\Delta r\,(-1)$$

$$= -\,q\,E\,\Delta r$$

Putting $E = \dfrac{1}{4\pi\varepsilon_o}\dfrac{Q}{r^2}$ and $\Delta r = r_{N-1} - r_N$, we get;

So, $$\Delta W_{N\to N-1} = -q\left(\frac{1}{4\pi\varepsilon_o}\frac{Q}{r^2}\right)(r_{N-1} - r_N) = q\left(\frac{1}{4\pi\varepsilon_o}\frac{Q}{r^2}\right)(r_N - r_{N-1})$$

As we know that r_N and r_{N-1} are very close to each other. So, for this step $r^2 = r_{N-1}r_N$, hence

$$\Delta W_{N\to N-1} = \frac{Qq}{4\pi\varepsilon_o}\left(\frac{r_N - r_{N-1}}{r_{N-1}r_N}\right)$$

$$\Delta W_{N\to N-1} = \frac{Qq}{4\pi\varepsilon_o}\left(\frac{1}{r_{N-1}} - \frac{1}{r_N}\right) \quad \text{——— (i)}$$

Similarly,

$$\Delta W_{N-1\to N-2} = \frac{Qq}{4\pi\varepsilon_o}\left(\frac{1}{r_{N-2}} - \frac{1}{r_{N-1}}\right) \quad \text{——— (ii)}$$

$$\vdots \qquad\qquad \vdots$$

$$\Delta W_{2\to 1} = \frac{Qq}{4\pi\varepsilon_o}\left(\frac{1}{r_1} - \frac{1}{r_2}\right) \quad \text{——— (iii)}$$

So total work done in bringing the charge from N to 1 is calculated by adding the work in each step, i.e.,

$$\Delta W_{N\to 1} = \Delta W_{N\to N-1} + \Delta W_{N-1\to N-2} + \ldots + \Delta W_{2\to 1}$$

$$\Delta W_{N\to 1} = \frac{Qq}{4\pi\varepsilon_o}\left(\frac{1}{r_{N-1}} - \frac{1}{r_N}\right) + \frac{Qq}{4\pi\varepsilon_o}\left(\frac{1}{r_{N-2}} - \frac{1}{r_{N-1}}\right) + \ldots + \frac{Qq}{4\pi\varepsilon_o}\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$$

$$\Delta W_{N\to 1} = \frac{Qq}{4\pi\varepsilon_o}\left[\left(\frac{1}{r_{N-1}} - \frac{1}{r_N}\right) + \left(\frac{1}{r_{N-2}} - \frac{1}{r_{N-1}}\right) + \ldots + \left(\frac{1}{r_1} - \frac{1}{r_2}\right)\right]$$

$$\Delta W_{N\to 1} = \frac{Qq}{4\pi\varepsilon_o}\left(\frac{1}{r_1} - \frac{1}{r_N}\right) \quad \text{——— (iv)}$$

If we bring the test charge q from infinity, then;

$$r_N = \infty \quad \text{and} \quad \frac{1}{r_N} = \frac{1}{\infty} = 0$$

So Eq. (iv) becomes

$$\Delta W_{N\to 1} = \frac{Qq}{4\pi\varepsilon_o}\left(\frac{1}{r_1} - 0\right)$$

or

$$\Delta W_{N\to 1} = \frac{Qq}{4\pi\varepsilon_o}\left(\frac{1}{r_1}\right)$$

We replace r_1 by r, to obtain a general expression for the electric potential energy 'U' at distance r from Q, i.e.,

$$U = \frac{Qq}{4\pi\varepsilon_o}\left(\frac{1}{r}\right) \quad \text{———} \quad (19.4)$$

The electric potential at any point in an electric field is equal to work done in bringing a unit positive charge from infinity to that point keeping it in equilibrium. The electric potential at distance r from Q is calculated by using formula:

$$V = \frac{U}{q}$$

Using Eq. (19.4), we get:

$$V = \frac{Qq}{4\pi\varepsilon_o}\left(\frac{1}{r}\right) \div q$$

$$V = \frac{1}{4\pi\varepsilon_o}\left(\frac{Q}{r}\right) \quad \text{———} \quad (19.5)$$

The Eq. (19.5) gives the potential at distance r from the charge Q. This equation shows that the potential is same at all the points that are equidistant from an isolated source charge Q.

19.3 CAPACITORS

Capacitors are a common component in most electronic devices. These are used to store energy. Typically, commercial capacitors have two identical, parallel conducting plates separated by a distance d, as shown in Fig. 19.3, is called a parallel plate capacitor. The medium between the plates is air or a sheet of some insulator. This medium is known as dielectric. When battery terminals are connected to an initially uncharged capacitor, equal amounts of positive and negative charge, +Q and -Q, are separated into its two plates. The capacitor remains neutral overall, but we refer to it as storing a charge Q in this circumstance. When a charge is

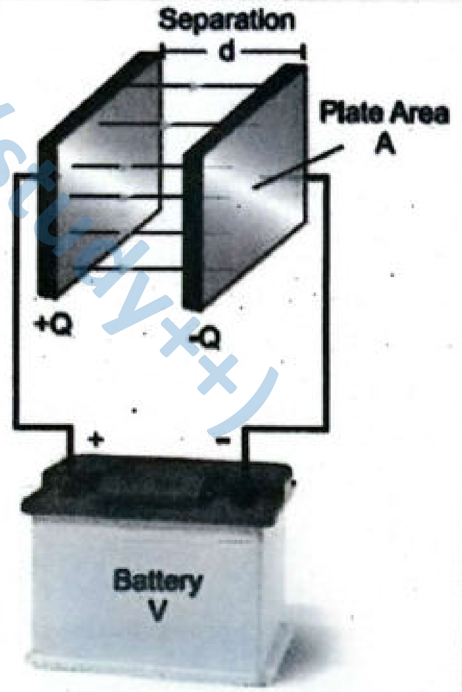


Figure 19.3: Parallel plate capacitor attached to a battery.

transferred to one of the plates say (A) due to electrostatic induction it would induce charge Q on the inner surface of the other plate (B). Mutual attraction between the charges keeps them bound on the inner surface of two plates and thus the charge remains stored in the capacitor even after removal of the battery.

Capacitance of a Capacitor

Experimentally it is seen that the charge stored on a capacitor is directly proportional to the voltage applied across it. If a charge 'Q' is transferred to one of the plates of a capacitor when the potential difference applied between the plates is 'V', then

$$Q \propto V$$

or $$Q = CV$$

or $$C = \frac{Q}{V} \quad \text{———} \quad (19.6)$$

Where 'C' is a constant of proportionality, called the capacitance of capacitor. Capacitance can be defined as:

The capability of a capacitor to store charges is called its capacitance.

The unit of capacitance is the farad (F), named for Michael Faraday (1791-1867), an English scientist who contributed to the fields of electromagnetism and electrochemistry. Since capacitance is charge per unit voltage, we see that a farad is a coulomb per volt.

If a capacitor stores a charge of 1 coulomb (a very large amount of charge) having the potential difference of 1 volt between the plates, then the capacitance is called 1farad.

One farad is, thus, a very large capacitance. Typical capacitors range from picofarad (1 pF = 10^{-12} F) to millifarad (1 mF = 10^{-3} F).

Capacitance of a Parallel Plate Capacitor

Consider a parallel plate capacitor connected to a voltage source, as shown in Fig. 19.4. Let the area of each plate is A and the separation between plates is d. It built +Q and –Q charges on the plates. If the positive plate is at potential V_1 and negative plate is at potential V_2, then the electric field strength E between the plates is:

$$E = -\frac{\Delta V}{\Delta r} = -\frac{(V_2 - V_1)}{d} = \frac{V_1 - V_2}{d}$$

$$E = \frac{V}{d} \quad \text{———} \quad (i) \qquad (\because V_1 - V_2 = V)$$

Where, d is separation between the plates. The strength of the electric field also depends on the number of charges on the plates. As, Q is the charge on either of the plate of area A, so the surface charge density on the plate is:

$$\sigma = \frac{Q}{A}$$

Also, as the electric field intensity E between the plates of capacitor is given by applying Gauss's law:

$$E = \frac{\sigma}{\varepsilon_o} \qquad \text{\_\_\_\_\_\_ (ii)}$$

substituting the value of σ in Eq. (ii), we get:

$$E = \frac{Q}{A\varepsilon_o} \qquad \text{\_\_\_\_\_\_ (iii)}$$

Combining Eqs. (i) and (iii), we get:

$$\frac{Q}{A\varepsilon_o} = \frac{V}{d}$$

or $$\frac{Q}{V} = \frac{A\varepsilon_o}{d}$$

Material Science and Technology

Capacitors are mainly made of ceramic, glass, or plastic, depending upon purpose and size. A computer chip contains number of capacitors.

As the medium between the plate is air, or vacuum, here we put $Q/V = C_{vac}$, hence we get:

$$C_{vac} = \frac{A\varepsilon_o}{d} \qquad \text{\_\_\_\_\_\_ (19.7 a)}$$

When an insulating material called dielectric is inserted between the plates of a capacitor, then capacitance of a capacitor is enhanced by a factor of ε_r, and is given as:

$$C_{med} = \frac{A\varepsilon_o\varepsilon_r}{d} \qquad \text{\_\_\_\_\_\_ (19.7 b)}$$

By dividing Eq. (19.7) by Eq. (19.6), we get:

$$\varepsilon_r = \frac{C_{med}}{C_{vac}} \qquad \text{\_\_\_\_\_\_ (19.8)}$$

The ratio of the capacitance of a capacitor with the dielectric medium between the plates to that of the capacitance of the same capacitor when the space is evacuated is called the relative permittivity ε_r of the material.

The equation $C_{med} = \frac{A\varepsilon_o\varepsilon_r}{d}$ shows that value of capacitance C depends upon:

- The area of the plates.
- The distance between the plates.
- The medium (dielectric) between the plates.

Example 19.2: What is the capacitance of a parallel plate capacitor with metal plates, each of area 1.00 m^2, separated by 1.00 mm? What charge is stored in this capacitor if a voltage of 3.00×10^3 V is applied to it?

Given: Area of the plate = A = 1.00 m^2

Distance between the plates = d = 1.00 mm $\Delta V = 3.00 \times 10^3$ V

To Find: a) C = ? b) Q = ?

Solution: a) Here we use: $C = A\epsilon_o/d$

Entering the given values into the equation, we get:

$$C = (1.00)(8.85\times10^{-12}) \div 1.00\times10^{-3}$$

or $$C = 8.85\times10^{-9}\ \text{F} = 8.85\ \text{nF}$$

b) Now the charge stored in the capacitor can be found by the equation: $Q = C\,V$

Entering the known values into this equation gives:

$$Q = (8.85\times10^{-9})(3.00\times10^{3}) = 26.6\ \mu\text{C}$$

Assignment 19.2

What is the capacitance of a large Van de Graff generator's terminal, given that it stores 8.00 mC of charge at a voltage of 12.0 mV?

19.4 COMBINATIONS OF CAPACITORS

Several capacitors can be connected together to be used in a variety of applications. Multiple connections of capacitors behave as a single equivalent capacitor. The total capacitance of this equivalent single capacitor depends both on the individual capacitors and how they are connected. Capacitors can be arranged in two simple and common types of connections, known as series and parallel combinations.

19.4.1 Series Combination of Capacitors

When the capacitors are connected plate to plate i.e., right plate of one capacitor is connected to left plate of the next capacitor, then it is called series combination. Fig. 19.4 shows series combination of three capacitors connected between points A and B.

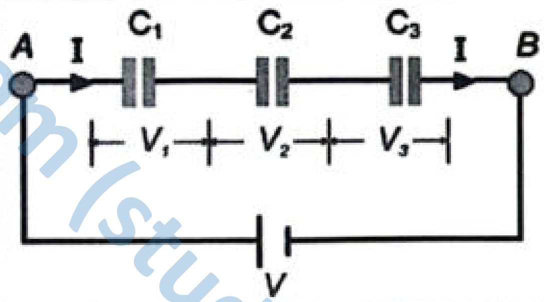


Figure 19.4: Series combination of capacitors.

If V is potential of the battery and V_1, V_2 and V_3 are potential of capacitors C_1, C_2 and C_3 respectively. Then;

$$V = V_1 + V_2 + V_3 \quad \text{\_\_\_\_\_\_ (19.9)}$$

As the battery will supply same amount of charge Q to each capacitor, so putting values of V_1, V_2 and V_3 in Eq. (19.9), we get:

$$V = \frac{Q_1}{C_1} + \frac{Q_2}{C_2} + \frac{Q_3}{C_3} \quad \text{\_\_\_\_\_\_ (i)}$$

Here, we use the formula Q = CV or C = Q/V. In series combination, due to *electrostatic induction* charge on each capacitor is same and equal to charge supplied by battery i.e.,

$$Q = Q_1 = Q_2 = Q_3$$

So, Eq. (i) becomes:

$$V = Q\left(\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}\right) \quad \text{or} \quad \frac{V}{Q} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$

Put $\frac{V}{Q} = \frac{1}{C_e}$, so above equation becomes:

$$\frac{1}{C_e} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$

Similarly, for n number of capacitors connected in series, we can write as:

$$\frac{1}{C_e} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \ldots \qquad \text{\_\_\_\_\_\_ (19.9)}$$

The Eq. (19.9) show that the equivalent capacitance of series capacitors is always less than any individual capacitance in the combination.

19.4.2 Parallel Combination of Capacitors

When two or more capacitors are connected between the same two points, then the combination is called parallel combination. Figure 19.5 shows parallel combination of three capacitors connected between points A and B.

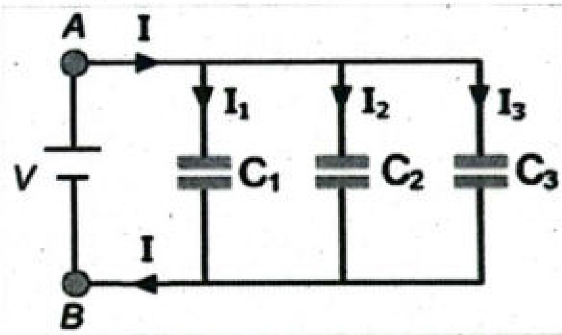


Figure 19.5: Parallel combination of capacitors.

If Q is charge supplied by the battery and Q_1, Q_2 and Q_3 are charges stored on each capacitor C_1, C_2 and C_3 respectively. Then

$$Q = Q_1 + Q_2 + Q_3 \qquad \text{\_\_\_\_\_\_ (i)}$$

As the battery will supply same amount of voltage V to each capacitor, so putting values of Q_1, Q_2 and Q_3 in Eq. (i), we get:

$$Q = C_1 V + C_2 V + C_3 V$$

Or

$$Q = V(C_1 + C_2 + C_3)$$

$$\frac{Q}{V} = C_1 + C_2 + C_3 \qquad \text{\_\_\_\_\_\_ (ii)}$$

Put $\frac{Q}{V} = C_e$ in Eq. (ii), we get:

$$C_e = C_1 + C_2 + C_3$$

Similarly, for 'n' number of capacitors connected in parallel, we can write as:

$$C_e = C_1 + C_2 + C_3 + \ldots \qquad \text{\_\_\_\_\_\_ (19.10)}$$

The Eq. (19.10) shows that the equivalent capacitance of parallel capacitors is larger than any of the individual capacitances.

For Your Information

If N identical capacitors of capacitance C are connected in series, then effective capacitance is C/N.
If N identical capacitors of capacitance C are connected in parallel, then effective capacitance is CN.

Example 19.3: Find the equivalent capacitance of two capacitors of capacitance 4 μF and 8 μF. (a) When connected in series (b) When connected in parallel.

Given: $C_1 = 4\ \mu F$ $\quad\quad$ $C_2 = 8\ \mu F$

To Find: a) For series combination, C_e = ? $\quad\quad$ b) For parallel combination, C_e = ?

Solution: a) For series combination

$$\frac{1}{C_e} = \frac{1}{C_1} + \frac{1}{C_2} \quad \text{or} \quad \frac{1}{C_e} = \frac{C_1 + C_2}{C_1 C_2}$$

or $$C_e = \frac{C_1 C_2}{C_1 + C_2}$$

Putting values, we get:

$$C_e = \frac{(4\mu)(8\mu)}{4\mu + 8\mu} = \frac{32 \times 10^{-12}}{12 \times 10^{-6}} = 2.66 \times 10^{-6}\ F = 2.66\ \mu F$$

b) For parallel combination

$$C_e = C_1 + C_2$$

or $$C_e = 4\ \mu F + 8\ \mu F = 12\ \mu F$$

Assignment 19.3

Find the equivalent capacitance of three capacitors of capacitance 4 μF, 6 μF and 8 μF, connected in series.

19.5 ENERGY STORED IN A CAPACITOR

Capacitor stores energy in the form of electric field. Energy stored in a capacitor is electrical potential energy, and it is thus related to the charge 'Q' and voltage 'V' on the capacitor. The capacitor starts with zero voltage and gradually comes up to its full voltage as it is charged. The first charge placed on a capacitor experiences a change in voltage $\Delta V = 0$, since the capacitor has zero voltage when uncharged. The final charge placed on a capacitor experiences $\Delta V = V$, since the capacitor now has its full voltage 'V' on it. The charge 'Q' on the capacitor is directly proportional to its potential difference 'V'. The graph of charge against potential difference is therefore a straight-line graph through the origin, as shown in the Fig. 19.6.

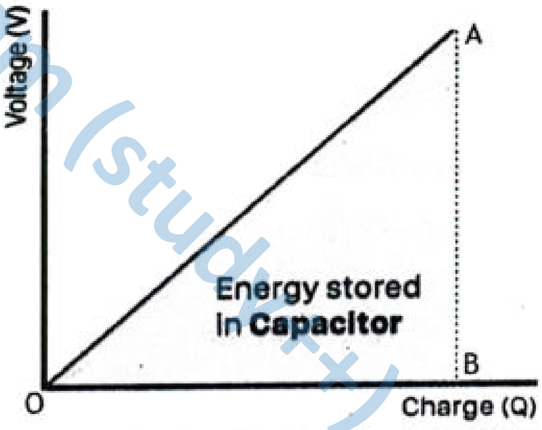


Figure 19.6: Graph of Potential-Difference verses Charge.

The electric potential energy (E) stored in a capacitor can be determined from the area under the potential-charge graph which is equal to the area of a right-angled triangle:

E = area of a right-angled triangle OAB

or $$E = \frac{1}{2} \times Base \times Height$$

Where the base is Q, and the height is V, i.e.,

$$E = \frac{1}{2} Q V \text{ \_\_\_\_\_\_ } (9.11\ a)$$

This energy is directly related to the amount of charge moved onto the plates and the voltage across them. Substituting Q = CV, the energy can also be defined as:

$$E = \frac{1}{2}(CV) \times V$$

or $$E = \frac{1}{2} CV^2 \text{ \_\_\_\_\_\_ } (9.11\ b)$$

By substituting the potential V=Q/C, the work done can also be defined in terms of just the charge and the capacitance:

or $$E = \frac{Q^2}{2C} \text{ \_\_\_\_\_\_ } (9.11\ c)$$

The Eqs. (19.11 a, b, c) are valid for calculating energy stores in a capacitor.

For Your Information

A battery stores electrical energy and releases it through chemical reactions. This means that it can be quickly charged but the discharge is slow. Unlike the battery, a capacitor temporarily stores electrical energy through distributing charged particles on plates to create a potential difference. A capacitor can take a shorter time than a battery to charge up and it can release all the energy very quickly.

Uses of Capacitor

Capacitors are very important circuit element in Electronics. Following are a few applications of capacitor in daily life:

- Capacitors are used for filtering unwanted frequencies in radio and TV set.
- Capacitors are used in camera flashes to store and quickly release a large amount of electrical energy.
- Refrigerator capacitors are usually used to keep the compressor running.
- Capacitor is used to increase speed of the fan.
- Capacitor is used in rectification circuit to act as a filter to reduce ripple voltage.

Example 19.4: Capacitance of a capacitor is 50 F. It is charged to a potential of 100 V. Calculate the energy stored in it.

Given: C = 50 F   V = 100 V
To Find: E = ?

Solution: Here we use the formula:

$$E = \frac{1}{2} CV^2$$

Putting values, we get:

$$E = \frac{1}{2}(50)(100)^2$$

$$E = 250 \times 10^3\ J \qquad \text{or} \qquad E = 250\ kJ$$

Assignment 19.4

Calculate the energy stored in a capacitor having 5 C of charge and a potential difference of 15 V.

19.6 CHARGING AND DISCHARGING OF A CAPACITOR

Consider an RC (resistor-capacitor) circuit for charging of a capacitor, as shown in Fig. 19.7 (a). When the switch is set at point 1, a battery of e.m.f 'ε' starts charging the capacitor through the resistor R. The charge builds up gradually on the plates to the maximum value of Q_o.

When a capacitor charges, electrons flow onto one plate and move off the other plate. This process will be continued until the potential difference across the capacitor is equal to the e.m.f of the battery.

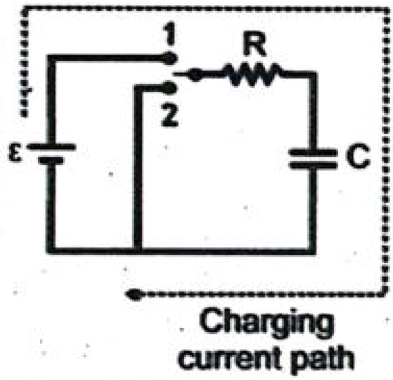


Figure 19.7 (a): Charging circuit for a capacitor.

The rate of flow of charge will not be linear. At the start, the current will be at its highest value but will gradually decrease to zero. Voltage on the capacitor is initially zero and rises rapidly at first, since the initial current is a maximum. Suppose at t = 0, charge on a capacitor is zero i.e. Q = 0. It can be shown experimentally that after time t, as charge builds up on the plates, it repels more charge that is arriving so the current drops as the charge on the plates increases. Charging will stop when the potential difference between the capacitor plates is equal to the e.m.f. of the battery.

Maximum charge on capacitor = Capacitance × e.m.f. of battery

Experiments show that the charging process of a capacitor is not linear but shows the exponential behavior. The exponential behavior for charging of capacitor is written in equation form as:

$$Q = Q_o\left(1 - e^{-t/RC}\right) \quad \text{\_\_\_\_\_\_ (19.12)}$$

Where, e = 2.7182 is a constant. The graph of charging process, between time t and charge Q, is shows in Fig. 19.7 (b). According to this graph, Q = 0 at t = 0 and increases gradually to its maximum value Q_o.

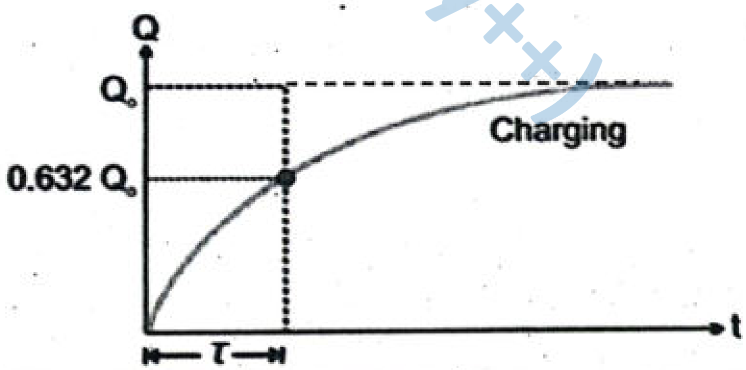


Figure 19.7 (b): Charging graph for a capacitor.

The time during which 63.2 % of its maximum value charge is deposited on the plates of the capacitor is called time constant (τ) of an RC circuit. This can be seen by putting t=RC in Eq. (19.12), i.e.,

$$Q = Q_o(1-e^{-1}) = Q_o\left(1-\frac{1}{2.718}\right) = Q_o(0.632)$$

or $\qquad \dfrac{Q}{Q_o} = 0.632 \quad$ or $\quad \dfrac{Q}{Q_o} = 63.2\ \%$

Smaller the resistance or the capacitance, the smaller the time constant, the faster the charging and the discharging rate of the capacitor, and vice versa.

When a capacitor is discharging, the current will be highest at the start. This will gradually decrease until reaching 0, when the current reaches zero, the capacitor is fully discharged as there is no charge stored across it. The circuit for discharging of a charged capacitor is shown in Fig. 19.8 (a).

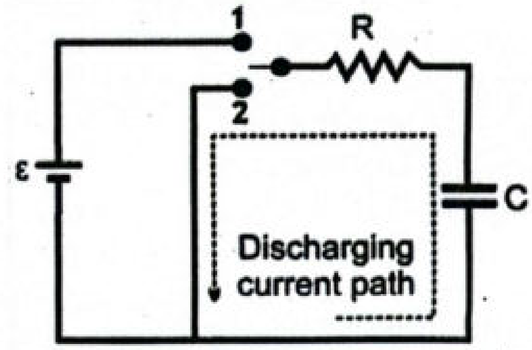


Figure 19.8 (a): Discharging circuit for a capacitor.

When the switch is set at terminal 2, the charge $-Q$ on the negative plate can now flow through the resistance and neutralize the charge $+Q$ on the positive plate of the capacitor. Assume that a fully charged capacitor begins discharging at time t = 0. It can be shown that charge left on either plate at time t is:

$$Q = Q_o e^{-t/RC} \qquad \text{\_\_\_\_\_\_ (19.13)}$$

The graph of discharging in Fig. 19.8 (b) shows that discharging begins at t = 0 when Q = Q_o, and decreases gradually. When t = RC, the magnitude of charge remaining on each plate is:

$$Q = Q_o\,(0.367)$$

or $\qquad \dfrac{Q}{Q_o} = 0.367$

or $\qquad \dfrac{Q}{Q_o} = 36.7\ \%$

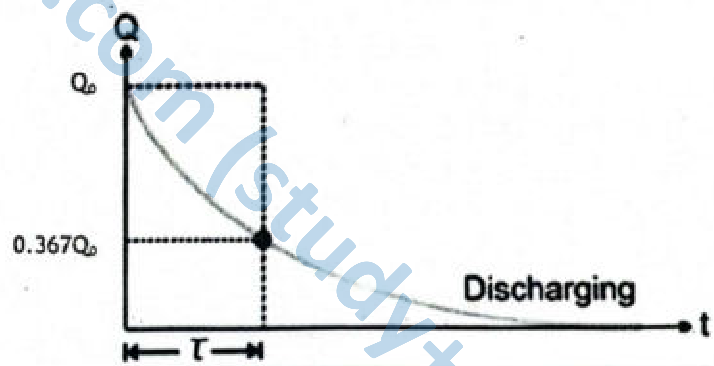


Figure 19.8 (b): Discharging graph for a capacitor.

A capacitor with smaller values of time constant (RC), lead to a more rapid discharge. The Eq. (19.12) and Eq. (19.13) can also be used for current and potential difference, if written as:

For Charging: $\qquad x = x_o\left(1-e^{-t/RC}\right) \qquad$ ______ (19.14)

For Discharging: $\qquad x = x_o\, e^{-t/RC} \qquad$ ______ (19.15)

Here x could represent current, charge or potential difference for a capacitor discharging through a resistor.

19.7 BIOELECTRICITY

Bioelectricity refers to the generation or action of electric currents or voltages in biological processes. Electric activity in living tissue is a cellular phenomenon that depends on the cell membrane. The cell membrane acts like a capacitor that stores energy as electrically charged ions on opposite sides of the cell membrane.

In most solutions, ions of a given electric charge are accompanied by ions of opposite charge, so that the solution itself has no net charge. If two solutions of different concentrations are separated by a membrane that allows one kind of ion to pass but block the other, producing equal and opposite net charges in the two solutions. This concentration imbalance gives rise to an electric-potential difference between the solutions.

In living cells, there are two types of solutions: those found inside and outside the cell. The cell membrane (as shown in Fig. 19.9), separating inside from outside, is semipermeable. Cell membrane allows certain ions to pass through while blocking others. In particular, nerve-cell and muscle-cell membranes are slightly permeable to positive potassium ions (K+), which diffuse outward, leaving a net negative charge in the cell. Ion channels allow specific inorganic ions (such as: Na^+, K^+, Ca^{2+}, or Cl^-) to diffuse rapidly down their electrochemical gradients across the lipid bilayer.

Cells control the flow of specific charged elements across the membrane with proteins that sit on the cell surface and create an opening for certain ions to pass through. These proteins are called ion channels. Ion channel spans the membrane and make hydrophilic tunnels across it, allowing their target molecules to pass through by diffusion. Channels are very selective and will accept only one type of molecule (or a few closely related molecules) for transport.

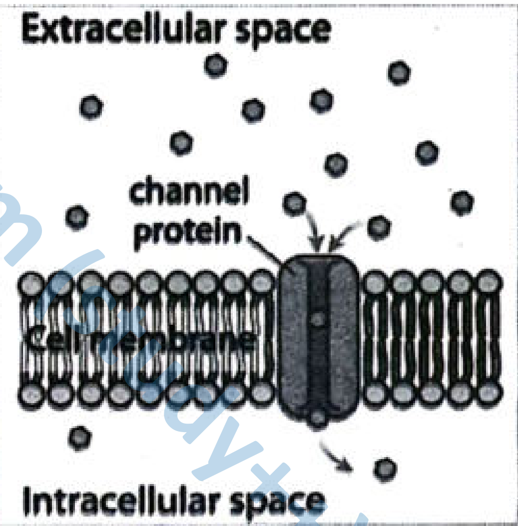


Figure 19.9: A channel protein, which forms a tunnel allowing a specific molecule to cross the membrane (down its concentration gradient).

When a cell is stimulated; it allows positive charges to enter the cell through open ion channels. The inside of the cell then becomes more positively charged, which triggers further electrical currents that can turn into electrical pulses, called action potentials. The bodies of many organisms use certain patterns of action potentials to initiate the correct movements, thoughts and behaviors.

The bioelectric potential across a cell membrane is typically about 50 millivolts; this potential is known as the resting potential. Bioelectric phenomena include fast signaling in nerves and the triggering of physical processes in muscles or glands.

There are several species of aquatic life, such as Electrophorus Electricus, that can naturally generate external electric shocks through internal biological mechanisms that act as batteries.

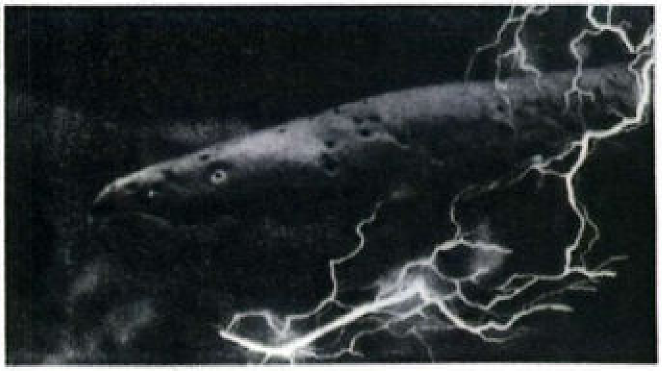
Such species are called electrogenic animals. Electric eels are probably the best known electrogenic animals. Some types of bacteria, yeast and fish are also electrogenic.

There are also animals that can detect electricity. They're called electroreceptive. Some electroreceptive animals are echidnas, platypuses, bees, spiders, dolphins, sharks and rays.

Most electrogenic animals are also electroreceptive. But there are many electroreceptive animals that are not electrogenic.

For Your Information

Electric eels are known for their ability to stun their prey by generating electricity, delivering shocks up to 860 volts.



SUMMARY

- The potential difference between two points is defined as "the work done in moving a unit positive charge from one point to another keeping the charge in electrostatic equilibrium".
- The potential difference between two points in an electric field is one volt if one joule of work is done in moving one coulomb of charge from one point to the other.
- The potential gradient represents the rate of change of potential with displacement.
- Capacitors are electronic devices used to store electric energy.
- The capability of a capacitor to store charges is called its capacitance.
- If a capacitor stores a charge of 1 coulomb (a very large amount of charge) having the potential difference of 1 volt between the plates, then the capacitance is called 1 farad.
- Equivalent capacitance of series capacitors is always less than any individual capacitance in the combination.
- The equivalent capacitance of parallel capacitors is larger than any of the individual capacitances.
- Bioelectricity refers to the generation or action of electric currents or voltages in biological processes.
- Electric eels are known for their ability to stun their prey by generating electricity, delivering shocks up to 860 volts.
- Electroreception is the ability of some animals to detect weak naturally occurring electrostatic fields in the environment.

Formula Sheet

$U_A - U_B = W_{BA}$

$\Delta U = q_o \Delta V U = \frac{Qq}{4\pi\varepsilon_o}\left(\frac{1}{r}\right)$

$V = \frac{1}{4\pi\varepsilon_o}\left(\frac{Q}{r}\right)$

$Q = CV$

$\sigma = \frac{Q}{A}$

$C_{vac} = \frac{A\varepsilon_o}{d}$

$C_{med} = \frac{A\varepsilon_o\varepsilon_r}{d}$

$\varepsilon_r = \frac{C_{med}}{C_{vac}}$

$\frac{1}{C_e} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \ldots$

$C_e = C_1 + C_2 + C_3 + \ldots$

$E = \frac{1}{2}CV^2$

$E = \frac{Q^2}{2C}$

$Q = Q_o\left(1 - e^{-t/RC}\right)$

$Q = Q_o\, e^{-t/RC}$

EXERCISE

Multiple Choice Questions

Encircle the correct option.

1) Which of the following is equivalent to 1.0 volt.

   A. newton/second B. joule/second

   C. joule/coulomb D. coulomb/joule

2) The quantity electric potential is defined as the amount of ______.

   A. electric potential energy B. force acting upon a charge

   C. potential energy per unit charge D. force per unit charge

3) A negatively charged particle is placed in a uniform electric field directed from South to North. In which direction will the particle move after it is released?

   A. East B. South C. North D. North-West

4) What is the charge that appears on the plates of the 10 pF capacitor if it is connected to a battery of 9 V.

   A. 90 pC B. 90 mC C. 90 kC D. 90 μC

5) What is the voltage that should be applied to the 5 μF capacitor to accumulate 1μC charge on its plates?

   A. 0.2 V B. 0.2 mV C. 0.2 kV D. 0.2 μV

6) Capacitor is a device used to__________

   A. store electrical energy B. vary the resistance

C. store magnetic energy D. dissipate energy

7) What is the total capacitance when three capacitors, C_1, C_2 and C_3 are connected in parallel?

A. $\frac{C_1}{C_2+C_3}$ B. $C_1+C_2+C_3$ C. $\frac{C_2}{C_1+C_3}$ D. $\frac{1}{C_1+C_2+C_3}$

8) If three capacitors of capacitances 1 F, 2 F and 10 F are connected in parallel then their equivalent capacitance will be:

A. 10 F B. 15 F C. 13 F D. 20 F

9) When capacitors are connected in parallel, the total capacitance is always ________ the individual capacitance values.

A. Greater than B. Less than C. Equal to D. Cannot be determined

10) What is the capacitance of a capacitor?

A. The ratio of charge to electric potential difference

B. The ratio of electric potential difference to charge

C. The product of charge and electric potential difference

D. The ratio of electric field strength to charge density

11) What is the electric potential difference between two points?

A. The work done in moving a unit charge between the two points

B. The force exerted on a unit charge at one of the points

C. The electric field strength between the two points

D. The distance between the two points

12) A 220 Ω resistor is in series with a 2.2 μF capacitor. The time constant is

A. 48 μs B. 480 μs C. 2.42 μs D. 24 μs

13) The formula for electrostatic potential is ___________.

A. Electrostatic potential = Work done × charge

B. Electrostatic potential = Work done/charge

C. Electrostatic potential = Work done + charge

D. Electrostatic potential = Charge Work/done

Short Questions

1) What is the relationship between electric potential and electric potential energy?

2) If you wish to store a large amount of energy in a capacitor bank, would you connect capacitors in series or parallel? Explain.

3) What are the units of (a) electric potential difference (b) electric potential energy (c) Capacitance?

4) What is the net amount of charge on a charged capacitor?

5) Write some applications of capacitors in real life.

6) Would you place the plates of a parallel-plate capacitor closer together or farther apart to increase their capacitance?

7) What is meant by electroreception?

8) If you were asked to design a capacitor in which small size and large capacitance were required, what would be the two most important factors in your design?

9) If a capacitor is fully charged and then left for discharging. How much charge will be left on the plate of the capacitor after time equal to one time constant?

Comprehensive Questions

1) Define and explain the term electric potential. Derive an expression for electric potential at a field point due to a source charge.

2) What is meant by potential gradient? Explain.

3) By using graph, derive an expression for the energy stored in a capacitor.

4) Derive the expression for the equivalent capacitance for parallel combination of capacitors.

5) Derive the formula for the combined capacitance of capacitors in series.

6) Explain the process of charging and discharging of a capacitor.

7) How bioelectricity is generated in animals? Explain.

Numerical Problems

1) A heart defibrillator delivers 4×10^2 J of energy by discharging a capacitor initially at 1×10^4 V. What is its capacitance?

(Ans: 8 μF)

2) Two capacitors of capacitance $C_1 = 6\ \mu$ F and $C_2 = 3\ \mu$ F are connected in series. Calculate the equivalent capacitance.

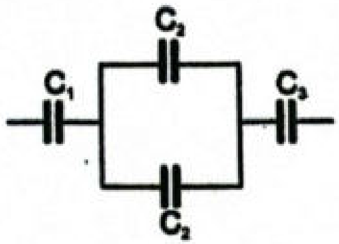
(Ans: 2μF)

3) A $2200\mu F$ capacitor is charged up with a 1.5 V cell. Calculate the charge and energy stored in the capacitor.

(Ans: 3.3 mC, 0.00248 J)

4) Find the equivalent capacitance for the following circuit if $C_1 = 1$ pF, $C_2 = 0.5$ pF, $C_3 = 1$ pF.

(Ans: 1/3 pF)



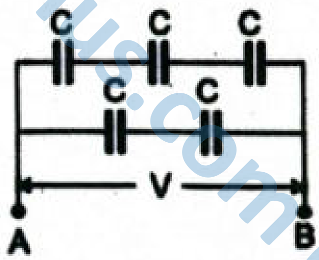
5) Suppose electrons in a TV tube are accelerated through a potential difference of 2.0×10^4 V from the heated cathode (where they are produced) toward the screen (which also serves as the anode), 25.0 cm away.

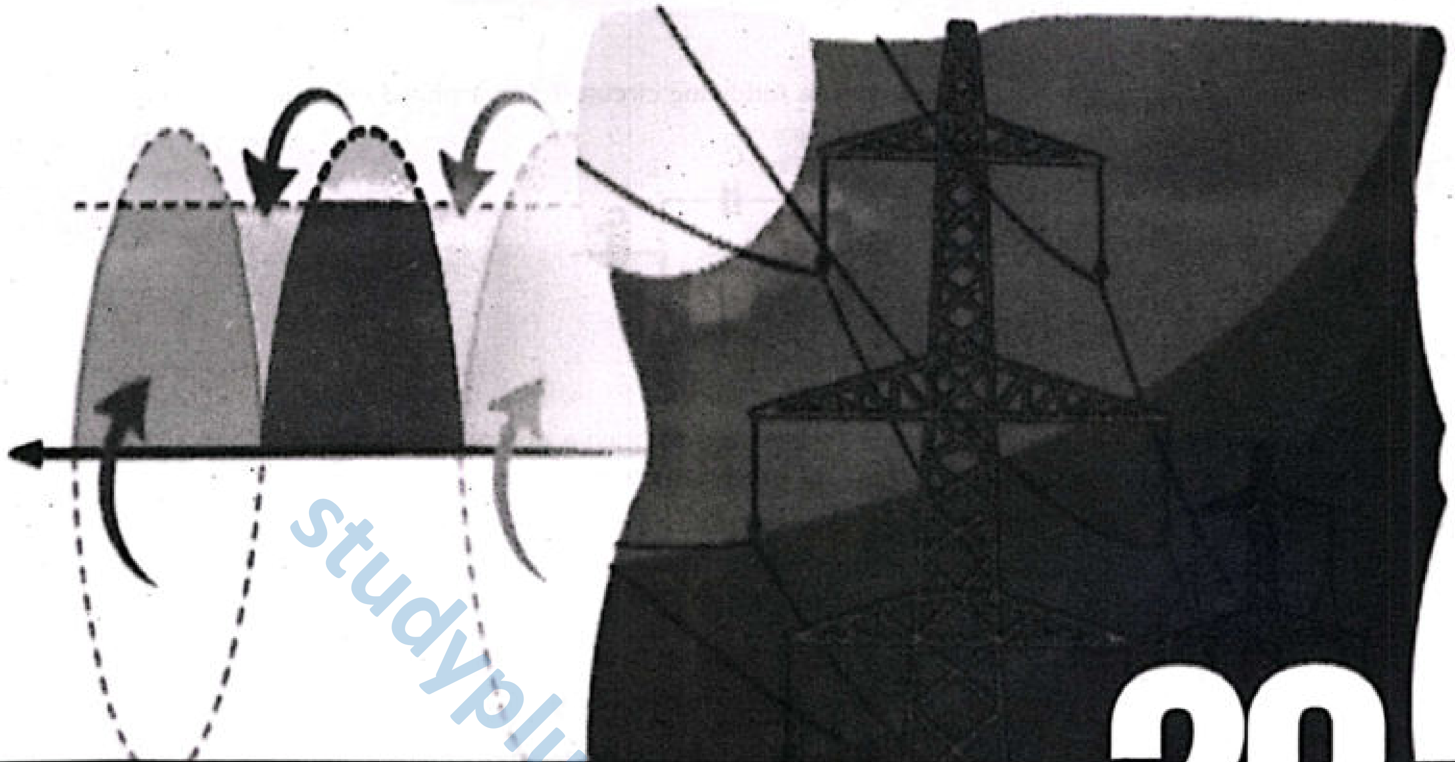
(a) At what speed would the electrons impact the phosphors on the screen? Assume they accelerate from rest and ignore relativistic effects.

(b) What's the magnitude of the electric field, if it is assumed constant?

(Ans: 8.38×10^7 m s^{-1} (b) 8.0×10^4 V m^{-1})

6) A network of five capacitors of capacitance C is connected to a 100 V supply, as shown in below figure. Determine the equivalent capacitance of the network. **(Ans: 5C/6)**





20 ALTERNATING CURRENT

Student Learning Outcomes (SLOs)

The student will

- use the terms period, frequency and peak value as applied to an alternating current, or voltage.
- use equations of the form $x = x_o \sin(\omega t)$ representing a sinusoidally alternating current or voltage.
- use the fact that the mean power in a resistive load is half the maximum power for a sinusoidal alternating current.
- distinguish between root-mean-square (r.m.s.) and peak values [including stating and using $I_{rms} = I_o/\sqrt{2}$ and $V_{rms} = V_o/\sqrt{2}$ for a sinusoidal alternating current]
- Distinguish graphically between half-wave and full-wave rectification.
- explain the use of a single diode for the half-wave rectification of an alternating current.
- explain the use of four diodes (bridge rectifier) for full-wave rectification of an alternating current.
- analyze the effect of a single capacitor in smoothing current flow [including the effect of the values of capacitance and the load resistance].
- define mutual inductance (M) and self-inductance (L), and their unit henry.
- describe the phase of A.C and how phase lags and leads in A.C circuits.
- identify inductors as important components of A.C circuits termed as chokes [devices which present a high resistance to alternating current]
- Calculate the reactances of capacitors and inductors.
- describe impedance as vector summation of resistances and reactances.

The electric current used in electrical devices have two forms: Direct Current (DC) and Alternating Current (AC). Direct current flows continuously in one direction while alternating current changes its polarity at regular interval of time. The direction of alternating current at any instant depends upon the polarity of the voltages. When an alternating voltage is applied in a circuit, the current flows first in one direction and then in the opposite direction.

From the lights in your home to the smartphone in your pocket, alternating current (AC) plays a vital role in powering our daily lives. It is the backbone of modern electricity, powering our homes, industries, and technologies. But what makes AC so efficient and widely used? Have you ever wondered how does it transmit power efficiently over long distances? What makes it possible for us to use a wide range of electrical devices in our daily lives?

In this unit, we'll discuss about alternating current and its behaviour with resistor, capacitor and inductor. Through interactive examples, diagrams, and real-life applications, you'll gain a deeper understanding of the principles and technologies that underlie our modern electrical infrastructure. Get ready to unlock the secrets of alternating current and discover how it powers the modern world!

20.1 ALTERNATING CURRENT AND VOLTAGE

An ac generator gives alternating current or voltage of the form:

$$x = x_o \sin(\omega t)$$

Where, x represents magnitude of alternating current or voltage corresponding to the time t, and x_o represents maximum value of alternating current or voltage. ω represents the angular frequency of generator and hence of current and voltage.

For alternating current: $I = I_o \sin(\omega t)$ ______ (20.1 a)

For alternating voltage: $V = V_o \sin(\omega t)$ ______ (20.1 b)

The graph of voltage or current against time is shown in Fig. 20.1. This is a sine wave. A sinusoidal alternating voltage/current can be produced by rotating a coil with a constant angular velocity (ω) in a uniform magnetic field.

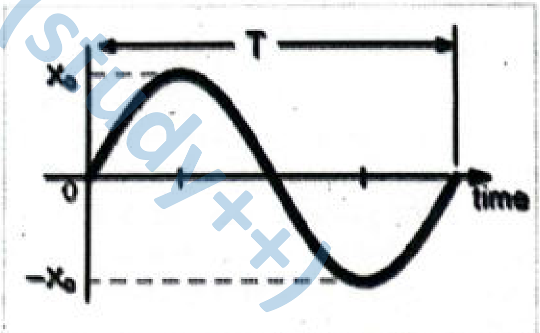


Figure 20.1: Sinusoidal wave form.

Graph shows that sinusoidal voltage or current:

- changes polarity (direction) after regular intervals.
- changes the magnitude continuously.
- the change from one polarity to the other is a smooth one.
- changing most rapidly at the zero (crossover) point and most slowly at its peak.

AC Terminologies

Some important A.C. terminology are defined below:

Cycle: One complete set of positive and negative values of an alternating quantity is known as a cycle. Figure 20.2 shows one cycle of an alternating voltage.

> **For Your Information**
> The frequency of A.C in Pakistan is 50 Hz.

Time Period (T): The time taken to complete one cycle of an alternating quantity is called its time period.

Frequency (f): The number of cycles that occurs in one second is called the frequency of the alternating quantity. It is measured in cycle/second or Hertz.

Average Value: The average value of a waveform is the average of all its values over a period of time. Finding an average value over time means adding all the values that occur in a specifying time interval and dividing the sum by that time.

The average value of a waveform from graph can be calculated by using the following formula:

$$\text{Average value} = \frac{\text{Total (net) area under the curve for time T}}{\text{Time T}}$$

The area above the time axis is taken as positive area and area below the time axis as negative area. In order to specify a sinusoidal voltage or current we do not use average value, because its value over one cycle is zero and cannot be used for power calculation.

Peak Value (x_o): Maximum value of alternating quantity (current or voltage) is called peak value, represents by x_o. The peak value of a sine wave occurs twice each cycle, once at the positive maximum value (x_o) and once at the negative maximum value ($-x_o$).

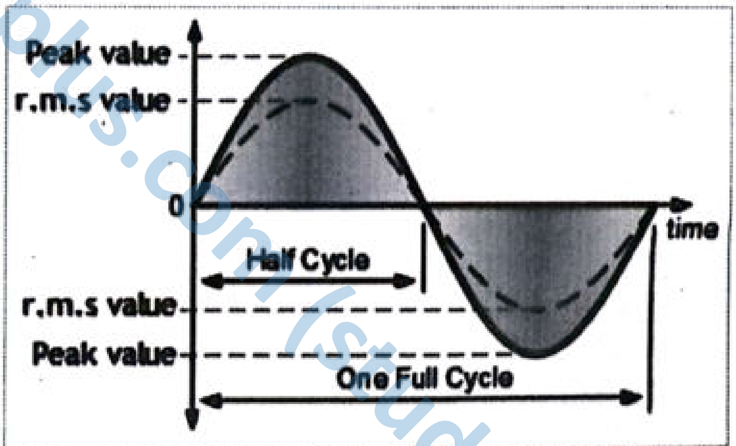


Figure 20.2: r.m.s and peak value.

Root-mean-square (r.m.s.) value: The r.m.s value of an alternating current is that steady current (d.c) which when flowing through a resistor produce the same amount of heat as that produced by the alternating current when flowing through the same resistance for the same time. The relation between root-mean-square (x_{rms}) and peak values (x_o) is given below:

r.m.s value for alternating current: $x_{rms} = x_o/\sqrt{2}$

The relation for current and voltage is given as:

r.m.s value for alternating current: $I_{rms} = I_o/\sqrt{2}$ ______ (20.2 a)

r.m.s value for alternating voltage: $V_{rms} = V_o/\sqrt{2}$ ______ (20.2 b)

For example, if the effective or r.m.s value of an alternating current is 7 A, then the alternating current will produce the same heating effect as that produced by 7 A direct current.

Mean Power and Maximum Power

As the equation of the alternating current varying sinusoidally is given by:

$$I = I_o \sin\omega t$$

If this current is passed through a resistance R, then power delivered at any instant is

$$P = I^2 R$$

$$P = (I_o \sin\omega t)^2 R$$

or $$P = I_o^2 R \sin^2 \omega t$$

For Your Information

An alternating current or voltage can also be represented as a cosine function of time, i.e.,

$$I = I_o \cos\omega t$$

$$V = V_o \cos\omega t$$

As the current is squared, so power is always positive. Since the value of $\sin^2\omega t$ varies between 0 and 1, its average value is 1/2. So, the average power delivered can be expressed as:

$$\langle P \rangle = \frac{1}{2} I_o^2 R \qquad \text{\_\_\_\_\_\_} (20.3)$$

This shows that the mean power in a resistive load is half the maximum power for a sinusoidal alternating current.

Example 20.1: An A.C. circuit consists of a pure resistance of 20 Ω and is connected across A.C. supply of 220 V, 50 Hz. Calculate (a) peak value of voltage (b) peak value of current (c) equation for voltage and current.

Given: R = 20 Ω $V_{rms} = 220$ V f = 50Hz
To Find: a) V_o = ? a) I_o = ? b) Equation for V and I = ?
Solution: a) For peak value of an alternating voltage, we use:

$$V_{rms} = V_o/\sqrt{2} \quad \text{OR} \quad V_o = \sqrt{2}\, V_{rms}$$

Putting values, we get:

$$V_o = \sqrt{2} \times 220 = 311.1\ \text{V}$$

b) For peak value of an alternating voltage, we use:

$$I_o = V_o/R$$

Putting values, we get:

$$I_o = 311.1/20 = 15.55\ \text{A}$$

c) As, $\omega = 2\pi f = 2\pi \times 50 = 314$ rad/s
So, equation for voltage is:

$$V = V_o \sin\omega t$$

$$V = 311.1 \sin(314t)$$

Equation for current is:

$$I = I_o \sin\omega t$$

$$I = 15.55 \sin(314t)$$

Assignment 20.1

The peak voltage of an ac supply is 320 V. What is the rms value of this voltage?

20.2 RECTIFICATION

Every electronic circuit needs a dc voltage for its functioning. This dc voltage has been obtained from the ac supply. For this purpose, the ac supply voltage has to be reduced (stepped down) first using a step-down transformer and then converted to dc by using a circuit called rectifier.

The process of converting ac voltage into dc voltage is called rectification.

A rectifier circuit uses diode for rectification. There are two types of rectification processes; half wave rectification and full wave rectification.

20.2.1 Half Wave Rectification

In a half-wave rectification process an AC signal is converted into DC by passing one half-cycle of the waveform and blocking the second-half. Half-wave rectifiers can be easily constructed using one diode. A diode D is connected in series with the load resistance R, as shown in Fig. 20.3.

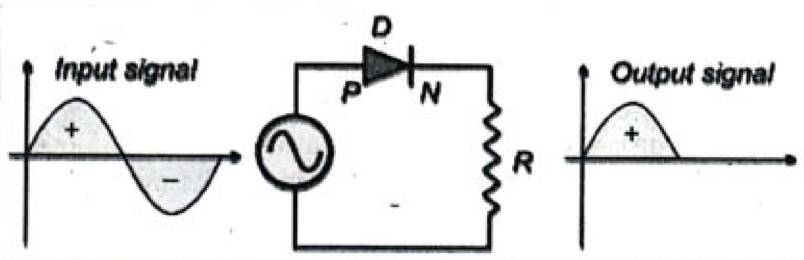


Figure 20.3: Half-wave rectifier circuit.

For the positive half cycle of the AC voltage, the diode D is forward biased, so it offers very low resistance and current flows through the resister R. Hence when the diode is forward biased, it acts as a closed switch, as shown below in the Fig. 20.4 (a).

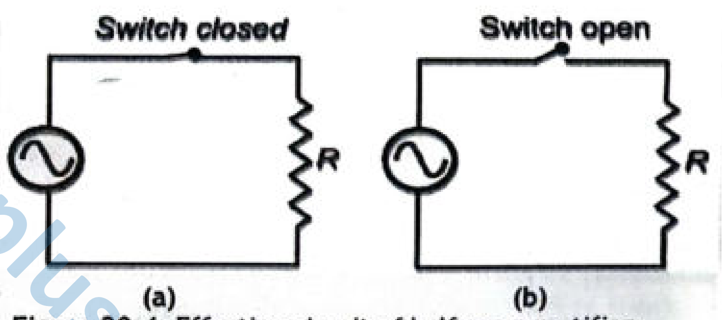


Figure 20.4: Effective circuit of half wave rectifier during: (a) forward biased (b) reversed biased.

For the negative half cycle of the AC voltage, the diode D is reversed biased, so it offers very high resistance and no current flows through the resister R (the output voltage is equal to zero). Hence when the diode is reversed biased, it acts as an open switch, as shown in the Fig. 20.4 (b). The half-wave rectifier's waveform before and after rectification is shown in the Fig. 20.5. The output waveform of a halfwave rectifier is a pulsating DC waveform. Filters in halfwave rectifiers are used to transform the pulsating DC waveform into constant DC waveforms. A capacitor can be used as a filter.

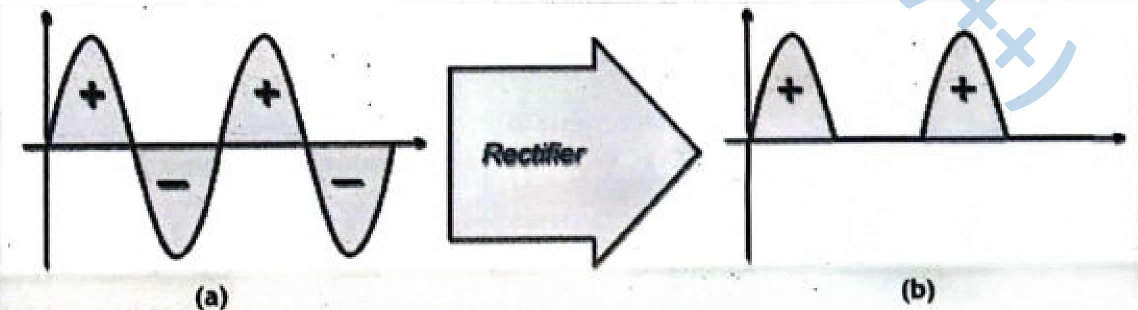
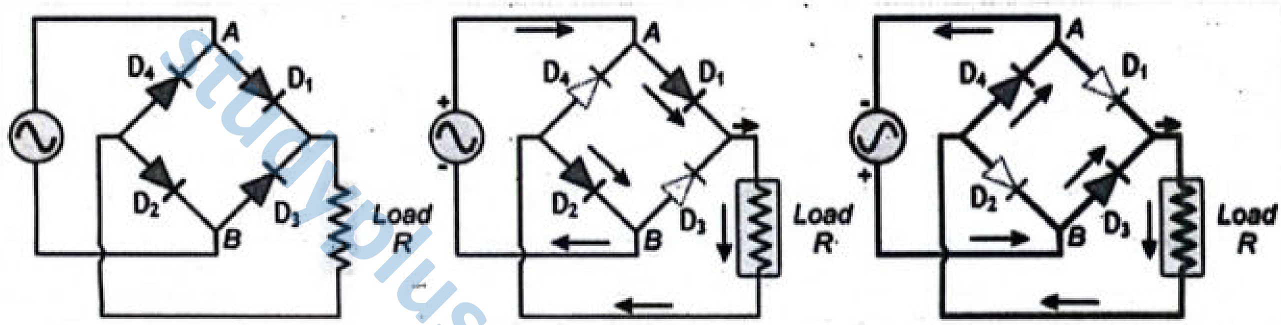


Figure 20.5: Half-wave rectification:
(a) input signal's waveform (b) output signal's waveform.

20.2.2 Full Wave Rectification

In full wave rectification, the complete cycle of AC signal is converted into pulsating DC. The circuit of the full wave rectifier can be constructed in two ways. The one method uses a centre-tapped transformer and two diodes. This arrangement is known as a centre-tapped full wave rectifier. Another method uses four diodes arranged as a bridge. This is known as a bridge rectifier. Here we will discuss only full wave bridge rectifier circuit.

Working of a Full Wave Bridge Rectifier: A full wave bridge rectifier is shown in the Fig. 20.6 (a). The circuit consists of four diodes D_1, D_2, D_3 and D_4 connected to form a bridge.



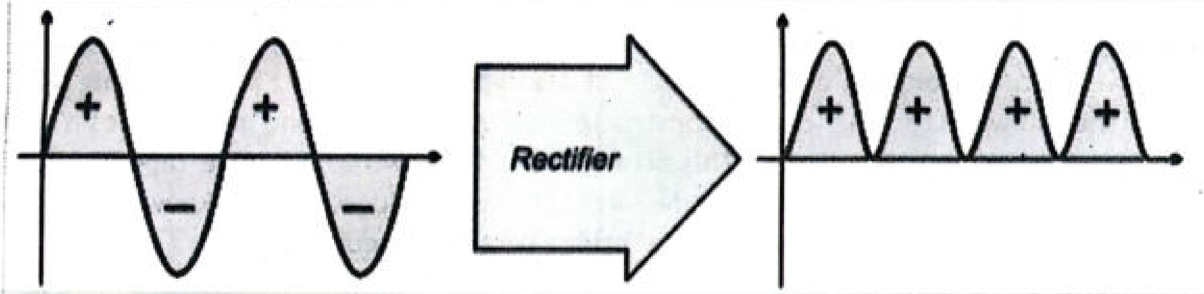
(a) Bridge rectifier (b) Positive half cycle (c) Negative half cycle

Figure 20.6: (a) Full wave bridge rectifier circuit. (b) Working of a full wave rectifier circuit during positive half cycle. (c) Working of a full wave rectifier circuit during negative half cycle. The arrows are showing the direction of the current flow.

During positive half cycle of secondary voltage, the end A becomes positive and end B is negative, as shown in Fig. 20.6 (b). This makes diodes D_1 and D_2 forward biased and diodes D_3 and D_4 reverse biased. Therefore, diodes D_1 and D_2 conduct while diodes D_3 and D_4 do not conduct. Thus, current (I) flows through diode D_1, load resistor R (from top to bottom), diode D_2 and to the negative terminal of input.

During negative half cycle, the end A becomes negative with respect to end B, as shown in Fig. 20.6 (c). This brings diodes D_3 and D_4 under forward bias and diodes D_1 and D_2 under reverse bias. Therefore, diodes D_3 and D_4 conduct while diodes D_1 and D_2 do not. Thus, current flows through diode D_3, load resistor R (from Top to bottom), diode D_4 and to the negative terminal of input, as shown in Fig. 20.6 (c).

It is obvious that one pair (D_1 and D_2) allows current flow during the positive half cycle of input voltage while the other pair (D_3 and D_4) allows current flow during the negative half cycle of input voltage. The current flowing through the load resistor R is in the same direction (top to bottom) during both half cycles. Hence, rectified output voltage is obtained across the load resistor R. The wave shape of input and output voltage is shown in Fig. 20.7.



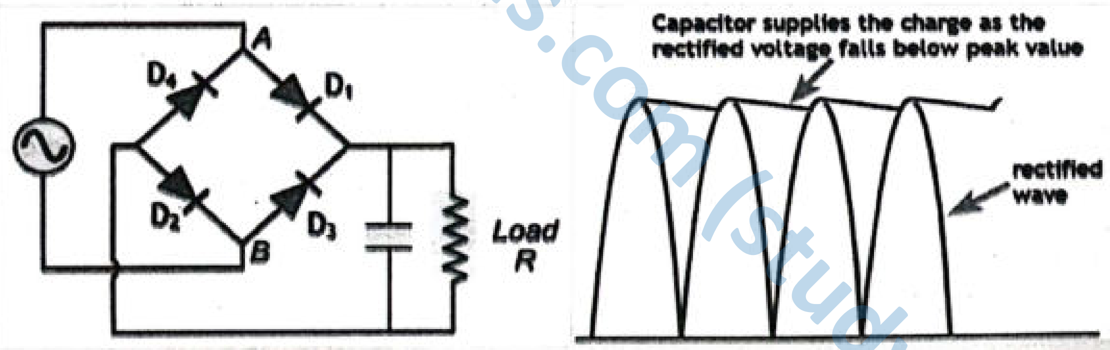
(a) (b)

Figure 20.7: Full-wave rectification:
(a) input signal's waveform (b) output signal's waveform.

Filtering

In a rectifier circuit (as shown in the Fig. 20.8-a), a capacitor smooths out the pulsating direct current (DC) into a more stable, constant output. This process is often referred to as 'filtering'.

As the voltage rises up to the peak of the input waveform, the capacitor charges to the peak voltage. When the input voltage drops, the capacitor discharges slowly by supplying charge to the load. The capacitor's discharge helps to fill in the gaps between the peaks of the rectified waveform, thereby reducing the voltage fluctuations and making the output smoother, as shown in the Fig. 20.8 (b).



(a) (b)

Figure 20.8: (a) Full-wave rectifier with capacitor: (b) smoothing action by capacitor.

The choice of capacitance value and load resistance depends on the specific application requirements. A higher capacitance value provides better smoothing, but may increase the charging current. A lower load resistance causes the capacitor to discharge faster, reducing its ability to smooth the output.

20.3 MUTUAL INDUCTANCE AND SELF-INDUCTANCE

Self-Induction: Consider a coil, which is connected to a source of emf say a battery, through a switch and a rheostat, as shown in Fig. 20.9 (a). If we move the rheostat quickly, the current through the coil will change with time. This change in current in the coil, changes the magnetic field, hence the flux through the coil changes, which finally induces an emf in the coil itself.

The phenomenon, in which a changing current induces an emf in the coil itself, is called self-induction.

Self-induced emf in a coil is proportional to time rate of change of current through the coil, i.e.,

$$\varepsilon_L = -L\frac{\Delta I}{\Delta t} \qquad \text{———— (i)}$$

Self-inductance of the coil 'L' can be defined as:

Self-inductance of a coil is the ratio of emf induced in the coil to the time rate of change of current through it.

Eq. (i) can also be written as:

$$L = \frac{-\varepsilon_L}{\Delta I/\Delta t} \qquad \text{———— (20.4)}$$

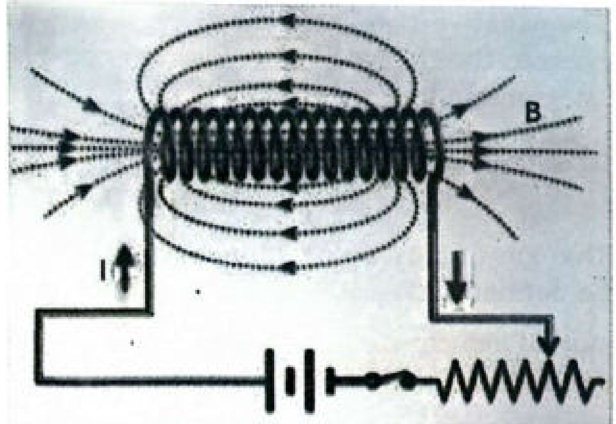


Figure 20.9 (a): Self-induction

The negative sign in eq. (20.4) is with accord of Lenz's law. It indicates that self-induced emf opposes the change, which produces it. due to this reason, self-induced emf is sometimes called as 'Back emf'. Due to self-inductance of coils, these are also known as 'Inductors' or '**choke**'. Inductors are widely used in electrical technologies. In A.C. circuits they behave like resistors.

From the eq. (20.4), the unit of self-inductance is henry (H),

$$H = V\,s\,A^{-1}$$

In case of self-inductance, one henry can be defined as:

Self-inductance of a coil will be one henry, if one volt emf is induced in the coil by the change of current at the rate of one ampere per second.

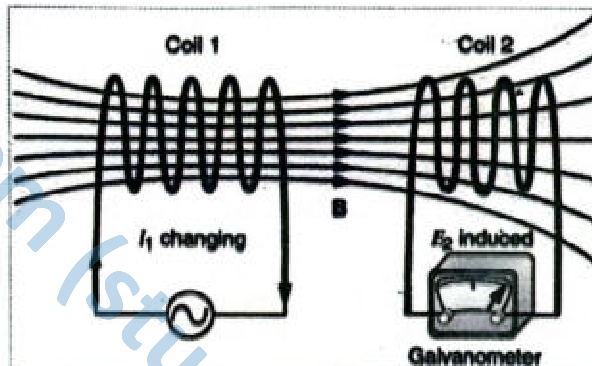


Figure 20.9 (b): Mutual induction between two coils

Mutual Induction: Consider two neighboring coils, as shown in Fig. 20.9 (b). One coil is connected to an emf source (A.C supply or D.C source with varying magnitude), while the other coil is connected to a galvanometer. The coils are placed near each other, such that the flux of coil-1 links with the coil-2. When A.C flows through coil-1 due to change of magnitude and direction of A.C, flux through the coil-1 changes, as this flux also links the coil-2, the changing flux through the coil-2 induces an emf in it, which can be seen by deflection of galvanometer's needle connected to the coil-2. This is called mutual induction, which can be defined as:

The phenomenon, in which changing current in one coil, induces an emf in neighboring coil, is called mutual induction.

The emf induced in coil-2 is proportional to time rate of change of current through the coil-1, i.e.,

$$\varepsilon_2 = -M\frac{\Delta I_1}{\Delta t} \qquad \text{———— (ii)}$$

The negative sign in the equation shows, that the direction of induced current is such that, it opposes the change of current in the coil-1. From above relation the value of mutual inductance 'M' can also be written as:

$$M = \frac{\varepsilon_2}{\Delta I_1/\Delta t} \qquad \text{———— (20.5)}$$

The unit of mutual-inductance is also henry (H). In case of mutual inductance, one henry can be defined as:

Mutual inductance between two coils is 1 henry if current changing at the rate of 1 A/s in one coil induces an emf of 1 V in the other coil.

20.4 PHASE OF A.C

In an AC circuit, the alternating current and voltage do not peak at the same time through capacitors or Inductors. Sometime voltage may be passing through its zero point while the current has passed or it is yet to pass through its zero point in the same direction. The angle between their zero points is the phase difference. The quantity which passes through its zero point earlier is said to leading while the other is said to be lagging. Since both alternating quantities have the same frequency, the phase difference between them remains the same.

The fraction of a period difference between the peaks expressed in degrees is said to be the phase difference.

It is represented by Φ. It is generally measured in degrees or radians. The phase difference is always less than or equal to 90° (It is customary to use the angle by which the voltage leads the current).

20.4.1 A.C. Through Resistor

Consider a circuit containing a resistor connected across an alternating voltage source as shown in Fig 20.10 (a).

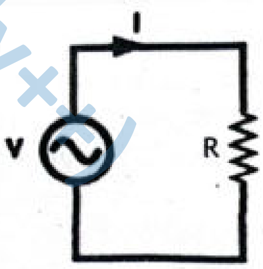


Figure 20.10 (a): A.C through a resistor.

The alternating voltage is be given by:

$$V = V_o \text{ Sin}\omega t \qquad \text{———— (i)}$$

Where, V_o is the peak value of the alternating voltage. As a result of this voltage, an alternating current 'I' will flow in the circuit. According to Ohm's law:

$$V = IR \qquad \text{or} \qquad I = \frac{V}{R}$$

or $$I = \frac{V_m \sin\omega t}{R}$$

As, $\frac{V_m}{R} = I_o$, so

$$I = I_o \sin\omega t \quad \_\_\_\_\_\_ \text{(ii)}$$

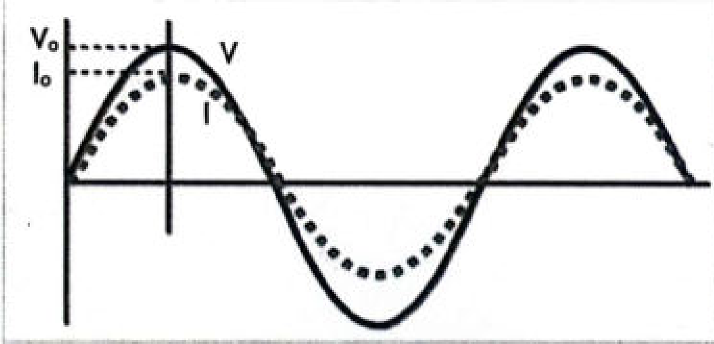


Figure 20.10 (b): Voltage and current are in phase.

Eqs. (i) and (ii) shows that:

In a resistor, applied voltage and the circuit current are in phase with each other.

Figure 20.10 (b) shows the waveform of current and voltage through a resistor. The applied voltage and current pass through their zero values at the same instant. The applied voltage and current attain their positive and negative peaks at the same instant. Hence current is in phase with the applied voltage. This is also indicated by the phasor diagram, as shown in Fig. 20.10 (c).

Figure 20.10 (c): Phasor diagram for resistor.

20.4.2 A.C Through an Inductor

An inductor is an electrical component which opposes changes in electric current passing through it. It consists of a conductor (such as a wire) usually wound into a coil.

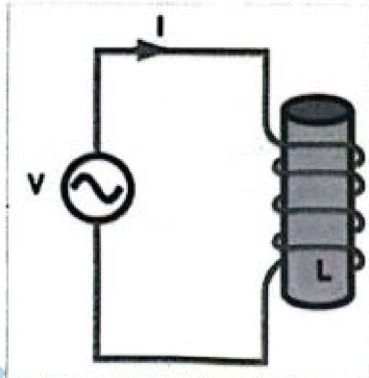


Figure 20.11 (a): A.C through an Inductor.

Consider an alternating voltage is applied across an inductor of inductance L, as shown in Fig. 20.11 (a). Let the equation for alternating current is:

$$I = I_o \sin\omega t \quad \_\_\_\_\_\_ \text{(i)}$$

When a sinusoidal current 'I' flows in time t then a back e.m.f. (=$L\Delta I/\Delta t$) is induced due to the inductance of the coil. This back e.m.f. at every instant opposes the change in current through the coil. As there is no drop in potential across the inductor (because resistance of an ideal inductor is zero), so the applied voltage has to overcome the back emf, i.e.,

$$\text{Applied alternating voltage} = \text{Back emf} \quad \_\_\_\_\_\_ \text{(ii)}$$

So, the energy which is required in building up current in inductance L, is returned back during the decay of the current. The changing current sets up a back e.m.f in the coil. The magnitude of back e.m.f is:

$$\varepsilon = L\frac{\Delta I}{\Delta t}$$

Using Eq. (i) and Eq. (ii), the magnitude of applied voltage is:

For Your Information

A choke is an inductor used in a circuit. It offers high reactance to frequencies above a certain frequency range, without appreciable limiting the flow of current.

$$V = L\frac{\Delta I}{\Delta t} = L\frac{\Delta(I_o \sin\omega t)}{\Delta t}$$

As, $\dfrac{\Delta(\sin\omega t)}{\Delta t} = \omega \cos\omega t,$

So, $$V = L\, I_o \frac{\Delta(\sin\omega t)}{\Delta t}$$

$$V = \omega L I_o \cos \omega t$$

or $$V = V_o \cos \omega t$$

or $$V = V_o \sin\left(\omega t + \frac{\pi}{2}\right) \quad \text{\_\_\_\_\_\_ (20.6)}$$

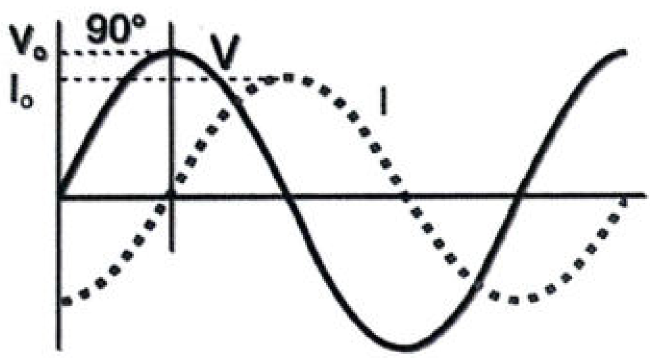


Figure 20.11(b): Voltage leads the current by 90°.

From Eq. (i) and Eq. (20.6), it is clear that:

In an inductor, voltage leads the current by π/2 radians or 90°.

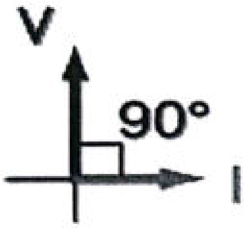


Figure 20.11 (c): Phasor diagram for inductor.

Figure 20.11 (b) also shows that current lags the voltage in an inductive coil. Inductance opposes the change in current and serves as increase or decrease of current in the circuit. This causes the current to lag behind the applied voltage which is indicated by the phasor diagram shown in Fig. 20.11 (c).

Inductive Reactance (X_L): The opposition offered by an inductor to the flow of A.C. is called inductive reactance. Therefore, in analogy to Ohm's law we can write:

$$V_o = I_o X_L$$

or $$X_L = \frac{V_o}{I_o}$$

or $$X_L = \frac{I_o \omega L}{I_o} \qquad \text{(using } V_o = I_o \omega L\text{)}$$

$$X_L = \omega L \qquad \text{or} \qquad X_L = 2\pi f L \quad \text{\_\_\_\_\_\_ (20.7)}$$

Hence, reactance of a coil depends upon frequency of A.C. In case of D.C, $f = 0$ so $X_L = 0$. Inductance has the same dimensions as resistance; therefore, it is measured in Ω.

20.4.2 A.C Through a Capacitor

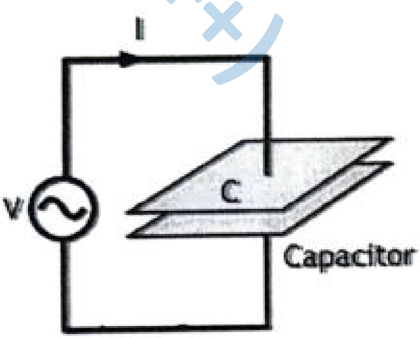


Figure 20.12 (a): A.C through a capacitor.

Consider an alternating voltage is applied across a capacitor of capacitance C, as shown in Fig. 20.12 (a). Due to the alternating voltage, the capacitor is charged in one direction and then in the other as the voltage reverses. The result is that electron move to-and-fro around the circuit, connecting the plates, thus constituting alternating current. The basic relation between the charge q on the capacitor and voltage V across its plates i.e. $q = CV$ holds at every instant. Let the equation for the applied alternating voltage is:

$$V = V_o \sin\omega t \quad \text{\_\_\_\_\_\_ (i)}$$

Then, at any instant charge on capacitor is:

$$q = CV = C V_o \sin\omega t$$

Now current 'I' flowing through capacitor is:

$$I = \frac{\Delta q}{\Delta t} = \frac{\Delta(CV_o \sin\omega t)}{\Delta t}$$

$$I = CV_o\,\omega \cos(\omega t)$$

$$I = CV_o\,\omega \sin\left(\omega t + \frac{\pi}{2}\right)$$

Here $I_o = CV_o\,\omega$, so

$$I = I_o \sin\left(\omega t + \frac{\pi}{2}\right) \quad \text{\_\_\_\_\_\_ (20.8)}$$

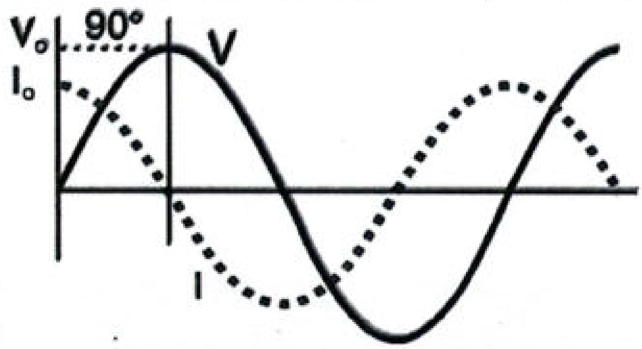


Figure 20.12 (b): Voltage lags the current by 90°.

From the Eq. (i) and Eq. (20.8), it is clear that:

In a capacitor, voltage lags the current by $\pi/2$ radians or 90°.

Capacitance opposes the change in voltage and serves to delay the increase or decrease of voltage across capacitor.

The phasor diagram in Fig. 20.12 (c) shows that in a capacitor the voltage lags behind the current.

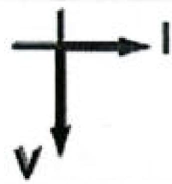


Figure 20.12 (c): Phasor diagram for capacitor.

Capacitive Reactance (X_C): The opposition offered by a capacitor to the flow of A.C is called capacitive reactance. Therefore, in analogy to Ohm's law, we can write:

$$V_o = I_o X_C$$

or $\quad X_C = \dfrac{V_o}{I_o} \quad$ or $\quad X_C = \dfrac{V_o}{CV_o\omega} \quad$ (As for capacitor, $I_o = C V_o \omega$)

$$X_C = \frac{1}{C\omega} \quad \text{or} \quad X_C = \frac{1}{2\pi f C} \quad \text{\_\_\_\_\_\_ (20.9)}$$

The capacitive reactance depends upon frequency of A.C. In case of D.C, X_C has infinite value. Capacitive reactance has the same dimensions as resistance; therefore, it is measured in Ω.

Mnemonic 'ELI the ICE man'

The mnemonic 'ELI the ICE man' helps to remember the phase difference of voltage and current in capacitor and inductor.

Voltage leads Current — Current leads Voltage

E L I the I C E man

in an inductor — in a capacitor

20.5 IMPEDANCE

Impedance is the combine effect of the resistance and the reactance present in an AC circuit. Thus, impedance can be broken down into resistance and reactance. Impedance is equivalent to resistance in AC circuit. These two provide opposition to the flow of alternating current in the circuit, and are generated due to the presence of capacitor and inductor. Let us explore impedance in RL and RC series circuits:

RL Series AC Circuit

Consider an AC circuit in which a resistor (R) is connected in series with a coil of pure inductance (L), as shown in Fig. 20.13 (a). Here, the voltage 'V' will be the phasor sum of the two component voltages, V_R and V_L. This means that the current flowing through the coil will still lag the voltage, but by an amount less than 90° depending upon the values of V_R and V_L. Taking current as the reference phasor, the phasor diagram of the circuit can be drawn, as shown in Fig. 20.13 (b).

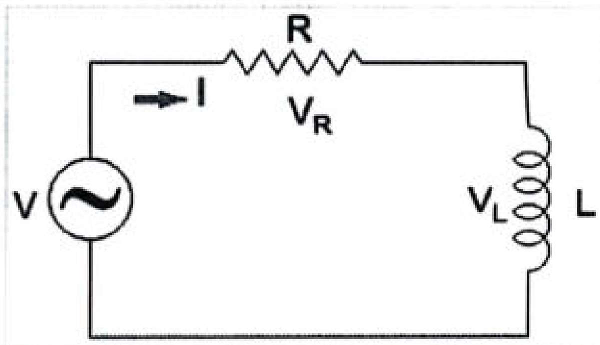


Figure 20.13 (a): RL series circuit.

- The voltage drops, $V_R = IR$ is **in phase** with current and is represented by the phasor OA.
- The voltage drops $V_L = IX_L$ leads the current by 90° and is represented the phasor AB.

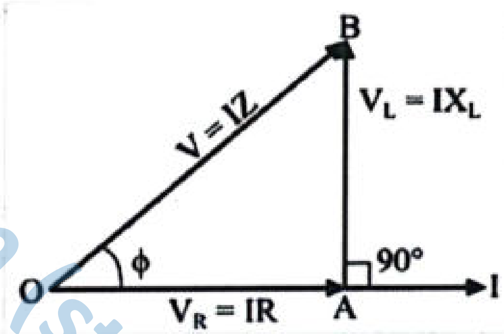


Figure 20.13 (b): Phasor diagram of RL series circuit.

The applied voltage V is the phasor sum of these two drops i.e.,

$$V^2 = V_R^2 + V_L^2$$

or

$$V = \sqrt{V_R^2 + V_L^2}$$

$$V = \sqrt{(IR)^2 + (IX_L)^2}$$

$$V = I\sqrt{R^2 + X_L^2}$$

$$\frac{V}{I} = \sqrt{R^2 + X_L^2}$$

As, V/I represent impedance and is denoted by Z. So,

$$Z = \sqrt{R^2 + X_L^2} \quad \text{\_\_\_\_\_\_} \quad (20.10)$$

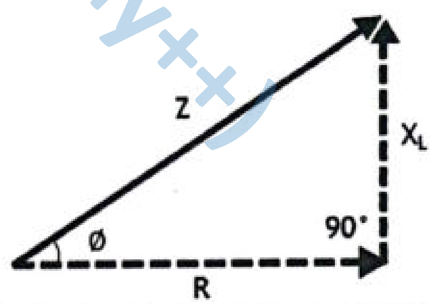


Figure 20.13 (c): RL Series Impedance triangle.

Where $X_L = 2\pi f L$. RL Series Impedance triangle is shown in Fig. 12.13 (c), having sides *R*, *X_L and Z*. The magnitude of impedance in R-L series circuit depends upon the values of resistance R, inductance L and supply frequency f.

RC Series AC Circuit

Consider an AC circuit in which a capacitor (C) and resistor (R) are connected in series with each other, as shown in Fig 20.14 (a).

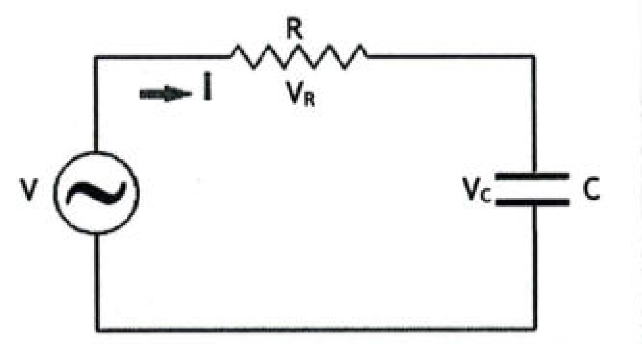


Figure 20.14 (a): RC series circuit.

The voltage 'V' across the combination is equal to the phasor sum of two component voltages, $V_R = IR$ & $V_C = IX_C$. Taking current as the reference phasor, the phasor diagram of the circuit can be drawn, as shown in Fig 20.14 (b).

The voltage drop V_R (= IR) is in phase with current and is represented by the phasor OA.

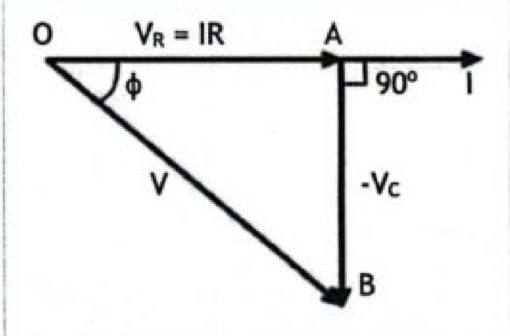


Figure 20.14 (b): Phasor diagram of RC series circuit.

The voltage drop V_C (= IX_C) lags behind the current by 90° and is represented by the phasor AB. The applied voltage V is the phasor sum of these two potential drops i.e.,

$$V^2 = V_R^2 + V_C^2$$

or

$$V = \sqrt{V_R^2 + V_C^2}$$

$$V = \sqrt{(IR)^2 + (-IX_C)^2}$$

$$V = I\sqrt{R^2 + X_C^2}$$

$$\frac{V}{I} = \sqrt{R^2 + X_C^2}$$

As, V/I represent impedance and is denoted by Z. So,

$$Z = \sqrt{R^2 + X_C^2} \qquad \text{\_\_\_\_\_\_ (20.11)}$$

Where $X_C = \frac{1}{C\omega}$. Impedance Z is measured in ohms (Ω).

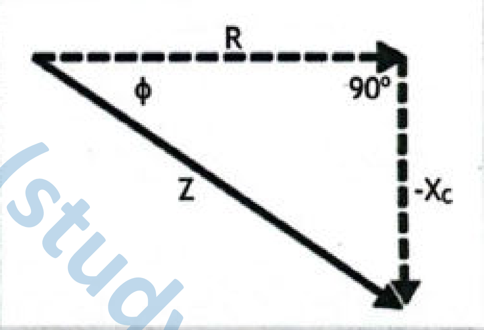


Figure 20.13 (c): RC series impedance triangle.

Example 20.2: A.C voltage across a 0.5 μF capacitor is 16sin(2×10³t) V. Find (a) the capacitive reactance (b) the peak value of current through the capacitor.

Given: $C = 0.5\ \mu F = 0.5\times 10^{-6}$ F $\qquad V = 16\sin(2\times10^3 t)$ V

To Find: a) $X_C = ?$ $\qquad$ b) $I_o = ?$

Solution: a) As alternating voltage is given by relation

$$V = V_o \sin(\omega t) \qquad \text{\_\_\_\_\_\_ (i)}$$

Also given that $\quad V = 16\sin(2\times10^3 t) \qquad$ ______ (ii)

Comparing (i) and (ii), we get:

$$V_o = 16\text{ V} \qquad \text{and} \qquad \omega = 2\times10^3$$

So, $X_C = \frac{1}{C\omega} = \frac{1}{(0.5\times10^{-6})(2\times10^{3})} = 1000\ \Omega$

b) Using ohm's law:

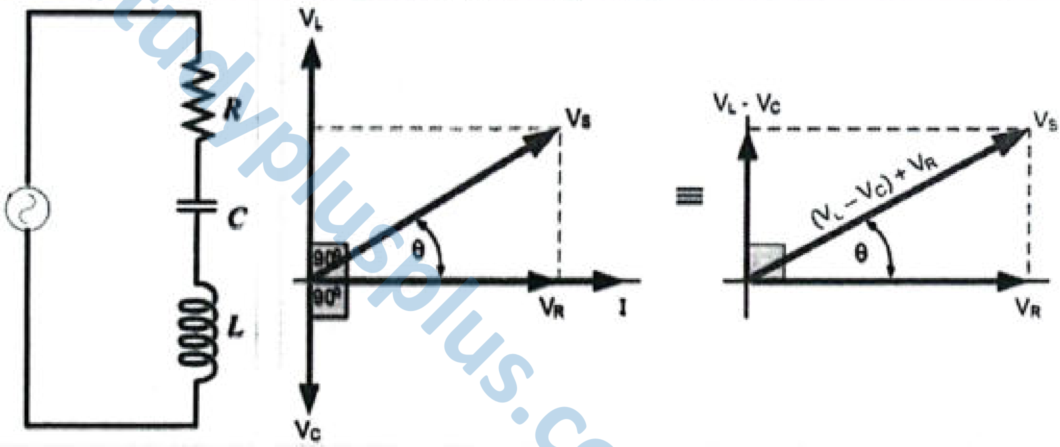
$$I_o = \frac{V_o}{X_c} = \frac{16}{1000} = 16\times10^{-3}\ \text{A} = 16\ \text{mA}$$

Assignment 20.2

The voltage across a 0.01 μF capacitor is 240 sin($1.25\times10^4 t$ - 30°) V. Write the mathematical expression for the current through it.

For Your Information

A circuit in which a resistor (R), an inductor (L), and a capacitor (C) are connected in series is called RLC series circuit. All the elements of RLC series circuit share the same current. The RLC series circuit and its Phasor diagram is given below:



SUMMARY

- One complete set of positive and negative values of an alternating quantity is known as a cycle.
- The time taken to complete one cycle of an alternating quantity is called its time period.
- The number of cycles that occurs in one second is called the frequency of the alternating quantity. It is measured in cycle/second or Hertz.
- The average value of a waveform is the average of all its values over a period of time.
- Maximum value of alternating quantity (current or voltage) is called peak value.
- The r.m.s value of an alternating current is that steady current (d.c.) which when flowing through a resistor produce the same amount of heat as that produced by the alternating current when flowing through the same resistance for the same time.
- The process of converting ac voltage into dc voltage is called rectification.
- In a half-wave rectification process an AC signal is converted into DC by passing one half-cycle of the waveform and blocking the second-half.
- In full wave rectification, the complete cycle of AC signal is converted into pulsating DC.

- In a rectifier circuit, a capacitor smooths out the pulsating direct current (DC) into a more stable, constant output. This process is often referred to as 'filtering'.
- The phenomenon, in which a changing current induces an emf in the coil itself, is called self-inductance. Self-inductance of a coil is the ratio of emf induced in the coil to the time rate of change of current through it.
- Self-inductance of a coil will be one henry, if 1 volt emf is induced in the coil by the change of current at the rate of one ampere per second.
- The phenomenon, in which changing current in one coil, induces emf in neighboring coil, is called mutual induction.
- Mutual inductance between two coils is 1 henry if current changing at the rate of 1 A/s in one coil induces an emf of 1 V in the other coil.
- The fraction of a period difference between the peaks expressed in degrees is said to be the phase difference.
- In an inductor, volage leads the current by $\frac{\pi}{2}$ radians or 90°.
- In a capacitor, volage lags the current by $\frac{\pi}{2}$ radians or 90°.
- A Choke is an inductor used in a circuit. It offers high reactance to frequencies above a certain frequency range, without appreciable limiting the flow of current.
- Impedance is the combine effect of the resistance and the reactance present in an AC circuit.

EXERCISE

Multiple Choice Questions

Encircle the Correct option.

1) A full wave bridge rectifier consists of
   A. no diode B. one diode C. Two diodes D. four diodes
2) Unit of reactance is
   A. henry B. ohms C. volt D. ampere
3) The unit of self-inductance, henry (H), is equal to:
   A. $A\,s^{-1}\,V^{-1}$ B. $V\,A\,s^{-1}$ C. $V\,s^{-1}\,A^{-1}$ D. $V\,s\,A^{-1}$
4) A capacitor is perfectly insulator for
   A. direct current
   B. alternating current
   C. direct as well as alternating current
   D. neither for direct current nor for alternating current
5) The peak value of alternating current is $5\sqrt{2}$ A. The mean square value of current will be
   A. 5 A B. 2.5 A C. $5\sqrt{2}$ A D. 5^2 A
6) In AC system we generate sine wave form because
   A. It can be easily draw B. It produces least disturbance in electrical circuits

C. It is nature standard D. Other waves cannot be produced easily.

7) An alternating voltage is given by 20sin(157t). The frequency of alternating voltage is
A. 50 Hz B. 25 Hz C. 100 Hz D. 75 Hz

8) Peak value of an alternating quantity (voltage or current) can be found if we know it's
A. phase B. r.m.s value C. frequency D. both (a) & (b)

Short Questions

1) Define (a) mutual-induction (b) self-induction.
2) Does the SI unit used for mutual-inductance and self-inductance are same? Explain briefly.
3) Draw the circuit for a half wave rectifier and full wave rectifier.
4) What is the use of a single diode for the half-wave rectification of an alternating current?
5) Distinguish graphically between half-wave and full-wave rectification.
6) Is the frequency content of the output of a half wave rectifier and full wave rectifier the same? Explain briefly.
7) In half wave rectifier, the half of the signal is blocked after the rectification. Why?
8) Distinguish between root-mean-square (r.m.s.) and peak values for a sinusoidal alternating current.
9) What is impedance? How is it related to resistance, reactance, and frequency? Also find its SI unit?
10) What is the difference between reactance and impedance?
11) Why does a capacitor block DC but allow AC to pass through?
12) How does the reactance of a capacitor change with an increase in frequency?
13) What is the phase difference between voltage and current in a (a) capacitor (b) Inductor?
14) How does the capacitance of a capacitor affect the current through it in an AC circuit?

Comprehensive Questions

1) What is half wave rectification? Explain.
2) With the help of a diagram, explain the operation of a bridge rectifier.
3) Analyze the effect of a single capacitor in smoothing current flow.
4) Explain the phenomenon of self-induction.
5) A sinusoidal alternating voltage of angular frequency ω is connected across an inductor. Find mathematical expression for instantaneous voltage and instantaneous current.
6) Explain the term impedance of an AC circuit. Find its expression for the RL series circuit.
7) In an RL series circuit will the current lag or lead the applied alternating voltage? Explain the answer with a phasor diagram.

8) Describe the phase of A.C and how phase lags and leads in A.C circuits.

9) Describe impedance as vector summation of resistances and reactances.

Numerical Problems

1) What is the rms value of the voltage of an ac supply having peak voltage 300 V? **(Ans: 212 V)**

2) An inductor with an inductance of 100 μH passes a current of 10 mA when its terminal voltage is 6.3 V. Calculate the frequency of A.C supply. **(Ans: 10^6 Hz)**

3) A coil of pure inductance 318 mH is connected in series with a pure resistance of 75 Ω. The voltage across resistor is 150 V and the frequency of power supply is 50 Hz. Calculate impedance of the circuit. **(Ans: 124.9 Ω)**

4) A resistor of resistance 30 Ω is connected in series with a capacitor of capacitance 79.5 μF across a power supply of 50 Hz and 100 V. Find impedance of the circuit. **(Ans: 50 Ω)**

Student Learning Outcomes (SLOs)

The student will

- state that electromagnetic radiation has a particulate nature.
- Explain and apply the photonic model of light to solve problems [use $E = hf$ to solve problems, and use the electronvolt (eV) as a unit of energy]
- Explain that a photon has momentum [including that the momentum is given by $p=E/c$ (connect with the idea that light can exert a force)]
- describe that photoelectrons may be emitted from a metal surface when it is illuminated by electromagnetic radiation.
- describe and use the terms threshold frequency and threshold wavelength.
- explain photoelectric emission in terms of photon energy and work function energy.
- state and apply $hf=\emptyset + 1/2mv_{max}^2$
- explain why the maximum kinetic energy of photoelectrons is independent of intensity, whereas the photoelectric current is proportional to intensity.
- Juxtapose the evidence for light as a wave and as a particle [Explain that the photoelectric effect provides evidence for a particulate nature of electromagnetic radiation while phenomena such as interference and diffraction provide evidence for a wave nature].
- Analyze qualitatively the evidence provided by electron diffraction for the wave nature of particles.
- Explain and apply the de Broglie wavelength to solve problems [use $\lambda= h/p$ to solve problems].
- State that there are discrete electron energy levels in isolated atoms (e.g. atomic hydrogen).
- explain the appearance and formation of emission and absorption line spectra.
- use $hf = \Delta E$ to solve problems.
- Describe the Compton effect qualitatively.
- Explain the phenomena of pair production and pair annihilation.
- Explain how electron microscopes achieve very high resolution.
- State and explain Heisenberg's uncertainty principle qualitatively.
- Use the uncertainty principle to explain why empirical measurements must necessarily have uncertainty in them.

Quantum theory is the foundation of Modern Physics. At the smallest scales, the classical laws for real cases no longer apply, so quantum physics explains the nature and behaviour of matter and energy at the atomic and subatomic levels. Quantum physics is the branch of physics that studies the behavior of matter and energy at the atomic and subatomic level. In this realm, the rules of classical physics are replaced by strange and counterintuitive phenomena. Quantum physics is not just a curiosity, but has led to countless technological innovations, from transistors and lasers to computer chips and medical imaging.

In this chapter, we'll study the fundamental principles and fascinating concepts of quantum physics that help us in exploring our universe.

21.1 ELECTROMAGNETIC WAVE

In 1864, the British physicist James Clerk Maxwell made the remarkable ideas that accelerated electric charges generate linked electric and magnetic disturbances that can travel through space, as shown in Fig. 21.1. These electric field (E) and magnetic field (B) can sustain each other, forming an electromagnetic wave that propagates through space with speed about 3×10^8 m s^{-1}. The direction of propagation is the direction of the vector product **E** $\times$ **B**. Electric and magnetic components of an electromagnetic wave are perpendicular to each other and to the direction of motion.

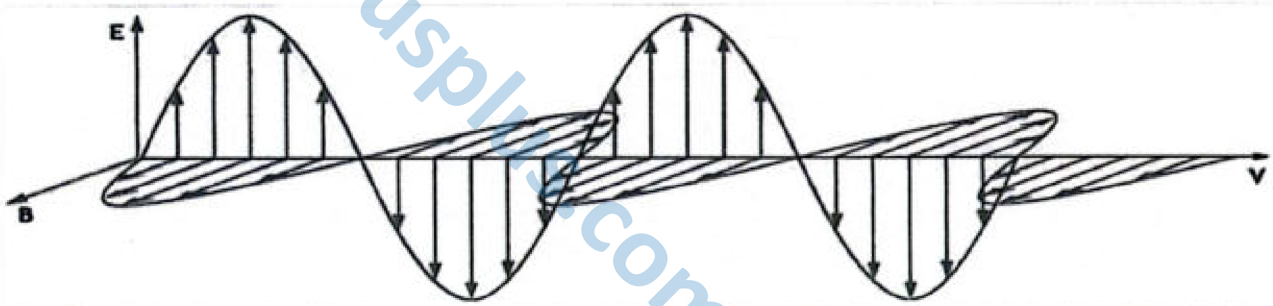


Figure 21.1: A pair of electric (red) and magnetic (blue) fields, propagating together as an electromagnetic wave in the direction indicated by the arrow at the speed of light.

Visible light emitted by the glowing filament of light bulb is an example of electromagnetic wave. There are many other waves which are electromagnetic in nature, e.g. radio waves, microwaves, infrared, ultraviolet, x-rays, and gamma rays, as shown in Fig. 21.2.

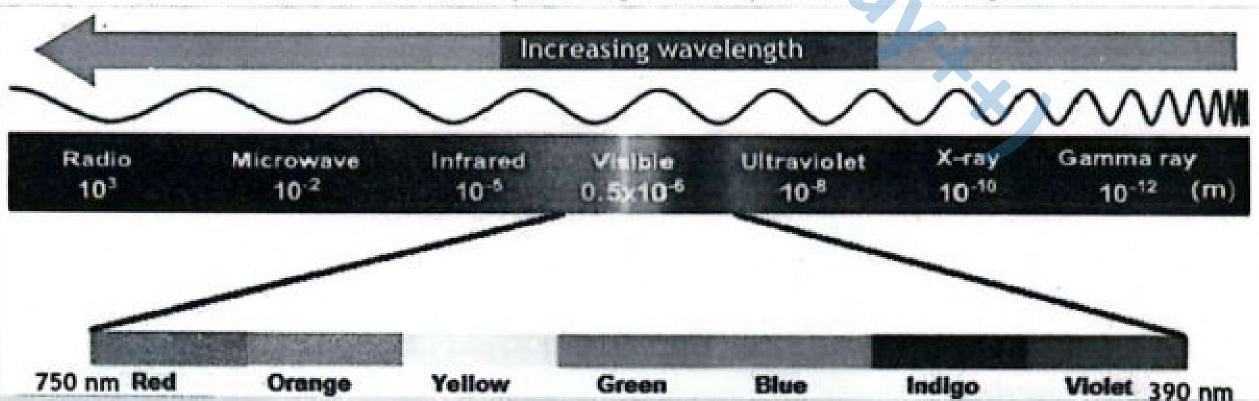


Figure 21.2: Electromagnetic radiation spectrum.

The visible wavelength spectrum of electromagnetic wave lies in between 750 nm (Red) and 390 nm (Violet). The electromagnetic wave travels through vacuum or through any specific medium from one point to another point depending on energy.

Plank's Quantum Theory

Planck's quantum theory is the fundamental theory of quantum mechanics. In 1900, Plank studied the electromagnetic radiation emitted from different atoms and molecules, and proposed a theory which was in complete agreement with experiments at all wavelengths. According to this theory:

1) Different atoms and molecules can emit or absorb energy (E) in discrete amount only, which is given by:

$$E = n\,hf \qquad \text{\_\_\_\_\_\_ (21.1)}$$

Here n = 1, 2, 3, ... is called quantum number, $h = 6.626\times10^{-34}$ J s is the plank's constant and f is the frequency of the radiation.

The energies of the molecules are said to be quantized, and the allowed energy state are called quantum states. Atoms or molecules emit or absorb energy only by jumping from one quantum state to another.

2) The smallest amount of energy that can be emitted or absorbed in the form of electromagnetic radiation is known as quantum (plural quanta).

The development of Plank's theory gave the birth to the Quantum Physics.

Example 21.1: What is the frequency of a photon whose energy is 66.3 eV?

Given: Energy of Photon = E = 66.3 eV = $66.3 \times 1.6 \times 10^{-19}$ J

To Find: Frequency = f = ?

Solution: Using the equation: $E = hf$

or $$f = \frac{E}{h} = \frac{66.3 \times 1.6 \times 10^{-19}}{6.63 \times 10^{-34}} = 1.6 \times 10^{16}\ \text{Hz}$$

Assignment 21.1

What will be the photon energy for a wavelength of 5000 angstroms, if the energy of a photon corresponding to a wavelength of 7000 angstroms is 4.23×10^{-19} J?

21.2 PHOTOELECTRIC EFFECT

The word photoelectric is the combination of two words Photo; means light and electric; means electron, it means that this process defines the interaction between photon and electron.

Photoelectric effect is the process of emitting the electrons from the metal surface when the metal surface is exposed to an electromagnetic radiation of sufficiently high frequency.

The emitted electrons are called **photoelectrons** because they are liberated by means of light.

For Your Information

The first discovery of the photoelectric effect was made by Hertz, who was also the first to produce the electromagnetic waves predicted by Maxwell.

The setup to observe the photoelectric effect is shown in Fig. 21.3. An evacuated glass tube contains a metal plate connected to the negative terminal of a battery. Another metal plate is maintained at a positive potential by the battery. When the tube is kept in dark, the ammeter (A) shows zero reading, indicating that there is no current in the circuit. However, when light of the appropriate frequency falls on the metal surface, a current is detected by the ammeter, indicating a flow of charges across the gap between the metal surface and the detector. The current associated with this process arises from electrons emitted from the negative plate and collected at the positive plate. Ultraviolet light is required in the case of emission of electrons from an alkali metal.

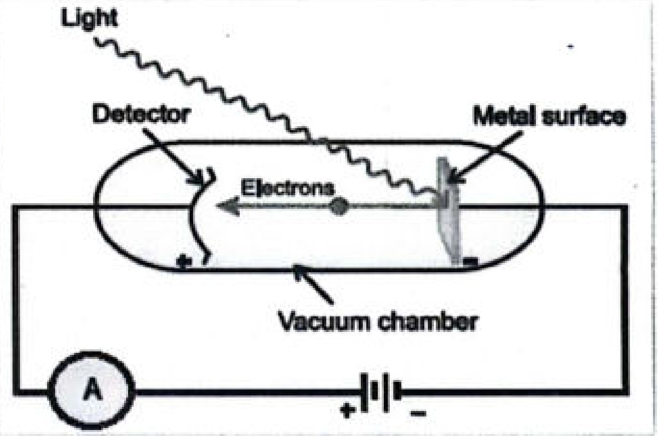


Figure 21.3: Schematic diagram of photoelectric effect setup.

Effect of Applied Voltage on Photoelectric Current

A graph of the photoelectric current versus the potential difference (V) between the metal plate and the detector for different light intensities is shown in Fig. 21.4. It can be noted from the graph that:

- The current increases as the incident light intensity increases.
- For large values of V, the current reaches a maximum value, corresponding to the case where all photoelectrons are collected at anode (detector).

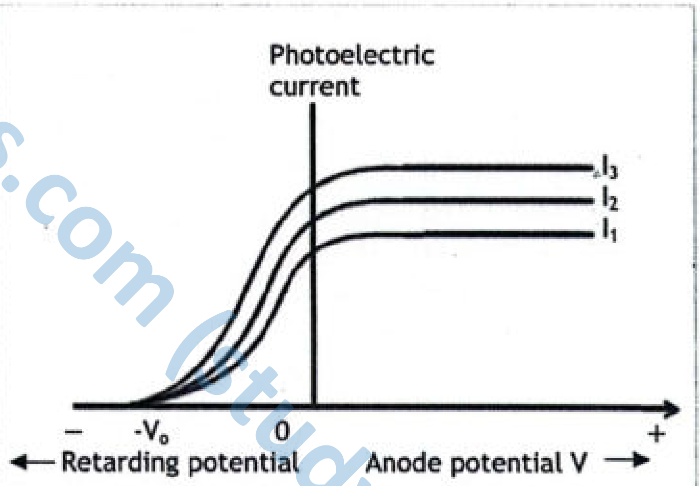


Figure 21.4: Graph of the photoelectric current versus the potential difference.

Thus, brighter light of constant frequency causes an increase in current (more electrons ejected) but does not cause the individual electrons to gain higher energies. It means that the maximum K.E of the electrons is independent of the intensity of the light. Classical physics says that more intense light has larger amplitude and thus delivers more energy. That should not only enable a larger number of electrons to escape from the metal; it should also enable the electrons emitted to have more K.E.

- When V is negative (i.e. when the battery in the circuit is reversed) the photoelectrons are repelled by the negative plate. Only those electrons having a K.E greater than 'eV' will reach the detector, where 'e' is the charge on electron.
- If V is greater than or equal to V_o, called the stopping potential, no electrons will reach the detector and the current will be zero. The stopping potential is independent of the radiation

intensity. The maximum K.E of the photoelectrons is related to the stopping potential through the relation.

$$K.E_{max} = eV_o \qquad (21.2)$$

Effect of Intensity of Incident Radiation on Photoelectric Current

If the frequency of the incident radiation and the potential difference (V) between the cathode and the anode is kept constant and the intensity of incident radiation is varied, then it is found that the photoelectric current increases linearly with the intensity of incident radiation, as shown in Fig. 21.5. Since the photoelectric current is directly proportional to the number of photoelectrons emitted per second, so it implies that:

The number of photoelectrons emitted per second is proportional to the intensity of incident radiation.

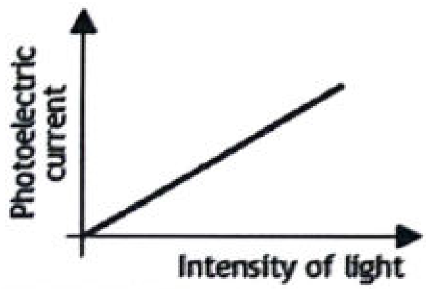


Figure 21.5: Graph of the photoelectric current versus intensity of light.

Effect of Frequency of Incident Radiation on K.E of Photoelectrons

The maximum K.E of photoelectrons depends on the frequency of the incident radiation, as shown in Fig. 21.6. If the incident light is very dim (low intensity) but high frequency, electrons with large K.E are released. Classical physics gives no explanation for the frequency dependence.

For a given metal, there is a threshold frequency 'f_o' below which no electrons are emitted, how intense the incident light may be. Classical physics has no explanation for the frequency dependence.

Work Function (Φ)

The minimum amount of energy required to escape the electron from metal surface is known as the work function (Φ) of the substance.

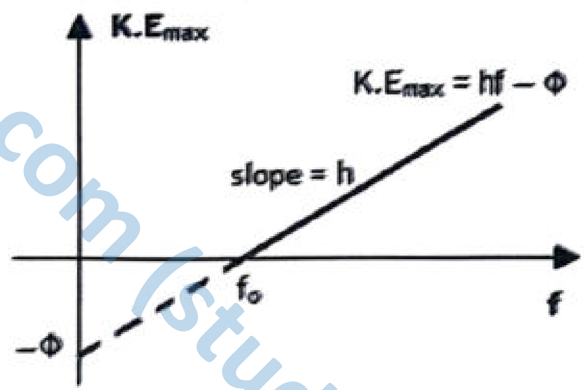


Figure 21.6: Plot between K.E of photoelectron and frequency of incident light.

Photon (Quantum) Theory of Photoelectric Effect

In the year 1905, Albert Einstein reinterpreted Planck's theory to further explain the photoelectric effect. Basically, Planck's work led Einstein in determining that light exists in discrete quanta of energy, called photons.

According to Einstein, the emission of photoelectron is the result of the interaction between a single photon of the incident radiation and an electron in the metal. When the photon's energy (E=hf) is transferred to an electron in a metal, a part of its energy (Φ) is used by the electron to break away from the metal and the rest appears as the maximum K.E of the electron. i.e.,

$$E = \Phi + K.E_{max} \qquad \text{\_\_\_\_\_\_} \quad (21.3)$$

This is called **Einstein's photoelectric equation.** Here, $\Phi = hf_o$ is the work function.

When K.E of the photoelectron is zero, the frequency 'f' is equal to threshold frequency f_o, hence the Einstein's photoelectric equation becomes:

$$hf_o = \Phi$$

Hence, we can also write Einstein's photoelectric equation as:

$$K.E_{max} = E - hf_o$$

- If $E < \Phi$, there will be no photoelectric effect.
- If $E = \Phi$, the photoelectric effect occurs, but the kinetic energy of the expelled photoelectron is 0.
- If $E > \Phi$, the photoelectric effect will occur and the expelled electron possesses kinetic energy.

There are so many practical utilizations of photoelectric effect; for example, photocells, photoconductive devices, and solar cell, etc.

Example 21.2: A metal whose work function is 4.2 eV is irradiated by radiation whose wavelength is 2000 Å. Find the maximum kinetic energy of emitted electron.

Given: Work function = Φ = 4.2 eV = $4.2 \times 1.6 \times 10^{-19}$ J = 6.72×10^{-19} J

Wavelength of radiation = λ = 2000 Å = 2000×10^{-10} m = 2×10^{-7} m

To Find: Maximum kinetic energy = $K.E_{max}$ = ?

Solution: By Einstein's photoelectric equation: $K.E_{max} = hf - \Phi$

or
$$K.E_{max} = \frac{hc}{\lambda} - \Phi$$

Putting values, we get:
$$K.E_{max} = \frac{6.63 \times 10^{-34} \times 3 \times 10^{8}}{2 \times 10^{-7}} - 6.72 \times 10^{-19} = 2.1015 \text{ eV}$$

Thus, maximum kinetic energy of emitted electron is 2.015 eV.

Assignment 21.2

Radiation of wavelength 3000 Å falls on a photoelectric surface for which work function is 1.6 eV. What is the stopping potential for emitted electron?

21.3 COMPTON'S EFFECT

Arthur H. Compton in 1923, conducted an experiment in which he made incident a beam of X-rays of wavelength λ, on a block of graphite. He found that the scattered X-rays had a slightly longer wavelength λ' than the incident X-rays, as shown in Fig. 21.7. The amount of energy reduction depended on the angle at which the X-rays were scattered. This phenomenon is known as Compton's effect.

Compton's effect is the phenomenon where X-rays rays scatter off electrons, transferring energy and momentum, and increasing the wavelength of the scattered radiation.

The change in wavelength $\Delta\lambda$, between a scattered X-ray and an incident X-ray is called the **Compton shift.**

In order to explain this effect, Compton assumed that if a photon behaves like particle, its collision with other particles is similar to that between two billiard balls. Hence, both energy and momentum must be conserved. If the incident photon collides with an electron initially at rest, the photon transfers some of its energy and momentum to the electron. Consequently, the energy and frequency of the scattered photon are lowered and its wavelength increases.

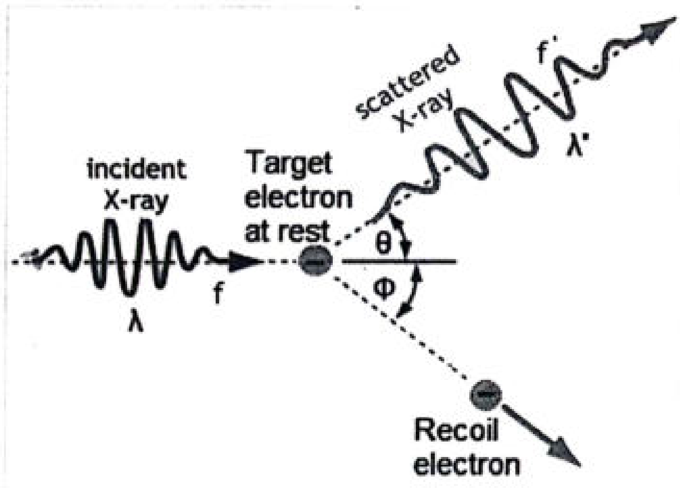


Figure 21.7: Compton's Effect.

By applying conservation of energy, we have

$$hf = K.E + hf' \qquad \_\_\_\_\_\_ (21.4)$$

Here, "hf" and "hf' " represent the energy of the incident and scattered photon, respectively, K.E is the kinetic energy given to the recoiling electron. By applying conservation of momentum, we have:

$$p = p_o + p' \qquad \_\_\_\_\_\_ (21.5)$$

Where p_o and p' represents the momentum of scattered photon and recoiling electron.

According to classical electromagnetic theory, **EM waves carry momentum** of magnitude E/c, where E is the energy of waves and c is the speed of light. Thus, the momentum of a photon is

$$p = \frac{E}{c} = \frac{hf}{c} = \frac{h}{\lambda} \qquad \_\_\_\_\_\_ (21.6)$$

Take the incident photon's direction along the x-axis, we can write the Eq. (21.6) in components as:

Along x-axis $$\frac{h}{\lambda} = p_e \cos\Phi + \frac{h}{\lambda'}\cos\theta \qquad \_\_\_\_\_\_ (21.7)$$

Along y-axis $$0 = -p_e \sin\Phi + \frac{h}{\lambda}\sin\theta \qquad \_\_\_\_\_\_ (21.8)$$

From Eq. (21.6), (21.7) and (21.8), Compton derived the following relationship

$$\lambda' - \lambda = \frac{h}{m_o c}(1-\cos\theta) \quad \text{or} \quad \Delta\lambda = \frac{h}{m_o c}(1-\cos\theta) \qquad \_\_\_\_\_\_ (21.9)$$

The quantity $\frac{h}{m_o c}$ = 0.00243 nm is constant and is known as the Compton wavelength because it has the dimension of a wavelength. It is cleared from this expression that Compton's shift depends only on scattering angle. Since $(1-\cos\theta)$ is always positive thus $\lambda' > \lambda$. Compton's shift does not depend upon; wavelength of incident photon and nature of scattering material.

Example 21.3: An X-ray of 0.20 nm is incident on a target. What is the Compton shift that can be expected at a 45°?

Given: $\lambda = 0.20$ nm $\quad \theta = 45^\circ$

To Find: $\lambda' - \lambda = ?$

Solution: The Compton shift can be calculated from the relation:

$$\lambda' - \lambda = \frac{h}{m_o c}(1 - \cos\theta)$$

Putting values, we get:
$$\lambda' - \lambda = \frac{6.63\times10^{-34}}{(9.11\times10^{-31})(3\times10^{8})}(1 - \cos 45^\circ) = 7.11\times10^{-13}\ \text{m}$$

Assignment 21.3

An X-ray of 71 pm is incident on a target. What is the smallest shift that can be expected at 60°? Find the wavelength of the scattered X-ray.

21.4 PAIR PRODUCTION

Pair production is the process of creation of a subatomic particle and its antiparticle. When a photon passes near a heavy nucleus, an electron-positron pair is created, as shown in Fig. 21.8. The photon is totally absorbed in this process.

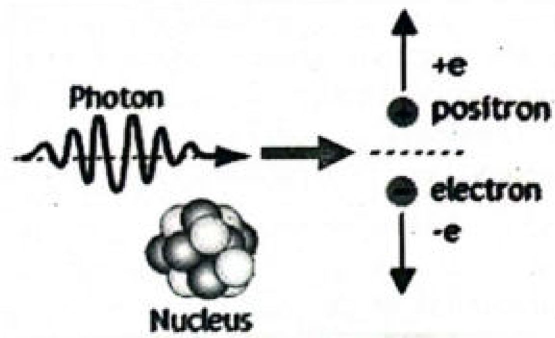


Figure 21.8: Pair production.

When a high energy photon interacts with matter, photon energy is changed into an electron-positron pair. This process is called pair production.

Phenomenon of pair production is evidence of conversion of energy into mass. Three parameters are conserved during the pair production; Charge, Momentum, and Energy.

Conservation of Charge: Charges must be conserved in this process. As electron and positron having opposite charge means that the sum of net charge of pairs is zero, which is actually equal to that of photon before the collision. Therefore, the conservation of electric charge will be observed in this process. It is impossible for a photon to produce a single electron, a single positron, two electrons, or two positrons, because the photon has zero charge and then charge will not be conserved.

Conservation of Momentum: Momentum must also be conserved in this process. If $\frac{hf}{c}$ is the momentum of photon, mv_{e^+} is the momentum of positron and mv_{e^-} is the momentum of electron, then apply conservation of momentum, we have:

$$\frac{hf}{c} = mv_{e^+} + mv_{e^-} \qquad \text{\_\_\_\_\_\_ (21.11)}$$

Conservation of Energy: Energy must be conserved in the process of pair production to occur, i.e.,

$$E_{Photon} = E_{electron} + E_{positron} \qquad \text{\_\_\_\_\_\_ (21.12 a)}$$

As the total energy of a particle with mass 'm' is the sum of its kinetic energy (K.E) and its rest mass energy ($m_o c^2$), so Eq. (21.12) can be written as:

$$hf = (K.E_{electron} + m_o c^2) + (K.E_{positron} + m_o c^2) \qquad \text{\_\_\_\_\_\_ (21.12 b)}$$

Where, $E = m_oc^2 = 0.511 MeV$. Thus, a photon must have energy of at least $2m_oc^2$ in order to create an electron-positron pair. If the photon energy is greater than $2m_oc^2$ (= 1.02 MeV) the excess energy appears as K.E of the electron and positron.

21.5 PAIR ANNIHILATION

Pair annihilation is the reverse process of pair production. Pair production is the process in which energy (photon) is converted into mass (electron-positron) and annihilation is the process in which mass (electron-positron) is converted into energy (photon).

Pair annihilation is a process in which an electron-positron pair combines to produce two photons.

It means during collision of electron and positron, these particles are disappeared and radiation is produced in term of gamma ray photons. Pair annihilation cannot create just one photon, because it is required to conserve both energy and momentum. The movement of photons in opposite direction justify the conservation of momentum.

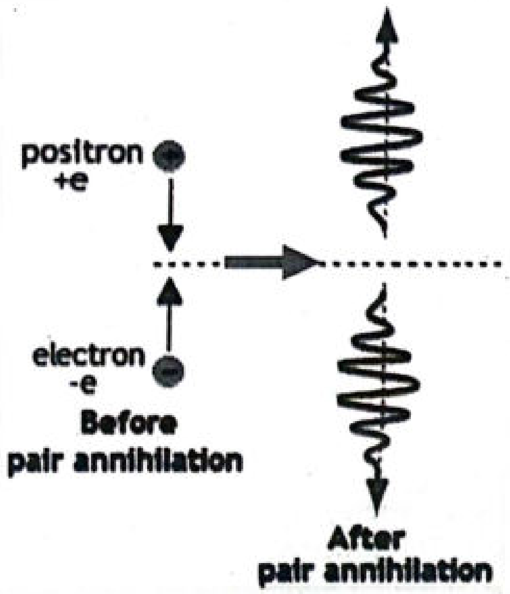


Figure 21.9: Pair Annihilation.

The total energy of the two photons must be equal to the total energy of the electron-positron pair. Annihilation of the pair then produces two photons, each with energy $E = hf = m_oc^2 = 0.511$ MeV, traveling in opposite direction, as shown in Fig. 21.9. In general;

$$2m_oc^2 + K.E_{electron} + K.E_{positron} = \text{Energy of two gamma photons} \qquad (21.13)$$

Some other examples of pair annihilation are proton-antiproton annihilation and Higgs production. Besides confirming the photon model of EM radiation, pair annihilation and pair production clearly illustrate Einstein's idea about mass and rest energy.

21.6 THE WAVE-PARTICLE DUALITY

Interference, diffraction and polarization are the basic properties of all electromagnetic waves. As light shows all the three effects, therefore, light can be considered as a wave. But Photoelectric effect can be explained only when we consider light as made by small energy packets (particles) called photon. It means photoelectric effect shows the particle nature of light. The light displays the dual nature, i.e., particle *characteristics as well as* wave characteristics.

In 1924, the French physicist Louis De Broglie predicted that if light (which characteristically demonstrated as pure wave properties) can have particle-like properties, then particle can also behave as a wave. So, particles of matter obey a wave equation just as photon does. Compton's investigation showed that the momentum p of a photon having wavelength λ is described by the equation;

$$p = \frac{h}{\lambda}$$

or $$\lambda = \frac{h}{p} \quad \text{\_\_\_\_\_\_ (21.14)}$$

Eq. (21.14) is called De Broglie relation and is a general formula that applies to material particles as well as to photons. This relation shows that a wavelength is associated with a moving particle, greater the particle's momentum the shorter its wavelength and vice versa. For macroscopic objects of heavy masses, their corresponding wavelengths are very small to be measured. For example, De Broglie wavelength of an object having mass 0.5 kg moving with a speed of 10 m s^{-1} is

$$\lambda = \frac{h}{p} = \frac{h}{mv} = \frac{6.63\times10^{-34}}{(0.5)(10)} = 1.3\times10^{-34}\ m$$

This wavelength is too small to be observed. De Broglie wavelength of an electron moving with a typical speed of 10^6 m s^{-1} is

$$\lambda = \frac{h}{mv} = \frac{6.63\times10^{-34}}{(9.1\times10^{-31})(10^6)} = 7.3\times10^{-10}\ m$$

This wavelength is comparable with the spacing between the atoms in a crystal, which is suitable for diffraction and interference. Thus, the wavelengths of very small particle of matter are observable.

There are so many practical applications of De-Broglie equations, for example; the theory behind the construction of electron microscope is the De-Broglie theory, which is commonly used now a day to study the deep observation of microscopic organisms like viruses, bacteria etc.

Example 21.4: A boy through a stone having mass 5.0 mg with the help of a catapult. If the stone moves with speed of 8.0 m s^{-1}, calculate De Broglie wavelength associated with the stone.

Given: $m = 50\ mg = 5.0\times10^{-6}\ kg$ $\qquad v = 8\ m\,s^{-1}$

To Find: $\lambda = ?$

Solution: The de-Broglie wavelength can be calculated by using: $\lambda = \frac{h}{mv}$

Putting values, we get: $$\lambda = \frac{6.63\times10^{-34}}{(5\times10^{-6})(8)} = 1.66\times10^{-29}\ m$$

Assignment 21.4

The speed of rifle bullet is 1500 m s^{-1}. If its mass is 20 g then find the De Broglie wavelength associated with it. Is this wavelength observable? Give reason.

21.7 ELECTRON MICROSCOPE

The electron microscope is a device that is based on the wave nature of electron. In microscope, accelerated electrons are used as illumination source. This microscope helps us to observe objects in detail even up to nano-scale with remarkably high resolution. Tungsten metal (material) and high potential source are used for excitation of electrons to form an electrons stream like a beam of light. Magnetic coils are used for focusing the electrons- beam on the target substance. The theoretical aspect of this set is; by increasing the applied potential, current is increased significantly and resultantly the strength of magnetic lens (magnetic coils) is increased.

Electron-microscope is basically practical usage and demonstration of wave nature of electrons, which is thousand times shorter than visible light which enables the electron microscope distinguish details not visible with optical microscope i.e. visible light wavelength is 4000 A° to 7000 A°, while electrons accelerated to 10,000 KeV and acquire wavelength about 0.12 A°. A beam of extremely fast-moving electrons is used instead of the light source of a conventional light microscope. The sample must be carefully prepared and placed in a vacuum chamber.

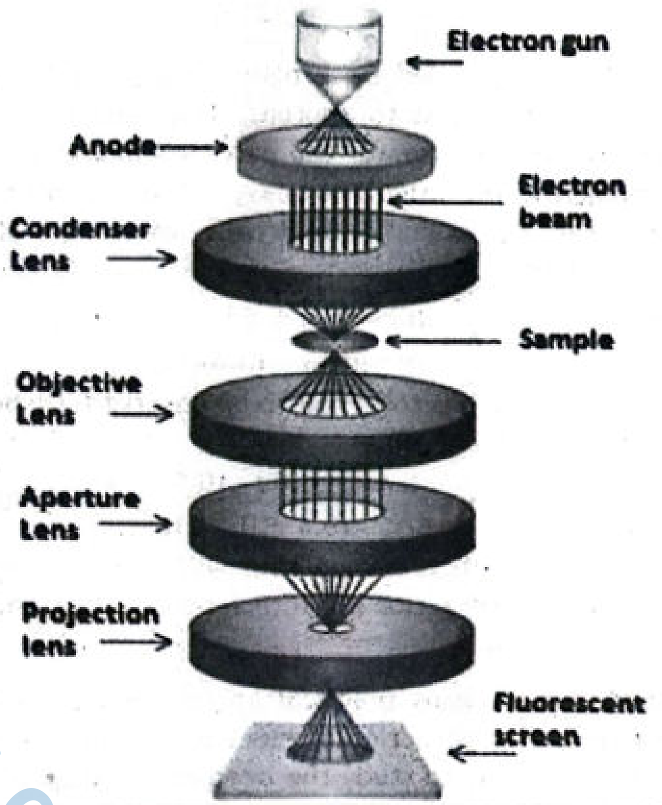


Figure 21.10: Electron microscope.

Ray diagram of electron microscope is shown in Fig. 21.10. The electron beam is passed through a series of coil-shaped electromagnets that replace conventional optical lenses. There are several components of electron microscope; electron gun, condenser lens, sample stage (magnetic object), objective lens, intermediate lens, projector lens, detector or screen and vacuum system. The electron gun and condenser lens are typically located in the upper part of the microscope. The sample stage and objective lens are situated in the middle section, the intermediate and projector lenses are located in lower part of microscope.

The detector or screen is usually located at the very bottom. The image may take the form of photograph, often referred to as an electron micrograph.

Any microscope is capable of detecting details that are comparable in size to the wavelength of radiation used to illuminate the object. Electron can be accelerated to very high kinetic energies, giving them a very short wavelength about 100 times shorter than those of the visible light (used in optical microscopes). As a result, electron microscope is able to provide details about 100 times smaller. The study of metals and crystals became easy with the *introduction* of an electron microscope.

21.8 HEISENBERG'S UNCERTAINTY PRINCIPLE

On microscopic level, we cannot measure any property of a particle without interacting with it in some way; this introduces unavoidable uncertainty into the result. One can never measure all the properties exactly. According to Planck's point of view; $E = hc/\lambda$, means a photon with a short wavelength has a large energy. Formulated by the German physicist and Nobel laureate Werner Heisenberg in 1927, the uncertainty principle states that we cannot find both the position and speed of a particle, such as a photon or electron, with perfect accuracy. It is a consequence of the dual nature of matter. According to Heisenberg uncertainty;

It is impossible to accurately find both, the position and momentum of an object simultaneously.

According to this principle; The product of uncertainties in determining the position and momentum of a particle at the same instant is approximately equal to $\hbar$ (read as 'h-cut'); whereas $\hbar = \frac{h}{2\pi} = 1.054 \times 10^{-34}$ J s.

If position of a particle has an uncertainty Δx and that of the corresponding momentum is Δp, then uncertainties are found to be related in general by the inequality as:

$$\Delta x . \Delta p \geq \hbar$$

It is impossible to know both the position and momentum exactly; i.e., if $\Delta x = 0$ and $\Delta p = \infty$. In quantum mechanics a particle is described by a wave, the term position refers; point where the wave is concentrated. The uncertainty in position means; the position is uncertain to the degree that the wave is spread out. While the uncertainty in momentum means; the momentum is uncertain to the degree that the wavelength is unclear. These uncertainties are inherent in the physical world and have nothing to do with the skill of observer; due to the very small value of angular Planck's constant, these uncertainties are not observable in normal everyday situations.

If we want accuracy in position, we must use photons of shorter wavelength because the best resolution we can get is about the wavelength of the radiation used. Short wavelength radiation implies high frequency, high energy photons. When these collide with the electrons, they transfer more momentum to the target. If we use longer wavelength i.e., less energetic photons, we compromise resolution and position.

If the energy is in the form of electromagnetic waves, the limited time available restricts the accuracy with which we can determine the frequency of the electromagnetic waves. Let's consider that the minimum uncertainty in the number of waves under observation is equal to number of them that we counted divided by the time interval, the uncertainty in the frequency can be measured as;

$$\Delta f \geq \frac{1}{\Delta t} \text{ i.e., } \Delta E = \hbar \Delta f.$$

Another kind of uncertainty principle concerns uncertainties in simultaneous measurements of the energy of a quantum state and its lifetime, if ΔE is the uncertainty in determining the energy of the system and Δt is the uncertainty in determining the time to which this determination refers, then the uncertainty principle for energy and time can be expressed as;

$$\Delta E . \Delta t \geq \hbar$$

A system that remains in metastable state for a very long time can have a well-defined energy, but if it remains in a state for only a short time the uncertainty in energy must be correspondingly greater. The uncertainty ΔE depends on the time interval Δt during which the system remains in the given state.

Example 21.5: Calculate the uncertainty in the momentum of an electron if uncertainty in its position is 1 Å (10^{-10} m).

Given: $\Delta x = 10^{-10}$ m

To Find: $\Delta p = ?$

Solution: **According to the uncertainty principle:**

$$\Delta x . \Delta p \geq \frac{h}{2\pi}$$

or

$$\Delta p \geq \frac{h}{2\pi \Delta x}$$

By putting values, we get:

$$\Delta p = \frac{6.63 \times 10^{-34}}{2(3.14)(10^{-10})} = 5.28 \times 10^{-25}\ \text{kg m s}^{-1}$$

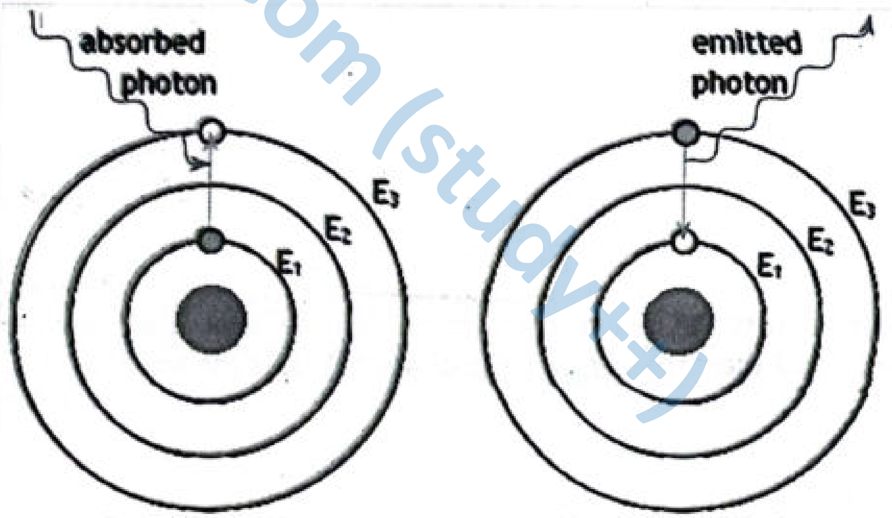
Assignment 21.5

Let's consider a ball with a mass of 0.1 kg moving at a speed of 30 m s^{-1}. The uncertainty in its momentum is 0.000001 of its actual momentum. Determine the uncertainty in its position?

21.9 DISCRETE ENERGY LEVELS IN AN ATOM AND ITS SPECTRA

The electrons orbiting an atom can only occupy certain allowed *orbits* (also called shells). Each orbit has a discrete energy state of the atom e.g. E_1, E_2, E_3 and so on, as shown in the Fig. 21.11. Electrons can move from one allowed energy level to another only by gain or loss of certain amount of energy.

- **Energy is required for an electron to move from a lower to a higher energy level. This transition is called an excitation.**
- **Energy is released if the electron moves from a higher to a lower energy level. This transition is called a de-excitation.**



Excitation of hydrogen atom **De-excitation of hydrogen atom**

Figure 21.11: Excitation and de-excitation of an isolated H-atom.

Electrons can move to a higher or lower energy level by absorbing or emitting electromagnetic radiation with a frequency f. This frequency depends on the difference of energy between the specific energy levels involved in the transition. The frequency of the radiation obeys the condition:

$$hf = E_n - E_p$$

Where E_n and E_p are higher and lower energy states respectively.

Photons emitted from such transitions, when allowed to pass through a narrow slit and a prism (or diffraction grating), will give rise to a set of line spectrum, unique to the element, as shown in Fig. 21.12. A spectrum is produced by using a prism or diffraction grating to separate the various wavelengths in a beam of light. A line spectrum contains a discrete set of wavelengths. It is a characteristic feature of an element.

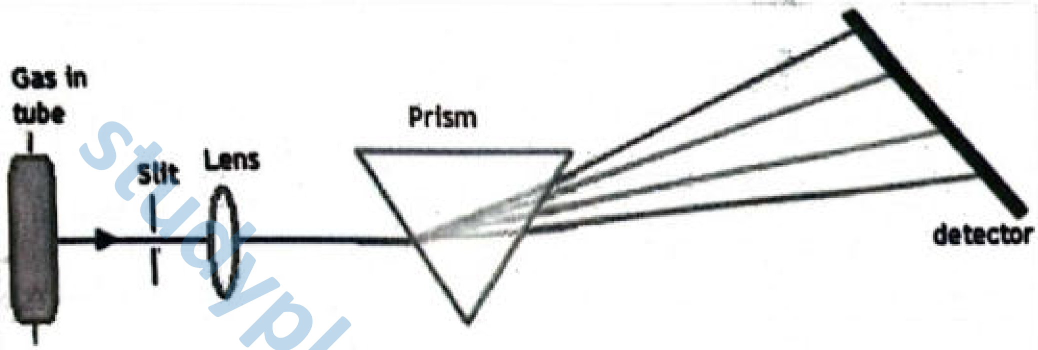


Figure 21.12: Setup for obtaining spectrum.

An **emission spectrum** of an element consists of a series of separate bright lines of definite frequencies (or wavelengths) on a dark background. It is produced when a stream of photons of different frequencies is passed through a narrow slit and normally through a diffraction grating. These photons are emitted randomly from transitions (from higher to lower excited states or the ground state) in the excited atoms of the element in a vapour or gas at low pressure. Emission spectra of Hydrogen is shown in Fig. 21.13 (a). Since each element has a unique set of orbital electrons, the emission line spectrum of an element is also unique, enabling it to be used as a means of identification of the element.

For Your Information

Different coloured lines in the spectrum is helpful in figuring out the kind of elements the substance is made of, as each element radiates a different amount of energy and has a unique emission level.

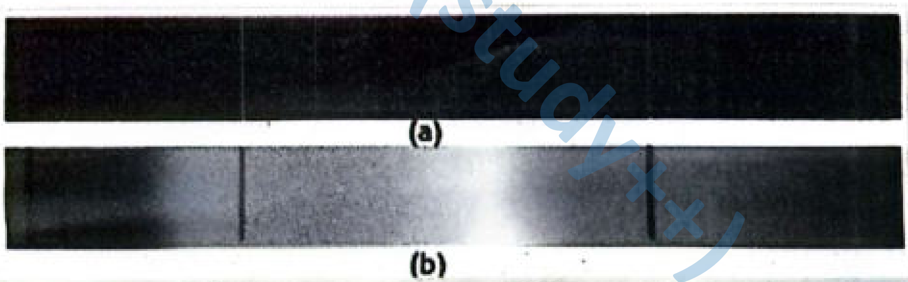


Figure 21.13: (a) H-emission spectrum (b) H-absorption spectrum.

An **absorption line spectrum** of an element consists of a series of separate dark lines of definite frequencies (or wavelengths) on a coloured background. The *coloured background is produced* when a stream of photons of different frequencies from a white light source (e.g. tungsten filament lamp) is passed through a narrow slit and a diffraction grating. The cool vapour of the element concerned is placed between the white light source and the narrow slit, such that its atoms may absorb excitation energy from photons incident from the white light source. The unabsorbed photons from the white light source will be incident on the screen with the original intensity. After the absorption, the excited atoms will eventually go back to the ground state by emitting the same photons absorbed earlier. Absorption spectra of Hydrogen are shown in Fig. 21.13 (b).

SUMMARY

- When electromagnetic radiation is absorbed by a material, electronically-charged particles are released from the material (or even within itself). This phenomenon is called the photoelectric effect.
- Conservation of momentum implies that if two or more bodies interact with each other in an isolated system (that is, no external force is acting upon them), then the total momentum of all the bodies remains constant.
- The electrons which are emitted from a metal surface upon the influence of light are called photoelectrons.
- Threshold energy is the minimum amount of energy needed by the electron to break free from the metal and eject from it.
- Threshold frequency is the lowest frequency of electromagnetic radiation that will produce a photoelectric effect in a material.
- The threshold wavelength is the largest possible wavelength of the incident radiation which allows photoemission to take place. No photoemission occurs if the wavelength is higher than the threshold.

Formula Sheet

$E = hf$ $\quad K.E_{max} = eV_o$ $\quad E = \Phi + K.E_{max}$

$\lambda = \frac{h}{p} = \frac{h}{mv}$ $\quad \Delta\lambda = \frac{h}{m_oc}(1-\cos\theta)$ $\quad \Delta x.\ \Delta p \geq \hbar$ $\quad \Delta E.\ \Delta t \geq \hbar$

EXERCISE

Multiple Choice Questions

Encircle the Correct option.

1) Which of the following spectral regions has the highest energy?
   A. Infrared B. Violet C. Red D. Blue
2) Which of the following statements is true about a photon?
   A. A photon has zero mass and zero momentum.
   B. A photon has finite mass and a finite value of momentum.
   C. A photon has zero mass but finite value of momentum.
   D. A photon has finite mass but zero momentum.
3) What happens to the kinetic energy of the emitted electrons when the light is incident on a metal surface?
   A. It varies with the frequency of light. B. It varies with the light intensity.
   C. It varies with the speed of light. D. It varies irregularly.

4) A photoelectric cell is a device which
A. Converts light energy into electricity. B. Converts electricity into light energy.
C. Stores light energy. D. Stores electricity.

5) Which property does the Compton Effect describe about photons?
A. Mass B. Momentum C. Wave properties D. Speed rates

6) What happens to a high energy photon after it strikes an electron?
A. decreases frequency B. decreases wavelength
C. increases energy D. increases momentum

7) Energy of gamma photon having a wavelength of 1A° is:
A. 12.4×10^4 eV B. 12.4×10^4 eV C. 12.4×10^4 eV D. 12.4×10^4 eV

8) If uncertainty in the position of an electron is zero, the uncertainty in its momentum will be
A. less than $h/4\pi$ B. greater than $h/4\pi$ C. zero D. infinite

9) What is the process by which a particle and its antiparticle annihilate each other?
A. Pair production B. Pair annihilation C. Nuclear fission D. Nuclear fusion

10) Which of the following particles is produced during pair annihilation?
A. Photon B. Electron C. Proton D. Neutron

Short Questions

1) Prove that energy and momentum of Photon are directly proportional to each other.
2) Convert 800 MeV into joules.
3) Write the two phenomena to describe photon-electron interaction.
4) In the interpretation of the photoelectric effect, how is it known that an electron does not absorb more than one photon?
5) How you can determine the work function from a plot of the stopping potential versus the frequency of the incident radiation in a photoelectric effect experiment? Can you determine the value of Planck's constant from this plot?
6) Speculate how increasing the temperature of a photo electrode affects the outcomes of the photoelectric effect experiment.
7) Which aspects of the photoelectric effect cannot be explained by classical physics?
8) What is the physical significance of Compton's effect?
9) Differentiate between Compton's shift and Compton's wavelength.
10) How can you say that scattering angle of photon plays effective role in measurement of Compton's shift?
11) What are the conditions required for pair production to occur?
12) What is meant by de Broglie wavelength?
13) Which phenomenon provides evidence for particulate nature of electromagnetic radiation?

Comprehensive Questions

1) What is photoelectric effect? How quantum physics explain its results.
2) What is Compton's effect? Discuss in detail.
3) State and discuss the Heisenberg Uncertainty Principle.

4) Discuss Pair production of electron as practical implementation of conversion of energy into mass correspondences to of Eisenstein energy-mass equation.
5) Explain Annihilation process as practical implementation of conversion of mass into energy correspondences to of Eisenstein energy-mass equation.
6) What do you understand by the term 'Dual nature of light'? Discuss.
7) Explain that electromagnetic radiation has a particulate nature.
8) Explain how electron microscopes achieve very high resolution.
9) There are discrete electron energy levels in isolated atoms. Explain this statement by the appearance and formation of emission and absorption line spectra.

Numerical Problems

1) In an experiment, the work function of potassium surface is found to be 2.1 eV. What should be the wavelength of incident radiation if the stopping potential for the electron is 0.43 V?

(Ans: 5.91×10^{-7} m)

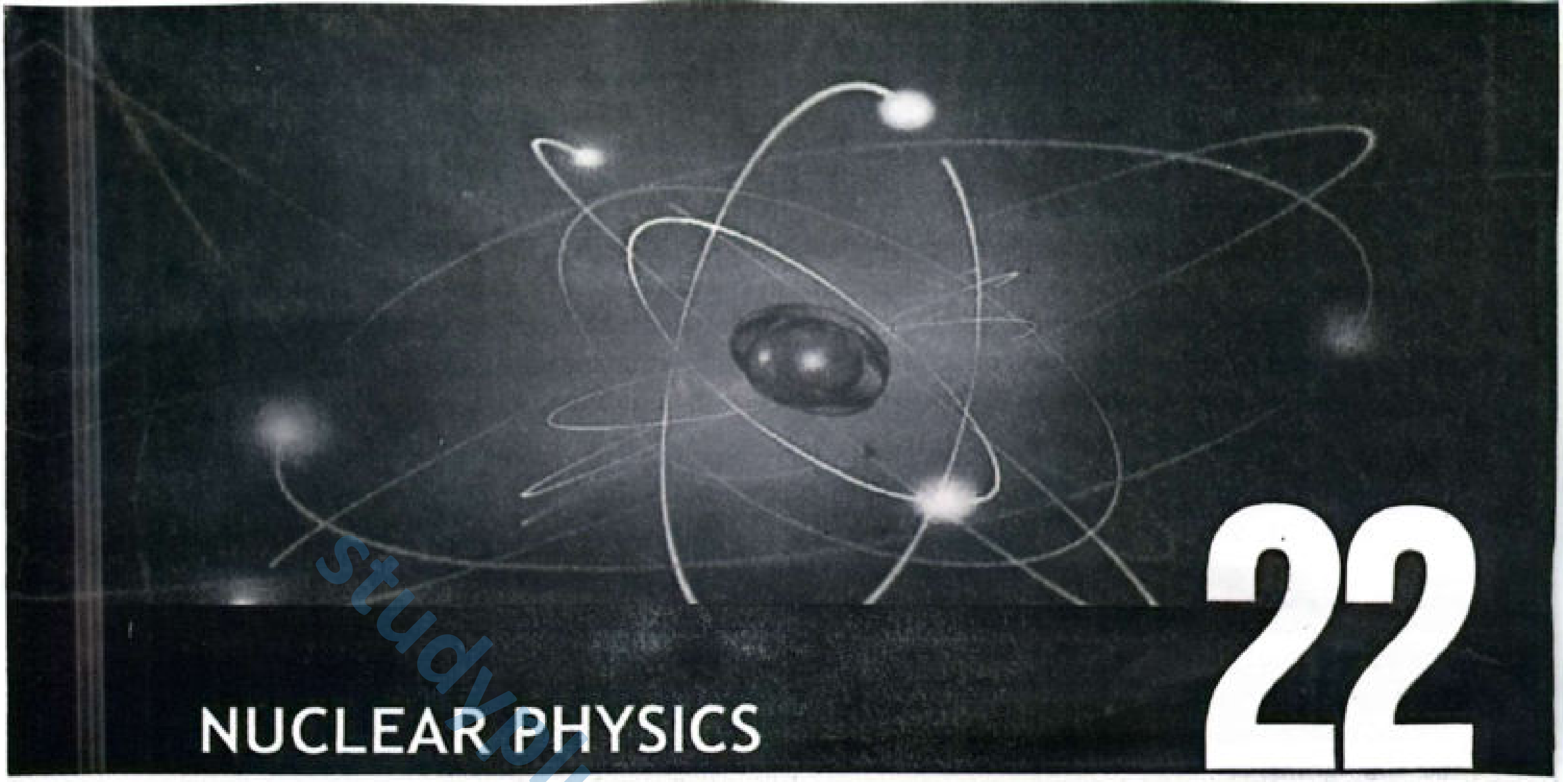
2) Light of wavelength 5000 Å falls on a metal surface having work function of 1.9 eV. Find (a) Energy of photon in electron-volt. (b) Kinetic energy of photo-electrons emitted. (c) Stopping potential. **(Ans: 2.48 eV, 0.58 eV, 0.58 V)**

3) Calculate the energies of the photons associated with the (i) violet light of 413 nm. (ii) X-rays of 0.1 nm. (iii) radio waves of 10 m. **(Ans: 3eV; 12424eV; 1.24×10^{-7} eV)**

4) Calculate the minimum energy required by a photon to transfer half of its energy to an electron at rest. **(Ans: 0.25 MeV)**

5) An electron is placed in a box about the size of an atom that is about 1 Å. What is the velocity of the electron? **(Ans: 7.29×10^{6} m s^{-1})**

6) A 50 keV X-ray is scattered through an angle of 90°. What is the energy of the X-ray after Compton scattering? **(Ans: 45.5 keV)**



22 NUCLEAR PHYSICS

Student Learning Outcomes (SLOs)

The student will

- Recognize the equivalence between energy and mass as represented by $E = \Delta mc^2$ and state and use this equation.
- define and use the terms mass defect and binding energy.
- sketch the variation of binding energy per nucleon with nucleon number.
- Recall what is meant by nuclear fusion and nuclear fission.
- Explain the relevance of binding energy per nucleon to nuclear reactions, including nuclear fusion and nuclear fission.
- Explain how the neutrons produced in fission create a chain reaction and that this is controlled in a nuclear reactor [including the action of coolant, moderators and control rods].
- calculate the energy released in nuclear reactions using $E = \Delta mc^2$.
- Explain that fluctuations in count rate provide evidence for the random nature of radioactive decay.
- explain that radioactive decay is both spontaneous and random.
- define activity and decay constant, and state and use $A = N\lambda$.
- Explain half-life with examples.
- use = $0.693/t_{1/2}$ to solve numerical problems.
- state the exponential nature of radioactive decay.
- use the relationship $x = x_o\, e^{-\lambda t}$ [where x could represent activity, number of undecayed nuclei or received count rate) to solve problems analytically and graphically].
- describe the function of the principle components of water moderated power reactor [core, fuel, rods, moderator, control rods, heat exchange, safety rods and shielding]
- explain why uranium fuel needs to be enriched before use.
- compare the amount of energy released in a fission reaction with the (given) energy released in a chemical reaction.
- Explain what is a medical tracer [a substance containing radioactive nuclei that can be introduced into the body and is then absorbed by the tissue being studied].
- Explain annihilation reactions [they occur when a particle interacts with its antiparticle and that mass-energy and momentum are conserved in the process].
- Illustrate how PET scanning works [positrons emitted by the decay of the tracer annihilate when they interact with electrons in the tissue, producing a pair of gamma-ray photons traveling in opposite directions]
- calculate the energy of the gamma-ray photons emitted during the annihilation of an electron-positron pair.
- Explain that the gamma-ray photons from an annihilation event travel outside the body and can be detected [including that an image of the tracer concentration in the tissue can be created by processing the arrival times of the gamma-ray photons].

Nuclear physics is a branch of physics that deals with the study of the atomic nucleus, its properties and interactions. We studied in grade-10, that the atomic nucleus is composed of protons and neutrons, which are collectively referred as nucleons. The atomic number 'Z' of an element corresponds to the number of protons in the nucleus, while the mass number 'A' is the sum of the number of protons and neutrons in the nucleus. Number of neutrons in an atom is represented by 'N' and is called neutron number. The relation between atomic number, mass number and neutron number can be given by the following equation: $A = Z + N$.

Nuclear physics has numerous applications, including nuclear power generation, nuclear medicine, and nuclear weapons. It also plays a crucial role in the study of astrophysics, as nuclear reactions are responsible for the energy production in stars and other celestial bodies.

22.1 MASS DEFECT

If we have a polythene bag of negligible mass containing 10 balls, each ball having a mass of 100 grams, then what should be the mass of the whole basket? Definitely it would be 1000 grams (1 kg) as it is our common observation that the mass of whole is the sum of masses of the constituents. But the same is not true for the nucleus. The total mass of nucleus is less than the sum of masses of its individual nucleons. For example, the mass of carbon should be greater than 12 u, as its constituent protons and neutrons have masses greater than 1 u but it has a mass of exactly 12 u which is less than the sum of masses of six protons and six neutrons. Similarly, mass defect for binding of one proton and one neutron in case of deuterium (^2H) can be shown in Fig. 22.1. Let '$m_{nucleus}$' be the mass of nucleus, 'm_p' is the mass of proton and 'm_n' is the mass of neutron then during the formation of the nucleus 'Δm' is the difference in mass which is known as the mass defect. Mathematically the mass defect can be given as:

For Your Information

To express the masses of atoms and nuclei in nuclear physics we use 'atomic mass unit' (amu). It is defined as one twelfth of the mass of a carbon-12 atom. It can be given as:

$$1 \text{ amu} = 1 \text{ u} = 1.66 \times 10^{-27} \text{ kg}$$

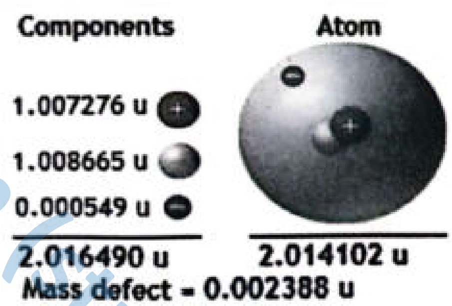


Figure 22.1: Mass defect in deuterium atom.

$$\Delta m = (m_n + m_p) - m_{nucleus}$$

The mass of deuterium is less than the sum of the masses of one proton and one neutron. This difference in mass of a nucleus is called as mass defect, and can be defined as:

The difference between the mass of the nucleus and sum of the masses of its constituent particles is called as mass defect.

In general, the equation for the mass defect can be given as:

$$\Delta m = [(A - Z)\, m_n + Z\, m_p] - m_{nucleus} \quad \text{______} \quad (22.1)$$

Example 22.1. Find the mass defect in the formation of Helium-4 nucleus. If the mass of He-4 is 4.002603u, the mass of proton is 1.007276u and the mass of neutron is 1.008665u.

Given: Mass of He-4: = m_{He} = 4.002603u Mass of proton = m_p = 1.007276u

Mass of neutron $= m_n = 1.008665u$

To Find: Mass defect $= \Delta m = ?$

Solution: To find the mass defect we use: $\Delta m = (A - Z)\ m_n + Z\ m_p - m_{nucleus}$

As we know that for helium-4 nucleus there are two protons and two neutrons hence Z=2 and (A–Z)= 4 – 2 = 2 now putting valeues, we get:

$$\Delta m = \left[(4-2)\ (1.008665) + 2\ (1.007276) - 4.002603\right]\ u = 0.029279\ u$$

Assignment 22.1

Find the mass defect of tritium nucleus. The mass of tritium is 5.0083×10^{-27} kg.

22.2 BINDING ENERGY

The mass defect remained a mystery for some time. But soon the scientists found that in formation of the nucleus, the missing mass is converted into energy. As during the formation of nucleus, energy is found to be released and conversely to break the nucleus into its parts. An equal amount of energy is to be given for this break. The mass converted into the energy is called as binding energy and can be defined as:

The minimum energy required to break an isolated nuclei into its constituent particles is called the binding energy.

The Einstein's famous equation relates this energy and mass which can be given as:

$$E = \Delta m\ c^2 \qquad \text{\_\_\_\_\_\_ (22.2)}$$

Here 'E' is the energy required to break the nucleus or the energy released during formation of the nucleus, 'Δm' is the mass defect and 'c' is the speed of light whose approximate value is $3\times10^8\ m\ s^{-1}$. This relation proves that the mass can be converted into energy and energy can be converted back into the mass i.e. mass and energy are inter-convertible quantities. The relation between binding energy and the mass can be shown in Fig. 22.2 (a). It is to note that the units for mass used in above equation should be in kilograms. For example, helium-4 nucleus containing two protons and two neutrons has a combined mass of 4.002603 amu while the sum of masses of two protons and two neutrons is 4.031882 amu. The mass defect can be given as:

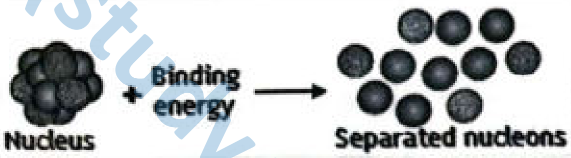


Figure 22.2 (a): Binding energy.

$$\Delta m = (4.031882 - 4.002603)\ amu = 0.029279\ amu$$

The equivalence of this mass in kg can be given as:

$$1\ amu = 1.66\times10^{-27} kg \qquad \text{or} \qquad 0.029279\ amu = (0.029279\times1.66\times10^{-27}) kg$$

$$\Delta m = 0.029279\ amu = (4.860314\times10^{-29}) kg$$

Using equation (22.3) we find:

$$E = (4.860314\times10^{-29} kg)\ (3\times10^8 ms^{-1})^2 = 4.374283\times10^{-12} J$$

This is the amount of energy required to break the Helium-4 nucleus. This is relatively a small amount of energy but it is enough to hold the nucleus together and give it its stability. It is often convenient in nuclear physics to express certain masses in energy units as they are simply interchangeable. According to Einstein's mass-energy equivalence relation: $E = mc^2$

For one amu:

Energy for 1 amu $= (1.66\times10^{-27}\text{kg})\ (3\times10^{8}\text{ms}^{-1})^2$

Energy for 1 amu $= 1.494\times10^{-10}$ J

As we know that: $1\ \text{eV} = 1.60\times10^{-19}$ J

Hence: $\frac{1}{1.60\times10^{-19}}\ \text{eV} = 1\ \text{J}$

Now energy equivalence for 1 amu mass in electron-volt units is:

$$1\ \text{amu} = 1.494\times10^{-10}\times\frac{1}{1.60\times10^{-19}}\ \text{eV}$$

$$= 9.315\times10^{8}\ \text{eV}$$

$$1\ \text{amu} = 931.5\ \text{MeV}$$

The rest mass and energy equivalent of some particles and nuclei is given in table 22.1.

Table 22.1: Masses of some particles

Particle	Mass (kg)	Mass (amu)	MeV
Electron	9.1164×10^{-31}	0.0005485	0.511
Proton	1.672×10^{-27}	1.007276	938.27
Neutron	1.675×10^{-27}	1.008665	939.57
Hydrogen	1.672×10^{-27}	1.007825	938.78
Deuterium	3.344×10^{-27}	2.014102	1876.12
Tritium	5.0083×10^{-27}	3.016049	2809.43
Helium	6.646×10^{-27}	4.002603	3728.40
Carbon-12	1.992×10^{-26}	12.000000	11177.9

Example 22.2. Find the energy required to break tritium nucleus (having mass 3.016049 u) in joules and in eV. If masses of proton and neutron are 1.007276 u and 1.008665u.

Given: Mass of tritium = M = 3.016049u Number of protons = Z = 1

Mass of proton $= m_p$ = 1.007276u Number of neutrons = A−Z = N = 2

Mass of neutron $= m_n$ = 1.008665u

To Find: Binding energy = E = ?

Solution: To find the binding energy first we find the mass defect:

$$\Delta m = (A - Z)\ m_n + Z\ m_p - M$$

Putting values, we get:

$$\Delta m = \left((2)\ 1.008665 + (1)\ 1.007276 - 3.016049\right)\ \text{amu} = 0.008557\ \text{amu}$$

To convert this value in kg:

$$\Delta m = (0.008557\times1.66\times10^{-27})\ \text{kg} = 1.420\times10^{-29}\ \text{kg}$$

Now to find the binding energy we use:

$$E = \Delta mc^2$$

$$E = (1.420\times10^{-29}\text{kg})\ (3\times10^{8}\text{ms}^{-1})^2 = 1.278\times10^{-12}\ \text{J}$$

To convert this energy into electron-volt we use:

$$E = 1.278\times10^{-12}\times\frac{1}{1.60\times10^{-19}}\ \text{eV} = 7.99\times10^{6}\ \text{eV}$$

Assignment 22.2

Find the binding energy of deuterium nucleus. The mass of deuterium is 3.344×10^{-27} kg.

Binding Energy per Nucleon

The binding energy of an atomic nucleus is the amount of energy required to completely separate all of its constituent nucleons (protons and neutrons). It is a measure of the strength of the nuclear force that holds the nucleus together. The binding energy is usually expressed in electron-volts or in joules. As from equation (22.2):

$$E = \Delta mc^2$$

Using the values of mass defect 'Δm' from equation (22.1) we get:

$$E = \left((A-Z)\, m_n + Z\, m_p - m_{nucleus}\right) c^2$$

Here 'E' is the binding energy. The absolute value of binding energy does not explain completely the stability of a nucleus. The more important thing to explain the stability of a nucleus is the binding energy per nucleon i.e. how much mass is converted by each nucleon to form a nucleus. The greater the binding energy per nucleon the greater will be the stability of the nucleus. The binding energy per nucleon also called as packing fraction (f) and can be given as:

Table 22.2: Binding energy of some nuclei in MeV

Nucleus	Binding energy	Binding energy per nucleon
^{2}H	2.23	1.12
^{4}He	28.3	7.07
^{12}C	89.1	7.4
^{56}Fe	486.24	8.68
^{206}Pb	1622	7.87
^{235}U	1786	7.6

$$f = \frac{E}{A} = \frac{\left((A-Z)\, m_n + Z\, m_p - m_{nucleus}\right) c^2}{A} \quad \text{\_\_\_\_\_\_ (22.3)}$$

The packing fraction also called the binding fraction is the measure of stability of a nucleus. Therefore, the rest energy of the bound system i.e. the nucleus is less than the combined rest energy of the separated nucleons. The binding energies of some commonly used nuclei are given in the table 22.2.

Example 22.3. Find packing fraction for C-12 nucleus. If mass of C-12 is 12.000u, mass of proton is 1.007276 u and mass of neutron is 1.008665 u.

Given: Mass of carbon-12 = $m_{nucleus}$ = 12.0 Number of protons = Z = 6
Mass of proton = m_p = 1.007276u Number of neutrons = A-Z = N = 6
Mass of neutron = m_n = 1.008665u

To Find: Packing fraction = f = ?

Solution: We have to find the binding energy as: $\Delta m = (A-Z)\, m_n + Z\, m_p - m_{nucleus}$

$$\Delta m = (6\times1.008665 + 6\times1.007276 - 12.000) = 0.095646 \text{ u}$$

To convert it into MeV we take:

$$E = 0.095646\times931.5 \text{ MeV} \quad \text{or} \quad E = 89.1 \text{ MeV}$$

For packing fraction, we use: $f = \frac{E}{A}$

Putting values, we get: $f = \frac{89.1 \text{ MeV}}{12}$ or $f = 7.4$ MeV / nucleon

Assignment: 22.3

Find the binding energy per nucleon for Nitrogen-13 nucleus having mass 2.3258×10^{-26} kg. mass of proton is 1.007276 u and mass of neutron is 1.008665 u.

Binding Energy Curve

The variation of binding energy per nucleon with mass number can graphically be represented by a curve known as the binding energy curve. This curve shows the relationship between the number of *nucleons in a nucleus* (represented by the mass number A) and the binding energy per nucleon. The graph for binding energy per nucleon (in MeV) plotted against the mass number (A) as we can see from Fig. 22.2 (b), the binding energy per nucleon increases as we move from the lightest nuclei (A = 1) towards the region of medium-sized nuclei (A = 50–100), reaching a peak at around A = 56.

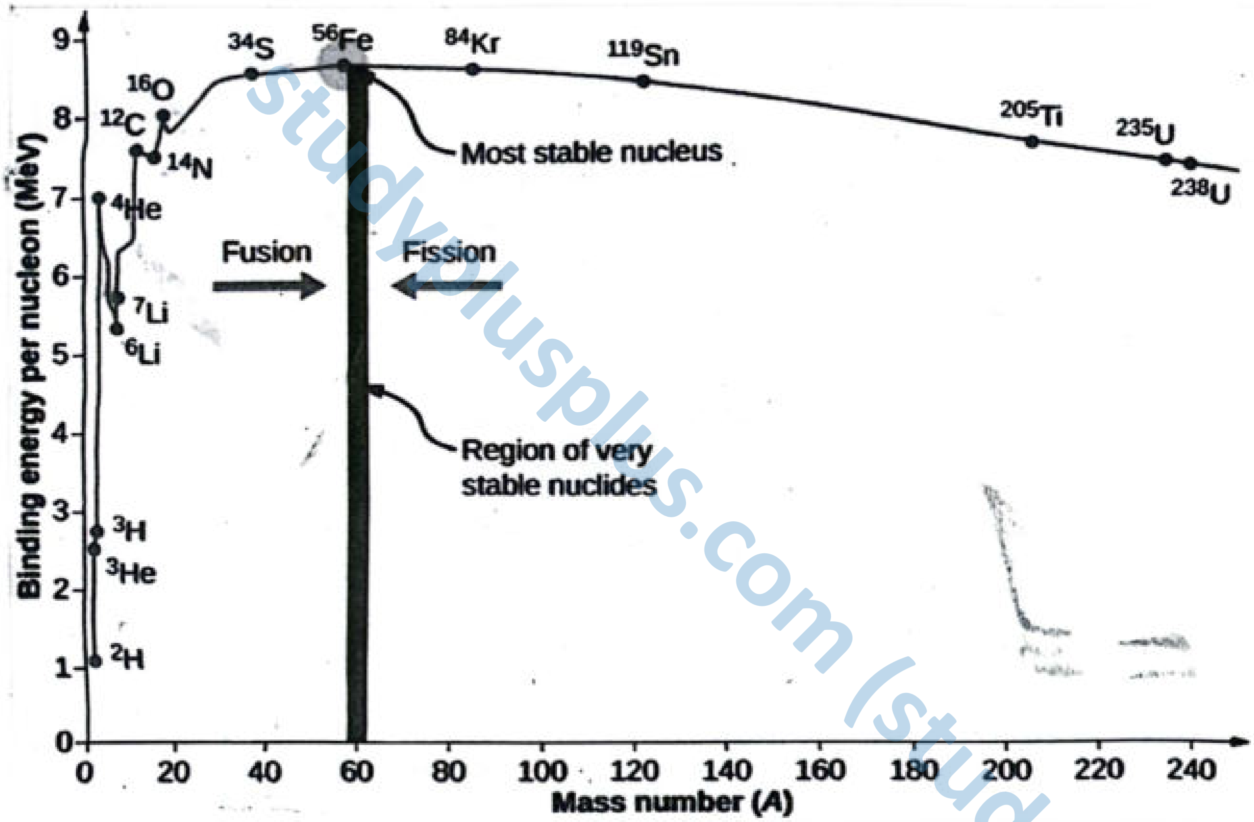


Figure 22.2 (b): Binding energy verses nucleon number

This region of medium-sized nuclei is known as the most stable nuclei or the valley of stability. Beyond the region of medium-sized nuclei, the binding energy per nucleon starts to decrease again, indicating that larger nuclei are less stable than smaller ones. This is because larger nuclei have a higher proportion of repulsive electrostatic forces between protons, which are not balanced by the attractive strong nuclear forces. The variation of binding energy per nucleon with mass number is an important factor in nuclear physics, as it helps to explain the stability and properties of atomic nuclei, and provides insight into nuclear reactions and nuclear energy production. Nuclei with a higher binding energy per nucleon are generally more stable and less likely to undergo fission. On the other hand, nuclei with a lower binding energy per nucleon, such as uranium-235, are more likely to undergo fission when they absorb a neutron, releasing energy and more neutrons, which can in turn cause further fission reactions. Fusion reactions tend to produce more stable nuclei with a higher binding energy per nucleon than the reactant nuclei. The fusion reactions release energy, as the product nucleus has a greater binding energy per nucleon than the reactant nuclei. For example, the fusion of hydrogen nuclei into helium releases a large amount of energy, as the product nucleus has a higher binding energy per nucleon than the reactant nuclei.

22.3 RADIOACTIVITY

Marie Curie studied radioactivity (nuclear radiations emission) shortly after Henri Becquerel discovered natural radioactivity. Marie Curie detected radioactivity in Uranium and Thorium using an electrometer, which showed that the air around radioactive samples became charged and conductive.

The natural emission of radiations from unstable nuclei is called radioactivity. Radioactivity is the phenomenon by which unstable atomic nuclei spontaneously decay into other nuclei, emitting radiation in the process. Spontaneous nuclear decay is the natural process by which an unstable atomic nucleus breaks down into a more stable configuration by emitting radiation. This process is spontaneous and occurs without any external influence or trigger. Radioactive decay can also result in the formation of a new element known as the daughter nuclei. Spontaneous nuclear decay is a natural process that occurs in many elements in the universe. While it can pose a significant health hazards if not properly managed, it also has many practical applications in fields such as energy production, medicine and materials sciences. There are three main types of radioactive decay alpha decay, beta decay, and gamma decay, along with these radiations some more radiations and particles are also produced like X-rays and neutrons etc.

22.3.1 Alpha Decay

Alpha decay occurs when an atomic nucleus emits an alpha particle, which is a helium nucleus consisting of two protons and two neutrons. This reduces the atomic number of the nucleus by two and the atomic mass by four. The general form of an alpha decay equation can be written as:

$$ {}^{A}_{Z}X \rightarrow {}^{A-4}_{Z-2}Y + {}^{4}_{2}He + Q $$

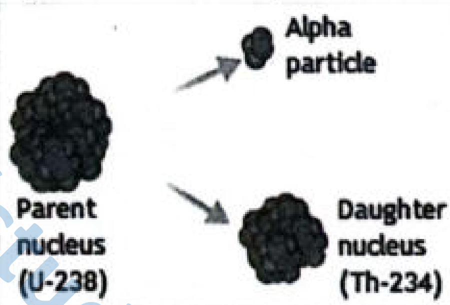


Figure 22.3: Alpha decay of U-238.

Here ${}^{A}_{Z}X$ is the parent nucleus which decays into ${}^{A-4}_{Z-2}Y$ known as daughter nucleus with emission of alpha-particle ${}^{4}_{2}He$ which is helium-4 nucleus and heat Q. For example, the alpha decay of uranium-238, as shown in Fig. 22.3 and can be represented as:

$$ {}^{238}_{92}U \rightarrow {}^{234}_{90}Th + {}^{4}_{2}He + Q $$

Here the uranium-238 nucleus (with 92 protons and 146 neutrons) decays into thorium-234 (with 90 protons and 144 neutrons) by emitting an alpha particle (with 2 protons and 2 neutrons).

22.3.2 Beta Decay

Beta decay occurs when an atomic nucleus emits a beta particle, which is an electron or a positron.

Beta decay can either increase or decrease the atomic number of the nucleus, but always conserves the total number of nucleons. Beta decays are of two types i.e. the negative beta decay and the *positive beta* decay.

Figure 22.4: Beta-minus decay.

i) Negative beta or beta-minus decay is the emission of an electron during nuclear activity. In an unstable nucleus, when a neutron decays into proton it emits an electron and an anti-neutrino. The equation for neutron decay can be given as:

$$n^0 = p^+ + {}^{0}_{-1}\beta + \bar{\nu}$$

The general equation for the nuclei emitting beta-minus decay can be given as:

$$^{A}_{Z}X = {}^{A}_{Z+1}Y + {}^{0}_{-1}\beta + \bar{\nu}$$

Beta-minus decay is shown in Fig. 22.4.

ii) Positive beta or beta-plus decay is the emission of positron during nuclear activity. In an unstable nucleus, when a proton decays into neutron it emits a positron and neutrino. The equation for proton decay can be given as:

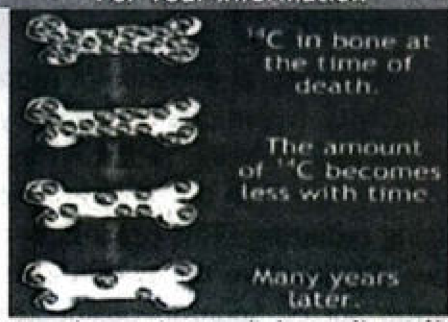
$$p^+ = n^0 + {}^{0}_{+1}\beta + \nu$$

The general equation for the nuclei emitting beta-plus decay can be given as:

$$^{A}_{Z}X = {}^{A}_{Z-1}Y + {}^{0}_{+1}\beta + \nu$$

Beta-plus decay is shown in Fig. 22.5.

For Your Information



Beta decay is used in radionuclide therapy (RNT) to treat cancer. Radioactive isotopes like lutetium-177 or yttrium-90 are used to target and destroy cancer cells. Beta decay of carbon-14 is the basis for radiocarbon dating which is a method used to determine the age of archaeological and geological samples.

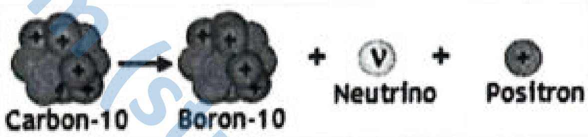


Figure 22.5: Beta-plus decay.

22.3.3 Gamma Decay

Gamma decay occurs when an atomic nucleus transitions from a higher energy state to a lower energy state, emitting a gamma ray in the process. Gamma rays, high-energy photons, are electromagnetic radiations which travel with the speed of light. Unlike alpha and beta radiation, gamma radiation does not involve the emission of massive particles. However, the emission of gamma radiation is subject to certain conservation laws that govern the properties of the nucleus. Specifically, the law of conservation of energy, momentum and

For Your Information

Gamma rays are used in radiation therapy to target and destroy cancer cells. In medical imaging *like* positron emission tomography (PET) scans, gamma rays are used to create detailed images of inside of the body. In non-destructive testing (NDT) gamma rays are employed to inspect and detect flaws in material without causing any damage.

angular momentum must be obeyed during the transition.

Even though no particles with mass are emitted during gamma decay, the emission of gamma radiation can result in the change in energy of the nucleus. For example, if a nucleus a gamma ray after undergoing alpha or beta decay it may end up in a different energy state. In such cases the nucleus may still be unstable and may continue to undergo further radioactive decay until it reaches a stable state. These laws imply that even though no particles with mass are emitted, the composition of the nucleus is certainly changed after emitting photons given by gamma decay. The gamma ray emission is shown in Fig. 22.6. The equation for gamma decay can be given as:

$$ {}^{A}_{Z}X^{*} \rightarrow {}^{A}_{Z}X + \gamma $$

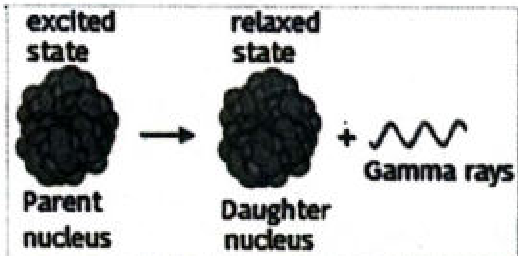


Figure 22.6: Gamma decay.

Here X^{*} represents the nucleus in an excited state. In an atom, nucleons (both protons and neutrons) have certain energy states. When they absorb energy during nuclear decay, they go up to the higher energy state and nucleus is said to be excited or in excited state. During de-excitation to the lower energy state the nucleus emits gamma radiations. Example of gamma ray emission is:

$$ {}^{60}_{27}Co^{*} \rightarrow {}^{60}_{27}Co + \gamma $$

The properties of three types of nuclear radiations are given in the Table 22.3.

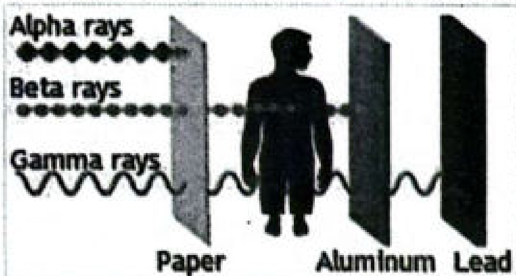


Figure 22.7: Types of radiation.

Table 22.3: Properties of nuclear radiations

Properties	Alpha	Beta	Gamma
Charge	+2	±1	0
Mass	4	Very small as compared to alpha particle	Negligible as compared to alpha particle and beta particle
Velocity	Slow (0.05 - 0.1 c)	Relatively fast (0.5 - 0.9 c)	Speed of light (c)
Penetration depth	Stopped by a few centimeters of air or a sheet of paper	Penetrate several meters of air, stopped by a few millimeters of aluminum	Penetrate several meters of air, stopped by several centimeters of lead or concrete
Typical source	Radon-222	Strontium-90	Cobalt-60

22.3.4 Spontaneous and Random Nature of Nuclear Decay

Radioactive decay is a random process i.e.; it is impossible to predict exactly when a particular unstable nucleus will decay. It may decay just on the moment you started observing it or it may not decay even for years. The decay is a natural process inherent to the unstable nucleus itself. It happens because the nucleus is seeking a more stable state. This randomness is reflected in the fluctuations observed in the count rate of radioactive emission. Count rate can be defined as:

The number of decays detected per unit time is called as count rate.

When measuring the count rate using a Geiger-Muller counter or any other counters it can be noted that the count rate varies over time even if the conditions remain constant. These fluctuations occur because each decay event is spontaneous, independent, random and unpredictable.

Each radioactive nucleus has a certain probability of decaying at any given moment but we cannot predict the exact time of decaying for any single nuclei, this property is known as "*spontaneity*". Although the individual decay events are random the average behavior of a large number of nuclei follows a predictable pattern described by the half-life of the substance. This is the reason that the count rate over longer periods becomes more stable and predictable in spite of showing fluctuations over short interval decays. These evidences provide clear evidence that decay events are not influenced by external factors and occur spontaneously. Spontaneous nuclear decay is a natural process that occurs in many elements in the universe. While it can pose a significant health hazard if not properly managed, it also has numerous practical applications in fields such as energy production, medicine, and materials science.

Example 22.4. Calculate the amount of energy 'Q' released during the following nuclear reaction. If mass of U-238 is 238.02891 u, mass of Th-234 is 234.04360 u and mass of alpha particle is 4.001506 u. $^{238}_{92}U \rightarrow ^{234}_{90}Th + ^{4}_{2}He + Q$

Given: Mass of uranium-238 = M_U = 238.02891 u Mass of Thorium-234 = M_T = 234.04360 u

Mass of alpha particle = M_α = 4.001506 u

To Find: Energy released = Q = ?

Solution: Energy in this nuclear reaction can be calculated as:

The total mass on L.H.S is: $M_U = 238.02891\ u$

The total mass on R.H.S is: $M_T + M_\alpha = 234.04360\ u + 4.001506\ u = 238.045106\ u$

Since the difference in mass is converted into energy 'Q' then we can write as:

$$Q = M_U - (M_T + M_\alpha) = 238.02891\ u - 238.045106\ u = -0.016196\ u$$

The negative sign shows that the energy is absorbed during the nuclear decay i.e. it is endothermic reaction. Now the amount of energy in MeV can be found as:

$$Q = -0.016196\ u \times 931.5\ MeV/u = 15.0866\ MeV$$

Hence 15.0866 MeV energy will be released when uranium-238 nuclei decays into thorium-234 while emitting an alpha particle.

Assignment: 22.4

Find the energy 'Q' in the following reaction. If mass of C-12 is 12.000 u, mass of N-14 is 14.00307 u and mass of electron is 0.0005486 u. $^{14}_{6}C = ^{14}_{7}N + e^{-} + Q$

22.4 HALF LIFE AND RATE OF DECAY

The half-life ($T_{1/2}$) of a radioactive material is a characteristic property of that material and is defined as:

The time it takes for half of the radioactive nuclei to decay in a given sample is called as half-life of that element.

For example, if we have 100 nuclei at time 't = 0' and after half-life the half of the nuclei will disintegrate and half would remain intact i.e., 50 nuclei of parent element will remain. After second half-life out of these 50 nuclei 25 will disintegrate and 25 nuclei of parent element will remain similarly after third and fourth half-lives the remaining nuclei of parent element will be 12.5 and 6.25 respectively. For example, the half-life of cobalt-60 is 5.27 years and its decay with half-lives is shown in Fig. 22.8. Although after every half-life the nuclei become half of their initial value but still it takes infinite time for the disintegration of all nuclei. The rate of disintegration of nuclei follows a law known as rate law which can be defined as:

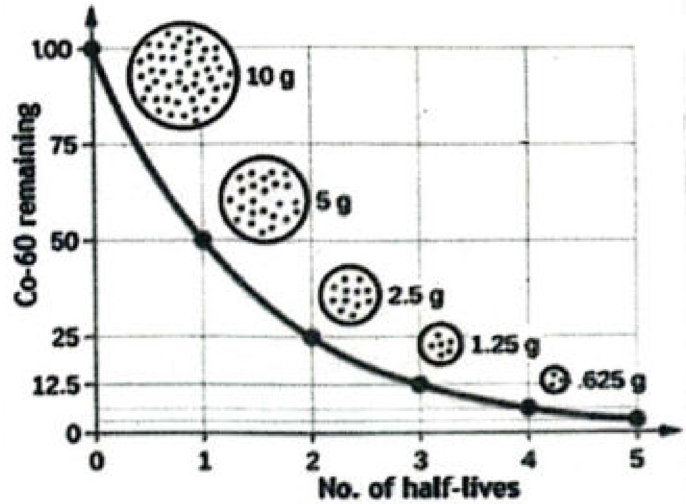


Figure 22.8: Decay of Co-60.

The rate at which a radioactive substance decays is directly proportional to the number of radioactive nuclei present at any given time, is called the rate law for nuclear decay.

Table 22.3: Half-lives of some elements		
Element	**Decays into**	**Half-life**
Carbon-14	Nitrogen-14	5,730 years
Aluminum-26	Magnesium-26	740,000 years
Iodine-129	Xenon-129	17 million years
Uranium-235	Lead-207	704 million years
Uranium-238	Thorium-234	4.5 billion years
Potassium-40	Argon-40	1.3 billion years
Rubidium-87	Strontium-87	49 billion years
Plutonium-239	Uranium-235	24,110 years
Cesium-137	Barium-130	30 years
Strontium-90	Yttrium-90	28.8 years
Radon-222	Polonium-218	3.82 days

Half-lives of some typical elements are given in Table 22.4. The rate of spontaneous nuclear decay is measured by the half-life. It can vary widely depending on the specific material, ranging from fractions of a second to billions of years. If 'ΔN' is the number of atoms which decay in time 'Δt' then according to decay law the time rate of spontaneous disintegration of a radioactive element is proportional to the number of nuclei 'N' present and can be given as:

$$\frac{\Delta N}{\Delta t} \propto -N$$

Here negative sign shows that 'ΔN' is decreasing with time. Now by changing the sign of proportionality into equation a constant is multiplied as:

$$\frac{\Delta N}{\Delta t} = -\lambda N$$

Here 'λ' is a constant and is known as decay constant which is the proportionality factor in the rate law of radioactive decay. It is equal to the fraction of the radioactive of the radioactive nuclei that decay per unit time. It can be given as:

$$\lambda = \frac{1}{N} \times \frac{\Delta N}{\Delta t}$$

While the ratio '$\Delta N / \Delta t$' is called the activity in the radioactive decay process which is the number of disintegrations per unit time. It is represented by 'A' and can be given as:

$$A = \frac{\Delta N}{\Delta t}$$

$$A = -\lambda N$$

The SI unit of activity is becquerel (Bq) which can be defined as:

When a nucleus disintegrates at the rate of one disintegration per second then the activity is equal to one becquerel.

In nature, there are some radiations present everywhere (called background radiation). Hence, we are constantly exposed to radiation in daily life, but the natural levels are typically low and safe for humans and other biological objects on Earth. Harmful effects require higher radiation doses. For example, the activity of radium used in the dial of a watch (to make it glow in the dark) is 4×10^4 Bq and for radiotherapy the dose range is 4×10^{13} Bq.

A more common unit of activity is the curie which can be defined as:

When there is an activity at the rate of 3.70×10^{10} decays per second then the activity is equal to one curie.

The inter conversion of these two units of activity can be given as:

$$1 \text{ Ci} = 3.70 \times 10^{10} \text{ Bq}$$

From the above equation, it is clear that the becquerel is very small unit as compared to the curie.

22.4.1 Exponential Nature of Nuclear Decay

Radioactive decay is exponential in nature. This means that the rate at which a radioactive substance decays is proportional to the number of un-decayed nuclei present at any given time. If 'N' be the number of nuclei at any instant which are undecayed, then

$$\frac{\Delta N}{\Delta t} = -\lambda N$$

For very small change we can take above relation as:

$$\frac{dN}{dt} = -\lambda N$$

By the separation of variable technique, we get:

$$\frac{1}{N} dN = -\lambda \, dt$$

Taking integration on both sides: $\int \frac{1}{N} dN = -\int \lambda \, dt$

$$\ln N = -\lambda\, t + \text{constant} \_\_\_\_\_ \text{(i)}$$

To find the value of this constant we can apply the initial conditions as: at $t = 0$, $N = N_o$

Using these values in above equation we get:

$$\ln N_o = -\lambda\,(0) + \text{constant} \quad \text{or} \quad \ln N_o = \text{constant}$$

Now Eq. (i) gets the form:

$$\ln N = -\lambda\, t + \ln N_o \quad \text{or} \quad \ln N - \ln N_o = -\lambda\, t$$

or $$\ln \frac{N}{N_o} = -\lambda\, t$$

Taking exponent on both sides, we get: $\frac{N}{N_o} = e^{-\lambda t}$

or $$N = N_o e^{-\lambda t}$$

This relation shows that the decay of nucleus is exponential with time.

Example 22.5. Find decay constant and activity of an element if its 100 out of 700 nuclei decay in 5 min.

Given: Original number of nuclei: 'N' = 700 Dacayed nuclei: 'ΔN' = 100

Time: 't' = 5 min = 300 s

To Find: Decay constant: λ = ? Activity: A = ?

Solution: To find decay constant we use: $\lambda = \frac{1}{N} \times \frac{\Delta N}{\Delta t}$

Putting values, we get: $\lambda = \frac{1}{700} \times \frac{100}{300} = 4.76 \times 10^{-4}\,s^{-1}$

Now to find activity, we use:

$$A = \lambda\, N$$

Putting values, we get: $A = 4.76 \times 10^{-4}\,s^{-1} \times 700 = 0.333\ \text{Bq} = 9 \times 10^{-12}\ \text{Ci}$

Assignment 22.5

Find decay constant and activity of an element if its 240 out of 780 nuclei decay in 12 seconds.

22.4.2 Relation between Decay Constant and Half-Life

The relation between decay constant and half-life can be deduced by considering a radioactive element that has 'N_o' number of nuclei at a certain instant. If the half-life of the element is four hours, then after four hours the number of radioactive element's nuclei left will be '$N_o/2$'. After eight hours i.e. after two half-lives the number of nuclei of the original element left will be '$N_o/4$'' similarly after twelve hours i.e. after three half-lives the number of nuclei of the original element left will be '$N_o/8$'' and so on. To represent the variation of un-decayed nuclei as a function of time a graphical method is more suitable, as shown in Fig. 22.9.

From the graph it is very easy to find the half-life of a material. To derive the relation for half-life, we use equation:

$$N = N_o e^{-\lambda t}$$

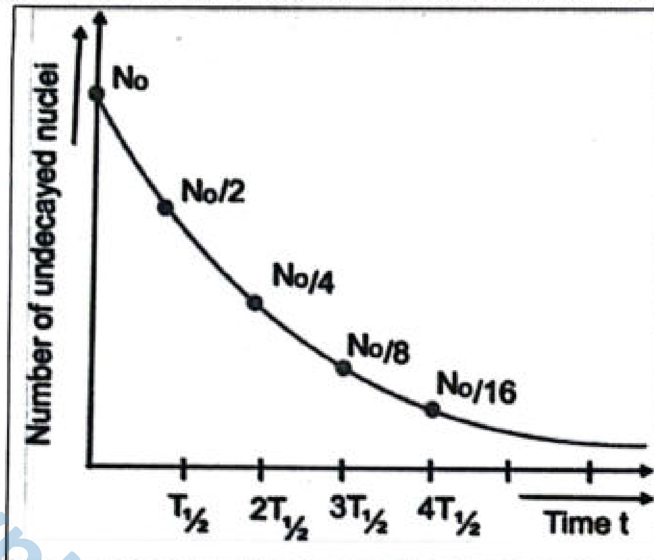
Now use the values for first half life of an element as: $N = \frac{N_o}{2}$ and $t = T_{1/2}$. Using these values in above equation we get:

$$\frac{N_o}{2} = N_o e^{-\lambda T_{1/2}}$$

or $$\frac{1}{2} = e^{-\lambda T_{1/2}}$$

Taking inverse of the above relation, we get:

$$2 = e^{\lambda T_{1/2}}$$



Time	N
0	10000
$T_{1/2}$	5000
$2T_{1/2}$	2500
$3T_{1/2}$	1250
$4T_{1/2}$	625

Figure 22.9: Graphical representation of nuclear decay

Now taking natural logarithm on both sides: $\ln 2 = \lambda\, T_{1/2}$

Now the relation of half-life of a radioactive element can be given as:

$$T_{1/2} = \frac{\ln 2}{\lambda} \qquad \text{or} \qquad T_{1/2} = \frac{0.693}{\lambda}$$

This equation shows the relationship between the decay constant and the half-life of an element. The half-life of a radioactive material is determined by measuring the activity of a given sample over a period of time. It is important to note that half-life is a characteristics property of each radioactive element and it is used to describe the rate at which a radioactive substance decays. The half-life of radioactive element is the duration it takes for half of the original amount of the substance to decay into a more stable form. The longer the half-life of a substance the slower is its rate of decay and less radiations it emits over time.

Example 22.6. Cesium-137 (a beta emitter used for treating cancer and in industrial devices to measure the thickness of materials) has a half-life of 9.5x10⁸ s. we have one mole of cesium at the start. What will be the decay constant and activity for this decay?

Given: Half-life of cesium-137 = $T_{1/2} = 9\times10^8$ s $\qquad N_o = 6.02\times10^{23}$

To Find: Decay constant = λ = ? $\qquad$ Activity = A = ?

Solution: The relationship between half-life and decay constant is given by:

$$T_{1/2} = \frac{0.693}{\lambda} \qquad \text{or} \qquad \lambda = \frac{0.693}{T_{1/2}}$$

Putting values, we get:

$$\lambda = \frac{0.693}{9.5\times10^8\,\text{s}} = 7.3\times10^{-10}\,\text{s}^{-1}$$

Now to find the activity, we use: $A = \lambda N$

Putting values, we get: $A = 7.3\times10^{-10}\,s^{-1}\times6.02\times10^{23} = 4.395\times10^{14}\,Bq = 1.188\times10^{4}\,Ci$

Assignment 22.6

The half-life of strontium (Sr-91) is 9.70 hours. Find its decay constant.

22.5 NUCLEAR REACTIONS

Nuclear reactions are processes in which one or more nuclides are produced from collision between two nuclei or one nucleus and a sub-atomic particle. A nuclear reaction is said to occur when an incident nucleus, particle or photon causes a change in the target nucleus.

Nuclear reactions are processes in which atomic nuclei interact, resulting in changes to the nucleus, such as fusion, fission, or radioactive decay, often releasing or absorbing energy.

A nuclear reaction changes the identity or characteristics of an atomic nucleus by bombarding it with an energetic particle. The bombarding particle may be an alpha particle, a gamma ray photon, a neutron, a proton or a heavy ion. These bombarding particles must have enough energy to approach the positively charged nucleus within the range of the strong nuclear force. Let the nucleus 'X' is bombarded by some light particle 'a' which products a nucleus 'Y' along with a light particle 'b' mathematically:

$$a + X \rightarrow Y + b$$

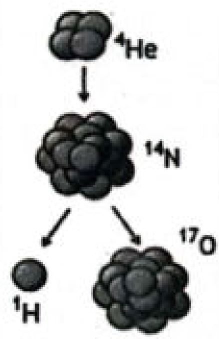


Figure 22.10: Nuclear reaction.

The first ever nuclear transmutation, as shown in Fig. 22.10, observed by Rutherford, is given below:

$$^{4}_{2}He + ^{14}_{7}N \rightarrow ^{17}_{8}O + ^{1}_{1}H + Q$$

Where Q is the energy released or absorbed equivalent to the difference of the rest masses of the elements on both sides of nuclear reaction. If 'Q' is positive the energy will be liberated in the reaction called exothermic reaction and if 'Q' is negative the energy is absorbed in the reaction called endothermic reaction. If thermal neutron (neutrons with energy equal to surrounding atoms i.e. about 0.025 eV) is bombarded on nitrogen-14 nucleus, the following reaction can be produced with release of a large amount of energy:

$$^{1}_{0}n + ^{14}_{7}N \rightarrow ^{14}_{6}C + ^{1}_{1}H + Q$$

The product in this case is the radio carbon-14, a proton and heat energy. Similarly, deuteron which has a unit positive charge and must have high energy to cause an induced nuclear reaction as:

$$^{2}_{1}H + ^{55}_{25}Mn \rightarrow ^{56}_{26}Fe + ^{1}_{0}n + Q$$

All the conservation laws must be followed in a nuclear reaction including law of conservation of mass-energy, momentum and charge etc. The main types of nuclear reactions due to low value of binding energy per nucleons are nuclear fusion and nuclear fission which are explained below.

22.5.1 Nuclear Fusion

Do you ever think the energy from the stars coming to us is obtained by which process? What is the fuel of the Sun and how it produces tremendous amount of energy? Answer to these questions lies in a simple nuclear reaction called the nuclear fusion reaction which can be stated as:

A process where two or more light atomic nuclei come close to each other to form a heavier nucleus by releasing a large amount of energy is called nuclear fusion.

Small nuclei combine to form heavier nuclei, releasing a large amount of energy in the form of heat. This process occurs naturally in stars where the high temperature and pressure causes the nuclei to collide and fuse. The Sun's core is the site of several nuclear reactions that powers its energy output. Scientists are currently working on developing nuclear fusion as a potential clean and sustainable source of energy. A simple fusion reaction is shown in Fig. 22.11 and can be written as:

$${}^{2}_{1}H + {}^{3}_{1}H \rightarrow {}^{4}_{2}He + {}^{1}_{0}n + Q \quad \text{------} \quad (22.1)$$

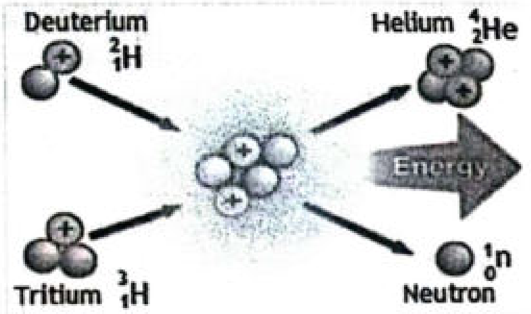


Figure 22.11: Fusion reaction.

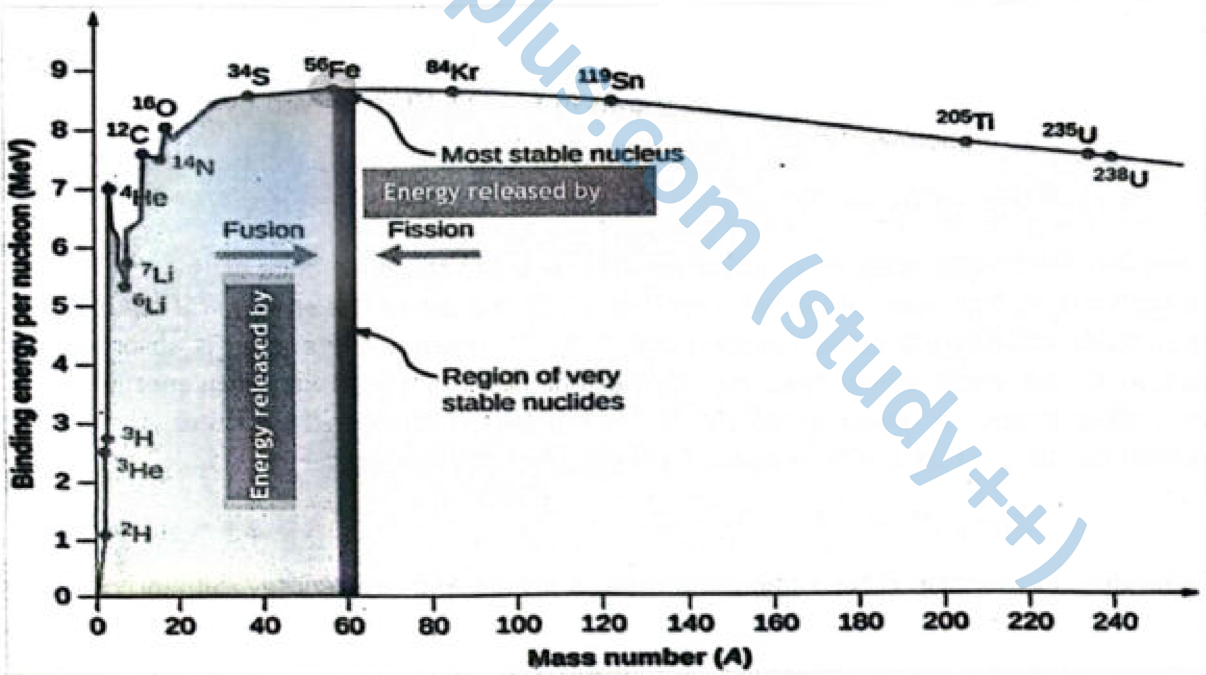


Figure 22.12: Fission and Fusion reaction with binding energy curve.

Why small nuclei tend to fuse with each other? This is due to their low binding energy per nucleon which makes them less stable as compared to the elements of region of greater stability. They tend to fuse with each other to get themselves more stable. As from Fig. 22.12 the binding energy per nucleon increases when the mass number increases and becomes maximum for '$^{56}_{26}Fe$' whose atomic mass is A = 56 which is 8.8 MeV per nucleon. The initial increase in binding energy per nucleon favors the fusion reaction.

Energy in Nuclear Fusion: We can calculate the amount of energy released during nuclear fusion reaction by using the concept of binding energy per nucleon as the total energy of fusing nuclei in equation (22.1) can be given:

Mass of Deutrium $= 2.014102$ u Mass of Tritium $= 3.016049$ u

Total Mass $= 5.030151$ u

To covert this mass in kilograms, we proceed as:

$$\text{Total Mass} = 5.030151\ \text{u} \times 1.660 \times 10^{-27}\,\text{kg/u} = 8.350 \times 10^{-27}\,\text{kg}$$

Energy of L.H.S of equation (22.1) is:

$$E = \Delta m\, c^2 = 8.350 \times 10^{-27}\,\text{kg} \times (3 \times 10^8\,\text{ms}^{-1})^2 = 7.515 \times 10^{-10}\,\text{J} = 4696.87\ \text{MeV} \quad \text{\_\_\_\_\_\_ (i)}$$

Now the total energy of product nuclei and particle in equation (22.1) can be given:

Mass of Helium $= 4.002603$ u Mass of Neutron $= 1.008665$ u

Total Mass $= 5.011268$ u

To covert this mass in kilograms we get:

$$\text{Total Mass} = 5.011268\ \text{u} \times 1.660 \times 10^{-27}\,\text{kg/u} = 8.319 \times 10^{-27}\,\text{kg}$$

Energy of L.H.S of equation (22.1) is:

$$E = \Delta m\, c^2 = 8.319 \times 10^{-27}\,\text{kg} \times (3 \times 10^8\,\text{ms}^{-1})^2 = 7.4871 \times 10^{-10}\,\text{J} = 4679.438\ \text{MeV} \quad \text{\_\_\_\_\_\_ (ii)}$$

Using these values in the equation (22.1), we get the amount of energy obtained/released in the above fusion reaction as:

$$4696.87\ \text{MeV} = 4679.438\ \text{MeV} + Q$$

or $$4696.87\ \text{MeV} - 4679.438\ \text{MeV} = Q$$

or $$Q = 17.43\ \text{MeV}$$

Here positive value of Q shows that it is an exothermic reaction and heat is released in this reaction. Hence a single event of fusion of deuterium and tritium forming helium and a neutron, releases 17.43 MeV energy.

22.5.3 Nuclear Fission

Otto Hahn and Fritz Strassmann while working upon the nuclear reaction find a startling discovery. They discovered a phenomenon called nuclear fission reaction which can be stated as:

A process where the nucleus of an atom splits into two or more smaller nuclei, with release of large amount of energy is called nuclear fission.

The *process is typically* initiated by bombarding the nucleus with a neutron, which causes it to become unstable and split apart. There are different possible nuclear reactions with different daughter nuclei they may be different nuclei or different isotopes of same nuclei. For example, a nuclear reaction is shown in Fig. 22.13 and can be given as:

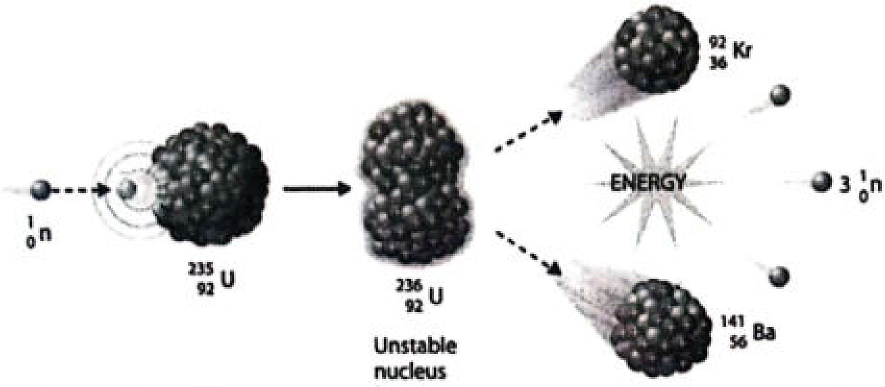


Figure 22.13: Fission reaction.

$$^{1}_{0}n + ^{235}_{92}U \rightarrow ^{236}_{92}U^{*} \rightarrow ^{141}_{56}Ba + ^{92}_{36}Kr + 3\,^{1}_{0}n + Q \quad \text{(i)}$$

In this reaction, when a slow neutron hits uranium-235, it forms an unstable nucleus (i.e., uranium-236 $^{236}_{92}U^{*}$) that breaks into barium, krypton, neutrons, and releases a lot of energy. Fission does not give the same product every time. Fission of uranium-235 can also yield different elements, such as shown in the following equation:

$$^{1}_{0}n + ^{235}_{92}U \rightarrow ^{236}_{92}U^{*} \rightarrow ^{132}_{50}Sn + ^{101}_{42}Mo + 3\,^{1}_{0}n + Q$$

The number of neutrons emitted during fission reaction is not necessarily to be three. There are some reactions which emit two neutrons per fission event. The product may be different isotopes of the same elements as in equation (i), which is shown in Fig. 22.14 and can be given as:

$$^{1}_{0}n + ^{235}_{92}U \rightarrow ^{236}_{92}U^{*} \rightarrow ^{139}_{56}Ba + ^{95}_{36}Kr + 2\,^{1}_{0}n + Q$$

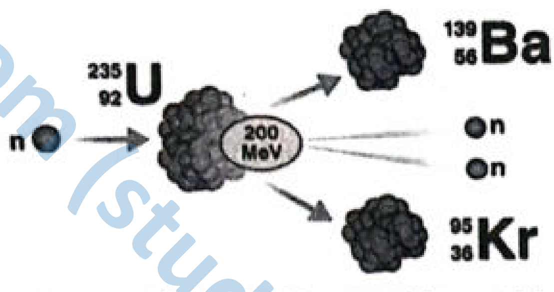


Figure 22.14: Fission reaction with different isotopes.

Nuclear fission has practical applications in nuclear power plants to generate electricity as well as in nuclear weapons. As from Fig. 22.12, we can see that the binding energy per nucleon is small for heavier nuclei and hence they are unstable and tend to get more stable condition. The fission reaction occurs due to instability of heavy nuclei such that they may get small fragments having larger stability.

Fission Chain Reaction: A single fission reaction may release two or three neutrons which can further cause fission reactions. That further reactions produce more neutrons that cause more fission reaction and so on.

A fission reaction in which every time at least one released neutron produces further fission reaction then such succession of fission reactions is called fission chain reaction.

The neutrons produced during fission reaction are fast neutrons and can escape out of the material without producing more fission reactions, as for fission, slow neutrons are required. To sustain the fission process, we need one neutron per fission to be capable of carrying out next fission. If the size of material is large, some of these neutrons may be captured to produce further fission. Hence, we need a minimum mass of material to sustain fission chain reaction which is called as critical mass and can be defined as:

The minimum mass of the material to sustain the fission chain reaction is known as the critical mass.

If the mass of the material is less than the critical mass the fission process would soon come to an end. But if the mass of the material is greater than the critical mass the fission process will get uncontrolled and would lead to cause the nuclear bomb. If the mass of the sample is equal to the critical mass (or the conditions are suitable for producing one more fission reaction per event) controlled nuclear energy will be produced as in case of nuclear reactors. This process is used in nuclear reactors and nuclear weapons. In a nuclear weapon, the nuclear fission is uncontrolled leading to a rapid and explosive release of energy. The potential for uncontrolled chain reactions is what makes nuclear weapons so dangerous and that's why strict safety protocols are to be followed to prevent any incident. In a nuclear reactor, the chain fission reaction is controlled using different techniques which enable us to produce energy. A fission chain reaction of uranium-235 is shown in Fig. 22.15. The volume occupied by the critical mass of a material is known as critical volume.

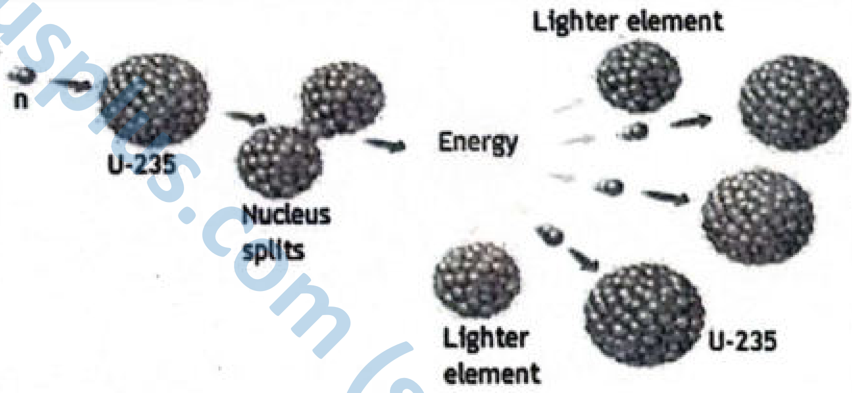


Figure 22.15: Fission chain reaction.

Energy in Fission Reaction: A huge amount of energy is released in nuclear reactions, particularly in nuclear fission reaction, which is much greater than the amount of energy released in a typical chemical reaction. For example, a single fission event like the splitting of uranium-235 nucleus releases about 200 MeV of energy. On the other hand, a chemical reaction like combustion of fossil fuels release energy on the order of a few electron volts (eV) per reaction. For example, energy released by one molecule of methane (CH_4) gas is about 9.2 eV.

The fission of 1 kg of uranium-235 can produce about 2.5 million times more energy than burning of 1 kg of coal. Hence, nuclear fission reaction releases energy that is millions of times greater than that of chemical reactions. Due to this huge energy difference nuclear power plants are so useful as compared to traditional chemical based energy sources.

Example 22.7. Find the amount of energy released by the following fission reaction if the masses of uranium, barium and krypton are 235.12142 u, 140.883 u and 91.9262 u, respectively. $\quad {}^{1}_{0}n + {}^{235}_{92}U \rightarrow {}^{141}_{56}Ba + {}^{92}_{36}Kr + 3{}^{1}_{0}n + Q$

Given: Mass of uranium-235: 235.12142 u
Mass of crypton-92: 91.9262 u
Mass of barium-141: 140.883 u
Mass of neutron: 1.008665 u

To Find: Energy: Q = ?

Solution: Total mass on L.H.S = 1.008665 u + 235.12142 u = 236.1301 u

So, Total energy on L.H.S = $236.1301\ u \times 931.5\ MeV/u\,c^2 = 219955.188\ MeV/c^2$

Now Total mass on R.H.S = $140.883\ u + 91.9262\ u + (3 \times 1.008665)\ u = 235.8352\ u$

So, Total energy on R.H.S = $235.8352\ u \times 931.5\ MeV/u\,c^2 = 219680.489\ MeV/c^2$

Now to find the energy 'Q' for the above fission, put values in reaction equation:

$$219955.188\ MeV/c^2 = 219680.489\ MeV/c^2 + Q$$

or $$Q = 219955.188\ MeV/c^2 - 219680.489\ MeV/c^2$$

$$Q = 274.699\ MeV/c^2$$

Positive sign shows that the energy is released during the fission reaction.

Assignment 22.7

Find the amount of energy released by the following fission reaction if the masses of uranium, Xenon, Strontium and neutron are 235.12142 u, 139.9055 u, 93.9064 u and 1.008665 u respectively constant.

$$^{1}_{0}n + ^{235}_{92}U \rightarrow ^{140}_{54}Xe + ^{94}_{38}Sr + 2^{1}_{0}n + Q$$

22.6 NUCLEAR REACTORS

Large amount of energy is released in a nuclear fission reaction; therefore, it is carried under controlled conditions. Then this energy is used for useful purposes like in the production of electricity and radioisotopes.

Nuclear reactor is a device used to initiate and control a nuclear fission chain reaction.

A nuclear reactor typically consists of the core, fuel rods, control rods or safety rods, moderator, coolant (heat exchanger) and shielding unit. A nuclear reactor is shown in Fig. 22.16.

Core of Reactor: The core is the main component of a nuclear reactor which contains the fuel in which the nuclear fission process takes place. The core is the heart of a reactor where the critical processes of nuclear fission and heat generation occur which enables the reactor to function as a source of energy. It is designed to sustain a controlled nuclear fission chain reaction. The main parts in the core of the nuclear reactor are:

Fuel Rods: The fuel used in nuclear reactor is typically Uranium-235 or Plutonium-239, which is in the form of small pallets and arranged into long rods of roughly 1 cm in diameter. Most reactors use uranium in which the amount of Uranium-235 is enriched to about 3%. These rods are placed into the reactor's core.

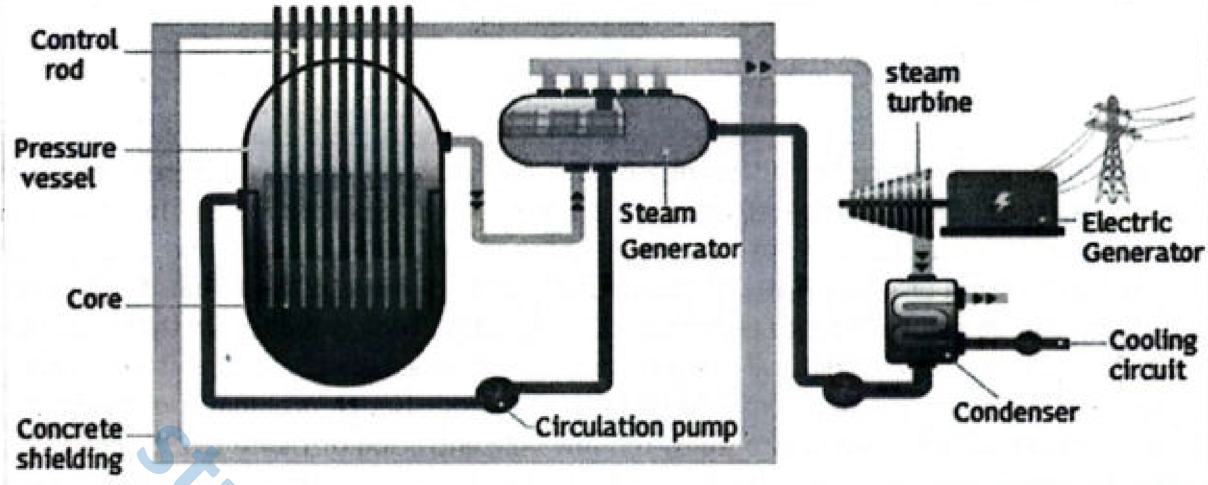


Figure 22.16: Schematic diagram of Nuclear Reactor.

Control Rods: Control rods also called as safety rods play very important role in the action of nuclear reactor as they are used to control the extent of reaction and do not allow the fuel to react at once. They are made of a material such as boron or cadmium that absorbs neutrons. These rods control the fission reaction by absorbing some of the neutrons and preventing them from causing additional fission reactions as for the output of a reactor to be constant only one neutron from each event should process further fission reaction. These rods can move into or out of the reactor's core as per need.

Moderator: Material that slows down the neutrons is called moderator. Neutrons with energies of about 0.025 eV are used in fission of uranium while the neutrons emitting from fission reaction are fast and have energies in the range of MeV which is not required for fission. The commonly used materials used as moderator are water and graphite that slows down the neutrons produced by fission as slowing down the neutrons make them more likely to cause additional fission.

Figure 22.17: Moderator.

Coolant: The coolant or heat exchanger is a fluid such as water or helium that circulates through the reactor's core and carries away the heat produced by the fission process. This heat is used to produce steam which in turns derives the turbines to generate electricity. It also serves the purpose to cool the fuel rods and moderators from excess heat. In the absence of a coolant the core can melt under the enormous heat produced by the fission process. After passing through the generator the coolant passes through the condenser which cooled down the material before entering again into the reactor's core.

Shielding Unit: The by-product of a fission reaction includes strong radiations and energetic particles which are harmful to human and the atmosphere. To keep human being safe, we have to shield those radiations and high energy particles within the reactor such that they would not release into the atmosphere. For the above said purpose we use a thick layer of concrete or other material that surrounds the reactor vessel. It protects workers and the environment from radiations emitted by the reactor.

Enrichment of Uranium

The uranium found in nature has different isotopes and each isotope has different properties and characteristics. Every isotope of uranium is not equally favorable for the nuclear fission chain reaction. Uranium usually has two main isotopes i.e., Uranium-235 and Uranium-238. About 99.3% of natural uranium is U-238 but unluckily it is not directly fissile but it can capture neutrons and can be converted into plutonium-239 (Pu-239) which is fissile. On the other hand, uranium-235 is about 0.7% of natural uranium and this isotope is fissile means it can sustain a chain reaction by absorbing neutrons and undergoes fission. Hence uranium fuel needs to be enriched because natural uranium contains too low concentration of fissile U-235. Enrichment increases the proportion of U-235 allowing reactors to achieve and maintain criticality, operate efficiently, manage reactor design and fuel usage effectively.

The process of converting non fissile material like uranium-238 into a fissile material like uranium-235 is called enrichment.

To sustain a controlled nuclear chain reaction a reactor needs a sufficient concentration of fissile material i.e. U-235 in nature its concentration is as low as 0.7% which is not sufficient and nuclear reactor cannot be run on this percentage. Enriched uranium typically contains 3-5% U-235 due to this higher concentration of U-235 it improves the efficiency of reactor making it more feasible to maintain a steady and controlled reaction.

Various methods can be used to enrich uranium including gaseous diffusion method, as shown in Fig. 22.18, gas centrifugation and laser enrichment. The most common method is gas centrifugation which uses rapidly spinning centrifuges to separate uranium isotopes based on difference in their masses.

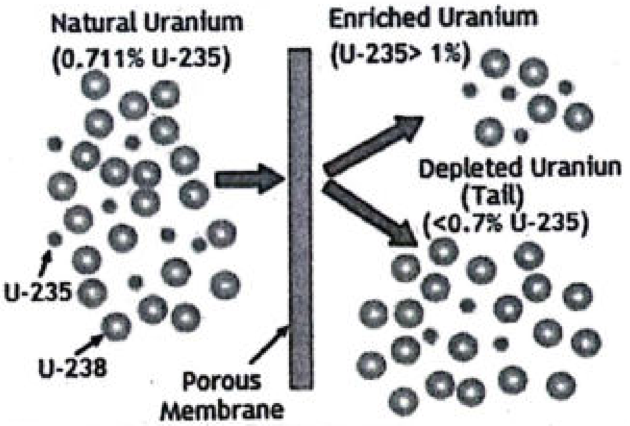


Figure 22.18: Gaseous diffusion uranium enrichment process.

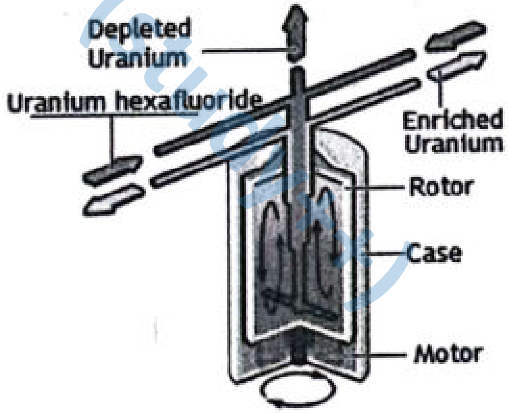


Figure 22.19: Gas Centrifuge.

The gas centrifugation process is shown in Fig. 22.19, in which a motor rotates the case which contained uranium hexafluoride. Gaseous diffusion enrichment is a method used to enrich uranium by converting it into a gas (Uranium hexafluoride) and passing it through porous barriers. The lighter U-235 molecules pass through more easily than the heavier U-238 molecules, gradually increasing the concentration of U-235. This process is repeated multiple times to achieve the desired level of enrichment.

22.7 ENERGY IN ANNIHILATION REACTIONS

Annihilation reactions are fundamental in particle physics and have practical applications in technologies like PET Scans in medical imaging. It occurs when a particle collides with its corresponding anti-particle resulting their mutual destruction and conversion of their mass into energy. This process is governed by the principles of conservation of momentum and mass-energy means total energy before and after the annihilation remains the same, as described by Einstein's equation $E = m c^2$. The total momentum also conserves in these reactions i.e. the total momentum of photons will be equal to the initial momentum of the particle and anti-particle. The most common example of annihilation reaction is electron-positron annihilation to produce two gamma ray photons and can be given as:

$$e^- + e^+ = \gamma + \gamma \qquad \text{\_\_\_\_\_\_ (22.2)}$$

Another annihilation reaction can be of proton and anti-proton in which they annihilate to produce photons or various other particles like pion and kaon, such reactions can be given as:

$$p^+ + p^- = \gamma + \gamma$$

or $$p^+ + p^- = \pi^+ + \pi^- + \pi^0$$

The annihilation reaction of proton and anti-proton is shown in Fig. 22.28.

Figure 22.19: Proton anti-proton annihilation.

The energy in electron-positron annihilation given by equation (22.2) can be calculated as:

The rest mass energy of an electron can be found by using equation $E = m_o c^2$ while the rest mass of an electron and positron is $9.1 \times 10^{-31}\,\text{kg}$. Now to calculate the rest mass energy of the electron and positron we can use:

$$E = m_o c^2 + m_o c^2 = 2hf$$

$$E = m_o c^2 = 9.1 \times 10^{-31}\text{kg} \times (3 \times 10^8 \text{ms}^{-1})^2$$

or $$E = m_o c^2 = 8.19 \times 10^{-14}\,\text{J}$$

This energy can be converted into electron-volt units as:

$$E = \frac{8.19 \times 10^{-14}}{1.6 \times 10^{-19}}\,\text{eV} = 511875\ \text{eV}$$

As positron has the same mass as that of electron hence it also has the same energy so total energy of the annihilating particles can be given as:

$$E = 2 \times 511875\ \text{eV} = 1.02\ \text{MeV}$$

This is minimum amount of energy produced by the annihilation reaction. Hence each gamma ray photon carries energy of 0.51 MeV. For the case when electron and positron are moving then their kinetic energy is also included in the total energy which determines the energy of the emitted gamma ray photons and can be written as:

$$K.E_{electron} + K.E_{positron} + 2m_oc^2 = 2hf \quad \_\_\_\_\_ \quad (22.3)$$

Example 22.8. Find the amount of energy released by the annihilation of electron and positron initially moving with kinetic energy of 0.12 MeV each, while the rest mass energy of electron and positron is 0.51 MeV.

Given: Rest mass energy of electron: $m_oc^2 = 0.51$ MeV

Rest mass energy of positron: $m_oc^2 = 0.51$ MeV

$K.E_{electron} = 0.12$ MeV $\quad\quad$ $K.E_{positron} = 0.12$ MeV

To Find: Energy of gamma ray photon: $hf = ?$

Solution: To find energy we use equation 22.15:

$$K.E_{electron} + K.E_{positron} + 2m_oc^2 = 2hf$$

Putting values, we get: $0.12 \text{ MeV} + 0.12 \text{ MeV} + 2\times(0.51 \text{ MeV}) = 2hf$

or $\quad 1.26 \text{ MeV} = 2hf$

or $\quad hf = 0.63 \text{ MeV}$

Hence the energy of each gamma ray photon will be 0.63 MeV.

Assignment: 22.8

Find the energy of gamma rays photon released by the proton and anti-proton annihilation reaction where the mass of proton and anti-proton is 1.007276 u. The reaction can be given as: $\quad p^+ + p^- = \gamma + \gamma$

22.8 MEDICAL USES OF RADIATIONS

Radiations have vast medical treatment and diagnostics applications. The most commonly used radiations you would ever use are the medical x-rays, gamma ray therapy and other radiations. These are widely used for different purposes in medical science. Medical tracer and PET scanner are some of the techniques commonly used in medical diagnostics.

22.8.1 Medical Tracer

A medical tracer is a substance which contains some radioactive nuclei. It is used in medical imaging to study the functions of tissues and organs. When we introduce this substance into the body the tracer is absorbed by the specific tissue being examined, as shown in Fig. 22.20. The radioactive nuclei emit gamma rays which can be detected by imaging equipment such as positron emission tomography (PET) or single photon emission computed tomography

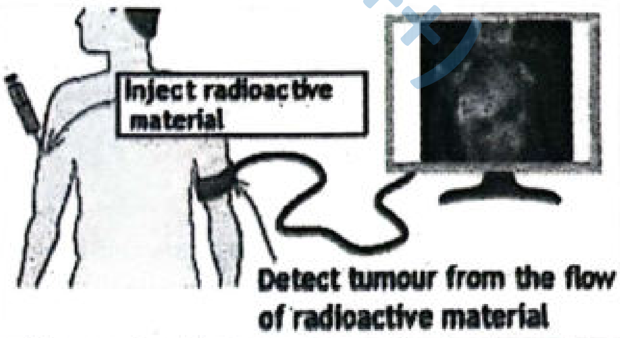


Figure 22.20: Medical Tracer.

(SPECT) scanners. These images help doctors diagnose and monitor various conditions such as cancer, heart disease and neurological disorders by providing detailed information about the processes occurring within the body. The tracer is usually introduced into the body through injection into the blood stream, ingestion or inhalation depending upon the type of study and the area of interest. When tracer enters into the body it is absorbed by specific tissues due to its affinity for certain biological processes. Tracers are valuable in research settings to study how diseases develop and progress as well as to explore new treatments and diagnostic methods. There are many radioactive materials which are used as medical tracers depending upon the need of study and their properties required. Some of the commonly used materials as medical tracers and their use are given in the table below:

Table 22.4: Medical tracers and their use

Element	Use	Half-life
Technetium-99	Bone scan, cardiac stress and thyroid	6 hours
Fluorodeoxyglucose	Metabolic activity and cancerous tumors	110 minutes
Iodine-131	Therapeutic applications, hyperthyroidism and thyroid cancer	8 days
Iodine-123	Thyroid gland, brain and heart scan	13 hours
Gallium-67	Infections, tumors, lymphoma and lung cancer	78 hours
Thallium-201	Myocardial perfusion imaging, heart function and coronary artery disease	73 hours
Radium-223	Bone metastases, prostate cancer and reduce cancer spread in bone tissues	11.4 days
Carbon-11	Neurodegenerative diseases and certain types of cancer	20 minutes
Oxygen-15	Measure blood flow and metabolism in tissues	2 minutes

22.8.2 PET Scanner

Positron emission tomography (PET) is a powerful imaging technique that allows us for detailed observation of metabolic physiological processes within the body. For diagnostic under PET scanner, we perform the following steps:

i) Inject the patient with some positron emitting radionuclide such as carbon-11. The tracer travels through the blood stream and accumulates in tissues or organs which are under studies.

ii) The radionuclide within the tracer undergoes radioactive decay emitting a positron. A positron is an anti-particle of electron which has the same mass as that of electron but of opposite polarity of charge.

iii) The emitted positron from C-11 travel a short distance and they encounter electrons in the surrounding tissues.

iv) When the electron and positron meet, they annihilate each other producing gamma ray photons, as shown in Fig. 22.21.

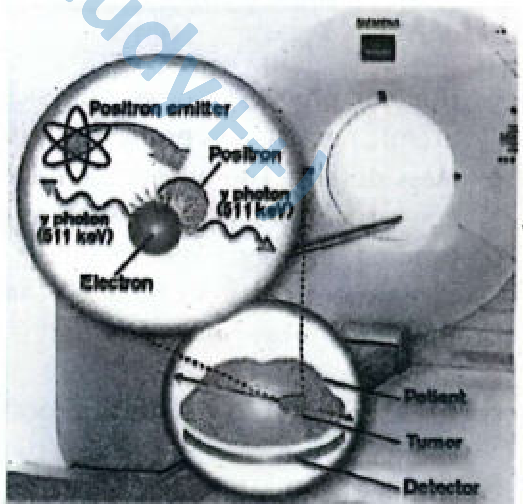


Figure 22.21: Annihilation within body.

v) The two gamma ray photons result from electron positron annihilation emitted with energy of 511 keV each and travel in opposite direction to conserve momentum.

vi) PET scanner consists of a ring of detectors that encircle the patient, as shown in Fig. 22.22. These detectors are sensitive for gamma ray detection when the gamma rays strike the detectors they are converted into an electric signal. In this process these gamma ray photons escape the body and travel outside of the body towards the ring detector that surrounds the body.

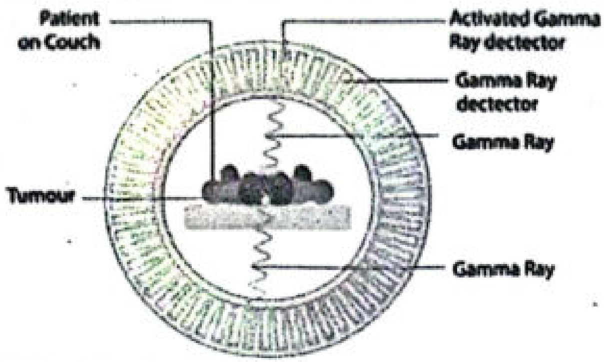


Figure 22.22: Detectors in PET.

vii) The scanner detects simultaneous arrival of many pairs of gamma rays travelling at 180°. Then it records their time of arrival and their spatial information. The detected signals are then processed by a computer to determine the location of annihilation event within the body. The computer then makes an image from the detected photon pair providing insight into the body.

The complete working diagram of positron emission tomography (PET) scanner is shown in Fig. 22.23.

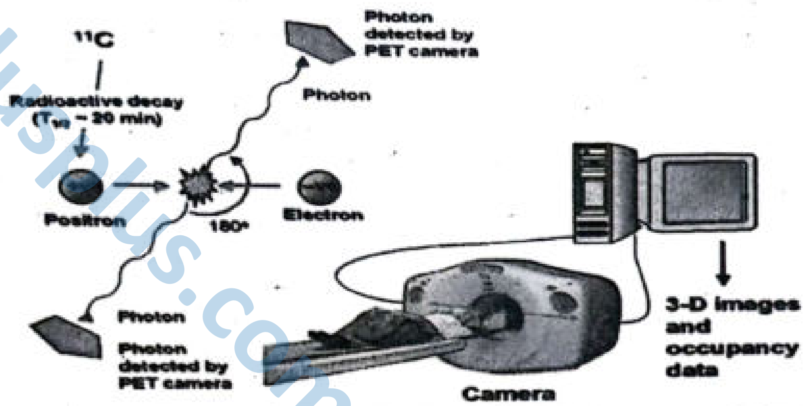


Figure 22.23: Schematic diagram of PET scanner.

SUMMARY

- **Nuclear physics:** Nuclear physics is a branch of physics that deals with the study of the atomic nucleus, its properties and interactions.
- **Mass defect:** The difference between the mass of the nucleus and sum of the masses of its constituent particles is called as mass defect.
- **Binding energy:** The minimum energy required to break an isolated nuclei into its constituent particles is called the binding energy.
- **Binding energy per nucleon:** It is the total binding energy of an atom divided by the number of nucleons.
- **Radioactivity:** The natural process of emission of radiations from unstable nuclei is called radioactivity.
- **Alpha decay:** Alpha decay occurs when an atomic nucleus emits an alpha particle, which is a helium nucleus consisting of two protons and two neutrons.

- **Beta decay:** Beta decay occurs when an atomic nucleus emits a beta particle, which is an electron or a positron.
- **Gamma decay:** Gamma decay occurs when an atomic nucleus transitions from a higher energy state to a lower energy state, emitting a gamma ray in the process.
- **Half-life:** The time it takes for half of the radioactive nuclei to decay is called as half-life of that element.
- **Curie:** When there is an activity at the rate of 3.70×10^{10} decays per second then the activity is equal to one curie.
- **Fusion:** A process where two or more light atomic nuclei come close to each other to form a heavier nucleus by releasing a large amount of energy is called nuclear fusion.
- **Fission:** A process where the nucleus of an atom splits into two or more smaller nuclei, some particles like neutron and a large amount of energy is called nuclear fission.
- **Fission chain reaction:** A fission reaction in which every time at least one released neutron goes further fission then such succession of fission reactions is called fission chain reaction.
- **Critical Mass:** The minimum mass of the material to sustain the fission chain reaction is called as the critical mass.
- **Critical volume:** The volume occupied by the critical mass of a material is known as critical volume.
- **Nuclear reactor:** Nuclear reactor is a device used to initiate and control a nuclear fission chain reaction.
- **Control rods:** Control rods are used to control the extent of reaction and do not allow the fuel to react at once.
- **Moderator:** Material that slows down the neutrons is called moderator.
- **Enrichment of uranium:** The process of converting non fissile material like uranium-238 into a fissile material like uranium-235 is called enrichment.
- **Gas diffusion method:** Gas diffusion method is used for enrichment of uranium in which the gas is allowed to diffuse from a porous wall.
- **Gas centrifuge method:** The most common method is gas centrifugation which uses rapidly spinning centrifuges to separate uranium isotopes based on difference in their masses.
- **Medical tracer:** A medical tracer is a substance which contains some radioactive nuclei.
- **PET scanner:** Positron emission tomography (PET) is a powerful imaging technique that allows us for detailed observation of metabolic physiological processes within the body.

Formula Sheet

$\Delta m = [(A - Z)\, m_n + Z\, m_p] - m_{nucleus}$

$E = \Delta m\, c^2$

$\dfrac{\Delta N}{\Delta t} = -\lambda N$

$f = \dfrac{E}{A} = \dfrac{((A-Z)\, m_n + Z\, m_p - m_{nucleus})\, c^2}{A}$

$A = \lambda N$

$T_{1/2} = \dfrac{0.693}{\lambda}$

$N = N_o e^{-\lambda t}$

EXERCISE

Multiple Choice Questions

Encircle the Correct option.

1) The process by which a heavy nucleus splits into two smaller nuclei is called:
   A. Fission B. Fusion C. Alpha decay D. Beta decay
2) Radiation with highest ionizing power is:
   A. Gamma rays B. X-rays C. Alpha particle D. Beta particle
3) Which of the following particles has almost the same mass as a proton but carries no charge:
   A. Neutron B. Proton C. Electron D. Beta particle
4) The charge on an alpha particle is:
   A. −1 B. −2 C. +1 D. +2
5) In the nucleus of uranium-235, the number of neutrons is:
   A. 92 B. 143 C. 235 D. 134
6) The half-life of radium is 1590 years. In how many years shall the Earth loss all its radium due to radioactive decay?
   A. 795 B. 1590 C. 3180 D. Infinite
7) The energy released in nuclear reactor is produced by:
   A. Fission B. Fusion C. Coal D. Gas
8) C-14 has a half-life 5730 years. The number of nuclei in a sample will drop to 1/8 of initial quantity in________ years:
   A. 1.44×10^4 B. 1.72×10^4 C. 2.58×10^4 D. 2.85×10^4

Short Questions

1) What are some of the potential benefits and drawbacks of using nuclear energy as a source of electricity compared to other forms of energy?
2) What happens to the atomic number and mass number of nucleus that (a) emits electron (b) undergoes electron capture (c) emits α particle?
3) Why does the low energy alpha particle not make physical contact with the nucleus when an alpha particle is headed directly towards the nucleus of an atom?
4) An alpha particle has twice the charge of a beta particle. Why does the former deflect less than the later when passing between electrically charged plates, assuming they both have the same speed?
5) Why uranium fuel needs to be enriched before use?

6) If U-238 undergoes alpha decay, what is the resulting nucleus?

7) How do chain reactions occur in nuclear fission?

8) How does the molecular weight difference between U-235 and U-238 hexafluoride molecules enable enrichment? What role do porous barriers play in the gaseous diffusion process?

Comprehensive Questions

1) Explain the terms mass defect and binding energy.

2) What is radioactivity? Explain.

3) How can you find energy from a nuclear decay?

4) Describe the importance of radiations in medical field?

5) What is nuclear fusion? Explain in detail. Also discuss some practical applications of this reaction.

6) What is nuclear fission? Explain in detail.

7) Discuss the function of the principle components of water moderated power reactor: such as: core, fuel, rods, moderator, control rods, heat exchange, safety rods and shielding.

8) Explain the exponential nature of radioactive decay.

9) Explain annihilation reaction in detail.

10) Illustrate how PET scanning works.

Numerical Problems

1) If an isotope has a half-life of 10 days and there are initially 5000 nuclei, how many will remain after 30 days?. **(Ans: 625)**

2) If a nucleus of carbon-14 has a decay constant of 0.00012 s^{-1}, what is its half-life? **(Ans: 5775 s)**

3) What is energy released when a nucleus of U-235 undergoes fission and release two neutrons? **(Ans: 1.793×10^{-11} J)**

4) Calculate the number of nuclei of C-14 remain un-decay after 40110 years if the initial atoms were 10,000 (The half-life of C-14 is approximately 5,730 years). **(Ans: 4670)**

5) Calculate the mass defect and binding energy per nucleon for silver (Ag) nucleus if it's atomic mass in 107.905949 u. **(Ans: 41.031×10^{-12} J/nucleon)**

6) Calculate energy released in the following reaction:

$${}^{1}_{0}n + {}^{238}_{92}U \rightarrow {}^{96}_{38}Sr + {}^{140}_{54}Xe + 3\,{}^{1}_{0}n + Q$$

(Ans: -6.474×10^{-11} J)

23 COSMOLOGY

Student Learning Outcomes (SLOs)

The student will

- Explain the term luminosity [as the total power of radiation emitted by a star].
- Apply the inverse square law for radiant flux intensity [F in terms of the luminosity L of the source $F = \frac{L}{4\pi d^2}$].
- Define and apply standard candles [Explain the use of standard candles to determine distances to galaxies].
- Explain blackbody radiation and apply Wien's displacement law to solve problems [$\lambda_{max}T$ = constant to estimate the peak surface temperature of a star]
- Apply the Stefan-Boltzmann law to solve problems [$L = 4\pi r^2 \times \sigma T^4$ to solve problems]
- estimate the radius of a star [applying Wien's displacement law and the Stefan-Boltzmann law]
- Explain that the lines in the emission and absorption spectra from distant objects show an increase in wavelength from their known values.
- explain why redshift leads to the idea that the Universe is expanding [include using $\frac{\Delta\lambda}{\lambda} \approx \frac{\Delta f}{f} \approx \frac{v}{c}$ for the redshift of electromagnetic radiation from a source moving relative to an observer to solve problems relating to the expanding universe].
- State and explain Hubble's law and how it leads to the Big Bang theory.

Electromagnetic radiations are playing a vital role in facilitating us during research. It can give us in depth information of a living or non-living body by penetrating into it. Electromagnetic radiations also give us information of far-off outer space or universe.

Electromagnetic energy travels in the form of waves and spans a broad spectrum ranging from very long radio waves to very short gamma rays. The human eye can only detect a small portion of this spectrum, called visible light. NASA's scientific instruments use the full range of the electromagnetic spectrum to study the Earth, the solar system, and the universe beyond.

Cosmology is the study of the origin, development, structure, history, and future of the entire universe. Radiation plays a significant role in cosmology, particularly in understanding residual radiation from the Big Bang. These aspects help us study the universe's origins, evolution, and structure.

23.1 BLACK BODY RADIATION

A black-body is an ideal object which is perfectly opaque and non-reflecting.

- It absorbs all the radiation that falls on it.
- It is also a good emitter.

Figure 23.1: An ideal blackbody.

A black-body is a theoretical object; however, stars are the best examples. The radiation emitted from a black-body has a characteristic spectrum that is determined by the temperature.

Wien's Displacement Law

This law helps understand thermal radiation, temperature and emission spectra. It shows that hotter objects emit shorter wavelengths (e.g., blue light), while cooler objects emit longer wavelengths (e.g., red light).

Wien's displacement law relates the observed wavelength of light emitted by a star to its surface temperature (T). It states that:

The black body radiation curve for different temperatures peaks at a wavelength, which is inversely proportional to the temperature.

This relation can be written as: $$\lambda_{max} \propto \frac{1}{T}$$

or $$\lambda_{max} = \text{constant} \times \frac{1}{T}$$

λ_{max} is the maximum wavelength emitted by the star at the peak intensity, as shown in Fig. 23.2. Here the constant is equal to 2.9×10^{-3} m K. Thus, the equation for Wien's displacement law is given by:

$$\lambda_{max} T = 2.9 \times 10^{-3} \text{ m K} \quad \text{(23.1)}$$

This equation shows that:

- The higher the temperature of a body, the shorter the wavelength at the peak intensity. Therefore, hotter stars tend to appear white or blue and cooler stars tend to appear red or yellow.
- The higher the temperature of a body, the greater the intensity of the radiation at each wavelength.

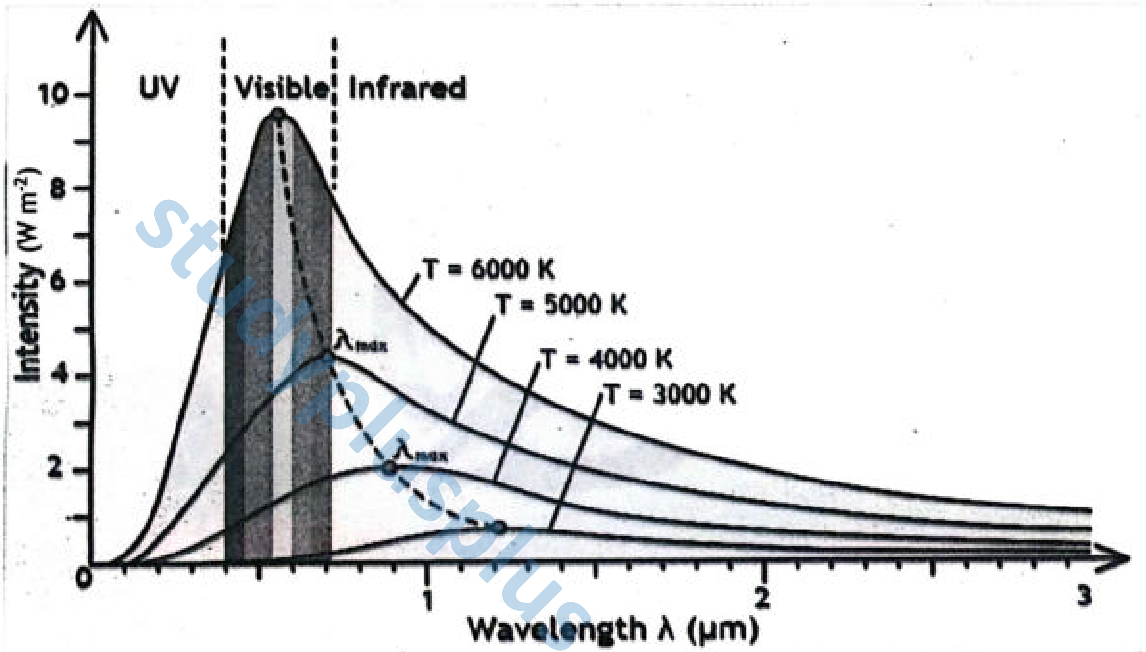


Figure 23.2: The intensity-wavelength graph shows the relation of temperature to the peak wavelength for four different stars.

23.2 LUMINOSITY AND RADIANT FLUX INTENSITY

Luminosity and radiant flux intensity are two different quantities related to electromagnetic radiation. Let us understand each term separately, before exploring the relationship between them.

Luminosity

In astronomy, luminosity is the total amount of electromagnetic energy emitted per unit time by a star or other celestial objects. This represents the power output of radiation emitted by a star.

Luminosity is a measure of the total power output of radiation emitted by a star.

In SI units, luminosity is measured in joule per second (J s^{-1}), or watt (W).

Luminosity depends on temperature and surface area of the star. The relationship between these two quantities is known as the **Stefan-Boltzmann Law**, which states that:

The total energy emitted by a blackbody per unit area per second is proportional to the fourth power of the absolute temperature of the body.

When considering a star to be a completely black body, the Stefan-Boltzmann law can be applied to find the value for luminosity (L) for a black body. i.e.,

$$L = A\sigma T^4 \qquad \text{\_\_\_\_\_\_ (23.2 a)}$$

Where A is the surface area, T is the temperature (in kelvins) and σ is the Stefan-Boltzmann constant, with a value of $5.67 \times 10^{-8}\ Wm^{-2}\ K^{-4}$. The surface area of a star (being sphere) is equal to $4\pi r^2$, so Eq. (23.2 a) can also be written as:

$$L = 4\pi r^2 \sigma T^4 \qquad \text{\_\_\_\_\_\_ (23.2 b)}$$

If two stars have same temperature, then the star with double radius will have approximately four times the luminosity.

Radiant Flux Intensity

Light sources (such as stars) that are further away from the Earth appear fainter because the light they emit is spread out over a larger area. The moment the light leaves the surface of the star, it begins to spread out uniformly through a spherical shell (with an area of $4\pi r^2$). The radius *r* of this sphere is equal to the distance *d* between the star and the Earth. By the time the radiation reaches the Earth, it has been spread over an area of $4\pi d^2$.

The luminosity per unit area measured on the surface of the Earth is known as radiant flux intensity.

Thus, the relationship between the luminosity (L) and radiant flux intensity (F) can be expressed as:

$$F = L/A \qquad \text{\_\_\_\_\_\_ (23.3 a)}$$

$$F = \frac{L}{4\pi d^2} \qquad \text{\_\_\_\_\_\_ (23.3 b)}$$

where A is the area of the illuminated surface. In SI units, radiant flux intensity is measured in $W\ m^{-2}$. Eq. (23.3) shows that the radiant flux follows an **inverse square law**: when light is twice as far away, it has spread over four times the area, resulting in a four-fold decrease in intensity.

Consider a point source of light of luminosity L that radiates equally in all directions, as shown in Fig. 23.3. A hollow sphere centered on the point would have its entire interior surface illuminated. As the radius increases, the surface area also increases, and the constant luminosity has more surface area to illuminate, leading to a decrease in observed brightness (radiant flux intensity).

For Your Information

Value of solar luminosity is: 3.84×10^{26} W

A greater radiant flux intensity (larger F) indicates that the star is closer to the Earth (smaller d).

The luminosity is the total power output of the star, whereas the radiant flux intensity is what is measured on Earth.

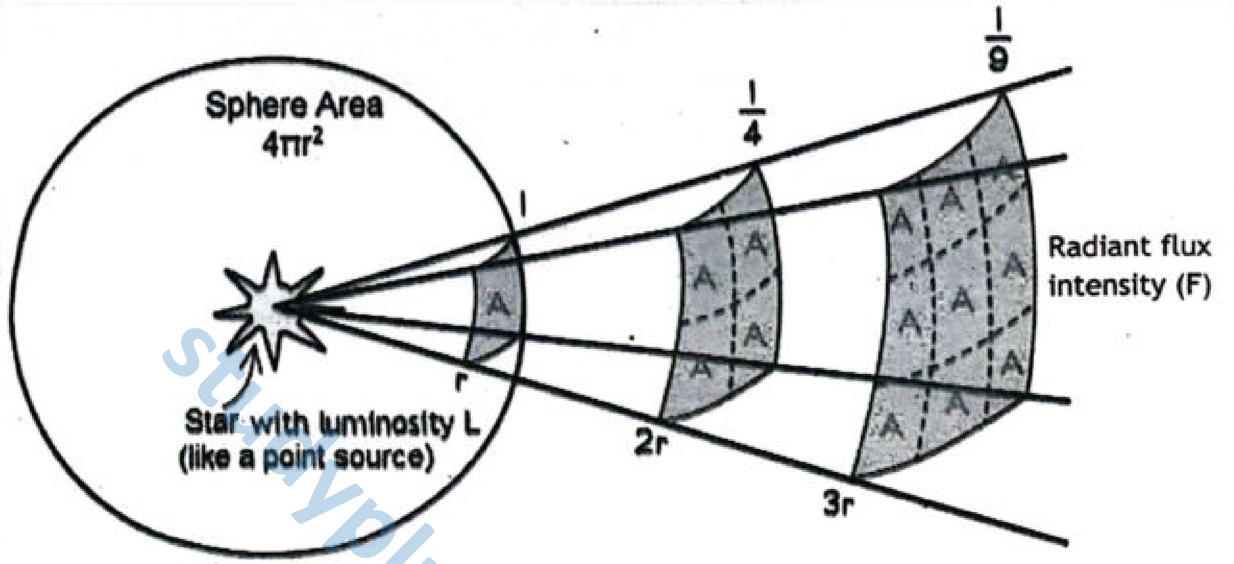


Figure 23.3: Inverse Square Law of radiant flux intensity.

Example 23.1: A star has a known luminosity of 9.7×10^{27} W. Observations of the star show that the radiant flux intensity of light received on Earth from the star is 114 nW m^{-2}. Determine the distance of the star from Earth.

Given: Luminosity = L = 9.7×10^{27} W

Radiant flux intensity = F = 114 nW m^{-2} = 114×10^{-9} W m^{-2}

To Find: Distance from the Earth = d = ?

Solution: We use the equation, $$F = \frac{L}{4\pi d^2}$$

By rearrange for distance d, we get: $$d = \sqrt{\frac{L}{4\pi F}}$$

By putting values, we get: $$d = 8.2 \times 10^{16} \text{ m}$$

Assignment 23.1

What is the relative luminosity of the objects A and B if: T_A = 100 K, T_B = 200 K, R_A = 10 m and R_B = 5 m.

23.3 STANDARD CANDLES: A Distance Indicator

Measuring astronomical distances accurately is an extremely difficult task. A direct distance measurement is only possible if the object is close enough to the Earth. For more distant objects, indirect methods must be used. Standard candles are useful for this purpose.

In astronomy, a standard candle is a source with a known luminosity. A standard candle is defined as:

An astronomical object which has a known luminosity due to a characteristic quality possessed by that class of object.

If the luminosity of a source (i.e., standard candle) is known, then the distance can be calculated by using the Eq. (23.3 b), which is based on how bright (i.e., the measured radiant flux intensity) it appears from Earth.

Examples of standard candles are: Cepheid variable stars and Type 1A supernovae. Each standard candle method can measure distances within a certain range. Organizing the data and measurements from each method allows astronomers to build up a larger picture of the universe, from nearby stars to distant galaxies. This is known as the cosmic distance ladder.

For Your Information

Edwin Hubble used Cepheid to determine the distances of "nebulae" and derive the Hubble law. Nebulae are interstellar clouds of gas and dust.

Estimating the Radius of Stars

The radius of a star can be estimated by combining Wien's displacement law and the Stefan-Boltzmann law. The procedure for this is as follows:

- Find the surface temperature of the star by using Wien's displacement law.
- Find the luminosity of the star by using the **inverse square law of flux** (if the radiant flux and stellar distance are given).
- Then, using the Stefan-Boltzmann law, the stellar radius can be obtained.

Example 23.2: The spectrum of the star Rigel in the constellation of Orion peaks at a wavelength of 263 nm, while the spectrum of the star Betelgeuse peaks at a wavelength of 828 nm. Find the surface temperature of these two stars. Which of these two stars is cooler?

Given: For the star Rigel: $\lambda_{max} = 263 \text{ nm} = 263 \times 10^{-9} \text{ m}$

For the star Betelgeuse: $\lambda_{max} = 828 \text{ nm} = 828 \times 10^{-9} \text{ m}$

To Find: Surface temperature for the star Rigel: T = ?

Surface temperature for the star Betelgeuse: T = ?

Which of the two stars is cooler = ?

Solution: We use the equation,

$$\lambda_{max} T = 2.9 \times 10^{-3} \text{ m K}$$

By rearrange for distance d, we get:

$$T = 2.9 \times 10^{-3} / \lambda_{max}$$

For the star Rigel: $T = 2.9 \times 10^{-3} / \lambda_{max} = 2.9 \times 10^{-3} / 263 \times 10^{-9} = 11026 \text{ K}$

For the star Betelgeuse: $T = 2.9 \times 10^{-3} / \lambda_{max} = 2.9 \times 10^{-3} / 828 \times 10^{-9} = 3502 \text{ K}$

As, Betelgeuse has surface temperature of 3500 K, therefore, it is cooler than Rigel.

Assignment 23.2

Betelgeuse is our nearest red giant star. It has a luminosity of 4.49×10^{31} W and emits radiation with a peak wavelength of 850 nm. Calculate the ratio of the radius of Betelgeuse r_B to the radius of the Sun r_S.

23.4 SPECTRA OF LIGHT

Astronomers are very limited in how they can investigate objects in the space. All of the techniques used by astronomers involve analysing the light emitted from the star, or galaxy. One of these techniques involves analysing the emission and absorption spectra of stars.

Elements in the star (mostly hydrogen and helium) absorb some of the emitted wavelengths. Therefore, characteristic lines are present when the spectrum is analysed. Every element has a unique set of absorption and emission lines. The pattern of lines is known as a spectral signature. The absorption and emission spectra of each element are inverses of each other: The wavelengths of a particular element's absorption lines are the same as the wavelengths of its emission lines. Astronomers can compare the spectrum of a celestial object or material with the spectra of known elements and molecules to figure out what the object or material is made of.

Absorption Spectra: When light passes through a gas, atoms and molecules in the gas absorb certain colors, or wavelengths, of that light. The result is an absorption spectrum: a rainbow with dark absorption lines.

Emission Spectra: The same gas can glow, giving off very specific colors to form an emission spectrum with bright lines known as emission lines.

Spectra of Light from Stars and Galaxies

When astronomers observe light from distant galaxies, they observe differences in the spectral lines to the light from the Sun. The lines in the emission and absorption spectra from distant objects show an increase in wavelength from their known values. These lines appear to be shifted sightly towards the red end of the spectrum but the lines have the same characteristic pattern, meaning the element can still be easily identified.

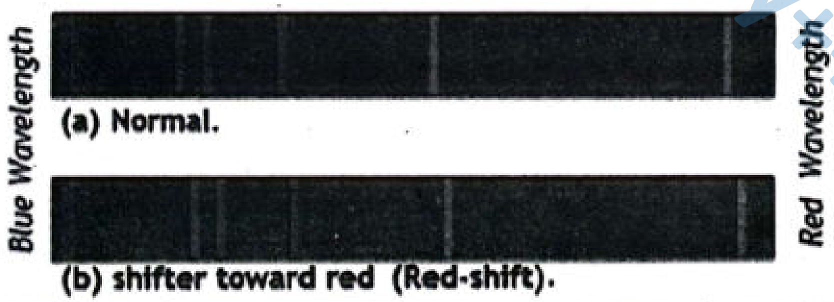


Figure 23.4: Emission spectra of hydrogen (a) spectral lines of hydrogen measured from a source in laboratory. (b) spectral lines of hydrogen measured from a distant galaxy.

Red-shift and Speed

Astronomers see red-shift in all galaxies. It is due to the reason that 'the space between the Earth and the galaxies is expanding'. This expansion has the effect of increasing the wavelength of the light from these galaxies, shifting them towards the red end of the spectrum. The more red-shifted the light from a galaxy is, the faster the galaxy is moving away from Earth. Recall that the Doppler effect is defined as:

There is apparent change in wavelength or frequency of the radiation from a source due to its relative motion toward or away from the observer.

Doppler effect of light can observe when spectra of distant stars and galaxies are observed, this is known as:

- Redshift, if the object is moving away from the Earth.
- Blueshift, if the object is moving towards the Earth.

Redshift is defined as:

The fractional increase in wavelength (or decrease in frequency) due to the source and observer receding from each other.

Red-shift can be calculated using:

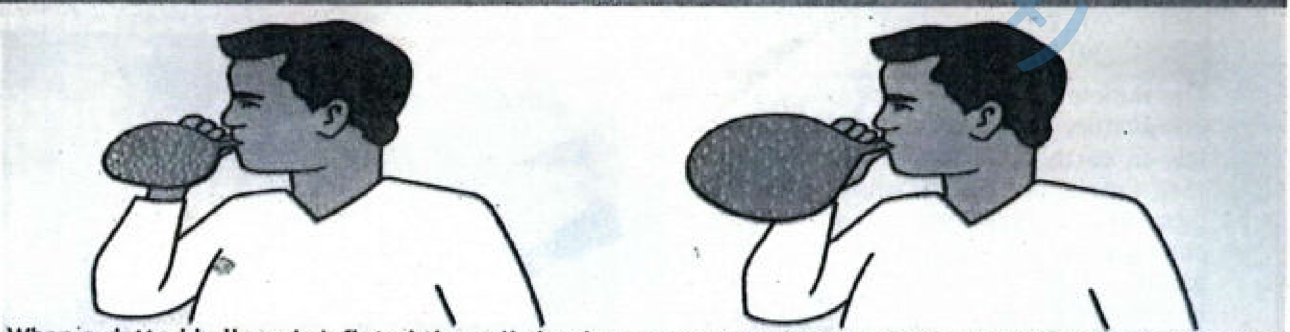
$$\frac{\Delta\lambda}{\lambda} = \frac{\Delta f}{f} = \frac{v}{c} \quad \text{———— (23.4)}$$

Where, $\Delta\lambda$ is shift in wavelength, λ is wavelength emitted from the source, Δf is shift in frequency, f is frequency emitted from the source, v is speed of recession and c is speed of light in a vacuum.

Expanding Universe

After the discovery of Doppler redshift, astronomers began to realise that almost all the galaxies in the universe are receding. This led to the idea that the space between the Earth and the galaxies must be expanding. This expansion stretches out the light waves as they travel through space, shifting them towards the red end of the spectrum. The more red-shifted the light from a galaxy is, the faster the galaxy is moving away from Earth.

Activity 23.1: A balloon inflating is similar to the stretching of the space between galaxies



Whan a dotted balloon is inflated then all the dots move away from each other as the rubber stretches. The expansion of the universe can be compared to dots on an inflating balloon. Just like the dots, the galaxies move away from each other.

23.5 HUBBLE'S LAW AND THE BIG BANG THEORY

Edwin Hubble investigated the light spectra emitted from a large number of galaxies. He used redshift data to determine the galaxy's recessional velocity (v), and standard candles to determine the distances (d) between the galaxy and Earth. From these measurements, he formulated a relationship, now known as Hubble's Law. Hubble's Law states:

Figure 23.5: The Big Bang Theory explains how the universe began 13.8 billion years ago.

The recession speed of a galaxy is directly proportional to its distance from the Earth.

Expression for the Hubble's Law can be written as:

$$v = H_0 d \qquad (23.5)$$

Where, H_o is Hubble's constant, or the rate of expansion of the universe (s^{-1}). The Eq. (23.5) shows that:

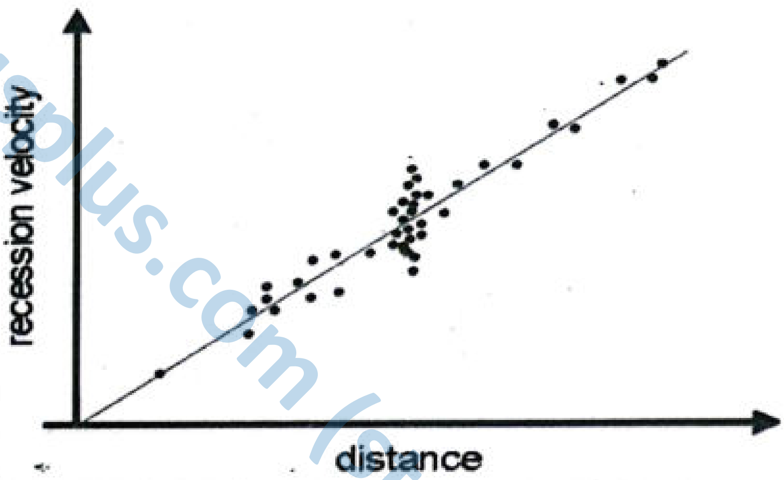
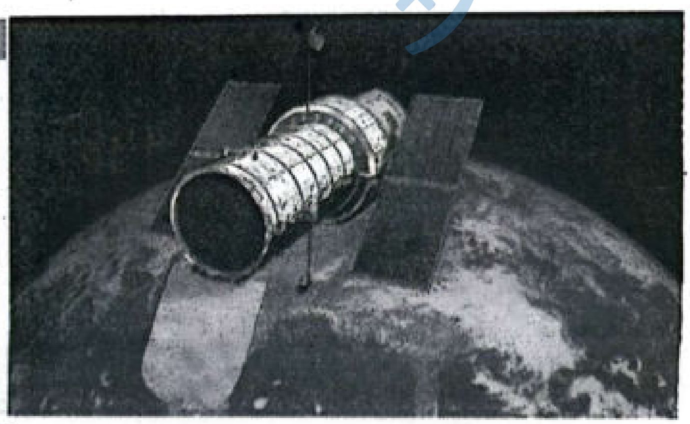


Figure 23.6: Graph of recession velocity against distance.

- The further away a galaxy, the faster it's recession velocity (v).
- The gradient of a graph of recession velocity against distance is equal to the Hubble constant (i.e., $v/d = H_0$), as shown in Fig. 23.6.

Hubble Space Telescope

The Hubble Space Telescope is a space-based observatory launched in 1990. It orbits very low in earth orbit, approximately 340 miles (540 km) above Earth's surface. It observes ultraviolet, visible, and near-infrared light. It contributed significantly to understanding galaxy evolution, star formation, and cosmology. It has also determining the rate of expansion of the universe.



Age of the Universe

If the galaxies are moving away from each other, then they must've started from the same point at some time in the past, as shown in Fig. 23.7. If this is true, the universe likely began in an extremely hot, dense singular point which subsequently began to expand very quickly. This idea is known as the Big Bang theory. According to the Big Bang theory:

About 13.8 billion years ago the whole universe was a very small, extremely hot and dense region. From this tiny point, the whole universe expanded outwards to what exists today.

Redshift of galaxies and the expansion of the universe are now some of the most prominent pieces of evidence to suggest this theory is true. The data from Hubble's law can be extrapolated back to the point where the universe started expanding, i.e., the beginning of the universe, as shown in Fig. 23.7. Therefore, the age of the universe T_o is equal to:

$$T_o = \frac{1}{H_o} \qquad (23.6)$$

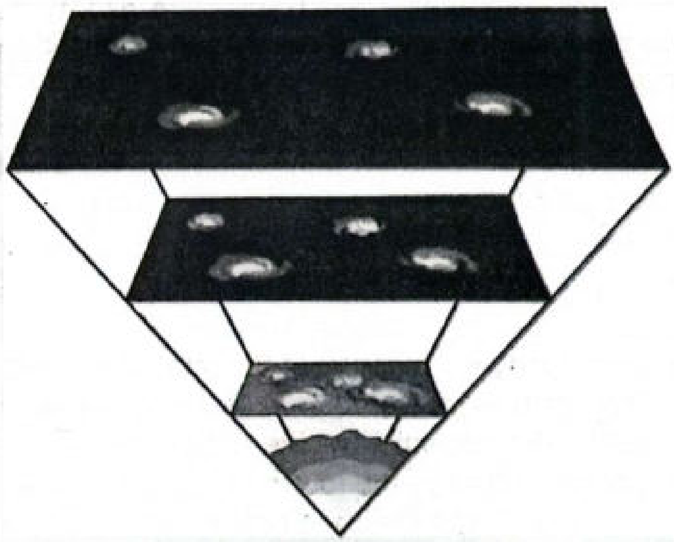


Figure 23.7: Tracing the expansion of the universe back to the beginning of time leads to the idea the universe began with a Big-Bang.

Presently estimated age of the universe ranges from 13 to 14 billion years. There is still some discussion about the exact age of the universe, therefore, finding accurate measurements for the Hubble's constant is a top priority for cosmologists.

James Web Space Telescope

It is launched in 2021. It orbits approximately 1.5 million kilometers from Earth.

It observes near-infrared and mid-infrared light, studying galaxy formation, star birth, and planetary systems. Its main objectives are to studying the universe's early stages, to understanding galaxy formation and evolution, observing star formation and planetary system development.

Example 23.3: A specific line in the spectrum of light obtained from a source in laboratory has a frequency of 4.570×10^{14} Hz. The same line in the spectrum of light from a distant galaxy has a frequency of 4.547×10^{14} Hz. What speed is the distance galaxy moving in relation to the Earth? Is it moving towards or away from the Earth?

Given: frequency of lab source: $f = 4.570 \times 10^{14}$ Hz
frequency of distant galaxy: $f' = 4.547 \times 10^{14}$ Hz

To Find: Speed of the galaxy: $v = ?$

Solution: Shift in frequency: $\Delta f = (4.570 - 4.547) \times 10^{14} = 2.3 \times 10^{12}$ Hz

Now we use the equation: $\frac{\Delta f}{f} = \frac{v}{c}$

Rearrange for speed v and putting values, we get:

$$v = \frac{c\Delta f}{f} = \frac{(3\times10^{8})(2.3\times10^{12})}{4.57\times10^{14}} = 1.5\times10^{6}\ \text{m s}^{-1}$$

The observed frequency is less than the emitted frequency (the light from a laboratory source), therefore, the source is receding from the Earth at 1.5×10^{6} m s^{-1}.

Assignment 23.3

A galaxy is found to be moving away with a speed of 2.1×10^{7} m s^{-1}. The galaxy is at a distance of 9.5×10^{24} m from Earth. Assuming the speed has remained constant, what is the age of the universe in years?

SUMMARY

- **Luminosity** is a measure of the total power output of radiation emitted by a star. In SI units, luminosity is measured in joule per second or watt.
- **Stefan-Boltzmann Law:** the total energy emitted by a black body per unit area per second is proportional to the fourth power of the absolute temperature of the body.
- **Blackbody** is an idealized object that is perfectly opaque and non-reflecting.
- Luminosity per unit area measured on the surface of the Earth is known as **radiant flux intensity.**
- **A standard candle** is defined as: an astronomical object that has a known luminosity due to a characteristic quality possessed by that class of object.
- **Wien's displacement law** states that: The black body radiation curve for different temperature peaks at a wavelength, which is inversely proportional to the temperature.
- **Redshift** is defined as: the fractional increase in wavelength (or decrease in frequency) due to the source and observer receding from each other.
- **Hubble's Law** states: the recession speed of galaxies moving away from Earth is proportional to their distance from the Earth.
- According to the **Big Bang theory**: about 13.8 billion years ago the whole Universe was a very small, extremely hot and dense region. From this tiny point, the whole Universe expanded outwards to what exists today.
- **Estimated age of the universe** ranges from 13 to 14 billion years.

Formula Sheet

$\lambda_{max} T = 2.9 \times 10^{-3}$ m K	$L = A\sigma T^4$	$F = L/A$
$\frac{\Delta\lambda}{\lambda} = \frac{\Delta f}{f} = \frac{v}{c}$	$v = H_0 d$	$T_o = \frac{1}{H_o}$

EXERCISE

Multiple Choice Questions

Encircle the Correct option.

1) SI unit of Luminosity is

   A. watts B. joules C. newtons/meter D. cd/m

2) Luminosity is associated with energy produced by

   A. objects B. celestial bodies C. light source D. fluorescent lamp

3) Luminosity depends on

   A. Size of the star B. Temperature of the star

   C. Colour of the star D. Option 1 and 2

4) Which law states that brightness of a source is inversely proportional to the square of its distance

   A. Law of brightness B. Direct Square Law of brightness

   C. Inverse Square Law of brightness D. Inverse Square Law of light

5) SI unit of Hubble's constant (H_o) is

   A. s^{-1} B. m^{-1} C. m D. $m\ s^{-1}$

6) Which statement about the Big-Bang theory is not correct:

   A. There was a giant explosion known as the big bang.

   B. This caused the universe to expand from a single point.

   C. Each point in the universe expands away from the others.

   D. The further away galaxies are the slower they are moving.

Short Questions

1) What is meant by blackbody radiation?
2) What is the relationship between luminosity and radiant flux intensity?
3) What is the inverse square law for radiant flux intensity?
4) What is meant by standard candles? Give any two examples.
5) What is Wien's displacement law?
6) Define Stefan-Boltzmann law.
7) How can we estimate the radius of a star by applying Wien's displacement law and the Stefan-Boltzmann law?
8) How redshift leads to the idea that the Universe is expanding?
9) How does the expansion of the universe support the Big Bang theory?

10) How do different types of spectra (emission, absorption, continuous) provide insights into the composition and properties of celestial objects?

Comprehensive Questions

1) Explain the term luminosity and radiant flux intensity.
2) Discuss the use of standard candles to determine distances to galaxies.
3) Explain that how the radius of a star can be estimated by applying Wien's displacement law and the Stefan-Boltzmann law.
4) Why redshift leads to the idea that the Universe is expanding? Discuss.
5) State and explain Hubble's law and how it leads to the Big Bang theory.

Numerical Problems

1) There are two stars, let named A and B. Which star has the greater temperature? Given that: $L_A = L_B = 10^4$ J/s; $R_A = 10^4$ m; $R_B = 10^5$ m. **(Ans: star A)**

2) The Sun has a surface temperature of 6000K produces peak radiation of 420 nm. Find out the temperature of the Sirius if peak radiation of Sirius is 72 nm. **(Ans: 35000 K)**

3) If a source has a luminosity of 3.826×10^{26} W and is at a distance of 3 lightyear, what is the radiant flux intensity? (Ans: 3.78×10^{-8} W m^{-2})

24 EARTH'S CLIMATE

Student Learning Outcomes (SLOs)

The student will

- Describe Earth's climate system as a complex system having five interacting components [the atmosphere (air), the hydrosphere (water), the cryosphere (ice and permafrost), the lithosphere (earth's upper rocky layer) and the biosphere (living things)]
- Relate ocean currents and wind patterns to the climate system [as the statistical characterization of the climate system, representing the average weather, typically over a period of 30 years, and is determined by a combination of processes in the climate system, such as ocean currents and wind patterns]
- Explain climate inertia as the phenomenon by which climate systems show resistance or slowness to changes in significant factors e.g. stabilization of greenhouse emissions might show a slow response due to the action of complex feedback systems]
- Explain that climate change can be categorized into internal variations and external forcing.
- Explain how global climate is determined by energy transfer from the Sun [with specific reference to the below factors and terms
  - state and use the term Earth energy budget
  - Explain how the energy imbalance between the poles and the equator can affect atmospheric circulation]
- Explain that due to the conservation of angular momentum, the Earth's rotation diverts the air to the right in the Northern Hemisphere and to the left in the Southern hemisphere, thus forming distinct atmospheric cells.
- Explain that ocean water that has more salt has a higher density and differences in density play an important role in ocean circulation.
- Explain how the thermohaline circulation transports heat from the, tropics to the polar regions.
- Explain how climate science is an example of a chaotic system, [using the metaphor of a butterfly's wing flaps may cause hurricanes in another part of the world, mathematics of chaos theory is not required; just the idea that with very complex systems it is very difficult to predict outcomes and they are very sensitive to initial conditions]

Understanding Earth's climate system is a complex interplay of various components that collectively determine the average weather conditions over time. Each component contributes its unique role to the global climate equilibrium. From the depths of oceans to the heights of the atmosphere, from the microscopic interactions of atoms and molecules to the grand *movements* of continents every facet of Earth's system influences and influenced by the climate. One of the key ingredients of this system is the radiant energy received from the Sun. This solar energy powers Earth's atmospheric circulations, ocean currents and the hydrological cycle shaping the distribution of heat and moisture across the planet. The composition of the atmosphere is primarily made up of gases like nitrogen, oxygen and greenhouse gases such as carbon dioxide and methane. Methane acts like a blanket, trapping some of incoming solar radiations and regulate the planet's temperature. The Earth's surface, with its diverse landscapes of oceans, ice caps, forests, deserts and urban areas play a significant role in modulating climate patterns. These surfaces absorb, reflect and emit radiations differently leading the complex patterns of temperature and precipitation across the globe. Meanwhile the biosphere which contains all living organism interact with the climate system through processes like photosynthesis, respiration and the release of greenhouse gases. In unrevealing the complexities of Earth's climate system, scientists employ a multitude of tools and techniques from satellite observations and computer models to field experiments and climate reconstructions. Through continuous advancements in technology our understanding of the Earth's climate system continues to evolve.

24.1 EARTH'S CLIMATE SYSTEM

Earth's climate system is a complex system with five interacting components the atmosphere, the hydrosphere, the cryosphere, the lithosphere and the biosphere.

Global climate is influenced by many factors including the Sun, Earth's position in space relative to the Sun and human made factors (such as greenhouse gas emission). The five interacting components of the Earth's climate system are shown in Fig. 24.1, and are discussed below:

Figure 24.1: Five components of Earth's climate system.

1) The Atmosphere (Air)

The atmosphere envelops the Earth and extends hundreds of kilometers from the surface. It mainly consists of nitrogen which is 78%, oxygen which is 21% and argon which is 0.9%. Additionally, there are trace gases such as water vapors (H_2O), carbon dioxide (CO_2), ozone (O_3) and methane (CH_4). These trace gases play a significant role in the climate system.

The role of the atmosphere is the shielding: as it protects life on Earth by shielding it from incoming ultraviolet radiations. It insulates the planet as it keeps the planet warm by acting as an insulating layer. The atmosphere prevents extreme temperature variations between day and night. It is responsible for convection and weather patterns as the solar heating causes layers of the atmosphere to convert, driving the air movement and influencing global weather patterns. Some trace gases like carbon dioxide and water vapors act as greenhouse gases. These gases allow visible light from the Sun to penetrate to the surface but block some of infrared radiations emitted by the Earth's surface which traps heat within the atmosphere contributing to the overall warming effect. The atmosphere is divided into different layers depending upon their altitude and functioning, as shown in Fig. 24.2.

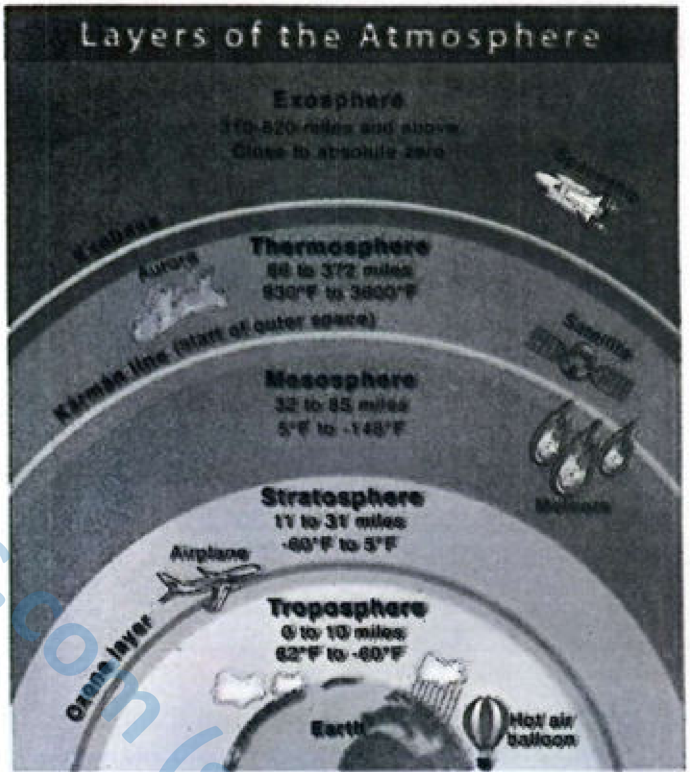


Figure 24.2: Layers of atmosphere.

- **Troposphere:** The lowest layer extending from the ground (the surface of the Earth) to about 16 km. It's the layer where weather occurs.
- **Stratosphere:** Above the troposphere, from 16 km to 50 km, is the stratosphere. The ozone layer is found here which prevents us from the harmful ultraviolet radiations coming from the Sun.
- **Mesosphere:** Above the stratosphere from 50 km to 136 km is the mesosphere. Meteors burn out in this part of the atmosphere.
- **Thermosphere:** Above the mesosphere from 136 km up to about 600 km is the thermosphere. It is where the auroras occur due to solar radiations.
- **Exosphere:** The exosphere is outer most layer into which extends from 600 km up to merging into the space. Satellites orbit in this region.

2) The Hydrosphere (Water)

The hydrosphere encompasses all the water on the Earth whether it is in liquid water, solid ice or the gaseous water (vapors form). The Earth contains 70 % water on its surface. Most of the hydrosphere consists of liquid water primarily found in the oceans. Our planet's abundant surface waters give Earth its distinct appearance as a blue globe and set it different from the all-other planets in the solar system. Some of the characteristics of hydrosphere are:

- The hydrosphere plays a critical role in regulating Earth's climate.
- It influences patterns of precipitation and affects the movement of energy throughout the climate system.
- Ocean currents, evaporation and condensation are all parts of hydrological cycle which redistributes the water around the planet.

The hydrological cycle is shown in Fig. 24.3. The water balance equation is the fundamental equation tool for understanding the movement and distribution of water in the hydrosphere. This equation relates inputs, outputs and changes in the water storage over time and can be given as:

$\Delta S = P - E - Q$ ____ (24.1)

Here 'ΔS' is the change in the storage of hydrosphere (which may be a specific region like lake sea etc.), 'P' is the precipitation (which is the amount of water falling from the atmosphere), 'E' is the evapotranspiration (which is the evaporation from the Earth's surface and plants) and 'Q' is the runoff (which is the water that moves across the surface to rivers or lakes and also the water which infiltrates into the groundwater).

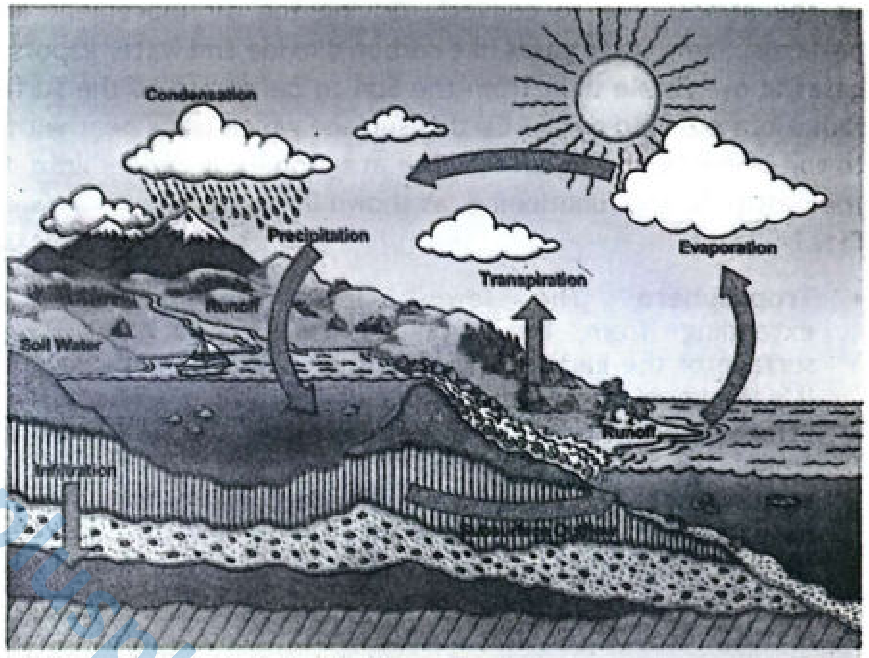


Figure 24.3: Hydrological cycle.

3) The Cryosphere (Ice)

The cryosphere refers to any place on Earth where water is in its solid form, where low temperatures freeze water and turn it into ice. The frozen water can be found in the form of solid ice or snow and occurs in many places around the Earth. The cryosphere exists in the polar-regions but is also found wherever snow, sea ice, glaciers, permafrost, ice sheets and icebergs exist. In Pakistan, the examples of cryosphere are the ice caped mountains and glaciers in the Himalaya, Karakorum and Hindu Kush Mountain ranges in the north. Snow and ice are the basic elements of the cryosphere as they interact throughout Earth's different environments to create sea ice, glaciers, ice shelves, icebergs and frozen ground. Although direct measurement of the cryosphere can be difficult to obtain due to the remote locations of many of these areas, satellite observations help scientists monitor changes in the global and regional climate by observing how regions of the Earth's cryosphere shrink and expand. Some of the components of the cryosphere are explained as:

- **Snow:** Snow is the precipitation that forms when water vapors freeze into ice crystals. It can form whenever there is high humidity and cold temperatures in the atmosphere. Because of its reflective property it regulates the climate by reflecting incoming sunlight back into the space and hence cooling the planet
- **Ice:** Ice forms when liquid water becomes a solid at temperatures below the freezing point. It is the part of sea ice, glaciers, ice shelves, icebergs and frozen ground. The ice in polar-regions has important impact on polar regions and Earth's climate. The sea ice layer

increases the salinity of sea water. It also restricts wind and wave action near coastlines. It also reflects much of the sunlight into the space back. Ice currently covers 10 percent of the Earth's surface and is disappearing rapidly

- **Permafrost:** Permafrost refers to permanently frozen ground found mainly in high altitude regions. It contains ice and soil that remain frozen for extended periods.

The components of cryosphere are shown in Fig. 24.4.

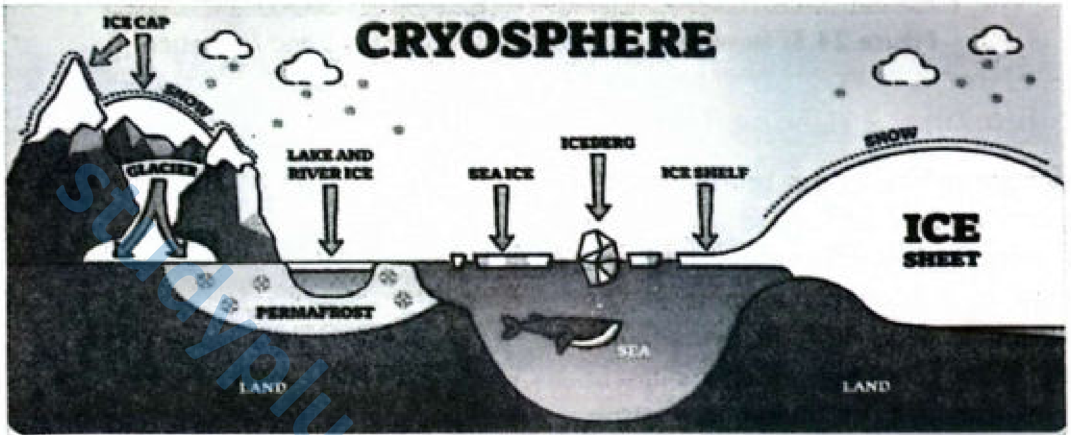


Figure 24.4: Cryosphere.

4) The Lithosphere (Earth's Upper Rocky Layer)

The lithosphere is the outermost solid shell of the Earth consisting of the crust and the uppermost part of the mental. It composed of various types of rocks and minerals. The lithosphere interacts with other components such as the atmosphere, hydrosphere and biosphere. Some important features of lithosphere are as:

- **Land surface:** The lithosphere provides the surface on which climate processes occur. Different types of land surfaces have different influences on the Earth's climate for example forests can moderate the temperatures while deserts tend to have extreme temperature fluctuations.
- **Carbon cycle:** Lithosphere plays a major role in carbon cycle which is essential for regulating Earth's climate. Rocks weather and erode over time releasing carbon dioxide into the atmosphere. On the other hand, some types of rocks like lime stone are carbon sinks which takes carbon from the atmosphere over a geological time scale.
- **Tectonic Activity:** Processes like tectonics, which involves the movement and interaction of lithosphere plates, can have significant impact on climate over a large timescale. Due to movement of tectonic plates mountain ranges are formed which affect atmospheric circulation patterns and precipitation distribution influencing regional climate.
- **Volcanic Activity:** Volcanic eruption which is related to tectonic activity can release large number of gases and particles into the atmosphere these emissions can temporarily cool the climate by blocking sunlight, leading to short term cooling events known as "*volcanic winters*"

The movement of tectonic plates and volcano formation is shown in Fig. 24.5.

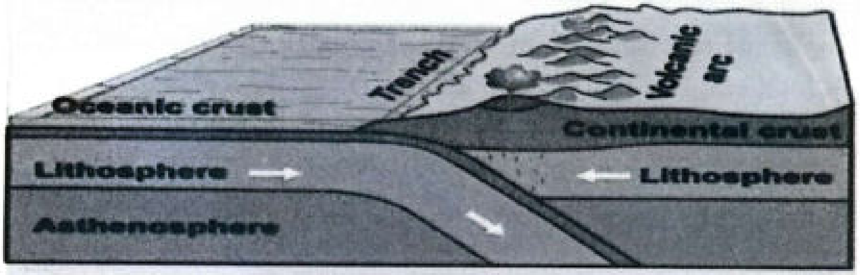


Figure 24.5: Movement of tectonic plates and volcano formation.

5) The Biosphere (Living Things)

The biosphere refers to the relatively thin life supporting layer of Earth's surface. It extends from a few kilometers into the atmosphere to the deep-sea vents in the ocean. Biosphere deals with the living things and their impact on the Earth's climate system. It consists of living organisms (plants, animals, fungi and microorganisms) and the non-living factors from which they derive energy and nutrients. The role of biosphere in the Earth's climate system can be given as:

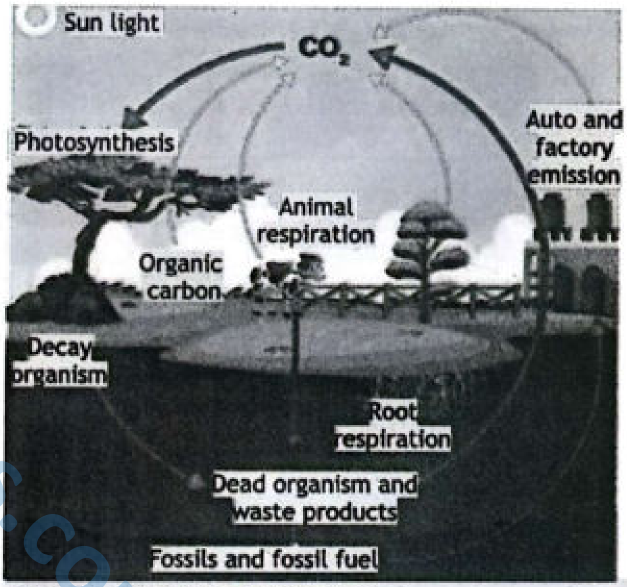


Figure 24.6 (a): Carbon cycle

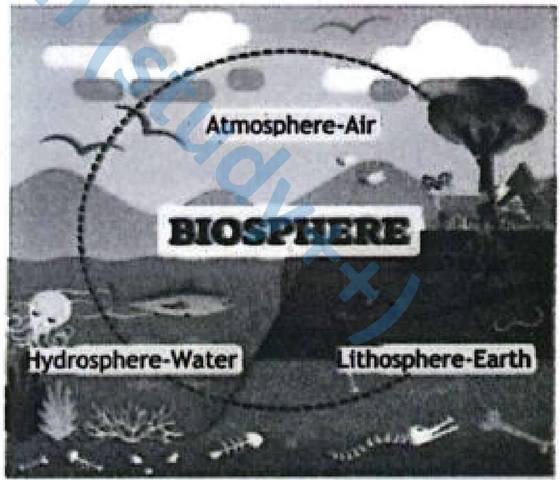


Figure 24.6 (b): Biosphere

- **Climate regulation:** The biosphere plays a crucial role in regulating the Earth's climate. Changes in the biosphere can directly impact climate patterns
- **Carbon cycle:** The biosphere is an integral reservoir in the carbon cycle. Through processes like photosynthesis living organisms absorb carbon dioxide (CO_2) from the atmosphere and convert it into organic matter. The carbon cycle is shown in Fig. 24.6 (a).
- **Energy flow:** Solar energy is captured by the plants during photosynthesis and this energy flows through the food chain as organisms consume each other. This energy transfer affects climate dynamics.
- **Water cycle:** The biosphere is closely linked to the water cycle. Evaporation, condensation, precipitation and transpiration all involve living organisms.
- **Nutrient cycles:** Elements like carbon (C), nitrogen (N_2) and phosphorous (P) cycle through the biosphere impacting both living organisms and the environment.

The biosphere is essential for maintaining species diversity, regulating climate and supporting life on the Earth. The biosphere is shown in Fig. 24.6 (b).

Example 24.1: Find the change in storage of hydrosphere in terms of volume whose area of the watershed is 120 km^2, with precipitation of 110 mm while evapotranspiration and runoff are 60 mm and 30 mm respectively.

Given: Area of watershed = A = 120 km^2 — Precipitation = P = 110 mm

Evapotranspiration = E = 60 mm — Runoff = Q = 30 mm

To Find: Change in storage: ΔS = ?

Solution: First of all we have to find the volumes of all these using:

Precipitation volume = P x A

$$P_v = 0.11 \text{ m} \times 120{,}000{,}000 \text{ m}^2 \quad \text{or} \quad P_v = 13{,}200{,}000 \text{ m}^3$$

Now — Evapotrans portation volume = E x A

$$E_v = 0.06 \text{ m} \times 120{,}000{,}000 \text{ m}^2 \quad \text{or} \quad E_v = 7{,}200{,}000 \text{ m}^3$$

Also — Roundoff volume = Q x A

$$Q_v = 0.03 \text{ m} \times 120{,}000{,}000 \text{ m}^2 \quad \text{or} \quad Q_v = 3{,}600{,}000 \text{ m}^3$$

Now to calculate the change in storage volume, we use:

$$\Delta S_v = P_v - E_v - Q_v$$

Putting values:

$$\Delta S_v = 13{,}200{,}000 \text{ m}^3 - 7{,}200{,}000 \text{ m}^3 - 3{,}600{,}000 \text{ m}^3$$

$$\Delta S_v = 2{,}400{,}000 \text{ m}^3$$

Assignment 24.1

Find the precipitation in mm if the change in storage of water is 20 mm, while evapotransportation is 50 mm and runoff is 30 mm.

24.2 OCEAN CURRENTS AND WINDS

Ocean currents and wind patterns are interconnected phenomenon that play crucial role in regulating the Earth's climate and distribute heat around the globe.

Ocean currents are the continuous, predictable and directional movement of seawater driven by gravity, wind and water density. Ocean water moves in two directions i.e. horizontally and vertically. Horizontal movements are called as currents while vertical changes are called up-welling and down-welling. The movement of ocean water is continuous and is broadly categorized into three types i.e., waves, tides and currents. Ocean currents can be defined as:

The streams of water that flow constantly on the ocean surface in definite directions are called ocean currents.

Ocean currents help distribute heat globally influencing climate patterns. They carry nutrients and food to marine organisms supporting sea life. Ocean currents can be categorized into two group i.e. surface ocean currents and deep ocean currents.

1) Surface Ocean Currents

The ocean currents which produced on shallower waters near the surface are called surface ocean currents. These currents are produced due to various factors like:

- **Wind:** Major surface ocean currents are primarily set in motion by the wind. As the wind blows across the ocean surface it drags the water along with it creating currents.
- **Subtropical Gyres:** Large rotating currents that start near the equator are called subtropical gyres. These gyres transfer heat from the equator towards the poles helping moderate climate and concentrating plastic trash in certain areas of the ocean.

Coriolis Effect

The rotation of Earth influences the path of these currents. A force resulting from the Earth's rotation is called coriolis force. In the northern hemisphere predictable winds called trade winds blow from east to west just above the equator. These winds pull surface water and the Coriolis effect deflects the currents. Then they bend to the right heading north. At about 30 degrees north latitude another set of winds called westerlies push the currents back to the east creating a closed clockwise loop. Similar patterns occur in the southern hemisphere but currents in this side are counter-clock wise i.e. bend to the left, as shown in Fig. 24.7.

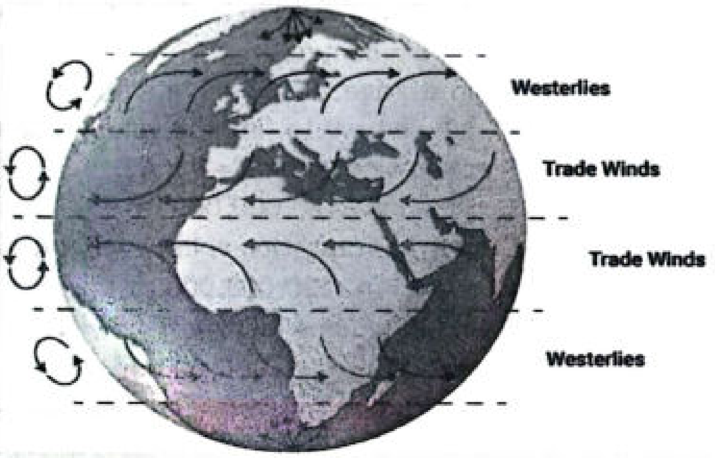


Figure 24.7: Coriolis force.

2) Deep Ocean Currents

The ocean currents beyond the depth of 500 meters are known as deep ocean currents. These currents are formed due to the differences in temperature and density of the water in different parts of the ocean. Some of them are explained below.

- **Warm Water Northward:** The surface currents carry warm water north from the equator as this water moves into higher northern latitudes it cools down
- **Density:** The density of sea water plays a major role in producing ocean currents and circulating heat as cooling process increases the water's density. Sea water density varies due to its salinity and temperature, high salinity makes water denser because more salt is packed into it. Denser water sinks and flows back toward the equator at deeper levels creating deep ocean currents. On the other hand, high temperature makes the water less dense. Near the ocean surface a combination of high salinity and low temperature makes water dense enough to sink into deep-ocean. The weight of

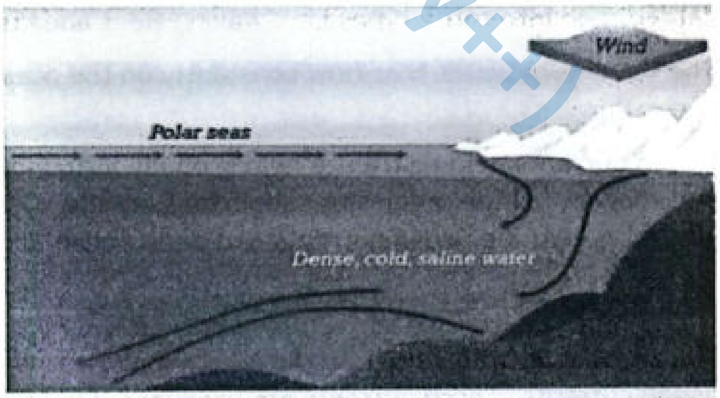


Figure 24.8: Water movement due to density difference.

water above pushes deep water molecules together making them denser. Deep circulation affects the entire ocean and significantly impacts abyssal properties (abyssal is the region in deep oceans where there is no light and very little oxygen with very high concentration of salts and density, this region has very few examples of life in it) where wind driven currents have no effect. The movement of water due to density is shown in Fig. 24.8.

Thermohaline Circulation

The Deep Ocean currents are caused by differences in water density. This process is called thermohaline circulation where "*thermo*" refers to temperature and "*haline*" to saltiness. Thermohaline circulation is a large-scale ocean circulation driven by density gradients created by surface heat and freshwater fluxes. The wind driven surface currents move toward poles from the equatorial region and cool the water during their movement and this cooled water sinks at high latitudes and this cycle continues which transport energy and mass. Thermohaline circulations act like a giant conveyor belt moving warm surface waters downward and forcing cold nutrient rich deep waters upward, as shown in Fig. 24.9.

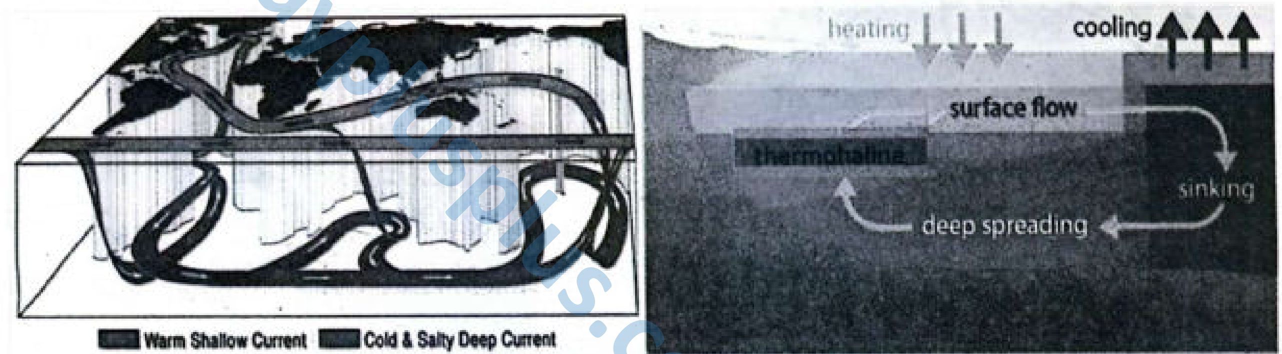


Figure 24.9: Thermohaline circulation.

Earth's Angular Momentum and Wind Patterns

The conservation of angular momentum plays a crucial role in determining the wind patterns on the Earth. This is closely related to the Earth's rotation and is responsible for the Coriolis effect which deflects the movement of air masses in different directions depending on the hemisphere. Angular momentum is a property of rotating objects depends upon velocity of rotation and the perpendicular distance from the axis of rotation. Due to this phenomenon the air moves from high latitudes (near the poles) to low latitudes (towards the equator). It experiences a change in velocity due to Earth's spherical shape and varying rotational speeds. The turning of winds due to the rotation of Earth is called as Coriolis effect, as shown in Fig. 24.7, and the force which bends the winds due to change in speeds is called as Coriolis force which can be given as:

$$F_c = 2mv\omega \sin\theta \qquad \text{\_\_\_\_\_\_ (24.2)}$$

Here 'F_c' is the Coriolis force, 'm' is the mass of the moving air packet, 'v' is the wind speed, 'ω' is the angular velocity of the Earth's rotation (which is approximately 7.292×10^{-5} rads^{-1}) and 'θ' is the angle with latitude (at equator it is 0^o while at poles it is $\pm 90^o$)

At the equator the Coriolis force is zero due to $\sin 0^\circ = 0$, while this force is maximum at the poles due to $\sin 90^\circ = 1$. The Coriolis effect causes winds to be deflected more strongly at the higher latitudes closer to the poles. It is important to note that the magnitude of wind speed is not directly influenced by the Coriolis force but the direction of the wind will change *significantly* as we move from equator towards the pole.

As from the equator where the distance of the surface of Earth from the axis of rotation is large i.e., it has large moment of inertia at this point, here the speeds of winds will be low but near poles the moment of inertia becomes small and hence increases the speed of winds. This difference in the speed of winds accounts for the weather patterns on Earth. The deflection of air due to Coriolis effect helps to create distinct atmospheric circulation cells as shown in Fig. 24.10.

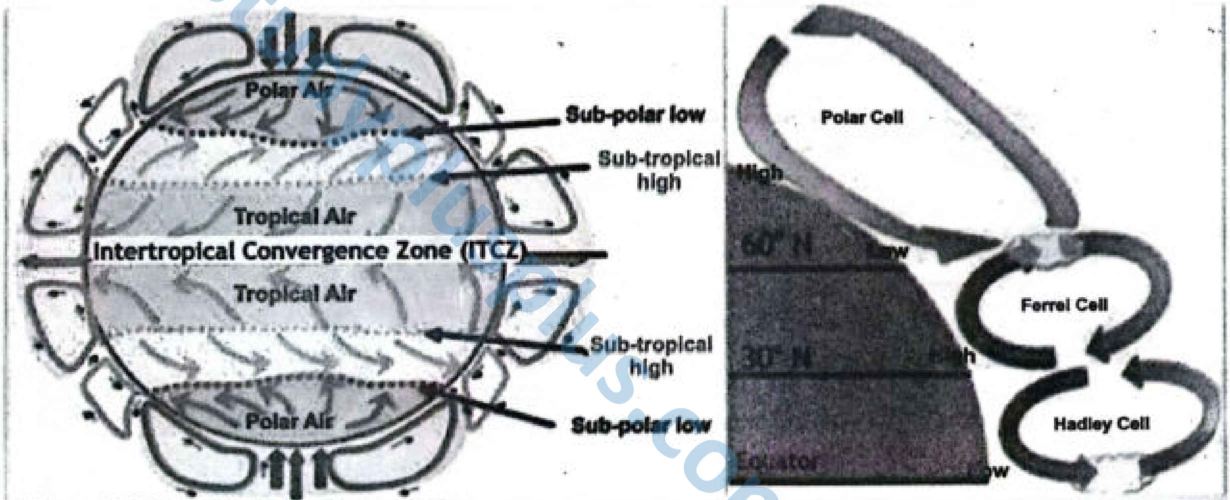


Figure 24.10: Wind circulation cells.

- **Hadley Cells:** Between the equator and 30° latitude, warm air rises at the equator due to intense heating and moves towards poles at high altitudes and finally descends around 30° latitude creating a cycle.
- **Ferrell Cells:** Between 30° to 60° latitude air descends around 30° and moves towards pole at the surface and rises at 60° latitude with movement towards equator at high altitudes.
- **Polar Cells:** Between 60° latitude and the poles air rises at 60° latitude and moves towards pole at high altitude. Finally, air descends at the pole.

These cells distribute heat and moisture around the globe which influences weather patterns and climate.

Wind patterns are integral components of the climate system playing a significant role in shaping regional and global climate. Wind patterns are the result of complex interactions between various atmospheric and oceanic processes. They are necessary for distribution of heat, moisture and energy across the Earth's surface influencing weather conditions and climate for both short and long timescales. The basic relationship between atmospheric pressure and horizontal wind can be described using the geostrophic balance.

This balance occurs when we disregard friction and changes in wind direction and speed. The geostrophic wind blows parallel to isobars and is influenced by two main forces as:

- Horizontal pressure gradient force (the change in pressure measured across a given distance is called a pressure gradient) derives the air movement from higher to lower pressure areas.
- Coriolis force results from the Earth's rotation and causes the deflection of wind to the right in the northern hemisphere and to the left in the southern hemisphere.

Wind speed increases as the distance between isobars decreases or pressure gradient increases. The impact of wind on climate can be given as:

- **Heat Transport:** Wind carries heat from one region to another. For example, ocean currents driven by wind transport warm water from the equator towards the poles.
- **Moisture Transport:** Wind transports moisture affecting precipitation patterns (precipitation is any liquid or frozen water that forms in the atmosphere and falls back to the Earth). Trade winds, westerlies and polar easterlies play essential roles in distributing moisture globally. These winds on globe are shown in Fig. 24.10.
- **Pollutant Dispersion:** wind disperses pollutants like dust, smoke and industrial emissions across the large area.
- **Pollen Distribution:** Wind help disperse pollen helping plant reproduction.
- **Daily weather Patterns:** Coastal regions experience daily changes in wind direction due to differential heating of land and water (the difference in how land and water surfaces absorb heat is called as differential heating).

Over a 30-year period these wind patterns exhibit statistical regularities that contribute to the climate of the region. Variations in these patterns over long-time scales can result in changes to regional climate and even global climate. Understanding and monitoring wind patterns are crucial for predicting weather events, mapping natural resources and assessing the impacts of climate change. Advanced modeling techniques and observational technologies continue to improve our understanding of these complex interactions within the climate system.

Do You Know?

As the air mass starts to move it is deflected to the right by the coriolis force. The deflection increases until the coriolis force is balanced by the pressure gradient force. At this point the wind will be blowing parallel to the isobars (having same pressures) when this happens the wind is referred to as the "geostrophic wind".

Example 24.2: Find the magnitude of the Coriolis force which bends the path of air mass 200,000 kg up to 25°, while the mass of air is moving at the speed of 80 km h^{-1}.

Given: Mass of air packet= $m = 200{,}000$ kg Speed of air = $v = 80$ km h^{-1}

Angle = $\theta = 25°$ Angular velocity of Earth = $\omega = 7.3\times10^{-5}$ rad s^{-1}.

To Find: Coriolis Force = $F_C = ?$

Solution: First we need to convert the velocity into SI units as:

$$v = 80 \text{ km h}^{-1} = \frac{80\times1000}{3600} \text{ m s}^{-1} \qquad \text{or} \qquad v = 22.2 \text{ m s}^{-1}$$

To find the Coriolis force, we use: $F_C = 2mv\omega\sin(\theta)$

Putting values, we get: $F_C = 2(200000)(22.2)(7.3\times10^{-5})\sin(25) = 274$ N

Assignment 24.2

Find the angle through which a Coriolis force of 200 N will deflect the air mass of 100,000 kg moving at the speed of 60 km h^{-1}.

24.3 CLIMATE INERTIA

Climate inertia refers to the tendency of the Earth's climate to resist or respond slowly to changes in external factors like greenhouse gas concentrations and solar radiations. This phenomenon occurs due to various complex feedback mechanisms and processes within the Earth's climate system. These feedback mechanisms include:

- **Positive feedback:** Some feedbacks amplify initial changes for example as the arctic ice melts due to warming it reduces the Earth's reflectivity leading to further warming.
- **Negative feedback:** Other feedbacks dampen changes for example increased carbon dioxide (CO_2) levels stimulate plant growth which absorbs some carbon acting as negative feedback.

The climate inertia is time dependent as for example if greenhouse gas emissions were to stabilize immediately the Earth's climate would continue to warm for some time due to heat already stored in the oceans. Climate processes operate on various timescales from days to centuries like:

- **Short-term Inertia:** The ocean and land surfaces absorb and release heat relatively slowly leading to short term inertia this is why daily temperature fluctuations occur
- **Long-term Inertia:** changes in greenhouse gas concentrations like carbon dioxide take decades to centuries to fully impact the climate system. When we stabilize greenhouse gas emission the climate system does not respond immediately.

Oceans play a main role in climate inertia as they have high heat capacity means they absorb and release heat slowly. Changes in ocean temperature occur gradually affecting climate patterns over extended periods. Similarly, the carbon cycle involves exchanges of carbon dioxide between the atmosphere, oceans, land and vegetation. It also takes time due to slow decomposition of organic matter. The melting of ice sheets and glaciers contributes to sea level rise but this process occurs over centuries leading to inertia in the response to climate change. Climate inertia underscores the importance of adaptation (adjusting to existing changes) and mitigation (reducing emission) strategies. Even if we take immediate actions the effects of past emissions will persist. Earth's climate change can be categorized into two main components i.e. internal variations and external forcing.

24.3.1 Internal Variations

Internal variations refer to natural fluctuations within the climate system itself. These variations occur due to interactions between different components of the Earth's climate system such as the atmosphere, oceans, land and ice etc. some of the examples of the internal variations are:

For Your Information

Jet streams are fast-moving bands of air in the upper atmosphere, typically between 6 km to 15 km above the Earth's surface. They play a significant role in shaping weather patterns and atmospheric circulation.

- **Atmospheric Circulation Patterns:** Changes in atmospheric circulations like jet streams, monsoons lead to regional climate variability.

- **Solar Radiations:** Variations in solar radiation due to sunspots and solar cycles impact climate over shorter time scales.
- **Volcanic Activity:** Volcanic eruptions release aerosols into the atmosphere affecting global temperatures temporarily. Internal variations are often cyclic and occur over decades to centuries.
- **Ocean Oscillations:** ocean currents and oscillations i.e. El-Nino southern oscillations (ENSO), Atlantic multi-decadal oscillations (AMO) and Pacific Decadal Oscillations (PDO) cause variations in the sea surface temperatures affecting weather patterns globally. Some oscillations are shown in Fig. 24.11.

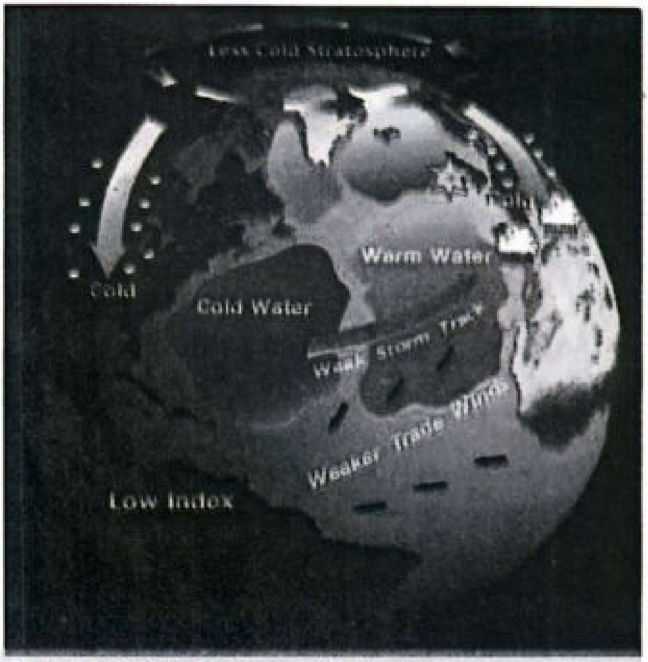


Figure 24.11: Ocean oscillations.

24.3.2 External Forcing

External forcing refers to factors outside the climate system that influence the behavior of the climate system. These factors can alter the Earth's energy balance leading to long term climate changes. Some of the examples of the external forcing are:

Greenhouse Gas Concentrations: Increased levels of greenhouse gases like carbon dioxide and methane etc. due to human activities enhance the greenhouse effect trapping more heat and causing global warming.

Solar Variability: Changes in solar output over longer time scales impact Earth's climate.

Orbital Variations: Earth's orbit around the Sun undergoes periodic variations. Eccentricity and axial tilt affect the distribution of solar energy, as shown in Fig. 24.12. But these effects are long time scales changes extended up to thousands of years.

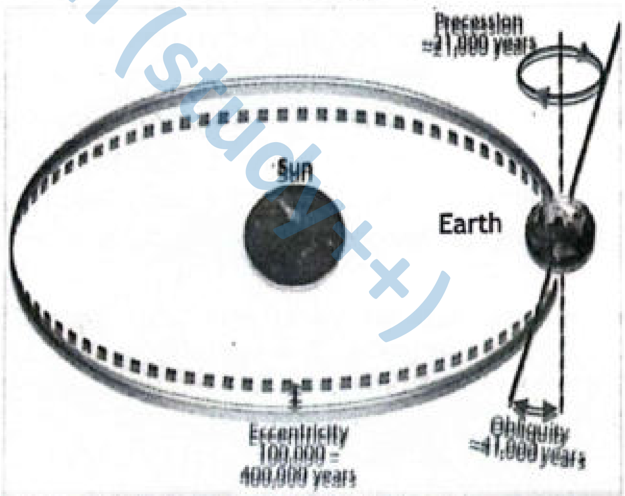


Figure 24.12: Eccentricity, tilt and precession of Earth.

Aerosols and Pollution: Anthropogenic aerosols (a suspension of particles or droplets in the air and includes dust, mist, fumes or smoke and anthropogenic mean such particles from the industrial sources or from combustion processes) can cool the climate by reflecting sunlight back into space.

External forcing factors operate over longer time frames like centuries and contribute to gradual climate shifts.

24.4 EARTH ENERGY BUDGET

The Earth's energy budget is a way of accounting for the balance between the energy that Earth receives from the Sun and the energy it radiates back into the space. It is simple the balance between the incoming solar radiation (insolation) and outgoing terrestrial radiations. The Earth's energy budget can have the components like:

- **Incoming Solar radiations:** The Sun emits energy in the form of sunlight. Approximately 49% of this solar energy is absorbed by the Earth's surface primarily by land, ocean and vegetation.
- **Atmospheric Absorption:** About 20% of solar radiation is absorbed by the atmosphere itself.
- **Outgoing Terrestrial Radiations:** The Earth which absorbs radiation can also re-emit these radiations as heat. This process occurs through infrared radiations emitted by the surface and the atmosphere.

The greenhouse effect also plays a role in the energy distribution in the Earth's climate in regulating the globe's temperature. It involves the interaction of solar radiation with greenhouse gases in the atmosphere. It can be explained in terms of steps like in first step sunlight penetrates the atmosphere and warms the Earth's surface. In second step the heated surface of the Earth emits infrared radiations. In third step greenhouse gases absorb some of these outgoing radiations preventing them from escaping directly into the space. Instead, they re-radiate part of this energy back towards the surface effectively trapping heat. Without the greenhouse effect Earth's average temperature would be well below freezing.

24.4.1 Solar Variability and Climate

While the Sun is essential for maintaining Earth's habitable conditions, its influence on climate change is relatively small compared to other factors. Some of the factors are as follows:

- **Solar Cycles:** The Sun undergoes 11-years solar cycles during which its activity varies. These cycles affect solar radiation, sunspots and solar flares.
- **Long-term Trends:** Over the past 40-years satellites have observed the Sun's energy output. However, changes in solar activity account for less than 0.1% of the warming observed since 1750.
- **Human Induced Warming:** Human activities particularly the burning of fossil fuels release greenhouse gases. The warming caused by these gases is over 270 times greater than any sunlight warming from the Sun itself during the same period.

24.4.2 Energy Imbalance

The uneven distribution of solar energy across the Earth's surface creates an energy imbalance. The energy imbalance on the Earth is the main cause of the atmospheric circulations. For example, equator receives more solar energy due to direct sunlight while the pole regions receive less solar energy due to oblique sunlight, as shown in Fig. 24.13.

This energy imbalance drives the atmospheric circulations as:

- Warm air rises at the equator creating low-pressure zones.
- Cooler air sinks at the poles forming high pressure zones.
- The resulting pressure gradients lead to winds like trade winds, westerlies and polar easterlies.
- Ocean currents also play a role in redistributing heat globally.

Changes in energy balance can impact atmospheric circulation patterns, affecting weather systems, ocean currents and climate zones. As the Sun's energy is vital for Earth's climate, human induced greenhouse gas emissions have a more significant impact on global warming. Understanding these processes helps us to address climate change and its consequences.

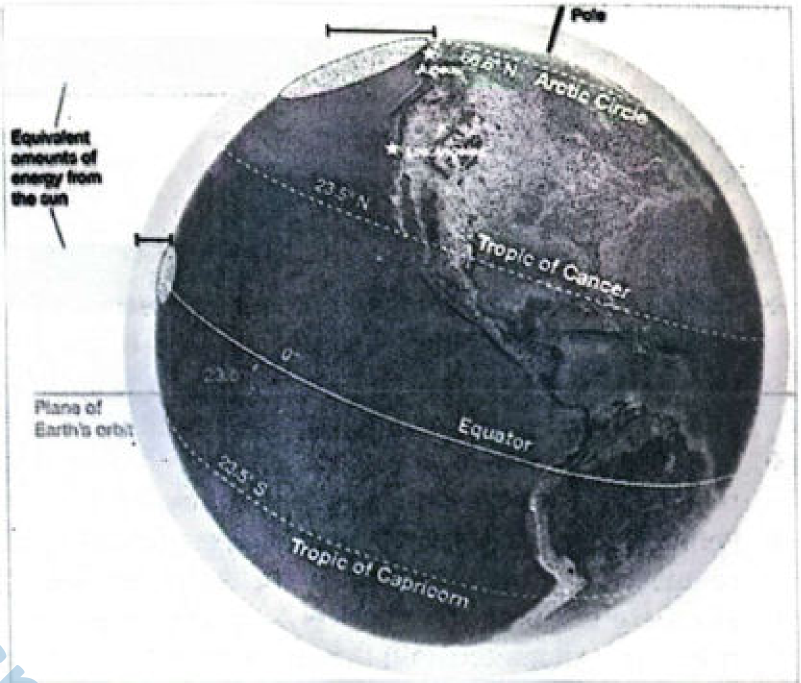


Figure 24.13: Energy imbalance between poles and equator.

24.4.3 Climate Science as a Chaotic System

Climate science provides a fascinating example of a chaotic system. In a chaotic system there is no equilibrium and no repeatable patterns emerge. Chaotic systems exhibit extreme sensitivity to initial conditions. This means that even small changes in the starting conditions can lead to vastly different outcomes over time. Climate being a complex and interconnected system can exhibit chaotic behavior as we often think of climate as having predictable seasons and weather patterns (like the news inform us about a rain today but it does not happen) it can also undergo sudden and rapid changes due to chaotic dynamics.

Butterfly Effect: Consider the metaphor of a butterfly flapping its wings in one part of the world then according to chaos theory this seemingly insignificant event could potentially set off a chain reaction of interactions that eventually lead to a hurricane forming in another part of world, this phenomenon is popularly known as the "*butterfly effect*". The butterfly effect illustrates how small perturbations can amplify and propagate through a complex system ultimately influencing large scale events.

Predictability and Uncertainty: Climate scientists sometimes refer to the effects of chaos as intrinsic or unforced variability. These are unpredictable changes arising from dynamic interactions within the climate system rather than being directly caused by external factors. Our atmosphere and oceans do not behave in simple easily predictable ways they are non-linear chaotic systems. As a result, we can only predict large-scale weather features with relative certainty and only for a few days in advance. In this scenario the weather forecasting for an extended time period is less reliable, these predictions may be more reliable only for short time scale afterward.

Climate science exemplifies the delicate balance between predictability and chaos. While we strive to understand and model climate dynamics the inherent complexity and sensitivity to initial conditions remind us that some aspects of our climate remain inherently uncertain and difficult to predict.

SUMMARY

- Earth's climate system is a complex system with five interacting components the atmosphere, the hydrosphere, the cryosphere, the lithosphere and the biosphere.
- Hydrosphere encompasses all the water on the Earth whether it is in liquid water, solid ice or the gaseous water vapors form.
- Cryosphere refers to any place on Earth where water is in its solid form, where low temperatures freeze water and turn it into ice.
- Permafrost refers to permanently frozen ground found mainly in high altitude regions. It contains ice and soil that remain frozen for extended periods.
- Biosphere refers to the relatively thin life supporting layer of Earth's surface. It extends from a few kilometers into the atmosphere to the deep-sea vents in the ocean.
- Nutrient cycles: Elements like carbon (C), nitrogen (N_2) and phosphorous (P) cycle through the biosphere impacting both living organisms and the environment.
- Large rotating currents that start near the equator are called subtropical gyres.
- Coriolis force: The rotation of Earth influences the path of these currents. A force resulting from the Earth's rotation is called coriolis force.
- The Deep Ocean currents are caused by differences in water density this process is called thermohaline circulation.
- Horizontal pressure gradient force is the change in pressure measured across a given distance is called a pressure gradient.
- Precipitation is any liquid or frozen water that forms the atmosphere and falls back to the Earth.
- Climate inertia refers to the tendency of the Earth's climate to resist or respond slowly to changes in external factors like greenhouse gas concentrations and solar radiations.
- Aerosol is a suspension of particles or droplets in the air and includes dust, mist, fumes or smoke.
- Chaotic system is system in which there is no equilibrium and no repeatable patterns emerge it exhibits extreme sensitivity to initial conditions.
- Butterfly Effect: the metaphor of a butterfly flapping its wings in one part of the world can lead to a hurricane forming in another part of world.

EXERCISE

Multiple Choice Questions

Encircle the Correct option.

1) Which of the following gases has the maximum contribution to global warming?
   A. Chlorofluorocarbons (CFCs) B. Methane (CH_4)
   C. Sulfur hexa-fluoride (SF_6) D. Carbon dioxide (CO_2)
2) Which of the following gas is not responsible for global warming?
   A. N_2O B. H_2O C. SO_2 D. CO_2
3) In global warming the temperature of _______________ increases:
   A. Troposphere B. Ionosphere C. Mesosphere D. Stratosphere
4) Which type of climate is found in our country Pakistan?
   A. Tundra B. Equatorial C. Monsoon D. Western
5) What is the height of troposphere near the poles and the equator in kilometers?
   A. 8 and 18 B. 6 and 16 C. 4 and 16 D. 10 and 20
6) What happens in the ionosphere?
   A. Meteors burns B. Airplanes fly C. Radio waves reflected D. Satellites move
7) Which atmospheric layer contains ions and helps in wireless communications?
   A. Troposphere B. Thermosphere C. Mesosphere D. Stratosphere
8) The part of the Earth atmosphere which contains about 70% of the total air in the atmosphere is:
   A. Troposphere B. Thermosphere C. Mesosphere D. Stratosphere
9) Hot and dry winds blowing in the northern plains of Punjab and Sindh are:
   A. Trade winds B. Loo C. Westerlies D. Easterlies
10) Pakistan is located in the region of _________ winds:
   A. Trade B. North easterlies C. Westerlies D. South easterlies

Short Questions

1) Where in the atmosphere is water vapor most concentrated and why?
2) What is the difference between the weather and the climate?
3) Warm air has a greater capacity to hold water vapors. Why?
4) How does the atmosphere retain itself?
5) Describe global latitudinal air movements due to uneven heating at different latitudes.
6) What would be the consequences if all the polar ice melts?
7) Is the lithosphere a part of mental? Justify your answer.
8) What are the four components of the biosphere?
9) Why stratosphere is called a calm and stable layer?

Comprehensive Questions

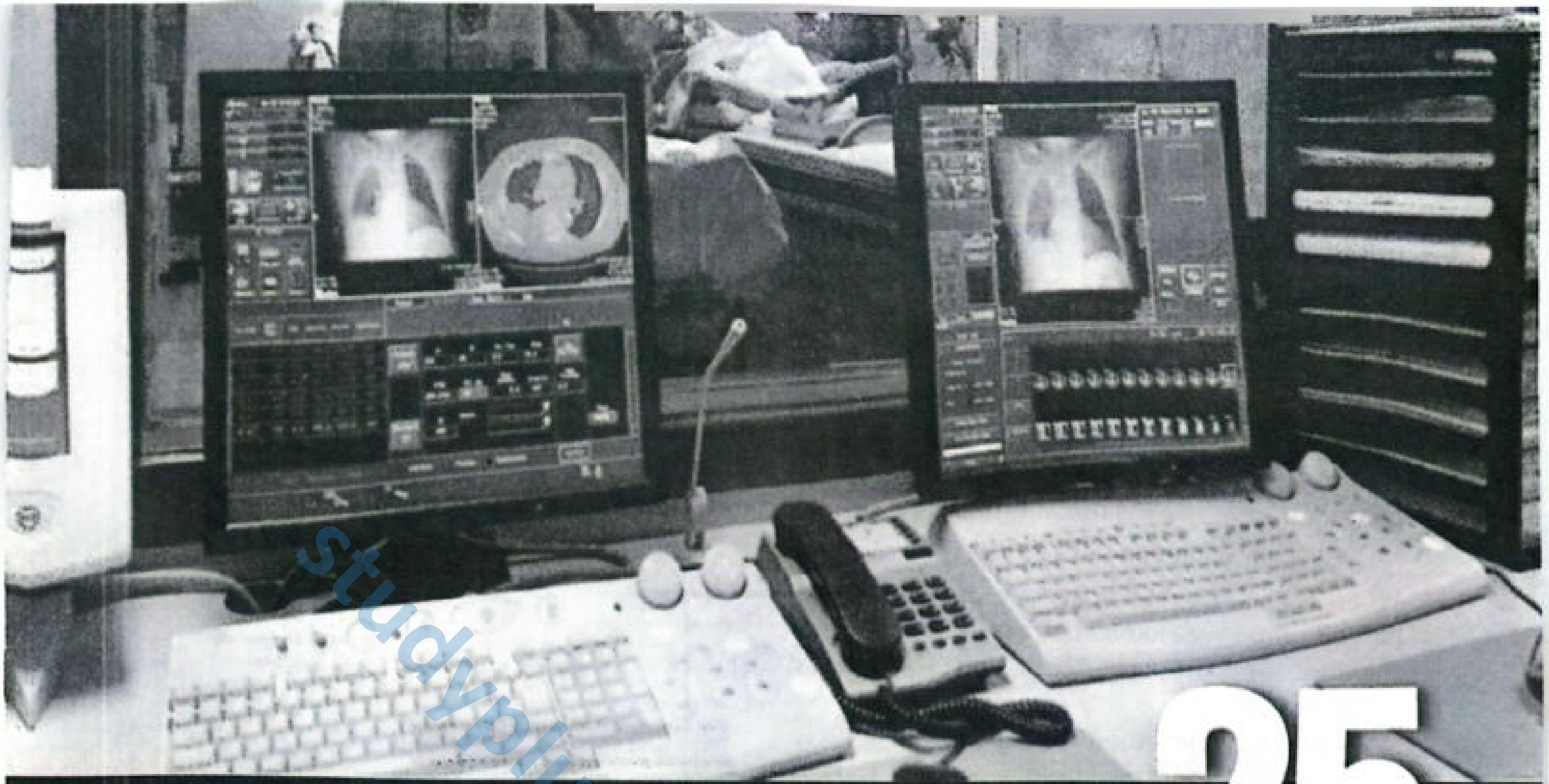
1) Describe Earth's climate system as a complex system having five interacting components.
2) What is climate inertia? Explain.
3) Explain how global climate is determined by energy transfer from the Sun.

4) Discuss the Earth energy budget in detail.
5) Explain how the energy imbalance between the poles and the equator can affect atmospheric circulation.
6) *Explain that due* to the conservation of angular momentum, the Earth's rotation diverts the air to the right in the Northern Hemisphere and to the left in the Southern hemisphere, thus forming distinct atmospheric cells.
7) Discuss that how differences in density of ocean water play an important role in ocean circulation.
8) How the thermohaline circulation transports heat from the, tropics to the polar regions? Explain.
9) Show that the climate science is a chaotic system in terms of butterfly effct.
10) Differentiate between surface ocean currents and deep ocean currents.
11) What are coriolis force and thermohaline circulations?

Numerical Problems

1) If the volume of change in storage of the hydrosphere, whose area is 30 km^2 is 600,000 m^3, find net precipitation (P) if the volume of evapotranspiration is 1,500,000 m^3 and runoff volume is 600,000 m^3. (Ans: 90 mm)

2) If a Coriolis force of 30 N deflects a mass of air at 30^0 moving with speed of 40 km h^{-1}. Find the mass of this air packet (where ω = 7.3×10^{-5} rad s^{-1}). (Ans: 369,900 kg)



25 MEDICAL IMAGING

Student Learning Outcomes (SLOs)

The student will

- Explain that piezo-electric effect and its application in medical science [ultrasound waves are generated and detected by a piezoelectric transducer]
- Explain how ultrasound can be used to obtain diagnostic information about internal body structures.
- Explain that X-rays are produced by electron bombardment of a metal target and calculate the minimum wavelength of X-rays produced from the accelerating p.d.
- Explain the use of X-rays in imaging internal body structures [including a describing of the term contrast in X-ray imaging]
- Explain how computed tomography (CT) scanning works [it produces a 3D image of an internal structure by first combining multiple X-ray images taken in the same section from different angles to obtain a 2D image of the section, then repeating this process along an axis and combining 2D images of multiple sections]

In medical, imaging is a range of tests used to create images of internal body parts. Medical imaging seeks to reveal internal structures hidden by the skin and bones, as well as to diagnose and treat disease. Medical imaging also creates a database of normal anatomy and physiology to make it possible to identify abnormalities.

There are many different types of imaging techniques used in modern medicine, such as ultrasound, X-rays, CT (computed tomography) scans, and MRI (magnetic resonance imaging). Each imaging type uses a different technology to create an image. In this unit, we'll discuss some of the imaging techniques in detail.

25.1 PIEZOELECTRIC EFFECT AND ULTRASONIC WAVES

Piezoelectric effect is the ability of certain materials to generate an electric charge in response to applied mechanical stress.

When piezoelectric material is placed under mechanical stress, a shifting of the positive and negative charges in the material takes place, which then results in an external electric field, as shown in Fig. 25.1 (a).

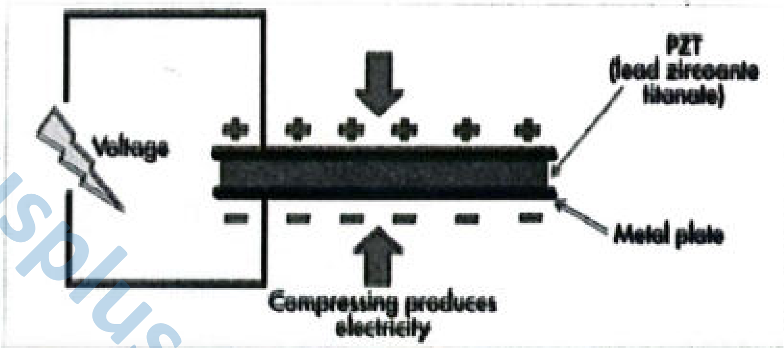


Figure 25.1 (a): Piezoelectric effect.

The piezoelectric effect is reversible, as shown in Fig. 25.1 (b), it means that:

The material can also generate stress when an electric field is applied. This is called inverse piezoelectric effect.

The piezoelectric effect is very useful within many applications that involve the production and detection of sound (especially ultrasound), generation of high voltages, electronic frequency generation and microbalances. It is also the basis of a number of scientific instrumental techniques with atomic resolution, such as scanning probe microscopes etc.

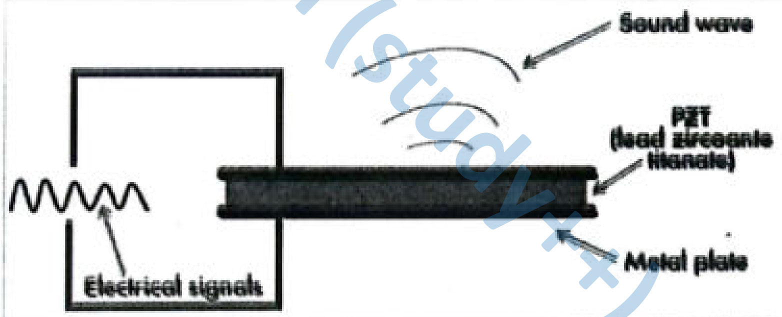


Figure 25.1 (b): Inverse piezoelectric effect.

Generation and Detection of Ultrasound

A normal human ear can hear a sound of frequency 20 Hz to 20,000 Hz.

The sound waves above 20,000 Hz are inaudible to normal ear and are called ultrasounds.

For Your Information

Lithium niobate ($LiNbO_3$) is a piezoelectric material used in optical and acoustic devices. Some other popular piezoelectric materials are *Lead zirconate titanate (PZT)* and Quartz.

Generation of ultrasound uses reverse piezoelectric effect. In ultrasound equipment, a piezoelectric transducer converts electrical energy into extremely rapid vibrations. These vibrations are so fast that it makes sounds of too high-pitched for our ears to hear.

When ultrasound waves hit an object and bounce back, creating an echo. After reflection or scattering, ultrasonic waves return to the detector causing the piezoelectric materials to vibrate, which generate an electric signal to be detected. In medical ultrasound, the detected signal determines the shape, size, and consistency of the object, generating a real-time image that appears on a screen. An ultrasound image of the pancreas is shown in Fig. 25.1 (c).

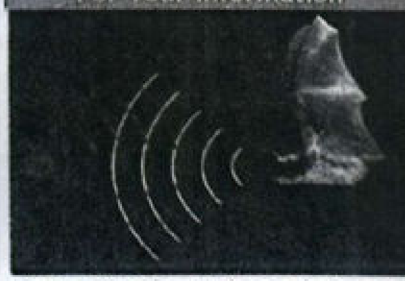
Figure 25.1 (c): Ultrasound image of the pancreas.

Applications of Ultrasound in Medical

Medical technology uses ultrasound for many purposes. Ultrasound is used to generate images of tissue and organs; this technique is known as sonography or echography. The great advantage of sonography over other imaging methods in medical technology is that sound waves are comparatively less harmful and can even be used for unborn babies.

The ability to focus very intense ultrasonic waves in a small region without disturbing the surrounding tissues provides a very effective tool in neurosurgery. Special types of ultrasonic equipment are in use for the treatment of arthritics, muscular, rheumatism and sciatica.

For Your Information



Few animals such as bats and frogs have the ability to use ultrasound to communicate with each other. Bats navigate and find their food by echolocation. Bats emit high pitched sound of short wavelength in order to navigate. These waves hit the surrounding and bounce back allowing bats to get an exact map of the surrounding.

An important industrial application is the ultrasonic flow meter. Ultrasonic flow meters measure the flow rate of a liquid by transmitting high-frequency sound waves into the flowing liquid. The waves reflect off particles or bubbles in the liquid, experiencing a frequency shift due to the liquid's flow velocity, known as the Doppler shift. This shift is measured and used to calculate the flow rate. The meters typically consist of transducers installed on the pipe, which transmit and receive the ultrasound signals. The signals are then processed to extract the Doppler shift information, and the flow rate is calculated based on the pipe's cross-sectional area.

Do You Know?

How ultrasonic waves provide the basis of the eye and vision systems for robots?

This technique has also been applied to monitor blood flow through arteries in the body. It provides information on blood clots, blocked arteries and cardiac function in adults and developing fetuses.

25.2 X-RAYS

X-rays originate from heavy atoms. Transitions of electrons between the shells in heavy atoms produce X-rays. X-rays can also be produced by Coulomb interaction of a fast-moving electron with orbital electrons as well as the positive nucleus. Let us see the production of X-rays in detail.

Inner shell Transition and Characteristic X-Rays

The inner shell electrons in heavy atoms are tightly bound. Large amount of energy is required for the displacement of these electrons from their normal energy levels. When a heavy target material is bombarded with a beam of electrons, that has been accelerated by several k eV, some of these electrons will collide with inner-shell electrons of the target and knock them out of their respective atoms (Fig. 25.2 a).

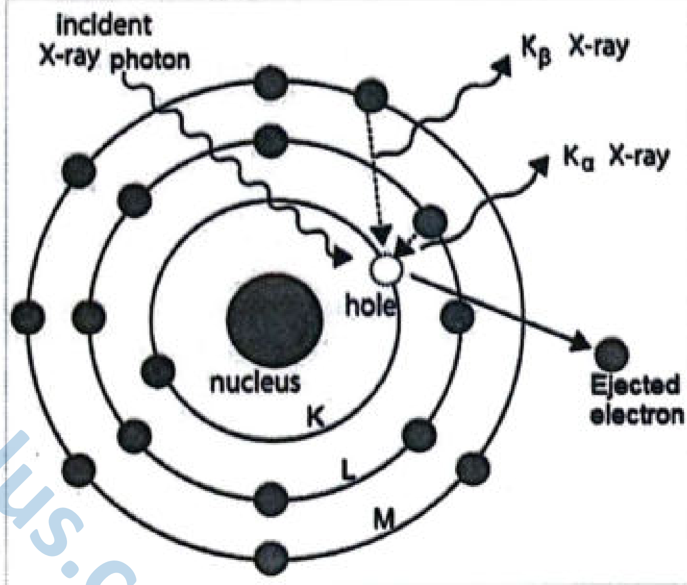


Figure 25.2 (a): Emission of characteristic X-rays.

Let a K-shell electron is knocked out from an atom creating a vacancy in K-shell. Then an electron from either, L, M, or N-shell will quickly jump down to fill the vacancy in the K-shell emitting the excess energy as X-ray photon. These X-rays consist of series of specific wavelengths or frequencies and hence are called characteristic X-rays.

An X-ray photon is emitted due to transition from L-shell to the vacancy in the K-shell, called K_α characteristic X-rays. An X-ray photon emitted due to transition from M-shell to the vacancy in the K-shell is called K_β characteristic X-rays. An X-ray photon due to transition from N-shell to the vacancy in the K-shell is called K_γ characteristic X-rays.

For Your Information

The study of characteristic x-rays spectra has played a very important role in the study of atomic structure and the periodic table of elements.

Continuous X-Rays

When a high energy electron travels towards a target nucleus in the X-rays tube, it has coulomb interaction with orbital electrons as well as the positive nucleus. The interaction with the nucleus is very strong due of the presence of concentrated positive charge in the nucleus. The force of attraction due to the large concentrated positive charge in the nucleus accelerates the electron, as shown in Fig. 25.2 (b). The electron begins to decelerate as it moves away from the nucleus. The decelerated electron emits radiation called Bremsstrahlung (a Germen word meaning braking radiation). This Bremsstrahlung is called continues X-rays. *Energy of the* electron decreases due to the emission of these radiations. According to quantum theory, these radiations must appear in the form of photons. Since the radiated photon carries energy, the electron must lose kinetic energy because of its encounter with the target nucleus.

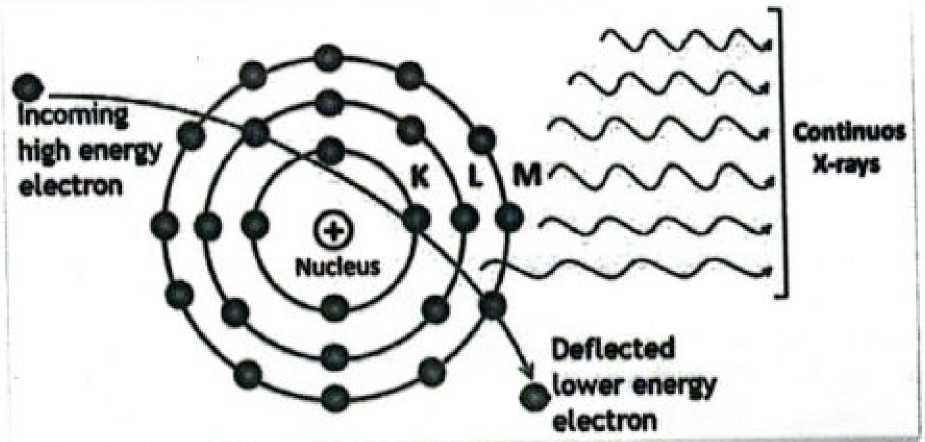


Figure 25.2 (b): Emission of continuous x-rays.

Let us consider an extreme example in which the initial energy of the electron (eV) is transformed completely into the energy of the photon (hf_{max}) during a single collision, i.e.,

$$eV = hf_{max}$$

or $$eV = \frac{hc}{\lambda_{min}}$$

or $$\lambda_{min} = \frac{hc}{eV} \qquad \text{———(25.1)}$$

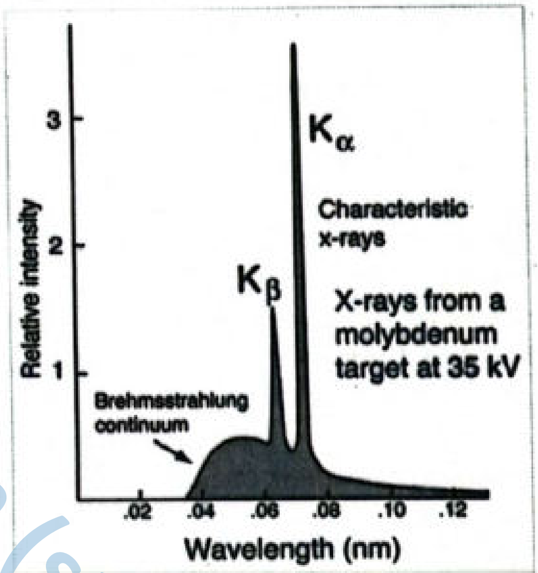


Figure 25.3: Graph of x-rays obtained from molybdenum target.

Actually, all radiation produced does not have the wavelength given in equation (25.1) because many of the electrons are not stopped in a single collision. This results in the production of the continuous spectrum of wavelength. Graph of X-rays obtained from molybdenum target is shown in Fig. 25.3.

Setup for Production of X-Rays

X-ray is a form of high-energy electromagnetic radiation that passes through most objects. An arrangement of producing X-Rays is shown Fig. 25.4.

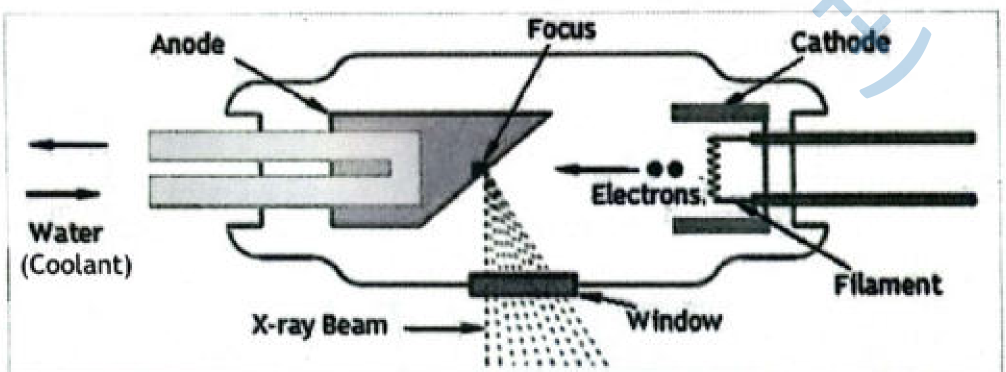


Figure 25.4: Production of X-rays in X-ray tube

The cathode is made of a filament which is heated by the current supplied from a battery. Filament emits electrons. The anode is made of a solid copper bar. A high melting-point metal like platinum or tungsten is embedded at end of the copper rod and it serve as a target. The cathode and anode are enclosed inside an evacuated glass chamber. A high DC voltage of the order of 50,000 V is maintained between cathode and anode.

The electrons emitted from the cathode are accelerated by the high potential difference. The energetic electrons strike the target and X-rays are produced. When such highly energetic electrons are suddenly stopped by target, an intense beam of X-rays is produced. A small part of the kinetic energy of the incident electrons is converted into X-rays, the rest is converted into heat. The target becomes very hot; therefore, it must have a high melting point. The heat generated in target is dissipated through the copper rod. Sometime the anode is cooled by water flowing behind the anode.

Applications of X-Rays in Imaging Internal Body Structures

X-ray imaging/photography is the most common and oldest diagnostic imaging technique. During an X-ray test, X-rays are allowed to pass through the body and are absorbed in different quantities by different organs. For example, the calcium content in bones means that they absorb x-rays better than other tissue, such as fat. An X-ray detector receives the X-rays after they pass through the body, generating an image that appears in shades of gray. The bones cast their shadow on the photographic plate, as shown in Fig. 24.5. It is because that bone is opaquer to X-rays than flesh. The X-ray photographs reveal fractures of bones or the presence of foreign bodies.

Figure 25.5: X-ray image of hand.

Conventional X-ray image is sufficient when looking at bones and other dense structures, but there is difficulty while looking x-ray image of soft tissues. For better evaluation of X-ray film of soft tissues, contrast is used in X-ray imaging. Contrast is the range of brightness, from lightest to darkest, in an image. Contrast in X-ray imaging allows the radiologist to evaluate structures that are not clearly evident on conventional X-ray exams.

25.3 CT SCAN

A CT scan (Computed Tomography scan) is a medical imaging method that is useful to get a detailed 3-D image of certain parts of the body, such as soft tissues, the blood vessels, the lungs, the brain, abdomen, and bones etc. A CT scan machine uses X-rays and computer to get image.

In the CT scanner there is an X-ray source and large number of detectors. Each detector records an image. The source and the detectors are mounted on a large ring, as shown in Fig. 25.6.

During CT scan, the patient lies down on the table (called couch) that slowly slides through the gantry, which is the circular part of the CT machine. Through the gantry, beams of x-rays are aimed at the body. The source and detectors, mounted on the ring, are rotated around the patient to give views from a variety of direction. This generates 2-D cross-sectional images called tomographic images. The couch and patient are then moved along the axis of the machine and another set of images is taken. These large numbers of images are then combined by a computer to give a merged detailed 3-D image of the organs under investigation.

Figure 25.6: CT scanner.

A CT scan of brain is shown in the Fig. 25.7. Here the CT scanner first combine multiple X-ray images of brain taken from different angles to obtain a 2-D image of the section. Repeating this process along an axis and combining 2-D images of multiple sections, a 3-D image of brain is obtained.

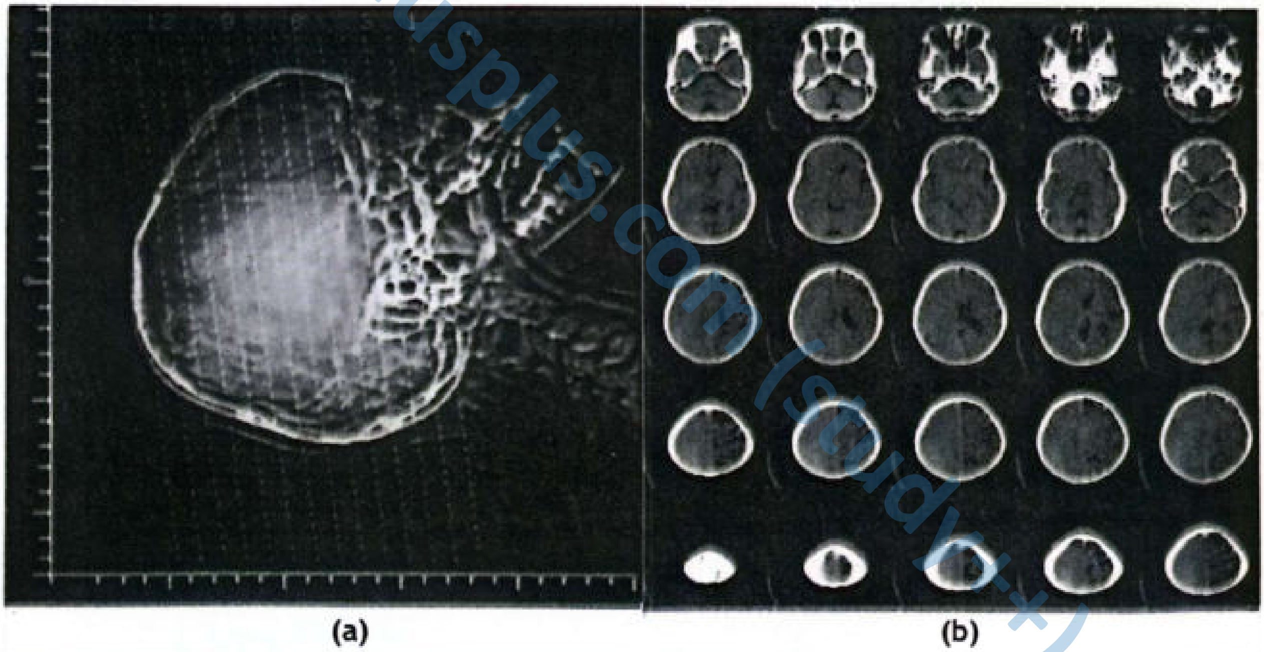


Figure 25.7: CT scan of brain (a) Dotted line shows the position of cross-section from where image is to be taken at different angles. (b) 2-D images of the different sections.

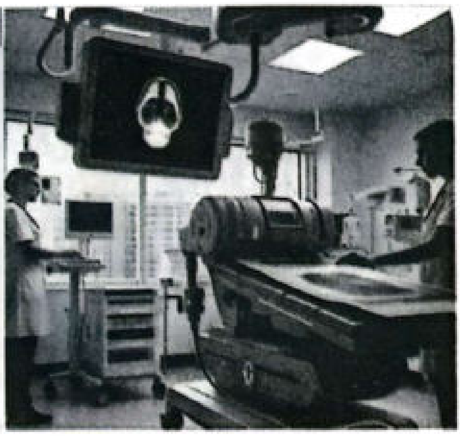
Importance of CT Scan Over Normal X-ray Film

CT scan gives a more detailed view than normal X-ray film. A normal X-ray film gives only limited information because it is like a shadow. Fine detail within the X-rays film may be invisible especially if one organ lies in front of the region of the body being studied. To give high-quality image CT scans are used. CT scanner helps in the study of the tumors in cancer patients where images of high quality are essential.

Medical Physics and Technology

Tumor detection involves various methods to identify abnormal cell growth or tumor presence. Some detection techniques include: imaging tests (such as X-rays, CT scans, MRI scans, PET scans, and ultrasound help visualize tumors), biopsy, blood tests, and physical examination. In physical examination doctors check for abnormalities, such as lumps or changes in skin.

Tumor detection is a complex process, and ongoing research aims to improve diagnostic accuracy and effectiveness. In future, AI-assisted imaging analysis and diagnostics would be used for tumor detection!



Example 25.1: When electrons are accelerated through a potential difference of 10^5 volts in an X -ray tube. Calculate the minimum wavelength produced.

Given: Potential difference: $V = 10^5$ volts

To Find: Minimum wavelength: $\lambda_{min} = ?$

Solution: We use the equation: $$\lambda_{min} = \frac{hc}{eV}$$

By putting values, we get: $$\lambda_{min} = \frac{(6.63\times10^{-34})(3\times10^{8})}{(1.60\times10^{-18})(10^{5})} = 1.24\times10^{-11}\ \text{m}$$

Assignment 25.1

What will be the minimum wavelength of x-rays produced, if the electrons were slowed down by the target. The potential difference across the x-ray tube is 4000 V.

SUMMARY

- In medical, **imaging** is a range of tests used to create images of internal body parts. There are many different types of imaging techniques used in modern medicine, such as ultrasound, X-rays and CT scans.
- **Piezoelectric effect** is the ability of certain materials to generate an electric charge in response to applied mechanical stress.
- The material can also generate stress when an electric field is applied. This is called **inverse piezoelectric effect.**
- The sound waves above 20,000 Hz are inaudible to normal ear and are called **ultrasound.**
- Due to transition of electrons between the shells in heavy atoms produce **characteristic x-rays.**
- An x-ray photon due to transition from L-shell to the vacancy in the K-shell is called **K_α characteristic x-rays.**

- An x-ray photon due to transition from M-shell to the vacancy in the K-shell is called K_β **characteristic x-rays.**
- An x-ray photon due to transition from N-shell to the vacancy in the K-shell is called K_γ **characteristic x-rays.**
- **Continuous x-rays** can also be produced by coulomb interaction of a fast-moving electron with orbital electrons as well as the positive nucleus.
- **Contrast in x-ray imaging** allows the radiologist to evaluate structures that are not clearly evident on conventional x-ray exams.
- A **CT scan (Computed Tomography scan)** is a medical imaging method that is useful to get a detailed 3-D image of certain parts of the body.

EXERCISE

Multiple Choice Questions

Encircle the Correct option.

1) Frequency of X-rays depends upon

   A. Number of electrons striking target B. Accelerating potential

   C. Nature of the target. D. Both B and C

2) Target material used in X-rays tube must have following properties:

   A. High atomic number and high melting point.

   B. High atomic number and low melting point.

   C. Low atomic number and low melting point.

   D. High atomic number only.

3) Which of the following is drawback while conducting a medical imaging test:

   A. Early detection of problem. B. Accurate diagnosis.

   C. Contribution to choose of effective treatment. D. Exposure to radiation.

4) In an experiment, high speed electrons emitted from the cathode are accelerated through a potential difference of 20 kV. Find the minimum wavelength of the X-rays produced. (take Planck's constant, $h = 6.6\times10^{-34}$ J s)

   A. 0.36782 A B. 0.61875 A C. 0.45867 A D. 0.72854 A

5) In an X-ray tube, electrons accelerated through a very high potential difference strike a metal target. If the potential difference is increased, the speed of the emitted X-rays:

   A. increases. B. decreases. C. is always equal to 3×10^{8} m s^{-1}.

   D. increases or decreases depending on the target material.

Short Questions

1) Differentiate between piezoelectric effect and inverse piezoelectric effect.
2) What is the distinction between a continuous X-ray and a characteristic X-ray?
3) What is the main purpose of X-rays?
4) X-rays can emit electrons from metal surface and X-rays can be diffracted. Comment.
5) Why X-rays have different properties from light even though both originate from orbital transition of electrons in excited atoms?
6) What is meant by breaking radiation?
7) What are K_β characteristic X-rays? Show with the help of diagram.

Comprehensive Questions

1) What is piezoelectric effect? How is it used to generate ultrasonic waves? Write its application in medical science.
2) How ultrasound can be used to obtain diagnostic information about internal body structures? Explain.
3) What are X-rays? Draw and explain the setup that produces X-rays.
4) Discuss the phenomenon of producing continuous X-rays. Also derive an expression for calculating the minimum wavelength of X-rays produced during the process.
5) Explain the use of X-rays in imaging internal body structures.
6) Explain how computed tomography (CT) scanning works?

Numerical Problems

1) If potential difference of 3000 V is applied across an X-ray tube. What will be the minimum wavelength of X-rays produced, if the electrons were slowed down by the target.

(Ans: 0.414 nm)

2) Find the voltage across the X-ray tube through which an electron must be accelerates in order to generate the continuous X-ray of exactly 0.1 nm. **(Ans: 12, 400 V)**

26 NATURE OF SCIENCE: A Debate

Student Learning Outcomes (SLOs)

The student will

Debates about Beauty in Physics:

- [SLO: P-12-G-01] Explain, with examples, what do thinkers who hold the view that there is inherent mathematical beauty in the natural world mean by: (i) elegance of simplicity (ii) symmetry.
- [SLO: P-12-G-02] Explain, with an example, a counterargument to the claim that physical truths must be inherently mathematically elegant or display symmetry.

Debates:

- [SLO: P-12-G-03] Describe the main pros and cons in the debate about: (i) whether humans should research whether there are aliens somewhere in the universe. (ii) whether research should continue on uncovering the secrets of subatomic particles, given the advent of nuclear weapons.

Thought experiments

- [SLO: P-11-G-04] Explain how the below thought experiments helped convey important physics concepts that Would have been impractical investigate empirically: (i) Newton's cannonball

The logic of shape, quantity, and order is the subject of mathematics. The use of mathematics in everything we do. In other words, it's everywhere. In everything the mathematics can be used on a daily basis; for examples: computers, software, hardware, art, money, engineering, sports, and even ancient and modern architecture is built around it.

A fascinating subject, mathematics includes formulas, branches, equations, and mathematical reasoning. Without realizing it, math's significance in daily life can be seen in a variety of ways. In daily duties, mathematics is essential, whether it is for gaming or computation. Whether we enjoy mathematics or not, it is a necessary part of daily life. As a result, we ought to acknowledge this situation and the importance of mathematics in day-to-day living.

26.1 ELEGANCE OF MATHEMATICS

Although mathematics is an extremely elegant field, understanding its elegance requires some knowledge. The beauty of mathematics is a topic that mathematicians like discussing. The synchronizations, patterns, and assemblies of numbers and forms, the traditional principles of symmetry and balance format, make the showcase of the beauty of mathematics.

The elegance of mathematics lies in how well everything fits together, unfortunately, the great majority of people who are frightened by a fear of difficulties with numbers sometimes fail to see its beauty. This is fixed by Mathematical Elegance. The author gives an interesting and approachable picture of the mathematical world through the use of hundreds of examples.

The natural sciences, engineering, health, economics, computer science, and social sciences all depend on mathematics. The fundamentals of mathematics are unaffected by scientific experimentation, despite the fact that they are often utilized to model a phenomenon. Logical reasoning and mental consistency are encouraged by mathematics, which also offers an efficient method of developing mental discipline.

Furthermore, a comprehension of mathematics is essential for understanding the material covered in other academic courses, including natural science, social studies, music, and even the arts. For the purpose of conveying relationships of any kind quantitatively, mathematics is a highly brief language.

Through the years, poets and authors have explored, with sometimes surprising, the link between the expressive richness of natural language and the thoroughness of mathematics.

Artists have used mathematics to create bold analogies and set tight parameters within which they can express their creativity. For example; Jorge Luis Borges, who is inspired by mathematical reasons in several of his works, like the "An Aleph is a point in space that encompasses all other points, offering a unified perspective where every location on Earth can be seen simultaneously from all angles, without spatial confusion or limitation".

The Magic of Mathematics

The majority of people may associate mathematics with intricate equations, chalkboards covered with numbers, and sometimes even recollections of past tests. Mathematics is a universal language that bridges the vast scales of the universe, describing the intricate patterns of a seashell and the majestic spirals of distant galaxies with equal precision and beauty.

Mathematics is the thread that connects everything in the universe, from the fascinating patterns of the Fibonacci sequence seen in sunflowers and pinecones to the basic ideas underlying the technological advancements we use on a daily basis.

Numbers are not the only thing in mathematics. It is the fundamental component of invention as well as the language of the cosmos and the beat of nature. You are experiencing the amazing magic of mathematics every time you are surprised by the design of a building, listen to a beautiful melody, or just esteem the symmetry of a snowfall.

Mathematicians behave like Poet; who conveys a wide spectrum of human experiences and emotions with the help of flowers' delicate petals, brilliant colours, and perfumed tracks. Similarly universal equation is very much similar to flower where different parameters, numbers play role like petals, colour and even perfume to design equation, for example:

The Euler's Equation;

$$e^{i\pi} + 1 = 0$$

Euler's identity shows that geometry, calculus, and complex numbers are intimately related. Everybody knows the value of π, which represents the ratio of a circle's diameter to its circumference. Scholars have researched this universal mathematical constant for millennia. It is fundamental to geometry and can be found everywhere. Another number that is at least as significant as π is e, which is approximately 2.718281. It is used to describe how something can increase or decrease in proportion to itself, which is a fundamental idea in calculus, and it plays a special role in the definition of exponentiation. Indeed, 'e' holds the same significance in calculus as π does in geometry.

Finally, there is '*i*', the hypothetical unit. In contrast to 'e' and 'π', '*i*' is not a number. It is absent from the standard number line. It is essentially a fictitious number: Although it was long since established that negative numbers had no square roots, someone questioned whether or not this would still hold true in practice. It turns out that complex numbers—numbers with both real and imaginary components—are produced when square roots of -1 are allowed. *i*, which is the positive square root of -1, is the fundamental unit of analysis in sophisticated mathematics.

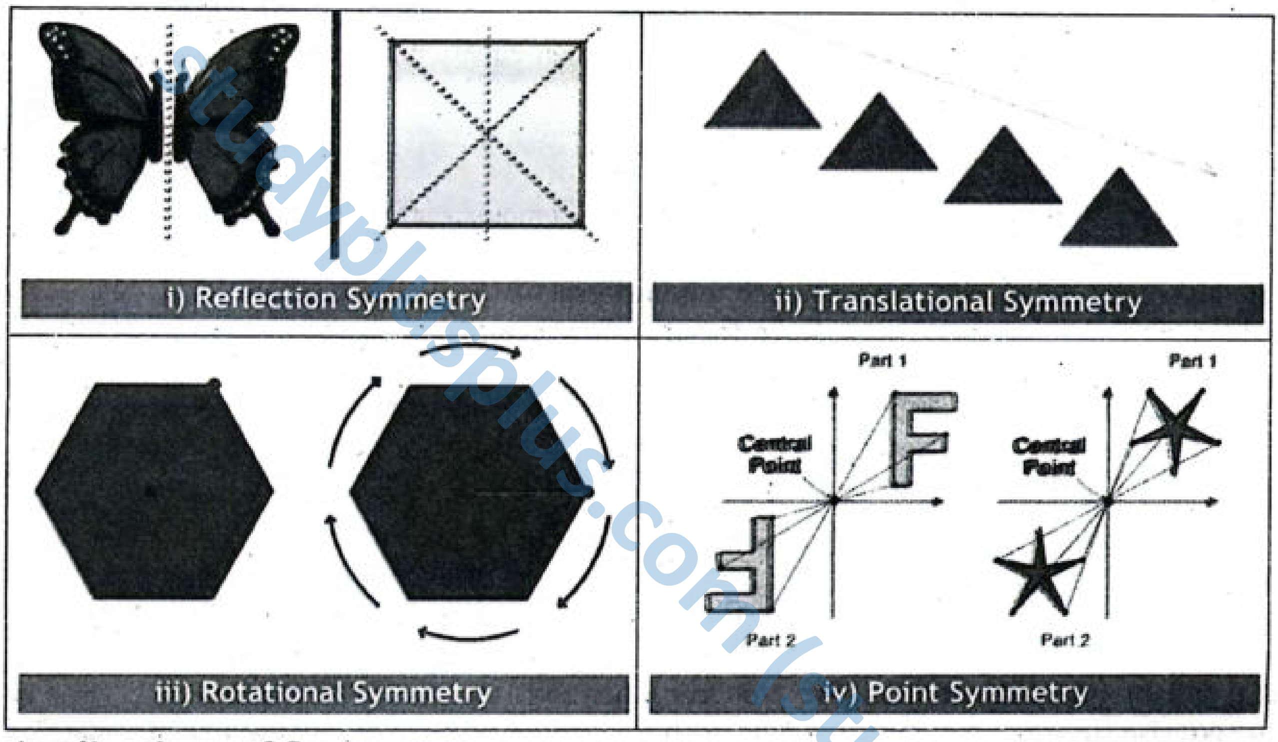
Indeed, 'e' holds the same significance in calculus as π does in geometry. π, e, *i*. the three integers that matter most in mathematics. They all appear to be quite unique, don't they? Throughout thousands of years of mathematical history, various individuals have studied them, they are all abstract, and they appear to have no clear connection to one another. They are also derived from quite distinct areas of mathematics. Complex numbers, geometry, and calculus are closely connected, as demonstrated by Euler's identity. After all, they may be explained in terms of one another, therefore they are not really distinct domains. Complex numbers can be thought of as the link between circular motion and arithmetic: The arithmetic of complex numbers provides information on angles, rotations, cycles, and periods, whereas the arithmetic of real numbers informs us about quantities.

Symmetry

In mathematics, symmetry is the balance and proportion that an object or geometric figure displays when one half reflects the other. It happens when specific points, lines, or forms perfectly match one another along a given axis. Group theory is an important area of algebra with applications in computer graphics, chemistry, and crystallography. The study of symmetry is essential to comprehend group theory.

People typically associate symmetry with reflecting a picture over a single plane to reveal the exact identical image, much like when they look in a mirror. However, that is merely one kind of symmetry. There are lots of alternative options. Rotational symmetry, for example, explains that an object's attributes remain unchanged as it is rotated around an axis, such as the Earth's orbit.

The word symmetry can be studied in the following four different phases;



Applications of Symmetry

Not only does symmetry exist in abstract mathematical ideas, but it also infuses everyday life and effects on many aspects of human existence.

Architecture: Structurally sound and aesthetically pleasing buildings are the result of architects using symmetry.

Art and Design: Artists frequently utilize symmetry in their works of art to attain harmony and balance.

Nature: The radial symmetry of starfish and the hexagonal symmetry of honeycombs are only two examples of the many examples of symmetrical wonders found in nature.

Music: Symmetry is a tool used by composers of melodies and harmonies to arouse feelings and produce enjoyable aural experiences.

The world in which we live is three-dimensional. While there are many advantages to this, according to Ivan Loseu, a mathematics associate professor at Northeastern University; it also makes for a challenging math problem. He claimed that minimizing the number of variables at play is the best route to a solution. Imagine that the Earth is rotating around the sun. Given that the motion takes place in three dimensions, the Earth's position can only be described by three variables. Given that the Earth never deviates from a specific plane, Newtonian physics allows us to further decrease the number of variables to only two. One variable is actually preferable to two, though. This is the reason why physicists track the Earth's elliptical journey using the characteristics of gravitational force.

According to Loseu,

The formula for gravity depends only on the distance between the sun and Earth. You can change the image, but the physical law doesn't change.

He said that symmetry is the single most important factor explaining why this problem can be resolved with just one variable.

According to Loseu, symmetry is any transformation that keeps your issue intact.

What Scientists say about Mathematics?

Pure mathematics is, in its way, the poetry of logical ideas.

(Albert Einstein, German theoretical physicist)

We will always have STEM with us. Some things will drop out of the public eye and go away, but there will always be science, engineering, and technology. And there will always, always be mathematics.

(Katherine Johnson, African-American mathematician)

Mathematics as an expression of the human mind reflects the active will, the contemplative reason, and the desire for aesthetic perfection. Its basic elements are logic and intuition, analysis and construction, generality and individuality.

(Richard Courant, German-American mathematician)

It is impossible to be a mathematician without being a poet in soul.

(Sofia Kovalevskaya, Russian mathematician)

Elegance of Mathematical Poetry

Pythagoras' Theorem

This equation represents the relationship between a right-angled triangle's sides, is a fundamental concept in geometry. Its simplicity and universal validity are reflected in rocket flight, building design, and a myriad of other uses.

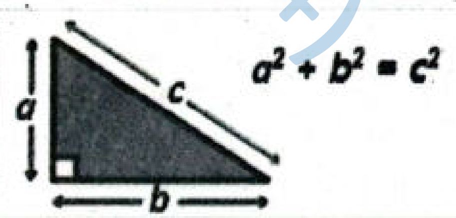


Figure 26.1: Pythagoras' Theorem.

Mass-Energy Equivalence

It is arguably the most well-known equation in physics, connecting mass (m), energy (E), and the speed of light (c). It reveals a significant truth about the inter convertibility of mass and energy with beauty and conciseness.

$$\underbrace{E}_{\text{energy}} = \underbrace{m}_{\text{mass}}\, c^{\overset{\text{squared}}{2}}$$

speed of light (constant)

Figure 26.2: Mass-Energy Equivalence.

Planck-Einstein Relation

A fundamental formula in quantum physics, the Planck-Einstein relation (E=hv), states that a quantum of energy (E), sometimes known as a photon, is equal to the Planck constant (h) times the oscillation frequency of an atomic oscillator. Planck's equation quantized energy is employed to liberate electrons from the photovoltaic material, resulting in the creation of an electric current.

$$E = h\nu = \frac{hc}{\lambda}$$

Figure 26.3: Planck-Einstein Relation.

Euler's Identity

This equation establishes an amazingly simple relationship between five of the most important numbers in mathematics: 0, 1, π, e, and i. It unites disparate mathematical domains in harmony, bridging them like a moving poem.

$$e^{i\pi} + 1 = 0$$

Figure 26.4: Euler's Identity.

26.2 ARE ALIENS REAL?

The prospect of life beyond Earth has long fascinated human curiosity. Since man discovered that Mars and other surrounding planets were distinct from other points of light in the night sky, people have speculated about the possibility of otherworldly life. The question of whether we are alone in the universe remains unanswered even though modern technology has made it possible for us to investigate those worlds up close and even look at (and hear from) planets orbiting other stars.

A very interesting comment on existences of aliens or aloneness of Earth;

> Two possibilities exist: either we are alone in the Universe or we are not. Both are equally terrifying.
>
> (Arthur C. Clarke, science fiction author and former Planetary Society board member)

Many people have an alien belief; however, this idea is mostly based on movies and reports of UFOs (unidentified flying objects). However, science sheds light on the possibility that we are not alone in the universe and can address some of the most pressing concerns regarding the nature of life that is most likely to exist.

Where might aliens exist?

There's a chance that life from beyond the solar system exists.

The comprehensive Mars exploration program of NASA (National Aeronautics and Space Administration) and other space organizations have taught us that the red planet was once a warm, wet planet that may have supported life. Even though scientists haven't discovered any evidence of it yet, tiny life might yet be present there. Given that, several of Jupiter's and Saturn's moons have liquid water beneath their ice crusts, there is a chance that these bodies could support otherworldly life. Certain frozen worlds in the outer solar system, such as Europa on Jupiter and Enceladus on Saturn, appear to have potentially functional underground waters. And that's the only thing found in our solar system. The number of divergent settings that potentially support life is increasing as scientists discover additional exoplanets around other stars and that is the thinking domain of future research for space seeker. According to Carl Sagan (American astronomer) point of view;

There is a lot of space in the universe. If we're alone, it feels like a terrible use of space. So, NASA will continue its search.

Main pros and cons in the debate about:

"Should Humans Research the Existence of Aliens?"

Pros:

- **Expanding Human Knowledge:** Researching the existence of aliens can lead to a deeper understanding of the universe, its origins, and the possibility of life beyond Earth.
- **Potential Benefits:** Discovering alien life could lead to new technologies, resources, and insights that could benefit humanity.
- **Inspiring Future Generations:** The search for extraterrestrial life can inspire young people to pursue careers in science, technology, engineering, and mathematics (STEM).

Cons:

- **Resource Allocation:** Researching alien life requires significant funding and resources, which could be allocated to more pressing problems on Earth, such as poverty, disease, and climate change.
- **Low Probability of Success:** The likelihood of finding alien life is currently unknown, and some argue that the probability of success is too low to justify the investment.
- **Risk of Unintended Consequences:** If we were to encounter alien life, there is a risk of unintended consequences, such as contamination or conflict.

Main pros and cons in the debate about:

"Should Research Continue on Subatomic Particles?"

Pros:

- **Advancing Fundamental Knowledge:** Researching subatomic particles can lead to a deeper understanding of the fundamental laws of physics and the nature of matter.
- **Potential Applications:** Discoveries in particle physics can lead to breakthroughs in fields like medicine, energy, and materials science.

- **Inspiring Innovation:** The pursuit of knowledge in particle physics can drive innovation and lead to new technologies.

Cons:

- **Nuclear Weapons:** The discovery of subatomic particles and the development of nuclear physics led to the creation of nuclear weapons, which pose a significant threat to global security.
- **Ethical Concerns:** Some argue that continuing research in particle physics could lead to new, potentially devastating technologies.
- **Opportunity Cost:** The significant resources required for particle physics research could be allocated to other areas of science or pressing global problems.

This debate highlights the complex trade-offs and ethical considerations involved in pursuing scientific knowledge.

26.3 THOUGHT EXPERIMENT: NEWTON'S CANNONBALL

It was common knowledge that objects had a tendency to fall to the ground, but no one was sure why. Yes, the term "gravitas," which simply means "having weight" or gravity, was applied to this unexplained effect. It was Newton who explained gravity, not who discovered it. Newton provided a clear mathematical law that could be tested to explain all of these occurrences and tie them together in a straightforward manner. The apple and the Moon are both connected to the Earth by gravity. The Moon moves sideways, which accounts for their different motion and the reason it doesn't land on Earth.

There were following two curiosity points of Newton's research domain;

1. Why doesn't the Moon fall from the sky and land on Earth, given that all objects experience gravitational acceleration towards Earth?
2. Why does the Moon move in a circle instead of a straight line, if, as Galileo said, objects move with constant speed and direction until acted upon by an external force?

Figure 26.5: Newton's Orbital Cannon.

Newton understood; due to the combination of its orbital motion and inertia, the Moon does not fall perfectly to Earth, nor does it fly away due to the acceleration Earthward that it experiences.

Newton devised a designed experiment in which cannon was positioned on a towering mountain and shot horizontally, as shown in Fig. 26.5. He was aware that the cannonball's horizontally travel distance in a time interval (t) could be calculated by multiplying its speed (v) by the interval like; horizontal distance covered by cannon ball can be measured as;

$$X = vt$$

While the vertical distance covered by cannon can be measured as;

$$Y = \frac{1}{2}gt^2$$

Newton therefore understood that, if he selected the ideal velocity, the cannonball's trajectory would curve at precisely the same rate as the Earth's, as the Earth is spherical, and the projectile would always remain at the same height above the earth.

By doing this, he strikes a balance between the cannonball's inertia, which causes it to desire to keep going in a straight path and away from the Earth, and the acceleration brought on by the Earth's gravity, which causes the cannonball to move towards the Earth's centre. As a result, the projectile orbits the planet, never getting any closer to it yet constantly increasing in speed, though acceleration is the change in velocity i.e., the change in both the speed and direction of an object. The projectile in this instance accelerates even if its speed is constant since its direction is changing. Newton discovered that the acceleration caused by Earth's gravity (a) and the orbit's radius (r), which is measured from the orbit's centre (i.e., the Earth's centre), were connected to the cannonball's speed in the following ways:

$$\text{Centripetal acceleration} = a = v^2/r$$

Newton discovered it for a cannonball orbiting the Earth, but it holds true for any circularly moving object as well. Because of inertia, objects always prefer to move in straight lines; some sort of acceleration is required to cause them to bend into circular motion. The acceleration for Newton's cannonball came from the Earth.

Results

Newton found the following results:

1) If velocity of cannon ball is less than the orbital velocity ball will fall on the Earth.

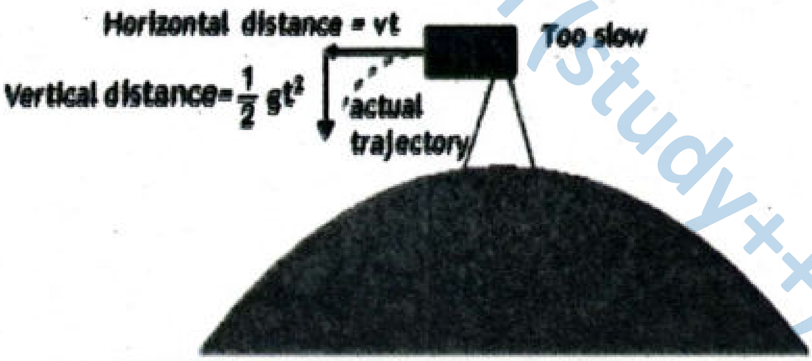


Figure 26.6: Velocity of cannon ball is less than the orbital velocity ball.

2) If velocity of cannon ball is equal to the orbital velocity ball will stay in orbit.

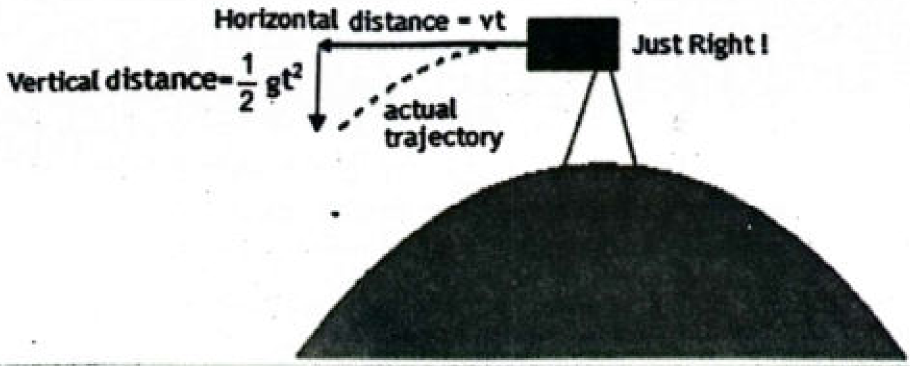


Figure 26.7: Velocity of cannon ball is equal the orbital velocity ball.

3) If velocity of cannon ball is greater than the orbital velocity ball will escape from the Earth.

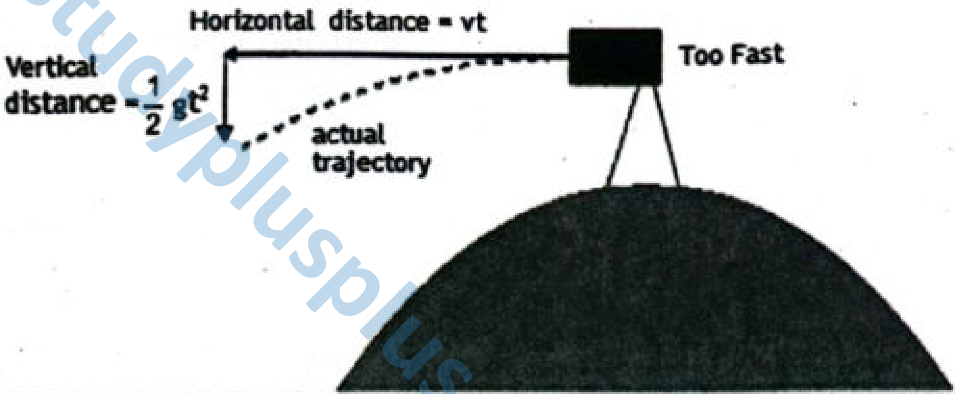


Figure 26.8: Velocity of cannon ball is greater than the orbital velocity ball.

Glossary

Absolute potential energy: The amount of work done in moving a body from Earth's surface to a point at infinite distance where the value of g is negligible.

Acoustic levitation: A method for suspending matter in air against gravity using acoustic pressure from high intensity sound waves.

Aerosol: A suspension of particles or droplets in the air and includes dust, mist, fumes or smoke.

Alpha decay: An atomic nucleus emits an alpha particle.

Amplitude: The maximum displacement of a vibrating body from mean position.

Artificial satellites: objects intentionally placed into orbit around the Earth or other celestial bodies.

Atomic mass: The number of protons and neutrons in the nucleus.

Atomic number: The number of protons in the nucleus.

Beta decay: An atomic nucleus emits a beta particle.

Big Bang theory: The whole Universe expanded outwards from this tiny point to what exists today.

Binding energy per nucleon: Total binding energy of an atom divided by the number of nucleons.

Binding energy: The minimum energy required to break an isolated nuclei into its constituent particles.

Bioelectricity: the generation or action of electric currents or voltages in biological processes.

Biosphere: Thin life supporting layer of Earth's surface. It extends from a few kilometers into the atmosphere to the deep-sea vents in the ocean.

Black body: An idealized object which is perfectly opaque and non-reflecting.

Bose-Einstein Condensation (BEC): A state of matter that forms at extremely low temperatures, such as close to absolute zero, causing particles kinetic energies to decrease significantly.

Butterfly Effect: The metaphor of a butterfly flapping its wings in one part of the world can lead to a hurricane forming in another part of world.

Capacitance: The capability of a capacitor to store charges.

Capacitor: An electric-devices used to store electric energy.

Carbon cycle: Lithosphere plays a major role in carbon cycle which is essential for regulating Earth's climate.

Cepheid variable stars: An example of standard candles.

Chaotic system: A system in which there is no equilibrium and no repeatable patterns emerge it exhibits extreme sensitivity to initial conditions.

Characteristic x-rays: X-rays produced due to transition of electrons between the shells in heavy atoms.

Choke: An inductor used in a circuit.

Climate inertia: The tendency of the Earth's climate to resist or respond slowly to changes in external factors like greenhouse gas concentrations and solar radiations.

Coherent Sources: Waves that have the same frequency and a constant phase relationship are considered coherent. This means the crests and troughs of the waves occur at the same time.

Constructive Interference: When the crests (peaks) of two or more waves overlap, they reinforce each other, producing a resultant wave with a higher intensity (brighter light, louder sound) compared to the individual waves.

Continuous x-rays: X-ray produced due to coulomb interaction of a fast-moving electron with orbital electrons or the positive nucleus.

Control Rods: Rods that are used to control the extent of reaction and do not allow the fuel to react at once.

Coriolis Force: A force resulting from the Earth's rotation.

Count Rate: The number of decays detected per unit time.

Critical Damping: When an oscillator comes to rest without any oscillation in the shortest time under damping force.

Critical Mass: The minimum mass of the material to sustain the fission chain reaction.

Critical temperature or superconducting transition temperature: The temperature below which the resistivity of substance become zero.

Critical volume: The volume occupied by the critical mass of a material.

Cryosphere: Any place on Earth where water is in its solid form.

CT scan: A medical imaging method used to get a detailed 3-D image of certain body parts.

Curie: Unit of activity.

Cycle: One complete set of positive and negative values of an alternating quantity.

Damped oscillations: Oscillations whose amplitude decreases under damping forces.

Degenerate matter: A state of matter where particles are so densely packed that quantum mechanical effects dominate over classical mechanics. This typically occurs in extremely high-pressure environments, such as the cores of massive stars like white dwarfs, neutron stars.

Destructive Interference: When the crest of one wave coincides with the trough of another wave, they cancel each other out partially or completely.

Diffraction Grating: A periodic structure with many closely spaced parallel slits or grooves.

Diffraction Pattern: The spatial distribution of intensity observed after a wave diffracts around an obstacle or through a slit. This pattern typically consists of alternating bright and dark bands due to constructive and destructive interference of the diffracted waves.

Diffraction: The bending of a wave around the edges of an obstacle or through a narrow slit.

Earth's Climate System: A complex system with five interacting components the atmosphere, the hydrosphere, the cryosphere, the lithosphere and the biosphere.

Electron microscope: A device that is based on the wave characteristics of electron.

Electroreception: ability of some animals to detect weak naturally occurring electrostatic fields in the environment.

EMF: energy supplied by a battery per unit charge

Energy imbalance: The uneven distribution of solar energy across the Earth's surface creates an energy imbalance.

Equilibrium position: position where the spring is neither stretched nor compressed.

Exosphere: The outer most layer merging into space is the exosphere which extends from 600 km up to merging into the space.

Farad: unit of capacitance.

Filtering: In a rectifier circuit, a capacitor smooths out the pulsating direct current into a more stable, constant output.

Fission chain reaction: A fission reaction in which every time at least one released neutron goes further fission.

Fission: A process where the nucleus of an atom splits into two or more smaller nuclei, some particles like neutron and a large amount of energy.

Force constant (k): Characteristic of a spring which is defined as the ratio of the force applied to the spring to the displacement caused by the force.

Forced oscillations: oscillations under the influence any external force.

Free oscillation: Oscillates under the influence of restoring force without any external force acting on it.

Frequency: The number of vibrations or oscillations per unit time.

Fringe Spacing: The distance between two consecutive bright and dark fringes.

Full wave rectification: the complete cycle of AC signal is converted into pulsating DC.

Fusion: A process where two or more light atomic nuclei come close to each other to form a heavier nucleus by releasing a large amount of energy.

Gamma decay: When an atomic nucleus transitions from a higher energy state to a lower energy state, emitting a gamma ray in the process.

Gas centrifuge method: A method uses rapidly spinning centrifuges to separate uranium isotopes based on difference in their masses.

Gas diffusion method: A method used for enrichment of uranium in which the gas is allowed to diffuse from a porous wall.

Geostationary Satellites: satellite that remain stationary above some point on Earth.

Grating Constant (d): The distance between the centers of two adjacent slits (or grooves) in a diffraction grating.

Gravitational field strength: It is a measure of the force exerted by gravity on a unit mass at a certain point in space.

Half-life: The time it takes for half of the radioactive nuclei to decay.

Half-wave rectification: process by which one half-cycle of the AC signal is converted into DC and blocking the second-half

Heisenberg uncertainty: It is impossible to accurately define both, the position and momentum of an object simultaneously.

henry (H): unit of inductance.

Hubble's Law: The recession speed of galaxies moving away from Earth is proportional to their distance from the Earth.

Hydrosphere: All the water on the Earth whether it is in liquid water, solid ice or the gaseous water vapors form.

Hydrostatic equilibrium: The net outward pressure in the core of stars that is counterbalanced by gravity.

Impedance: combine effect of the resistance and the reactance in an AC circuit.

In phase: When the phase difference between two oscillating systems is 0° or 360°.

Interference: The interaction of two or more waves propagating through the same medium, resulting in a superposition effect that can lead to constructive or destructive interference.

Inverse piezoelectric effect: material generate stress when an electric field is applied.

Isotopes: Atoms of same element which have different mass but same atomic number.

K_α characteristic x-rays: x-ray produced due to transition from L-shell to the vacancy in the K-shell.

K_β characteristic x-rays: x-ray produced due to transition from M-shell to the vacancy in the K-shell.

K_γ characteristic x-rays: x-ray produced due to transition from N-shell to the vacancy in the K-shell.

Light Damping: The damping is said to be light when the amplitude of oscillations decreases gradually with time.

Lithosphere: The outermost solid shell of the Earth consisting of the crust and the uppermost part of the mental. It composed of various types of rocks and minerals.

Long-term Inertia: Changes in greenhouse gas concentrations like carbon dioxide take decades to centuries to fully impact the climate system.

Luminosity: Measure of the total power output of radiation emitted by a star.

Mass defect: The difference between the mass of the nucleus and sum of the masses of its constituent particles.

Medical Imaging: Range of tests used to create images of internal body parts.

Mesosphere: Above the stratosphere from 50 km to 136 km (85 miles) is the mesosphere.

Moderator: Material that slows down the neutrons in a reactor.

Monochromatic Waves: The waves having a single wavelength.

Mutual induction: Changing current in one coil induces an emf in neighboring coil.

Newton's law of universal gravitation: Every object in the universe attracts every other object with a force which is directly proportional to the product of their masses and inversely proportional to the square of the distance between their centres.

Nuclear physics: A branch of physics that deals with the study of the atomic nucleus, its properties and interactions.

Nuclear reactor: A device used to initiate and control a nuclear fission chain reaction.

Nutrient cycles: Elements like carbon (C), nitrogen (N_2) and phosphorous (P) cycle through the biosphere impacting both living organisms and the environment.

Ocean currents: The streams of water that flow constantly on the ocean surface in definite directions.

Orbital velocity: constant tangential speed of satellite in orbit.

Oscillation or vibration: A complete round trip of an oscillating or vibrating body about the mean position.

Oscillatory or vibratory motion: A body moves to and fro about its mean position.

Out of phase: When the phase difference between two oscillating systems is 180°.

Pair annihilation: A process in which an electron-positron pair produces two photons.

Pair production: When a high energy photon interact with matter photon energy is changed into an electron-positron pair.

Parallel plate capacitor: Two identical conducting plates separated by a distance.

Pauli Exclusion Principle: Two electrons cannot occupy same quantum state.

Peak value: Maximum value of alternating quantity.

Permafrost: Permanently frozen ground found mainly in high altitude regions. It contains ice and soil that remain frozen for extended periods.

PET scanner: A powerful imaging technique that allows us for detailed observation of metabolic physiological processes within the body.

Phase difference: The fraction of a period difference between the peaks expressed in degrees.

Phase of the motion: The angle which specifies the displacement as well as the direction of a point executing SHM.

Photoelectric effect: is the process of emitting the electrons from the metal surface when the metal surface is exposed to an electromagnetic radiation of sufficiently high frequency.

Photoelectrons: The electrons which are emitted from a metal surface upon the influence of light.

Piezoelectric effect: Ability of certain materials to generate an electric charge in response to applied mechanical stress.

Potential gradient: Rate of change of potential along with displacement.

Precipitation: Any liquid or frozen water that forms the atmosphere and falls back to the Earth.

Predictability: unpredictable changes arising from dynamic interactions within the climate system rather than being directly caused by external factors.

Pressure gradient: The change in pressure measured across a given distance.

Pressure of gas: The pressure exerted by a gas molecule is a measure of the force exerted by gas molecules per unit area as they collide with the walls of their container.

Quanta: The smallest amount of energy that can be emitted or absorbed in the form of electromagnetic radiation.

r.m.s value: value of an alternating current is that steady current (d.c.) which when flowing through a resistor produce the same amount of heat as that produced by the alternating current when flowing through the same resistance for the same time.

Radiant flux intensity: Luminosity per unit area measured on the surface of the Earth.

Radioactivity: The natural process of emission of radiations from unstable nuclei.

Rectification: process of converting ac voltage into dc voltage.

Redshift: the fractional increase in wavelength (or decrease in frequency) due to the source and observer receding from each other.

Resonance: The phenomena in which the amplitude of vibration of an oscillator attains maximum value when deriving frequency equals the natural frequency of oscillator.

Root Mean Square (rms) Speed: The square root of the mean square speed of the gas molecules.

Rubens tube: A tube used for demonstrating acoustic standing waves.

Satellite: any object that orbits another object due to the force of gravity, maintaining a stable path around it.

Self-inductance: The phenomenon of changing current induces an emf in the coil itself.

Short-term inertia: The ocean and land surfaces absorb and release heat relatively slowly leading to short term inertia this is why daily temperature fluctuations occur.

Simple harmonic motion: Oscillatory motion in which acceleration of the particle is directly proportional to its displacement from the mean position and is always directed towards the mean position.

Simple pendulum: A small but heavy bob of mass m which is suspended by a light and inextensible string

Spontaneous decay: The natural process by which an unstable atomic nucleus breaks down into a more stable configuration by emitting radiation.

Standard candle: An astronomical object which has a known luminosity due to a characteristic quality possessed by that class of object.

Statistical mechanics: A branch of physics that connects the microscopic details of a system (such as motion, energy, and the interaction of individual particles) with the macroscopic observables measure (such as temperature, pressure, volume, and entropy).

Stefan-Boltzmann Law: The total energy emitted by a black body per unit area per second is proportional to the fourth power of the absolute temperature of the body.

Stopping potential or Cut off potential: In the photoelectric experiments, at certain negative voltage at which the current becomes zero.

Stratosphere: Above the troposphere from 16 kilometers to 50 kilometers (31 miles) it is called the stratosphere.

Subtropical Gyres: Large rotating currents that start near the equator are called subtropical gyres.

Super conductivity: In certain conditions, Bose-Einstein Condensation leads to super-conductivity, where electrical resistance drops to zero, allowing current to flow without resistance.

Super fluidity: A notable property of Bose-Einstein Condensation, where condensate shows zero viscosity, allowing it to flow without resistance.

Superconducting transition temperature or critical temperature: The temperature below which the resistivity of substance become zero.

Superconductors: Those substances whose resistivity become zero at very low temperatures.

Tectonic Activity: Processes like tectonics which involves the movement and interaction of lithosphere plates can have significant impact on climate over a large timescale.

Thermohaline circulation: The deep ocean currents are caused by differences in water density.

Thermosphere: Above the mesosphere from 136 kilometers up to about 600 kilometers (372 miles) is the thermosphere.

Threshold frequency: The threshold frequency is the lowest frequency of electromagnetic radiation that will produce a photoelectric effect in a material.

Threshold wavelength: The Threshold wavelength is the largest possible wavelength of the incident radiation which allows photoemission to take place.

Time Period: The time taken to complete one oscillation or vibration.

Troposphere: The lowest layer extending from the ground (the surface of the Earth) to about 16 kilometers (10 miles).

Type 1A supernovae: An example of standard candles.

Ultrasound: Sound waves above 20,000 Hz.

Uranium Enrichment: The process of converting non fissile material like uranium-238 into a fissile material like uranium-235.

Van Der Waals Equation: Equation that describe the behavior of real gases, but it can also be used for ideal gases as well.

Vibration or oscillation: A complete round trip of an oscillating or vibrating body about the mean position.

Vibratory or oscillatory motion: A body moves to and fro about its mean position.

Volcanic eruption: A tectonic activity that release gases and particles into the atmosphere.

Volt: unit of electric potential/e.m.f.

Voltage: The potential difference across a cell, electrical supply or electrical component.

Wave front: A surface connecting points in a wave that are in the same phase (at the same point in their cycle).

Wavelength (λ): The distance between two consecutive crests (or troughs) of a wave.

Wien's displacement law: The black body radiation curve for different temperatures peaks at a wavelength, which is inversely proportional to the temperature.

Work Function: The minimum amount of energy required to escape the electron from metal surface.

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