Based on National Curriculum of Pakistan 2022-23

# Model Textbook of

# Physics

## Grade

## 11



**National Curriculum Council**
Ministry of Federal Education and Professional Training

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**A Model Textbook of Physics for Grade 11**
based on National Curriculum of Pakistan (NCP) 2022-23

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and promotional purposes, benefiting the public interest.

# Preface

This Model Textbook for Physics Grade 11 has been developed by NBF according to the National Curriculum of Pakistan 2022-2023. The aim of this textbook is to enhance learning abilities through inculcation of logical thinking in learners, and to develop higher order thinking processes by systematically building the foundation of learning from the previous grades. A key emphasis of the present textbook is creating real life linkage of the concepts and methods introduced. This approach was devised with the intent of enabling students to solve daily life problems as they grow up in the learning curve and also to fully grasp the conceptual basis that will be built in subsequent grades.

After amalgamation of the efforts of experts and experienced authors, this book was reviewed and finalized after extensive reviews by professional educationists. Efforts were made to make the contents student friendly and to develop the concepts in interesting ways.

The National Book Foundation is always striving for improvement in the quality of its textbooks. The present textbook features an improved design, better illustration and interesting activities relating to real life to make it attractive for young learners. However, there is always room for improvement, the suggestions and feedback of students, teachers and the community are most welcome for further enriching the subsequent editions of this textbook.

May Allah guide and help us (Ameen).



Dr. Kamran Jahangir
Managing Director

# Practical Applications of Physics-XI in Everyday Life.

Physics is the foundation of our understanding of the natural world, therefore learning physics empowers students to:

- Comprehend the underlying principles governing our universe.
- Develop critical thinking and problem-solving skills.
- Foster curiosity, creativity, and innovation.
- Address real-world challenges and technological advancements.
- Prepare for innovative careers in science, technology, engineering, and mathematics (STEM).

Unit wise relevance of this book to our natural world is briefly given below:

**Unit 1: Physical Quantities and Measurements**

Understanding physical quantities and measurements is crucial in everyday life. Accurate measurements are essential in medicine, engineering, and architecture. This unit lays the foundation for problem-solving and critical thinking, skills vital in professions like science, technology, engineering, and mathematics (STEM).

**Unit 2: Vectors**

Vectors play a significant role in navigation systems (GPS), video games, and weather forecasting. Mastering vectors helps engineers design safer buildings, optimize traffic flow, and predict natural disasters. You'll see vectors in action in fields like aviation, oceanography, and computer graphics.

**Unit 3: Translatory Motion**

Translatory motion principles govern vehicle movement, projectile trajectories, and sports performance. This unit's concepts are applied in transportation systems, aerospace engineering, and athletic training. Understanding translatory motion enhances safety, efficiency, and innovation.

**Unit 4: Rotational and Circular Motion**

Rotational motion is integral to machinery, gears, and engines. This unit's concepts are crucial in designing amusement park rides, bicycle gears, and satellite orbits. You'll find applications in mechanical engineering, robotics, and renewable energy systems.

**Unit 5: Work and Kinetic Energy**

Understanding work and kinetic energy helps optimize energy consumption in industries like manufacturing, transportation, power generation and construction. This unit's principles are applied in designing more efficient machines and renewable energy systems. and

**Unit 6: Fluid Mechanics**

Fluid mechanics governs water supply systems, ocean currents, blood circulation and atmospheric circulation. Mastering fluid mechanics improves irrigation systems, water treatment processes, and weather forecasting. Applications extend to aviation, chemical engineering, naval architecture, and environmental science.

**Unit 7: Physics of Solids**

The physics of solids underlies material science, structural engineering, and architecture. This Unit's concepts help develop stronger, lighter materials for construction, aerospace, and biomedical applications.

**Unit 8: Heat and Thermodynamics**

Thermodynamics principles govern heating and cooling systems, refrigeration, engines. Understanding heat transfer enhances energy efficiency in buildings, industries, and transportation systems.

**Unit 9: Waves**

Wave phenomena are essential in music, telecommunications, and medical imaging. This unit's concepts are applied in sonar technology, wireless communication, and radar sensation.

**Unit 10: Electrostatics**

Electrostatics principles govern lightning protection, static electricity safety, and high-voltage transmission. Mastering electrostatics enhances electrical engineering, materials science, and nanotechnology.

**Unit 11: Electricity**

Electricity powers our daily lives. This unit's concepts underlie electrical circuits, electronic devices, and power distribution systems. Understanding electricity enables innovation in fields like renewable energy, electronics, and telecommunications.

**Unit 12: Magnetism**

Magnetism is crucial in electric motors, generators, and medical imaging (MRI). This unit's principles are applied in materials science, electromagnetism, and advanced technologies like magnetic levitation trains.

**Unit 13: Relativity**

Einstein's relativity revolutionized our understanding of space, time, and gravity. Relativity's implications extend to GPS technology, particle physics, and cosmology, inspiring breakthroughs in fields like astrophysics and quantum mechanics.

**Unit 14: Particle Physics**

Particle physics reveals the fundamental nature of matter and energy. This unit's concepts underlie cutting-edge technologies like particle accelerators, quantum computing, medical imaging, and radiation therapy.

By learning these fundamental physics concepts, you'll gain a deeper understanding of the world around you and develop problem-solving skills essential for innovative careers in science, technology, engineering, and mathematics (STEM).

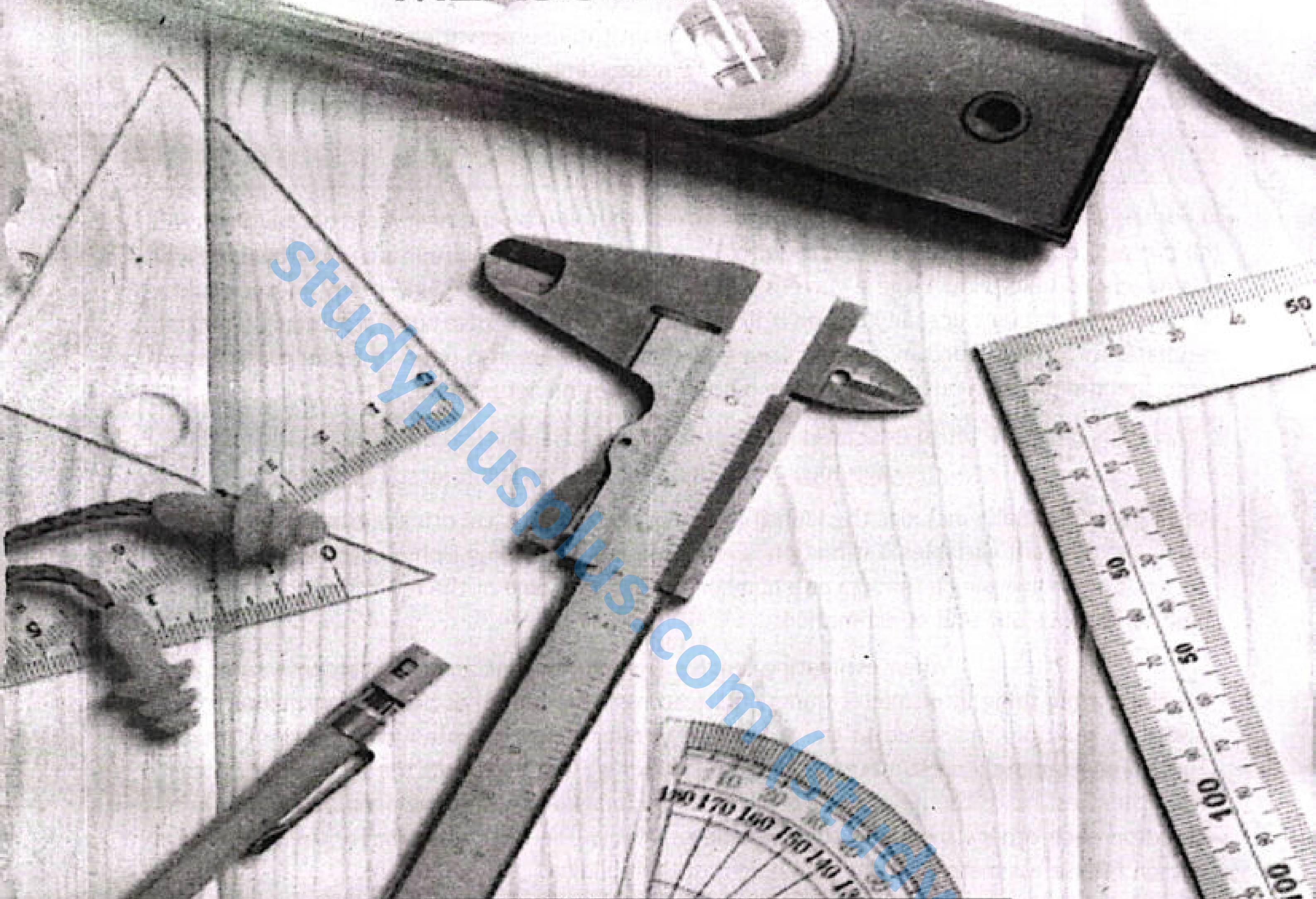
**Managing Author**

Physics-XI

## Contents

UNIT 1

# PHYSICAL QUANTITIES AND MEASUREMENTS





## Student Learning Outcomes (SLOs)

**The students will:**

- Make reasonable estimates of physical quantities [of those quantities that are discussed in the topics of this grade].
- Express derived units as products or quotients of the SI base units.
- Analyze the homogeneity of physical equations [Through dimensional analysis].
- Derive formulae in simple cases [Through using dimensional analysis].
- Analyze and critique the accuracy and precision of data collected by measuring instruments.
- Assess the uncertainty in a derived quantity [By simple addition of absolute, fractional or percentage uncertainties].
- Justify why all measurements contain some uncertainty.

# PHYSICAL QUANTITIES AND MEASUREMENTS

Physics is based on experimental observations. Observations may be qualitative or quantitative. Qualitative observations have no associated numbers. This deal with facts that can be observed with our five senses: sight, smell, taste, touch and hearing. Colors, shapes and textures of objects are examples of qualitative observations. Observations, like 'water keeps its level', is also an example of qualitative observation. A quantitative observation includes numbers, and is also called a measurement. We can measure mass, time, distance, speed, pressure, force, torque, momentum, and energy. Quantitative observations are useful to a scientist.

## 1.1 ESTIMATION OF PHYSICAL QUANTITIES

In our daily life, we may face some situations like: What will be the height of this building? Will the piece of equipment fit in the back of our car, or do we need to rent a truck? How long will this download take? How large a current will be there in this circuit? How many houses could a proposed power plant actually power if it is built? Usually, we solve such problems by making estimations. In many circumstances, scientists and engineers also need to make estimates of some specific physical quantity with the help of little or no actual data.

An estimation is a rough educated guess of the value of a physical quantity by using prior experience and sound physical reasoning.

An estimation usually includes the identification of correct physical principles and a good guess about the relevant variables. Estimation is very useful in developing a physical sense. Estimation does not mean guessing a formula or a number at random. Some of the following strategies may help to improve our skill of estimation:

**Estimation of length:** When estimating lengths, remember that anything can be a ruler. For breaking a big thing into smaller things, first estimate the length of one of the smaller things and then multiply this value to the number of smaller things to obtain the length of the big thing. For example, to estimate the height of building, we first count the number of floors it has. Then, estimate the height of a single floor by imagining how many people would have to stand on each other's shoulders to reach the ceiling. In the last, we estimate the height of a person. These estimates give you the height of the building.

Sometimes it also helps to do this in reverse, i.e., to estimate the length of a small thing which in bulk making up a bigger thing. For example, to estimate the thickness of a sheet of paper, estimate the thickness of a stack of paper and then divide it by the number of pages in the stack. These same strategies of breaking big things into smaller things or aggregating smaller things into a bigger thing can sometimes be used to estimate other physical quantities, such as mass and time. In such situations, some of the length, mass and time scales, as shown in Table 1.1 may be helpful.

**Estimate Areas and Volumes from Lengths:** While dealing with area or volume of a complex object, introduce a simple model of the object, such as a sphere or a box. Estimate the linear dimensions (such as the radius of the sphere or the length, width, and height of the box) and then use standard geometric formulas to find the area or volume. If you have an *estimate of* area or volume, you can also do the reverse; that is, use standard geometric formulas to get an estimate of its linear dimensions.

| Table 1.1: The estimation of some physical quantities. | | |
|---|---|---|
| **Length (m)** | **Mass (kg)** | **Time (s)** |
| Diameter of proton = $10^{-15}$ | Mass of electron = $10^{-30}$ | Mean lifetime of unstable nucleus= $10^{-22}$ |
| Diameter of large nucleus = $10^{-14}$ | Mass of proton = $10^{-27}$ | Time for single floating-point operating in a supercomputer = $10^{-17}$ |
| Diameter of H-tom = $10^{-10}$ | Mass of bacterium = $10^{-15}$ | Time period of visible light = $10^{-15}$ |
| Diameter of typical virus = $10^{-9}$ | Mass of mosquito = $10^{-6}$ | Time period of an atom in solid = $10^{-13}$ |
| Width of pinky fingernail = $10^{-2}$ | Mass of hummingbird = $10^{-2}$ | Time period of nerve impulse = $10^{-3}$ |
| Height of 4-years old child = $10^{0}$ | Mass of 1 liter water = $10^{0}$ | Time for 1 heartbeat = $10^{0}$ |
| Length of football ground = $10^{2}$ | Mass of a Motorcycle = $10^{2}$ | One day = $10^{5}$ |
| Diameter of Earth = $10^{7}$ | Mass of atmosphere = $10^{19}$ | One year = $10^{7}$ |
| Diameter of solar system = $10^{12}$ | Mass of Moon = $10^{22}$ | Human lifetime = $10^{9}$ |
| 1 light-year = $10^{16}$ | Mass of Earth = $10^{25}$ | Recorded human history = $10^{11}$ |
| Diameter of Milky-Way = $10^{21}$ | Mass of Sun = $10^{30}$ | Age of Earth = $10^{17}$ |
| Distance between edges of observable universe = $10^{26}$ | Mass of known universe = $10^{53}$ | Age of universe = $10^{18}$ |

**Estimate Mass from Volume and Density:** To estimate the mass of an object, it is helpful first to estimate its volume, and then to determine its mass using an estimate of its average density (recall, that density has dimension of mass/volume, so mass = density × volume). For this estimation, it helps to remember that the density of air is about 1 kg $m^{-3}$, the density of water is $10^3$ kg $m^{-3}$, and the densest everyday solids has a maximum value around $10^4$ kg $m^{-3}$. Asking yourself whether an object floats or sinks in either air or water can give you a rough estimate of its density. You can also do the reverse: if you have an estimate of an object's mass and its density, you can use them to get an estimate of its volume.

**Example 1.1: Estimate the energy required for an adult man to walk up through stairs from ground floor to 1st floor?**

Solution:

As, the energy required = $m\,g\,h$

We have to take the following estimations:

Mass of an adult = m ≈ 70 kg

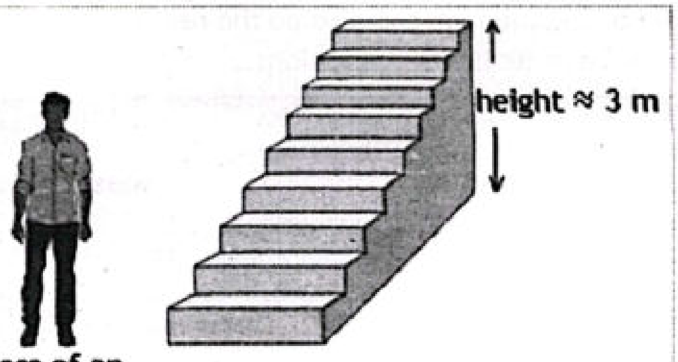
Distance between 2 floors ≈ 3 m

So,

Energy required ≈ 70 kg × 10 m $s^{-2}$ × 3 m

= 2100 kg $m^2$ $s^{-2}$

= 2100 J




Assignment 1.1

**Estimate that how many floating-point operations can a supercomputer do in 1 day? Time for single floating-point operating in a supercomputer is $10^{-17}$ s.**

## 1.2 DERIVED UNITS IN TERMS OF BASE UNITS

In Grade 9, we have studied about base and derived physical quantities and their units. We know that derived units can be expressed in terms of base units and are obtained by multiplying or dividing base units with each other. Here, we will express some more derived units as products or quotients of the SI base units. Let us first take force as an example:

$$\text{Force} = \text{mass} \times \text{acceleration}$$

$$\text{Force} = \text{mass} \times \frac{\text{velocity}}{\text{time}}$$

$$\text{Force} = \text{mass} \times \frac{\text{displacement}}{\text{time}^2}$$

Now, we put SI units for each physical quantity i.e., N for force, kg for mass, m for displacement and s for time, so we get:

$$N = kg \times m\,s^{-2}$$

For work we proceed as:

$$\text{Work} = \text{Force} \times \text{displacement}$$

$$\text{Work} = \text{mass} \times \text{acceleration} \times \text{displacement}$$

$$\text{Work} = \text{mass} \times \frac{\text{velocity}}{\text{time}} \times \text{displacement}$$

$$\text{Work} = \text{mass} \times \frac{\text{displacement}}{\text{time}^2} \times \text{displacement}$$

$$\text{Work} = \text{mass} \times \frac{\text{displacement}^2}{\text{time}^2}$$

Now, we put SI units for each physical quantity i.e., J for work, kg for mass, m for displacement and s for time, so we get:

$$J = kg \times \frac{m^2}{s^2}$$

Similarly, we can express other derived units as products or quotients of the SI base units. Some examples are shown in the Table 1.2.

Table 1.2: Some derived units as products or quotients of the SI base units.

| Name of Derived Quantity | SI Unit | Symbol | In terms of base units |
|---|---|---|---|
| Force | newton | N | $kg\ m\ s^{-2}$ |
| Work | joule | J | $N\ m = kg\ m^2 s^{-2}$ |
| Power | watt | W | $J\ s^{-1} = kg\ m^2 s^{-3}$ |
| Pressure | pascal | Pa | $N\ m^{-2} = kg\ m^{-1}s^{-2}$ |
| Electric Charge | coulomb | C | A s |

## 1.3 DIMENSIONS OF PHYSICAL QUANTITIES

Dimension denotes the qualitative nature of a physical quantity. For example, length, width, height, distance, displacement, radius etc. all are measured in meters because they have the same nature and thus share the same dimensions.

Dimension of a physical quantity is often represented by capital letter enclosed in square brackets [ ]. Dimensions for base quantities are given in the Table 1.3.

Table 1.3: Dimensions of Base Quantities.

| Sr. No | Physical Quantity | Dimensions |
|---|---|---|
| 1 | mass | [M] |
| 2 | length | [L] |
| 3 | time | [T] |
| 4 | electric current | [I] |
| 5 | temperature | [θ] |
| 6 | intensity of light | [J] |
| 7 | amount of substance | [N] |

Dimensions of derived quantities are obtained by multiplication or division of the dimensions of base quantities, from which these quantities are derived. For example, the dimension for area, volume, velocity and acceleration are $[L^2]$, $[L^3]$, $[LT^{-1}]$ and $[LT^{-2}]$ respectively.

Thus, dimensions give the relation of a given physical quantity with base quantities i.e. mass, length, time etc. There are the following essential terms used in dimensional analysis:

Dimensional Variables: Those physical quantities that have dimensions and variable in magnitude are called dimensional variables. Some dimensional variables are length, velocity, acceleration, force, energy and acceleration etc.

Dimensional Constants: Those physical quantities that have dimensions and a constant magnitude are called dimensional constants. Some examples of dimensional constants are Planck's constant (h), gravitational constant (G), speed of light in vacuum (c) and ideal gas constant (R) etc.

Dimensionless Variables: Those physical quantities that have no dimensions and have variable magnitudes are called dimensionless variables. Some examples of dimensionless variables are plane angle, solid angle, strain and coefficient of friction etc.

Dimensionless Constants: Those physical quantities that have no dimensions and have constant magnitude are called dimensionless constants. The pure numbers (1, 2, 3, ....), the exponential constant (e = 2.718) and π are some examples of dimensionless constants.

## 1.3.1 Advantages of Dimensions

Using the method of dimensions (called dimensional analysis), we can check the homogeneity of an equation, derive a possible formula and determine units of physical quantities. Dimensional analysis makes use of the fact that dimensions can be treated as algebraic quantities. That is, quantities can be added or subtracted only if they have the same dimensions. Furthermore, the terms on both sides of an equation must have the same dimensions.

(i) The Homogeneity of an Equation

In order to check the correctness of an equation, we must ensure that both sides of the equation have the same dimensions: otherwise, the equation cannot be considered as a physically correct equation. This is called the principle of homogeneity of dimensions.

Let us check whether the equation $v_f = v_i + at$ is dimensionally correct.

Dimensions of L.H.S. = $[LT^{-1}]$

Dimensions of R.H.S. = $[LT^{-1}] + [LT^{-2}][T]$

$= [LT^{-1}] + [LT^{-1}]$

$= 2[LT^{-1}]$

As 2 is dimensionless constant, therefore

Dimensions of L.H.S = Dimensions of R.H.S

Hence, the equation is dimensionally correct.

(ii) To Derive a Possible Formula

Deriving a relation for a physical quantity depends on the correctly guessing various factors on which the physical quantity depends. Let us derive the formula for wavelength of matter waves using dimensional analysis.

As wavelength (λ) of matter waves may depend upon Planck's constant (h), velocity (v) and mass (m) of the particle.

So, the relation for the wavelength (λ) will be of the form:

$$\lambda \propto h^a m^b v^c$$

$$\lambda = (\text{constant})\ h^a\ m^b\ v^c \quad \text{\_\_\_\_\_\_ (1)}$$

We have to find the values of powers i.e. a, b and c.

Using dimension on both sides, we get:

$$[L] = \text{constant}\ [M L^2 T^{-1}]^a\ [M]^b\ [LT^{-1}]^c$$

OR
$$[M^0 L^1 T^0] = \text{constant}\ [M]^{a+b}\ [L]^{2a+c}\ [T]^{-a-c} \quad \text{\_\_\_\_\_\_ (2)}$$

Equating the powers of M on both sides of equation (2), we get:

$$a + b = 0 \quad \text{\_\_\_\_\_\_ (3)}$$

Equating the powers of L on both sides of equation (2), we get:

$$2a + c = 1 \quad \text{\_\_\_\_\_\_ (4)}$$

Equating the powers of T on both sides of equation (2), we get:

$$-a - c = 0 \quad \text{\_\_\_\_\_\_ (5)}$$

On solving equations (3), (4) and (5), we get:

$$a = 1,\ b = -1 \text{ and } c = -1$$

Put the values of a, b, and c in (1), we get:

$$\lambda = (\text{constant})\ h^1\ m^{-1}\ v^{-1}$$

OR
$$\lambda = (\text{constant}) \times \frac{h}{mv}$$

### 1.3.2 Limitations of Dimensional Analysis

Some limitations of dimensional analysis are:

1) Dimensional analysis does not distinguish between the physical quantities having same dimensions. For example, if the dimensional formula of a physical quantity is $[ML^2T^{-2}]$ it may represent work, or energy, or torque.
2) Dimensional analysis cannot be used to derive formulas containing trigonometric functions, exponential functions, logarithmic functions, etc.
3) Dimensional analysis cannot determine the dimensionless constant when deriving a possible formula.
4) Dimensional analysis doesn't always prove that a relation is physically correct although relation is dimensionally correct. However, a dimensionally wrong equation is always wrong.

**Example 1.2:** Derive the formula for the time period of a simple pendulum using dimensional analysis.

Solution: The time period of a simple pendulum is possibly depending on the mass of the bob (m), the length of the pendulum (l), the angle which the string makes with vertical (θ) and the acceleration due to gravity (g). So, the relation for the time period T will be of the form:

$$T \propto m^a\ l^b\ \theta^c\ g^d$$

$$T = (\text{constant})\ m^a\ l^b\ \theta^c\ g^d \quad \text{\_\_\_\_\_\_ (1)}$$

We have to find the values of powers i.e. a, b, c and d:

Using the dimension on both sides, we get:

$$[M^0 L^0 T] = \text{constant}\ [M]^a\ [L]^b\ [LL^{-1}]^c\ [LT^{-2}]^d$$

$$[M^0 L^0 T] = \text{constant}\ [M]^a\ [L]^{b+d}\ [T]^{-2d} \quad \text{\_\_\_\_\_\_ (2)}$$

Equating the powers of M on both sides of equation (2), we get:

$$a = 0 \quad \text{\_\_\_\_\_\_ (3)}$$

Equating the powers of L on both sides of equation (2), we get:

$$b + d = 0 \qquad \text{——— (4)}$$

Equating the powers of T on both sides of equation (2), we get:

$$-2d = 1$$

or $\quad d = -1/2 \qquad$ ——— (5)

Put d = -1/2, in (4), we get:

$$b = 1/2$$

Put the values of a, b, c and d in (1):

$$T = (\text{constant})\ m^{o}\ l^{1/2}\ g^{-1/2}$$

$$T = (\text{constant}) \times \sqrt{\frac{l}{g}}$$

Where the constant, found by experiment, is $2\pi$.

Assignment 1.2

Which of the following relationships is dimensionally consistent with an expression yielding a value for acceleration? In these equations, x is a distance, t is time, and v is velocity.

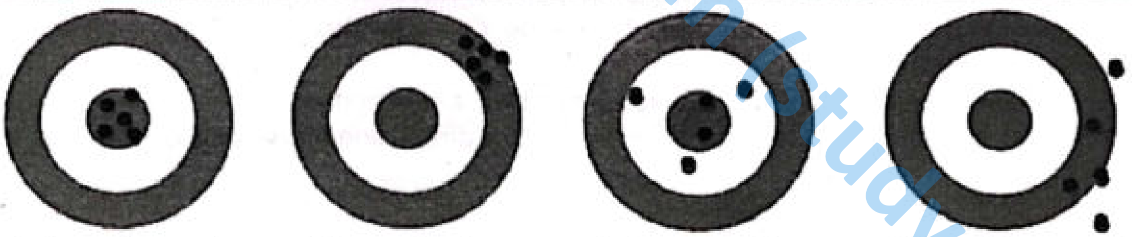
(a) $v/t^2$ (b) $v/x^2$ (c) $v^2/t$ (d) $v^2/x$

## 1.4 PRECISION AND ACCURACY

Science is based on observations and experiments, which involve measurements. Precision is measurements of the same physical quantity agree with each other. Accuracy refers to how closely a measurement agrees with a standard or true value. Hence:

Precision refers to the closeness of measured values to each other, while accuracy refers to the closeness of a measured value to a standard or true value.

For Your Information



(a) precise and accurate (b) precise but not accurate (c) not precise but accurate (d) neither precise nor accurate

Several independent trials of shooting at a bullseye target illustrate the difference between being accurate and being precise. Accuracy is how close an arrow gets to the bull's-eye center. Precision is how close a second arrow is to the first one (regardless of closeness to the target).

To understand the concept of precision and accuracy, consider a person who weighs exactly 160.0 pounds and he weights himself three times on three different scales. Results of the scales are:

| | |
|---|---|
| Scale A: | 170.1, 169.9 and 170.0 pounds. |
| Scale B: | 161, 162 and 158 pounds. |
| Scale C: | 159.9. 160.0 and 160.1 pounds. |

In this case, the weight measured by scale A is very precise, but not accurate. The weight measured by scale B is fairly accurate but not precise. The weight measured by scale C is both precise and accurate.

The precision of a measurement is associated with least count of the measuring instrument. The smaller the least count of the measuring instrument, the greater its precision. Precision is indicated by the absolute uncertainty in measurement. Accuracy is indicated by the fractional or percentage uncertainty or error in measurement. The smaller the magnitude of fractional or percentage uncertainty or error, the greater its accuracy.

## 1.5 UNCERTAINTIES

In Grade 9, we have studied about the sources of human error, systematic error and random error in experiments. The difference between the true value and observed value of a measurement is called error. i.e.,

Error = true value – observed value

In measurement the error may occur due to:

- Negligence or inexperience of a person.
- Using a faulty apparatus.
- Inappropriate method or technique.

Errors can be divided into the following three types:

- Personal Error
- Systematic Error
- Random Error

Here we will study about uncertainty.

**Tip for Solving Numerical Problems Symbolic Solutions!**

When solving problems, it is very useful to perform the solution completely in algebraic form and wait until the very end to enter numerical values into the final symbolic expression. This method will save many calculator keystrokes, especially if some quantities cancel so that you never have to enter their values into your calculator! In addition, you will only need to round once, on the final result.

Uncertainty is the range of possible values within which the true value of the measurement lies.

For example, a measurement of 3.06 mm ± 0.02 mm means that the experimenter is confident that the actual value for the quantity being measured lies between 3.04 mm and 3.08 mm.

Uncertainty is a quantitative measurement of variability in the data.

All measurements have a degree of uncertainty. This is caused by two factors, the limitations of the measuring instrument (systematic error) and the skills of the experimenter.

Absolute uncertainty is equal to the least count of a measuring instrument, for example the length of a glass slab measured with a meter rod is 37.5 cm. The least count of meter rod is 1 mm = 0.1 cm, then the absolute uncertainty in the measured value will be ± 0.1 cm i.e., ± 0.05 cm uncertainty develops at each end. For example, if one end of the slab coincides with 20.5 cm mark and the other end with 58.0 cm mark of meter rule, the length of the slab along with uncertainty is given by:

$$(58.0 \pm 0.05)\text{ cm} - (20.5 \pm 0.05)\text{ cm} = (37.5 \pm 0.1)\text{ cm}$$

It means that the length of slab is between 37.4 cm and 37.6 cm.

In the above measurement, precision is ± 0.1 cm, which is equal to the magnitude of absolute uncertainty.

The accuracy in the measurement is indicated by the magnitude of fractional error. Here:

$$\text{Fractional Uncertainty} = \frac{\text{Absolute Uncertainty}}{\text{Measured Value}}$$

$$\text{Fractional Uncertainty} = \frac{\pm 0.1}{37.5} = \pm 0.003$$

Percentage uncertainty = fractional uncertainty × 100 %

**Remember!**

If $x + \Delta x = (2.0 \pm 0.1)$ mm, then Actual/Absolute uncertainty is
$\Delta x = \pm 0.1$ mm
Fractional uncertainty is:
$\frac{\Delta x}{x} = 0.05$
Percentage uncertainty is:
$\frac{\Delta x}{x} \times 100\% = 5\%$

The smaller the magnitude of fractional (relative) uncertainty or error, the greater will be the accuracy of measurement.

## 1.5.1 Rules for Calculating Uncertainties in Final Result

There are some rules for calculating uncertainties in different cases but we need to be very careful whether we use the absolute or percentage uncertainty in each case.
Let x and y are two different physical quantities with uncertainties Δx and Δy respectively. If z is a physical quantity which is obtained by operating x and y, then the propagated uncertainty Δz in the result can be calculated by using the following rules.

a) Rule for Addition and Subtraction
If two or more than two measured quantities are added or subtracted, then their absolute uncertainties are added to get uncertainty in the result.

If: $z = x + y$ or $z = x - y$
then; $\Delta z = \pm(\Delta x + \Delta y)$

For example, if: $x \pm \Delta x = (24.0 \pm 0.1)$ cm
and $y \pm \Delta y = (30.0 \pm 0.1)$ cm
then; $\Delta z = \pm 0.2$ cm

b) Rule for Multiplication and Division
If two or more than two quantities are multiplied or divided, then their percentage uncertainties are added to get uncertainty in the result.

If: $z = xy$ or $z = x/y$,
then; % uncertainty in z = % uncertainty in x + % uncertainty in y

c) Rule for Power of a Quantity
The total uncertainty in power of a quantity is equal to the percentage uncertainty multiplied with that power.

If: $z = x^3$,
then; percentage uncertainties in z = ± 3 (percentage uncertainty in x)

d) Uncertainties in Average Values of Many Measurements
The uncertainty in the average value is calculated by adopting the following steps:

i) Find the average of measured values.
ii) Find the deviation of each value from the average.

iii) The mean deviation is the uncertainty in the average.

For example, three readings are recorded for the radius of a small cylinder as:

$$r_1 = 1.50\text{ cm},\ r_2 = 1.51\text{ cm} \text{ and } r_3 = 1.52\text{ cm}$$

The uncertainty in the average radius is calculated as:

Finding average: $\bar{r} = \dfrac{r_1 + r_2 + r_3}{3}$

$$= \frac{1.50\ cm + 1.51\ cm + 1.52\ cm}{3} = 1.51\ cm$$

Finding deviation: $\Delta r_1 = \bar{r} - r_1 = 1.51 - 1.50 = 0.01$ cm

$\Delta r_2 = \bar{r} - r_2 = 1.51 - 1.51 = 0$ cm

$\Delta r_3 = \bar{r} - r_3 = 1.51 - 1.52 = 0.01$ cm

Finding mean deviation: $\overline{\Delta r} = \dfrac{\Delta r_1 + \Delta r_2 + \Delta r_3}{3}$

$$= \frac{0.01\ cm + 0\ cm + 0.01\ cm}{3} = 0.0067\text{ cm} = 0.007\text{ cm}$$

e) Uncertainty in Timing Experiment

The time period T of a vibrating body can be found by dividing time of multiple vibrations by the number of vibrations.

$$T = \frac{\text{Time of multiple vibrations}}{\text{No. of vibrations}}$$

The uncertainty in time period $\Delta T$ is found by dividing the least count (L.C) of the time recording device by the number of vibrations.

$$\Delta T = \frac{\text{L.C}}{\text{No. of vibrations}}$$

For example, the time recorded for 20 vibrations of a pendulum is t = 35.2 s. Let the least count of stop watch used is 0.1 s (1/10 s). So, the uncertainty in measured time is (35.2 s ± 0.1 s).

Then the time period of the pendulum is obtained as:

$T = 35.2/20 = 1.76$ s

Uncertainty in time period is $\Delta T = 0.1/20 = 0.005$ s

So, $T + \Delta T = (1.76 \pm 0.005)$ s

**Example 1.3: If voltage measured across a conductor is V ± ΔV = (7.3 ± 0.1) volts and current is I ± ΔI = (2.73 ± 0.051) ampere. Find the resistance and uncertainty in it.**

Given: $V \pm \Delta V = (7.3 \pm 0.1)$ volts $\quad I \pm \Delta I = (2.73 \pm 0.051)$ ampere

To Find: $R \pm \Delta R = ?$

Solution: According to ohm's law, R is calculated as:

$R = V/I = 7.3/2.73 = 2.7\ \Omega$

Percentage uncertainty in $V$ is:

$$= \frac{\Delta V}{V} \times 100\% = \frac{0.1}{7.3} \times 100\ \% = 1.37\ \% = 1\ \%$$

Percentage uncertainty in $I$ is:

$$= \frac{\Delta l}{l} \times 100\% = \frac{0.05}{2.73} \times 100\,\% = 1.83\,\% = 2\,\%$$

Thus, the total uncertainty in R is:

$$= 1\,\% + 2\,\% = 3\,\%$$

So, $R \pm \Delta R = 2.7 \pm 3\,\%$

$$= 2.7\,\Omega \pm \left(\frac{3}{100} \times 2.7\right)\Omega$$

$$= (2.7 \pm 0.08)\,\Omega$$

**Example 1.4: If radius of a circular disc is measured as 2.25 cm with uncertainty ± 0.01 cm. Find its surface area with uncertainty in it.**

Given: $r = 2.25$ cm, $\Delta r = \pm 0.01$ cm

To Find: $A \pm \Delta A = ?$

Solution: As, $A = \pi r^2 = 3.14 \times 2.25^2 = 15.90\ cm^2$

Percentage uncertainty in $r = \frac{\Delta r}{r} \times 100\,\% = \frac{0.01}{2.25} \times 100\,\% = 0.4\,\%$

Percentage uncertainty in area is $= 2 \times 0.4\,\% = 0.8\,\%$

So, $\Delta A = 0.8\,\% \times 15.90\ cm^2 = 0.13\ cm^2$

Thus $A \pm \Delta A = (15.90 \pm 0.13)\ cm^2$

**Assignment 1.3**

**The radius of a circle is measured to be (10.5 ± 0.2) m. Calculate (a) the area and (b) the circumference of the circle, also give the uncertainty in each value.**

## SUMMARY

- **Estimation** does not mean guessing a formula or a number at random. An estimation is a rough educated guess to the value of a physical quantity by using prior experience and sound physical reasoning.
- **Derived units** can be expressed in terms of base units and are obtained by multiplying or dividing base units.
- **Dimension** denotes the qualitative nature of a physical quantity.
- In order to check the correctness of an equation, we have to show that both sides of an equation have the same dimensions; otherwise, the equation cannot be physically correct. This is called the **principle of homogeneity of dimensions.**
- **Uncertainty** is the range of possible values within which the true value of the measurement lies.
- **Absolute uncertainty** is equal to the least count of a measuring instrument.
- **Precision** refers to the closeness of measured values to each other.
- **Accuracy** refers to the closeness of a measured value to a standard or true value.

# EXERCISE

## Multiple Choice Questions

Encircle the correct option.

1) The mean diameter of a wire is found to be (0.50 ± 0.02) mm. The percentage uncertainty in the diameter is:

A. 2 % B. 4 % C. 6 % D. 8 %

2) A reaction takes place that is expected to yield 171.9 g of product, but it only yields 154.8 g. What is the percent error for this experiment?

A. 17.1 % B. 90.1 % C. 111.0 % D. 9.9 %

3) Three different people weigh a standard mass of 2.00 g on the same balance. Each person obtains a reading of exactly 7.32 g for the mass of the standard. These results imply that the balance is:

A. both accurate and precise
B. neither accurate nor precise
C. accurate but not precise
D. precise but not accurate.

4) Dimension of universal gravitational constant (G) is:

A. $[M^{-2}L^3T^{-2}]$ B. $[M^3L^{-1}T^{-2}]$ C. $[M^{-1}L^3T^{-2}]$ D. $[M^{-3}L^3T^{-2}]$

5) A measurement, which on, repetition gives same or nearly same result is called:

A. accurate B. average C. precise D. estimated

6) A student is measuring the time of an event by using stopwatch. He takes 5 measurements as: 3.0 s, 3.2 s, 3.4 s, 2.8 s, 3.1 s. What is the uncertainty in the results?

A. ±0.3 s B. ±0.6 s C. ±3.1 s D. ±7.75 s

7) Which of the following quantity has a different dimension?

A. force B. weight C. modulus of elasticity D. tension

8) If the dimensions of a physical quantity are given by $[L^a M^b T^c]$, then the physical quantity will be:

A. force, if a = -1, b = 0, c = -2
B. pressure, if a = -1, b = 1, c = -2
C. velocity, if a = 1, b = 0, c = 1
D. acceleration, if a = 1, b = 1, c = -2

9) Order of magnitude of $(10^6 + 10^3)$ is:

A. $10^{18}$ B. $10^9$ C. $10^6$ D. $10^3$

10) Which of the following may be used as a valid formula to calculate the speed of ocean waves? [v = speed, g = acceleration due to gravity, λ = wavelength, ρ=density, h= depth].

A. $v = \sqrt{\lambda g}$ C. $v = \rho g h$ C. $v = gh/\lambda$ D. $v = \lambda g h$

# PHYSICAL QUANTITIES AND MEASUREMENTS

## Short Questions

Give short answers of the following questions.

1.1 Draw a table to show a reasonable estimate of some physical quantities.

1.2 Express the units of the following derived quantities in terms of base units. (a) Force (b) Work (c) Power (d) Pressure (e) Electric charge.

1.3 Why is it important to use an instrument with the smallest resolution?

1.4 What is the importance of increasing the number of readings in an experiment?

1.5 What is the difference between prècision and accuracy?

1.6 What is the principle of homogeneity of dimensions?

1.7 A ball is thrown in the air and 5 different students are individually measuring the time it takes to fall back down using stopwatches. The times obtained by each student are the following: 6.2 s, 6.0 s, 6.4 s, 6.1 s, 5.8 s. (i) What is the uncertainty of the results? (ii) How should the resulting time be expressed?

1.8 The energy of a photon is given by $E = hf$, where f is frequency. Find the dimensions of Planck's constant $h$.

1.9 Justify why all measurements contain some uncertainty.

## Comprehensive Questions

Answer the following questions in detail.

1.1 Define and explain the term uncertainty.

1.2 Discuss the rules for calculating uncertainty propagation in the final results in different cases.

1.3 What does the dimension of a physical quantity mean? What are its advantages? Explain with examples?

1.4 What is meant by estimation of a physical quantity? Explain with examples.

## Numerical Problems

1.1 Estimate number of heartbeats in a lifetime of 60-years? **(Ans: $10^9$)**

1.2 Determine the dimensions of each of the following quantities.

a) $\frac{v^2}{ax}$ b) $\frac{at^2}{2}$ **(Ans: (a) No, (b) [L])**

1.3 If $A = \frac{X^2}{Y^2 Z}$, then find the percentage uncertainty in A. The percentage uncertainties in X, Y and Z are 1 %, 1 % and 2 % respectively. **(Ans: 6 %)**

1.4 A spherical ball of radius $r$ experiences a resistive force $F$ due to the air as it moves through it at speed v. The resistive force F is given by the expression

$$F = c\,r\,v$$

Where c is constant. By using dimensions, derive the SI base unit of the constant c.

**(Ans: kg $m^{-1}s^{-1}$)**

1.5 The pressure (P) at a depth (h) in an incompressible fluid of density ($\rho$) is given by

$$P = \rho g h$$

Where g is acceleration due to gravity. Check the homogeneity of this equation.

1.6 Estimate how many protons are there in a bacterium? (Take mass of bacterium as $10^{-15}$ kg and mass of proton $10^{-27}$ kg). **Ans: $10^{12}$ protons)**

1.7 Estimate how many hydrogen atoms does it take to stretch across the diameter of the Sun? (Take diameter of the Sun as $10^{5}$ km and diameter of proton $10^{-14}$ km).

**(Ans: $10^{19}$ hydrogen atoms)**

1.8 The current passing through a resistor R = (13 ± 0.5) Ω is I = (3 ± 0.1) A.

a) Calculate the power consumed (correct to one significant figure).

b) Find the percentage uncertainty of the current passing through the resistor.

c) Find the percentage uncertainty of the resistance.

d) Find the absolute uncertainty of the electrical power.

**(Ans: 117 W, 3 %, 3.84 %, 11.7 W)**

# VECTORS

UNIT 2




## Student Learning Outcomes (SLOs)

The students will

- Represent a vector in 2-D as two perpendicular components.
- Describe the product of two vectors (dot and cross-product) along with their properties.

In our daily life, we used to deal with physical quantities, such as mass, time, length, velocity, acceleration, force, work and torque etc. Some physical quantities, known as scalars, can be completely understood by a number and a proper unit only. On the other hand, some physical quantities require direction along with a number and a unit; such quantities are known as vectors. Displacement, force, acceleration, torque and momentum etc., are the examples of vector quantities. Vectors are very useful in our daily life. For example, a signboard provides information about distances and directions of other places relative to the location of the signboard. Distance is a scalar quantity. Knowing the distance alone is not enough to get to the desired location. We must also know the direction from the signboard to the destination. The direction along with the distance, is a vector quantity called the displacement. Therefore, signboards give information about displacement from the signboard to the destination.

Graphically, a vector can be completely expressed by a line with an arrow head. Length of the line represents the magnitude according to the proper scale and the arrow head represents the direction. We have studied about the addition of vectors in grade 9; here we shall study about the components of vectors and about the products of vectors.



## 2.1 VECTOR RESOLUTION

Components are the effective parts of a vector in different directions. In 2-D, we take x-component and y-component. The component along x-axis is called the horizontal or the x-component and the component along the y-axis is called the vertical or the y-component. Both the x-component and y-component are perpendicular to each other and are called rectangular components.

The components of a vector which are mutually perpendicular are called rectangular components.

Consider a vector 'A' making an angle 'θ' with the horizontal, as shown in the Fig. 2.1. Perpendiculars are drawn from the head of vector A on the x-axis and y-axis. The projection OQ of vector A along x-axis is x-component of vector A and is represented by $A_x$. The projection OS of vector A along y-axis is y-component of vector A and is represented by $A_y$. So, the vector A in its component form can be written as:

$$\mathbf{A} = \mathbf{A}_x + \mathbf{A}_y \qquad \text{\_\_\_\_\_\_ (2.1)}$$

OR

$$\mathbf{A} = A_x\,\hat{\imath} + A_y\,\hat{\jmath} \qquad \text{\_\_\_\_\_\_ (2.2)}$$

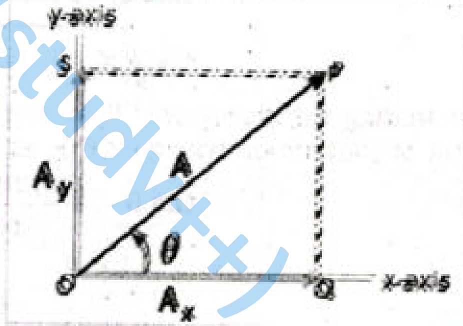



**Figure 2.1:** Rectangular components of a vector.

Here, $\hat{\imath}$ and $\hat{\jmath}$ are representing directions along +x-axis and +y-axis, respectively, and are called unit vectors. Unit vectors specify the direction of any vector and has magnitude equal to one.

### 2.1.1 Finding Magnitude of Rectangular Components

In order to find the magnitude of rectangular components $A_x$ and $A_y$ of a vector **A**, we apply the trigonometric ratios. For this, consider the triangle OPQ, as shown in Fig. 2.1. As:

$$\sin\theta = \frac{perpendicular}{hypotenuse} = \frac{\overline{QP}}{\overline{OP}} = \frac{\overline{OS}}{\overline{OP}}$$

or $$\sin\theta = \frac{A_y}{A}$$

or $$A_y = A\sin\theta \quad \text{\_\_\_\_\_\_ (2.3)}$$

and,

$$\cos\theta = \frac{base}{hypotenuse} = \frac{\overline{OQ}}{\overline{OP}}$$

or $$\cos\theta = \frac{A_x}{A}$$

or $$A_x = A\cos\theta \quad \text{\_\_\_\_\_\_ (2.4)}$$

From equation (2.3) and (2.4), we can find the magnitude of perpendicular components $A_x$ and $A_y$.

### 2.1.2 Finding a Vector from its Components

If the rectangular components $A_x$ and $A_y$ of a vector **A** are given, then its magnitude 'A' and direction 'θ' can be found. Again, we consider the triangle OPQ, as shown in Fig. 2.1. By applying Pythagoras theorem, we get:

$$(OP)^2 = (OQ)^2 + (QP)^2 \quad \text{\_\_\_\_\_\_ (2.5)}$$

As QP = OS, so equation (2.5) can be written as:

$$(OP)^2 = (OQ)^2 + (OS)^2 \quad \text{\_\_\_\_\_\_ (2.6)}$$

Putting OP = A, OQ = $A_x$ and OS = $A_y$, in equation (2.6), we get:

$$A^2 = (A_x)^2 + (A_y)^2$$

or $$A = \sqrt{(A_x)^2 + (A_y)^2} \quad \text{\_\_\_\_\_\_ (2.7)}$$

**For Your Information**

A vector which specifies the location of a point P (a, b) with respect to origin is called position vector, and can be given as: $r = a\hat{i} + b\hat{j}$ and its magnitude is given as: $r = \sqrt{a^2 + b^2}$.

For finding the direction 'θ' of a vector, we apply trigonometric ratio of tangent on triangle OPQ, as:

$$\tan\theta = \frac{perpendicular}{base}$$

or $$\tan\theta = \frac{A_y}{A_x}$$

or $$\theta = \tan^{-1}\frac{A_y}{A_x} \quad \text{\_\_\_\_\_\_ (2.8)}$$

By using equations (2.7) and (2.8), we can find the magnitude and direction of a vector.

**Example 2.1: A peddler is pushing a trolley on a horizontal road with a force of 50 N making an angle 30° with the road. Find the horizontal and vertical components of the force.**

*Given:* Magnitude of force = F = 50 N

Angle of the force with horizontal = θ = 30°

To Find: $F_x$ = ? $F_y$ = ?

**Solution:** To find x-component of the force F, we use the equation:

$$F_x = F \cos\theta$$

Putting values, we get:

$$F_x = 50 \cos 30^\circ$$

or, $$F_x = 43.3 \text{ N}$$

To find y-component of the force F, we use the equation:

$$F_y = F \sin\theta$$

Putting values, we get:

$$F_y = 50 \sin 30^\circ$$

or, $$F_y = 25 \text{ N}$$

**Assignment 2.1**

**Fatima is pulling her trolley bag while climbing up the ramp at her school gate. Find the force with which she is pulling her bag, if the x-component and y-component of her force are 12 N and 5 N, respectively.**

## 2.2 PRODUCT OF VECTORS

When two vector quantities are multiplied, then the product may be a scalar quantity or a vector quantity. The product obtained depends upon the nature of the given vectors. Let us discuss the two types of products in the following sections.

### 2.2.1 Scalar Product or Dot Product:

When the product of two vector quantities gives a scalar quantity then the product is called scalar product.

The scalar product of two vectors is represented by putting a dot (·) between the symbols of the two vectors; therefore, it is also known as dot product.

Let us take two vectors **A** and **B** that are making an angle θ with each other, as shown in the Fig. 2.2. The dot product between these two vectors can be denoted by **A.B** and is defined as:

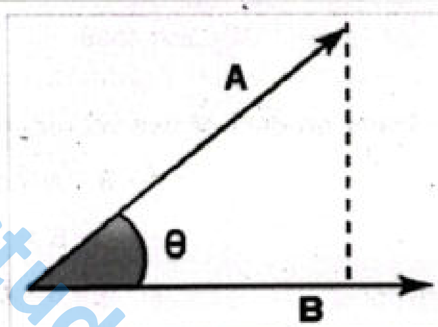



**Figure 2.2:** Two vectors **A** and **B** that are making an angle θ with each other.

$$\mathbf{A} \cdot \mathbf{B} = A\,B \cos\theta \quad \text{\_\_\_\_\_\_} \quad (2.9)$$

Here, A and B are the magnitudes of the vectors **A** and **B**. Thus, the scalar product of two vectors is obtained by multiplying their magnitudes with the cosine of the angle between them.

Equation (2.9) can also be written as:

$$\mathbf{A} \cdot \mathbf{B} = (A \cos\theta)\,B \quad \text{\_\_\_\_\_\_} \quad (2.10)$$

It is shown in the Fig. 2.3 that (A cos θ) is the component of vector **A** in the direction of vector **B**. So, equation (2.10) can also be written as:

$$\mathbf{A} \cdot \mathbf{B} = (\text{component of vector } \mathbf{A} \text{ along } \mathbf{B})\,B \quad \text{\_\_\_\_\_\_} \quad (2.11)$$

Similarly, it can be shown that:

$\mathbf{A} \cdot \mathbf{B} = A$ (component of vector **B** along **A**) ______ (2.12)

Hence, from equation (2.11) and (2.12), we can also define scalar product as:

The product of magnitude of either vector with the component of other vector in the direction of first vector.

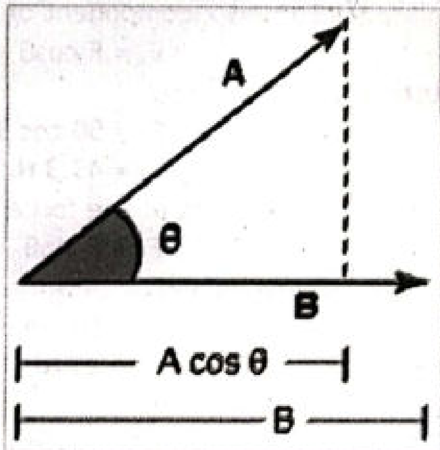



**Figure 2.3:** component of vector **A** in the direction of vector **B**.

Examples of Scalar Product:

- Work: Work (W) is an example of scalar product. Work is a scalar quantity which is the scalar product of displacement **d** and force **F**. Mathematically, it can be written as:

$$W = \mathbf{F} \cdot \mathbf{d}$$

or $$W = F\,d \cos\theta$$

- Power: Power (P) is also an example of scalar product. It is equal to the scalar product of force **F** and velocity **v**. Mathematically; it can be written as:

$$P = \mathbf{F} \cdot \mathbf{v}$$

or $$W = F\,v \cos\theta$$

- Electric Flux: Electric flux (Φ) is a scalar quantity which is the scalar product of electric field intensity **E** and vector area **A**. Mathematically, it can be written as:

$$\Phi = \mathbf{E} \cdot \mathbf{A}$$

or $$W = E\,A \cos\theta$$

Properties of Scalar Product:

1) Scalar product of two vectors obeys the commutative property, i.e.,

$$\mathbf{A} \cdot \mathbf{B} = A\,B \cos\theta$$

$$= B\,A \cos\theta = \mathbf{B} \cdot \mathbf{A}$$

Hence, $$\mathbf{A} \cdot \mathbf{B} = \mathbf{B} \cdot \mathbf{A}$$

2) Scalar product of two orthogonal (perpendicular) vectors is equal to zero, i.e.,

$$\mathbf{A} \cdot \mathbf{B} = A\,B \cos 90^\circ = 0$$

Similarly, for mutually perpendicular unit vectors, we can show:

$$\hat{i} \cdot \hat{j} = \hat{j} \cdot \hat{k} = \hat{k} \cdot \hat{i} = 0$$

Note that the dot product of a vector with the null vector is also zero.

3) Scalar product of two parallel vectors has maximum value, and is equal to the product of their magnitudes only. Hence:

$$\mathbf{A} \cdot \mathbf{B} = A\,B \cos 0^\circ = AB$$

Scalar product of a vector with itself is equal to the square of its magnitude, i.e.,

$$\mathbf{A} \cdot \mathbf{A} = A\,A \cos 0^\circ = A^2$$

Similarly, we can show that the dot product of a unit vector with itself is unity, i.e.,

$$\hat{\imath}\cdot\hat{\imath} = \hat{\jmath}\cdot\hat{\jmath} = \hat{k}\cdot\hat{k} = 1$$

4) Scalar product of two antiparallel vectors is negative, i.e.,

$$\mathbf{A}\cdot\mathbf{B} = AB\cos 180^\circ = -AB$$

5) Scalar product of two vectors in terms of their rectangular components can be given as:

$$\mathbf{A}\cdot\mathbf{B} = (A_x\hat{\imath} + A_y\hat{\jmath} + A_z\hat{k})\cdot(B_x\hat{\imath} + B_y\hat{\jmath} + B_z\hat{k})$$

OR $$\mathbf{A}\cdot\mathbf{B} = A_xB_x + A_yB_y + A_zB_z \qquad \text{\_\_\_\_\_\_ (2.13)}$$

As, $$\cos\theta = \frac{\mathbf{A}\cdot\mathbf{B}}{AB}$$

So, in components form: $$\cos\theta = \frac{A_xB_x + A_yB_y + A_zB_z}{AB} \qquad \text{\_\_\_\_\_\_ (2.14)}$$

**Example 2.2: Find the power delivered by the engine for attaining velocity (3, 4) m s$^{-1}$, while it exerts a force (8, -2) N.**

Given: Velocity = $\mathbf{v} = (3\hat{\imath} + 4\hat{\jmath})$ m s$^{-1}$ Force = $\mathbf{F} = (8\hat{\imath} - 2\hat{\jmath})$ N

To Find: Power Delivered = P = ?

Solution: As we know that power is the scalar product of force and velocity hence:

$$P = \mathbf{F}\cdot\mathbf{v}$$

$$P = (3\hat{\imath} + 4\hat{\jmath})\cdot(8\hat{\imath} - 2\hat{\jmath})$$

$$P = 24 - 8 = 16\text{ W}$$

Assignment 2.2

**If vectors A= 5î + ĵ and B = 2î + 4ĵ, then find:**
**(i) projection of A on B. (ii) projection of B on A.**

## 2.2.2 Vector Product or Cross Product:

When the product of two vector quantities gives a vector quantity then the product is called vector product.

vector product of two vectors is represented by putting a cross (x) between the symbols of the two vectors, therefore it is also known as cross product.

Let us take two vectors **A** and **B** that are making an angle θ with each other, as shown in the Fig. 2.4. The cross product between these two vectors can be denoted by **A**×**B** and is defined as:

$$\mathbf{A}\times\mathbf{B} = AB\sin\theta\,\hat{n} \qquad \text{\_\_\_\_\_\_ (2.15)}$$

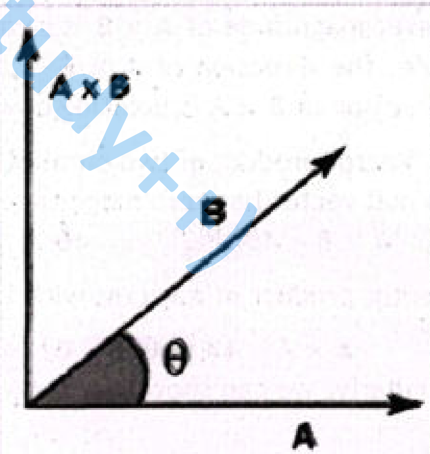



**Figure 2.4:** Cross product of two vectors **A** and **B** is shown by **A×B**.

Here, A and B are the magnitudes of the vectors **A** and **B**. Thus, the vector product of two vectors is obtained by multiplying their magnitudes with the sine of the angle between them.

In equation (2.15), $\hat{n}$ represents the direction of the vector product, which is always perpendicular to the plane containing **A** and **B**. The direction of cross product ( $\hat{n}$ ) can be found by right hand rule of vector product, as shown in Fig. 2.5. It can be stated as:

Rotate the fingers of your right hand in the direction from first vector to the second vector through smaller angle of the two possible angles with erect thumb. The direction of the product will be along the direction of erect thumb.

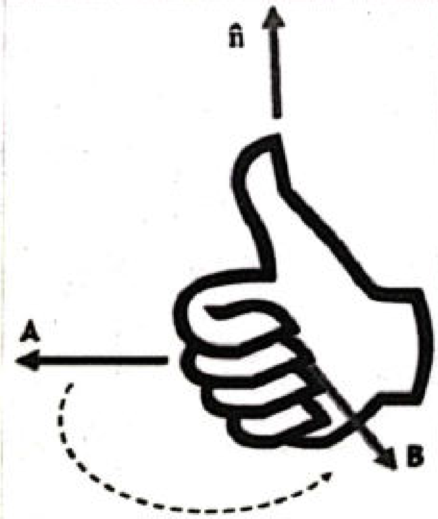



**Figure 2.5:** Right hand rule.

Examples of Vector Product:

- Torque ( $\tau$ ) is an example of vector product. It is the cross product of force (**F**) and position vector (**r**).

$$\tau = r \times F = r F \sin\theta \, \hat{n}$$

- Angular momentum (L) is also an example of vector product. It is the cross product of linear momentum **P** and position vector **r**, i.e.,

$$L = r \times P = r p \sin\theta \, \hat{n}$$

Properties of Vector Product:

1) Vector product is not commutative but anti-commutative. It means that by changing the order of vectors, the direction of vector product gets reversed, as shown in Fig. 2.6. i.e.,

$$\mathbf{A} \times \mathbf{B} \neq \mathbf{B} \times \mathbf{A} \qquad \mathbf{A} \times \mathbf{B} = -\mathbf{B} \times \mathbf{A}$$

Here, magnitude of **A** × **B** is same as **B** × **A**. According to right hand rule, the direction of **A** × **B** is pointing upward given by $\hat{n}$ and the direction of **B** × **A** is pointing downward given by $-\hat{n}$.

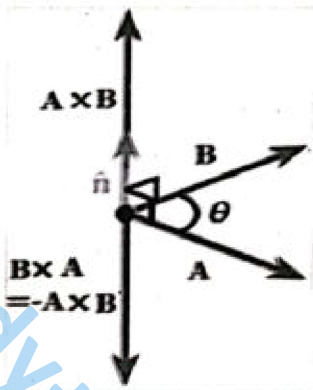



**Figure 2.6:** By changing the order of vectors, the direction of vector product gets reversed.

2) Vector product of two parallel or anti-parallel vectors is null vector (a null vector has zero magnitude and arbitrary direction). i.e.,

$$\mathbf{A} \times \mathbf{B} = AB \sin 0^\circ \hat{n} = \mathbf{0} \quad \text{and} \quad \mathbf{A} \times \mathbf{B} = AB \sin 180^\circ \hat{n} = \mathbf{0}$$

Vector product of a vector with itself is equal to null vector. i.e.,

$$\mathbf{A} \times \mathbf{A} = AA \sin 0^\circ \hat{n} = \mathbf{0}$$

Similarly, we can show that the vector product of a unit vector with itself is null vector. i.e.,

$$\hat{i} \times \hat{i} = \hat{j} \times \hat{j} = \hat{k} \times \hat{k} = 0$$

3) Vector product of two perpendicular vectors has a maximum value, and is equal to the product of their magnitudes only. Hence:

$$\mathbf{A} \times \mathbf{B} = A B \sin 90^0 \hat{n} = A B \hat{n}$$

Similarly, for mutually perpendicular unit vectors, we can show:

$$\hat{i} \times \hat{j} = \hat{k}, \qquad \hat{j} \times \hat{k} = \hat{i}, \qquad \hat{k} \times \hat{i} = \hat{j}$$

4) Vector product of two vectors in terms of their rectangular components can be given as:

$$\mathbf{A} \times \mathbf{B} = (A_x\,\hat{i} + A_y\,\hat{j} + A_z\,\hat{k}) \times (B_x\,\hat{i} + B_y\,\hat{j} + B_z\,\hat{k})$$

By using the determinant and solving, we get:

$$\mathbf{A} \times \mathbf{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix} \qquad \text{\_\_\_\_\_\_ (2.16)}$$

OR $$\mathbf{A} \times \mathbf{B} = (A_yB_z - A_zB_y)\,\hat{i} + (A_zB_x - A_xB_z)\,\hat{j} + (A_xB_y - A_yB_x)\,\hat{k} \qquad \text{\_\_\_\_\_\_ (2.17)}$$

**Physical Significance of Vector Product:**

Vector product gives us the area of a plane. For example, if two vectors **A** and **B** represent adjacent sides of a plane, as shown in the Fig. 2.7, then the magnitude of **A** × **B** gives us the magnitude of that area (the area may be rectangle or parallelogram).

Area of parallelogram = |**A** × **B**|

The unit vector $\hat{n}$ gives the direction of that area, which is normal to the plane and can be found by right hand rule, as shown in Fig. 2.5.

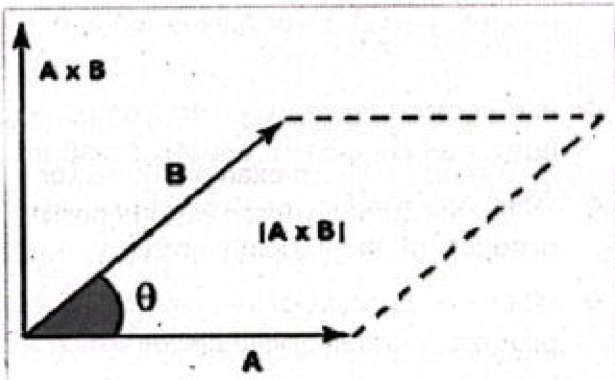



**Figure 2.7:** Magnitude of **A x B** gives area of parallelogram.

**Example 2.3: Find the area of the parallelogram whose adjacent sides are given by the vectors: A = ( $\hat{i}$ + 6 $\hat{j}$ + 2 $\hat{k}$ ) m and B = ( 7 $\hat{i}$ + $\hat{j}$ + 5 $\hat{k}$ ) m.**

Given: **A** = ( $\hat{i}$ + 6 $\hat{j}$ + 2 $\hat{k}$ ) m  **B** = ( 7 $\hat{i}$ + $\hat{j}$ + 5 $\hat{k}$ ) m

To Find: **A** × **B** = ?

Solution: As we know that vector product gives us the area of parallelogram. So,

$$\mathbf{A} \times \mathbf{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 6 & 2 \\ 7 & 1 & 5 \end{vmatrix}$$

$$\mathbf{A} \times \mathbf{B} = \hat{i}\begin{vmatrix} 6 & 2 \\ 1 & 5 \end{vmatrix} - \hat{j}\begin{vmatrix} 1 & 2 \\ 7 & 5 \end{vmatrix} + \hat{k}\begin{vmatrix} 1 & 6 \\ 7 & 1 \end{vmatrix}$$

$$\mathbf{A} \times \mathbf{B} = 28\,\hat{i} + 9\,\hat{j} - 41\,\hat{k} \ \ m^2$$

**Assignment 2.3**

**Two vectors A and B having magnitudes 3.2 unit and 5.2 unit respectively, making an angle of 60° with each other. What is the magnitude of their cross products?**

## SUMMARY

- **Vectors** are those quantities, which require direction along with their magnitude for complete description.
- A vector with unit magnitude is called **unit vector**, it is required for the information of direction of any vector.

- A vector with zero magnitude and arbitrary direction is called **null vector.**
- A vector which specifies the location of a point P (a, b) with respect to origin is called **position vector**, and can be given as: $r = a\,\hat{\imath} + b\,\hat{\jmath}$ and its magnitude is given as: $r = \sqrt{a^2 + b^2}$.
- The components of a vector which are mutually perpendicular are called **rectangular components** of that vector.
- A vector can be found, if its rectangular components are given by relations: $A = \sqrt{(A_x)^2 + (A_y)^2}$ for magnitude and $\theta = tan^{-1}\dfrac{A_y}{A_x}$ for direction.
- If a vector is given then its components can be found by the relation, $A_x = A\cos\theta$ for horizontal component and $A_y = A\sin\theta$ for vertical component.
- When the product of two vectors gives us scalar quantity, such product is called **scalar product**, mathematically given as: $\mathbf{A} \cdot \mathbf{B} = A\,B\cos\theta$.
- When the product of two vectors gives us vector quantity, such product is called **vector product**, mathematically given as: $\mathbf{A} \times \mathbf{B} = A\,B\sin\theta\,\hat{n}$.

## EXERCISE

### Multiple Choice Questions

Encircle the correct option.

1) The number of perpendicular components of a force (in 2-D) are:

A. 1 B. 2 C. 3 D. 4

2) $\hat{\jmath} \times \hat{\imath} =$ _____

A. 0 B. 1 C. $\hat{k}$ D. $-\hat{k}$

3) A force of 10 N is making an angle of 30° with the horizontal. Its x-component will be:

A. 4 N B. 5 N C. 7 N D. 8.7 N

4) If two forces of magnitude 3 N and 4 N are acting at right angle to each other than their resultant force will be:

A. 7 N B. 5 N C. 1 N D. Null vector

5) Angle between two vectors A and B can be easily determined by:

A. dot product B. cross product C. head to tail rule D. right hand rule

6) For which angle the equation $|\mathbf{A} \cdot \mathbf{B}| = |\mathbf{A} \times \mathbf{B}|$ is correct?

A. 30° B. 45° C. 60° D. 90°

### Short Questions

Give short answers of the following questions.

2.1 If the cross product of two vectors vanishes, what will you say about their orientation?

2.2 Find the dot product of unit vectors with each other at (a) 0° and (b) 90°.

2.3 Show that scalar product obeys commutative property.

2.4 Solve by using the properties of dot and cross product: (a) $\hat{i} \cdot (\hat{j} \times \hat{k})$ (b) $\hat{j} \times (\hat{j} \times \hat{k})$?

2.5 If both the dot product and the cross product of two vectors are zero. What would you conclude about the individual vectors?

2.6 What are rectangular components of a vector? How they can be found?

2.7 Give two examples for each of the scalar and vector product.

2.8 Show that: $\mathbf{A} \cdot \mathbf{B} = A_x B_x + A_y B_y + A_z B_z$.

2.9 What units are associated with the unit vectors $\hat{i}$, $\hat{j}$, and $\hat{k}$?

## Comprehensive Questions

Answer the following questions in detail.

2.1 Explain the resolution of a vector in 2-D as two rectangular components.

2.2 Describe the dot product of two vectors along with their properties.

2.3 Describe the cross product of two vectors along with their properties.

2.4 What geometric interpretation does the cross product have? Explain with the help of a diagram.

## Numerical Problems

2.1 If the magnitude of cross product between two vectors is $\sqrt{3}$ times the dot product, find angle between them. **(Ans: 60°)**

2.2 A force is acting on a body making an angle of 30° with the horizontal. The horizontal component of the force is 20 N. Find the force. **(Ans: 23.1 N)**

2.3 A vector **F** having magnitude 5.5 N makes 10° with x-axis and a vector **r** with magnitude 4.3 m makes 80° with x-axis. What is the magnitude of their dot and cross products?
**(Ans: 8.1 N m and 22.2 N m)**

2.4 The magnitude of dot and cross product of two vectors are $6\sqrt{3}$ and 6, respectively. Find the angle between the vectors. **(Ans: $\theta = 30°$)**

2.5 A force of 200 N is making an angle of 30° with the x-axis. Find the horizontal and vertical components of this force. **(Ans: 173.2 N and 100 N)**

# TRANSLATORY MOTION

UNIT 3

## Student Learning Outcomes (SLOs)



**The students will:**

- Derive the equations of motion [For uniform acceleration cases only. Derive from the definitions of velocity and acceleration as well as graphically].
- Solve problems using the equations of motion [For the cases of uniformly accelerated motion in a straight line, including the motion of bodies falling in a uniform gravitational field without air resistance. This also includes situations where the equations of motion need to be resolved into vertical and horizontal components for 2-D motion].
- Evaluate and analyse projectile motion in the absence of air resistance [This includes solving problems making use of the below facts:

  (i) Horizontal component ($V_H$) of velocity is constant.

  (ii) Acceleration is in the vertical direction and is the same as that of a vertically free falling object.

  (iii) The horizontal motion and vertical motion are independent of each other. Situations may require students to determine for projectiles:

  - How high does it go?

  - How far would it go along the level land?

  - Where would it be after a given time?

  - How long will it remain in flight?

  Situations may also require students to calculate for a projectile launched from ground height the

  - launch angle that results in the maximum range.

  - relation between the launch angles that result in the same range.
- Predict qualitatively how air resistance affects projectile motion [This includes analysis of both the horizontal component and vertical component of velocity and hence predicting qualitatively the range of the projectile.]
- Apply the principle of conservation of momentum to solve simple problems.
- [Including elastic and inelastic interactions between objects in both one and two dimensions. Knowledge of the concept of coefficient of restitution is not required.
- Examples of applications include:

  - karate chops to break a pile of bricks

  - car crashes

  - ball & bat

  - the motion under thrust of a rocket in a straight line considering short thrusts during which the mass remains constant]
- Predict and analyse motion for elastic collisions [This includes making use of the fact that for an elastic collision, total kinetic energy is conserved and the relative speed of approach is equal to the relative speed of separation]
- Justify why though the momentum of a closed system is always conserved, some change in kinetic energy may take place.

The cover photo of this chapter shows an anti-ship missile being fired by Pakistan Navy in the North Arabian Sea. The motion of a missile through the air can be described by its range, velocity, and acceleration. When it flies, a short patch of its path can be considered a straight line without any change in direction. However, its motion is not straight for long; instead, it is a 2-dimensional motion under the action of gravity, called projectile motion. Similarly, all the objects in this universe are in motion. Thus, understanding motion is also key to understanding other concepts in physics. For example, an understanding of velocity is crucial to the study of momentum. There are three types of motion: translational motion, rotational motion and vibrational motion. In this chapter, we will concern ourselves only with translational motion, such as motion along a straight line and projectile motion.

We begin with kinematics which is the branch of mechanics that deals with the study of motion without considering the causes of motion. This includes the terms such as displacement, velocity and acceleration in straight line. Using these terms, we study the motion of objects undergoing constant acceleration. We begin with derivation of the equations of motion and then we use them to solve problems of uniformly accelerated motion in a straight line, including the motion of bodies falling in a uniform gravitational field without air resistance. It is also extended to objects moving along curved paths, such as projectile motion under the action of gravity only and its applications. Next, we deal with dynamics, which is the branch of mechanics which deals with the study of forces that cause the objects and systems to move. This includes momentum and associated law of conservation.

## 3.1 EQUATIONS OF UNIFORMLY ACCELERATED MOTION

There are three equations of motion which shows the relation between initial velocity, final velocity, acceleration, displacement and time. These equations describe and predict the motion of objects under constant acceleration. To simplify the derivation of these equations, the following **assumptions** are made:

- The object is moving along the straight line.
- Acceleration is constant.
- Only magnitudes of vectors such as displacement, velocity and acceleration are considered.
- The direction of initial velocity and all the quantities in the direction of initial velocity are taken as positive.

Following are the three equations of motion:

$$v_f = v_i + a\,t$$
$$S = v_i\,t + \frac{1}{2}a\,t^2$$
$$2\,a\,S = v_f^{\,2} - v_i^{\,2}$$

Let us derive these equations of motion.

### 3.1.1 First Equation of Motion

Consider a body is moving with initial velocity $v_i$ in a straight line with constant acceleration $a$. Its velocity becomes $v_f$ after time t. The velocity-time graph for the motion of the body is shown in the Fig. 3.1. As we know that the slope of velocity-time graph gives the acceleration of body. From the graph:

$a$ = slope of line AB

or $a = \frac{BC}{AC}$

or $a = \frac{BD - CD}{OD}$ ______ (3.1)

From the graph, it can be noted that BD represents the final velocity $v_f$, CD represents the initial velocity $v_i$ and OD represents the time t. Hence, we can write the equation (3.1) as:

or $a = \frac{v_f - v_i}{t}$

or $at = v_f - v_i$

or $v_f = v_i + at$ ______ (3.2)

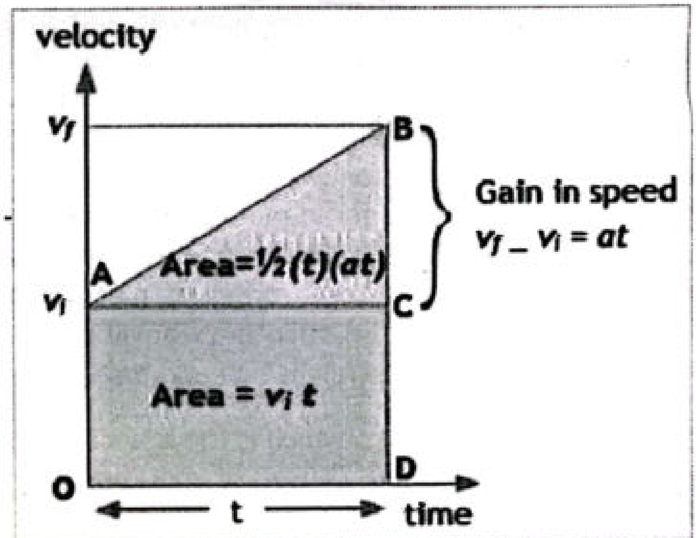



**Figure 3.1:** Velocity-time graph for a body moving with constant acceleration.

Equation (3.2) is the first equation of motion. It represents the relation between initial velocity, final velocity, acceleration and time.

## 3.1.2 Second Equation of Motion

Consider a body is moving with initial velocity $v_i$ in a straight line with constant acceleration $a$. Its velocity becomes $v_f$ after time t. The velocity-time graph for the motion of the body is shown in the Fig. 3.1.

As the total distance S travelled by the body is equal to the total area under the velocity-time graph. Hence:

Total distance S = Area of trapezium OABD

$$S = \frac{1}{2}\begin{pmatrix}\textit{sum of}\\ \textit{parallel sides}\end{pmatrix} \times \begin{pmatrix}\textit{distance between}\\ \textit{the parallel sides}\end{pmatrix}$$

As shown in the graph that OA and BD are parallel sides, and OD is the distance between these parallel sides. Therefore, we can express the total distance in terms of the above equation as:

$$S = \frac{1}{2}(OA + BD) \times OD$$

Putting OA = $v_i$, BD = $v_f$ and OD = t, we get:

$$S = \frac{1}{2}(v_i + v_f) \times t$$

Now, using first equation of motion $v_f = v_i + at$, the above equation may become:

$$S = \frac{1}{2}(v_i + v_i + a\,t) \times t$$

or $S = \frac{1}{2}(2v_i + at) \times t$

or $S = \frac{1}{2}(2v_i t + a t^2)$

Finally, we get:

$$S = v_i t + \frac{1}{2} a t^2 \quad \text{\_\_\_\_\_\_} \quad (3.3)$$

Equation (3.3) is the second equation of motion, it represents the relation between initial velocity, acceleration, distance and time.

### 3.1.3 Third Equation of Motion

Consider a body is moving with initial velocity $v_i$ in a straight line with constant acceleration $a$. Its velocity becomes $v_f$ after time t. The velocity-time graph for the motion of the body is shown in the Fig. 3.1.

As the total distance S travelled by the body is equal to the total area under the graph. Hence:

Total distance S = Area of trapezium OABD

$$S = \frac{1}{2}\left(\begin{matrix}\textit{sum of}\\ \textit{parallel sides}\end{matrix}\right) \times \left(\begin{matrix}\textit{distance between}\\ \textit{the parallel sides}\end{matrix}\right)$$

As shown in the graph that OA and BD are parallel sides, and OD is the distance between these parallel sides. Therefore, we can express the total distance in terms of the above equation as:

$$S = \frac{1}{2}(OA + BD) \times OD$$

or $2S = (OA + BD) \times OD$

Multiplying both sides by $\frac{BC}{OD}$, we get:

$$2S \times \frac{BC}{OD} = (OA + BD) \times OD \times \frac{BC}{OD}$$

As, $\frac{BC}{OD} = a$, so we get:

$$2S \times a = (OA + BD) \times BC$$

Putting $OA = v_i$, $BD = v_f$ and $BC = v_f - v_i$, we get:

$$2aS = (v_i + v_f) \times (v_f - v_i)$$

or $2aS = v_f^2 - v_i^2$ ______ (3.4)

Equation (3.4) is the third equation of motion, it represents the relation between initial velocity, final velocity, acceleration and distance.

**Example 3.1: A bus is travelling at 10 m $s^{-1}$ accelerates uniformly at 2 m $s^{-2}$. Calculate the velocity of the bus after 5 s.**

Given: Initial velocity of the bus = $v_i$ = 10 m $s^{-1}$ Acceleration = $a$ = 2 m $s^{-2}$

Time = t = 5 s

To Find: Final velocity of the bus = $v_f$ = ?

Solution: Using 1st equation of motion:

$$v_f = v_i + a\,t$$

*Putting values and solving, we get:*

$$v_f = 10 + 2(5)$$

$$v_f = 20 \text{ m s}^{-1}$$

Example 3.2: A car is travelling initially with a velocity of 4 m $s^{-1}$ then it accelerates at 1 m $s^{-2}$ for 10 s. Find the distance travelled by the car during this time.

Given: Initial velocity of the car = $v_i$ = 4 m $s^{-1}$ Acceleration = $a$ = 1 m $s^{-2}$

Time = t = 10 s

To Find: Distance travelled = S = ?

Solution: Using 2[nd] equation of motion:

$$S = v_i t + \frac{1}{2} a\,t^2$$

*Putting values and solving, we get:*

$$S = 4(10) + \frac{1}{2}(1)(10)^2$$

$$S = 40 + \frac{1}{2}(100) = 40 + 50 = 90 \text{ m}$$

Assignment 3.1

1) A train slows down from 80 km $h^{-1}$ with a uniform retardation of 2 m $s^{-2}$. How long will it take to attain a speed of 20 km $h^{-1}$?

2) A car travels with a velocity of 5 m $s^{-1}$. It then accelerates uniformly and travels a distance of 50 m. If the velocity reached is 15 m $s^{-1}$, find the acceleration and the time to travel this distance.

### 3.1.4 Free-Fall Motion

An example of uniformly accelerated motion is the motion of a free-falling body. In the absence of air resistance, all objects (lighter or heavier) fall freely near the surface of the Earth with same acceleration, independent of their masses. This acceleration is called acceleration due to gravity, denoted by $g$. It has a value of 9.81 m $s^{-2}$ and is directed towards centre of the Earth. By substituting $a = g$ and $S = h$ in equations (3.2), (3.3) and (3.4), we get the equations of motion for free-fall as given below:

$$v_f = v_i + g\,t$$

$$h = v_i\,t + \frac{1}{2} g\,t^2$$

$$2\,g\,h = (v_f)^2 - (v_i)^2$$

Using these equations, we can solve problems for the motion of bodies falling in a uniform gravitational field without air resistance.

Example 3.3: A kangaroo can jump over an object 2.50 m high. (a) Considering just its vertical motion, calculate its vertical speed when it leaves the ground. (b) How long a time is it in the air?

Given: Height = h = 2.50 m Final velocity = $v_f$ = 0 m $s^{-1}$ (At highest point)

To Find: (a) Initial velocity = $v_i$ = ? (b) Total time in the air = T = ?

Solution: (a) From 3[rd] equation of motion:

$$2\,g\,h = (v_f)^2 - (v_i)^2$$

$$2(-9.8)(2.5) = 0 - v_i^2$$
$$v_i^2 = 49\ m^2\ s^{-2}$$
$$v_i = 7\ m\ s^{-1}$$

(b) Now using 1st equation of motion:

$$v_f = v_i + g\,t$$

Where t is the time to reach the maximum height, g will be negative for moving up, so

$$0 = 7 - 9.8\,t$$
$$t = 0.71\ s$$

So, the total time in air = T = 2 (0.71 s) = 1.42 s

Assignment 3.2

A 1 kg ball is dropped from top of the leaning tower of Pisa. The ball reaches the ground in 3.34 s. Find the (a) height of the tower (b) velocity of the ball when it strikes the ground.

## 3.2 PROJECTILE MOTION

Till now, we have studied the motion of bodies in a straight line, either horizontal or vertical. There are many situations in which a body moves along a curved path in a plane having both vertical and horizontal components. For example, when a player kicks football to another player, it moves in a curved path, as shown in Fig. 3.2. Similarly, when a stone thrown horizontally in air from the top of a building, or a long jumper leaving the ground at an angle, etc., are the examples of projectile motion.

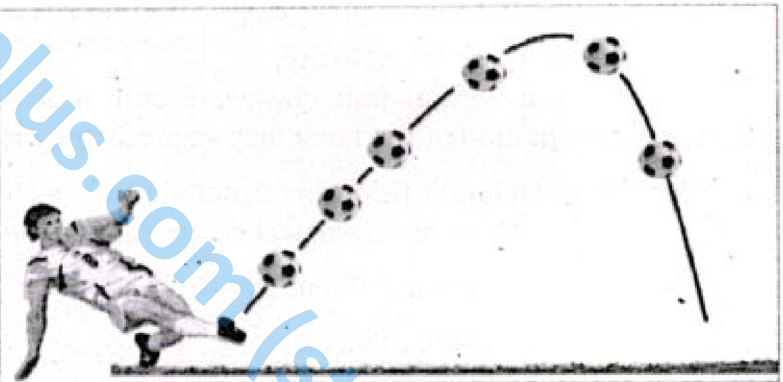


**Figure 3.2:** Two-dimensional motion of a football.

Projectile motion is a two-dimensional motion of an object thrown in the air under the action of gravitational force only.

In our discussion of projectile motion, the effects of other forces, such as air friction and rotation of the Earth, are neglected. The object that is thrown is called projectile and its path is called its trajectory. A football, a cricket ball or a baseball when thrown in air are examples of projectiles.

Experimentally, it can be proved that the horizontal and vertical motions are completely independent of each other.

Consider the motion of two different coloured balls, blue and green, as shown in Fig. 3.3. The blue ball is dropped vertically downward while the green ball is thrown horizontally at the same time from a cliff. Neglecting air resistance, both the balls hit the ground at the same time. The key to analyse two-dimensional projectile motion is to break the motion into two components, one along the horizontal axis and other along vertical axis.

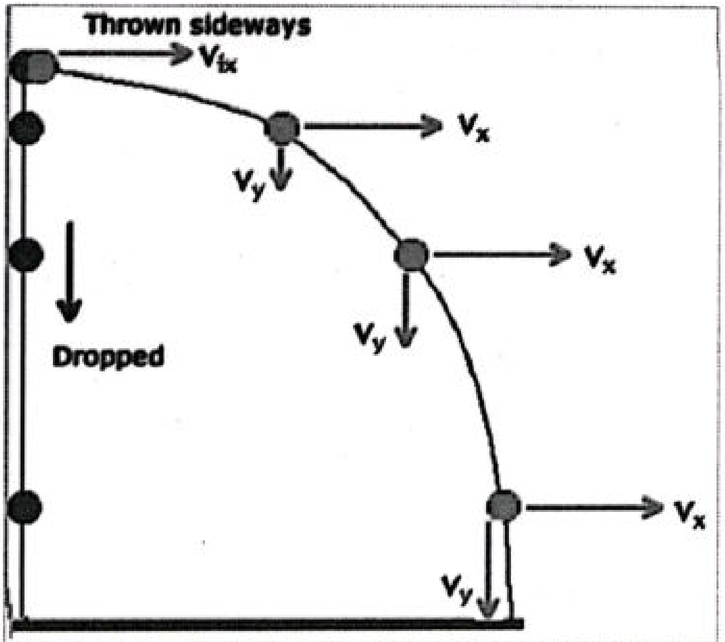


**Figure 3.3:** The two balls, one is thrown horizontally and other is thrown vertically down, hit the ground at the same time.

### 3.2.1 Projectile Motion for an Object Launched Horizontally

In many situations, when a projectile is thrown horizontally from a certain height with some initial velocity, it travels forward as well as falls downward until it strikes the ground. Hence, neglecting air resistance, the motion of the body follows projectile motion.

Consider the green ball is thrown horizontally from a cliff of certain height with velocity $v_i$, as shown in Fig. 3.3. There is no vertical component of the initial velocity, i.e., $v_{iy} = 0$. Hence:

Initial horizontal component of velocity = $v_{ix} = v_i$

Initial vertical component of velocity = $v_{iy} = 0$

Thus, there is **no horizontal acceleration**, while the **vertical acceleration is the acceleration due to gravity**, i.e.,

$$a_x = 0, \qquad a_y = g$$

Acceleration due to gravity g is taken positive when the ball is falling downward and negative when the ball is moving upward.

The horizontal component of velocity remains constant, while the vertical component of velocity increases. So, at any instant $t$, its velocity components are:

**Horizontal component of velocity is:**

$$v_{fx} = v_{ix} = v_i \cos\theta$$

**Vertical component of velocity is:**

$$v_{fy} = v_{iy} + a_y t$$

$$v_{fy} = 0 + g\,t$$

$$v_{fy} = g\,t$$

Hence, the magnitude of instantaneous velocity $t$ is given by:

$$v_f = \sqrt{(v_{fx})^2 + (v_{fy})^2} \qquad \text{\_\_\_\_\_ (3.9)}$$

The direction $\theta$ of instantaneous velocity $v_f$ is determined as:

$$\theta = \tan^{-1}\left(\frac{v_{fy}}{v_{fx}}\right) \qquad \text{\_\_\_\_\_ (3.10)}$$

At instant t, the **horizontal displacement** $X$ covered by the body is given by:

$$X = v_{fx}\,t \qquad \text{\_\_\_\_\_ (3.11)}$$

And the **vertical displacement**, as the body moves downward from the height is given by:

$$Y = \frac{1}{2}gt^2 \qquad \text{\_\_\_\_\_ (3.12)}$$

### 3.2.2 Projectile Motion for an Object Launched at Some Angle with Horizontal

There are many situations in which a projectile (object) is thrown with velocity $v_i$ at an angle $\theta$ with the horizontal. After given an initial push, it moves under the action of force of gravity, as shown in Fig. 3.4. Here, the air resistance is neglected.

There is no horizontal acceleration, while the vertical acceleration is the acceleration due to gravity, i.e.,

$$a_x = 0, \qquad a_y = -g$$

Resolve the initial velocity into its horizontal and vertical components:

$$v_{ix} = v_i \cos\theta$$

$$v_{iy} = v_i \sin\theta$$

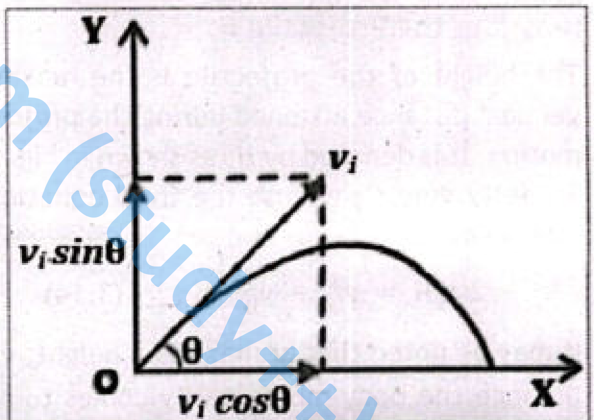



**Figure 3.4:** Rectangular components of initial velocity.

The horizontal component of velocity remains constant, while the vertical component of velocity changes. So, at any instant $t$, its velocity components are expressed as:

**Horizontal component of velocity is:**

$$v_{fx} = v_{ix} = v_i \cos\theta$$

**Vertical component of velocity is:**

$$v_{fy} = v_{iy} + a_y t$$

$$v_{fy} = v_i \sin\theta - g\,t$$

Hence, magnitude of instantaneous velocity at any instant t is given by:

$$v_f = \sqrt{(v_{fx})^2 + (v_{fy})^2}$$

The direction $\theta$ of instantaneous velocity $v_f$ is determined as:

$$\theta = \tan^{-1}\left(\frac{v_{fy}}{v_{fx}}\right)$$

At instant t, the **horizontal displacement** $x$ covered by the body is given by:

$$x = v_{fx}\, t$$

$$x = v_i \cos\theta \times t$$

And the **vertical displacement**, as the body moves upward, is given by:

$$y = v_{iy}\, t + \frac{1}{2} a_y t^2$$

$$y = (v_i \sin\theta)\, t - \frac{1}{2} g\, t^2 \qquad \text{\_\_\_\_\_ (3.13)}$$

We will describe the maximum height, the range and time of flight of a projectile in the coming sections.

### Height of the Projectile H:

The height of the projectile is the maximum vertical distance attained during the projectile motion. It is denoted by $H$, as shown in Fig. 3.5. To determine $H$, we use the third equation of motion as;

$$2\, a_y H = v_{fy}^2 - v_{iy}^2 \qquad \text{\_\_\_\_\_ (3.14)}$$

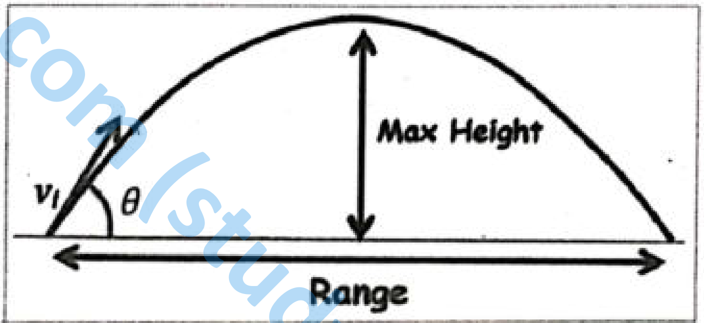


**Figure 3.5:** Height and range of a projectile.

It may be noted that at maximum height, $v_{fy} = 0$, since the body momentarily comes to rest. Also, for a projectile moving upward, $a_y = -g$, and equation (3.14) becomes:

$$2(-g)H = 0 - v_{iy}^2$$

Since, $v_{iy} = v_i \sin\theta$ so, we get: $\quad -2\,g\,H = 0 - v_i^2 \sin^2\theta$

By arranging, we get:

$$H = \frac{v_i^2 \sin^2\theta}{2g} \qquad \text{\_\_\_\_\_ (3.15)}$$

Using this equation, we can find the height of the projectile if the magnitude and direction of the initial velocity is known.

Time of flight T:

The time taken by the projectile from the point of projection to the point where it hits the ground at same level is called the time of flight. It is denoted by T. To determine T, we use the second equation of motion:

$$s = v_i t + \frac{1}{2} a t^2 \qquad \text{\_\_\_\_\_\_ (3.16)}$$

For y-direction, equation (3.16) becomes:

$$H = v_{iy} t + \frac{1}{2} a_y t^2$$

Since, the projectile returns to ground (same height), so we take $H = 0$. Substituting $a_y = -g$, $v_{iy} = v_i \sin\theta$ and $t = T$ in above equation, we get:

$$0 = (v_i \sin\theta)\,T - \frac{1}{2} g T^2$$

By rearranging, we get:

$$T = \frac{2v_i \sin\theta}{g} \qquad \text{\_\_\_\_\_\_ (3.17)}$$

If magnitude and direction of the initial velocity is known, then this equation can be used to find the time of flight of the projectile.

The time taken by the projectile to reach the highest point is called the time of summit. It is denoted by T′ and is given by:

$$T' = \frac{T}{2} = \frac{v_i \sin\theta}{g} \qquad \text{\_\_\_\_\_\_ (3.18)}$$

Range of the Projectile R:

The horizontal distance travelled by a projectile is called range. It is denoted by R. To determine R, we use the relation, $S = vt$. As the range is horizontal distance, so we can write:

$$X = v_x t$$

Substituting $X = R, v_x = v_i \cos\theta$ and $t = T = \frac{2v_i \sin\theta}{g}$ in above equation, we get:

$$R = v_i \cos\theta \times \frac{2v_i \sin\theta}{g}$$

or

$$R = \frac{v_i^2 \sin 2\theta}{g} \qquad \text{\_\_\_\_\_\_ (3.19)}$$

This equation can be used to find range of a projectile, if magnitude and direction of initial velocity are known. Thus, by knowing two quantities; magnitude and direction of initial velocity, we can find height, range and time of flight for the projectile.

Maximum Range $R_{max}$: The greater the initial speed, the greater the range. For a given value of initial velocity $v_i$, the range of the projectile is maximum if $\sin 2\theta = 1$, which occurs when

$2\theta = 90^\circ$ or $\theta = 45^\circ$. By substituting the value of $\theta = 45^\circ$ in equation (3.19), we get maximum range, i.e.,

$$R_{max} = \frac{v_i^2}{g} \qquad (3.20)$$

Same Range: If the initial speed of the projectile $v_i$ and acceleration due to gravity g remain constant, then there are always two angles for which the projectile has the same range, as shown in Fig. 3.6 (a). These angles are complementary to each other i.e., $\theta$ and $90^\circ - \theta$. Hence, a projectile has the same range for pairs of angles such as (75°, 15°), (60°, 30°) and (70°, 20°), etc.

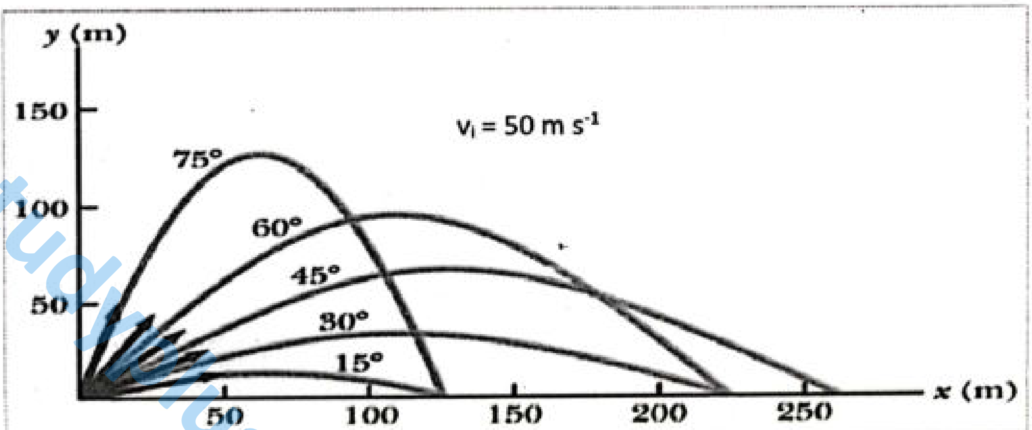


Figure 3.6 (a): Height verses range at different angles of projectile.

Effect of Air Resistance on Projectile Motion:

Air resistance affects the motion just like friction. Due to air resistance, particle's energy decreases. Generally, air resistance decreases the velocity of projectile. As a result, both the horizontal and vertical components of velocity decreases.

Air resistance affects the parabolic motion of a projectile by reducing its range and maximum height. Hence, air resistance can significantly alter the trajectory of the motion, as shown in Fig. 3.6 (b). Air resistance also increases the time of flight of the projectile.

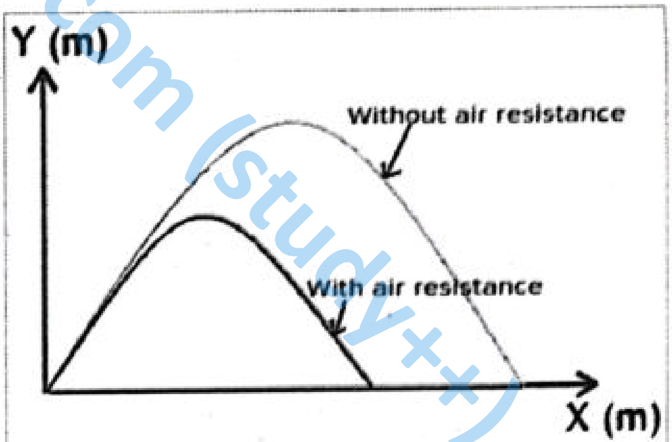


Figure 3.6 (b): Trajectory of a projectile with and without air resistance.

**Example 3.4: If a projectile is launched horizontally with a speed of 12.0 m s⁻¹ from the top of a 24.6 m high building. Determine the horizontal displacement of the projectile.**

Given: $v_{ix} = 12.0\ m\ s^{-1}$ $\qquad y = 24.6\ m$

To Find: $x = ?$

Solution: First, we need to find time t, for this we use second equation of motion in y-direction.

$$y = v_{iy}\,t + \frac{1}{2}a_y t^2$$

For a projectile launched horizontally:

$v_{iy} = 0$, $a_y = g = 9.8\ m/s^2$, so we get:

$$24.6 = (0)t + \frac{1}{2}(9.8)\,t^2$$

$$t = 2.24\ s$$

Now using second equation of motion in x-direction, we get:

$$x = v_{ix}\,t + \frac{1}{2}a_x\,t^2$$

For a projectile launched horizontally $a_x = 0\ m/s^2$, so we get:

$$x = (12)\,(2.24) + \frac{1}{2}(0)t^2$$

$$x = 26.9\ m.$$

**Example 3.5: A projectile is launched with an initial speed of 200.0 m s$^{-1}$ at an angle of 30° with the horizontal.**

**(a) Determine the time of flight of the projectile.**

**(b) Determine the peak height of the projectile.**

**(c) Determine the horizontal displacement of the projectile.**

Given: $v_i = 200\ m\ s^{-1}$ $\quad \theta = 30°$

To Find: (a) Time of flight = T = ? (b) Maximum height = H = ?

(c) Range of projectile = R = ?

Solution: (a) Time of flight = T = ?

$$T = \frac{2v_i \sin\theta}{g}$$

$$T = \frac{2(200)\sin 30°}{9.8} = 20.41\ s$$

(b) Maximum height = H = ?

$$H = \frac{v_i^2 \sin^2\theta}{2g}$$

$$H = \frac{(200)^2 \sin^2 30°}{2(9.8)} = 510.2\ m$$

(c) Range of projectile = R = ?

$$R = \frac{v_i^2 \sin 2\theta}{g}$$

$$R = \frac{(200)^2 \sin 2(30°)}{9.8} = 3534.7\ m$$

**Assignment 3.3**

**A ball is thrown with a speed of 30 m $s^{-1}$ at an angle of 30° with the horizontal. Determine: (a) the height to which it rises, (b) the time of flight and (c) the horizontal range.**

## 3.3 LAW OF CONSERVATION OF MOMENTUM

If a system consists of particles, each moving with different velocities, then the total momentum of an isolated system is the sum of the momenta of the individual particles. A system is said to be isolated if and only if the net external force, such as the gravitational force or friction, acting on the system is zero. There is no ideally isolated system in the universe, but we consider an isolated system that does not interact with its environment. The law of conservation of momentum states as:

**The total momentum of an isolated system of interacting particles is conserved.**

If $p_i$ and $p_f$ are initial and final momenta of an isolated system, then according to law of conservation of momentum:

$$P_f = P_i$$

or $$P_f - P_i = 0$$

or $$\Delta p = 0 \quad (3.21)$$

The law of conservation of momentum is useful in collision problems. When a collision occurs in an isolated system due to internal forces, the momentum of each particle changes. Such forces do not contribute to the net force, which remains zero. Hence, the total momentum of the system does not change with the passage of time. It is also applicable in explosions.

### 3.3.1 Applications of Conservation of Momentum

In an explosion, chemical energy (stored in the bonds of the atoms) is transformed into the kinetic energy of the fragments. Using the principles of conservation of momentum, numerical values can be predicted for the fragments after the explosion.

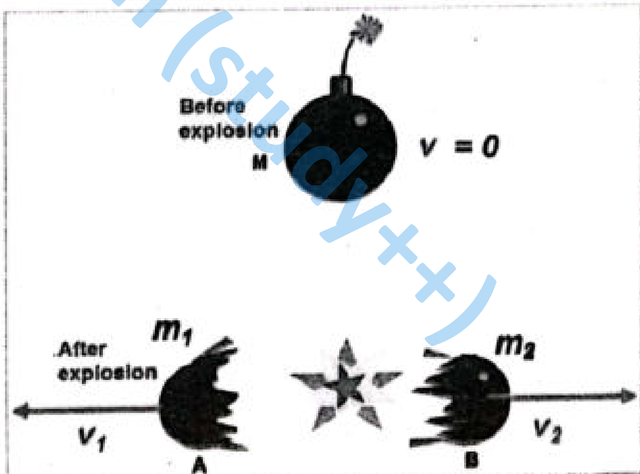



Figure 3.7 (a): Explosion of a bomb into two fragments A and B.

All objects before an explosion are generally considered at rest. After and during the explosion, the objects fly away in different directions with different speeds. Momentum is always conserved; in this case, the initial momentum of everything is zero. Knowing this, the final momentum should be zero too. Consider the explosion of a bomb into two fragments identified as A and B, as shown in Fig. 3.7 (a). The initial momentum of the system before the explosion is zero. As the momentum must be conserved, so sum of the final momentum of the two fragments must be zero.

Therefore, the fragments of same masses must move in opposite directions with equal speed for their momentum to be conserved.

An explosion in a cannon also follows conservation of momentum. For example, consider a cannon on a frictionless ground shooting a cannon ball, as shown in Fig. 3.7 (b).


**Figure 3.7 (b):** Momentum conservation in cannon.

The sum of initial momentum is zero, because nothing is moving. After explosion inside the cannon, the cannon ball will be shot forward at very high speed, while the cannon recoils at a much slower speed due to its heavy mass. Sum of the final momentum will be zero, this makes the initial momentum and the final momentum equal.

**Figure 3.8 (a):** A karate player strikes the pile with his hand very fast and break it with a single blow.

In explosions, objects move apart instead of coming together like in collision.

**A karate player** can break a pile of tiles with a single blow because he strikes the pile with his hand very fast, as shown in Fig. 3.8 (a). In doing so, the large momentum of his hand is reduced to zero in a very short time interval. This exerts a large force on the pile of tiles which is sufficient to break them apart.

Conservation of momentum is also applied on **ball and bat**. When a ball hits on the bat, the total momentum before and after the collision must be equal (You have to add up the momentum of ball and bat). So, the momentum of ball and bat before and after the collision must be equal, as shown in Fig 3.8 (b).

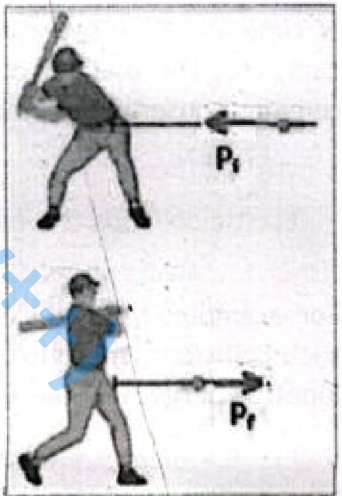



**Figure 3.8 (b):** The momentum of ball and bat before and after the collision must be equal.

**Rockets and jet engines** also work on the law of conservation of momentum. In these machines, hot gases produced by burning of fuel rush out with large momentum. As a result, the machines gain an equal and opposite momentum. This enables them to move with very high velocity.

Conservation of momentum is also applied when two car hits each other. Net momentum of cars before and after the collision must be equal.

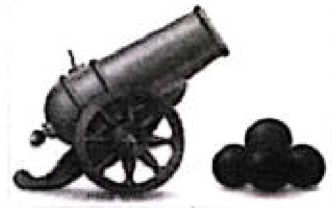
**Example 3.6: A 46 g tennis ball is launched from a 1.35 kg homemade cannon. If the cannon recoils with a speed of 2.1 m $s^{-1}$, determine the muzzle speed of the tennis ball.**

Given: Mass of the ball = $m_b$ = 46 g = 0.046 kg

Mass of the cannon = $m_c$ = 1.35 kg

Recoil speed of the cannon = $v_c = -2.1$ m $s^{-1}$

To Find: Muzzle speed of the ball = $v_b$ = ?



Solution: Initially, the ball is in the cannon and both objects are at rest. The total momentum of system is initially 0. i.e.,

$$p_i = 0$$

After the explosion, the total momentum of system must also be 0. Thus, the cannon's backward momentum must be equal to the ball's forward momentum.

$$p_f = 0$$

$$m_c v_c + m_b v_b = 0$$

$$(1.35)(-2.1) + (0.046)v_b = 0$$

$$-2.8 + (0.046)\ v_b = 0$$

Solving for $v_b$, we get:

$$v_b = 2.8/0.046 = 60.87 \text{ m s}^{-1}$$

**Assignment 3.4**

**A bullet of mass 20 g is fired from a gun with a muzzle velocity of 100 m $s^{-1}$. Find the recoil of the gun if its mass is 5 kg.**

## 3.4 ELASTIC AND INELASTIC COLLISIONS

A collision occurs when two bodies come in physical contact with each other for a short interval of time and then separate. For example, the collision of ball with a bat, the collision of two cars, etc. There are two types of collisions: Elastic collision and inelastic collision. Momentum remains conserved in all types of collisions, but kinetic energy may change.

### Elastic Collision

Elastic collision is a collision in which both the momentum and the kinetic energy of the system are conserved.

For example, the collisions between atomic and subatomic particles are elastic in nature. In such collisions, the two objects collide and return to their original shapes with no loss of total kinetic energy, i.e. the kinetic energy does not change into other types of energy.

### Inelastic Collision

Inelastic collision is a collision in which the momentum of system is conserved but kinetic energy is not conserved.

For example, a meteorite falls on the Earth. During inelastic collisions, the kinetic energy is transformed into other forms of energy, such as heat energy, sound energy, and material deformation etc.

### 3.4.1 Elastic Collision

Consider an isolated system consisting of two spherical bodies such as billiard balls of masses $m_1$ and $m_2$ initially moving towards right with velocities $u_1$ and $u_2$ along a straight line without rotation. The speed of body having mass $m_1$ is greater than speed of body having mass $m_2$ and is approaching to it. So, the two bodies encounter head-on elastic collision. After collision they move with velocities $v_1$ and $v_2$ along the same straight line, as shown in Fig. 3.9.

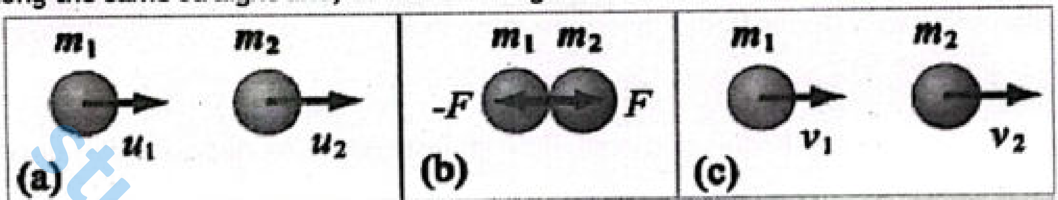



**Figure 3.9:** Two balls: (a) before collision (b) during collision (c) after collision.



We take the velocity as positive if a body is moving towards right and negative if it is moving towards left. Since the collision is elastic, therefore, we have two conservation laws in this case. According to the **law of conservation of momentum:**

$$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2 \qquad \text{(3.22)}$$

After rewriting the above equation, we have:

$$m_1u_1 - m_1v_1 = m_2v_2 - m_2u_2$$

$$m_1(u_1 - v_1) = m_2(v_2 - u_2) \qquad \text{(3.23)}$$

Similarly, according to the **law of conservation of kinetic energy**, we get:

$$\frac{1}{2}m_1u_1^2 + \frac{1}{2}m_2u_2^2 = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2$$

After cancelling the factor 1/2, rewrite the above equation:

$$m_1(u_1^2 - v_1^2) = m_2(v_2^2 - u_2^2)$$

$$m_1(u_1 - v_1)(u_1 + v_1) = m_2(v_2 - u_2)(v_2 + u_2) \qquad \text{(3.24)}$$

Dividing equation (3.24) by (3.23), we get:

$$u_1 + v_1 = v_2 + u_2$$

or $$u_1 - u_2 = v_2 - v_1$$

or $$u_1 - u_2 = -(v_1 - v_2) \qquad \text{(3.25)}$$

This equation shows that relative speed of two bodies before collision is equal but opposite to relative speed after collision. Hence, for two bodies colliding elastically, the relative speed of approach before collision is equal to the relative speed of separation after collision. From equation (3.25), we get: $v_2 = u_1 + v_1 - u_2$ ______ (3.26)

By putting the value of $v_2$ from equation (3.25) into the equation (3.22), we get:

$$m_1u_1 + m_2u_2 = m_1v_1 + m_2(u_1 + v_1 - u_2)$$

or $$m_1u_1 + m_2u_2 = m_1v_1 + m_2u_1 + m_2v_1 - m_2u_2$$

or $$m_1v_1 + m_2v_1 = m_1u_1 + m_2u_2 - m_2u_1 + m_2u_2$$

or $$(m_1 + m_2)v_1 = (m_1 - m_2)u_1 + 2m_2u_2$$

or $$v_1 = \frac{(m_1 - m_2)}{(m_1 + m_2)}u_1 + \frac{2m_2}{(m_1 + m_2)}u_2 \qquad \text{(3.27)}$$

Similarly, from equation (3.25)

$$v_1 = u_2 + v_2 - u_1 \qquad \text{(3.28)}$$

By putting the value of $v_1$ from equation (3.28) into the equation (3.22), we get:

$$m_1u_1 + m_2u_2 = m_1(u_2 + v_2 - u_1) + m_2v_2$$

or $$m_1u_1 + m_2u_2 = m_1u_2 + m_1v_2 - m_1u_1 + m_2v_2$$

or $$m_1v_2 + m_2v_2 = m_1u_1 + m_2u_2 - m_1u_2 + m_1u_1$$

or $$(m_1 + m_2)v_2 = 2m_1u_1 - (m_1 - m_2)u_2$$

or $$v_2 = \frac{2m_1}{(m_1 + m_2)}u_1 - \frac{(m_1 - m_2)}{(m_1 + m_2)}u_2 \qquad \text{------ (3.29)}$$

Equations (3.27) and (3.29) give the velocities of two bodies after collision.

Case 1: When the two colliding bodies have the same mass i.e., $m_1 = m_2$, then from equations (3.27) and (3.29), we get:

$$v_1 = u_2 \qquad \text{and} \qquad v_2 = u_1$$

This shows that velocities of bodies exchange during elastic collision of two bodies having equal masses.

Case 2: When bodies have the same mass i.e., $m_1 = m_2$, and the second body (target) is at rest ($u_2 = 0$), then from equations (3.27) and (3.29), we get:

$$v_1 = 0 \qquad \text{and} \qquad v_2 = u_1$$

This shows that when the first body comes to rest the second body moves with the initial velocity of the first body.

Case 3: When a lighter body ($m_1$) collides with a massive body ($m_2 \gg m_1$) at rest ($u_2 = 0$), then under such condition $m_1$ can be neglected i.e., $m_1 = 0$, so from equations (3.27) and (3.29), we get:

$$v_1 = -u_1 \qquad \text{and} \qquad v_2 = 0$$

Hence the first body (which is lighter) rebounds with the same initial velocity but in opposite direction. The second body (which is heavier) remain at rest even after collision. For example, if a ball is thrown at a fixed wall, the ball will bounce back with the same velocity, it had initially, but in the opposite direction.

Case 4: When a massive body ($m_1$) collides with a lighter body ($m_1 \gg m_2$) at rest ($u_2 = 0$), then under such condition $m_2$ can be neglected i.e., $m_2 = 0$, so from equations (3.27) and (3.29), we get:

$$v_1 = u_1 \qquad \text{and} \qquad v_2 = 2\,u_1$$

Hence, the first body (which is heavier) continues to move with the same initial velocity. While the second body (which is lighter) will move with twice the initial velocity of the first body.

**Example 3.7: An object of mass 5 kg moving at 10 m s$^{-1}$ collides with another object of mass 10 kg moving in the same direction at 5 m s$^{-1}$. Assume that the collision is a one-dimensional elastic collision. What will be the speed of both objects after the collision?**

Given: $m_1 = 5$ kg $u_1 = 10$ m s$^{-1}$ $m_2 = 10$ kg $u_2 = 5$ m s$^{-1}$

To Find: $v_1 = ?$ $v_2 = ?$

Solution: Using the relation:

$$v_1 = \frac{(m_1 - m_2)}{(m_1 + m_2)}u_1 + \frac{2m_2}{(m_1 + m_2)}u_2$$

Putting values: $$v_1 = \frac{(5 - 10)}{(5 + 10)}10 + \frac{2(10)}{(5 + 10)}5$$

$$= 50/15 = 3.3 \text{ m s}^{-1}$$

Similarly,
$$v_2 = \frac{2m_1}{(m_1+m_2)}u_1 - \frac{(m_1-m_2)}{(m_1+m_2)}u_2$$

$$v_2 = \frac{2(5)}{(5+10)}10 - \frac{(5-10)}{(5+10)}5 = 8.33 \text{ m s}^{-1}$$

Hence, velocity of the lighter object will decrease to 3.3 m $s^{-1}$ and heavier object will move with the velocity i.e., 8.33 m $s^{-1}$.

## SUMMARY

- **Equations of motion** show the relation between initial velocity, final velocity, acceleration, displacement and time.
- **Projectile motion** is a two-dimensional motion of an object thrown in the air under the action of gravitational force only. In projectile motion horizontal component of velocity remains constant, while the vertical component of velocity changes.
- The **height of the projectile** is the maximum vertical distance attained by it during projectile motion.
- The time taken by the projectile to reach the maximum height and then return to the ground is called the **time of flight.**
- The horizontal distance travelled by a projectile is called **range.** A projectile has the same range for complementary angles.
- The **law of conservation of momentum** states, the total momentum of an isolated system of interacting particles is conserved.
- An **elastic collision** is a collision in which both the momentum and the kinetic energy of the system are conserved.
- An **inelastic collision** is a collision in which the momentum is conserved but kinetic energy is not conserved.



## EXERCISE

### Multiple Choice Questions

Encircle the correct option.

1) A projectile thrown upward moves in its parabolic path, the velocity and acceleration vectors for the projectile are perpendicular to each other at:

A. no where B. the highest point C. the launch point D. the landing point

2) A truck driving along a highway road has a large amount of momentum. If it moves at the same speed but has twice as much mass, its momentum is _______.

A. zero B. quadrupled C. doubled D. unchanged

3) A 5 N force is applied to a 3 kg ball to change its velocity from 9 m $s^{-1}$ to 3 m $s^{-1}$. This impulse causes the momentum change of the ball to be ____ kg m $s^{-1}$.

A. −2.5 B. −10 C. −18 D. −45

4) Which of the following statements is true about the projectile motion?

A. Projectile motion is the motion of an object projected vertically upward into the air and moving under the influence of gravity.

B. Projectile motion is the motion of an object projected into the air and moving independently of gravity.

C. Projectile motion is the motion of an object projected into the air and moving under the influence of gravity.

D. Projectile motion is the motion of an object projected horizontally into the air and moving independently of gravity.

5) The vertical component of velocity of a projectile is smallest at:

A. The instant it is thrown. B. Halfway to the top.
C. The top. D. The landing point.

6) A 4 kg object has a momentum of 12 kg m $s^{-1}$. The object's speed is:

A. 3 m $s^{-1}$ B. 4 m $s^{-1}$ C. 12 m $s^{-1}$ D. 48 m $s^{-1}$

7) A bomb of mass 9 $kg$ explodes into 2 pieces of masses 3 kg and 6 kg. The velocity of mass 3 kg is 1.6 m $s^{-1}$, the kinetic energy of mass 6 kg is:

A. 3.84 J B. 9.6 J C. 1.92 J D. 2.92 J

8) What is the force experienced by a projectile after the initial force that launched it into the air, in the absence of air resistance?

A. The gravitational force B. The nuclear force

C. The contact force D. The electromagnetic force

9) If a projectile is launched on level ground, what launch angle maximizes the range of the projectile?

A. 0° B. 30° C. 45° D. 90°

## Short Questions

**Give short answers of the following questions.**

**3.1** What are the conditions for using the equations of motion?

**3.2** You through a small ball vertically up in the air. How are the velocity and acceleration of the ball oriented with respect to one another (a) when the ball is moving upward (b) when the ball is moving downward?

**3.3** For a projectile motion, is the velocity ever zero? Is the acceleration ever zero?

**3.4** Draw a diagram showing the velocity and acceleration of a projectile at several points along its path, assuming (a) the projectile is launched horizontally and (b) the projectile is launched at an angle θ with the horizontal.

**3.5** An aeroplane while flying horizontally drops a bomb when reaches exactly above the target, but misses it. Explain why?

**3.6** How air resistance affects projectile motion?

**3.7** Why do a slow-moving loaded truck and a speeding rifle bullet each have a large momentum?

**3.8** What is the difference between elastic and inelastic collision?

**3.9** An object that has a small mass and an object that has a large mass have the same momentum. Which object has the largest kinetic energy?

**3.10** Can objects in a system have momentum while the momentum of the system is zero? Explain your answer.

**3.11** For any specific velocity of projection, prove that the maximum range is equal to four times of the corresponding height.

**3.12** Is momentum conserved when a bat hits a ball? How?

## Comprehensive Questions

**Answer the following questions in detail.**

**3.1** Derive the equations of motion for uniformly accelerated objects.

**3.2** Define the law of conservation of momentum. Explain how it applies to a handball bouncing off a wall.

**3.3** Explain elastic collision and prove that for two bodies colliding elastically, relative speed of approach before collision is equal to relative speed of separation after collision.

**3.4** Discuss the motion of a projectile in the absence of air resistance.

**3.5** Derive mathematical equations for (a) Maximum height attained, (b) time of flight, (c) range of a projectile.

**3.6** Discuss that why though the momentum of a closed system is always conserved, but some change in kinetic energy may take place.

## Numerical Problems

**3.1** On dry concrete, a car can decelerate at a rate of 7.00 m $s^{-2}$, whereas on wet concrete it can decelerate at only 5.00 m $s^{-2}$. Compare the distances necessary to stop the car moving at 30.0 m $s^{-1}$ (about 110 km $h^{-1}$) (a) on dry concrete and (b) on wet concrete.

**(Ans: 64.3 m / 90.0 m)**

**3.2** Find the angle of projection of a projectile for which the maximum height and corresponding range are equal. **(Ans: 76°)**

**3.3** A jet plane comes in for a landing with a speed of 100 m $s^{-1}$, and its acceleration can have a maximum magnitude of 5.00 m $s^{-2}$ as it comes to rest.

(a) From the instant the plane touches the runway, what is the minimum time interval needed before it can come to rest?

(b) Can this plane land on a small tropical island airport where the runway is 0.8 km long? Explain your answer.

**(Ans: 20 s, it cannot, it would require a runway of minimum length 1 km)**

3.4 A projectile is launched with an initial speed of 21.8 m $s^{-1}$ at an angle of 35°above the horizontal. Determine:
(a) the time of flight of the projectile.
(b) the peak height of the projectile.
(c) the horizontal displacement of the projectile. **(Ans: 2.55 s, 7.98 m, 45.6 m)**

3.5 A projectile is launched horizontally from the top of a 45.2 m high cliff and lands a distance of 17.6 m from the base of the cliff. Determine the magnitude of the launch velocity.

**(Ans: 5.79 m $s^{-1}$)**

3.6 Two equal-mass carts roll towards each other on a level, low-friction track. One cart rolls rightward at 2 m $s^{-1}$ and the other cart rolls leftward at 1 m $s^{-1}$. After the carts collide, they couple and roll together. Ignoring resistive forces, find their combined speed.

**(Ans: 0.5 m $s^{1}$)**

3.7 A 0.5 kg ball traveling at a speed of 4 m $s^{-1}$ to the right collides elastically with another ball of 3.5 kg which is initially at rest. Find velocities of both the balls after collision?

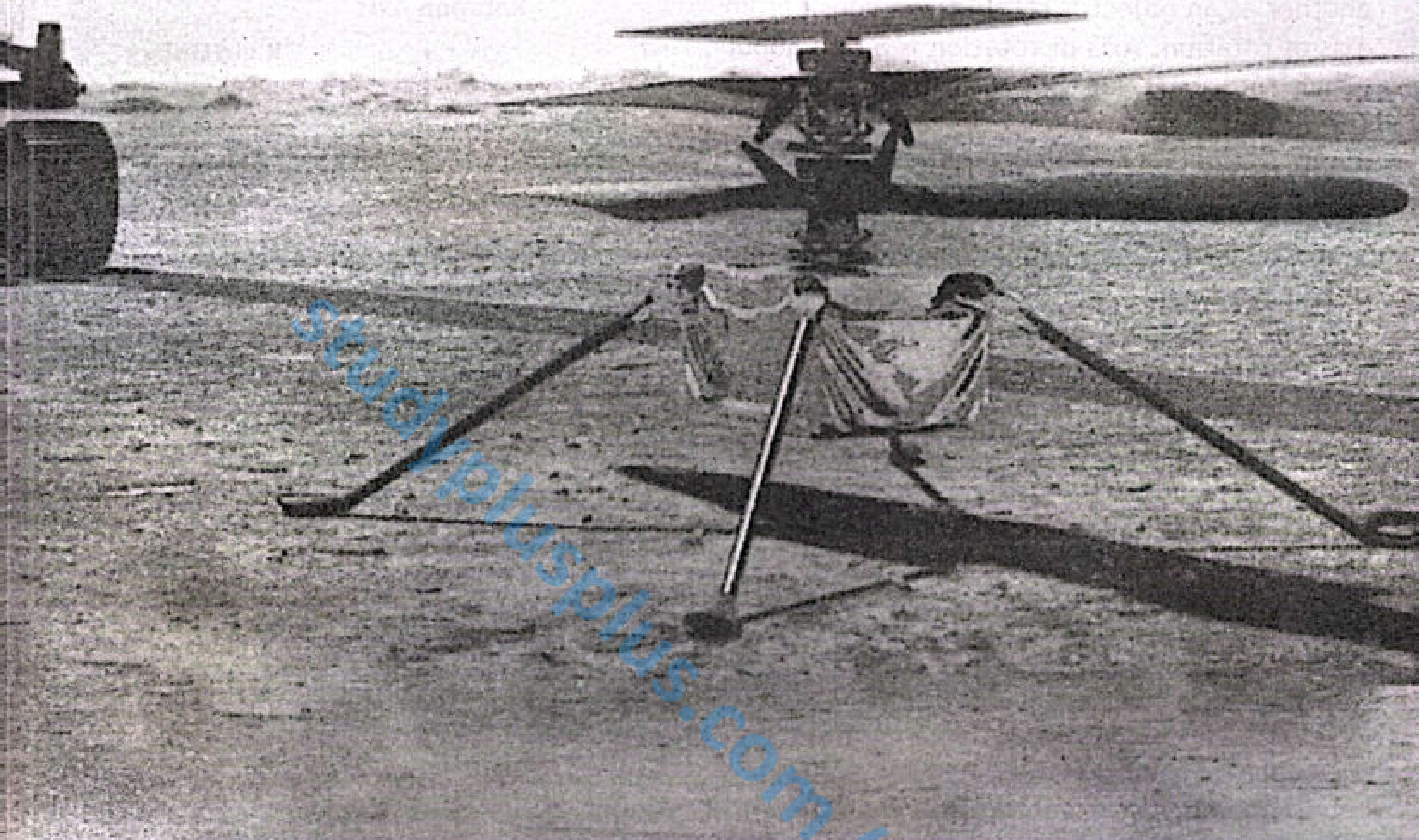
**(Ans: −3.0 m $s^{-1}$, 1.0 m $s^{-1}$)**

3.8 A 17.5 g bullet is fired at a muzzle velocity of 582 m $s^{-1}$ from a gun with a mass of 8.0 kg and a barrel length of 75.0 cm.
(a) How long is the bullet travelled in the barrel?
(b) What is the force on the bullet while it is in the barrel?
(c) Find the impulse exerted on the bullet while it is in the barrel.
(d) Find the bullet's momentum as it leaves the barrel.

**(Ans: 0.00258 s, 3950 N, 10.2 kg m $s^{-1}$, 10.2 kg m $s^{-1}$)**

UNIT 4

# ROTATIONAL AND CIRCULAR MOTION





## Student Learning Outcomes (SLOs)

**The students will:**

- Express angles in radians.
- Define and calculate angular displacement, angular velocity and angular acceleration [This involves use of $S = r\theta$, $v = r\omega$, $\omega = 2\pi/T$, $a == r\omega^2$ and $a = v^2/r$ to solve problems].
- Use **equations of angular motion** to solve problems involving rotational motions.
- Analyse qualitatively motion in a curved path due to a perpendicular force.
- Define and calculate centripetal force [Use $F = mr\omega^2$, $F = mv^2/r$].
- Analyze situations involving circular motion in terms of centripetal force [e.g. situations in which centripetal acceleration is caused by a tension force, a frictional force, a gravitational force, *or a normal force*].
- Define and calculate moment of inertia of a body and angular momentum.
- State and apply the law of conservation of angular momentum. Illustrate the applications of conservation of angular momentum in real life. [Such as by flywheels to store rotational energy, by gyroscopes in navigation systems, by ice skaters to adjust their angular velocity].
- Justify how a centrifuge is used to separate materials using centripetal force.
- Derive and apply the relation between torque, moment of inertia and angular acceleration.
- Explain why the objects in orbiting satellites appear to be weightless.
- Describe how artificial gravity is created to counter weightlessness.

Rotational motion is the turning or spinning motion of an object about an axis that passes through it. For rotational motion of rigid objects, which are non-deformable and the particles forming it stay in fixed positions relative to one another as an object is rotated, we consider an axis of rotation. Axis of rotation is a line about which rotation takes place. This line remains fixed during rotational motion, while the other points of the body move in circles about it.

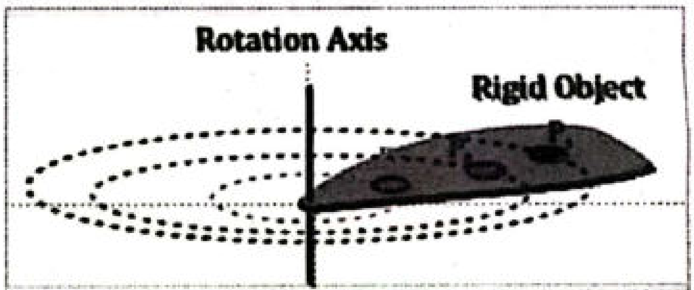


**Figure 4.1:** Axis of rotation.

The axis of rotation may be a pivot, hinge or any other support. Every point in a rotating rigid object moves in a circle (shown dashed in Fig. 4.1 for points $P_1$, $P_2$ and $P_3$) with the center on the axis of rotation. A straight line drawn from the axis to any point in the object sweeps out the same angle in the same time interval.



## 4.1 ROTATIONAL KINEMATICS

Rotational kinematics deals with motion of objects along a circular path without any reference to forces or torques.

### 4.1.1 Angular Position (θ)

The angle through which position vector of a moving object is displaced with respect to some chosen reference direction is called angular position 'θ'.

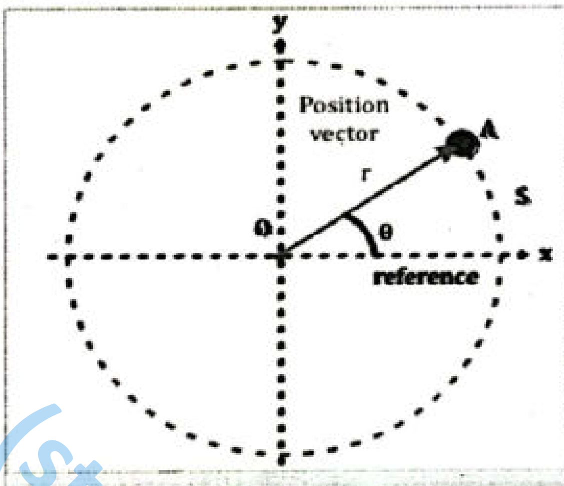


**Figure 4.2: Angular Position.**

Let an object 'A' is rotated through arc length 'S' from a certain reference axis, along a circle of radius 'r' as shown in Fig. 4.2. The angular position of the rigid object is the angle 'θ' between this radial line (represented by position vector 'r') and the fixed reference line in space (often chosen as the + x axis). Mathematically,

$$\theta = \frac{S}{r} \qquad \text{\_\_\_\_\_\_ (4.1)}$$

This resembles the way we identify the position of an object in translational motion as the distance x between the object and the reference position, which is the origin (x = 0).

### 4.1.2 Angular Displacement (Δθ)

The change in angular position with respect to chosen reference direction is termed as angular displacement.

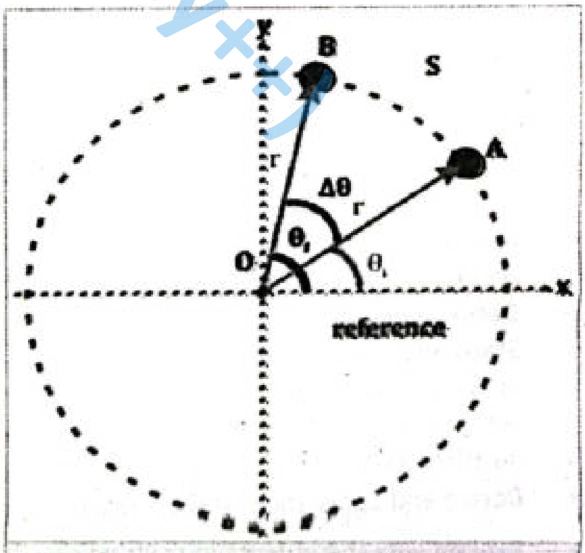


**Figure 4.3: Angular Displacement.**

As a particle on a rigid object travels from position A to position B in a time interval t, as in Fig. 4.3, the reference line fixed to the object sweeps out an angle, given by:

$$\Delta\theta = \theta_f - \theta_i \qquad \text{\_\_\_\_\_\_ (4.2)}$$

Conventionally, positive angular displacements represent anti-clockwise motion and negative represents clockwise motion.

**Units of Angular Displacement:** The SI unit of angular displacement is radian. Other units are degrees and revolutions.

**Relation between radian and degree:**

In one complete rotation, there are 360°.

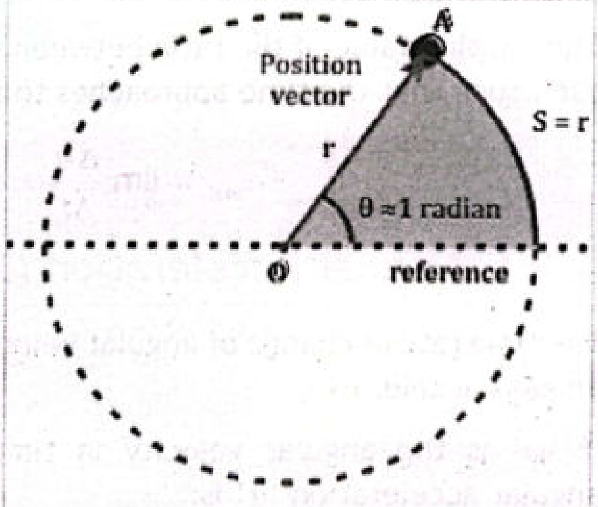
Number of degrees in one revolution = 360°

To find the number of radians in one revolution, we put S as circumference of circle, which is $2\pi r$, in equation 4.1, we get:

number of radians in one revolution = $2\pi$ rad

As for one complete revolution the number of radians must be equal to the number of degrees, therefore:

$$2\pi\, rad = 360^0 \text{ or } 1rad = \frac{360^0}{2\pi} = \frac{360^0}{2\times 3.14} = 57.3^0$$

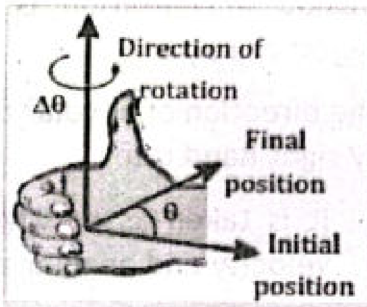



One radian (1 rad) is the angle subtended at the center of a circle by an arc with a length equal to the radius of the circle.

**Figure 4.4:** Radian Measurement.

**Direction of Angular Displacement:** Angular displacement is a vector quantity, having both magnitude and direction. The right hand rule is used to specify the direction.

Holding the axis of rotation in right hand with fingers curling in the direction of rotation; the thumb gives the direction of angular displacement.




**Right Hand Rule**

### 4.1.3. Angular Velocity (ω)

The time rate of change of angular displacement of a body is called angular velocity.

If '$\Delta\theta$' is the small angular displacement in time '$\Delta t$', then angular velocity '$\omega$' is:

$$\omega = \frac{\Delta\theta}{\Delta t} \qquad \text{\_\_\_\_\_\_ (4.3)}$$

**Units of Angular Velocity:** The SI unit of angular velocity is radian per second (rad $s^{-1}$).

Other units are deg/s or rev/s or rev /min (rpm). The direction of angular velocity is same as that of angular displacement.

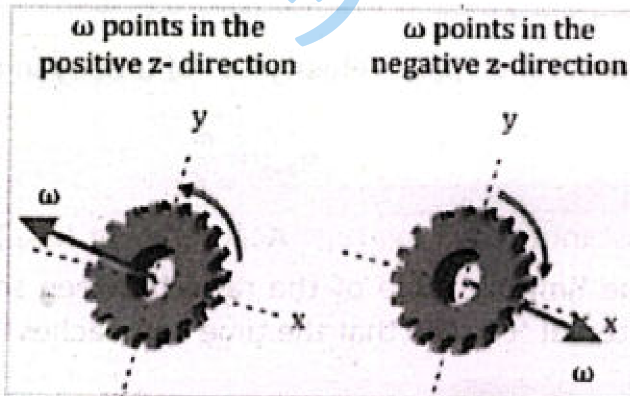



**Figure 4.5:** Angular velocity.

**Average Angular Velocity ($\omega_{av}$):**

The total angular displacement '$\theta$' of a body during time '$t$' is called average angular velocity.

$$\omega_{av} = \frac{\theta}{t} \quad \text{\_\_\_\_\_ (4.4)}$$

**Instantaneous Average Velocity ($\omega_{inst}$):**

The limiting value of the ratio between small angular displacement '$\Delta\theta$' and small time interval '$\Delta t$', such that the time approaches to zero, is called instantaneous angular velocity.

$$\omega_{inst} = \lim_{\Delta t \to 0} \frac{\Delta\theta}{\Delta t} \quad \text{\_\_\_\_\_ (4.5)}$$

## 4.1.4. Angular Acceleration ($\alpha$)

The time rate of change of angular velocity is called angular acceleration.

If '$\omega$' is the angular velocity in time 't', then angular acceleration '$\alpha$' is:

$$\alpha = \frac{\Delta\omega}{\Delta t} \quad \text{\_\_\_\_\_ (4.6)}$$

**Units of Angular Acceleration:** The SI unit of angular acceleration is rad/s$^2$. Other units are deg/s$^2$ or rev/s$^2$.

The direction of angular acceleration is determined by right hand rule.

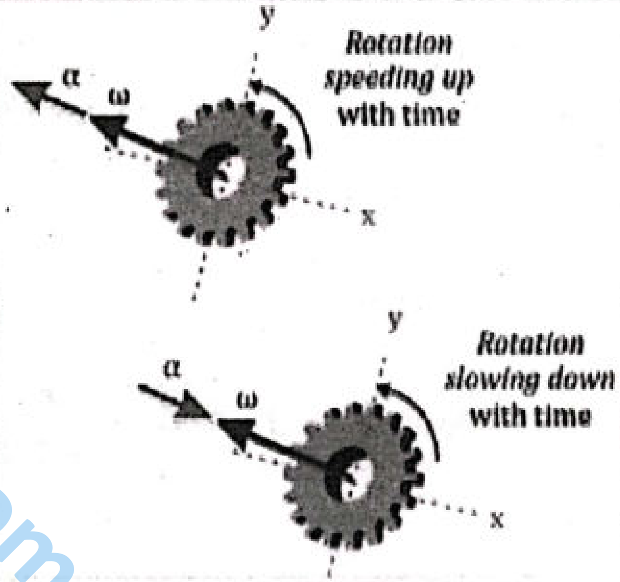


**Figure 4.6:** Angular acceleration.

- It is taken as positive when angular velocity of a body increases. In such case angular velocity and angular acceleration have same direction.
- It is taken as negative when angular velocity of a body decreases. In such case angular velocity and angular acceleration are anti-parallel.

**Average Angular Acceleration ($\alpha_{av}$)**

The total angular velocity 'ω' of a body in time 't' is called average angular acceleration.

$$\alpha_{avg} = \frac{\omega}{t} \quad \text{\_\_\_\_\_ (4.7)}$$

**Instantaneous Average Acceleration ($\alpha_{inst}$)**

The limiting value of the ratio between small change in angular velocity 'ω' and small time interval 't', such that the time approaches to zero, is called instantaneous angular acceleration.

$$\alpha_{inst} = \lim_{\Delta t \to 0} \frac{\Delta\omega}{\Delta t} \quad \text{\_\_\_\_\_ (4.8)}$$

### 4.1.5 Relationship between Linear and Angular Kinematic Quantities

Linear kinematic quantities like displacement, velocity and acceleration can be related with their rotational analogue.

**A. Relation between Linear and Angular Displacement:** Consider the Fig. 4.7, in which a particle that moves in circle of radius 'r' with center at 'O'. Let the particle moves from point A to B, and there is another point C such that ∠AOC = 1 radian, therefore Arc AC must be equal to radius 'r'. By using simple geometry, we can write:

$$\frac{\text{Arc AB}}{\text{Arc AC}} = \frac{\angle AOB}{\angle AOC}$$

Here, Arc AB is linear displacement 'S' and ∠AOC is the angular displacement 'θ'. As Arc AC = r and ∠AOC = 1 radian, so the above equation becomes:

$$\frac{S}{r} = \frac{\theta}{1rad} \quad \text{or} \quad \theta = \frac{S}{r}$$

For angular displacement θ in radians, we can write:

$$S = r\theta \qquad \text{\_\_\_\_\_ (4.9)}$$

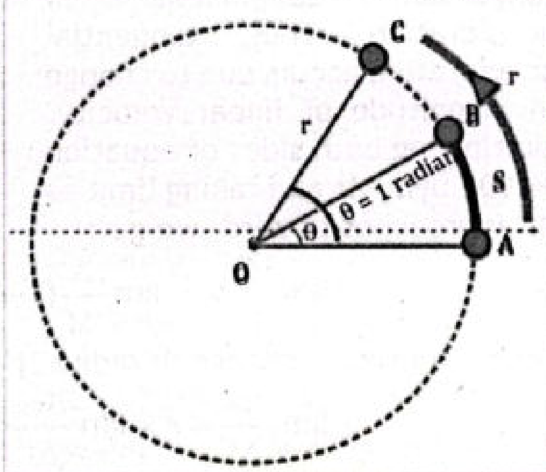



**Figure 4.7:** Linear and angular acceleration.

**B. Relation between Linear and Angular Velocity:** Multiplying both sides of equation (4.9) by $\Delta/\Delta t$ and taking limit $\Delta t$ approaches to zero, we get:

$$\lim_{\Delta t\to 0}\frac{\Delta}{\Delta t}S = \lim_{\Delta t\to 0}\frac{\Delta}{\Delta t}(r\theta)$$

Since there is no change in radius 'r' with respect to time, therefore:

$$\lim_{\Delta t\to 0}\frac{\Delta S}{\Delta t} = r\times\lim_{\Delta t\to 0}\frac{\Delta\theta}{\Delta t} \qquad \text{\_\_\_\_\_ (4.10)}$$

Now by definitions of linear and angular velocities:

$$v = \lim_{\Delta t\to 0}\frac{\Delta S}{\Delta t} \quad \text{\_\_\_\_\_ (4.11)} \qquad \text{and} \qquad \omega = \lim_{\Delta t\to 0}\frac{\Delta\theta}{\Delta t} \quad \text{\_\_\_\_\_ (4.12)}$$

Putting values from equation (4.11) and equation (4.12) in equation (4.10), we get:

$$v = r\omega \qquad \text{\_\_\_\_\_ (4.13)}$$

The points A and B move closer together as $\Delta t$ approaches to zero. And the direction of linear velocity is along the tangent to the circle. Therefore, this velocity is also called as tangential velocity.

**C. Relation between Linear and Angular Acceleration:** In angular motion, the linear acceleration has two components, tangential component and the radial component, as shown in the Fig. 4.8.

In vector form: $\mathbf{a} = \mathbf{a}_t + \mathbf{a}_R$

In magnitude $a = \sqrt{a_T^2 + a_R^2}$

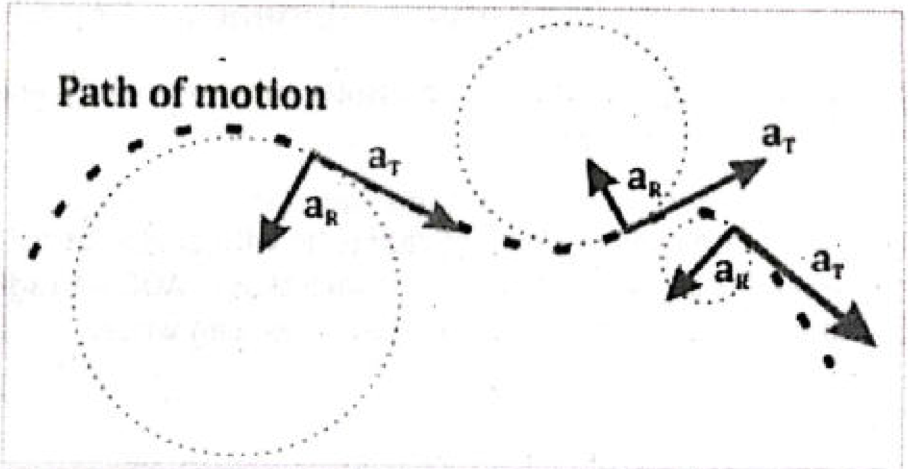



Figure 4.8: Acceleration components.

Tangential Component:
The component of angular acceleration which is parallel to linear instantaneous velocity is tangential component of acceleration. Thus, tangential acceleration occurs due to change in magnitude of linear velocity. Multiplying both sides of equation (4.10) by $\Delta/\Delta t$ and taking limit as $\Delta t$ approaches to zero, we get:

$$\lim_{\Delta t \to 0} \frac{\Delta}{\Delta t} v = \lim_{\Delta t \to 0} \frac{\Delta}{\Delta t}(r\omega)$$

Since there is no change in radius 'r' with respect to time, therefore:

$$\lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t} = r \times \lim_{\Delta t \to 0} \frac{\Delta \omega}{\Delta t} \quad \text{\_\_\_\_ (4.14)}$$

Now by definitions of linear and angular accelerations:

$$a = \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t} = a_T \quad \text{\_\_\_\_ (4.15)}$$

and
$$\alpha = \lim_{\Delta t \to 0} \frac{\Delta \omega}{\Delta t} \quad \text{\_\_\_\_ (4.16)}$$

Putting values from equations (4.15) and (4.16) in equation (4.14), we get:

$$a_T = r\alpha \quad \text{\_\_\_\_ (4.17)}$$

This enables us to write all the kinematic equations in rotational form, as shown in the Table 4.1. Kinematics for rotational motion is similar to translational kinematics.

**Table 4.1: Kinematic Equations for Rotational Motion**

| Equations for Linear Motion | Equations for Angular Motion |
|---|---|
| $S = v\,t$ | $\theta = \omega\,t$ |
| $v_f = v_i + at$ | $\omega_f = \omega_i + at$ |
| $2aS = v_f^2 - v_i^2$ | $2\alpha\theta = \omega_f^2 - \omega_i^2$ |
| $S = v_i t + ½ at^2$ | $\theta = \omega_i t + ½ \alpha t^2$ |

Radial Component: The component of acceleration in angular motion which is along radius of the circular path is radial component of acceleration. This acceleration arises due to change in direction of linear instantaneous velocity. For an object moving in a circular path with constant speed, there is only the radial acceleration, also called centripetal acceleration.

**Example 4.1: In a workshop, a bicycle tyre of radius 33.1 cm is rolled across the level floor with an initial velocity of 6.80 m s$^{-1}$. Assuming constant angular acceleration, the tyre comes to rest at a distance of 74.8 m. Determine (a) initial angular velocity of the tyre; (b) the total number of revolutions it made before coming to rest; (c) the angular acceleration of the tyre; and (d) the time it took before coming to rest.**

Given: Initial velocity '$v_i$' = 6.80 m $s^{-1}$
Radius 'r' = 33.1 cm = 0.331 m
Final angular velocity '$\omega_f$' = 0.00 rad $s^{-1}$
Distance 'S' = 74.8 m

To Find: (a) Angular velocity '$\omega i$' = ?
(b) Number of revolutions 'N' = ?
(c) Angular acceleration '$\alpha$' = ?
(d) Time 't' = ?

Solution: (a) The relation between linear and angular velocity is $v = r\omega$ or $\omega = \frac{v}{r}$

Putting values: $\omega = \frac{6.80}{0.331}$ therefore, $\omega = 20.54\, rad\, s^{-1}$

(b) When the tyre completes one revolution, it moves a distance equal to the circumference of the tyre ($2\pi r$), as long as there is no slipping or sliding. The number of revolutions will be the total distance divided by distance covered during each revolution ($2\pi r$). $N = \frac{S}{2\pi r}$

Putting values: $N = \frac{74.8}{2 \times 3.14 \times 0.331}$ therefore, $N = 35.9\, rev$

(c) In one revolution there are 2π radians, the total angular displacement θ will be 35.9 × 2π radians = 225.6 radians (θ = 225.6 radians). To find angular acceleration we would use the equation independent of time (3rd equation) i.e.

$$2\alpha\theta = \omega_f^2 - \omega_i^2 \quad \text{or} \quad \alpha = \frac{\omega_f^2 - \omega_i^2}{2\theta}$$

Putting values: $\alpha = \frac{(0)^2 - (20.54)^2}{2 \times 225.6}$ or $\alpha = -0.94\, rad\, s^{-2}$

(d) To find '$t$', we can use any of the equation involving time, however the simpler equation $\omega_f = \omega_i + \alpha t$, by rearranging this equation for time, we get:

$$t = \frac{\omega_f - \omega_i}{\alpha}$$

putting values: $t = \frac{0 - 20.54}{-0.94}$ therefore, $t = 21.9 s$

So, the tyre will take about 22 seconds before coming to rest.

**Assignment 4.1**

**The front wheel of a tractor travels 700 revolutions while the rear wheel 280 in a time interval of 40 seconds. Find their angular velocities.**

## 4.2 CENTRIPETAL ACCELERATION AND CENTRIPETAL FORCE

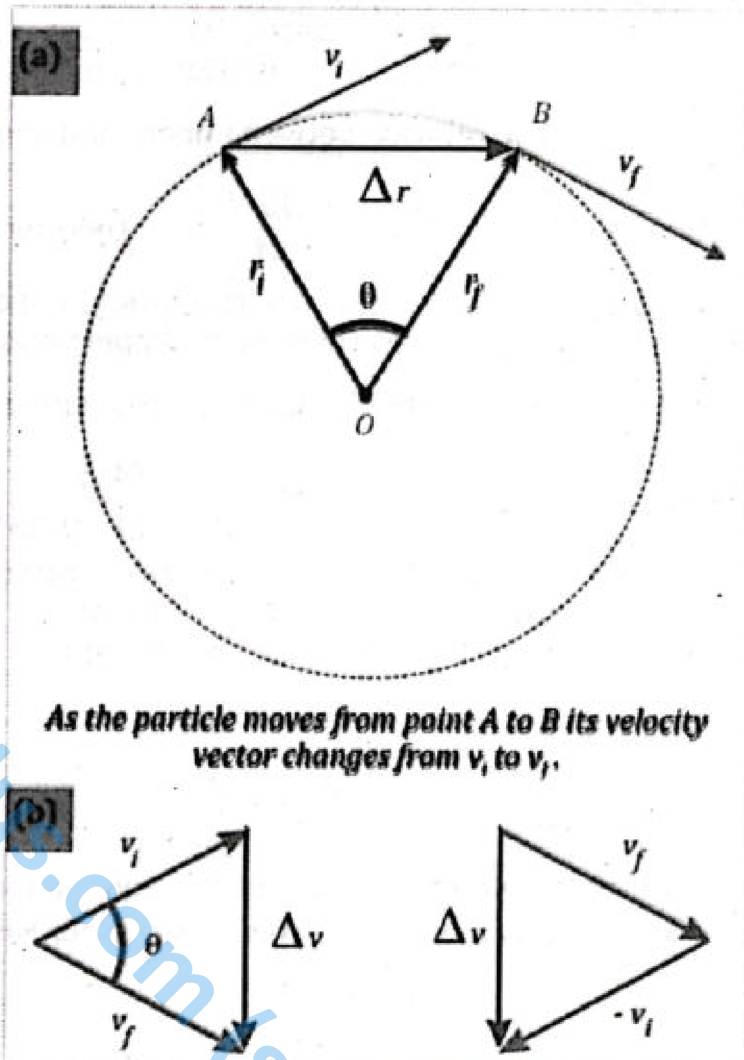
Consider a particle is moving in a circular path of radius r with constant speed, this means that direction of velocity is changing. This change in velocity of the particle produces acceleration which is directed towards the center of the circle, this type of acceleration is called centripetal acceleration.

Consider the Fig. 4.9 (a) in which a particle follows a circular path. The particle is at point A at time $t_i$ with velocity $v_i$. It reaches at point B at a later time $t_f$ with velocity $v_f$. For uniform circular motion $v_i$ and $v_f$ differ only in direction; their magnitudes are same, i.e. $|v_i| = |v_f| = |v|$

In Fig. 4.9 (b) velocity vectors have been redrawn tail to tail. The vector $\Delta v$ joins the heads of two vectors, representing vector addition,

$$v_f = v_i + \Delta v$$

The angle '$\Delta\theta$' between the two position vector '$r_i$' and '$r_f$' is the same as the angle '$\Delta\theta$' between the two velocity vectors '$v_i$' and '$v_f$'. This is because the velocity vector is perpendicular to the position vector, thus the two angles must be same. This allows us to write a relation for the lengths of the sides of the two triangles.




*As the particle moves from point A to B its velocity vector changes from $v_i$ to $v_f$.*

Figure 4.9: Centripetal Acceleration.

$$\frac{\Delta v}{v} = \frac{\Delta r}{r}$$

Where $|r_i| = |r_f| = |r|$ and $|v_i| = |v_f| = |v|$

or $$\Delta v = v\frac{\Delta r}{r}$$

Dividing both sides by $\Delta t$, we get:

$$a_{av} = \frac{v}{r}\frac{\Delta r}{\Delta t}$$

Now imagine the points 'A' and 'B' in the figure are extremely close together. As 'A' and 'B' approach each other, '$\Delta t$' approaches to zero. The acceleration at this stage will now be instantaneous acceleration.

$$a = \frac{v}{r}\lim_{\Delta t \to 0}\frac{\Delta r}{\Delta t}$$

Since, $v = \lim_{\Delta t \to 0}\frac{\Delta r}{\Delta t}$. Therefore, $a = \frac{v}{r} \times v$

This acceleration is referred to as centripetal acceleration $a_c$.

$$a_c = \frac{v^2}{r} \qquad \text{or} \qquad a_c = \left(\frac{v^2}{r}\right)\hat{r}$$

As centripetal acceleration $a_c$ is directed towards the center of the circle, the radial vector r is directed outwards from the center of the circle, thus a negative sign can be added to the equation.

$$a_c = -\left(\frac{v^2}{r}\right)\hat{r} \qquad \text{\_\_\_\_\_\_ (4.18)}$$

as v = r ω, therefore, $$a_c = -\left(\frac{r^2\omega^2}{r}\right)\hat{r}$$

Hence, $$a_c = -r\omega^2\hat{r} \qquad \text{\_\_\_\_\_\_ (4.19)}$$

According to Newton's second law, an object that is accelerating must have a net force acting on it. for example, to open a door, force must be applied to produce tangential acceleration thereby creating torque. Similarly, for an object to move in a circle, a force must be applied to it to keep it moving in that circle. Thereby giving it necessary radial (centripetal) acceleration.

## 4.2.1 Centripetal Force

The net force that causes the particle to undergo centripetal acceleration is called centripetal force $F_c$.

When Newton's second law is applied to a particle moving in a uniform circular motion, we can write: $F_c = m\, a_c$ ______ (4.20)

Putting equation (4.18) or equation (4.19) in equation (4.20), we can write centripetal force $F_c$ as:

$$F_c = -\left(\frac{mv^2}{r}\right)\hat{r}$$

and $F_c = -mr\omega^2\hat{r}$ ______ (4.21)

The direction of the centripetal force is always directed towards the center of the circle. Centripetal force is not a new force, but any net force that makes an object move towards the center of the circle can be termed as centripetal force. For example, to swing a ball in a circle at the end of a string, the tension in the string act as centripetal force. For a moon revolving around the Earth, or planets revolving around the Sun, gravity act as centripetal force. In other situation, it can be a normal force, or even an electric force (as in CD players and computer hard disks).

**Frictional force and normal force as centripetal force:**

When a car travels without skidding around an un-banked curve, the static frictional force between the tyres and the road provides the necessary centripetal force. The reliance on friction can be eliminated completely for a given speed, if the roads are banked at an angle relative to the horizontal while making a turn (Fig. 4.10).

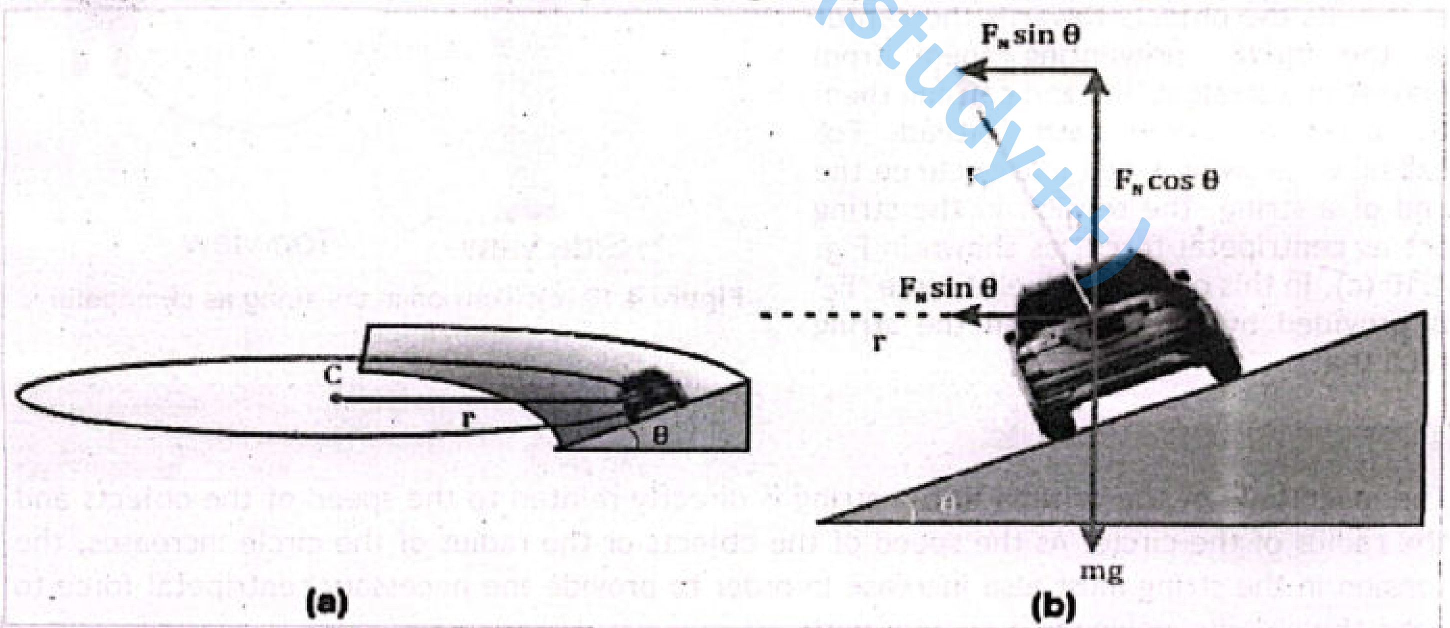



**Figure 4.10:** Friction and normal force as centripetal force.

Because the roadbed makes an angle with respect to the horizontal, the normal force has a component '$F_N \sin\theta$' that points toward the center 'C' of the circle and provides the centripetal force.

In the Fig. 4.10 (a) part shows a car going around a friction-free banked curve. The radius of the curve is 'r', where 'r' is measured parallel to the horizontal. Part (b) of the figure shows the normal force '$F_N$' that the road applies on the car, the normal force is perpendicular to the road. Because the roadbed makes an angle 'θ' with respect to the horizontal, the normal force has a component '$F_N \sin\theta$' that points toward the center C of the circle and provides the centripetal force.

$$F_c = F_N \sin\theta = \frac{mv^2}{r} \quad \text{\_\_\_\_\_ (4.22)}$$

Since the car does not accelerate along the component of normal force '$F_N \cos\theta$', this component only balances the weight 'mg' of the car. Therefore, '$F_N \cos\theta = mg$'.

Dividing equation (4.22) by this equation, we get: $\frac{F_N \sin\theta}{F_N \cos\theta} = \frac{mv^2 / r}{mg}$

or $$\tan\theta = \frac{v^2}{rg} \quad \text{\_\_\_\_\_ (4.23)}$$

This equation indicates that for a given speed v, the centripetal force needed for a turn of radius 'r' at an angle 'θ' is independent of the mass of the vehicle. higher speeds and smaller radii require more steeply banked curves—that is, larger values of 'θ'. At a speed that is too low for a given 'θ', a car would slide down a frictionless banked curve; at a speed that is too higher, a car would slide off the curve.

**Tension force as centripetal force:** When objects are connected by a string or rope and moving in a circle, the tension in the string acts as the centripetal force. The tension in the string is responsible to provide the necessary centripetal force, as it pulls the objects towards the center of the circle, preventing them from moving in a straight line and causing them to follow a curved path instead. For example, to swing a ball in a circle on the end of a string, the tension in the string act as centripetal force, as shown in Fig. 4.10 (c). In this case centripetal force '$F_c$' is provided by tension 'T' in the string such that:

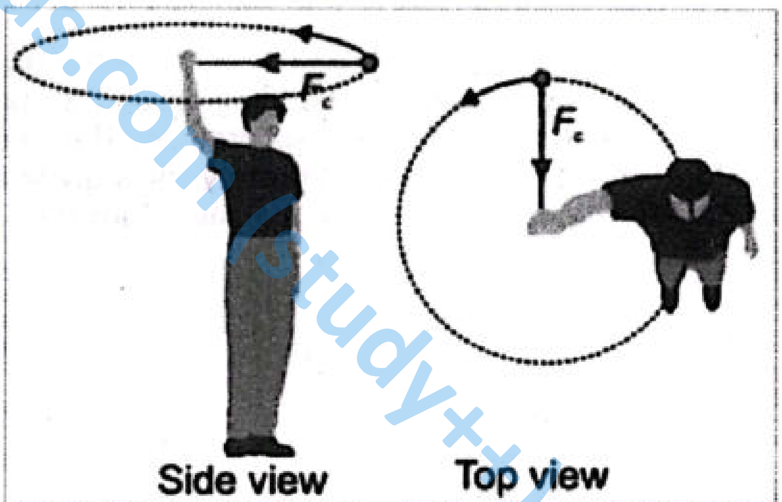



**Figure 4.10 (c): Tension in the string as centripetal force.**

$$T = \frac{mv^2}{r}$$

The magnitude of the tension in the string is directly related to the speed of the objects and the radius of the circle. As the speed of the objects or the radius of the circle increases, the tension in the string must also increase in order to provide the necessary centripetal force to keep the objects moving in a circular path.

**Gravitational force as centripetal force:** The force of gravity keeps planets in orbit around the sun and satellites in orbit around Earth, serving as the centripetal force. Without the force of gravity acting as the centripetal force, planets and satellites would not be able to maintain their orbits and would instead drift off into space. This force is essential for maintaining the balance between the inward force of gravity and the outward force of the object's inertia. This is why the sun's gravitational pull keep planets in orbit around it, and Earth's gravitational pull is able to keep satellites in orbit around it, as shown in Fig. 4.10 (d). In this case, the gravitational force '$F_g$' is responsible for providing the centripetal acceleration required for the circular motion. Mathematically:

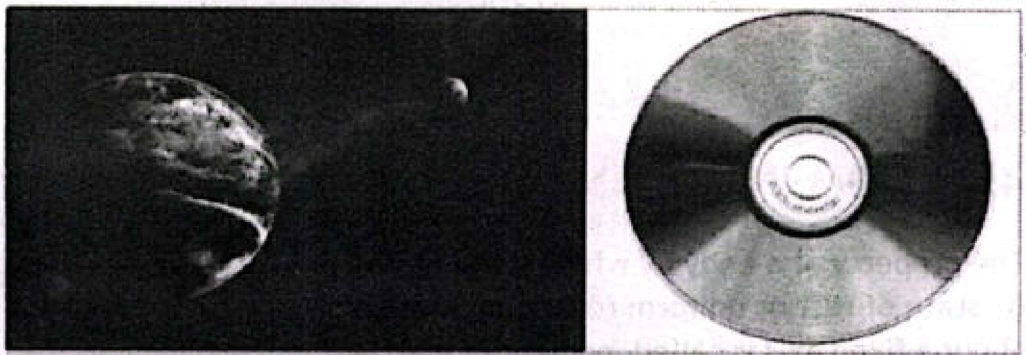


**Figure 4.10 (d):** Examples of centripetal force.

$$F_g = \frac{mv^2}{r}$$

**Centrifuge:** A centrifuge is a device that separates substances suspended in a liquid mixture by spinning a sample of liquid mixture very quickly around an axle. Any small denser particles found in the liquid travel in a straight line inside the test tube, obeying Newton's first law. The liquid in the test tube applies a centripetal force on these particles to keep them moving in a circle. After running the centrifuge at high speed for a period of time, the particles become clumped together at the bottom of the test tube, which can be collected and the sample is analyzed, as shown in Fig. 4.11.

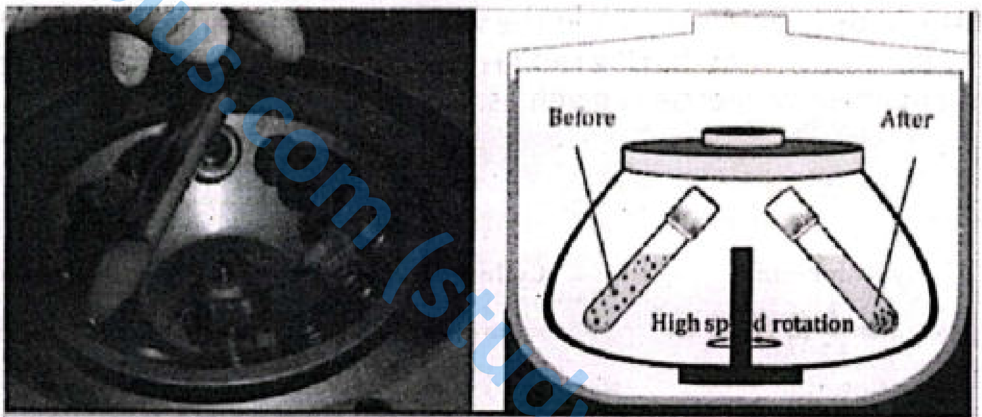



**Figure 4.11:** Centrifuge

The same centrifugation principle can be applied in the following commonly used devices.

**Cream Separator** is a centrifugal device that separates milk into cream and skimmed milk.

**Washing Machine Dryer** consists of a long cylinder with small holes on its walls. Wet clothes are placed in this cylinder, and then rotated rapidly to dry it.

**Example 4.2: The centripetal force on a car of mass 856 kg moving along a curve is 7250 N. If its speed is 12.0 m s$^{-1}$, what is the radius of the curve?**

Given: Mass 'm' = 856 kg  Centripetal force '$F_c$' = − 7250 N.  Speed 'v' = 12.0 m s$^{-1}$,

Solution:  Radius 'r' = ?

Solution: The centripetal force is given as:

$$F_c = -\frac{mv^2}{r} \quad \text{or} \quad r = -\frac{mv^2}{F_c}$$

Putting values: $r = -\frac{856 \times (12)^2}{-7250}$ or $r = 17\,m$

Assignment 4.2

A car, with its centre of gravity 0.4 m above the Earth's surface, is passing through a curve whose speed limit is 15 m s$^{-1}$. Find radius of the curve.

## 4.3 MOMENT OF INERTIA

The property of a body by which it maintains its state of rest or uniform rotational motion about a fixed axis is called moment of inertia (or rotational inertia).

The moment of inertia (or rotational inertia) is the rotational equivalent of mass. Objects with larger mass have a larger inertia, meaning that they are harder to accelerate linearly. Similarly, an object with a larger moment of inertia is harder to angularly accelerate. The moment of inertia is given by:

$$I = mr^2 \quad (4.24)$$

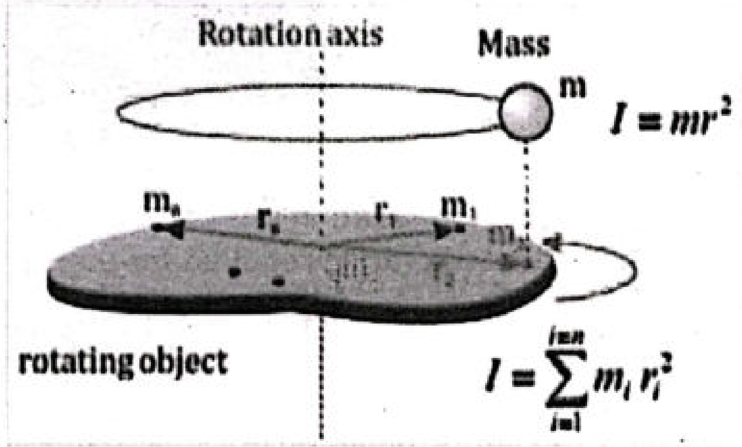



Figure 4.12: Moment of inertia.

If the body is rigid, we divide the whole body into large number of small portions having masses $m_1, m_2, m_3, \ldots, m_n$ having radii $r_1, r_2, r_3, \ldots, r_n$ from its axis of rotation, as shown in Fig. 4.12, and moment of inertia is given as:

$$I = \sum_{i=1}^{i=n} m_i r_i^2 \quad (4.25)$$

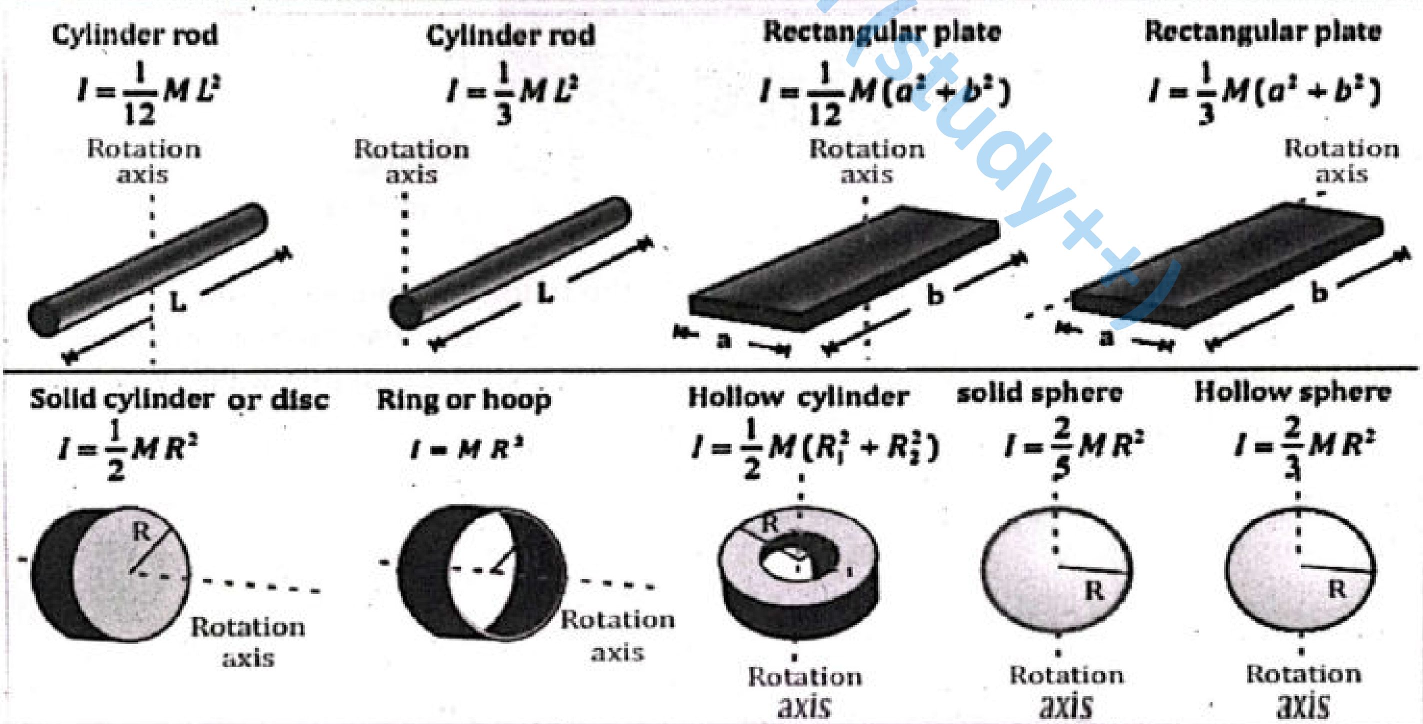



Figure 4.13: Moment of inertia for uniform objects.

Fig. 4.13 shows the calculated moments of inertia for various objects of uniform composition, each with mass 'M'.

## 4.4 ANGULAR MOMENTUM

The angular momentum 'L' of an object is defined as:

The cross product of position vector 'r' with respect to axis of rotation and linear momentum 'p' of an object. $\vec{L} = \vec{r} \times \vec{p}$

The SI unit of angular momentum is kg $m^2 s^{-1}$, and dimensions are $[ML^2T^{-1}]$.

### 4.4.1. For a Point Mass

Consider a mass 'm' rotating at distance 'r' from the axis of rotation, as shown in Fig. 4.14. By definition of angular momentum:

$$\vec{L} = \vec{r} \times \vec{p} \quad \text{or} \quad \vec{L} = r\,p \sin\theta\, \hat{n}$$

Since $\theta = 90^\circ$ and $\sin 90^\circ = 1$, therefore, magnitude of angular momentum is given by:

$$L = rp \qquad \text{\_\_\_\_\_ (4.26)}$$

From the definition of linear momentum:

$$p = mv \qquad \text{\_\_\_\_\_ (4.27)}$$

The relation between linear and angular velocity is:

$$v = r\omega \qquad \text{\_\_\_\_\_ (4.28)}$$

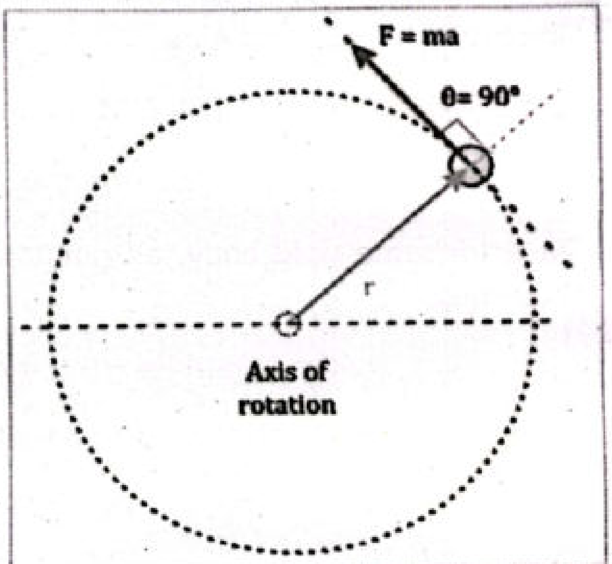


Figure 4.14: Mass 'm' rotating at distance 'r' from axis of rotation.

Putting equation (4.28) in equation (4.27), we get:

$$p = mr\omega \qquad \text{\_\_\_\_\_ (4.29)}$$

Now putting equation (4.29) in equation (4.26), we get: $p = r(mr\omega)$

Hence $L = mr^2\omega = I\omega$ _____ (4.30)

From Eq. (4.30), the angular momentum of an object can also be defined as the product of moment of inertia and its angular velocity, just like linear momentum is defined as product of mass and velocity.

### 4.4.2. For a Rigid Body

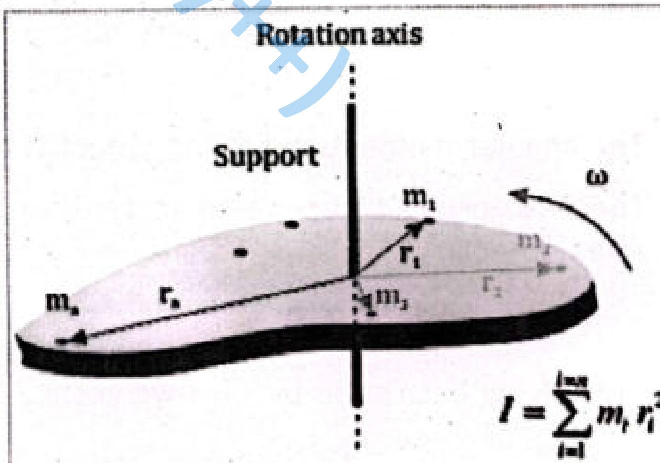


Figure 4.15: Rigid body rotating about axis of rotation.

Consider a rigid body and divide it into large number of small masses '$m_1$, $m_2$, $m_3$, ..., $m_n$' having distances '$r_1$, $r_2$, $r_3$, ........ $r_n$' from the axis of rotation, as shown in the Fig. 4.15. The net angular momentum will be sum of all the individual angular momenta:

$$L_{net} = L_1 + L_2 + L_3 + L_4 + - - - + L_n \quad \text{\_\_\_\_ (4.31)}$$

The angular momentum about point 1 will be:

$$L_1 = m_1 r_1^2 \omega_1 \quad \text{\_\_\_\_\_ (4.32)}$$

Similarly, the angular momentum about point 2 will be:

$$L_2 = m_2 r_2^2 \omega_2 \quad \text{\_\_\_\_\_ (4.33)}$$

and $$L_3 = m_3 r_3^2 \omega_3 \quad \text{\_\_\_\_\_ (4.34)}$$

Similarly, $$L_n = m_n r_n^2 \omega_n \quad \text{\_\_\_\_\_ (4.35)}$$

Putting equations (4.32), (4.33), (4.34) and (4.35) in equation (4.31), we get:

$$L_{net} = m_1 r_1^2 \omega_1 + m_2 r_2^2 \omega_2 + m_3 r_3^2 \omega_3 + - - - - - + m_n r_n^2 \omega_n \quad \text{\_\_\_\_\_ (4.36)}$$

Since for same rigid body, all points on the body rotate with the same angular velocity 'ω', therefore,

$$\omega_1 = \omega_2 = \omega_3 = - - - - = \omega_n = \omega$$

Therefore, equation (4.36) can be written as:

$$L_{net} = m_1 r_1^2 \omega + m_2 r_2^2 \omega + m_3 r_3^2 \omega + - - - - - + m_n r_n^2 \omega$$

or $$L_{net} = \left(m_1 r_1^2 + m_2 r_2^2 + m_3 r_3^2 + - - - - - + m_n r_n^2\right)\omega$$

The term in parenthesis in above equation is moment of inertia of a rigid body, so,

$$L_{net} = \left(\sum_{i=1}^{i=n} m_i r_i^2\right)\omega = I\omega \quad \text{\_\_\_\_\_ (4.37)}$$

### 4.4.3. Relation between Torque and Angular Momentum

The angular momentum L of an object is defined as:

The cross product of position vector r with respect to axis of rotation and linear momentum P of an object.

$$\mathbf{L} = \mathbf{r} \times \mathbf{p}$$

Multiplying both sides by $\frac{\Delta}{\Delta t}$, we get: $\frac{\Delta}{\Delta t}\mathbf{L} = \frac{\Delta}{\Delta t}(\mathbf{r} \times \mathbf{p})$

or $$\frac{\Delta \mathbf{L}}{\Delta t} = \mathbf{r} \times \frac{\Delta \mathbf{p}}{\Delta t} \quad \text{\_\_\_\_\_ (4.38)}$$

According to Newton's second law of motion in terms of momentum:

$$F = \frac{\Delta p}{\Delta t} \qquad \text{———— (4.39)}$$

Putting equation (4.39) in equation (4.38), we get:

$$\frac{\Delta L}{\Delta t} = r \times F \qquad \text{———— (4.40)}$$

By the definition of torque: $\tau = r \times F$ ———— (4.41)

Therefore, $\frac{\Delta L}{\Delta t} = \tau$ ———— (4.42)

## 4.4.4. Conservation of Angular Momentum

In the absence of any external torque, the angular momentum of a system remains constant.

i.e., $\frac{\Delta L}{\Delta t} = 0$

Therefore, $\Delta L = 0$

Or $L_f - L_i = 0$

Hence, $L_f = L_i$

or $I_f\,\omega_f = I_i\,\omega_i$ ———— (4.43)

Equation (4.43) implies that

The final angular momentum should be equal to initial angular momentum.

A spinning ice skater is an interesting example of conservation of angular momentum. When the skater's arms are extended, the rotational inertia 'I' is relatively large and the angular velocity 'ω' is relatively small, as shown in Fig. 4.16. Often at the end of the spin, the skater pulls his arms close to his body resulting in a much faster spin (larger angular velocity) because of a much smaller rotational inertia. When a rotating body contracts, its angular velocity increases; and when a rotating body expands, its angular velocity decreases. This phenomenon is the

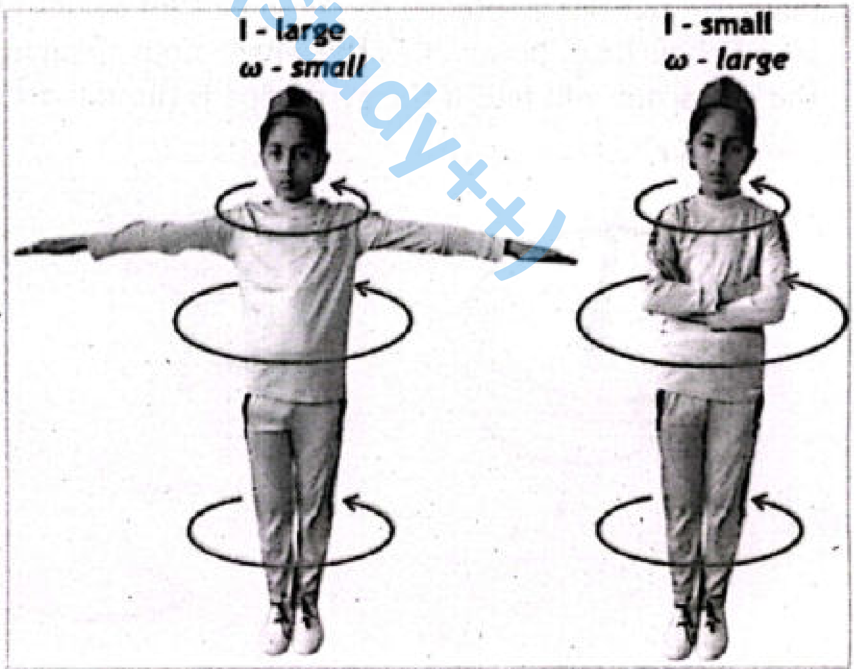


Figure 4.16: Spinning ice skater.

result of the conservation of angular momentum. As:

$$L_f = L_i \quad \text{or} \quad I_f\,\omega_f = I_i\,\omega_i$$

And moment of inertia is given by: $I = mr^2$

Therefore, $m_f r_f^2\,\omega_f = m_i r_i^2\,\omega_i$

As $r_f < r_i$ and $m_f = m_i$ therefore his rotational speed will increase to compensate for the decrease in rotational inertia.

Similarly, gymnasts and divers generate their spins (torque) from a solid base or a diving board after which the angular momentum remains unchanged, as shown in Fig. 4.17. The usual somersaults and twists result by making variations in their rotational inertia.

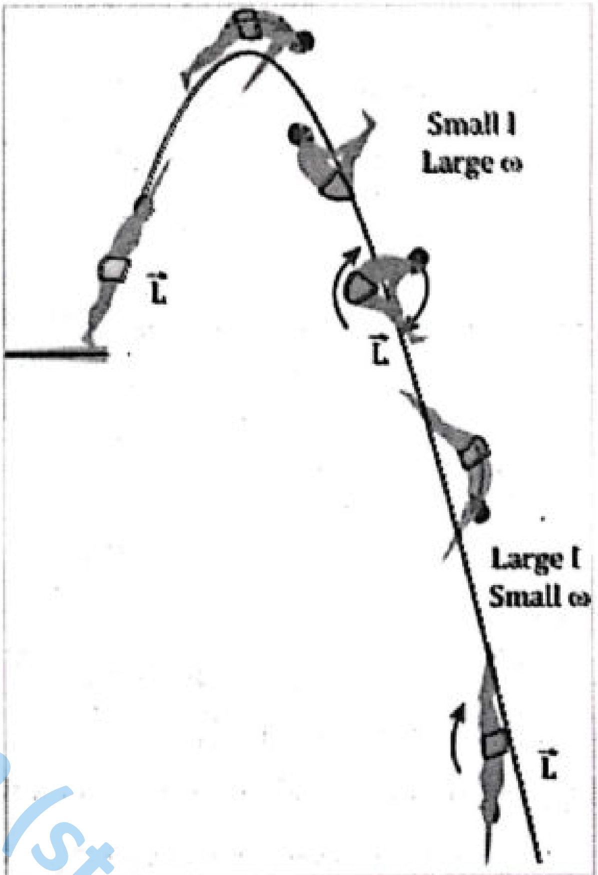


Figure 4.17: Board divers.

A **gyroscope** is a device that utilizes the principle of angular momentum to maintain its orientation relative to the Earth's axis or resist changes in its orientation. A very unusual and fascinating type of motion you probably have observed is that of a gyroscope, which utilize the principle of angular momentum.

Gyroscope usually consists of a wheel mounted on an axle which can rotate freely and is secured in a metal frame, as shown in Fig. 4.18 (a). When the wheel is made to spin the gyroscope can be balanced mounted on a flat surface, however as the wheel stops spinning the gyroscope will fall. If the gyroscope is tilted it also

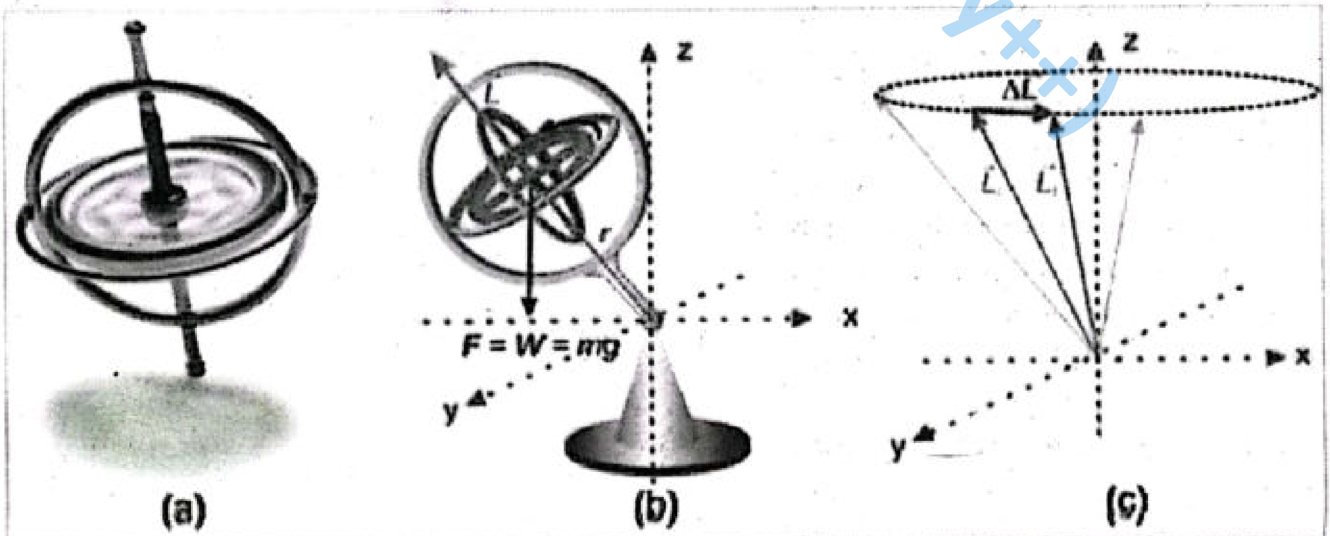


Figure 4.18: Spinning Gyroscope.

keep levitated without falling, but will start precession about gravitational force axis. This is the gravity-defying part of a gyroscope, as shown in the Fig. 4.18 (b).

This unusual behaviour can be explained by the vector nature of angular momentum, the change in direction of gyroscope will require a torque. The torque is provided by gravitational force as its weight towards the ground. The angular momentum will start to follow the torque, as shown in the Fig. 4.18 (c), the change in angular momentum '$\Delta L$' is:

$$\Delta L = \tau \times \Delta t$$

Where the torque has the same direction as '$\Delta L$' and '$\Delta t$' is the duration of time. The same effects can also be observed even if it is lifted by string looped around its lower end.

A **flywheel** (as shown in Fig 4.19) is a mechanical component that stores energy by spinning a heavy disc or wheel about an axle. When torque is applied, the rotational speed increases, storing kinetic energy that can be used for different tasks. The concept of a flywheel is based on the principle of conservation of energy, where the energy input is stored in the form of rotational motion. This stored energy can then be released when needed, such as during power outages or to provide additional power for machinery.

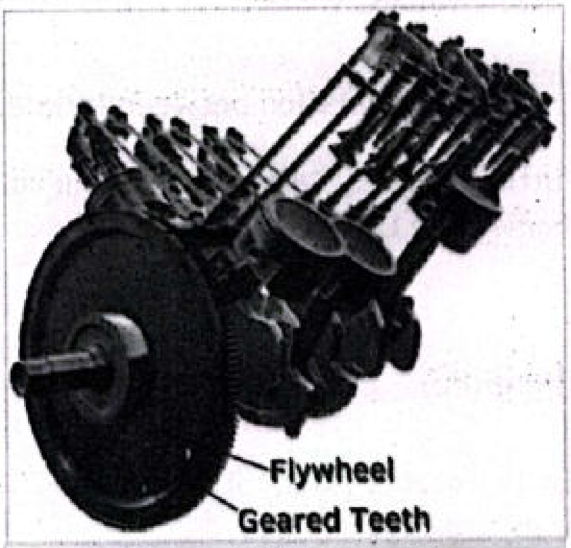


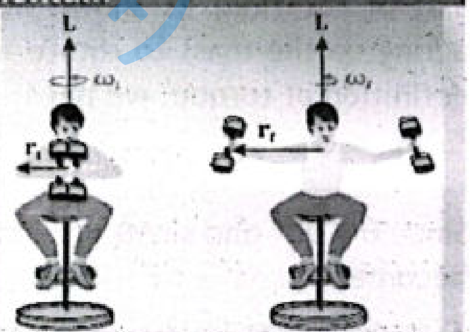
Figure 4.19: Flywheel.

Flywheels are commonly used in various applications, such as in engines, industrial machinery, and energy storage systems. In vehicles, flywheels can help smooth out the power delivery and improve fuel efficiency. In industrial settings, they can provide backup power and help regulate the speed of machinery.

The design of a flywheel involves careful consideration of the material, size, and shape of the disc or wheel, as well as the bearings and axle to minimize friction and maximize energy storage. Additionally, the speed and torque at which the flywheel operates must be carefully controlled to ensure safe and efficient operation.

**Activity - Conservation of Angular Momentum**

Hold pair of dumbbells in your hand and find a turntable to rotate at full speed by holding dumbbells close to your body. As soon as you extend your arms your rotation speed (angular velocity) will decrease. Again, upon drawing your hands nearer towards your chest the angular velocity will increase.
Can you explain why does this happens?




In recent years, there has been growing interest in using flywheels as a form of energy storage for renewable energy sources, such as wind and solar power. By storing excess energy generated during peak production times, flywheels can help balance the supply and demand of electricity on the grid.

**Example 4.3:** What is the angular momentum of a 3.6 kg uniform cylindrical grinding wheel of radius 31 cm rotating at 1150 rpm? (b) How much torque is required to stop it in 7.8 s?

Given: Mass 'm' = 3.6 kg Radius 'R' = 31 cm = 0.31 m

Initial angular velocity '$\omega_i$' = 1150 rpm = 120.4 rad $s^{-1}$ Time duration '$\Delta t$' = 7.8 s

To Find: (a) Angular momentum $L = ?$ (b) Torque $\tau = ?$

Solution: (a) The angular momentum is given as: $L = I\omega$

Since, moment of inertia for disk is $I = \frac{1}{2}mR^2$, therefore $L = \frac{1}{2}mR^2\omega$

Putting values, we get: $L = \frac{1}{2} \times 3.6 \times (0.31)^2 \times 120.4 = 20.83$ J s

(b) From the relation between torque and angular momentum: $\tau = \frac{L_f - L_i}{\Delta t}$

Putting values, where initial angular momentum $L_i$ is 20.83 kg $m^2$ $s^{-1}$ and final angular momentum $L_f$ is zero (0 kg $m^2$ $s^{-1}$).

$$\tau = \frac{0 - 20.83}{7.8}$$

Therefore, $\tau = -2.67\,kg\,m^2\,s^{-2} = -2.67\,Nm$

**Assignment 4.3**

Earth rotates about its own axis. What will be its angular momentum when its average angular speed around its axis is $7.29 \times 10^{-5}$ rad $s^{-1}$?

## 4.5 TORQUE AND ANGULAR ACCELERATION

Relationship exists between torque and angular acceleration, just like force and acceleration as in Newton's second law of motion.

### 4.5.1 For a Point Mass

Consider a mass 'm' rotating at a distance 'r' from the axis of rotation, as shown in Fig. 4.20. The force 'F' acting on the mass to rotate it is a tangential force. By definition of torque, we have:

$$\tau = r\,F\sin\theta\,\hat{n} \quad \text{\_\_\_\_\_\_ (4.44)}$$

Since $\theta = 90°$ and $\sin 90° = 1$, magnitude of equation 1 will become: $\tau = rF$ ______ (4.45)

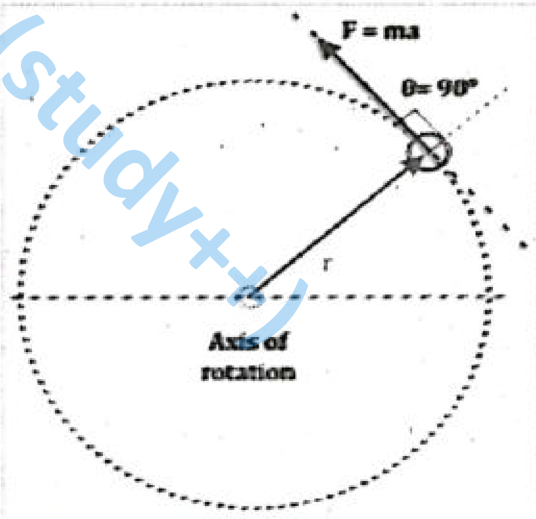



**Figure 4.20:** Mass 'm' rotating at distance 'r' from axis of rotation.

According to Newton's second Law, $F = ma$______ (4.46)

The relation between tangential and angular acceleration is given by:

$$a = r\alpha \quad \text{\_\_\_\_\_\_ (4.47)}$$

Putting equation (4.47) in equation (4.46), we get:

$$F = mr\alpha \qquad \text{\_\_\_\_\_\_} \ (4.48)$$

Putting equation (4.48) in equation (4.45), we get:

$$\tau = r(mr\alpha) \quad \text{or} \quad \tau = mr^2\alpha \ \text{\_\_\_\_\_\_} \ (4.49)$$

Since, the term $mr^2$ is moment of inertia, therefore,

$$\tau = I\alpha \qquad \text{\_\_\_\_\_\_} \ (4.50)$$

Equations (4.50) states that torque is moment of inertia times angular acceleration. This statement is similar to Newton's second law of motion F = ma, which gives force as equal to inertia (mass) times acceleration.

## 4.6 WEIGHTLESSNESS IN SATELLITES

Weightlessness occurs when the feeling of weight is completely or almost completely absent, meaning there is zero apparent weight. This occurs during free-fall, when the force of gravity is balanced out by the inertial force from orbital flight, like centrifugal force.

In a weightless environment, objects and individuals float freely, as there is no force pulling them towards the ground. This phenomenon is commonly experienced by astronauts in space, where they appear to be float inside their spacecraft with objects around them.

The term zero gravity is often used incorrectly to describe weightlessness, as astronauts in space stations are not in gravity free environment high above the Earth, 250 miles out in space, where most space stations orbit, the gravitational field is still quite strong there roughly 95% of its at on surface of the Earth. Weightlessness can be achieved in two ways. One that to travel millions of miles from gravitational force of large object, where the gravitational force reduces to nearly zero. Or the second and much more practical is to create weightless environment through act of free fall. The space stations are in constant free fall, having the right speed and at right altitude. Inside the space station the astronaut is also falling free, so they appear to float, as shown in Fig. 4.21, and physicists call it weightlessness.

**Figure 4.21:** Weightlessness in satellites.

Weightlessness can be enjoyed in amusement parks momentarily; it can also be simulated on Earth through techniques such as parabolic flights or neutral buoyancy tanks, allowing researchers to study the effects of microgravity on the human body and various materials. Living in a space station is not easy, besides the dangers of space travel and time spent away from family in isolation, astronauts feel many health issues related to microgravity. Their bones and muscles get weakened, cardiovascular system is affected and immune system is compromised.

Apart from all these health challanges some everyday activities become near-impossible. Their basic necessities like eating, sleeping, and showering habits are modified. They even can't cry; they face difficulty in digesting food and even in urination and excretion. Rotational simulated gravity has been proposed as a solution in human spaceflight to the adverse health effects caused by prolonged weightlessness.

Even though things and people may feel weightless in a weightless setting, they still possess mass and inertia, causing them to maintain their straight-line motion unless an outside force intervenes.

## 4.7 ARTIFICIAL GRAVITY

The gravity produced artificially in the satellites to counteract the effect of weightlessness is called artificial gravity.

It can be generated by rotating a space-station around its own axis, as shown in Fig. 4.22. The surface of the rotating space station exerts a force on objects with in contact with it and thereby provides the centripetal force that keeps the object moving on a circular path. In space stations, the astronauts feel weightless and cannot work effectively. In order to overcome this difficulty artificial gravity can be provided by rotating it about its own axis.

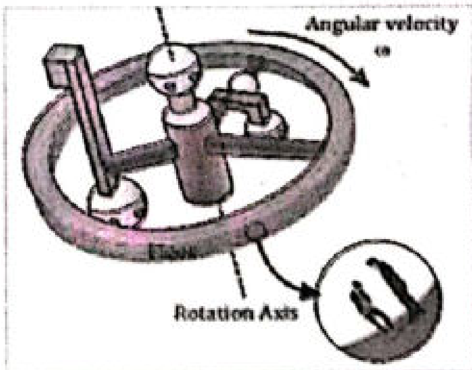


**Figure 4.22:** Artificial gravity.

To describe artificial gravity, consider a circular tube shaped part of the space station in which artificial gravity will be provided to the occupants of the space station. Let it have the radius 'R' and rotate with velocity 'v' as shown in the Fig. 4.21. The centripetal acceleration experienced at any point on the outer rim is:

$$a_c = \frac{v^2}{R} \qquad \text{------ (4.52)}$$

Linear Velocity: From equation (4.52): $v^2 = a_c R$

Therefore, $v = \sqrt{a_c R}$

To provide the same force as the force of gravity this centripetal acceleration, and hence centripetal force, must be equal to the acceleration due to gravity i.e. $a_c = g$

Hence $$v = \sqrt{gR} \qquad \text{------ (4.53)}$$

Angular Velocity of Satellites: The relation between linear and angular velocity is

$$v = \omega R \qquad \text{------ (4.54)}$$

By comparing equations (4.53) and (4.54), we get:

$$\omega R = \sqrt{gR} \qquad \text{or} \qquad \omega = \frac{\sqrt{gR}}{R} = \sqrt{\frac{gR}{R^2}}$$

Therefore, $$\omega = \sqrt{\frac{g}{R}} \qquad \text{\_\_\_\_\_ (4.55)}$$

Time Period of Satellites: Time period is the time required for the satellite to complete one rotation, i.e., $T = \dfrac{2\pi R}{v}$

Since $v = \omega R$, therefore: $T = \dfrac{2\pi R}{\omega R}$

or $$T = \frac{2\pi}{\omega} \qquad \text{\_\_\_\_\_ (4.56)}$$

Putting the values from equation (4.55) in equation (4.56), we get:

$$T = 2\pi\sqrt{\frac{R}{g}} \qquad \text{\_\_\_\_\_ (4.57)}$$

Frequency of Satellites: Since frequency is the reciprocal of time period, i.e., $f = \dfrac{1}{T}$, so, from equation (4.57) we get:

$$f = \frac{1}{2\pi}\sqrt{\frac{g}{R}} \qquad \text{\_\_\_\_\_ (4.57)}$$

**EXAMPLE 4.4: An 80.0 kg astronaut stands on the rim of rotating ring-shaped space station providing him sufficient artificial gravity g = 9.8 m s$^{-2}$. If the radius of the space station is 1.5 km. Calculate his (a) angular velocity, (b) time period and (c) frequency of rotation.**

Given: Mass of astronaut = m = 80.0 kg Radius of space ship = R = 1.50 km = 1500 m

To Find: (a) Angular velocity 'ω' =? (b) Time period 'T' = ? (c) Frequency 'f' = ?

SOLUTION: (a) The angular velocity for artificial satellite is:

$$\omega = \sqrt{\frac{g}{R}}$$

Putting values, we get: $\omega = \sqrt{\dfrac{9.8}{1500}}$ or $\omega = 0.08\,rad\,s^{-1}$

(b) The time period for artificial satellite is:

$$T = 2\pi\sqrt{\frac{R}{g}}$$

Putting values, we get: $T = 2\times 3.14\sqrt{\dfrac{1500}{9.8}}$ or $T = 77.73\,s$

(c) Since frequency is the reciprocal of time period $f = \dfrac{1}{T}$

Putting values, we get: $f = \dfrac{1}{77.73}$ or $f = 0.013\,Hz$

**Assignment 4.4**

**A space ship, having cylindrical shape, is rotated at a speed of 20 rpm about its axis in order to provide artificial gravity to its inhabitants. If the spaceship has a diameter of 8 m, find the artificial gravity it provides.**

## SUMMARY

- **Angular velocity:** The rate at which an object changes the angle while moving on a circular path.
- **Tangential acceleration:** The acceleration in a direction tangent to the circle at the point of interest in circular motion.
- **Angular acceleration:** The rate of change of angular velocity with respect to time.
- **Centripetal acceleration:** The acceleration of an object moving in a circle, directed toward the center.
- **Centripetal force:** Any net force causing uniform circular motion.
- **Moment of inertia:** Mass times the square of perpendicular distance from the rotation axis; for a point mass, it is $I = mr^2$ and, because any object can be built up from a collection of point masses, this relationship is the basis for all other moments of inertia.
- **Torque:** The turning effectiveness of a force and is defined as product of moment of inertia and angular acceleration ($\tau = I\alpha$).
- **Angular momentum:** The product of moment of inertia and angular velocity ($L = I\omega$).
- Angular momentum is conserved, i.e., the initial angular momentum is equal to the final angular momentum when no external torque is applied to the system.



## EXERCISE

### Multiple Choice Questions

**Encircle the correct option.**

1) The term "centrifugation" means separation

A. through spinning
B. of components at higher temperature
C. through evaporation
D. of components at lower temperature

2) A car turns around a curve at 30 km h$^{-1}$. If it turns at double the speed, the tendency to overturn is:

A. doubled
B. quadrupled
C. halved
D. unchanged

3) The moment of inertia of a spinning body about a certain axis, doesn't depend on:

A. distribution of mass around the axis
B. orientation of the axis
C. mass of the body
D. angular velocity of the body

4) The change in angular momentum of a rod, when a torque of 2.5 N m is acted upon it for 2 s, is:

A. 1.25 J s
B. 2.5 J s
C. 5 J s
D. 0 J s

5) If size (length) of the wings of a fan is increased, its rotational speed, for the same voltage and current, will:

A. increase B. decrease C. remain constant D. may increase or decrease

6) In a body, angular acceleration is produced by:

A. net force B. power C. pressure D. net torque

7) An astronaut feels weightless inside the International Space Station. It is because the International Space Station is:

A. outside the gravitational field of Earth B. freely falling

C. at rest D. in motion

8) How many radians account for circumference of a circle?

A. 1 rad B. 2 rad C. π rad D. 2π rad

## Short Questions

Give short answers of the following questions.

4.1 What is the value of angular acceleration of the minute hand of your wrist watch?

4.2 Is there a real force that removes water from wet clothes in a washing machine? Explain how the water is removed.

4.3 Determine the relation between (a) linear and angular displacement. (b) linear and angular velocity. (c) linear and angular acceleration.

4.4 Is centripetal force a fundamental force or a force provided by any of the fundamental forces? Can any combination of the fundamental forces provide centripetal force?

4.5 There are generally double tyres in heavy vehicles on one side of an axle. Will its moment of inertia be different from that of a single tyre?

4.6 Why is it best to have the blades rotate in opposite directions for a helicopter having two sets of lifting blades?

4.7 If diameter of Earth becomes half and there is no change in its mass, what affect will be there on the rotational speed of Earth around its own axis?

4.8 Why does in circular motion, a tangential acceleration can change the magnitude of the velocity but not its direction?

4.9 Why does usually the value of artificial gravity is smaller than 9.8 m $s^{-2}$?

4.10 Why is a gyroscope used in aeroplanes?

4.11 How does the rotation of a flywheel helps to even out the power delivery from the engine?

4.12 A wall clock's arms show time as 09:15. Express the angle between the arms in radians.

## Comprehensive Questions

Answer the following questions in detail.

4.1 What is centripetal force? Explain. Write down at least two applications where centripetal force plays its role.

4.2 What is moment of inertia? Derive its relation for rigid body.

4.3 Derive the expression for angular momentum of a body. Also deduce the relation between angular momentum and torque.

4.4 Explain conservation of angular momentum using practical life examples.

4.5 Derive the relation between torque and angular acceleration.

4.6 Justify how a centrifuge is used to separate materials using centripetal force.
4.7 Explain why the objects in orbiting satellites appear to be weightless.
4.8 Describe how artificial gravity be produced in a satellite to counter weightlessness.
4.9 Analyse motion in curved path due to perpendicular forces.

## Numerical Problems

4.1 What will be the angular velocity of fly wheel of an engine if it completes 3000 revolutions in a minute? **(Ans. 314 rad $s^{-1}$)**

4.2 A car is passing through a turn that is in the form of an arc of a circle of radius 14.5 m. What will be the maximum speed limit (the speed at which the car can cross the bridge without losing contact with the road) if the centre of gravity of the car is 0.5 m from the ground? **(Ans. 12.1 m $s^{-1}$)**

4.3 A PT teacher rotates his stick at the axis that passes through its centre. If mass of the stick is 200 g and its length is 0.8 m, find its moment of inertia. **(Ans. 0.01 kg $m^2$)**

4.4 A football of mass 450 g rotates with an angular speed of 10 rev $s^{-1}$. If its radius is 11 cm, compute its angular momentum. **(Ans. 0.137 J s)**

4.5 A merryman in a circus is standing with his arms extended on a turn table rotating with angular velocity 10 rad/s. He brings his arms closer to his body so that his moment of inertia is reduced to one third of the initial value. Find his new angular velocity. **(Ans. 30 rad $s^{-1}$)**

4.6 A boy exerts a force of 200 N at the edge of the 30.0 kg merry-go-round, which has a 2.0 m radius. Calculate the angular acceleration produced (a) when no one is on the merry-go-round and (b) when the boy, having 20.0 kg weight, sits 1.5 m away from the center. (ignore friction). **(Ans. 6.67 rad $s^{-2}$, 3.8 rad $s^{-2}$)**

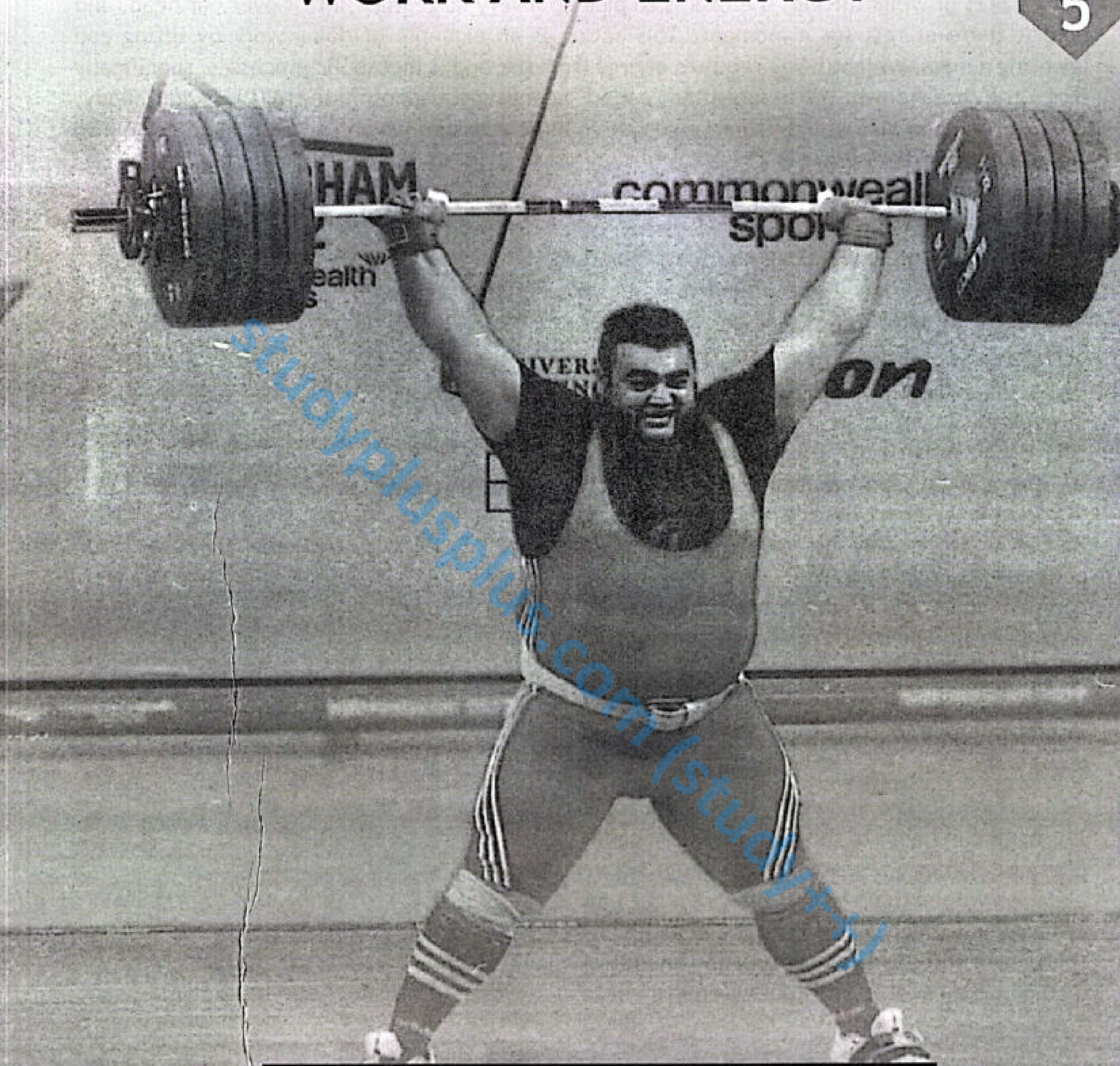
4.7 A wheel-shape space station provides an artificial gravity of 5.00 m $s^{-2}$ to its inhabitants. If it has a diameter of 100 m, find its angular speed in rpm. **(Ans. 3 rpm)**

4.8 The minute hand of a watch is 2 cm long. If it travels 9.4 cm of length, how many radians will it travel? **(Ans. 4.7 rad)**

4.9 How many revolutions does the fidget spinner make after being flicked with an initial angular velocity of 10 revolutions per second and coming to rest in 5 seconds? *(ignore air resistance)* **(Ans. 25 revolutions)**

# WORK AND ENERGY

UNIT 5






## Student Learning Outcomes (SLOs)

**The students will:**

- Derive the formula for kinetic energy [Using the equations of motion].
- Deduce the work done from force-displacement graph.
- Differentiate between conservative and non-conservative forces.
- Utilize the work - energy theorem in a resistive medium to solve problems.

In the sport of weightlifting, the task is to pick up a very large mass, lift it over our head, and hold it there at rest for a moment. This action is an example of doing work by lifting and lowering a mass. Weightlifting requires energy from the body's metabolic processes, specifically from the breakdown of a molecule called adenosine triphosphate (ATP). Additionally, weightlifting can also utilize stored glycogen in muscle tissue, which can be broken down into glucose to provide energy.

## 5.1 WORK

The work done on an object is the scalar (or dot) product of force **F** and displacement **d**.

$$W = \mathbf{F} \cdot \mathbf{d}$$

or $$W = Fd\cos\theta \quad (5.1)$$

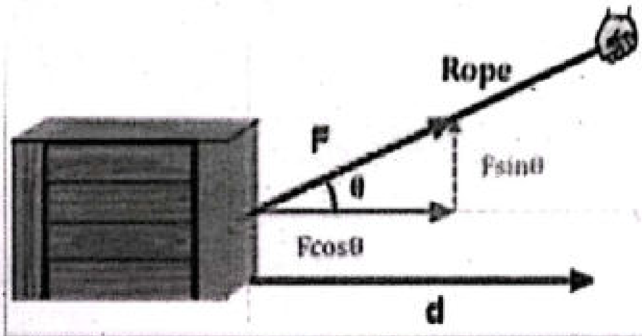



**Figure 5.1:** Work done on an object when a force 'F' produces displacement d.

In equation (5.1), 'F' is the magnitude of force, 'd' is the magnitude of displacement, and '$\theta$' is the angle between force and the displacement. From the Fig. 5.1, we can see that the force can be resolved into two components *Fcosθ* and *Fsinθ*. Here, *Fcos θ* is the component of force along the direction of displacement which is the effective component. Whereas, *Fsinθ* is the component of force which is perpendicular to the direction of displacement and therefore plays no role in doing work.

**Units of Work:** The SI unit of work is the joule (J) (named in honor of the 19th-century English physicist James Prescott Joule). From above equation we see that the unit of work is the unit of force multiplied by the unit of distance. In SI the unit of force is the newton and the unit of distance is the metre, so 1 joule is equivalent to 1 newton-metre (N m).

$$1\ \text{J} = 1\ \text{N m}$$

**Dependence of Work:** The work done depends upon force 'F', displacement 'd' and the angle 'θ' between them, as shown in Fig. 5.2.

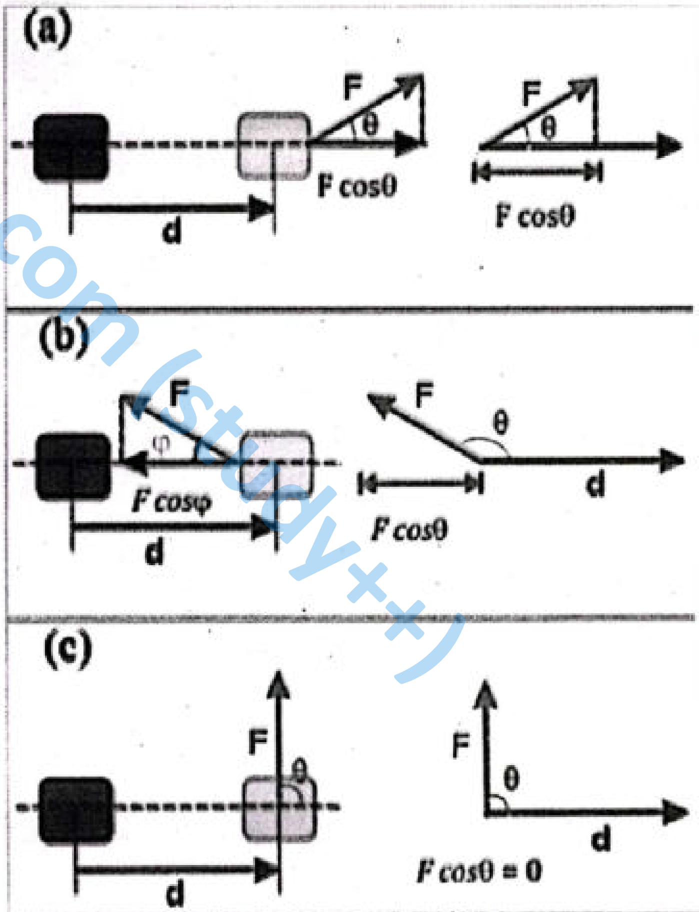



**Figure 5.2:** (a) positive work (b) negative work (c) zero work.

a) **Positive Work:** When the force has a component in the same direction as the displacement (0° ≤ θ < 90°), *cos* θ in above equation is positive and the work W is positive.

b) Negative Work: When the force has a component opposite to the displacement ($90^\circ < \theta \leq 180^\circ$), $\cos\theta$ is negative and work done by the force will be negative.

c) Zero Work: When the force is perpendicular to the displacement, $\theta = 90^\circ$ then the work done by the force is zero.

**Work Done from Force-Displacement Graph:** The area under the force-displacement graph gives the work done.

A. Work Done by Constant Force: In Fig. 5.3, graph for a constant force to produce net displacement is shown, here the blue shaded area of rectangle represent work done by the constant force.

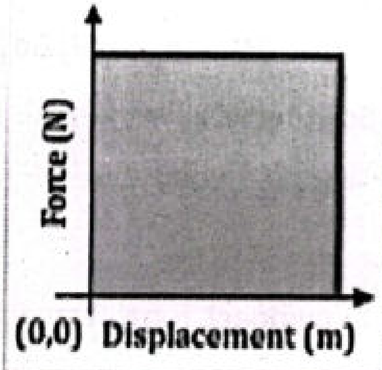


**Figure 5.3:** Work done by constant force.

B. Work Done by Variable Force: In Fig. 5.4, graph with increasing force to produce net displacement is shown, here the blue shaded area of triangle represent work done by the increasing force. However, in many situations of daily life, the force is variable in many different ways, and therefore we get different graphs for area under the curve for work done by such forces.

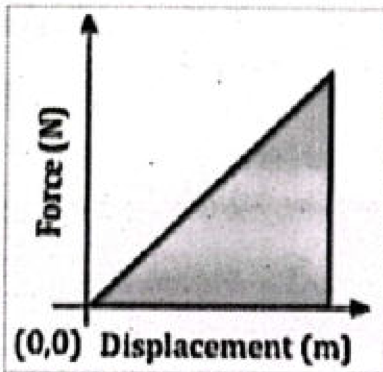


**Figure 5.4:** Work done by variable force.

For example, when we stretch a spring, the more we stretch it, the harder we have to pull, so the force we exert is not constant as the spring is stretched. When a rocket moves away from Earth the work is done against the force of gravity which varies as inverse of the square of distance from the center of Earth.

Consider a graph in Fig. 5.5, is drawn between $F\cos\theta$ and d. To find the total work done, we divide the displacement into number of small displacements $\Delta d_1$, $\Delta d_2$, $\Delta d_3$, ..., $\Delta d_n$, with corresponding effective component of forces are $F_1\cos\theta_1$, $F_2\cos\theta_2$, $F_3\cos\theta_3$, ........., $F_n\cos\theta_n$.

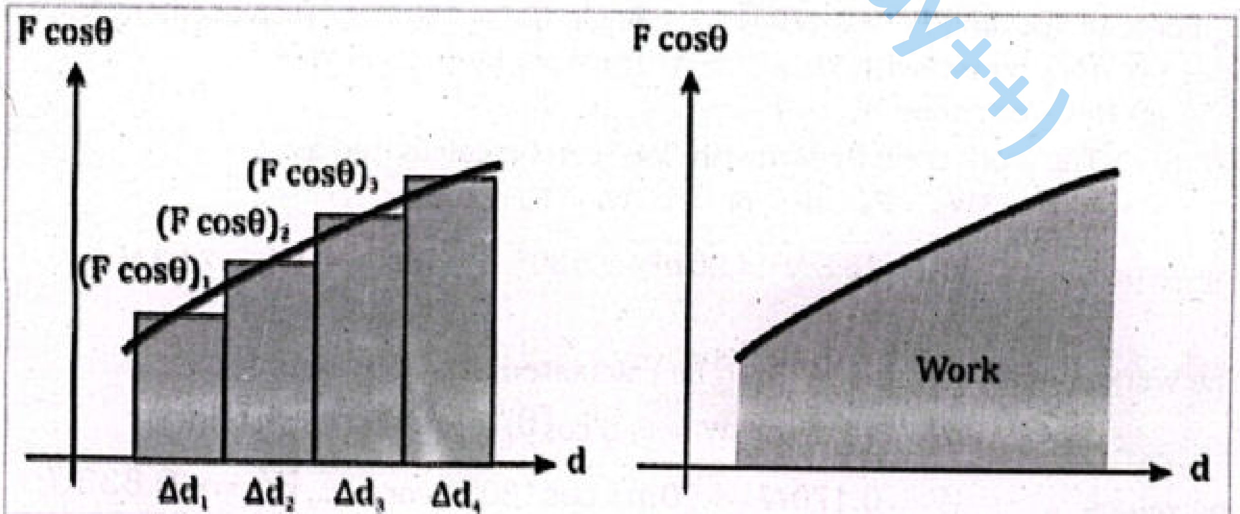


**Figure 5.5:** Graphical analysis of work done by variable force.

The total work done will now be the sum of all the individual work done.

$$W_{Total} = W_1 + W_2 + W_3 + \ldots + W_n$$

As, $W_1 = F_1 \cos\theta_1 \Delta d_1$, $W_2 = F_2 \cos\theta_2 \Delta d_2$,

$W_3 = F_3 \cos\theta_3 \Delta d_3$, and $W_n = F_n \cos\theta_n \Delta d_n$.

So, the total work done is:

$$W_{Total} = F_1 \cos\theta_1 \Delta d_1 + F_2 \cos\theta_2 \Delta d_2 + F_3 \cos\theta_3 \Delta d_3 + \ldots + F_n \cos\theta_n \Delta d_n \quad \text{\_\_\_\_\_\_ (5.2)}$$

In compact form, the above equation can be written as:

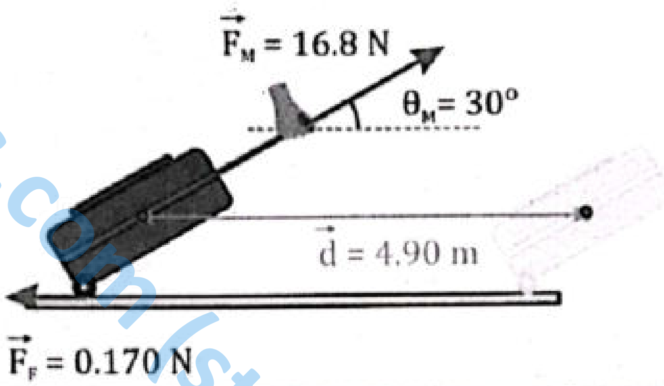
$$W_{Total} = \sum_{i=1}^{n} F_i \cos\theta_i \Delta d_i \quad \text{\_\_\_\_\_\_ (5.3)}$$

James Prescott Joule
(1818 – 1889)

Joule showed that heat was not a substance but, instead, the transfer of energy. He found that thermal energy produced by stirring water or mercury is proportional to the amount of energy transferred in the stirring.

Thus, the work done by a variable force is equal to the area under the *Fcos θ* and *d* curve.

**EXAMPLE 5.1:** Mehwish is dragging a suitcase at an airport and pulls it through a distance of 4.90 m along level ground. She is applying a constant force of 16.8 N in 30° with the horizontal. A 0.170 N friction force opposes the suitcase's motion. Find the work done by (a) Mehwish (b) frictional force and (c) net work done by all the forces acting on the suitcase.




Given: Force by Mehwish '$F_M$' = 16.8 N  Angle '$\theta_M$' = 30°

Force of friction '$F_f$' = 0.170 N  Angle '$\theta_F$' = 180°  Displacement 'd' = 4.90 m

To Find: (a) Work by Mehwish $W_M$ = ?  (b) Work by friction $W_f$ = ?

(c) Net work done $W_{net}$ = ?

Solution: (a) The work done by Mehwish '$W_M$' can be calculated as:

$$W_M = F_M \cdot d \quad \text{or} \quad W_M = F_M\, d \cos\theta_M$$

Putting values: $W_M = 16.8 N \times 4.90 m \times \cos 30°$ or $W_M = 71.3 Nm$

Hence, $W_M = 71.3 J$

(b) The work done by friction '$W_f$' can be calculated as:

$$W_f = F_f \cdot d \quad \text{or} \quad W_f = F_f\, d \cos\theta_f$$

Putting values: $W_f = 0.170 N \times 4.90 m \times \cos 180°$ or $W_f = -0.833 J$

(c) Although there is force of gravity due to weight of the suitcase acting on it. But as it is perpendicular to the direction of motion, it does not have an effective component in work. The net work done '$W_{net}$' is the sum of work done by Mehwish '$W_M$' and friction '$W_f$'.

$$W_{Total} = W_M + W_f$$

Putting values: $W_{Total} = 71.3\ J + (-0.833\ J)$

Therefore, $W_{Total} = 70.467\ J = 70.5\ J$

**Assignment5.1**

**A box having 40 kg mass is dragged on a frictionless inclined surface to a height of 8 m. If the inclined plane makes an angle of 20° with the Earth, find the work done against gravity.**

## 5.2 CONSERVATIVE & NON-CONSERVATIVE FIELDS



The region around a body where it can influence other bodies by a force associated with it is called field of force or simply force field. Examples include electric field, viscous field and gravitational field.

### 5.2.1 Conservative Field

A field is said to be conservative if it has two important properties:

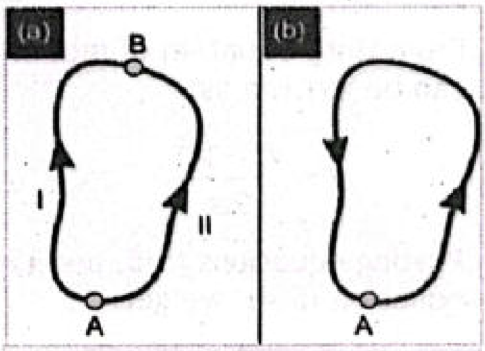



**Figure 5.6:** (a) An object moves between points A and B via two different paths 1 and 2. (b) The object makes a round trip back to A.

1. Work Done is Independent of the Path Taken: If the work done on a particle moving between any two points A and B is same for path I and II, as shown in Fig. 5.6 (a), the field will be conservative.

2. Work Done around a Closed Path is Zero: If the work done on a particle moving through any closed path (the path in which the beginning and end points are same) gives zero, as shown in Fig. 5.6 (b), such a field will be conservative.

Gravitational and electric fields are examples of conservative fields and the associated forces are conservative forces.

### 5.2.2 Non-Conservative Field

A non-conservative field is that field in which work done depends upon the path followed or the work done along a closed path is not zero.

Frictional field is a non-conservative field, and frictional or drag forces are non-conservative forces because when an object is moved in frictional field, the work done against frictional force depends upon the path followed.

Frictional force, viscous drag and air resistance are all examples of non-conservative forces and the fields where they act are termed as non-conservative fields.

## 5.3 KINETIC ENERGY

The energy possessed by a body due to its motion is called kinetic energy.

A boy kicks a football; it moves because it possess Kinetic energy. Now think a tennis ball and a football moving with same speed. Which possess greater ability to do work? Of course, the football with larger mass, which is difficult to stop. Similarly, now two footballs are approaching you with different speeds, which can do more work? Again, the football with greater speed is difficult to stop. Thus, the object's mass and its speed contribute to its Kinetic energy. Like all energies, Kinetic energy is also a scalar quantity.

Consider a situation in which all the work done transfers only kinetic energy to a cricket ball. Let the cricket ball is initially at rest $v_i = 0$ m/s, and a horizontal force F is applied to move it through a displacement 'd' and achieve a final velocity $v_f = v$, as shown in Fig. 5.7. This work done W appears as the kinetic energy K.E. Such that:

$$W = K.E = F\,d \quad \text{\_\_\_\_\_\_ (5.4)}$$

By Newton's Second Law of motion

$$F = ma \quad \text{\_\_\_\_\_\_ (5.5)}$$

From third equation of motion, distance d can be written as:

$$d = \frac{v_f^2 - v_i^2}{2a} \quad \text{\_\_\_\_\_\_ (5.6)}$$

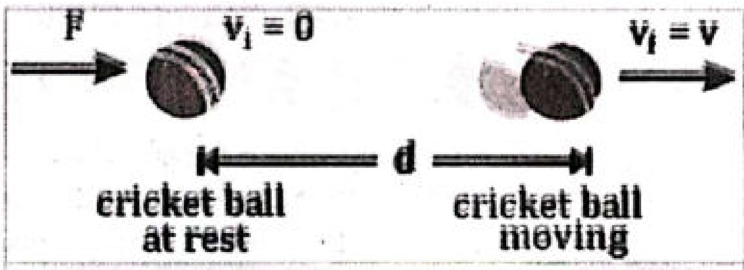


**Figure 5.7:** Work done transfers kinetic energy to a cricket ball.

Putting equations (5.5) and (5.6) in equation (5.4), we get:

$$K.E = \frac{1}{2}m\left(v_f^2 - v_i^2\right) \quad \text{\_\_\_\_\_\_ (5.7)}$$

As the object (cricket ball) started from rest therefore $v_i = 0$ and $v_f = v$.

Therefore, $$K.E = \frac{1}{2}mv^2 \quad \text{\_\_\_\_\_\_ (5.8)}$$

Although derived for cricket ball, the equation (5.8) in general shows the relation for the kinetic energy of any moving object. For example, an iron ball of 1.0 kg moving with a speed of 2.0 m $s^{-1}$ has a Kinetic energy of 2 J.

Equally important, it demonstrate the work kinetic energy theorem which states that work done on an object is equal to change in energy i.e. W = Δ E, where 'W' is the work done and ΔE is the change in energy.

## 5.4 WORK - ENERGY PRINCIPLE

Since work is defined as the movement of an object through a distance, energy can also be described as the ability to move an object through a distance. The net work done on an object is equal to the change in the object's kinetic energy, i.e.,

$$W_{net} = K.E_f - K.E_i = \Delta K.E \quad \text{\_\_\_\_\_\_ (5.9)}$$

Where, the change in the kinetic energy is due to the object's change in speed.

Work-Energy Theorem is also valid for potential energy. However, potential energy cannot be defined only for conservative forces.

In situations where potential energy can be defined, change in potential energy is exactly equal to the **negative** of change in kinetic energy, in which case the Work-Energy theorem becomes

$$W_{net} = \Delta K.E = -\Delta U \quad \text{\_\_\_\_\_\_} \quad (5.10)$$

## 5.4.1 Work - Energy Theorem in Resistive Medium

Why is a skydiver quite confident when using a parachute? Probably she relies on balance between resistive forces and her weight. A resistive force on a moving object opposes the motion of the object or prevent a stationary object from moving, for example friction, viscous force etc.

When forces are acting on an object, energy transformations occur, which means the work is being done. The work done '$W$' on an object by an applied force '$F$' is the sum of the gain in kinetic energy '$\Delta K.E$' of the object and the work done by the object against the resistive force '$W_r$'. That is:

$$W = \Delta K.E + W_r$$

The work done by the object against the resistive force takes energy away from the object, decreasing its kinetic energy. Mathematically:

$$W_r = W - \Delta K.E \quad \text{\_\_\_\_\_\_} \quad (5.11)$$

Both Newton's laws and work energy theorem can be used to solve problems. However, if the forces are not constant, Newton's laws will be difficult to apply.

## 5.4.2 Implications of Energy Loses in Practical Devices and Efficiency

**Input of Mechanical Machine:** The energy supplied to a mechanical machine is called input. Input is equal to the product of effort 'p' and the distance 'd', through which effort acts (Fig. 5.8).

$$input = p \times d$$

**Output of Mechanical Machine:** The work done by a mechanical machine is called output. Output is the product of load 'W' and the distance 'D' through which the load lifts.

$$output = W \times D$$

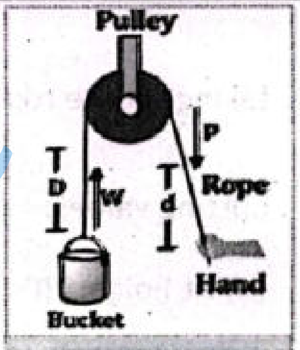


Figure 5.8: Input and out of pulley.

**Efficiency:** The efficiency is the ratio of work output to the work input, and can be expressed in percentage as:

$$efficiency = \frac{\text{useful energy output}}{\text{energy input}}$$

Efficiency is the ratio of similar quantities and therefore has no unit.
The efficiency cannot be greater than 1 (or 100%), in fact it cannot be equal to 1 (or 100%) for real machines. In case of pulley system, the efficiency is only 40%, rest of the 60% goes into waste and converted into unwanted forms of energy.

**Ideal Machine:** For an ideal machine only, the input will be equal to output, this means that no energy is wasted and all energy is converted into useful work. Therefore, the efficiency of an ideal machine is 100 %.

$$output = input \quad \text{or} \quad efficiency = 1$$

Almost all energy transformation technologies operate at efficiencies less than 100 %, most of the wasted energy becomes thermal energy.

For example, automobiles are highly inefficient. Suppose that an amount of fuel containing 1000 J of chemical potential energy is used by an automobile's engine. Only 10 % of the energy is available to do work.

Table 5.1: Typical efficiencies of energy transformation technologies.

| Device | Efficiency (%) |
|---|---|
| Electric generator | 98 |
| Hydroelectric power plant | 95 |
| Large electric motor | 95 |
| Home gas furnace | 85 |
| Wind generator | 55 |
| Fossil fuel power plant | 40 |
| Automobile engine | 25 |
| Fluorescent light | 20 |
| Incandescent light | 5 |

$$\text{Efficiency} = \frac{\text{energy output}}{\text{energy input}} \times 100$$

or $$\text{Efficiency} = \frac{100\,\text{J}}{1000\,\text{J}} \times 100 = 10\ \%$$

Only about 10% (or 100 J) of useful work is done in producing the kinetic energy of the moving car; rest of the energy is wasted in engine and cars transmission. Some other examples of energy losses in practical devices are given in Table 5.1.

**EXAMPLE 5.2: In a science experiment, a 3.00 kg water rocket is launched from ground. The rocket's total energy at the top of its flight is 2352 J. (a) What was the rocket's launching speed? (b) What height did the rocket reach? (c) What are the kinetic energy and potential energy of the rocket 2.5 s after its launch?**

Given: Mass of rocket 'm' = 3.00 kg; Total energy '$E_T$' = 2352 J

Acceleration due to gravity 'g' = 9.8 m $s^{-2}$; Time 't' = 2.5 s

To Find: (a) Initial speed 'v' = ?; (b) Height 'h' = ?

(c) 'K.E' of rocket after 2 s = ?; 'P.E' of rocket after 2 s = ?

Solution: (a) By law of conservation of energy, when the rocket is launched it K.E is converted into P.E, and at height 'h' this potential energy is equal to the total energy.

$$K.E = E_T \quad \text{or} \quad \frac{1}{2}mv^2 = E_T \quad \text{or} \quad v^2 = \frac{2 \times E_T}{m}$$

taking square root on both sides: $$v = \sqrt{\frac{2 \times E_T}{m}}$$

putting values: $$v = \sqrt{\frac{2 \times 2352}{3.00}} \quad \text{or} \quad v = 39.6\,\text{m s}^{-1}$$

(b) At height 'h', the potential energy is equal to the total energy.

$$P.E = E_T \quad \text{or} \quad mgh = E_T \quad \text{or} \quad h = \frac{E_T}{mg}$$

Putting values: $$h = \frac{2352}{3.00 \times 9.80} \quad \text{or} \quad h = 80\,m$$

(c) To find 'K.E' after 2.5 seconds we will first have to calculate the speed of rocket by using first equation of motion along y-axis

$$v_f = v_i - g \times t$$

Putting values: $v_f = 39.6 - 9.8 \times 2.5$

Therefore, $v_f = 15.1\ ms^{-1}$

Putting this value in the kinetic energy equation $K.E = \frac{1}{2}mv_f^2$

Putting values: $K.E = \frac{1}{2} \times 3.00 \times (15.1)^2$ or $K.E = 342 J$

To find the potential energy, we will use law of conservation of energy

$$E_T = K.E + P.E \quad \text{Or} \quad P.E = E_T - K.E$$

putting values: $P.E = 2352 - 342$

Hence, $P.E = 2010\ J$

**Assignment 5.2**

**A crane holding 5000 kg of mass at a height of 12 m. Suddenly, the crane un-holds the mass. Find the kinetic energy of the mass just before striking the ground.**

## SUMMARY

- **Work:** The work done on a body by a constant force is defined as the product of the displacement and the components of the force in the direction of the displacement.
- The energy possessed by a body due to its motion is called **Kinetic energy.**
- **Gravitational field as conservative field:** Gravitational field is conservative field as work done is independent of the path followed and work done along the closed path is zero.
- A **non-conservative field** is that field in which work done depends upon the path followed or the work done along a closed path is not zero.
- **Efficiency:** The efficiency is the ratio of work output to the work input.

## EXERCISE

### Multiple Choice Questions

**Encircle the correct option.**

1) If the unit of force and displacement travelled each be increased five times, then the unit of work will be increased by:

A. 25 times  B. 10 times  C. 5 times  D. 0 times

2) The force that acts on a body but does no work is:

A. gravitational force  B. frictional force  C. elastic force  D. centripetal force

3) The odd force from the following is:

A. gravitational force  B. elastic force  C. frictional force  D. electric force

4) The work done by a body while covering a vertical height of 10 m is 500 J. By how much amount does the energy of the body change?

A. 500 J  B. - 500 J  C. 50 J  D. - 50 J

5) Two objects, A and B, have the same mass. Object A is moving at twice the speed of object B. The kinetic energy of object A as compared to object B is

A. $K.E_A = 4\ K.E_B$. B. $K.E_A = 2\ K.E_B$ C. $K.E_A = K.E_B$ D. $K.E_A = \frac{1}{2}\ K.E_B$

6) If velocity of a body reduces to half of its initial value, then the kinetic energy of the body compared to its initial value is:

A one fourth B. double C. four times D. half

7) A moving hockey ball is hit, such that its speed doubles. Its kinetic energy will become

A. double B. quadruple C. halve D zero

8) An engine converts 600 J of input energy into 150 J of useful work. The efficiency of the engine is

A. 25 % B. 50 % C. 75 % D. 100 %

9) Two engines, A and B, are rated for the same power output. Engine A has a higher efficiency than engine B. What can you conclude?

A. Engine A requires less fuel to produce the same power.
B. Engine A is physically larger and heavier than engine B.
C. Engine B produces more waste heat than engine A.
D. The efficiency rating is unrelated to fuel consumption.

10) Consider two machines, A and B. Machine A has an efficiency of 40 % and Machine B has an efficiency of 60 %. If both machines are provided with 500 J of energy, how much more useful work does Machine B perform compared to Machine A?

A. 50 J B. 100 J C. 200 J D. 300 J

11) A wind turbine converts 30 % of the wind's kinetic energy into electrical energy. If the kinetic energy of the wind is 900 J. The electrical energy produced by the turbine is

A. 90 J B. 270 J C. 600 J D. 700 J



## Short Questions

Give short answers of the following questions.

5.1 Is it possible that a force is acting on a body and the body is in motion due to this force but the work done after certain time is zero?

5.2 Some non-conservative forces are acting on a body. Can they change the total mechanical energy of the body?

5.3 Kinetic energy and work are related. Can kinetic energy ever be negative? Can work ever be negative?

5.4 Differentiate between conservative and non-conservative forces.

5.5 What is the work done by the moon as it revolves around the Earth?

## Comprehensive Questions

Answer the following questions in detail.

5.1 Derive the formula for kinetic energy by using the equations of motion.

5.2 Explain work done by a constant and variable force using force-displacement graph.

5.3 What is work-energy theorem? Explain in detail. Also, write some implications of energy loses in practical devices and efficiency.

## Numerical Problems

5.1 A car carrying truck unloads a 1500 kg car using a plank, as shown in figure. If the plank makes an angle of $30^0$ with the ground and its upper end is at 2 m height, what will be the work done by gravitational force? Also, draw the force-displacement graph for.

**(Ans: 29.4 kJ)**

5.2 A car, having a total mass of 1500 kg (including the driver), is travelling at a speed of 40 kph through a straight path. How much work will be required to stop the said car if its brakes fail and engine turns off? **(Ans: 92.6 kJ)**

5.3 A ball of mass 100 g is released from a height of 30 m. If the ball encounters an air resistance of 0.4 N, find the kinetic energy of the ball just before striking the ground.

**(Ans: 17.4 J)**

5.4 (a) What is the kinetic energy of a car with a mass of 1200 kg traveling at a speed of 20 m $s^{-1}$? (b) If the car comes to a stop over a distance of 50 meters due to braking, what is the average force exerted by the brakes? **(Ans: 240 kJ, 4800 N)**

5.5 A roller coaster cart weighing 500 kg is at the peak of a 30 m tall hill and is traveling at 5 m $s^{-1}$. Find the total mechanical energy at the top of the hill. If it descends to the bottom without any friction losses, what will be its speed at the bottom? **(Ans: 153,250 J, 24.75 m $s^{-1}$)**

5.6 A car engine operates with 25 % efficiency and produces 50 kJ of useful work. How much fuel energy did the car consume? **(Ans: 200 kJ)**

5.7 A refrigerator uses 600 W of electrical power and has an efficiency of 15 %. How much cooling power (useful energy) does the refrigerator produce in one hour? **(Ans: 324 kJ)**

# FLUID MECHANICS

UNIT 6


## Student Learning Outcomes (SLOs)

**The students will:**

- Justify and use Archimedes's principle of flotation.
- Justify how ships are engineered to float in the sea.
- Define and apply the terms: steady (streamline or laminar) flow, incompressible flow and non-viscous flow as applied to the motion of an ideal fluid.
- Use equation of continuity to solve problems.
- Explain that squeezing the end of a rubber pipe results in increase in flow velocity.
- Justify that the continuity is a form of the principle of conservation of mass.
- Justify that the pressure difference can arise from different rates of flow of a fluid [Bernoulli effect].
- Explain and apply Bernoulli's equation for horizontal and vertical fluid flow.
- Explain why real fluids are viscous fluids.
- Describe how viscous forces in a fluid cause a retarding force on an object moving through it.
- Describe super fluidity [As the state in which a liquid will experience zero viscosity. Students should know the implications of this state e.g. this allows for super fluids to creep over the walls of containers to 'empty' themselves. It also implies that if you stir a superfluid, the vortices will keep spinning indefinitely.]
- Analyze the real world applications of the Bernoulli effect [For example, atomisers in perfume bottles, the swinging trajectory of a spinning cricket ball and the lift of a spinning golf ball (the magnus effect), the use of Ventur ducts in filter pumps and car engineers to adjust the flow of fluid, etc.

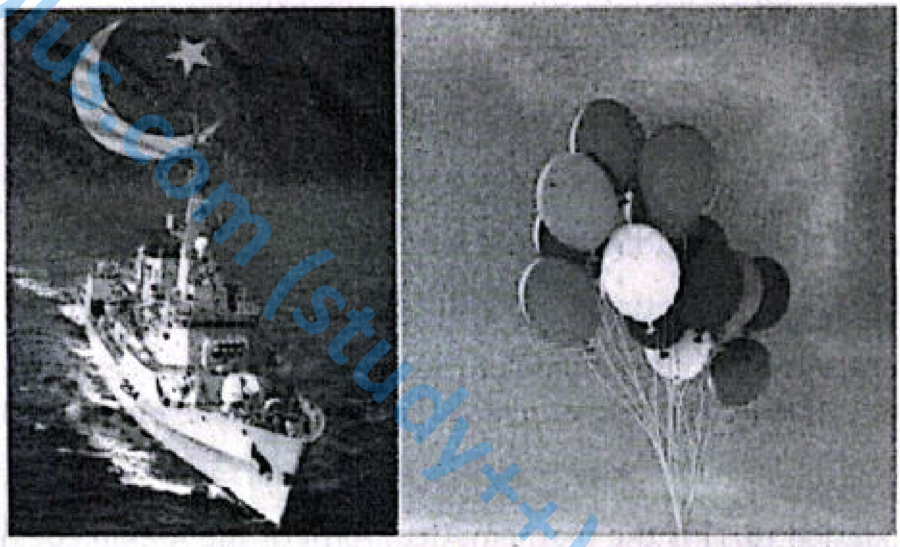
It is well known that liquids and gases belong to fluids. The air we breathe and the water we drink are fluids. An important fluid, blood is flowing in our body's veins, flow of which is essential for life. Life would not exist without fluids and without the behavior that fluids exhibit. Why does rising smoke curl and twist? How does a nozzle increase the speed of water emerging from a hose? How does an airplane gain lift while accelerating on the runway? Fluid mechanics allows us to answer these and many other questions.

In Grade-IX, we have dealt with many situations in which fluids are static. In this chapter, we will deal mainly with the flow of fluid. Fluid mechanics is the branch of physics in which we study about the fluid, either static or dynamic. Studying the fluid in motion is called fluid dynamics. The study of fluids in motion is relatively complex, but analysis can be simplified by making few assumptions such as fluid under consideration is non-viscous, incompressible and its motion is steady. The analysis is further simplified by the use of two important conservation principles, the conservation of mass and conservation of energy. The law of conservation of mass gives us the equation of continuity, while the law of conservation of energy forms the basis of Bernoulli's equation. The equation of continuity and the Bernoulli's equation along with their numerous applications in everyday life, including sports, transportation and technology are discussed in this unit.



## 6.1 UPTHRUST AND ARCHIMEDES PRINCIPLE

Have you ever thought that: Why do heavy ships float on water surface? How does the balloon fly in sky? Why does a mug filled with water feels lighter under water, but feel heavy as soon as we take it out of water? Why do some objects float while others do not?



(a) (b)

**Figure 6.1:** (a) A heavy ship float on the surface of water. (b) Helium-filled balloons are flying off in the sky.

Answers to all these questions are based on the fact that pressure increases with depth in a fluid. So, the upward force on the bottom of an object in a fluid is greater than the downward force on the top of the object. There is a net upward force, called buoyant force or up-thrust, on any object in any fluid. If the up-thrust is greater than the object's weight, the object will rise to the surface and float. If the up-thrust is less than the object's weight, the object will sink. If the up-thrust equals the object's weight, the object will remain suspended at that depth. The up-thrust is always present whether the object floats, sinks, or is suspended in a fluid. Due to up-thrust an air-filled balloon immediately shoots up to the surface when released under water.

Archimedes principle help us to calculate magnitude of up-thrust. It states that:

When an object is totally or partially immersed in a liquid, an up-thrust acts on it equal to the weight of the liquid it displaces.

Consider a solid cylinder of cross-sectional area $A$ and height $h$ immersed in a liquid, as shown in Fig. 6.2. If $h_1$ and $h_2$ be the depth of top and bottom surfaces of the cylinder respectively from the surface of the liquid, then: $h_2 - h_1 = h$

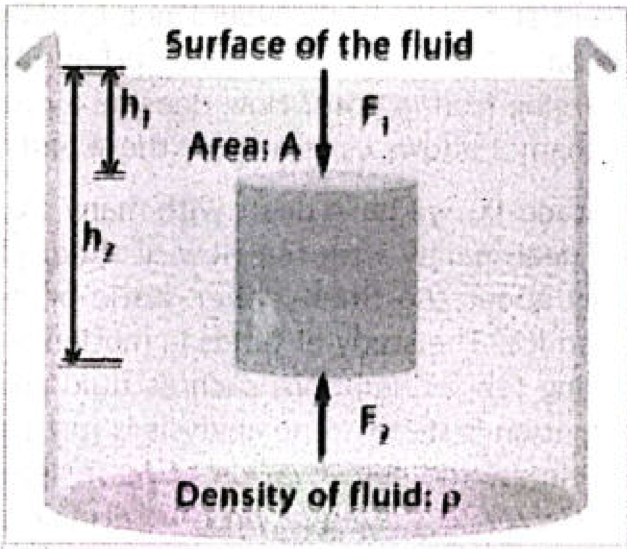


**Figure 6.2:** Up-thrust acts on the cylinder immersed in liquid

If $P_1$ and $P_2$ are the liquid pressure at depths $h_1$ and $h_2$ respectively, then:

$$P_1 = \rho g h_1$$

and
$$P_2 = \rho g h_2$$

If $F_1$ and $F_2$ are the forces exerted by liquid on the top and bottom surfaces of the cylinder respectively, then in terms of pressure:

$$F_1 = P_1 A = \rho g h_1 A$$

and
$$F_2 = P_2 A = \rho g h_2 A$$

The net up-thrust F of the liquid on the cylinder is:

$$F = F_2 - F_1$$

or
$$\text{Upthrust} = \rho g h_2 A - \rho g h_1 A$$

or
$$\text{Upthrust} = \rho g A (h_2 - h_1)$$

or
$$\text{Upthrust} = \rho g A h$$

or
$$\text{Upthrust} = \rho g V \qquad (6.1)$$

**For Your Information**

Nearly two thousand years ago Archimedes noticed up-thrust and found that: there is an apparent loss in the weight of the object when immersed in a liquid due to up-thrust of liquid. He stated his findings in his famous principle i.e., Archimedes principle.

Here, Ah = V is the volume of the cylinder and is equal to the volume of the liquid displaced by the cylinder. Therefore, $\rho g V$ is the weight of the liquid displaced.

So, Upthrust = weight of the liquid displaced

Equation (6.1) is the mathematical form of the Archimedes' principle. Archimedes principle is also helpful to calculate the density of an object.

Ships: Ships are engineered to float in the sea by employing several fundamental principles and design elements: buoyancy, stability and structural reliability.

**a) Buoyancy:** Ships are designed to displace a volume of water equal to their weight, allowing them to float. This is achieved by creating a hull shape that maximizes displacement while minimizing weight. The hull is designed to withstand water pressure, which increases with depth. The hull is shaped to displace water, creating an upward buoyant force ($F_b$) equal to the weight of the ship (W)., i.e.,

$$F_b = \rho V g = W$$

**b) Stability:** Ships are engineered to resist overturning by distributing weight low, using ballast tanks, and designing a stable hull shape.

**c) Structural Reliability:** The ship's structure is engineered to withstand stresses, loads, and vibrations using strong, lightweight materials like steel and aluminum.

By combining these principles and design elements, ships are engineered to float and operate safely in the sea.

**Submarines:** Submarine can travel over as well as under water. It floats over water when the weight of water equal to its volume is greater than its weight. Under this condition, it is similar to a ship and travel over water. It has a system of tanks which can be filled with and emptied of seawater. When these tanks are filled with seawater, the weight of the submarine increases. As soon as the weight becomes greater than the up-thrust, it sinks and remains under water. To return to the surface, the tanks are emptied from seawater.

**Example 6.1: A cubic block of wood is completely dipped in water. Calculate the up-thrust of water acting on it if each side of the block is 10 cm long.**

**Given:** $L = 10\text{ cm} = \frac{10}{100}\text{ m} = 0.1\text{ m}$

**To Find:** Upthrust = ?

**Solution:** According to the Archimedes' principle, upthrust is given by the formula:

$$\text{Upthrust} = \rho g V$$

**For cube,** $V = L^3$, so above equation becomes:

$$\text{Upthrust} = \rho g (L)^3$$

Here for water, we use $\rho = 1000\text{ kgm}^{-3}$, putting values, we get:

$$\text{Upthrust} = 1000 \times 9.8 (0.1)^3$$

$$\text{Upthrust} = 9.8\text{ N}$$

**Assignment 6.1**

**An iron object with density $7.8\text{ g cm}^{-3}$ appears 200 N lighter in water than in air. Calculate:**

**(a) The volume of the object. (b) The weight of the object in the air.**

## 6.2 VISCOUS DRAG AND TERMINAL VELOCITY

Viscosity is measure of fluid's resistance to flow. A more viscous fluid pours slowly while a less viscous pours easily. Imagine a Styrofoam cup with a hole in the bottom. If you pour honey into the cup, the cup drains very slowly. If you fill the same cup with water, the cup will drain more quickly. That is because honey's viscosity is much higher as compared to that of water. Hence, viscosity determines the *rate of flow* of a liquid.

**The resistance offered by different layers of a fluid to its flow is called viscosity.**

| Table 6.1: Viscosity of some fluids at 20°C | |
|---|---|
| **Substance** | **Viscosity (mPa s)** |
| Water | 1.0016 |
| Whole milk | 2.12 |
| Honey | 2000-10000 |
| Glycerin | 1410 |
| Mercury | 1.55 |
| Methanol | 0.5940 |
| Ethanol | 1.144 |

Gases also have viscosity, although it is a little harder to notice it in ordinary circumstances. In most liquids, viscosity decreases as temperature increases, whereas in most gases, viscosity increases as temperature increases. Therefore, it is important to always measure the temperature of a fluid when determining its viscosity. The coefficient of viscosity is represented by $\eta$. Its unit is $N\ s\ m^{-2}$ or $kg\ m^{-1}\ s^{-1}$ or $Pa\ s$. Substances that flow easily have small coefficient of viscosity, while those that do not flow easily have large coefficient of viscosity. For example, water and air have low coefficient of viscosity, whereas honey and engine oil have large high coefficient of viscosity.

Drag Force: A drag force acts on a solid object when it is moving in a fluid (either a gas or a liquid). You feel the drag force when you move your hand through water. If you stretch out your hand out of the window of a fast-moving car, you can easily recognize that considerable drag force has to be exerted on your hand as it moves through the air. You feel a smaller drag force when you tilt your hand. The drag force become larger as the speed of the car increases.

The retarding force (resistance) experienced by an object moving through a fluid is called drag force or viscous drag.

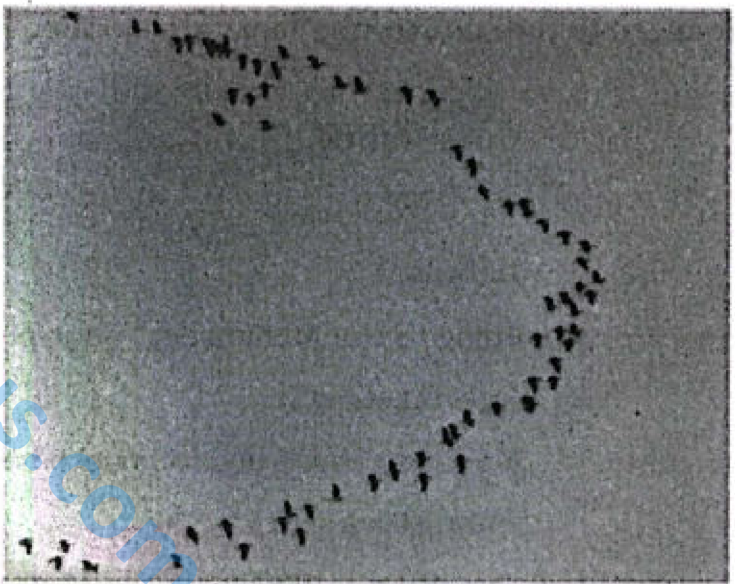


**Figure 6.3:** Geese fly in a V-formation during their long migratory travels. This shape reduces drag and energy consumption for individual birds.

When the fluid is a gas (like air), it is called aerodynamic drag or air resistance. When the fluid is a liquid (like water), it is called hydrodynamic drag.

**Swimmers and Skydivers**

**Swimmers and skydivers change the effective size and orientation of their bodies by bending, twisting or stretching their arms and legs in and out. This allows them to control their speed and direction of motion by using the drag.**

The drag force depends upon various factors such as size, shape and orientation of the object, viscosity of the fluid and the relative speed of the object with respect to fluid.

For small objects, the drag force is given by Stoke's law. According to Stoke's Law, the drag force ($F_d$) on a spherical object is directly proportional to radius of object r, velocity v and coefficient of viscosity $\eta$ of fluid. Thus, the mathematical form of Stoke's law is:

$$F_d = 6\pi\eta r v \qquad (6.2)$$

## 6.2.1 Terminal Velocity

**The maximum constant velocity acquired by a freely falling object in a viscous medium is called terminal velocity.**

Terminal velocity is attained when the weight of the object is balanced by the upward drag force. For example, in case of a raindrop, initially, it accelerates due to the gravity (Fig. 6.4). At first, there will be no drag force. As the velocity increases, the retarding force also increases. As the object falls faster and faster, the drag force increases, So,

$$\text{Net Force} = \text{weight} - \text{drag force}$$

Finally, when viscous drag is equal to the force due to gravity, the net force becomes zero and so does the acceleration. The raindrop then falls at constant velocity (i.e. terminal velocity $v_t$).

$$0 = w - F_d$$

$$F_d = w$$

or

$$6\pi\eta r v_t = mg$$

$$v_t = \frac{mg}{6\pi\eta r} \qquad \text{———} \quad (6.3)$$

Where $r$ is radius of droplet and $\eta$ is the viscosity of the air. For sphere of uniform density $\rho$, mass $m$ is given by:

$$m = \rho V$$

As the volume of sphere is $\frac{4}{3}\pi r^3$, Hence

$$m = \rho\left(\frac{4}{3}\pi r^3\right) \qquad \text{———} \quad (6.4)$$

Putting equation (6.4) in equation (6.3), we get:

$$v_t = \frac{\rho\left(\frac{4}{3}\pi r^3\right)g}{6\pi\eta r}$$

$$v_t = \frac{2\rho g r^2}{9\eta} \qquad \text{———} \quad (6.5)$$

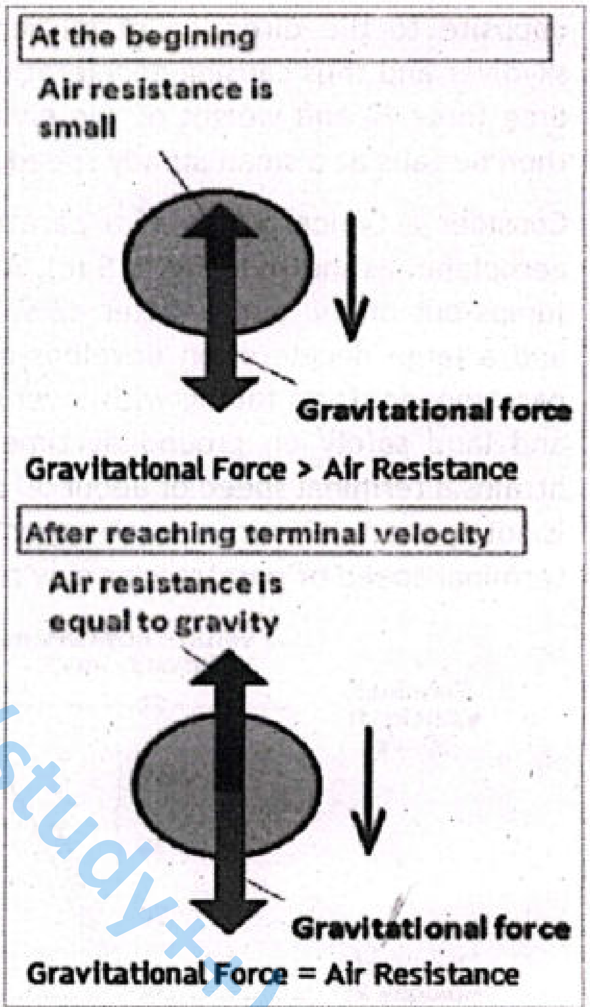



Figure 6.4: A rain droplet falling freely under the action of gravity.

As $\rho, g, \eta$ are constants, so from equation (6.5) we can deduce that $v_t \propto r^2$. This indicates that the smaller mass will attain terminal speed sooner than larger mass. Thus, from equation (6.5), we can conclude that the terminal velocity depends on the square of the radius of the sphere and inversely proportional to the viscosity of the medium.

## 6.2.2 Paratrooper's Jump

A paratrooper initially falls with large acceleration after jumping out of the plane without opening his parachute, and attains a high terminal velocity, as shown in Fig. 6.5 (a).

**Figure 6.5 (a):** A paratrooper, before opening his parachute.

To land safely on ground, he opens his parachute, as shown in Fig. 6.5 (b). Opening the parachute provides a large surface area to produces a large drag force (greater than weight) opposite to the direction of motion. This slows down the skydiver and thus causing deceleration. After some time, the drag force $F_d$ and weight of the paratrooper become equal, then he falls at a small steady speed to the ground level.

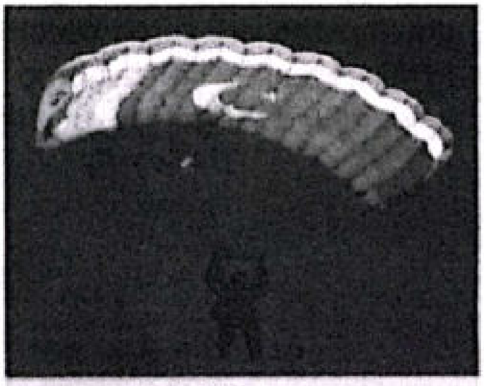


**Figure 6.5 (b):** A paratrooper, after opening his parachute.

Consider a typical graph of a paratrooper jumping from an aeroplane, as shown in Fig. 6.5 (c). At $t = 0$ s, the paratrooper jumps out of the plane. After 42 s, he opens the parachute and a large deceleration develops and within next 5 s, the paratrooper starts falling with a very moderate steady speed and land safely on ground at time 70 s. The paratrooper attains a terminal speed of about 50 m s$^{-1}$ when the parachute is not opened. When the paratrooper opens his parachute, the terminal speed of paratrooper now reaches 8 m s$^{-1}$.

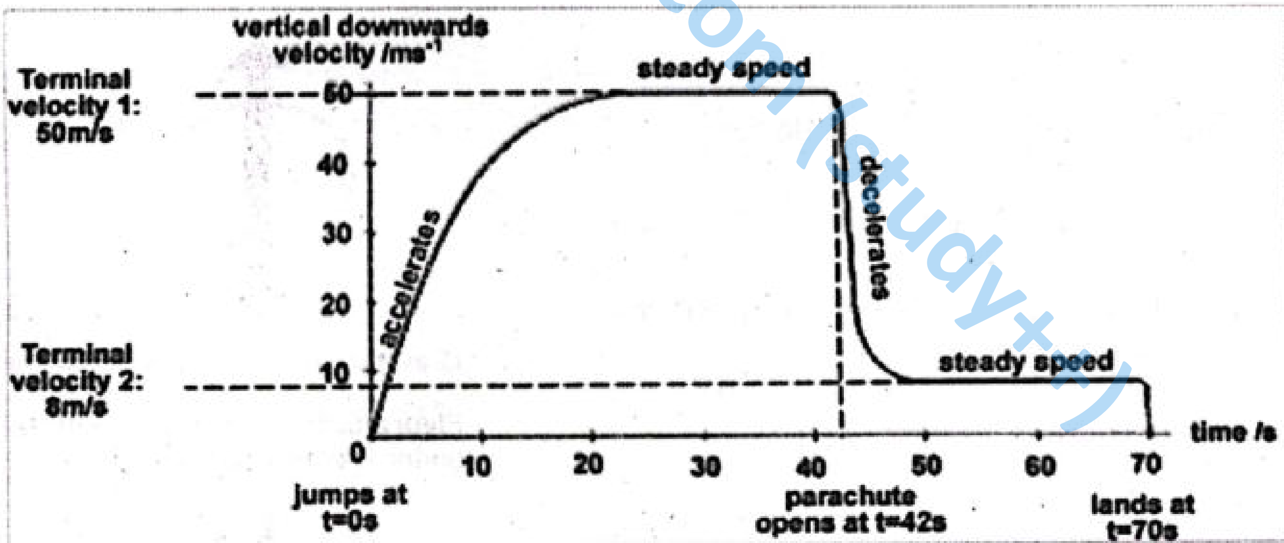



**Figure 6.5 (c):** A typical graph of *velocity against time* for a paratrooper.

**Examples 6.2: A spherical body of radius 2 mm is passing through air with velocity 2 m s$^{-1}$. Find the drag force on the body due to the air. Viscosity of air is $1.9\times10^{-5}$ kg m$^{-1}$ s$^{-1}$.**

Given: Radius = $r$ = 2 mm Velocity = $v$ = 2 m s$^{-1}$

Viscosity of air = $\eta$ = $1.9 \times 10^{-5}$ kg m$^{-1}$ s$^{-1}$.

To Find: Drag force = $F_d$ = ?

Solution: Here, we will use Stoke's law:

$$F_d = 6\pi \eta r v$$

Putting values, we get:

$$F_d = 6 \times 3.14 \times 1.9 \times 10^{-5} \times 2 \times 10^{-3} \times 2$$

$$F_d = 143.2 \times 10^{-8}\ \text{N}$$

**Assignment 6.2**

**Find the terminal velocity obtained by a raindrop of radius 0.3 mm falling through air of viscosity $1.8\times10^{-5}$ kg $m^{-1}s^{-1}$.**

## 6.3 FLUID FLOW

A fluid is a substance that can flow, such as a liquid or a gas. The flow rate of a fluid is the volume of fluid passing a given point in a pipe per unit time. The fluid can flow in two ways: streamline flow or turbulent flow.

If every particle that passing a particular point moves along exactly the same smooth path followed by previous particles that have passed that point earlier, then such flow is called streamline flow.

Streamline flow, also known as steady or laminar flow, is characterized by smooth flow of a fluid through a tube, as shown in Fig. 6.6 (a). By smooth flow, we mean that all particles of the fluid follow the same uniform path, called streamline. The streamlines (paths of different particles) do not cross each other and every fluid particle arriving at a given point has the same velocity. It usually occurs at lower velocities and with streamlined objects. In laminar flow, the middle layer of fluid tends to flow faster.

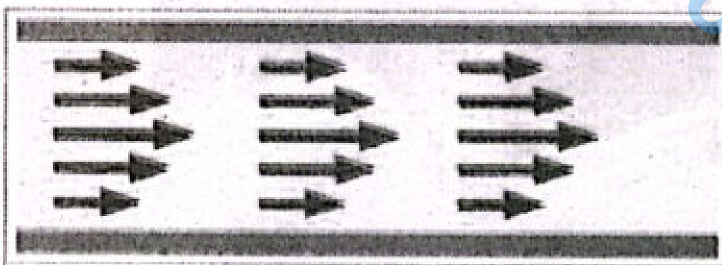


**Figure 6.6: (a) Streamline flow.**

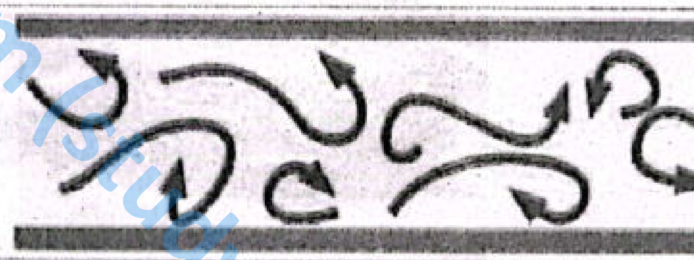


**Figure 6.6: (b) Turbulent flow.**

Above a certain critical speed, streamline flow becomes turbulent. Turbulent flow is irregular flow characterized by small whirlpool-like regions as shown in Fig. 6.6 (b). Turbulent flow, also known as non-laminar flow, is the unpredictable flow of a fluid resulting from excessive speed of the flow or sudden changes in direction or size of the tube or pipe. Turbulent flow occurs at higher velocities or with non-streamlined objects, where the flow lines become disordered and mixed.

Irregular flow of fluid is called turbulent flow.

Remember that the motion of real fluids is very complex and not fully understood, we make some simplifying assumptions in our approach. In model of ideal fluid flow, we make the following assumptions:

- The fluid is non-viscous. In a non-viscous fluid, internal friction is neglected.
- The flow is laminar. In laminar flow, all particles passing through a point have the same velocity.
- The fluid is incompressible. The density of an incompressible fluid is constant. Liquids are generally incompressible, while gases are compressible.
- The flow is irrotational, means it travels in straight lines.

In real fluids, there is always some viscosity. It means that real fluids are viscous. Ideal fluids do not have any viscosity or can be said to have zero viscosity. When a fluid is viscous, it essentially refers to the thickness of the fluid or the friction the fluid faces while it flows. Therefore, ideal fluids do not experience the opposing force and have a non-viscous flow, while real fluids have a viscous flow.

**For Your Information**

The streamlined bodies of dolphins assist their movement in water by reducing the pressure of water against their skin as well as reducing friction. The streamline shape of dolphins enables them to move very speedily.

**Streamlined Design of Fast-Moving Bodies**

As we have studied that the drag force depends on various factors such as size, shape and orientation of the object. The drag force increases significantly with vehicle speed, which reduces the performance of vehicles. Therefore, in **Auto Engineering**, the fast-moving objects are designed as streamlined to improve their performance. A wind tunnel is used for studying the interaction between a solid-stationary model and an airstream. A wind tunnel simulates this interaction by producing a high-speed visible airstream (as shown in the picture) which flows across a model being tested.

## 6.4 EQUATION OF CONTINUITY

Equation of continuity is an important concept of fluid dynamics. Common applications where equation of continuity is used are pipes, tubes, ducts with flowing fluids or gases, rivers and power plants etc.

To derive this equation, consider the steady flow of the fluid through a tube of varying cross-sectional areas, as shown in the Fig. 6.7. The tube has a single entry and single exit. In short interval of time $\Delta t$, the fluid will cover a distance $\Delta x_1$ with a velocity $v_1$ at the lower end of the pipe. At this time, the distance covered by the fluid will be:

$$\Delta x_1 = v_1 \Delta t$$

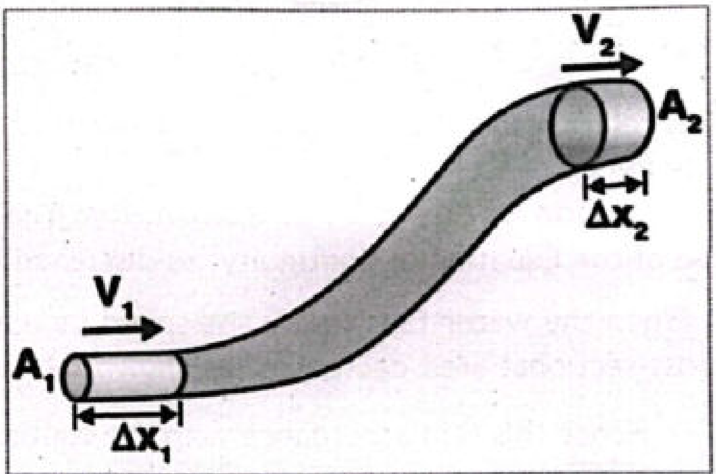


**Figure 6.7:** Fluid flowing through a tube of varying cross-sectional areas.

Now, at the lower end of the pipe, the volume of the fluid that will flow into the pipe will be:

$$V = A_1 \Delta x_1 = A_1 v_1 \Delta t$$

It is known that: mass (m) = Density ($\rho$) × Volume (V). So, the mass of the fluid in $\Delta x_1$ region will be:

$$\Delta m_1 = \rho \times V$$

$$\Delta m_1 = \rho_1 A_1 v_1 \Delta t \qquad \_\_\_\_\_ \ (6.6)$$

Similarly, the mass of the fluid at the upper end in $\Delta x_2$ region will be:

$$\Delta m_2 = \rho_2 A_2 v_2 \Delta t \qquad \_\_\_\_\_ \ (6.7)$$

Here, $v_2$ is the velocity of the fluid through the upper end of the pipe i.e., through $\Delta x_2$, in $\Delta t$ time and $A_2$ is the cross-sectional area of the upper end. As in case of ideal fluid flow, total mass of fluid is conserved; so equating (6.6) and (6.7), we get:

$$\Delta m_1 = \Delta m_2$$

$$\rho_1 A_1 v_1 = \rho_2 A_2 v_2 \qquad \_\_\_\_\_ \ (6.8)$$

Moreover, if the fluid is incompressible, the density will remain constant for steady flow, i.e., $\rho_1 = \rho_2$.

Thus, equation (6.8) can be written as:

$$A_1 v_1 = A_2 v_2 \qquad \_\_\_\_\_ \ (6.9)$$


**Figure 6.8 (a):** As the water falls, its speed increases and cross-sectional area decreases, in accordance with the continuity equation.

Equation (6.9 a) is known as continuity equation and can be written in a more general form as:

$$A v = \text{Constant} \qquad \_\_\_\_\_ \ (6.10)$$

This equation states that for an ideal fluid, the product of cross-sectional area of the pipe and the fluid speed at any point along the pipe is always constant (if there is no source or sink present). This product is equal to the volume flow per second or simply the flow rate. From equation of continuity, we can show that:

$$v \propto \frac{1}{A}$$

This relation shows that smaller the cross-sectional area, the greater the velocity of fluid, and vice versa.

**Application of Equation of Continuity:** There are many phenomena in real word that make use of the Equation of Continuity, as discussed below:

i) When the water falls freely, its speed increases and so its cross-sectional area decreases (as shown by the relation $v \propto \frac{1}{A}$). Hence this is in accordance with the continuity equation.

ii) By squeezing the end of a rubber pipe (as shown in Fig. 6.8 b), its cross-sectional area (A) decreases. As a results velocity of water increases (as shown by the relation $v \propto \frac{1}{A}$). Hence according to the continuity equation, reducing the area of the opening will cause the velocity to increase.

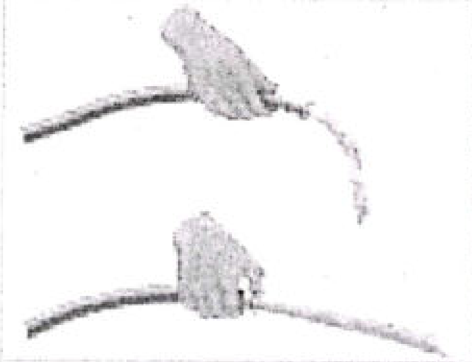


**Figure 6.8 (b):** Squeezing the end of a rubber pipe results in increase in flow velocity.



**Example 6.3:** In a normal adult, the average speed of the blood through the aorta (radius r = 0.8 cm) is 0.33 m s$^{-1}$. From the aorta, the blood goes into major arteries, which are 30 in number, each of radius 0.4 cm. Calculate the speed of the blood through the arteries.

**Given:** Radius of aorta = $r_1$ = 0.8 cm = $0.8 \times 10^{-2}$ m

Velocity of blood in aorta = $v_1$ = 0.33 m s$^{-1}$

Radius of artery = $r_2$ = 0.4 cm = $0.4 \times 10^{-2}$ m

**To Find:** Velocity of blood in arteries = $v_2$ = ?

**Solution:** We use equation of continuity at the aorta and arteries, such as:

$$A_1 v_1 = 30 A_2 v_2$$

The factor of 30 on R.H.S appears because blood from aorta distributed into 30 equal-sized arteries, each with an internal cross-sectional area $A_2$. As $A = \pi r^2$ (internal cross-sectional area being circular), thus above equation becomes:

$$\pi r_1^2 v_1 = 30 \pi r_2^2 v_2$$

$$r_1^2 v_1 = 30 r_2^2 v_2$$

$$v_2 = \frac{1}{30}\left(\frac{r_1}{r_2}\right)^2 v_1$$

Putting values, we get:

$$v_2 = \frac{1}{30}\left(\frac{0.8\ cm}{0.4\ cm}\right)^2 0.33$$

$v_2 = 0.044\ m\ s^{-1}$

Thus, the blood flows through the arteries at a speed of $0.044\ m\ s^{-1}$. This speed is much lower than the speed of blood in aorta. This is due to the fact that the total combined internal cross-sectional area of 30 arteries is greater than that of the aorta. The slower speed of the blood in arteries is favorable for the exchange of gasses.

**Assignment 6.3**

**Why does the deep water flows slowly as compared to shallow water in rivers?**

## 6.5 BERNOULLI'S EQUATION

The Bernoulli's equation is an approximate relation between pressure, velocity and elevation for flow of an ideal fluid. In this section, we will derive the Bernoulli's equation by applying the conservation of energy principle for flowing fluids.

Let us consider the two different regions, as shown in Fig. 6.9. An ideal fluid flows through the pipe in time interval $\Delta t$. If the speed of fluid at lower point is $v_1$ and at upper point is $v_2$, then the distance $\Delta x_1$ covered by fluid in time $\Delta t$ is $v_1 \Delta t$.

$$\Delta x_1 = v_1 \Delta t \quad \text{——— (6.11)}$$

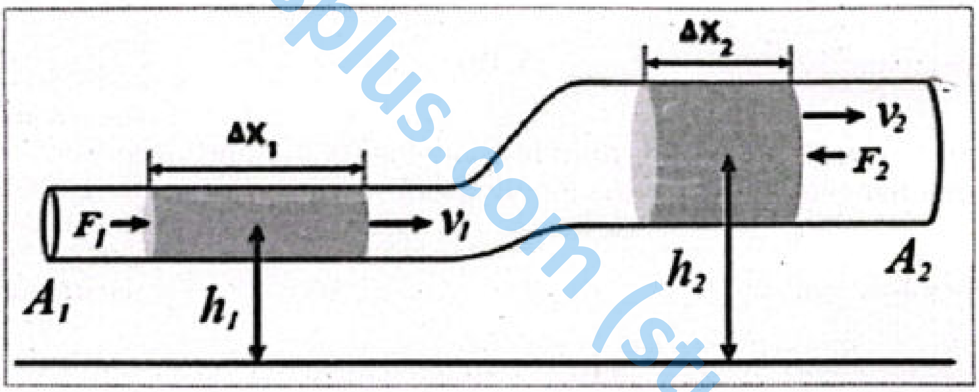



**Figure 6.9: Fluid flowing through a tube of varying cross-sectional area.**

Similarly, during the same interval of time $\Delta t$, the distance $\Delta x_2$ travelled by fluid is $v_2 \Delta t$.

$$\Delta x_2 = v_2 \Delta t \quad \text{——— (6.12)}$$

At lower end, the work done on the fluid in moving through a distance $\Delta x_1$, will be:

$$W_1 = F_1 \Delta x_1 \quad \text{——— (6.13)}$$

using $P = F/A$ or $F = PA$ in (6.13), we get:

$$W_1 = P_1 A_1 \Delta x_1 \quad \text{——— (6.14)}$$

The work done on the fluid at the upper end is negative because $F_2$ is opposite to $F_1$. Thus:

$$W_2 = -P_2 A_2 \Delta x_2 \quad \text{——— (6.15)}$$

Net work done W is obtained by adding equation (6.14) and equation (6.15), i.e.

$$W = W_1 + W_2$$

$$W = P_1 A_1 \Delta x_1 - P_2 A_2 \Delta x_2 \quad \text{\_\_\_\_\_ (6.16)}$$

If $v_1$ and $v_2$ are the velocities of fluid at lower and upper ends respectively, then:

$$W = P_1 A_1 v_1 t - P_2 A_2 v_2 t \quad \text{\_\_\_\_\_ (6.17)}$$

Moreover, if we consider the equation of continuity, then:

$$A_1 v_1 = A_2 v_2 = \frac{V}{t}$$

Where V is the volume of the fluid that flows in time t. Hence:

$$A_1 v_1 t = A_2 v_2 t = V,$$

So, the equation (6.17) becomes:

$$W = P_1 V - P_2 V$$

$$W = (P_1 - P_2) V \quad \text{\_\_\_\_\_ (6.18)}$$

As $V = \frac{m}{\rho}$, so equation (6.18) becomes:

$$W = (P_1 - P_2)\frac{m}{\rho} \quad \text{\_\_\_\_\_ (6.19)}$$

Daniel Bernoulli
(1700-1782)

The name Bernoulli's equation was set on the name of a Swiss physicist Daniel Bernoulli (1700-1782). He derived this equation in 1738.

A part of this work W is utilized by the fluid in changing its K.E and a part is used in changing its P.E, so applying conservation of energy, we get:

$$W = \Delta K.E + \Delta P.E$$

$$(P_1 - P_2)\frac{m}{\rho} = \left(\frac{1}{2}mv_2^2 - \frac{1}{2}mv_1^2\right) + (mgh_2 - mgh_1)$$

Where $h_1$ and $h_2$ are the heights of pipe at lower and upper ends, respectively. Cancelling m from both sides we get:

$$(P_1 - P_2)\frac{1}{\rho} = \left(\frac{1}{2}v_2^2 - \frac{1}{2}v_1^2\right) + (gh_2 - gh_1)$$

Rearranging, we get:

$$P_1 + \frac{1}{2}\rho v_1^2 + \rho g h_1 = P_2 + \frac{1}{2}\rho v_2^2 + \rho g h_2 \quad \text{\_\_\_\_\_ (6.20)}$$

The equation (6.20) is the well-known Bernoulli's equation. However, the subscripts with various parameters on both sides of the equation represent two different points along the pipe. Thus, the general equation can be written as:

$$P + \frac{1}{2}\rho v^2 + \rho g h = \text{constant} \quad \text{\_\_\_\_\_ (6.21)}$$

Thus, Bernoulli's equation states that sum of the pressure, the potential energy per unit volume and the kinetic energy per unit volume will remain constant. Bernoulli's equation is an important tool in investigating many technical systems such as power generation stations, plumbing systems and flight of an aeroplane.

## 6.6 APPLICATIONS OF BERNOULLI'S EQUATION

According to Bernoulli's equation, slow air flow creates a high pressure while fast air flow creates low pressure. This effect is used in many fields of life, such as in filter pump, venturi meter, atomizers, the flow of air over an aerofoil and in blood flow. In this section, we will discuss all of these applications.

**Activity: Bernoulli's Theorem**

Take two A4 size sheets and position them as shown in the picture. When a continuous stream of air is blown between them what do you think will happen to the positions of the papers? Will the sheets separate or move closer to one another? Test your prediction by doing this activity. Was your prediction correct?

The sheets will move closer to one another. This is due to the Bernoulli's principle, which states that fast airflow creates low pressure between the two sheets. Thus, sheets will move towards the low-pressure central region.

### 6.6.1 Atomizer

Atomizer is a device that is used to emit liquid droplets as fine spray. It works on Bernoulli's principle.

When you squeeze the rubber bulb (Fig. 6.10), high speed air passes horizontally over a vertical tube and creating a lower pressure than that inside the container. This pressure difference pushes the liquid from the reservoir up through the vertical narrow tube and into the moving stream of air. Atomizer has a nozzle at the end of the horizontal tube, which causes the liquid to break up into small drops, mixing it with the air and carried away with the stream of air.

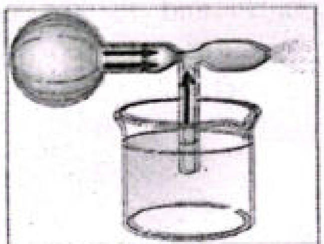


**Figure 6.10: A simple Atomizer.**

Atomizer nozzles are used for spraying perfumes, applying paint and in engine carburetor etc.

### 6.6.2 Filter Pump

A filter pump is designed and can be explained on the basis of Bernoulli's effect. A filter pump has a jet in the middle (Fig. 6.11). When water from tap reaches the jet, its speed increases and hence causes a pressure drop near it. According to Bernoulli's Principle, the pressure of the moving air decreases as the speed of the air increases.

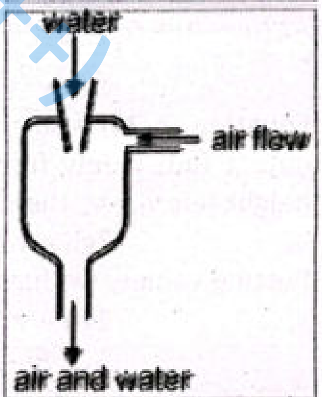



**Figure 6.11: Filter pump.**

The air thus flows in from the side tube to which the vessel is connected. The air and water together are forced to the bottom of the filter pump.

### 6.6.3 Torricelli's Theorem

Torricelli's Theorem states that

The speed of flow of fluid from an orifice at depth h below the top surface of a liquid is equal to speed gained by the fluid in free falling through the height h.

Consider a tank filled with some fluid and with an orifice (slit) near the bottom, as shown in Fig. 6.12. The top surface of fluid is at height $h_1$ and the opening near bottom is at height $h_2$. According to Bernoulli's equation:

$$P_1 + \frac{1}{2}\rho v_1^2 + \rho g h_1 = P_2 + \frac{1}{2}\rho v_2^2 + \rho g h_2 \quad \text{——— (6.22)}$$

Here velocity $v_1$ at top surface is negligible as compared to velocity of efflux $v_2$. By neglecting second term from L.H.S in the equation (6.22), we get:

$$P_1 + \rho g h_1 + 0 = P_2 + \frac{1}{2}\rho v_2^2 + \rho g h_2 \quad \text{——— (6.23)}$$

If we put $v_2 = v$, then equation (6.22) becomes as:

$$\frac{1}{2}\rho v^2 = P_1 - P_2 + \rho g h_1 - \rho g h_2$$

As $P_1$ and $P_2$ are equal being the atmospheric pressures at top surface and at the orifice respectively, so $P_1 - P_2 = 0$. Hence, we get:

$$\frac{1}{2}\rho v^2 = \rho g (h_1 - h_2)$$

or

$$\frac{1}{2} v^2 = g (h_1 - h_2)$$

$$v = \sqrt{2g(h_1 - h_2)}$$

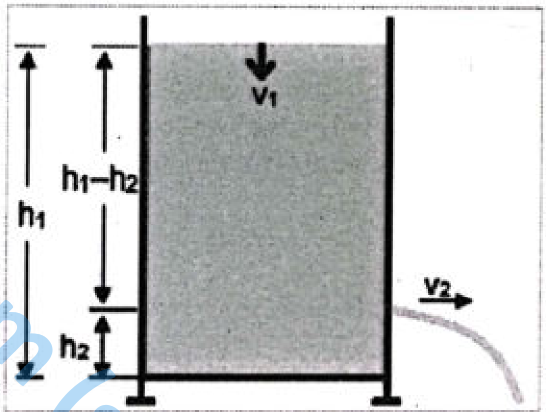



Figure 6.12: A tank filled with water and has an orifice near the bottom. The top surface of water comes down with almost zero speed.

As $h_1 - h_2 = h$, in general, we can write:

$$v = \sqrt{2gh} \quad \text{——— (6.24)}$$

Equation (6.24) also true for the velocity gained by an object in falling from height h. If an object falls freely from height h, its initial velocity $v_i = 0$ and its final velocity after falling height h is $v_f = v$, then using third equation of motion, we get:

$$2gh = v_f^2 - v_i^2$$

Putting values, we have:

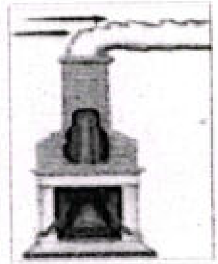
$$2gh = v^2 - 0^2$$

$$v = \sqrt{2gh}$$

This is the same result as in equation (6.24), indicating that the speed of flow of fluid from an orifice at depth $h$ below the top surface of a liquid is equal to the speed gained by the fluid in free fall from the height $h$.

**Do You Know?**

In the summer you can enjoy by getting heat from the fireplace without the room filling up with smoke! This is again due to the Bernoulli's effect. Can you explain how?



## 6.6.4 Venturi Meter

A venturi meter (or flow meter) is used to measure the flow speed of liquid through a tube. The liquid having high pressure and low velocity gets converted to the low pressure and high velocity at a particular point and again reaches to high pressure and low velocity. The point where the fluid has low pressure and high velocity is the place where the venturi flow meter is used.

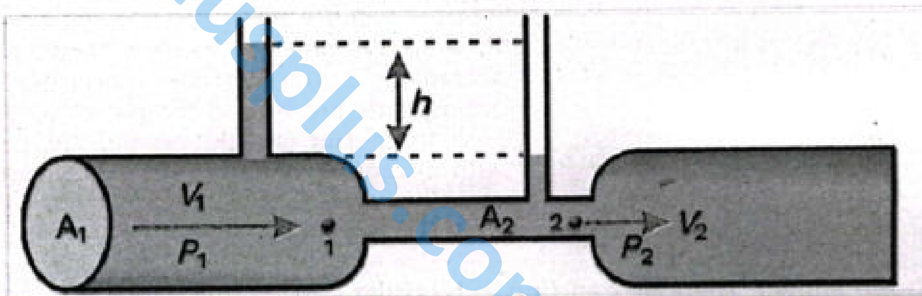



**Figure 6.13:** Venturi meter.

The venturi meter is constructed, as shown in Fig. 6.13. Venturi meter is so designed as to ensure the stream line flow of liquid through a pipe section with area $A_1$, a flow velocity $v_1$ and pressure $P_1$. In the narrow section (the throat) with area $A_2$, the fluid flows with flow speed $v_2$, and has pressure $P_2$. As a result, the pressure difference at two points 1 and 2 is appeared as a height difference h.

To derive an expression for the pressure difference, we use the Bernoulli's equation and the continuity equation. First applying Bernoulli's equation to the portion 1 and 2, we have:

$$P_1 + \frac{1}{2}\rho v_1^2 + \rho g h_1 = P_2 + \frac{1}{2}\rho v_2^2 + \rho g h_2$$

As $h_1 = h_2$, so $\rho g h$ term will cancel out from both sides and we get:

$$P_1 + \frac{1}{2}\rho v_1^2 = P_2 + \frac{1}{2}\rho v_2^2$$

$$P_1 - P_2 = \frac{1}{2}\rho v_2^2 - \frac{1}{2}\rho v_1^2$$

$$P_1 - P_2 = \frac{1}{2}\rho (v_2^2 - v_1^2) \qquad \text{\_\_\_\_\_\_ (6.25)}$$

From equation of continuity, we have:

$$A_1 v_1 = A_2 v_2$$

or $$v_2 = \frac{A_1}{A_2} v_1 \qquad \text{——— (6.26)}$$

Putting (6.26) in (6.25), we get:

$$P_1 - P_2 = \frac{1}{2}\rho\left[\left(\frac{A_1}{A_2}v_1\right)^2 - v_1^2\right]$$

$$P_1 - P_2 = \frac{1}{2}\rho v_1^2\left(\frac{A_1^2}{A_2^2} - 1\right) \qquad \text{——— (6.27)}$$

*Also,* $P_1 - P_2 = \rho g h$ ——— (6.28)

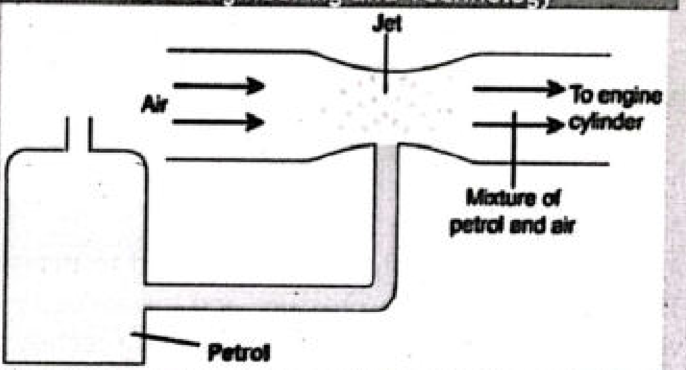
Putting equation (6.28) in (6.27), we get:

$$\rho g h = \frac{1}{2}\rho v_1^2\left(\frac{A_1^2}{A_2^2} - 1\right)$$

$$2gh = v_1^2\left(\frac{A_1^2}{A_2^2} - 1\right)$$

$$v_1 = \sqrt{\frac{2gh}{\left(\frac{A_1^2}{A_2^2} - 1\right)}} \qquad \text{——— (6.29)}$$

**Auto Engineering and Technology**




The carburetor used in engines contains a venture tube. When the engine is switched on, air is sucked into the venture tube. Fast moving air in the narrow throat of the tube causes a low pressure at this region according to the Bernoulli's principle. The higher atmospheric pressure in the petrol compartment will push the petrol into the region. The petrol will mix with air before it flows into the cylinder engine for combustion.

If $A_1$, $A_2$ and $h$ are known, then we can find $v_1$ by using equation (6.29) and hence rate of flow can be calculated using equation of continuity.

## 6.6.5 Aerofoil

An aerofoil is the term used to describe the cross-sectional shape of an object that creates a lift, when moved through a fluid such as air.

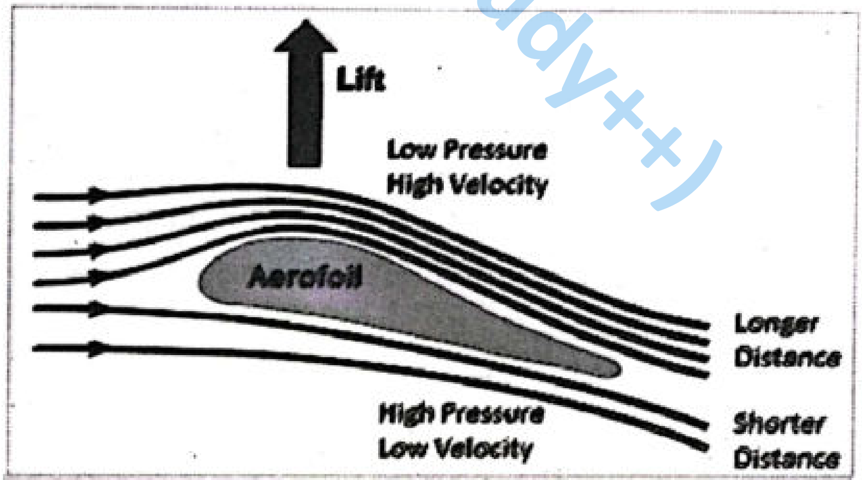



Figure 6.14: Cross-section of aeroplane's wing.

Aerofoils are employed on aircraft as wings to produce lift perpendicular to the air flow (Fig. 6.14). When a wing shaped like an aerofoil, moves in air, the flow of air over the top travels faster creating a region of low pressure. The flow of air below the wing is slower resulting in a region of higher pressure. The pressures difference between the upper-side and lower-side of the wing causes a net upward force, called lift, which helps the plane to take-off.

### 6.6.6 The Magnus Effect

The Magnus effect is a phenomenon related to a spinning object moving through a fluid. In Magnus effect, a spinning ball curves away from its path of flight, as shown in the Fig. 6.15. The path of the spinning object is deflected in a manner that is not present when the object is not spinning. The Magnus Effect depends on the rotational speed of the object. The deflection is caused by the pressure difference of the fluid on opposite sides of the spinning object due to the varying fluid velocities.

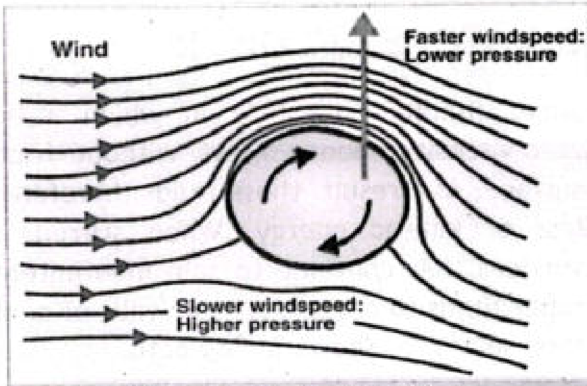



**Figure 6.15: Magnus Effect**

In the case of a ball spinning through the air, the spinning ball drags some of the surrounding air with it. A spinning object moving through a fluid deviates from its straight path because of pressure differences that develop in the fluid as a result of velocity changes induced by the spinning body.

**Example 6.4:** Water is flowing streamline through a closed pipe system. The speed of water at one point is 4 m $s^{-1}$ and the pressure is 47.1 kPa, while at another point 3 m lower, the speed is 3 m $s^{-1}$. Find the pressure at the lower point.

Given: At higher point:

Speed = $v_1$ = 4 m $s^{-1}$ height = $h_1$ = 3.0 m

Pressure = $P_1$ = 47.1 kPa = $47.1\times10^3$ Pa

At lower point:

Speed = $v_2$ = 3 m $s^{-1}$ height = $h_2$ = 0 m

To Find: Pressure = $P_2$ = ?

Solution: Apply Bernoulli's theorem:

$$P_1 + \frac{1}{2}\rho v_1^2 + \rho g h_1 = P_2 + \frac{1}{2}\rho v_2^2 + \rho g h_2$$

By rearranging above equation, we get:

$$P_2 = P_1 + \frac{1}{2}\rho(v_1^2 - v_2^2) + \rho g(h_1 - h_2)$$

Putting the given values, and taking $\rho$ = 1000 kg/$m^3$ for water, we get:

$$P_2 = (47.1\times10^3) + \frac{1}{2}(1000)\,[(4)^2-(3)^2] + (1000)(9.8)(3-0)$$

After using calculator, we get:

$$P_2 = 47.1 + 3.5 + 29.4\times10^3$$

$$P_2 = 80{,}000 \text{ Pa} \quad \text{or} \quad 80 \text{ k Pa}$$

**Assignment 6.4**

**The speed of air across the top and bottom of an aeroplane's wing is 450 m $s^{-1}$ and 410 m $s^{-1}$ respectively. Calculate the lift on the wing if the wing is 15 m long and 3 m wide ($\rho$ = 1.29 kg $m^{-3}$, for air).**

## 6.7 SUPER FLUIDITY

Super fluidity is a state in which a liquid experiences zero viscosity, hence flows without friction through any surface. As a result, these fluids therefore flow without any loss of kinetic energy. When stirred, superfluid forms vortices that continue to spin indefinitely. This allows for super fluids to creep over the walls of containers to 'empty' themselves, as shown in Fig. 6.16.

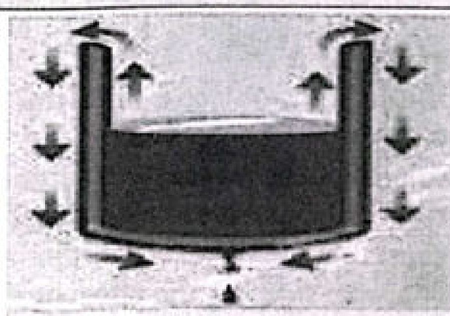


**Figure 6.16:** Super fluids creeps over the walls of containers to 'empty' themselves.

Super fluidity is observed in liquid helium at temperatures near absolute zero, as well as electrons within a superconducting solid. The neutron fluid in a neutron star may also be a superfluid. The unusual behavior of superfluid arises from quantum mechanical effects.

To create superfluid states, helium gas cooled to a few degrees above absolute zero, as shown by the graph in Fig. 6.17. This is achieved by compressing the gas, and then expelling it through a small nozzle. As the gas expands, it rapidly cools.

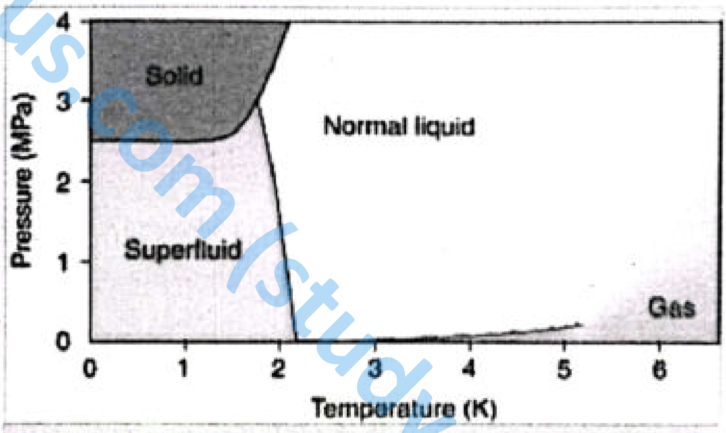



**Figure 6.17:** Pressure-Temperature graph for He.

If a superfluid is placed in a rotating container, instead of rotating uniformly with the container, the rotating state consists of quantized vortices. That is, when the container is rotated below a certain velocity (known as the critical angular velocity), the liquid remains perfectly stationary. Once the critical angular velocity is achieved, the superfluid will form a vortex.

The vortex strength of super fluid is quantized, i.e., a superfluid can only spin at certain 'allowed' angular velocities. Rotation in a normal fluid (like water) is not quantized. If the rotation speed is increased more and more quantized vortices will be formed which arrange in wonderful patterns.

**For Your Information**

Helium-4 ($^4He$) becomes a superfluid at a temperature below 2.17 K. Helium-3 ($^3He$) becomes a superfluid below 0.0025 K.

Application of Superfluids: Superfluids are used in high-precision devices, such as gyroscopes, which allow the measurement of some theoretically predicted gravitational effects. Recently, superfluids have been used to trap and slow the speed of light.

## Fluid Mechanics and Medical Technology

Medical technology uses heartbeat and sound of blood flow to find the blood pressure. A fluid exerts pressure on the walls of its container. As a fluid, blood also exerts pressure on the walls of the heart, arteries, vessels, and capillaries that make up the circulatory system. The sphygmomanometer is a well-known medical equipment used for the measurement of blood pressure of a person. The sphygmomanometer cuff (air bag) is placed around the upper arm of a person, inflated, and then deflated while the meter measures the pressure of blood passing through those arteries in the arm. The external pressure is increased by pressing the air bulb repeatedly. The sound of the blood flow is heard by the stethoscope.

The gradual increase in pressure squeezes the arm and compresses the blood vessels under the air bag. When the pressure in the air bag becomes greater than the systolic pressure (120 torr), then the vessels collapse, cutting off the flow of blood. At this instant stethoscope detect no sound. The pressure in the bag is then decreased by slowly opening the release valve of air bulb. When the external pressure becomes equal to systolic pressure, the blood flows (turbulent) with high speed through narrow contracted vessel. Stethoscope detects sound at this instant and dial of the barometer gives systolic pressure. By further opening the release valve, the pressure in the air bag drops enough that the vessels no longer remain compressed and the blood is now in laminar flow. At this instant there are no gurgle's sound heard by stethoscope, so this is the signal to record diastolic pressure.

For a normal person the blood pressure varies between systolic (120 torr) to diastolic (75-80 torr). These values change with age.

## SUMMARY

- **Archimedes principle** states that: When an object is totally or partially immersed in a liquid, an up-thrust acts on it equal to the weight of the liquid it displaces.
- The resistance offered by different layers of a fluid to its flow is called **viscosity**. Its unit is $N\ s\ m^{-2}$ or $kg\ m^{-1}\ s^{-1}$ or Pa s.
- The retarding force (resistance) experienced by an object moving through a fluid is called **drag force or viscous drag.**
- The maximum constant velocity acquired by a freely falling object in a viscous medium is called **terminal velocity.**
- If every particle that passes a particular point moves along exactly the same smooth path followed by previous particle passing that has passed that point, then such flow is called **Streamline.**
- Irregular flow of fluid is called **turbulent flow.**
- **Equation of continuity** is defined as: For an ideal fluid the product of cross-sectional area of the pipe and the fluid speed at any point along the pipe is always constant. This product is equal to the volume flow per second or simply the flow rate.
- **Bernoulli's equation** states that sum of the pressure, the potential energy per unit volume and the kinetic energy per unit volume will remain constant. According to Bernoulli's equation slow air flow creates a high pressure, while fast air flow creates low pressure.

- The speed of flow of fluid from an orifice at depth h below the top surface of a liquid is equal to speed gained by the fluid in free falling through the height h. This statement is called **Torricelli's Theorem.**
- A **venturi meter (or flow meter)** is used to measure the flow rate of liquid through a tube.
- An **aerofoil** is the term used to describe the cross-sectional shape of an object that creates a lift, when moved through a fluid such as air.
- In **Magnus effect,** a spinning ball curves away from its path of flight.
- **Super fluidity** is a state in which a liquid experiences zero viscosity, hence flows without friction past any surface.

## EXERCISE

### Multiple Choice Questions

Encircle the correct option.

1) Principle of floatation helps us to find:

A. Density B. Velocity C. Area D. Pressure

2) According to Archimedes' principle, the up-thrust on an object equal to:

A. weight of the displaced liquid B. volume of the displaced liquid
C. mass of the displaced liquid D. density of the displaced liquid

3) In the equation for Stoke's law, the sphere must be:

A. moving with a low, non-zero acceleration. B. moving with a constant velocity.
C. at rest in the fluid. D. of the same density as the fluid.

4) Two identical spherical drops of water are falling through air with a steady velocity of 20 cm $s^{-1}$. If the drops combine to form a single drop, what would be the terminal velocity of the single drop?

A. 10 cm $s^{-1}$ B. 20 cm $s^{-1}$ C. 32 cm $s^{-1}$ D. 40 cm $s^{-1}$

5) Spherical balls of radius 'r' are falling in a viscous fluid of viscosity '$\eta$' with a velocity 'v'. The retarding viscous force acting on the spherical ball is:

A. inversely proportional to 'r' but directly proportional to velocity 'v'
B. directly proportional to both radius 'r' and velocity 'v'
C. inversely proportional to both radius 'r' and velocity 'v'
D. directly proportional to 'r' but inversely proportional to 'v'

6) Two rain drops reach the Earth with their terminal velocities in the ratio 4:9. The ratio of their radii is:

A. 4 : 9 B. 2 : 3 C. 3 : 2 D. 9 : 4

7) A liquid flows through a pipe with a diameter of 10 cm at a velocity of 9 cm/s. If the diameter of the pipe decreases to 6 cm, the new velocity of the liquid will be:

A. 12 cm $s^{-1}$ B. 15 cm $s^{-1}$ C. 21 cm $s^{-1}$ D. 25 cm $s^{-1}$

8) If a pipe with flowing water has a cross-sectional area nine times greater at point 2 than at point 1, what would be the relation of flow speed at the two points?

A. The flow speed at point 2 is nine times that at point 1
B. The flow speed at point 2 is three times that at point 1
C. The flow speed at point 1 is nine times that at point 2
D. The flow speed at point 1 is three times that at point 2

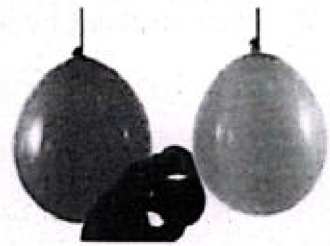
9) With an increase in temperature, the viscosity of liquids and gases, respectively will

A. increase and increase B. increase and decrease
C. decrease and increase D. decrease and decrease

10) According to equation of continuity, when water falls its speed increases, while its cross-sectional area ________

A. remains same B. decreases C. increases D. can't be predicted

11) The figure shows two balloons are hung by a string. When the air is blown harder through the straw, the balloons will:



A. remain stationary
B. move closely to each other
C. move far apart to each other
D. burst

12) Which will produce the greatest increase in flow velocity through a tube?

A. halving the tube radius B. doubling the viscosity of the liquid
C. doubling the tube area D. doubling the tube radius

13) Where does the Venturi effect specifically occur?

A. Where a stream is expanded
B. Where a stream flows downwards with gravity
C. Where a stream constricts
D. In the exact center of a flowing stream

14) The end of a hose has a diameter of 4 cm. If one wants the velocity of the water coming out to be 4 times higher, what should be the diameter of the nozzle on the end?

A. 1/4 cm B. 4 cm C. 2 cm D. 1/2 cm

## Short Questions

Give short answers to the following questions:

6.1 Why do athletes, such as swimmers and bicyclists, wear body suits in competition.
6.2 Distinguish between turbulent and streamline flow.

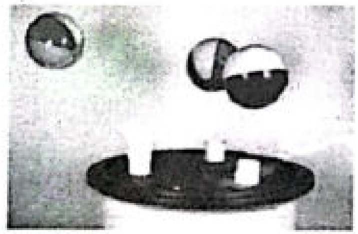
6.3 When there is a change in the width of the river. The speed of the water decreases in wider regions whereas the speed of water increases in the narrower regions. Why?

6.4 It is dangerous to stand close to rail tracks when a rapidly moving train passes. Explain why?

6.5 Verify that pressure has units of energy per unit volume.

6.6 A perfume bottle or atomizer sprays a fluid from inside the bottle when it is pressed. How does the fluid rise up in the vertical tube in the bottle?

6.7 If you lower the window on a car while moving, an empty plastic bag can sometimes fly out the window. Why does this happen?



6.8 Explain how an upthrust is produced when an aeroplane is running on runway?

6.9 Why does a ball placed in a vertical air jet become suspended (as shown in figure)?

6.10 Can you increase the flow velocity of water in a rubber pipe by squeezing? Explain briefly.

6.11 What is meant by superfluid? How is superfluid made?

## Comprehensive Questions

Answer the following questions in detail:

6.1 What is up thrust? Explain with the help of examples.

6.2 Explain Archimedes' principle of flotation.

6.3 What is viscosity? Explain with the help of examples.

6.4 What is viscous drag? Explain with the help of examples.

6.5 Explain Stoke's law. Derive its mathematical formula by using dimensional analysis.

6.6 Explain terminal velocity and derive its equation by using Stoke's law.

6.7 State the equation of continuity and derive its mathematical form. Also write one example from daily life, where equation of continuity applies.

6.8 Derive the Bernoulli's equation. Also, discuss its application in daily life.

6.9 What is Torricelli's theorem. Explain.

6.10 What is venturi meter? Explain.

6.11 Describe super fluidity.

## Numerical Problems

6.1 The settling speed of a small fog droplet in air is found to be 2.98 mm $s^{-1}$. Find out the radius of the droplet. The coefficient of viscosity of air is $1.9\times10^{-5}$ kg $m^{-1}$ $s^{-1}$. **(Ans: $5.1\times10^{-6}$ m)**

6.2 Water is flowing at a velocity of 2.00 m $s^{-1}$ through a hose with an internal diameter of 1.60 cm. (a) What is the flow rate in liters per second? (b) The fluid velocity in this hose's nozzle is 15.0 m $s^{-1}$. What is the nozzle's inside diameter? **(Ans: 0.402 L $s^{-1}$, 0.584 cm)**

6.3 Blood is pumped from the heart at a rate of 5.0 L $min^{-1}$ into the aorta (of radius 1.0 cm). Determine the speed of blood through the aorta. **(Ans: 27 cm $s^{-1}$)**

6.4 Water flows through a pipe with an internal diameter 4 cm at a speed of 2 m $s^{-1}$. What should be the diameter of the nozzle, if the water is to emerge out with a speed of 4 m $s^{-1}$. **(Ans: 2.8 cm)**

6.5 A hose lying on the ground has water coming out of it at a speed of 5.4 m $s^{-1}$. You lift the nozzle of the hose to a height of 1.3 m above the ground. At what speed does the water now come out of the hose? **(Ans: 1.9 m $s^{-1}$)**

6.6 A pipe has two different cross-sectional areas, $A_1$ = 25 $cm^2$ and $A_2$ = 4 $cm^2$. The volumetric flow rate of water through the pipe is $5\times10^{-3}$ $m^3$ $s^{-1}$. Determine:

(a) the speed of water in both cross-sectional areas of the pipe.

(b) the pressure difference between them. ($\rho = 10^3$ kg $m^{-3}$)

**(Ans: 2 m $s^{-1}$, 12.5 m $s^{-1}$, $7.6 \times 10^4$ Pa)**

6.7 The speed of water in a hose increases from 1.96 m $s^{-1}$ to 25.5 m $s^{-1}$ as it moves from the hose to the nozzle. Calculate the pressure in the hose, given that the absolute pressure in the nozzle is $1.01\times 10^5$ N $m^{-2}$ (atmospheric, as it must be) and assuming level and frictionless flow.

**(Ans: $4.24\times 10^5$ N $m^{-2}$)**

6.8 A hole is drilled at the bottom of a full bucket of water of height h. What will be the velocity of water exiting from the hole? **(Ans: $\sqrt{2gh}$ )**

6.9 The efflux speed of oil from a narrow hole of a tank is 33.8 m $s^{-1}$, at pressure of $4.12\times10^5$ Pa. Calculate the density of the oil? **(Ans: 0.72 kg $m^{-3}$)**

6.10 If the speed of air across the top and bottom of a small aeroplane's wing is 30 m $s^{-1}$and 20 m $s^{-1}$ respectively. Calculate the pressure difference between the top and bottom of wings if $\rho$ = 1.29 kg $m^{-3}$, for air. **(Ans: 322.5 Pa)**

6.11 An empty metrological balloon has a weight of 80 N. How much maximum contents the balloon can lift besides its own weight if it is filled with 10 $m^3$ of hydrogen? (Density of hydrogen = 0.09 kg $m^{-3}$, Density of air = 1.3 kg $m^{-3}$). **(Ans: 38.58 N)**

# PHYSICS OF SOLIDS

UNIT 7



## Student Learning Outcomes (SLOs)

**The students will:**

- Distinguish between the structures of crystalline, glassy, amorphous and polymeric solids.
- Describe that deformation of solids in one dimension [that it is caused by a force and that in one dimension, the deformation can be tensile or compressive].
- Define and use the terms stress, strain and the Young's modulus.
- Describe an experiment to determine the Young modulus of a metal wire.
- Describe and use the terms elastic deformation, plastic deformation and elastic limit.
- Justify why and apply the fact that the area under the force-extension graph represents the work done.
- Determine the elastic potential energy of a material [that is deformed within its limit of proportionality from the area under the force-extension graph. Also state and use $E_P = \frac{1}{2}Fx = \frac{1}{2}kx^2$ for a material deformed within its limit of proportionality].

Materials have specific uses depending upon their properties and characteristics, such as hardness, ductility, brittleness, malleability and response to the applied pressure. What makes steel hard and lead soft? It depends on the structure, the particular order and the bonding of atoms in a material. This clue has made it possible to design and create materials with unique properties for their use in the modern technology.

## 7.1 CLASSIFICATION OF SOLIDS

Solids are the materials which are incompressible and have a fixed shape and volume. On the basis of atomic arrangement (the structure), solids may be classified into three types: crystalline, polycrystalline (polymeric) and amorphous (glassy) solids.

### 7.1.1 Crystalline Solids

In solids, particles are closely packed together but their packing may have different patterns. The arrangement of particles can be studied by X-rays diffraction technique.

Crystalline solids are those in which the constituent atoms, ions or molecules, have a regular and well-defined arrangement.

Molecular and ionic structures of crystals are shown in Fig. 7.1.

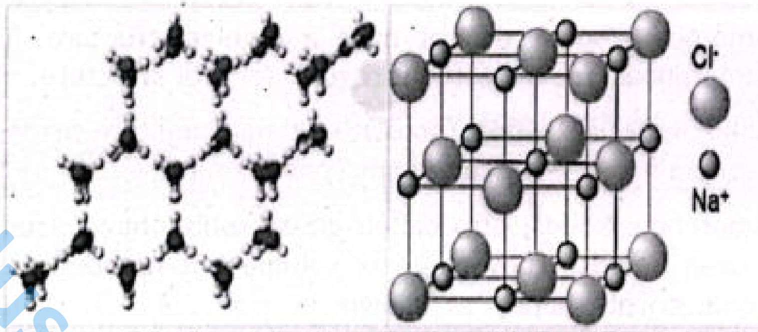



**Figure 7.1:** The molecular (left) and ionic (right) crystal structure of NaCl.

Examples of crystalline solids include salts (sodium chloride, and potassium chloride), metals (copper, iron and zinc), non-metals (diamond, sulphar and mica), ionic compounds (NaCl and copper sulphate), ceramic (zirconium), sand and quartz. Crystalline solids have sharp melting points. Crystalline solids are anisotropic for their physical property measurement. On cutting, they give sharp cleaves.

### 7.1.2 Polycrystalline Solids

The solids having structure in between order and disorder, are called polycrystalline or polymeric solids. Hence, these are partially or poorly crystalline.

A solid material made up of many small crystals with random orientation, is called polycrystalline solid.

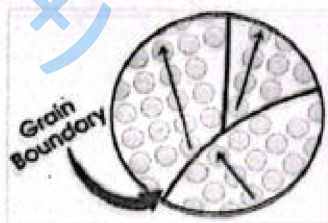



**Figure 7.2:** Polycrystalline structure.

The small crystals in polymeric solids are known as crystallites or grains which are oriented in different directions, and have distinct grain boundaries, as shown in Fig. 7.2. Normally, grains of the polycrystalline solids have sizes in the range of $10^2$ nm to $10^3$ nm. Polycrystalline materials are isotropic and exhibit the same properties in all directions.

Polymers are of two types, i.e. natural and synthetic. Polymers occur in nature include rubber with formula $(C_5H_6)_n$, resin and wood. Whereas, synthetic polymers including polythene $(C_2H_4)_n$, polystyrene $(C_8H_8)_n$, polyvinyl chloride $(C_2H_3Cl)_n$ etc. Polymers are made by the repetition of small molecules *consisting mainly* carbon, oxygen, hydrogen and nitrogen, to make large chains of molecules by a process called *polymerization*. The properties of these materials include, low specific gravity, exhibit a good strength to weight ratio, toughness, resistive to corrosion, poor conductivity (heat and electricity) and low cast.

**Material Science and Space Technology**

Zylon is a synthetic polymer, which is stiffer than steel, has high strength and has excellent thermal stability. It has various medical applications like artificial muscles, it is used in electric batteries and even used in Martian rover sent by NASA to the planet Mars.

### 7.1.3 Amorphous Solids

Amorphous solids do not have a regular structure. The term 'amorphous' means without regular form or structure.

Solid materials whose constituent particles are arranged in a random manner are called amorphous solids.

Amorphous solids, also called glassy solids, have structure like frozen liquids. They have fix volume but not definite regular geometrical shape, as shown in Fig. 7.3 (the structure of beryllium fluoride $(BeF_2)$). Latest technique called '*atomic electron tomography*', a type of 3-D imaging showed 85 % of atoms were in a disorder arrangement in amorphous solids but there were pockets, where a fraction of atoms found into ordered super-clusters, hence, they are called short range ordered materials. They have a range of melting temperature, i.e., if we heat a glass rod, it gradually softens into a paste like state before it becomes viscous liquid at 800 °C.

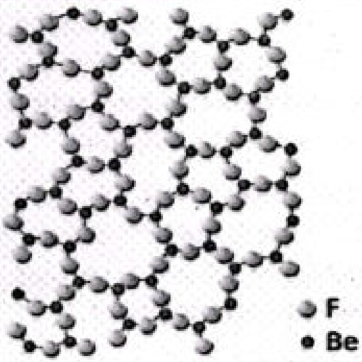



**Figure 7.3:** Amorphous Solid structure.

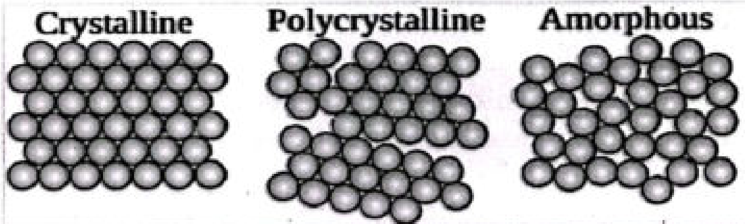



**Figure 7.4:** The crystalline, polycrystalline and amorphous solids.

**Engineering and Technology**

Atomic electron tomography (AET) has become a powerful tool for atomic scale structural characterization in 3-D and 4-D. It provides the ability to correlate structures and properties of materials at the single atom level.

Examples of amorphous solids include, glass (sometimes called super-cooled liquid), rubber, thin film systems deposited on a substrate at low temperatures, glues and many polymers. Amorphous solids can withstand higher temperatures, without affecting their *efficiency* as compared to crystalline and polycrystalline solids. If these solids are melted and then cooled slowly, they can be converted into crystalline solids. Amorphous solids are used almost in every field of life: like construction, household wares and laboratory wares, manufacturing of tyres and production of shoes. Amorphous silica is one of the best materials for converting sunlight into electricity, used in solar panels.

**Material Science and Technology**

Graphene, a hexagonal carbon crystalline structure, only a single atom thick, is the thinnest known material. It is the most revolutionary material to be developed in the 21st century. It is used in carbon nanotubes. In proportion to its thickness, it is the strongest material. It is an extraordinary conductor of heat and electricity and 100% transparent to light.

## 7.1.4 Unit Cell and Crystal Lattice

The crystal structure of solid is analyzed by using X-ray diffraction and Bragg's law. Solids are characterized by their structure, which is made up of some basic unit, that can be defined as:

A unit cell is the smallest basic portion of a crystal, which repeatedly stacked together in three dimensions, to make the entire crystal structure.

A unit cell has all the characteristics of the whole crystal. The atoms, molecules or ions which make the crystal, are called basis and are arranged in regular pattern. An imaginary geometrical structure to join the basis is called lattice. A lattice basically tells us about the basic structure of the points (basis). The crystal structure is obtained by placement of atoms on basis.

The structure of a crystal, obtained by the repetition of the unit cell, is known as crystal lattice.

A unit cell and its corresponding crystal lattice are shown in Fig. 7.5 (a). There are three types of unit cells:

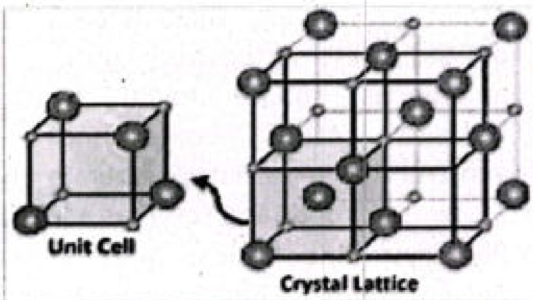



**Figure 7.5 (a):** Crystal lattice and unit cell.

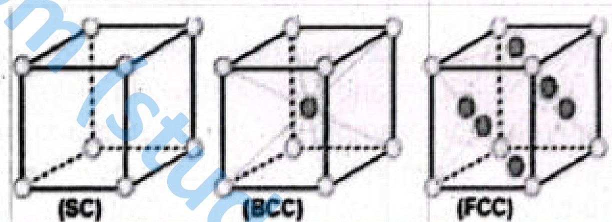



**Figure 7.5 (b):** Types of unit cells.

i) Simple cubic (SC) contains eight atoms at each corner. Hence, there is one atom per unit cell in SC, as shown in Fig. 7.5 (b).
ii) Body centered cubic (BCC) have eight atoms at corners and one atom at the center of body. Hence, there are two atoms per unit cell in BCC.
iii) Face-centered cubic (FCC) have eight atoms at corners and one atom at each face. Hence, there are four atoms per unit cell in FCC.

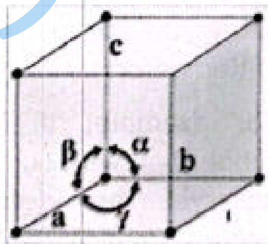



**Figure 7.5 (c):** Unit cell.

A unit cell has three sides for a space lattice as, (a, b, c) having three angles $(\alpha, \beta, \gamma)$, as shown in Fig. 7.5 (c). In terms of different relations between sides and angles of a unit cell, it can be divided into seven distinct shapes, as shown in Fig. 7.6. Some of the daily life applications of solids include:

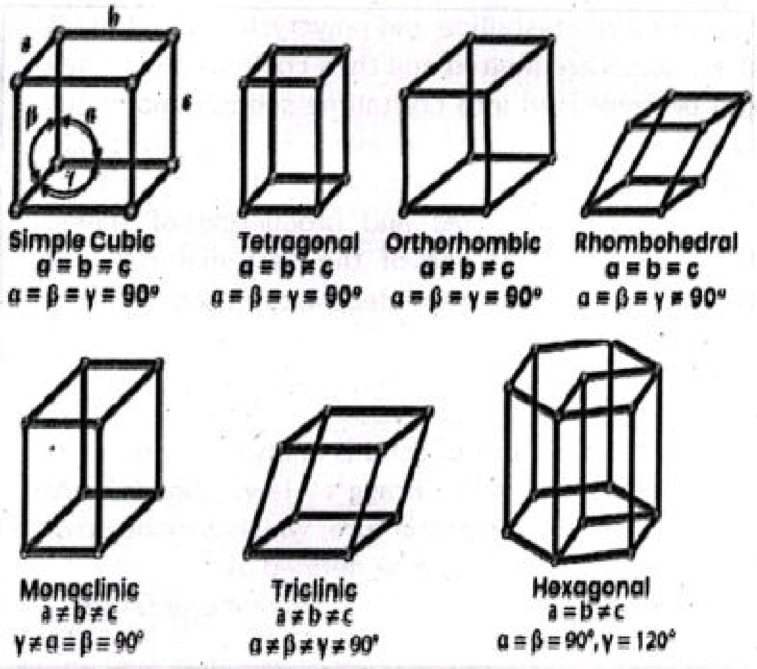



**Figure 7.6:** Seven shapes of unit cells.

- **Crystalline solids:** Diamond is the most decent example, which is used in jewelry and various industries. Similarly, quartz is extensively used in manufacturing of watches and clocks. Many crystalline solids are also used as a raw material in various industries.
- **Amorphous solids:** Glass is one of the most extensively used amorphous solids, found in utensils, bottles, boxes and construction material. Rubber is another widely used amorphous solid, found in tyres, footwears, ropes and serve as a material in many industries.
- **Polycrystalline solids:** Most inorganic metals, many ceramics, rocks and ice are polycrystalline, which have vast applications in our daily life.

## 7.2 DEFORMATION IN SOLIDS

A solid has definite shape and volume, possesses rigidity, is not compressed easily, has closely packed particles and cannot flow. In order to change the shape or size of a solid, a force is required. For example, if you stretch a spring by pulling its ends, the length of the spring increases. When you release the ends of the spring, it regains its original shape and size. To study the mechanical properties of solids, we usually check the behavior of solids under applied force. Application of force on a solid may bring change in its dimension, i.e., it deforms a solid.

**The change in shape, length or volume of a solid when it is subjected to an external force is called deformation.**

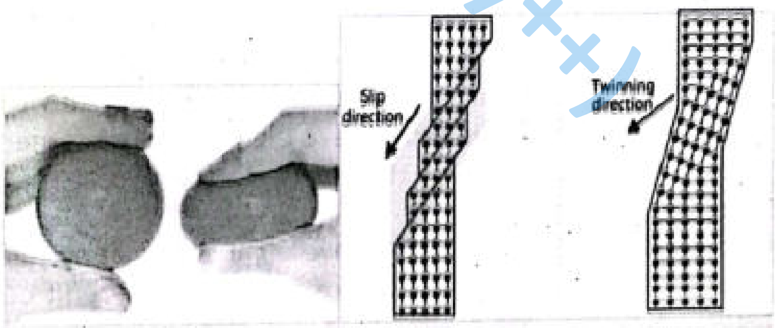


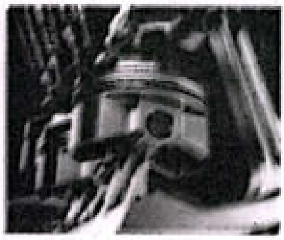

**Figure 7.7:** Deformation in a ball and in crystals.

For example, if we squeeze a rubber ball with our fingers, we can observe the deformation that occur. Similarly, at atomic level, a crystal subjected to applied pressure, may deform mainly in two forms, a slip of one cleave over the other or twinning of crystal, both of them are shown in Fig.

7.7. The results of mechanical tests are usually expressed in terms of stress and strain, which can be defined in terms of applied force and resulting deformation.

**Auto Engineering and Technology**

What makes the moving parts of engine frictionless? Due to very low coefficient of friction and shine, Teflon (an amorphous solid) is used for coating the materials used in automobiles, industries and manufacturing. Teflon coating is used in high efficiency engines to reduce the friction, as it has good efficiency even at higher temperatures. It is also used in nonstick cooking utensils and moving parts of machines.



## 7.2.1 Stress and Strain

The internal resistance offered by a body to resist deformation is called stress. It is a physical quantity that measures the magnitude of force causing the deformation. Stress is directly proportional to the applied force and inversely proportional to the area of cross section over which the force applied. It can be defined as:

**The applied force per unit area is called stress.**

Mathematically, it can be written as:

$$Stress\ (\delta) = \frac{Force\ (F)}{Area\ (A)}$$

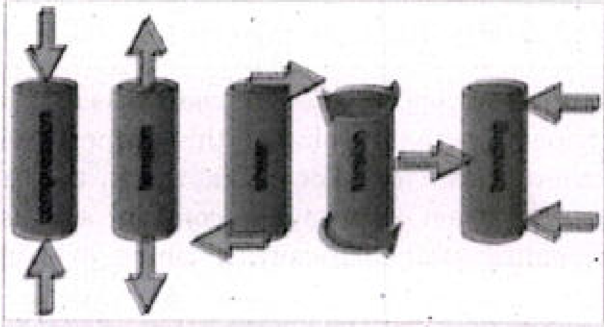


Figure 7.8: There are five types of stresses, i.e. compression, tension, shear, torsion and bending.

The SI unit of stress is newton per square meter (N $m^{-2}$), also known as pascal (Pa) and its dimensions are $[ML^{-1}T^{-2}]$. Generally, stress can be divided into three types.

- The one-dimensional stretching force acting on the area of cross section of a body which produces linear deformation in it, is called *tensile stress*.
- The force which has a component acting tangentially to the area of a body produces deformation in the shape of a body, is called *shear stress*.
- The force which acts uniformly from all directions on the area producing a deformation in the volume of a body, is called *volume stress*.

Due to action of a force on a body, a deformation is produced in it.

**The quantitative measure of deformation is called the strain.**

**For Your Information**

Amorphous solids have two defining properties. They create pieces of odd, often twisted surfaces when cleaved or broken. They have poorly described patterns, when exposed to x-rays because their components are not organized in a typical sequence.

Strain can be found by the ratio of the change to the actual value of some parameter (length, shape or volume); hence it has no units and a dimensionless quantity. On the basis of stress, strain can also be subdivided into three types.

- The fractional change in length, i.e., change in length divided by the original length, is called tensile strain. It causes due to tensile stress and is given by:

$$\text{Tensile strain } (\varepsilon) = \frac{\text{Change in length } (\Delta L)}{\text{Original length } (L)}$$

- The tangential force acting on an area (shear stress), produces a change in the shape of a body, called shear strain. Shear strain is given by:

$$\text{Shear strain } (\varepsilon) = \frac{\text{Displacement of sheared face}}{\text{distance from fixed face}} = \frac{\Delta X}{Y}$$

- The force acting at right angle to an area from all directions produces deformation in the volume of a body, called volume strain and can be given as:

$$\text{Volume strain } (\varepsilon) = \frac{\text{Change in volume } (\Delta V)}{\text{Original volume } (V)}$$

### 7.2.2 Modulus of Elasticity

After removing stress, some materials regain their original shape, length or volume; this property of materials is called '*elasticity*'. For elastic solids, the ratio of the stress to the strain is always a constant and is called '*elastic modulus*'. Mathematically, it can be given as:

$$\text{Elastic modulus} = \frac{\text{Stress}}{\text{Strain}}$$

Since strain has no units, hence modulus of elasticity has the same units as that of the stress i.e. N $m^{-2}$ (or Pa) and dimensions as $[ML^{-1}T^{-2}]$. Depending upon type of stress, modulus of elasticity is of three types, as under:

**Young's Modulus:** In case of linear deformation, the ratio of the tensile stress to the tensile strain is called 'Young's modulus', and is represented by 'Y'. Mathematically it can be given as:

$$Y = \frac{F/A}{\Delta L/L} \qquad \text{———} \quad (7.1)$$

Here, 'F' is the applied force on area '$A$' of the material (say a rod) of length 'L'. The force changes the length by '$\Delta L$', as shown in Fig. 7.9, for both increase and decrease in length.

**Bulk Modulus:** For three dimensional forces, the deformation occurs in all directions, as shown in Fig. 7.10. In this case, the ratio of the volume stress to the volume strain is called 'bulk modulus'. Mathematically, it is expressed as:

**For Your Information**

Objects can often experience both compressive and tensile stress simultaneously. Example includes a long shelf loaded with heavy books. The top surface of the shelf is in compressive stress and the bottom surface of the shelf is in tensile stress. Similarly, long and heavy beams sag under their own weight, in modern building construction, such bending strains can be eliminated with the use of L-beams.

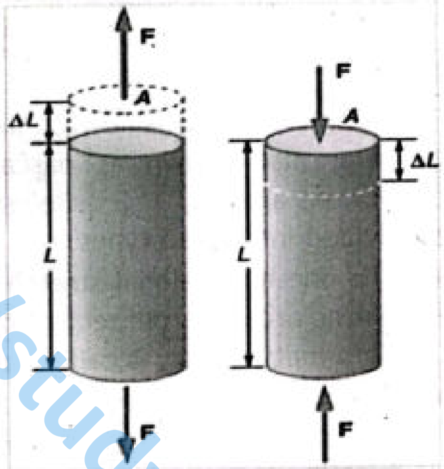



Figure 7.9: Young's modulus.

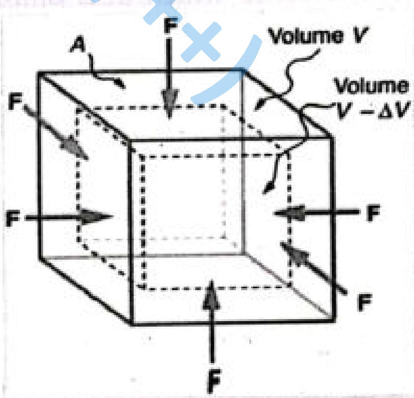



Figure 7.10: Bulk modulus.

$$B = \frac{F/A}{(-\Delta V/V)} \quad \text{———} \quad (7.2)$$

Here, the negative sign shows the decrease in volume, but bulk modulus is always positive.

**Shear Modulus:** Due to tangential force, the deformation is produced in the shape of a body, as shown in Fig. 7.11. In this case, the ratio of the shear stress to the shear strain is called 'shear modulus'. Mathematically, it is expressed as:

$$S = \frac{F/A}{(\Delta X/Y)} \quad \text{———} \quad (7.3)$$

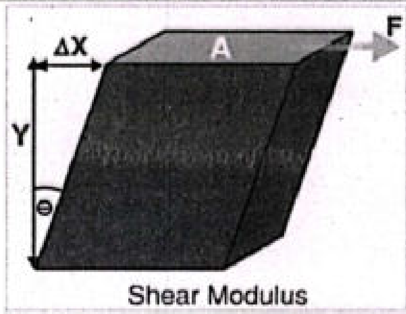



**Figure 7.11:** Shear modulus.

Here, strain is the ratio of the displacement of the sheared face to the height of fix face. With reference to the triangle, shown in Fig. 7.11, the shear strain it is expressed as:

$$\tan\theta = \frac{\Delta X}{Y}$$

For very small angle 'θ': $\tan\theta \approx \theta$ hence, $\theta = \Delta X/Y$. Now equation (7.3) gets the following form:

$$S = \frac{F}{A\theta} \quad \text{———} \quad (7.4)$$

Here, the angle 'θ' is called 'the angle of shear'.

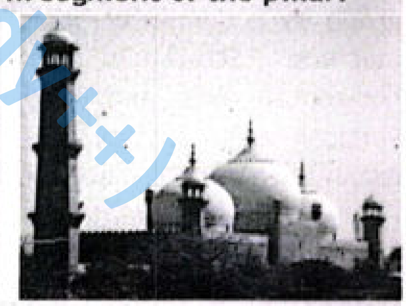
Table 7.1: Table for values of the modulus of elasticity of different materials

| Material | Young's Modulus ($10^{10}$ N m$^{-2}$) | Bulk Modulus ($10^{10}$ N m$^{-2}$) | Shear Modulus ($10^{10}$ N m$^{-2}$) |
|---|---|---|---|
| Steel | 20.0 | 15.8 | 8.0 |
| Aluminum | 7.0 | 7.0 | 2.5 |
| Copper | 12.0 | 12.0 | 4.0 |
| Iron | 19.0 | 8.0 | 5.0 |
| Brass | 1.0 | 14.0 | 3.6 |

**Example 7.1: A Masjid's minar having area of cross-section 0.2 m², the upper dome's weight 10,000 N, is made up of a material, with Young's modulus of $4.5\times10^{10}$ Pa and mass density of 2700 kg m$^{-3}$. Find the compressive stress at the cross-section located 3.0 m below the top of the pillar and value of compressive strain of the top 3.0 m segment of the pillar.**

**Given:** Pillar segment height 'h' =3.0
Area of cross-section 'A' = 0.20 m$^2$
Density '$\rho$' = 2700 kg m$^{-3}$
Weight of upper dome '$w_d$' = 10,000 N

**To Find:** Compressive stress '$\delta$' = ?
Compressive strain '$\varepsilon$' = ?



**Solution:** To find the mass, we have to find volume of the pillar's segment, as:

$$V = A\,h$$

$$V = (0.20\ m^2)(3.0\ m) = 0.60\ m^3$$

$$m = \rho V \Rightarrow \quad m = (2700\ kg\,m^{-3})(0.60\ m^3) = 1.60\times10^3\,kg$$

The weight of the pillar segment is:

$$w_p = m\,g \Rightarrow \quad w_p = (01.60\times10^3\,kg)(9.8\ ms^{-2}) = 1.568\times10^4\,N$$

The normal force is the sum of weight of the upper dome and the pillar segment.

$$F = w_d + w_p \qquad \Rightarrow \qquad F = (1.0\times10^4 N) + (1.568\times10^4 N)$$

$$F = 2.568\times10^4 N$$

Now the compressive stress: $$\delta = \frac{2.568\times10^4 N}{0.20\ m^2}$$

$$\delta = 128.4\ kPa$$

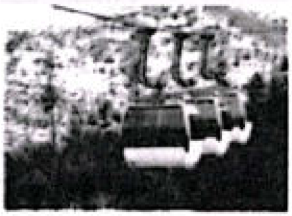
And the compressive strain: $$Strain = \frac{Stress}{Y} = \frac{128.4\ kPa}{4.5\times10^7 kPa}$$

$$Strain = 2.85\times10^{-6}$$

**Discussion:** The normal force acting on the cross-sectional area is not constant along its length, but varies from the smallest value at the top to the largest value at the bottom of the pillar. So if the pillar has constant area of cross section along its lenth, the stress is the largest at the bottom. That is why the base portion of pillar has gradual increase in cross section, as can be seen from given figure.

**.Assignment 7.1**

**The suspension cable to use cable cars at Murree has an unsupported span of 3.25 km. Calculate the amount of stretch in the steel cable of diameter of 6 cm for its maximum load. The caution on weight is given by the engineers to not more than $4\times10^6$ N.**



## 7.2.3 Stress-Strain Curve

To study the behavior of a material under applied stress, we use a tensile test machine which strains the material at a fixed linear rate and records the stress. The values are then plotted on a graph by a computer attached to the machine. Strain is plotted along horizontal axis and stress on the vertical axis. A typical stress-strain graph for a ductile material is shown in Fig. 7.12. In the initial stage of deformation, the stress increases linearly with the strain, as shown by the portion (OA) in the graph. This portion has linear proportionality between stress and strain and obeys Hooke's law. In this region, if the applied stress is *removed* the body regains its original shape.

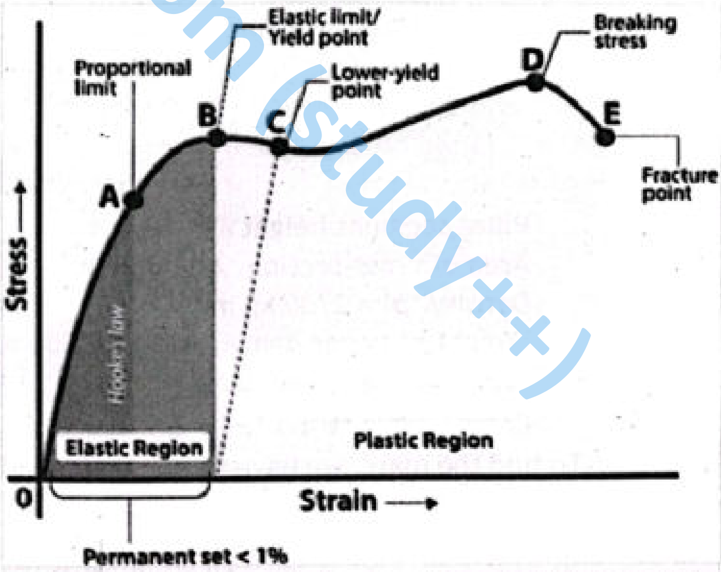
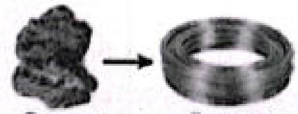



Figure 7.12: Stress-strain curve for a ductile material

Point 'A' is called the proportional limit corresponding to stress $\delta_p$ and defined as the greatest stress, which a material can endure without losing straight line proportionality between stress and strain. After crossing point 'A' the region is shown by the portion (AB) in the graph, where straight line proportionality between stress and strain ends, but still if stress is removed, the body will regain its original shape. Hence, the portion (O-B), where the body has the ability to regain its original shape, is called '*elastic region*'. The maximum limit of the stress for elasticity of a material, is called yield point or elastic limit of the material, corresponding to point 'B' in the graph whose stress can be given as $\delta_y$. Some material has lower yield point represented by 'C'. If stress is increased beyond the yield point, the body gets permanently changed and does not recover its original shape, even if stress is removed. This behavior of materials is called 'plasticity', where if the stress is zero still strain is not zero. This region is represented by the portion after point 'B' and onward. Stress is increased further at point 'D', called ultimate tensile strength ($\delta_u$), the maximum stress that a material can withstand. After crossing point 'D' the body breaks at point 'E' representing the fracture stress ($\delta_f$). Substances, which undergo plastic deformation until they break are known as ductile materials. Lead, copper and wrought iron are examples of ductile materials. Other materials, which break just after elastic limit are called brittle materials, like glass and high carbon steel.

**For Your Information**




Metals, with high ductility such as, gold copper and steel can be drawn into long and thin wires without breaking. Gold is the most ductile material. A wire of about 2 km in length can be drawn from one gram of gold.

**Example 7.2: Calculate the change in length of the upper leg bone of a soccer player, having mass of 70.0 kg, supporting 62.0 kg of his mass on this bone, assuming the bone acts as a uniform rod, 40.0 cm long and with a radius of 2.00 cm.**

Given: Mass 'm' = 62.0 kg Young's modulus for bone 'Y' = $9\times10^9$ Pa

Radius of the bone 'r' = 0.020 m Length of the bone 'L' = 0.400 m

To Find: Change in length 'ΔL' = ?

Solution: The force is equal to the weight supported by the bone:

$$F = m\,g \qquad F = (62.0\ \text{kg})(9.80\ \text{ms}^{-2}) = 607.6\ \text{N}$$

The area of the cross-section is:

$$\pi r^2 = (3.14)(0.020\ \text{m})^2 = 1.257\times10^{-3}\text{m}^2$$

Now by using equation: $\Delta L = \dfrac{F\,L}{Y\,A}$

Putting values, we get: $\Delta L = \dfrac{(607.6\ \text{N})(0.400\ \text{m})}{(9\times10^9\text{N/m}^2)(1.257\times10^{-3}\text{m}^2)} = 2\times10^{-5}\text{m}$

**Discussion:** This change in length is very small, consistent with our experience that bones are rigid. Even the large forces during haevy physical activities (like weight lifting) of a person do not compress or bend bones significantly.

**Assignment 7.2**

**Find the strain in your own leg's bone while lifting a 50 kg box. Relate your finding with the example given above. Do the strain in your leg is greater or lesser than the strain in above case? Explain the reason.**

## 7.3 ELASTIC POTENTIAL ENERGY

As works done on bodies store some amount of energy in them, similarly work done on elastic bodies for deformation also stores energy in the body which is known as elastic potential energy.

### 7.3.1 Modulus of Metallic Wire

To study the work done on a body to produce strain, consider a metallic wire which can be strained by the application of force on it. Measure the length and area of cross section of the wire and connect a vernier scale to it. Take another wire as reference wire with a scale on it for precise measurement of change in length of the metallic wire upon adding the load (weight) with it, as shown in Fig. 7.13. We will find the extension in the wire with the help of vernier scale due to the increasing force on test wire. By using equation 7.1 with average force, original length, change in length and area of the metallic wire, we can find the Young's modulus of the metallic wire.

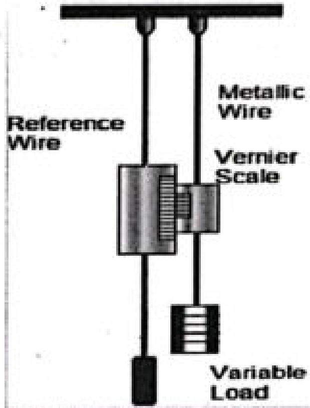



Figure 7.13: Young's modulus for metallic wire.

### 7.3.2 Work from Force-Extension Graph

To find the work done by the variable load on the metallic wire to produce extension, draw a graph by taking weight on x-axis and extension on y-axis. Initially, no weight is attached to the wire and it has zero extension. By continuously increasing the weight, the wire got extension, as shown in the Fig. 7.14. The area under the curve from O to A can be found by finding the area of triangle AOF, which can be given as:

$$Area = \frac{1}{2} F x \quad \_\_\_\_ \quad (7.5\ a)$$

The average force applied on the wire is given as,

$$F_{Avg} = \frac{0+F}{2} = \frac{1}{2} F$$

We know that the work done can be found by the product of average force and displacement. Therefore, work done is:

$$W = F_{Avg}\, x \quad \Rightarrow \quad Work = \frac{1}{2} F x \quad \_\_\_\_ \quad (7.5\ b)$$

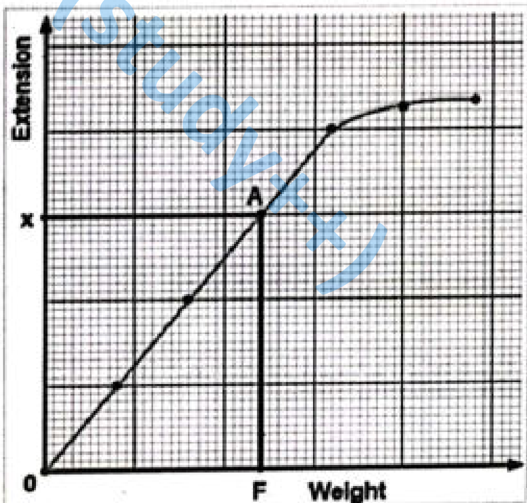



Figure 7.14: Force-extension graph.

From equations (7.5 a) and (7.5 b), it is clear that the work done during the application of a load for extension in elastic material, within elastic limit, is equal to the area under the force-extension graph.

### 7.3.3 Derivation of Elastic Potential Energy

The energy stored in a material due to strain produced in it is called the elastic potential energy. This energy can be calculated by using the relation for work done during extension. For an elastic material, the force can be given by the Hook's law as:

$$F = k\,x$$

The elastic potential energy in deformed material, within elastic limit is equal to the work done in producing that deformation. Hence, the potential energy of a deformed material is expressed as:

$$\text{Elastic Potential Energy} = \text{Work Done} = \frac{1}{2}F\,x$$

$$P.E_{elastic} = \frac{1}{2}(k\,x)x$$

or $$P.E_{elastic} = \frac{1}{2}k\,x^2 \quad \text{\_\_\_\_\_} \quad (7.6)$$

The energy of deformed material can also be written in terms of Young's modulus. If we use the value of force from equation (7.1) with extension 'x' then it gets the form:

$$F = \frac{YAx}{L} \qquad \because y = \frac{F/A}{x/L}$$

**Civil Engineering**

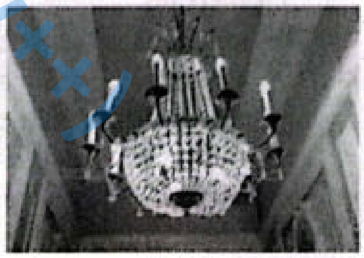
Engineers while making the suspension bridges test the materials for their elastic properties. They use some mechanical testing machines apply stress and calculate strain. This helps them to choose the best material to be used. The wires used in suspension bridges possess large strain energy and can withstand the huge load of traffic.

Where 'A' is the area of cross-section and 'L' is the original length of the wire. substituting above value of force in equation (7.5) we get:

$$P.E = \frac{1}{2}\left[\frac{YA}{L}\right]x^2 \quad \text{\_\_\_\_\_} \quad (7.7)$$

This elastic potential energy is also called the strain energy.

**Example 7.3: Calculate the strain energy stored in a 1 m long steel wire connected to the roof, holding a chandelier of weight 980 N in the main lounge of your home. The diameter of steel wire is 0.004 m and the Young's modulus of steel is $2.0 \times 10^{11}\ N\,m^{-2}$.**



Given: Length of steel wire 'L' = 1 m
Young's modulus for steel 'Y' = $2.0 \times 10^{11}$ Pa
Diameter of the steel wire 'r' = 0.004 m
Weight of chandelier 'W' = 980 N
To Find: The change in length 'ΔL' = ?
The Strain energy stored in wire 'U' = ?
Solution: The force on the wire is equal to the weight supported by it.

$$F = W = 980 \quad N$$

The area of the cross-section of the steelwire is:

$$\pi r^2 = \pi (d/2)^2 = (3.14)(0.002\ m)^2 = 1.257\times10^{-5} m^2$$

Now by using equation (7.1):

$$\Delta L = \frac{F\,L}{Y\,A} = \frac{(980\ N)(1\ m)}{(20\times10^{10} N/m^2)(1.257\times10^{-5} m^2)}$$

$$\Delta L = 3.90\times10^{-4} m$$

Now, the strain energy U stored in the steel wire can be found by using the formula.

$$P.\,E = \frac{1}{2}\left[\frac{Y\,A}{L}\right]x^2$$

$$U = \frac{1}{2}\left[\frac{(20\times10^{10} N\ m^{-2})\ (1.257\times10^{-5} m^2)}{1m}\right](3.90\times10^{-4} m)^2$$

$$U = 0.191\ J$$

This change in length in steel wire is very small, and hence the strain energy stored in the wire is also small, which shows the suitability of material (steel) for holding heavey masses.

**Assignment: 7.3**

**Calculate the work done in stretching a wire of length 5 m with cross-sectional area 1 mm$^2$. When the force applied by you on the edges of wire produces change in the length of wire by 1 mm. Young's modulus of the wire is 2x10$^{11}$ Pa.**

## SUMMARY

- Solids can be classified into crystalline, polycrystalline and amorphous.
- **Crystalline solids** have a regular three dimensional structure. Polycrystalline solids have a structure that is intermediate between order and disorder, while amorphous solids have an irregular structure.
- **A unit cell** is the basic building block of the crystalline solids whose repetition in all directions makes a whole crystal.
- **Stress** is the force applied on unit area of a solid and the deformation produces due to it is called strain.
- **The modulus of elasticity** of a solid is a constant, and is the ratio of stress to strain.
- **Tensile stress** produces a change in length and the ratio of this change in length to the original length is called tensile strain. The ratio of the tensile stress to the tensile strain is called Young's modulus.
- **Volume stress** produces a change in volume of a solid and the ratio of this change in volume to original volume is called the volume strain. The ratio of the volume stress to the volume strain is called bulk modulus.
- **Shear stress** produces change in shape and the ratio of the sheared face to the fix face is called shear strain. The ratio of the shear stress to the shear strain is called shear modulus.
- **Strain energy** of a material is represented by the area under the stress-strain curve.
- **Elastic materials** break shortly after elastic limit, while those who break after ultimate tensile strength, are called plastic materials.

# EXERCISE

## Multiple Choice Questions

Encircle the correct option.

1) Glue is an example of:

A. crystalline solid B. amorphous solid C. poly-crystalline solid D. ductile solid

2) Which of the following properties is generally exhibited by amorphous solids?

A. anisotropy B. glass-transition C. orderliness D. geometry

3) Silicon is found in nature in the forms of:

A. SC structure B. BCC structure C. FCC structure D. network structure

4) One end of a uniform wire of length 'L' and of weight 'W' is attached rigidly to a point in the roof and a weight '$W_1$' is suspended from its lower end. If 'S' is the area of cross-section of the wire, the stress in the wire at a height '3L/4' from its lower end is:

A. $\frac{W_1}{S}$ B. $\frac{W_1+(W/4)}{S}$ C. $\frac{W_1+(3W/4)}{S}$ D. $\frac{W_1+W}{S}$

5) On suspending a weight 'mg', the length '$l$' of an elastic wire with area of cross-section 'A' becomes double of its initial length. The instantaneous stress on the wire is:

A. $\frac{mg}{2A}$ B. $\frac{mg}{A}$ C. $\frac{2mg}{A}$ D. $\frac{4mg}{A}$

6) The diameter of a brass rod is 4 mm and Young's modulus of brass is $10\times10^{10}$ N m$^{-2}$. The force required to stretch by 0.1% of its length is:

A. $36\pi$ N B. $40\pi$ N C. $360\pi$ N D. $400\pi$ N

7) A cantilever beam has a load at the free end. The strain energy in the beam is due to:

A. bending B. shearing C. stretching D. elongation

8) When a pressure of 100 atmospheres is applied on a spherical ball of rubber and its volume reduces to 0.01%. The bulk modulus of the material of rubber in dynes cm$^{-2}$ is:

A. $1\times10^{12}$ B. $10\times10^{12}$ C. $20\times10^{12}$ D. $100\times10^{12}$

9) The property of a material to store strain energy is called:

A. ductility B. hardening C. resilience D. stiffness

10) Gradient of the force-extension graph has:

A. increasing value B. decreasing value C. variable value D. constant

## Short Questions

Give short answers to the following questions:

7.1 Why do most of the solids prefer to be in the crystalline state? What is glass transition in amorphous solids?

7.2 Why the window glasses of old buildings show milky appearance over time?

7.3 In the elastic body, which one is more fundamental: the stress or the strain? Explain.

7.4 Can tensile stress produce volumetric strain? Explain.

7.5 What does the slope of the force-extension graph represents? Explain.

7.6 Is strain energy always positive or can it be negative? Justify your answer.

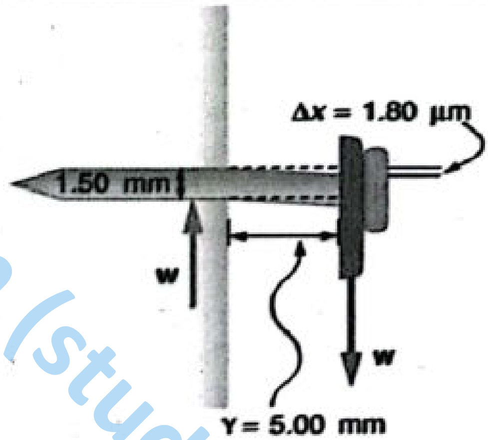
7.7 On which types of loads the strain energy depends? Explain.
7.8 Why are suspension bridges assign a period of use? Is it dangerous to use the bridge after that period?
7.9 If we want to break a wire, why we use repeated bending? How does it affect the wire?
7.10 What is the significance of modulus of elasticity?
7.11 What is meant by crystal lattice.

## Comprehensive Questions

Answer the following questions in detail.
7.1 Explain the three types of solids: crystalline, poly-crystalline and amorphous solids.
7.2 What is unit cell? Explain its types.
7.3 Explain the terms: stress, strain and modulus of elasticity.
7.4 Draw and discuss the stress-strain curve for a ductile material.
7.5 Explain plastic and elastic deformation by drawing stress-strain curve.
7.6 Justify that 'the area under the force-extension curve is equal to the work'.
7.7 Derive a relation for the elastic potential energy of a material.
7.8 Describe an experiment to determine the Young's Modulus of metallic wire.

## Numerical Problems

7.1 Calculate the mass of picture frame hanging from a steel nail ($Y_S = 20 \times 10^{10}$ Pa), which is sufficient to produce a force providing shear strain, as shown in figure here. Given that the nail bends only 1.80 μm. Also discuss the result.
**(Ans: 5.2 kg,** Discussion: This is a fairly massive picture, and it is impressive that the nail flexes only 1.80 μm, amount undetectable to the unaided eye, which shows strength of the material of nail i.e. the steel.)




7.2 A 5 m long aluminum wire ($Y_A = 7 \times 10^{10}$ Pa) of diameter 3 mm, supports a 40 kg mass. To achieve same elongation in a copper wire ($Y_C = 12 \times 10^{10}$ Pa) of the same length under the same weight, find the diameter of copper wire. **(Ans: 2.29 mm)**

7.3 A 4 kg mass is hanging with the help of a 1.5 m long steel wire ($Y_S = 20 \times 10^{10}$ Pa) another mass of 6 kg is hanging from the 4 kg mass using 1 m long brass wire ($Y_B = 1.0 \times 10^{10}$ Pa). Both wires have diameter 0.25 cm. Calculate the total elongation in the whole system of wires, also calculate the total strain energy. **(Ans: $2.79 \times 10^{-4}$ m, 11.02 mJ)**

7.4 A rod with a cross-sectional area of 90 mm$^2$ and a length of 3 m. If a stress of 300 MPa is applied to stretch the rod, then find the strain energy if Young's modulus of the rod is 200 GPa. **(Ans: 12.15 J)**

7.5 A force of 500 N is applied to one end of a cylindrical steel rod of diameter 50 cm. What is the tensile stress? **(Ans: $2.5 \times 10^5$ N m$^{-2}$)**

UNIT 8

# HEAT AND THERMODYNAMICS

How thermodynamics plays its role in the human bodies, in terms of work and energy?

## Student Learning Outcomes (SLOs)

**The students will:**

- State that regions of equal temperatures are in thermal equilibrium.
- Relate a rise in temperature of an object to an increase in its internal energy.
- Apply the equation of state for an ideal gas.
- State that the Boltzmann constant is given by: $k = R/N_A$
- Describe the basic assumptions of the kinetic theory of gases.
- Use $W = p\Delta V$ for the work done when the volume of a gas changes at constant pressure.
- Describe the difference between the work done by a gas and the work done on a gas.
- Define and use first law of thermodynamics.
- Explain qualitatively, in terms of particles, the relationship between the pressure, temperature and volume of a gas.
- Use the equation, including a graphical representation of the relationship between pressure and volume for a gas at constant temperature.
- Justify how the first law of thermodynamics expresses the conservation of energy.
- Relate a rise in temperature of a body to an increase in its internal energy.
- State the working principle of a heat engine.
- Describe the concept of reversible and irreversible processes.
- State and explain the second law of thermodynamics.
- State the working principle of Carnots' engine.
- Describe that refrigerator is a heat engine operating in reverse as that of an ideal heat engine.
- Explain that an increase in temperature increases the disorder of the system.
- Explain that increase in entropy means degradation of energy.
- Explain that energy is degraded during all natural processes.
- Identify that system tends to become less orderly over time.
- Explain that Entropy, S, is a thermodynamic quantity that relates to the degree of disorder of the particles in a system.
- State that the Carnot cycle sets a limit for the efficiency of a heat engine at the temperatures of its heat reservoirs given by: $\text{Efficiency} = 1 - \frac{T_{\text{cold reservoir}}}{T_{\text{hot reservoir}}}$

# UNIT 8 HEAT AND THERMODYNAMICS

The concept of temperature is rooted in qualitative ideas of "hot" and "cold" based on our sense of touch. A body that feels hot usually has a higher temperature than a similar body that feels cold. However, many properties of matter that we can measure depend on temperature. The length of a metal rod, steam pressure in a boiler, the ability of a wire to conduct an electric current, and the color of a very hot glowing object—all these depend on temperature. Temperature is also related to the kinetic energies of the molecules of a material.

Heat is the form of energy associated with the kinetic energies of all the molecules in a substance and its unit is joule. Temperature describes the degree of hotness and coldness of an object and it is the measure of average kinetic energy of the molecules and its unit is kelvin.

## 8.1 THERMAL EQUILIBRIUM

**Two systems are in thermal equilibrium with each other, if they are at the same temperature.**

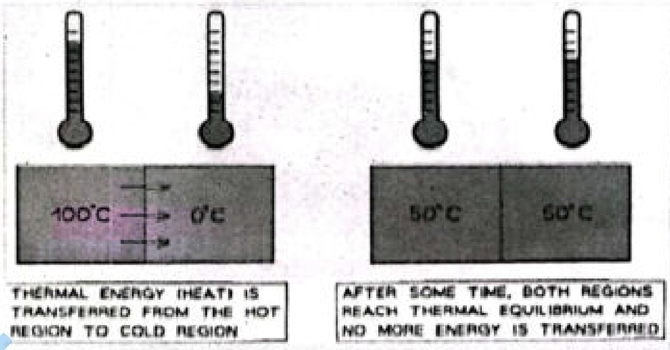



**Figure 8.1(a): Thermal equilibrium.**

The temperature is the main property that determines whether two systems will be in thermal equilibrium or not. When two bodies at different temperatures are connected by a **diathermic substance**, heat starts flowing between them from the higher temperature body to the lower temperature body until their temperatures become equal and after that no net heat flow takes place between them, as shown in the Fig 8.1 (a). They are now said to be in thermal equilibrium. Thus, we can state that when two bodies are in thermal equilibrium (at same temperature), no net heat flow takes place between them. A system is said to be in thermal equilibrium with the surroundings, if each part of the system and the surrounding are at the same temperature.

**Do You Know?**

Diathermic substances are those substances that allow heat to pass through them and the process is called a diathermic process.

**For Your Information**

**System:** The matter or space region that is being studied is called a "system." For example, a gas in a cylinder, etc.

**Surroundings of the system:** The term "surroundings" refers to everything outside the system.

**Boundary of the system:** A system is separated from the surroundings by its boundary. All energy changes, caused by the work done or heat exchange between the system and its surroundings, take place through this boundary.

**Types of the systems:**

**Closed system:** It is a system in which heat energy can enter or leave the system but mass cannot do either. An example of a closed system is a container sealed on all sides.

**Open system:** A system in which both heat energy and mass can enter or leave it, is called open system. An example of open system is a glass beaker with an open top which allows matter (i.e., water) to be added or removed, as well as, heat to be added or removed.

**Isolated system:** A system in which there is no transfer of mass and heat energy across its boundary is called an isolated system. An example of it is a thermos flask filled with hot water.

## 8.1.1 Internal Energy

The sum of all the forms of molecular energies (such as kinetic and potential energy) of a substance is called internal energy. An ideal gas is one in which all collisions between atoms or molecules are perfectly elastic and there are no intermolecular attractive forces between them. The internal energy of an ideal gas system is generally the translational K.E of its molecules. Molecules of a gas are always in random motion. According to the kinetic theory of gases, the average kinetic energy of gas molecules is given by:

$$\left\langle \frac{1}{2}mv^2 \right\rangle = \frac{3}{2}kT \quad \text{———} \quad (8.1)$$

Where k is Boltzmann's constant. If temperature increases, the average kinetic energy of gas molecules also increases. Therefore, the internal energy increases with rise in temperature.

**Do You Know?**
When a substance melts or boils, energy is required to break the bonds holding the particles together, which increases the potential energy.

Since temperature is directly proportional to average kinetic energy of gas molecules, i.e. $T \propto \langle K.E \rangle$. Therefore, the internal energy of an ideal gas is directly proportional to its temperature.

The internal Energy of a system can be increased by heating and doing mechanical work. When we heat a substance, energy associated with its atoms or molecules increases. i.e., heat is converted into internal energy. Similarly, when two objects are rubbed together, their internal energy increases due to the mechanical work. The increase in temperature of the object indicates an increase in its internal energy. Likewise, when an object slides over a surface and comes to rest due to frictional forces, the mechanical work done on or by the system is partially converted into internal energy. In study of heat changes and heat flow, we focus on the changes in the internal energy of a substance with changes in temperature indicating these changes in internal energy.

**DO YOU KNOW?**
A piece of metal is hammered. Does its internal energy increases?

In thermodynamics, internal energy is function of state. Consequently, it does not depend on the path but depends on initial and final states of the system.

Consider a system which undergoes a pressure and volume change from $P_1$ and $V_1$ to $P_2$ and $V_2$, regardless of the process by which the system changes from initial to final state, as shown in the Fig. 8.1 (b). The change in internal energy is the same and is independent of the paths $C_1$ and $C_2$ and it depends on the temperature difference between two points. Like the gravitational P.E we take the change in internal energy and not its absolute value, which is important.

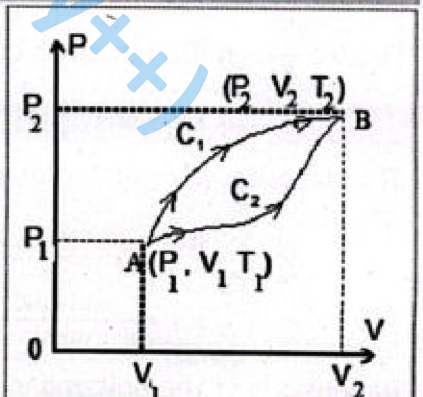



**Figure 8.1 (b): PV graph of a system.**

In case of real gases, the molecules exert mutual force of attraction on one another. The internal energy of real gas is partially kinetic energy and partially intermolecular potential energy and hence it is function of temperature and volume.

**Sign Conventions:** If initial internal energy is $U_1$ at initial temperature $T_1$ and final internal energy $U_2$ at final temperature $T_2$, then change in internal energy = $\Delta U = U_2 - U_1$.

- When the temperature of a system increases then its internal energy increases ($U_2 > U_1$) and the change in internal energy $\Delta U$ is positive.
- When the temperature of a system decreases then its internal energy decreases ($U_2 < U_1$) and the change in internal energy $\Delta U$ is negative.
- When the temperature of a system remains constant then its internal energy remains constant (i.e., $U_2 = U_1$) and the change in internal energy $\Delta U$ is zero.

## 8.1.2 Ideal Gas Equation and its Applications

It is an equation of state of an ideal gas that relates pressure, volume, quantity of gas, and temperature. The ideal gas equation is written as:

$$PV = nRT \quad \text{\_\_\_\_\_\_} \quad (8.2)$$

In this equation, P refers to the pressure of the ideal gas, V is its volume, n is its number of moles, R is the universal gas constant (R = 8.314 $J\,mol^{-1}K^{-1}$), and T is its temperature.

**Do You Know?**

**What are ideal gas conditions?**

There are some assumptions that must hold true for a gas to be "ideal.":

- The gas particles have negligible volume.
- The gas particles are equally sized and do not have intermolecular forces (attraction or repulsion) with other gas particles.
- The gas particles undergo perfectly elastic collisions with no energy loss.

If $N_A$ is Avogadro number and N is total number of gas molecules, then number of moles can be expressed as:

$$n = \frac{N}{N_A}$$

Hence, equation (8.2) can be written as:

$$PV = \frac{NRT}{N_A} \quad \text{or} \quad PV = N\left(\frac{R}{N_A}\right)T$$

**For Your Information**

Unit of Boltzmann constant is same as that of entropy $J\,K^{-1}$.

Putting $\frac{R}{N_A}$ = k (Boltzmann's constant), we get:

$$PV = NkT \quad \text{\_\_\_\_\_\_} \quad (8.3)$$

The numerical value of Boltzmann constant k can be estimated as:

$$k = \frac{R}{N_A}$$

$$k = \frac{8.314\,J\,K^{-1}mol^{-1}}{6.022140857\times10^{23}mol^{-1}} = 1.3806452 \times 10^{-23}\,J\,K^{-1}$$

Dimensions of the Boltzmann constant are $[ML^2T^{-2}K^{-1}]$.

**Example: 8.1: Calculate the average translational kinetic energy of molecules in a gas at room temperature.**

**Given:** Temperature = T = 25 C° = 25 + 273 = 298 K

**To Find:** Average translational kinetic energy = <K.E> =?

**Solution:** As $\langle K.E.\rangle = \frac{3}{2}kT$

Where k = Boltzmann constant = $1.38 \times 10^{-23}\ J\ K^{-1}$

Thus, $\langle K.E.\rangle = \frac{3}{2} \times 1.38 \times 10^{-23} \times 298$

$\langle K.E.\rangle = 6.17 \times 10^{-21}\ J$

**Assignment: 8.1**

**Estimate the total number of air molecules (including molecules of oxygen, nitrogen, water vapour and other constituents) in a room of capacity 20.0 m³ at a temperature of 27 °C and 1 atm pressure.**

## 8.2 KINETIC THEORY OF GASES

In the 19th century, James Clark Maxwell, Rudolph, Clausius and others developed the kinetic theory of gases to explain the behaviour of gases. According to the theory, a gas is composed of a number of tiny, hard spheres (molecules) that collide with one another and with the container walls. These molecules move in accordance with Newton's laws of motion. It explains how the motion of molecules influences the macroscopic properties of gases, such as temperature and pressure. It relates macroscopic properties (T, P, and V etc.) of gases to microscopic properties (K.E etc.).

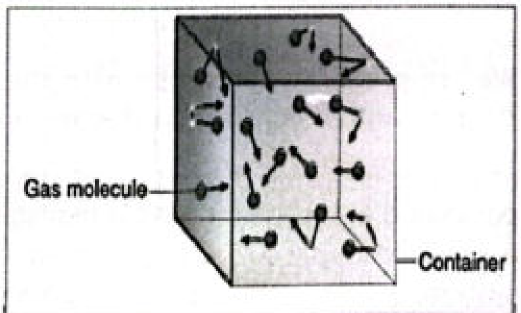



Figure 8.2: Gas molecules in a container.

**Assumptions of Kinetic Theory of Gases:** The following are the assumptions of the kinetic theory of gases:

- A finite volume of gas consists of a very large number of molecules.
- The size of the molecules is much smaller than the separation between them.
- The gas molecules are in random motion and may change their direction of motion after every collision. Collisions between gas molecules and with the walls of the container are assumed to be perfectly elastic.
- Molecules exert no force on each other except during a collision.

**Condition for Real Gas to Behave as an Ideal Gas:** According to general gas equation:

$$PV = nRT$$

Putting, $n = \frac{Mass\ of\ gas}{moler\ mass} = \frac{m}{M}$, we get:

$$PV = \left(\frac{m}{M}\right)RT \quad \text{or} \quad PM = \left(\frac{m}{V}\right)RT$$

Putting, $\frac{m}{V} = \rho$ (density of gas), we get:

$$PM = \rho RT$$

**Do You Know?**

Work is a form of energy, and it has units of joule (where 1 J = kg m² s⁻²). You may also see other units used, such as atmospheres for pressure and liters (l) for volume, resulting in l-atm (liter atmosphere) as the unit for work.

or $\rho = \frac{PM}{RT}$ ______ (8.4)

For given mass of the gas, we can write above equation as: $\rho \propto \frac{PM}{T}$

This equation suggests that at low pressure and high temperature, the density of gas will be low. Moreover, the distance between gas molecules becomes large, leading to the negligible intermolecular forces. In this situation, a real gas behaves as an ideal gas. Conversely, at high pressure and low temperature, the density of gas will be high while the distance between gas molecules will be small and intermolecular forces will be stronger; therefore, a real gas cannot behave as an ideal gas.

## 8.2.1 Work in Thermodynamics

Both heat and work correspond to the transfer of energy by some means. One of the main ways, energy enters or leaves a system is through work. To calculate the work done by a constant force, we can use the following general equation:

Work = Force × Displacement

We are primarily concerned with the work that gases do when they expand or contract in thermodynamics.

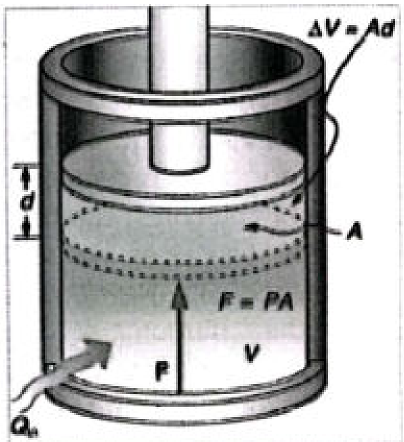



**Figure 8.3:** Work done by the gas

**Pressure-Volume Work:** Let us consider a gas that is contained in a cylinder with a movable, frictionless piston with a cross-sectional area A, as shown in the Fig. 8.3. When the system is in equilibrium, it occupies a volume "V" and exerts pressure "P" on the cylinder's walls and the piston, i.e.,

$P = F/A$ Or $F = PA$

This is the force exerted by the gas on the piston. The gas gradually expands by ΔV in order to maintain equilibrium. A little distance = d = Δy is covered by the piston as it rises. The work done by the gas is given by: $W = F\Delta y$

Putting F = PA, we get: $W = PA\Delta y$

Since $A\Delta y = \Delta V$ (change in volume), so

$W = P\Delta V$ ______ (8.5)

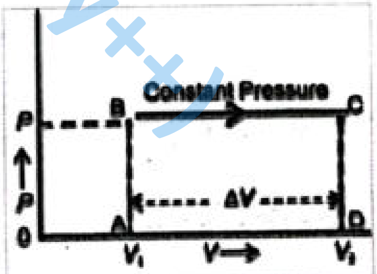



**Figure 8.4:** Area of P-V graph equals Work done.

This is the work done by gas on piston. Where P is the external pressure (as opposed to the pressure of the gas in the system) and ΔV is the change in the volume of the gas, which can be calculated from the initial and final volumes of the gas:

$\Delta V = V_{final} - V_{initial}$

**Graphical Representation:** Work done can be calculated from the area under a P-V graph. From Fig. 8.4 the area under the P-V graph = P ΔV = work.

**Example 8.2:** A 2 kg of argon gas is heated to a temperature of 30 °C. If the gas is contained in a cylinder of radius 1 m and height 2 m, what is the pressure of the gas?

**Given:** mass = m = 2 kg  Temperature = T = 30 °C = 30 + 273 = 303 K

r = 1 m  h = 2 m

**To Find:** Pressure = P = ?

**Solution:** Volume = V = $(\pi r^2)$h

$V = 3.142\,(1)^2 \times 2 = 6.284\ m^3$

$$n = \frac{mass\ of\ gas}{molar\ mass} = \frac{2000\ g}{40\ g\ mol^{-1}} = 50\ \text{mol}$$

From general gas equation  PV= n RT  (where, $R = 8.314\ JK^{-1}\ mol^{-1}$)

$$P = \frac{nRT}{V} \quad \text{or} \quad P = \frac{50 \times 8.314 \times 303}{6.284} = 20044\ \text{Pa}$$

**Assignment 8.2**

A sample of gas is compressed to one third of its initial volume at constant pressure of $2.25 \times 10^5\ N\ m^{-2}$. During the compression, 200 J of work is done on the gas. Determine the final volume of the gas.

## 8.2.2 Difference between Work Done by the Gas and on the Gas

In thermodynamics, work done is given by:

$$W = P\Delta V$$

**Positive Work Done:** When gas expands, it is said that work is done by the gas (system) on the surroundings. Due to the expansion of gas, the final volume will be greater than the initial volume. So, the change in volume of the gas is positive, i.e., $\Delta V > 0$, as shown in the Fig. 8.5. Therefore,. the work done by the gas on the surroundings is positive. Mathematically,

$$W = +P\Delta V$$

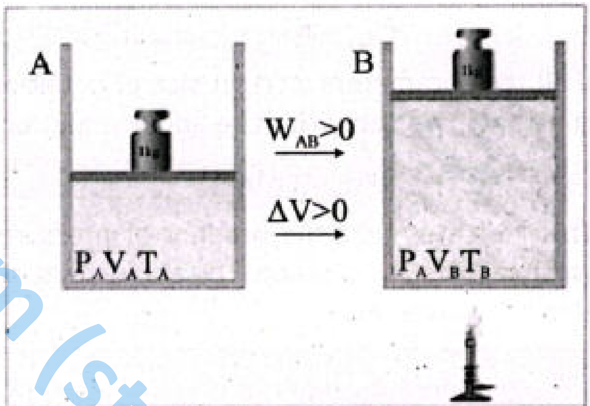



**Figure 8.5:** Work done by the system.

Therefore, when a gas expands, we can say that the work done by the system on the surroundings is positive.

**Negative Work Done:** When gas contracts, the final volume of the gas is smaller than the initial volume. So, the change in volume of the gas is negative, i.e. $\Delta V < 0$, as shown in the Fig. 8.6. Therefore, the work done on the gas by the surroundings is negative. Mathematically,

$$W = -P\Delta V$$

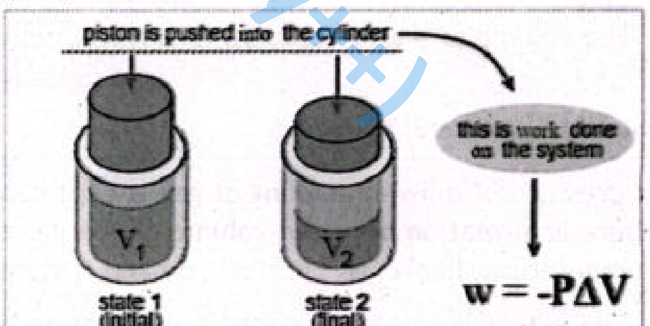



**Figure 8.6:** Work done on the system.

Therefore, when a gas contracts, we can say that the work done by the surroundings on the system is negative.

**Zero Work Done:** If volume remains constant, then change in volume is zero (i.e., $\Delta V = 0$), and the work done is zero.

$$W = P\Delta V$$

or $$W = P(0) = 0.$$

## 8.3 THE GAS LAWS

It is a common observation that whenever we change the external conditions like temperature and pressure, the volume of a given quantity of any gas is affected. Gas laws provide the relationships between the volume of given amount of a gas and variables like temperature or pressure.

### 8.3.1 Boyle's Law

For an ideal gas system with fixed quantity at constant temperature, the pressure and volume are related by the Boyle's law. This relationship can be derived from ideal gas equation:

$$PV = nRT \qquad (8.6)$$

If all the parameters on right side of equation are constants, we can write the above equation as:

$$PV = \text{Constant}$$

This indicates that the product of pressure and volume remains constant. The above relation can also be written as:

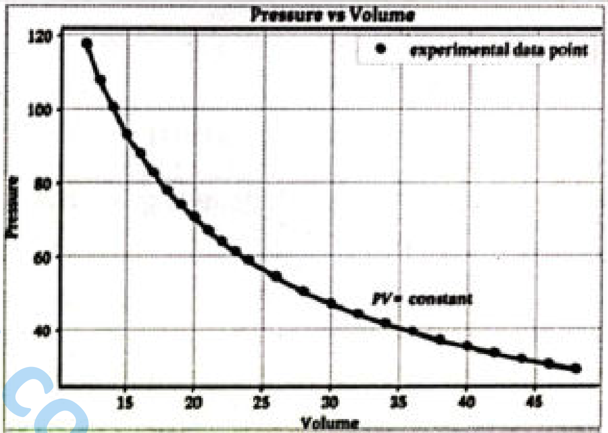



Figure 8.7: Boyle's law.

$$V \propto \frac{1}{P}$$

As the pressure increases, volume of the gas decreases and vice versa, graphically it is shown in Fig. 8.7. Therefore, Boyle's law can be stated as:

**The volume of a given amount of a gas at constant temperature is inversely proportional to the pressure applied.**

### 8.3.2 Charles's Law

If pressure of a given amount of gas is kept constant, there is a relation between volume of the gas and its temperature, known as Charles' law and is stated as:

**The volume of a given amount of a gas at constant pressure is directly proportional to the absolute temperature.**

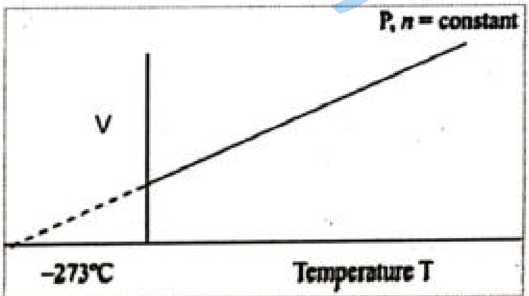



Figure 8.8: Charles's law.

The general gas law can then be expressed as:

$$V \propto T \qquad \text{or} \qquad \frac{V}{T} = \text{constant}$$

Graphically, it is shown in Fig. 8.8. This graph also indicates that at temperature -273 °C the volume of all gases reduces to zero showing that a gas cannot exist at such a low temperature.

### 8.3.3 Gay Lussac's Law

To find the relation between pressure and temperature of given amount of a gas at constant volume, we refer Amontons's law, also called as Gay Lussac's law, which can be stated as:

The applied pressure of a given amount of a gas at constant volume is directly proportional to the absolute temperature.

Mathematically:

$$P \propto T \qquad \text{or} \qquad \frac{P}{T} = \text{constant}$$

Graphically, the Gay Lussac's law is shown in Fig. 8.9. For all gas laws, the temperature is expressed in Kelvin scale.

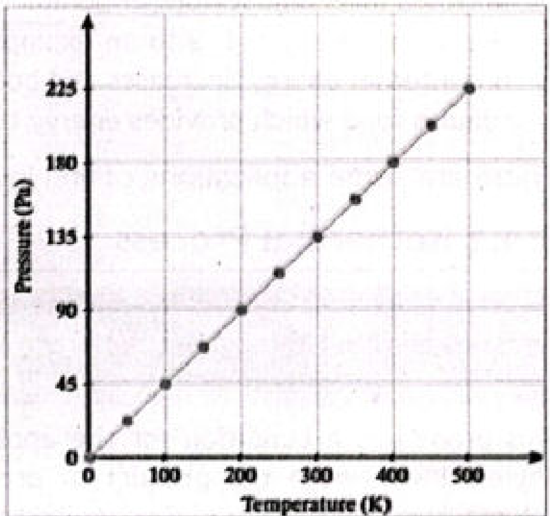



**Figure 8.9:** Gay Lussac's law.

## 8.4 FIRST LAW OF THERMODYNAMICS

Whenever heat is added to a system, the internal energy of the system increases due to the rise in temperature of the system. At the same time, if the system is allowed to expand, it may do work on its environment. First law of thermodynamics is based on law of conservation of energy applied on thermodynamic process. The first law of thermodynamics can be stated as:

When heat (Q) is added to a system, it increases the internal energy ($\Delta U$) of the system along with doing work done (W) by the system on the environment.

Mathematically, it is written as:

$$Q = \Delta U + W \qquad (8.7)$$

Similarly, when heat is removed from the system, its internal energy decreases and the work will be done on the system due to compression in a gas (system).

The change in internal energy $\Delta U = U_f - U_i$ of the system depends only on the initial and the final states of the system; it is independent of the path followed. It is the change in internal energy which is important, that is why we use change in internal energy in spite of absolute value of internal energy.

**Do You Know?**

How can you measure the temperature of core of Earth?

**Examples:** First law of thermodynamics dictates the behavior of heat energy:

- Melting of ice cubes is an example of first law of thermodynamics.
- Sweating in a crowded room is also an example of first law of thermodynamics, in which heat from a person's body transfers to sweat, which take away heat of the body but warms the room by increasing the temperature of atmosphere in the room.
- Filling the air in the tube of a car wheel is also an example of first law of thermodynamics, in which mechanical work of pumping the gas increases internal energy of system.
- Human metabolism is also an example of first law of thermodynamics, when we do work our internal energy decreases and body temperature falls. To maintain internal energy, we require food which provides energy to our bodies.

There are some applications of first law of thermodynamics, which are given below.

## 8.4.1 Isothermal Process

**A thermodynamic process in which temperature remains constant throughout the process is called isothermal process.**

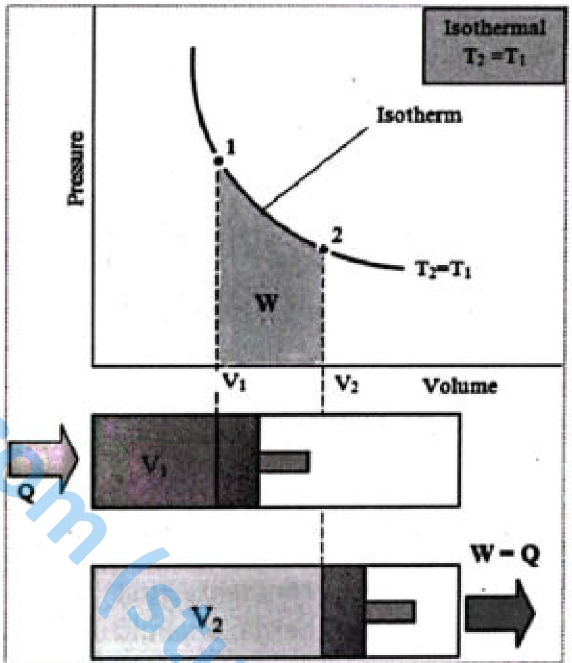


Figure 8.10: Iso-thermal process.

This process is a condition for the application of Boyle's law, where the product of pressure and volume remains constant. For understanding the process, consider an ideal gas, as shown in Fig. 8.10. Initially at pressure $P_1$ and volume $V_1$. After heating the system, the moveable piston moves outward by increasing the volume to $V_2$ and decreasing the pressure to $P_2$, while temperature of gas remains constant. Hence, such that product of pressure and volume will remain same for both initial and final values. In this example, as the gas expands, work is done by the gas, if gas compresses the work will be done on the gas. Furthermore, the internal energy of the system remains constant, i.e., $\Delta U = 0$, due to constant temperature. Hence, for isothermal process, the first law of thermodynamics is written as:

$$Q = 0 + W$$

or $$Q = W \qquad (8.8)$$

The above equation shows that for a gas to expand isothermally, some amount of heat Q is to be given to the system to do work. As heat takes time to move from one place to another, hence isothermal process should be carried out slowly to keep the temperature constant.

**The curve representing isothermal process is called isotherm.**

**Do You Know?**

Humans learnt to use of fire to raise the temperature much earlier than use of heat for mechanical work, e.g., like in steam engines. Do you know of a reason why this could be true?

Isotherm is represented in the graph of Fig. 8.10, whereas the shaded area represents work done. Some common examples of isothermal process include:

- Melting of ice at zero degrees centigrade is an example of isothermal process.
- Changing states of different liquids through the process of melting and evaporation.
- Reaction in a heat pump is also an example of isothermal process.

Do You Know?

How clouds are formed in the atmosphere?

## 8.4.2 Adiabatic Process

If we do not supply or remove heat from the system, can the system still perform work? Yes, when a system expands without addition of heat, it still does work.

A process in which no heat or mass transfers between a thermodynamic system and its surroundings is called an adiabatic process.

In adiabatic process, the system transfers energy only to the surroundings as work. Hence, for adiabatic process, the first law of thermodynamics is written as:

For $Q = 0$, $W = -\Delta U$

This equation shows that if no heat is supplied to the system and system still expands and perform work, this work is done at the expense of internal energy.

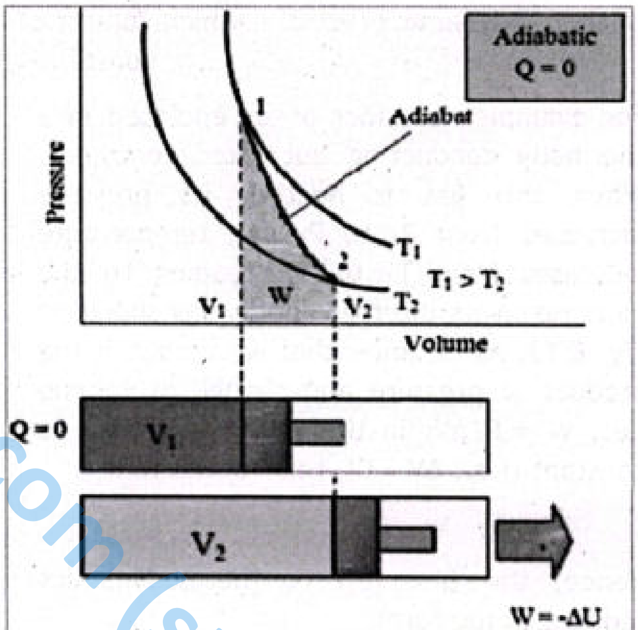



**Figure 8.11:** Adiabatic process.

In adiabatic process, if gas expands and does work, its temperature and internal energy decreases. While if gas is compressed adiabatically, its temperature and internal energy increases. In an adiabatic process, the gas expands or compresses rapidly.

The curve representing adiabatic process is called adiabat.

Adiabat is represented in the graph in Fig. 8.11. Some examples of adiabatic process are:

- The rapid compression and expansion of air as sound wave passes through the it.

For Your Information

Blow air onto your hand with your mouth wide open and then with your lips puckered. When does the air feel cooler and why? Assume the process is so fast that Q = 0.

- The vertical flow of air in the atmosphere, air expands and cools as it rises, and contracts and gets warms as it descends.
- Expansion and contraction of gas cloud in interstellar regions.

In case of adiabatic process, Boyle's law gets the form:

$$PV^{\gamma} = \text{Constant}$$

### 8.4.3 Isochoric Process

When you heat a gas in closed container, what will happen? There are some thermodynamic processes in which system does not change its volume; such a process is called isochoric process.

**A thermodynamic process in which volume of the system remains constant is called isochoric or constant-volume process.**

For example, consider a gas enclosed in a thermally conducting but fixed container. When this gas is heated, its pressure increases from $P_1$ to $P_2$ and temperature increases from $T_1$ to $T_2$, leading to the increase in its internal energy, as shown in Fig. 8.12. As we know that work done is the product of pressure and change in volume i.e., $W = P\,\Delta V$. In this case, as volume is constant (i.e., $\Delta V = 0$), this implies that

$$W = 0$$

Hence, the first law of thermodynamics reduces to the form:

$$Q = \Delta U$$

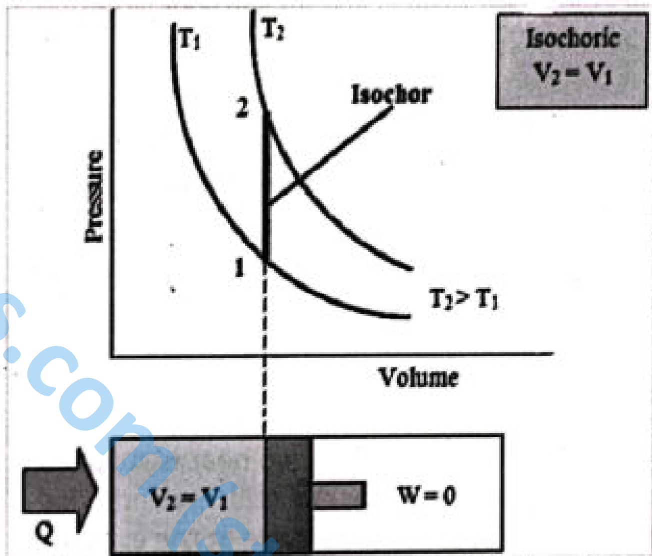



Figure 8.12: Iso-choric process.

This means that in isochoric process, all the heat given to the system will be utilized in increasing its internal energy. On the other hand, if heat is removed from the system in isochoric process, both temperature and pressure of the system decrease. The P-V graph of isochoric process is a vertical line and called "isochor". Examples of isochoric process include:

- Cooking food in a pressure cooker: a closed container where when heat is supplied the temperature and pressure get increased.
- When gasoline-air mixture is burnt in the engine of a car, it increases the pressure and temperature of the gas but volume of a gas remains constant.

### 8.4.4 Isobaric Process

The term isobaric is derived from the Greek word 'iso' meaning same and 'baros' meaning pressure. In isobaric process, when heat is transferred to the system, some work is done, and there is also a change in internal energy of the system. It is the process in which no quantities in the first law of thermodynamics becomes zero.

**Do You Know?**

How a balloon under varying atmospheric temperature may be in isobaric state?

**A thermodynamic process in which pressure of the system remains constant is called isobaric.**

When a gas expands slowly due to heat being supplied while keeping the pressure constant, it is an example of isobaric process.

Let's consider a gas in the container with a cross-sectional area A, as shown in Fig. 8.13, with initial pressure, temperature and volume P, $T_1$ and $V_1$ respectively. When this gas is supplied a certain amount of heat Q, its temperature and volume changes to $T_2$ and $V_2$ respectively, while pressure remains constant. As the gas expands, it does work and its internal energy also increases. Mathematically, the force is expressed as:

$$F = P A$$

If the gas undergoes a displacement $\Delta y$, then work done by the gas is expressed as:

$$\text{Work} = (\text{Force})\,(\text{Displacement})$$

$$W = P A \Delta y = P \Delta V$$

$$Q = \Delta U + W$$

or

$$Q = \Delta U + P \Delta V$$

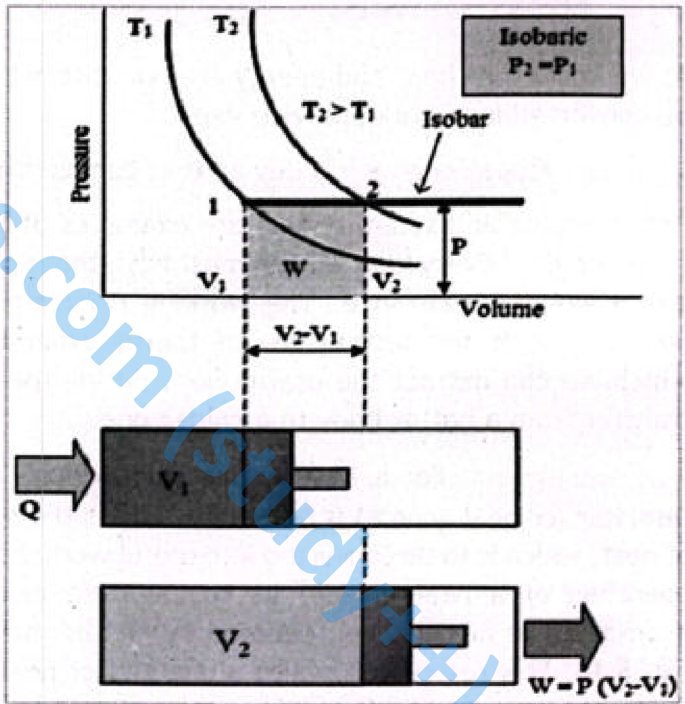



Figure 8.13: Iso-baric process.

The P-V graph of an isobaric process is a horizontal line, called 'isobar'. The area under isobar is equal to work done. Examples of isobaric process include:

- Boiling of water to steam in an open container.
- Solid moth ball (made of solid material which, at room temperature slowly changes to gas and becomes fumes in air) in your linen closet sublimating into an insect repelling gas.
- Ice formation in nature in which volume changes to compensate energy removal, while pressure remains constant.

**Example 8.3: If the internal energy of a gas decreased by 200 J, while 50 J work is done by the gas. How much energy is transferred as heat? Is the work done positive or negative?**

**Given:** $\Delta U = 200$ kJ $\quad W = 50$ J

**To Find:** Heat Q = ?

**Solution:** By using first law of thermodynamics, we have:

$$Q = \Delta U + W = -200\ J + 50\ J = -150\ J$$

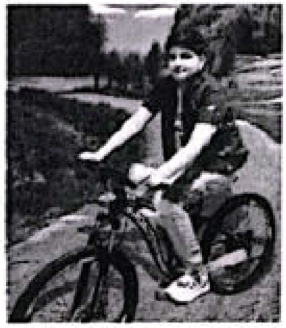
Negative sign shows that heat is given out of the system.

Assignment 8.3

A cup is filled with hot tea and a spoon is used for stirring purpose. Initially the tea has an internal energy of 200 kJ. On cooling, it loses 150 kJ of heat. The spoon does 25 kJ of work while stirring. Find the change in the internal energy of the tea and the final internal energy of the tea.

**Do You Know?**

How thermodynamics plays a role in the human bodies, in terms of work and energy? Thermodynamics also applies to the living bodies like human. This forms the basis of the biological thermodynamics. As in the cover picture of this chapter, a boy is eating an apple and also riding a bicycle. When he rides a bicycle, heat (Q) is transferred out of the body and work (W) is done by him which removes the internal energy (U). Do you know from where human gets the energy for all this process? Human and other living things get energy from the food intake which may be considered as work done on the body (system).



## 8.5 HEAT ENGINE

As we know that heat and energy are two distinct quantities but are closely related; heat can be converted into work and vice versa.

**A heat engine is a device that converts heat energy into mechanical work.**

Petrol engine and steam engine are examples of heat engine. The earliest heat engine was a steam engine. Every heat engine must have three parts, namely a hot reservoir, a cold reservoir and a working substance. The working principle of a heat engine is the second law of thermodynamics in which we can extract the useful work during the heat transfer from a hotter body to a colder one.

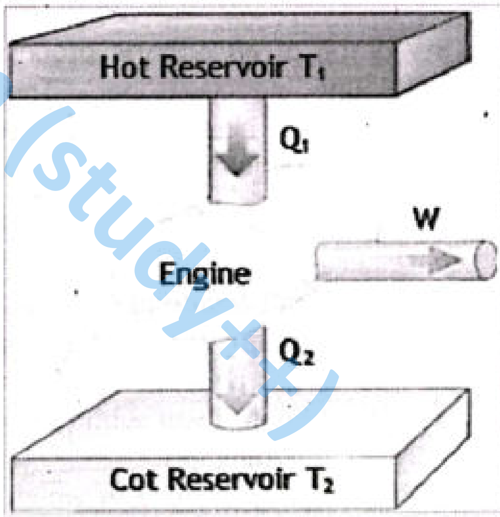



Figure 8.14: Heat engine.

**Hot Reservoir:** For a heat engine to function, a hot reservoir (or heat source) is necessary. This is the source of heat, which is to be converted into useful work. For the operation of a heat engine, its temperature must be maintained at higher level, denoted by '$T_1$' as shown in Fig. 8.14. This reservoir supplies a quantity of heat '$Q_1$' flowing towards low temperature.

**Cold Reservoir:** For a heat engine, a cold reservoir (or heat sink) is another necessary part. The difference between the temperatures of both reservoirs is responsible for the heat flow. Heat sink absorbs heat '$Q_2$' at temperature '$T_2$' from heat source. Cold reservoir is essential for the operation of heat engine, as heat can only flow when there is a difference.

**Working Substance:** In a heat engine, a working substance is the main part which practically converts heat into useful mechanical work. Working substance in most of the heat engines is a gas. It performs work due to the heat flowing from heat source to the heat sink. Mathematical relation for work done for heat engine is given as:

$$W = Q_1 - Q_2$$

For continues operation of the heat engine, it is made cyclic. Leonard Sadi Carnot invented the first heat engine. It was a type of hot air engine.

**Thermodynamics and Auto-engine**

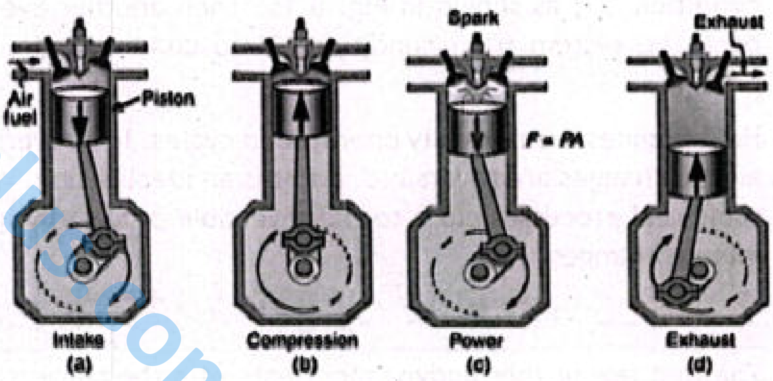
A petrol engine of a car extracts heat from burning fuel and converts some of its energy into mechanical work. The engine expels rest of energy to the atmosphere. A petrol engine roughly has efficiency of about 25% to 30%. Motorbikes normally has two cylinders whereas cars have four cylinders on the same crankshaft timed to fire turn by turn in succession for smooth running of car. It has four steps of working.

**Intake Stroke:** In this stroke, the piston moves downward and petrol-air mixture is drawn through inlet valve, as shown in figure (a).

**Compression Stroke:** In this stroke, piston moves upward, inlet valve closed and petrol-air mixture compresses adiabatically, as shown in figure (b).

**Power Stroke:** A spark ignites the mixture, causing a rapid increase in temperature and pressure, expands adiabatically, and hence forces the piston to move downwards. This is the stroke which delivers the power to the engine and hence called power stroke, as shown in figure (c).

**Exhaust Stroke:** in this stroke piston moves upward. Outlet valve opens and residual gases are expelled out into the atmosphere as shown in figure (d).




## 8.6 REVERSIBLE AND IRREVERSIBLE PROCESSES

In thermodynamics, a process occurs, when exchange of energy takes place between the system and its surroundings. There are mainly two types of processes in thermodynamics: reversible and irreversible processes.

### Reversible Process:

**A process which can be retraced in exactly reverse order without causing any change to its surroundings is called reversible process.**

For example, if heat is absorbed in a direct process, it will be evolved in the reverse process. If work is done by the system in direct process, it will be done on the system in reverse process. In reality, no actual change is exactly reversible but some changes can be taken as reversible if done slowly. Some examples of reversible process are:

- The transformation between water and ice.
- Liquefaction and evaporation of a substance performed slowly.

**Thermodynamics and Thermometry**

A thermodynamic scale of temperature is a scale which has two fixed points. One is absolute zero (0 K) and the other is the triple point of water. The triple point of water is the temperature at which water co-exists in all three states i.e. solid (ice), liquid and gas (vapors). Its value is 273.16 K.

- Melting, freezing, boiling, and condensing are the reversible processes.

Irreversible Process:

A process which cannot be retraced in exactly the reverse order without causing a change into its surroundings is called irreversible process.

Examples of irreversible processes include:

- Any process involving friction or dissipation of energy.
- Decaying of animals and plants.
- Convection, conduction and radiation are irreversible process.

Cyclic Process:

A succession of events which bring the system back to its initial condition is called cyclic process.

For example, an event 'A' brings the system from condition '1' to condition '2', as shown in Fig. 8.15. Then another event B brings back the system from condition '2' to condition '1', is a cyclic process.

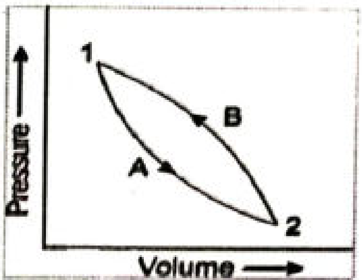


Figure 8.15: A cyclic process.

Heat engines are normally operated in cycles. In a reversible cycle, all the changes are reversible, but it is an idealization. We can take some real processes close to the reversible process by ignoring the minute changes.

## 8.7 SECOND LAW OF THERMODYNAMICS

The first law of thermodynamics deals with the conversion of heat into work and vice versa; it is simply law of conservation of energy. However, it does not address the conditions under which this conversion takes place. For finding this condition, we require another law, the second law of thermodynamics. This law deals with the circumstances under which heat can be converted into work and also the direction heat flow. It can be stated in two different ways as:

Lord Kelvin's Statement:

It is impossible for a heat engine to convert heat taken from single reservoir entirely into work without leaving any change in the working system.

**Do You Know?**
What may be the net change in internal energy in cyclic process?

It is clear that in any heat engine, a single heat reservoir, no matter how much energy it contains, cannot be made to perform any work. This is why heat content of our atmosphere cannot be utilized for work, as it is a single source and there is no sink relative to it.

Rudolf Clausius's Statement:

It is impossible to cause heat to flow from cold body to hot body without the expenditure of work.

Second law of thermodynamics explains that both a hot and a cold reservoir are essential for a working substance to do work. As from Fig. 8.14, a heat source $Q_1$ rejects heat towards a heat sink $Q_2$ at lower temperature, while some of the heat is converted into mechanical work W, which can be given as:

$$W = Q_h - Q_c$$

In a cyclic process, the substance eventually returns to its initial state, so the change in internal energy is equal to zero i.e., $\Delta U = 0$.

**For Your Information**

The second law of thermodynamics is a fundamental principle in physics that addresses the direction of natural processes and even the concept of entropy. This law explains the heat naturally flows from hot objects to cold ones and not the opposite way i.e. from cold to hot one. This can be explained if you put a cup of hot tea in a room with air-conditioner is running the tea will cool down due to low temperature of the room but when you switch off the A.C and after sometime the temperature of the room increases but the tea will not get warm up to the initial level. This shows that naturally heat can only flow in one direction i.e. from hotter body to colder one.

## 8.8 CARNOT'S ENGINE



A Carnot heat engine is a theoretical engine that operates on the principle of Carnot cycle. This concept was introduced by Sadi Carnot in 1824. A heat engine operating in an ideal reversible cycle between a heat source and a heat sink would be the most efficient engine. For the operation of Carnot engine, we consider Carnot cycle using an ideal gas as working substance. The working principle of Carnot's engine is the isothermal and adiabatic processes. Carnot's engine works in a cycle on the isothermal expansion, adiabatic expansion, isothermal compression and finally adiabatic compression to reach the initial state while taking some useful work at output. This working process in the form of P-V diagram is shown in Fig. 8.16. This cycle can be explained in four steps as:

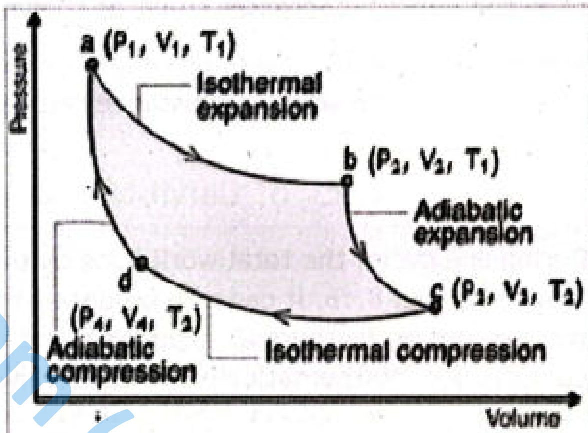



**Figure 8.16:** P-V diagram for Carnot cycle.

**1) Isothermal Expansion:** In the first step, the gas is allowed to expand isothermally at pressure $P_1$ and volume $V_1$. The temperature of the gas remains high at this moment, represented by $T_1$. Absorbing heat $Q_1$ from hot reservoir, the pressure of gas decreases to $P_2$ and volume increases to $V_2$. Hence, work is done by the gas, while temperature remains constant. The isotherm expansion is represented by portion 'ab' in Fig. 8.16 and mechanically shown in Fig. 8.17 (A).

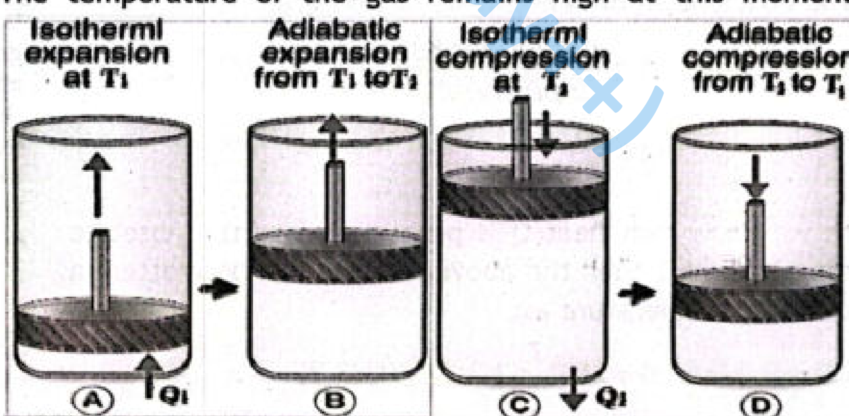



**Figure 8.17:** Working of Carnot engine.

2) Adiabatic Expansion: In this step, the cylinder is insulated so that gas neither absorbs nor releases the heat, as shown in Fig. 8.17 (B). The gas expands adiabatically, causing the temperature of the gas drops from high temperature $T_1$ to a low temperature $T_2$. As shown in adiabat (the portion of cycle represented by 'bc' in Fig. 8.16), the pressure further decreases to $P_3$ and volume further increases to $V_3$, causing the piston moves outwards by doing work, as shown in Fig. 8.17 (B).

3) Isothermal Compression: In this process, the gas is kept at low temperature $T_2$ and it compresses by rejecting heat $Q_2$, as shown in Fig. 8.17 (C). The pressure in this case increases to $P_4$ while volume decreases to $V_4$ and hence work is done on the gas. The isotherm for this case is represented by curve 'cd' in Fig. 8.16.

Adiabatic Compression: Finally, the gas is once again placed in insulated cylinder, ensuring that no heat enters or leaves the gas. The gas is compressed adiabatically to increase its temperature from $T_2$ to $T_1$, as shown in Fig. 8.17 (D). During this step, the pressure increases to $P_1$ and volume decreases to $V_1$, as shown in Fig. 8.16 represented by curve 'da'.

Thermal and mechanical equilibrium is maintained throughout the process to make it perfectly reversible. As the working substance returns to its initial state, so there is no change in internal energy i.e., $\Delta U = 0$.

## 8.8.1 Efficiency of Carnot Engine:

During one cycle, the total work done by the gas can be found by the area enclosed by the path 'abcda' in Fig. 8.16. It can also be determined by the difference of heat absorbed $Q_1$ (input) by the gas during isothermal expansion and the heat rejected $Q_2$ by the gas during isothermal compression. Mathematically, it can be given as:

$$W = Q_1 - Q_2 \quad \text{\_\_\_\_\_ (8.8)}$$

The above expression comes from the fact that in the first law of thermodynamics, $Q = \Delta U + W$, while $\Delta U = 0$, it simplifies to:

$$W = Q \qquad W = Q_1 - Q_2$$

The efficiency $\eta$ of heat engine can be given as:

$$\eta = \frac{output(work)}{input(heat)}$$

$$\eta = \frac{Q_1 - Q_2}{Q_1} = 1 - \frac{Q_2}{Q_1}$$

As we know that heat Q is proportional to the absolute temperature T then the above relation can be written in terms of temperature as:

$$\eta = (1 - \frac{T_2}{T_1}) \quad \text{\_\_\_\_\_ (8.9)}$$

Efficiency is usually expressed in percentage, so:

**Do You Know?**

Does internal energy of an ideal gas depend on pressure? What about a real gas?

Out of solid, liquid and gas of same mass and temperature, which one has greatest internal energy? And which one has the least internal energy?

$$\text{Percentage Efficiency} = \eta = (1 - \frac{T_2}{T_1}) \times 100\ \%$$

**Conclusion:** From above result, it is clear that efficiency of Carnot engine is independent of nature of working substance. It depends only on the temperatures of hot and cold reservoirs. For the efficiency to be 100 % the ratio of temperatures should be zero. This will only happen if $T_2 = 0$ or $T_1 = \infty$, which is impossible because we do not have reservoir at absolute zero temperature or infinite temperatures. Therefore, it is impossible for a real engine to have an efficiency of 100 %.

### 8.8.2 Carnot Theorem

Carnot's theorem states that no heat engine can be more efficient than a Carnot engine operating between the same two temperatures.

In most of the cases, the cold reservoir is our atmosphere. In order to increase the efficiency of heat engine, we have to increase the temperature of hot reservoir. All real heat engines are less efficient than Carnot's engine due to heat losses.

**Example 8.4:** Find the efficiency of a heat engine operating between 400 °C and 30 °C.

**Given:** $T_1 = 400 + 273 = 673\text{ K}$ $\quad T_2 = 30 + 273 = 303\text{ K}$

**To Find:** Percentage Efficiency = ?

**Solution:** Percentage Efficiency $= (1 - \frac{T_2}{T_1}) \times 100\ \%$

$$\eta = (1 - \frac{303}{673}) \times 100\ \% = (1 - 0.45) \times 100\ \% = 55\ \%$$

**Assignment: 8.4**

A heat engine is working with thermal efficiency of 40%. It receives 4 kJ of heat from a furnace. What amount of waste heat does it reject?

## 8.9 REFRIGERATOR

We know that in nature, heat flows from hotter body towards colder body. If we want to move heat from a colder to a hotter body, we need to reverse the working of heat engine, and work is to be done on the system.

A refrigerator is a device designed to remove heat from a region that is at lower temperature than its surroundings.

The same device can be used to heat a space that is at higher temperature than the surroundings. In this case, the device is called **Heat Pump**, as shown in Fig. 8.18. In refrigerators, heat flows from lower temperature source to higher temperature sink. In other words, we can also define refrigerator as:

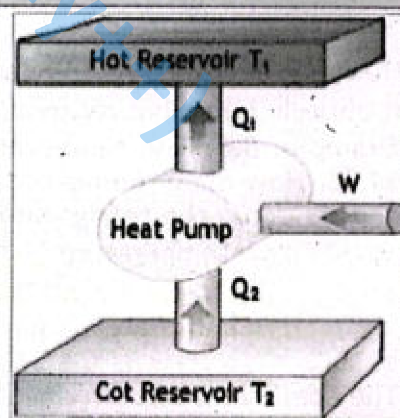


**Figure 8.18:** Schematic diagram of heat pump.

**A device in which the working substance performs cycle in a direction opposite to that of a heat engine is called refrigerator.**

Refrigerator removes some amount of heat $Q_2$ from a cold reservoir at low temperature $T_2$. Some work $W$ is performed by the compressor of the refrigerator on the working substance. The quantity of heat $Q_1$ is rejected to a hot reservoir at higher temperature $T_1$. The working substance in this case is usually a gas called the refrigerant.

Co-efficient of Performance (CP) of Refrigerator: The coefficient of performance for cooling or cooling energy ratio can be defined as:

**The ratio of the amount of heat removed from the cold reservoir to the work required to do so.**

Mathematically, it can be written as:

$$\text{Coefficient of performance for cooling} = CP_{\text{Cooling}} = \frac{Q_2}{W}$$

As we know that: $W = Q_1 - Q_2$ by using this value in above equation, we get:

$$CP_{\text{Cooling}} = \frac{Q_2}{Q_1 - Q_2} \quad \text{\_\_\_\_\_\_ (8.10)}$$

As we know that heat Q is proportional to temperature T, we can express the $CP_{\text{Cooling}}$ as:

$$CP_{\text{Cooling}} = \frac{T_2}{T_1 - T_2}$$

Similarly, coefficient of performing for heating, or heating energy ratio can be expressed as:

$$\text{Coefficient of Performance for Heating} = CP_{\text{Heating}} = \frac{Q_1}{W}$$

As we know that: $W = Q_1 - Q_2$ by using this value in above equation, we get:

$$CP_{\text{Heating}} = \frac{Q_1}{Q_1 - Q_2} \quad \text{\_\_\_\_\_\_ (8.11)}$$

As we know that heat Q is proportional to temperature T.

$$CP_{\text{Heating}} = \frac{T_1}{T_1 - T_2}$$

No cyclic device has ever been built that can extract heat '$Q_2$' from a cold reservoir and reject it entirely into a hot reservoir without expenditure of work.

**Example: 8.5: The temperature inside a refrigerator is 6 °C and the room temperature is 37 °C. How many joules of heat will be delivered to the room for each joule of electricity consumed by the refrigerator? Assume the refrigerator is ideal.**

Given: Low temperature '$T_2$' = 6 °C High temperature '$T_1$' = 37 °C

To Find: Quantity of heat pumped out in the room '$Q_1$' = ?

Solution: First of all, we have to convert the temperatures into the Kelvin scales as:

$T_1 = 37°C + 273°C = 310\ K$ $T_2 = 6°C + 273°C = 279\ K$

The coefficient of performance of the refrigerator for cooling is:

$$CP_{cooling} = \frac{T_2}{T_1 - T_2}$$

$$CP_{cooling} = \frac{279}{310-279} = 9$$

The coefficient of performance of the refrigerator for cooling can also be expressed as:

$$CP_{cooling} = \frac{Q_2}{W} \quad \Rightarrow \quad Q_2 = W\, CP_{cooling}$$

$$Q_2 = 9\ W$$

Where 'W' is the work done by the pump, and then heat can be given as:

$$Q_1 = Q_2 + W$$

$$Q_1 = 9W + W = 10W$$

So, for each joule of work done (for each joule of electricity consumed), the quantity of heat pumped out in the room will be:

$$Q_1 = 10\ J$$

This shows that the refrigerator has to perform significant work to expel heat from its interior into the room on a hot summer day when the outside temperature is 37 °C.

Assignment 8.5

The temperature inside a refrigerator is $t_2$ °C and the room temperature is $t_1$ °C. Find the amount of heat delivered to the room for each joule of electricity consumed ideally.

## 8.10 ENTROPY

The concept of entropy was introduced into the study of thermodynamics by Rudolph Clausius in 1856 to provide a quantitative statement to second law of thermodynamics. It introduced another state variable, along with temperature, pressure, volume and internal energy. Entropy can be defined in many ways.

A thermodynamic quantity that represents the unavailability of a system's thermal energy for conversion into mechanical work is called entropy.

It is simply **the degree of disorder or randomness in a system**. For mathematical derivation of entropy, consider a system undergoes a reversible process during which it absorbs an amount of heat $\Delta Q$ at an absolute temperature T. The increase in state variable called entropy S of the system is given by:

$$\Delta S = \frac{\Delta Q}{T} \quad \text{——— (8.12)}$$

From this equation, we find that:

Entropy is the measure of a system's thermal energy per unit temperature that is unavailable for doing useful work.

Like potential energy and internal energy, it is change in entropy which is important rather than the absolute value of entropy. Examples of entropy changes include:

- Burning of wood, as wood converts into ash, smoke and gases, releasing the stored energy within the wood.

- Melting of ice, as the molecules of ice lose their order during melting, as shown in Fig. 8.19. If we place some ice in the bucket for some time in a room, we notice that with the passage of time the ice is converting into liquid water by releasing the heat. As we know that molecules are ordered in ice while in water the molecules are moving randomly. This activity shows that with time the entropy of the system decreases in this natural process. In all natural processes the entropy of the system as well as the universe is increasing and hence the universe is going from an order to disorder state with time.

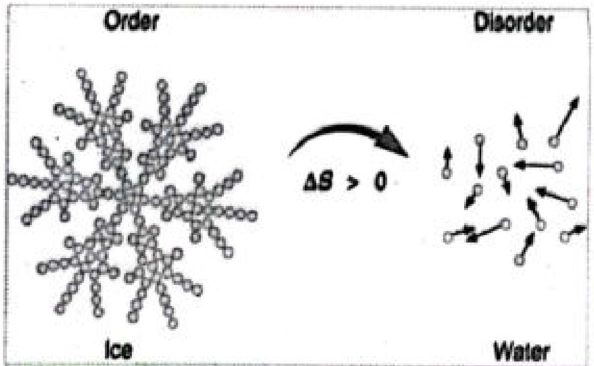


Figure 8.19: From order to disorder.

## 8.10.1 Sign Convention for Entropy

During a change, the sign of entropy is important consider. It depends on the direction of heat flow, i.e., whether heat is added to the system or removed from the system.

- The change in entropy is positive if heat is added to the system.
- The change in entropy is negative if heat is removed from the system.

Suppose heat Q flows from a heat source at temperature $T_1$ to a heat sink at temperature $T_2$. The change in entropy of the heat source can be given as $-Q/T_1$, as the source loses heat, so its entropy is negative. The entropy of sink is given as $+Q/T_2$. As $T_1 > T_2$, the net change in entropy will be positive:

$$\Delta S = \frac{Q}{T_2} - \frac{Q}{T_1}$$

This equation shows that in all natural processes where heat flows from hotter body to colder body, there is a net increase in entropy. We can define second law of thermodynamics in terms of entropy as follows:

**If a system undergoes a natural process, it will proceed in the direction that causes the entropy of the system plus the environment to increase.**

It is observed that all natural processes tend to proceed towards a state of greater disorder. There is a relation between entropy and molecular disorder, as shown in the Fig. 8.20.

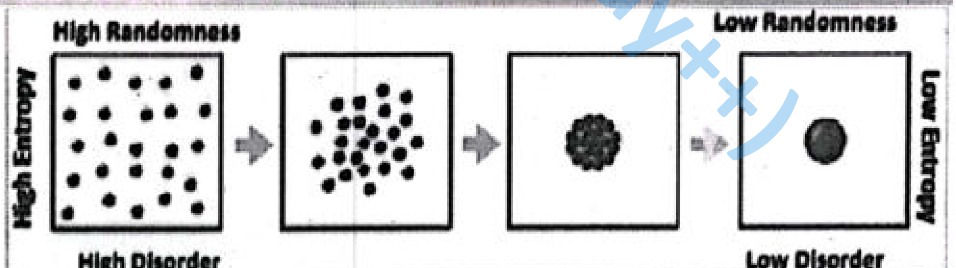


Figure 8.20: Randomness of molecules.

Entropy requires a particular direction for time, known as **"Arrow of Time"**. As one goes 'forward' in time the entropy of isolated system increases, hence entropy measurement is a way of distinguishing the past from the future.

## 8.10.2 Entropy and Energy

**Do You Know?**

Does the entropy of a star increase or decrease as it radiates? What will be the effect of this radiation on space around it and on the universe?

In nature only those processes are more likely to occur where the entropy of the system increases or remains constant.

- For all reversible processes, entropy of the system remains constant.
- Entropy increases for irreversible process.

Every time entropy increases the opportunity to convert some heat into work is lost.

**An increase in entropy means the degradation of energy.**

For example, in a car engine the burning of fuel produces some work while the remaining heat energy is released into the atmosphere. The heat energy released into the atmosphere is no longer available for work. Before entropy change, all the heat from this process was available for work, but after the process a major part of heat energy is lost into the atmosphere.

Even in the case of reverse processes such as in refrigerator or air conditioner, the temperature of one system decreases at the expense of an increase in the temperature of another system. When these systems are considered together as universe, the entropy of universe always increases.

**Example: 8.6: Calculate entropy change when 700 g of ice melts at 0 °C. Latent heat of fusion for ice is $L_f = 3.36\times10^5$ J kg$^{-1}$. Also discuss the sign of the entropy changes.**

Given: m = 700 g = 0.7 kg  T = 0 °C = 273 K  $L_f = 3.36\times10^5$ J kg$^{-1}$

To Find: $\Delta S$ = ?

Solution: The relation for heat '$\Delta Q$' in this case can be given as:

$$\Delta Q = m L_f$$

Here 'm' is the mass and '$L_f$' is the latent heat of fusion.

$$\Delta S = \frac{\Delta Q}{T} = \frac{mL_f}{T}$$

$$\Delta S = \frac{0.7\ \text{kg} \times 336000\ \text{J kg}^{-1}}{273\ K} = 861.5\ \text{J K}^{-1}$$

The positive value of entropy indicates that entropy increases in melting of ice, as molecules are loosing order to become orderless liquid.

**Assignment 8.6:**

**A body at 150 °C undergoes a reversible isothermal process. The heat energy removed in the process is 7373 J. Find the entropy change of the body.**

**For Your Information**

Second law of thermodynamics gives us clues to tackle with environmental crisis we are facing right now. In fact, environmental crisis arises due to our efforts to bring order to the nature for our comforts, which is against law of nature that is why we can say that environmental crisis is entropy crisis.

The energy processes we use are not efficient, as a result most of the energy is lost to the environment, causing thermal pollution, as heat is the ultimate death of any form of energy i.e., whenever any type of energy is converted completely into heat energy it is now no longer available for doing useful work. With thermal pollution most of the heat engines we use cause air pollution.

UNIT 8

# HEAT AND THERMODYNAMICS

## SUMMARY

- **Thermodynamics** is that branch of physics in which we study the transformation of heat into other forms of energy.
- **Thermal energy** is the energy possessed by a system due to movement of its particles.
- **Internal energy** is the energy associated with the random and disordered motion of molecules of a substance.
- In thermodynamics, **work** is the product of pressure and change in volume of the system.
- **First law of thermodynamics** states that when heat is added to a system, it increases the internal energy of the system plus work is done on the environment.
- An **isothermal process** is a thermodynamic process in which temperature remains constant throughout the process.
- An **adiabatic process** is a thermodynamic process in which no heat or mass transfers between the system and the surroundings.
- An **isochoric** or **constant volume process** is a thermodynamic process in which volume of the system remains constant.
- An **isobaric** or **constant pressure process** is a thermodynamic process in which pressure of the system remains constant.
- A **heat engine** is a device that converts heat energy into mechanical work.
- **Second law of thermodynamics** states that it is impossible for a heat engine to convert heat taken from a single reservoir entirely into work without leaving any change in the working substance.
- **Carnot's theorem** states that, no heat engine can be more efficient than a Carnot engine operating between the same two temperatures.
- **Entropy** is a measure of the degree of disorder or randomness in a system.

## EXERCISE

### Multiple Choice Questions

Encircle the Correct option.

1) A real gas can be approximated to as ideal gas at:

A. low density B. high pressure C. high density D. low temperature

2) A thermos flask containing milk as a system is shaken vigorously, temperature of milk rises due to process called:

A. isochoric B. isothermal C. isobaric D. adiabatic

3) Considering your metabolism as a system, during fasting in Ramadan, it is a/an _______ process.

A. isochoric B. isothermal C. isobaric D. adiabatic

4) Change in internal energy of a gas kept in rigid container when 'Q' J energy is added to it is:

A. $\frac{Q}{2}$ B. $\frac{Q}{3}$ C. Q D. 2Q

5) The volume of an ideal gas increases from 5 $m^3$ to 20 $m^3$ under a constant pressure of $6 \times 10^5$ Pa. work done by the gas is:

A. $5 \times 10^5$ J  B. $6 \times 10^6$ J  C. $9 \times 10^6$ J  D. $9 \times 10^7$ J

6) Which process involves an increase in entropy?

A. crystallization  B. sublimation  C. freezing  D. condensation

7) A Carnot engine working between 300 K and 600 K has work output 800 J/cycle, the heat supplied is:

A. 1000 J/cycle  B. 1600 J/cycle  C. 1800 J/cycle  D. 2000 J/cycle

8) A Carnot engine working between 0 $^0$C to 200 $^0$C has efficiency $\eta_1$. Then same engine working between 0 $^0$C to -200 $^0$C has efficiency $\eta_2$. Ratio of its efficiencies is.

A. 0.577  B. 0.638  C. 0.733  D. 0.85

9) Maximum work is obtained by a process called:

A. isochoric  B. isothermal  C. isobaric  D. adiabatic

10) The coefficient of performance of a refrigerator operating at room temperature is 5. The temperature inside the refrigerator is:

A. 150 K  B. 200 K  C. 250 K  D. 300 K

11) If the temperature of heat source is doubled than before, the efficiency increases by:

A. 1.5times  B. 2 times  C. 2.5 times  D. 3 times

12) Thermodynamics is that branch of physics in which we study:

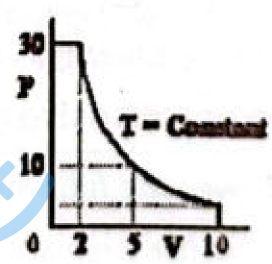
A. No heat transfers to the environment.
B. Neither heat nor any mass are transferred to the environment.
C. No dissipated energy and heat transferred to the environment.
D. No mass transfers to the environment.

13) If two objects are in thermal equilibrium with each other, they cannot:

A. be moving.
B. be undergoing an elastic collision.
C. have different pressures.
D. be at different temperatures.

14) What is change in internal energy in the figure?

A. zero  B. 20 J  C. 40 J  D. 160 J




15) The change in internal energy is independent of pathstaken, similar to:

A. K.E.  B. wave energy  C. gravitational P.E.  D. solar energy

16) Boltzmann constant k is equal to:

A. $\frac{N_A}{R}$  B. $\frac{R}{N_A}$  C. $\frac{R}{N_o}$  D. $\frac{N_o}{R}$

17) Two spheres of same size are made of the same material, but one is hollow and the other is solid. They are heated to the same temperature. Then:

A. Both spheres will expand equally.
B. The hollow sphere will expand more than the solid one.
C. The solid sphere will expand more than the hollow one.

D. No conclusion can be drawn about their relative expansions unless the nature of the material is known.

18) Expansion during heating:

A. occurs at the same rate for all materials.
B. increases the weight of a material.
C. occurs at different rates for different materials.
D. increases the density of material.

## Short Questions

### Give short answers to the following questions:

8.1 Why does the gas pressure in the tires rise when a car runs on a road through some distance?

8.2 When the internal energy of a system rises, can its temperature remain the same?

8.3 Can the temperature of an isolated system change? Explain.

8.4 The internal energy of a system is increased as a result of some process, how can one tell that the increase was due to orderly macroscopic work W or due to the flow of disorderly microscopic energy 'Q'?

8.5 At what temperature (in °C) does the volume of a gas at 0°C becomes double, while pressure remains constant?

8.6 Should the internal energy of a system necessarily increase if heat is added to it?

8.7 Why is the slope of an adiabatic curve steeper than the slope of isothermal curve?

8.8 Why isothermal processes typically slow and adiabatic processes fast?

8.9 Can a process be both adiabatic and isothermal? Explain.

8.10 Does the entropy of a Carnot engine increases for each cycle?

8.11 Does entropy of a system increases or decreases due to friction?

8.12 What are the similarities and dissimilarities in the working of refrigerator and air conditioner?

8.13 When a coffee filled thermos is vigorously shaken: (i) is the work done on coffee? (ii) Does the internal energy of coffee increase? (iii) Does the temperature of coffee rise? (iv) Does the coffee get heat from outside?

## Comprehensive Questions

### Answer the following questions in detail.

8.1 Define internal energy of a system and find its relation with absolute temperature.

8.2 Explain the gas laws.

8.3 Explain first law of thermodynamics.

8.4 Define and explain: (a) Isothermal process (b) Isobaric process (c) adiabatic process (d) isochoric process.

8.5 Explain the working of a heat engine.

8.6 Define and explain second law thermodynamics.

8.7 Define and explain Carnot's Engine, also discuss the relation for the efficiency of Carnot's engine.

8.9 Explain the working of a refrigerator.

8.10 What is entropy? Explain.

8.11 Explain that entropy is the degradation of energy.

## Numerical Problems

8.1 Calculate number of moles of air in an inflated balloon of radius 10 cm and pressure 180 kPa at room temperature. **(Ans: 0.3 moles)**

8.2 A heat engine operating between the freezing and boiling point of water. Find its efficiency. **(Ans: 27%)**

8.3 One mole of a perfect gas in a cylinder with a piston has pressure P, volume V and temperature T. If its temperature is increased by 1 K. Find the increase in volume, if initial volume was 100 $m^3$. **(Ans: 0.37 $m^3$)**

8.4 The high temperature of Carnot engine is 600 K. If engine absorbs 600 J of heat at low temperature of 400 K. Find the work done by the engine. **(Ans: 200 J)**

8.5 A 220 W electric immersion heater is used to boil 136 g of water. Calculate the time required to bring this water from 23.5 $^0$C to its boiling point, ignoring any heat loss.

**(Ans: 198 s)**

8.6 If a heat engine at room temperature has an efficiency 50 %, you change the heat source to increase the efficiency up to, 70 %. Find the temperature difference between the two heat sources. **(Ans: 397 K)**

8.7 A Carnot engine absorbs an amount Q of heat at temperature T. Find the amount of heat rejected at temperature T/3. **(Ans: Q/3)**

8.8 Energy 500 J is required to melt 2 g of ice at 0 $^0$C. Find the change in entropy of 70 g water at 0 $^0$C, if it changes into ice in a refrigerator. **(Ans: -64 J $K^{-1}$)**

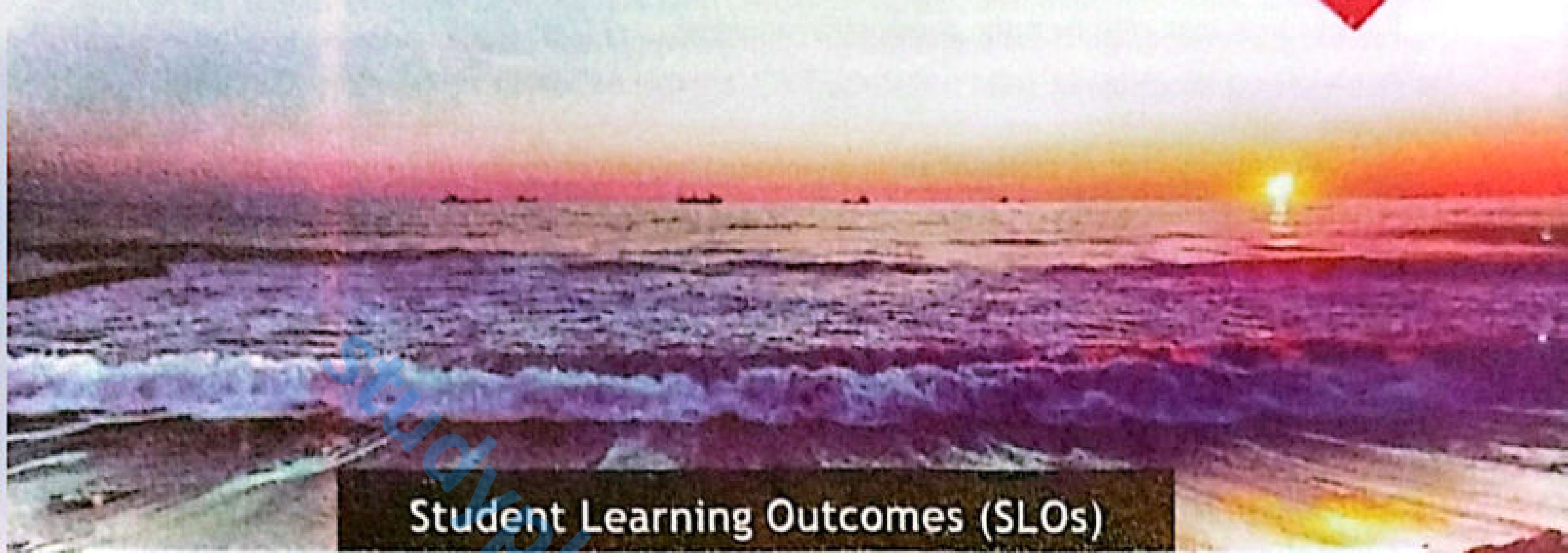
8.9 A gas expands from volume $1m^3$ to $2m^3$ at constant atmospheric pressure.

(a) Calculate the work done by the gas.

(b) Represent the work done in PV diagram. **(Ans: $1.013\times 10^5$ J)**

# WAVES

UNIT 9



## Student Learning Outcomes (SLOs)

**The students will:**

- Use intensity = power/area to solve problems. Use *intensity* $\propto$ *(amplitude)*$^2$ for a progressive wave to solve problems.
- Explain that when a source of sound waves moves relative to a stationary observer, the observed frequency is different from the source frequency [describing of the Doppler effect for a stationary source and a moving observer is not required]
- Use the expression $f' = \frac{v}{v \pm v_s} f$ for the observed frequency when a source of sound waves moves relative to a stationary observer.
- Explain the applications of the Doppler effect [such as radar, sonar, astronomy. satellite, radar speed traps and studying cardiac problems in humans].
- Explain that polarization is a phenomenon associated with transverse waves.
- Define and apply Malus's law [$I = I_o \cos^2 \theta$ to calculate the intensity of a plane-polarized electromagnetic wave after transmission through a polarizing filter or a series of polarizing filters. (calculation of the effect of a polarizing filter on the intensity of an un-polarized wave is not required)].
- Use the principle of superposition of waves to solve problems.
- Differentiate between constructive and destructive interference.
- Apply the principle of superposition to explain the working of noise canceling headphones.
- Illustrate experiments that demonstrate stationary waves [using microwaves, stretched strings and air columns (it will be assumed that end corrections are negligible; knowledge of the concept of end corrections is not required)].
- Explain the formation of a stationary wave using graphical representation.
- Explain the formation of harmonics in stationary waves.
- Analyze experiments that demonstrate diffraction [including the qualitative effect of the gap width relative to the wavelength of the wave; for example, diffraction of water waves in a ripple tank].
- Explain the term coherence.
- Explain beats [as the pulsation caused by two waves of slightly different frequencies interfering with each other].
- Illustrate examples of how beats are generated in musical instruments.
- Explain the use of polaroids in sky photography and stress analysis of materials.
- Describe qualitatively gravitational waves [as waves of the intensity of gravity generated by the accelerated masses of an orbital binary system that propagate as waves outward from their source at the speed of light].
- State that as a gravitational wave passes a body with mass the distortion in space-time can cause the body to stretch and compress periodically.
- State that gravitational waves pass through the Earth due to far off celestial events, but they are very minute amplitude.
- Describe the use of interferometers in detecting gravitational waves [Interferometers are very sensitive detection devices that make use of the interference of laser beams (working and set up details are not required) and were used to first detect the existence of gravitational waves].

The world is full of waves: sound waves, waves on a string, seismic waves, and electromagnetic waves, such as visible light, radio waves, television signals, and x-rays. All these waves have a source: a vibrating object. The importance of radio and television signals and other forms of electromagnetic waves cannot be ignored. Communication using these waves is the backbone of modern civilization.

Waves transfer energy from one location to another. In this chapter, we will study about different phenomenon of waves such as Doppler's effect, beats, interference, stationary waves and their applications.

## 9.1 INTENSITY OF WAVES

All waves carry energy and sometimes their energy can be directly observed. For example, earthquakes can shake whole cities, performing the work of thousands of wrecking balls. Loud sounds can damage nerve cells in the inner ear, causing permanent hearing loss. Ultrasound is used for deep-heat treatment of muscle strains. A laser beam can burn away a malignancy. Water waves abolish beaches.

**Energy carried by a wave per unit area in unit time is called intensity.**

The amount of intensity in a progressive wave is directly related to square of its amplitude, as shown in the following relation:

$$\text{Intensity} \propto (\text{amplitude})^2$$

So, an earthquake having large amplitude produce large ground displacements. The energy of a wave depends on time as well. For example, the longer deep-heat ultrasound is applied, the more energy it transfers. Waves can also be concentrated or spread out. For example, sunlight can be focused to burn wood. Earthquakes spread out, so they do less damage as farther they get from the source. All these pertinent factors are included in the definition of *intensity I* as power per unit area:

$$I = \frac{P}{A} \quad \text{\_\_\_\_\_ (9.1 a)}$$

where $P$ is the power carried by the wave through area $A$. As, power is energy per unit time ($P = E/t$), equation (9.1 a) can be written as:

$$I = \frac{E}{A \times t} \quad \text{\_\_\_\_\_ (9.1 b)}$$

The definition of intensity is valid for any energy in transit, including that carried by progressive waves. The SI unit for intensity is watts per square meter ($W\ m^{-2}$).

EXAMPLE 9.1:

1) The average intensity of sunlight on Earth's surface is about 500 $W\ m^{-2}$. Calculate the amount of energy that falls on a solar collector with an area of 0.50 $m^2$ in 4.0 h.

Given: Intensity of sunlight on the Earth = I = 500 $W\ m^{-2}$

Area of solar collector = A = 0.5 $m^2$ Time = t = 4.0 h

**To Find:** Energy fall on solar collector = E = ?

**Solution:** Using equation:

$$I = \frac{E}{A \times t}$$

or $$E = I \times A \times t$$

Substitute the values into the equation, we get:

$$E = (500\ \text{W m}^{-2})(0.50\ \text{m}^2)(4.0 \times 3600\ \text{s})$$

$$E = 3.6 \times 10^6\ \text{J}$$

2) If amplitude of the wave is doubled then how much energy is increased?

**Solution:**

Energy depends on the intensity of the wave. Intensity is proportional to the square of the amplitude. i.e.,

$$\text{Intensity} \propto (\text{amplitude})^2$$

If amplitude is increased to double then energy will be increased to 4 times because 4 is the square of 2.

**Assignment 9.1**

To increase intensity of a wave by a factor of 50, by what factor should the amplitude of the wave be increased?

## 9.2 DOPPLER'S EFFECT

Possibly you have noticed how the sound of a vehicle's horn changes as the vehicle moves away from you. The frequency of the sound you hear appears higher as the vehicle approaches you and appears lower as it moves away from you. This phenomenon is known as Doppler's Effect, named after an Austrian physicist Christian Doppler (1803-1853), who described it in 1842.

**The apparent change in the frequency of a wave due to the relative motion between the observer and the source is called Doppler's effect.**

The Doppler's effect is most often associated with sound, however it's common to all waves, including water and light. In deriving the Doppler's effect, we assume the air is stationary and all speed measurements are made relative to this stationary medium.

Motion of a source of sound toward an observer increases the rate at which he or she receives the vibrations. The velocity of each vibration is the speed of sound whether the source is moving or not. Each vibration from an approaching source has a shorter distance to travel. The wavelength is shortened when the source is moving towards the observer and is lengthened when it is moving away from the observer. The vibrations are therefore received at a higher frequency than they are sent. Similarly, sound waves from a receding source are received at a lower frequency than they are sent.

Consider a source of sound S emitting sound wave having magnitude of velocity v, frequency f and wavelength $\lambda$. When source S and listener L are at rest then listener will receive f number of waves per second, according to relation:

$$v = f\lambda$$

$$f = \frac{v}{\lambda}$$

Following are the possible situations regarding source of sound and the listener:

## (1) The Source is at Rest and Listener is Moving

**(a) Listener Moves Towards Stationary Source:** If the listener *L* is moves with speed $v_L$ towards the stationary sounding source S, as shown in Fig. 9.1 (a). The speed of sound relative to listener increases to $(v + v_L)$ and wavelength remains unchanged.

The apparent frequency f′ is:

$$f' = \frac{(v + v_L)}{\lambda}$$

Putting $\lambda = \frac{v}{f}$, we get:

$$f' = \frac{v + v_L}{v} f \quad \text{\_\_\_\_\_\_ (9.2)}$$

As $\frac{v + v_L}{v} > 1$

So, $f' > f$

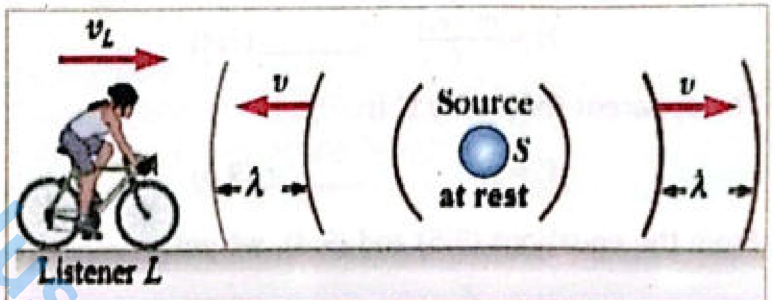



Figure 9.1 (a): The listener is moving toward the stationary source.

Hence, the frequency of sound increases, as a result its pitch also increases.

**(b) Listener Moves Away from the Stationary Source:** If the listener L is moves away from the stationary sounding source S with speed $v_L$, as shown in Fig. 9.1 (b), then speed of sound relative to the listener decreases to $(v - v_L)$ and wavelength remains unchanged.

The apparent frequency f′ is:

$$f' = \frac{v - v_L}{\lambda}$$

Putting $\lambda = \frac{v}{f}$, we get

$$f' = \frac{v - v_L}{v} f \quad \text{\_\_\_\_\_\_ (9.3)}$$

As $\frac{v - v_L}{v} < 1$

So, $f' < f$

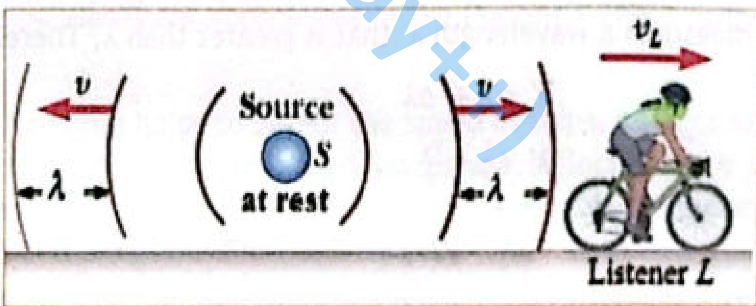



Figure 9.1 (b): Listener moves away from the stationary source.

Hence, the frequency of sound decreases, as a result its pitch also decreases.

## (2) The Source is Moving and Listener is at Rest

**(a) When the Source Moves Toward a Stationary Listener:** If the source is moving with speed $v_S$ toward the stationary listener A, as shown in Fig. 9.1 (c), then each new wave is emitted from a position to the right of the origin of the previous wave. As a result, the wave fronts heard by the listener A are closer together as compared to when the source is stationary. As a result, the wavelength $\lambda'$ measured by listener A is shorter than the wavelength $\lambda$ of the source. During each vibration, which lasts for a time interval T (the period), the source moves a distance $v_S T = v_S/f = \Delta\lambda$ causing the wavelength to be shortened by this amount. Therefore, the apparent wavelength $\lambda'$ is:

$$\lambda' = \lambda - \Delta\lambda$$

$$\lambda' = \frac{v}{f} - \frac{v_S}{f}$$

$$\lambda' = \frac{(v - v_S)}{f} \qquad \text{------ (9.4)}$$

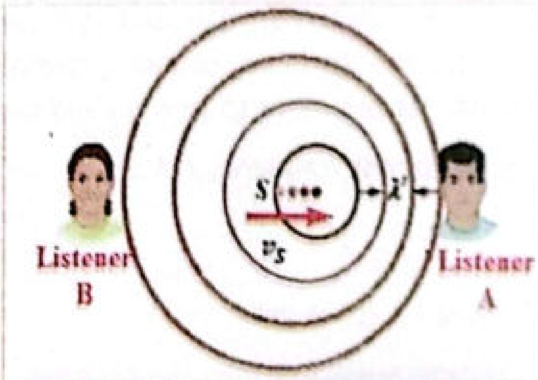



**Figure 9.1 (c): The source is moving and listener is at rest.**

The apparent frequency f′ is:

$$f' = \frac{v}{\lambda'} \qquad \text{------ (9.5)}$$

From the equations (9.5) and (9.4), we get:

$$f' = \frac{v}{v - v_S} f \qquad \text{------ (9.6)}$$

As $\quad \frac{v}{v - v_S} > 1$

So, $\quad f' > f$

Thus, when the source moves toward stationary listener, the frequency of sound increases, as a result its pitch also increases.

**(b) When the Source Moves Away from a Stationary Listener:** In this case, the observer measures a wavelength $\lambda'$ that is greater than $\lambda$, Therefore, the apparent wavelength $\lambda'$ is:

$$\lambda' = \lambda + \Delta\lambda$$

$$\lambda' = \frac{v}{f} + \frac{v_S}{f}$$

$$\lambda' = \frac{(v + v_S)}{f} \qquad \text{------ (9.7)}$$

The apparent frequency f′ is:

$$f' = \frac{v}{\lambda'} \qquad \text{------ (9.8)}$$

From the equation (9.7) and (9.8), we get:

$$f' = \frac{v}{v + v_S} f \qquad \text{------ (9.9)}$$

As $\quad \frac{v}{v + v_S} < 1$

**For Your Information**

Picture shows Doppler's effect for waves moving on the surface of water. The circular wave crests are closer together in front of a swimming duck, than those behind it and reach a receiver more frequently.

So, $f' < f$

Hence, when the source moves away from a stationary listener, the frequency of sound decreases, as a result its pitch also decreases.

**Example 9.2:** A sound source is receding from a stationary observer at 22 m s$^{-1}$ and have frequency 354 Hz. If the speed of sound is 332 m s$^{-1}$, then what frequency does the observer hear?

**Given:** Frequency of sound wave = $f$ = 354 Hz  Speed of sound source = $v_S$ = 22 m s$^{-1}$

Speed of sound = $v$ = 332 m s$^{-1}$

**To Find:** Apparent frequency = $f'$ = ?

**Solution:** When a source is receding from a stationary observer, then the apparent frequency is:

$$f' = \frac{v}{v + v_S} f$$

$$f' = \frac{332}{332 + 22}(354) = 332 \text{ Hz}$$

**Assignment 9.2**

A bus is moving at 20 m s$^{-1}$ along a straight road with its 500 Hz horn sounding. You are standing at the road side. What frequency do you hear as the bus is:

a) Approaching you?  b) Receding from you? (Take the speed of sound = 340 m s$^{-1}$)

**Activity: 9.1**

Take a tub full of water and sweep an object such as glass, on the surface of water. What do you observe? Share your experience with your class fellows.

## 9.2.1 Applications of Doppler's Effect

Doppler's effect has many interesting applications. It is applied in weather observation to characterize cloud movement and weather patterns, and has other applications in aviation and radiology. We will discuss some of the most significant applications of the Doppler's effect in the following:

- **SONAR (Sound Navigation and Ranging)** helps to detect the speed of ships, aeroplanes and submarines using the Doppler's effect. When sound waves are reflected from a moving body such as submarine, their frequency changes. This change in frequency allows us to calculate the speed and direction of the submarine.
- The velocity of **Earth's satellites** is determined from the Doppler shift in frequency of the radio waves they transmit.
- Doppler's Effect can be used in radar **system** to find the speed and direction of aeroplanes. Radar systems send out radio waves. If the frequency of the reflected radio waves from an aeroplane decreases, then the aeroplane is moving away from the radar. If the frequency of the reflected radio waves from an aeroplane increased, then the aeroplane is moving toward the radar.

- The Doppler's effect is of great interest to astronomers, who use shift in the frequency of electromagnetic waves produced by moving stars in our galaxy and beyond in order to gather information about those stars and galaxies. In 1929, Edwin Hubble observed that all galaxies appeared to be red-shifted (i.e. moving away from us and each other), leading him to propose that the universe is expanding. Specific information about stars within galaxies can be determined by using the Doppler's effect.

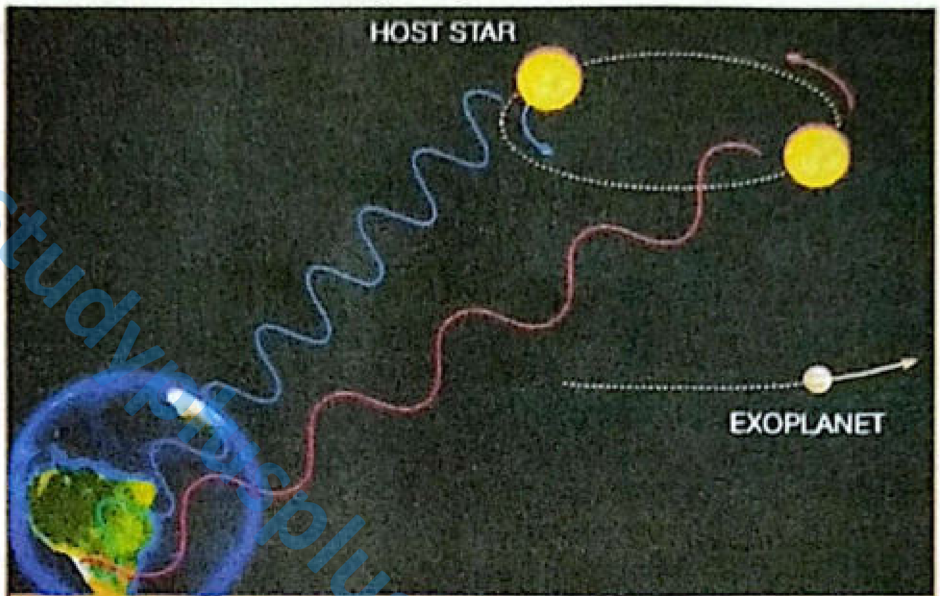



Figure 9.2: Blue shift and red shift.

Fig. 9.2 shows electromagnetic radiation emitted by stars in a distant galaxy would appear to be shifted to lower in frequencies (a red shift) if the star is moving away from the Earth. On the other hand, there is an upward shift in frequency (a blue shift) of such observed radiation if the star is moving towards the Earth.

- Doppler effect is used to measure speed in RADAR sensors. This is known as **radar speed traps.** When a fixed-frequency radio wave sent by the sender continuously strikes an object moving towards or away from the sender, the frequency of the reflected radio wave will be changed due to the object's motion. Thus, analyzing the frequency of the reflected wave one can find the speed of moving object.
- Doppler effect is used in the **diagnosis of cardiac diseases.** Doppler ultrasound (or Doppler echocardiography) is a test in which very high frequency sound waves are bounced off the heart and blood vessels. The returning sound waves are picked up and turned into pictures showing blood flow through the arteries or the heart. Doppler ultrasound

**Doppler Effect and Medical Technology**


Doppler ultrasound is used to assess blood flow through the coronary arteries (the blood vessels supplying the heart), the carotid artery (the main artery in the neck), the major arteries in the arms and legs, or in the heart itself (echocardiography).

testing allows doctors to clearly see flow of blood through the heart and blood vessels. It also allows them to see and measure obstructions in arteries and measure the degree of narrowing or leakage of heart valves.

## 9.3 SUPERPOSITION OF WAVES

Various wave phenomena in nature require two or more waves passing through the same region of space at the same time. Two travelling waves can meet and pass through each other without being vanished or even altered.

When two waves of same nature pass through the same medium, their interaction may cause to the formation of a new wave. In the region of overlap, the resultant wave is found by adding the displacements of the individual waves. For such analysis, the superposition principle applies. According to this principle:

**When two or more waves are passing through the same region at the same time, the total displacement at the point where they interact is equal to the vector sum of the individual displacements due to each pulse at that point.**

**For Your Information**

When two pebbles are thrown into a pond, the expanding circular waves don't cancel each other. In fact, the ripples pass through each other.

If a particle of medium is simultaneously acted upon by $n$ number of waves, such that its displacement due to each of the individual $n$ waves is $y_1$, $y_2$, $y_3$, ..., $y_n$, then resultant displacement $y$ of the particle is:

$$y = y_1 + y_2 + y_3 + ... + y_n$$

In Fig. 9.3, you can see two waves (X and Y) superposing to form the resultant wave (Z). The diagram shows only three points where it displays how the displacement of the resultant wave is calculated.

For example, when time t = 0, the displacement of the Y wave is -0.9 (signs must be taken into account as displacement is a vector quantity) and the displacement of the X wave was -2.1. As a result, the displacement of the resultant wave will be the vector sum of those, (-0.9) + (-2.1) = -3. This is simply the principle of superposition.

The superposition of waves can lead to the following three effects:

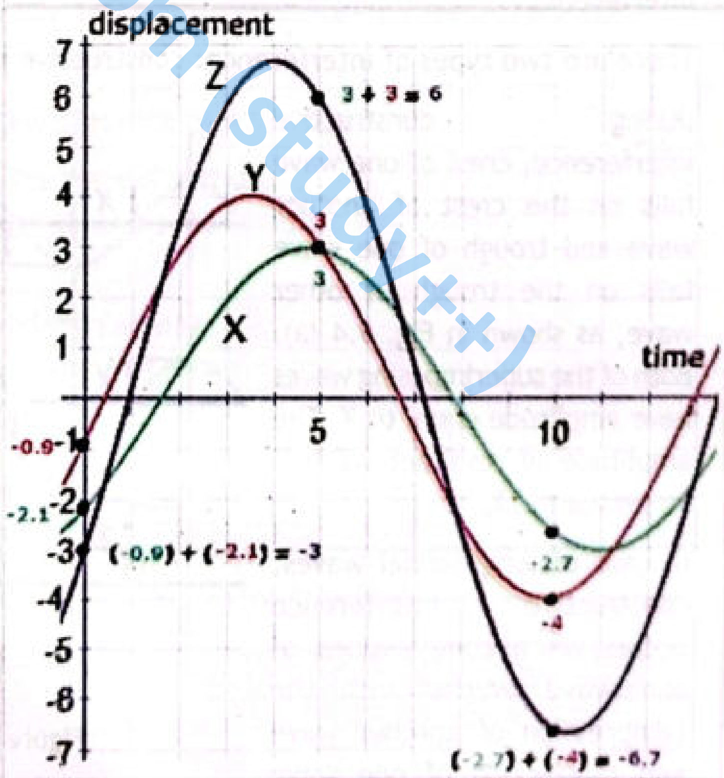



**Figure 9.3:** Two waves (X and Y) superposing to form the resultant wave (Z).

1. When two waves having the same frequency travel with the same speed along the same direction in a specific medium, then they superpose and create an effect known as the interference of waves.
2. When two waves having same frequencies travel with the same speed along opposite directions in a specific medium, then they superpose to produce stationary waves.
3. When two waves having slightly different frequencies travel with the same speed along the same direction in a specific medium, they superpose to produce beats.

**Coherent Waves**

When two or more light waves are traveling together in such a way that their phase difference is constant, then such waves are known as the coherent waves. This is possible if the waves have same frequency and wavelength.
The source which emits a light wave with the same frequency, wavelength and phase (or having a constant phase difference) is known as a coherent source. The laser is a highly coherent light source. Coherent light waves can also be produces by Young's Double Slit experiment.

## 9.4 INTERFERENCE

The waves which have constant phase difference and same frequency are called coherent waves.

**The effect produced due to the superposition of waves from two coherent sources is known as interference.**

There are two types of interference: constructive interference and destructive interference.

During constructive interference, crest of one wave falls on the crest of another wave and trough of one wave falls on the trough of other wave, as shown in Fig. 9.4 (a). Both of the superimposing waves have amplitude equal to $X$. The amplitude of resultant wave is increased to $2X$.

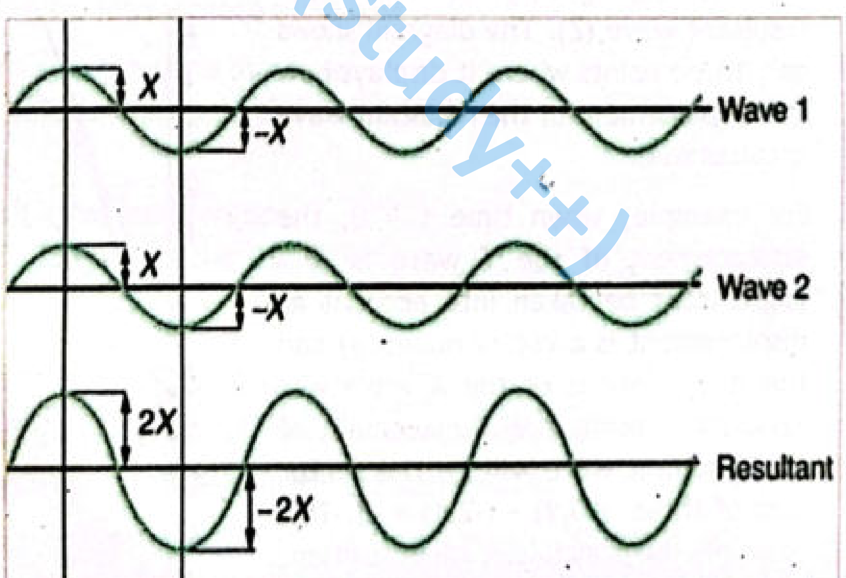


**Figure 9.4 (a): Constructive Interference.**

In case of longitudinal waves, constructive interference occurs when compressions of one wave overlap with the compression of another wave and rarefaction of one wave overlap with the rarefaction of other wave.

**When two waves overlap at a point in the same phase then two waves reinforce each other. This is called constructive interference.**

During destructive interference, crest of one wave falls on the trough of another wave. The amplitude of resultant wave is equal to the difference between the amplitude of the individual waves as i.e. $X - X = 0$, as shown in Fig. 9.4 (b).

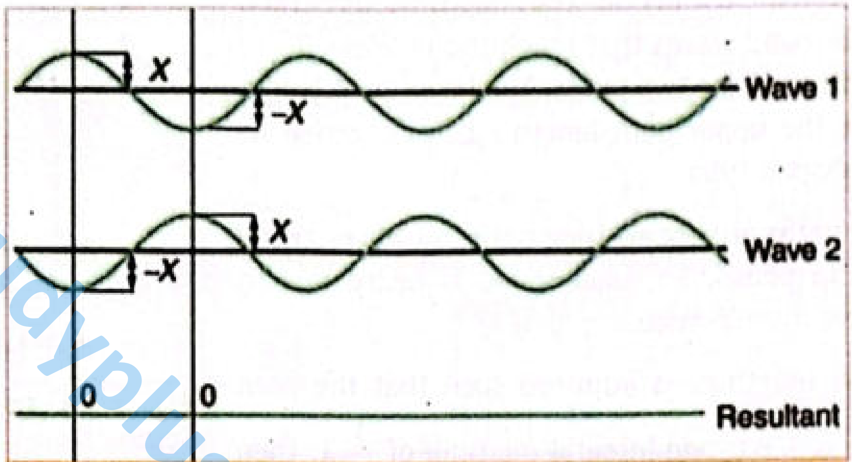



**Figure 9.4 (b): Destructive Interference**

In case of longitudinal waves, destructive interference occurs when compressions of one wave overlaps with the rarefaction of another wave.

**When two waves overlap at a point out of phase (180°) then two waves cancel each other. This is called destructive interference.**

## 9.4.1 Conditions for Interference

Following conditions are necessary for interference of waves:

i) waves must be travelling in the same direction.
ii) waves must arrive at the same place at the same time.
iii) waves must be,

(a) in phase, for *constructive interference*.
(b) out of phase (180°), for *destructive interference*.

iv) path difference must be,

(a) integral multiple of wavelength λ, for *constructive interference*.

$$d = m\lambda \quad \text{(where } m = 0, 1, 2, 3, \ldots)$$

(b) odd integral multiple of $\frac{1}{2}\lambda$, for *destructive interference*.

$$d = (m + \frac{1}{2})\lambda \quad \text{(where } m = 0, 1, 2, 3, \ldots)$$

## 9.4.2 Interference of Sound Waves

One simple device for demonstrating interference of sound waves is illustrated in Fig. 9.5 (a). Sound from a loudspeaker S is sent into a tube at point P, where there is a T-shaped junction. Half the sound energy travels in one direction, and half travels in the opposite direction. Therefore, the sound waves that reach the receiver R can travel along either of the two paths. The lower path length $r_1$ is fixed, but the upper path length $r_2$ can be varied by sliding the U-shaped tube.

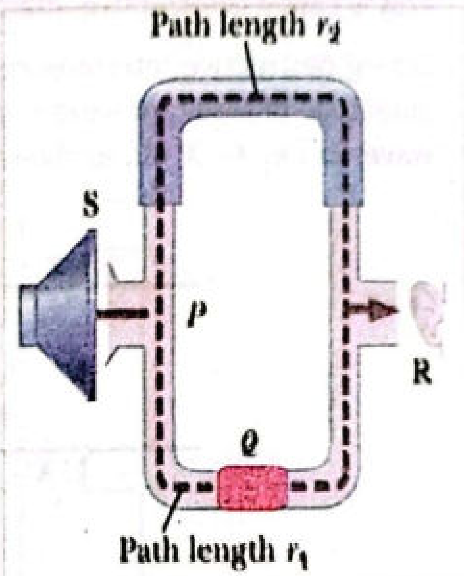


Figure 9.5 (a): Interference of Sound.

- If the two paths are equal, then both the waves arrive at R, are in phase. So, loud sound is heard due to constructive interference.
- If the path length $r_2$ is adjusted such that the path difference $r_2 - r_1$ is odd integral multiple of $\frac{1}{2}\lambda$, then both waves arriving at R may be out of phase. So, no sound is heard due to destructive interference.
- If the sound waves are stopped at Q by pinching the rubber portion of tube, then loud sound is heard again. This proves that silence is due to destructive interference of the two sound waves.

### Noise-canceling Headphones

Noise-canceling headphones (as shown in Fig. 9.5 b) uses the principle of superposition to attenuate the environmental noise inside the ear cup of the headphone. The basic idea is to sense the unwanted sound wave coming from the noisy environment and produce an appropriate canceling sound wave through a loudspeaker. The canceling wave should match the noise in amplitude and has the opposite phase (i.e., 180° out of phase).

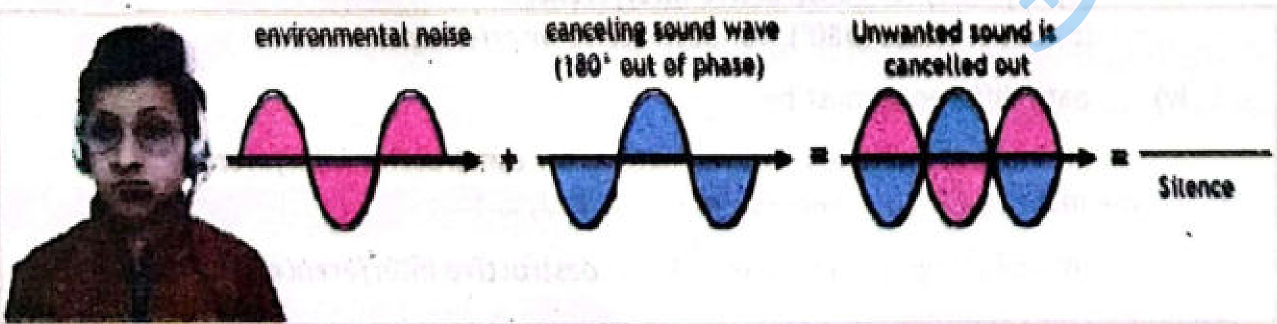


Figure 9.5 (b): Noise-canceling headphones.

In this way, the cancelling sound wave will destructively interfere with the environmental noise and the noise component will be effectively attenuated.

## 9.5 Beats

Beats are the interference effect results from the superposition of two waves with slightly different frequencies. When two sound waves having slightly different frequencies encounter each other, then the amplitude of sound waves is added and subtracted alternatively through a given period. Hence, the sound grows louder and softer through the given period and fluctuating sound (alternate soft and loud sound) is heard. It is important to note that, the loud sensation of sound is produced by the constructive interference. On the other hand, the soft sound is instigated due to destructive interference.

When two waves of slightly different frequencies are played simultaneously then periodic alternations of sound between maximum and minimum loudness are produced which are called beats.

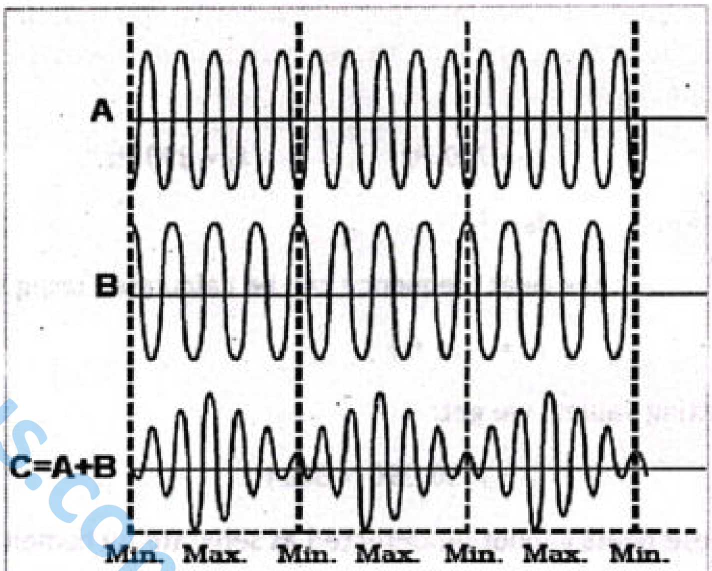



Figure 9.6: Generation of beats.

Let us consider two sound waves, A and B of slightly different frequencies but having similar amplitude propagating in the same medium. A resultant wave C is obtained by the superposition of A and B, as shown in Fig. 9.6. When these two sound waves encounter, a fluctuating sound can be heard. Note that for a certain time, the crest of A overlaps the crest of B. Hence, this causes constructive interference. Therefore, the sound intensity rises for this certain period. However, for an interval of time, the crest of B overlaps the trough of A. This causes destructive interference.

### Beats Frequency

The number of beats generated per second is called beats frequency.

The beats frequency is equal to the absolute value of the difference in frequencies of the two waves. It is denoted by $f_b$.

$$f_b = |f_1 - f_2| \qquad (9.10)$$

So, if two sound waves with frequencies of 32 Hz and 30 Hz are played simultaneously, a beat frequency of 2 Hz will be detected. If the numbers of beats per second are more than 10, then the beats can't be heard as separate.

### Application of Beats

Beats are very useful to compare frequencies, finding the unknown frequency, tuning the musical instrument, in ultrasonic imaging and radar speed traps.

Musicians utilizes the phenomenon of beats to tune a piano string. They pluck the string and tap a tuning fork at the same time. If the two sound sources: the piano string and the tuning fork, produce detectable beats then their frequencies are not identical. They will then adjust the tension of the piano string and repeat the process until the beats can no longer be heard. As the piano string becomes more in tune with the tuning fork, the beat frequency will be reduced and approach 0 Hz. When beats are no longer heard, the piano string is tuned to the tuning fork; that is, they play the same frequency as the tuning fork. This process allows a musician to match the strings' frequency to the frequency of a standardized set of tuning forks.

**Example 9.3:** Two musical instruments are sounded together to produce beats. Evaluate the beat frequency of these two sound waves having frequencies 750 Hz and 390 Hz respectively?

Given: $f_1 = 750\ Hz$ $\quad f_2 = 390\ Hz$

To Find: $f_b = ?$

Solution: The beat frequency can be calculated using the formula:

$$f_b = |f_1 - f_2|$$

Putting values, we get:

$$f_b = |750 - 390| = 380\ Hz$$

These beats cannot be detected as separate by human ear. Can you tell why?

## 9.6 STATIONARY WAVES (OR STANDING WAVES)

Sometimes waves do not seem to move; rather, they just vibrate about mean position. Such standing waves can be seen on the rubber band. If a rubber band is tied at both ends and plucked from the middle then stationary waves are produced due to the reflections of waves from the ends of the rubber band.

**When two identical waves having same speed, amplitude and frequency traveling in opposite direction are superposed then a wave obtained is called stationary wave.**

Stationary wave is formed by the superposition of two or more identical waves, moving in opposite directions, each having the same amplitude and frequency, as illustrated in Fig. 9.7. The wave reflected at the boundary interferes constructively or destructively with the incoming wave. The two waves move through each other with their amplitude adding as they go by. The displacement of the particles of waves are shown at seven different positions at t = 0, (1/4)T, (1/2)T, (3/4)T and T. It can be noted from the Fig. 9.7 (c) that displacement of points 1, 3, 5 and 7 are always zero, these points of the medium do not vibrate at all and are called nodes.

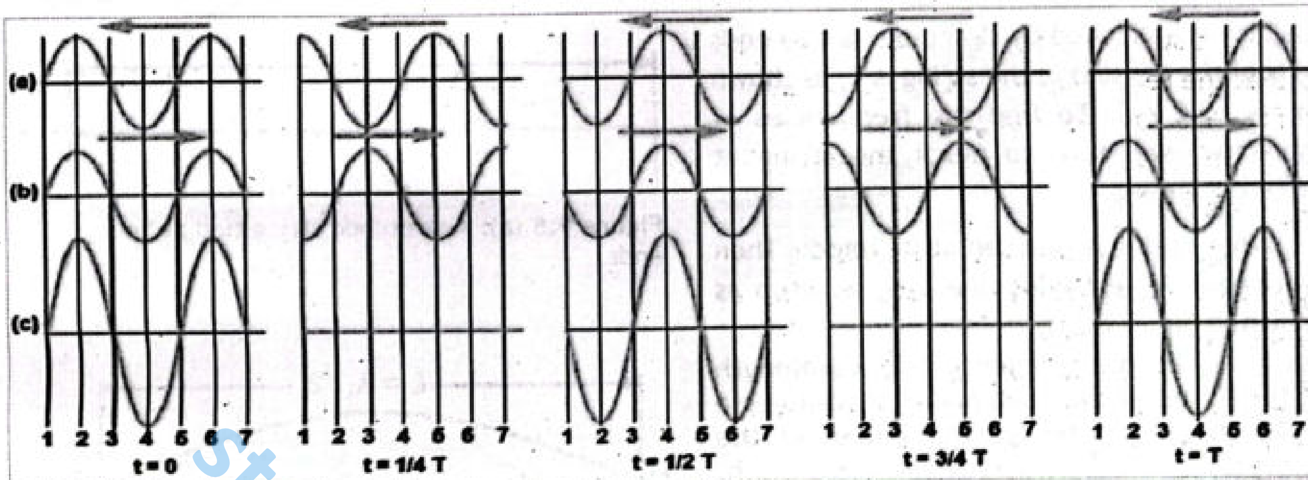



**Figure 9.7:** Standing wave created by the superposition of two identical waves moving in opposite directions.

The displacement of points 2, 4 and 6 has maximum amplitude but rapidly oscillating up and down, these points are called antinodes. Antinodes are oscillating with amplitude which is equal to the sum of amplitude of the component waves. Antinodes are usually represented by A and node are usually represented by N. This pattern of nodes and antinodes form a stationary wave because they appear to be standing, not moving through space like progressive waves. So, standing wave is confined in a given region of space.

**Activity 9.2**

The picture shows stationary wave on the surface of some liquid placed on a vibrator. The waves are visible in the photo due to the reflection of light from a lamp.

You can try this experiment at home. Take a bowl of milk and place it on a common box fan. Vibrations from the fan will produce circular standing waves on the surface of the milk.

Characteristics of Stationary Waves

A stationary wave has the following important characteristics:

(i) No energy is transferred from particle to particle in stationary waves.

(ii) The distance between two successive nodes or anti-nodes is equal to $\lambda/2$.

(iii) The distance between adjacent node and anti-node is equal to $\lambda/4$.

## 9.6.1 Stationary Waves in a Stretched String

Stationary waves can be setup in any media which do not transmit energy from one point to another point. Standing waves are also found on the strings of musical instruments and are produced due to the reflections of waves from the ends of the string. Consider a string of length

L which is stretched by clamping its two ends so that the tension in the string is T, as shown in Fig. 9.8 (a). To find the frequencies of vibration, we have to pluck the string at different points.

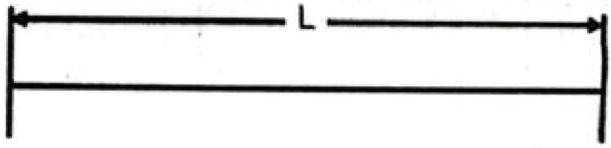



**Figure 9.8 (a):** A stretched string tied at both ends.

(1) When string is plucked at its middle then stationary wave having one loop is setup as shown in Fig. 9.8 (b).

Let $f_1$, $\lambda_1$ be the frequency and wavelength of the either of the component of transverse wave then from figure, the length of the string is equal to one-half of the wavelength. Therefore,

$$L = \frac{\lambda_1}{2}$$

$$\lambda_1 = 2L$$

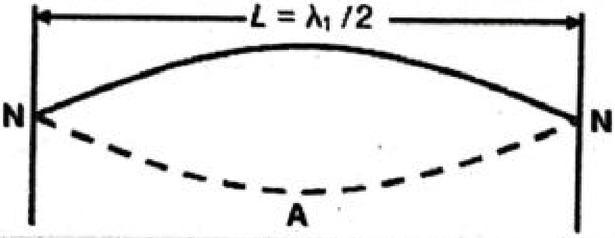



**Figure 9.8 (b):** Stationary wave having one loop.

If M is the total mass of the string, then speed $v$ of progressive wave along the string is:

$$v = \sqrt{\frac{T \times L}{M}}$$

As $$v = f_1 \lambda_1$$

$$f_1 = \frac{v}{\lambda_1} = \frac{v}{2L}$$

$$f_1 = \frac{1}{2L}\sqrt{\frac{T \times L}{M}} \qquad \text{———— (9.11 a)}$$

If $m$ is mass per unit length $M/L$, then

$$f_1 = \frac{1}{2L}\sqrt{\frac{T}{m}} \qquad \text{———— (9.11 b)}$$

This is called fundamental frequency, first harmonic or first mode of vibration.

(2) When string is plucked at one quarter of its length then stationary wave having two loops are setup, as shown in Fig. 9.8 (c). Let $f_2$, $\lambda_2$ be the frequency and wavelength of the either of the component transverse wave then from figure it can be seen that, the length of the string is equal to two-halves of the wavelength, since each loop is equivalent to one-half of a wavelength. Therefore,

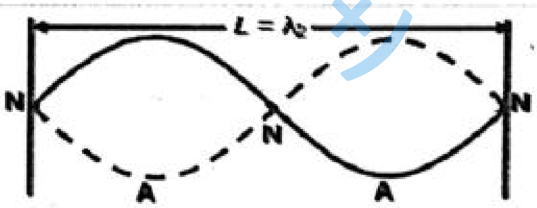



**Figure 9.8 (c):** Stationary wave having two loops.

$$L = \frac{\lambda_2}{2} + \frac{\lambda_2}{2} = \lambda_2$$

or $\lambda_2 = L$

As $v = f_2 \lambda_2$

so, $f_2 = \dfrac{v}{\lambda_2} = \dfrac{v}{L}$

$$f_2 = 2\left(\frac{v}{2L}\right)$$

because $\left(\dfrac{v}{2L}\right) = f_1$, so

$$f_2 = 2 f_1 \quad \text{\_\_\_\_\_\_ (9.12)}$$

Hence when the string vibrates in two loops its frequency of vibration will be doubled as compared to its fundamental frequency, which is called second harmonic or second mode of vibration.

(3) When string is plucked at one sixth of its length then stationary wave having three loops is setup, as shown in Fig. 9.8 (d). Let $f_3$, $\lambda_3$ be the frequency and wavelength of the either of the component transverse wave then from figure it can be seen that, the length of the string is equal to three-halves of the wavelength, since each loop is equivalent to one-half of a wavelength:

$$L = \frac{\lambda_3}{2} + \frac{\lambda_3}{2} + \frac{\lambda_3}{2} = \frac{3\lambda_3}{2}$$

$$\lambda_3 = \frac{2L}{3}$$

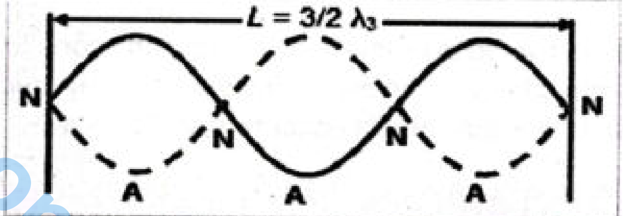



**Figure 9.8 (d):** Stationary wave having three loops.

As $v = f_3 \lambda_3$

so, $f_3 = \dfrac{v}{\lambda_3} = \dfrac{3v}{2L}$

because $\left(\dfrac{v}{2L}\right) = f_1$, so

$$f_3 = 3 f_1 \quad \text{\_\_\_\_\_\_ (9.13)}$$

Hence when the string vibrates in three loops its frequency of vibration will be three times as compared to its fundamental frequency, which is called third harmonic or third mode of vibration. Similarly, if frequency is increased to $4f_1$, $5f_1$, $6f_1$, ..., then stationary waves of 4th, 5th, 6th..., harmonic are setup and the cord will vibrate in 4, 5, 6, ..., loops respectively.

When string is vibrating having n loops, and $f_n$, $\lambda_n$ are the frequency and wavelength of the either of the component transverse wave then:

$$L = \frac{\lambda_n}{2} + \frac{\lambda_n}{2} + \frac{\lambda_n}{2} + \dots = \frac{n\lambda_n}{2}$$

or $$\lambda_n = \frac{2L}{n} \quad \text{———} \quad (9.14)$$

The lowest frequency, called the fundamental frequency, corresponds to the longest wavelength. Also, frequency $f_n$ of n-loop is:

$$f_n = n f_1 \qquad \text{OR} \qquad f_n = \frac{n}{2L}\sqrt{\frac{T}{m}} \quad \text{———} \quad (9.15)$$

Thus, it can be concluded that:

(i) The string resonates only if it is a whole number multiple of half wavelength, i.e.,

$$L = \frac{\lambda}{2}, 2\frac{\lambda}{2}, 3\frac{\lambda}{2}, \ldots$$

(ii) Stationary waves on the string can be setup only with discrete set of frequencies $f_1$, $f_2$, $f_3$, ...$f_n$. This is called quantization of frequency.

(iii) The lowest frequency $f_1$ is called fundamental frequency and its higher multiples are called overtones or harmonics.

(iv) As the string vibrate in more and more loops its frequency goes on increasing but the wavelength gets correspondingly decreasing, such that product of frequency and wavelength is always constant and is equal to speed of wave.

## 9.7 STATIONARY WAVES IN AIR COLUMN

Standing waves can also be generated in air columns such as organ pipes. Organ pipe is a musical instrument. It consists of a long tube which produces sound by mean of vibrating air column. Flute, trombone, clarinet, etc. are familiar examples of organ pipes. Organ pipes can be open or closed. If both ends of an organ pipe are open, then it is called open organ pipe. If one end of an organ pipe is closed, then it is called closed organ pipe.

Suppose we have a tube that is closed at one end and open at the other. If we hold a vibrating tuning fork near the open end of the tube, an incident sound wave travels through the tube and reflects off the closed end. The reflected sound has the same frequency and wavelength as the incident sound wave, but is traveling in the opposite direction. At the closed end of the tube, the molecules of air have very little freedom to oscillate, and a node arises. At the open end, the molecules are free to move, and at the right frequency, an antinode occurs and the air column in the tube resonates loudly. The standing wave formed in the tube has an antinode at the open end and a node at the closed end. A portion of the sound wave is reflected back into the tube even at an open end. The distance from a node to an antinode is one-fourth of a wavelength, and *this equals* the length of the tube thus, $L = 1/4\ \lambda_1$.

### 9.7.1 Modes of Vibration in Open Pipe

Let a vibrating tuning fork be held at one end of open pipe of length L. There are anti-nodes at both the ends with a node at the middle. The modes of vibration in open organ pipe are given below.

(1) Fundamental Frequency: Let $f_1$, $\lambda_1$ be the frequency and wavelength of stationary wave, as shown in Fig. 9.9 (a), the length L and wavelength are related as:

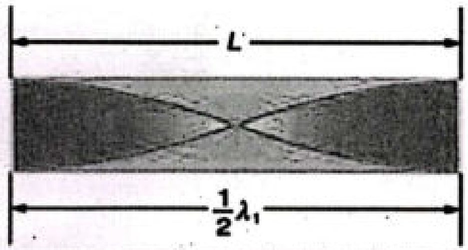



Figure 9.9 (a): Fundamental Frequency

$$L = \frac{\lambda_1}{4} + \frac{\lambda_1}{4} = \frac{\lambda_1}{2}$$

$$\lambda_1 = 2L$$

As $v = f_1\lambda_1$

So, $f_1 = \frac{v}{\lambda_1}$

or $f_1 = \frac{v}{2L}$ ——— (9.16)

This is called fundamental frequency or first harmonic for open pipe.

(2) Second Harmonic: Let $f_2$, $\lambda_2$ be the frequency and wavelength of stationary wave, as shown in Fig. 9.9 (b), the length L and wavelength are related as:

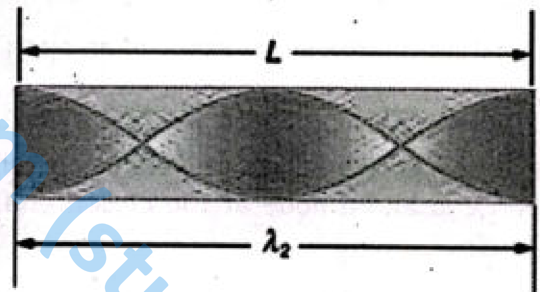



Figure 9.9 (b): Second harmonic.

$$L = \lambda_2$$

$$\lambda_2 = L$$

As $v = f_2\lambda_2$

or $f_2 = \frac{v}{\lambda_2} = \frac{v}{L}$

or $f_2 = 2\left(\frac{v}{2L}\right)$

Since, $\left(\frac{v}{2L}\right) = f_1$, so

$f_2 = 2f_1$ ——— (9.17)

This is called second harmonic for open pipe.

(3) Third Harmonic: Let $f_3$, $\lambda_3$ be the frequency and wavelength of stationary wave, as shown in Fig. 9.9 (c). The length L and wavelength are related as:

$$L = 3\frac{\lambda_3}{2}$$

$$\lambda_3 = \frac{2L}{3}$$

As $\quad v = f_3 \lambda_3$

or $\quad f_3 = \frac{v}{\lambda_3} = \frac{3v}{2L}$

or $\quad f_3 = 3\left(\frac{v}{2L}\right)$

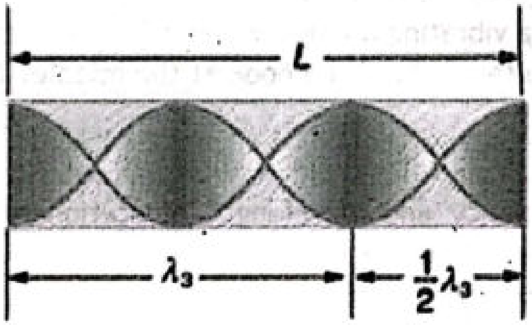



**Figure 9.9 (c): Third harmonic.**

Since, $\left(\frac{v}{2L}\right) = f_1$, so

$$f_3 = 3 f_1 \quad \text{\_\_\_\_\_ (9.18)}$$



This is the formula for third harmonic for open pipe. Similarly, from the above results, the wavelength $\lambda_n$ and frequency $f_n$ for nth harmonic is given by:

$$\lambda_n = \frac{2L}{n} \quad \text{\_\_\_\_\_ (9.19)}$$

and $\quad f_n = n f_1 \quad$ _____ (9.20) $\quad$ where $n = 1,2,3, \ldots$

Hence all harmonics are present in an open pipe.

### 9.7.2 Modes of Vibration in Closed Pipe

Let a vibrating tuning fork be held at one end of closed pipe of length L. There is an anti-node at open end and a node at the closed end. The modes of vibration in closed organ pipe are given below.

**(1) Fundamental Frequency or First Harmonic:** Let $f_1$, $\lambda_1$ be the frequency and wavelength of stationary wave, as shown in Fig. 9.10 (a), the length L and wavelength are related as:

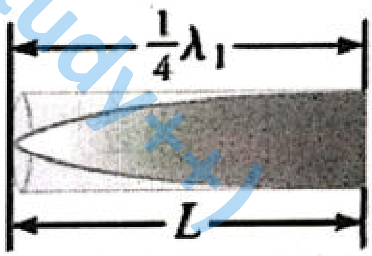



**Figure 9.10 (a): First harmonic.**

$$L = \frac{\lambda_1}{4}$$

or $\quad \lambda_1 = 4L$

As

$$v = f_1 \lambda_1$$

so, $\quad f_1 = \frac{v}{\lambda_1}$

$$f_1 = \frac{v}{4L} \quad \text{\_\_\_\_\_ (9.21)}$$

(2) Second Harmonic: Let $f_2$, $\lambda_2$ be the frequency and wavelength of stationary wave, as shown in Fig. 9.10 (b). The length L and wavelength are related as:

$$L = \frac{\lambda_2}{2} + \frac{\lambda_2}{4} = 3\frac{\lambda_2}{4}$$

or $$\lambda_2 = \frac{4L}{3}$$

As

$$v = f_2 \lambda_2$$

or $$f_2 = \frac{v}{\lambda_2}$$

or $$f_2 = 3\frac{v}{4L}$$

Since, $\frac{v}{4L} = f_1$, so

$$f_2 = 3 f_1 \quad \text{\_\_\_\_\_ (9.22)}$$

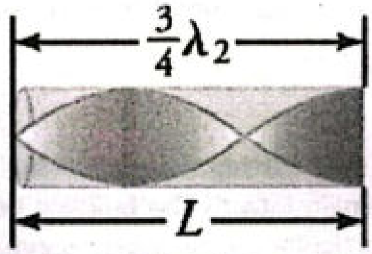



Figure 9.10 (b): Second harmonic.

(3) Third Harmonic: Let $f_3$, $\lambda_3$ be the frequency and wavelength of stationary wave, as shown in Fig. 9.10 (c), the length L and wavelength are related as:

$$L = \lambda_3 + \frac{\lambda_3}{4} = 5\frac{\lambda_3}{4}$$

or $$\lambda_3 = \frac{4L}{5}$$

As $v = f_3 \lambda_3$

so $$f_3 = \frac{v}{\lambda_3}$$

or $$f_3 = 5\frac{v}{4L}$$

Since, $\frac{v}{4L} = f_1$, so

$$f_3 = 5 f_1 \quad \text{\_\_\_\_\_ (9.23)}$$

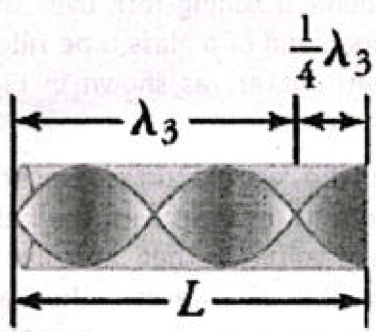



Figure 9.10 (c): Third harmonic.

Similarly, from the above results, the wavelength $\lambda_n$ and frequency $f_n$ for $nth$ harmonic is given by

$$\lambda_n = \frac{4L}{2n-1} \quad \text{\_\_\_\_\_ (9.24)}$$

and $$f_n = (2n-1) f_1 \quad \text{or} \quad f_n = (2n-1)\frac{v}{4L} \quad \text{\_\_\_\_\_ (9.25)}$$

where n = 1, 2, 3, .... . Thus, it can be seen that in an open pipe, fundamental frequency is $\frac{v}{2L}$ and all harmonics are present while in a closed pipe, fundamental frequency is $\frac{v}{4L}$ and only odd harmonics are present.

**For Your Information**

**During earthquakes, often buildings of a certain height are devastated while other taller buildings remain intact. The building height matches the condition for setting up a standing wave for that particular height provides evidence for conditions appropriate for resonance, and constructive and destructive interference. As the earthquake waves travel along the surface of the Earth and reflect off denser rocks, constructive interference occurs at certain points. A building may be vibrated for several seconds with a driving frequency matching that of the natural frequency of vibration of the building, producing a resonance resulting in one building collapsing while neighboring buildings do not. Often areas closer to the epicenter are not damaged while areas farther away are damaged.**

## 9.7.3 Experiment to Demonstrate Stationary Waves

Stationary wave can be setup in air column by holding a sounded tuning fork over the open end of a glass tube filled with water, as shown in Fig. 9.10 (d).

If the water surface in the tube is lowered with the help of reservoir, then at certain height of water, the air column in the tube resonates loudly. There will be several other heights at which the air column in the tube resonates.

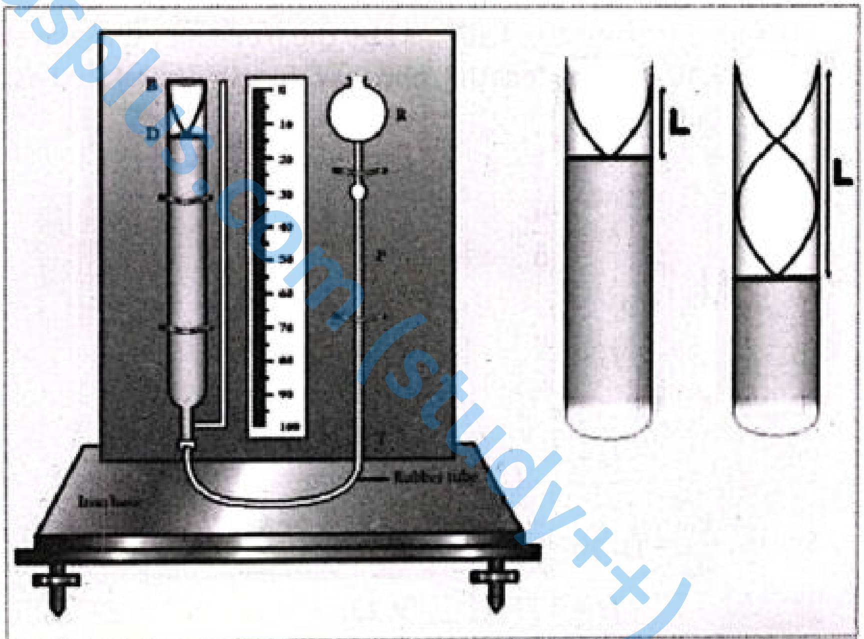



**Figure 9.10 (d): Resonance tube.**

**Example 9.4: A guitar string with a length of 80.0 cm is plucked. The speed of a wave in the string is 400 m s$^{-1}$. Calculate the frequency of the first, second, and third harmonics?**

Given: L = 0.80 m $\quad v$ = 400 m s$^{-1}$

To Find: First harmonic = $f_1$ = ? $\quad$ Second Harmonic = $f_2$ = ?

Third harmonic = $f_3$ = ?

Solution: For first harmonic

$$f_1 = \frac{v}{2L}$$

$$= \frac{400}{2(0.8)} = 250 \text{ Hz}$$

Second harmonic: $f_2 = 2(f_1) = 500$ Hz

Third harmonic: $f_3 = 3\,(f_1) = 750$ Hz

**Assignment 9.3**

**A frequency of the first harmonic is 587 Hz is sounded out by a vibrating guitar string. The speed of the wave is 600 m $s^{-1}$. Find the length of the string.**

## 9.8 POLARIZATION

Polarization is a phenomenon associated to transverse waves. A light wave is an electromagnetic wave. An electromagnetic wave is a special type of transverse wave that has both an electric and a magnetic component, as shown in Fig. 9.11 (a).

Ordinarily, a ray of light consists of a number of waves vibrating in all the directions perpendicular to its line of propagation. Light emitted by the sun, a lamp, or by a candle flame are created by electric charges that vibrate in various directions, thus creating a variety of electromagnetic wave that vibrates in several planes perpendicular to its line of propagation, as shown in Fig. 9.11 (b). A light wave that is vibrating in more than one plane is referred to as unpolarized light. It is possible to transform unpolarized light into polarized light.

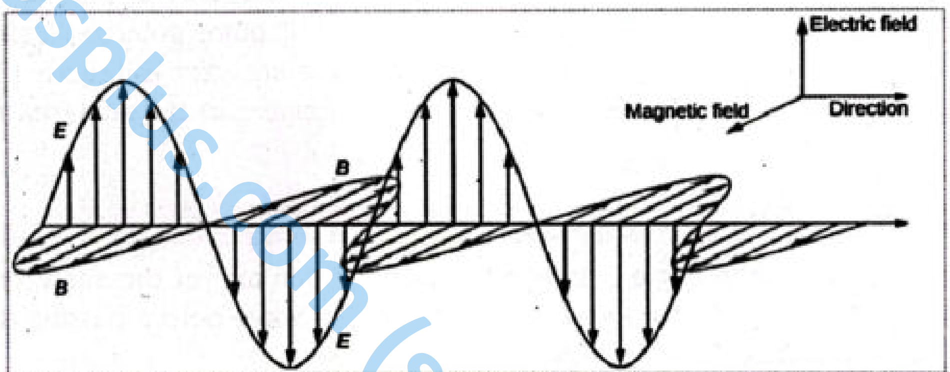



**Figure 9.11 (a): An electromagnetic wave.**

Polarized light waves are light waves in which the vibrations occur in a single plane.

The process of transforming unpolarized light into polarized light is known as polarization.

There are a variety of methods of polarizing light, as discussed below.

### 9.8.1 Polarization by a Polaroid Filter

The one most common method of polarization includes the use of a Polaroid filter.

Polaroid filters are made of a special material that is capable of blocking all the planes of vibration of an electromagnetic wave and transmit only one plane of vibration, as shown in Fig. 9.11 (b). So, a Polaroid serves as a device that filters out one plane of the vibrations upon transmission of the light through the filter. When unpolarized light is transmitted through a Polaroid filter, it emerges as polarized light, i.e., its intensity reduces to half and with vibrations in a single plane.

A Polaroid filter is able to polarize light because of the chemical composition of the filter's material. In the same manner, two Polaroid filters oriented with their polarization axis perpendicular to each other will block all the light. This observation could never be explained by a particle nature of light.

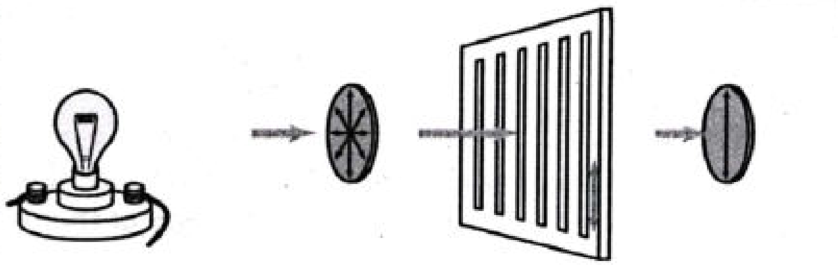



Figure 9.11 (b): Polaroid filters is blocking all the planes of vibration of an electromagnetic wave and transmit only one plane of vibration.

When unpolarized light passes through a polaroid filter its intensity reduces and only light parallel to the grid axis within the polarizing filter is allowed to pass.

The axis of polarization of transmitted light is the same as the filter that polarizing it. It is then perfectly plane polarized light. When perfectly plane polarized light is incident on an analyzer, the intensity I of the light transmitted by the analyzer is directly proportional to the square of the cosine of angle between the transmission axis of the analyzer and the polarizer.

i.e., $I \propto \cos^2\theta$

or $I = I_o \cos^2\theta$ ______ (9.26)

Where θ is the angle between the transmission axis of the analyzer and the polarizer. $I_o$ is the intensity unpolarized wave i.e., intensity of wave before passing through the polarizing filter. The equation (9.26) is known as Malus's law.

## 9.8.2 Polarization by Reflection

Unpolarized light can also polarized by reflection from nonmetallic surfaces, as shown in Fig. 9.11 (c). The amount to which polarization occurs is dependent upon the angle at which the light incident to the surface and material of the surface. Metallic surfaces reflect light with a variety of vibrational directions; such reflected light is unpolarized. However, nonmetallic surfaces such as water and asphalt roadways, reflect light such that there is a large concentration of vibrations in a plane parallel to the reflecting surface. A person viewing objects by means of light reflected off of nonmetallic surfaces will often perceive a glare if the extent of polarization is large. Fishermen are familiar with this glare since it prevents them from seeing fish that lie below the water surface. Light reflected

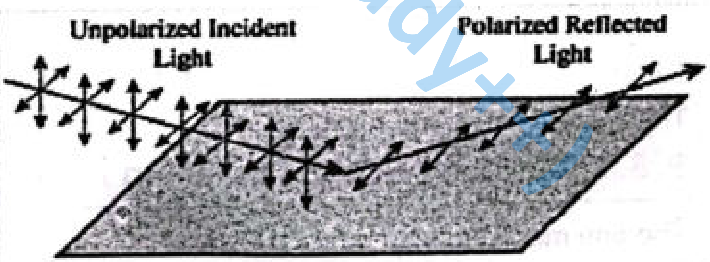



Figure 9.11 (c): Polarization by reflection.

off a lake is partially polarized in a direction parallel to the water's surface. Fishermen know that the use of glare-reducing sunglasses with the proper polarization axis allows for the blocking of this partially polarized light. By blocking the plane-polarized light, the glare is reduced and the fisherman can more easily see fish located under the water.

### 9.8.3 Uses of Polaroids

**Polaroids in Sky Photography:** Polaroid sunglasses are familiar to most of us. They have a special ability to reduce the glare of light reflected from water or sky. A camera used to photograph the clouds is fitted with a polaroid before the camera lens. The light coming from sky is polarized by polaroid. Thus, the background becomes sufficiently dark against which a clear photograph is obtained.

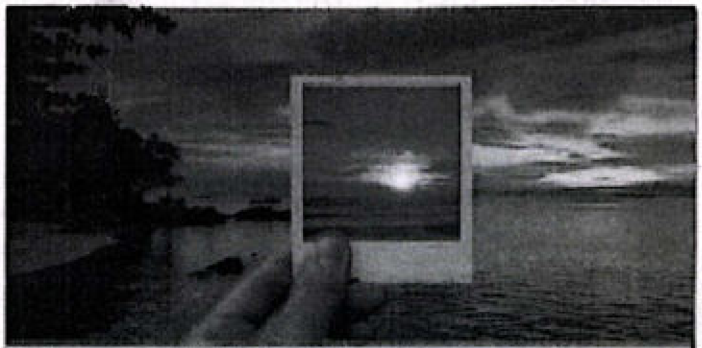


Figure 9.11 (d): Polaroids can be used in Sky Photography.

**Polaroids in Stress Analysis:** In plastic industry, polaroid is used to perform stress analysis tests on transparent plastics. As light passes through a plastic, each color of visible light is polarized with its own direction. If such a plastic is placed between two polarizing plates, a colorful pattern is observed. As the top plate is turned, the color pattern changes as new colors become blocked and the formerly blocked colors are transmitted. Structural stress in plastic is signified at locations where there is a large concentration of colored bands. This location of stress is usually the location where structural failure will most probably occur.

Polaroids are also used in the entertainment industry to display 3-D movies.

**Example 9.5: An Unpolarized light having intensity of = 16 W $m^{-2}$ is incident on a pair of polarizers. The first polaroid filter has its transmission axis at 50° from the vertical. The second polaroid filter has its transmission axis at 20° from the vertical. Calculate the intensity of the light transmitted through both the filters.**

Given: Intensity unpolarized light = $I_o$ = 16 W $m^{-2}$

Transmission axis of 1st polarizer from the vertical = 50°

Transmission axis of 2nd polarizer from the vertical = 20°

To Find: Intensity of the light transmitted = I = ?

Solution: First, we calculate the intensity of the light when it emerges from the first polarizer. As the light is unpolarized, so

$$I_1 = I_o / 2 = 16/2 = 8 \text{ W m}^{-2}$$

Now we calculate the intensity of the light when it emerges from the second polaroid filter. As, when it emerges from the first polaroid filter the light is linearly polarized at 50°. The angle between this light and the transmission axis of the second polaroid filter is 50°- 20° = 30°

Therefore, $$I_2 = I_1 \cos^2 30° = 8 \times (0.866)^2 = 8 \times 0.75 = 6 \text{ W m}^{-2}$$

**Assignment 9.4**

**What angle is required between the direction of polarized light and the axis of a polaroid filter to reduce its intensity by 90% ?**

## 9.9 DIFFRACTION

The bending of waves around the sharp edges or corners of an obstacles (or slits) and spreading into its geometrical shadow is called diffraction.

To observe the diffraction effect in water, we generate plane waves of water in a ripple tank and place two obstacles in line in such a way that separation between them is equal to the wavelength of water waves, as shown in Fig. 9.12. After passing through the small slit between the two obstacles, the water waves will spread around the slit and change into almost semicircular pattern (Fig. 9.12 a).

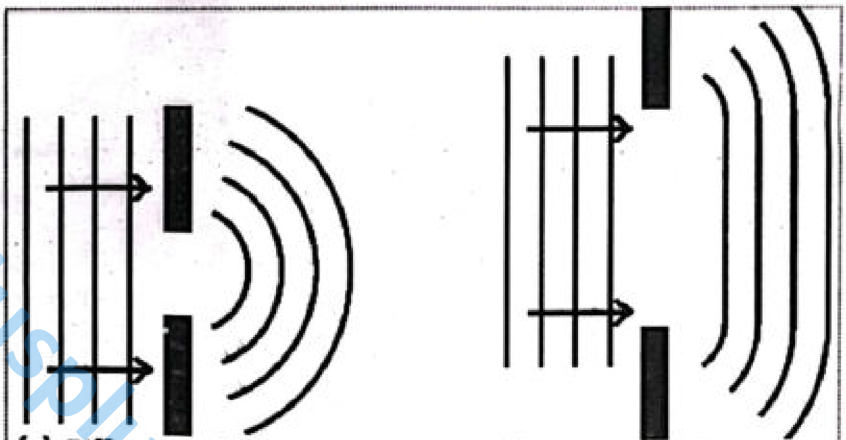



Figure 9.12: For observable diffraction separation between obstacles must be equal to the wavelength.

Diffraction of waves can only be observed clearly if the size of the obstacle is comparable with the wavelength of the wave. If the size of the obstacle is larger than the wavelength of the wave, then only a small diffraction occurs near the corners of the obstacle, as shown in Fig. 9.12 (b).

Diffraction can also be observed in light waves, when a beam of monochromatic light passes through a narrow slit or beam of monochromatic light passes over a knife edge. The diffraction of light can be observed only if size of the opening or obstacle is small enough to be comparable with wavelength of light used.

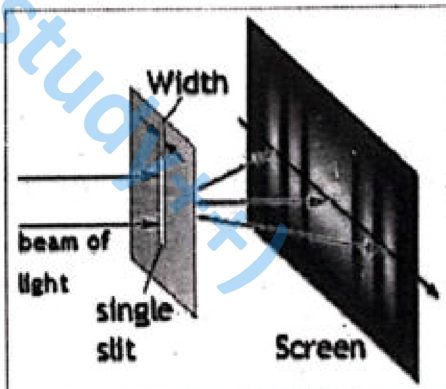



Figure 9.13: Diffraction of light.

Consider a beam of light passing through a single slit and falls on the photographic film, as shown in Fig. 9.13. The central bright band (also called fringe) is of high intensity and very wider than the slit and other surrounding bands. These bands are the result of interference effect.

## 9.10 GRAVITATIONAL WAVE

Albert Einstein predicted the existence of gravitational waves in 1916.

A gravitational wave is a stretch and compress of space and so can be found by measuring the change in length between two objects.

Gravitational waves are actually the ripples in spacetime. When objects move, the curvature of spacetime changes and these changes move away (like ripples on water surface) as gravitational waves.

> **For Your Information**
> Spacetime is a mathematical model that fuses the three dimensions of space and the one dimension of time into a single four-dimensional continuum.

Every physical object, that accelerates, produces gravitational waves. This includes humans, vehicles, airplanes etc. But the masses and accelerations of objects on Earth are too much small to make gravitational waves big enough to be detected with our instruments. To find big enough gravitational waves, we have to see far away outside of our solar system. The Universe is filled with extremely massive objects undergoing quick accelerations that generate gravitational waves which can be detected.

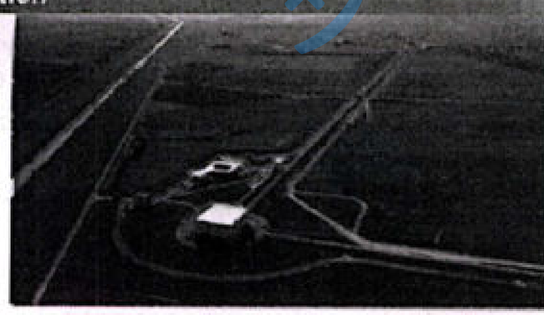
Examples of some events that could cause a gravitational wave are:

- When a star explodes asymmetrically, called a supernova.
- When two big stars orbit each other.
- When two black holes orbit each other and collide to merge.

### 9.10.1 Interferometer

Interferometer are tools used for investigation in many fields of science and engineering. They are called interferometers because they work on interference of two or more light to create an interference pattern. This pattern can be measured and analyzed. The interference patterns generated by interferometers contain information about the object or phenomenon being studied. They are often used to make very small measurements that are not possible by any other way. That is why they are so powerful for detecting gravitational waves. LIGO's interferometers are designed to measure a distance of 1/10,000th the width of a proton, small enough!

> **For Your Information**



The Laser Interferometer Gravitational-Wave Observatory (LIGO) is a large-scale physics experiment and observatory designed to detect gravitational waves and to develop gravitational-wave observations as an astronomical tool. This project is a collaboration between USA, India, Germany, Australia and U.K. LIGO consists of two instruments called interferometers, each with two 4 km (2.5 mile) long arms arranged in the shape of an "L". The interferometers act as 'antennae' to detect gravitational waves. Gravitational waves cause space itself to stretch in one direction and simultaneously compress in a perpendicular direction. In LIGO, this causes one arm of the interferometer to get longer while the other gets shorter, then vice versa, back and forth as long as the wave is passing. Since the arms are simultaneously changing lengths in opposing ways. This effect is measured by laser beam in interferometer. Hence gravitational waves are detected.

## SUMMARY

- **Superposition of waves** occur two or more waves are passing through the same region at the same time, the total displacement at the point where they interact, is equal to the vector sum of the individual displacements due to each pulse at that point.
- The effect produced due to the superposition of waves from two coherent sources is known as **interference.**
- When two waves meet at a point in the same phase then two waves reinforce each other. This is called **constructive interference.**
- When two waves meet at a point out of phase (180°) then two waves cancel each other. This is called **destructive interference.**
- When two waves of slightly different frequencies are played simultaneously then periodic alternations of sound between maximum and minimum loudness are produced which are called **Beats.**
- The number of beats generated per second is called **beats frequency.**
- When two identical waves having same speed, amplitude and frequency traveling in opposite direction superpose then a wave obtain is called **stationary wave.**
- **The distance between two successive nodes or anti-nodes** is equal to $\lambda/2$. The distance between adjacent node and anti-node is equal to $\lambda/4$.
- The apparent change in the frequency of sound due to the relative motion between the listener and source of sound is called **Doppler's effect.**
- **Coherent Light:** Light made up of waves with the same wavelength that are in phase with each other.
- **Polarized light waves** are light waves in which the vibrations occur in a single plane.
- The process of transforming un-polarized light into polarized light is known as **polarization.**

## EXERCISE

### Multiple Choice Questions

**Encircle the Correct option.**

1) On doubling amplitude of the wave, intensity is increased:

A. 1/2 times B. 2 times C. 4 times D. 3 times

2) Two waves, each with amplitude of 0.5 m are superimposed with constructive interference such that they are in phase. What is the resultant amplitude?

A. 0.25 m B. 0.5 m C. 0 m D. 1 m

3) What happens when two sound waves of frequencies differing by more than 10 Hz reach our ear simultaneously?

A. beats are not produced. B. the waves destroy each other's effect.

C. interference of sound does not take place.

D. beats are produced but cannot be heard by human ear.

4) A car has two horns; one is emitting a frequency of 199 Hz and the other is emitting a frequency of 203 Hz. What beat frequency do they produce?

A. 4 Hz B. 199 Hz C. 201 Hz D .203 Hz

5) Which of the following frequencies are higher harmonics of a string with fundamental frequency of 150 Hz?

A. 200 Hz, 300 Hz B. 300 Hz, 600 Hz C. 250 Hz, 450 Hz D. 250 Hz, 500 Hz

6) What is the wavelength of the third harmonic (n=3) of a standing wave established on a string of length 3 m fixed at both ends?

A. 1 m B. 1.5 m C. 2 m D. 3 m

7) A node is a point located along the medium where there is always ____.

A. a double crest. B. constructive interference.

C. destructive interference . D. a double rarefaction.

8) Which phenomenon is produced when two or more waves passing simultaneously through the same medium meet up with one another?

A. refraction B. diffraction C. interference D. reflection

9) A standing wave is formed by waves of frequency 256 Hz. The speed of the waves is 128 m/s. The distance between the nodes must be:

A. 2.00 m B. 1.00 m C. 0.500 m D. 0.250 m

10) An air column that is closed at one end is used to determine the speed of sound. The frequency of the tuning fork used is 329.6 Hz. The length of the shortest air column producing the resonance is 25.0 cm. The speed of the sound must be:

A. 380.6 $m\ s^{-1}$ B. 282.4 $m\ s^{-1}$ C. $3.30 \times 10^{2}\ m\ s^{-1}$ D. $3.50 \times 10^{2}\ m\ s^{-1}$

11) The intensity of beam observed through polarizer:

A. is increased B. is decreased C. is always zero D. remains unchanged

## Short Questions

Give short answers of the following questions.

9.1 Two identical waves undergo pure constructive interference. Is the resultant intensity twice that of the individual waves? Explain your answer.

9.2 Circular water waves decrease in amplitude as they move away from where a rock is dropped. Explain why.

9.3 Differentiate between constructive and destructive interference?

9.4 The frequency of a stretched string depends on its length. Give two other factors that affect the frequency of a stretched string.

9.5 The pitch of the sound emitted by the siren of a moving fire engine appears to change as it passes a stationary observer. (i) Name this phenomenon. (ii) Will the crew in the fire engine notice this phenomenon? Give a reason for your answer. (iii) Give an application of this phenomenon.

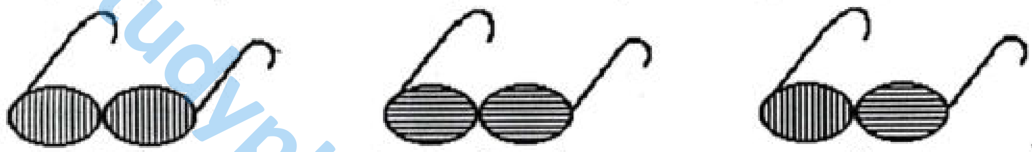
9.6 How should a sound source move with respect to an observer so that the frequency of its sound does not change?

9.7 Can sound waves be polarized? Explain.

9.8 Suppose that light passes through two Polaroid filters whose polarization axis are parallel to each other. What would be the result?

9.9 Give any three examples of a gravitational wave?

9.10 Consider the three pairs of sunglasses below. Which pair of the glasses is capable of eliminating the glare resulting from sunlight reflecting off the calm waters of a lake? Explain you option. (The polarization axes are shown by the straight lines)



A B C

## Comprehensive Questions

Answer the following questions in detail.

9.1 What is superposition principle? Discuss.

9.2 What is interference? Discuss its types and corresponding conditions.

9.3 What are stationary waves? How stationary wave is formed? Discuss.

9.4 Discuss the formation of stationary waves in (a) stretched string (b) open pipe (c) closed pipe.

9.5 What are beats? Explain.

9.6 Explain the use of polaroids in (a) sky photography (b) stress analysis of materials.

9.7 What is meant by the Doppler's effect? Discuss its cases. Also give some application of the Doppler's effect.

9.8 Describe an experiment to demonstrate the interference of sound.

9.9 What is polarization? How is plane polarized light produced and detected?

9.10 What is meant by the term 'Gravitational Waves'? Explain in detail.

9.11 Explain the use of interferometer in detecting gravitational waves.

## Numerical Problems

9.1 Ultrasound of intensity $1.50 \times 10^2$ W m$^{-2}$ is produced by the rectangular head of a medical imaging device measuring 3.00 cm by 5.00 cm. What is its power output? **(Ans: 0.225 W)**

9.2 Two tuning forks of frequencies 440 Hz and 437 Hz are sounded together. How many beats will be heard over a period of 10 seconds? **(Ans: 30 beats)**

9.3 Suppose that a string is 1.2 m long and vibrates in first, second and third harmonic standing wave patterns. Determine the wavelength of the waves for each of the three patterns.

**(Ans: 2.4 m, 1.2 m, 0.8 m)**

9.4 The string is 6.0 m long and is vibrating at the third harmonic. The string vibrates up and down with 45 complete vibrational cycles in 10 seconds. Determine the frequency, period, wavelength and speed for this wave. **(Ans: 4.5 Hz, 0.22 sec, 4.0 m, 18 m $s^{-1}$)**

9.5 An air column closed at one end resonates at the second maximum or the second resonant length. The frequency of the sound wave is 1024 Hz. The air temperature is 18.6°C.

a) Draw a diagram of the displacement wave pattern inside the column.

b) Calculate the speed of sound in air.

c) Calculate the wavelength.

d) Calculate the length of the closed air column. **(Ans: 343 m $s^{-1}$, 0.335 m, 0.251 m)**

9.6 Suppose a train that has a 150 Hz horn is moving at 35.0 m $s^{-1}$ in still air on a day when the speed of sound is 340 m $s^{-1}$. What frequencies are observed by a stationary person at the side of the tracks

(a) as the train approaches. (b) after it passes.

(c) What frequency is observed by the train's engineer traveling on the train?

**(Ans: 167 Hz, 136 Hz, same frequency as emitted by the horn)**

9.7 A polarized light of intensity $I_o$ is passed through another polarizer whose axis makes an angle of 60° with the axis of the polaroid filter. What is the intensity of transmitted polarized light from second polaroid filter? **(Ans: $I_o/4$)**

UNIT 10

# ELECTROSTATICS



## Student Learning Outcomes (SLOs)

**The students will:**

- state that an electric field is an example of a field of force.
- Define and calculate electric field strength [Use F = q E for the force on a charge in an electric field. Use E = ΔV/Δr to calculate the field strength of the uniform field between charged parallel plates].
- Represent an electric field by means of field lines.
- Describe the effect of a uniform electric field on the motion of charged particles.
- state that, for a point outside a spherical conductor, the charge on the sphere may be considered to be a point charge at its center.
- Explain how a Faraday cage works [by inducing internal electric fields that work to shield the inside from the influence of external electric fields].
- State and apply Coulomb's law [$F = k\frac{q_1 q_2}{r^2}$ for the force between two point charges in free space, where $k = \frac{1}{4\pi\varepsilon_0}$]
- Use $E = k\,Q/r^2$ for the electric field strength due to a point charge in free space.
- Describe how ferrofluids work [they make use of temporary soft magnetic materials suspended in liquids to develop fluids that react to the poles of a magnet and have many applications in fields such as electronics].

We know that all matter is made up of atoms. These atoms are composed of electrons, protons, and neutrons. Each proton has one unit of positive charge and each electron has one unit of negative charge. A neutron has no charge. Protons and neutrons are tightly packed in the nucleus while electrons can be thought of as small charged clouds that surround the nucleus of atoms. An atom normally has an equal number of electrons and protons making it electrically neutral. If an electron is removed from an atom, then the atom acquires a net positive charge. If an extra electron is added to an atom, it acquires a net negative charge. In the cover picture, it is shown that when we rub a comb in our hairs, the comb gets charged. When this charged comb is brought to the pieces of papers, these pieces get attracted due to induction. The study of electric charges at rest and the forces between them is referred to as electrostatics.

In this chapter, we investigate electric forces and discuss about the various phenomena associated with the positive and negative charges.

## 10.1 COULOMB'S LAW

A French physicist Charles Coulomb (1736-1806) performed experiments for the electrostatic force and proposed a formula to calculate it. The mathematical formula for the electrostatic force is called Coulomb's law. This law is defined as:

**The force of attraction or repulsion between two charged bodies is directly proportional to the product of magnitude of charges and inversely proportional to the square of distance between them.**

If we consider two point charges $q_1$ and $q_2$ separated by distance r, as shown in Fig. 10.1. According to Coulomb's law, the magnitude of the force F is given by:

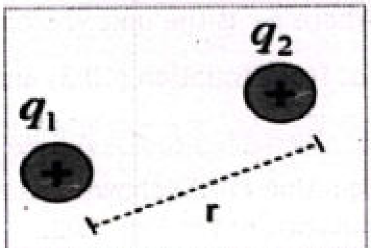



**Figure 10.1:** Two similar charges separated by a distance r.

$$F \propto q_1 q_2$$

$$F \propto \frac{1}{r^2}$$

By combining above two relations, we get:

$$F \propto \frac{q_1 q_2}{r^2}$$

or $$F = k\frac{q_1 q_2}{r^2} \qquad \text{\_\_\_\_\_\_ (10.1)}$$

Here $k$ is the constant of proportionality, known as Coulomb's constant. It is typically expressed in terms of permittivity of fee space $\varepsilon_o$. i.e. $k = \frac{1}{4\pi\varepsilon_o}$

Here $\varepsilon_o = 8.85\times10^{-12}\ C^2\ N^{-1}\ m^{-2}$.

So, $$F = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r^2} \qquad \text{\_\_\_\_\_\_ (10.2)}$$

This is mathematical form of coulomb's law. The equation (10.2) represents the force between the two charges when there is vacuum between them.

The value of $k$ depends upon system of units used and medium between the charges. In SI system, $k = 9 \times 10^9\ N\ m^2\ C^{-2}$. This value of k is applicable if there is vacuum between the charges. Coulomb's law is applicable to point charges and uniform spherical charge distribution. A point charge is an imaginary charge located at a single point in space. An electron can be considered a point charge.

**A charged sphere also behaves as a point charge for a point outside the spherical conductor.** This is due to the reason that the electric field at any point outside a spherical charge distribution is the same as the field that would be produced by a point charge located at its center. This is true only if the spherical conductor is isolated. If another charged object is brought near the spherical conductor, it will induce surface charges in the spherical conductor that are not spherically symmetric, and therefore, we can no longer treat it as a point charge.

When using Coulomb's force law, remember that force is a vector quantity and must be treated accordingly. Coulomb's law in vector form for the electric force exerted on a charge $q_1$ by a second charge $q_2$, is written as:

$$\boldsymbol{F}_{12} = k\frac{q_1 q_2}{r^2}\hat{r}_{12} \qquad \text{\_\_\_\_\_\_ (10.3)}$$

Where, $\hat{r}_{12}$ is unit vector in the direction of $\boldsymbol{F}_{12}$. Similarly, the electric force exerted on a charge $q_2$ by a charge $q_1$, is written as:

$$\boldsymbol{F}_{21} = k\frac{q_1 q_2}{r^2}\hat{r}_{21} \qquad \text{\_\_\_\_\_\_ (10.4)}$$

Where $\hat{r}_{21}$ is the unit vector in the direction of $\boldsymbol{F}_{21}$. Since $\hat{r}_{12} = -\hat{r}_{21}$

So, from equation (10.3) and (10.4), we can write:

$$\boldsymbol{F}_{21} = -\boldsymbol{F}_{12} \qquad \text{\_\_\_\_\_\_ (10.5)}$$

Equation (10.5) shows that two forces are equal in magnitude but opposite in direction which is illustrated in Fig. 10.2.

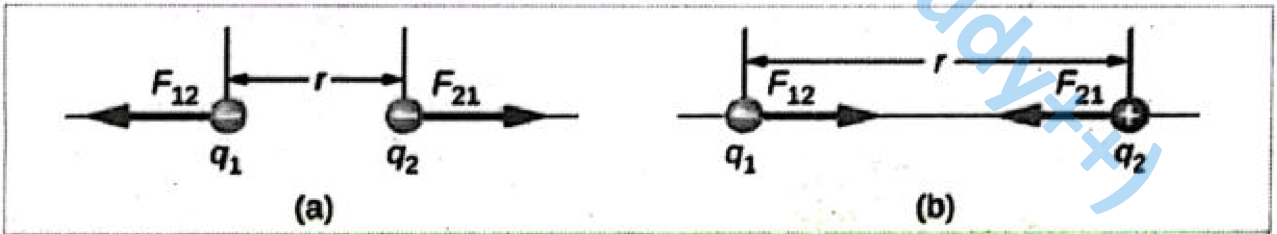



Figure 10.2: (a) Charges with the same sign repel each other. (b) Charges with opposite signs attract each other

For like charges, the product $q_1 q_2$ will be positive resulting in a force of repulsion between these two charges. For unlike charges, the product $q_1 q_2$ will be negative resulting in a force of attraction between these two charges.

Like other forces, electric forces obey Newton's third law; hence, the forces $F_{12}$ and $F_{21}$ are equal in magnitude but opposite in direction.

A comparison of coulomb's force and gravitational force shows that both act at a distance without direct contact, such force is called field force. Both are inversely proportional to the distance squared, with the force directed along a line connecting the two bodies. The mathematical form is the same, with the masses $m_1$ and $m_2$ in Newton's law replaced by charges $q_1$ and $q_2$ in Coulomb's law and with Newton's constant G replaced by Coulomb's constant k. There are two important differences: electric forces can be either attractive or repulsive, whereas gravitational forces are always attractive. Additionally, the electric force between charged elementary particles is far stronger than the gravitational force between the same particles.

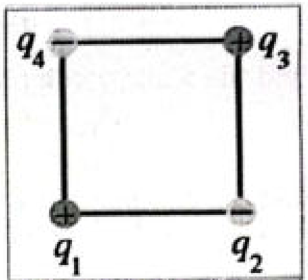



Figure 10.2 (c): Four charges $q_1$, $q_2$, $q_3$ and $q_4$ placed near each other.

The Superposition Principle of Force: When we have number of charges in a certain region, then each charge experiences a net force due to all other charges placed at suitable distance. These electric forces can all be computed separately, one at a time, then added as vectors. This is an example of the superposition principle.

For example, if four charges $q_1$, $q_2$, $q_3$ and $q_4$ are placed near each other, as shown in Fig. 10.2 (c), the resultant force exerted by charges $q_2$, $q_3$ and $q_4$ on charge $q_1$ is:

$$F = F_{12} + F_{13} + F_{14}$$

The following example illustrates this procedure in two dimensions.

**Example 10.1: Three point charges are located at the corners of a right triangle, as shown in figure 10.3, where $q_1 = q_3 = 5.00\ \mu C$, $q_2 = -2.00\ \mu C$, and a = 0.100 m. Find the resultant force exerted on $q_3$.**

Given: $q_1 = q_3 = 5.00\ \mu C$, $q_2 = -2.00\ \mu C$, a = 0.100 m.

To Find: $F_3$ = ?

Solution: As charge $q_3$ is near two other charges, it will experience two electric forces. These forces are exerted in different directions, as shown in Fig. 10.3. Because two forces are exerted on charge $q_3$, we solve this example by using superposition principle.

The force $F_{32}$ exerted on $q_3$ by $q_2$ is attractive because $q_2$ and $q_3$ have opposite signs. In the coordinate system, the attractive force $F_{32}$ is to the left (in the negative x-direction). The force $F_{31}$ exerted on $q_3$ by $q_1$ is repulsive because both charges are positive. The repulsive force $F_{31}$ makes an angle of 45° with the x-axis.

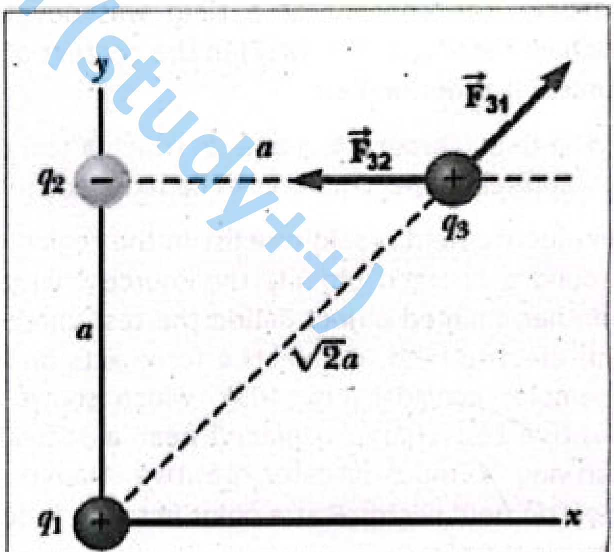



Figure 10.3: Three point charges located at the corners of a right triangle.

Using Coulomb's law to find the magnitude of $F_{32}$

$$F_{32} = \frac{1}{4\pi\varepsilon_o}\frac{q_2 q_3}{r^2}$$

$$F_{32} = 9\times10^{9}\frac{(2\times10^{-6})(5\times10^{-6})}{a^2}$$

$$= 9\times10^{9}\frac{(2\times10^{-6})(5\times10^{-6})}{(0.100)^2} = 8.99\text{ N}$$

To find the magnitude of $F_{31}$ is

$$F_{31} = \frac{1}{4\pi\varepsilon_o}\frac{q_3q_1}{r^2}$$

$$F_{31} = 9\times10^{9}\frac{(5\times10^{-6})(5\times10^{-6})}{(\sqrt{2}a)^2}$$

$$=9\times10^{9}\frac{(2\times10^{-6})(5\times10^{-6})}{2(0.100)^2} = 11.2\text{ N}$$

Find the x and y-components of the force $F_{31}$:

$$F_{31x} = F_{31}\cos 45° = 7.94\text{ N}$$

$$F_{31y} = F_{31}\sin 45° = 7.94\text{ N}$$

Find the components of the resultant force acting on $q_3$:

$$F_{3x} = F_{31x} + F_{32x} = 7.94\text{ N} + (-8.99\text{ N}) = -1.04\text{ N}$$

$$F_{3y} = F_{31y} + F_{32y} = 7.94\text{ N} + 0 = 7.94\text{ N}$$

In vector form, the resultant force acting on $q_3$ is:

$$F_3 = (-1.04\,i + 7.94\,j)\text{ N}$$

**Assignment 10.1**

**What is the magnitude of the force of attraction between an iron nucleus bearing charge q=26e and its innermost electron, if the distance between them is $1\times10^{-12}$ m.**

## 10.2 ELECTRIC FIELD INTENSITY

An electric field, like other fields (e.g., gravitational or magnetic), is a vector field that surrounds the charged object. Electric fields are found around electric charges and help determine the direction and magnitude of force the charge exerts on a nearby charged particle. The concept of a field was developed by Michael Faraday (1791-1867) in the context of electric forces. It is defined as:

> The region around a charge in which a test charge can feel an electric force is called electric field.

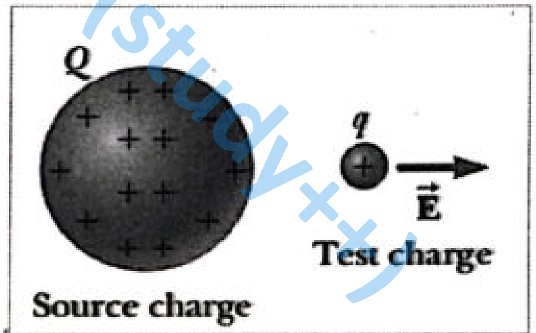



**Figure 10.4:** A small positive test charge q placed near an object carrying a much larger positive charge Q, experiences an electric field $\vec{E}$ established by the source charge Q. We always assume that the test charge is so small that the field of the source charge is unaffected by its presence.

An electric field is said to exist in the region of space around a charged object, the source charge. When another charged object called the test charge enters this electric field, an electric force acts on it. As an example, consider Fig. 10.4, which shows a small positive test charge q placed near a second object carrying a much greater positive charge Q. The electric field vector $E$ at a point in space is defined as the electric force $F$ acting on a positive test charge q placed at that point, divided by the magnitude of test charge. If test charge q is moved away from the source charge Q, the force will decrease till a certain distance, and after that the force will practically reduce to zero. This is because of the fact that now q is not in the field of Q.

Electric field intensity $\boldsymbol{E}$ is a single vector quantity containing information about the field strength and its direction at a point. The strength of the field at a point in space determines the amount of force that a charge will experience if it is placed at that point. The direction in which this unit positive test charge will move or tends to move is the direction of the electric field. The test charge is so small that it does not distort the original field due to the primary source.

**The force per unit positive test charge at any point is called electric field intensity at that point.**

If $\boldsymbol{F}$ is the force experienced by a test charge q placed inside the field, then the value of electric intensity $\vec{E}$ at that point is given as:

$$\boldsymbol{E} = \frac{\boldsymbol{F}}{q} \qquad \text{\_\_\_\_\_\_ (10.6)}$$

If we use coulomb's law, $\boldsymbol{F} = k\frac{Qq}{r^2}\hat{r}$, then the above equation appears as:

$$\boldsymbol{E} = k\frac{Q}{r^2}\hat{r} \qquad \text{\_\_\_\_\_\_ (10.7)}$$

The SI unit of $E$ is newton per coulomb ($N\,C^{-1}$) or volt per metre ($V\,m^{-1}$). The strength of the field is proportional to the magnitude of the source charge. Its strength decreases as the test charge moves away from source charge. $\boldsymbol{E}$ is always directed from positive to negative charges.

**Example 10.2: Determine electric field at the surface of a sphere of radius 3.0 m if a point charge of 9 μC is placed at the centre.**

Given: Electric field = E = ? Radius of sphere = r = 3.0 m

Point charge at the centre = q = 9 μC = $9\times10^{-6}$ C

To Find: Electric field intensity = ?

Solution: Using the formula for electric field intensity:

$$E = k\frac{q}{r^2}$$

By putting the above values in the equation, we get:

$$E = 9\times10^9 \times \frac{9\times10^{-6}}{(3)^2} = 9\times10^{9-6}$$

$$E = 9\times10^3\ N\,C^{-1}$$

**Assignment 10.2**

**A conducting sphere carries a charge of 200 μC. Find the electric field intensity at a distance 60 cm from the centre of the sphere.**

## 10.3 ELECTRIC FIELD LINES

A way of visualizing electric field patterns is to draw lines, called electric field lines. Electric field lines were first introduced by Faraday.

The *direction* and *intensity* of electric field in the vicinity of a charge body can be represented by drawing imaginary lines called electric lines of force.

It is important to remember that electric fields are three-dimensional. Two-dimensional representation of electric field lines for the field due to a single positive point charge is shown in Fig. 10.5. This two-dimensional drawing shows only the field lines that lie in the plane containing the point charge.

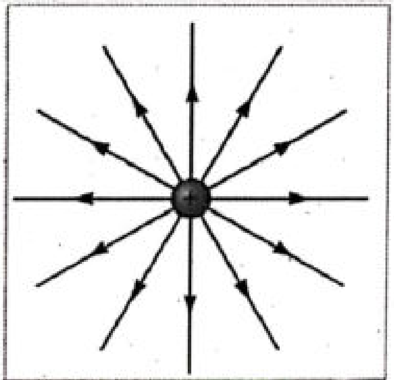


**Figure 10.5:** Electric field lines of isolated positive charge in 2-dimension.

The electric field lines are actually directed radially outward from positive charge in all directions; therefore, instead of the flat "wheel" of lines shown, you should picture an entire spherical distribution of lines.

The electric field lines representing the field due to a single negative point charge are directed toward the charge (Fig. 10.6).

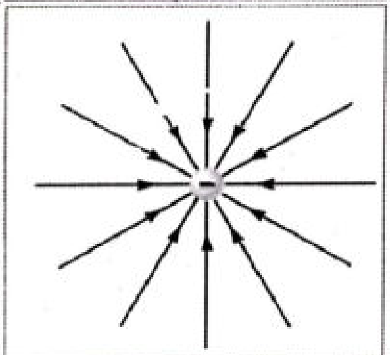


**Figure 10.6:** Electric field lines of isolated negative charge in 2-dimension.

In either case, the lines are along the radial direction and extend all the way to infinity. Notice that the lines become closer as they approach the charge, indicating that the strength of the field increases as we move towards the source charge.

Figure 10.7 shows the symmetric electric field lines for two point charges of equal magnitude but opposite sign. This charge configuration is called an electric dipole. Note that the field lines start from positive charge and ends at negative charge. The high density of lines between the charges indicates a strong electric field in this region. Thus, field is stronger in the region between the charges because the resultant intensity is equal to the sum of the intensities due to positive and negative charges. The number of lines that begin at the positive charge must equal the number of lines that terminate at the negative charge.

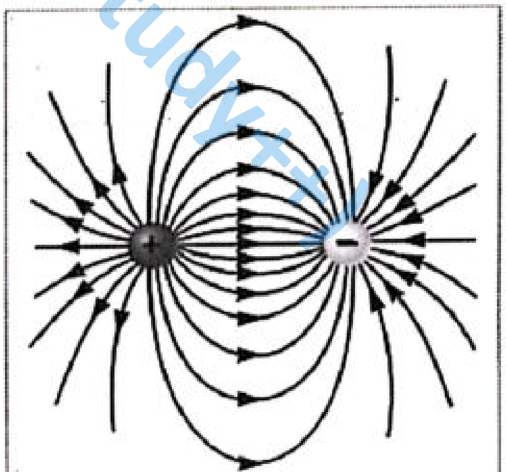


**Figure 10.7:** Two-dimensional drawing of electric field lines of two opposite charges placed near each other.

Figure 10.8 shows the field lines of two positive charges. At some points, situated between the two charges, the resultant intensity is zero, which creates a field free region. Two negative charges also produce the fields very similar to the field produced by two positive charges, except that the directions are reversed.

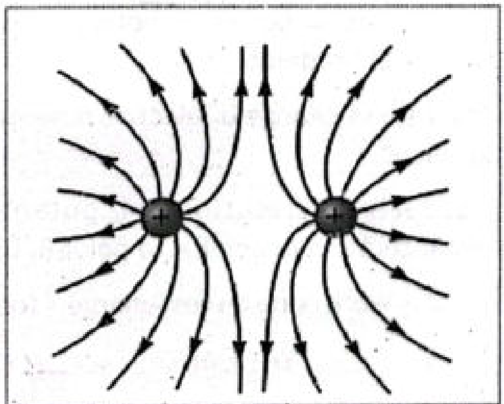


Figure 10.8: Two-dimensional drawing of electric field lines of two similar charges placed near each other.

The electric lines of force between two oppositely charged parallel plates separated by a small distance is shown in Fig. 10.9. One plate has a uniform positive charge distribution while the other has a uniform negative charge distribution. The field lines start from the positive plate and end at negative plate. The field between the plates is uniform in strength. If the plates have finite length, then field lines at the ends of plates will curve, as shown in Fig. 10.9. Such curved field at the ends of plates is called "fringing field". Fringing field shows that field is not uniform at the ends.

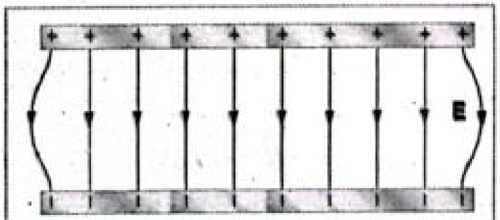



Figure 10.9: Fringing field at the ends of oppositely charged plates.

From the above discussion, we conclude that:

- The field lines never intersect each other.
- To represent field lines starting or ending at infinity, a single charge must be used.
- Electric lines of force originate from a positive charge and terminate at a negative charge.
- The tangent drawn to any point on a field line indicates the direction of electric intensity at that point. The arrows indicate the direction of electric field at various points.
- The intensity of electric field is proportional to the number of electric field lines per unit area.
- Electric lines of force are close in the region where field is strong and far apart in the weaker regions.
- The electric lines of force have the tendency to contract in length. This explains attraction between oppositely charged bodies.

## 10.4 POTENTIAL GRADIENT

The potential gradient represents the rate of change of electric potential with respect to displacement. In other words, it represents the slope along which potential is changing.

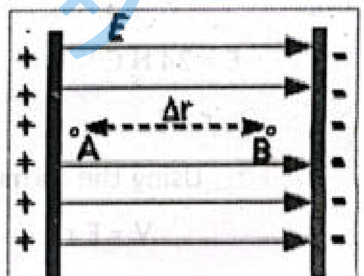



Figure 10.10 (a): Potential difference between two points A and B.

Potential difference between two points is the difference between the electric potential at points A and the electric potential at point B, as shown in Fig. 10.10 (a). If $V_A$ and $V_B$ is measured potential at these two points, then $V_A - V_B$ is the potential difference. If $\Delta V$ is the change in potential between two points separated by a distance $\Delta r$, then, $\Delta V/\Delta r$ will be potential gradient.

**The rate of change of electric potential $\Delta V$ with respect to displacement $\Delta r$ is known as potential gradient.**

To derive a relation for potential gradient, consider a test charge q moving a small distance $\Delta r$ from point A to point B. Using definition of work:

Work done on test charge = force acting on test charge× displacement moved by test charge

$$W = F\,\Delta r \qquad \text{(10.8)}$$

For a charge q is in an electric field E, force is given by F = q E.

So, equation (10.8) becomes:

$$W = q\,E\,\Delta r \qquad \text{(10.9)}$$

Also, the work done on the test charge q moving through potential difference $\Delta V$ is equal to the decrease of electric potential energy, i.e.,

$$W = -\,q\,\Delta V \qquad \text{(10.10)}$$

Negative sign indicates that the work done on q is against field force. Comparing equation (10.9) and equation (10.10), we get:

$$q\,E\,\Delta r = -\,q\,\Delta V$$

$$E\,\Delta r = -\,\Delta V$$

$$E = -\frac{\Delta V}{\Delta r} \qquad \text{(10.11)}$$

**Potential Gradient in Electrical Engineering and Technology**

During the manufacturing of cables, the value of dielectric insulation provided is kept higher than the potential gradient of the conductor, else the cable is not safe. If the value of potential gradient is kept high in power systems, then it may affect the person on touching.

Equation (10.11) shows that strength of the electric field E is equal to the potential gradient $\Delta V/\Delta r$. The negative sign indicates that the electric potential increases in the opposite direction of the electric field vector. Also, equation (10.11) shows that the unit of electric field E is volt per metre.

**Example 10.3: The electric field at a point due to a point charge is 24 N $C^{-1}$ and the electric potential at that point is 6 J $C^{-1}$. Calculate the distance of the point from the charge.**

Given: E = 24 N $C^{-1}$ V = 6 J $C^{-1}$

To Find: r = ?

Solution: Using the formula for potential gradient, we have:

$$V = E\,r$$

or $$r = V/E$$

Putting values, we get:

$$r = 6/24 = 0.25 \text{ m}$$

**Assignment 10.3**

Show that: $\frac{\text{volt}}{\text{metre}} = \frac{\text{newton}}{\text{coulomb}}$

### 10.4.1 Motion of a Charged Particle in an Electric Field

When a charge +q is placed in an electric field E, as shown in the Fig. 10.10 (b), it experiences a force in the electric field E.

- If the charge is allowed to move freely in the electric field, it will move from positive plate to the negative plate and gain kinetic energy.
- If the charge is moved against the electric field, an external force (F = +q E) is required. This increases the potential energy of the charge. When the charge is released from its position, it will move back and gain an equivalent amount of K.E.

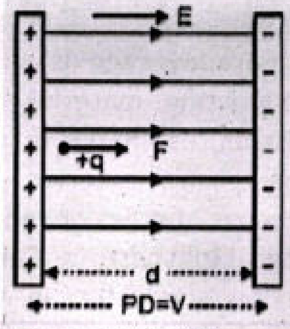



**Figure 10.10 (b):** Charged Particle in an electric field.

## 10.5 FERROFLUID

A ferrofluid is a liquid which becomes highly magnetized in the presence of a magnetic field. Ferrofluid is composed of very small, nanometer-sized particles (diameter usually 10 nm or less) of magnetite, hematite or some other compound containing iron, and a liquid (usually oil). These particles are suspended in liquid. In the absence of magnetic field, ferrofluid acts like a liquid. The magnetite particles move freely in the fluid. In the presence of magnetic field, the particles are temporarily magnetized that react to the poles of a magnet. Removing the external field will lead to the disappearance of induced magnetic field.

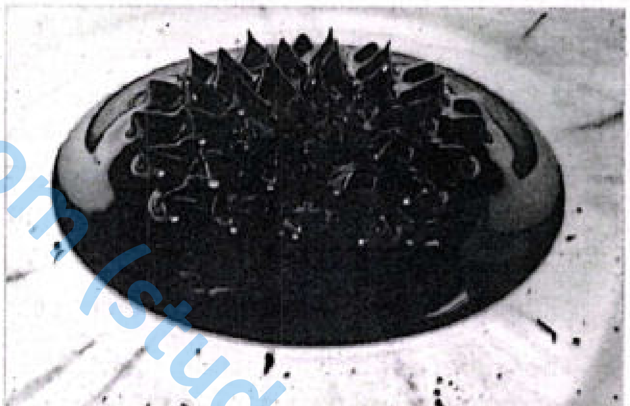


**Figure 10.11:** Magnified photograph of ferrofluid when influenced by a magnetic field.

Ferrofluids are colloidal suspensions of magnetic nanoparticles in a liquid.

The nano-particles are usually iron oxide ($Fe_3O_4$) and liquids are water or an organic solvent like kerosene.

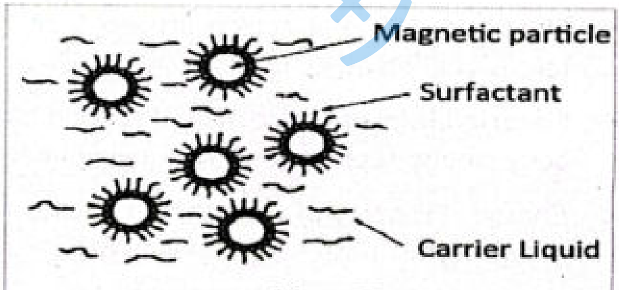
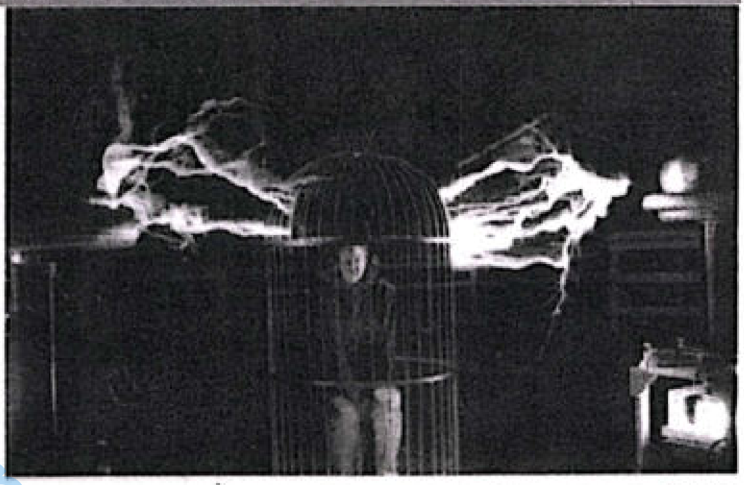



**Figure 10.12:** Microscopic view of a ferrofluid.

Ferrofluids are fascinating materials with a range of applications in different fields such as electronics, optics and medical physics. These materials are used in magnetic resonance imaging (MRI), bio-sensing, medical imaging, medicinal therapy, dynamic loudspeakers, magneto-optic sensors, heat transfer/dissipation and many more. In the near future, ferrofluids may be used to carry medications to specific locations in the body.

**For Your Information**



A **Faraday cage** is a cage made of a conducting material. A Faraday cage distributes charge or radiation around the cage's exterior, it cancels out electric charges or radiation within the cage. In short, a Faraday cage is a hollow conductor, in which the charge remains on the external surface of the cage.

The fields within a conductor cancel out with any external fields, so the electric field within a conductor is zero. The Faraday cages act as big hollow conductors. We can put things into the Faraday cage to shield them from electrical fields.

## SUMMARY

- **Coulomb's law:** The force of attraction or repulsion between two charged bodies is directly proportional to the product of the magnitude of charges and inversely proportional to the square of distance between them.
- **Coulomb's law is applicable** to point charges and uniform spherical charge distribution.
- **The Superposition Principle of Force:** When a number of separate charges act on the charge of interest, each exerts an electric force. These electric forces can all be computed separately, one at a time, then added as vectors. This is known as the superposition principle of force.
- **Electric Field:** The region around a charge where a test charge experiences an electric force is called electric field.
- **Electric Lines of Force:** The direction and intensity of the electric field around a charged body can be represented by drawing imaginary lines called electric lines of force.
- **Charge:** Property of matter that causes a force when near another charge. Charge comes in two forms: positive and negative.
- **Potential Gradient:** The rate of change of electric potential $\Delta V$ with respect to displacement $\Delta r$ is known as potential gradient.
- **Ferrofluids:** Ferrofluids are colloidal suspensions of magnetic nanoparticles in a liquid. The

nano-particles are usually iron oxide ($Fe_3O_4$). The liquids are usually water or an organic solvent like kerosene.

# EXERCISE

## Multiple Choice Questions

**Encircle the Correct option.**

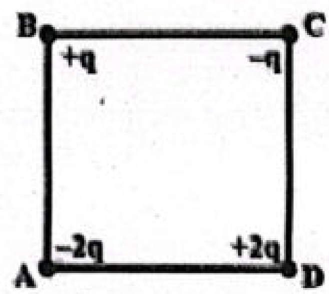
1) Which of the following is equivalent to 1 volt?

A. newton/second B. joule/second C. joule/coulomb D. coulomb/joule

2) Two positive charges are placed on a screen. Which statement describes the electric field produced by the charges?

A. It is constant everywhere. B. It is zero near each charge.
C. It is zero halfway between the charges. D. It is strongest halfway between the charges.

3) Four charges are placed at the corners of a square ABCD as shown. The force on a positive charge placed at the centre of the square is




A. 0
B. Along diagonal AC
C. Along diagonal BD
D. perpendicular to side AB

4) The bob of a pendulum with mass m, length l and a charge q is in the rest position in a uniform horizontal electric field of E. The tension in the string of the pendulum is:

A. mg B. qE C. $[(mg)^2 + (qE)^2]^{1/2}$ D. $[(mg)^2 + (qE)^2]^{1/4}$

5) A negatively charged particle is placed in a uniform electric field directed from South to North. In which direction will the particle move after it is released?

A. East B. South C. North D. North-West

6) If two points are at the same potential, are there any electric field lines connecting them?

A. yes B. No C. may or may not D. insufficient information.

7) Coulomb law is usually applied in the area of:

A. Magnetism B. Electrostatics C. Electromagnetism D. Gravitation

8) Let F be the force between two equal point charges at some distance. If the distance between them is doubled and individual charges are also doubled, what will be the force acting between the charges?

A. F B. 2F C. 4F D. $\frac{F}{2}$

## Short Questions

**Give short answers to the following question:**

10.1 When an air passenger touches the knob of the toilet door during a flight, he may get an electric shock. Why it is so?

10.2 How will the radius of a flexible ring change if it is given a positive charge?

10.3 Write some applications of electrostatics in real life.

10.4 What are the limitations of coulomb's law?
10.5 If two points are at the same potential, are there any electric field lines connecting them?
10.6 What is the difference between electric field and electric field strength?
10.7 Draw electric field lines for (a) an isolated positive charge (b) oppositely charged parallel plates.
10.8 How can we say that 'a charged sphere also behaves as a point charge for a point outside the spherical conductor'.
10.9 Describe the effect of a uniform electric field on the motion of charged particles.

## Comprehensive Questions

Answer the following questions in detail.

10.1 Define and explain Coulomb's law?
10.2 What is electric field strength? Explain with the help of examples.
10.3 What is meant by the term potential gradient? Show that $E = \Delta V / \Delta r$.
10.4 Describe the motion of a charged particle in a uniform electric field.
10.5 What are ferrofluids? Explain.
10.6 Explain how a Faraday cage works?
10.7 Describe the effect of a uniform electric field on the motion of charged particle.



## Numerical Problems

10.1 Charges of magnitude 100 microcoulomb each are located in vacuum at the corners A, B and C of an equilateral triangle measuring 4 meters on each side. If the charge at A and C are positive and the charge at B negative, what is the magnitude and direction of the total force on the charge at C? **(Ans:5.625 N, force is parallel to AB)**

10.2 What is the magnitude of a point charge that would create an electric field of 1.00 N $C^{-1}$ at a points 1.00 m away? **(Ans: $1.11 \times 10^{-10}$ C)**

10.3 Consider a point charge +q placed at the origin and another point charge -2q placed at a distance of 9 m from the charge +q. Determine the point between the two charges at which the electric potential is zero. **(Ans:3 m from +q )**

10.4 An object with a net charge of 24 μC is placed in a uniform electric field of 610 N $C^{-1}$ directed vertically. What is the mass of this object if it floats in the field? **(Ans:1.5 g )**

10.5 What will be the electric field at a distance of 30 cm from a 3 μC point charge.

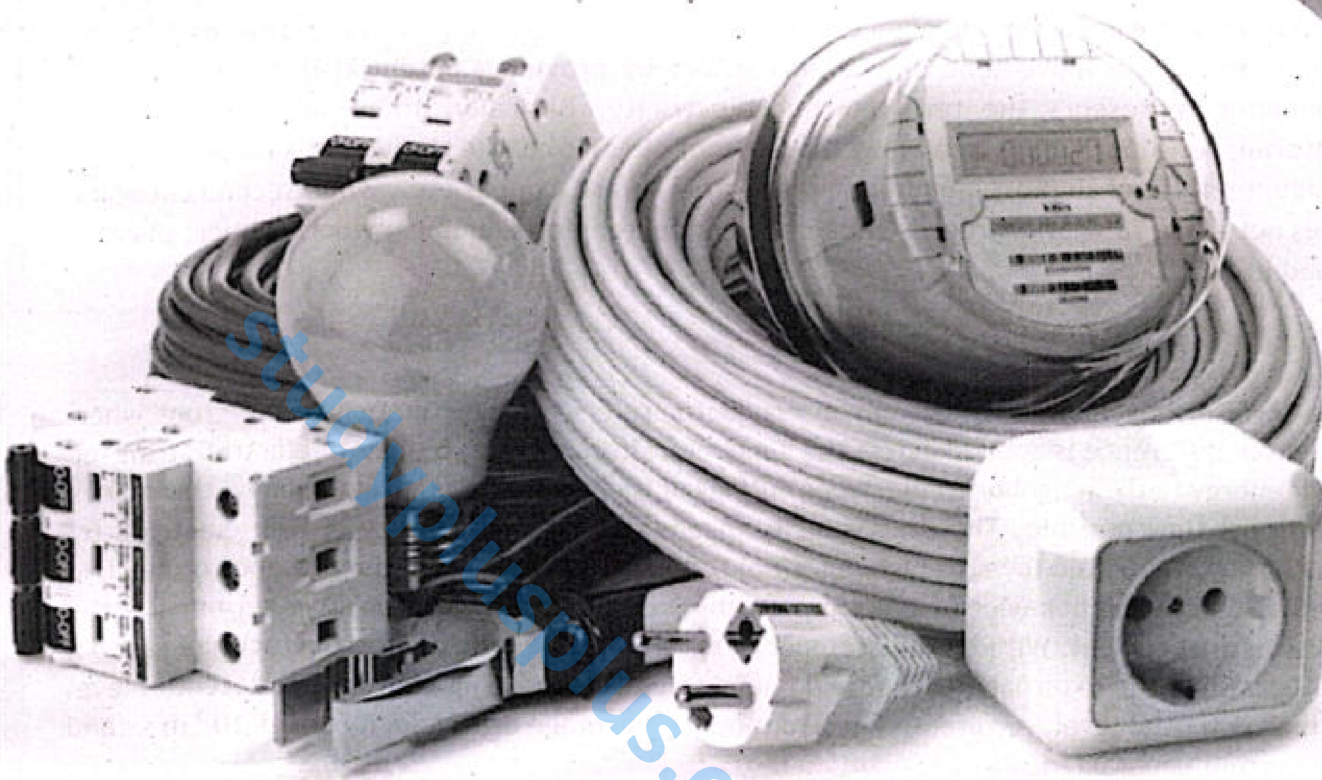
**(Ans: $3\times10^5$ N $C^{-1}$)**

10.6 The electric field at a point due to a point charge is 26 N $C^{-1}$ and the electric potential at that point is 13 J $C^{-1}$. Calculate the distance of the point from the charge. **(Ans: 0.5 m)**

10.7 The electric field at a point 0.5 m from a charge is 2.5 N $C^{-1}$. Find the value of electric potential at that point. **(Ans: 1.25 V)**

# ELECTRICITY





## Student Learning Outcomes (SLOs)

**The students will:**

- Use, for a current-carrying conductor, the expression $I = Anvq$ [where n is the number of charge carriers per unit volume.]
- state and use $V = W/Q$.
- state and use $P = IV$, $P = I^2 R$ and $P = V^2/R$.
- state and use $R = \rho L/A$.
- State that the resistance of a light-dependent resistor (LDR) decreases as the light intensity increases.
- Define and use the electromotive force (e.m.f.) [of a source as energy transferred per unit charge in driving charge around a complete circuit].
- Distinguish between e.m.f. and potential difference (p.d.) in terms of energy considerations.
- Explain the effects of the internal resistance of a source of e.m.f. on the terminal potential difference.
- state Kirchhoff's first law and describe that it is a consequence of conservation of charge.
- state Kirchhoff's second law and describe that it is a consequence of conservation of energy.
- Derive, using Kirchhoff's laws, a formula for the combined resistance of two or more resistors in series.
- Derive and apply a formula for the combined resistance of two or more resistors in parallel.
- Use Kirchhoff's laws to solve simple circuit problems.
- State and use the principle of the potentiometer as a means of comparing potential differences.
- Explain the use of a galvanometer in null methods.
- Explain the use of thermistors and light-dependent resistors in potential dividers. [to provide a potential difference that is dependent on temperature and light intensity].
- Explain the internal resistance of sources and its consequences for external circuits.
- Explain how inspectors can easily check the reliability of a concrete bridge with carbon fibers as the fibers conduct electricity.

Electricity is the branch of physics in which we deal with dynamic state of charges. There are two categories of electricity; static electric city and current electricity. The discharging of electrical pulse due to imbalance of charges (Positive & Negative) refer the term Static electricity, while movement of charge carriers by providing potential difference across conductor represents the term current electricity. Basically, there are two types of materials; one category which allow the flows of electric charge through it is named as good conductors, for examples; aluminium, gold, steel, brass, copper etc., while second category does not allow flow of charges is called bad conductor, for examples; wool, rubber, plastic, wood etc.

## 11.1 DRIFT VELOCITY

The term drift velocity of electrons in conductor refers the slow motion of electrons when potential difference is applied across the conductor. In conductor; billions of electrons transfer their energy to the neighboring electrons during collision when they move randomly, this makes electrons' flow possible. The drift velocity means the average velocity of free electrons by which they are drifted towards high potential terminal of conductor in the presence of electric field. This drifting behavior of electrons (within conductor) towards positive terminal is the cause of drift current. Without the presence of external electric field, electrons acquire random motion, while the external electric field bounds all electrons to move from low potential level to high potential level, resultantly electrons acquire average drift velocity about $10^{-3}$ m $s^{-1}$ and net current is obtained.

The term "Current" represents the dynamic state of charges and the categories of current based upon the direction of the flow of charges; means the electric and conventional flow depends upon the nature of charges. Actually, electrons do not move parallel to the conductor's length like trucks on a road. In conducting material; billions of electrons are ready to push neighboring electrons within inter-atomic spaces.

The average velocity of electrons within conducting material without the existence of an external electric field is shown in Fig. 11.1 (a). The concept of drift velocity cannot be described without the presence of an external electric field. The average value of Fermi velocity of electrons in metals is comparatively very high about $10^6$ m $s^{-1}$ (It does not depend upon current and applied voltage) while the drift velocity $10^{-3}$ m $s^{-1}$.

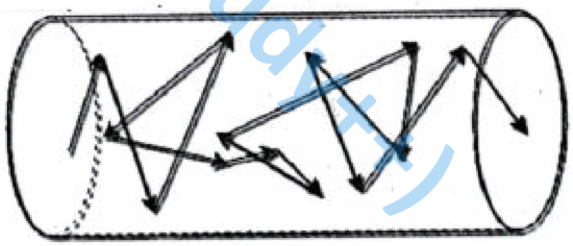


**Figure 11.1 (a):** Motion of electrons in the absence of electric field.

In metals, during motion, electrons acquire energy, lose it when collide with each other, as a result metal becomes hot. For the continuation of the flow of charges in metals, external electric field is mandatory which helps to drift electrons.

During the elastic collision of electrons, the thermal velocities become high in the presence of the external electric field. When the emf source is attached across the terminals of conducting material, all electrons are aligned in a well-organized way in a particular direction. Basically, emf is the measurement of work done per unit charge for conversion of non-electrical energy to electrical energy. Potential difference is the effect of that emf. The emf and potential difference can be measured with the same scales of units. The external electrical field applied can be measured in terms of potential differences across the conducting material per unit length.

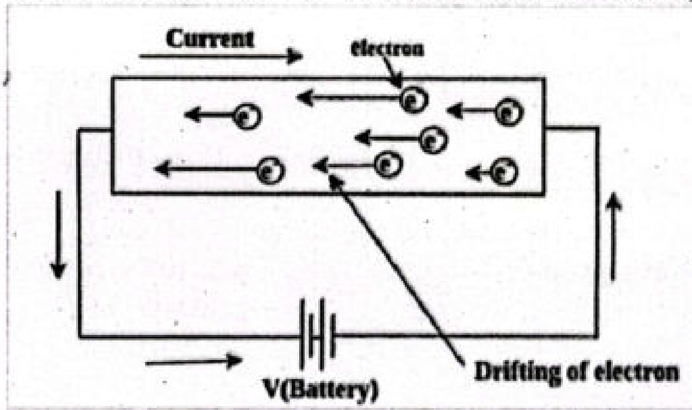



Figure 11.1 (b): Motion of electrons in the presence of an electric field.



**Table: 11.1: Comparison of drift velocity and mobility.**

| Drift Velocity | Mobility |
|---|---|
| Average velocity acquired by charge carriers within conducting material in the presence of external electric field. | Measurement of magnitude of drift velocity of charge carriers per unit electric field. |
| Formula: $v = -\mu E$ | Formula: $\mu = v/E$ |
| Unit: meter per second ($m\ s^{-1}$) | Unit: $m^2 s^{-1} V^{-1}$ |

Drift velocity factors: Let's consider an emf source of potential "V" is attached across the metallic conductor (wire) having length "L" and uniform cross-section area "A" [as shown in Fig. 11.1(b)]. The external electric field intensity can be written as;

$$\text{Electric field intensity (E)} = \frac{\text{Potenatial across conductor (V)}}{\text{the length of conductor (L)}}$$

If Number of free electrons are "N" the free electron density within metallic conductor is:

$$\text{Free electron density (n)} = \frac{\text{Number of free electrons (N)}}{\text{Volume of the conductor (AL)}}$$

Here, number of free electrons = N = nAL. If charge on one electron is "e" then the charge quantization is "ne". The total charge on conducting material becomes;

$$Q = nALe$$

As, the current "I" passes through conductor is:

$$I = neAv \qquad \text{\_\_\_\_\_\_ (11.1)}$$

So, the drift velocity v of electrons is: $v = \frac{I}{nAe}$

Now the current density can be measured as: $J = nev$

Drift velocity and current have direct relation. The particular directive flow of charge carriers depending upon provision of external electric field, otherwise electrons acquire speeds in *random directions.*

**Example 11.1:** A current of 3 A is flowing in a copper conductor with a cross-section of 1 mm$^2$. Find the drift velocity of the electrons. (For copper, n = 8.5 x $10^{28}$ per m$^3$).

Given: I = 3 A A = 1 mm$^2$ = $1\times10^{-6}$ m$^2$ n = 8.5 x $10^{28}$ per m$^3$

To Find: v = ?

Solution: Here we use the formula I = nAvQ

Putting values, $3 = 8.5\times10^{28} \times 1\times10^{-6} \times v \times 1.6\times10^{-19}$

Solving for v, we get: $v = 2.20\times10^{-4}$ m s$^{-1}$

Assignment 11.1

A wire with diameter 0.02 m contains $10^{28}$ free electrons per m$^3$. Find the drift velocity for free electrons in the wire with an electric current 100 A.

## 11.2 ELECTRIC POTENTIAL

Electric Potential Energy: Any charged particle experiences a force when it moves within the region of constant electric field. i.e. $F = Eq$. It means that if we want to move any charge carrier within a constant electric field, work is required which changes the electric potential energy of the charge carrier similar to the movement of any massive object in a uniform gravitational field that requires work, which changes the potential energy of an object. When an object moves naturally, there will be no requirement for external force or in other words no need for expenditure of energy, but when an object moves against the natural forces, we must require external energy, similar concept is used in electric potential energy. Electric potential energy is basically energy required by any charge carrier to move against the electric field.

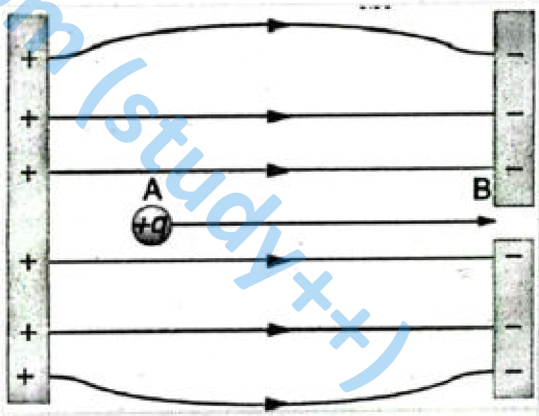


**Figure 11.2 (a):** A charge q experiences a force in an electric field.

Let's consider a charge carrier q experiences a force in constant electric field E, this positive charge will move from the left plate to the right plate, i.e., from a positively charged plate to a negatively charged plate, as shown in Fig. 11.2 (a). If the distance covered by charge is S in the direction of the electric field, as shown in Fig. 11.2 (b), then work done is qES.

The electrostatic force is conservative in nature, so it means that work done on charge q does not depend upon the path followed. Basically, charged particle possesses electric potential energy when it is placed in an electric field, so electric potential energy is the measurement of work done to place a charged particle in an electric region from any infinite region.

Relation between Electric Potential energy and Electric Potential: As electric potential energy is *qES*, so the term electric potential refers the amount of Electric potential energy per unit charge to move charged object from any infinite point to the uniform or non-uniform electric field region against the directions of electric field lines.

$$\text{Electric potential} = \frac{\text{Electric potential energy}}{\text{Charge under observation}} = \frac{Work}{Charge}$$

$$V = U/q$$

So, $U = V\,q$

Electric potential is measured in volt (V), and is given as V = J $C^{-1}$. Electric potential is work done per unit charge against the electric field. Let's consider any referenced point "a" has a distance "r" from a referenced point charge "Q". So, the electric potential with respect to the referenced point can be studied as;

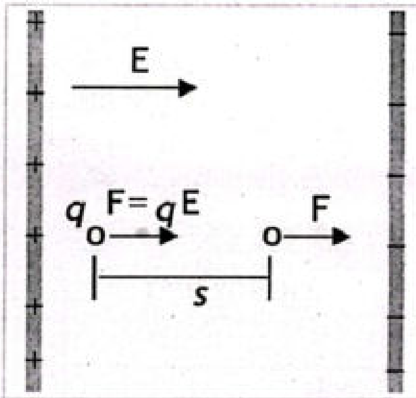



Figure 11.2 (b): Charge cover a distance S.

$$V = \frac{W}{Q}$$

$$V = \frac{F.r}{q} = \frac{Frcos\theta}{q} = \frac{Eqrcos\theta}{q} = -\,E\,r$$

$\theta$ =180° represents the work done against the electric field lines. The electric potential can also be expressed as;

$$\text{Electric potential} = k\frac{q}{r}$$

Electric field intensity and electric potential are field quantities. For more than one charges electric potential can also be measured as;

$$V = k\sum_{i=1}^{N}\frac{q_i}{r_i}$$

Where, k is coulomb's constant; 9 x $10^9$ $Nm^2$ $C^{-2}$. In the field of Atomic Physics and nuclear physics, the charged objects of interest are electrons ($q_e$ = 1.6 x $10^{-19}$ C for one electron as well as one proton), the electrostatic potential energy of charged objects can be expressed in terms of "Electron-Volt" (1 eV = 1.6 x $10^{-19}$ J).

The electric potential is very similar to gravitational potential because both can be considered relative parameters, so electric potential cannot be measured without measuring the difference in potentials across the terminals of the battery. In any electric circuit, the term electric potential tells about the energy conversion phenomenon from electrical form to any other form of energy with respect to every point charge that moves between any two defined points within an electric circuit.

Example 11.2: What is the potential difference between two points in an electric field if it takes 600 J of energy to move a charge of 2 C between these two points?

Given: $\Delta U$ = 600 J    Q = 2 C

To Find: $\Delta V$ = ?

Solution: Using the relation

$$\Delta V = \frac{\Delta U}{Q}$$

Putting values, we get: $\Delta V = \frac{600}{2} = 300$ volt

**Assignment 11.2**

Calculate the current passing through a conducting material having cross-section area 1 $cm^2$ and after applying constant potential across conductor electrons acquire $1.5 \times 10^{-2}$ m $s^{-1}$ and free electron density is $10^{24}$ electrons per cubic meter.

**Table 11.2: Conversion factors for energy.**

| ENERGY UNIT | CONVERSION INTO | | |
|---|---|---|---|
| | Joule | Kilo Joule per mole | Electron-volt |
| Joule | 1 | $6.66 \times 10^{20}$ | $6..24 \times 10^{18}$ |
| Kilo joule per mole | $1.66 \times 10^{-21}$ | 1 | 0.01 |
| Electron-volt | $1.6 \times 10^{-19}$ | 96.5 | 1 |

## 11.3 ELECTROMOTIVE FORCE AND MAXIMUM POWER OUTPUT

We must require some kind of source named as emf source to accelerate charge carriers which will be able to provide potential difference across the conducting material. The emf source helps to convert chemical energy into electrical energy. The term electromotive force is not representing the mechanical concept of force rather it represents work per unit charge.

The potential difference of a cell can be expressed by the equation:

$$V = \varepsilon - Ir$$

whereas r is the internal resistance battery, as shown in Fig. 11.3. The direction of flow of charges can be taken positive or negative depending upon terminals of emf source and the load of electric circuit can be studied as;

The electric current can also be measured in term of load resistance R, emf and internal resistance r; i.e.,

$$I = \frac{\varepsilon}{R+r} \quad \text{------} \quad (11.2)$$

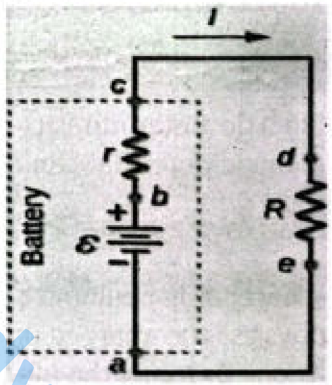



**Figure 11.3:** emf of source helps to convert chemical energy into electrical energy.

In many electronic circuits and systems, it is important to have maximum transfer of power from the source to the load. For example, in radio or TV transmitting systems, we want maximum power transfer from the transmitting medium to the antenna systems. We want maximum power transfer from amplifier to speaker system. This is accomplished by proper matching of load resistance R and source resistance r.

Consider the circuit as shown in fig. If V is the P.D. across R, the loss of potential energy per second is known as power delivered to R by the current I. As for electrical power

$$P_{out} = I^2 R$$

Using equation (11.2), we get:

$$P_{out} = \frac{\varepsilon^2 R}{(R+r)^2}$$

or $$P_{out} = \frac{\varepsilon^2 R}{R^2 + r^2 + 2Rr}$$

or $$P_{out} = \frac{\varepsilon^2 R}{R^2 + r^2 - 2Rr + 4Rr}$$

or $$P_{out} = \frac{\varepsilon^2 R}{(R-r)^2 + 4Rr}$$

**For Your Information**

A fresh Energizer E91 AA alkaline primary battery drops from about 0.9 Ω at -40 °C, when the low temperature reduces ion mobility, to about 0.15 Ω at room temperature and about 0.1 Ω at 40 °C.

When R = r, the denominator of the expression for $P_{out}$ is minimum and so $P_{out}$ is maximum. Thus, it can be concluded that:

Maximum power is delivered to a load R when the internal resistance of the source of emf is equal to the load resistance.

This statement is called maximum power transfer theorem. The value of the maximum output power is

$$(P_{out})_{max} = \frac{\varepsilon^2}{4r} \quad \text{OR} \quad (P_{out})_{max} = \frac{\varepsilon^2}{4R} \qquad \text{\_\_\_\_\_\_ (11.3)}$$

If the load resistance is less or greater than the source resistance, then the power delivered to the load will be low.

## 11.4 VARIATION OF RESISTANCE WITH TEMPERATURE

The following equation shows that how variation of temperature effects resistance of any conducting material;

$$R_t = R_o [1 + \alpha (T - T_o)] \qquad \text{\_\_\_\_\_\_ (11.4)}$$

**Table 11.3: Electrical material Vs. Temperature coefficient of resistance (α)**

| | |
|---|---|
| **Metals** | Greater than zero; resistance increases with temperature. |
| **Solid non-metals** | Equal to zero; resistance is independent of temperature. |
| **Semi-conductors** | Less than zero; resistance decreases with increase in temperature. |
| **Alloys** | Has small value greater than zero. |
| **Superconductors** | At low temperature, the resistance of certain substances becomes exactly zero. |

The resistance of conducting material basically depends upon physical factors such as length, thickness (cross-section area) of conductor and non-physical factor like temperature of the conductor. Resistance (R) of conducting material is directly proportional to length (L) of conductor and inversely proportional to cross-section area (A) of conductor. i.e.,

$$R \propto L \qquad \text{and} \qquad R \propto \frac{1}{A}$$

So, $$R \propto \frac{L}{A} \qquad \text{or} \qquad R = \text{constant}\ \frac{L}{A}$$

Resistance of conducting material can also be expressed in terms of specific resistance also named as Resistivity.

$$R = \rho \frac{L}{A}$$

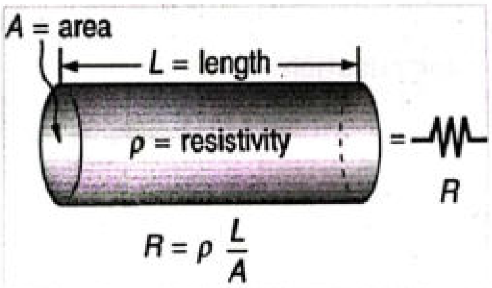



Figure 11.4 (a): A cylindrical shaped conductor.

The SI- Unit of resistivity is Ohmmeter (Ω m). Let's consider a cylindrical shaped conductor having different dimensional structures, as shown in Fig. 11.4 (a). We can have a look at the conductor with length "l" and cross-section-area "a" with resistance $R_1$, as shown in Fig. 11.4 (b).


Figure 11.4 (b): Conductor-1

If conducting object's length is changed from l to 2l but area of cross-section is taken constant, the resistance of conductor will be increased from $R_1$ to $2R_1$, as shown in Fig. 11.4 (c).

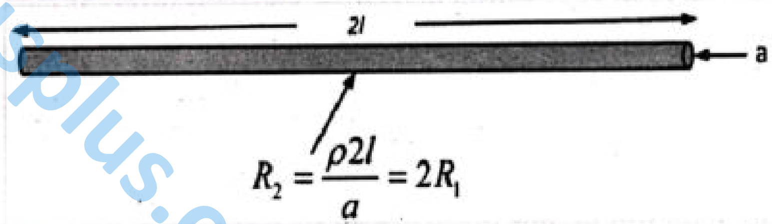



Figure 11.4 (c): Conductor-2

If the length of conducting object is not going to change but wire is replaced with another wire having same length but double cross-section area, then the resistance of conducting material becomes $R_3 = R_1/2$.

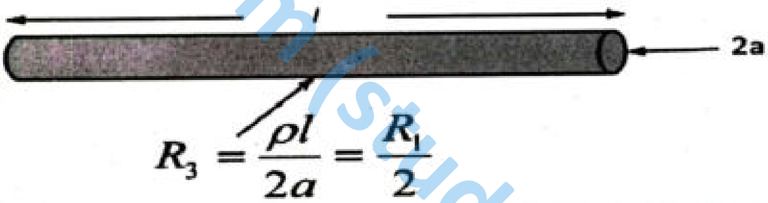



Figure 11.4 (d): Conductor-3

The reciprocal of resistivity is called conductivity, so $\rho = \sigma^{-1}$. The SI-Unit of σ is $(\text{ohm.m})^{-1}$. Another unit is "Siemens" whereas; 1 Sie = $(\text{ohm.m})^{-1}$, basically, resistivity is the physical property of conducting materials, which shows the ability of the atoms of that particular material to impede the flow of electrons. The higher the resistivity the stronger the electrons will be attracted to the atoms, and they will experience more difficulty in moving through the conducting material. Resistivity is the intrinsic property while resistance is the extrinsic property. The resistivity of the conducting material does not depend upon the physical structure of the conductor. It does not change by changing temperature until the material phase transition occurs. In other words, we can say that resistivity is a composition function, while resistance depends upon the physical dimensions of the conducting material and the temperature variation of the conducting material which plays a very effective role in increasing and decreasing resistance.

**Table 11.4: Resistivity.**

| Material | Resistivity (Ω m) |
|---|---|
| Aluminum | $2.7 \times 10^{-8}$ |
| Copper | $1.7 \times 10^{-7}$ |
| Graphite | $8 \times 10^{-6}$ |
| Quartz | $5 \times 10^{16}$ |
| Silicon | $2.3 \times 10^{3}$ |

It means that thicker wire resists the electric flow of charges less as compared to thiner wire. As the resistance of copper is less than the resistance of rubber; that's the reason, that copper is used for the conduction of charge carriers while for insulating purposes rubber is used.

Example 11.3: Calculate the resistance of wire 10 m long that has a diameter of 2 mm and resistivity of $2.63\times10^{-2}\ \Omega$ m.

Given: $L = 10$ m $\quad r = \frac{2\text{ mm}}{2} = 1\text{ mm} = 1\times 10^{-3}$ m $\quad \rho = 2.63\times10^{-2}\ \Omega$ m

To Find: R = ?

Solution: Using the formula $R = \frac{\rho L}{A}$ OR $R = \frac{\rho L}{\pi r^2}$

Putting values, we get: $$R = \frac{(2.63\times10^{-2})(10)}{(3.14)(1\times10^{-3})^2} = 0.83758\times10^{5} = 83{,}758\ \Omega$$

Assignment 11.3

A wire has length 10 m with resistance 100 Ω. If wire is stretched to 3 times of its original length, how much the resistance of the wire will be increased?

## 11.5 LIGHT-DEPENDENT RESISTORS (LDRs)

Light dependent resistor is also called Photo-resistor, this device is used to detect light levels, like; Security system based on light detection, the basic mechanism behind the working of LDR is the mutual variation between resistance of material and intensity of light. Here we can observe that when intensity of light increases the resistance of conducting material decreases.

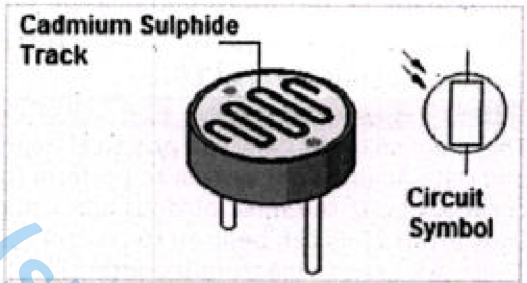



Figure 11.5 (a): LDR.

The basic principle of work of LDR is photoconductivity.

The conducting behavior of LDR can be better explained with the help of energy band theory. Actually, the material reduces its conductance when incident light intensity reduces. Declining graphical representation indicates inverse behavior of resistance at different luminance scales and darkness level, as shown in Fig. 11.5 (b).

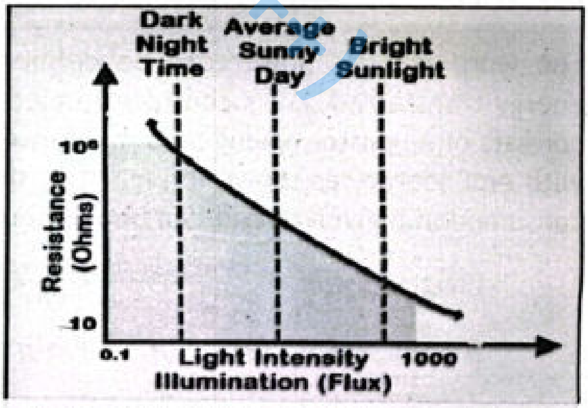



Figure 11.5 (b): Inverse behavior of resistance.

Commonly, two types of LDR exists, i.e. Intrinsic and extrinsic photo resistors. Intrinsic LDRs are related to pure semiconducting material like germanium or silicon etc. while extrinsic LDRs are based upon modified semiconducting materials by doping process, mostly these LDRs cannot be used for shorter wavelength lights and effective only for longer wavelength. LDRs have lot of practical applications like electronic eye security system, street light controlling system, light switching for sunset or sunrise conditions etc.

LDR device is basically part of an automation systems, which helps to reduce the efforts of human in so many fields in daily life. Especially, for the case of power saving, the loss of electricity should be minimal. There are so many characteristics of LDR like maximum power dissipated device within given range of temperature. As LDR is a sensitive device, that's why its sensitivity changes with change of wavelength of light, this device can detect in the range from $4 \times 10^3$ A° to $10^4$ A°.

In dark environment, resistance of LDR acquires its maximum value in MΩ, while in lighted environment it reduces to few 100 Ω. Photo-resistors have lower photoelectric sensitivity at high temperatures, but higher sensitivity at lower temperatures. LDRs have sharp resistance recovery rate.

In potential divider circuit, LDR can be used because its resistance can be increased or decreased by changing the intensity of that light which falls on it, as shown in Fig. 11.5 (c). It means that by increasing or decreasing intensity of light the resistance of circuit can also be changed, which affect the input potential dropped. Hence this is the one of the best practical application of LDR.

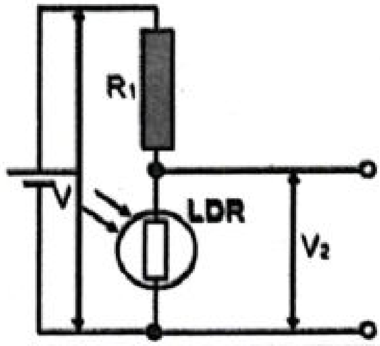


Figure 11.5 (c): LDR based potential divider circuit.

**Engineering and Technology**

There are so many daily life practical applications of LDRs. Used as part of a SCADA (Supervisory Control and Data Acquisition) system to perform functions such as counting the number of packages on a moving conveyor belt; the most obvious application for an LDR is to automatically turn on a light at a certain light level. LDRs can be used to control the shutter speed on a camera. These are used as light sensors. These are used to measure the intensity of light. Their latency property is used in audio compressors and outside sensing. Infrared astronomy and Infrared Spectroscopy also use photo resistors for measuring mid-infrared spectral region. Photo resistors are available in small size; it is easy to carry from one place to another place. Low cost, used in street lighting design, Alarm clocks, Burglar alarm circuits, Light intensity meters.

## 11.6 ELECTRIC POWER AND ELECTRIC ENERGY

The word electric power can be defined as the rate of energy transferred. Let's consider an electric circuit which consists of a resistor having load resistance (R) is connected with emf source, as shown in Fig. 11.6, the electric power consumed in a given circuit can be measured as;

$$\text{Electric power} = \frac{\text{energy consumed by the electric circuit}}{\text{time}}$$

$$P = \frac{\text{energy}}{\text{time}} = \frac{VQ}{t} = VI = I^2R = \frac{V^2}{R}$$

The term power dissipation in resistor is due to energy loses in term of heat energy. SI unit of electric power is watt.

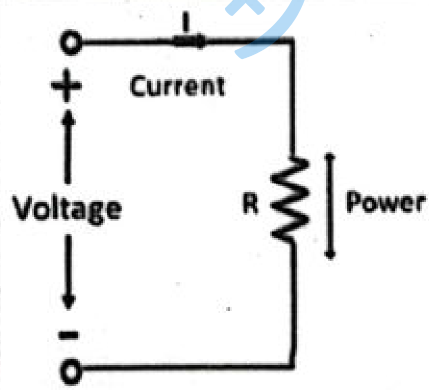


Figure 11.6: An electric circuit.

1 watt = 1 volt x 1 ampere

The energy required or acquired by charge carriers to move from one position to another position is called electrical energy. When electrons move through resistor by provision of potential difference across the ends of conducting material, the concept of energy dissipation can be analyzed in term of heat energy. In other words, one coulomb charge per unit time (Q/t) drops due to potential difference (V), the rate of energy dissipation can be studied as VI. So, Energy dissipation can be calculated with the help of power dissipation.

$$\text{Energy dissipation} = VQ = VIt = I^2Rt = V^2t/R$$

The commercial unit of electrical energy is 'kilowatt-hour'.

$$1\text{kilo watt hour} = 1000 \text{ watt} \times 3600 \text{ seconds} = 3.6 \times 10^6 \text{ Joule}$$



**Example 11.4:** Calculate 1-month cost of using 50 W energy saver for 8 hours daily in your study room. Assume that the price of a unit is Rs. 20.

Given: Power (in watt) = 50 watt Time (in hours) for 30 days = 8×30 = 240 h
Cost per units = Rs. 20

To Find: Cost of electricity = ?

Solution: As No of units = $\frac{power\ (watt) \times time\ of\ use\ in\ hours}{1000}$

Putting values, we get: $= \frac{50 \times 240}{1000} = 12$ unit

Cost of electricity = number of units × cost of 1 unit
= 12×20 = Rs. 240

**Assignment 11.4**

Estimate the cost of electricity consumed for a month, if the following devices are used as specified:

i) 10 bulbs of 40 watts for 10 hours.
ii) 10 fridges of 250 watts for 24 hours.
iii) 40 tube lights of 25 watts for 10 hours.
(Given the rate of electricity is Rs. 50/unit).

## 11.7 KIRCHHOFF'S LAWS

Gustav Robert Kirchhoff introduced laws related to two electrical physical quantities "current" and "voltage" as modified form of Ohm's work. Kirchhoff formulated a pair of laws as practical applications of law of conservation of charge and energy. These laws help in calculation of current in complex combinations of resistances with more than one emf sources.

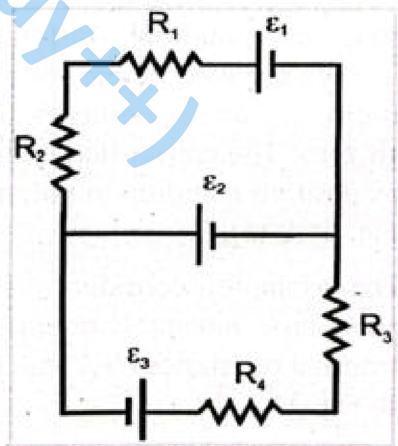



**Figure 11.7:** A multi-loop circuit.

For example, the circuit in Fig. 11.7 shows a multi-loop circuit, which consists of junctions (also known as a node). Here the three resistors are connected and three emf sources are attached to produce current. As the circuit is complex, the equivalent resistance cannot be found here. The circuit cannot be solved by applying Ohm's law, so Kirchhoff's rules can be used instead of Ohm's.

### 11.7.1 Kirchhoff's First Law OR Kirchhoff's Current Law (KCL)

According the Kirchhoff's First law:

**The currents entering the node always equal to currents leaving the node.**

Let's consider five wires are connected at junction point, representing the currents $I_1$, $I_2$, $I_3$, $I_4$ and $I_5$, where $I_1$, $I_2$, $I_3$ indicating the flow of charges entering the node taken positive while $I_4$ and $I_5$ indicating the flow of charges leaving from the node are taken negative, as shown in Fig. 11.8.

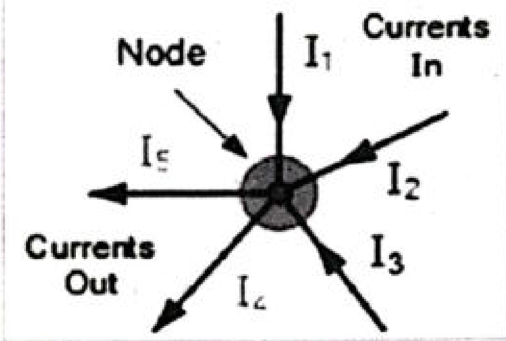



Figure 11.8: Kirchhoff's First law.

$$I_1 + I_2 + I_3 - I_4 - I_5 = 0 \quad \text{hence} \quad \sum I = 0$$

Since charge can neither be created nor destroyed, so the charge that has entered the junction as current must leave in equal amount from the junction. Thus, Kirchhoff's Current law is just a specific way of stating law of conservation of charge.

### 11.7.2 Kirchhoff's Second Laws OR Kirchhoff's Voltage Law (KVL)

According the Kirchhoff's Second Law:

**The algebraic sum of all the changes in potential around any closed path (loop) must be zero.**

Mathematically, $\Sigma V = 0$

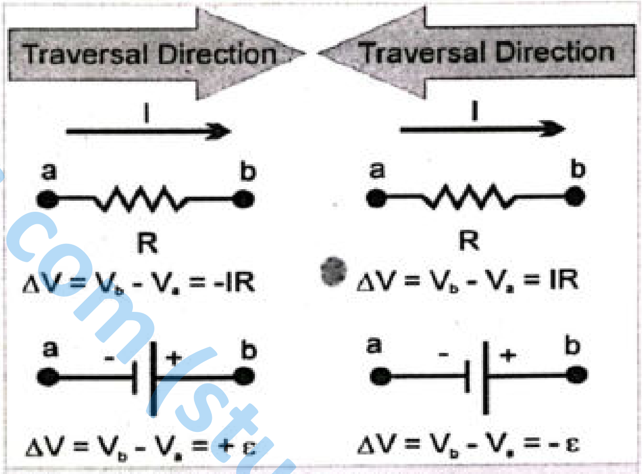



Figure 11.9 (a): Sign convention for Kirchhoff's law.

According to Kirchhoff's law, the algebraic sum of potential differences, including voltage supplied by the voltage sources (emf) and voltage drops in resistive elements (IR), in any loop must be equal to zero. The convention is that, rise in potential is taken as positive and drop in potential as negative, as shown in Fig. 11.9 (a).

For example, consider a simple loop ABCDA with no junctions, having two emf sources '$\varepsilon_1$' and '$\varepsilon_2$' with internal resistances '$r_1$' and '$r_2$' and a resistor $R_1$, as shown in Fig. 11.9 (b).

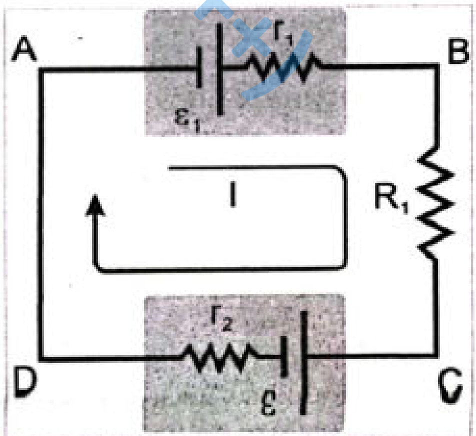



Figure 11.9 (b): A simple loop.

Let's us assume clockwise current 'I', and we travel the circuit in the direction of assumed current, starting from 'A'. At '$\varepsilon_1$' the direction is from negative to positive terminal of battery thus there is rise in potential and is taken as positive. Since at internal resistor '$r_1$' there is drop in potential $\Delta Vr_1 = Ir_1$ is taken as negative. At '$\varepsilon_2$' the direction is from positive to negative terminal of battery and there is drop in potential and is taken as negative. Through internal resistor '$r_2$' again there is drop in potential $\Delta V_{r2} = IR_2$ and is taken as negative. Thus, according to Kirchhoff's Voltage law (KVL), we have:

$$\Delta V_{\varepsilon_1} + \Delta V_{r_1} + \Delta V_{R_1} + \Delta V_{\varepsilon_2} + \Delta V_{r_2} = 0$$

or

$$+\varepsilon_1 + (-Ir_1) + (-IR_1) + (-\varepsilon_2) + (-Ir_2) = 0$$

$$+\varepsilon_1 - Ir_1 - IR_1 - \varepsilon_2 - Ir_2 = 0$$

This law is based on the conservation of energy whereby voltage is defined as the energy per unit charge ($V = \Delta U/q_o$). The total amount of energy gained per unit charge must equal the amount of energy lost per unit charge, as energy and charge are both conserved. Thus, Kirchhoff's voltage law is just a specific way of stating law of conservation of energy.

### 11.7.3 Applying Kirchhoff's Laws

In order to solve problems by using Kirchhoff's Laws consider the circuit shown in Fig. 11.10. Let us take two closed loops:

- *Loop 1* (ABCFA), assuming current '$I_1$' to be flowing through it.
- *Loop 2* (CDEFC), assuming current '$I_2$' to be flowing through it.

The choice of loops is quite arbitrary, but it should be such that each resistance is included at least once in the selected loop.

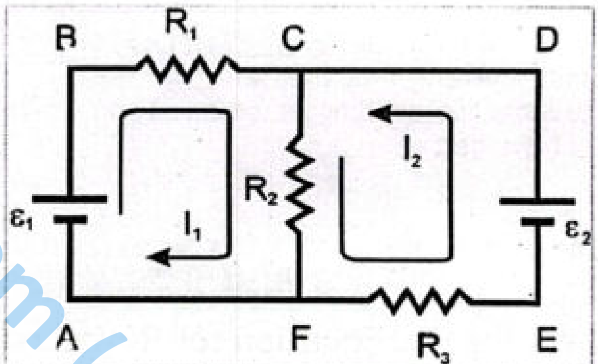



**Figure 11.10:** Complex circuit.

The direction of assumed current is not important; a wrong direction of assumed current will only indicate negative sign in the result. If we go around the loop along the direction of assumed current $I_1$, starting from point 'A'. The battery '$\varepsilon_1$' is taken as positive because we are going from negative to positive terminal in the branch from 'A' to 'B'. At resistor '$R_1$' the drop in potential is taken as negative $V_{R1} = -I_1R_1$. At resistor $R_2$ the drop in potential is taken as negative $V_{R2} = -(I_1 + I_2)R_2$. Therefore, by applying Kirchhoff's Loop rule, we get:

$$\varepsilon_1 + V_{R1} + V_{R2} = 0$$

$$\varepsilon_1 - I_1R_1 - I_1R_2 - I_2R_2 = 0 \qquad \text{——— (11.5)}$$

If we go around the loop (CFEDC) along the direction of assumed current $I_2$, starting from point C. At resistor $R_2$ the drop in potential is taken as negative $V_{R2} = -(I_1 + I_2)R_2$. At resistor $R_3$ the drop in potential is taken as negative $V_{R3} = -I_2R_3$. The battery '$\varepsilon_2$' is taken as positive because again we are going from negative to positive terminal in the branch from 'E' to 'D'. Therefore, by applying Kirchhoff's Loop rule. Mathematically,

$$V_{R2} + \varepsilon_2 + V_{R3} = 0$$

$$-I_1R_2 - I_2R_2 + \varepsilon_2 - I_3R_3 = 0 \quad \_\_\_\_\_ \ (11.6)$$

By simultaneously solving the equations (11.5) and (11.6), the branch currents in the circuit can be determined.

## Deriving the Equation for Resistors in Series

Consider two resistors $R_1$ and $R_2$ connected in series, as shown in Fig. 11.11. A single resistor R is equivalent to them.

From **Kirchhoff's first law**, the current I through each resistor is the same. Since they're connected in series (i.e., having no junctions).

From **Kirchhoff's second law**, the total p.d in a closed loop must equal the sum of the p.d both resistors:

$$V = V_1 + V_2 \quad \_\_\_\_\_ \ (11.7)$$

From Ohm's Law, potential difference is given by the product of current and resistance, i.e.,

$$IR = IR_1 + IR_2 \quad \_\_\_\_\_ \ (11.8)$$

Since current **I** is the same for all resistors, so dividing equation (11.8) by I, we get:

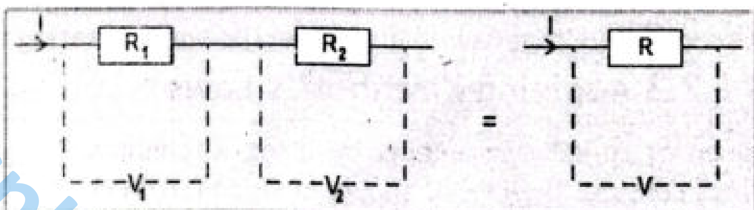



**Figure 11.11:** Resistors connected in series.

$$R = R_1 + R_2$$

The equivalent resistor R of several resistors connected in series is given by:

$$R = R_1 + R_2 + R_3 + ... \quad \_\_\_\_\_ \ (11.9)$$

## Deriving the Equation for Resistors in Parallel

Consider two resistors $R_1$ and $R_2$ connected in parallel, having equivalent resistance R, as shown in Fig. 11.12. From **Kirchhoff's first law**, the current through each resistor will be different because it splits at the junctions. The current through the equivalent resistor R will be the total current I.

$$I = I_1 + I_2 \quad \_\_\_\_\_ \ (11.10)$$

From **Kirchhoff's second law**, the p.d across resistors in different branches must be same and the resistor R will have the same p.d across it too:

$$V = V_1 = V_2 \quad \_\_\_\_\_ \ (11.11)$$

From Ohm's Law, potential difference is given by the product of current and resistance. So, equation (11.10) becomes:

$$\frac{V}{R} = \frac{V}{R_1} + \frac{V}{R_2} \quad \_\_\_\_\_ \ (11.12)$$

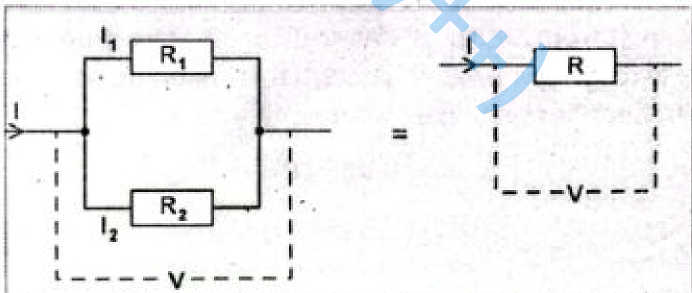



**Figure 11.12:** Resistors connected in parallel.

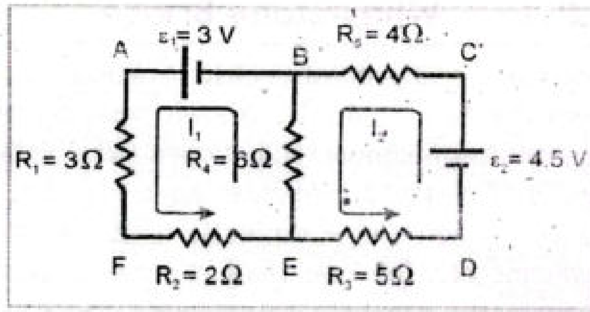
since potential difference V is the same for all resistors, so equation (11.12) becomes:

$$\frac{1}{R}=\frac{1}{R_1}+\frac{1}{R_2} \quad \text{\_\_\_\_\_\_ (11.13)}$$

The equivalent resistor R of several resistors connected in parallel is given by:

$$\frac{1}{R}=\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3}+.... \quad \text{\_\_\_\_\_\_ (11.14)}$$

**Example 11.5: Find current flowing through each resistor, using Kirchhoff's law, in the given circuit.**




**Solution:**

Applying Kirchhoff's law to loop BAFEB:

$$\varepsilon_1 - I_1R_1 - I_1R_2 - (I_1 - I_2)R_4 = 0$$

$$3 - 3I_1 - 2I_1 - 6(I_1 - I_2) = 0$$

$$3 - 5I_1 - 6I_1 + 6I_2 = 0$$

$$11I_1 - 6I_2 = 3 \quad \text{\_\_\_\_\_\_ (1)}$$

Now applying Kirchhoff's law to loop DCBED:

$$\varepsilon_2 - I_2R_5 - (I_2 - I_1)R_4 - I_2R_3 = 0 \quad \text{or} \quad 4.5 - 4I_2 - 6(I_2 - I_1) - 5I_2 = 0$$

$$4.5 - 9I_2 - 6I_2 + 6I_1 = 0$$

or $-6I_1 + 15I_2 = 4.5$ ______ (2)

Multiplying equation (1) by 5 and equation (2) with 2 and adding:

$$55I_1 - \cancel{30I_2} = 15$$

$$-12I_1 + \cancel{30I_2} = 9$$

$$43I_1 \quad = 24$$

$$I_1 = 0.56\,A$$

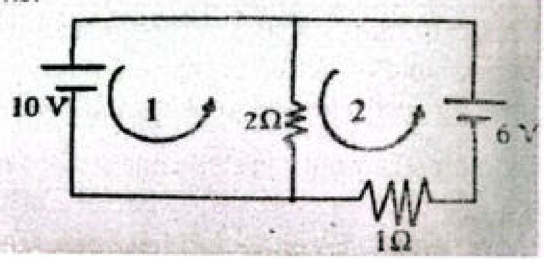
Putting value of $I_1$ in equation (1):

$$11(0.56) - 6I_2 = 3$$

Current flowing through resistances $R_1$ and $R_2$ is $I_1$ = 0.56 A. Current flowing through resistances $R_3$ and $R_5$ is $I_2$ = 0.53 A. Current flowing through resistance $R_4$ is $I_1 - I_2$ = 0.56 A - 0.53 A = 0.03 A in the upward direction. As both the values of currents $I_1$ and $I_2$ are positive, therefore the supposed directions of current are their actual directions.

**Assignment 11.5**

Find the current flowing through each resistor in the circuit having two loops, as shown in the figure.



## 11.8 NULL METHOD OF MEASUREMENT

This method basically shows null reading on galvanometer which is used for comparison between any known and unknown quantities. Null reading on galvanometer is used to find resistance in a circuit named as Wheatstone bridge. Potentiometer is also one of the electrical devices in which we use null method for finding potentials.

### 11.8.1 Wheatstone Bridge

Wheatstone bridge consists of four-resistors (three-known resistors while 1-unknown resistor) and a galvanometer is connected between two points, say b and d (Fig. 11.13). Wheatstone bridge becomes balance when no deflection is seen on galvanometer. The null reading measurement on galvanometer can help to calculate the value of unknown resistance.

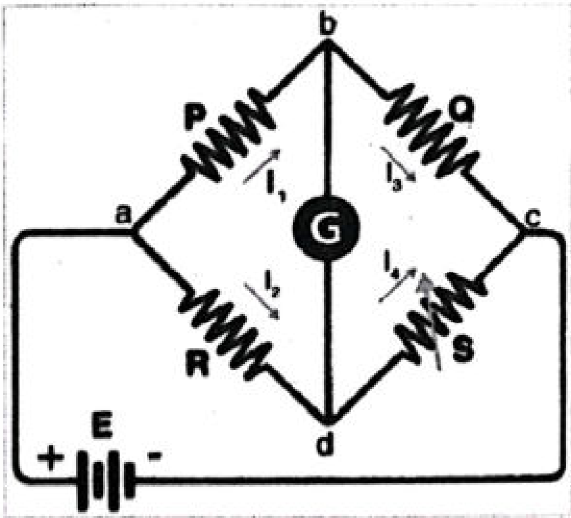



Figure 11.13: Wheatstone bridge circuit.

Let $I_1$ and $I_2$ be the currents through P and R respectively when the bridge is balanced. Since there is no current through Galvanometer, the current in Q and S are also $I_1$ and $I_2$ respectively. As the Galvanometer reads zero, points B and D are at the same potential. Hence for balanced bridge:

*Potential drop across ab = Potential drop across ad*

i.e. $I_1 P = I_2 R$ ______ (11.15)

Similarly,

*Potential drop across bc = Potential drop across dc*

$I_1 Q = I_2 S$ ______ (11.16)

Dividing (11.15) by (11.16), we get:

$$\frac{P}{Q} = \frac{R}{S}$$

or $R = \frac{PS}{Q}$ ______ (11.17)

Hence, if three resistances on the right side of equation are known, the fourth resistance R can be calculated.

### 11.8.2 Potentiometer

A potentiometer is a null type resistance network device for measuring potential differences. Its principle of action is that an unknown emf or P.D. is measured by balancing it, wholly or in part, against a known potential difference.

A simplest potentiometer consists of wire AB of uniform cross-section, stretched alongside a scale and connected across battery of potential V, as shown in Fig. 11.14. A standard cell of known emf $\varepsilon_1$ is connected between A and terminal 1 of a two-way switch S.

Slider N is pressed momentarily against wire AB and its position is adjusted until the galvanometer deflection is zero. Let $l_1$ be the corresponding distance between A and N. The fall of potential over length $l_1$ of the wire is then the same as the emf $\varepsilon_1$. i.e.

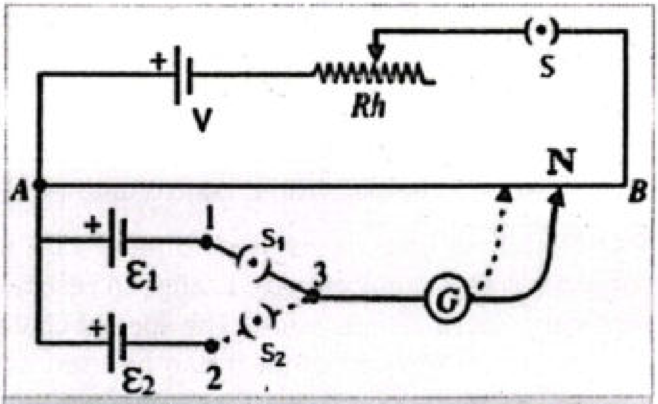


Fig: 11.14: Potentiometer.

$$\varepsilon_1 \propto l_1 \qquad \text{\_\_\_\_\_\_ (11.18)}$$

Then move the switch to 2, thereby replacing the standard cell by another cell, the emf $\varepsilon_2$ of which is to be measured. Adjust the slider N again to give zero deflection on G. If $l_2$ be the new distance between A and N, then

$$\varepsilon_2 \propto l_2 \qquad \text{\_\_\_\_\_\_ (11.19)}$$

From the equations (11.18) and (11.19), we get:

$$\frac{\varepsilon_2}{\varepsilon_1} = \frac{l_2}{l_1}$$

$$\varepsilon_2 = \left(\frac{l_2}{l_1}\right)\varepsilon_1 \qquad \text{\_\_\_\_\_\_ (11.20)}$$

Basic purposes of this electrical instrument are;

- Determination of electromotive force.
- Measurement of internal resistance.
- Comparison of potential of two emf sources.
- Better output in measuring potential difference as compare to voltmeter.

## 11.9 THERMISTOR

A thermistor (short for thermal resistor) is a heat sensitive resistor; it means that the change of temperature has direct relation with change of resistance. Samuel Ruben was the first achiever who invented the first thermistor, which later on used for commercial purposes.

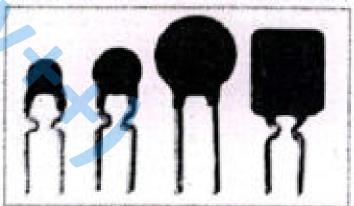


Figure 11.15: Thermistor.

Thermistors are usually made of a semiconductor material (semiconductor oxides of iron, nickel and cobalt). Pair of platinum leads is attached at the two ends for electrical connections. The arrangement is enclosed in a very small glass bulb and sealed. They are generally in the form of discs, rods, beads, etc., as shown in Fig. 11.15.

Thermistor can be categorized on the basis of numeric values of temperature co-efficient as: negative temperature coefficient (NTC) and positive temperature coefficient (PTC). The difference between NTC and PTC is inverse and direct relation of temperature and resistance respectively. Positive temperature co-efficient of resistance means that the resistance increases with the rise in temperature, whereas negative temperature co-efficient of resistance that the resistance decreases with the rise in temperature.

There are so many practical applications of thermistor like; temperature's compensation, controlling, etc.

Polycrystalline material is used to make LDR because this material is very supportive to change in resistance even by small change of temperature. The special characteristic of this electrical device is that it can be used as a potential divider. As thermistor's resistance changes with change of temperature, if this resistor is connected with potential divider circuit with known value of resistance, then desired voltage can be measured on the basis of temperature variation.

**For Your Information**

Metals (e.g. copper, aluminum) have positive temperature co-efficient of resistance because the resistance of metals increases with the rise in temperature. Electrolytes, insulators (e.g. glass, mica, rubber etc.) and semiconductors (e.g. germanium, silicon etc.) have negative temperature co-efficient of resistance because their resistance decreases with the rise in temperature.

In potential divider circuit, thermistor can be used because its resistance can be increased or decreased by changing temperature (as shown in Fig. 11.16). By increasing temperature, the resistance of thermistor falls and by decreasing temperature the resistance of thermistor increases depending upon which type of thermistor is connected NTC or PTC. If we feel thermistor's temperature high, then it means potential will be dropped because its resistance will also be decreased and vice versa.

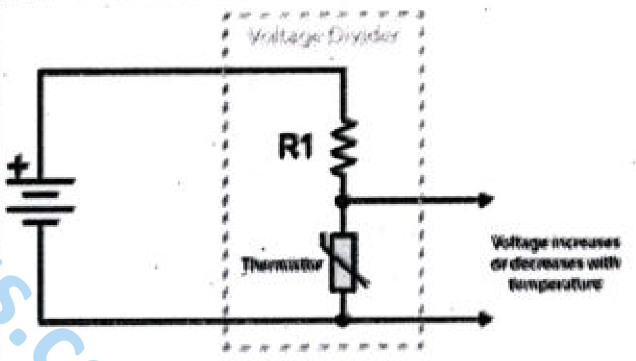



Figure 11.16: Temperature based potential divider circuit.

## 11.10 CARBON FIBERS IN CONCRETE BRIDGE

Carbon fibre-reinforced concrete has been used in construction projects due to its ability to improve compressive strength, tensile strength, elasticity modulus, chemical stability, abrasion resistance and corrosion resistance; reduce shrinkage cracks and brittleness; light weightiness; good thermal conductivity. Carbon fiber was introduced into concrete bridges (like shown in Fig. 11.18) to improve the flexural strength and reduce the crack width. Carbon fibers contain mainly carbon atoms and are commonly used in civil engineering works.

Figure 11.18: Carbon fibre-reinforced concrete.

The optimum carbon fiber amount should be 0.3 % by volume of concrete. Inspector can easily check the reliability of concrete bridge made by carbon fibres because fiber conducts electricity if sensor shows the electric resistance increasing over time the fibres are separating because of cracks.

Carbon is a very competence material as conductor because its molecular structure is very similar to graphite. There are so many electrical applications of carbon fibres, for example: carbon electrodes for electrolysis, electromagnetic interference shielding, sensing, electrical switching etc.

## SUMMARY

- **Drift velocity:** The average velocity with which electrons 'drift' in the presence of an electric field.
- **Resistance:** A measurement of the opposition to flow of charge carriers in an electrical circuit. Its unit is ohms.
- **Resistivity:** It is a fundamental specific property of a material that measures its electrical resistance.
- **Conductivity:** It is the reciprocal of resistivity its unit is $MHO.m^{-1}$
- **Electromotive force:** The electric potential produced by either an electrochemical cell or by changing the magnetic field.
- **Potential gradient:** The slope of potential distance graph is called potential gradient.
- **Kirchhoff's first rule (KCL):** The algebraic sum of all currents inward and outward with respect to node must equal zero.
- **Kirchhoff's second rule:** For any closed loop in a circuit, the sum of the potential differences across all components present in a closed loop is zero.
- **Light dependent resistor (LDR):** A special type of resistor that works on the photoconductivity principle, it means that resistance increases or decreases with the variation of intensity of light.
- **Thermistor:** A special kind of thermometer, which shows the inverse relation between temperature and resistance.
- **Potential divider:** A voltage divider is a circuit that takes a larger voltage and divides it down by a fixed ratio according to the electronic components to give a smaller output voltage. Thermistor and LDR can be used within potential divider circuit.
- **Null Measurement Method:** Measuring method which can be used in Wheatstone bridge electric circuit arrangements, in which the quantity to be measured is balanced by an opposing known quantity that is varied until the resultant of the two is zero, which can be seen on electrical device like galvanometer.



## EXERCISE

### Multiple Choice Questions

**Encircle the Correct option.**

1) On which of the following factor the drift velocity does not depend: ________.

A. length of the wire. B. cross-section of the wire.

C. number of free electrons. D. magnitude of the electric field.

2) Drift velocity of electrons is independent of ________.

A. time consumed. B. the length of the wire.

C. the number of free electrons. D. the Strength of the electric field.

3) A 10 Ω resistor is connected across the terminals of a 10 V battery. The power dissipation of the resistor is

A. 100 W B. 50 W C. 25 W D. 10 W

4) A heavy duty refrigerator is used about 1000 W, if it is allowed to run continuously for 10 hours daily, how many kilowatt-hours of energy does it consume in 10 days?

A. 100 kWh B. 200 kWh C. 300 kWh D. 350 kWh

5) What will be the value of emf if 3A current within 5mins perform work about 900J?

A. 10 V B. 3 V C. 1 V D. 5 V

6) Which one of the following electrical devices helps us to measure the accurate value of emf?

A. meter Bridge B. voltmeter C. multimeter D. potentiometer

7) The unit of dynamic charge is:

A. coulomb B. ohm C. ampere D. volt

8) The resistance of a steel wire of particular length and thickness is 100 Ω. If the length and the diameter of the wire both are doubled, then the resistivity in Ω m will be: ______.

A. $1\times10^{-7}$ Ω m B. $2\times10^{-7}$ Ω m C. $4\times10^{-7}$ Ω m D. $5\times10^{-7}$ Ω m

9) How much power dissipation will be increased if current passing through the circuit is increased 100 % (let keeping temperature constant)?

A. 100 % B. 200 % C. 300 % D. 500 %

10) There are 2 resistors of having values 2 *Ω* and 4 *Ω* respectively are connected in series to a 6 V battery. The heat dissipated by the 4 *Ω* resistor in 5 *s* will be:

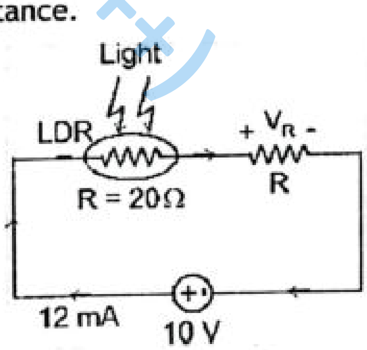
A. 5 J B. 10 J C. 20 J D. 30 J

11) The algebraic sign of an IR drop primarily depends upon the

A. direction of flow of current. B. battery connections.

C. magnitude of current flowing through it. D. value of resistance.

12) What will be the nearest value of protected resistance, if LDR having resistance 20 Ω is operated at specific intensity of light is to be protect through a series resistance in such a way that up to 12 mA current in the provision of 10 V potential across the circuit as shown in following figure?




A. 681 Ω B. 773 Ω

C. 813 Ω D. 973 Ω

13) Which of the following electrical instrument can be used as a null detector in the Wheatstone bridge?

A. galvanometer B. ammeter C. voltmeter D. multimeter

14) Potentiometer can be used for________________________

A. comparing two voltages. B. comparing two currents.
C. measuring a voltage. D. measuring a current.

15) In order to achieve high accuracy, the slide wire of a potentiometer should be:
A. as long as possible. B. as short as possible.
C. neither too small not too large. D. very thick.

16) There are ___________types of thermistor.
A. one B. two C. three D. four

17) Thermistor is special kind of resistor named as:
A. laser resistor B. photo resistor C. thermal resistor D. electric resistor

## Short Questions

Give short answers of the following questions.

11.1 Why it is not possible to measure the drift speed for electron by timing their travel along a conductor?

11.2 What is the difference between e.m.f. and potential difference?

11.3 Why we use Kirchhoff's law for circuit problems solution.

11.4 The Kirchhoff's current rule is based on conservation of charge. Explain how?

11.5 While analyzing a circuit, the internal resistance of e.m.f. sources is ignored. Why?

11.6 Why rise in temperature of a conductor, with positive temperature coefficient, is accompanied by a rise in the resistance?

11.7 Under what circumstances can the terminal potential difference of a battery exceed its e.m.f.?

11.8 What is the working principle of Galvanometer?

11.9 How inspectors can easily check the reliability of a concrete bridge with carbon fibers.

11.10 Could electronic devices charge themselves without being plugged into an electricity source?

11.10 Does a source of electricity ever run out of electrons?

11.11 Two wires have equal length, one is made of copper and the other of manganin and they have the same resistance. Which wire here will be thicker between these two given wires?

## Comprehensive Questions

Answer the following questions in detail.

11.1 What is LDR? Discuss in detail.

11.2 Explain the use of thermistors and light-dependent resistors in potential dividers.

11.3 Explain the following terms. (A) Drift velocity, (B) emf, (C) internal resistance.

11.4 What is potential divider? Explain.

11.5 What is Wheatstone bridge? Explain in detail.

11.6 What is a potentiometer? Explain.

11.7 Explain Kirchhoff's first law and describe that it is a consequence of conservation of charge.

11.8 Explain Kirchhoff's second law and describe that it is a consequence of conservation of energy.

**11.9** Using Kirchhoff's laws, derive a formula for the combined resistance of two or more resistors in series.

**11.10** Using Kirchhoff's laws, derive a formula for the combined resistance of two or more resistors in parallel.

**11.11** Explain the electronic current in a metallic wire due to the drift of free electrons in the wire.

## Numerical Problems

**11.1** If 1A current flows through a copper wire having 1 $cm^2$ area of cross-section and 10 km length; calculate the time required by charge carrier to travel from one end to other end of the conductor (Free electron density of copper is $8.5 \times 10^{28}$ per $m^3$). **(Ans: 431 years)**

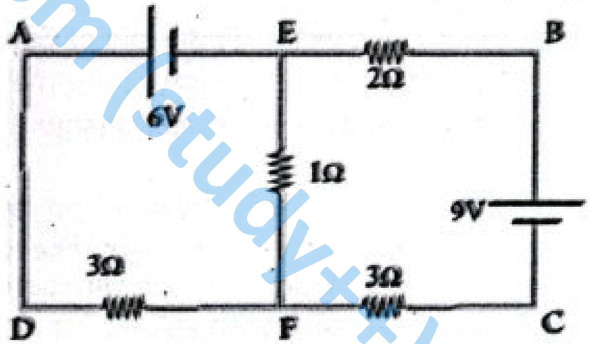
**11.2** Calculate the flow of charges per unit time inside the emf source having potential 2 V and terminals are connected with each other having internal resistance 0.02 Ω? **(Ans: 100 A)**

**11.3** Calculate the terminal potential of the emf source if internal resistance of the battery is 10 Ω and current and internal potential are 1 A and 10 V respectively? **(Ans: 0 V)**

**11.4** There is a copper coil having 2000 number of turns with 0.8 $mm^2$ cross-section are, the length per turn is 80 cm. Calculate the resistance of the coil? (Resistivity of copper is 0.02 μΩm)? **(Ans: 40 Ω)**

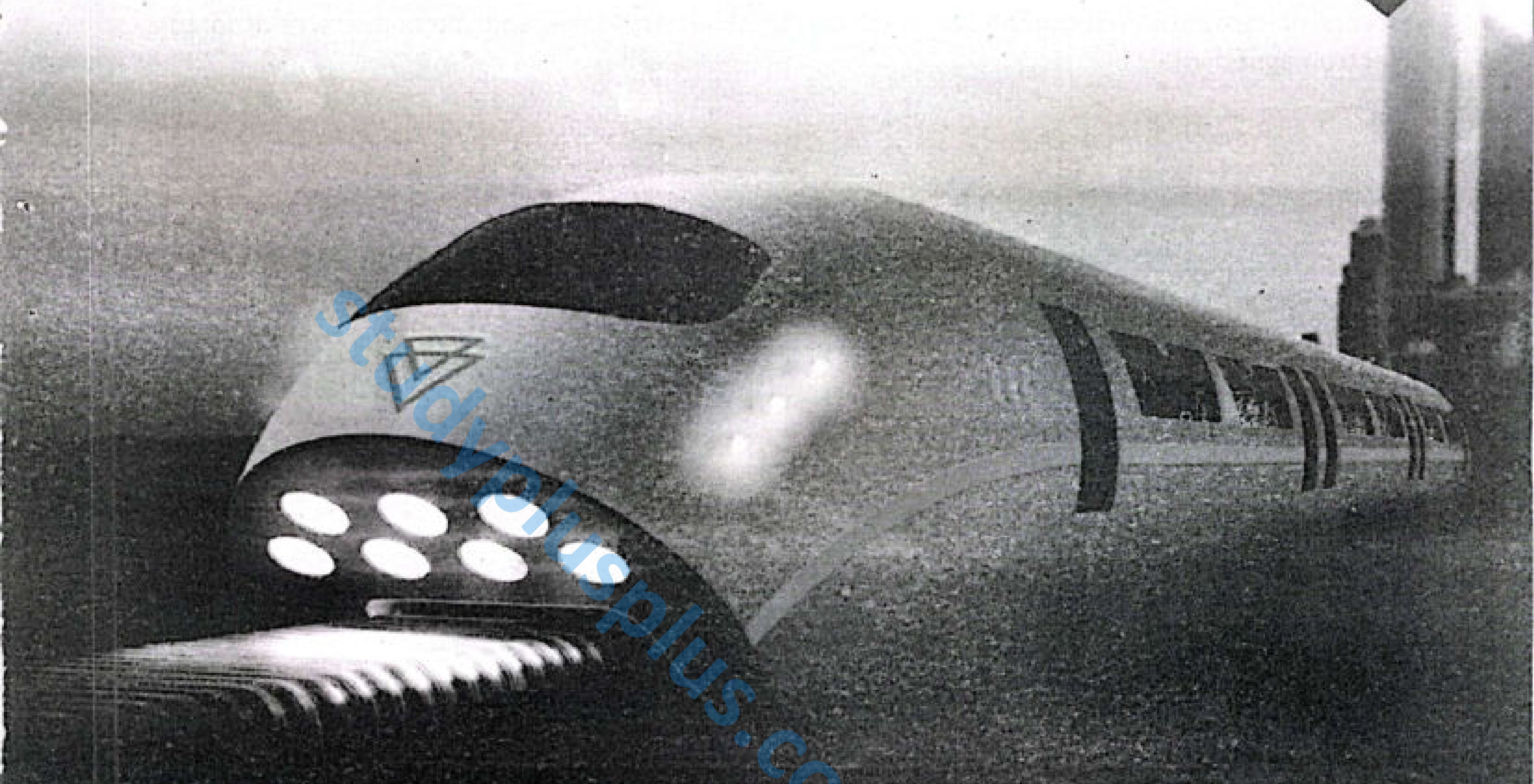
**11.5** A piece of silver wire has a resistance of 1 Ω. What will be the resistance of magnum wire of one third of the length and one third of diameter, if the specific resistance of wire is 30 times that of silver? **(Ans: 90 Ω)**

**11.6** Calculate the current that flows in the 1 Ω resistor in the following circuit. **(Ans:0.13 A)**



UNIT 12

# ELECTROMAGNETISM





## Student Learning Outcomes (SLOs)

**The students will:**

- Define and explain magnetic fields.
- state that a force might act on a current-carrying conductor placed in a magnetic field.
- use the equation $F = BIL \sin\theta$ [with directions as interpreted by Fleming's left-hand rule to solve problems].
- Define magnetic flux density [as the force acting per unit current per unit length on a wire placed at right angles to the magnetic field]
- use $F = BQV \sin\theta$ to solve problems.
- describe the motion of a charged particle moving in a uniform magnetic field perpendicular to the direction of motion of the particle.
- explain how electric and magnetic fields can be used in velocity selection.
- sketch magnetic field patterns due to the currents in a long straight wire, a flat circular coil and a long solenoid.
- state that the magnetic field due to the current in a solenoid is increased by a ferrous core.
- explain the origin of the forces between current-carrying conductors and determine the
- direction of the forces.
- define magnetic flux [as the product of the magnetic flux density and the cross-sectional area perpendicular to the direction of the magnetic flux density].
- use $\phi = BA$ to solve problems.
- use the concept of magnetic flux linkage.
- explain experiments that demonstrate Faraday's and Lenz's laws. [(a) that a changing magnetic flux can induce an e.m.f. in a circuit, (b) that the induced e.m.f. is in such a direction as to oppose the change producing it, (c) the factors affecting the magnitude of the induced e.m.f.
- Use Faraday's and Lenz's laws of electromagnetic induction to solve problems.
- explain how seismometers make use of electromagnetic induction to the earthquake detection [specifically in terms of: (i) any movement or vibration of the rock on which the seismometer rests (buried in a protective case) results in relative motion between the magnet and the coil (suspended by a spring from the frame.) (ii) the emf induced in the coil is directly proportional to the displacement associated].

# UNIT 12 ELECTROMAGNETISM

Magnetism is the study of how a magnetic field, generated by a moving charged particle, affects other charged particles or permanent magnets. Electromagnetism is the study of the magnetic effects of current. In this unit, we shall study about the laws and phenomena related to electromagnetism.

## 12.1 MAGNETIC FIELDS

The magnetic field is the region or space surrounding a magnet in which a compass needle, a small magnet, or a piece of ferromagnetic material, and a moving charged particle can experience a magnetic force. A permanent bar magnet, a current-carrying conductor, a fluctuating electric field and a moving charge can create a magnetic field. The magnetic force between magnets acts over a distance. They apply force due to interaction of their magnetic fields (i.e. non-contact force).

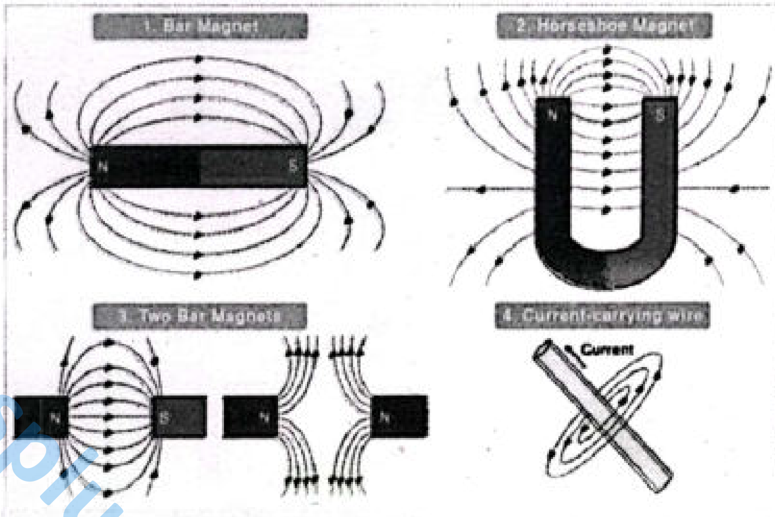



**Figure 12.1 (a):** Magnetic field.

Michael Faraday proposed an alternative explanation. He suggested that each magnet sets up a magnetic field in space around it, as shown in Fig. 12.1 (a). When you place another magnet in that field, it responds to the field at its own location. This interaction of fields is the cause of force between them. Interaction of field lines can be drawn in the form of picture. As we have already discussed in earlier classes, gravitational fields exert the force of gravity on other masses. An electric field exists around charges, so that other charges in this region will experience an electric force. Similarly, magnetic fields exert force on other magnets, like compass needles, magnetic materials, and moving electric charges.

> **For Your Information**
>
> The magnetic field was studied in 1269 by Petrus Peregrinus de Maricourt, with John Mitchell claiming magnetic poles repel each other in 1750. Charles-Augustin de Coulomb confirmed Earth's magnetic field in 1785, and Simeon Denis Poisson presented the first magnetic field model in 1824.

You can easily visualize the magnetic field of a magnet by sprinkling iron filings near the magnet, as shown in the Fig. 12.1 (b). The iron particles become magnetized in a magnetic field and stick together along magnetic field lines. One key distinction between magnetic field lines and electric field lines is that the magnetic field lines

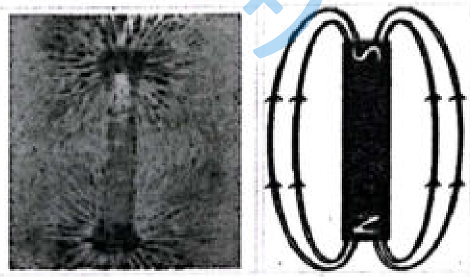



**Figure 12.1 (b):** Magnetic field of a magnet by sprinkling iron filings.

form complete loops. Magnetic field lines exit from the north pole and enter the south pole, as shown in Fig. 12.1 (b). But inside the magnet, the direction of magnetic field lines goes from the south pole to the north pole. This is the reason why there are no magnetic monopoles, which means that the North and South poles do not exist independently. The magnetic field at the poles of the bar magnet is strongest.

**Hans Christian Oersted (1777-1851)**

In April 1820, Orsted discovered that a magnetic needle aligns itself perpendicularly to a current-carrying wire. That was definite experimental evidence of the relationship between electricity and magnetism.

### Properties of Magnetic Field Lines

1. Magnetic field lines never cross each other.
2. The density of the magnetic field lines indicates the strength of the magnetic field.
3. Magnetic field lines always form closed loops.
4. Magnetic field lines always start from the north pole and enter at the south pole.
5. Like magnetic poles repel, and unlike magnetic poles attract each other.

## 12.1.1 Magnetic Field Due to Current in a Long Straight Wire

In 1820, Hans Oersted first described the magnetic field due to current in a wire.

### Experiment

Take a piece of copper wire that passes vertically through a horizontal piece of card board, as shown in Fig. 12.2. Place small magnetic compass needles on the card board along a circle with the centre at the wire. All the compass needles point in the direction of north-south. When a heavy current passes through a wire, the compass needles set themselves along the tangent to the circle. Circular magnetic field in anti-clockwise direction is produced around current carrying conductor, as shown in Fig 12.3 (a).

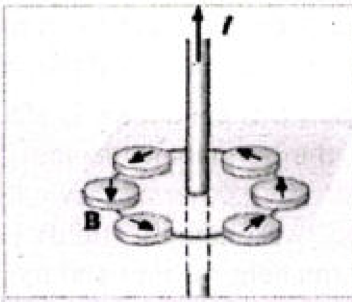



Figure 12.2: Compass needle pointing in the direction of magnetic field.

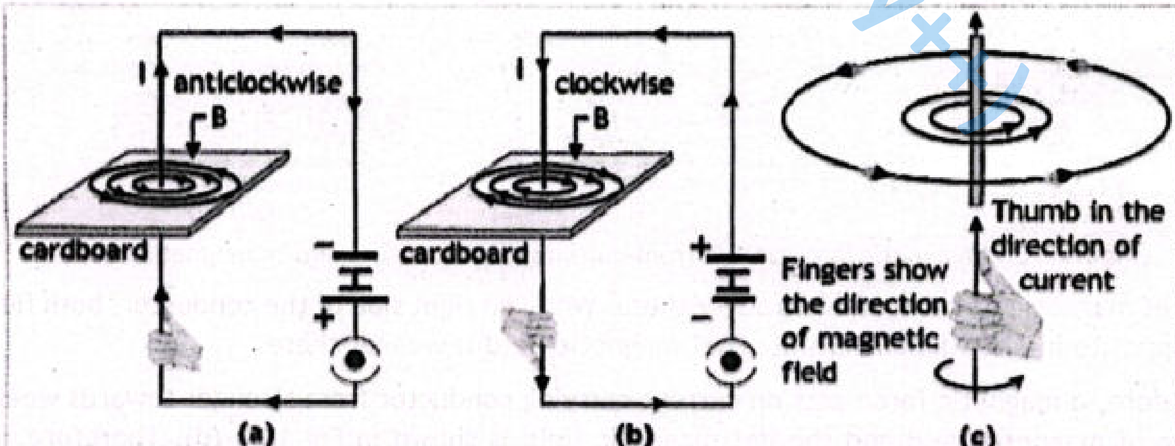



Figure 12.3: Circular magnetic field is produced around current carrying conductor.

When we reverse the direction of the current, the direction of **B** is also reversed in clockwise direction, as shown in Fig 12.3(b). If the current through the wire is stopped, all the needles will again point in the north-south direction. We conclude that

- A magnetic field is set up only around a current-carrying conductor.
- The lines of forces are circular, and their direction depends upon the direction of the current.

Direction of Magnetic Field: The direction of the magnetic field can be determined by the right-hand rule, as shown in Fig. 12.3 (c). It can be stated as:

**If the wire is grasped in the fist of the right hand with the thumb pointing in the direction of (conventional) current, then the curled fingers indicate the direction of magnetic field.**

## 12.2 MAGNETIC FORCE ON A CURRENT-CARRYING CONDUCTOR IN UNIFORM MAGNETIC FIELD

Consider a straight-current carrying conductor, as shown in Fig. 12.4 (a) having concentric circular magnetic field lines in clockwise direction due to the current flow into the page. Fig. 12.4 (b) is representing magnetic field of a permanent magnet (From North to South pole).

When this conductor is placed in external magnetic field of the permanent magnet, their magnetic fields interact with each other as shown in Fig. 12.4 (c). We can see that the two magnetic fields i.e. external magnetic field of permanent magnet and magnetic field of current carrying conductor support each other on left side of the conductor

> **For Your Information**
>
> A current-carrying wire in a magnetic field moves because a force acts on it. The magnetic field making the wire move is called a catapult field. The catapult field is due to the combined effect of the magnetic field of current-carrying wire and magnetic field of bar magnet. Fig. 12.4 shows the separate fluxes, and how they combine to form a catapult field.

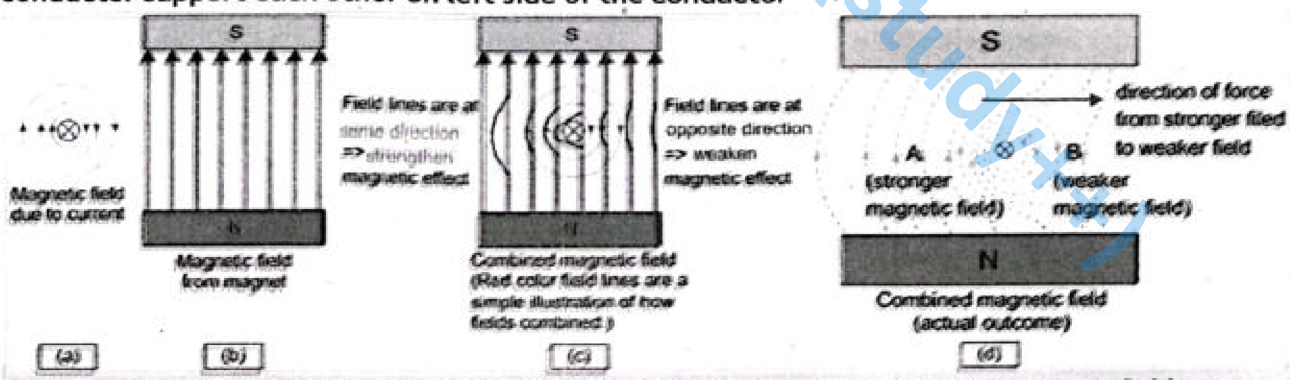



**Figure 12.4:** Magnetic force on a current-carrying conductor in uniform magnetic field.

and net magnetic field becomes stronger there. While on right side of the conductor, both fields are opposite in direction. Therefore, net magnetic field is weaker there.

Therefore, a magnetic force acts on current carrying conductor from stronger towards weaker region of magnetic field and the net magnetic field is shown in Fig 12.4 (d). Therefore, the conductor tends to move towards the right where the net magnetic field is weak.

The magnetic force on the conductor will be directed toward the right, perpendicular to both the length of conductor and the external magnetic field.

**Derivation:** Consider a conductor with length L carrying current I placed on two conducting rails in uniform magnetic field B, as shown in Fig. 12.5. When magnetic force acts on the conductor, it begins to move on the conducting rails in the direction of force. From different experiments, we conclude following points for force experienced by the conductor.

i) Magnetic force on the conductor is directly proportional to current flowing in it.

$$F \propto I$$

ii) Magnetic force on the conductor is directly proportional to it length.

$$F \propto L$$

iii) Magnetic force on the conductor is directly proportional to strength of external magnetic field.

$$F \propto B$$

iv) Magnetic force on the conductor is directly proportional to sine of angle between length of the conductor and magnetic field.

$$F \propto \sin\theta$$

Where θ is angle between B and L, combining the above four relations, we get:

$$F \propto I\,LB\sin\theta$$

Or $$F = K\,I\,L\,B\sin\theta$$

Where, k is the proportionality constant and in SI system its value is equal to 1.

$$F = BIL\sin\theta \quad \text{\_\_\_\_\_\_} \quad (12.1\ a)$$

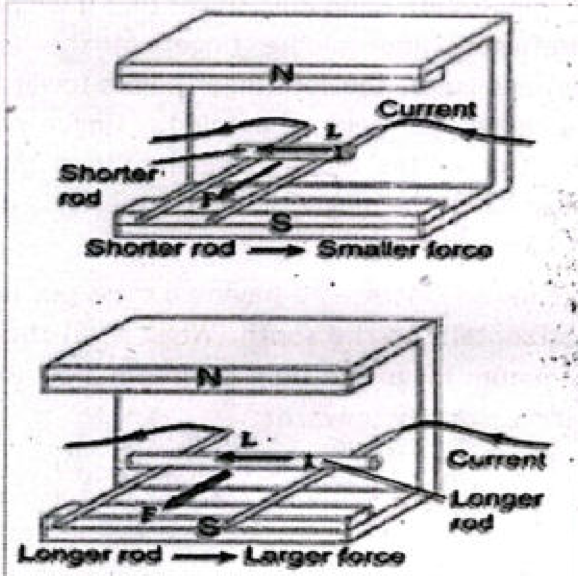



**Figure 12.5:** Conductor carrying a current, placed on two conducting rails in uniform magnetic field.

In vector form, it can be expressed as:

$$F = I\,L\,B\sin\theta\,\hat{n}$$

$$F = I\,(L \times B) \quad \text{\_\_\_\_\_\_} \quad (12.1\ b)$$

The magnetic force is perpendicular to both the length of the conductor and magnetic field.

**Maximum Force:** This magnetic force F is maximum when $\theta = 90^0$, i.e., when the conductor is placed perpendicular to the magnetic field.

Then, $F_{max} = I\,L\,B\sin 90^0$

$F_{max} = I\,L\,B\,(1) = I\,L\,B$

$F_{max} = I\,L\,B$

**Convention to represent direction**

To show the direction, sometime we use Dot (•) and cross(×):

Dot (•) represents **out of the plane.**

cross (×) represents **into the plane.**

**Minimum Force:** This magnetic force F is minimum (zero) when the conductor is placed parallel (i.e., $\theta = 0^o$) or antiparallel (i.e., $\theta = 180^o$) to the magnetic field.

For $\theta = 0^o$, $F_{min} = I\,L\,B\sin 0^o$

$$F_{min} = I\,L\,B\,(0) = 0$$

For $\theta = 180^{o}$, $F_{min} = I\,L\,B\sin 180^{o} = 0$

Thus, it is concluded that a current-carrying conductor will not experience force in a magnetic field if the angle between **B** and **L** is $0^{o}$ or $180^{o}$.

## FLEMING'S LEFT-HAND RULE

Fleming's Left-Hand Rule states that if we position our thumb, forefinger, and middle finger of the left-hand mutually perpendicular, the forefinger points towards the direction of the magnetic field, the middle finger points towards the direction of the electric current, then thumb indicates the direction of the magnetic force experienced by the conductor, as shown in Fig. 12.6.

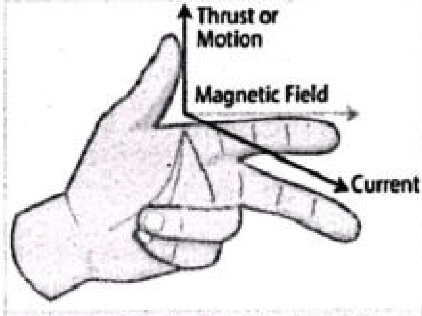
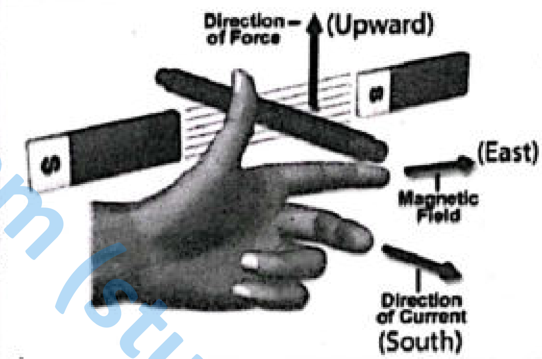



**Figure 12.6: Fleming's Left-Hand Rule.**

**Example 12.1: A wire having a mass per unit length of 0.400 g cm$^{-1}$ carries a 5.0 A current horizontally to the south. What is (a) the direction of force and (b) the magnitude of the minimum magnetic field perpendicular to the length of the conductor needed to lift this wire vertically upward?**

Given: $\frac{m}{L} = 0.400\ \text{g cm}^{-1} = \frac{0.400\ \ 10^{3}}{1\ \ 10^{2}} = 0.04\ \text{kg m}^{-1}$

$I = 5.0\ \text{A}$




Solution: According to given condition, the magnetic force must be upward to lift the wire. For current in the south direction, the magnetic field must be towards east to produce an upward magnetic force, as shown by the Fleming's left-hand rule in the figure.

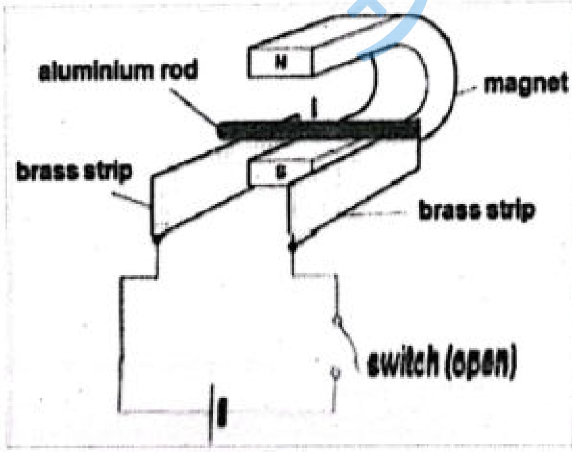
$F_B = ILB\sin\theta$ with $w = mg$

In order to lift the wire, the magnetic force must be equal to the weight of the wire.

$$F_B = F_g$$

$$ILB\sin\theta = mg \quad \text{or} \quad B = \left(\frac{m}{L}\right)\frac{g}{I\sin\theta}$$

$$B = (0.04)\left(\frac{9.8}{5.0\,\text{A} \times \sin 90^{o}}\right) = 0.0784\ \text{T}$$




**Assignment 12.1**

**The figure shows a light aluminium rod resting between the poles of a magnet. A current is passed through the rod from two brass strips connected to a power supply. (a) On the figure, draw the direction of the current when the switch is closed (b) State which way the rod**

**moves when the switch is closed. Give a reason for your answer. (c) State the effect on the movement of the rod when:**
**(i) the current is increased.**
**(ii) the current is reversed.**

## 12.3 MAGNETIC INDUCTION or MAGNETIC FLUX DENSITY

A current-carrying conductor positioned at right angle to a magnetic field will experience a magnetic force that can be used to quantify magnetic flux density.

The formula $B = \frac{F}{IL}$ determines the flux density B for a uniform magnetic field. Where F is the force acting on a current carrying conductor, I is the magnitude of current, and L is the length of the conductor in the uniform magnetic field with magnetic flux density B.
The SI unit of magnetic flux density (or) magnetic induction is tesla (T).

Since $B = \frac{F}{IL}$ and $1\,T = \frac{1N}{(1A)(1m)\sin 90^0}$

If a 1 m-long conductor carrying a current of 1 A, placed perpendicular to a magnetic field, experiences a force of 1 newton, then the magnetic induction is one tesla.
Another CGS unit used for B is gauss (G). The relation between tesla and gauss is

$1\,T = 10000\,G$ OR $1\,T = 10^4\,G$ OR $1\,G = 10^{-4}\,T$

Magnetic field is a vector quantity.

## 12.4 MAGNETIC FORCE ACTING ON MOVING CHARGE IN UNIFORM MAGNETIC FIELD

It is experimentally verified that moving charges produce magnetic fields around them. When moving charged particles enter in an external magnetic field of the permanent magnet, the magnetic field of the moving charge interacts with the magnetic field of the permanent magnet. Due to interaction of both fields, net magnetic field becomes stronger on one side of the charge than its other side (just like explained earlier for force on current carrying conductor placed in magnetic field). Therefore, magnetic force acts on the moving charge particle from stronger to weaker magnetic field region and deflect from its path, as shown in Fig 12.7. It has been seen that the charged particle deflects from its path, perpendicular to both magnetic fields and velocity, because a net force is acting on it perpendicular to B and v. When a charge enters into the magnetic field, it experiences a force, provided that the following conditions are fulfilled:

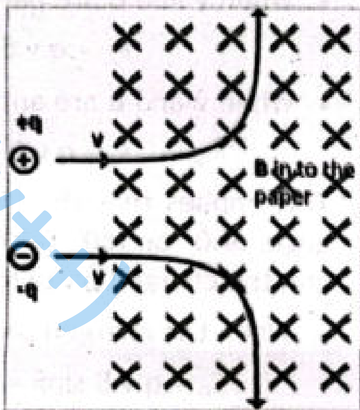



**Figure 12.7:** Trajectories of charged particles in a magnetic field.

- The charge must be moving, because no magnetic force acts on a stationary charge.
- The velocity of the moving charge must have a component that is perpendicular to the direction of the magnetic field.

Consider a positive point charge '+q' moving with velocity 'v' making an angle '$\theta$' with the magnetic field 'B', as shown in the Fig. 12.8.

The force experienced by a positive point charge is mathematically expresses as:

$$F_B = q\,(v \times B) \quad \text{———} \quad (12.2)$$

Magnetic force $F_B$ acting on negative charge is:

$$F_B = -q\,(v \times B) \quad \text{———} \quad (12.3)$$

The negative sign in equation (12.3) shows that the direction of magnetic force acting on negative charge is opposite to that of the positive charge. The magnitude of magnetic force acting on a charge 'q' moving with velocity 'v' in the magnetic field of strength 'B' is given by:

$$F_B = q\,v\,B\sin\theta$$

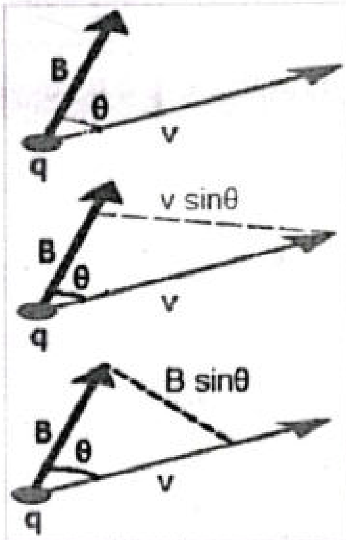



**Figure 12.8:** Velocity of charge is making an angle '$\theta$' with the magnetic field.

Where $\theta$ is angle between **v** and **B**.

**Maximum Force:** The force will be maximum when the charged particle moves perpendicular to B ($\theta = 90^\circ$).

$$F_B = q\,v\,B\sin 90^\circ$$

$$F_{B(max)} = q\,v\,B$$

$F_B$ is maximum and charged particle moves in circular path.

**Minimum Force:**

- When v and **B** are parallel, ($\theta = 0^\circ$)

$$F_B = q\,v\,B\sin 0^\circ = q\,v\,B\,(0) = 0$$

- When v and **B** are anti-parallel ($\theta = 180^\circ$)

$$F_B = q\,v\,B\sin 180^\circ = q\,v\,B\,(0) = 0$$

This implies that the charged particle moves at an angle of 0° or 180° to the magnetic field, then F = 0 and the charged particle moves in a straight path.

- When the charged particle is at rest (v = 0)

$$F_B = q\,v\,B\sin\theta = q\,(0)\,B\sin\theta = 0$$

- If q = 0, then $F_B = (0)\,v\,B\sin\theta = 0$

If the angle between magnetic field B and the velocity v of charged particle is other than 0°, 90° and 180° then charged particle moves in helical path (spiral path), as shown in Fig. 12.9. This particle's motion has both parallel and perpendicular components.

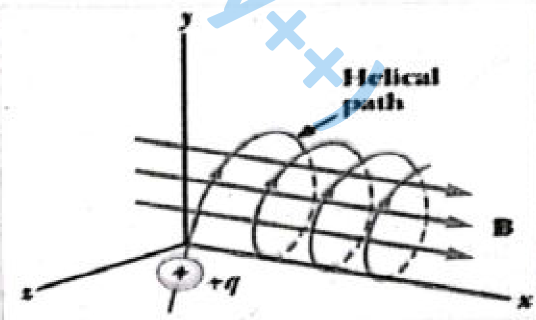



**Figure 12.9:** Other than angle 0°, 90°, 180° Charged particle move in helical path.

The direction of the force experienced by the moving charge is perpendicular to the both magnetic field B and its velocity v. For a positive charge, Right-Hand Rule-II and Fleming's Left-Hand Rule for force on a current-carrying conductor are used to determine the direction of force. where the direction of current is replaced by the velocity of the charged particle.

For instance, if a positive particle moves from the bottom of the setup shown in Fig. 12.10, an outward force acts on the charged particle. The direction of magnetic force on negatively charged particles will be inward. Magnetic force on a moving charged particle has a direction perpendicular to both velocity and magnetic field B.

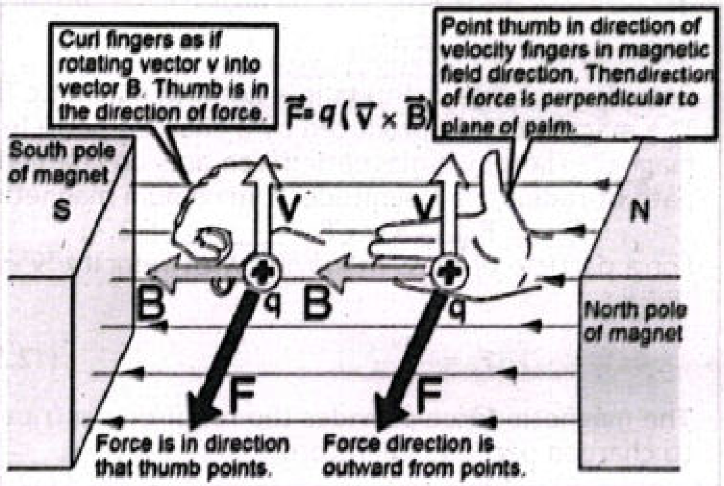



**Figure 12.10:** Magnetic force on a moving charged particle has a direction perpendicular to both velocity and magnetic field.

**Example 12.2:** An electron is accelerated through 3600 V from rest and then enters a uniform 2.70 T magnetic field. What are (a) the maximum and (b) the minimum values of the magnetic force this particle experiences?

Given: $\Delta V = 3600$ V $\qquad$ B = 2.70 T

To Find: $F_{B_{max}} = ?$ $\qquad$ $F_{B_{min}} = ?$

Solution: For Speed of electron: $\frac{1}{2}$ m v² = e ΔV $\qquad$ or $\qquad v = \sqrt{\dfrac{2e\Delta V}{m}}$

Putting values, we get: $\qquad v = \sqrt{\dfrac{2\times1.6\times10^{-19}\times3600}{9.1\times10^{-31}}} = 3.556 \times 10^{7}\ \text{m s}^{-1}$

(a) $\quad F_{B_{max}} = e\,v\,B = (1.60\times10^{-19}\ C)(3.556\times10^{7}\ m/s)(2.70\ T) = 1.54\ \times 10^{-11}\ N$

(b) $\quad F_{B_{min}} = e\,v\,B\sin0^{0} = 0$ $\qquad$ or $\qquad F_{B_{min}} = e\,v\,B\sin180^{0} = 0$

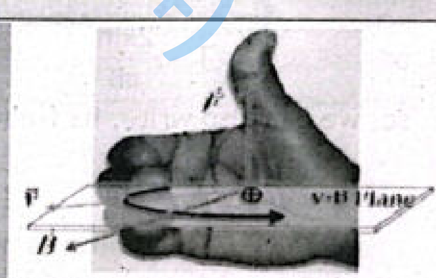
Assignment 12.2

Prove that magnetic force is not responsible to do work on a charged particle moving in circular path in a magnetic field.

**For your information**

**Right hand rule to find direction of magnetic force on charged particle in magnetic field:**

**Place the velocity and magnetic field vectors in one plane and sweep from velocity towards magnetic field vector with right hand through smaller angle then magnetic force on charge is directed towards the thumb and perpendicular to the plane of velocity and magnetic field.**



## 12.5 MOTION OF A CHARGED PARTICLE IN A UNIFORM MAGNETIC FIELD

Consider a charge q enters in a uniform magnetic field B with velocity v, as show in Fig. 12.11. The magnetic field produced by a charged particle interacts with magnetic field of permanent magnet. Thereby a magnetic force acts on charged particle and it begins to move in a circular path of radius 'r'. Magnitude of maximum magnetic force is:

$$F_{B(max)} = q\,v\,B \quad \text{\_\_\_\_\_\_\_} (12.4)$$

For a particle of mass m moving with velocity 'v' in a circle of radius 'r' the centripetal force '$F_c$' is:

$$F_c = \frac{mv^2}{r} \quad \text{\_\_\_\_\_\_\_} (12.5)$$

The magnetic force provides the required centripetal force to charged particle, therefore,

$$F_c = F_{B(max)} \quad \text{\_\_\_\_\_\_\_} (12.6)$$

Putting values from equations (12.4) and (12.5) in (12.6):

$$\frac{mv^2}{r} = q\,v\,B$$

$$\frac{mv}{r} = q\,B \quad \text{\_\_\_\_\_\_\_} (12.7)$$

Radius of circular path is $r = \frac{mv}{qB}$ _______ (12.8)

Putting $v = r\omega$ in equation (12.7):

$$\frac{mr\omega}{r} = q\,B \quad \text{or} \quad m\,\omega = q\,B$$

$$\omega = \frac{qB}{m} \quad \text{\_\_\_\_\_\_\_} (12.9)$$

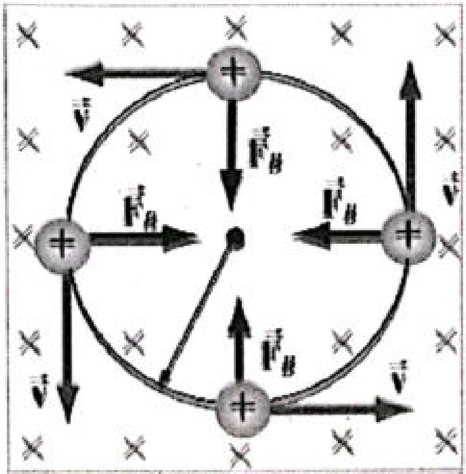


Figure 12.11: A magnetic force acts on charged particle and move it into a circular path.

This is the cyclotron frequency of a charge particle moving in a circle in a magnetic field in terms of angular motion. The time period T of the charged particle is:

$$T = \frac{2\pi}{\omega} \quad \text{\_\_\_\_\_\_\_} (12.10)$$

Putting value of ω from equation (12.9) in (12.10), we get: $T = \frac{2\pi}{qB/m}$

or $$T = \frac{2\pi m}{qB} \quad \text{\_\_\_\_\_\_\_} (12.11)$$

so, we can also write, its frequency as:

$$f = \frac{qB}{2\pi m} \quad \text{\_\_\_\_\_\_\_} (12.12)$$

**Example 12.3: A proton is moving in a circular orbit of radius 12 cm in a uniform 0.40 T magnetic field perpendicular to the velocity of the proton. Find the speed of the proton.**

Given: Radius = r = 12 cm = 0.12 m  Uniform magnetic field= B= 0.40 T

Charge of proton= e = $1.60 \times 10^{-19}$C  Mass of proton = $m_p = 1.67 \times 10^{-27}$ kg

To Find: Velocity of proton = v = ?

**Solution:** As, $$v = \frac{eBr}{m_p}$$

Substitute numerical values: $$v = \frac{(1.6\quad 10^{19})(0.040)(0.12)}{1.67\quad 10^{27}}$$

$$v = 4.59 \times 10^{6}\ m\ s^{-1}$$

**Assignment 12.3**

**In a uniform magnetic field with a strength of $1.20 \times 10^{-3}$ T, calculate the radius of an orbit of circular path of an electron moving at a speed of $2.0 \times 10^{7}\ m\ s^{-1}$.**

## 12.6 VELOCITY SELECTOR METHOD

Velocity selector is a device used as velocity filter for charged particles. It has mutually perpendicular electric and magnetic fields. When charged particle enters with velocity v perpendicular to both electric and magnetic fields then both electric and magnetic forces act on charged particle. At a particular velocity the electric and magnetic force acting on charged particle are equal in magnitude but opposite in direction cancel out their net effect and charged particle moves in straight path undeflected, as shown in Fig 12.12.

then $F_E = F_B$

Putting $F_E$ = q E and $F_B$ = q v B,

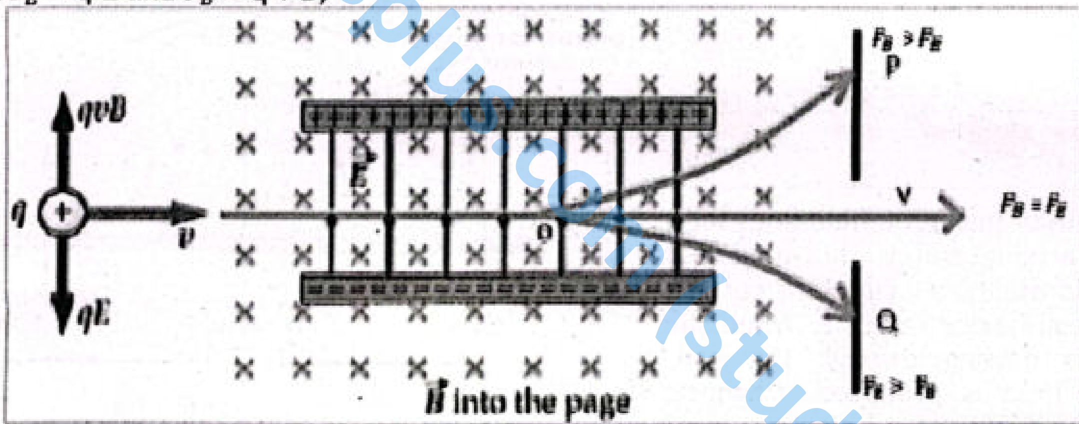



**Figure 12.12:** No net force acting on the charge particle.

Therefore, q E = q v B

or $$v = \frac{E}{B} \qquad \text{\_\_\_\_\_\_\_\_ (12.13)}$$

If the beam of charged particles, moving at different velocities, passes through the velocity selector, only those particles will pass straight through the device, which have velocities equal to the ratio of the electric field to the magnetic field. Particles moving slower than this speed will be deflected in the direction of electric force because magnetic force will be less than electric force on it. On contrary, those charge particles having greater speeds will be deflected in the direction of magnetic force because magnetic force will be greater than electric force on it.

The vector sum of electric and magnetic forces acting on a charged particle in electric and magnetic fields applied in same region is called Lorentz force.

$$F = F_E + F_B$$

$$F = qE + q(v \times B)$$

## 12.7 MAGNETIC FIELD PATTERNS

### 12.7.1 Magnetic Field Due to Current Carrying Straight Conductor

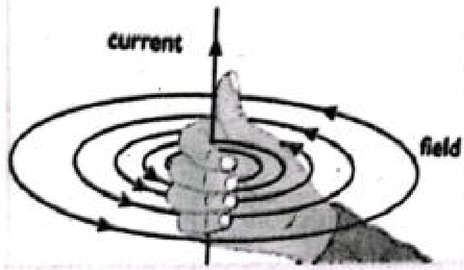



Figure 12.13: Magnetic field lines for a current carrying straight wire.

The electric current flowing through a wire produces a magnetic field around it in the form of concentric circle, as shown in Fig. 12.13. The strength of the magnetic field depends upon the magnitude of the current and the distance of the point from the current-carrying conductor. The strength of the magnetic field produced by current in a long, straight wire is given by:

$$B = \frac{\mu_0 I}{2\pi r}$$

Where 'I' is the current and 'r' is the distance of any point from the wire, and the constant is $\mu_0 = 4\pi \times 10^{-7}$ Wb $A^{-1}\,m^{-1}$.

### 12.7.2 Magnetic Field Due to Current Carrying Flat Circular Coil

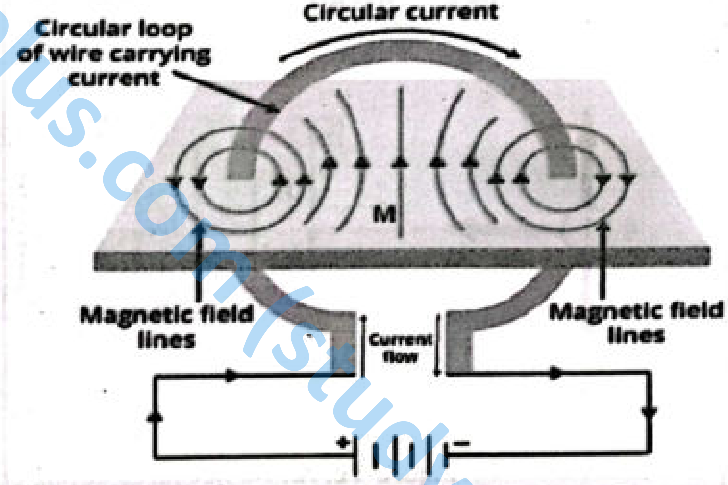



Figure 12.14: Magnetic field lines for a current carrying coil.

The pattern of magnetic field lines for a current carrying coil is shown in Fig. 12.14. Consider a circular current carrying coil having radius r. When the current is passing through the coil, magnetic field is produced. Magnetic field at the center of the single current carrying circular loop is given by:

$$B = \frac{\mu_0 I}{2r}$$

### 12.7.3 Magnetic Field Due to Current Carrying Solenoid

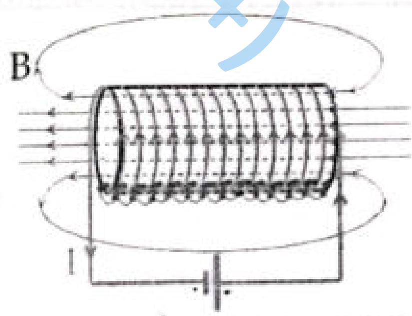



Figure 12.15: Solenoid.

A solenoid is a current-carrying coil that produces magnetic field, as shown in Fig. 12.15. The magnetic field of a solenoid is similar to the magnetic field of a bar magnet, with the north pole at one end of the coil and the south pole at the other end, depending on the direction of current 'I'. The field inside the solenoid is strong and uniform as compare to the outside. The mathematical expression for magnetic field of solenoid is given by:

$$B = \mu_0 n I$$

Where n is number of turns (N) per unit length (L) of solenoid, (i.e., n = N/L).

Right-Hand Rule for a Solenoid: Curl the fingers of right hand in the direction of current, the erect thumb will point in the direction of north pole.

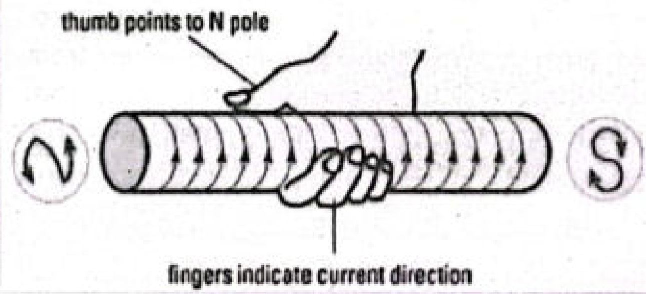



**Figure 12.16:** Right hand rule for solenoid.

Example 12.4: A horizontal power line carries a current of 100 A in an east to west direction. What is the magnitude and direction of the magnetic field due to the current 2.0 m below the line?

Given: I = 100 A, R = 2.0 m, $\mu_o = 4\pi \times 10^{-7}$ Wb/A m

To Find: Magnetic field (B) = ?

Solution: By using formula $B = \frac{\mu_0 I}{2\pi r}$

$$B = \frac{4\pi \times 10^{-7} \times 100}{2\pi \times 2} = 1.0 \times 10^{-5} T$$

By using the right-hand thumb rule, we can infer that magnetic field B acts in south direction.

Assignment 12.4

A 15.0 cm long solenoid has 300 turns of wire and 5.0 A current flow through it. How strong a magnetic field is there inside the solenoid?

## 12.8 MAGNETIC FIELD DUE TO THE CURRENT IN A SOLENOID IS INCREASED BY A FERROUS CORE

Ferrous materials are those that contain iron. The term "ferrous" comes from the Latin word "ferrum," which means iron. Iron can be easily magnetized when placed in external magnetic fields.

In the absence of a core, the magnetic lines of forces start to diverge by curving sharply and immediately outside the coil.

**Figure 12.17:** solenoid

If a ferrous material (such as iron rod) is placed inside the solenoid (Fig.12.17), the strength of the magnetic field increases significantly. This occurs because the ferrous core becomes magnetized due to the magnetic field generated by the solenoid. This process is called induction. If wire of solenoid is tightly wound on the iron core, then magnetic field at the cross section remain parallel and close to each other, that's why new magnetic field is uniform and strong there.

The relative permeability $\mu_r$ of a ferrous core (iron core) is very high. Therefore, when iron core is inserted in solenoid, it increases the strength of magnetic field many times, given by the formula

$$B_{(\text{Iron cored solenoid})} = \mu_r B_{(\text{air cored})} = \mu_r \mu_0 n I$$

Common ferrous materials used in cores include iron, silicon steel, and various iron alloys. Each of these materials has its own characteristic relative permeability.

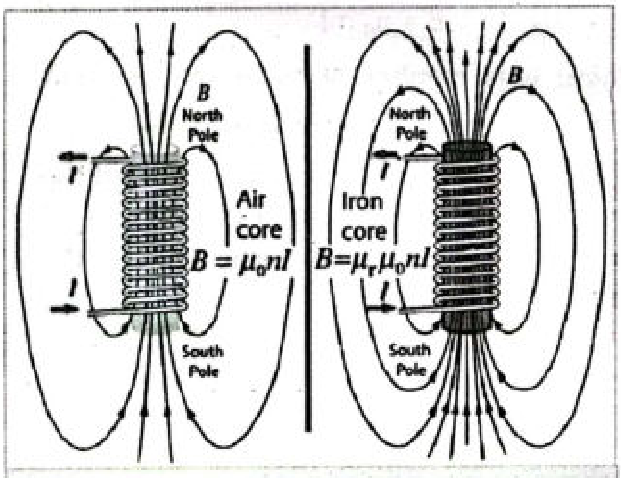



**Figure 12.18:** Magnetic field of solenoid with iron core is stronger.

## 12.9 MAGENATIC FORCES BETWEEN CURRENT-CARRYING CONDUCTORS

The attractive force between the two parallel wires carrying current in the same direction is magnetic in nature. When two parallel current-carrying conductors are placed close to each other, each conductor produces its own magnetic field. Each wire is in the magnetic field of the other, causing them to experiences a force that is at the right angle to both the current and the magnetic field. In the region between two parallel wires, the magnetic fields orient in opposite directions, which weakens the net magnetic field.

Whereas, the magnetic field on the outer sides of wires is stronger than in between the wires. Therefore, magnetic force acts on the current-carrying conductor from a stronger to a weaker magnetic field region, and two wires attract each other, as shown in Fig. 12.19 (a).

Note that, in the case of attractive force (currents in the same direction), there is a neutral point between the wires where the magnetic fields are equal in magnitude but opposite in direction. If the currents in two

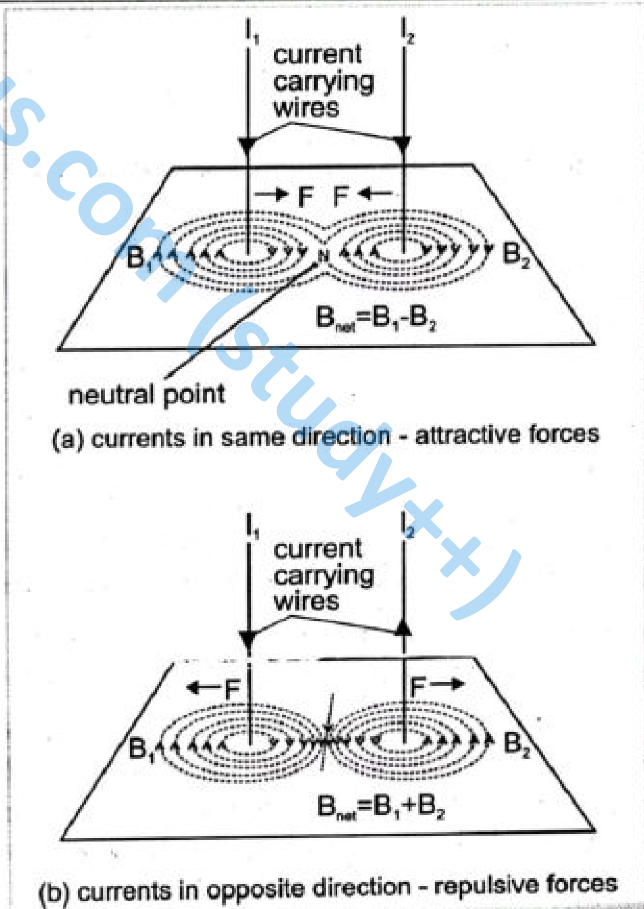



**Figure 12.19:** Interaction of two current-carrying conductor.

wires are equal in magnitude, the neutral point will be located midway between the wires (assuming the medium has uniform permeability). Otherwise, the neutral point would be closer to the wire with the smaller current.

The magnetic force acting per unit length on each current-carrying wire is repulsive in nature when current flowing in both wires is in the opposite direction.

In between two parallel wires carrying current in opposite directions, as shown in Fig. 12.19 (b), the magnetic fields of the two wires are in the same direction and support each other; therefore, the net magnetic field is stronger. The magnetic field on the outer sides of the wire is weaker as compared to the field in between the wires. Therefore, magnetic force acts on the current-carrying conductor from a stronger to a weaker magnetic field region, and two wires repel each other. Consider two wires of length L, carrying currents $I_1$ and $I_2$ placed at distance r from each other. Each wire is in the magnetic field of the other, as shown in Fig. 12.19 (c).

Magnetic field of 1st wire is given by:

$$B_1 = \frac{\mu_0 I_1}{2\pi r} \quad \text{———} \quad (12.14)$$

Magnetic field of 2nd wire is given by:

$$B_2 = \frac{\mu_0 I_2}{2\pi r} \quad \text{———} \quad (12.15)$$

Force exerted by first wire on the second wire is:

$$F_{12} = B_1 I_2 L \quad \text{———} \quad (12.16)$$

Putting value from equation (12.14) in (12.16), we get:

$$F_{12} = \left(\frac{\mu_0 I_1}{2\pi r}\right) I_2 L$$

$$\frac{F_{12}}{L} = \frac{\mu_0 I_1 I_2}{2\pi r} \quad \text{———} \quad (12.17)$$

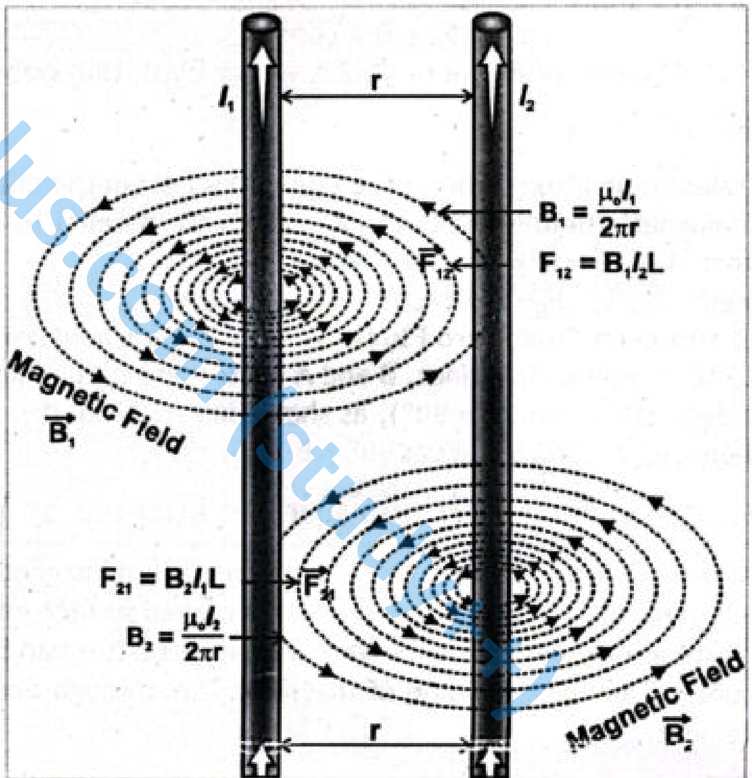



Figure 12.19 (c): Magnitude of force between two current-carrying conductor.

This is the expression for magnetic force acting per unit length of a current carrying conductor.

Similarly, force exerted by 2nd wire on the first wire is given by:

$$F_{21} = B_2 I_1 L \quad \text{———} \quad (12.18)$$

Putting value from equation (12.15) in (12.18), we get:

$$F_{21} = \left(\frac{\mu_0 I_2}{2\pi r}\right) I_1 L \quad \text{or} \quad \frac{F_{21}}{L} = \frac{\mu_0 I_1 I_2}{2\pi r}$$

So, these forces make the action and reaction pair.

## 12.10 MAGNETIC FLUX

**The number of magnetic lines of force passing through certain element of area is called magnetic flux.**

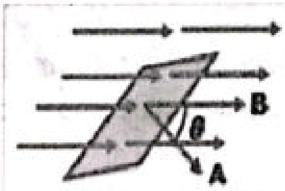



**Figure 12.20 (a):** Magnetic Flux.

Magnetic flux $\Phi_B$ is the scalar product of uniform magnetic field **B** and vector area **A**.

Mathematically: $\Phi_B$ = **B.A**

If $\theta$ is angle between magnetic field B and vector area A, as shown in Fig 12.20 (a), then magnitude of magnetic flux is:

$$\Phi_B = B\,A\cos\theta$$

**Unit:** The unit of magnetic flux is weber (Wb). One weber is given by 1 Wb = 1 N m $A^{-1}$.

### Special Cases

**(i) Maximum Flux:** If the surface (plane) is held perpendicular to magnetic field lines, **B** and **A** vectors are parallel to each other (i.e., $\theta = 0^o$), as shown in Fig. 12.20 (b).

Then, $\Phi_B = B\,A\cos 0^0 = B\,A\,(1) = B\,A$

**(ii) Minimum Flux (Zero Flux):** If the surface is held parallel to the magnetic field lines, **B** and **A** vectors are perpendicular to each other (i.e., $\theta = 90^o$), as shown in Fig. 12.20 (b).

Then, $\Phi_B = B\,A\cos 90^0 = B\,A\,(0) = 0$

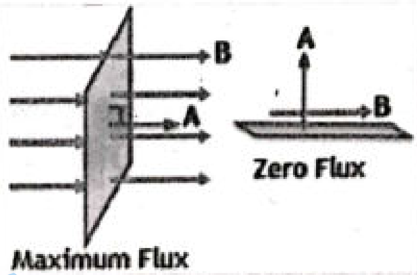



**Figure 12.20 (b): Maximum and minimum magnetic flux.**

### Magnetic Flux Through Curved Surface or Non-Uniform Magnetic Field

When a curved surface is placed in a non-uniform magnetic field (Fig. 12.20 c), then, we divide the curved surface into a number of small elements. The net magnetic flux can be found by adding the value of magnetic flux through each element.

Thus $\Phi_B = \Sigma B.\Delta A$

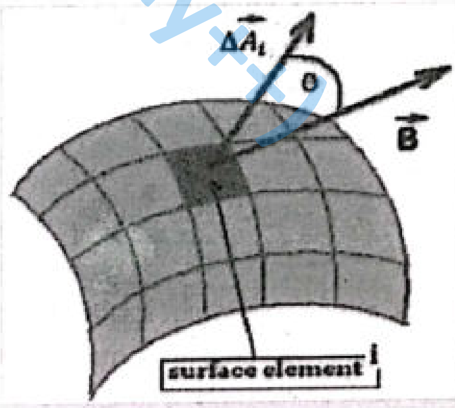



**Figure 12.20 (c): Magnetic flux** through curved surface.

**Magnetic Flux Density:** Magnetic flux density (B) is also defined as:

**Maximum magnetic flux per unit area is called magnetic flux density.**

$$B = \frac{\Phi}{A}$$

Unit of magnetic flux density is Wb $m^{-2}$ which is equal to tesla (T).

**Example12.5:** In a certain region, the magnetic field is given by $B = (4\,\hat{i} + 6\,\hat{k})$ Wb $m^{-2}$. How much magnetic flux passes through a 2.0 $m^2$ area loop in this region if the loop lies flat in the x y-plane?

Given: Magnetic induction = $B = (4\,\hat{i} + 6\,\hat{k})$ Wb $m^{-2}$ Area of the loop = $\Delta A = (2.0\,\hat{k})$ $m^2$

To Find: Magnetic flux = $\Phi_B$ = ?

Solution: As we know that: $\Phi_B = B \cdot \Delta A$

Putting the values, we get: $\Phi_B = (4\,\hat{i} + 6\,\hat{k}) \cdot (2\,\hat{k})$ = 12 Wb

Assignment: 12.5

A magnetic field of 0.8 T passes perpendicular to a disc with a radius of 2 cm. Find the magnetic flux through the disc.

## 12.11 MAGNETIC FLUX LINKAGE

Magnetic flux linkage is defined as:

**The product of the magnetic flux and the number of turns of the coil.**

It is a quantity commonly used for solenoids and coils, which are made of N turns of wire.

The magnetic flux density by a single wire is usually very low. Magnetic flux of single turn can be increased by increasing magnetic flux density and by increasing area of loop.

The magnetic flux linkage refers to the number of turns (N) on a coil multiplied by the magnetic flux (Φ) of one turn. This gives the equation:

$$N\Phi = NBA$$

If the cross-sectional area (A) of loop and magnetic flux density (B) are not perpendicular, the equation becomes:

$$\text{Magnetic flux linkage} = BAN\cos\theta$$

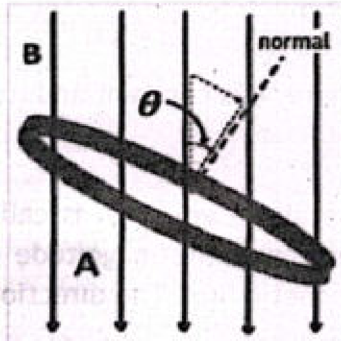



**Figure 12.21:** Magnetic flux linkage.

Where, θ is the angle between the vector area (A) and the magnetic flux density (B), as shown in Fig. 12.21.

**Units:** The flux linkage ΦN has the units of Weber turns (Wb turns).

**Example 12.6:** A solenoid with a circular cross-sectional area of 0.80 $m^2$ and 300 turns is facing perpendicular to a magnetic field with magnetic flux density of 4 mT. Determine the magnetic flux linkage for this solenoid.

Given: Cross-sectional area, A = 0.80 $m^2$ Magnetic flux density, B = 4 mT= $4 \times 10^{-3}$ T

Number of turns of the coil, N = 300 turns

To Find: ΦN = ?

Solution: Using the formula ΦN = BAN

Substitute in values: $\Phi N = (4 \times 10^{-3}\ T) \times 0.80\ m^2 \times 300 = 0.96$ Wb turns

Assignment 12.6

A solenoid contains 200 turns of wire. Each piece of wire has a cross-sectional area of $0.004\ m^2$ If the magnetic flux density is 12.0 mT calculate the magnetic flux linkage.

## 12.12 FARADAY'S LAW

The phenomenon in which the change in magnetic flux causes an induced emf in a conductor is called electromagnetic induction.

The basic requirement for electromagnetic induction is the change in magnetic flux linking the conductor (or coil).

### Faraday's Laws of Electromagnetic Induction

It states that:

The average e.m.f induced in a conducting coil of N turns is directly proportional to the rate of change of magnetic flux through the conductor.

If N is the number of turns of the coil and $\varepsilon$ is the induced e.m.f. then:

$$\text{Induced e.m.f} \propto \text{rate of change of magnetic flux}$$

$$\text{Induced e.m.f} \propto \frac{\textit{Total change in magnetic flux}}{\textit{Total time}}$$

For N turns of coil:

$$\text{Induced e.m.f} \propto N\frac{\Delta\phi}{\Delta t}$$

$$\varepsilon = k N \frac{\Delta\phi}{\Delta t}$$

Where, k is constant and its value is '1' in SI units. So,

$$\varepsilon = N\frac{\Delta\phi}{\Delta t}$$

The above equation is called Faraday's law of electromagnetic induction, and it is used to determine the magnitude of induced emf. The induced emf always opposes the change in magnetic flux. The direction of induced emf is given by Lenz's law.

$$\varepsilon = -N\frac{\Delta\phi}{\Delta t}$$

The negative sign indicates that the direction of the induced current is such that it opposes the change in flux. Faraday's law is the fundamental law that describe the production of emf due to change of magnetic flux in various circuits and devices. For example, AC devices like induction motors, induction generators or transformers, as well as DC devices like DC motors, DC generators.

**Experiment 1:** In this experiment, we use a bar magnet and a coil connected to a sensitive galvanometer, as shown in the Fig. 12.22.

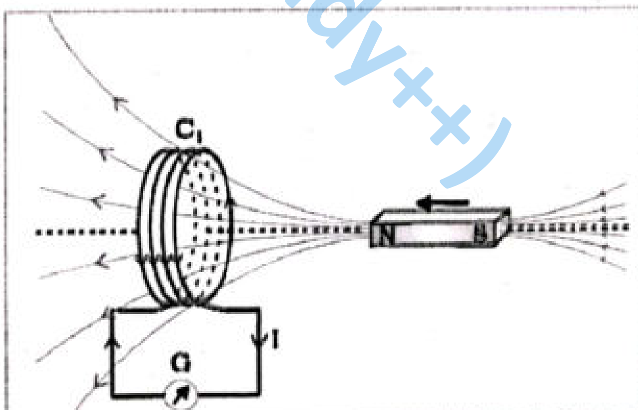



**Figure 12.22:** Magnet is moving and coil is at rest.

**Case I:** When there is no relative motion between the magnet and coil, the galvanometer shows no current.

**Case II:** When the bar magnet is moved towards the coil, the magnetic flux changes and induces emf. As a result, induced current flows in the coil.

**Case- III:** When the bar magnet is moved away from the coil, the magnetic flux again changes, inducing an emf and an induced current flow in the coil but in the opposite direction.

Experiment 2: Emf is Induced by relative motion of a coil with respect to magnetic field. In this experiment, the coil is rotated in a magnetic field. As the change in magnetic flux is given by:

$$\Delta\Phi = \Delta B(A\cos\theta)$$

During the rotation of the coil, the angle $\theta$ between the magnetic field and the vector area A of the coil changes. Therefore, the magnetic flux of the coil changes, inducing an emf in the coil. The induced current flows through the circuit, and which produces deflection in the galvanometer. This is the basic principle of an electric generator (Fig. 12.23).

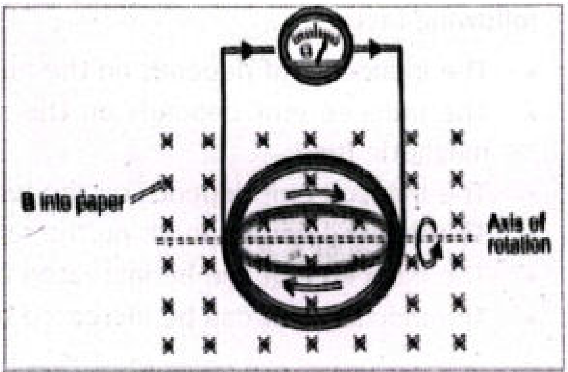



**Figure 12.23:** Coil is rotated in magnetic field.

Experiment 3: Emf is also Induced by changing a magnetic field (the transformer effect). In this experiment, two coils are placed closed to each other.

The primary coil 'P' is connected in series with the battery through a switch and rheostat, while the secondary coil S is connected only to a galvanometer. If the switch of coil P is closed, a momentary current is induced in the coil S. When the switch P is opened, a current is produced in the coil S in opposite direction. When we open or close the switch then there will be a change of current, which causes to changing magnetic field of coil P and the magnetic flux through coil S changes. The changing magnetic flux induces emf in coil S, as shown in Fig. 12.24.

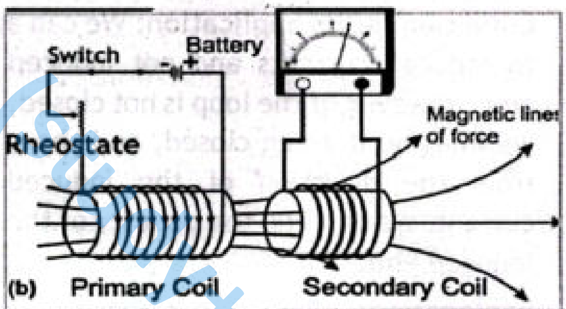



**Figure 12.24:** By changing magnetic field of primary coil, the magnetic flux through secondary coil changes.

If the current in the primary coil is varied with the help of a rheostat, then the magnetic field of coil P and the magnetic flux through coil S change. The changing magnetic flux induces the emf in it. The induced emf and induced current will remain in the coil as long as the magnetic flux through it changes. This is the basic principle of the working of a transformer.

- The electromotive force, or emf is induced when the magnetic flux linking with a coil changes (the magnetic flux either increases or decreases).
- A constant magnetic flux cannot produce emf in a conductor. The cause of induced emf is a change in magnetic flux.

- The magnitude of induced emf depends on the rate of change of magnetic flux.
- The induced emf is independent of the resistance and resistivity of the circuit and depends *only on the rate of* change of the magnetic flux through the conductor.
- Induced current ($I = \varepsilon/R$) depends on the resistance and resistivity of the circuit.
- When the magnetic flux through a conductor changes, an induced emf always produces, but induced current only flows when the circuit is closed.

Factors Affecting the Magnitude of the Induced emf: The induced emf is affected by the following factors:

- The induced emf depends on the number of turns in a coil.
- The induced emf depends on the speed of the movement of the conductor through the magnetic field.
- The induced emf depends on the length of the conductor inside the magnetic field.
- The induced emf depends on the rate of change of magnetic flux through the conductor.
- The induced emf can be increased by increasing the strength of external magnetic field.
- The induced emf can be increased by increasing the area of coil.

## 12.13 LENZ'S LAW

In 1834, Russian physicist Heinrich Lenz discovered that the polarity of induced emf always leads to an induced current that opposes the change which induces the emf.

The direction of the induced current is always such as to oppose the change that causes the current.

**Condition for its application:** We can apply Lenz's law directly to closed loops because it refers to induced currents and not induced emf. However, if the loop is not closed, we imagine it being closed, and then, from the direction of the induced current, we can find the polarity of the induced emf.

**Explanation:** When a bar magnet is pushed towards a coil connected to a galvanometer then emf is induced due to change in its magnetic flux and induced current flows through it, as shown in Fig. 12.25 (a).

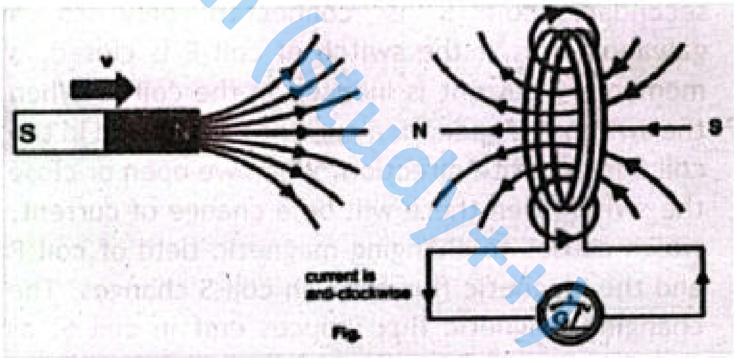



**Figure 12.25 (a):** The bar magnet is pushing towards the coil.

Direction of current in the coil is determined by Lenz' s law. It tells us that induced current opposes its cause. In this experiment, cause of induced current is push of the N- pole of magnet towards the coil (Which increases the magnetic flux through the coil). Induced current of the coil induces such magnetic field which opposes the push of the magnet towards the coil and hence oppose the increase of magnetic flux through the coil. It will be only possible if left side of the coil acts as N-pole and right side acts as S-pole of a bar magnet, as shown in Fig. 12.25(a). Now, by applying Right Hand Rule, we see that direction of current in the coil is anti-clockwise.

Similarly, if we 'pull' the magnet away from the coil, the induced current opposes the 'pull' by creating the south pole towards the bar magnet according to Lenz's law, as shown in Fig. 12.25 (b). When a magnet is moved away from the coil, flux decreases through the coil. To oppose this pull of the magnet and decrease in magnetic flux through the coil, the coil induces such current in it that attracts the bar magnet. For this, the left side of the coil acts as the S-pole and the right-side acts as N-pole. The S-pole of the coil attracts the N-pole of the bar magnet and opposes its motion. For this, direction of induced current in the coil reverses, i.e., current flows clockwise as according to Right Hand Rule.

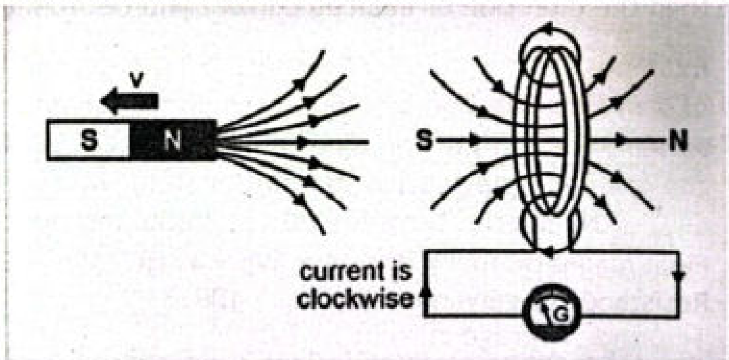


**Figure 12.25 (b):** The bar magnet is pulling away from the coil.

## Lenz's Law and Law of Conservation of Energy

The law of conservation of energy states that energy can neither be created nor destroyed, but it can be changed from one form to another.

Lenz's law states that the direction of current is such that it opposes the change in the magnetic flux. So, an effort is required to do work against opposing forces. This work leads to changes in magnetic flux; hence, electric current is induced. Thus, mechanical energy is converted into equivalent amount of electrical energy, which is in accordance with the law of conservation of energy.

Let's exaplain further with the help of an example: consider that N-pole of a magnet is approaching the coil, as shown in Fig. 12.26, the repulsive force acts on the bar magnet due to the current induced in the coil. The result is that the motion of the magnet is opposed. The mechanical energy spent in overcoming this opposition is converted into electrical energy, which appears in the coil. We have to spend mechanical energy to induce electrical energy. Thus, Lenz's law is in accordance with the law of conservation of energy.

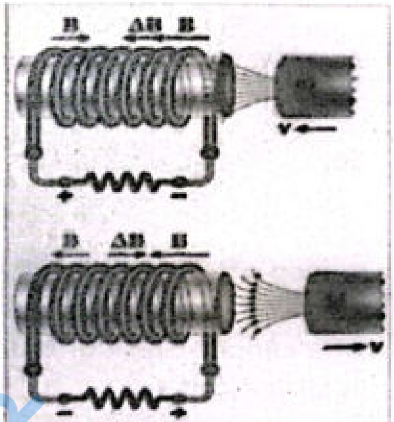


**Figure 12.26:** Lenz's law.

**Fleming's Right Hand Rule:** To find the direction of the induced current, Fleming's right-hand rule may be used, as shown in Fig. 12.27. It is stated as:

**Stretch out of forefinger, middle finger and thumb of your right hand so that they are at right angles to one another.**

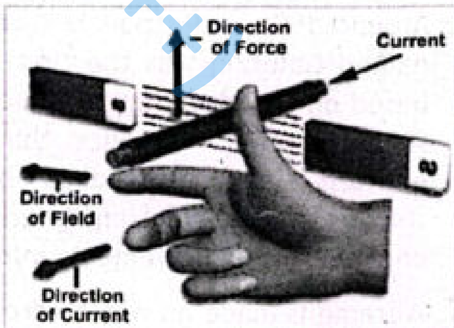


**Figure 12.27:** Fleming's right-hand rule to find the direction of the induced current.

**If the forefinger points in the direction of magnetic field, thumb in the direction of motion of the conductor, then the middle finger will point in the direction of induced current.**

When the conductor moves upwards at right angle to a stationary magnetic field then the direction of induced current is from right to left. If the motion of the conductor is downward, then the direction of induced current will be from left to right.

**Example 12.7: A coil of resistance 50 Ω is placed in magnetic field and initial magnetic flux of 2 m Wb. The coil has 50 turns and a galvanometer of 100 Ω resistance is connected in series with it. Find the average emf and the current if the coil is moved in 1/10th s from the given magnetic field to another field where final magnetic flux of 4 m Wb.**

Given: Number of turns N = 50 Initial magnetic flux = $\Phi_1$ = 2m Wb = $2 \times 10^{-3}$ Wb

Final magnetic flux = $\Phi_2$ = 4 m Wb = $4 \times 10^{-3}$ Wb Resistance of coil = $R_1$ = 50 Ω

Resistance of galvanometer $R_2$ = 100 Ω Time taken for change $\Delta t$ = 1/10 s = 0.1 s

To Find: Average current I = ?

Solution: By Faraday's law magnitude of induced emf is

$$\varepsilon = N\frac{\Delta\phi}{\Delta t} \quad \text{or} \quad \varepsilon = N\left(\frac{\phi_2-\phi_1}{\Delta t}\right)$$

Putting values: $$\varepsilon = 50 \times \left(\frac{4\times10^{-3}-2\times10^{-3}}{0.1}\right) = 1\text{ V}$$

As, galvanometer is connected in series. So $R_{eq} = R_1 + R_2$

$$R_{eq} = 50 + 100 = 150\ \Omega$$

So, $$I = \frac{V}{R_{eq}} = \frac{\varepsilon}{R_{eq}} \quad \text{or} \quad I = \frac{1}{150} = 6.67\times10^{-3}\text{ A} = 6.67\text{ mA}$$

**Assignment 12.6**

**A loop of wire is placed in a uniform magnetic field that is perpendicular to the plane of the loop. The strength of the magnetic field is 0.6 T. The area of the loop begins to shrink at a constant rate of 0.8 $m^2 s^{-1}$. What is the magnitude of emf induced in the loop while it is shrinking?**

## 12.14 SEISMOMETER

Around 1906, a Russian Empire nobleman and inventor named Gallitzin was the first to create a seismometer based on Faraday's law of electromagnetic induction. A seismometer is a device that records seismic waves emitted during earthquakes, explosions, or other earth-shaking events. Electromagnetic sensors in seismographs convert ground movements into electrical signals.

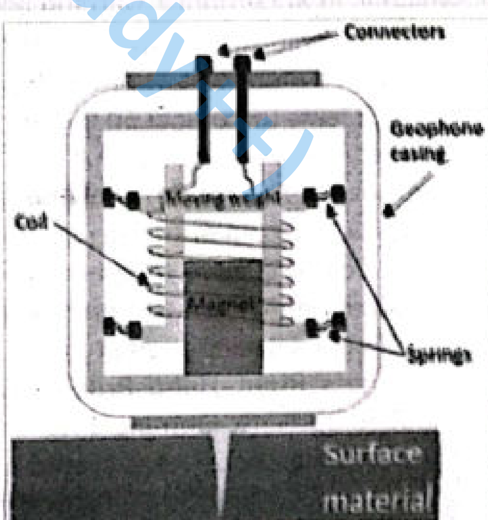



Figure 12.28: Seismometer.

A frame is made on which wire is wound and the ends of this frame are connected with springs tied with side walls, as shown in Fig. 12.28. A permanent magnet is placed between this frame and its coil in such a way that coil can oscillate freely.

When the Earth vibrates due to earthquake, there is relative motion between magnet and the coil. This causes change in magnetic flux through the coil. It induces emf and current in the coil according to Faraday's law of electromagnetic induction. This induced current is given to external circuit where values are recorded and graph is plotted.

During a high-intensity earthquake, the relative velocity between the coil and magnet increases, and the amplitude of the vibration of the coil increases.
The induced emf in the coil is directly proportional to the displacement of the coil within the magnetic field.
When the coil moves faster, the rate of change of displacement of the coil in a magnetic field increase; as a result, the rate of change of magnetic flux of the coil increases, and induced emf increases.

## SUMMARY

- The force on a charge moving inside a magnetic field result from interaction between the magnetic field produced by the moving charge (which is equivalent to current) and the applied magnetic field.
- When a charge moves in magnetic field in circular path, its speed and K.E remain same but its velocity and momentum change due to change in direction of motion
- For a point on the axis of a circular coil carrying current, magnetic field is maximum at the centre of the coil.
- The magnetic field is the region or space surrounding a magnet in which a compass needle, a small magnet, another ferromagnetic material, and a moving charged particle experiences a magnetic force.
- Magnetic flux linkage is defined as the product of the magnetic flux and the number of turns of the coil.
- Faraday's law states that 'the magnitude of induced e.m.f. is directly proportional to the rate of change of magnetic flux linking the coil and the number of turns of the coil N.
- Lenz's Law states that the direction of the induced current is always such as to oppose the change that causes the current.

## EXERCISE

### Multiple Choice Questions

Encircle the correct option.

1) Which of the following is the SI unit of magnetic induction?

A. henry B. tesla C. weber D. farad

2) A 0.5 m long straight wire carrying a current of 3.2 A kept at right angle to a uniform magnetic field of 2.0 tesla. The force acting on the wire will be:

A. 2 N B. 2.4 N C. 1.2 N D. 3.2 N

3) An electron and proton enter a magnetic field with same velocity, which of the following is the ratio of their acceleration?

A. $\frac{m_e}{m_p}$ B. $\frac{m_p}{m_e}$ C. $\left(\frac{m_e}{m_p}\right)^{\frac{1}{2}}$ D. $\left(\frac{m_p}{me}\right)^{\frac{1}{2}}$

4) If charge particle enters a magnetic field an angle $60^o$, then its path will be:

A. elliptical B. straight line C. helical D. circular

5) An electron is revolving in circular path of radius r in a magnetic field. If the magnetic field is halved, then new radius will be:

A. r/2 B. r/4 C. 2r D. 4r

6) Which of the following particles in motion cannot be deflected by magnetic field?

A. electron B. proton C. alpha-particle D. neutron

) A proton and an $\alpha$-particle moving with same velocity enter a magnetic field perpendicularly. Which one of the following is the ratio of radius of circular path of proton to $\alpha$-particle?

A. 1/2 B. 1 C. 2 D. 4

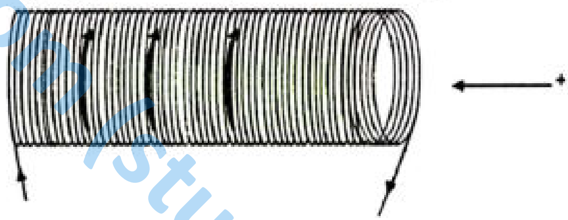
8) A charged particle is moving in uniform magnetic field in a circular path. The radius of circular path is R. If the energy of the particle is doubled, then the new radius will be:

A. $\frac{R}{\sqrt{2}}$ B. $\frac{R}{2}$ C. $\sqrt{2}\,R$ D. 2R

9) A positively charged particle moving with velocity of $3.1\times 10^5$ m/s normal to uniform magnetic field of magnitude $5.4\times10^{-5}T$. Particle experiences force of $8.1\times10^{-16}$ N, the charge on particle is:

A. $4.8\times10^{-10}$ C B. $1.6\times10^{-19}$ C C. $6.4\times10^{-17}$ C D. $4.8\times10^{-17}$ C

10) A proton is moving along the axis of a solenoid carrying a current, as shown in figure. The magnetic force on proton will be:




A. radially inward. B. radially outward.

C. no force acts. D. radially outward.

11) Magnetic flux will be maximum when:

A. magnetic field is perpendicular to the plane area.

B. magnetic field lies parallel to the plane area.

C. area is held at an angle of $45^o$.

D. area is held at an angle of $60^o$.

12) The cause of induced emf is:

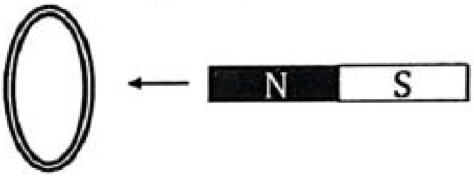
A. constant magnetic flux. B. increase in magnetic flux only.

C. decrease in magnetic flux only. D. change in magnetic flux.

13) Lenz's law is in accordance with the law of conservation of:

A. charge B. energy C. momentum D. angular momentum

14) A metallic circular ring is suspended by a string and is kept in a vertical plane. When a magnet is pushed towards the ring, then it will:




A. get displaced towards the magnet.

B. remains stationary.

C. get displaced away from the magnet.

D. nothing can be said.

## Short Questions

Give short answers of the following questions.

12.1 What are the various ways to create a magnetic field.

12.2 In what way is a magnetic field different from an electric field?

12.3 Why does a current carrying conductor may experience force in magnetic field?

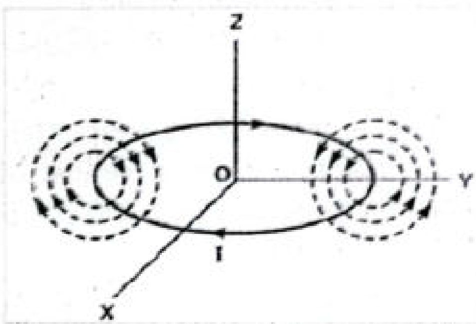
12.4 Can a charged particle move in a straight line through some region of space, in which magnetic field is non-zero? Explain.

12.5 When is the magnetic force on a moving charge in a magnetic field maximum, and when is it minimum?

12.6 In a region with a homogeneous magnetic field B directed normal to the plane into the paper, an alpha particle and a proton are travelling in its plane. What will be the ratio of the radii of the two particles' field trajectories if they have equal linear momenta?

12.7 The kinetic energy of a charged particle moving in uniform magnetic field does not change. Why? Explain

12.8 Describe the changes in the magnetic field inside a solenoid carrying a constant current I under the conditions of (a) doubling the length of the solenoid while maintaining the same number of turns, and (b) doubling the number of turns while maintaining the same length.




12.9 Explain how 1 ampere is defined using concept of force between two long parallel current-carrying wires.

12.10 A circular loop of radius R carrying current I lies in the XY-plane with its centre at origin (as shown in figure). What is the magnetic flux through the XY-plane?

12.11 A suspended magnet is vibrating freely in a horizontal plane. When a metal plate is brought beneath the magnet, the oscillations of the magnet are significantly dampened. Describe the cause of damping by using Lenz's law.

12.12 A thin metallic ring is dropped into a vertical bar magnet. Does current in the ring flow clockwise or counterclockwise when seen from above?

12.13 Show that $\varepsilon$ and $\frac{\Delta\phi}{\Delta t}$ have the same units.



## Comprehensive Questions

Answer the following questions in detail.

12.1 Explain that magnetic field is an example of a field of force produced either by current-carrying conductors or by permanent magnets.

12.2 Explain that a magnetic force act on a current-carrying conductor placed in a magnetic field. Derive an expression for it.

12.3 Explain that a magnetic force act on a charged particle in a uniform magnetic field.

12.4 Describe the concept of magnetic flux ($\Phi_B$) as scalar product of magnetic field (B) and area (A) using the relation $\Phi_B$ = B.A

12.5 Describe quantitatively the path followed by a charged particle shot into a magnetic field in a direction perpendicular to the field.

12.6 Describe the concept of magnetic force between current-carrying conductors. Derive an expression for the magnetic force acting per unit length on each current-carrying parallel conductor when current is flowing through them.

12.7 State and explain Faraday's law of electromagnetic induction.

12.8 Describe how the direction of an induced current can be predicted by Lenz's law and how Lenz's law obeys the principle of conservation of energy.

## Numerical Problems

12.1 A horizontal power line carries a current of 600 A from south to north. Earth's magnetic field $5\times 10^{-5} T$ is directed toward the north and inclined downward at 70° to the horizontal. Find the magnitude of the magnetic force on 100 m of the line due to Earth's magnetic field.

**(Ans: 2.82 N)**

12.2 A proton is accelerated in a cyclotron in which the magnetic induction is 0.75 Wb $m^{-2}$. Find the cyclotron frequency. **(Ans: $11.4\times 10^6$ Hz)**

12.3 A velocity selector consists of electric and magnetic fields described by the expressions E = E $\hat{k}$ and B = B $\hat{j}$. With B = 18.0 mT. Find the value of E such that electron of energy 850 eV moving in the negative $x$ direction is undeflected. **(Ans: $311\times 10^3$ N $C^{-1}$)**

12.4 Two long straight parallel wires are 0.6 m apart and carry current 4 A and 8 A. Find the point lying between two wires at perpendicular distance where resultant magnetic field will be zero (neutral point). **(Ans: 0.2 m and 0.4 m respectively)**

12.5 A coil of wire has 100 loops. Each loop has an area of $2.0\times 10^{-3} m^2$. A magnetic field is perpendicular to the surface of each loop at all times. If the magnetic field is changed from 0.2 T to 0.6 T in 0.1 s, find the average emf induced in the coil during this time. **(Ans: 0.8 V)**

12.6 A 2 m long wire carrying a current of 15 A is placed in a uniform magnetic field of 0.50 T. If the wire makes an angle of 60° with the direction of magnetic field, find the magnitude of the magnetic force acting on the wire. **(Ans: 12.99 N)**

12.7 A particle carrying a charge of $1\,\mu C$ and moving with velocity of $2\times 10^6$ m $s^{-1}$ enters in a magnetic field of 0.5 T at angle of $60^0$ with the field. Calculate the magnitude of magnetic force acting on it. **(Ans: 0.866 N)**

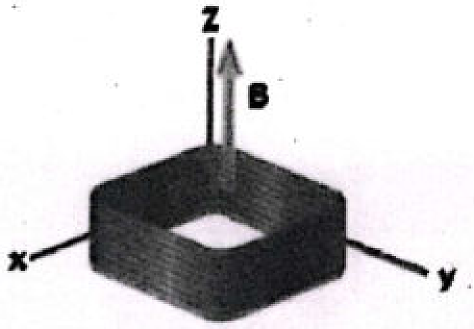
12.8 A single circular loop has radius 2 cm. The plane of the loop lies at 40° to a uniform magnetic field of 0.2 T. Find the magnitude of the magnetic flux through the loop.

**(Ans: $1.62\times 10^{-4}$ Wb)**

12.9 Find the magnetic force acting on a charged particle of charge $2\,\mu C$ in a magnetic field of 2 T acting in y-direction when particle velocity is $(2\hat{i}+3\hat{j})\times 10^6 ms^{-1}$.

**(Ans: 8 N, +ve Z-axis)**

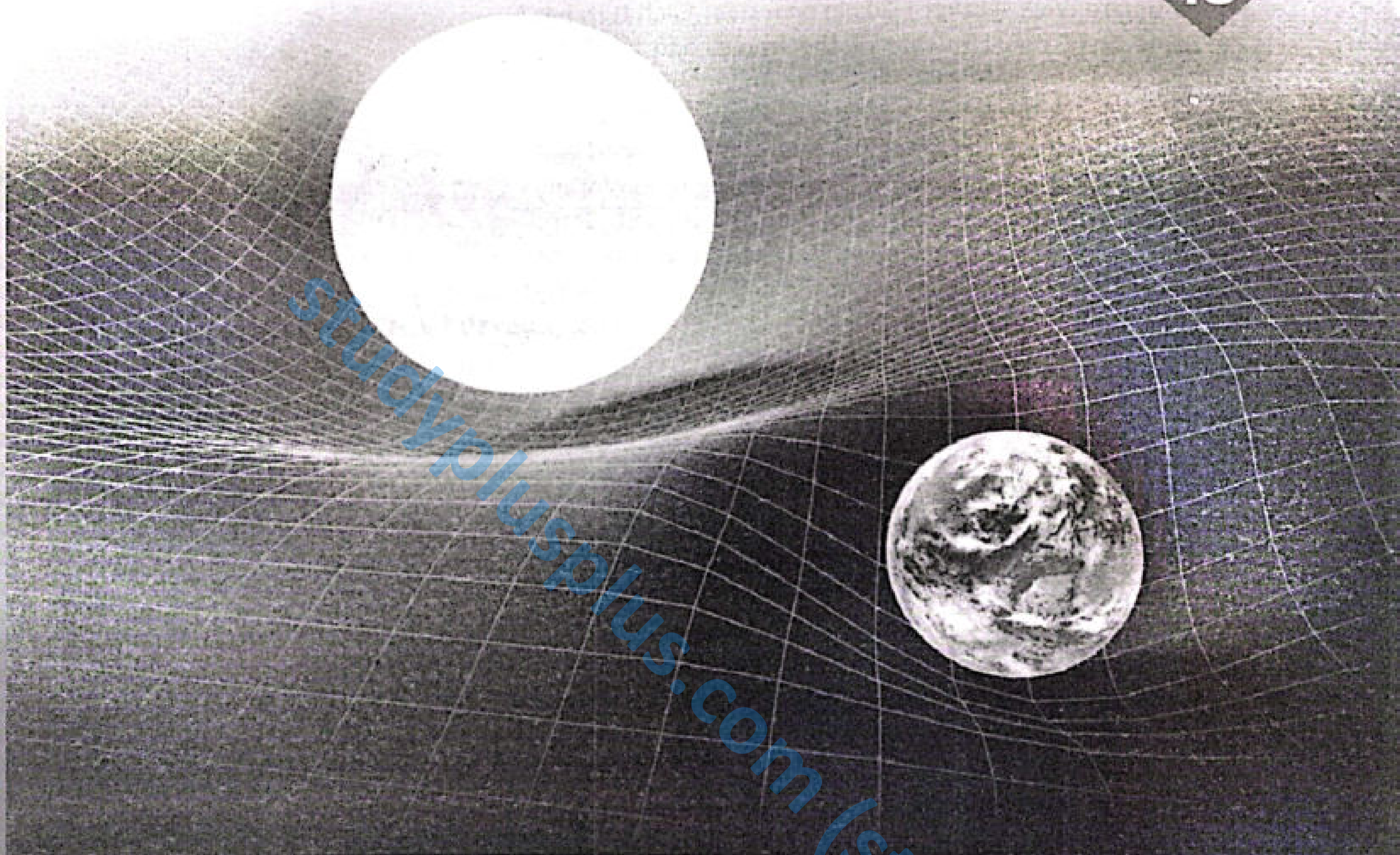
12.10 A coil with 25 turns of wire is wrapped on a frame with a square cross section 1.80 cm on a side. Each turn has the same area, equal to that of the frame, and the total resistance of the coil is 0.350 ohm. An applied uniform magnetic field is perpendicular to the plane of the coil, as in the Fig. (a) Applying Faraday's law find the induced emf in the coil when the magnetic field is changing uniformly from zero to 0.50 T in 0.80 s. (b) find the magnitude of induced current and (c) Applying Lenz's law find the direction of the induced current in the coil while the field is changing.




**(Ans: $5.06 \times 10^{-3}$ V, $1.45 \times 10^{-2}$ A, clockwise)**

# RELATIVITY

UNIT 13





## Student Learning Outcomes (SLOs)

**The students will:**

- distinguish between inertial and non-inertial frames of reference.
- describe the significance of Einstein's assumption of the constancy of the speed of light.
- Describe that if c is constant then space and time become relative.
- State the postulates of special relativity.
- Explain qualitatively and quantitatively the consequences of special relativity.
- [Specifically in the case of:
  a- the relativity of simultaneity.
  b- the equivalence between mass and energy.
  c-length contraction.
  d- time dilation.
  e-mass increase.]
- State that spacetime is a mathematical model in relativity that treats time as a fourth dimension of the traditional three dimensions of space. (It can be thought of as a metaphorical sheet of paper that can bend, and when it bends it can cause effects such as stretching and compression seen when gravitational waves pass through objects).

Albert Einstein's theory of relativity transformed the theoretical physics and astronomy during the 20th century by superseding a 200-years-old theory of mechanics created by Isaac Newton. The ideas behind relativity might seem mysterious, as they introduced new concepts such as four-dimensional spacetime, relativity of simultaneity, time dilation, and length contraction.

The theory of relativity, given by Albert Einstein, includes two theories: special relativity and general relativity. Albert Einstein published special theory of relativity and general theory of relativity in 1905 and 1915, respectively. Special relativity applies to all physical phenomena in the absence of gravity. General relativity explains the law of gravitation and its relation to the forces of nature. Einstein explained the situations in which Newtonian physics might fail to deal successfully with phenomena, and in doing so proposed revolutionary changes in human concepts of time, space and gravity.

Over the last century, many experiments have confirmed the validity of both special relativity and general relativity. It includes the observation that light deflect from distant stars as the starlight passed by our sun, proving that gravity distort or curve space. Instead of being an invisible force that attracts objects to one another, gravity is a curving or warping of space. The more massive an object, the more it warps the space around it, as shown in the cover picture of this unit.

The special theory of relativity became an important and necessary tool in the new fields of nuclear physics, atomic physics and quantum mechanics. Nuclear power plants and nuclear weapons etc., would be impossible without the knowledge that matter can be transformed into energy. So, the theory has an enormous impact on the modern world. In this unit, we shall discuss some important aspects of special relativity.



## 13.1 FRAME OF REFERENCE

A frame of reference is a coordinate system that can be used to determine positions and velocities of objects with in that frame.

For example, when a ball rolls in a street, you can say that the ball is moving because the frame of reference is the streets, whatever may be on the side of the street or the Earth itself. The origin, orientation and scale of a reference frame are specified by a set of geometric points whose positions are identified both mathematically and physically. All measurements of motion are made relative compared to a frame of reference. Different frames of reference can move relative to one another.

Consider two persons A and B are seated in a bus and the bus is moving with a velocity v, as shown in Fig. 13.1. If we ask to A about the velocity of B, he will say that B is at rest. But if we ask the same question to a person C standing on ground, road side. He will say that B is moving with a velocity v in the positive X direction. So, before specifying the velocity we have to specify the frame of reference.

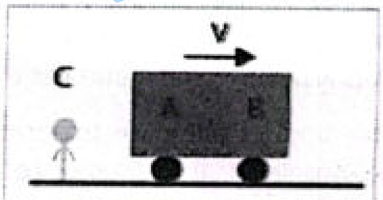



**Figure 13.1:** two persons A and B are seated in a bus and the bus is moving with a velocity v.

Frame of reference are of two types: Inertial frame of reference and non-inertial frame of reference.

## Inertial Reference Frame

An inertial reference frame is a frame of reference in which Newton's law holds true: i.e., an object at rest remains at rest, and object in motion remains in motion with a constant velocity on a straight line, unless acted upon by an external force.

A frame of reference that is at rest or moving with a uniform velocity along a straight line is called an inertial frame of reference.

An inertial frame of reference does not accelerate. Any reference frame that moves with constant velocity relative to an inertial frame is itself inertial. The Earth can be considered an inertial reference frame for many experiments. The interior of a car moving along a road at constant velocity and the interior of a stationary house are examples of inertial reference frames.

## Non-inertial Reference Frame

In a non-inertial frame of reference, objects experience acceleration even in the absence of applied forces because the reference frame itself is accelerating relative to an inertial frame. Newton's laws do not hold in a non-inertial frame of reference.

A frame of reference that is accelerating is called a non-inertial frame of reference.

An accelerating car is example of non-inertial reference frame.

# 13.2 SPECIAL THEORY OF RELATIVITY

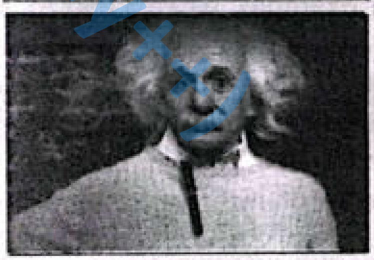
In 1905, Albert Einstein introduced his famous theory of relativity. This theory deals with the structure of space-time. Einstein's special theory of relativity is based on two postulates:

### 1) The Principle of Relativity

The laws of physics are the same for all observers in any inertial frames of reference. As long as an object is moving in a straight line at a constant speed (i.e., with zero acceleration), the laws of physics are the same for every observer.

This implies that the experiments performed in stationary and moving inertial frames of reference yield the same results. For example, it is impossible to determine experimentally whether an inertial reference frame is stationary or moving without observing it from an external frame of reference.

Albert Einstein
(1879-1955)



Albert Einstein was a German-born theoretical physicist. He was one of the greatest and most influential scientists of all time.

Consider two individuals; one standing in a stationary train and the other is standing in a train moving at constant velocity v, as shown in Fig. 13.2. Both are observing the free fall motion of a ball under the action of gravity.

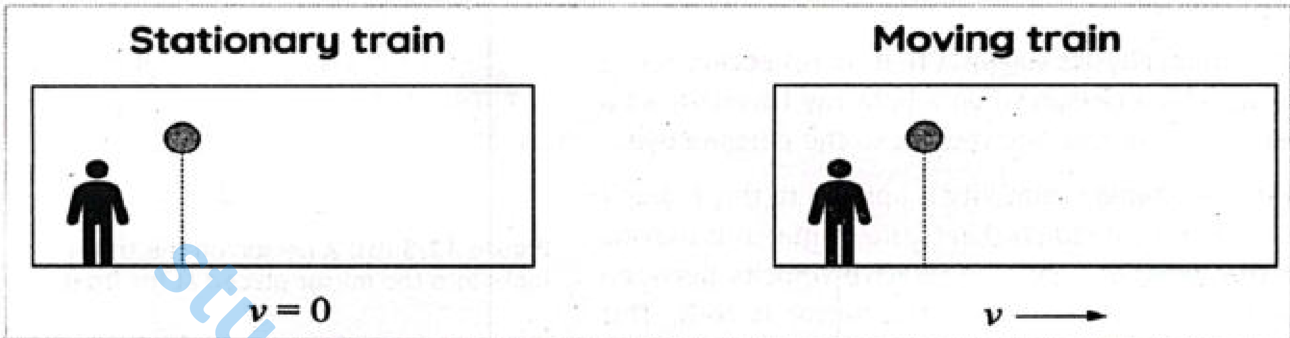



**Figure 13.2: Freely falling ball under the action of gravity in inertial frame.**

Both the individuals are in inertial frames of reference, where Newton's laws apply equally for both of them. Both persons will observe the ball falling in a vertical path. If the trains are windowless, the passengers will have no way of knowing whether their train is stationary or moving at constant velocity.

### 2) The Principle of the Constancy of the Speed of Light

The speed of light in vacuum is always constant for all observers, regardless of the relative velocity between the source of light and the observer measuring it. No matter how fast an observer or the light source is moving, measured speed of light (c) always yields the same value i.e., c = 299,792,458 m $s^{-1}$.

This postulate has profound implications, particularly as it contradicts Newtonian mechanics. For instant, you were travelling at 0.5c in the same direction as the beam of light, Newtonian mechanics predicts that you would observe the speed of the light beam to be 0.5c. However, according to special relativity, you would still observe the speed of light to be at c. This leads to a range of fascinating effects.

When a stationary person measures the speed of light emitted from a train at rest, the speed is simply c. However, the speed of light remains c even when the train is moving relative to the person, as shown in the Fig. 13.3 (a).

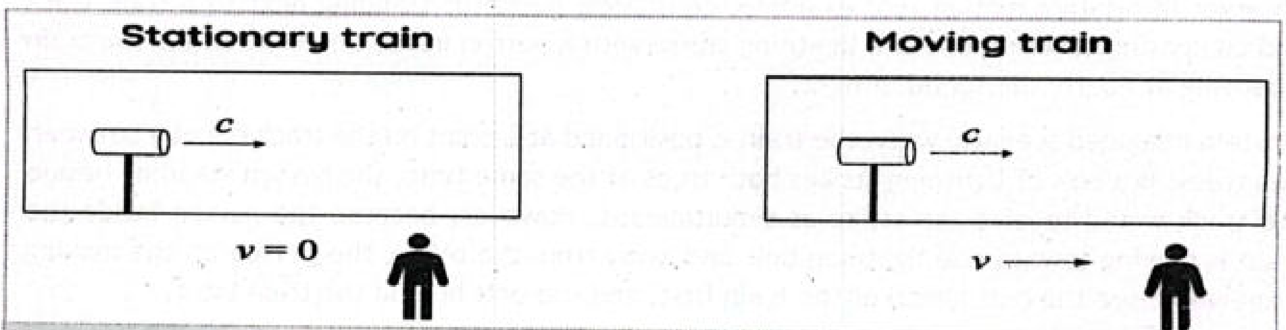



**Figure 13.3 (a): The principle of the constancy of speed of light.**

If a person travelling on this moving train looks into the mirror placed at his front, as shown in Fig. 13.3 (b). Will this person see a reflection of themselves in the mirror?

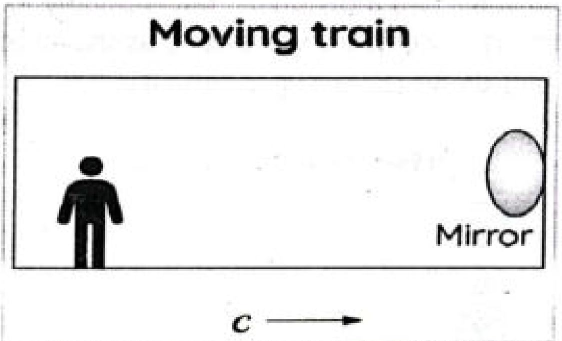



**Figure 13.3 (b):** A person on the train looks into the mirror placed at his front.

Newtonian Physics suggests that no reflection would be seen by a person when a light ray travels from a person to a mirror and returns to the person's eyes.

When Newtonian relativity is applied to this thought experiment as depicted in figure, a mirror is moving at the speed of light, the relative velocity between light from the person and the mirror is zero. This implies the light will never reach the mirror and thus person would not see a reflection.

However, this outcome contradicts the first postulate of special relativity which states that a person in an inertial frame of reference, such as this one, will not be able to identify whether the frame of reference is stationary or moving at a constant speed. If the person cannot see their reflection in the mirror, it would suggest that the train must not be stationary, as a reflection would be visible if were the case.

Special relativity, however, asserts that a reflection is indeed seen. The contradiction derived from Newtonian mechanics in this thought experiment supports the constancy of light's speed. The second postulate of special relativity states that light's speed is constant in a vacuum for all inertial frames of reference. This means that from the perspective of the person and mirror in the moving train, light's relative velocity is still c. as a result, the light will reach the mirror and be reflected back to the person's eyes. Therefore, the person will see a reflection of themselves in the mirror.

## 13.3 CONSEQUENCES OF THE SPECIAL THEORY OF RELATIVITY

There are other surprising consequences of the special theory of relativity. These consequences of the special theory of relativity are summarized in the following.

### Relativity of simultaneity

Two events that are simultaneous for one observer, may not be simultaneous for another observer in relative motion. For example, consider a person is standing next to a train track and comparing observations of a lightning storm with a person inside a moving train. The train is moving at nearly the speed of light.

Einstein imagined scenario when the train is positioned at a point on the track equally between two trees. If a bolt of lightning strikes both trees at the same time, the person standing beside the track would receive the strike as simultaneous. However, because the person inside the train is moving toward one lightning bolt and away from the other, the person on the moving train would see the bolt ahead of the train first, and the bolt behind the train later.

Einstein concluded that simultaneity is not absolute. In other words, events that appears simultaneous to one observer may occur at different times for another observer. He realized that it is not light speed that changes, but time itself that is relative. Time moves differently for objects in motion compared to those at rest. However, the speed of light remains constant and is observed to be the same by anyone, anywhere in the universe, regardless of their motion.

### Mass-Energy Equivalence

Energy and mass are equivalent and transmutable, expressed by the equation:

$$E = mc^2 \quad \text{\_\_\_\_\_ (13.1)}$$

where E is Energy, m stands for mass and c for the velocity of light. Many scientists observed that the object's mass increases with the velocity but never knew how to calculate it. This equation is the answer to their problem, which explains that the increased relativistic weight of the object is equal to the kinetic energy divided by the square of the speed of light. As, the speed of light is too high, so a tiny amount of mass is equivalent to a very large amount of energy. That's why atomic and hydrogen bombs are so powerful. The concept of mass defect in atomic nucleus is also justified by the expression of mass-energy equivalence.

### Length Contraction

Objects appear shorter in the direction of their motion relative to the observer. The length of an object measured in its rest frame is called the ***proper length*** ($L_o$). Other observers in different reference frames, which are in relative motion will always measure the length (L) to be shorter. This phenomenon is described by the following equation:

$$L = L_o\sqrt{1-\frac{v^2}{c^2}} \quad \text{\_\_\_\_\_ (13.2)}$$

This effect, known as ***length contraction*** occurs only in direction of motion.

### Time Dilation

One of the key consequences of Einstein's special relativity is that time is experienced differently by moving objects. An object in motion undergoes time dilation, meaning that it experiences time more slowly compared to when it is at rest. Moving clocks are observed to tick more slowly than clocks that are stationary from the observer's perspective.

The time taken for an event to occur within its rest frame is called ***proper time*** ($t_o$). Observers in different reference frames in a relative motion will always measure the lapsed time taken (t) to be longer. This is expressed by the equation:

$$t = \frac{t_o}{\sqrt{1-\frac{v^2}{c^2}}} \quad \text{\_\_\_\_\_ (13.3)}$$

### Mass Variation

**The mass of a moving object increases as its velocity increases. This phenomenon is known as mass variation, is another expression of mass-energy equivalence. It is represented mathematically as:**

$$m = \frac{m_o}{\sqrt{1-\frac{v^2}{c^2}}} \quad \text{——— (13.4)}$$

Where, m is the relativistic mass, $m_o$ is the rest mass of particle, c is the speed of light and v is the velocity of the particle relative to a stationary observer.

This effect becomes noticeable only at relativistic speeds (speed close to the speed of light). As an object is accelerated closer to the speed of light, its mass increases. The more massive it becomes, the more energy is required to achieve the same acceleration, making further acceleration more and more difficult. The energy that is put into attempting acceleration is instead converted into mass. The total energy of an object is kinetic energy plus the energy embodied in its mass. To accelerate even the smallest object to the speed of light would require an infinite amount of energy. Therefore, material objects are restricted to speeds less than the speed of light.

**Example 13.1: If a 0.5 kg body is moving at a speed of equivalent to 90 % of the speed of light. What will be its mass in this situation?**

Given: **v = 90 % of c = 0.9c** **$m_o$ = 0.5 kg**

To Find: **m = ?**

Solution: **Using the relation:** $m = \frac{m_o}{\sqrt{1-\frac{v^2}{c^2}}}$

**Putting values, we get:** $m = \frac{0.5}{\sqrt{1-\frac{(0.9c)^2}{c^2}}} = 1.15 \text{ kg}$

**Example 13.2: At what speed would the mass of proton be tripled? The rest mass of a proton is 1.673×l 0–27 kg.**

Given: **$m_o = 1.673\times10^{-27}$ kg**

To Find: **v = ?**

Solution: **Using the relation:** $m = \frac{m_o}{\sqrt{1-\frac{v^2}{c^2}}}$

**Putting the given values, we get:** $3\,m_o = \frac{m_o}{\sqrt{1-\frac{v^2}{c^2}}}$

$$3 = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$$

$$\sqrt{1-\frac{v^2}{c^2}} = \frac{1}{3}$$

squaring both sides, we get:

$$1-\frac{v^2}{c^2} = \frac{1}{9} \quad \text{or} \quad \frac{v^2}{c^2} = 1-\frac{1}{9} = \frac{8}{9}$$

$$v^2 = \frac{8}{9}c^2 \quad \text{or} \quad v = \frac{\sqrt{8}}{3}c = 0.9428c$$

**Assignment 13.1**

**1) What is the speed of a rod relative to the observer, if the length of rod is measure to be half of its proper length?**

**2) The time period of a pendulum is measured to be 3 s in the inertial frame. What is the period when measured by an observer moving with a speed of 0.95c with respect to the pendulum?**



## 13.4 TIME AS A FOURTH DIMENSION IN THE SPACETIME MODEL

Until the 20th century, it was thought that the three-dimensional geometry of the universe (its description in terms of locations, shapes, distances, and directions) was distinct from time (the measurement of when events occur within the universe). However, according to Einstein, you need to describe the location of an object not only in three-dimensional space (length, width and height) but also in time. Hence time is the fourth dimension.

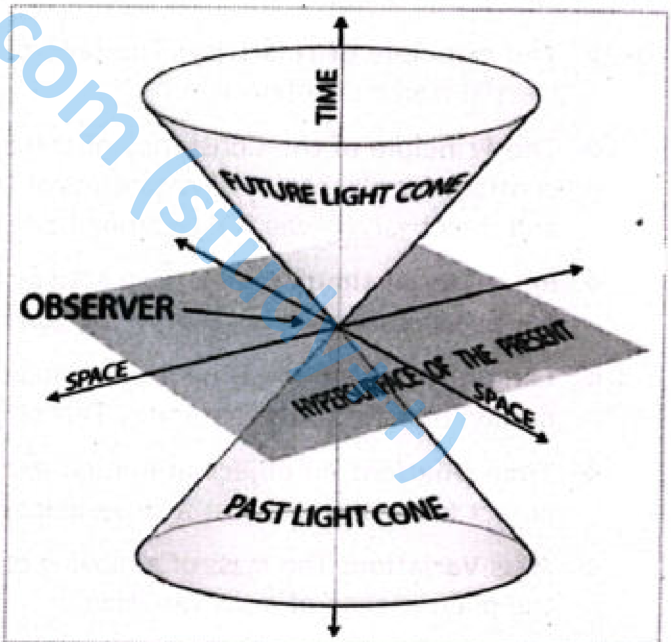


Figure 13.4: Minkowski space.

To know where you are, you have to know what time it is. The history of an object's location through time traces out a line or curve on a spacetime diagram. Each point in a spacetime diagram represents a unique position in space and time.

Spacetime is a **mathematical model**, as shown in the Fig. 13.4, that unites the three dimensions of space with the single dimension of time into a single four-dimensional continuum. Spacetime diagrams are useful in visualizing and understanding relativistic effects such as how different observers perceive where and when events occur.

A spacetime diagram is a graphical representation of locations in space at various times, as shown in figure 13.4. Spacetime diagrams can depict the geometry underlying phenomena like time dilation and length contraction without any mathematical equations.

**For Your Information**

In 1908, Hermann Minkowski presented a geometric interpretation of special relativity that combined time and the three spatial dimensions of space into a single four-dimensional continuum now known as Minkowski space. This interpretation proved vital to the general theory of relativity, wherein spacetime is curved by mass and energy.

## SUMMARY

- A **frame of reference** is a coordinate system that can be used to determine the positions and velocities of objects in that frame.
- A frame of reference which is accelerating is called a **non-inertial frame of reference.**
- A frame of reference which is at rest or moving with a uniform velocity along a straight line is called an **inertial frame of reference.**
- **The principle of relativity:** The laws of physics are the same for all observers in any inertial frame of reference.
- **The Principle of the Constancy of the Speed of Light:** The speed of light in vacuum is constant for all observers, regardless of the relative velocity between the source of light and the observer who is measuring its velocity.
- **Relativity of simultaneity:** Two actions that appears simultaneous for one person may not be simultaneous for another person in relative motion.
- **Length Contraction:** Objects are measured and appear shorter in the direction of motion relative to the observer. This effect of relativity is called length contraction.
- **Time Dilation:** An object in motion experiences time dilation, meaning that when an object is moving very fast it experiences time more slowly than when it is at rest.
- **Mass Variation:** The mass of a moving objects increases as its velocity increases. This is the phenomenon of mass variation.
- A **spacetime diagram** is a graphical illustration of locations in space and times.
- **Minkowski space** is a geometric interpretation of special relativity that combines time and the three spatial dimensions of space into a four-dimensional continuum.

# EXERCISE

## Multiple Choice Questions

**Encircle the correct option.**

1) Theory of Relativity was formulated by:

A. Isaac Newton B. Stephen William Hawking

C. Albert Einstein D. Ernest Rutherford

2) A frame of reference is:

A. a graph plotted between distance and time.

B. a graph plotted between speed and time.

C. the velocity of an object through empty space without regard to its surroundings.

D. an arbitrarily fixed point with respect to which motion of other points is measured.

3) A pendulum has a time period of 5.0 s relative to the pendulum's frame of reference. What is the speed of an observer relative to the pendulum's frame of reference if the period measured by the observer is 10.0 s? (here, c is the speed of light).

A. 0.606c B. 0.779c C. 0.866c D. 0.693c

4) An airliner traveling at 200 m $s^{-1}$ emits light from the front of the plane. Which statement describes the speed of the light?

A. It travels at a speed of c + 200 m $s^{-1}$. B. It travels at a speed of c - 200 m $s^{-1}$.

C. It travels at a speed c, like all light. D. It travels at a speed slightly less than c.

5) An inertial frame of reference is the one:

A. which is at rest.

B. which is moving with a uniform velocity along a straight line.

C. which has zero acceleration.

D. All of these.

## Short Questions

Give short answers of the following questions.

**13.1** What is the difference between inertial and non-inertial frames of reference?

**13.2** State the postulates of special theory of relativity?

**13.3** Explain why is it impossible for a particle with mass to move faster than the speed of light.

13.4 Imagine a train travelling at the speed of light. A person on this train looks into the mirror placed in front of him. Will this person see a reflection of himself in the mirror?

13.5 What happens to the density of an object as its speed increases?

13.6 What is meant by the relativity of simultaneity?

## Comprehensive Questions

Answer the following questions in detail.

13.1 What is meant by frames of reference? Explain with the help of examples. Also discuss its types.

13.2 Discuss the postulate of the special theory of relativity.

13.3 Explain the following consequences of the special theory of relativity:

a) Relativity of simultaneity b) Length Contraction

c) Time Dilation d) Mass Variation e) Mass-Energy Equivalence.

13.4 State and explain how time is considered the fourth dimension alongside the traditional three dimensions of space.

## Numerical Problems

13.1 Calculate the equivalent energy of an electron with rest mass $9.11\times10^{-31}$ kg.

**(Ans: 0.512 MeV)**

13.2 The length of a spaceship is 100 m. What will be its length if the spaceship moves at a speed of 0.99c? **(Ans: 14 m)**

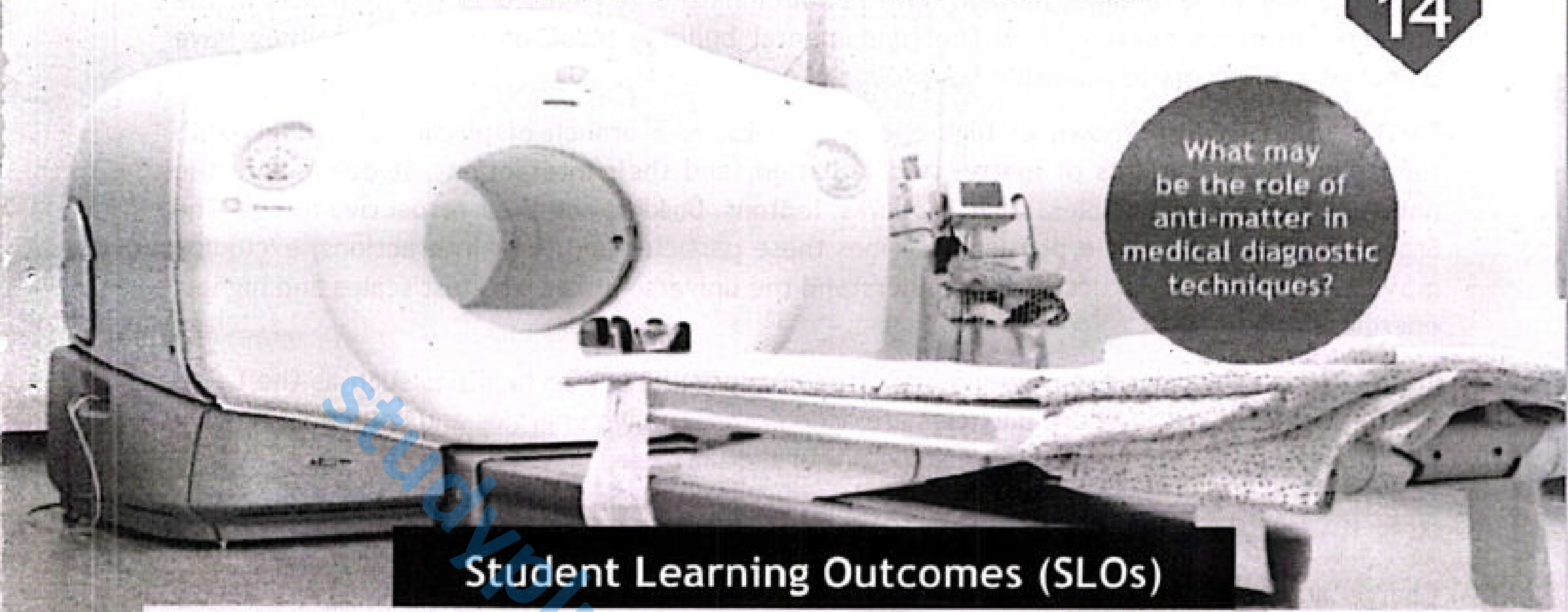
13.3 Particles called π-mesons are produced by accelerator beams. If these particles travel at $2.70\times10^{8}$ m $s^{-1}$ and live $2.60\times10^{-8}$ s when at rest relative to an observer. How long do they live as viewed in the laboratory? **(Ans: $5.96\times10^{-8}$ s)**

13.4 A neutral π-meson lives $1.40\times10^{-16}$ s as measured in the laboratory, and $0.840\times10^{-16}$ s when at rest relative to an observer. What is its velocity relative to the laboratory?

**(Ans: 0.800 c)**

# PARTICLE PHYSICS

UNIT 14




## Student Learning Outcomes (SLOs)

**The students will:**

- State that nucleon number and charge are conserved in nuclear processes.
- Describe the composition, mass and charge of alpha, beta and gamma radiations [both β- (electron) and β+ (positron) are included].
- Explain that an antiparticle has the same mass but opposite charge to the corresponding particle [give the example that a positron is the anti-particle of an electron].
- State that electron-antineutrinos are produced during β- decay and electron-neutrinos are produced during β+ decay.
- Explain that α-particles have discrete energies but that β-particles have a continuous range of energies because anti-neutrinos are emitted in β- decay.
- Describe quarks and anti-quarks are fundamental particles [including that there are six flavors (types) of quark: up, down, strange, charm, top and bottom].
- Describe protons and neutrons in terms of their quark composition.
- State that a hadron may be either a baryon (consisting of three quarks) or a meson (consisting of one quark and an anti-quark).
- Describe the changes to quark composition that take place during β- and β+ decay.
- State that electrons and neutrinos are fundamental particles called leptons.
- State W, Z, gluon and photons as fundamental particles called exchange particles or force carriers.
- State that Higgs Boson as a fundamental particle which is responsible for the particle's mass.
- Explain that every subatomic particle has a corresponding anti-particle [that has a same mass as a given particle but opposite electric or magnetic properties according to the Standard Model of Particle Physics].
- Describe protons and neutrons in terms of their quark composition.
- State that a hadron may be either a baryon or meson.
- Explain that there are various contending theories about what 'mass' and 'force' are generated from [e.g. that these are generated from quantum fields when they are energized, or from multidimensional 'strings' that vibrate in higher dimensions to give rise to particles].
- Explain the working principle of particle accelerators and also their uses.
- Explain that anti-matter is the counter part of matter [e.g. a positron is the anti-matter counterpart to an electron].
- Illustrate that the anti-particles usually have the same weight, but opposite charge, compared to their matter counter parts.
- State that most of the matter in the observable universe is matter.
- Describe the asymmetry of matter and anti-matter in the universe as an unresolved mystery.
- Describe annihilation reactions [a particle meets its corresponding anti-particle, they undergo annihilation reaction in which either all the mass is converted to heat and light energy, or some mass is left over in the form of new subatomic particles].

From the beginning of time, humans have had an innate urge to discover the mysteries of the universe. In other quest to find the fundamental building block of the cosmos, they have unlocked many doors to scientific knowledge.

Particle physics, also known as high-energy physics, is a branch of physics that deals with fundamental constituents of matter and radiation, and their interactions. It delves into the nature of subatomic particles, such as quarks, leptons, bosons, and their respective forces. The Standard Model of particle physics describes these particles and their interactions, excluding gravity. This field of physics aims to understand the universe at the smallest scales and highest energies.

Today, particle physics experiments are often conducted at large facilities such as the Large Hadron Collider (LHC), where particles are accelerated to high energies and collided, allowing physicists to probe the fundamental structure of the universe.



## 14.1 CONSERVATION LAWS IN NUCLEAR REACTIONS

Charge and nucleon conservation are fundamental principles in nuclear processes, deeply rooted in the laws of physics and the interactions among subatomic particles. These conservation laws play a crucial role in maintaining the integrity of atomic nuclei and ensuring that the fundamental properties of matter are preserved during nuclear reactions and decays.

### 14.1.1 Conservation of Charge

The conservation of charge is a fundamental property of particle interactions. In any closed system, the total charge before and after an interaction or decay must remain the same. Consider the example of α-decay:

$$_{88}Ra^{226} \longrightarrow {}_{86}Rn^{222} + {}_{2}He^{4} + Q$$

Here in this reaction, total charge on L.H.S is equal to the total charge on R.H.S. This is because charge is a conserved quantity i.e. it cannot be created or destroyed. It can only be transferred from one particle to another. In nuclear processes, the charges of the particles involved (such as protons, electrons, positrons) are carefully balanced to maintain overall charge neutrality.

### 14.1.2 Conservation of Nucleons

Nucleons, comprising both protons and neutrons, are the building blocks of atomic nuclei. The conservation of nucleons refers to the principle that the total number of nucleons remains constant before and after a nuclear reaction or decay. Consider the following nuclear reaction

$$_{7}N^{14} + {}_{2}He^{4} \longrightarrow {}_{8}O^{17} + {}_{1}H^{1} + Q$$

The number of nucleons on the L.H.S = 14 + 4 = 18

The number of nucleons on the R.H.S = 17 + 1 = 18

Hence, total number of nucleons on both the sides are same, i.e., 18. This conservation law is a consequence of the strong nuclear force, that binds nucleons together in the nucleus. The strong force is a short-range force that acts between quarks (the constituents of protons and neutrons), ensuring that the nucleons are not spontaneously created or destroyed during nuclear processes.

The conservation of charge and nucleons is upheld in various nuclear reactions, such as alpha decay, beta decay, and fusion reactions. These laws have been extensively tested and confirmed through experiments and observations in particle physics and nuclear science. They provide a deep insight into the underlying symmetries and interactions within the subatomic world and have far-reaching implications for understanding the behavior of matter at the smallest scales.

Alpha particles, beta particles, and gamma rays are three types of radiation commonly associated with radioactive decay processes. These particles and rays have distinct properties regarding mass and charge:

**1) Alpha Particle:**

**Composition:** An alpha particle consists of two protons and two neutrons, which are bound together as a single entity.

**Mass:** The total mass of an alpha particle is approximately 4 atomic mass units (u) or unified atomic mass units (u), equivalent to about $6.64 \times 10^{-27}$ kilograms.

**Charge:** Alpha particles carry a positive charge of +2e, where "e" represents the elementary charge. This charge corresponds to twice the charge of a single proton.

**2) Beta Particle:**

**Composition:** Beta particles are high-energy electrons ($\beta^-$) or positrons (beta-plus) emitted during certain types of radioactive decay.

**Mass:** β-particles have a much smaller mass than α-particles. Electrons have a mass of approximately $9.11 \times 10^{-31}$ kg, while positrons have the same mass but with a positive charge.

**Charge:** $\beta^-$ particles (electrons) carry a negative charge of -1e. $\beta^+$ particles (positrons) carry a positive charge of +1e, where $e = 1.6\times10^{-19}$ C.

**3) Gamma Ray:**

**Composition:** Gamma rays are electromagnetic waves, similar to X-rays. They do not consist of particles in the traditional sense.

**Mass:** Gamma rays have no mass because they are composed of photons, which are massless particles.

**Charge:** Gamma rays are electrically neutral, meaning they carry no electric charge.

These differences in mass and charge are essential for understanding their behavior, interactions, and effects in various contexts, including radioactivity and radiation protection.

## 14.2 PARTICLE'S DECAYS

Antiparticles are a unique concept in particle physics, representing the counterparts to regular particles. They share the same mass as their corresponding particles but have opposite electric charges. This phenomenon is a consequence of the symmetries in the fundamental laws of physics. Here are a couple of examples to illustrate this concept:

**Electron-Positron:**

**Electron:** An electron is a subatomic particle with a negative electric charge of -1e and a mass of approximately $9.11 \times 10^{-31}$ kg.

**Positron:** The positron is the antiparticle counterpart of the electron. It has the same mass as an electron (approximately $9.11 \times 10^{-31}$ kg) but carries a positive electric charge of +1e. This pairing of electron and positron is an example of matter-antimatter annihilation. When an electron and a positron collide, they annihilate each other, converting their masses into energy in the form of gamma rays. This process is known as matter-antimatter annihilation.

**Proton-Antiproton:**

**Proton:** A proton is a subatomic particle found in the nucleus of an atom. It carries a positive electric charge of +1e and has a mass of approximately $1.67 \times 10^{-27}$ kg.

**Antiproton:** The antiproton is the antiparticle counterpart of the proton. It shares the same mass as a proton (approximately $1.67 \times 10^{-27}$ kg) but carries a negative electric charge of -1e. Antiprotons can be produced artificially in high-energy particle accelerators and are used in scientific experiments to study the fundamental properties of matter.

These examples highlight the principle of charge conservation in particle-antiparticle pairs, where the total charge remains conserved while exhibiting opposite signs. Additionally, the conservation of mass is also evident as the masses of particles and their corresponding antiparticles are identical.

## 14.2.1 Alpha Decays

In **alpha decay,** atomic number of the parent nucleus reduces by 2 and the mass number reduces by 4 units. The decay product is called the daughter nucleus. The daughter nucleus may also remain unstable and undergo further disintegration till it attains stability. Alpha decay of $_{88}Ra^{226}$ is shown below:

$$_{88}Ra^{226} \longrightarrow {}_{86}Rn^{222} + {}_{2}He^{4} + Q$$

Q is the disintegration energy, which is always positive (as the process is spontaneous).

## 14.2.2 Beta Decays

Beta-minus (β-) decay and beta-plus (β+) decay are two types of radioactive decay processes involving the emission of beta particles. These processes occur in atomic nuclei to achieve a more stable nuclear configuration. Here's how each process works:

In **beta-minus decay,** a neutron (n) within the nucleus is transformed into a proton (p), an electron (beta-minus particle $\beta^-$), and an antineutrino ($\bar{\nu}_e$). The antineutrino is a nearly massless, electrically neutral particle that carries away some of the energy released during the decay. The emitted beta-minus particle is an electron. The process can be summarized as follows:

$$n = p^+ + \beta^- + \bar{\nu}_e \qquad (14.1)$$

The newly formed proton remains in the nucleus, increasing the atomic number by 1, while beta-minus particle and antineutrino are emitted from the nucleus.

In **beta-plus decay**, a proton within the nucleus is transformed into a neutron (n), a positron (beta-plus particle $\beta^+$ ), and a neutrino ($\nu_e$). The emitted beta-plus particle is a positron, which is the antiparticle of an electron. The process can be summarized as follows:

$$p^+ = n + \beta^+ + \nu_e \quad \text{\_\_\_\_\_\_} \quad (14.2)$$

The newly formed neutron remains in the nucleus, reducing the atomic number by 1, while beta-plus particle and neutrino are emitted from the nucleus. Both beta minus and beta plus decay processes are governed by the weak nuclear force, one of the fundamental forces in nature responsible for mediating certain types of particle interactions. These processes play a crucial role in the overall stability and transformation of atomic nuclei, allowing them to achieve more balanced configurations and release excess energy.

## 14.2.3 Energies for Alpha and Beta Decay

Alpha and beta particles are products of radioactive decay processes, and they carry different amounts of energy due to their distinct properties and mechanisms of emission.

Alpha particles have discrete energies in nuclear processes due to the specific nature of the quantum mechanical interactions within atomic nuclei. These energies are determined by the energy levels and quantum states of the particles within the nucleus, as well as the binding forces that hold the nucleus together. Inside a nucleus, protons and neutrons are arranged in energy levels or shells, similar to how electrons are arranged in energy levels around an atom's nucleus. These energy levels are quantized, meaning they can only take a certain discrete value. When an alpha particle is formed during radioactive decay, it involves the rearrangement of nucleons within the nucleus, which results in specific energy changes.

Quantization of Energy Levels: Just as electrons in an atom can only exist only at certain discrete energy levels, nucleons within a nucleus also occupy quantized energy levels. When nucleons rearrange themselves to form an alpha particle, the energy change corresponds to the difference between the initial and final energy states.

Binding Energies: The formation of an alpha particle involves the strong nuclear force, which binds nucleons together in the nucleus. As nucleons combine to form an alpha particle, changes occur in the binding energies of the particles involved. These changes result in the release of energy, which is carried away by the alpha particle.

Conservation of Energy: The conservation of energy is a principle in all physical processes. The total energy of the system before and after the alpha particle emission must be conserved. This conservation leads to the emission of alpha particles with specific, well-defined energies. Consequently, the energies of alpha particles emitted in nuclear processes are quantized and discrete. These specific energy levels of alpha particles provide valuable information about the structure of atomic nuclei, the binding forces involved, and the dynamics of nuclear reactions.

**Energy of beta particles:** The continuous energies spectrum of beta particles in nuclear processes can be attributed to the probabilistic nature of quantum mechanics, along with the conservation of energy and momentum. Unlike alpha particles, which have discrete energies, the energies of beta particles are spread out over a range of values. This continuous energy spectrum arises from several factors:

**i) Quantum Uncertainty:** In quantum mechanics, particles are described by wave functions that exhibit inherent uncertainty in certain properties, including energy. This uncertainty allows for a range of possible energies for a given particle. When a beta particle is emitted during a nuclear decay, its specific energy is not predetermined but subject to quantum uncertainty.

**ii) Neutrinos in Beta Decay:** In beta decay processes, such as beta-minus (β-) and beta-plus (β+) decay, an additional particle is involved: the neutrino or antineutrino. These particles are extremely lightweight and have negligible mass, which means they carry away some of the energy released during the decay. This leads to a spread of energies in the emitted beta particles, as the neutrinos can take varying amounts of energy, leaving less for the beta particles.

**iii) Conservation Laws:** While the total energy before and after beta decay must be conserved, there is flexibility in how that energy is distributed between the emitted beta particle and the accompanying neutrino. This flexibility results in a continuum of possible energy values for the beta particles.

**iv) Final State Interactions:** In some cases, the emitted beta particles can interact with other particles or fields as they travel through matter. These interactions can affect the energy and momentum of the beta particles, contributing to the continuous energy spectrum observed in experiments.

## 14.3 MATTER AND ANTI-MATTER

In 1928, a British physicist Paul Dirac, while studying the behavior of electrons at very high speeds (speeds comparable to light) derived an equation for the energy of an electron by combining special relativity and quantum theory. Generally, the relationship for the energy and momentum of a particle can be expressed as:

$$E = \frac{p^2}{2m} \qquad \text{\_\_\_\_\_ (14.2)}$$

Solving this relation for relativistic speed by using special relativity and quantum theory, he derived an equation which can be given as:

$$E = \pm\sqrt{m^2c^4 + p^2c^2} \qquad \text{\_\_\_\_\_ (14.4)}$$

Here 'E' is the energy of the particle, 'm' is the mass, 'p' is the momentum and 'c' is the speed of light. Positive and negative solutions for energy give a striking idea for anti-particles.

If positive solution of the Dirac's equation is for electron (the particle) then negative solution for energy must be for a particle which should have mass and momentum equal to that of electron but of opposite charge and magnetic moment. Dirac said this energy solution is for positron (the anti-particle of electron). The energy cones for electron and positron are shown in Fig. 14.1. Soon after the prediction of Dirac on solving his equation for the existence of anti-particles, James Anderson discovered the positron. In 1932, while studying the tracks of cosmic ray particles in a cloud chamber at California institute of technology, he discovered a particle which has the mass equal to the mass of an electron but has the opposite charge to that of electron i.e. the positive charged particle. This was the first experimental discovery of an anti-particle. After that a lot of anti-particles were discovered. Electron, proton and neutron along with their respective anti-particles are listed here in table 14.1.

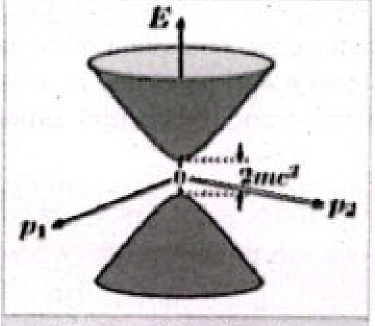


**Figure 14.1:** Electron-positron energy.

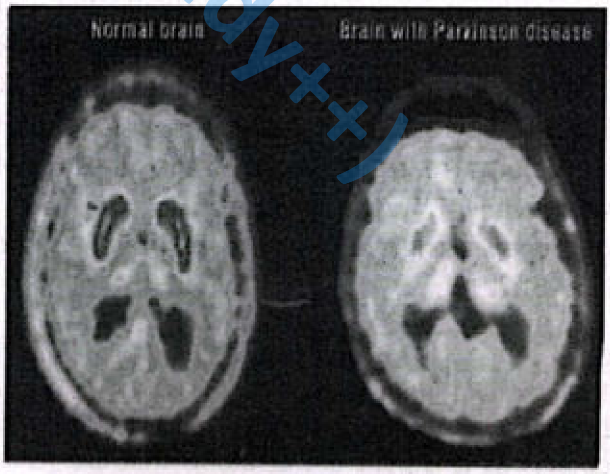
Table 14.1 (Particles and anti-particles)

| Property | Electron | Positron | Proton | Anti-proton | Neutron | Anti-neutron |
|---|---|---|---|---|---|---|
| Mass (kg) | $9.1\times10^{-31}$ | $9.1\times10^{-31}$ | $1.67\times10^{-27}$ | $1.67\times10^{-27}$ | $1.67\times10^{-27}$ | $1.67\times10^{-27}$ |
| Mass (MeV/$c^2$) | 0.51 | 0.51 | 938.28 | 938.28 | 939.57 | 939.57 |
| Charge | -1e | +1e | +1e | -1e | 0 | 0 |
| Spin | 1/2 | 1/2 | 1/2 | 1/2 | -1/2 | -1/2 |
| Family | fermion | fermion | fermion | fermion | fermion | Fermion |

Particles are divided into two groups depending upon their spins. Particles with integer spins (0, 1, 2 ...) are called 'boson' and the particles with half integer spin (1/2 etc.) are called 'fermions'.

**Anti-matter and Medical Technology**

Antimatter is used in the medical technology, for example, anti-protons are used to treat some types of cancers, positrons are used in positron emission topography (PET scan) to create images of internal body part. In this process positrons are emitted by the decay of radio nuclei. Photons are produced when positrons collide with electrons, converting the electron-positron pair into energy. Scanner detects the annihilated photons at an angle of 180° apart from each other. Using these photons an image of the internal body is obtained. PET scan is used to evaluate organs like the heart and brain and are also used for detection and evaluation of cancers.



**For Your Information**

Radioactive potassium-40 present in bananas, Brazil nuts and even human body produce anti-particles. It releases positrons in beta decay. The amount of such reaction and the positron production is so small that it has no health threats.

**Example 14.1: The energy of a particle system is 2 MeV moving with relativistic speed, approach each other. If their masses are 0.511 MeV/c² find their momentum. Also justify why momentum has two values i.e. positive and negative.**

Given: $E = 2\ MeV$ OR $E = (2\times10^{6}\times1.6\times10^{-19})J = 3.2\times10^{-13}J$

$m = 0.511\ MeV/c^{2}$ OR $m = (\dfrac{0.511\times10^{6}\times1.6\times10^{-19}}{9\times10^{16}})kg = 9.11\times10^{-31}kg$

To Find: Momentum: p = ?

Solution: To find momentum of the system we use equation 14.4.

$$E = \pm\sqrt{m^{2}c^{4} + p^{2}c^{2}}$$

Squaring on both sides, we get: $E^{2} = m^{2}c^{4} + p^{2}c^{2}$

Re-arranging the above: $p^{2} = \dfrac{E^{2} - m^{2}c^{4}}{c^{2}}$

Using values: $p = \sqrt{\dfrac{(3.2\times10^{-13}J)^{2} - (9.11\times10^{-31}kg)^{2}(3\times10^{8}ms^{-1})^{4}}{(3\times10^{8}ms^{-1})^{2}}}$

$$p = \pm1.03\times10^{-21}kg\ ms^{-1}$$

There are two values of momentum i.e. positive and negative, corresponding to particle and anti-particle.

**Assignment 14.1**

**Find the momentum of a positron, moving with relativistic speed. The energy of the positron is 0.47 MeV.**

Physicists at the Relativistic Heavy Ion Collider (RHIC) in New-York created the anti-nuclei of helium-4. Anti-helium-4 is the heaviest anti-particle produced. Scientists at CERN (the European research laboratory for particle physics), artificially created an atom of anti-matter, which has opened the doors of further research in the field of anti-matter. Hydrogen along with its anti-hydrogen is shown here in the Fig. 14.2.

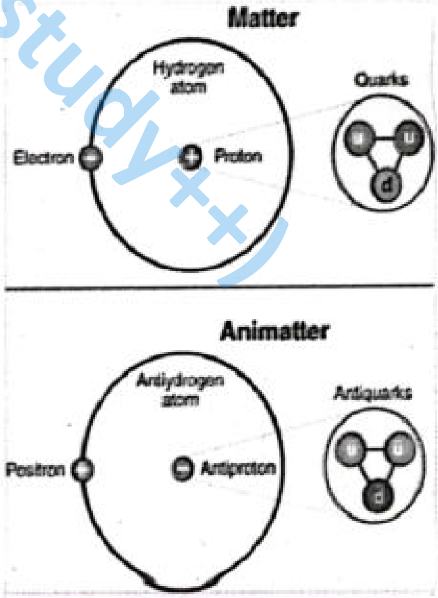



**Figure 14.2:** Hydrogen and anti-hydrogen.

**For Your Information**

The cost of 1 gram of anti-matter is 62.5 trillion dollars, making it the most expensive material on Earth. Anti-matter can create objects as it is built out of atoms that are '*charge conjugate*' of ordinary atoms, i.e., the anti-atom.

## 14.3.1 Asymmetry of Matter and Anti-matter

Particle and anti-particles mirror each other almost perfectly; for every particle in the universe, there ought to be a particle of anti-matter. Our Earth, the Sun and the dust between the galaxies are made up of normal matter. It looks like the whole universe is made up of matter, so when we look around, we don't see any anti-matter, why? The asymmetry in the universe is essential for the existence of stars and even life itself. Hence asymmetry is good thing in itself.

A particle and anti-particle are charge conjugate of each other (i.e., a positive and negative pair), hence whenever in any reaction, a pair of particle and anti-particle is made law of conservation of charge requires this conjugation, which is known as C-symmetry. Also, the particle and anti-particle are like '*mirrors*' to each other, as the inversion of nuclear spatial coordinates; the state of being equal with reference to some coordinate is called 'parity' (the space inversion). Collectively, charge conjugation and parity are called CP-symmetry. A simple depiction of charge symmetry and parity is shown in Fig. 14.3.

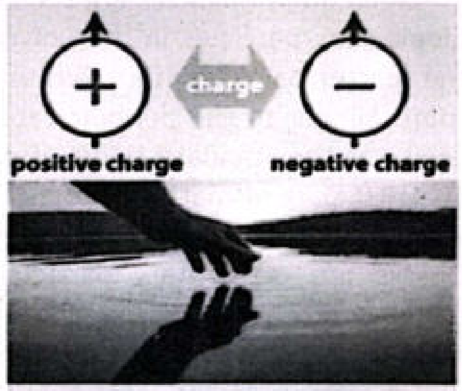



**Figure 14.3:** A simple depiction of C-symmetry and P-symmetry

Then why the anti-matter is so rare if the matter and anti-matter are perfectly balanced. What happened to the anti-matter stuff? The answer lies to the early universe.

The most accepted theory about the creation of universe is the big bang theory. According to this theory, about 13.7 billion years ago an explosion took place with creation of matter and antimatter in perfectly equal amount with CP-symmetry. But with this symmetry all the matter would annihilate leaving behind an empty universe. But the real universe has the excess matter which formed by surviving this annihilation, is due to CP-violation.

> CP-violation is a phenomenon where the same decay process has a different probability for a particle than for an anti-particle.

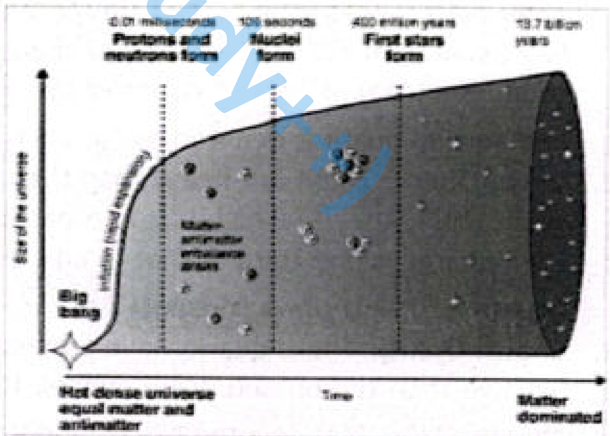



**Figure 14.4:** The timeline of universe.

In particle physics, it is the violation of the combined conservation laws i.e. charge conjugation and parity by the weak force. The CP-violation seems to be the main cause of this asymmetry in the universe. In early universe, equal amount of matter and anti-matter were produced, but after first second when nuclei formed the matter started dominating. As in the nucleus two forces are prominent the strong nuclear force which binds the nucleus and the weak force responsible for the nuclear decays, both violate the CP. Evidences in early universe for strong force's CP-violation are less as compare to the weak force.

## 14.4 QUARKS

In the early 20$^{th}$ century, protons and neutrons were considered as elementary particles. However, in 1964 Murray Gell-Mann and George Zweig independently proposed the quark theory, which described the protons and neutrons as being composed of quarks. The existence of quark was first confirmed in a deep inelastic scattering experiment at the Stanford Linear Accelerator Center (SLAC) in 1968.

Elementary particles are those particles which are not composed of the other particles.

Elementary particles in the universe can be broadly categorized mainly into two groups i.e., the matter particles (fermions with half integer spin) and force carriers (bosons with integer spin). Similar to the periodic table of elements, elementary particles are arranged in a table known as the Standard Model of elementary particles, as shown in Fig. 14.5.

The first part of matter particles in the table is quarks, which have a size of roughly $10^{-18}$ m. Quarks are elementary particles and basic building blocks of meson and baryons, the main constituent of all the matter. Each quark has a corresponding anti-quark with same properties but opposite charge. These quarks combine to form composite particles called hadrons. Quarks can be divided into six flavors (flavors are some properties of the particles) i.e. up, down, charm, strange, top and bottom. All the quarks have half spin (1/2).

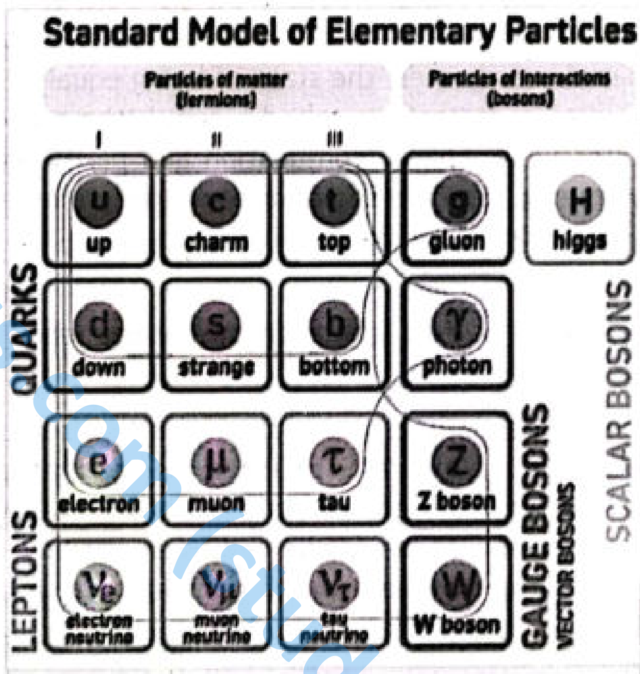



Figure 14.5: The standard model of particles.

1. **Up Quark:** These are the lightest among all the quarks that is why they are the most stable among all the six quark flavors. Their mass ranges from 1.7-3.1 MeV/c$^2$ and has a charge of +2/3e (two-third of charge of an electron). They are denoted by '$u$' and anti-up quark is denoted by '$\bar{u}$'. It is first generation quark.
2. **Down Quark:** These are heavy than the up quark but lighter than rest of all quarks. Due to small mass, they are highly stable particles. Their mass ranges from 4.1-5.7 MeV/c$^2$ and their charge is one-third of that of an electron (-1/3 e). They are denoted by '$d$' and their counterpart anti-down is denoted by '$\bar{d}$'. It is first generation quark.
3. **Charm Quark:** These quarks have a mass about 1280 MeV/c$^2$. It is hundreds of times more massive than the up and down quarks. It is the second generation of quarks. They have a charge +2/3e. It is denoted by '$c$' and its anti-charm quark is denoted by '$\bar{c}$'.
4. **Strange Quark:** These quarks have a mass about 96 MeV/c$^2$. It is the second generation of quarks. They have a charge -1/3e. It is denoted by '$s$' and its anti-strange quark is denoted

by '$\bar{s}$'. They have very long life even they can live 100,000 times longer than the similar particles.

5. **Top Quark:** These quarks are the heaviest among all the quarks and have a mass about 173.1 GeV/$c^2$. It is the third generation of quarks. They have a charge +2/3e. It is denoted by '$t$' and its anti-top quark is denoted by '$\bar{t}$'. It is not a stable particle even it decays before it has an opportunity to form hadronic bound states.
6. **Bottom Quark:** These quarks have a mass about 4.18 GeV/$c^2$. It is the third generation of quarks. They have a charge -1/3e. It is denoted by '$b$' and its anti-bottom quark is denoted by '$\bar{b}$'. Bottom quark decays mostly into a charm quark but rarely into an up quark.

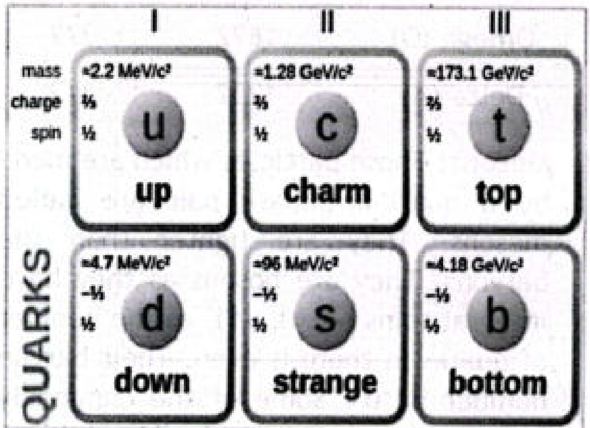


Figure 14.6: The quarks.

The quarks are shown in figure 14.6. Another property of quarks and gluons is the color charge, which is related to particle's strong interaction in the theory of quantum chromo dynamics (QCD). Quantum chromo dynamics is the model which governs the behavior of quarks. This color property is totally different from common meanings of color in everyday life. Quarks can be found in three color charges i.e. the red, the blue and the green along with the three anti color quarks i.e. the anti-red, the anti-blue and the anti-green.

## 14.4.1 Hadrons:

The subatomic particles which are made up of quarks and anti-quarks held together by the strong force with the help of gluons are called hadrons. They are the heaviest particles. As we studied that quarks have fractional charges, but hadrons which are made up of quarks have integer charges.

Some examples of hadrons include kaons, protons, neutrons, anti-protons and anti-neutrons. Hadrons experience strong nuclear force. Properties of hadrons are as follow:

- They carry no net color charge, although their constituent particles quarks to carry colour charge.
- Hadrons are unstable particles and they can decay. Only hadron which is stable even in free-state is the proton while the neutron is stable only within the nucleus.

Due to participating quarks, hadrons are mainly divided into two groups i.e. baryons and mesons.

1. **Baryons:** Those particles which are approximately equal in mass to that of a proton or greater are called baryon. These hadrons are made up of three quarks and are classified as fermions, as they have half integer spin due to the odd number of quarks. Proton and neutron are well known baryons. A quantum number which is equal to the number of baryons minus the number of anti-baryons in a system of subatomic particles is called 'baryon

number'. Baryons have a baryon number +1 while anti-baryons have a baryon number of −1. Some baryons along with their properties are listed in the Table 14.2.

**Table 14.2: Some baryons with their properties.**

| Particle | Rest Mass (GeV/C$^2$) | spin | Electric charge | Quarks Composition | Baryon Number | Lepton Number |
|---|---|---|---|---|---|---|
| Proton (p) | 938.3 | 1/2 | +1 | $u\ u\ d$ | +1 | 0 |
| Neutron (n) | 939.6 | 1/2 | 0 | $d\ d\ u$ | +1 | 0 |
| Omega (Ω) | 1672 | 3/2 | -1 | $s\ s\ s$ | +1 | 0 |
| Delta (Δ) | 1232 | 3/2 | +2 | $u\ u\ u$ | +1 | 0 |

2. **Mesons:** Those particles which are made by a quark-antiquark pair are called mesons. They are lighter than the baryons. They are bosons as they have integral spins (-1, 0, +1), as the number of quarks in them is even. Their baryon number is '0'. Some of the commonly known mesons are pion, kaon and rho. A typical Baryon and a meson are shown in Fig. 14.7. Some of mesons along with their properties are listed in the Table 14.3.

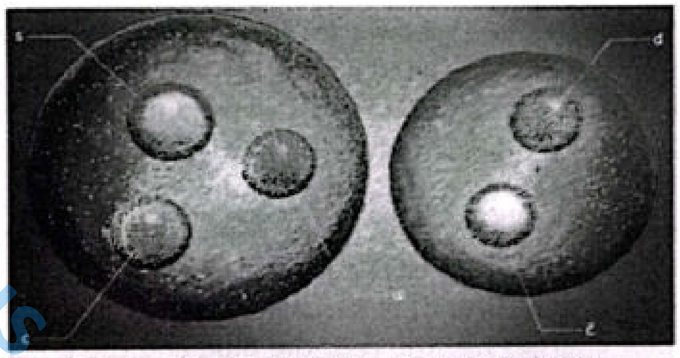


**Figure 14.7: Left (Baryon), right (Meson).**

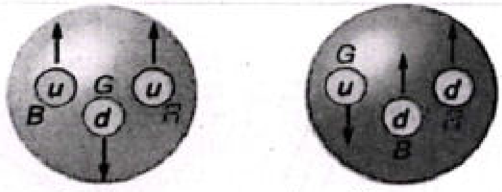
**Table 14.3: Some mesons with their properties.**

| Particle | Rest Mass (GeV/C$^2$) | spin | Electric charge | Quarks Composition | Baryon Number | Lepton Number |
|---|---|---|---|---|---|---|
| Pion$^+$ ($\pi^+$) | 0.140 | 0 | +1 | $u\ \bar{d}$ | 0 | 0 |
| Kaon$^-$ ($K^+$) | 0.494 | 0 | -1 | $s\ \bar{u}$ | 0 | 0 |
| Kaon$^0$ ($K^0$) | 0.498 | 0 | 0 | $d\ \bar{s}$ | 0 | 0 |
| Rho$^+$ ($\rho^+$) | 0.770 | 1 | +1 | $u\ \bar{d}$ | 0 | 0 |

## 14.4.2 Composition of Proton and Neutron

Protons and neutrons are made up of quarks. A proton is made up of two up quarks and one down quark. As we know that up quark has a charge of (+2/3)e and down quark has a charge of (-1/3)e. By adding the charge of these three quarks we get +1e charge for a proton, i.e., (+2/3 +2/3-1/3 = +1). Similarly, the anti-proton consists of the anti-quarks of the same type. A proton in terms of color charge consists of a blue-up, a red-up and a green-down quark. The composition of both proton and the neutron along with type and color charge is shown in Fig. 14.8.

Similarly, a neutron is made up of two down quarks and one up quark. Both down quarks have a charge of (-1/3)e and one up quark have a charge of (+2/3)e. By adding the charge of these three quarks we get '0' charge for a neutron, i.e. (+2/3 -1/3 -1/3 = 0). Similarly, the anti-neutron consists of the anti-quarks of the same type. A neutron in terms of color charge consists of a blue-down, a red-down and a green-up quark. The composition of both the proton and the neutron, along with their counter anti-particles i.e., the anti-proton and the anti-neutron is shown in Fig. 14.9.




| | Proton | Neutron |
|---|---|---|
| Spin | $\frac{1}{2}+\frac{1}{2}-\frac{1}{2}=\frac{1}{2}$ | $-\frac{1}{2}+\frac{1}{2}+\frac{1}{2}=\frac{1}{2}$ |
| Charge | $+\frac{2}{3}+\frac{2}{3}-\frac{1}{3}=1$ | $+\frac{2}{3}-\frac{1}{3}-\frac{1}{3}=0$ |

**Figure 14.8:** Composition of proton and neutron.

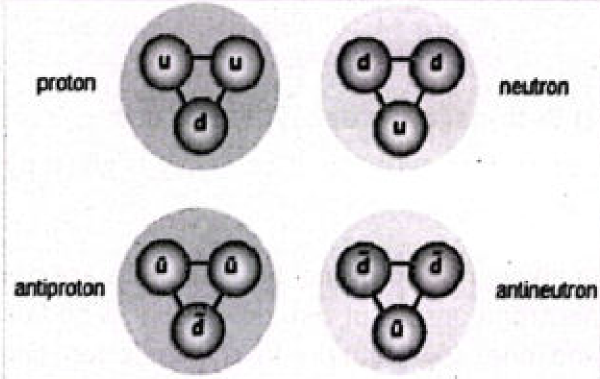



**Figure 14.9:** Composition of the anti-proton and the anti-neutron.

**For Your Information**

In electromagnetic force, charge is the basic property that allows a particle to experience this force. Neutral particles cannot experience the electromagnetic force. Similarly, in the strong interactions of the strong nuclear force (color force), color is the basic property of a particle to experience this force. A colorless particle cannot feel the strong nuclear force. This property is sometime referred as 'color charge'. There are three color charges i.e. red, blue and green and three anti-color charges i.e. anti-red, anti-blue and anti-green.

## 14.4.3 Change in Quarks during Beta Decays

As we have studied in topic 14.2.1 that whenever a proton decays into a neutron or a neutron decay into a proton, these processes are called beta decay. We now need to study how quarks change their types during these decays.

**1) Beta-positive Decay:** In this decay, a proton is converted into a neutron, a positron ($\beta^+$) and a neutrino. This decay can be represented by equation 14.2, as:

$$p^+ = n + e^+ + \nu_e$$

In above relation, only two particles are made up of quarks i.e. the proton and the neutron. A proton consists of two up-quarks and one down-quark, while a neutron consists of two down-quarks and one up-quark. So, in this reaction one of the up-quark turns into a down-quark with production of weak boson $W^+$, which is very short lived and soon decays into a positron and a neutrino due to the weak interaction. Beta positive decay, when an up-quark turns into a down-quark is shown in Fig. 14.10.

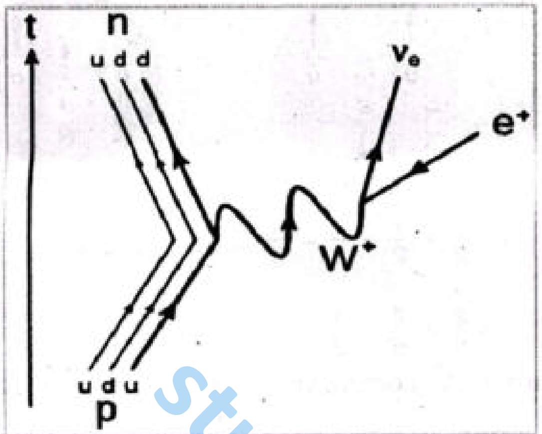



Figure 14.10: Beta-positive decay.

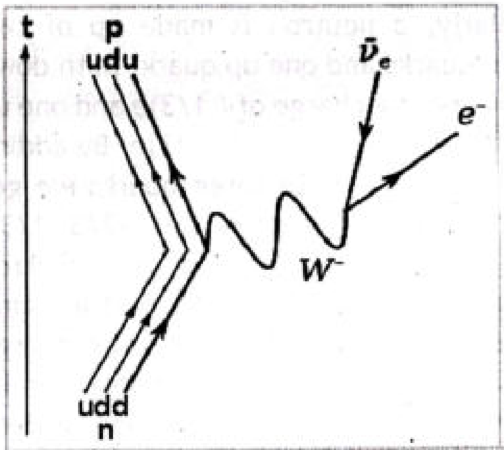



Figure 14.11: Beta-negative decay.

**2) Beta-negative decay:** In this decay, a neutron is converted into a proton, an electron ($\beta^-$), and an anti-neutrino. This decay is given by equation 14.1, as:

$$n = p^+ + e^- + \bar{\nu}_e$$

In above relation, only two particles are made up of quarks i.e., the proton and the neutron. A neutron consists of two down-quarks and one up-quark, while proton consists of two up-quarks and one down-quark. In this reaction one of the down-quark turns into an up-quark with production of a weak boson W⁻, which is very short-lived and soon decays into an electron and an anti-neutrino due to the weak interaction. Beta negative decay, where a down-quark turns into an up-quark, is shown in Fig. 14.11.

## 14.5 LEPTONS

In the standard model of particle physics, the second group of matter particles is lepton. Leptons are also elementary particles; they are not composed of other particles. Leptons can have a unit positive, unit negative electric charge or may be neutral. The smallest lepton is the electron, which is most stable among all leptons. Other leptons, such as muon and tau exist only at high energy collisions and lasts for a very short period.

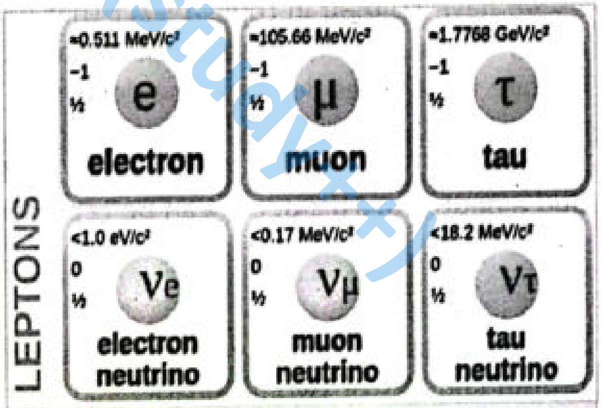



Figure 14.12: Leptons.

Leptons have spin ½ and do not take part in strong interactions. The strong nuclear force is blind for leptons. Leptons cannot be found within nucleus or nucleons; rather if they produced in nucleus, they escape with large kinetic energies. Just like baryon number, the lepton number is also a concept in particle physics. A lepton number is a conserved quantum number. For leptons this number is '+1' while for anti-leptons its value is '-1'. For any reaction, this number can be calculated as difference between the number of leptons and the number of anti-leptons. Some leptons and their properties are given in the Table 14.4.

Table 14.4: Leptons with their properties.

| Particle | Rest Mass ($MeV/C^2$) | spin | Electric charge | Baryon Number | Lepton Number |
|---|---|---|---|---|---|
| Electron) $e^-$ | 0.511 | 1/2 | -1 | 0 | +1 |
| Muon ($\mu$) | 105.66 | 1/2 | -1 | 0 | +1 |
| Tau ($\tau$) | 1776.8 | 1/2 | -1 | 0 | +1 |
| Electron neutrino ($\nu_e$) | <1.0 ($eV/C^2$) | 1/2 | 0 | 0 | +1 |
| Muon neutrino ($\nu_\mu$) | <0.17 | 1/2 | 0 | 0 | +1 |
| Tau neutrino ($\nu_\tau$) | <18.2 | 1/2 | 0 | 0 | +1 |

## 14.5.1 Conservation Laws of Lepton and Baryon

Like conservation laws of mass, energy, momentum and charge in physics, we need some other conservation laws like the lepton-number conservation and the baryon-number conservation. Every particle interaction should necessary be according to these laws of conservations. The interaction which is not satisfying any of these laws cannot take place in real world. The law of conservation of lepton-number says that

The lepton-number of reaction on both sides should essentially be same.

Similarly, the law of conservation of baryon-number says that

The baryon-number of reaction on both sides should essentially be same.

**Example 14.2: Show that the beta-positive decay and the beta negative decay satisfy the laws of conservation of charge, lepton-number and baryon-number.**

Given: Beta negative decay: $n = p^+ + e^- + \overline{\nu_e}$ Beta positive decay: $p^+ = n + e^+ + \nu_e$

To Find: charge , lepton-number and baryon-number are same on both sides.

Solution:

**For law of conservation of charge:** As we know that charge on each particle is.

$n = \nu_e = \overline{\nu_e} = 0$ $\quad p = e^+ = +1$ $\quad e^- = -1$

Beta negative decay: $0 = (+1) + (-1) + (0) \Rightarrow 0 = 0$ Satisfied.

Beta positive decay: $+1 = (0) + (+1) + (0) \Rightarrow +1 = +1$ Satisfied.

**For law of conservation of lepton-number:** As we know that lepton number of each particle is. $n = p = 0$ $\quad e^+ = \overline{\nu_e} = -1$ $\quad e^- = \nu_e = +1$

Beta negative decay: $0 = (0) + (+1) + (-1) \Rightarrow 0 = 0$ Satisfied.

Beta positive decay: $0 = (0) + (-1) + (+1) \Rightarrow +1 = +1$ Satisfied.

**For law of conservation of baryon-number:** As we know that baryon number of each particle is. $n = p = +1$ $\quad e^+ = \overline{\nu_e} = e^- = \nu_e = 0$

| | | | | |
|---|---|---|---|---|
| Beta negative decay: | $+1=(+1)+(0)+(0)$ | $\Rightarrow$ | $+1=+1$ | Satisfied. |
| Beta positive decay: | $+1=(+1)+(0)+(0)$ | $\Rightarrow$ | $+1=+1$ | Satisfied. |

As all the basic conservation laws of particals are satisfied, hence beta-positive and beta-negative decays are real.

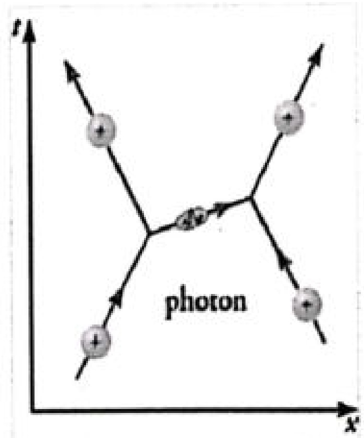
**Assignment 14.2**

Show that the particle reactions given below satisfy the laws of conservation of energy, charge, lepton- number and baryon-number.

i) $\pi^- + p = \pi^0 + n + \pi^- + \pi^+$ ii) $\mu^+ = e^+ + \nu_e + \overline{\nu_\mu}$

**Point To Ponder**

Hideki Yukawa introduced the quantum idea of fields (which is studied in quantum field theory QFT). He proposed that the nuclear forces are transmitted by the exchange of particles called "mediators or carrier particles". Later, we found carrier particles for all natural forces. A fermion (matter particle) produces a boson (force carrier) which is absorbed by another fermion to exert force on each other. The interaction between two approaching protons' repulsion by exchanging a photon (force carrier for electromagnetic interaction) is shown here by the Feynman diagram.




## 14.6 FORCE CARRIERS

In standard model of particle, there is a group of particles which do not make the matter but they are responsible for binding matter together. The interactions between matter-particles are mainly due to four types of fundamental forces i.e. the strong nuclear force, the weak nuclear force, the electromagnetic force and the gravitational force. Each force interacts through the exchange of particles called 'force carriers' or 'mediators'. The force carriers, are shown in Fig. 14.13, and are given in detailed below.

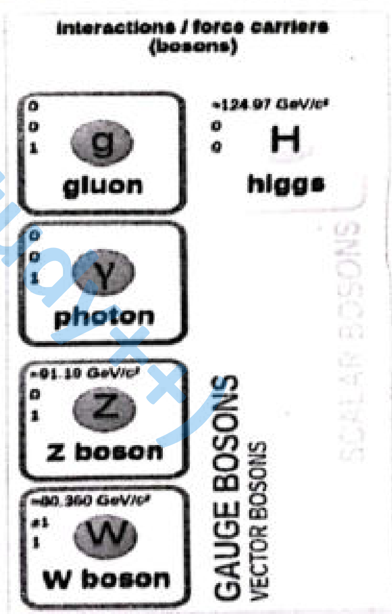



Figure 14.13: Force carriers.

### 14.6.1 The Photon

In interactions involving electromagnetic force, the exchange particle is photon. A photon is the smallest quantum of electromagnetic radiation and is the basic unit of light. Photons can travel through vacuum with a speed of $3\text{x}10^8$ m $s^{-1}$. According to Max Planck, the energy of photon is the product of its oscillation frequency and the Planck's constant, expressed as:

$$E = hf \quad \_\_\_\_ (14.5)$$

Photons have zero rest mass energy; they exist only as moving particles. They are neutral and stable particles. They have a spin of '1' hence belong to the boson family. They are also elementary particles which have energy, momentum and frequency of oscillations. According to quantum field theory, they are exchanged between the charged particles when they exert electromagnetic force on each other. Hence, they are the mediators of electromagnetic force, as shown in Fig. 14.14, where two approaching electrons (shown in blue color) repel each other (shown in red color) by exchanging a photon. During electromagnetic interactions every photon is emitted by one charged particle and absorbed by the another, so photon carries the electromagnetic force in this context.

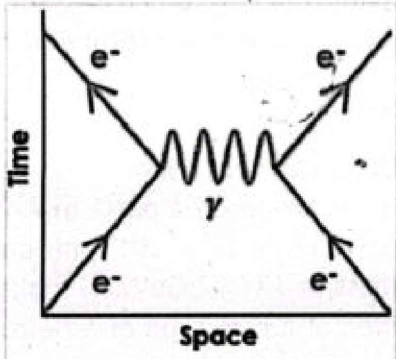


**Figure 14.14:** Photons mediators of electromagnetic force.

## 14.6.2 The Gluon

A particle that is exchanged between quarks to bind them together to form particles like protons and neutrons is called gluon.

Gluons are massless particles, travel at the speed of light and are vector boson, meaning they have a spin of 1. They are the exchange particles for the color force (the strong nuclear force). Like quarks, gluons also exhibit the property of color. During strong interactions, one quark or anti-quark emits a gluon and other anti-quark or quark absorbs the gluon; hence they are called the quanta of strong force. The gluons are capable of exchanging color for conservation of color charge. They have no electric charge. The exchange of a gluon between two quarks is shown in Fig. 14.15.

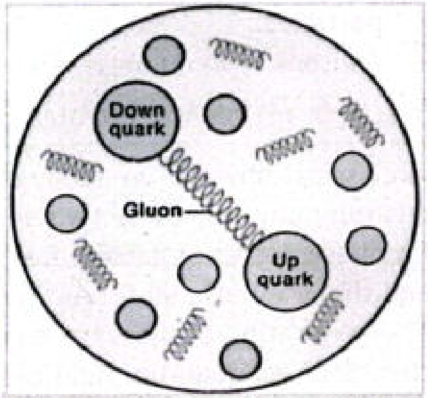


**Figure 14.15:** exchange of gluon between two quarks.

## 14.6.3 The Z-Boson

Z-boson was discovered in 1983 at CERN. Z-boson is electrically neutral and has its own anti-particle. It has a mass of 91.19 GeV/$c^2$, which is nearly hundred times that of the proton, making it one of the heaviest elementary particles. Bosons are very short-lived particles, with a lifetime of order of $10^{-25}$ seconds. It is the carrier of the weak nuclear force and is a partner to $W^+$ and $W^-$ bosons, that mediate radioactive decay processes. In weak interactions, W and Z bosons interact with each other as well as with quarks and leptons.

## 14.6.4 The W-Boson

W-bosons found in two different charge compositions i.e. $W^+$ and $W^-$. They can have both positive and negative electric charges. W and Z are gauge bosons that mediate the weak nuclear force. They have a mass of 80.360 GeV/$c^2$ and 91.2 GeV/$c^2$ and spin 1. They are responsible for many nuclear processes such as the reactions that power stars. They are very short-lived particles, with a lifetime of order of $3x10^{-25}$ seconds. They do not have their anti-particles.

### 14.6.5 The Higgs Boson

Up to the year 1964, the mass of a particle was considered as intrinsic property. In 1964, Peters Higgs proposed a particle which is responsible for the mass of matter, which he called it Higgs particle. In July, 2012 this particle was experimentally confirmed at CERN. Higgs particle has a mass of 124.97 GeV/$c^2$, making it 130 times more massive than a proton. Peter Higgs gave the idea of a new type of a field, called the Higgs field that fills the universe and gives mass to all elementary particles. It is a scalar field and the Higgs boson is a wave in that field. It has a zero spin, zero electrical charge and no strong force interactions.

All the above four particles were gauge bosons that is a form of force carrier, while Higgs boson is an elementary particle which is produced due to quantum excitation of Higgs field.

- Quarks interact strongly with the Higgs field, gaining relatively large mass.
- Electrons (lepton) interact only slightly with the Higgs field, making them extremely light particles.
- Photons have no mass, and hence do not interact with the Higgs field.

### 14.6.6 Theories about Generation of Force and Mass

In classical physics, we study only the macroscopic concepts of mass and forces. However, in quantum physics, these two entities have very different meanings. As we know that there are four basic forces of nature, i.e. strong nuclear force, weak nuclear force, electromagnetic force and the gravitational force. Two main theories about the force generation are discussed here. The generation of mass, as explained above involves Higgs field. For force we have two main theories i.e., quantum field theory and the string theory.

Quantum Field Theory (QFT):

Quantum field theory is a set of physical principles that combine the elements of quantum mechanics with the theory of relativity to explain the behavior of sub atomic particles and their interactions due to a variety of force fields. Three different quantum field theories address three of the four fundamental forces in nature. Matter interacts through these forces. Electromagnetism explains how atoms hold together, strong nuclear force explains the stability of nucleus and the weak nuclear force explains why some atoms undergo radioactive decay. QFT describes the particles as excited states of quantum fields, which are more fundamental than particles themselves. Interaction between the particles is basically an interaction between the fields. Each interaction can be represented by a Feynman diagram. Similarly, according to QFT, mass is not the strength of attraction of two bodies or the quantity of matter; rather, it is the tendency of an object to resist changes in its speed and position. In precise, at the smallest level, everything is made up of some entities or fluid-like substance call quantum field. This field sometimes behaves like particles, sometimes like waves, and they can even interact with each other.

### String Theory:

String theory suggest that, the whole universe is made up of tiny vibrating strings. These strings are smaller than the smallest particles. These strings can twist, vibrate and fold. By twisting, folding and vibrating they create matter, energy and forces. Phenomenon such as electromagnetism and gravity arise from these strings. This theory replaces point-like particles to one dimensional string. According to this theory strings have characteristics like charge and mass and the characteristics of strings are controlled by their vibrations. According to string theory, the matter composition is shown in Fig. 14.16.

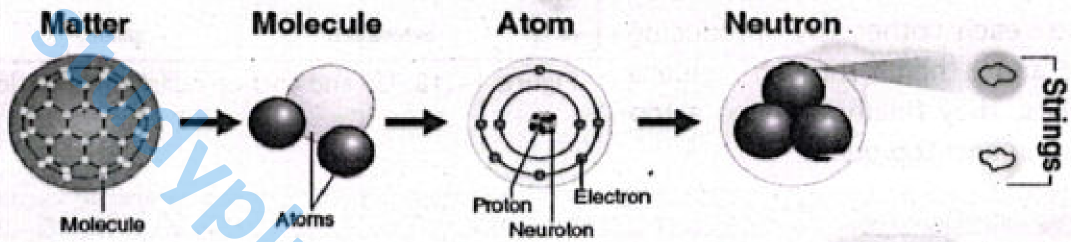



Figure 14.16: Composition of matter according to string theory.

## 14.7 PARTICLE ANNIHILATION

Einstein's famous mass-energy equation indicates that mass can be converted into energy and vice versa. As we studied earlier, that soon after Big Bang matter and anti-matter were formed. What will happen if matter comes close to its anti-matter? They annihilate each other and convert their mass into energy in accordance with Einstein's mass-energy equation. A particle and its anti-particle cannot exist in the same place, because when they come close to each other, they annihilate each other. After annihilation process, the product is the energy in the form of gamma ray photons. If an electron and a positron come in a close proximity, they annihilate each other and forms two gamma ray photons.

$$e^- + e^+ \rightarrow \gamma + \gamma$$

Production of two photons is necessary in this reaction to conserve the momentum of the system. The annihilation of electron-positron is shown in Fig. 14.17. The above reaction of annihilation is that in which the product is energy, but this energy can further be converted into matter particles, in Large Hadron Collider at CERN and Tevatron at Fermi-Lab, the energy from such annihilations has been converted into matter particles which were previously undiscovered.

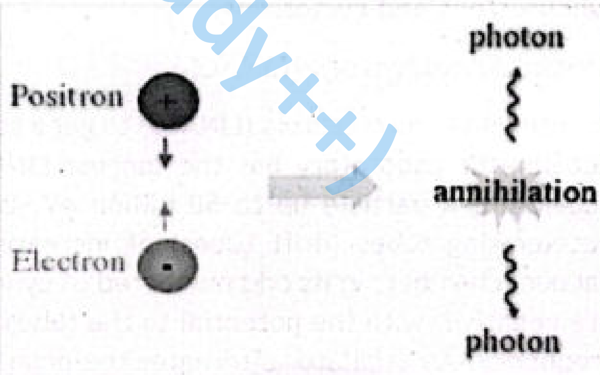



Figure 14.17: Electron-positron annihilation.

### 14.7.1 Annihilation of Matter into Matter

Annihilation not only gives us energy by totally destroying the particles, but it can also produce some new particles. If quarks with their own flavored anti-quark come close to each other, they annihilate each other, producing energy and a new pair of quark and anti-quark. The annihilation of an up quark and an anti-up quark is shown in Fig. 14.18, in which at first, they annihilate each other by producing energy, and then through gluons interactions, they finally produce a top quark and an anti-top quark.

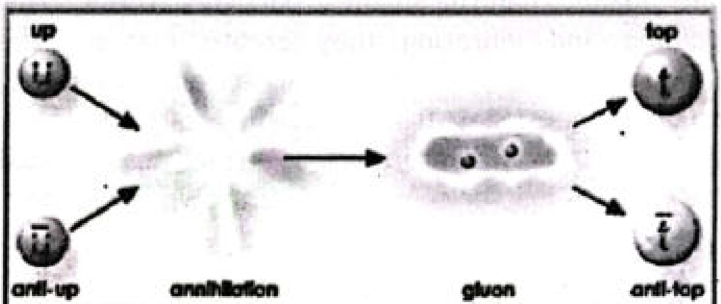


Figure 14.18: Up and anti-up quark annihilation.

**For Your Information**

Resistive plate chamber (RPC) is a particle detector widely used in high energy physics for muon's detection. It is relatively new and has become standard technology. It is known for very high detection efficiency (>95%), excellent sub-nanosecond time resolution, high counting rates and relatively low cost. This detector is easy to build, covers large areas and is extensively used in nuclear facilities like BESIII, ALICE and CMS at CERN. It operates on the principle of ionizing a gas when particles traverse through it.

## 14.8 PARTICLE ACCELERATORS

In High Energy Physics (Particle Physics), we need to accelerate particles to very high speeds to study their behavior at high energies. We use to collide these high energy particles to produce new particles. For such high energies to achieve, we need projectiles that can accelerate the particles to higher velocities. The instruments used to accelerate the particles up to a desired speed and energy are called accelerators. There are two basic types of particle accelerators i.e. linear accelerators and circular accelerators, including synchrotrons and cyclotrons.

### Linear Accelerators (LINACS)

We use linear accelerators (LINACS) to get a steady, intense beam of particle. The SLAC National Accelerator Laboratory has the longest LINAC in the world, which is 2 miles long and can accelerate a particle up to 50 billion eV. Linear accelerators consist of a set of cylindrical accelerating tubes (drift tubes) of increasing length which are connected alternately in a vacuum chamber. In its odd numbered of cylinders are positive and even numbered of cylinders are negative, with the potential to the tubes reversing periodically according to the oscillating frequency. An oscillator alternates the polarity of the tubes. An ion source is used to produce ions which are to be accelerated. An accelerated beam of charge particles leaves the drift tubes and moves towards the target. A schematic diagram of a linear accelerator is shown in Fig. 14.19.

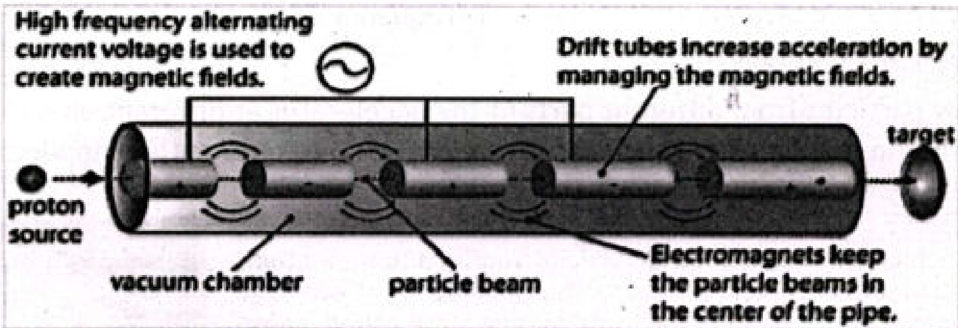



Figure 14.19: Linear accelerator.

## Synchrotron Accelerators

Synchrotron accelerators, also called the 'heavy lifters', are circular accelerators. These are the highest energy accelerators in the world. These accelerators have a closed pathway that moves particles in a ring.

When a particle enters the accelerator it moves repeatedly in the circular path, which is enclosed in a vacuum pipe. Radiofrequency cavities at intervals around the ring increase their speeds. Several magnets create electromagnetic fields to bend the particles' path. The maximum energy obtained by the synchrotron accelerator is 13.6 TeV which was produced during the proton collision in the Large Hadron Collider (LHC) at CERN.

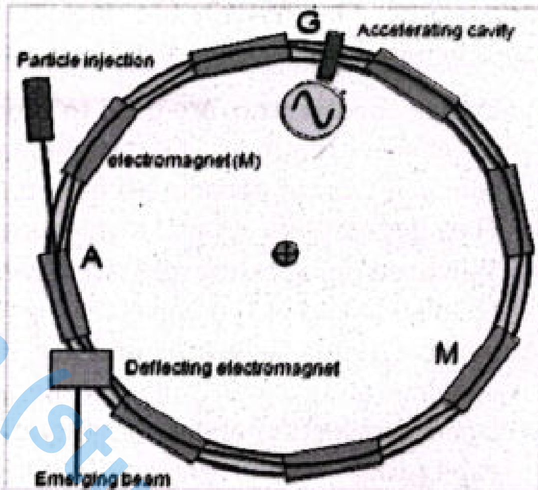



Figure 14.20 (a): Synchrotron accelerator.

Researchers send different beams to collide at required energy at fixed points, where particle detectors (like Compact Muon Solenoid CMS) are positioned to detect the particles produced by these collisions. Synchrotron accelerators are used to study the basic building blocks of the universe. Synchrotron accelerators like LHC can accelerate heavy ions and nuclei like lead. At RHIC accelerator in USA, the heavy ions like uranium and gold to create quark-gluon plasma. A schematic diagram of a synchrotron accelerator is shown in Fig. 14.20 (a).

## Cyclotron Accelerators

Cyclotron accelerators, accelerate the particles in a spiral pattern, starting from the center of spiral. They use one large electromagnet to bend the path of particles. To move particles in increasing larger circles, they use metal electrodes. Cyclotrons can accelerate particles to 520 million eV. Cyclotron accelerators consist of two circular metal boxes called 'dees', which are separated from each other

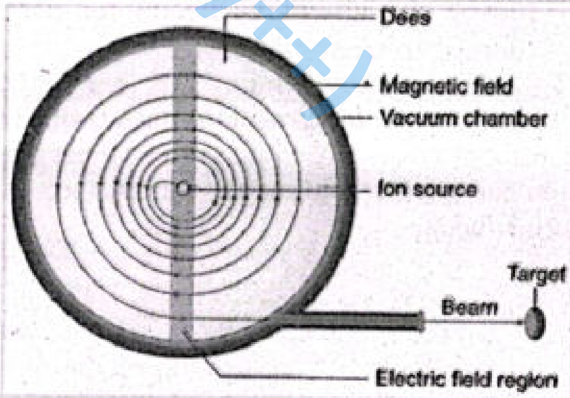


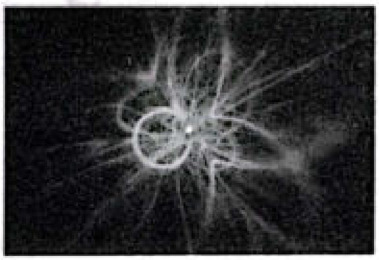

Figure 14.20 (b): Cyclotron accelerator.

and connected to a radiofrequency oscillator of frequency $10^7$ Hz, and alternating potential of 10 kV is applied between dees.

They can draw particles from different parts of the accelerator at different energies. Source of ion is placed in the middle from which ions move outward in spiral path in applied electric and magnetic fields.

**Particle Physics and Early Universe**

Just like at very high energies, electrons are not confined to their atoms i.e., electrons and ions are found in plasma state. Similarly, at very high energies, the baryonic matter gets the same plasma state called quark-gluon plasma, the high temperature soup that made up the universe just after the Big Bang. In this state of baryonic matter, quarks and gluons are no longer confined to each other by the strong force. Instead, they become free from their mutual attraction.



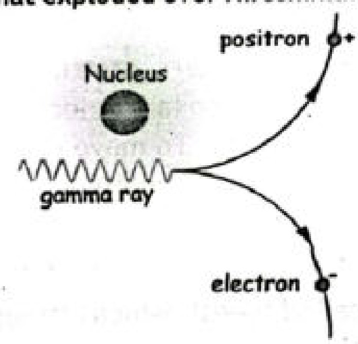
## 14.8.1 Uses of Particle Accelerators

Particle accelerators are not only used for scientific research but they can also be used in various other field of daily life, including medical field. Some uses of particle accelerators are listed below:

- **Linear accelerators** are used to produce medical isotopes, and create beams of radiations for cancer treatment. Electron linear accelerator (LINACS) for cancer therapy is the most common type of particle accelerator. It serves as injectors for higher energy accelerators. They generate x-rays and high energy electrons for radiation therapy.
- **Synchrotron accelerators** are used to produce high energy environment to study the building blocks of the universe. Higgs boson was discovered using these accelerators. In the medical field, they are used for studying cell biology, crystallography, high resolution imaging, and cancer radiation therapy. In environmental sciences they use in toxicology, clean combustion and atmospheric research. They are also used in agriculture, mineral exploration, engineering and forensics.
- **Cyclotron accelerators** are used to produce large number of certain types of particles, such as muons and neutrinos. They are also used to produce medical isotopes. They are the best source of high-energy beams for particle physics experimentation. They use in particle therapy to treat cancer.

**For Your Information**

If 1 kg of anti-matter came into contact with 1 kg of matter, the resulting explosion would be equivalent to 43 megatons of TNT or about 3,000 times more powerful than the bomb that exploded over Hiroshima. Even one gram of anti-matter can produce an explosion of the size of a nuclear bomb.




Example 14.3: A 3.0 MeV photon interacts with a lead nucleus and creates an electron-positron pair. The pair moves perpendicular to the initial direction of travel of the photon. (a) Find the kinetic energies of the electron and positron, assuming the nucleus is at rest after the collision. (b) Find the kinetic energy of lead nucleus to ensure momentum conservation. Would

this amount of K.E greatly affect the results in part (a)? The mass of lead atom is 193007 MeV.

Given: $E_{photon} = 3\ MeV$ $\quad m_{Lead} = 193007\ MeV$

To Find: (a) Kinetic energy of electron: $E_{electron} = ?$

Kinetic energy of positron: $E_{positron} = ?$

(b) Kinetic energy of lead nucleus: $E_{nucleus} = ?$

Solution: (a) According to law of conservation of energy (ignoring energy of nucleus):

$$E_{photon} = E_{electron} + E_{positron}$$

$$3\ MeV = E_{electron} + E_{positron} \quad \text{-------(1)}$$

Now the y-component of momentum (ignoring nucleus):

$$0 = -pc_{electron} + pc_{positron} \quad \text{or} \quad pc_{electron} = pc_{positron}$$

As the pair have equal momentum, they must have equal energy. Hence equation (1) gets the form:

$$E_{electron} = E_{positron} = E$$

$$3\ MeV = 2E$$

Energy of either of the pair is: $E = \frac{3}{2} MeV = 1.5 MeV$

The rest mass energy of each particle is 0.511 MeV, hence:

$$K.E = (1.5 - 0.511)\ MeV$$

$$K.E_{electron} = K.E_{positron} = 0.989\ MeV$$

(b) To ensure x-component of momentum to be conserved (as positron and electron are moving in y-direction): $pc_{photon} = pc_{nucleus}$

$$3MeV = pc_{nucleus}$$

Using: $E_{nucleus} = \sqrt{(pc)^2 + (mc^2)^2}$

Putting values: $E_{nucleus} = \sqrt{(3MeV)^2 + (193007MeV)^2}$

$$E_{nucleus} = 193007MeV\sqrt{1 + \frac{9}{(193007)^2}}$$

Using the binomial expansion, we get:

$$E_{nucleus} = 193007MeV\left(1 + \frac{1}{2}\frac{9}{(193007)^2}\right)$$

$$E_{nucleus} = 2.33\times10^{-5} MeV \quad \text{or} \quad E_{nucleus} = 2.3.3\ eV$$

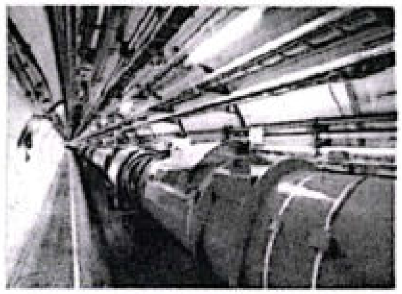
As the nuclear energy has a very small value, the nucleus can preserve momentum conservation while taking a very small portion of the total energy available. So we can ignore the presence of nucleus while dividing the energy of incoming photon between electron and positron.

Assignment 14.3

A gamma-ray photon produces an electron-positron pair. If the rest mass energy of electron is 0.51 MeV and the total K.E of the pair is 0.78 MeV, then find the energy of gamma-ray photon in MeV.

**For Your Information**

The Large Hadron Collider (LHC) at CERN is the most powerful synchrotron accelerator in the world. It moves the particles like protons and other matter at extremely high-speed close to the speed of light, before colliding them. These collisions produce massive particles like the Higgs boson and the top quark. It has the ability to accelerate particles to energy of 6.5 trillion eV.



## SUMMARY

- **Nuclear radiation** includes three types of particles: alpha particle with double positive charge and heaviest among all, beta particle with negative charge and medium mass and the gamma photon which is neutral and massless.
- **Conservation laws** are the universal principles which must be followed during every interaction and decay of particles.
- **Matter and anti-matter** are the two distinct forms of particles, identical in all respects except charge and magnetic moment.
- **Quarks** are the elementary particles which make up all matter in the universe, these are of six types.
- **Hadrons** are the particles which are made up of quarks.
- **Baryons** are the particles which are made up of three quarks, proton and neutron are baryons.
- **Mesons** are the particles which are made up of a quark and an anti-quark, pions are the examples of mesons.
- **Beta positive decay** is the process in which proton converts into a neutron by changing one of its up quarks into down quark, producing a positron and an electron neutrino through the emission of W-boson.
- **Beta negative decay** is the process in which neutron converts into a proton by changing one of its down quarks into up quark, by producing an electron and an anti-neutrino through W-boson.
- **Leptons** are the lightest particles among the fermions; electron and muon are examples of the leptons.
- **Photon, gluon, W and Z bosons** are the force carriers responsible for the exchange of electromagnetic, strong and weak force respectively.
- **Higgs boson** is the particle associated with the Higgs fields which particle interact with to acquire masses.
- **Particle and anti -particle** whenever come close to each other, they annihilate each other by releasing energy or by forming a set of another particle and anti-particle pair.
- **Particle accelerators** are instruments used for accelerating the particle up to very high speed and high energies for research and technology.

## EXERCISE

### Multiple Choice Questions

Encircle the correct option.

1) Which fundamental conservation principle states that the total number of nucleons (protons and neutrons) remains constant in a closed system?

A. charge B. energy C. nucleon D. momentum

2) According to the principle of charge conservation, in any physical process, the total electric charge before and after the process must?

A. decreases B. increases C. become zero D. becomes neutral

3) In a nuclear reaction, if a nucleus emits an alpha particle (helium nucleus), what happens to the nucleon number (A) of the original nucleus:

A. decreases by 1 B. decreases by 2 C. remains same D. increases by 1

4) During beta-minus ($\beta$-) decay, a neutron is transformed into a proton, emitting an electron (beta particle) in the process. What happens to the total charge during this decay, the total charge:

A. increases B. decreases C. remains same D. becomes neutral

5) Positrons in our atmosphere can be produced during the natural process of:

A. spinning of Earth B. lightning C. Earth's magnetic field D. winds

6) Which type of carrier particle has not been found yet?

A. W-boson B. Z-boson C. pion D. graviton

7) What type of hadron is always constructed partially of an anti-quark?

A. baryon B. lepton C. meson D. photon

8) Analysis of which particles resulted into the search of Higgs boson?

A. W and Z boson B. up and down quark
C. mesons and baryons D. neutrinos and photons

9) Which device is used for proton decay?

A. Cyclotron B. Synchrotron C. super-kamiokande D. Large Hadron Collider

10) Lepton number for 'delta' particle is:

A. -1 B. 0 C. +1/2 D. +1

11) Which particle has a baryon number of +1?

A. muon B. tau C. kaon D. omega

12) The particle which has the color property is:

A. pion B. kaon C. gluon D. proton

### Short Questions

Give short answers of the following questions.

14.1 Why does beta-minus ($\beta$-) decay lead to an increase in the atomic number of the nucleus, while beta-plus ($\beta$+) decay results in a decrease in the atomic number?

14.2 Explain how the mass-energy equivalence principle, as expressed by Einstein's famous equation $E=mc^2$, is relevant to understanding the energy release in nuclear processes such as alpha decay.
14.3 How do the principles of nucleon conservation and charge conservation in nuclear reactions reflect deeper symmetries and conservation laws present in the fundamental interactions of subatomic particles?
14.4 Can anti-matter burn? Explain.
14.5 List at least three applications of anti-matter.
14.6 Why are gluons short range exchange particles? Explain.
14.7 Why do gluons interact only with particles in the first two rows of the standard model?
14.8 What are the Feynman diagrams? What information about particles these diagrams give?
14.9 What are the differences between quarks and leptons?
14.10 What advantages do circular particle accelerators have over the linear particle accelerators?
14.11 What is the difference between quantum field theory and the string theory?
14.12 How do we determine if a particle reaction or decay occurs?

## Comprehensive Questions

Answer the following questions in detail.
14.1 Explain how the principles of nucleon conservation and charge conservation play a fundamental role in understanding nuclear processes. Provide examples to illustrate the application of these conservation laws in nuclear reactions.
14.2 Describe the process of beta-minus ($\beta^-$) decay and beta-plus ($\beta^+$) decay. How do these decay processes violate the conservation of nucleon number within a nucleus while still respecting the conservation of charge? How is the energy released during these processes related to the mass difference between the initial and final particles?
14.3 Discuss the nature of gamma radiation in the context of nuclear decay. How is gamma radiation different from alpha and beta particles? Explain how the emission of gamma rays is related to the de-excitation of an excited nucleus and the transition to a lower energy state.
14.4 Discuss the fundamental differences between alpha, beta, and gamma radiation in terms of their origins, properties, interactions with matter, and potential biological hazards. How do these types of radiation impact various applications, such as medical imaging and nuclear energy production?
14.5 What is matter and anti-matter? How did Dirac propose the existence of anti-matter?
14.6 Why is there asymmetry between matter and anti-matter in the known universe?
14.7 Explain the quark family of particles.
14.8 What are hadrons, baryons, mesons and leptons?
14.9 What are the force carriers? Explain.
14.10 What is particle annihilation? Discuss its two cases.
14.11 Explain the particle accelerators. Also, give working of its three types.

14.12 Give in detail the uses of linear and circular particle accelerators.

## Numerical Problems

14.1 A uranium-238 nucleus undergoes alpha decay, emitting an alpha particle (helium nucleus) with a mass of 4.00151 atomic mass units (u). Calculate the energy released in this alpha decay process. The speed of light (c) is approximately $3.00 \times 10^8$ meters per second, and 1 atomic mass unit (u) is approximately equal to 931.5 MeV/$c^2$. **(Ans: 4.25 MeV)**

14.2 A carbon-14 nucleus undergoes beta-minus (β-) decay, emitting a beta particle (electron) with a negligible mass. Calculate the energy released in this beta-minus decay process. The speed of light (c) is approximately $3.00 \times 10^8$ meters per second. **(Ans: 0.17MeV)**

14.3 A nucleus undergoes a gamma decay, emitting a gamma ray photon with energy of 2.00 MeV. Calculate the frequency and wavelength of this gamma ray.

**(Ans: $3.02 \times 10^{20}$ Hz, $9.94 \times 10^{-13}$ m)**

14.4 If the negative solution of Dirac equation is for electron, then prove that the positive solution for same equation is not for a proton of the same momentum.

14.5 Show that the following particle reactions satisfy the laws of conservation of charge, lepton-number and baryon-number.

i) $\pi^- + p = K^+ + K^- + n$ ii) $\pi^+ = \mu^+ + \nu_\mu$

# Glossary

| | |
|---|---|
| **Accuracy** | The closeness of a measured value to a standard or true value. |
| **Addition Of Vectors** | Vectors are added by the head to tail rule, rather than simple algebraic addition. |
| **Adiabatic Process** | A thermodynamics process in which no heat enters or leaves the system. |
| **Amorphous Solids** | Solids in which constituent particles are arranged in a random manner i.e. irregular structures. |
| **Amps** | unit of electric current. |
| **Angular Acceleration** | The time rate of change of angular velocity is called angular acceleration. |
| **Angular Displacement** | It is the angle of the movement of a body in a circular path. |
| **Angular Momentum** | The cross product of position vector and linear momentum. |
| **Angular Velocity** | The rate of change of angular displacement. |
| **Antinode** | The point of maximum displacement on a Stationary wave. |
| **Artificial Gravity** | The rotation of a spaceship about its own axis to produce gravity like effect produced in orbiting space ship to overcome weightlessness. |
| **Atom** | The smallest unit of matter. |
| **Average Velocity** | The average velocity of an object is its total displacement divided by the total time taken |
| **Baryons** | Are the particles which are made up of three quarks like proton and neutron. |
| **Base Quantities** | Seven physical quantities such as length, mass, time, electric current, temperature, amount of substance and intensity of light. |
| **Battery** | A group of connected electric cells. |
| **Boyle's Law** | The volume of a given amount of gas in inversely proportional to the pressure of the gas while temperature is kept constant. |
| **Brittle Materials** | Materials which break after the elastic limit and do not go into the plastic region. |
| **Carnot's Theorem** | No heat engine can be more efficient than a Carnot's engine operating between the same two temperatures. |
| **Centre of Mass** | The point at which all the mass of the body is assumed to be concentrated. |
| **Centripetal Force** | The force which bends the straight path of a body into a circular path. |
| **Charles's Law** | The volume of a given amount of gas in directly proportional to the temperature of the gas while pressure is kept constant. |
| **Circuit** | In electronics, a circuit is a complete circular path that electricity flows through. |
| **Compression** | The region of a longitudinal wave where particles are closer than the average. |
| **Conductor** | A substance or material that allows electrons, or electrical current, to flow through it. |
| **Conservation Laws** | In all natural phenomenon some basic laws are to be conserved like charge, mass-energy, momentum etc. |
| **Conservative Field** | The field in which work done along a closed path is zero. |
| **CP-Violation** | It is a phenomenon of violation of combined conservation laws of charge and parity by the weak force. |
| **Crest** | The portion of a transverse wave above the mean level. |
| **Crystal Lattice** | It is the repetition of unit cells in a crystal. |
| **Crystalline Solids** | Solids which have three dimensional regular patterns throughout their body. |
| **Current** | The movement or flow of electricity through a conductor. |
| **Cyclic Process** | A process which come back to initial state after succession of events. |
| **Cyclotron** | A cyclotron is a machine that accelerates charged particles or ions to high energies |

| | |
|---|---|
| **Cyclotron Accelerators** | These are devices to accelerate the atomic sized particles in spiral path. |
| **Damping** | A process whereby energy is dissipated from the oscillatory system. |
| **Deformation** | The change in shape, length or volume of a body subjected to a force is called deformation. |
| **Denser Medium** | The medium which has greater density, like solids. |
| **Derived Quantities** | The physical quantities defined in terms of base quantities. |
| **Destructive Interference** | The interference between two waves when they are out of phase i.e. 180 degree phase difference. |
| **Diffraction** | Bending of light around obstacles. |
| **Dimension** | A measurable extent of a particular kind, such as length, breadth, depth, or height. |
| **Displacement** | The Directed change in the position of a body from its initial position to its final position. |
| **Doppler Shift** | The apparent change in the frequency of a wave due to relative motion of source and observer. |
| **Drag Force** | A retarding force experienced by an object moving through a fluid. |
| **Drift Velocity** | The average velocity with which free electrons drift towards the positive end of the wire in the presence of external electric field is known as drift velocity. |
| **Ductile Materials** | Materials which goes into plastic region before breakage. |
| **Elastic Collision** | The interaction of two or more bodies in which both momentum and kinetic energy conserve. |
| **Electric Potential** | Electric potential is the work done per unit charge to bring the charge from infinity to a point in an electric field. |
| **Electricity** | The flow of electrons. |
| **Electron** | A negatively charged particle that orbits the nucleus of an atom. The flow of electrons produce electricity. |
| **Energy** | The ability to do work. |
| **Entropy** | It is the measure of disorder of the system. |
| **Entropy** | A thermodynamic quantity representing the unavailability of a system's thermal energy for conversion into mechanical work, often interpreted as the degree of disorder or randomness in the system. |
| **Equilibrium** | The condition of a system when neither its state of motion nor its internal energy state tends to change with time. |
| **First Condition of Equilibrium** | When the body has zero linear acceleration. |
| **First Law of Thermodynamics** | When heat (Q) is added to the system it increases the internal energy (U) of the system plus work (W) is done by the system. |
| **Force Carriers** | These are the particles which carry anyone of the natural forces. |
| **Forced Oscillations** | The oscillations of a body subjected to an external force. |
| **Free Oscillations** | The oscillation of a body or system with its own natural frequency and under no external influence other than the impulse that initiated the motion. |
| **Freely Falling Body** | A body moving under the action of gravity only. |
| **Gay Lussac's Law** | The pressure of a given amount of gas in directly proportional to the temperature of the gas while volume is kept constant. |
| **Geo-Stationary Satellite** | The satellite whose orbital motion is synchronized with the rotation of the Earth. |
| **Gluon** | It is the mediator for strong nuclear force. |
| **Hadrons** | The subatomic particles which are made up of quarks. |
| **Harmonics** | An overtone accompanying a fundamental tone at a fixed interval, produced by vibration of a string, column of air, etc. |
| **Heat** | The amount of thermal energy in a system. |

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| Heat Engine | It is a device which converts heat energy into the mechanical energy. |
| Higgs Particle | It is a vector boson which has the field called Higgs's field, particles acquire their mass by the interaction with this field. |
| Ideal Fluid | An ideal fluid is a fluid that is incompressible and no internal resistance to flow (zero viscosity). |
| Impulse | The product of force and time for which it acts on a body. |
| Inelastic Collision | The interaction in which kinetic energy does not conserve, however total energy and momentum remains conserved. |
| Instantaneous Acceleration | The acceleration of a body at a particular instant of time. |
| Instantaneous Velocity | The velocity of a body at a particular instant of time. |
| Insulator | Any material that will not allow electricity to easily flow through. |
| Internal Energy | The energy possessed by the molecules of a body. |
| Irreversible Process | A process which cannot be retraced in backward direction without bringing change in its surroundings. |
| Isobaric Process | A thermodynamic process in which the pressure of the system remains constant. |
| Isochoric Process | A thermodynamic process in which the volume of the system remains constant. |
| Isothermal Process | A thermodynamic process in which the temperature of the system remains constant. |
| Kilowatt | A unit for measuring electrical energy. |
| Kilowatt Hour (Kwh) | One kilowatt of electrical energy produced or used in one hour. |
| Kinetic Energy | Energy possessed by a body due to its motion. |
| Laminar Flow | The type of fluid flow in which the fluid travels smoothly or in regular paths. |
| Leptons | Particles which are light in mass and do not feel the strong nuclear force, like electron and neutrinos. |
| Linear Accelerators | These are devices to accelerate the atomic sized particles in straight lines for different experiments. |
| Longitudinal Wave | The wave in which the particles of the medium vibrate parallel to the propagation of the wave. |
| Magnetic Field | The region around a magnet or region around the current carrying conductor in which magnetic force acts on magnetic material. |
| Magnetic Flux | The magnetic lines of force passing through area held perpendicular to field lines. |
| Magnetic Flux Density | The magnetic lines of force passing through a unit area |
| Magnetic Flux Linkage | It is the product of magnetic flux and number of turns of coil. |
| Mesons | Particles which are made up of two quarks i.e. a quark and an anti-quark. |
| Modulus Of Elasticity | The ratio of stress to strain in a body. |
| Molar Specific Heat at Constant Pressure | Amount of heat required to raise the temperature of one mole of a gas through 1 K keeping the pressure of the system constant. |
| Molar Specific Heat at Constant Volume | Amount of heat required to raise the temperature of one mole of a gas through 1K keeping the volume of the system constant. |
| Moment Arm | Perpendicular distance between the axis of rotation and line of action of the force. |
| Moment of Inertia | A quantity expressing a body's tendency to resist angular acceleration. |
| Momentum | The product of mass and velocity of an abject. |
| Node | The point on a stationary wave having zero displacement. |
| Null Vector | A vector having zero magnitude and arbitrary direction. |

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| **Orbital Velocity** | The tangential velocity to put a satellite in orbit around the Earth. |
| **Oscillatory Motion** | The to and fro motion of an object from its mean position. |
| **Pair Annihilation** | Particle and anti-particle when come close to each other annihilate each other by producing energy. |
| **Particle Accelerators** | These are devices to accelerate the atomic sized particles. |
| **Periodic Motion** | The motion repeated in equal intervals of time. |
| **Permeability** | It is the measure of the ability of a material to support the formation of a magnetic field within the material |
| **Phase** | A quantity which indicates the state and direction of motion of a vibrating particle. |
| **Photon** | It is the mediator for electromagnetic force. |
| **Physical Quantity** | The quantity which can be measured. |
| **Physics** | The branch of science that describes the motion, energy and their mutual relationship. |
| **Pitch** | The quality of a sound governed by the rate of vibrations producing it. The degree of highness or lowness of a tone. |
| **Plane Wave Front** | When the small part of the spherical or cylindrical wave front originates from a distant source, then the wave front which is obtained is known as a plane wave front. |
| **Polarization** | The orientation of vibration along a particular direction. |
| **Polycrystalline Solids** | Solids that have intermediate structure between crystalline and amorphous. |
| **Position Vector** | A vector that describes the location of a point. |
| **Potential Difference** | It is the difference between the potentials between two points in the electric field. |
| **Potential Energy** | Energy possessed by a body due to change in its position in some force field. |
| **Power** | The ability or capacity of a body to do work. |
| **Precision** | The closeness of measured values to each other. |
| **Progressive Wave** | The wave which transfers energy away from the source. |
| **Projectile** | An object thrown into the space or air and allowed to move free under the influence of gravity and air resistance. |
| **Quarks** | The elementary particles which are the basic building blocks of all the matter. |
| **Radian** | The angle subtended at the centre of the circle by an arc length equal to the radius of the circle |
| **Random Error** | An error in measurement caused by factors which vary from one measurement to another. |
| **Range of a Projectile** | The horizontal distance covered by the projectile from the point where it is launched to the point where it hits the ground at same level. |
| **Rarer Medium** | A medium in which speed of light is more and has less density is known as rarer medium. |
| **Rays** | Radial lines leaving the point source in all directions. |
| **Rectangular Components** | The components of a vector which are mutually perpendicular. |
| **Refrigerator** | A device which extract heat from colder body and rejects it into hotter body. |
| **Resistance** | The opposition to flow of electricity through a material. |
| **Restoring Force** | The force that brings the body back to its equilibrium position. |
| **Resultant Vector** | A single vector which has the same effect as all individual vectors may have. |
| **Reversible Process** | A process which can be retraced back under same conditions. |
| **Rotational Equilibrium** | If a body rotating with constant angular velocity, it has zero angular acceleration and said to be in rotational equilibrium. |
| **Scalar Product** | The product of two vectors which gives, as a result a scalar quantity. |

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| **Scalar Quantity** | A physical quantity that has magnitude only. |
| **Second Condition of Equilibrium** | The condition in which a body has zero angular acceleration. |
| **Second Law of Thermodynamics** | To extract work from a system two reservoirs at different temperatures are necessary so that heat can flow between them. |
| **Significant Figures** | The number of accurately known digits and first doubtful digit are called significant figures. |
| **Simple Harmonic Motion** | A motion of a body in which acceleration is directly proportional to displacement from mean position and is always directed towards the mean position. |
| **Solenoid** | A solenoid is a coil of insulated or enameled wire. |
| **Spherical Wave Front** | When the disturbance is propagated in all directions from a point source. |
| **Static Electricity** | An electrical charge built up due to friction between two dissimilar materials. |
| **Stationary Wave** | The superposition of two waves of same frequency but moving in opposite direction. |
| **Steradian** | The solid angle subtended at the centre of a sphere by an area of its surface equal to the square of the radius of the sphere |
| **Strain** | The quantitative measure of deformation in a body. |
| **Strain Energy** | The area under the force-extension graph represents the strain energy in the deformed material. |
| **Stress** | It is the applied force per unit area of the solid. |
| **Switch** | An electrical component used for connecting, breaking, or changing the connections in an electrical circuit. |
| **Synchrotron Accelerators** | These are devices to accelerate the atomic sized particles in circular path. |
| **System** | A thermodynamic system is the portion of universe under study. |
| **System International** | The international agreed system of units used almost over. |
| **Systematic Error** | Systematic error is associated with faulty equipment or a flawed experiment design. |
| **Terminal Velocity** | Maximum constant velocity of an object falling vertically downward. |
| **Tesla** | It is SI unit of magnetic flux density. |
| **Thermal Energy** | The energy of a system possessed due average kinetic energy of its molecules. |
| **Thermal Equilibrium** | When two systems are at same temperature they are called in thermal equilibrium with each other. |
| **Thermal Equilibrium** | The state in which net flow of heat is zero. |
| **Thermodynamic Work** | In thermodynamics when gas is the working substance the work is equal to the product of pressure and the change in volume of the system. |
| **Thermodynamics** | The study of transfer of heat energy. |
| **Torque** | Torque is the twisting force that tends to cause rotation. |
| **Total Internal Reflection** | When the angle of incidence increases by the critical angle, then the incident light in spite of leaving the medium is reflected back in the same medium. |
| **Trajectory** | The path followed by a projectile. |
| **Translational Equilibrium** | A body moving with uniform velocity, hence moving with zero linear acceleration. |
| **Transverse Wave** | The wave in which the particles of the medium vibrate perpendicular to the propagation of wave. |
| **Trough** | The portion of a transverse wave where the particles are below mean position. |
| **Turbulent Flow** | Disorderly and changing flow pattern of fluids. |
| **Uncertainty** | The range of possible values within which the true value of the measurement lies |

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| **Unit Cell** | It is the basic unit of a crystalline solid which has all the properties of the entire crystal. |
| **Unit Vector** | A vector of zero magnitude used for the direction. |
| **Vector Product** | The product of two vectors that result into a vector quantity. |
| **Vector Quantity** | A physical quantity which requires both magnitude and direction for its complete discerption. |
| **Velocity Selector** | Velocity selector is a device with a perpendicular arrangement of electric and magnetic fields which is used as a velocity filter for charged particles. |
| **Volt** | The unit of measurement of force used to produce an electric current. |
| **W And Z Bosons** | They are the mediator for weak nuclear force. |
| **Watt** | A unit for measuring electric power. |
| **Wave-Front** | A surface over which the phase of the wave is constant. |
| **Wavelength** | The distance between successive crests of a transverse wave. |
| **Weber** | It is SI unit of magnetic flux. |
| **Work** | Work is the energy transferred to or from an object via the application of force along a displacement. |

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