Finding the Radius of a Circle Using Bayesian Probability

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Abstract

Bayesian probability was used to determine the radius of a circle on a standard letter sheet with 15 circles, all of same size and non-overlapping. Out of a total of 321 throws, 130 landed in the cirlce. The radius of a circle using Bayesian probability was found to be 22.77mm+/-3.45mm. This value was compared to the measured value of 21.1mm+/-.2mm.

Key Words
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1 Introduction

It is not always possible to directly find the measurement or existence of an object. For example, the Rutheford Gold Foil Expirement. Ernest Rutheford shot α particles at foil and measured its deflection. He noticed that very few deflected and from there discovered the nucleus.

A similar technique was employed to find the radius of a circle.

Similar to Rutheford's experiment, pens were randomly dropped on a piece of paper containing non overlapping circles (all of same size). Using Bayesian probability, the ratio of the number of dots in the circle to out the circle assists in determing the area of the circle and from there, the radio of the circle.

The method of Bayesian Probability can be compared to direct measurement to see if such an indirect method is valid.

2 Experimental Design

Two methods were used to find the radius of the cirlce. The first was indirectly by using Bayesion probability (theoretical), the second was through direct measurement (experimental).

In this expirement, a standard letter sheet measuring 215.9mm x 279.4mm with 15 non over-lapping evenly sized circles with a thin sheet of cardboard placed under it [1] was placed on a horizontal surface.

A pen with a .4mm tip was arbitrarily droppoed orthogonal to the paper. Each drop was considered an independent event.

The total number of dots (N) was the number of drops inside or on the perimeter of the paper.

The number of dots inside the circle (M) was the number of dots inside or on the perimeter of the circle [2].

Bayes Theorem states that the posterior is proportional to the prior multiplied by the likelihood of an event:

$$P(S|A, M, K, I) = P(S|I)P(M|A, N, S, K, I) \tag{1}$$

[1] to prevent drag mark from the pen

[2] here the perimeter means the outer edge of the black line

where S is area of the circle (mm), A is the area of the paper (mm), K is the number of circles, I is prior information, M is number of dots that landed in the circle, and N is the total number of dots.

From this equation, an equation that would find the most likely area for the circles wa derived [3]:

$$P(S|A,M,N,K,I) = \frac{N!}{M!(N-M)!}(\frac{KS}{A})^{M}(1-\frac{KS}{A})^{N-M} \tag{2}$$

The log of the equation was used to find probable areas

$$Log(P(S|A,M,N,K,I)) = Log(\frac{N!}{M!(N-M)!})+Mlog(\frac{KS}{A})+(N-M)Log(1-\frac{KS}{A}) \tag{3}$$

By taking the derivative and setting it to S, the most probable area was calculated:

$$\frac{d}{dS}Log(P(S|A,M,N,K,I)) = \frac{M}{S} - \frac{(N-M)K}{A-KS} \tag{4}$$

$$\frac{MA}{NK} = S_0 \tag{5}$$

The permimeter of the cirle to the other permiter of the circle was measured for the diameter [4].

3 Experimental Results

A pen was dropped on the paper 321 times, of which 130 landed in the circles and 191 landed outside the cirlce.

Symbol	Description	Value
M	Number of dots in Circle	130
N	Total number of dots	321
A	Area of Paper (mm^2)	160322.46
K	Number of Circles	15
S	Area of Circle	N/A

Table 1:

The number of dots that landed in the circle was about 1/3 of the total dots, which seemed to be in agreement that the sum of the area of all the circles would be roughly 1/3 of the total area of the paper.

From this,a graph of Log(P(S—A,N,I,M,K) vs S was plotted using equation 3:

[3] derivation at end of paper

[4] again, it is defined as the outer edge of the black line

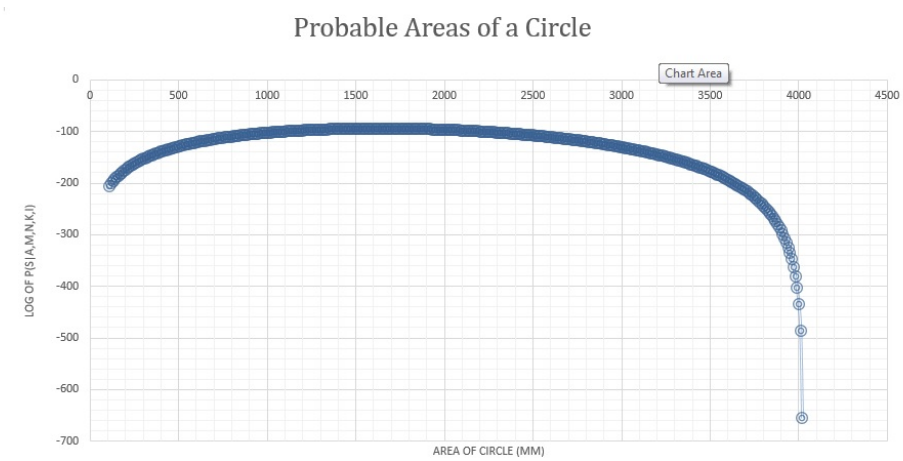

Figure 1: The peak is around 1500m-1800mm

From equation 5, the most likely area (S_0) was 1628.24mm^2+/-3.54mm^2. This value was in agreement with the graph as being the most likely value. With this value, the radius of the circle was calculated:

$$\begin{aligned} r &= \sqrt{\frac{S_0}{\pi}} \\ &= \sqrt{\frac{1628.24}{\pi}} \\ &= 22.77mm + / - 3.45mm^2 \end{aligned}$$

To see if these values are reasonable, direct measurement was taken. The diameter of the radius was measured to be 42.1mm+/-.2mm. From this, the radius was calculated:

$$\begin{aligned} r &= \frac{d}{2} \\ &= 21.1mm^2 + / - .4mm^2 \end{aligned}$$

Where d was distance

The area was then calculated:

$$\begin{aligned} A &= \pi r^2 \\ &= \pi 21.1^2 \\ &= 1392.0 + / - .8mm \end{aligned}$$

From here, it can be observed that the area obtained using Bayesian probability is quite off from from the measured value-16.9% off.

However, the radius obtained from Bayesian was $22.7mm^2$+/-$3.45mm^2$ while the measured was $21.1mm^2$+/-$.8mm^2$.

Finally, the two radii were averaged to produce a more refined estimate of $21.9mm^2$+/-$1.93mm^2$

4 Conclusion

The value for the radius using Bayesian Probability did not differ greatly from measure values although the area (as expected) did.

However, 321 dots was not a large enough quantity for this expirement. A much greater sample would yield data that is closer to the measure and also would produce a smaller uncertainty.

5 Acknowledgements

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6 Citation

D Sivia, *Data Analysis:A Bayesian Tutorial* Cambridge:Oxford University Press pp 2-27

7 Derivation

Symbol	Description
M	Number of dots in Circle
N	Total number of dots
A	Area of Paper (mm^2)
K	Number of Circles
S	Area of Circle

Table 2:

The number of dots that landed in the circle was about 1/3 of the total dots, which seemed to be in agreement that the sum of the area of all the circles would be roughly 1/3 of the total area of the paper.

The probability of landing in one circle is S/A so for 2 it is 2S/A and so on up to KS/A. The probability of not landing in the circle is (A-KS)/A).

The likelyhood of all hits being in the circle is very low. It is more likely that some will fall in and some will fall out.

Each individual hit is treated as an independent event and as points so the order for how they fall does not matter.

N-M will give the number that did not fall in the circle. Taking the product of the likely hood of landing in the circle and not landing in the circle wil tell how likely it is to land in the circle.

$$P(S|A,M,N,K,I) = \frac{N!}{M!(N-M)!}(\frac{KS}{A})^M(1-\frac{KS}{A})^{N-M} \tag{6}$$