Physical conjugate(?) prior

[redacted]

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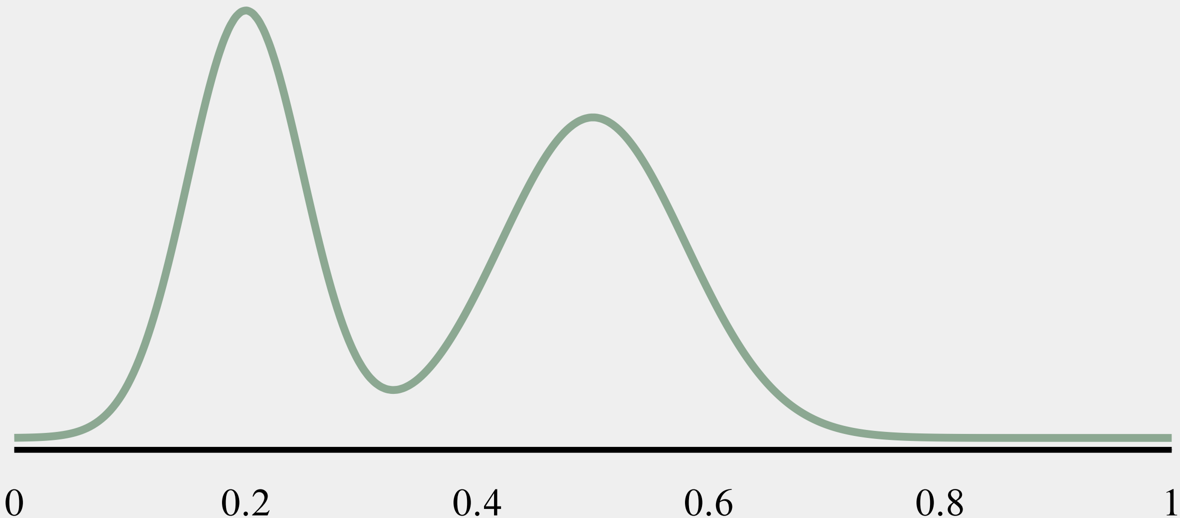
Gaussian mixture

$$p(\psi \mid \Theta) = \sum_{i=1}^{k} w_i \mathcal{N}\left(\psi \mid \mu_i, \sigma_i^2\right)$$



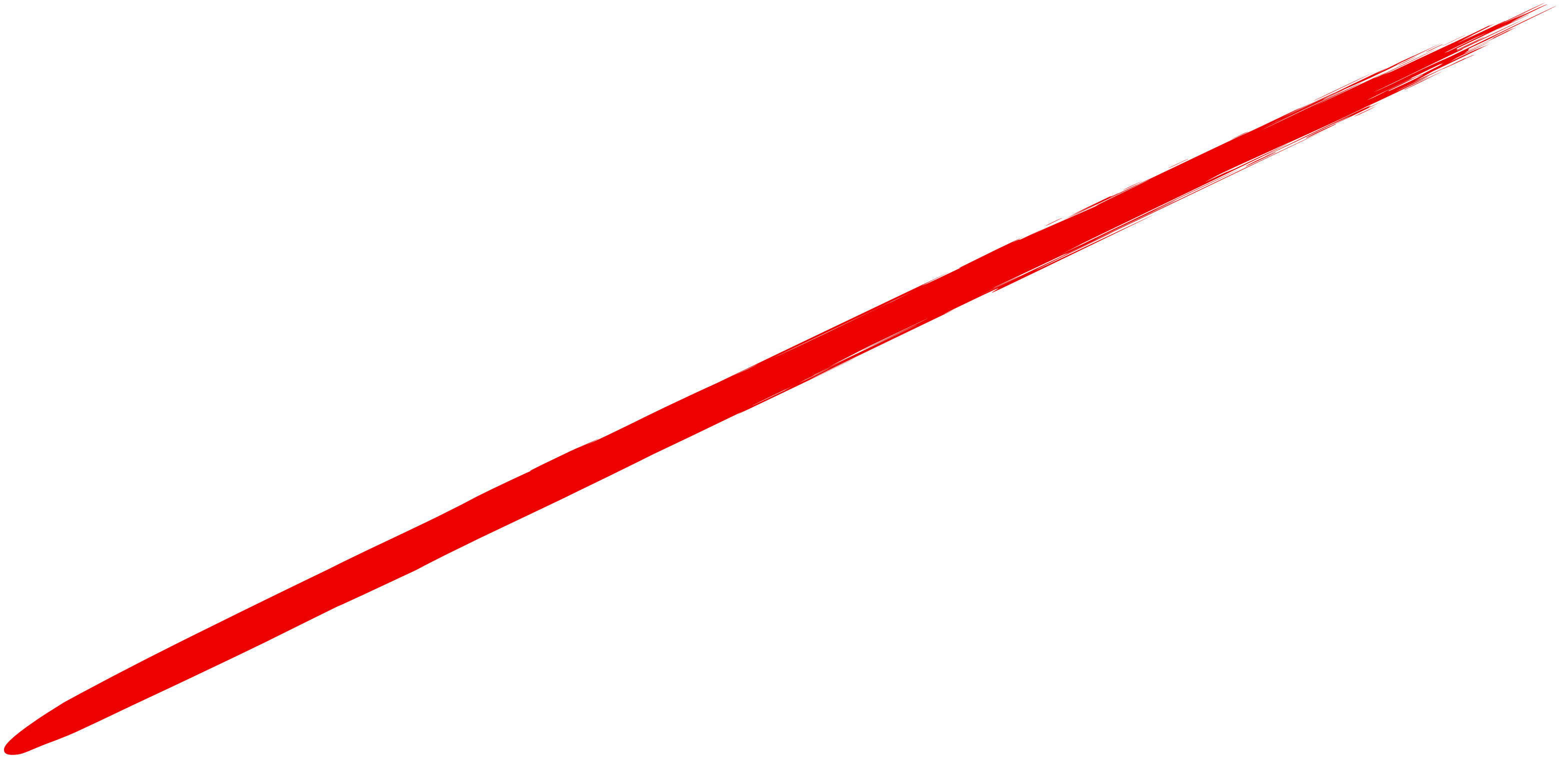
$\mathcal{N}\Gamma^{-1}$ is a conjugate prior for $\mathcal{N}$ with unknown mean and variance.

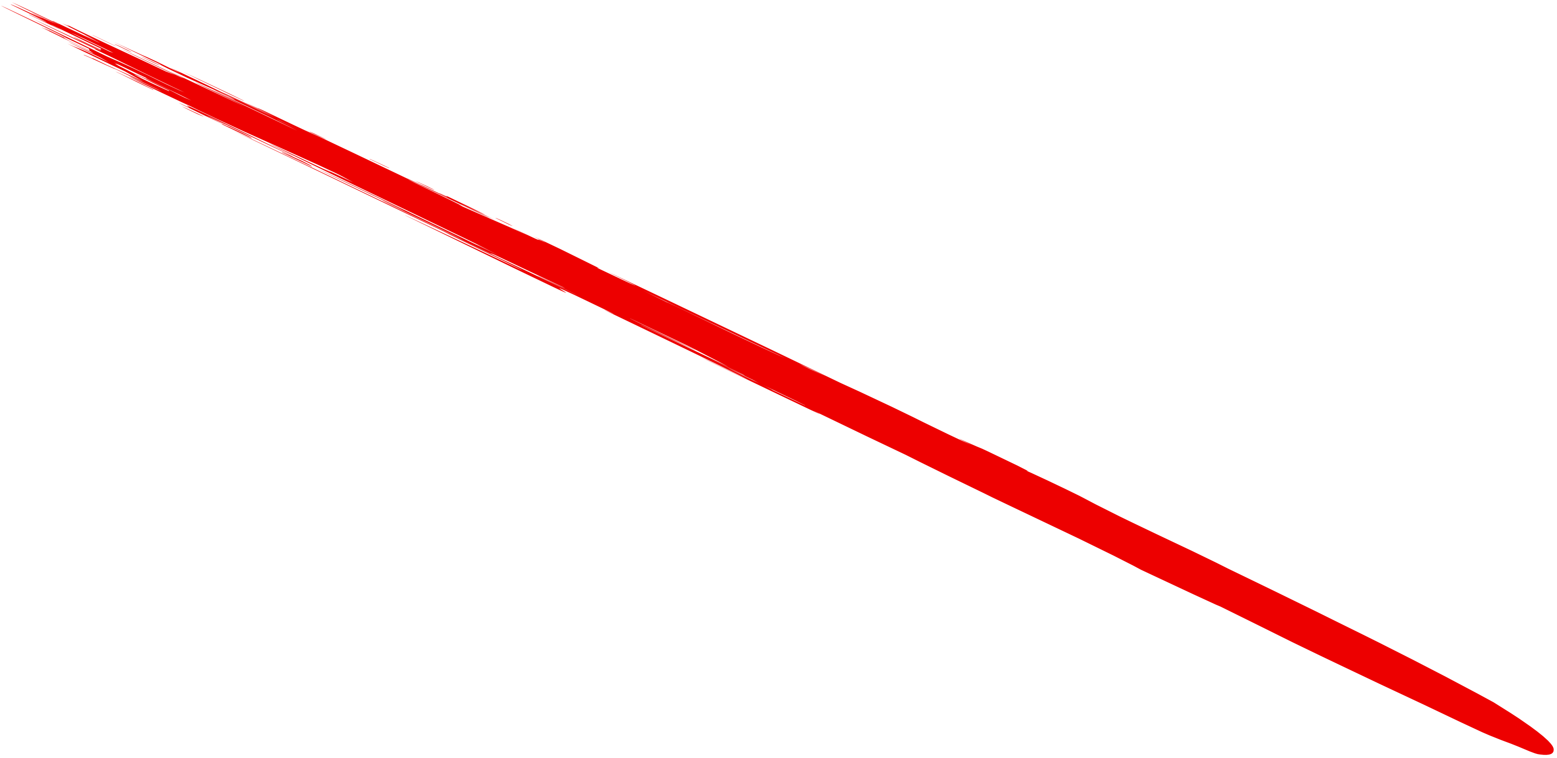
Dirichlet Normal-Inverse-Gamma

$$p(\Theta \mid \Psi) = \mathrm{Dir}(\boldsymbol{w} \mid \boldsymbol{a}) \cdot \prod_{i=1}^{k} \mathcal{N}\Gamma^{-1}\left(\mu_i, \sigma_i^2 \mid \tau_i, \kappa_i, \beta_i, \gamma_i\right)$$

Posterior (???)

$$p(\Theta \mid \psi, \Psi) = \frac{1}{M}\Bigg[\sum_{j=1}^{k} c_j \operatorname{Dir}\left(\boldsymbol{w} \mid \tilde{\boldsymbol{a}}_j\right) \cdot \mathcal{N}\Gamma^{-1}\left(\mu_j, \sigma_j^2 \mid \tilde{\tau}_j, \tilde{\kappa}_j, \tilde{\beta}_j, \tilde{\gamma}_j\right) \cdot \prod_{i \neq j}^{k} \mathcal{N}\Gamma^{-1}\left(\mu_i, \sigma_i^2 \mid \tau_i, \kappa_i, \beta_i, \gamma_i\right)\Bigg]$$





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Insight: Updating the beliefs imply updating the semantics weights w_i, which in hand are updated using the previous slide!

Thm 3.

```
funct moments(measurements) {
```

```
Array moments_hist = [] \\ Parameters for the predictive post.
```

```
for ψ in measurements do {
```

$$\text{posterior} = p(\theta|\psi,\Psi)$$

```
\\ Calculate sufficient moments of the posterior
```

$$\mathit{Array}\ \mathtt{moments} = \mathtt{E}[\mathtt{g}_\mathtt{j}(\theta)] = \mathtt{M}_{\mathtt{g}_\mathtt{j}(\theta)}(\mathtt{posterior})$$

```
moments_hist.push(moments)
```

```
approx_post = p(moments|θ) \\ Approximate posterior as DNIG
```

}

```
predictive_posterior = p(ψ| E[moments_hist]) \\ Gaussian mixture
```

```
return approx_post, predictive_posterior
```

Physical conjugate(?) prior

Dirichlet Normal-Inverse-Gamma

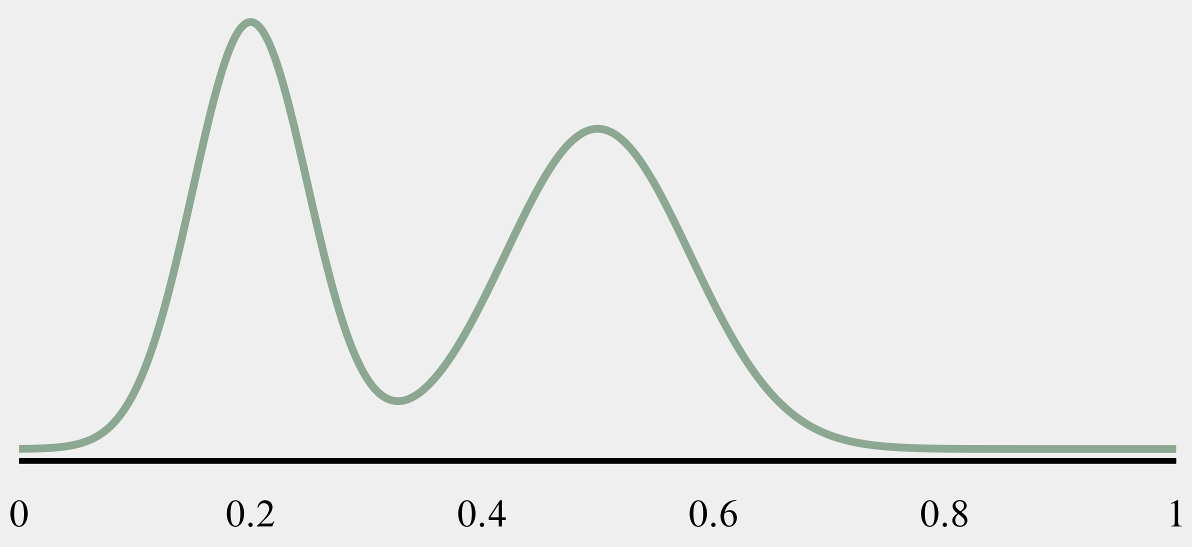
$$p(\Theta \mid \Psi) = \mathrm{Dir}(\boldsymbol{w} \mid \boldsymbol{a}) \cdot \prod_{i=1}^{k} \mathcal{N}\Gamma^{-1}\left(\mu_i, \sigma_i^2 \mid \tau_i, \kappa_i, \beta_i, \gamma_i\right)$$

$\mathcal{N}\Gamma^{-1}$ is a conjugate prior for $\mathcal{N}$ with unknown mean and variance.

$\times$

Gaussian mixture

$$p(\psi \mid \Theta) = \sum_{i=1}^{k} w_i \mathcal{N}\left(\psi \mid \mu_i, \sigma_i^2\right)$$



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Posterior (???)

$$p(\Theta \mid \psi, \Psi) = \frac{1}{M}\left[\sum_{j=1}^{k} c_j \,\mathrm{Dir}\left(\boldsymbol{w} \mid \tilde{\boldsymbol{a}}_j\right) \cdot \mathcal{N}\Gamma^{-1}\left(\mu_j, \sigma_j^2 \mid \tilde{\tau}_j, \tilde{\kappa}_j, \tilde{\beta}_j, \tilde{\gamma}_j\right) \cdot \prod_{i \neq j}^{k} \mathcal{N}\Gamma^{-1}\left(\mu_i, \sigma_i^2 \mid \tau_i, \kappa_i, \beta_i, \gamma_i\right)\right]$$

Thm 3.

```
funct moments(measurements) {
  Array moments_hist = [] \\ Parameters for the predictive post.
  for ψ in measurements do {
    posterior = p(θ|ψ,Ψ)
    \\ Calculate sufficient moments of the posterior
    Array moments = E[gⱼ(θ)] = M_gⱼ(θ)(posterior)
    moments_hist.push(moments)
    approx_post = p(moments|θ) \\ Approximate posterior as DNIG
  }
  predictive_posterior = p(ψ| E[moments_hist]) \\ Gaussian mixture
  return approx_post, predictive_posterior
}
```

Insight: Updating the beliefs imply updating the semantics weights w_i, which in hand are updated using the previous slide!

The Big Picture