**Understanding Data and Statistical Design (60117)**

**Lab 4: Two factor experiments**

This lab is marked from 21.

Please submit via Canvas.

**Due by the conclusion of the lab class**

In this week’s lab we look at two way ANOVA -tests.

**QUESTION 1 [21 marks]**

In this question we analyse the effect of sugar content and chocolate type on the flavour of chocolate chip cookies. The variables we consider are summarised in the table below.

|  |  |  |
| --- | --- | --- |
| **Name** | **Type** | **Description** |
|  | numerical | flavour rating of cookie: 1-10 |
|  | factor | sugar content: 0 (0.5 cups), 1 (0.375 cups), 2 (0.25 cups) |
|  | factor | chocolate type: 1 (milk), 2 (semi-sweet), 3 (dark) |

The sample data consists of 11 observations of for each combination of levels of and (data available in lab4.csv on Canvas) for a total of 99 observations.

The statistical model for this experiment is

where

* is flavour rating of the -th cookie with and
* is global mean
* is the treatment effect on of
* is the treatment effect on of
* is the interaction effect on of and
* is the random effect on of -th cookie with and

The components of the factorial design are:

* **factor A** – sugar content (variable ) which has 3 **levels**: 0 (0.5 cups), 1 (0.375 cups), 2 (0.25 cups)
* **factor B** – chocolate type (variable ) which has 3 **levels**: 1 (milk), 2 (semi-sweet), 3 (dark)
* **treatments** – the 9 combinations of levels of the factors
* **experimental units** – the 9 groups of 11 cookies each prepared with a different treatment
* **measurement units** – the 99 cookies used in the experiment
* **response variable** – flavour rating (variable ).

Note that the response variable is a discrete variable, so we know it cannot be normally distributed. Irrespective of this, we will apply -tests to the data and determine from the residual analysis whether application of these tests seems appropriate.

**Plots**

To start we produce some boxplots of by levels of factors and and combinations of these.

A graph of a box plot

Description automatically generated

A graph of a box plot

Description automatically generated

A graph of a box plot

Description automatically generated

The first chart show the between-treatment variation is dominated by the within-treatment variation, so it is not clear from the chart whether the factor will turn out to have a statistically-significant effect on . Similar comments can be made about the second chart and the effect of the factor on .

The third chart does show some large between-treatment variation for some combinations of levels of factors and , so it appears likely that a significant interaction effect between these factors will be found.

We can investigate interaction effects using the interaction plots below.

A graph of sugar and sugar

Description automatically generated

A graph of lines and a line

Description automatically generated with medium confidence

The non-parallel trace lines in both plots suggest the interaction between factors could be significant.

1. Referring to the interaction plots above, rank the average flavour ratings of cookies made with the 3 types of chocolate using 0.5 cups of sugar **[3 marks]**.

Analysing the plot, the chip factor 1 has the highest mean score, followed by the factor 3 and last factor 2.

**ANOVA and F-tests**

Perform two-factor ANOVA (with interaction) and associated *F*-tests to determine the significance of the two factors.

1. Using significance level , document the interaction term *F*-test from the two way ANOVA. Write down the null and alternative hypotheses, the test statistic and associated p-value, the test decision (with reason) and a conclusion using a minimum of mathematical language **[3 marks]**.

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Description automatically generated

**F-test:** 2.579

**P\_value:** 0.0426

**Ho:** There is no significance difference between the groups of the interaction between sugar:chips

**Ha:** There is significance difference between the groups of the interaction between sugar:chips

**Test decision:** We have strong evidence to reject the null hypotheses, since the p\_value <

**Conclusion:** There is a least one of the groups with a significance difference.

1. Consider again the interaction -test from (b). Write down R code that takes as input the interaction test statistic and returns the p-value **[3 marks]**.

**pf(q = 2.579, df1=4, df2=90, lower.tail = FALSE)**

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**Tukey post hoc analysis**

From the results of the *F*-tests we know that both factors are significant, as is their interaction. Now we perform post-hoc analysis for each factor. Due to the interaction term, we need to do this for each level of the other factor.

1. Using significance level , perform Tukey post-hoc analysis for factor by each level of . Summarise the findings **[3 marks]**.

chip = 1:

contrast estimate SE df lower.CL upper.CL t.ratio p.value

sugar0 - sugar1 1.364 0.821 90 -0.594 3.321 1.660 0.2262

sugar0 - sugar2 2.273 0.821 90 0.315 4.230 2.767 0.0187

sugar1 - sugar2 0.909 0.821 90 -1.048 2.867 1.107 0.5124

chip = 2:

contrast estimate SE df lower.CL upper.CL t.ratio p.value

sugar0 - sugar1 -1.818 0.821 90 -3.776 0.139 -2.213 0.0743

sugar0 - sugar2 -0.364 0.821 90 -2.321 1.594 -0.443 0.8977

sugar1 - sugar2 1.455 0.821 90 -0.503 3.412 1.771 0.1853

chip = 3:

contrast estimate SE df lower.CL upper.CL t.ratio p.value

sugar0 - sugar1 1.091 0.821 90 -0.867 3.048 1.328 0.3834

sugar0 - sugar2 1.727 0.821 90 -0.230 3.685 2.103 0.0950

sugar1 - sugar2 0.636 0.821 90 -1.321 2.594 0.775 0.7195

Confidence level used: 0.95

There is a significant difference between one group for the factor 1.

For the factor 1, the contrast between sugar0-2 has a p\_value of 0.0187

For the rest there is no strong evidence to reject the null hypothesis.

This can be appreciated in the following graph:

![A graph with multiple rows of dots and numbers

Description automatically generated with medium confidence]()

1. Using significance level , perform Tukey post-hoc analysis for factor by each level of . Summarise the findings **[3 marks]**.

A screen shot of a computer

Description automatically generated

There is a significant difference between one group for the factor 0.

For the factor 0, the contrast between chip1-2 has a p\_value of 0.013

For the rest there is no strong evidence to reject the null hypothesis.

This can be appreciated in the following graph:

![A graph with a grid of lines and dots

Description automatically generated with medium confidence]()

**Assumptions**

The conclusions drawn from the -tests rely on certain assumptions being satisfied.

1. Using diagnostic plots, determine if the assumptions of normality, independence and constant variance appear to have been metand from this decide if the conclusions drawn in (b)-(e) are sound **[3 marks]**.

A group of graphs with numbers

Description automatically generated with medium confidence

**Residual vs Fitted:** It doesn’t seem to have a pattern and it appears to be randomly spread, indicating constant variance.

**Normality:** The Q-Q plot suggest a normal distribution since almost all dots lie in the line, however both tails have some points out the line, suggesting that a normality test is necessary

**Scale-Location:** As the residual vs fitted, there is no pattern, suggesting that there is a constant variance for the error.

**Cook’s distance plot:** Suggest that there may be some observations interacting strongly with the model results.

**Constant leverage plot:** It does not seems to be observations with strong interaction with model (outliers).

After reviewing the assumptions using the autoplot, the drawing conclusions from b-e sounds good, however its better to conduce a normality test like the Shapiro test.

1. Using significance level , test if the residuals are normally distributed. Write down the null and alternative hypotheses, the test statistic and associated p-value, the test decision (with reason) and a conclusion using a minimum of mathematical language **[3 marks]**.

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**P\_value:** 0.7373

**Ho:** The distribution of the sample is normal

**Ha:** The distribution of the sample is not normal

**Test decision:** We don’t have evidence to reject the null hypotheses, since the p\_value>

**Conclusion:** We can’t treat the sample distribution as normal.