Math227B Final Project
Due Fri Mar 22

1. Consider the Gierer-Meinhardt reaction-diffusion equations:

$$u_t = D_u\left(u_{xx} + u_{yy}\right) + a\frac{u^2}{v} + \bar{u} - \alpha u, \quad v_t = D_v\left(v_{xx} + v_{yy}\right) + au^2 - \beta v$$

where u and v are concentrations of the activator and inhibitor, a is a (constant) production rate, $\bar{u}$ is a constant parameter representing sources, and α and β are constant decay rates.

a. Find the constant equilibrium solution.

b. Determine the conditions under which patterns will form. That is, find the conditions for instability about the equilibrium solution. These conditions will place constraints on the parameter choices.

c. What is the maximum growing mode? How does it depend on the parameters?

d. Develop a forward-time, centered-space discretization of the Gierer-Meinhardt system.

e. Develop a backward-time, centered-space discretization of the Gierer-Meinhardt system. Only discretize the diffusion term implicitly, not the nonlinear term.

f. Solve the Gierer-Meinhardt system in 1D using as an initial condition, a small perturbation of the steady state solution. Take two sets of parameter choices: (i) parameters for which the steady-state is unstable, (ii) parameters for which the steady states are stable. Be sure to precisely state your parameter choices. Then, solve the system numerically in these two regimes using the methods you developed in parts c and d. Explain the differences in results that you obtain.

g. Compare your linear and nonlinear (numerical) solutions and demonstrate the effectiveness and limitations of the linear stability analysis. That is, over what regions of time does the linear theory predict the nonlinear evolution? You can also use a machine-learning approach!

h. Solve the system in 2D using the RDL lab. Explain some of the differences you observe in 2D compared to 1D.