System of interest:

$$\frac{\partial}{\partial t}y(x,t) = D\frac{\partial^2}{\partial x^2}y(x,t) - ky(x,t)$$

Initial conditions:

$$y(x,0) = 1, y(0,t) = 0$$

Boundary conditions:
Problem 4:

$$\frac{\partial}{\partial x}y(0,t) = \frac{\partial}{\partial x}y(1,t) = 0$$

Problem 5:

$$\frac{\partial}{\partial x}y(0,t) = 1, \frac{\partial}{\partial x}y(1,t) = 0$$

First, apply separation of variables technique to the system in order to get time and space components separate from one another.

$$y(x,t) = U(x)T(t)$$

$$U(x)\frac{\partial}{\partial t}T(t) = DT(t)\frac{\partial^2}{\partial x^2}U(x) - kU(x)T(t)$$

For simplicity, rewrite the above expression as:

$$U(x)T'(t) = DT(t)U''(x) - kU(x)T(t)$$

Divide both sides by $U(x)T(t)$

$$\frac{T'(t)}{T(t)} = D\frac{U''(x)}{U(x)} - k$$

Define $\lambda = \frac{T'(t)}{T(t)}$:

$$\lambda = D\frac{U''(x)}{U(x)} - k$$

$$\lambda U(x) = DU''(x) - kU(x)$$

$$U''(x) = \frac{k-\lambda}{D}U(x)$$

Solving this equation we get:

$$U(x) = Ae^{\sqrt{\frac{\lambda+k}{D}}x} + Be^{-\sqrt{\frac{\lambda+k}{D}}x}$$

Using conditions from problem 4:

$$U(0) = 0 = Ae^0 + Be^0 = A + B$$
$$U(1) = 0 = Ae^{-\sqrt{\frac{\lambda+k}{D}}}(e^{2\sqrt{\frac{\lambda+k}{D}}} - 1)$$

$$2\sqrt{\frac{\lambda+k}{D}} = 2\pi il$$

$$U(1) = A(2i\sin(\pi lx))$$

Using conditions from problem 5:

$$U(0) = 1 = Ae^0 + Be^0$$
$$U(1) = 0 = Ae^{-\sqrt{\frac{\lambda+k}{D}}}(e^{2\sqrt{\frac{\lambda+k}{D}}} - 1)$$