In [107]:
```
%matplotlib inline
import numpy as np
import pandas as pd
import scipy.stats
from matplotlib import pyplot as plt
from IPython.display import HTML
```

In [2]:
```
plt.style.use('ggplot')
```

Systems Cell Biology

Motor simulations assignment

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In [109]:
```
HTML('''<script>
code_show=true;
function code_toggle() {
 if (code_show){
 $('div.input').hide();
 } else {
 $('div.input').show();
 }
 code_show = !code_show
}
$( document ).ready(code_toggle);
</script>
Toggle code <a href="javascript:code_toggle()">here</a>.''')
```

Out[109]: Toggle code here.

A:

Simulate a single kinesin-type motor stepping along a microtubule (MT). At t=0 (the start of a 'run') it should attach to a microtubule. At each subsequent timestep it should have a probability of stepping, or falling off the MT (which ends the run). Each successful step should advance it 8 nm. You should adjust the stepping frequency so that the average velocity is 800 nm/sec, and should adjust the off rate (probability of falling off the MT for a given step) so that the mean travel distance is 800 nm. Important: the 'timestep' of the simulation should be relatively small compared to the frequency of stepping, so on many 'iterations' of the simulation there will be no step; the probability of a step must be the same on each iteration.

In [3]:
```
def simulate_microtubule(n, ts, off_thresh, step_thresh):

    # initialize simulation
    step = np.random.randint(1,ts)
    output = []

    # simulate n runs
    for i in range(n):

        n_ts = 0 #number of timesteps
        motor_steps = 0
        detach = off_thresh + 1

        while detach >= off_thresh:

            n_ts += 1
            step = np.random.randint(1,ts)

            if step < step_thresh:
                detach = np.random.randint(1,ts)
                if detach < off_thresh:
                    output.append([motor_steps, 8*motor_steps, n_ts, mot
or_steps*8/(n_ts/ts), n])
                else:
                    motor_steps += 1

    return output
```

In [5]:
```
df = pd.DataFrame(data=simulate_microtubule(10000, 10000, 99, 102),
                  columns=["motor_steps","travel_distance","time_steps",
"velocity", 'runs'])
```

In [241]:
```
# display statistics
df.describe()
```

Out[241]:

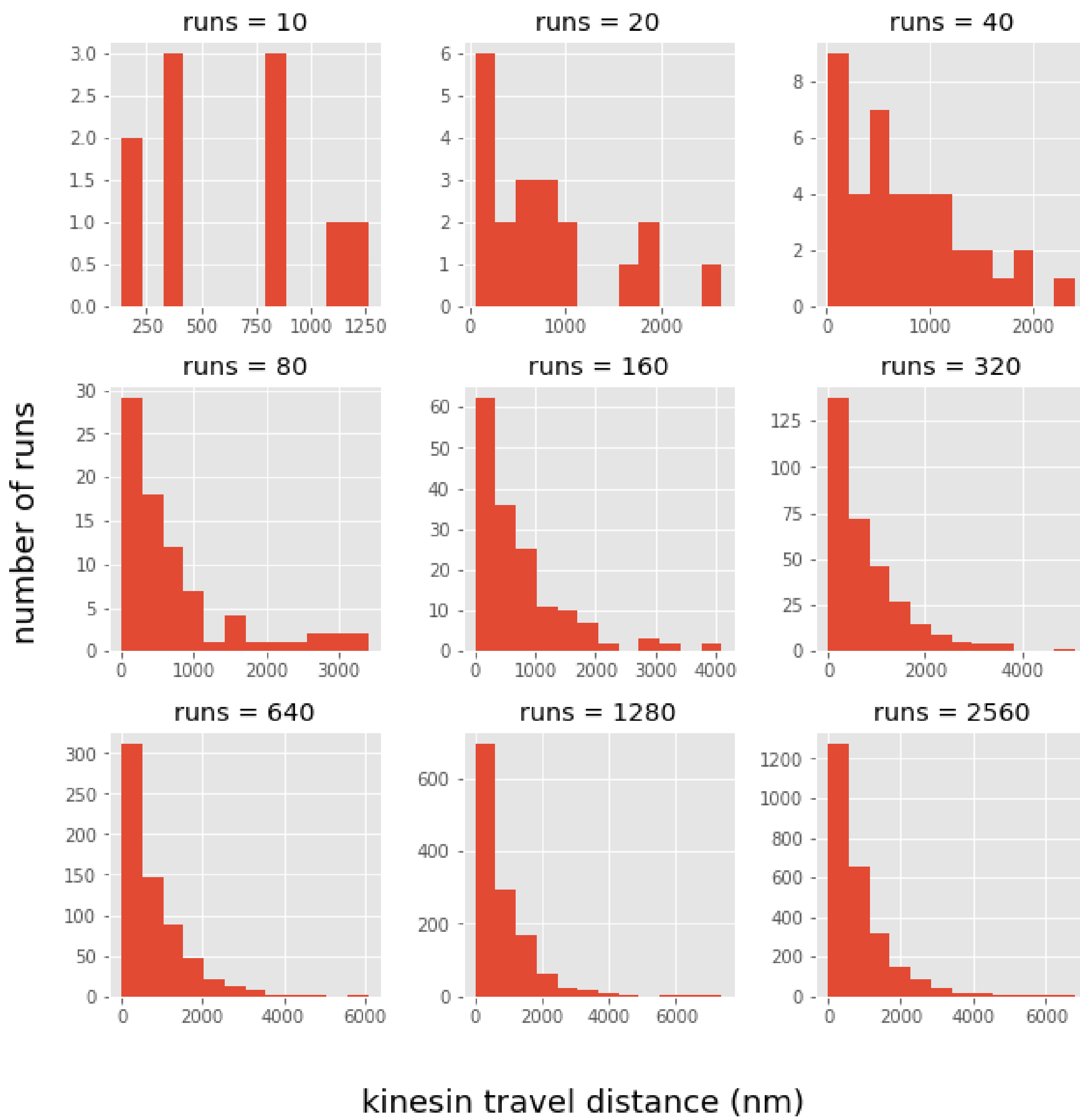
	motor_steps	travel_distance	time_steps	velocity	runs
count	15110.000000	15110.000000	15110.000000	15110.000000	15110.000000
mean	100.622237	804.977895	10061.803044	798.290861	7196.432826
std	101.777890	814.223119	10136.944969	208.093369	3957.611198
min	0.000000	0.000000	1.000000	0.000000	10.000000
25%	28.000000	224.000000	2853.000000	733.307956	2560.000000
50%	70.000000	560.000000	6977.000000	796.634154	10000.000000
75%	138.000000	1104.000000	13809.750000	861.546886	10000.000000
max	1053.000000	8424.000000	111867.000000	8000.000000	10000.000000

B:

Simulate different numbers of trials(e.g. 10, 20, 40, 80, 160, 1000 individual ‘runs’) and make corresponding histograms showing the distribution of run lengths.

In [251]:
```
fig, axarr = plt.subplots(3,3, figsize=(10,10))
font = {'weight': 'normal',
        'size': 18}

runs = 10
for i in range(3):
    for j in range(3):
        cur_df = pd.DataFrame(data=simulate_microtubule(runs, 10000, 99, 
102),
                      columns=["motor_steps","travel_distance","time_ste
ps","velocity", "runs"])
        df = pd.concat([df, cur_df])
        cur_df.hist(column="travel_distance", ax=axarr[i,j], bins=12)
        axarr[i,j].set_title("runs = {}".format(runs))
        runs *= 2


fig.text(0.5, 0.04, 'kinesin travel distance (nm)', ha='center', va='cen
ter', fontdict=font)
fig.text(0.06, 0.5, 'number of runs', ha='center', va='center', rotation
='vertical', fontdict=font)
```

```
Out[251]: Text(0.06,0.5,'number of runs')
```



In [253]:
```
df.describe()
```

Out[253]:

	motor_steps	travel_distance	time_steps	velocity	runs
count	40660.000000	40660.000000	40660.000000	40660.000000	40660.000000
mean	100.868224	806.945794	10095.639621	798.296161	3748.858829
std	101.422947	811.383572	10107.545885	206.695202	3656.499826
min	0.000000	0.000000	1.000000	0.000000	10.000000
25%	28.000000	224.000000	2883.000000	731.629714	1280.000000
50%	70.000000	560.000000	7010.500000	796.204616	2560.000000
75%	140.000000	1120.000000	13950.500000	861.605283	2560.000000
max	1053.000000	8424.000000	111867.000000	8888.888889	10000.000000

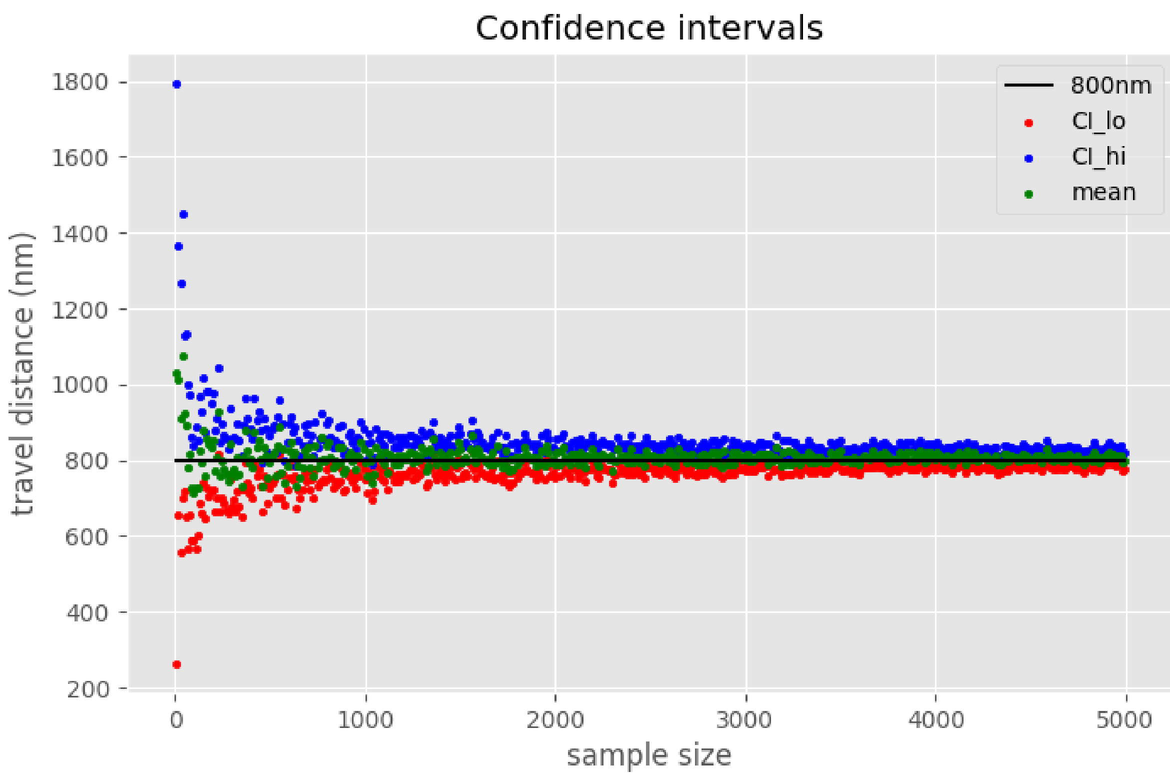
C:

Determine what sample size is required for reasonable estimate of the distribution's properties. First, confirm that your simulation is working correctly—generate a really large number of runs (eg 10,000, and confirm that the mean travel is about 800 nm. If not, there is a problem in your simulation). Next, for each choice of sample size from B, generate many samples of that size, and determine a 95% CI (using standard t-test approach discussed in class) and for each sample determine whether the calculated CI contains the actual mean of the population (which you know to be 800 nm, by construction). Then, for that sample size, determine the proportion of correct events, i.e. the number of tests that included 800, divided by the total number of tests. Plot a graph of the proportion, as a function of the number of elements in the sample. At some point (i.e. for some sample size N), the proportion should get close to 0.95. When is that? Finally, for each sample size, use the statistic we discussed in class (on proportions) : for each sample size, does the 95% CI include 0.95?

In [6]:
```
def confidence_interval(df, num_runs, num_subsets, confidence=0.95):
    subsets = df.sample(num_runs)
    m, se = subsets.mean()["travel_distance"], scipy.stats.sem(subsets[
"travel_distance"])
    h = se * scipy.stats.t.ppf((1 + confidence) / 2., num_runs-1)
    return m, m-h, m+h
```

In [7]:
```
lower, means, upper = [],[],[]

for i in np.arange(10,5000,10):
    m,l,u = confidence_interval(df,i,6)
    lower.append(l)
    means.append(m)
    upper.append(u)

fig, ax = plt.subplots(1,1,figsize=(8,5),dpi=100)
ax.scatter(np.arange(10,5000,10), lower,s=10,color="red",label="CI_lo")
ax.scatter(np.arange(10,5000,10), upper,s=10,color="blue",label="CI_hi")
ax.scatter(np.arange(10,5000,10), means,s=10,color="green",label="mean")
ax.plot(np.arange(10,5000,10), [800 for i in range(len(means))],color="b
lack",label="800nm")
ax.legend()
ax.set_xlabel("sample size")
ax.set_ylabel("travel distance (nm)")
ax.set_title("Confidence intervals")
```

Out[7]: `Text(0.5,1,'Confidence intervals')`



In [12]:
```
trials = 100
sample_sizes = [10*(2**i) for i in range(8)]
correct = {sample: 0 for sample in sample_sizes}

for sample in sample_sizes:
    print(sample)
    for t in range(trials):
        cur_df = pd.DataFrame(data=simulate_microtubule(sample, 10000, 9
9, 102),
                              columns=["motor_steps","travel_distance",
"time_steps","velocity", "runs"])
        if 800 in cur_df["velocity"]:
            correct[sample] += 1
```

```
10
20
40
80
160
320
640
1280
```

In [13]:
```
correct
```

Out[13]:
```
{10: 0, 20: 0, 40: 0, 80: 0, 160: 0, 320: 0, 640: 0, 1280: 100}
```

---

2 (velocity variation)

A:

How does velocity variation change with the size of the window used to calculate velocity? Show distributions

In [43]:
```
velo_df = pd.DataFrame(data=simulate_microtubule(100, 20000, 98, 102),
                       columns=["motor_steps","travel_distance","time_steps",
"velocity", 'runs'])
```

In [44]:
```
velo_df.describe()
```

Out[44]:

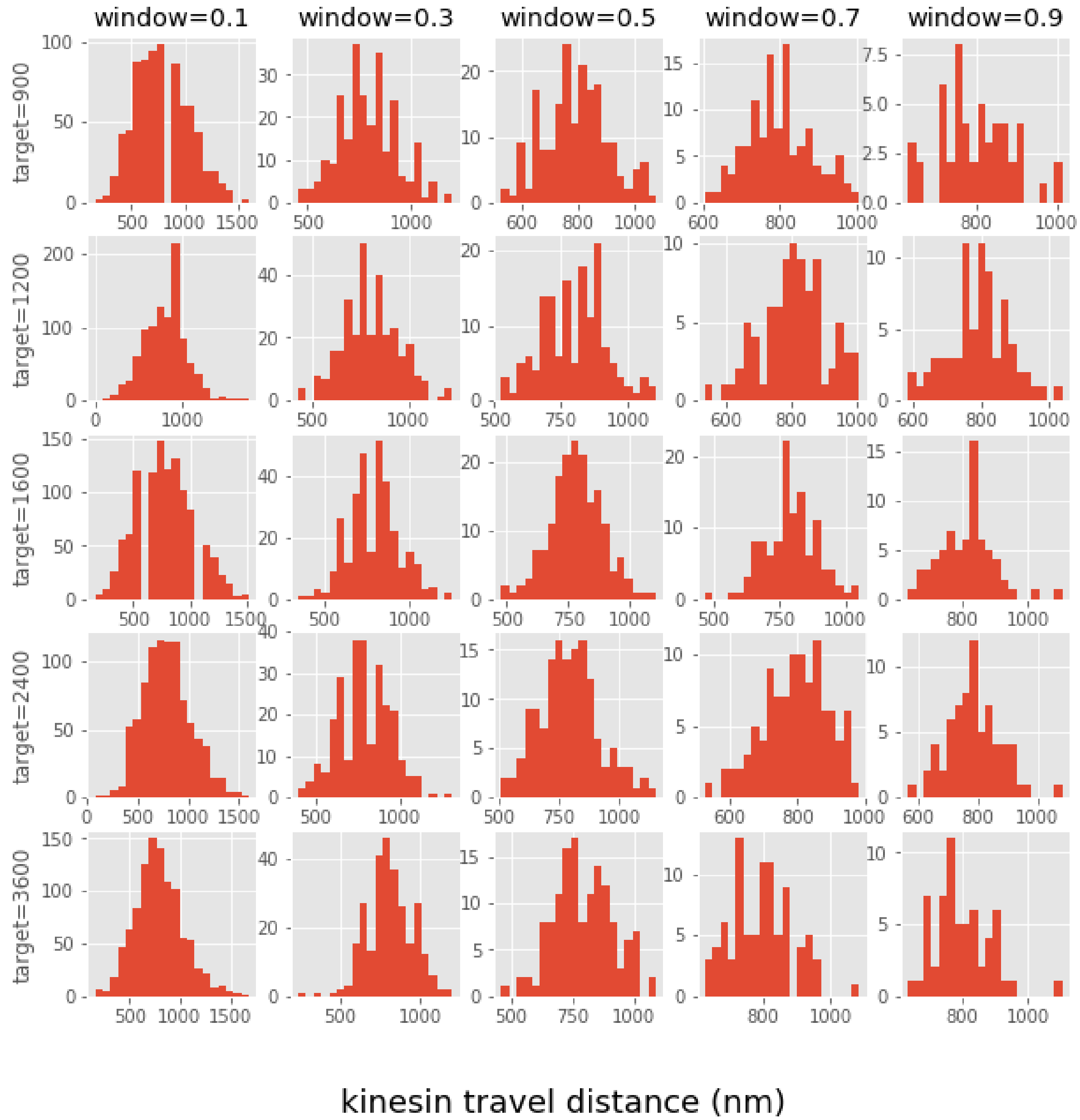
	motor_steps	travel_distance	time_steps	velocity	runs
count	100.000000	100.00000	100.000000	100.000000	100.0
mean	201.620000	1612.96000	40383.210000	797.079306	100.0
std	207.724798	1661.79838	41383.873752	114.415199	0.0
min	5.000000	40.00000	1515.000000	464.609800	100.0
25%	53.500000	428.00000	11249.750000	741.601014	100.0
50%	136.500000	1092.00000	25695.500000	790.153795	100.0
75%	301.250000	2410.00000	57664.750000	836.625694	100.0
max	1115.000000	8920.00000	217315.000000	1236.051502	100.0

In [29]:
```
def simulate_microtubule_window(n, ts, off_thresh, step_thresh, w):

    # initialize simulation
    step = np.random.randint(1,ts)
    output = []
    # simulate n runs
    for i in range(n):

        n_ts = 0 #number of timesteps
        motor_steps = 0
        previous_steps = 0
        detach = off_thresh + 1

        while detach >= off_thresh:

            n_ts += 1
            step = np.random.randint(1,ts)

            if step <= step_thresh:
                detach = np.random.randint(1,ts)
                if detach <= off_thresh:

                    break
                else:
                    motor_steps += 1

            if n_ts == ts*w:
                velo = (motor_steps - previous_steps)*8/w
                output.append(velo)
                n_ts = 0
                previous_steps = motor_steps

    return output
```

In [53]:

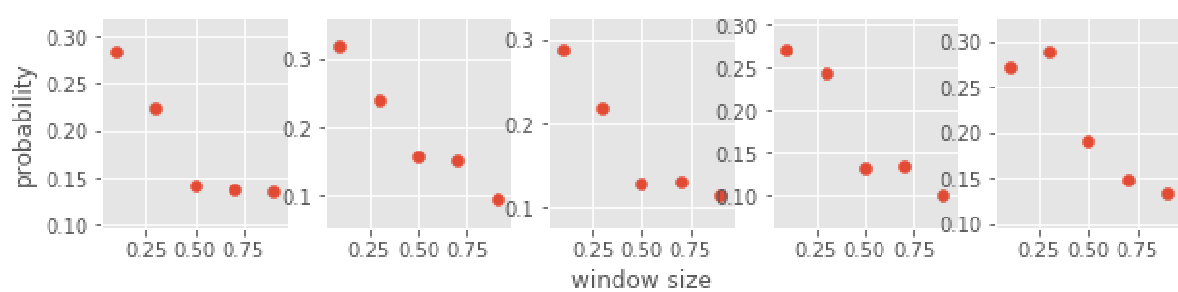
```
fig, axarr = plt.subplots(5,5, figsize=(10,10))
font = {'weight': 'normal',
        'size': 18}
targets = [900, 1200, 1600, 2400, 3600]
windows = [0.1, 0.3, 0.5, 0.7, 0.9]
probs = []
for i, target in enumerate(targets):
    sub_probs= []
    for j,w in enumerate(windows):
        v = simulate_microtubule_window(100, 10000, 100, 100, w)
        sub_probs.append(len([ve for ve in v if ve >= 900])/len(v))
        axarr[i][j].hist(v, bins=20)
        #plt.hist(v)
    probs.append(sub_probs)

for ax, target in zip(axarr[:,0], targets):
    ax.set_ylabel("target={}".format(target))
for ax, w in zip(axarr[0], windows):
    ax.set_title("window={}".format(w))

fig.text(0.5, 0.04, 'kinesin travel distance (nm)', ha='center', va='cen
ter', fontdict=font)
```

Out[53]: Text(0.5,0.04,'kinesin travel distance (nm)')



In [71]:
```
fig, axarr = plt.subplots(1,5, figsize=(10,2))
for i in range(5):
    axarr[i].scatter(windows, probs[i])
axarr[0].set_ylabel("probability")
axarr[2].set_xlabel("window size")
```

Out[71]: Text(0.5,0,'window size')



3:

In [104]:
```
def two_motor_simulation(n, ts, off_thresh, step_thresh, attach_thresh):

    #initialize simulation:
    step = np.random.randint(1,ts)
    output = []

    for i in range(n):

        d1, d2, m1, m2 = 0,0,True,True
        while m1 == True or m2 == True:
            #both motors attached:
            if m1 == True and m2 == True:
                # decide if motors will detach
                m1_detach = np.random.randint(0,ts)
                m1_step = np.random.uniform(0,ts)
                m2_detach = np.random.randint(0,ts)
                m2_step = np.random.uniform(0,ts)

                if m1_step <= step_thresh:
                    if m1_detach <= off_thresh: m1 = False
                    else: d1 += 1
                if m2_step <= step_thresh:
                    if m2_detach <= off_thresh: m2 = False
                    else: d2 += 1

                #did both fall off?
                if m1 == False and m2 == False:
                    output.append(max(d1,d2))

            #motor 1 attached only:
            elif m1 == True:
                m1_detach = np.random.randint(0,ts)
                m1_step = np.random.uniform(0,ts)
                m2_attach = np.random.randint(0,ts)
                # will motor 1 step?
                if m1_step <= step_thresh:
                    # will motor 1 detach?
                    if m1_detach <= off_thresh:
                        m1 = False
                        output.append(d1)
                    else:
                        d1 += 1
                if m2_attach <= attach_thresh: m2 = True

            #motor 2 attached only:
            elif m2 == True:
                m2_detach = np.random.randint(0,ts)
                m2_step = np.random.uniform(0,ts)
                m1_attach = np.random.randint(0,ts)

                # will motor 1 step?
                if m2_step <= step_thresh:
                    # will motor 1 detach?
                    if m2_detach <= off_thresh:
                        m2 = False
                        output.append(d2)
```

```
                    else:
                        d2 += 1
                if m1_attach <= attach_thresh: m1 = True
    return output
```

In [105]:

```
fig, axarr =plt.subplots(1, 4, figsize = (12,3))
for velo, step, ax in zip([100,200,400,800], [99.875, 99.75, 99.5, 99.],
axarr):
    o = two_motor_simulation(100, 10000, 100, step, 10)

    pi = 2 # fixed on-rate, 2 per second
    analytic = 800*(1+(pi*800/(2*velo))) # calculate analytic solution f
rom Lecture 3 slide #36, Analytic Mean-field theory
    ax.hist(np.array(o)*8, bins=12)
    ax.axvline(x=np.mean(np.array(o)*8),color ='black',linewidth=2, line
style='dashed')
#     plt.text(scipy.mean(np.array(distance)*8) + 500., hist[0].max()*0.
75, 'Mean= %1.1f' % scipy.mean(np.array(distance)*8), fontsize=15)
#     plt.text(scipy.mean(np.array(distance)*8) + 500., hist[0].max()*0.
60, 'Analytic mean= %1.1f' % analytic, fontsize=15)
    ax.set_title('velocity:{}'.format(velo))
    ax.set_xlabel('mean:{}'.format(np.mean(o)) + "\n" + 'analytic mean:
{}'.format(analytic))
axarr[0].set_ylabel('num runs')
```

Out[105]: Text(0,0.5,'num runs')