

Attention

CISC 7026: Introduction to Deep Learning

University of Macau

Agenda

1. GNN Review
2. VAE Review and Coding
3. Attention
4. Keys and Queries
5. Self Attention
6. Homework

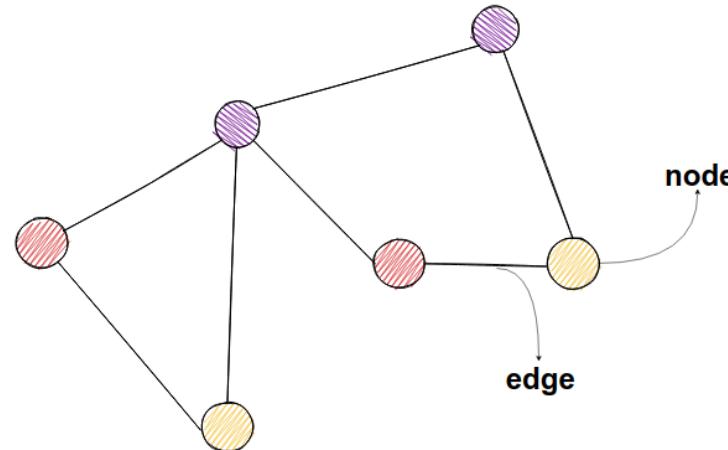
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Graph is a structure of nodes (vertices) and edges

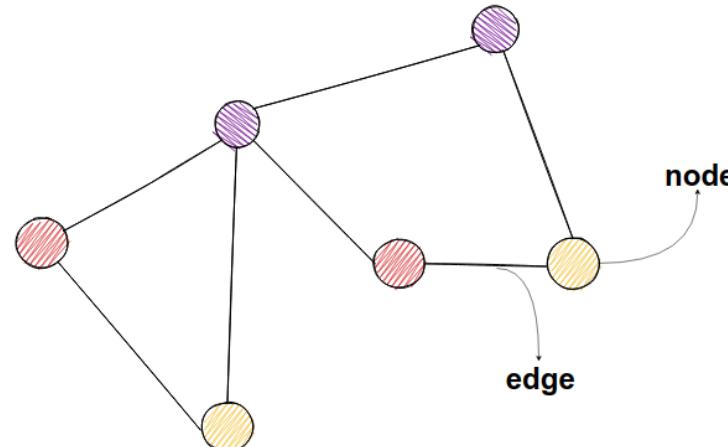
GNN Review

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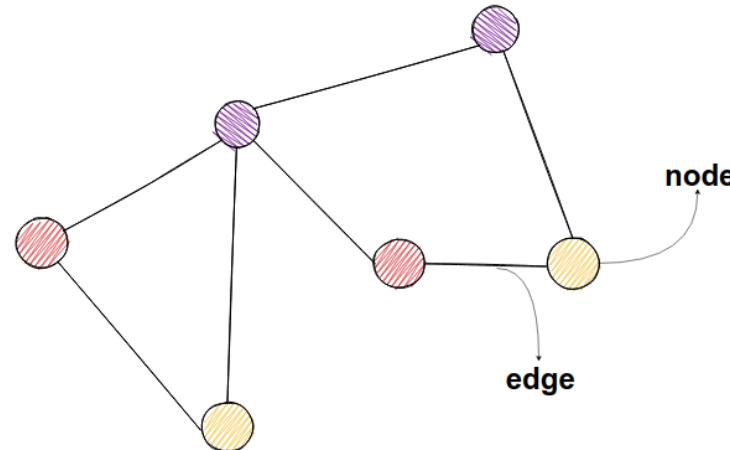
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A **node** is a vector of information

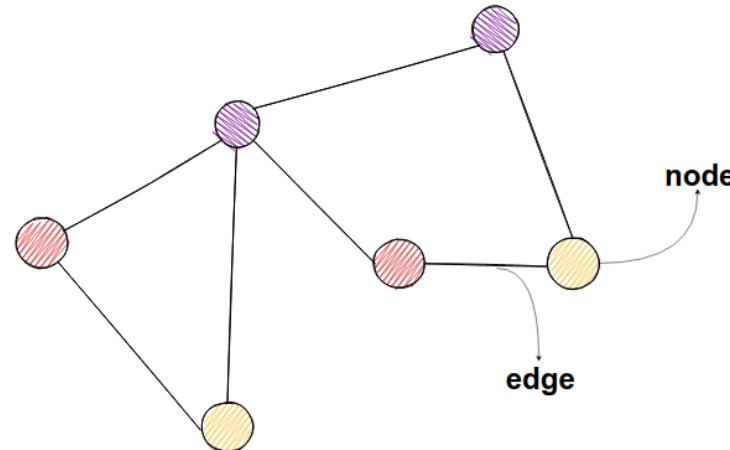
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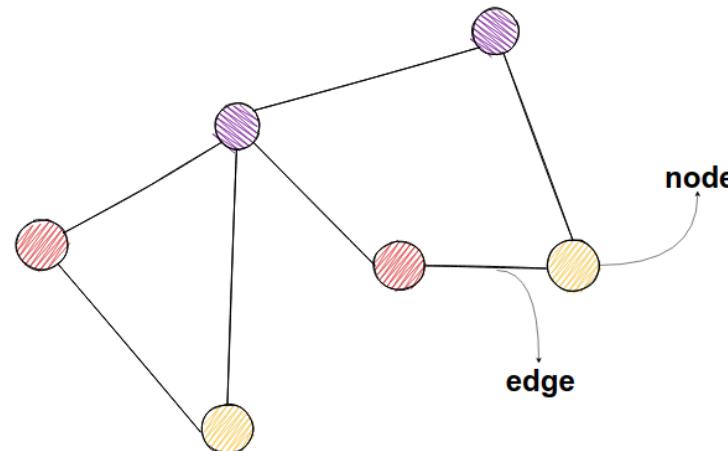
An **edge** connects two nodes

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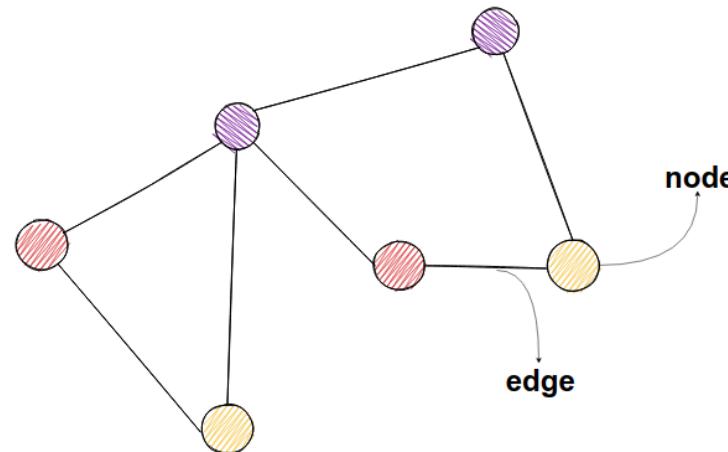
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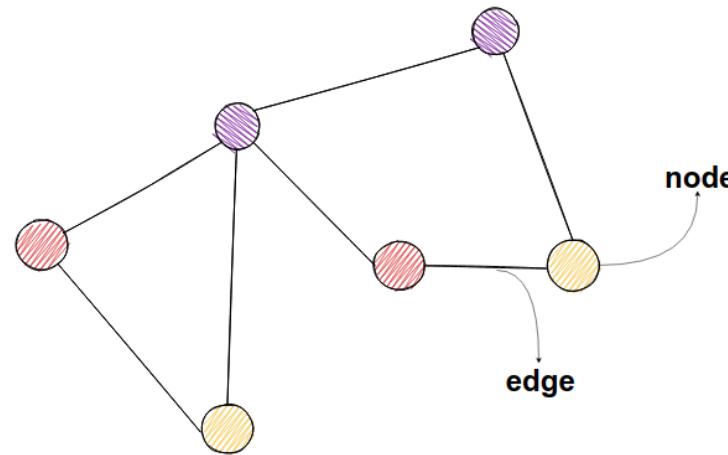
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The **neighborhood** $N(i)$ contains all neighbors of node i

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$$\text{Node } i \quad \mathbf{x}(i) \in \mathbb{R}^{d_x}; \quad i \in 1, \dots, T$$

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Rather than time t or space u, v , graphs are a function of index i

Node i $\mathbf{x}(i) \in \mathbb{R}^{d_x}; \quad i \in 1, \dots, T$

Neighborhood of i $\mathbf{N}(i) = \begin{bmatrix} i \\ j \\ k \\ \vdots \end{bmatrix}; \quad \mathbf{N}(i) \in \mathcal{P}(i); \quad i \in 1, \dots, T$

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This is just one node, we use this graph layer for all nodes in the graph

Apply graph convolution over all nodes in the graph

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$$f(\mathbf{x}, \mathbf{N}, \boldsymbol{\theta}) = \begin{bmatrix} f(\mathbf{x}, \mathbf{N}, \boldsymbol{\theta})(1) \\ f(\mathbf{x}, \mathbf{N}, \boldsymbol{\theta})(2) \\ \vdots \\ f(\mathbf{x}, \mathbf{N}, \boldsymbol{\theta})(T) \end{bmatrix} = \begin{bmatrix} \sigma\left(\sum_{j \in \mathbf{N}(1)} \boldsymbol{\theta}^\top \bar{\mathbf{x}}(j)\right) \\ \sigma\left(\sum_{j \in \mathbf{N}(2)} \boldsymbol{\theta}^\top \bar{\mathbf{x}}(j)\right) \\ \vdots \\ \sigma\left(\sum_{j \in \mathbf{N}(T)} \boldsymbol{\theta}^\top \bar{\mathbf{x}}(j)\right) \end{bmatrix}$$

Apply graph convolution over all nodes in the graph

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How does this compare to regular convolution (images, sound, etc)?

Standard 1D convolution

$$\begin{bmatrix} \sigma\left(\sum_{j=1}^k \theta^\top \bar{x}(j)\right) \\ \sigma\left(\sum_{j=2}^{k+1} \theta^\top \bar{x}(j)\right) \\ \vdots \\ \sigma\left(\sum_{j=T-k}^T \theta^\top \bar{x}(j)\right) \end{bmatrix}$$

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Question: What is the output size of standard convolution?

Answer: $(T - k - 1) \times d_h$

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Answer: $T \times d_h$

We can use pooling with graph convolutions too

$$\text{SumPool} \left(\begin{bmatrix} \sigma\left(\sum_{j \in \mathcal{N}(1)} \boldsymbol{\theta}^\top \bar{\mathbf{x}}(j)\right) \\ \sigma\left(\sum_{j \in \mathcal{N}(2)} \boldsymbol{\theta}^\top \bar{\mathbf{x}}(j)\right) \\ \vdots \\ \sigma\left(\sum_{j \in \mathcal{N}(T)} \boldsymbol{\theta}^\top \bar{\mathbf{x}}(j)\right) \end{bmatrix} \right) = \sigma\left(\sum_{j \in \mathcal{N}(1)} \boldsymbol{\theta}^\top \bar{\mathbf{x}}(j)\right) + \sigma\left(\sum_{j \in \mathcal{N}(2)} \boldsymbol{\theta}^\top \bar{\mathbf{x}}(j)\right) + \dots + \sigma\left(\sum_{j \in \mathcal{N}(T)} \boldsymbol{\theta}^\top \bar{\mathbf{x}}(j)\right)$$

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A generative model learns the structure of data

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If you like generative models, you should study Bayesian statistics

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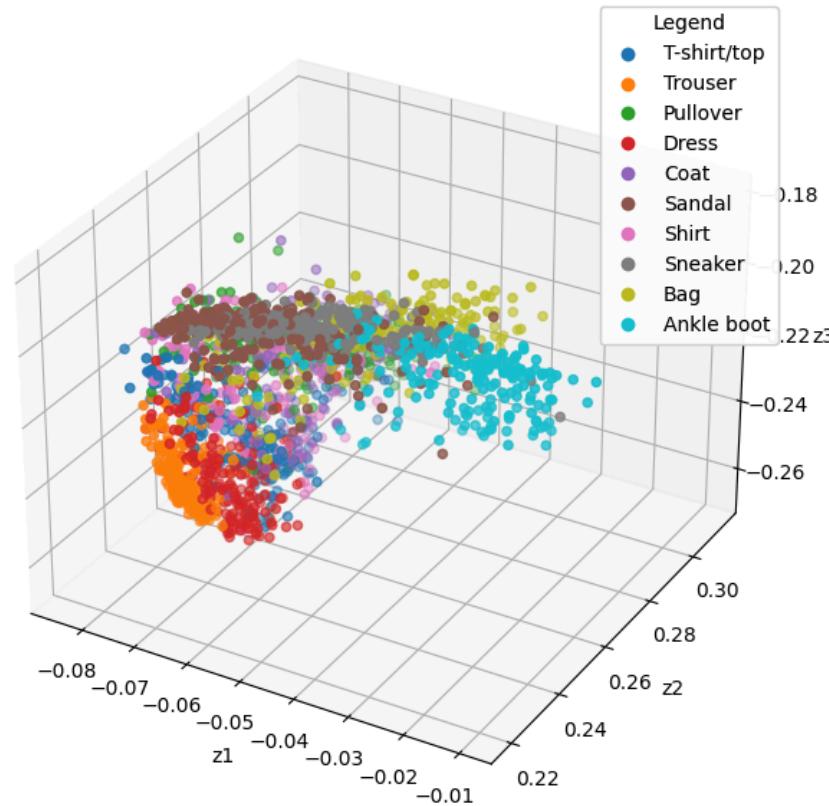
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Back to the variational autoencoder

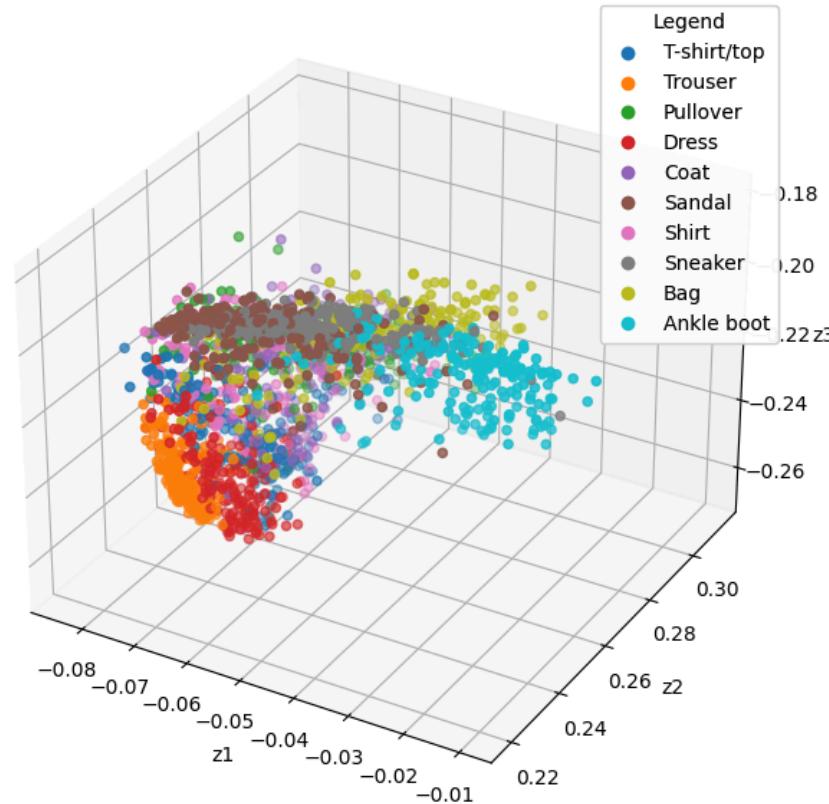
VAE Review and Coding

Latent space Z for autoencoder on the clothes dataset with $d_z = 3$

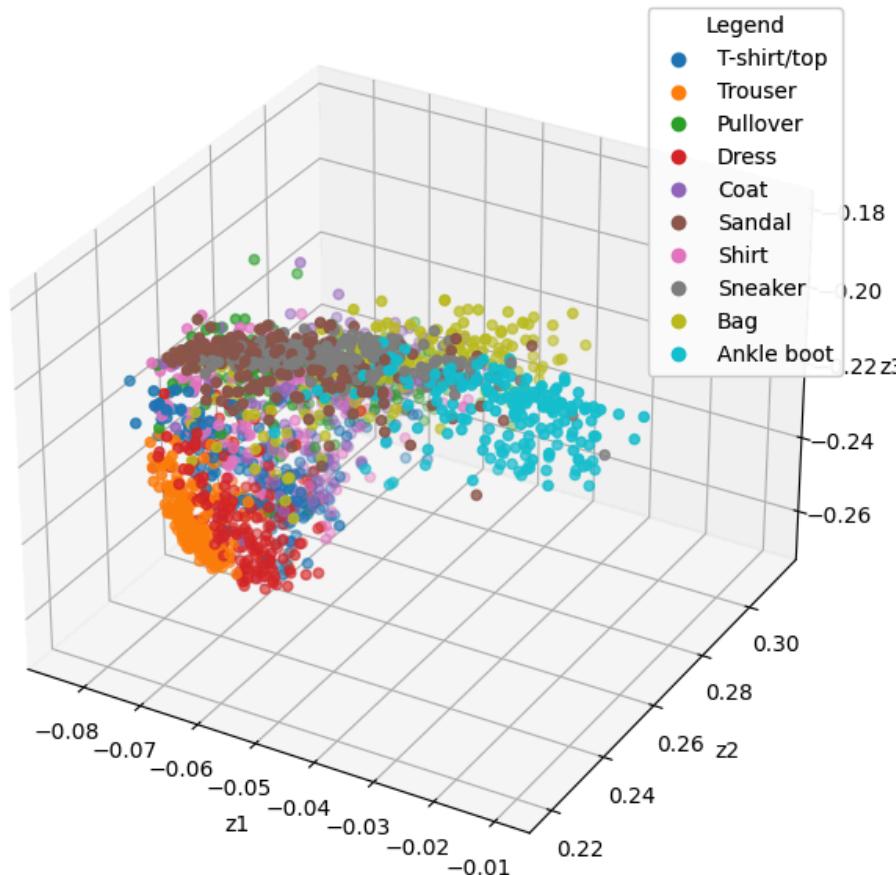


VAE Review and Coding

What happens if we decode a new point?

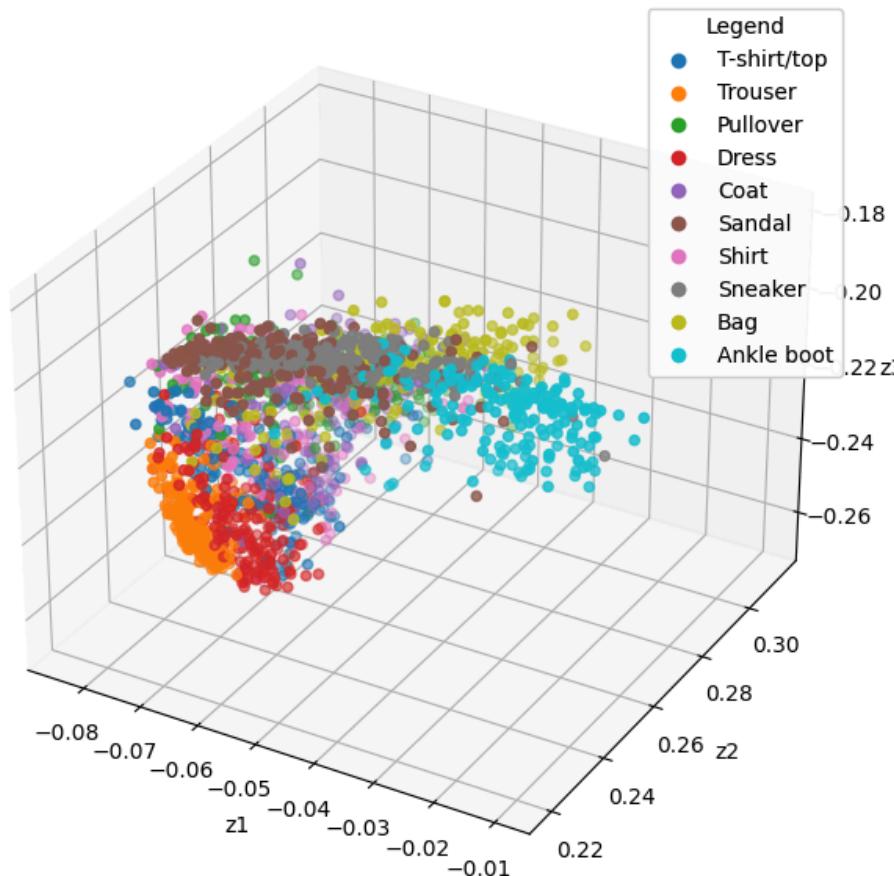


VAE Review and Coding



Autoencoder generative model:

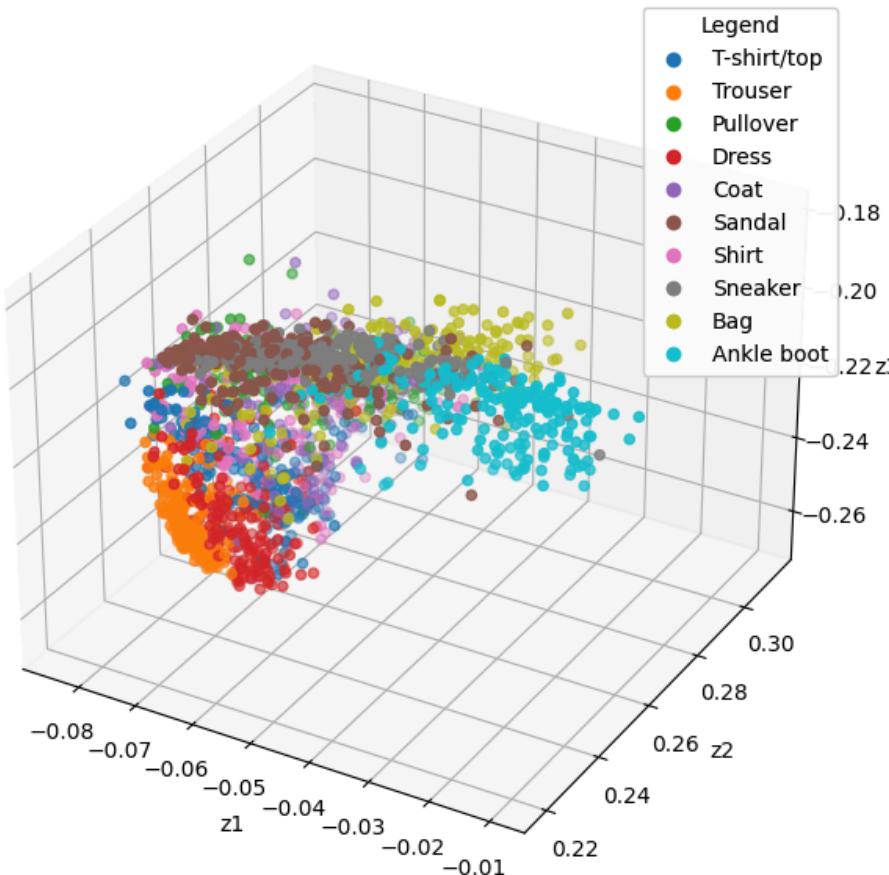
VAE Review and Coding



Autoencoder generative model:

1. Encode $\begin{bmatrix} \mathbf{x}_{[1]} \\ \vdots \\ \mathbf{x}_{[n]} \end{bmatrix}$ into $\begin{bmatrix} \mathbf{z}_{[1]} \\ \vdots \\ \mathbf{z}_{[n]} \end{bmatrix}$

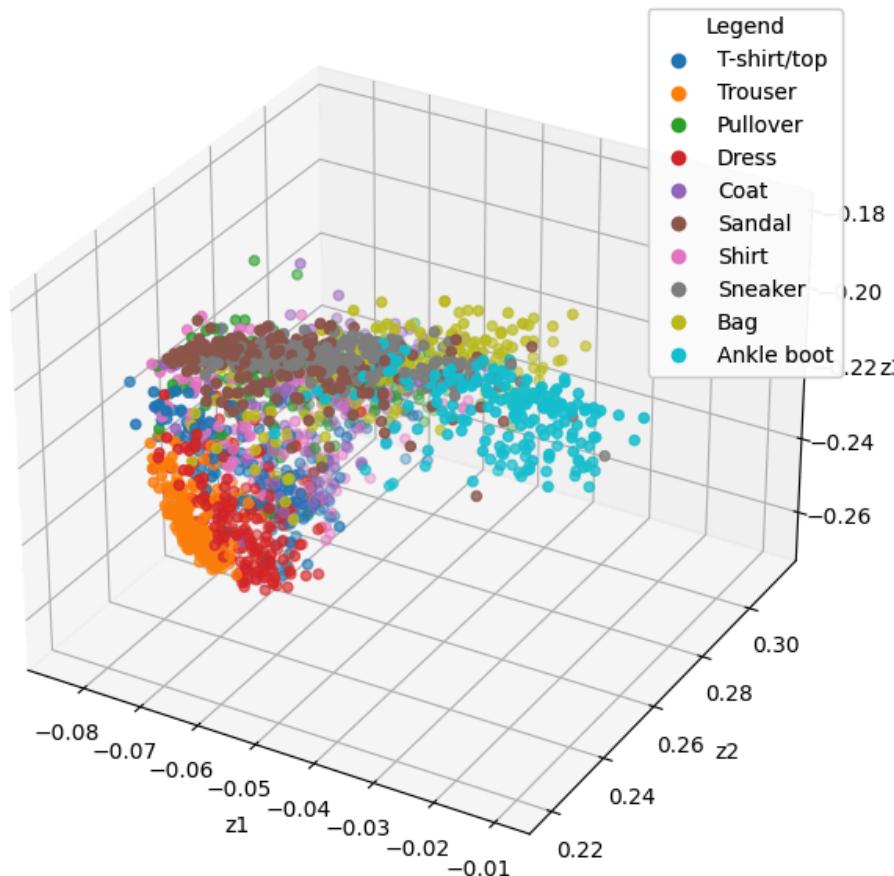
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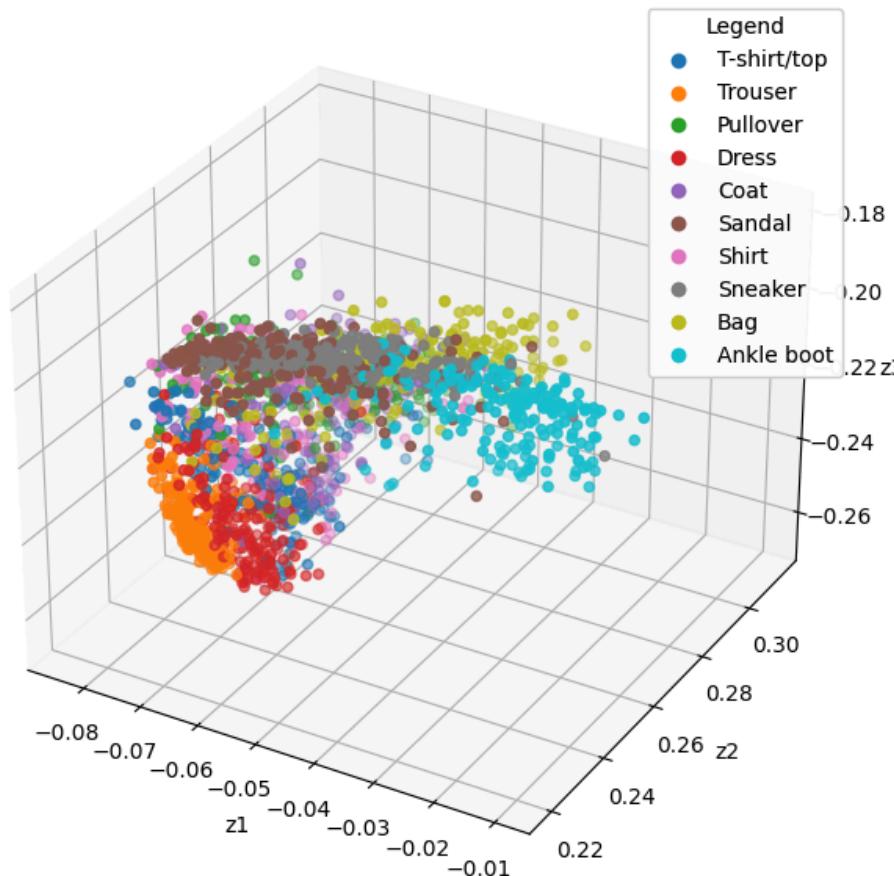
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4. Decode \mathbf{z}_{new} into \mathbf{x}_{new}

VAE Review and Coding



VAE Review and Coding



$$f^{-1}(z_k + \epsilon, \theta_d)$$

But there is a problem, the **curse
of dimensionality**

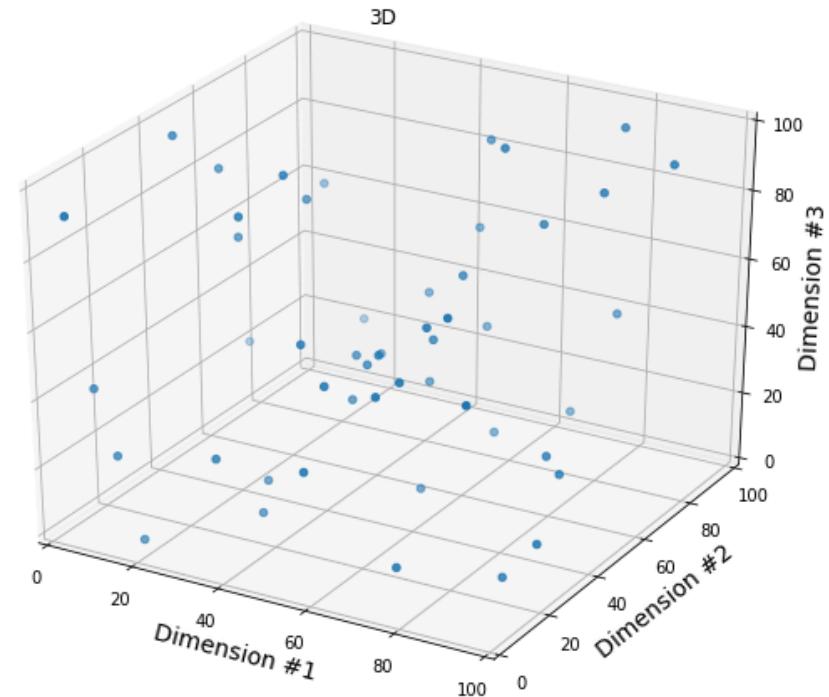
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VAE Review and Coding

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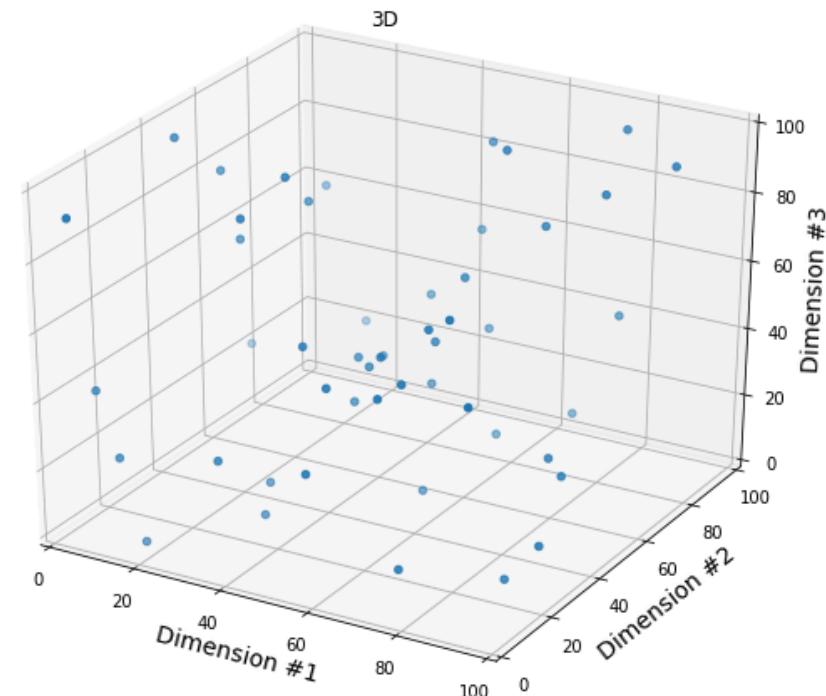
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VAE Review and Coding

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$f^{-1}(z + \varepsilon)$ will produce either garbage, or z

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This keeps the point close together

We can sample new points $z_{\text{new}} \sim N(\mathbf{0}, \mathbf{1})$

VAE Review and Coding

Key idea 1: We want to model the distribution over the dataset X

$$P(x; \theta), \quad x \sim X$$

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Large $P(x; \theta)$



$P(x; \theta) \approx 0$



VAE Review and Coding

Given a prior distribution

$$P(\mathbf{z}) = \mathcal{N}(\mathbf{0}, \mathbf{1})$$

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By approximating the marginal likelihood (from Bayes rule)

$$P(\mathbf{x}; \boldsymbol{\theta}) = \int_{\mathbf{z}} P(\mathbf{x} \mid \mathbf{z}; \boldsymbol{\theta}) P(\mathbf{z}) \mathrm{d}\mathbf{z}$$

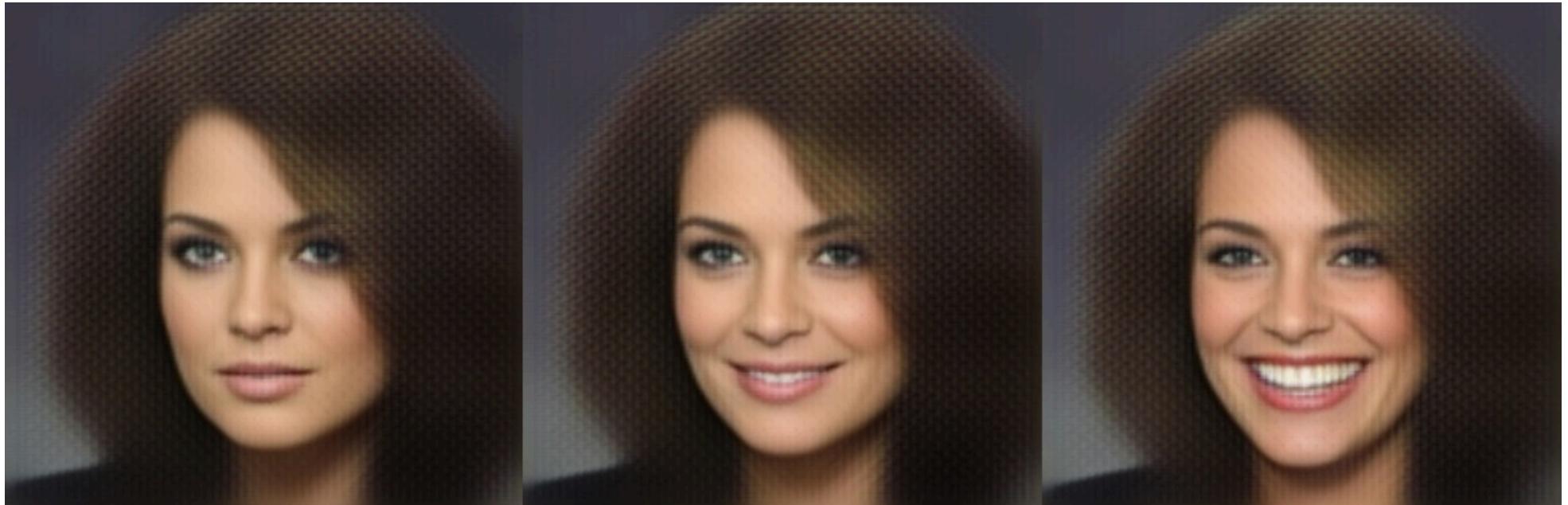
VAE Review and Coding

Key idea 2: There is some latent variable z which generates data x

VAE Review and Coding

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$x :$



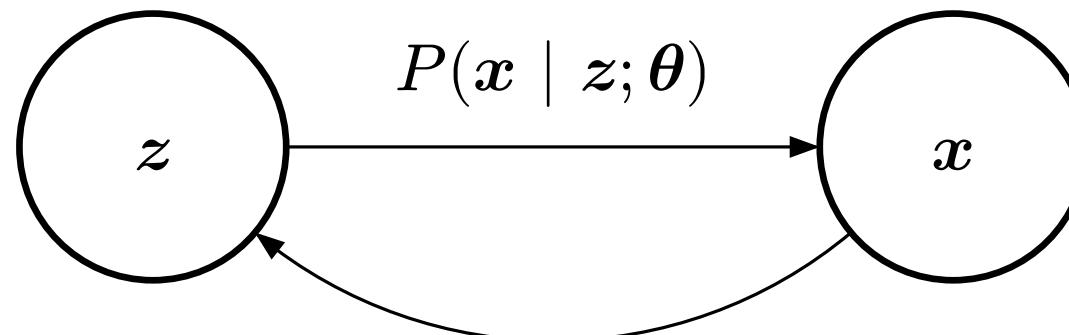
$z : [\text{woman brown hair (frown} \mid \text{smile)}]$

VAE Review and Coding

We cast the autoencoding task as a **variational inference** problem

VAE Review and Coding

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Decoder

$$P(x | z; \theta)$$

Generate new x

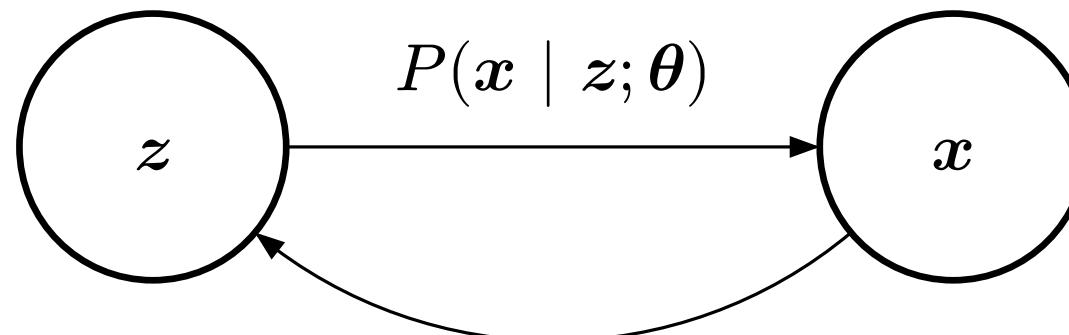
Encoder

$$P(z | x; \theta)$$

Learn meaningful z

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$$P(x | z; \theta)$$

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VAE Review and Coding

How do we implement f (i.e., $P(z \mid x; \theta)$)?

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$$f : X \times \Theta \mapsto \Delta Z$$

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Our encoder should output d_z means and d_z standard deviations

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Our encoder should output d_z means and d_z standard deviations

$$f : X \times \Theta \mapsto \mathbb{R}^{d_z} \times \mathbb{R}_+^{d_z}$$

VAE Review and Coding

```
core = nn.Sequential(...)  
mu_layer = nn.Linear(d_h, d_z)  
# Neural networks output real numbers  
# But sigma must be positive  
# Output log sigma, because e^(sigma) is always positive  
log_sigma_layer = nn.Linear(d_h, d_z)  
# Alternatively, one sigma for all data  
log_sigma = jnp.ones((d_z,))  
  
tmp = core(x)  
mu = mu_layer(tmp)  
log_sigma = log_sigma_layer(tmp)  
distribution = (mu, exp(log_sigma))
```

VAE Review and Coding

We covered the encoder

$$f : X \times \Theta \mapsto \Delta Z$$

We can use the same decoder as a standard autoencoder

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Encoder outputs a distribution ΔZ but decoder input is Z

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We can use the same decoder as a standard autoencoder

$$f^{-1} : Z \times \Theta \mapsto X$$

Encoder outputs a distribution ΔZ but decoder input is Z

Solution: Sample a vector z from the distribution ΔZ

VAE Review and Coding

Put it all together

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Step 1: Encode the input to a normal distribution

$$\mu, \sigma = f(x, \theta_e)$$

VAE Review and Coding

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Step 1: Encode the input to a normal distribution

$$\mu, \sigma = f(x, \theta_e)$$

Step 2: Generate a sample from distribution

$$z = \mu + \sigma \odot \varepsilon$$

VAE Review and Coding

Put it all together

Step 1: Encode the input to a normal distribution

$$\mu, \sigma = f(x, \theta_e)$$

Step 2: Generate a sample from distribution

$$z = \mu + \sigma \odot \varepsilon$$

Step 3: Decode the sample

$$x = f^{-1}(z, \theta_d)$$

VAE Review and Coding

```
# Create normal distribution from input
mu, sigma = model.f(x)

# Randomly sample a z vector from our distribution
epsilon = jax.random.normal(key, x.shape[0])
z = mu + sigma * epsilon

# Decode/reconstruct z back into x
pred_x = model.f_inverse(z)
```

VAE Review and Coding

Now, all we must do is find θ that best explains the dataset distribution

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VAE Review and Coding

From the KL divergence, we derived the **ELBO** loss for the VAE

$$\mathcal{L}(\mathbf{X}, \boldsymbol{\theta}) = \underbrace{\frac{m}{n} \sum_{i=1}^n \sum_{j=1}^{d_z} \left(x_{[i],j} - f^{-1}\left(f\left(\mathbf{x}_{[i]}, \boldsymbol{\theta}_e\right), \boldsymbol{\theta}_d\right)_j \right)^2}_{\text{Reconstruct } \mathbf{x}} - \underbrace{\beta \left(\sum_{i=1}^n \sum_{j=1}^{d_z} \mu_{[i],j}^2 + \sigma_{[i],j}^2 - \log(\sigma_{[i],j}^2) - 1 \right)}_{\text{Make } \mathbf{z} \text{ normally distributed}}$$

VAE Review and Coding

```
def L(model, x, m, n, beta, key):
    mu, sigma = model.f(x) # Encode input into distribution
    # Sample from distribution
    z = mu + sigma * jax.random.normal(key, x.shape[0])
    # Reconstruct input
    pred_x = model.f_inverse(z)
    # Compute reconstruction and kl loss terms
    recon = jnp.sum((x - pred_x) ** 2)
    kl = jnp.sum(mu ** 2 + sigma ** 2 - jnp.log(sigma ** 2) -
1)
    # Loss function contains reconstruction and kl terms
    return m / n * recon + beta * kl
```

VAE Review and Coding

https://colab.research.google.com/drive/1UyR_W6NDIujaJXYlHZh6O3NfaCAMscpH#scrollTo=nmyQ8aE2pSbb

VAE Review and Coding

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<https://openai.com/index/glow/>

Agenda

1. GNN Review
2. VAE Review and Coding
3. Attention
4. Keys and Queries
5. Self Attention
6. Homework

Agenda

1. GNN Review
2. VAE Review and Coding
3. **Attention**
4. Keys and Queries
5. Self Attention
6. Homework

Attention

Attention and transformers are the “hottest” topic in deep learning

Attention

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People use them for almost every task (even if they shouldn’t!)

Attention

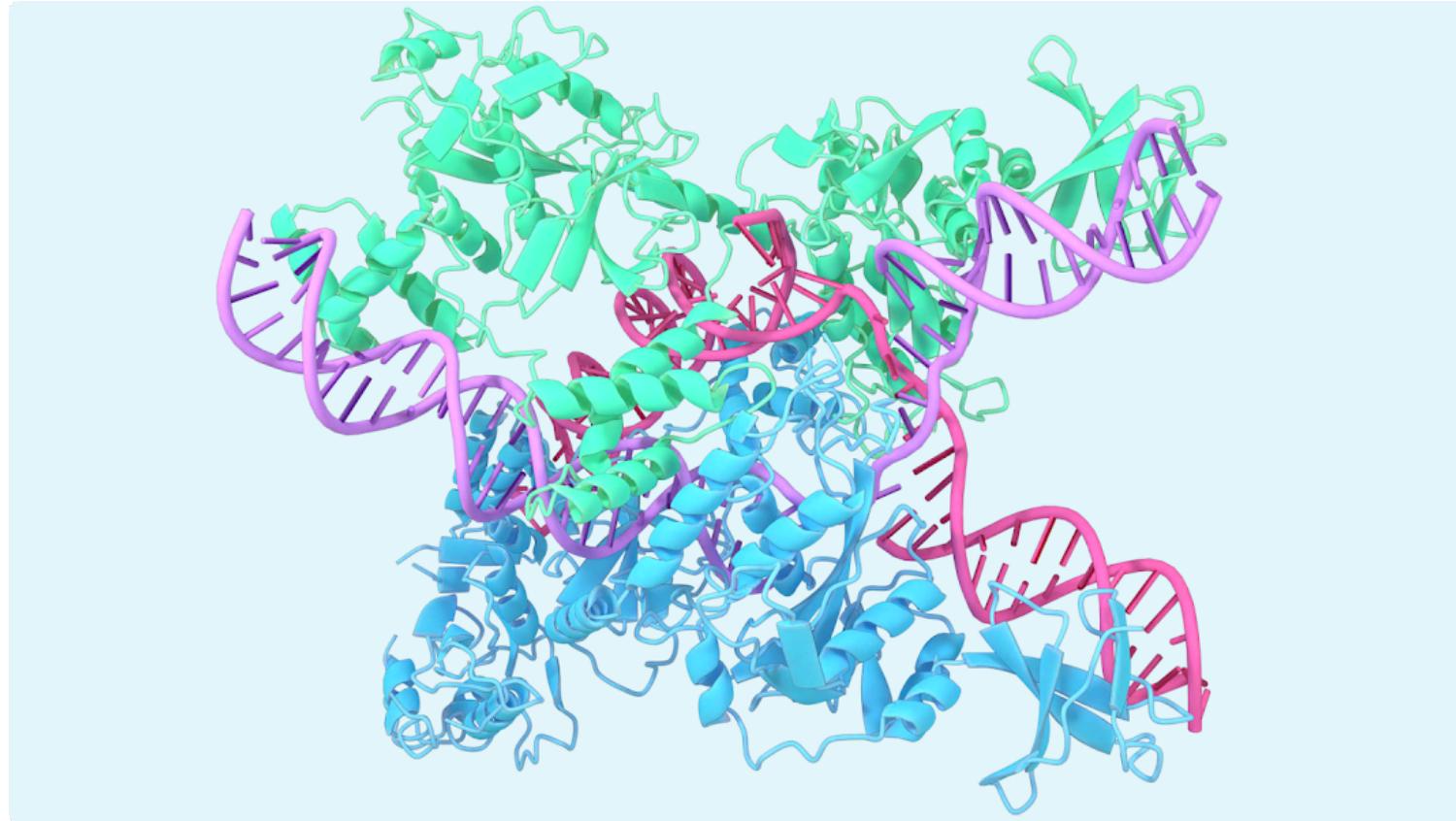
Attention and transformers are the “hottest” topic in deep learning

People use them for almost every task (even if they shouldn’t!)

Let’s review some projects based on attention

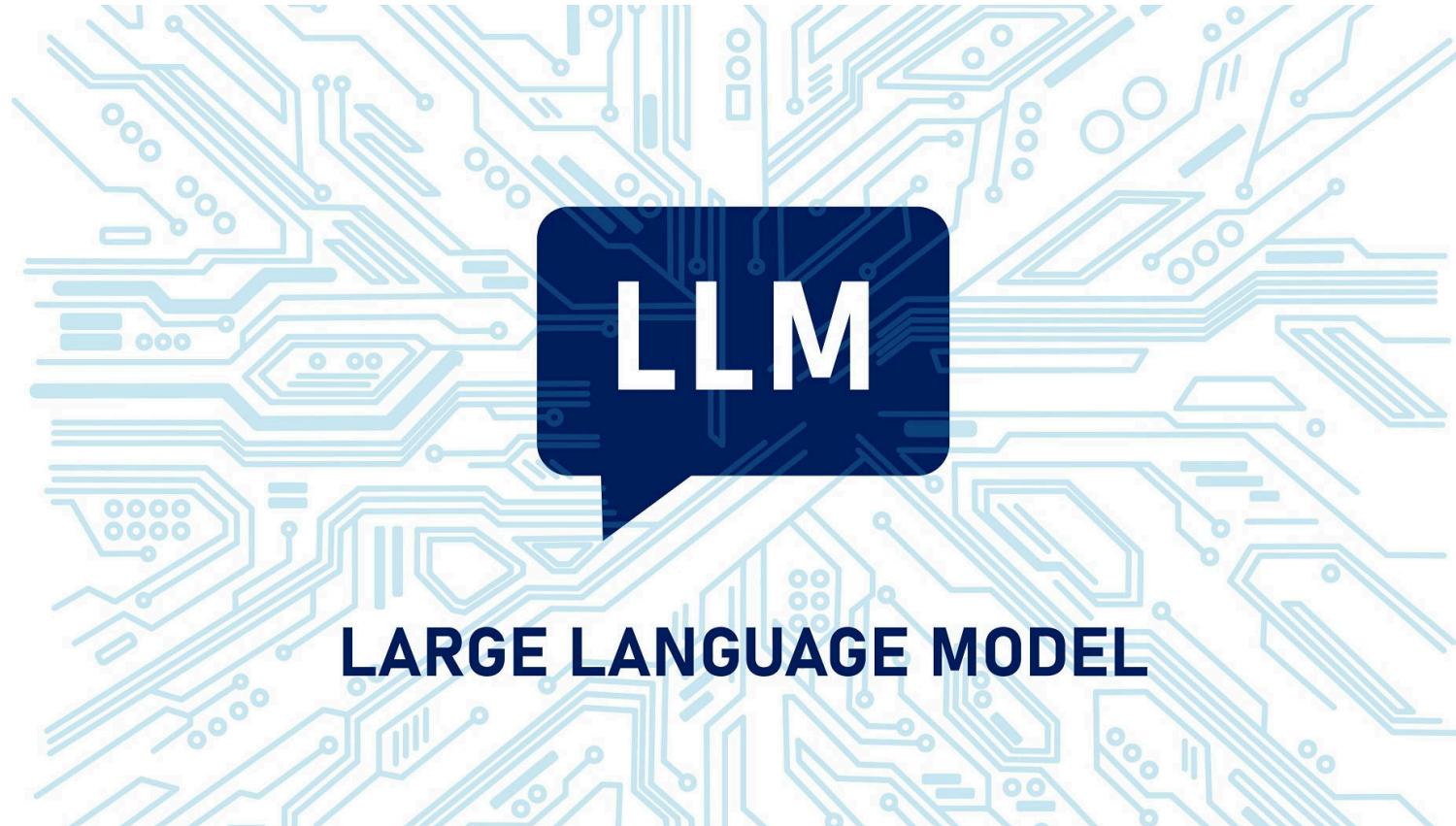
Attention

AlphaFold (Nobel prize)



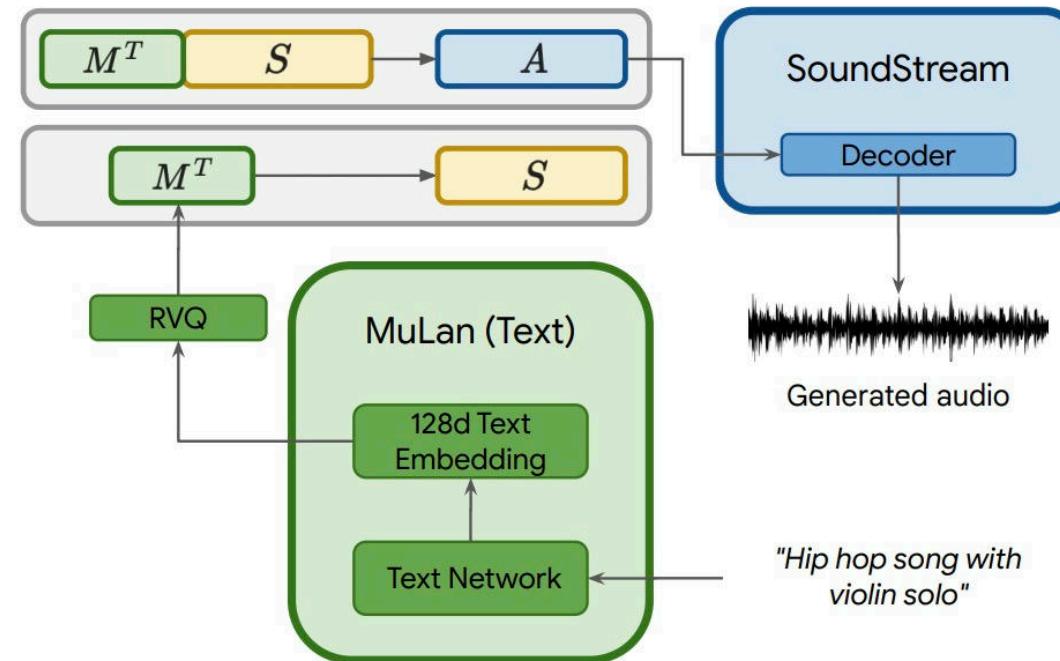
Attention

ChatGPT, Qwen, LLaMA, Mistral, Doubou, Ernie chatbots



Attention

MusicTransformer, MuLan



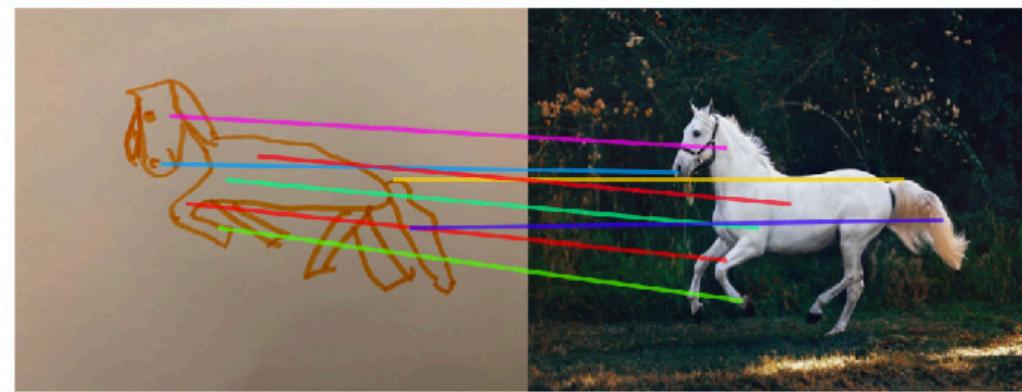
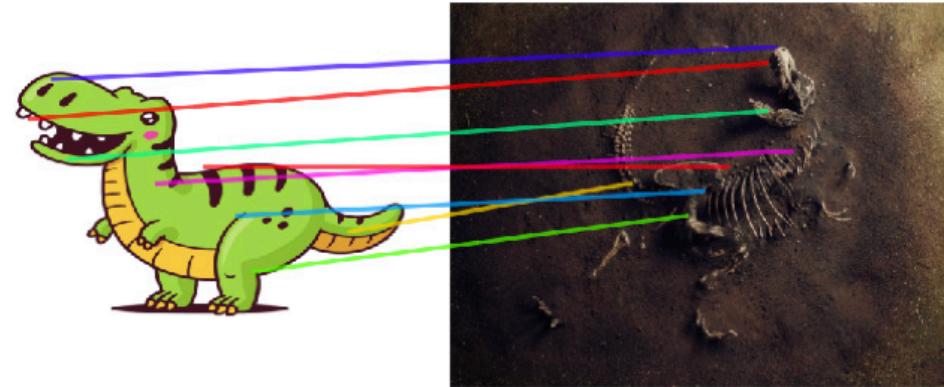
Attention

Google Translate, Baidu Translate, Apple Translate



Attention

ViT, DinoV2



Attention

All these models are **transformers**

Attention

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At the core of each transformer is **attention**

Attention

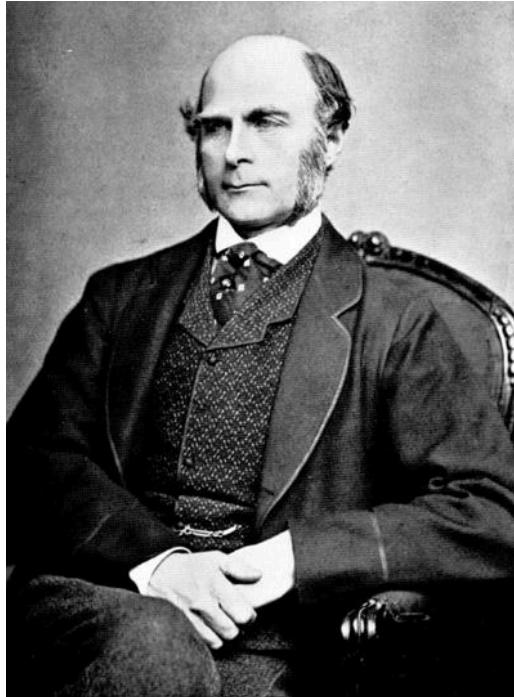
All these models are **transformers**

At the core of each transformer is **attention**

We can derive attention from composite memory

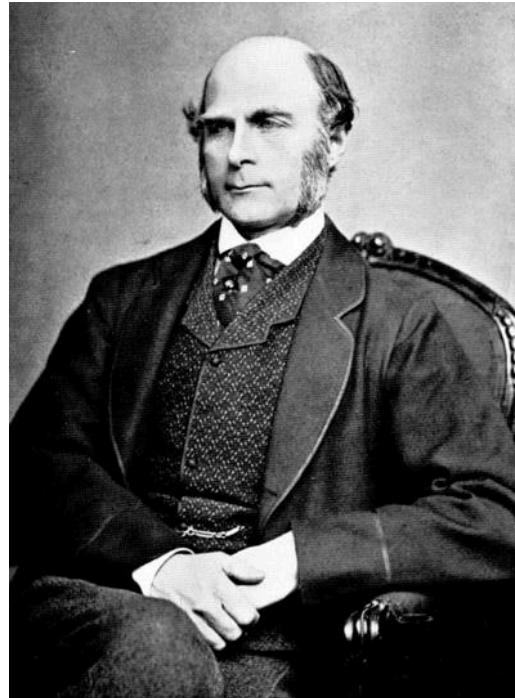
Attention

Francis Galton (1822-1911)
photo composite memory

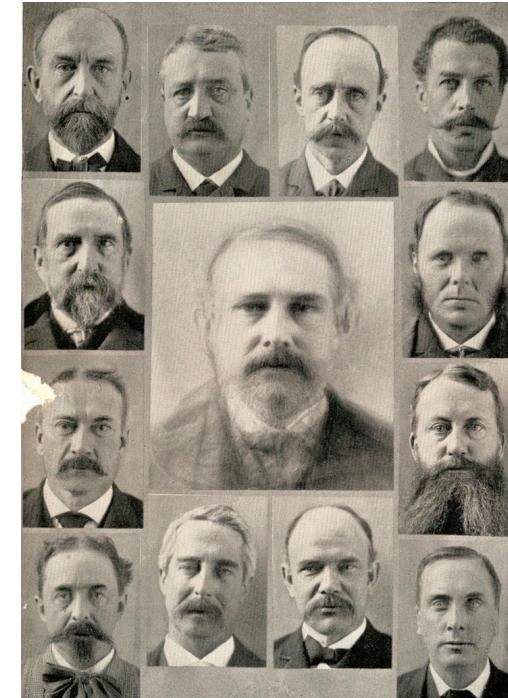


Attention

Francis Galton (1822-1911)
photo composite memory



Composite photo of members of a party



Attention

Task: Find a mathematical model of how our mind represents memories

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$$X : \mathbb{R}^{h \times w} \quad \text{People you see at the party}$$

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$f : X^T \times \Theta \mapsto H$

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Composite photography/memory uses a weighted sum

Attention

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$$f(\mathbf{x}, \boldsymbol{\theta}) = \sum_{i=1}^T \boldsymbol{\theta}^\top \overline{\mathbf{x}}_i$$

Attention

Limited space, cannot remember everything

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Introduced forgetting term $\gamma \in [0, 1]$

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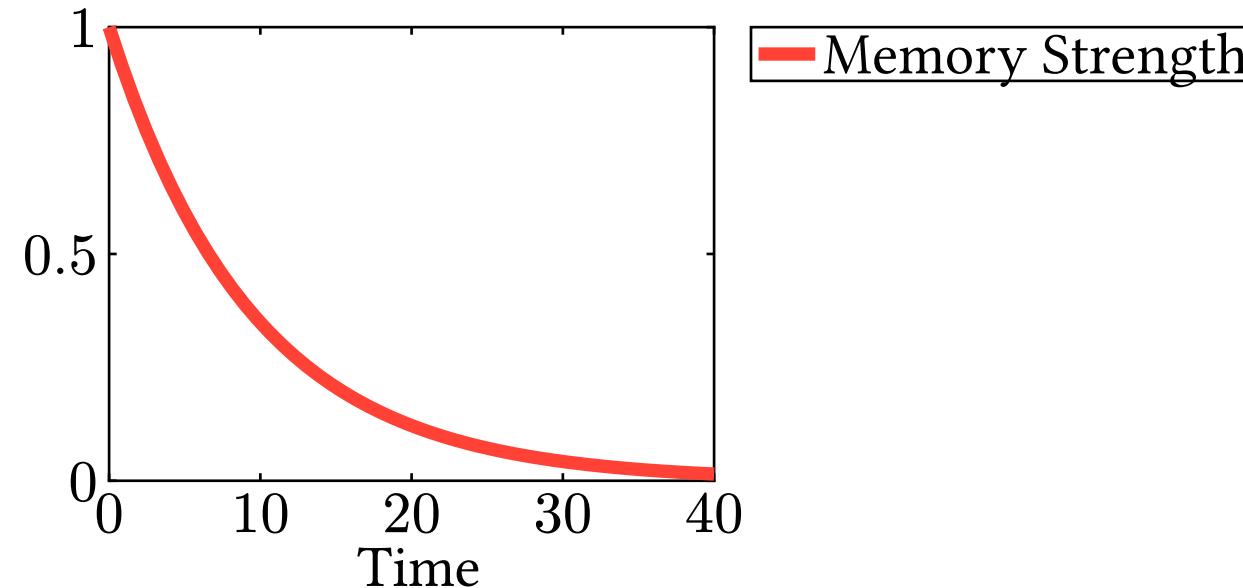
$$f(\mathbf{x}, \boldsymbol{\theta}) = \sum_{i=1}^T \gamma^{T-i} \cdot \boldsymbol{\theta}^\top \overline{\mathbf{x}}_i$$

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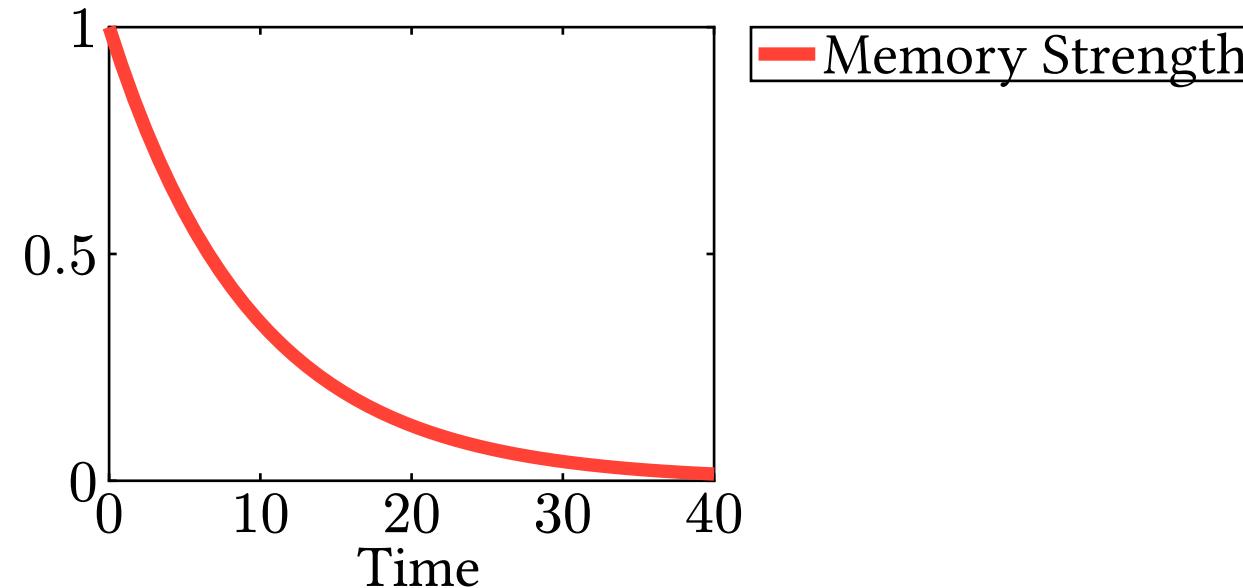


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Introduced forgetting term $\gamma \in [0, 1]$

$$f(\mathbf{x}, \boldsymbol{\theta}) = \sum_{i=1}^T \gamma^{T-i} \cdot \boldsymbol{\theta}^\top \bar{\mathbf{x}}_i$$



Question: Does this accurately model what **you** remember?

Attention

Example: We attend a party in 1850s

Attention

Example: We attend a party in 1850s

We talk with many people at this party

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We talk with many people at this party



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10 PM

Attention

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11 PM

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10 PM

11 PM

12 AM

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Example: We attend a party in 1850s

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10 PM

11 PM

12 AM

1 AM

Attention

According to forgetting, the memories should fade with time

Attention

According to forgetting, the memories should fade with time



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$$\gamma^3 \boldsymbol{\theta}^\top \overline{\boldsymbol{x}}_1$$

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$$\gamma^1 \boldsymbol{\theta}^\top \bar{\boldsymbol{x}}_3$$

$$\gamma^0 \boldsymbol{\theta}^\top \bar{\boldsymbol{x}}_4$$

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$$\gamma^0 \boldsymbol{\theta}^\top \bar{\boldsymbol{x}}_4$$

Attention

Any questions before moving on?

Attention

Consider another party, with one more guest

Attention

Consider another party, with one more guest



Attention

Consider another party, with one more guest



$$\gamma^4 \theta^\top \bar{x}_1$$

$$\gamma^3 \theta^\top \bar{x}_2$$

$$\gamma^2 \theta^\top \bar{x}_3$$

$$\gamma^1 \theta^\top \bar{x}_4$$

$$\gamma^0 \theta^\top \bar{x}_5$$

Attention

Consider another party, with one more guest



$$\gamma^4 \boldsymbol{\theta}^\top \overline{\mathbf{x}}_1$$

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$$\gamma^1 \boldsymbol{\theta}^\top \overline{\mathbf{x}}_4$$

$$\gamma^0 \boldsymbol{\theta}^\top \overline{\mathbf{x}}_5$$

Question: What will happen to Taylor Swift?

Attention



$$\gamma^4 \theta^\top \bar{x}_1$$

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We will forget meeting her!

Attention



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We will forget meeting her!

Question: Would you forget meeting Taylor Swift?

Attention

Our model of memory is incomplete

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Memories are not created equal, some are more important than others

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Important memories persist longer than unimportant memories

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Our model of memory is incomplete

Memories are not created equal, some are more important than others

Important memories persist longer than unimportant memories

We will **pay more attention** to certain memories

What does human memory actually look like?



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$$1.0 \cdot \theta^\top \bar{x}_1$$

What does human memory actually look like?



$$1.0 \cdot \theta^\top \bar{x}_1 \quad 0.1 \cdot \theta^\top \bar{x}_2$$

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Question: How can we achieve this forgetting?

Attention

In our composite model, forgetting is a function of time

Attention

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Question: Any forgetting mechanism that is not a function of time?

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Answer: Forgetting in recurrent neural network is function of input!

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$$f_{\text{forget}}(x, \theta) = \sigma(\theta_\lambda^\top \bar{x})$$

$$f(h, x, \theta) = f_{\text{forget}}(x, \theta) \odot h + \theta_x^\top \bar{x}$$

Attention

First, write our forgetting function with slightly different notation

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$$\lambda(x, \theta_\lambda) = \sigma(\theta_\lambda^\top \bar{x}); \quad \theta_\lambda \in \mathbb{R}^{(d_x+1) \times 1}$$

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Only pay attention to important inputs

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Only pay attention to important inputs

Attention

We can use this simple form of attention to remember Taylor Swift

Attention

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Attention

$$\begin{array}{ccccc} \lambda(x_1, \theta_\lambda) & \lambda(x_2, \theta_\lambda) & \lambda(x_3, \theta_\lambda) & \lambda(x_4, \theta_\lambda) & \lambda(x_5, \theta_\lambda) \\ \cdot \theta^\top \bar{x}_1 & \cdot \theta^\top \bar{x}_2 & \cdot \theta^\top \bar{x}_3 & \cdot \theta^\top \bar{x}_4 & \cdot \theta^\top \bar{x}_5 \end{array}$$

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Question: What do the images look like now?

Attention

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This form of attention will learn to pay attention to everyone!

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Not a good model of attention!

Attention

We should normalize $\lambda(x, \theta_\lambda)$ to model finite (human) attention span

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For example, normalize attention to sum to one

$$\sum_{i=1}^T \lambda(x_i, \theta_\lambda) = 1$$

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Now the model must choose who to remember!

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Question: How can we ensure that the attention sums to one?

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Question: How can we ensure that the attention sums to one?

Answer: Softmax!

Attention

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$$\text{softmax} : \mathbb{R}^k \mapsto \Delta^{k-1}$$

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$$\text{softmax} : \mathbb{R}^k \mapsto \Delta^{k-1}$$

$$\text{softmax} \left(\begin{bmatrix} x_1 \\ \vdots \\ x_k \end{bmatrix} \right) = \frac{\exp(\mathbf{x})}{\sum_{i=1}^k \exp(x_i)} = \begin{bmatrix} \frac{\exp(x_1)}{\exp(x_1)+\exp(x_2)+\dots+\exp(x_k)} \\ \frac{\exp(x_2)}{\exp(x_1)+\exp(x_2)+\dots+\exp(x_k)} \\ \vdots \\ \frac{\exp(x_k)}{\exp(x_1)+\exp(x_2)+\dots+\exp(x_k)} \end{bmatrix}$$

Attention

Let us rewrite attention using softmax

Attention

Let us rewrite attention using softmax

The attention we pay to person i is

$$\lambda \left(\begin{bmatrix} \mathbf{x}_1 \\ \vdots \\ \mathbf{x}_T \end{bmatrix}, \boldsymbol{\theta}_\lambda \right)_i = \text{softmax} \left(\begin{bmatrix} \boldsymbol{\theta}_\lambda^\top \bar{\mathbf{x}}_1 \\ \vdots \\ \boldsymbol{\theta}_\lambda^\top \bar{\mathbf{x}}_T \end{bmatrix} \right)_i = \frac{\exp(\boldsymbol{\theta}_\lambda^\top \bar{\mathbf{x}}_i)}{\sum_{j=1}^T \exp(\boldsymbol{\theta}_\lambda^\top \bar{\mathbf{x}}_j)}$$

Attention



Attention



$$\lambda \left(\begin{bmatrix} \mathbf{x}_1 \\ \vdots \\ \mathbf{x}_5 \end{bmatrix}, \theta_\lambda \right)_1 \cdot \theta^\top \overline{\mathbf{x}}_1$$

Attention



$$\lambda \left(\begin{bmatrix} \mathbf{x}_1 \\ \vdots \\ \mathbf{x}_5 \end{bmatrix}, \theta_\lambda \right)_1 \lambda \left(\begin{bmatrix} \mathbf{x}_1 \\ \vdots \\ \mathbf{x}_5 \end{bmatrix}, \theta_\lambda \right)_2 \\ \cdot \theta^\top \bar{\mathbf{x}}_1 \quad \cdot \theta^\top \bar{\mathbf{x}}_2$$

Attention



$$\lambda \left(\begin{bmatrix} \mathbf{x}_1 \\ \vdots \\ \mathbf{x}_5 \end{bmatrix}, \theta_\lambda \right)_1 \lambda \left(\begin{bmatrix} \mathbf{x}_1 \\ \vdots \\ \mathbf{x}_5 \end{bmatrix}, \theta_\lambda \right)_2 \lambda \left(\begin{bmatrix} \mathbf{x}_1 \\ \vdots \\ \mathbf{x}_5 \end{bmatrix}, \theta_\lambda \right)_3 \\ \cdot \theta^\top \bar{\mathbf{x}}_1 \quad \cdot \theta^\top \bar{\mathbf{x}}_2 \quad \cdot \theta^\top \bar{\mathbf{x}}_3$$

Attention



$$\lambda \left(\begin{bmatrix} \mathbf{x}_1 \\ \vdots \\ \mathbf{x}_5 \end{bmatrix}, \theta_\lambda \right)_1 \lambda \left(\begin{bmatrix} \mathbf{x}_1 \\ \vdots \\ \mathbf{x}_5 \end{bmatrix}, \theta_\lambda \right)_2 \lambda \left(\begin{bmatrix} \mathbf{x}_1 \\ \vdots \\ \mathbf{x}_5 \end{bmatrix}, \theta_\lambda \right)_3 \lambda \left(\begin{bmatrix} \mathbf{x}_1 \\ \vdots \\ \mathbf{x}_5 \end{bmatrix}, \theta_\lambda \right)_4 \\ \cdot \theta^\top \bar{\mathbf{x}}_1 \quad \cdot \theta^\top \bar{\mathbf{x}}_2 \quad \cdot \theta^\top \bar{\mathbf{x}}_3 \quad \cdot \theta^\top \bar{\mathbf{x}}_4$$

Attention



$$\lambda \left(\begin{bmatrix} x_1 \\ \vdots \\ x_5 \end{bmatrix}, \theta_\lambda \right)_1 \lambda \left(\begin{bmatrix} x_1 \\ \vdots \\ x_5 \end{bmatrix}, \theta_\lambda \right)_2 \lambda \left(\begin{bmatrix} x_1 \\ \vdots \\ x_5 \end{bmatrix}, \theta_\lambda \right)_3 \lambda \left(\begin{bmatrix} x_1 \\ \vdots \\ x_5 \end{bmatrix}, \theta_\lambda \right)_4 \lambda \left(\begin{bmatrix} x_1 \\ \vdots \\ x_5 \end{bmatrix}, \theta_\lambda \right)_5 \\ \cdot \theta^\top \bar{x}_1 \quad \cdot \theta^\top \bar{x}_2 \quad \cdot \theta^\top \bar{x}_3 \quad \cdot \theta^\top \bar{x}_4 \quad \cdot \theta^\top \bar{x}_5$$

Attention



$$\lambda \left(\begin{bmatrix} x_1 \\ \vdots \\ x_5 \end{bmatrix}, \theta_\lambda \right)_1 \lambda \left(\begin{bmatrix} x_1 \\ \vdots \\ x_5 \end{bmatrix}, \theta_\lambda \right)_2 \lambda \left(\begin{bmatrix} x_1 \\ \vdots \\ x_5 \end{bmatrix}, \theta_\lambda \right)_3 \lambda \left(\begin{bmatrix} x_1 \\ \vdots \\ x_5 \end{bmatrix}, \theta_\lambda \right)_4 \lambda \left(\begin{bmatrix} x_1 \\ \vdots \\ x_5 \end{bmatrix}, \theta_\lambda \right)_5 \\ \cdot \theta^\top \bar{x}_1 \quad \cdot \theta^\top \bar{x}_2 \quad \cdot \theta^\top \bar{x}_3 \quad \cdot \theta^\top \bar{x}_4 \quad \cdot \theta^\top \bar{x}_5$$

Attention



Attention



$$0.70 \cdot \theta^\top \bar{x}_1$$

Attention



$$0.70 \cdot \theta^\top \bar{x}_1 - 0.04 \cdot \theta^\top \bar{x}_2$$

Attention



$$0.70 \cdot \theta^\top \bar{x}_1 \quad 0.04 \cdot \theta^\top \bar{x}_2 \quad 0.03 \cdot \theta^\top \bar{x}_3$$

Attention



$$0.70 \cdot \theta^\top \bar{x}_1 \quad 0.04 \cdot \theta^\top \bar{x}_2 \quad 0.03 \cdot \theta^\top \bar{x}_3 \quad 0.20 \cdot \theta^\top \bar{x}_4$$

Attention



$$0.70 \cdot \theta^\top \bar{x}_1 \quad 0.04 \cdot \theta^\top \bar{x}_2 \quad 0.03 \cdot \theta^\top \bar{x}_3 \quad 0.20 \cdot \theta^\top \bar{x}_4 \quad 0.03 \cdot \theta^\top \bar{x}_5$$

Attention



$$0.70 \cdot \theta^\top \bar{x}_1 \quad 0.04 \cdot \theta^\top \bar{x}_2 \quad 0.03 \cdot \theta^\top \bar{x}_3 \quad 0.20 \cdot \theta^\top \bar{x}_4 \quad 0.03 \cdot \theta^\top \bar{x}_5$$

$$0.70 + 0.04 + 0.03 + 0.20 + 0.03 = 1.0$$

Attention

$$\lambda \left(\begin{bmatrix} x_1 \\ \vdots \\ x_T \end{bmatrix}, \theta_\lambda \right)_i = \frac{\exp(\theta_\lambda^\top \bar{x}_i)}{\sum_{j=1}^T \exp(\theta_\lambda^\top \bar{x}_j)}$$

Attention

$$\lambda \left(\begin{bmatrix} x_1 \\ \vdots \\ x_T \end{bmatrix}, \theta_\lambda \right)_i = \frac{\exp(\theta_\lambda^\top \bar{x}_i)}{\sum_{j=1}^T \exp(\theta_\lambda^\top \bar{x}_j)}$$

Compute attention for all inputs at once

Attention

$$\lambda \left(\begin{bmatrix} \mathbf{x}_1 \\ \vdots \\ \mathbf{x}_T \end{bmatrix}, \boldsymbol{\theta}_\lambda \right)_i = \frac{\exp(\boldsymbol{\theta}_\lambda^\top \bar{\mathbf{x}}_i)}{\sum_{j=1}^T \exp(\boldsymbol{\theta}_\lambda^\top \bar{\mathbf{x}}_j)}$$

Compute attention for all inputs at once

$$\lambda \left(\begin{bmatrix} \mathbf{x}_1 \\ \vdots \\ \mathbf{x}_T \end{bmatrix}, \boldsymbol{\theta}_\lambda \right) = \begin{bmatrix} \frac{\exp(\boldsymbol{\theta}_\lambda^\top \bar{\mathbf{x}}_1)}{\sum_{j=1}^T \exp(\boldsymbol{\theta}_\lambda^\top \bar{\mathbf{x}}_j)} \\ \vdots \\ \frac{\exp(\boldsymbol{\theta}_\lambda^\top \bar{\mathbf{x}}_T)}{\sum_{j=1}^T \exp(\boldsymbol{\theta}_\lambda^\top \bar{\mathbf{x}}_j)} \end{bmatrix}$$

Attention

This is a simple form of attention

Attention

This is a simple form of attention

Next, we will investigate the attention used in transformers

Agenda

1. GNN Review
2. VAE Review and Coding
3. **Attention**
4. Keys and Queries
5. Self Attention
6. Homework

Agenda

1. GNN Review
2. VAE Review and Coding
3. Attention
4. **Keys and Queries**
5. Self Attention
6. Homework

Keys and Queries

The modern form of attention behaves like a database

Keys and Queries

The modern form of attention behaves like a database

We label each person at the party with a **key**

Keys and Queries

The modern form of attention behaves like a database

We label each person at the party with a **key**

The key describes the content of each x

Keys and Queries

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Keys and Queries

The modern form of attention behaves like a database

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Musician

Keys and Queries

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Musician

Lawyer

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Musician

Lawyer

Shopkeeper

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Musician

Lawyer

Shopkeeper

Chef

Keys and Queries

The modern form of attention behaves like a database

We label each person at the party with a **key**

The key describes the content of each x



Musician

Lawyer

Shopkeeper

Chef

Scientist

Keys and Queries

We can search through our keys using a **query**

Keys and Queries

We can search through our keys using a **query**

Query: Which person will help me on my exam?

Keys and Queries

We can search through our keys using a **query**

Query: Which person will help me on my exam?



Keys and Queries

We can search through our keys using a **query**

Query: Which person will help me on my exam?



Musician

Lawyer

Shopkeeper

Chef

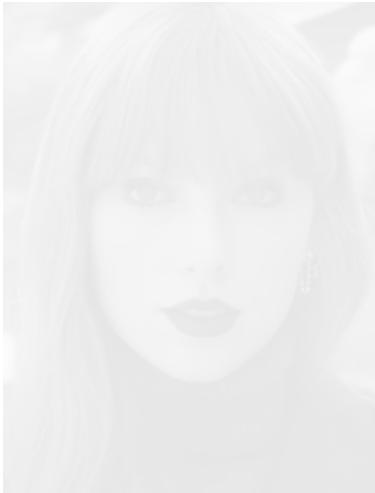
Scientist

Keys and Queries

We can search through our keys using a **query**

Query: Which person will help me on my exam?

Musician



Lawyer



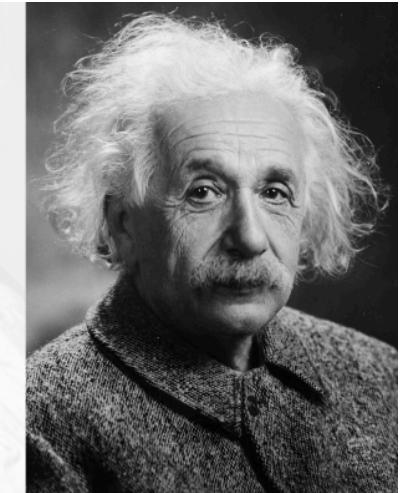
Shopkeeper



Chef



Scientist



Keys and Queries

Query: I want to have fun

Keys and Queries

Query: I want to have fun



Keys and Queries

Query: I want to have fun



Musician

Lawyer

Shopkeeper

Chef

Scientist

Keys and Queries

Query: I want to have fun

Musician



Lawyer



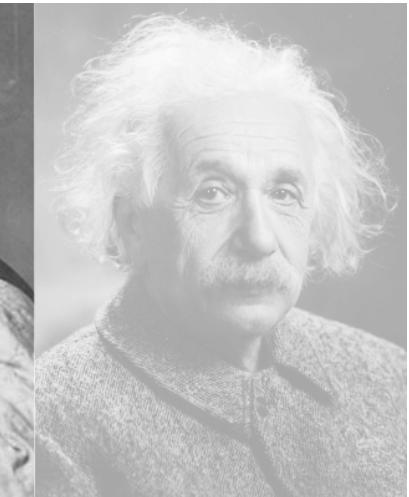
Shopkeeper



Chef



Scientist



Keys and Queries

Query: I want to have fun

Musician



Lawyer



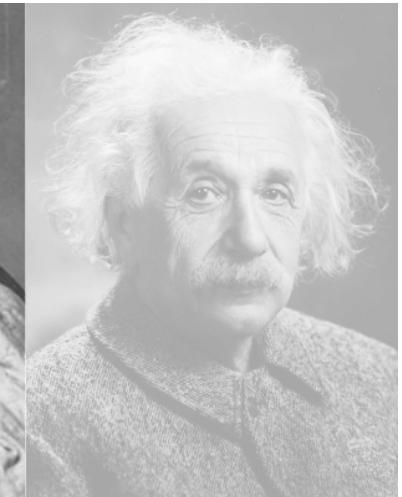
Shopkeeper



Chef



Scientist



How do we represent keys and queries mathematically?

Keys and Queries

For each input, we create a key k

Keys and Queries

For each input, we create a key \mathbf{k}

$$\mathbf{k}_j = \theta_K^\top \mathbf{x}_j, \quad \theta_K \in \mathbb{R}^{d_x \times d_h}, \quad \mathbf{k}_j \in \mathbb{R}^{d_h}$$

Keys and Queries

For each input, we create a key \mathbf{k}

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Create keys for all inputs

Keys and Queries

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Keys and Queries

Now, create a query from some x_q

Keys and Queries

Now, create a query from some x_q

$$q = \theta_Q^\top x_q, \quad \theta_Q \in \mathbb{R}^{d_x \times d_h}, \quad q \in \mathbb{R}^{d_h}$$

Keys and Queries

Now, create a query from some x_q

$$q = \theta_Q^\top x_q, \quad \theta_Q \in \mathbb{R}^{d_x \times d_h}, \quad q \in \mathbb{R}^{d_h}$$

To determine if a key and query match, we will take the dot product

Keys and Queries

Now, create a query from some \mathbf{x}_q

$$\mathbf{q} = \boldsymbol{\theta}_Q^\top \mathbf{x}_q, \quad \boldsymbol{\theta}_Q \in \mathbb{R}^{d_x \times d_h}, \quad \mathbf{q} \in \mathbb{R}^{d_h}$$

To determine if a key and query match, we will take the dot product

$$\mathbf{q}^\top \mathbf{k}_i = (\boldsymbol{\theta}_Q^\top \mathbf{x}_q)^\top (\boldsymbol{\theta}_K^\top \mathbf{x}_i)$$

Keys and Queries

Now, create a query from some \mathbf{x}_q

$$\mathbf{q} = \boldsymbol{\theta}_Q^\top \mathbf{x}_q, \quad \boldsymbol{\theta}_Q \in \mathbb{R}^{d_x \times d_h}, \quad \mathbf{q} \in \mathbb{R}^{d_h}$$

To determine if a key and query match, we will take the dot product

$$\mathbf{q}^\top \mathbf{k}_i = (\boldsymbol{\theta}_Q^\top \mathbf{x}_q)^\top (\boldsymbol{\theta}_K^\top \mathbf{x}_i)$$

Question: What is the shape of $\mathbf{q}^\top \mathbf{k}_i$?

Keys and Queries

Now, create a query from some x_q

$$q = \theta_Q^\top x_q, \quad \theta_Q \in \mathbb{R}^{d_x \times d_h}, \quad q \in \mathbb{R}^{d_h}$$

To determine if a key and query match, we will take the dot product

$$q^\top k_i = (\theta_Q^\top x_q)^\top (\theta_K^\top x_i)$$

Question: What is the shape of $q^\top k_i$?

Answer: $(1, d_h) \times (d_h, 1) = 1$, the output is a scalar

Keys and Queries

Now, create a query from some \mathbf{x}_q

$$\mathbf{q} = \boldsymbol{\theta}_Q^\top \mathbf{x}_q, \quad \boldsymbol{\theta}_Q \in \mathbb{R}^{d_x \times d_h}, \quad \mathbf{q} \in \mathbb{R}^{d_h}$$

To determine if a key and query match, we will take the dot product

$$\mathbf{q}^\top \mathbf{k}_i = (\boldsymbol{\theta}_Q^\top \mathbf{x}_q)^\top (\boldsymbol{\theta}_K^\top \mathbf{x}_i)$$

Question: What is the shape of $\mathbf{q}^\top \mathbf{k}_i$?

Answer: $(1, d_h) \times (d_h, 1) = 1$, the output is a scalar

Large dot product \Rightarrow match! Small dot product \Rightarrow no match.

Keys and Queries

Example:

Keys and Queries

Example:

$$k_i = \theta_K^\top$$



Keys and Queries

Example:

$$k_i = \theta_K^\top$$



$$q = \theta_Q^\top \text{ Musician}$$

Keys and Queries

Example:

$$k_i = \theta_K^\top$$



$$q = \theta_Q^\top \text{ Musician}$$

$$q^\top k_i = (\theta_Q^\top \text{ Musician})^\top \left(\theta_K^\top \begin{array}{c} \\ \text{Musician} \\ \end{array} \right) = 100$$

Keys and Queries

Example:

$$k_i = \theta_K^\top$$



$$q = \theta_Q^\top \text{ Musician}$$

$$q^\top k_i = (\theta_Q^\top \text{ Musician})^\top \left(\theta_K^\top \begin{array}{c} \text{Taylor Swift portrait} \\ \vdots \end{array} \right) = 100$$

Large attention!

Keys and Queries

Example:

$$k_i = \theta_K^\top$$



Keys and Queries

Example:

$$k_i = \theta_K^\top$$



$$q = \theta_Q^\top \text{ Mathematician}$$

Keys and Queries

Example:

$$k_i = \theta_K^\top$$



$$q = \theta_Q^\top \text{ Mathematician}$$

$$q^\top k_i = (\theta_Q^\top \text{ Mathematician})^\top \left(\theta_K^\top \begin{array}{c} \text{Taylor Swift portrait} \\ \vdots \end{array} \right) = -50$$

Keys and Queries

Example:

$$k_i = \theta_K^\top$$



$$q = \theta_Q^\top \text{ Mathematician}$$

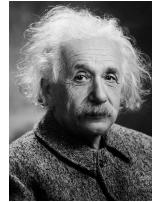
$$q^\top k_i = (\theta_Q^\top \text{ Mathematician})^\top \left(\theta_K^\top \begin{array}{c} \text{Taylor Swift portrait} \\ \vdots \end{array} \right) = -50$$

Small attention!

Keys and Queries

Example:

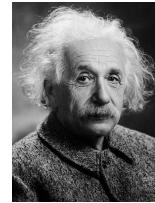
$$k_i = \theta_K^\top$$



Keys and Queries

Example:

$$k_i = \theta_K^\top$$

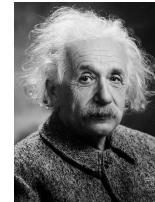


$$q = \theta_Q^\top \text{ Mathematician}$$

Keys and Queries

Example:

$$k_i = \theta_K^\top$$



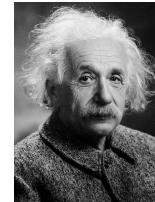
$$q = \theta_Q^\top \text{ Mathematician}$$

$$q^\top k_i = (\theta_Q^\top \text{ Mathematician})^\top \left(\theta_K^\top \begin{array}{c} \text{Mathematician} \\ \text{Albert Einstein} \end{array} \right) = 90$$

Keys and Queries

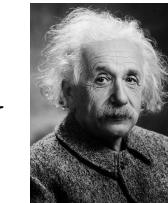
Example:

$$k_i = \theta_K^\top$$



$$q = \theta_Q^\top \text{ Mathematician}$$

$$q^\top k_i = (\theta_Q^\top \text{ Mathematician})^\top \left(\theta_K^\top \right) = 90$$

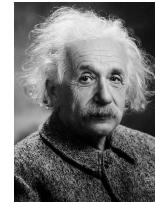


Large attention!

Keys and Queries

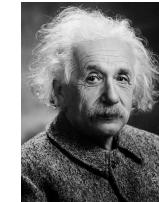
Example:

$$k_i = \theta_K^\top$$



$$q = \theta_Q^\top \text{ Mathematician}$$

$$q^\top k_i = (\theta_Q^\top \text{ Mathematician})^\top \left(\theta_K^\top \right) = 90$$



Large attention!

Remember, there are multiple inputs to pay attention to

Keys and Queries

We compute a key for each input

$$\mathbf{K} = [\mathbf{k}_1 \ \mathbf{k}_2 \ \dots \ \mathbf{k}_T] = [\boldsymbol{\theta}_K^\top \mathbf{x}_1 \ \boldsymbol{\theta}_K^\top \mathbf{x}_2 \ \dots \ \boldsymbol{\theta}_K^\top \mathbf{x}_T], \quad \mathbf{K} \in \mathbb{R}^{d_h \times T}$$

Keys and Queries

We compute a key for each input

$$\mathbf{K} = [\mathbf{k}_1 \ \mathbf{k}_2 \ \dots \ \mathbf{k}_T] = [\boldsymbol{\theta}_K^\top \mathbf{x}_1 \ \boldsymbol{\theta}_K^\top \mathbf{x}_2 \ \dots \ \boldsymbol{\theta}_K^\top \mathbf{x}_T], \quad \mathbf{K} \in \mathbb{R}^{d_h \times T}$$

Keys and Queries

Then compute attention for each key

$$\mathbf{q}^\top \mathbf{K} = \mathbf{q}^\top [\mathbf{k}_1 \ \mathbf{k}_2 \ \dots \ \mathbf{k}_T] = [q^\top \mathbf{k}_1 \ q^\top \mathbf{k}_2 \ \dots \ q^\top \mathbf{k}_T]$$

Keys and Queries

Then compute attention for each key

$$\mathbf{q}^\top \mathbf{K} = \mathbf{q}^\top [\mathbf{k}_1 \ \mathbf{k}_2 \ \dots \ \mathbf{k}_T] = [\mathbf{q}^\top \mathbf{k}_1 \ \mathbf{q}^\top \mathbf{k}_2 \ \dots \ \mathbf{q}^\top \mathbf{k}_T]$$

$$\mathbf{q}^\top \in \mathbb{R}^{1, d_h}, \quad \mathbf{K} \in \mathbb{R}^{d_h \times T}$$

Keys and Queries

Then compute attention for each key

$$\mathbf{q}^\top \mathbf{K} = \mathbf{q}^\top [\mathbf{k}_1 \ \mathbf{k}_2 \ \dots \ \mathbf{k}_T] = [\mathbf{q}^\top \mathbf{k}_1 \ \mathbf{q}^\top \mathbf{k}_2 \ \dots \ \mathbf{q}^\top \mathbf{k}_T]$$

$$\mathbf{q}^\top \in \mathbb{R}^{1, d_h}, \quad \mathbf{K} \in \mathbb{R}^{d_h \times T}$$

Question: What is the shape of $\mathbf{q}^\top \mathbf{K}$?

Keys and Queries

Then compute attention for each key

$$\mathbf{q}^\top \mathbf{K} = \mathbf{q}^\top [\mathbf{k}_1 \ \mathbf{k}_2 \ \dots \ \mathbf{k}_T] = [\mathbf{q}^\top \mathbf{k}_1 \ \mathbf{q}^\top \mathbf{k}_2 \ \dots \ \mathbf{q}^\top \mathbf{k}_T]$$

$$\mathbf{q}^\top \in \mathbb{R}^{1, d_h}, \quad \mathbf{K} \in \mathbb{R}^{d_h \times T}$$

Question: What is the shape of $\mathbf{q}^\top \mathbf{K}$?

Answer: T

Keys and Queries

Then compute attention for each key

$$\mathbf{q}^\top \mathbf{K} = \mathbf{q}^\top [\mathbf{k}_1 \ \mathbf{k}_2 \ \dots \ \mathbf{k}_T] = [\mathbf{q}^\top \mathbf{k}_1 \ \mathbf{q}^\top \mathbf{k}_2 \ \dots \ \mathbf{q}^\top \mathbf{k}_T]$$

$$\mathbf{q}^\top \in \mathbb{R}^{1, d_h}, \quad \mathbf{K} \in \mathbb{R}^{d_h \times T}$$

Question: What is the shape of $\mathbf{q}^\top \mathbf{K}$?

Answer: T

Do not forget to normalize with softmax!

Keys and Queries

Normalize, only pay attention to important matches

Keys and Queries

Normalize, only pay attention to important matches

$$\begin{aligned}\text{softmax}(\mathbf{q}^\top \mathbf{K}) &= \text{softmax}(\mathbf{q}^\top [\mathbf{k}_1 \ \mathbf{k}_2 \ \dots \ \mathbf{k}_T]) \\ &= \text{softmax}([\mathbf{q}^\top \mathbf{k}_1 \ \mathbf{q}^\top \mathbf{k}_2 \ \dots \ \mathbf{q}^\top \mathbf{k}_T])\end{aligned}$$

Keys and Queries

Normalize, only pay attention to important matches

$$\begin{aligned}\text{softmax}(\mathbf{q}^\top \mathbf{K}) &= \text{softmax}(\mathbf{q}^\top [\mathbf{k}_1 \ \mathbf{k}_2 \ \dots \ \mathbf{k}_T]) \\ &= \text{softmax}([\mathbf{q}^\top \mathbf{k}_1 \ \mathbf{q}^\top \mathbf{k}_2 \ \dots \ \mathbf{q}^\top \mathbf{k}_T])\end{aligned}$$

We call this **dot-product attention**

Keys and Queries

Query: Which person will help me on my exam?

Keys and Queries

Query: Which person will help me on my exam?



$$q^\top k_1$$

$$q^\top k_2$$

$$q^\top k_3$$

$$q^\top k_4$$

$$q^\top k_5$$

Keys and Queries

Query: Which person will help me on my exam?



$$q^\top k_1$$

$$q^\top k_2$$

$$q^\top k_3$$

$$q^\top k_4$$

$$q^\top k_5$$

−1.71

0.60

−1.01

−0.61

2.73

softmax

Keys and Queries

Query: Which person will help me on my exam?



$$q^\top k_1$$

$$q^\top k_2$$

$$q^\top k_3$$

$$q^\top k_4$$

$$q^\top k_5$$

−1.71

0.60

−1.01

−0.61

2.73

softmax

0.01

0.10

0.02

0.03

0.84

Keys and Queries

Query: Which person will help me on my exam?



$$q^\top k_1$$

$$q^\top k_2$$

$$q^\top k_3$$

$$q^\top k_4$$

$$q^\top k_5$$

$$-1.71$$

$$0.60$$

$$-1.01$$

$$-0.61$$

$$2.73$$

softmax

$$0.01$$

$$0.10$$

$$0.02$$

$$0.03$$

$$0.84$$

Keys and Queries

Put dot product attention into our composite model

Keys and Queries

Put dot product attention into our composite model

$$\lambda \left(\begin{bmatrix} \mathbf{x}_1 \\ \vdots \\ \mathbf{x}_T \end{bmatrix}, \theta_\lambda \right)_i = \text{softmax}(\mathbf{q}^\top [\mathbf{k}_1 \ \mathbf{k}_2 \ \dots \ \mathbf{k}_T])_i$$

Keys and Queries

Put dot product attention into our composite model

$$\lambda \left(\begin{bmatrix} \mathbf{x}_1 \\ \vdots \\ \mathbf{x}_T \end{bmatrix}, \theta_\lambda \right)_i = \text{softmax}(\mathbf{q}^\top [\mathbf{k}_1 \ \mathbf{k}_2 \ \dots \ \mathbf{k}_T])_i$$

Then, write our composite memory model with dot product attention

Keys and Queries

Put dot product attention into our composite model

$$\lambda \left(\begin{bmatrix} \mathbf{x}_1 \\ \vdots \\ \mathbf{x}_T \end{bmatrix}, \theta_\lambda \right)_i = \text{softmax}(\mathbf{q}^\top [\mathbf{k}_1 \ \mathbf{k}_2 \ \dots \ \mathbf{k}_T])_i$$

Then, write our composite memory model with dot product attention

$$f \left(\begin{bmatrix} \mathbf{x}_1 \\ \vdots \\ \mathbf{x}_T \end{bmatrix}, \theta \right) = \sum_{i=1}^T \theta^\top \mathbf{x}_i \cdot \lambda \left(\begin{bmatrix} \mathbf{x}_1 \\ \vdots \\ \mathbf{x}_T \end{bmatrix}, \theta_\lambda \right)_i$$

Keys and Queries

Put dot product attention into our composite model

$$\lambda \left(\begin{bmatrix} \mathbf{x}_1 \\ \vdots \\ \mathbf{x}_T \end{bmatrix}, \theta_\lambda \right)_i = \text{softmax}(\mathbf{q}^\top [\mathbf{k}_1 \ \mathbf{k}_2 \ \dots \ \mathbf{k}_T])_i$$

Then, write our composite memory model with dot product attention

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Keys and Queries

$$f\left(\begin{bmatrix} x_1 \\ \vdots \\ x_T \end{bmatrix}, \theta\right) = \sum_{i=1}^T \theta^\top x_i \cdot \lambda\left(\begin{bmatrix} x_1 \\ \vdots \\ x_T \end{bmatrix}, \theta_\lambda\right)_i$$

Keys and Queries

$$f\left(\begin{bmatrix} \mathbf{x}_1 \\ \vdots \\ \mathbf{x}_T \end{bmatrix}, \boldsymbol{\theta}\right) = \sum_{i=1}^T \boldsymbol{\theta}^\top \mathbf{x}_i \cdot \lambda\left(\begin{bmatrix} \mathbf{x}_1 \\ \vdots \\ \mathbf{x}_T \end{bmatrix}, \boldsymbol{\theta}_\lambda\right)_i$$

We relabel $\boldsymbol{\theta}$ to $\boldsymbol{\theta}_V$

$$f\left(\begin{bmatrix} \mathbf{x}_1 \\ \vdots \\ \mathbf{x}_T \end{bmatrix}, \boldsymbol{\theta}\right) = \sum_{i=1}^T \boldsymbol{\theta}_{\textcolor{red}{V}}^\top \mathbf{x}_i \cdot \lambda\left(\begin{bmatrix} \mathbf{x}_1 \\ \vdots \\ \mathbf{x}_T \end{bmatrix}, \boldsymbol{\theta}_\lambda\right)_i$$

Keys and Queries

$$f\left(\begin{bmatrix} \mathbf{x}_1 \\ \vdots \\ \mathbf{x}_T \end{bmatrix}, \boldsymbol{\theta}\right) = \sum_{i=1}^T \boldsymbol{\theta}^\top \mathbf{x}_i \cdot \lambda\left(\begin{bmatrix} \mathbf{x}_1 \\ \vdots \\ \mathbf{x}_T \end{bmatrix}, \boldsymbol{\theta}_\lambda\right)_i$$

We relabel $\boldsymbol{\theta}$ to $\boldsymbol{\theta}_V$

$$f\left(\begin{bmatrix} \mathbf{x}_1 \\ \vdots \\ \mathbf{x}_T \end{bmatrix}, \boldsymbol{\theta}\right) = \sum_{i=1}^T \boldsymbol{\theta}_V^\top \mathbf{x}_i \cdot \lambda\left(\begin{bmatrix} \mathbf{x}_1 \\ \vdots \\ \mathbf{x}_T \end{bmatrix}, \boldsymbol{\theta}_\lambda\right)_i$$

In dot-product attention, we call $\boldsymbol{\theta}_V^\top \mathbf{x}_i$ the **value**

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2. VAE Review and Coding
3. Attention
4. **Keys and Queries**
5. Self Attention
6. Homework

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Self Attention

Previously, we chose our own query $x_q = \text{Musician}$

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We can also create queries from all the inputs

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$$Q = [q_1 \ q_2 \ \dots \ q_T] = [\theta_Q^\top x_1 \ \theta_Q^\top x_2 \ \dots \ \theta_Q^\top x_T], \quad Q \in \mathbb{R}^{T \times d_h}$$

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We call this dot-product **self** attention

Self Attention

Attention looks messy, but we can rewrite it in matrix form

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$$Q = [q_1 \ q_2 \ \dots \ q_T] = [\theta_Q^\top x_1 \ \theta_Q^\top x_2 \ \dots \ \theta_Q^\top x_T]$$

$$K = [k_1 \ k_2 \ \dots \ k_T] = [\theta_K^\top x_1 \ \theta_K^\top x_2 \ \dots \ \theta_K^\top x_T]$$

$$V = [v_1 \ v_2 \ \dots \ v_T] = [\theta_V^\top x_1 \ \theta_V^\top x_2 \ \dots \ \theta_V^\top x_T]$$

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$$\mathbf{Q} = [\mathbf{q}_1 \ \mathbf{q}_2 \ \dots \ \mathbf{q}_T] = [\theta_Q^\top \mathbf{x}_1 \ \theta_Q^\top \mathbf{x}_2 \ \dots \ \theta_Q^\top \mathbf{x}_T]$$

$$\mathbf{K} = [\mathbf{k}_1 \ \mathbf{k}_2 \ \dots \ \mathbf{k}_T] = [\theta_K^\top \mathbf{x}_1 \ \theta_K^\top \mathbf{x}_2 \ \dots \ \theta_K^\top \mathbf{x}_T]$$

$$\mathbf{V} = [\mathbf{v}_1 \ \mathbf{v}_2 \ \dots \ \mathbf{v}_T] = [\theta_V^\top \mathbf{x}_1 \ \theta_V^\top \mathbf{x}_2 \ \dots \ \theta_V^\top \mathbf{x}_T]$$

$$\text{attn}\left(\begin{bmatrix} \mathbf{x}_1 \\ \vdots \\ \mathbf{x}_T \end{bmatrix}, \theta\right) = \text{softmax}\left(\frac{\mathbf{Q}\mathbf{K}^\top}{\sqrt{d_h}}\right)\mathbf{V}$$

Self Attention

Attention looks messy, but we can rewrite it in matrix form

$$\mathbf{Q} = [\mathbf{q}_1 \ \mathbf{q}_2 \ \dots \ \mathbf{q}_T] = [\theta_Q^\top \mathbf{x}_1 \ \theta_Q^\top \mathbf{x}_2 \ \dots \ \theta_Q^\top \mathbf{x}_T]$$

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$$\text{attn}\left(\begin{bmatrix} \mathbf{x}_1 \\ \vdots \\ \mathbf{x}_T \end{bmatrix}, \theta\right) = \text{softmax}\left(\frac{\mathbf{Q}\mathbf{K}^\top}{\sqrt{d_h}}\right)\mathbf{V}$$

This operation powers today's biggest models

Self Attention

$$\mathbf{Q} \in \mathbb{R}^{T \times d_h} \quad \mathbf{K} \in \mathbb{R}^{T \times d_h} \quad \mathbf{V} \in \mathbb{R}^{T \times d_h}$$

$$\underbrace{\text{attn}\left(\underbrace{\begin{bmatrix} \mathbf{x}_1 \\ \vdots \\ \mathbf{x}_T \end{bmatrix}}_{\mathbb{R}^{T \times d_h}}, \boldsymbol{\theta}\right)}_{\mathbb{R}^{T \times d_h}} = \text{softmax}\left(\overbrace{\frac{\mathbf{Q}\mathbf{K}^\top}{\sqrt{d_h}}}^{\mathbb{R}^{T \times T}}, \underbrace{\mathbf{V}}_{\mathbb{R}^{T \times d_h}}\right)$$

Self Attention

$$\mathbf{Q} \in \mathbb{R}^{T \times d_h} \quad \mathbf{K} \in \mathbb{R}^{T \times d_h} \quad \mathbf{V} \in \mathbb{R}^{T \times d_h}$$

$$\underbrace{\text{attn}\left(\underbrace{\begin{bmatrix} \mathbf{x}_1 \\ \vdots \\ \mathbf{x}_T \end{bmatrix}}_{\mathbb{R}^{T \times d_h}}, \boldsymbol{\theta}\right)}_{\mathbb{R}^{T \times d_h}} = \text{softmax}\left(\overbrace{\frac{\mathbf{Q}\mathbf{K}^\top}{\sqrt{d_h}}}^{\mathbb{R}^{T \times T}}, \underbrace{\mathbf{V}}_{\mathbb{R}^{T \times d_h}}\right)$$

Self Attention

```
class Attention(nn.Module):
    def __init__(self):
        self.theta_K = nn.Linear(d_x, d_h, bias=False)
        self.theta_Q = nn.Linear(d_x, d_h, bias=False)
        self.theta_V = nn.Linear(d_x, d_h, bias=False)

    def forward(self, x):
        A = softmax(
            self.theta_Q(x) @ self.theta_K(x).T / d_h, axis=0
        )
        return A @ self.theta_V(x)
```

Self Attention

Next lecture, we will create the transformer

Self Attention

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We will also discuss foundation models

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Homework

<https://colab.research.google.com/drive/18VBb7sz0u8ul5vsFEJnQaepn0pQy4cUa?usp=sharing>