Recurrent Neural Networks

CISC 7026: Introduction to Deep Learning

University of Macau

Admin

Makeup lecture Saturday October 26, 13:00-16:00

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Assignment 5 is on Moodle, due in 2 weeks

Agenda

1. Review
2. Sequence Modeling
3. Composite Memory
4. Linear Recurrence
5. Scans
6. Output Modeling
7. Recurrent Loss Functions
8. Backpropagation through Time
9. Recurrent Neural Networks
10. Coding

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Review

In perceptrons, each neuron in a layer is independent

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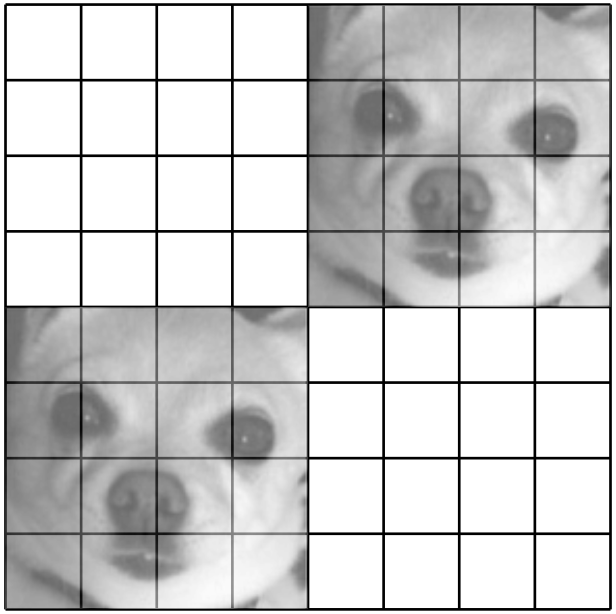

These images are equivalent to a neural network

Review

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It is a miracle that our neural networks could classify clothing!

Review

A **signal** represents information as a function of time, space or some other variable

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$$x(t) = 2t + 1$$

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$$x(u, v) = \frac{u^2}{v} - 3$$

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In **signal processing**, we analyze the meaning of signals

Review

$$x(t) = \text{stock price}$$

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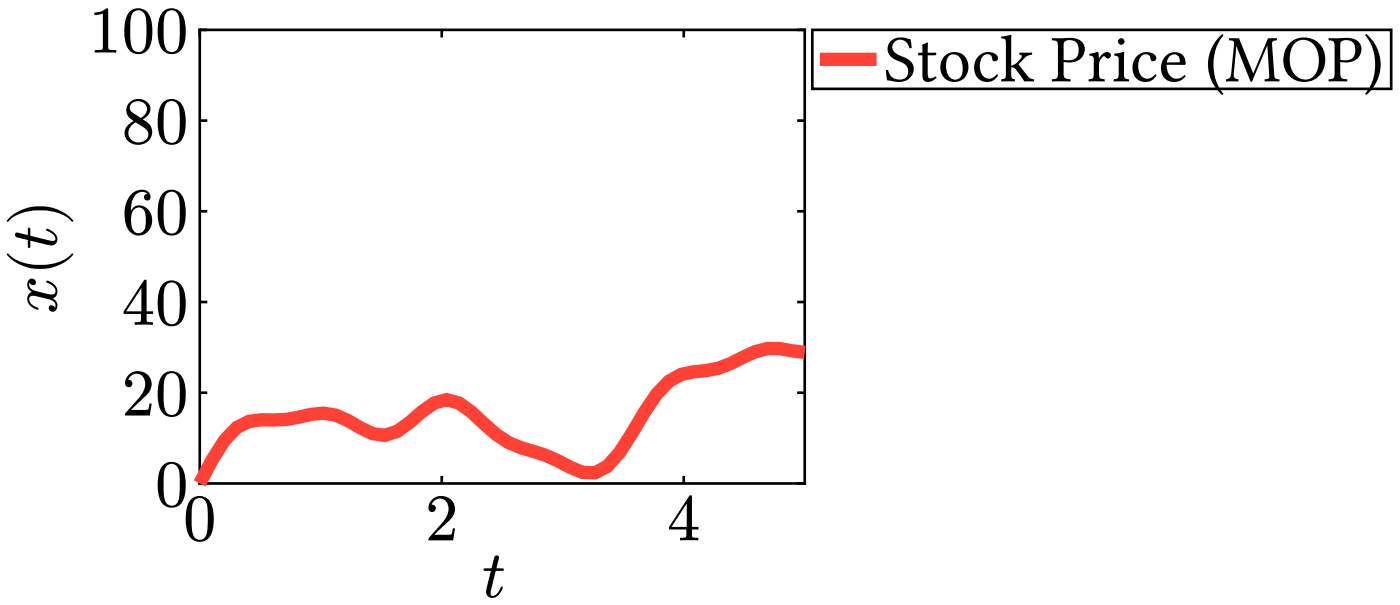

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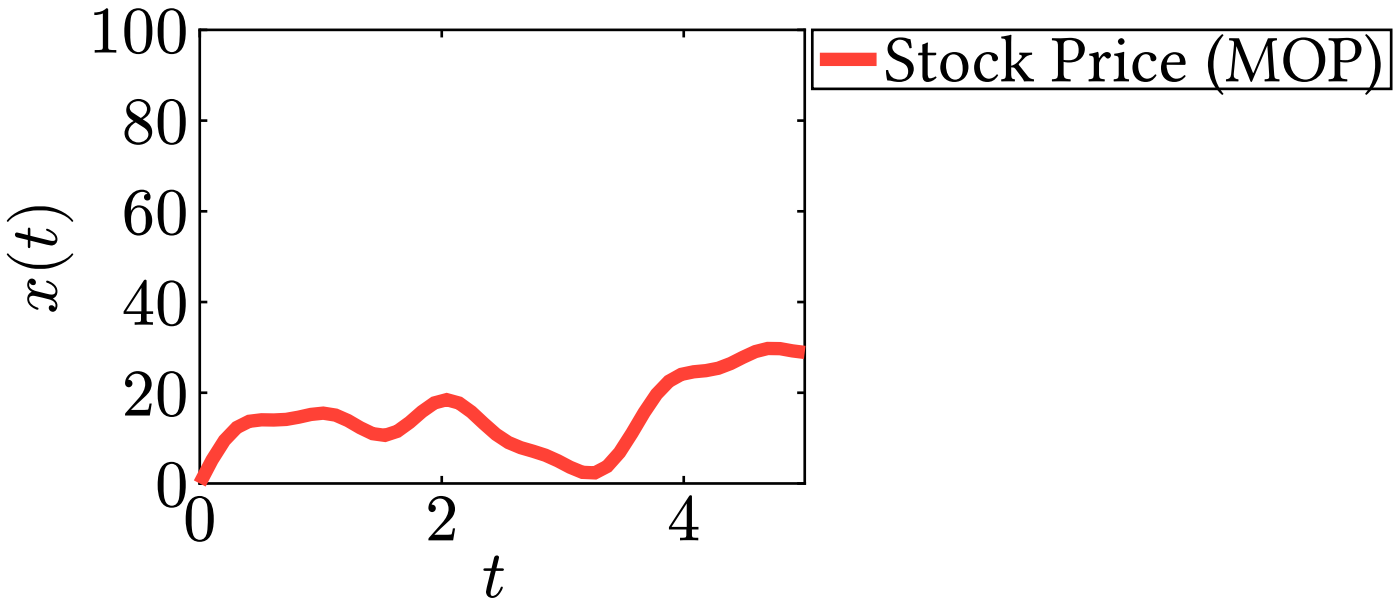

There is an underlying structure to $x(t)$

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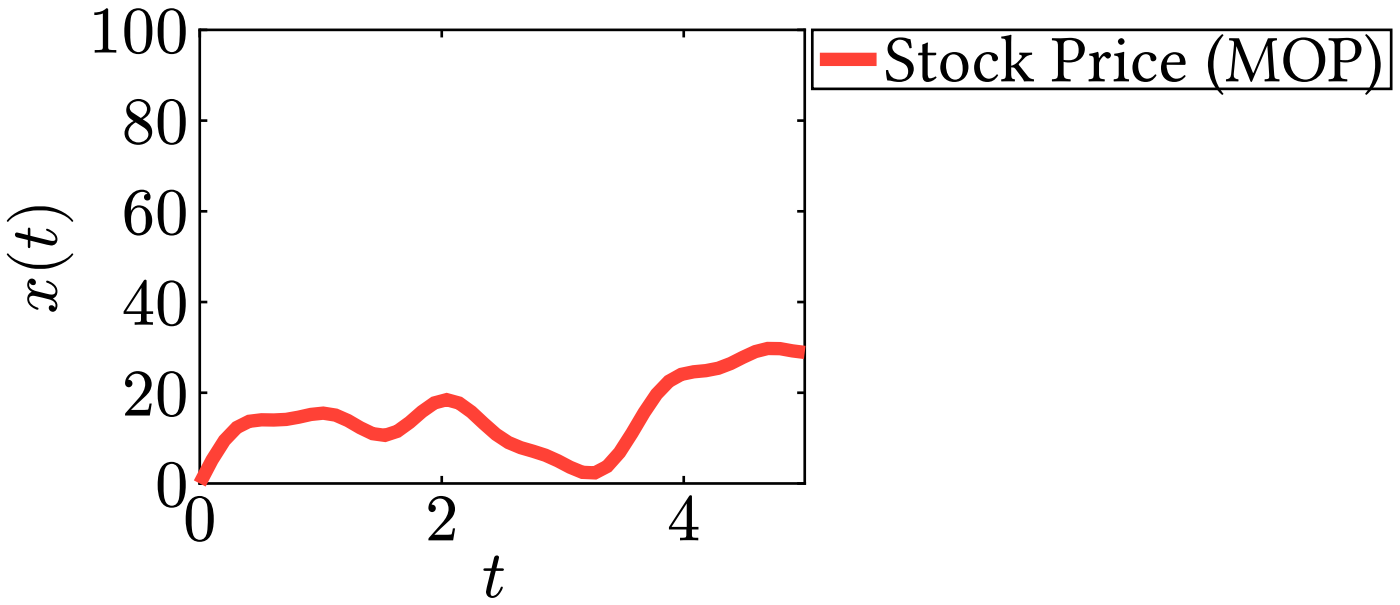

There is an underlying structure to $x(t)$

Structure: Tomorrow's stock price will be close to today's stock price

Review

$$x(t) = \text{audio}$$

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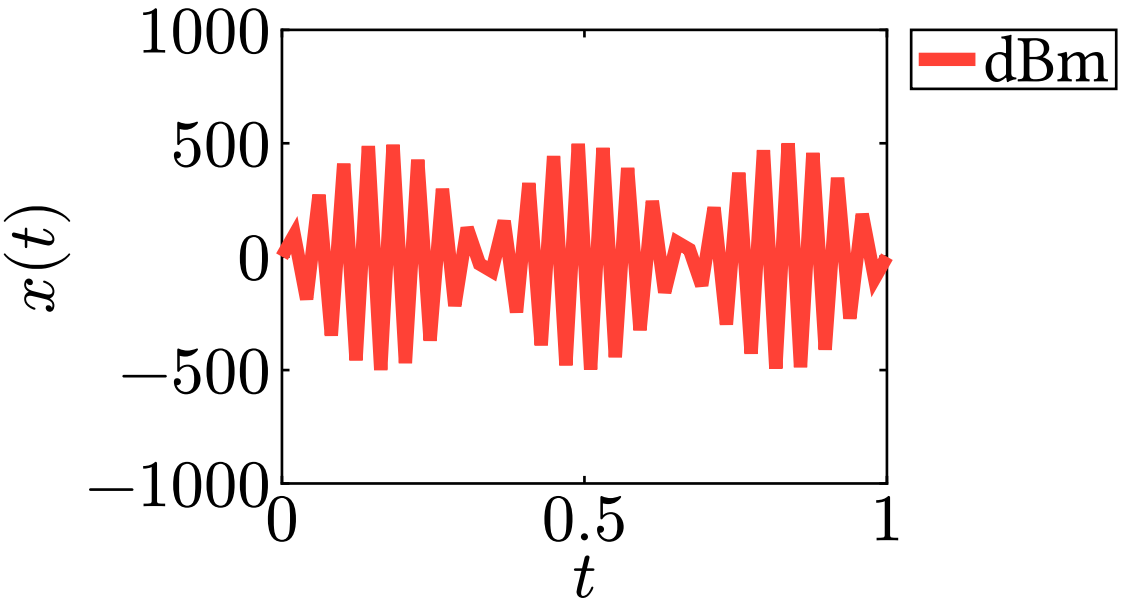

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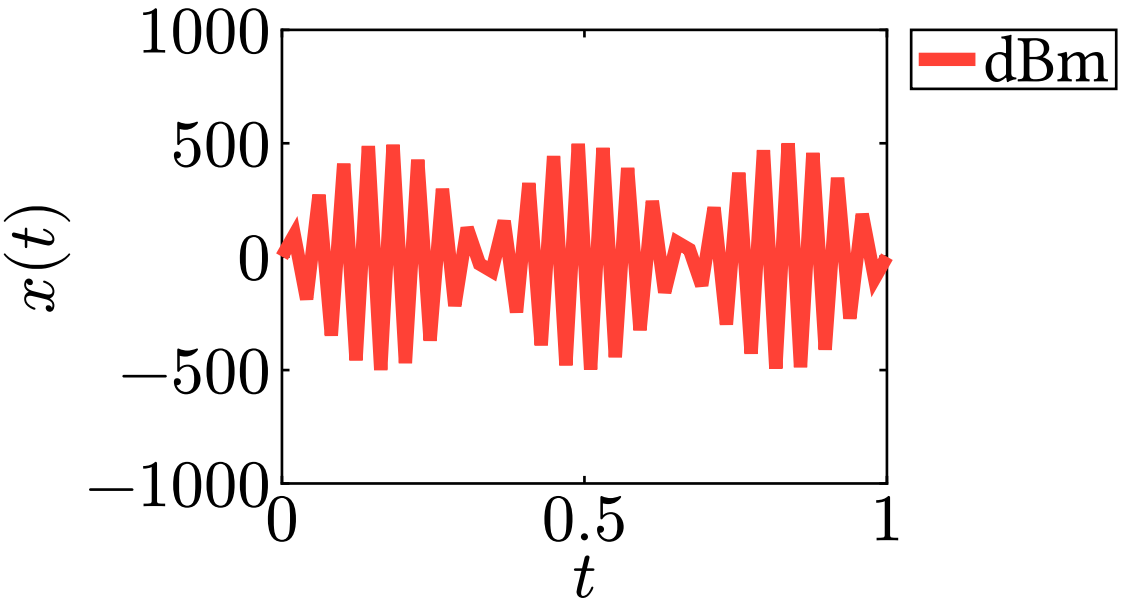

Structure: Nearby waves form syllables

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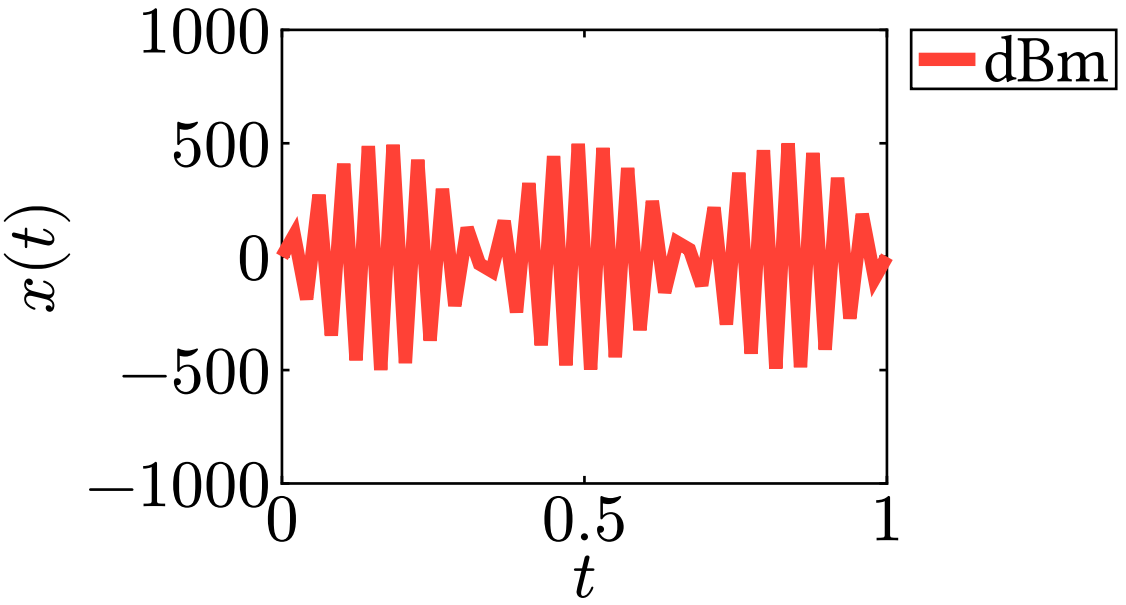

Structure: Nearby waves form syllables

Structure: Nearby syllables combine to create meaning

Review

$$x(u, v) = \text{image}$$

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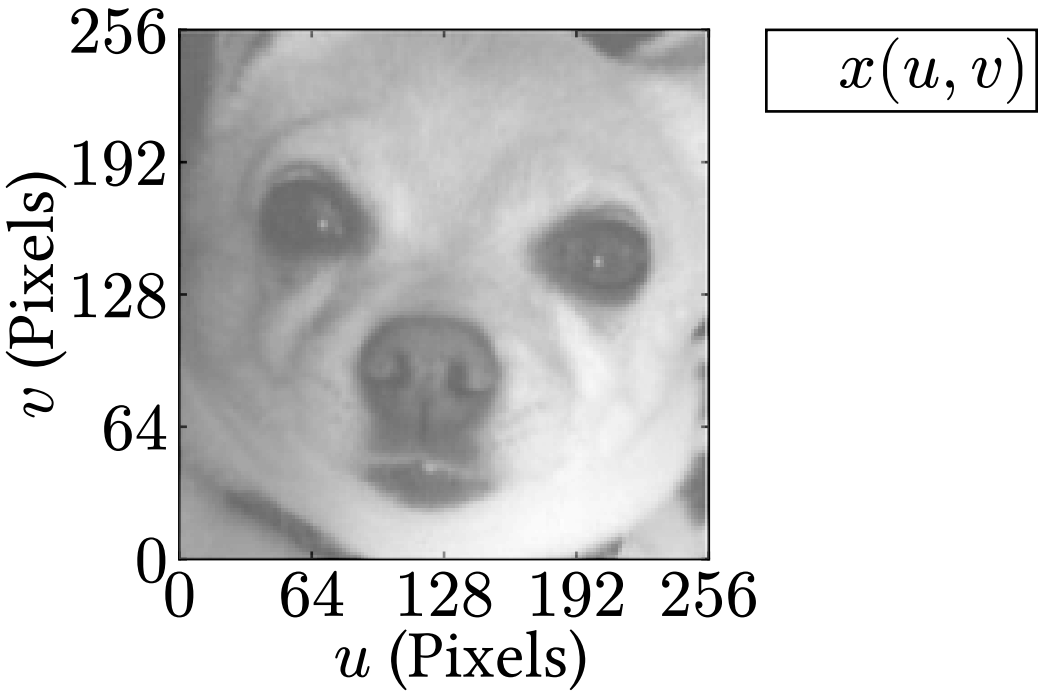

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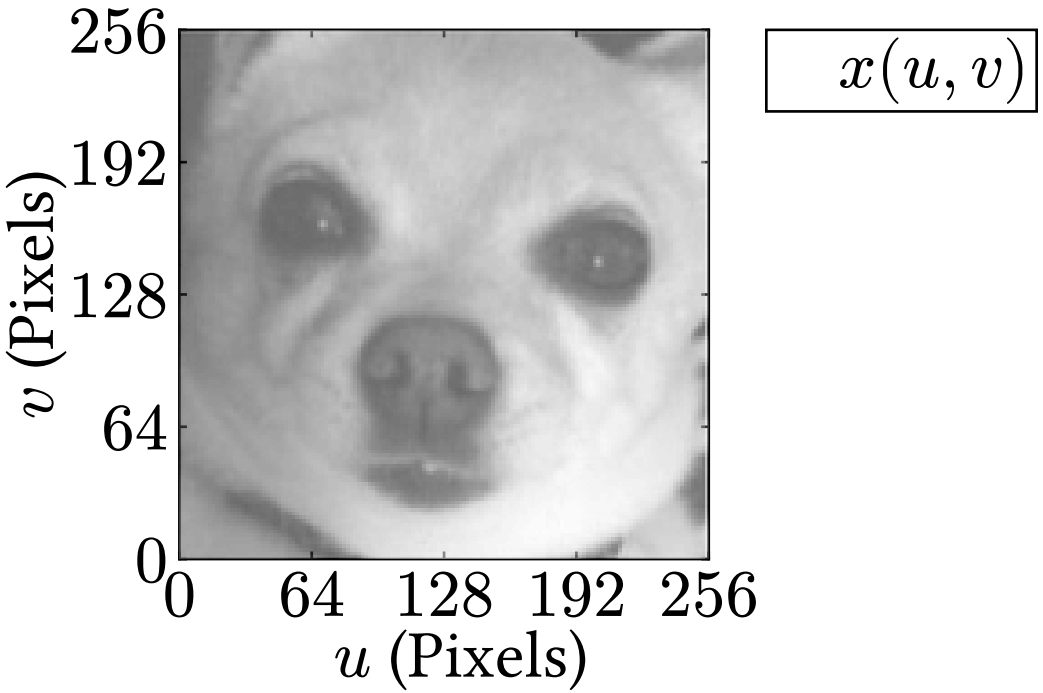

Structure: Repeated components (circles, symmetry, eyes, nostrils, etc)

Review

Two common properties of signals:

Review

Two common properties of signals:

Locality: Information concentrated over small regions of space/time

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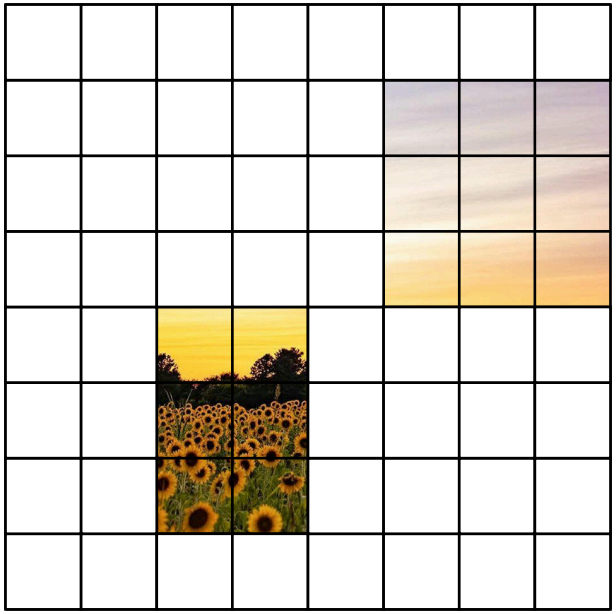
Two common properties of signals:

Locality: Information concentrated over small regions of space/time

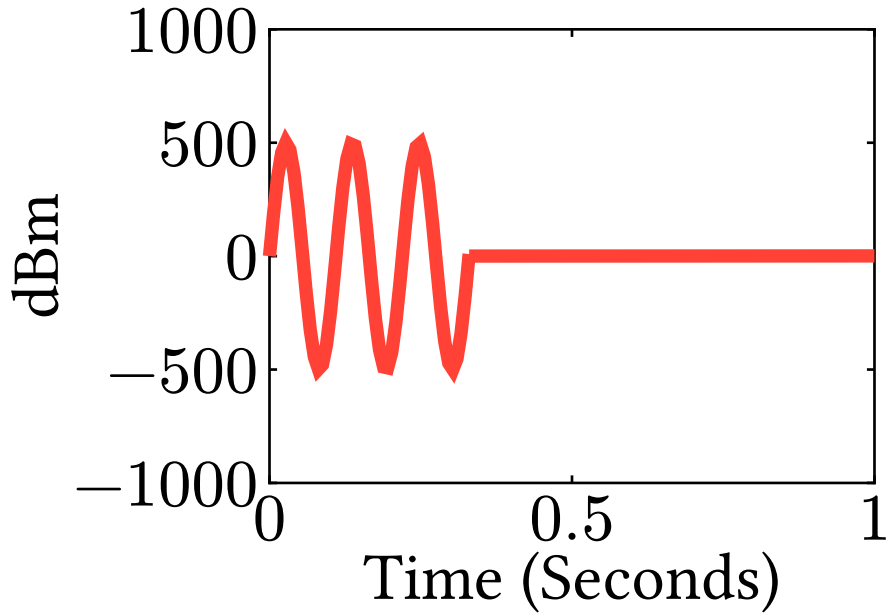
Translation Equivariance: Shift in signal results in shift in output

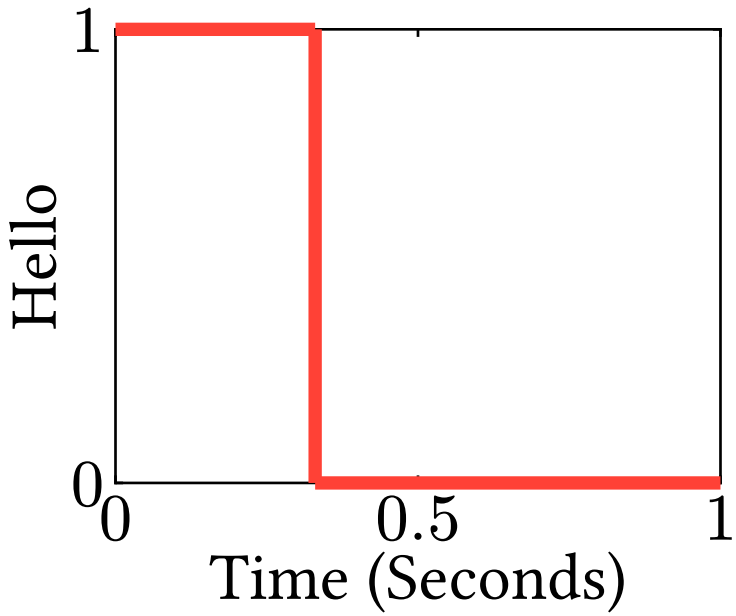
A more realistic scenario of locality and translation equivariance



Review

We use convolution to turn signals into useful signals





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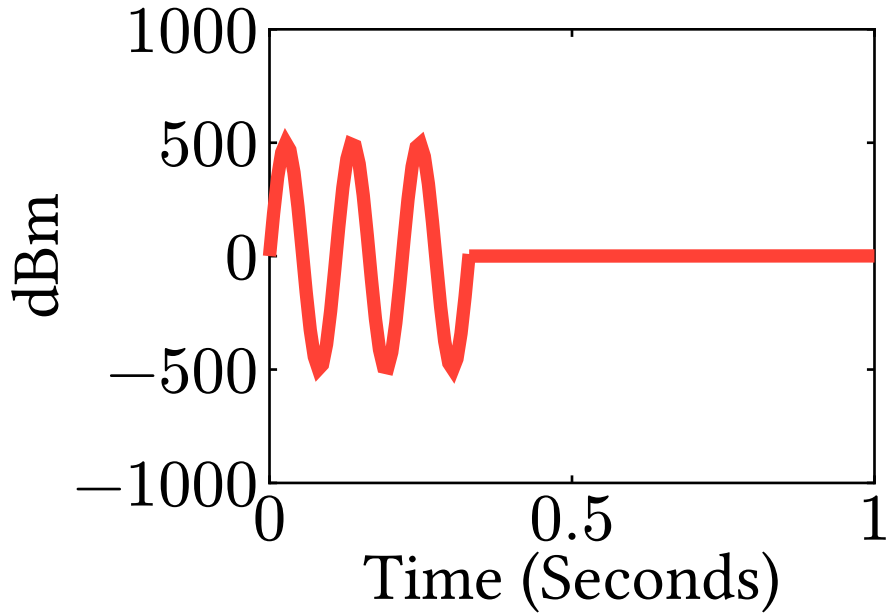

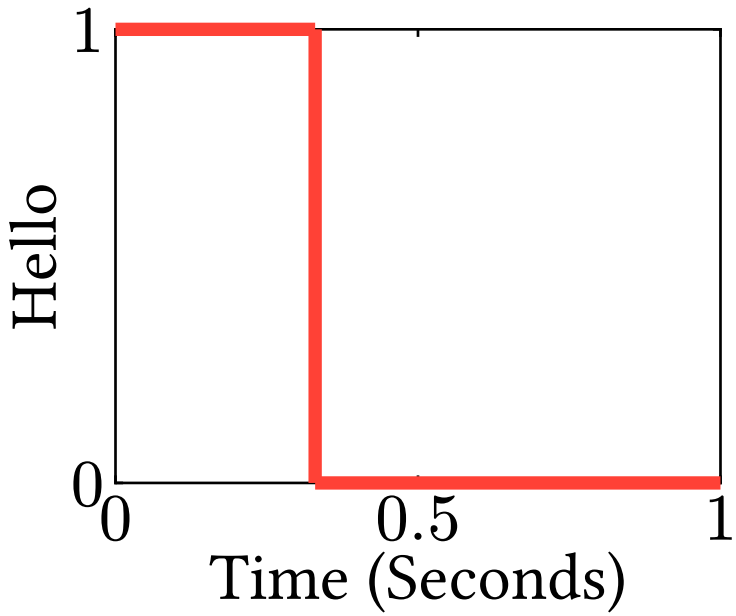

Convolution is translation equivariant and local

Review

Convolution is the sum of products of a signal $x(t)$ and a **filter** $g(t)$

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If the t is continuous in $x(t)$

$$x(t) * g(t) = \int_{-\infty}^{\infty} x(t-\tau)g(\tau)d\tau$$

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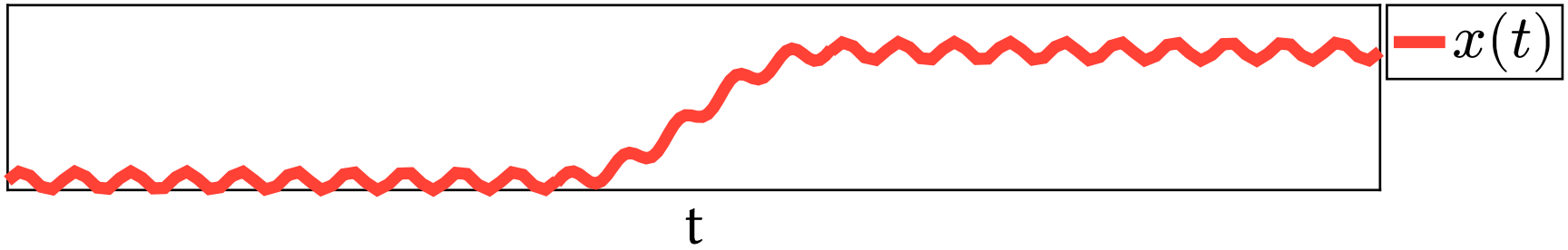
We slide the filter $g(t)$ across the signal $x(t)$

Review



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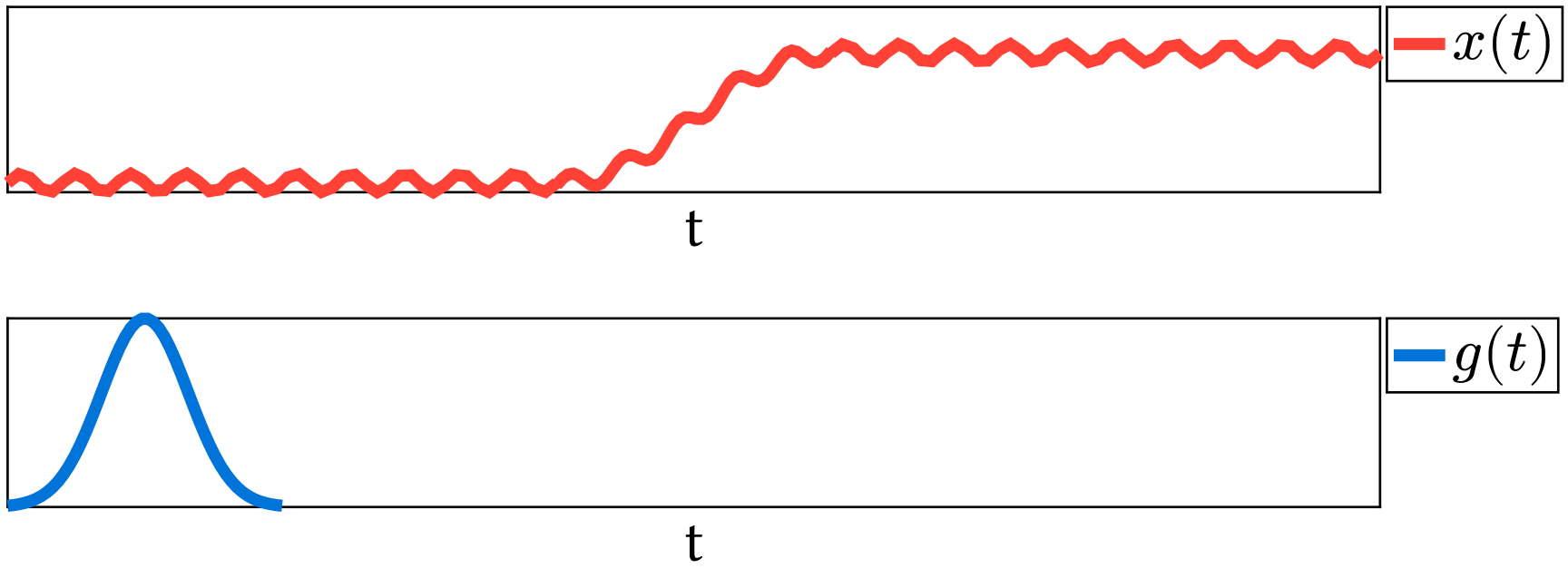

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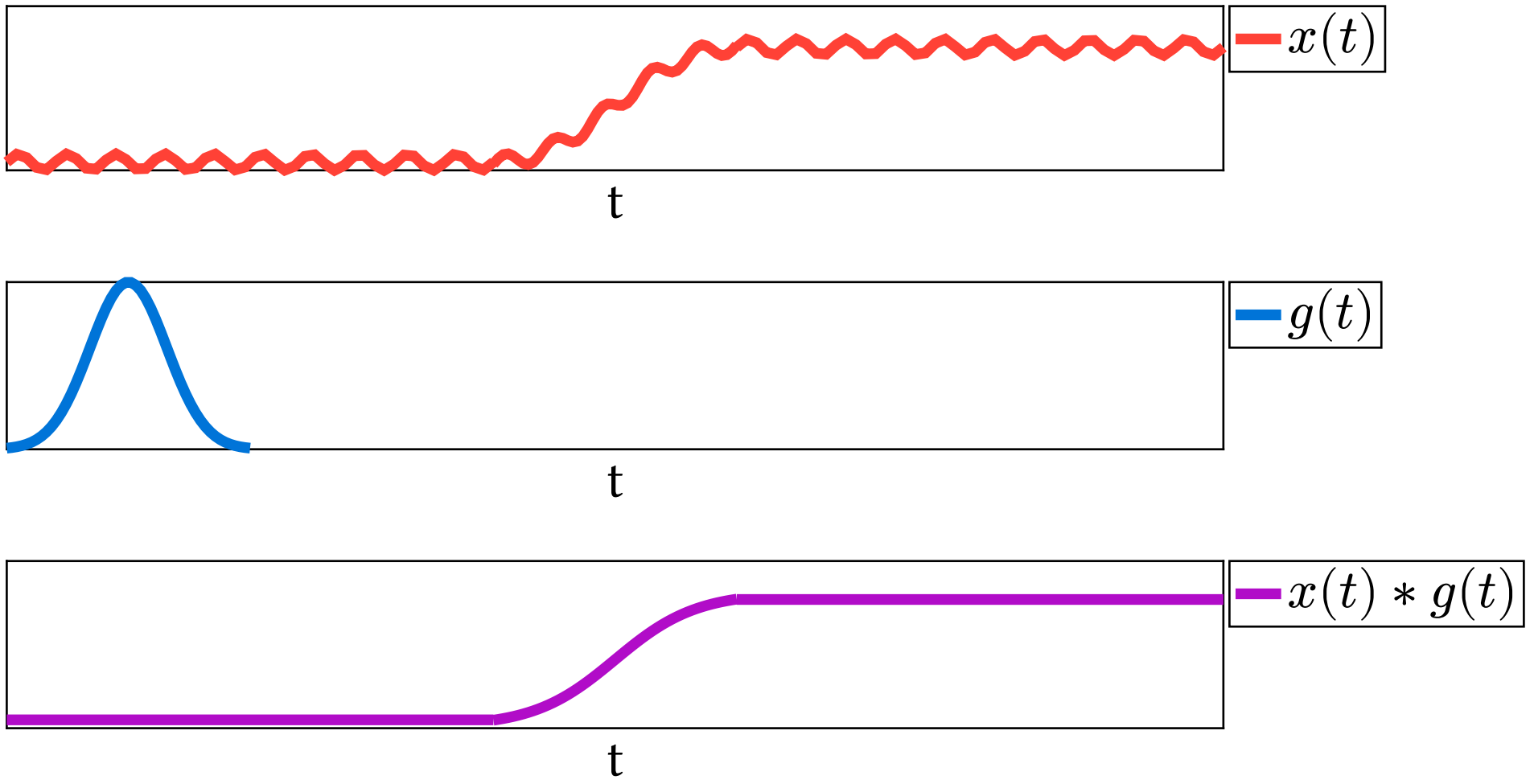

Review

$$\begin{bmatrix} x(t) \\ g(t) \\ y(t) \end{bmatrix} = \begin{bmatrix} 1 & 2 & 3 & 4 & 5 \\ 2 & 1 & & & \\ & & & & \end{bmatrix}$$

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To make a convolution layer, we make the filter with trainable parameters

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We can write both a perceptron and convolution in vector form

$$f(x(t), \boldsymbol{\theta}) = \sigma\left(\boldsymbol{\theta}^\top \begin{bmatrix} 1 \\ x(0.1) \\ x(0.2) \\ \vdots \end{bmatrix}\right)$$

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A convolution layer applies a "mini" perceptron to every few timesteps

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The output size depends on the signal length

Review

If we want a single output, we should **pool**

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$$z(t) = f(x(t), \boldsymbol{\theta}) = \left[\sigma\left(\boldsymbol{\theta}^\top \begin{bmatrix} 1 \\ x(0.1) \\ x(0.2) \end{bmatrix} \right) \quad \sigma\left(\boldsymbol{\theta}^\top \begin{bmatrix} 1 \\ x(0.2) \\ x(0.3) \end{bmatrix} \right) \quad \ldots \right]^\top$$

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$$\text{SumPool}(z(t)) = \sigma\left(\boldsymbol{\theta}^\top \begin{bmatrix} 1 \\ x(0.1) \\ x(0.2) \end{bmatrix} \right) + \sigma\left(\boldsymbol{\theta}^\top \begin{bmatrix} 1 \\ x(0.2) \\ x(0.3) \end{bmatrix} \right) + \ldots$$

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$$\text{MeanPool}(z(t)) = \frac{1}{T-k+1}\,\text{SumPool}(z(t)); \quad \text{MaxPool}(z(t)) = \max(z(t))$$

Review

Our examples considered:

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- 1 dimensional variable t

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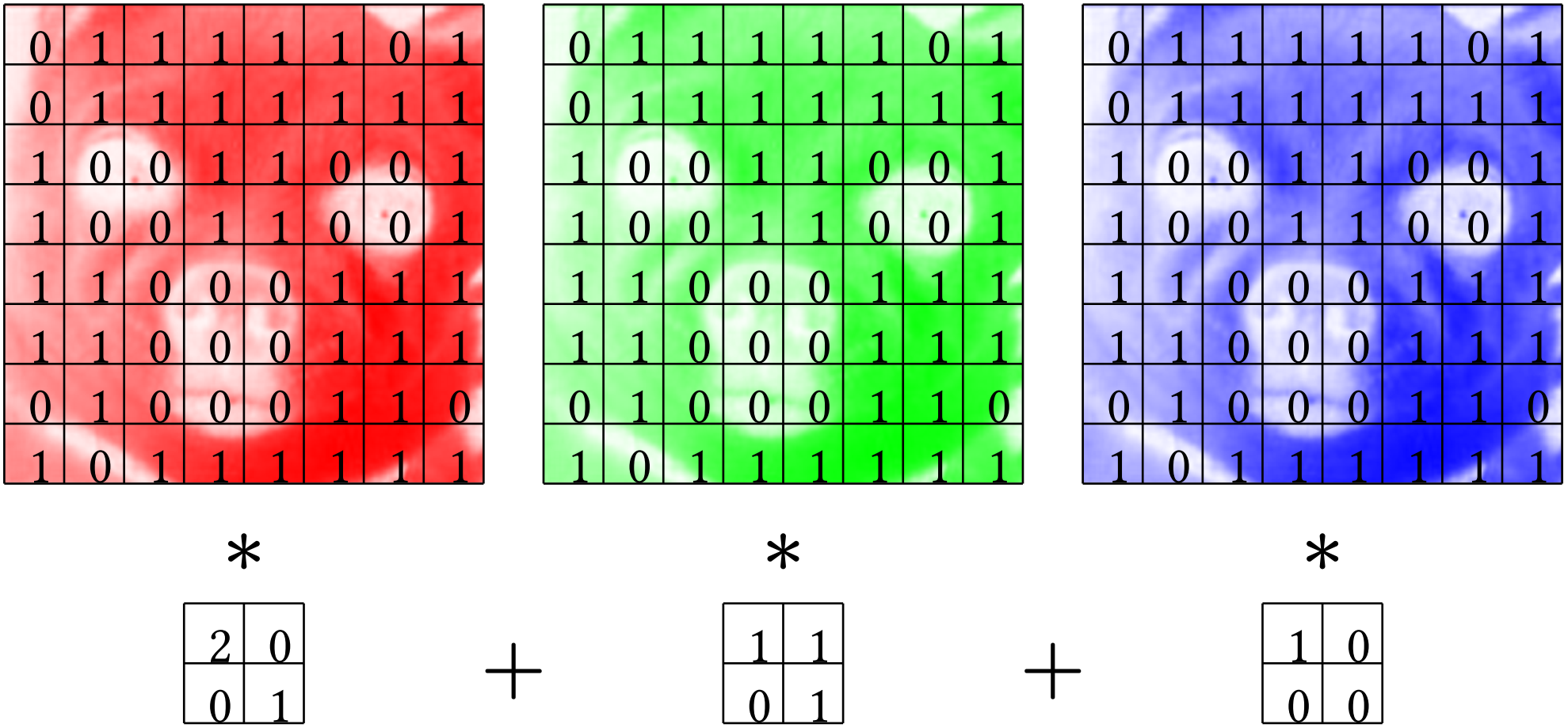
The idea is exactly the same

Review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0 1 1 1 1 1 1 1
1 0 0 1 1 0 0 1
1 0 0 1 1 0 0 1
1 1 0 0 0 1 1 1
1 1 0 0 0 1 1 1
0 1 0 0 0 1 1 0
1 0 1 1 1 1 1 1
*
2 0
0 1
+
1 1
0 1
+
1 0
0 0

Review

One last thing, **stride** allows you to "skip" cells during convolution

Review

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This can decrease the size of image without pooling

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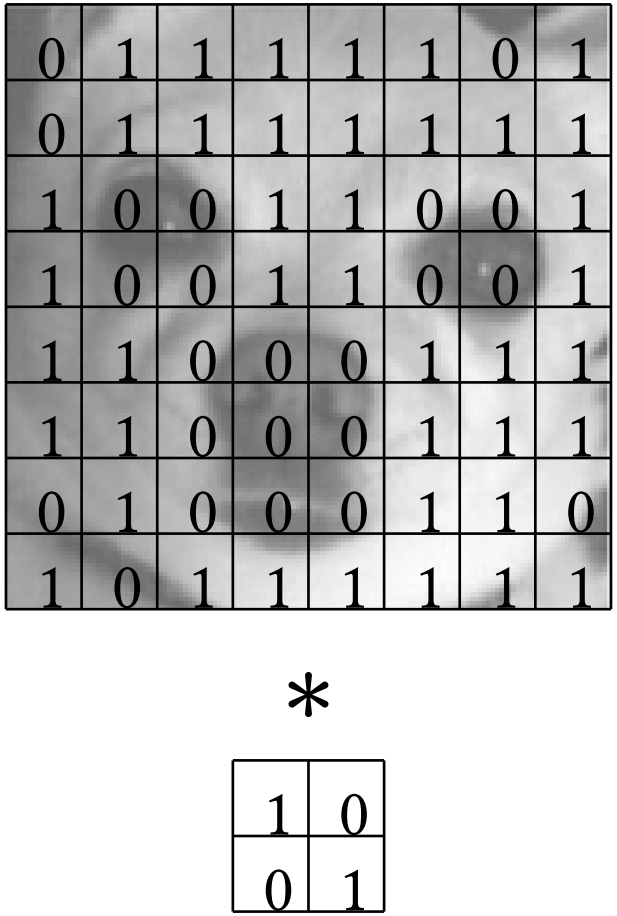
This can decrease the size of image without pooling

Padding adds zero pixels to the image to increase the output size



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Now, we will discuss a neuroscience approach to sequence modeling

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You can solve temporal tasks using either convolution or RNNs

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Now, we will discuss a neuroscience approach to sequence modeling

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You can solve temporal tasks using either convolution or RNNs

So what is the difference between convolution and recurrent models?

Sequence Modeling

Convolution works over inputs of any variables (time, space, etc)

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Recurrent models do not assume locality or equivariance

Equivariance and locality make learning more efficient, but not all problems have this structure

Let us examine some real life signals, and see if these properties hold

Sequence Modeling

Example 1: You like dinosaurs as a child, you grow up and study dinosaurs for work

Sequence Modeling

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Question: Is this local?

Sequence Modeling

Example 1: You like dinosaurs as a child, you grow up and study dinosaurs for work

Question: Is this local?

Answer: No, two related events separated by 20 years

Example 2: Your parent changes your diaper

Sequence Modeling

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Question: Translation equivariant?

Sequence Modeling

Example 2: Your parent changes your diaper

Question: Translation equivariant?

No! Ok if you are a baby, different meaning if you are an adult!

Example 3: You hear a gunshot then see runners

Sequence Modeling

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Sequence Modeling

Example 3: You hear a gunshot then see runners

Question: Translation equivariant?

Answer: No! (1) gunshot, (2) see runners, enjoy the race. (1) see runners, (2) hear gunshot, you start running too!

Sequence Modeling

Example 3: You hear a gunshot then see runners

Question: Translation equivariant?

Answer: No! (1) gunshot, (2) see runners, enjoy the race. (1) see runners, (2) hear gunshot, you start running too!

Question: Any other examples?

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Problems without locality and translation equivariance are difficult to solve with convolution

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For these problems, we need something else!

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How do humans experience time and process temporal data?

Sequence Modeling

Problems without locality and translation equivariance are difficult to solve with convolution

For these problems, we need something else!

How do humans experience time and process temporal data?

Can we design a neural network based on human perceptions of time?

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Composite Memory

How do humans process temporal data?

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See dog $\rightarrow$ photoreceptors fire $\rightarrow$ neurons fire in the brain

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Composite Memory

How do humans process temporal data?

We only perceive the present

See dog $\rightarrow$ photoreceptors fire $\rightarrow$ neurons fire in the brain

No dog $\rightarrow$ no photoreceptors fire $\rightarrow$ no neurons fire

We know there was a dog, even if we no longer see it

Composite Memory

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Without the ability to reason over memories, we would only react to stimuli like bacteria

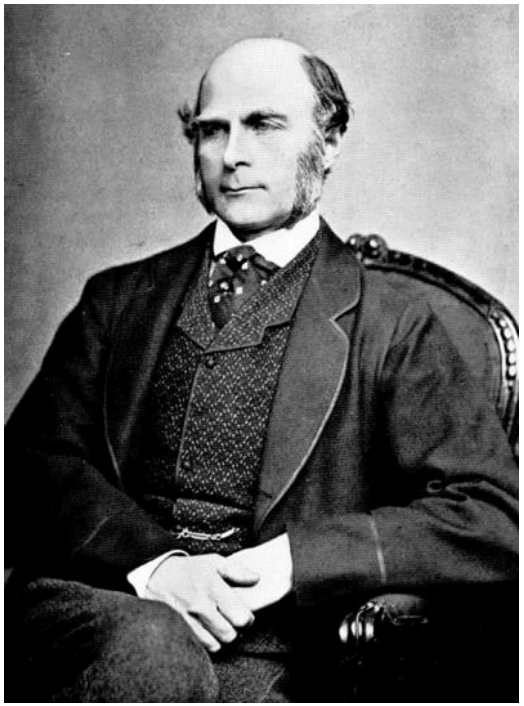
Composite Memory

So how do we model memory in humans?

Composite Memory

Francis Galton (1822-1911)

photo composite memory



Composite Memory

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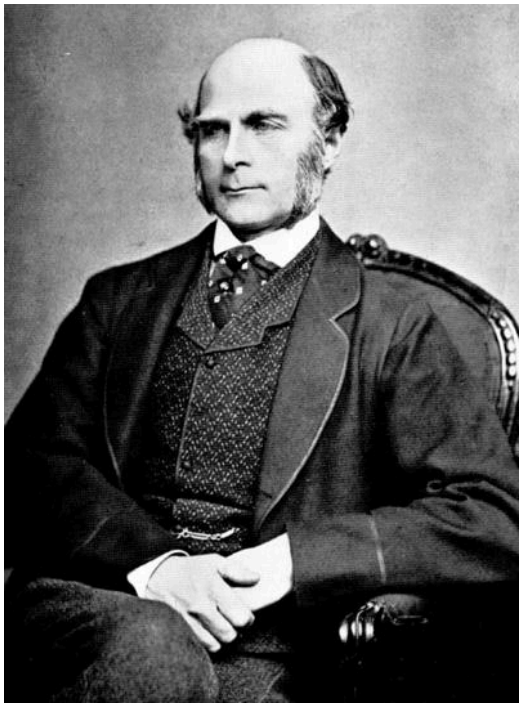
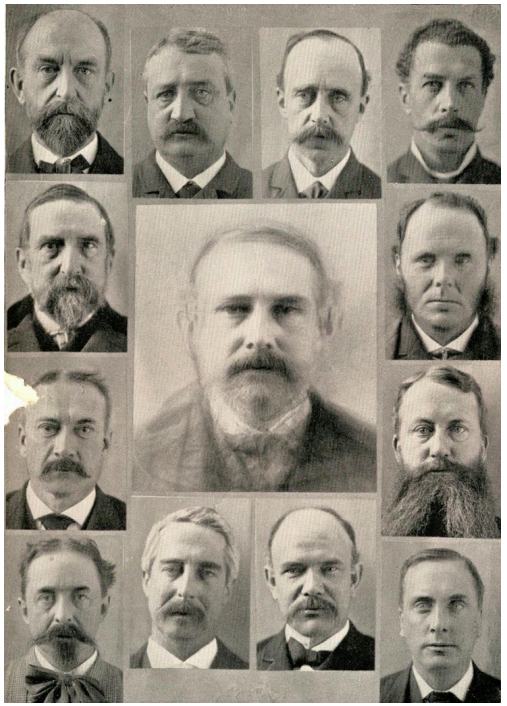


Composite photo of members of a party



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We can rewrite f as a **recurrent function**

Agenda

1. Review
2. Sequence Modeling
3. **Composite Memory**
4. Linear Recurrence
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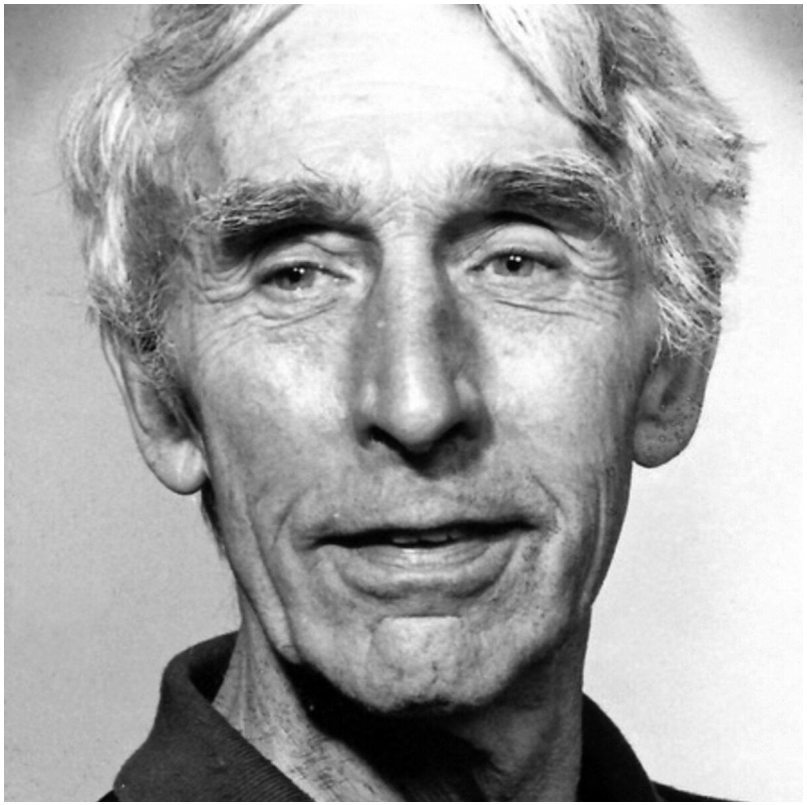

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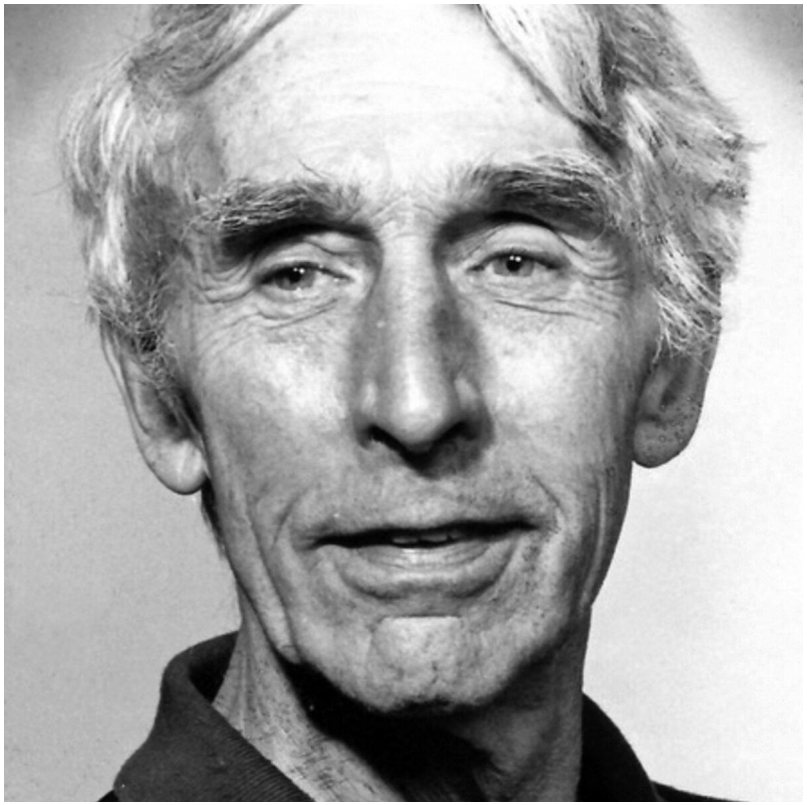

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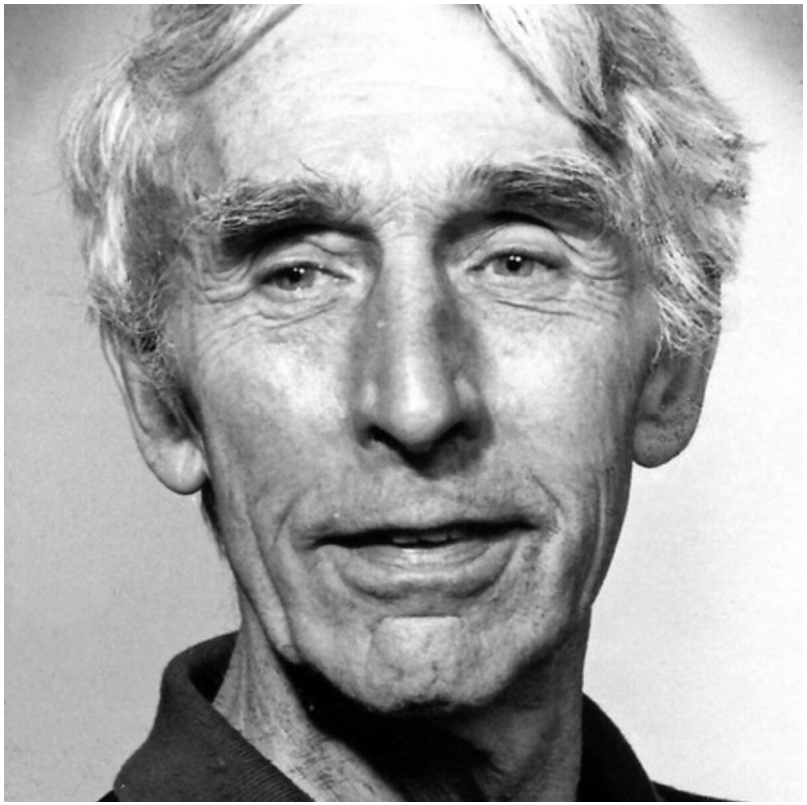

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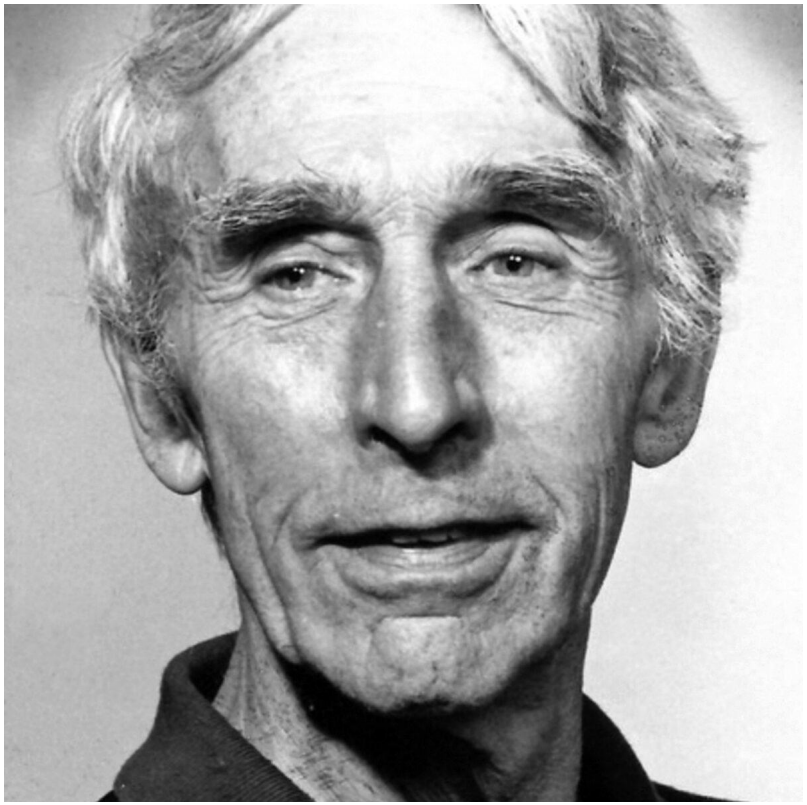

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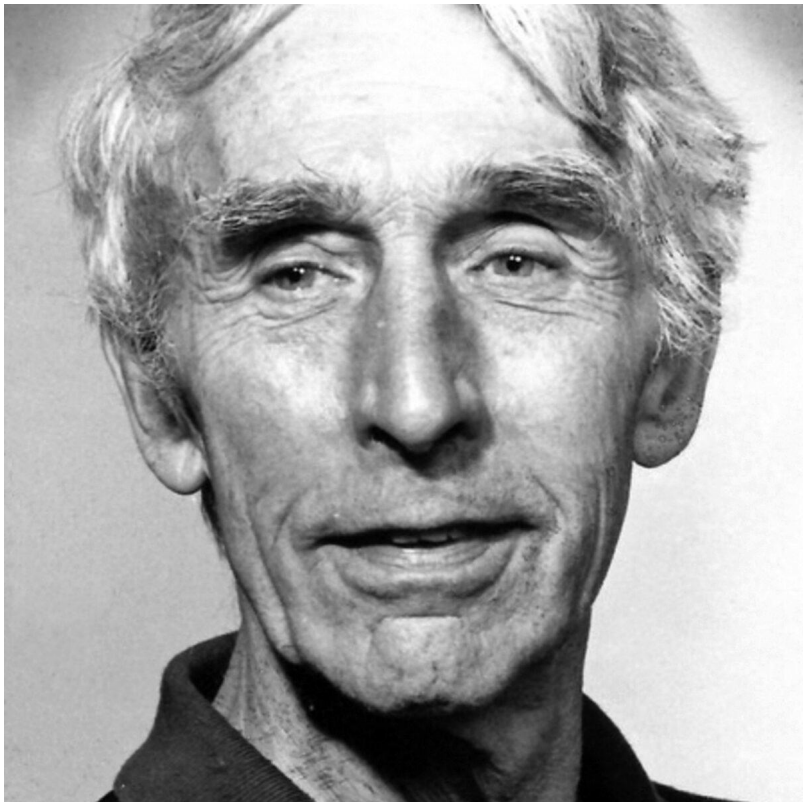

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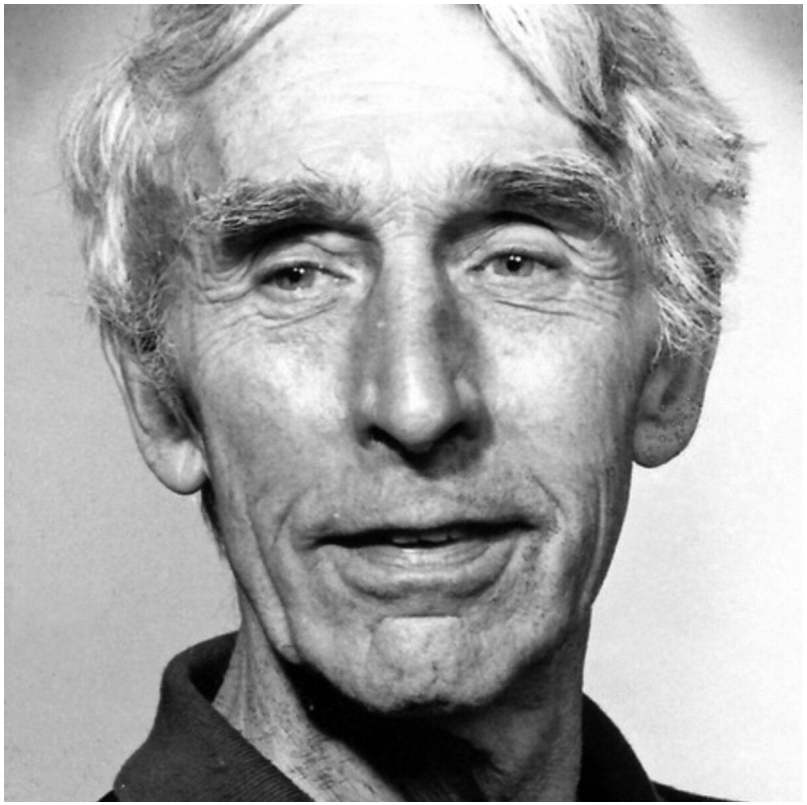

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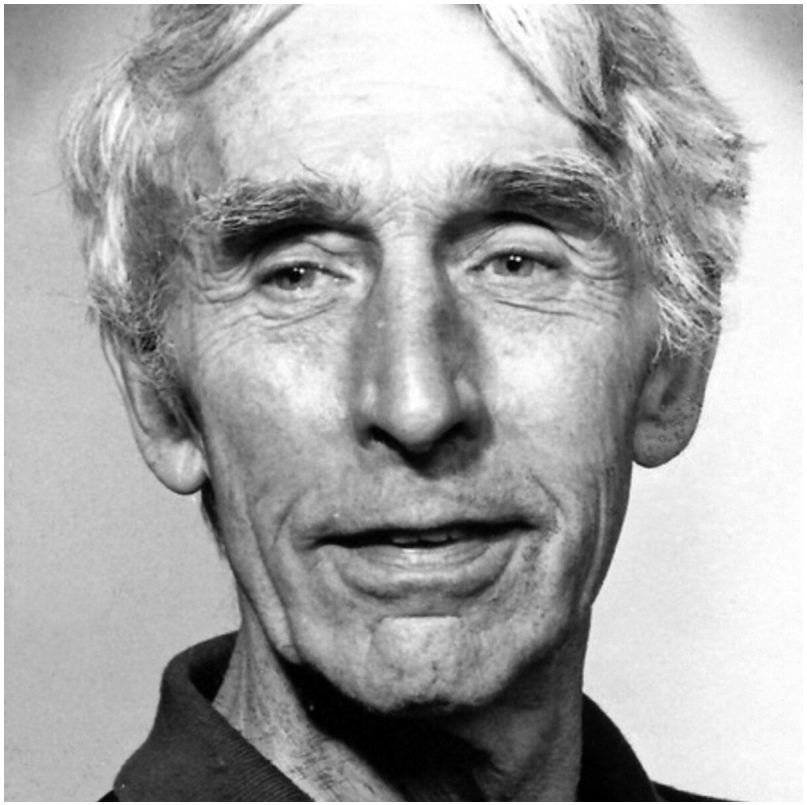

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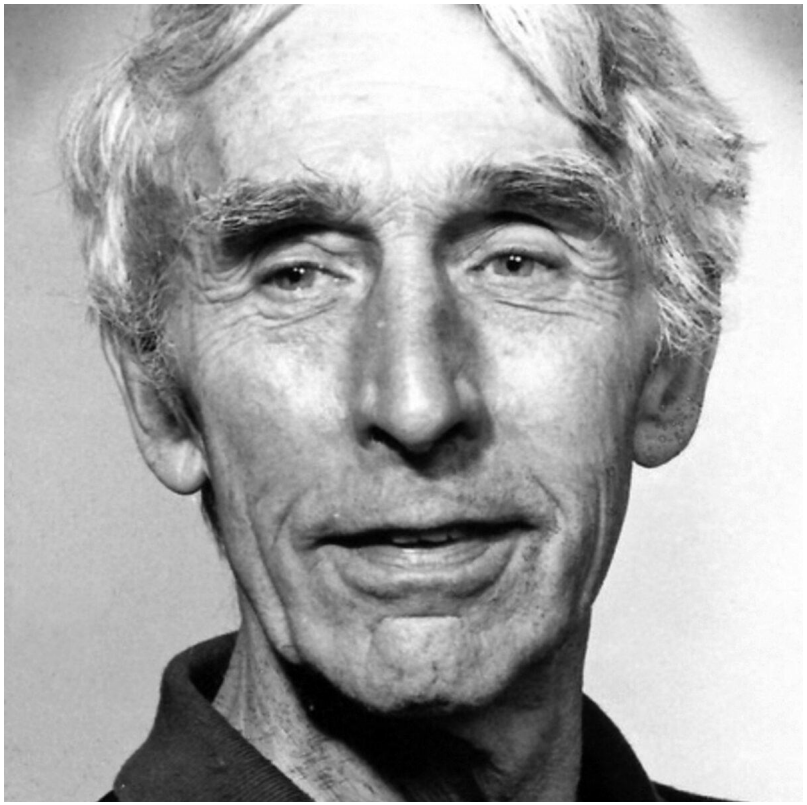

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Let us work out how forgetting works

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$$\boldsymbol{h}_T = \gamma^3 \boldsymbol{h}_{T-3} + \gamma^2 \boldsymbol{\theta}^\top \overline{\boldsymbol{x}}_{T-2} + \gamma \boldsymbol{\theta}^\top \overline{\boldsymbol{x}}_{T-1} + \boldsymbol{\theta}^\top \overline{\boldsymbol{x}}_T$$

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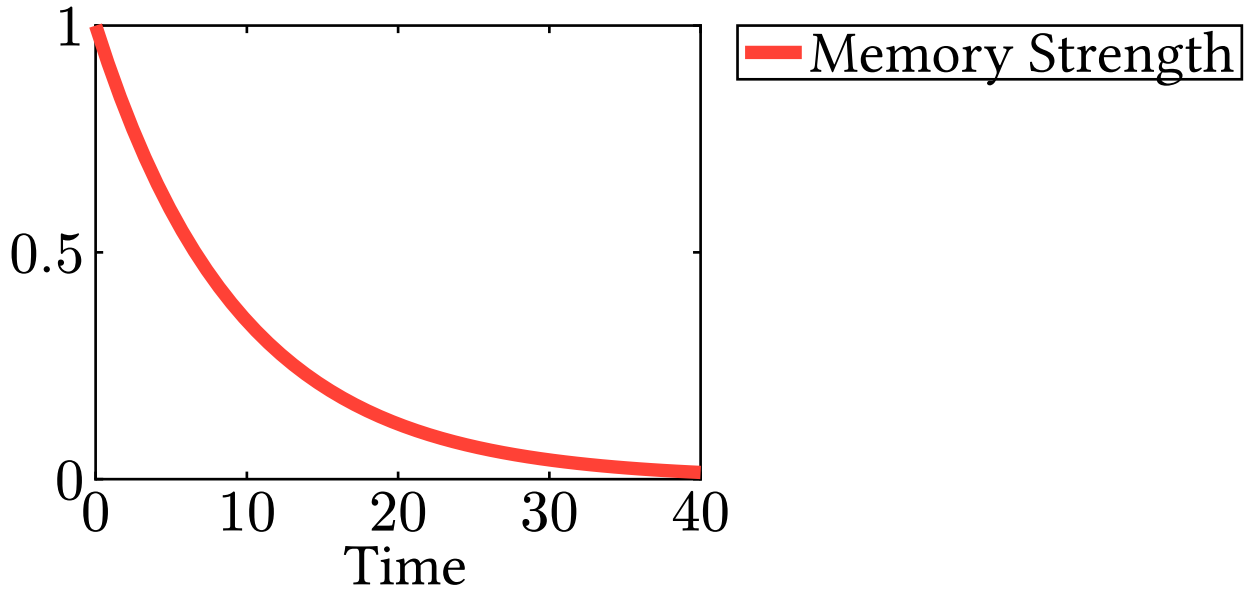

Linear Recurrence

As T increases, we add new information $\boldsymbol{x}_T$

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The memory decay is smooth and differentiable

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We can learn the parameters $\gamma, \boldsymbol{\theta}$ using gradient descent

Linear Recurrence

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Our models learn to play board games and computer games

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Our models learn to play board games and computer games

Outperforms other recurrent models (LSTM, GRU, etc)

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Scans

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How do we compute $\boldsymbol{h}_1, \boldsymbol{h}_2, ..., \boldsymbol{h}_T$ on a computer?

We use an algebraic operation called a **scan**

Scans

Our function f is just defined for a single X

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Scans

```
import jax
import jax.numpy as jnp

T, d_x, d_h = 10, 2, 4
xs, h0 = jnp.ones((T, d_x)), jnp.zeros((d_h,))
theta = [jnp.ones((d_h,)), jnp.ones((d_x, d_h))] # (b, W)

def f(h, x):
    b, W = theta
    result = h + (W.T @ x + b)
    return result, result # return one, return all

hT, hs = jax.lax.scan(f, init=h0, xs=xs) # Scan f over x
```

Scans

`torch` does NOT have built-in scans, and is very slow compared to `jax`

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```
def scan(f, h, xs):
    # h shape is (d_h,)
    # xs shape is (T, d_x)
    hs = []
    for x in xs:
        h = f(h, x, theta)
        hs.append(h)
    # output shape is (T, d_h)
    return torch.stack(hs)
```

Scans

```
import torch
T, d_x, d_h = 10, 2, 4

xs, h0 = torch.ones((T, d_x)), torch.zeros((d_h,))
theta = (torch.ones((d_h,)), torch.ones((d_x, d_h)))

def f(h, x):
    b, W = theta
    result = h + (W.T @ x + b)
    return result # h

hs = scan(f, h0, xs)
```

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Answer: Yes, linear operations obey the associative property

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We are almost done defining recurrent models

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Example: I ask you your favorite ice cream flavor

You recall previous times you ate ice cream, but not your phone number

We will model this recall of memories using a function g

Output Modeling

Let g define our memory recall function

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g searches your memories h using the input x, to produce output y

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$\boldsymbol{x}$: "What is your favorite ice cream flavor?"

$\boldsymbol{h}$: Everything you remember (hometown, birthday, etc)

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Now, we will combine f and g

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Step 1: Perform scan to find recurrent states

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$$\begin{bmatrix} \boldsymbol{h}_1 \\ \vdots \\ \boldsymbol{h}_T \end{bmatrix} = \text{scan}(f) \left(\boldsymbol{h}_0, \begin{bmatrix} \boldsymbol{x}_1 \\ \vdots \\ \boldsymbol{x}_T \end{bmatrix}, \boldsymbol{\theta}_f \right)$$

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Step 2: Perform recall on recurrent states

$$\begin{bmatrix} \boldsymbol{y}_1 \\ \vdots \\ \boldsymbol{y}_T \end{bmatrix} = \begin{bmatrix} g(\boldsymbol{h}_1, \boldsymbol{x}_1, \boldsymbol{\theta}_g) \\ \vdots \\ g(\boldsymbol{h}_T, \boldsymbol{x}_T, \boldsymbol{\theta}_g) \end{bmatrix}$$

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Questions? This is on the homework

Output Modeling

To summarize, we defined:

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- Recurrent function f

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- Scanned recurrence $\text{scan}(f)$

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- Execute $\text{scan}(f)$ over inputs to make recurrent states

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To run our model:

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- Execute g over recurrent states to make outputs

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Relax

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Recurrent Loss Functions

Let us examine some example tasks:

Recurrent Loss Functions

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- Clock

Recurrent Loss Functions

Let us examine some example tasks:

- Clock
- Explaining a video

Recurrent Loss Functions

Task: Clock – keep track of time

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Every minute, the minute hand ticks

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Count/remember the ticks to know the time

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$$X \in \{0, 1\}, \quad Y \in \mathbb{R}^2$$

Recurrent Loss Functions

Example input sequence:

$$\begin{bmatrix} 1 \\ 1 \\ \vdots \\ 1 \end{bmatrix}$$

Recurrent Loss Functions

Example input sequence:

$$\begin{bmatrix} 1 \\ 1 \\ \vdots \\ 1 \end{bmatrix}$$

Desired output sequence

$$\begin{bmatrix} 0 & 1 \\ 0 & 2 \\ \vdots & \vdots \\ 2 & 13 \end{bmatrix}$$

We have a corresponding label $\boldsymbol{y}$ for each input x

Recurrent Loss Functions

Can use square error

Recurrent Loss Functions

Can use square error

First, scan f over the inputs to find h

$$\boldsymbol{h}_{[i],j} = \text{scan}(f)\left(\boldsymbol{h}_0, \begin{bmatrix} \boldsymbol{x}_{[i],1} \\ \vdots \\ \boldsymbol{x}_{[i],T} \end{bmatrix}, \boldsymbol{\theta}_f\right)$$

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$$\mathcal{L}(\boldsymbol{X}, \boldsymbol{Y}, \boldsymbol{\theta}) = \sum_{i=1}^{n}\sum_{j=1}^{T}\left[g\left(\boldsymbol{h}_{[i],j}, \boldsymbol{x}_{[i],j}, \boldsymbol{\theta}_g\right) - \boldsymbol{y}_{[i],j}\right]^2$$

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Onto the next task

Recurrent Loss Functions

Task: Watch a video, then explain it

$$X \in \mathbb{Z}^{3\times 32\times 32}, \quad Y \in \{\text{comedy show}, \text{action movie}, \ldots\}$$

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Example input sequence:

$$\begin{bmatrix} I_1 \\ I_2 \\ \vdots \\ I_T \end{bmatrix}$$

Example output: "dancing dog"

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Example output: "dancing dog"

Unlike before, we have many inputs but just one output!

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We will use the classification loss

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$$\mathcal{L}(\boldsymbol{X}, \boldsymbol{Y}, \boldsymbol{\theta}) = \sum_{i=1}^{n} \sum_{j=1}^{d_y} y_{[i],j} \log g\Big(\boldsymbol{h}_{[i],T}, \boldsymbol{x}_{[i],T}, \boldsymbol{\theta}_g\Big)_j$$

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We only care about the $\boldsymbol{h}_T$

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Backpropagation through Time

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Step 2: Update parameters using gradient

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Just like all other models, we train recurrent models using gradient descent

Step 1: Compute gradient

Step 2: Update parameters using gradient

How do we compute gradients for recurrent functions?

Backpropagation through Time

First, compute gradient of f

$$f(\boldsymbol{h}, \boldsymbol{x}, \boldsymbol{\theta}) = \gamma \boldsymbol{h} + \boldsymbol{\theta}^\top \overline{\boldsymbol{x}}$$

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Question: What is $\nabla_{\boldsymbol{\theta}} f$?

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$$\nabla_{\boldsymbol{\theta}} f(\boldsymbol{h}, \boldsymbol{x}, \boldsymbol{\theta}) = \overline{\boldsymbol{x}}^\top$$

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$$\nabla_{\boldsymbol{\theta}} f(\boldsymbol{h}, \boldsymbol{x}, \boldsymbol{\theta}) = \overline{\boldsymbol{x}}^\top$$

Too easy, now let us find the gradient of $\text{scan}(f)$

Backpropagation through Time

$$\text{scan}(f)\left(\boldsymbol{h}_0, \begin{bmatrix}\boldsymbol{x}_1 \\ \vdots \\ \boldsymbol{x}_T\end{bmatrix}, \boldsymbol{\theta}\right) = \begin{bmatrix} f(h_0, x_1, \boldsymbol{\theta}) \\ f(h_1, x_2, \boldsymbol{\theta}) \\ \vdots \\ f(h_{T-1}, x_T, \boldsymbol{\theta}) \end{bmatrix} = \begin{bmatrix} f(h_0, x_1) \\ f(f(h_0, x_1), x_2) \\ \vdots \\ f(\ldots f(h_0, x_1) \ldots, x_T) \end{bmatrix}$$

Question: What is $\nabla_{\boldsymbol{\theta}} \text{scan}(f)$?

Hint: Chain rule

Backpropagation through Time

$$\text{scan}(f)\left(\boldsymbol{h}_0, \begin{bmatrix}\boldsymbol{x}_1 \\ \vdots \\ \boldsymbol{x}_T\end{bmatrix}, \boldsymbol{\theta}\right) = \begin{bmatrix} f(h_0, x_1, \boldsymbol{\theta}) \\ f(h_1, x_2, \boldsymbol{\theta}) \\ \vdots \\ f(h_{T-1}, x_T, \boldsymbol{\theta}) \end{bmatrix} = \begin{bmatrix} f(h_0, x_1) \\ f(f(h_0, x_1), x_2) \\ \vdots \\ f(...f(h_0, x_1)..., x_T) \end{bmatrix}$$

Question: What is $\nabla_{\boldsymbol{\theta}} \text{scan}(f)$?

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Question: Any issues with this?

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What if $\nabla_{\boldsymbol{\theta}} f$ is $\ll 1$ or $\gg 1$?

Agenda

1. Review
2. Sequence Modeling
3. Composite Memory
4. Linear Recurrence
5. Scans
6. Output Modeling
7. Recurrent Loss Functions
8. **Backpropagation through Time**
9. Recurrent Neural Networks
10. Coding

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Recurrent Neural Networks

Until now, f was a linear function

Recurrent Neural Networks

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If we make f a neural network, then we have a **recurrent neural network** (RNN)

Recurrent Neural Networks

The simplest recurrent neural network is the **Elman Network**

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$\boldsymbol{h}$ grows large and causes exploding gradients, σ should be sigmoid!

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Question: Anything missing from our linear model?

Answer: Forgetting!

Add forgetting

$$f(\boldsymbol{h}, \boldsymbol{x}, \boldsymbol{\theta}) = \sigma\left(\boldsymbol{\theta}_1^\top \boldsymbol{h} \odot f_{\text{forget}}(\boldsymbol{h}, \boldsymbol{x}, \boldsymbol{\theta}) + \boldsymbol{\theta}_2^\top \overline{\boldsymbol{x}}\right)$$

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Through gradient descent, neural network learns which memories to forget

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Left term forgets old, right term replaces forgotten memories

Recurrent Neural Networks

There are even more complicated models

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- Long Short-Term Memory (LSTM)

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LSTM has 6 different functions! Too complicated to review

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MGU is simpler and performs similarly to LSTM and GRU

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Recall the gradient for the linear recurrence

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Question: What is the gradient for $\operatorname{scan}(f)$ of Elman network?

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All RNNs suffer from either exploding gradient (ReLU) or vanishing gradient (sigmoid). Active area of research!

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Coding

Jax RNN https://colab.research.google.com/drive/147z7FNGyERV8oQ_4gZmxDVdeoNt0hKta#scrollTo=TUMonlJ1u8Va

Homework https://colab.research.google.com/drive/1CNaDxx1yJ4-phyMvgbxECL8ydZYBGQGt?usp=sharing

Makeup lecture Saturday October 26, 13:00-16:00