# Recurrent Neural Networks

## CISC 7026: Introduction to Deep Learning

University of Macau

# Admin

Makeup lecture Saturday October 26, 13:00-16:00

# Agenda

1. Review
2. Sequence Modeling
3. Composite Memory
4. Linear Recurrence
5. Scans
6. Output Modeling
7. Recurrent Loss Functions
8. Backpropagation through Time
9. Recurrent Neural Networks
10. Coding

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# Review

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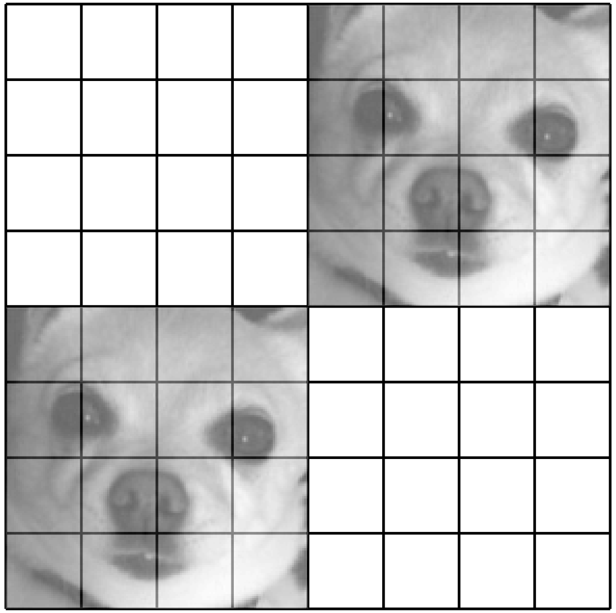

# Review

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It is a miracle that our neural networks could classify clothing!

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In **signal processing**, we analyze the meaning of signals

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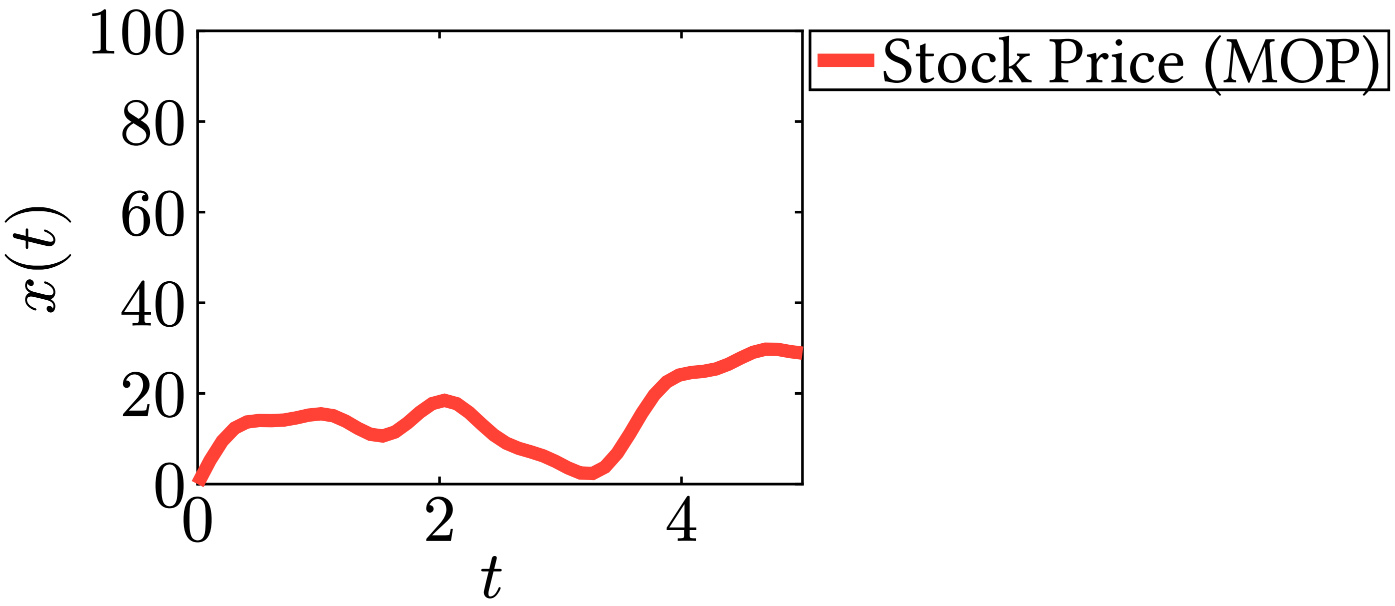

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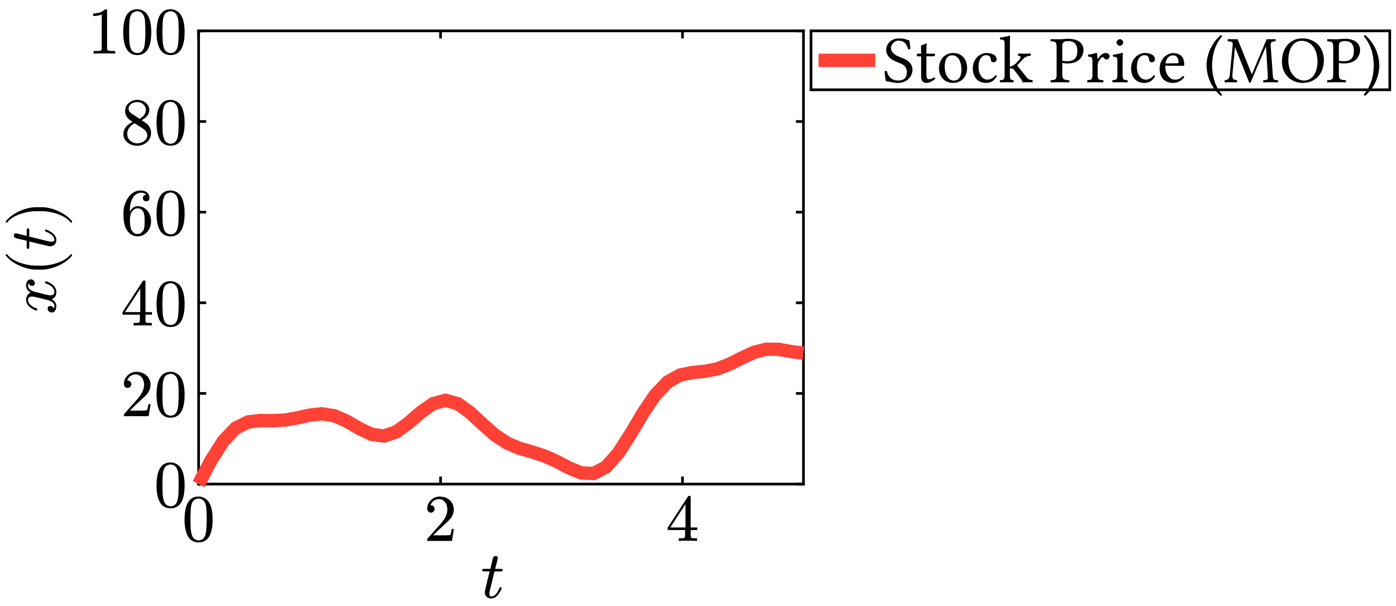


There is an underlying structure to $x(t)$

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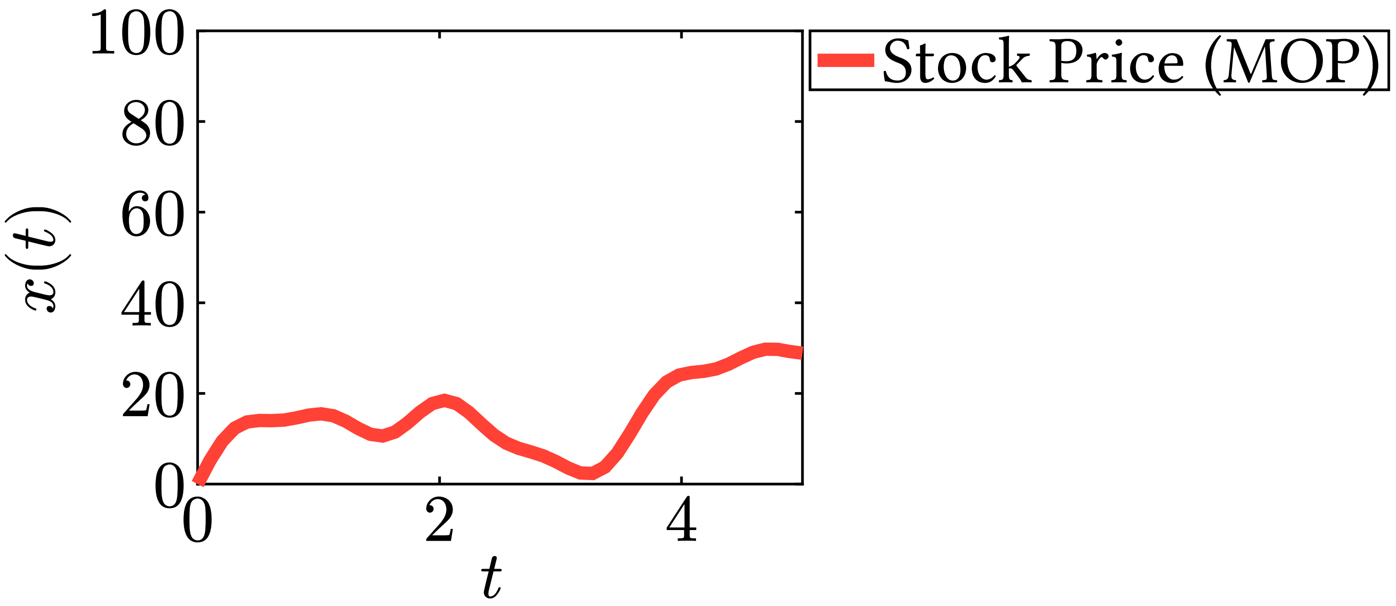


There is an underlying structure to $x(t)$

**Structure:** Tomorrow's stock price will be close to today's stock price

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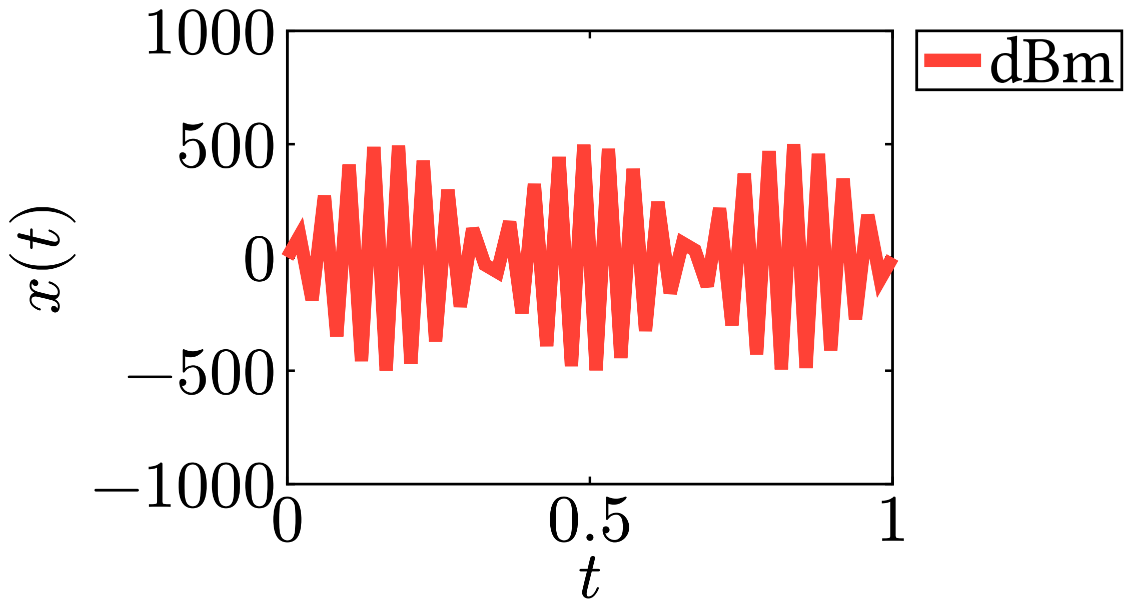

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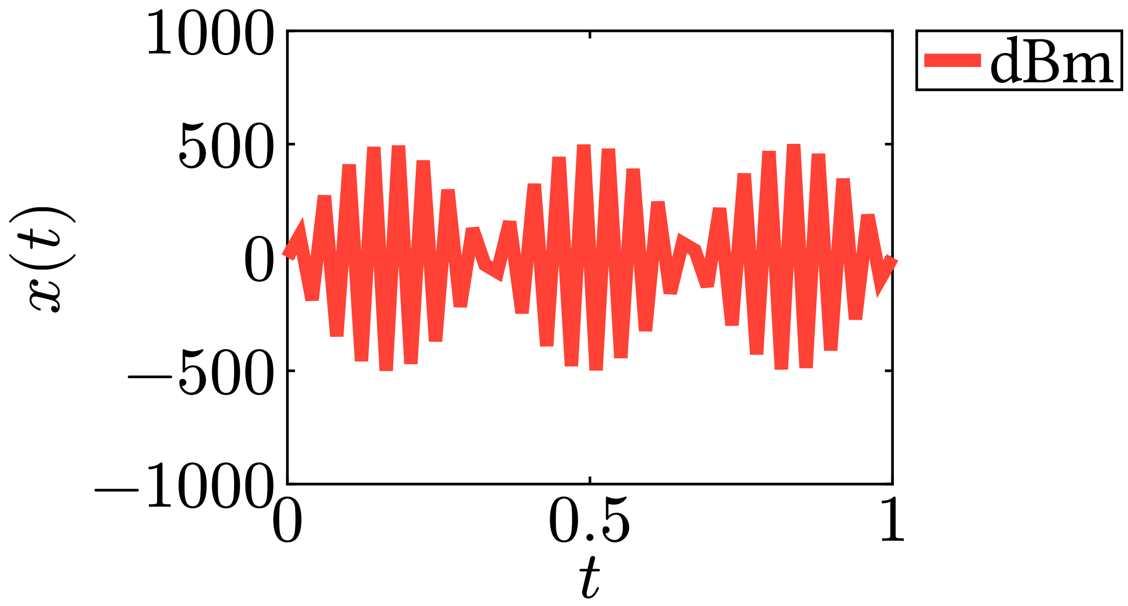


**Structure:** Nearby waves form syllables

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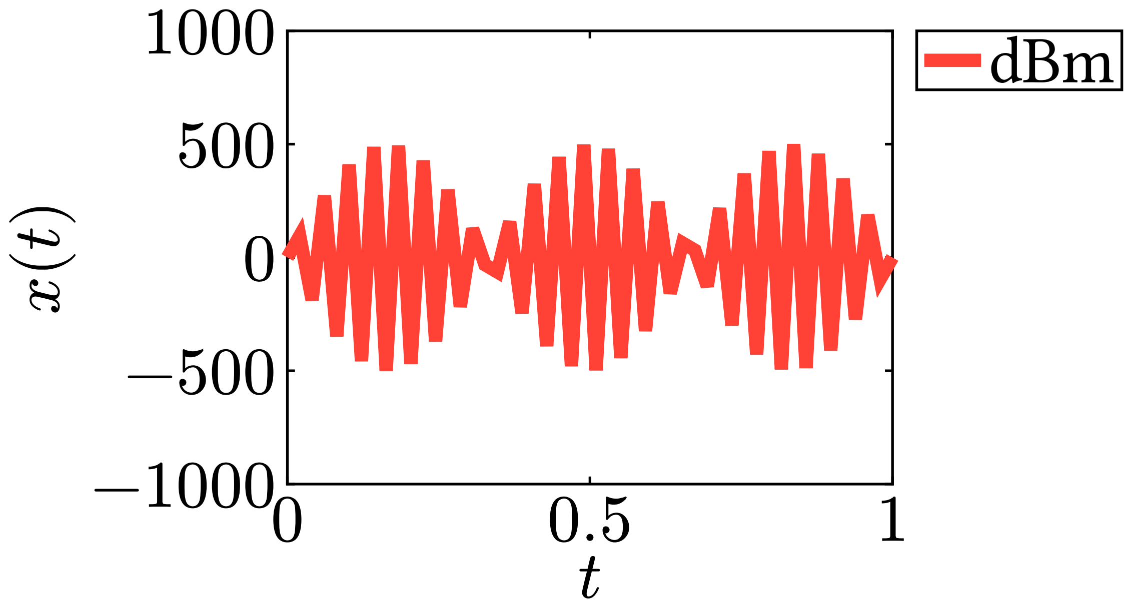


**Structure:** Nearby waves form syllables

**Structure:** Nearby syllables combine to create meaning

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$$x(u, v) = \text{image}$$

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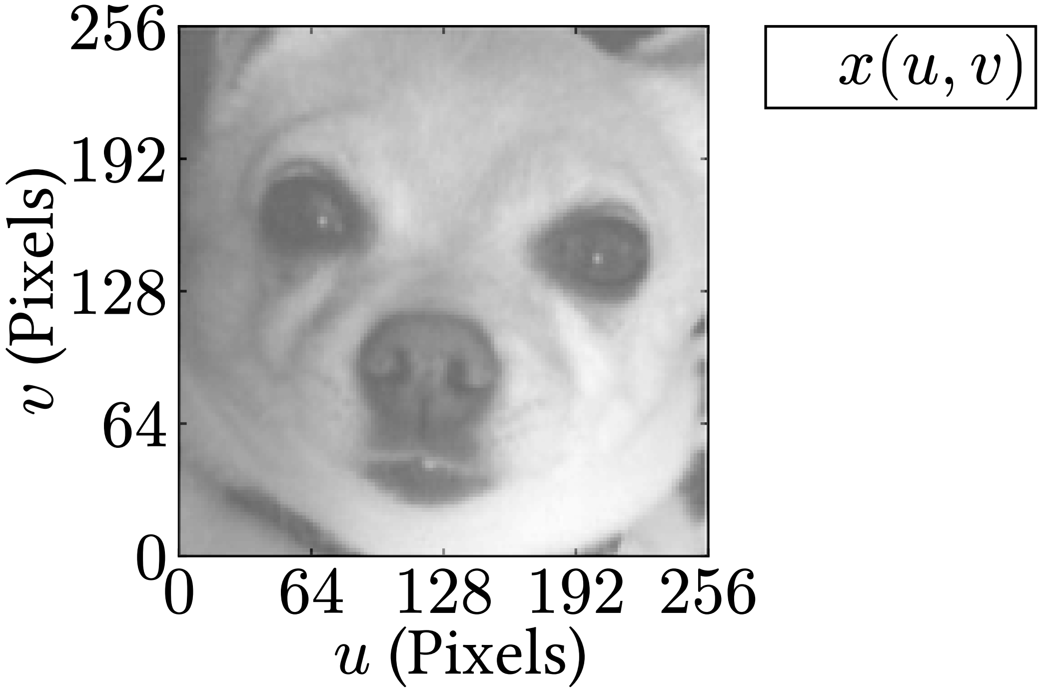

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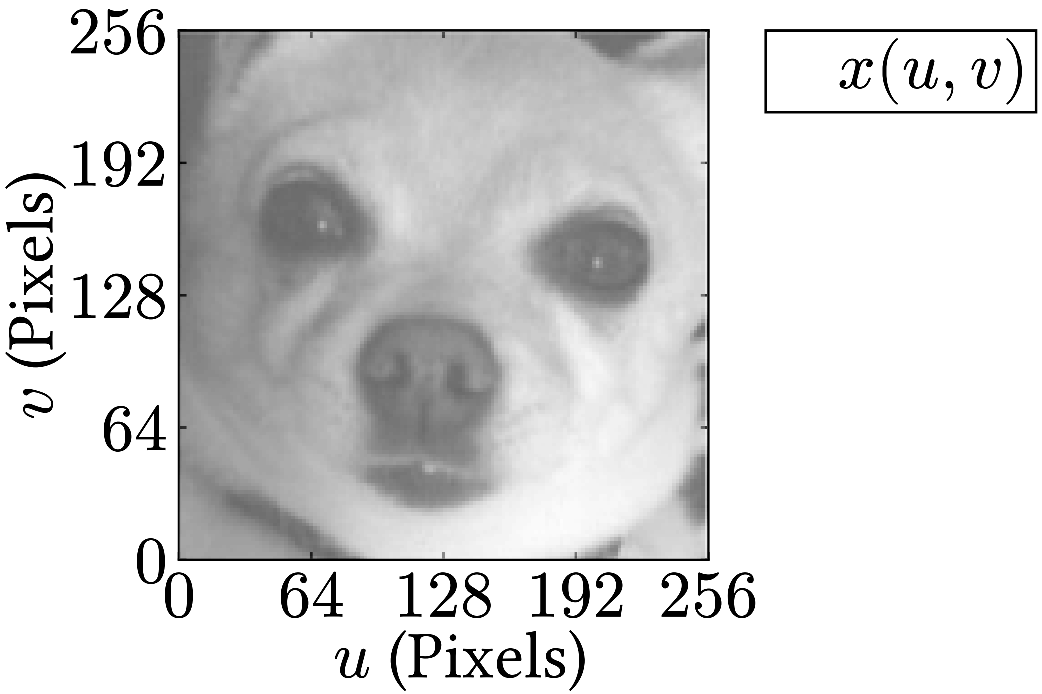


**Structure:** Repeated components (circles, symmetry, eyes, nostrils, etc)

# Review

**Locality:** Information concentrated over small regions of space/time

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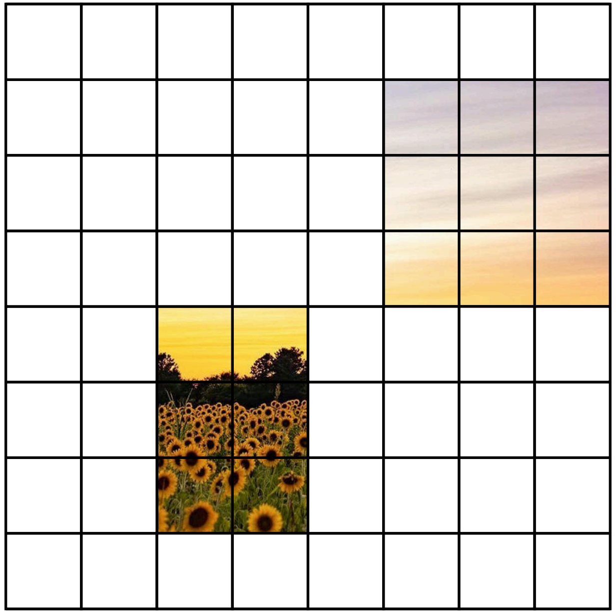
**Translation Equivariance:** Shift in signal results in shift in output

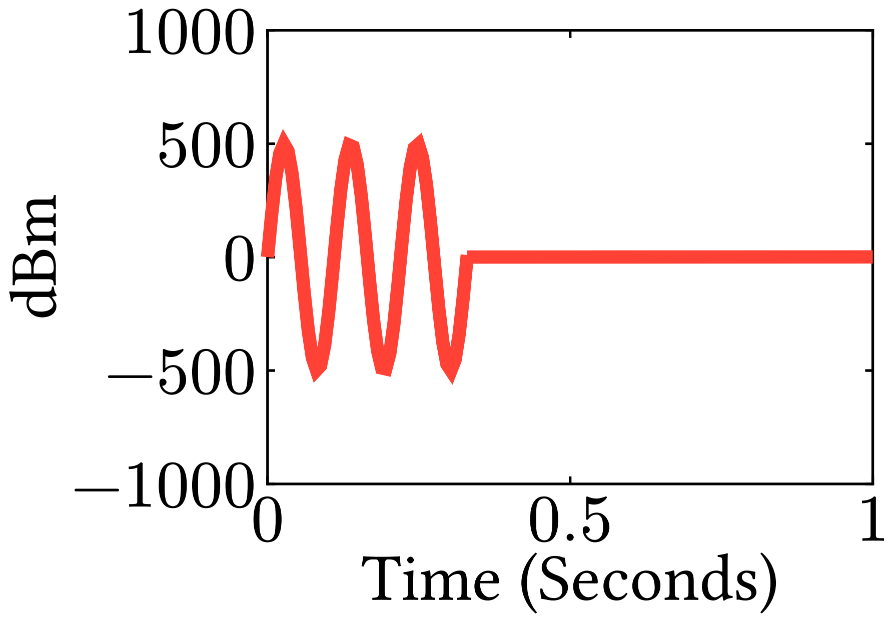
# Review

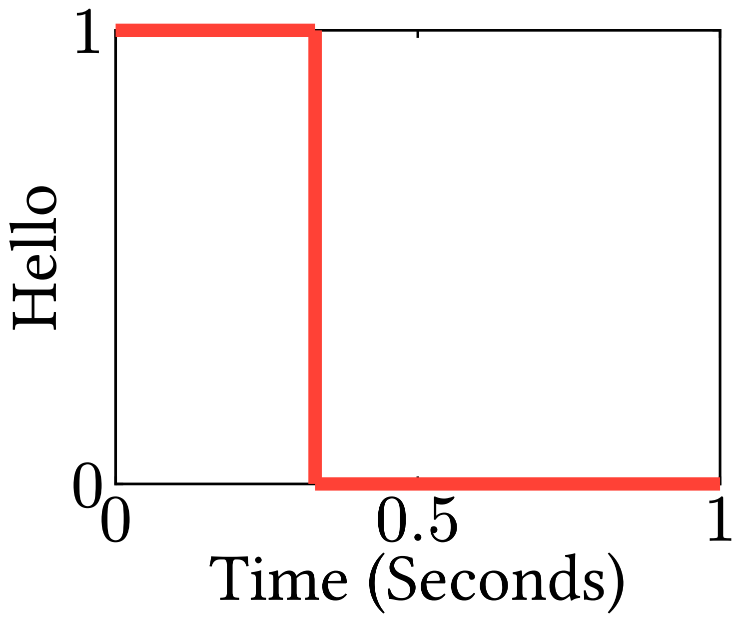
A more realistic scenario of locality and translation equivariance



# Convolution

We use convolution to turn signals into useful signals



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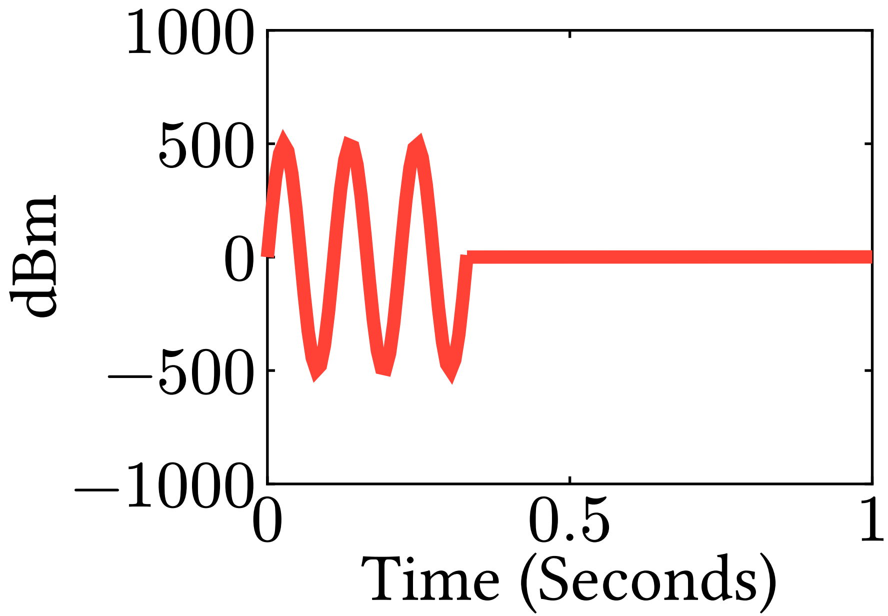


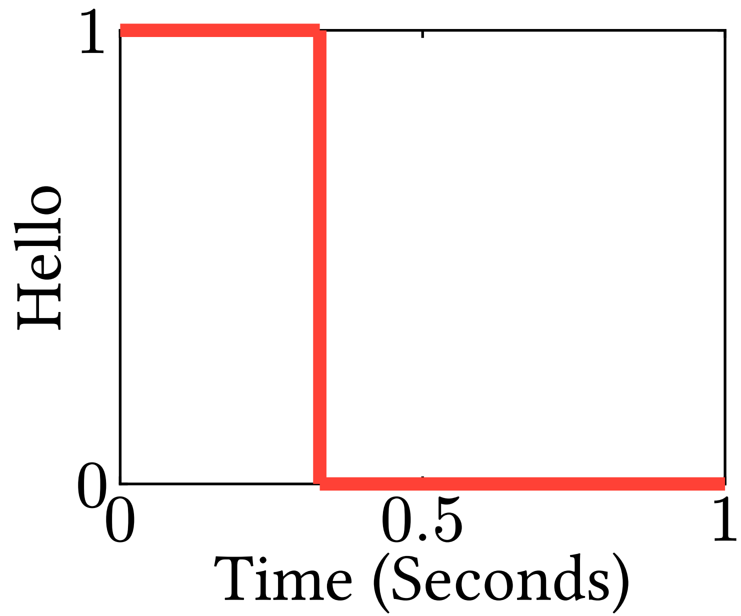


Convolution is translation equivariant and local

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If the t is continuous in $x(t)$

$$x(t) * g(t) = \int_{-\infty}^{\infty} x(t - \tau) g(\tau) d\tau$$

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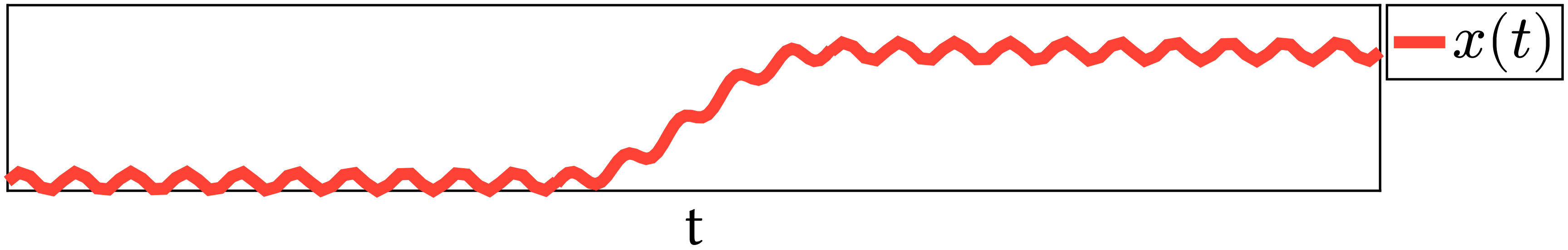
We slide the filter $g(t)$ across the signal $x(t)$

# Convolution



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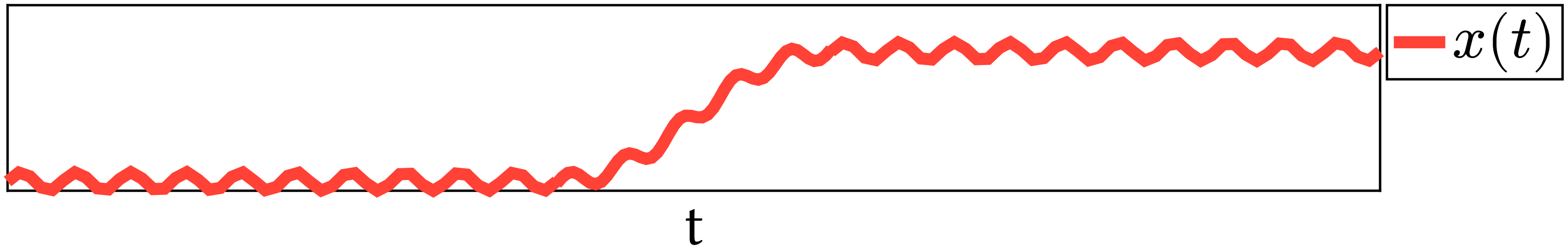


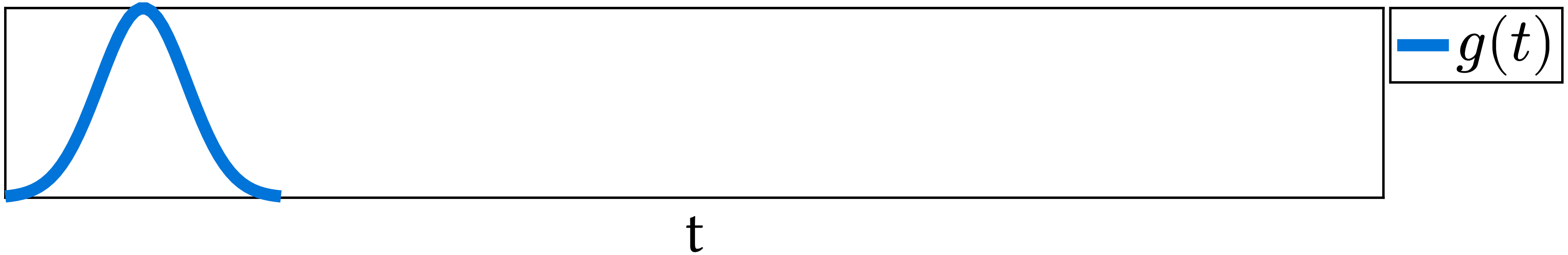

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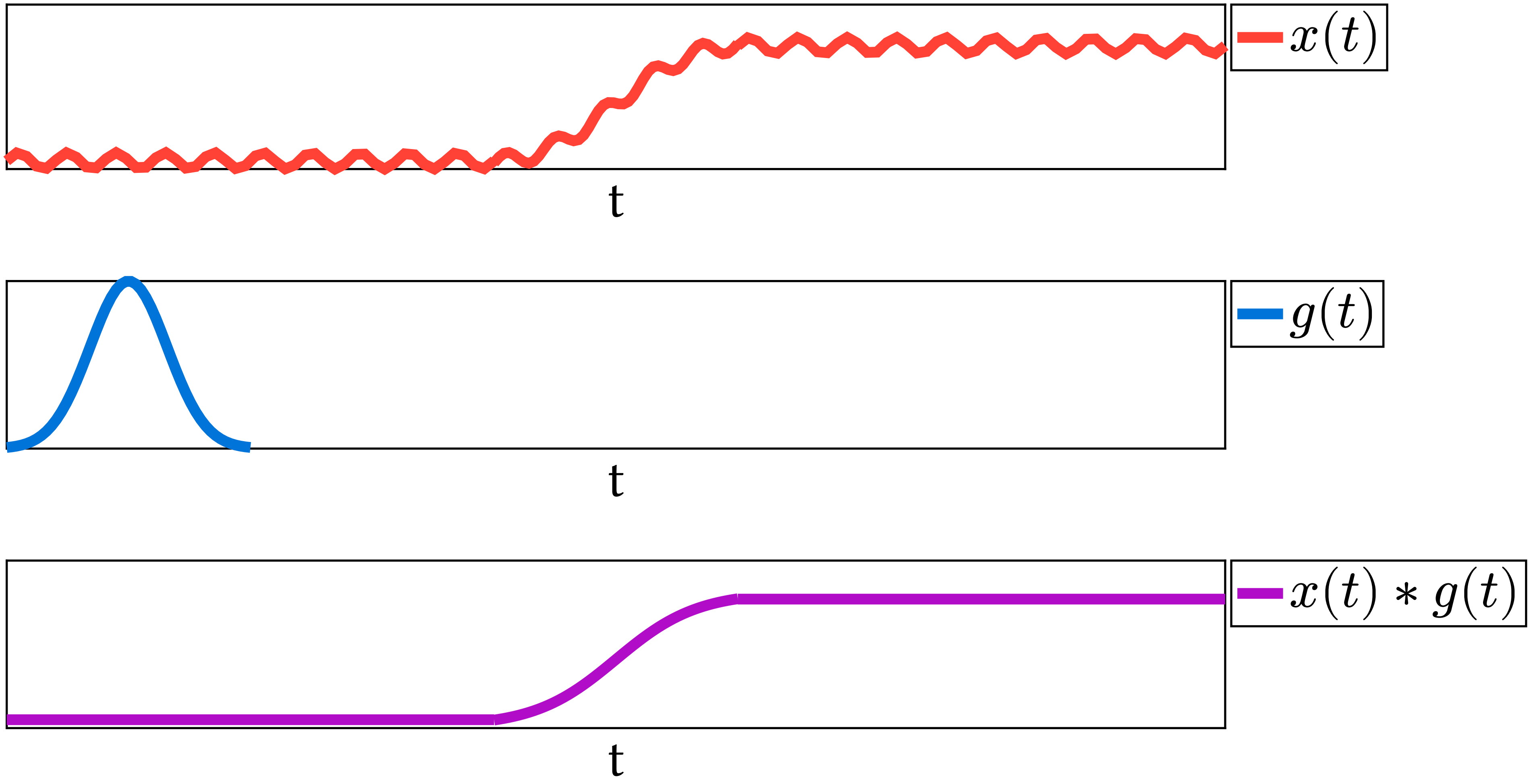

# Convolution

$$\begin{bmatrix} x(t) \\ g(t) \\ y(t) \end{bmatrix} = \begin{bmatrix} 1 & 2 & 3 & 4 & 5 \\ 2 & 1 & & & \\ & & & & \end{bmatrix}$$

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$$\begin{bmatrix} x(t) \\ g(t) \\ y(t) \end{bmatrix} = \begin{bmatrix} \color{red}{1} & \color{red}{2} & 3 & 4 & 5 \\ \color{red}{2} & \color{red}{1} & & & \\ \color{red}{4} & & & & \end{bmatrix}$$

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$$\begin{bmatrix} x(t) \\ g(t) \\ y(t) \end{bmatrix} = \begin{bmatrix} 1 & 2 & 3 & 4 & 5 \\ & 2 & 1 & & \\ 4 & 7 & & & \end{bmatrix}$$

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# Review

We can write both a perceptron and convolution in vector form

$$f(x(t), \boldsymbol{\theta}) = \sigma\left( \boldsymbol{\theta}^\top \begin{bmatrix} 1 \\ x(0.1) \\ x(0.2) \\ \vdots \end{bmatrix} \right)$$

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A convolution layer applies a “mini” perceptron to every few timesteps

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The output size depends on the signal length

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If we want a single output, we should **pool**

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$$z(t) = f(x(t), \boldsymbol{\theta}) = \begin{bmatrix} \sigma\left( \boldsymbol{\theta}^\top \begin{bmatrix} 1 \\ x(0.1) \\ x(0.2) \end{bmatrix} \right) & \sigma\left( \boldsymbol{\theta}^\top \begin{bmatrix} 1 \\ x(0.2) \\ x(0.3) \end{bmatrix} \right) & \dots \end{bmatrix}^\top$$

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$$\mathrm{SumPool}(z(t)) = \sigma\left(\boldsymbol{\theta}^\top \begin{bmatrix} 1 \\ x(0.1) \\ x(0.2) \end{bmatrix}\right) + \sigma\left(\boldsymbol{\theta}^\top \begin{bmatrix} 1 \\ x(0.2) \\ x(0.3) \end{bmatrix}\right) + \dots$$

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$$\mathrm{MeanPool}(z(t)) = \frac{1}{T - k + 1} \mathrm{SumPool}(z(t)); \quad \mathrm{MaxPool}(z(t)) = \max(z(t))$$

# Review

$$\begin{bmatrix}0&1&1&1&1&1&0&1\\0&1&1&1&1&1&1&1\\1&0&0&1&1&0&0&1\\1&0&0&1&1&0&0&1\\1&1&0&0&0&1&1&1\\1&1&0&0&0&1&1&1\\0&1&0&0&0&1&1&0\\1&0&1&1&1&1&1&1\end{bmatrix} * \begin{bmatrix}2&0\\0&1\end{bmatrix} + \begin{bmatrix}0&1&1&1&1&1&0&1\\0&1&1&1&1&1&1&1\\1&0&0&1&1&0&0&1\\1&0&0&1&1&0&0&1\\1&1&0&0&0&1&1&1\\1&1&0&0&0&1&1&1\\0&1&0&0&0&1&1&0\\1&0&1&1&1&1&1&1\end{bmatrix} * \begin{bmatrix}1&1\\0&1\end{bmatrix} + \begin{bmatrix}0&1&1&1&1&1&0&1\\0&1&1&1&1&1&1&1\\1&0&0&1&1&0&0&1\\1&0&0&1&1&0&0&1\\1&1&0&0&0&1&1&1\\1&1&0&0&0&1&1&1\\0&1&0&0&0&1&1&0\\1&0&1&1&1&1&1&1\end{bmatrix} * \begin{bmatrix}1&0\\0&0\end{bmatrix}$$

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This can decrease the size of image without pooling

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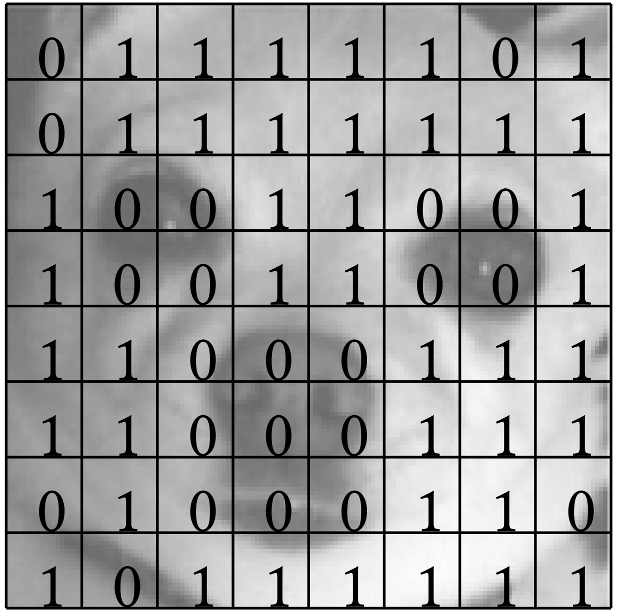
**Padding** adds zero pixels to the image to increase the output size




$$*$$

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

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You can solve temporal tasks using either convolution or RNNs

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Equivariance and locality make learning more efficient, but not all problems have this structure

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**Example 1:** You like dinosaurs as a child, you grow up and study dinosaurs for work

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# Sequence Modeling

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**Answer:** No! (1) gunshot, (2) see runners, enjoy the race. (1) see runners, (2) hear gunshot, you start running too!

**Question:** Any other examples?

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Humans experience time and process temporal data

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Humans experience time and process temporal data

Can we design a neural network based on human perceptions of time?

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# Composite Memory

How do humans experience time?

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Humans create memories

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How do humans experience time?

Humans create memories

We experience time when we reason over our memories

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If all your memories were erased, you would be a different person

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If all your memories were erased, you would be a different person

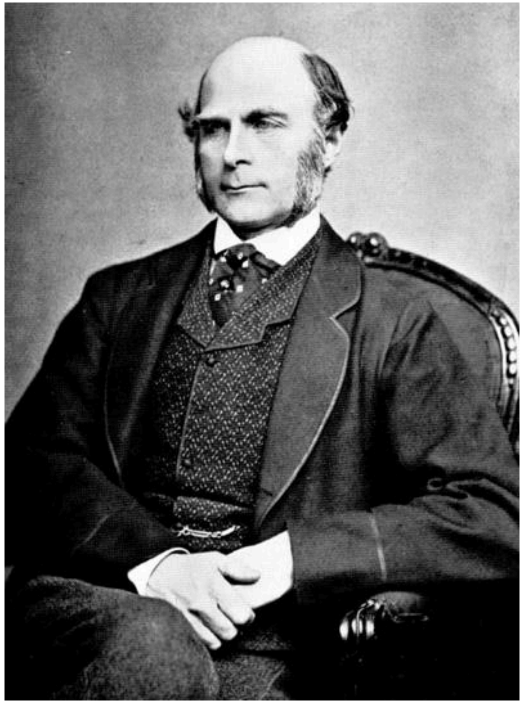
Without the ability to reason over memories, we would simply react to stimuli like bacteria

# Composite Memory

So how do we model memory in humans?

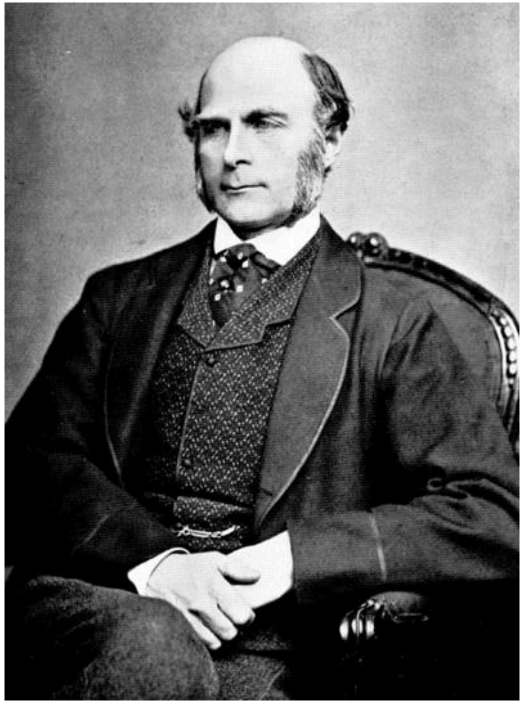
# Composite Memory

Francis Galton (1822-1911),
composite memory

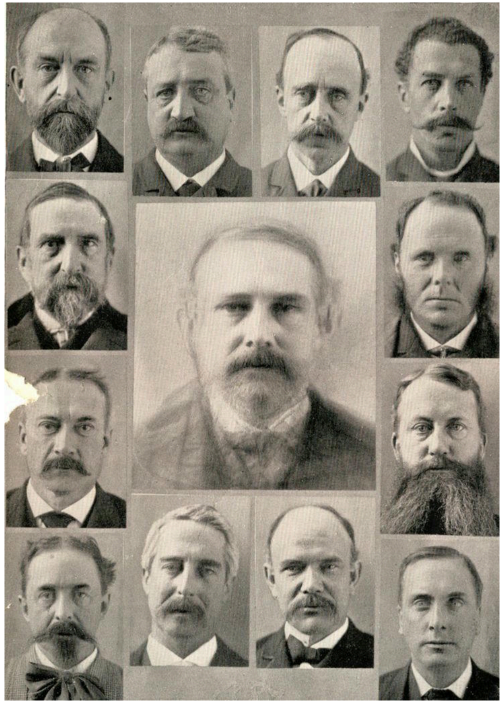


# Composite Memory

Francis Galton (1822-1911), composite memory



Composite photo of members of a party



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$$f : X \times \Theta \mapsto Y$$

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$$f : X \times \Theta \mapsto Y$$

$$f(\boldsymbol{x}, \boldsymbol{\theta}) = \sum_{i=1}^{T} \boldsymbol{\theta}^\top \overline{\boldsymbol{x}}_i$$

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# Composite Memory

$$f(\boldsymbol{x}, \boldsymbol{\theta}) = \sum_{i=1}^{T} \boldsymbol{\theta}^\top \overline{\boldsymbol{x}}_i$$

**Question:** What if we see a new face?

$$f(\boldsymbol{x}, \boldsymbol{\theta}) = \left( \sum_{i=1}^{T} \boldsymbol{\theta}^\top \overline{\boldsymbol{x}}_i \right) + \boldsymbol{\theta}^\top \overline{\boldsymbol{x}}_{\text{new}}$$

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3. **Composite Memory**
4. Linear Recurrence
5. Scans
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**Question:** What is the meaning of $\boldsymbol{h}$ in humans?

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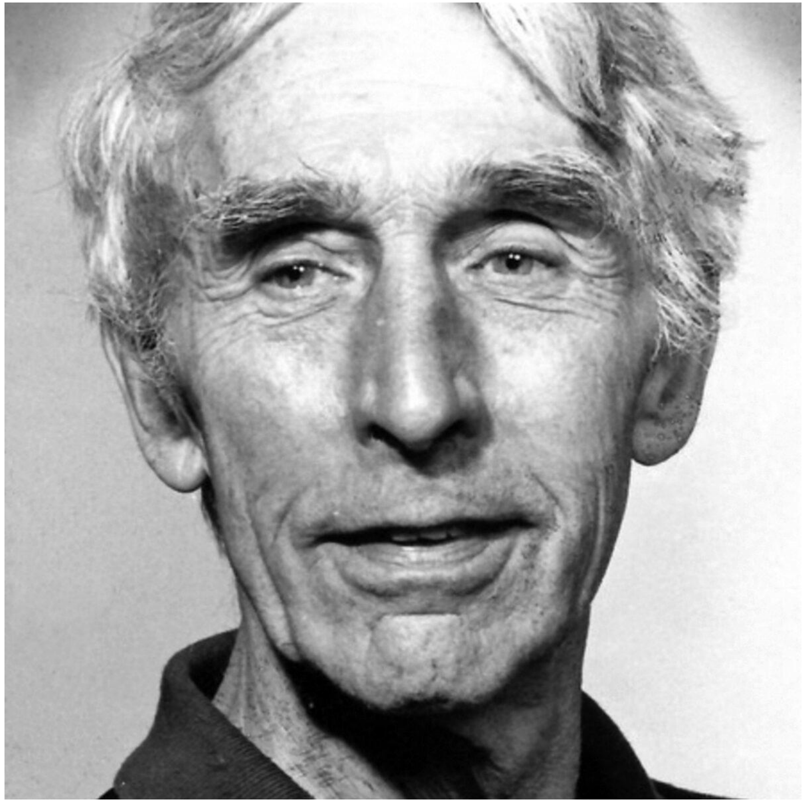

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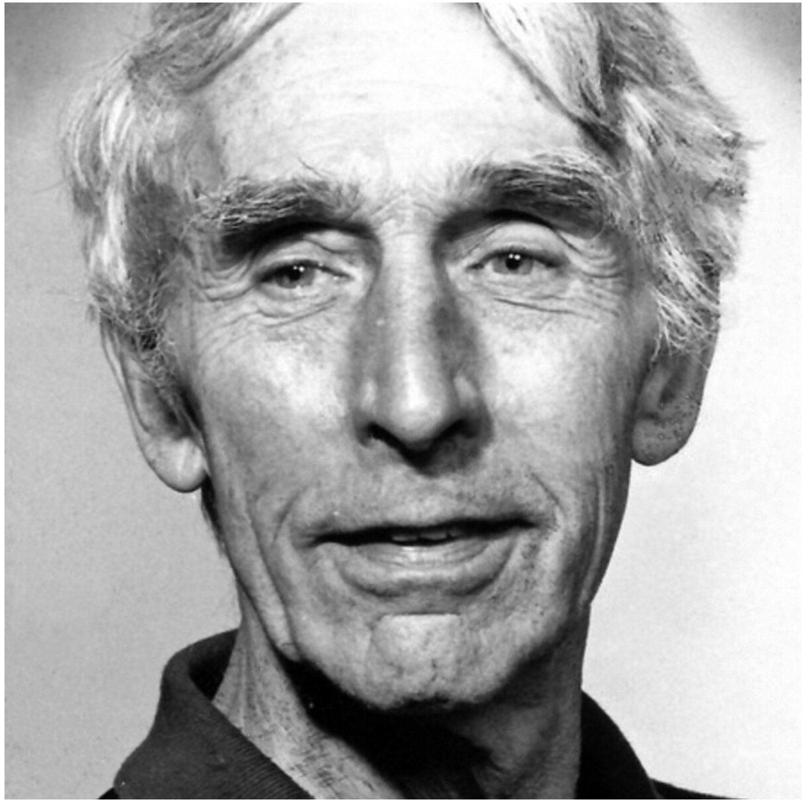

$$f(\boldsymbol{h}, \boldsymbol{x}, \boldsymbol{\theta}) = \gamma \boldsymbol{h} + \boldsymbol{\theta}^\top \overline{\boldsymbol{x}}; \quad 0 < \gamma < 1$$

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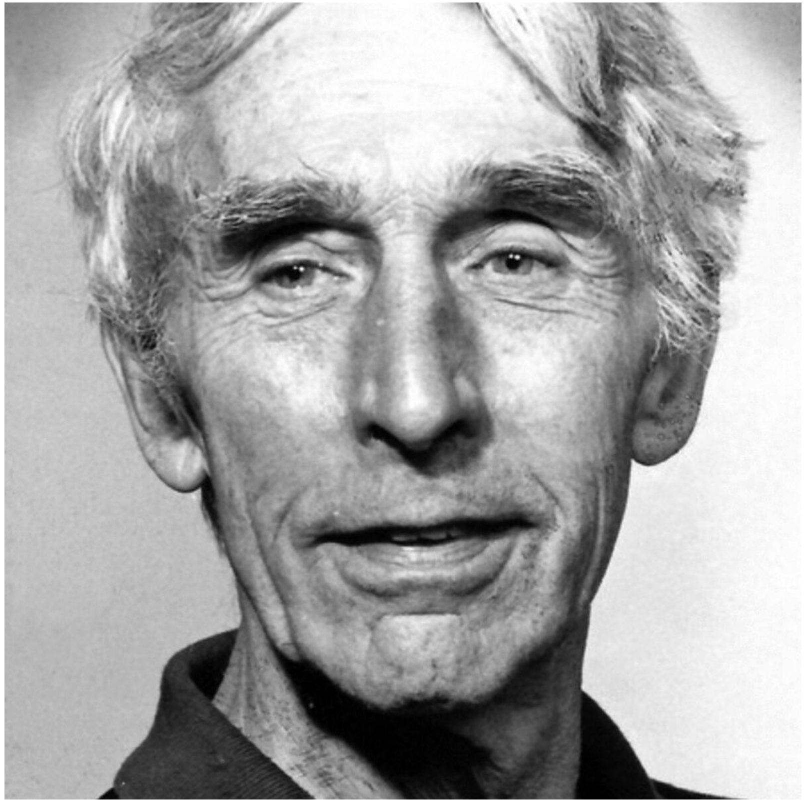

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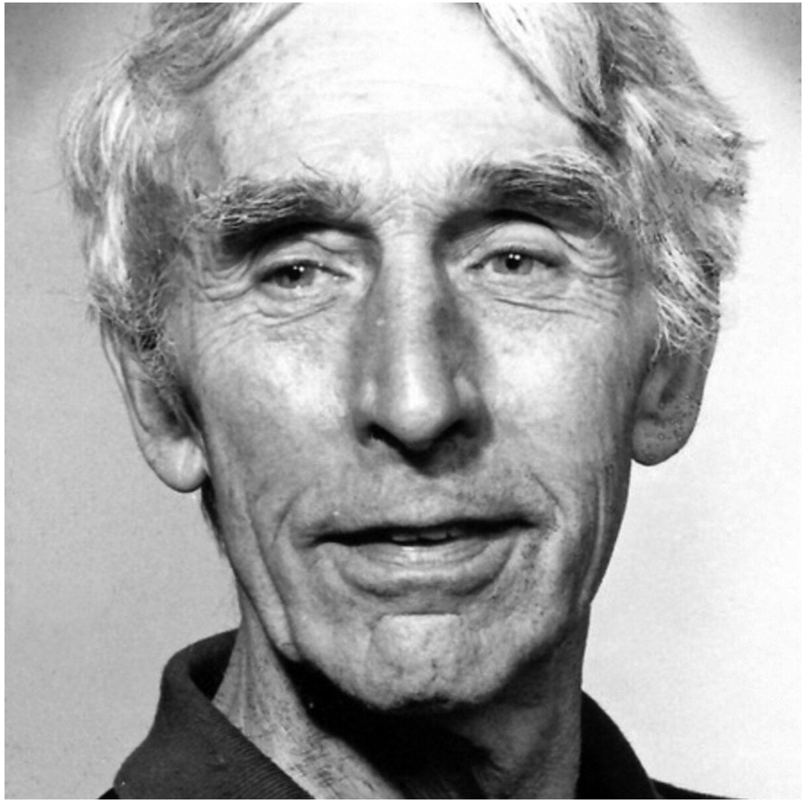

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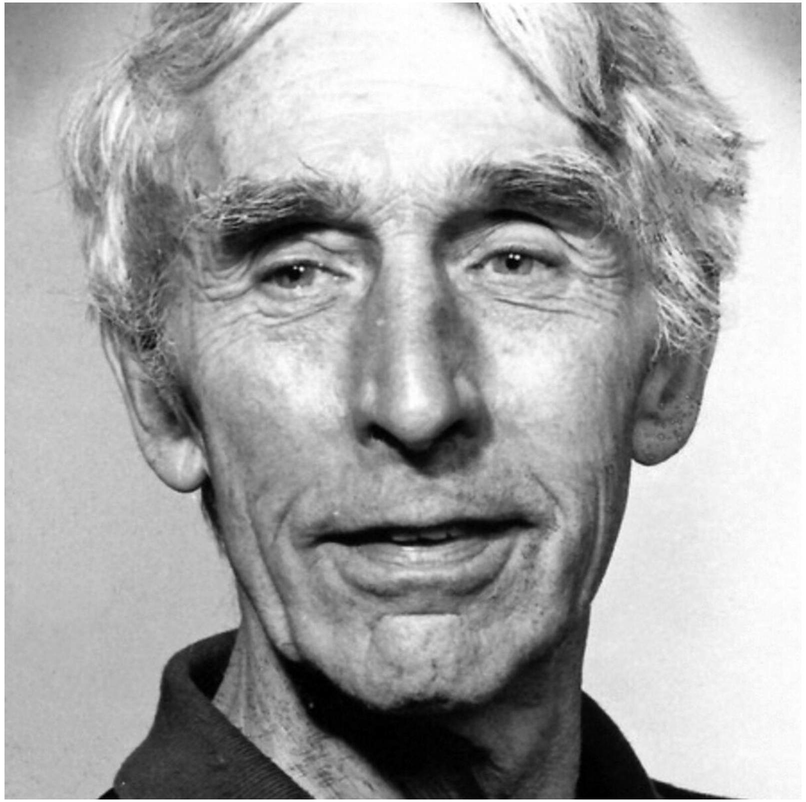

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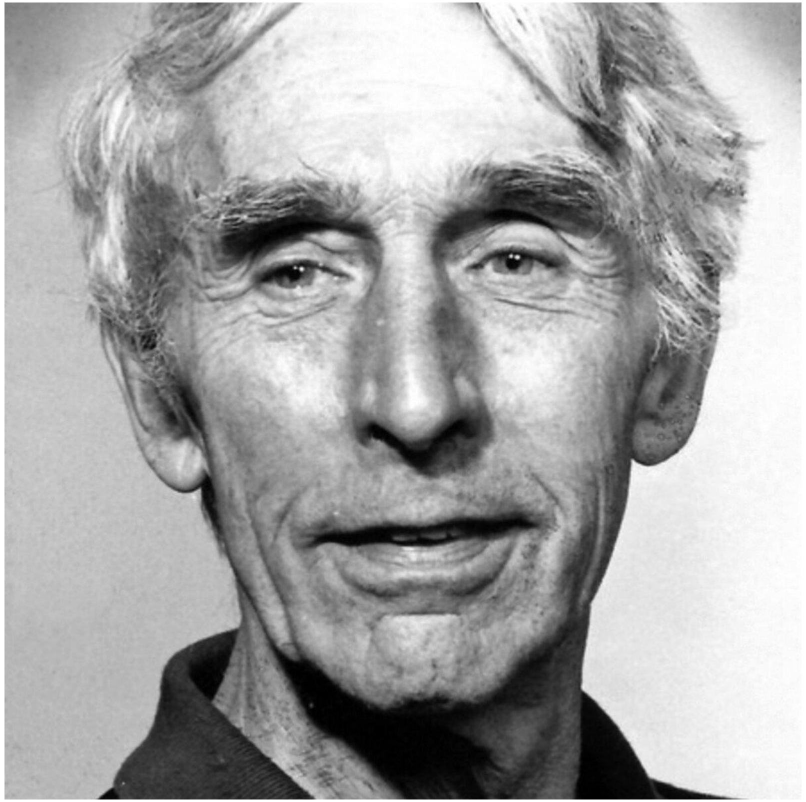

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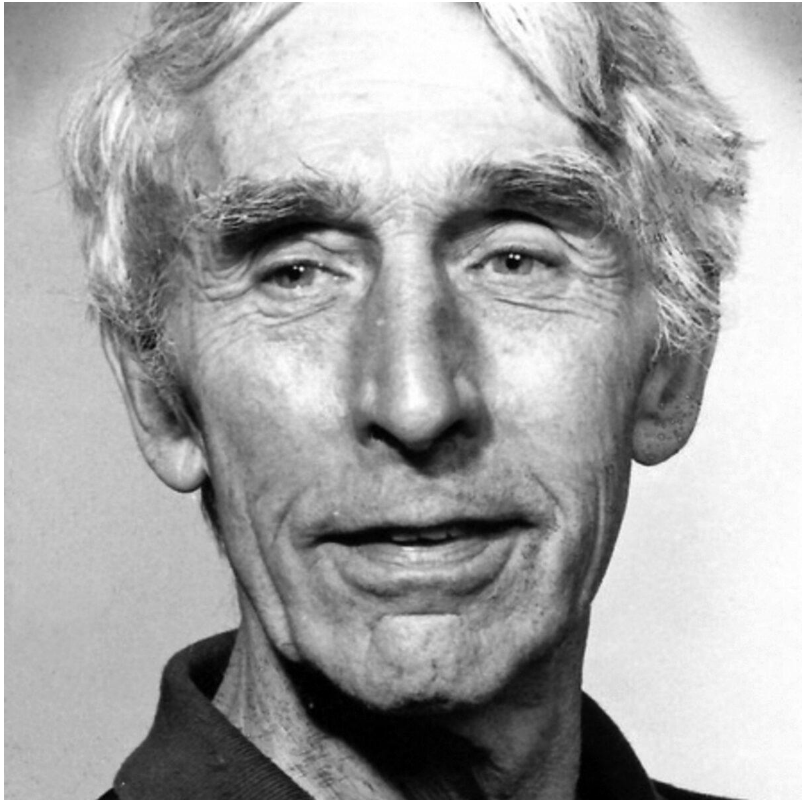

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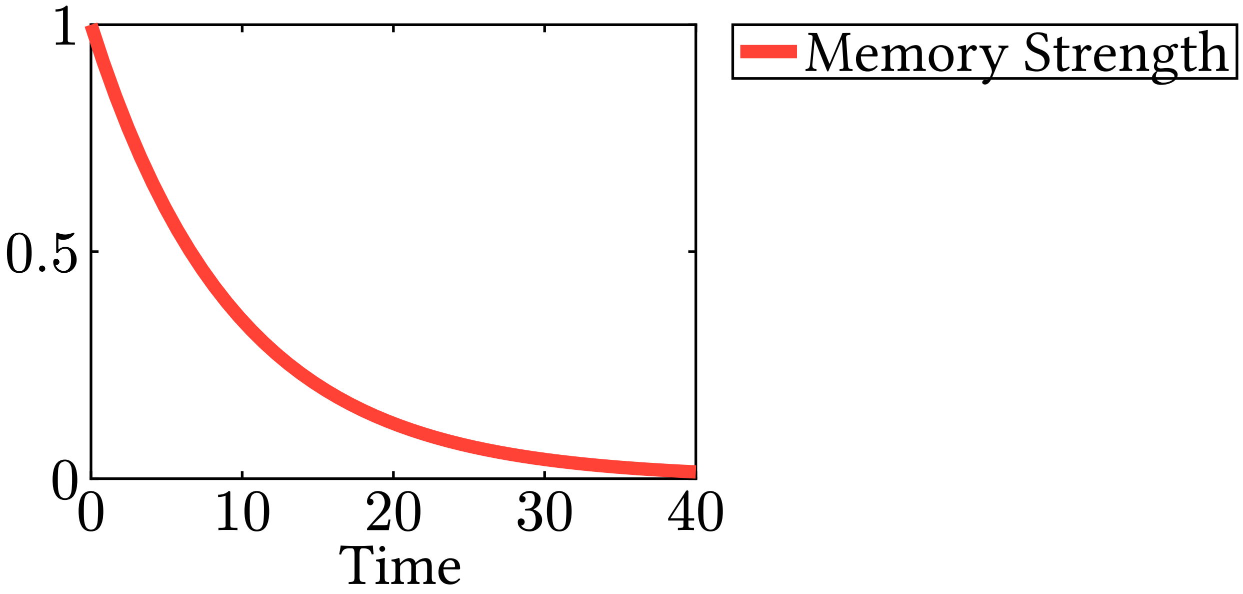

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We use an algebraic operation called a **scan**

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$$\text{scan}(f)\left(\boldsymbol{h}_0, \begin{bmatrix} \boldsymbol{x}_1 \\ \vdots \\ \boldsymbol{x}_T \end{bmatrix}, \boldsymbol{\theta}\right) = \begin{bmatrix} f(h_0, x_1, \boldsymbol{\theta}) \\ f(h_1, x_2, \boldsymbol{\theta}) \\ \vdots \\ f(h_{T-1}, x_T, \boldsymbol{\theta}) \end{bmatrix} = \begin{bmatrix} f(h_0, x_1) \\ f(f(h_0, x_1), x_2) \\ \vdots \\ f(\ldots f(h_0, x_1) \ldots, x_T) \end{bmatrix}$$

# Scans

```python
import jax
import jax.numpy as jnp

T, d_h = 10, 4
xs, h0 = jnp.ones((T, d_h)), jnp.zeros((d_h,))
theta = (jnp.ones((d_h,)), jnp.ones((d_h, d_h))) # (b, W)

def f(h, x):
    b, W = theta
    result = h + W.T @ x + b
    return result, result # return one, return all

_, hs = jax.lax.scan(f, init=h0, xs=xs) # Scan f over x
```

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```
def scan(f, h, xs):
    # h shape is (d_h,)
    # xs shape is (T, d_x)
    hs = []
    for x in xs:
        h = f(h, x, theta)
        hs.append(h)
    # output shape is (T, d_h)
    return hs
```

# Scans

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import torch
T, d_h = 10, 4
xs, h0 = torch.ones((T, d_h)), torch.zeros((d_h,))
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    return result # h

hs = scan(f, h0, xs)
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OpenAI, Google, etc are currently experimenting with LLMs that use **associative scans**

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**Question:** Does $f(\boldsymbol{h}, \boldsymbol{x}, \boldsymbol{\theta}) = \gamma \boldsymbol{h} + \boldsymbol{\theta}^\top \overline{\boldsymbol{x}}$ obey the associative property?

**Answer:** Yes, linear operations obey the associative property

# Agenda

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2. Sequence Modeling
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# Output Modeling

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Now, combine $f$ and $g$

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$$\begin{bmatrix} \boldsymbol{y}_1 \\ \vdots \\ \boldsymbol{y}_T \end{bmatrix} = \begin{bmatrix} g(\boldsymbol{h}_1, \boldsymbol{x}_1, \boldsymbol{\theta}_g) \\ \vdots \\ g(\boldsymbol{h}_T, \boldsymbol{x}_T, \boldsymbol{\theta}_g) \end{bmatrix}$$

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Questions? This is on the homework

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# Recurrent Loss Functions

Let us examine some example tasks:

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- Explaining a video

# Recurrent Loss Functions

**Task:** Clock – keep track of time

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$$X \in \{0, 1\}^2, \quad Y \in \mathbb{R}^2$$

# Recurrent Loss Functions

Example input sequence:

$$\begin{bmatrix} 0 & 1 \\ 0 & 1 \\ \vdots & \vdots \\ 1 & 1 \end{bmatrix}$$

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Desired output sequence

$$\begin{bmatrix} 0 & 1 \\ 0 & 2 \\ \vdots & \vdots \\ m & s \end{bmatrix}$$

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Onto the next task

# Recurrent Loss Functions

**Task:** Watch a video, then explain it to me

$$X \in \mathbb{Z}^{3 \times 32 \times 32}, \quad Y \in \{\text{comedy show}, \text{action movie}, \dots\}$$

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Example input sequence: $\begin{bmatrix} I_1 \\ I_2 \\ \vdots \\ I_T \end{bmatrix}$

Example output: "dancing dog"

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Example input sequence: $\begin{bmatrix} I_1 \\ I_2 \\ \vdots \\ I_T \end{bmatrix}$

Example output: "dancing dog"

Unlike before, we have many inputs but just one output!

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We only care about the $\boldsymbol{h}_T$

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**Question:** What is $\nabla_{\boldsymbol{\theta}} f$?

$$\nabla_{\boldsymbol{\theta}} f(\boldsymbol{h}, \boldsymbol{x}, \boldsymbol{\theta}) = \overline{\boldsymbol{x}}^\top$$

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**Question:** What is $\nabla_{\boldsymbol{\theta}} \operatorname{scan}(f)$?

**Hint:** Chain rule

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# Recurrent Neural Networks

If $f$ is a neural network, then we have a recurrent neural network

# Recurrent Neural Networks

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**Question:** Anything missing from our linear model?

**Answer:** Forgetting!

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Add forgetting

$$f_{\text{forget}}(\boldsymbol{h}, \boldsymbol{x}, \boldsymbol{\theta}) = \sigma\left(\boldsymbol{\theta}_1^\top \overline{\boldsymbol{x}} + \boldsymbol{\theta}_2^\top \overline{\boldsymbol{h}}\right)$$

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When $f_{\text{forget} < 1}$, we forget!

**Minimal gated unit** (MGU)

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$$f_2(\boldsymbol{h}, \boldsymbol{x}, \boldsymbol{\theta}) = \sigma\left(\boldsymbol{\theta}_3^\top \overline{\boldsymbol{x}} + \boldsymbol{\theta}_4^\top \left(f_{\text{forget}}(\boldsymbol{h}, \boldsymbol{x}, \boldsymbol{\theta}) \odot \boldsymbol{h}\right)\right)$$

$$f(\boldsymbol{h}, \boldsymbol{x}, \boldsymbol{\theta}) = f_{\text{forget}}(\boldsymbol{h}, \boldsymbol{x}, \boldsymbol{\theta}) \odot \boldsymbol{h} + \left(1 - f_{\text{forget}}(\boldsymbol{h}, \boldsymbol{x}, \boldsymbol{\theta})\right) \odot f_2(\boldsymbol{h}, \boldsymbol{x}, \boldsymbol{\theta})$$

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Left term forgets old, right term replaces forgotten memories

# Recurrent Neural Networks

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LSTM has 6 different functions! Too complicated to review

# Recurrent Neural Networks

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Elman network $f$: $$f(\boldsymbol{h}, \boldsymbol{x}, \boldsymbol{\theta}) = \sigma\left(\boldsymbol{\theta}_1^\top \overline{\boldsymbol{h}} + \boldsymbol{\theta}_2^\top \overline{\boldsymbol{x}}\right)$$

**Question:** What is the gradient for scan$(f)$?

# Recurrent Neural Networks

Elman network $f$: $$f(\boldsymbol{h}, \boldsymbol{x}, \boldsymbol{\theta}) = \sigma\left(\boldsymbol{\theta}_1^\top \overline{\boldsymbol{h}} + \boldsymbol{\theta}_2^\top \overline{\boldsymbol{x}}\right)$$

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$$\nabla[\sigma] \cdot \nabla[\sigma] \cdot \ldots = 0$$

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All RNNs suffer from either exploding gradient (ReLU) or vanishing gradient (sigmoid). Active area of research!

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2. Sequence Modeling
3. Composite Memory
4. Linear Recurrence
5. Scans
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7. Recurrent Loss Functions
8. **Backpropagation through Time**
9. Recurrent Neural Networks
10. Coding

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# Recurrent Neural Networks

Jax RNN https://colab.research.google.com/drive/147z7FNGyERV8oQ_4gZmxDVdeoNt0hKta#scrollTo=TUMonlJ1u8Va

Homework https://colab.research.google.com/drive/1CNaDxx1yJ4-phyMvgbxECL8ydZYBGQGt?usp=sharing

Makeup lecture Saturday October 26, 13:00-16:00