Reinforcement Learning

CISC 7026 - Introduction to Deep Learning

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University of Macau

Admin

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The final assignment is just an assignment

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Your lowest assignment score will not count

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Your lowest assignment score will not count

If you already have 100% on all other assignments, the final project will not change your score

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Lecture Goal: Provide a proper understanding of the theoretical foundations of reinforcement learning

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Lecture Goal: Give you enough information to begin learning RL on your own

What is RL?

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How does reinforcement learning (RL) differ from supervised or unsupervised learning?

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$$f^{-1}(f(\boldsymbol{x}, \boldsymbol{\theta}), \boldsymbol{\theta}) = \boldsymbol{x}$$

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What does this mean?

What is RL?

Example: You train a model f to play chess

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$$f : X \times \Theta \mapsto Y$$

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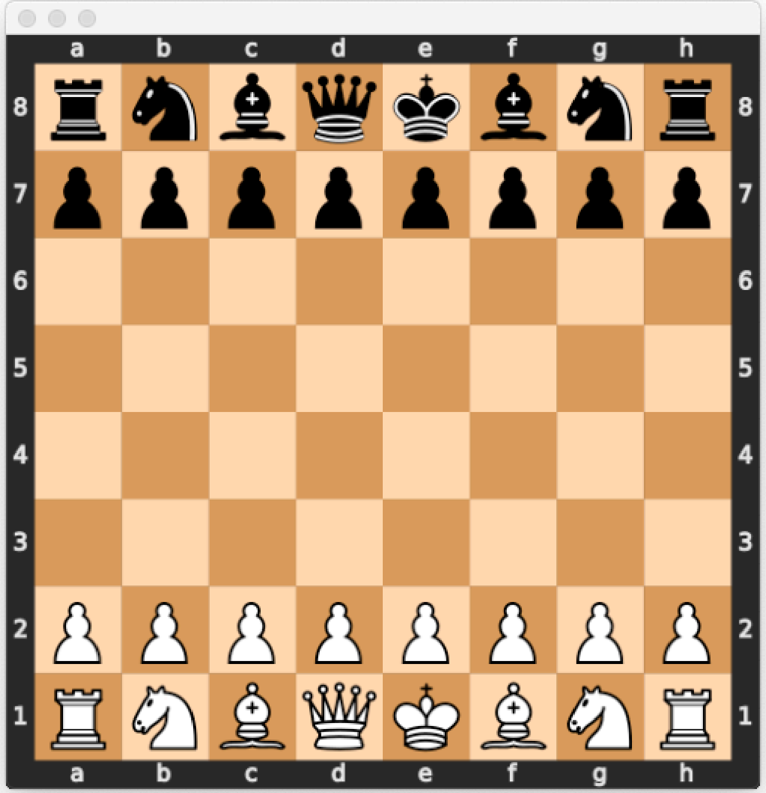

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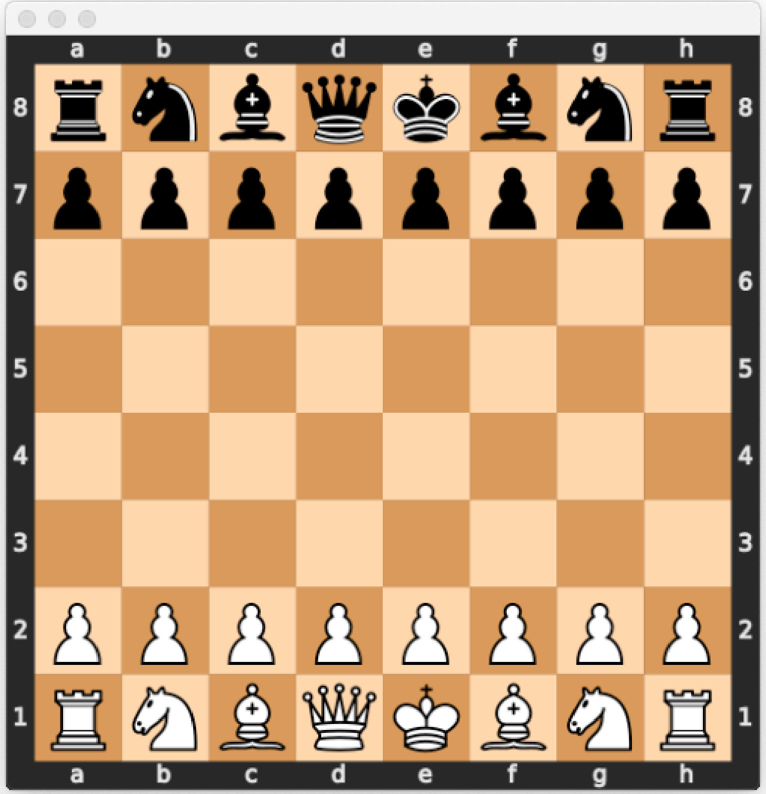

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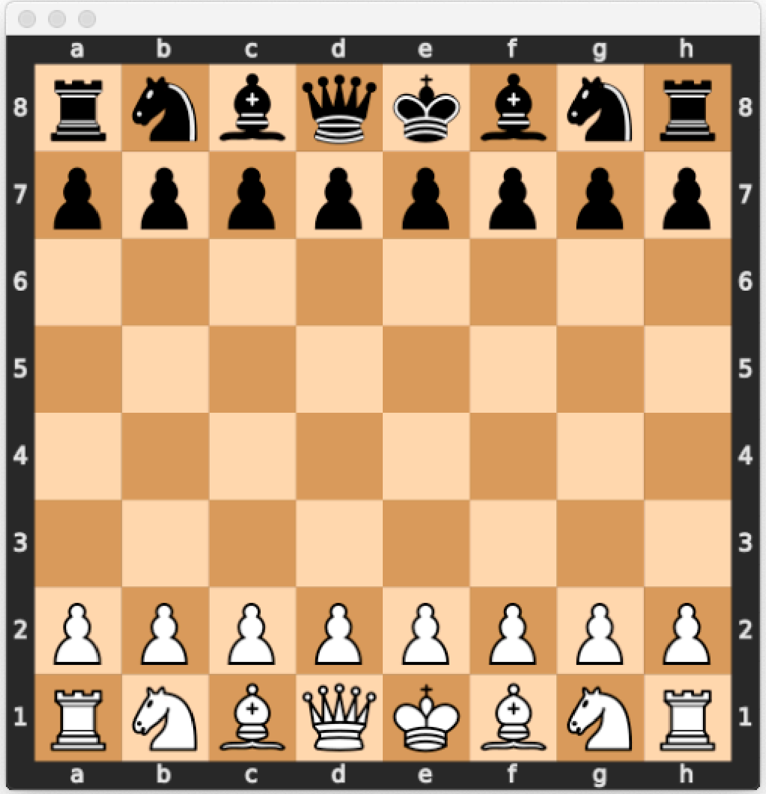

What is the correct answer?

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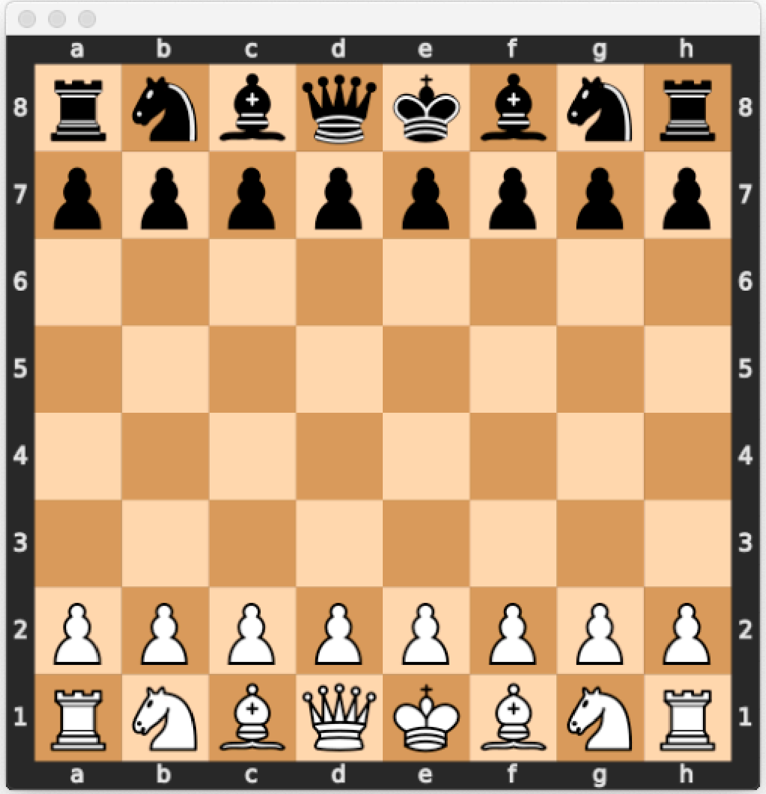

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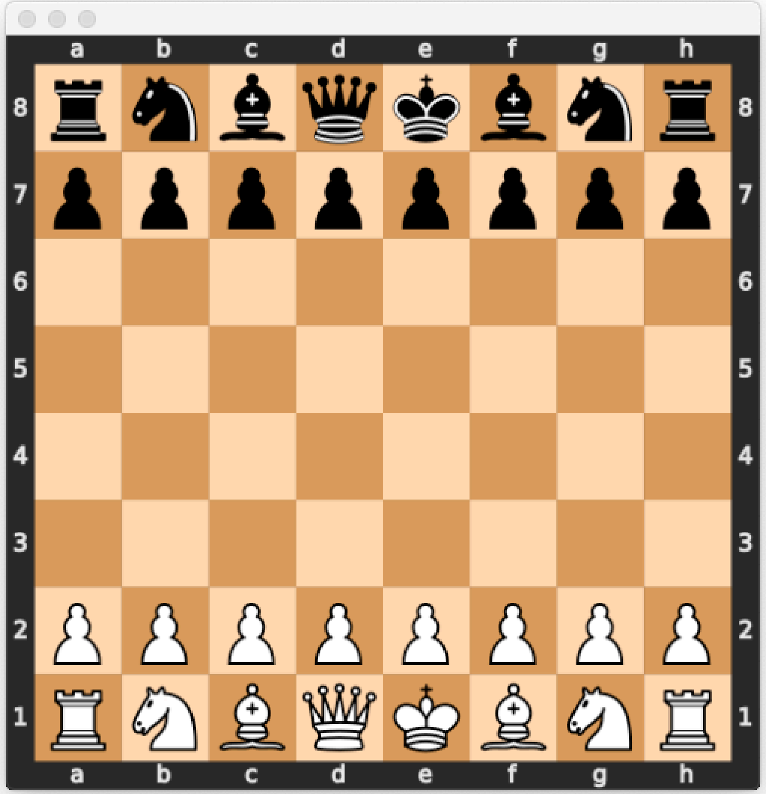

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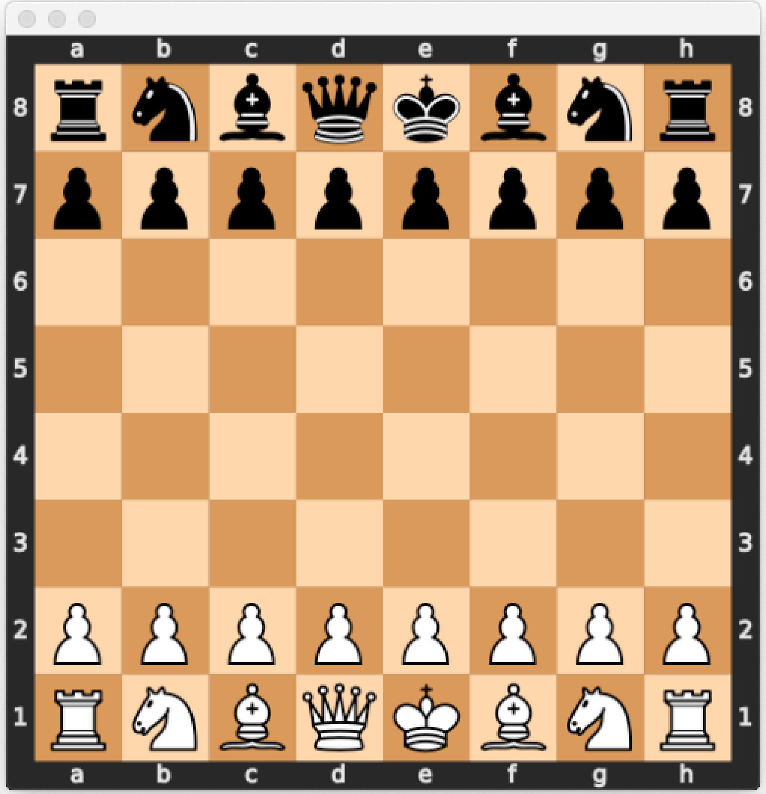

No answer, no supervised learning

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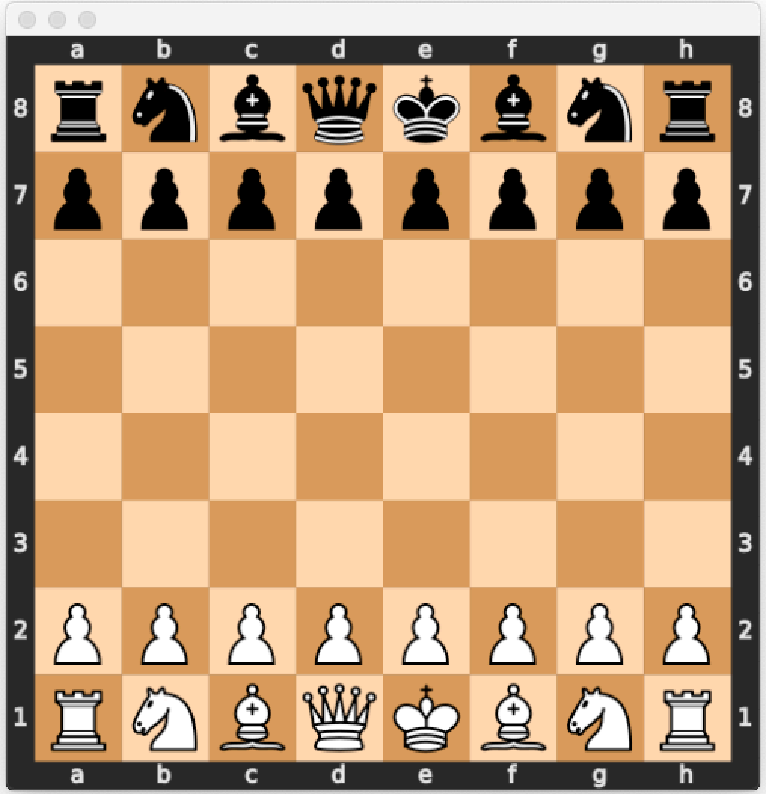

No answer, no supervised learning

RL can train without the answer!

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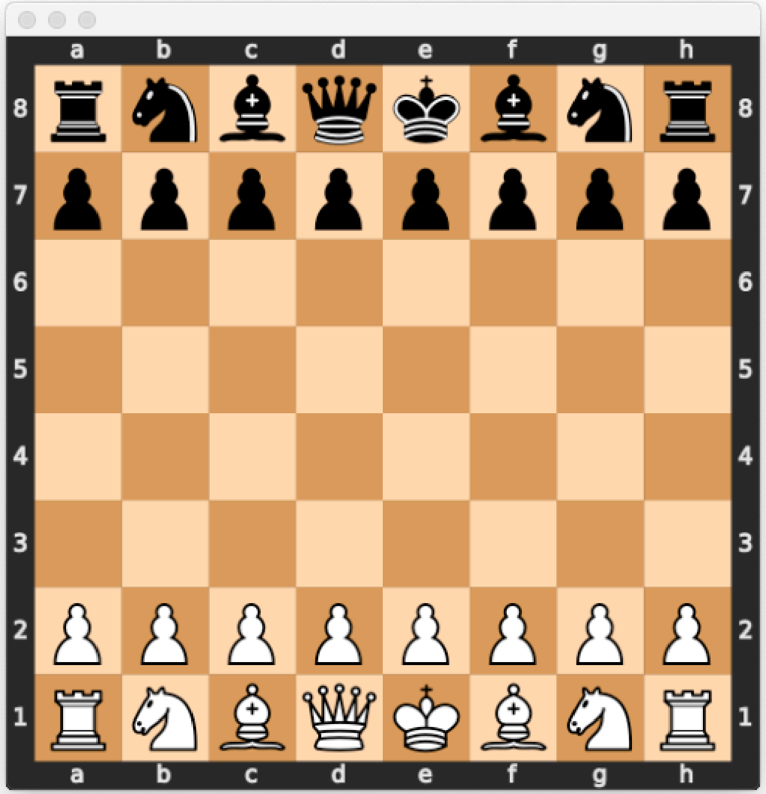

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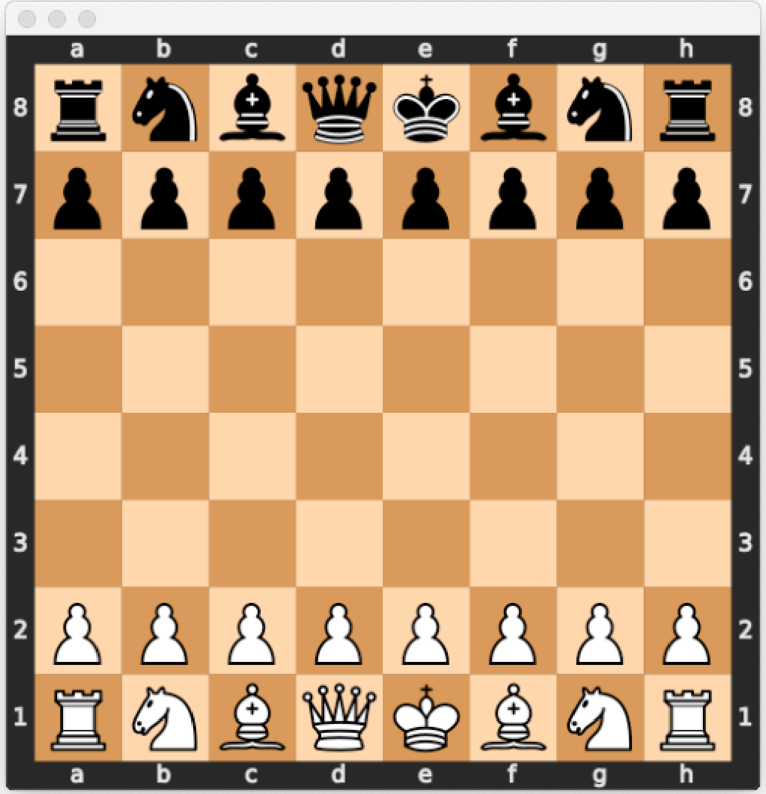

An answer gives us just one move

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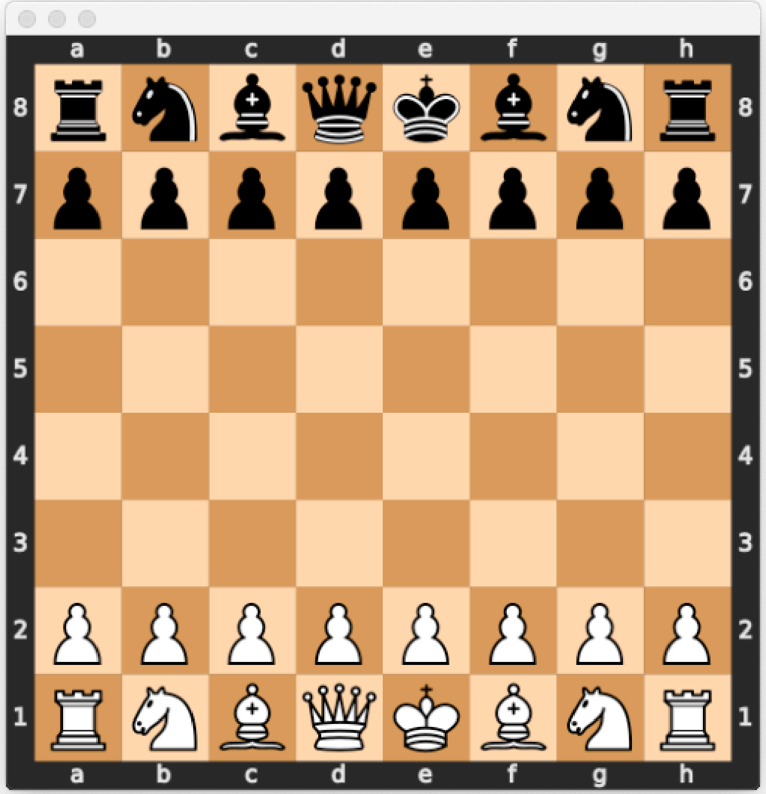

An answer gives us just one move

We need many moves to win

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RL gives us the best **sequence** of moves to achieve a result

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- Win a game of chess

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- Reduce human suffering

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RL gives us the best **sequence** of moves to achieve a result

- Win a game of chess
- Drive a customer to the store
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- Treat a sick patient
- Prevent climate change
- Reduce human suffering
- Find your own purpose (achieve conciousness)

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Real applications of RL:

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https://www.youtube.com/watch?v=Zeyv1bN9v4A	GT
https://www.youtube.com/watch?v=kopoLzvh5jY&t=1s	H&S
https://www.youtube.com/watch?v=eHipy_j29Xw	DoTA

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Other real applications of RL:

- Autonomous vehicles
- Video game NPCs
- Behavior modeling in psychology/ecology/biology
- Material and drug design
- Finance
- Alignment in large language models
  - Artificial General Intelligence?
- Anywhere with cause and effect
  - Where you **change** the world by **interacting** with it

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Instead of a model and dataset, we have an **agent** and **environment**

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Agent

Environment

What is RL?

The agent receives a positive reward for doing good

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The agent receives a positive reward for doing good

And a negative reward for doing bad

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And a negative reward for doing bad

Eventually, the agent only does good behaviors

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When the baby cries, they will receive hugs (reward)

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So the baby will learn to cry to get more hugs!

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Enough about the agent, let us talk about the environment

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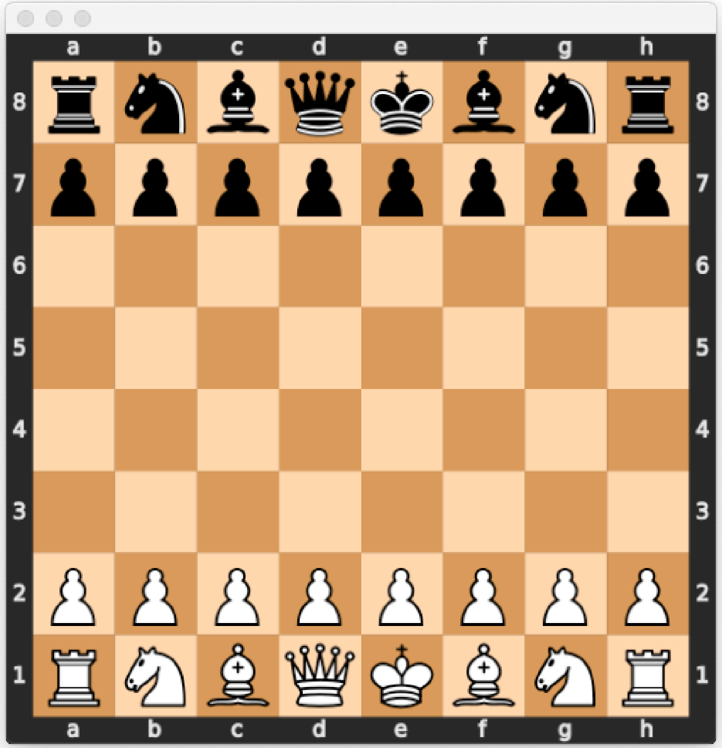

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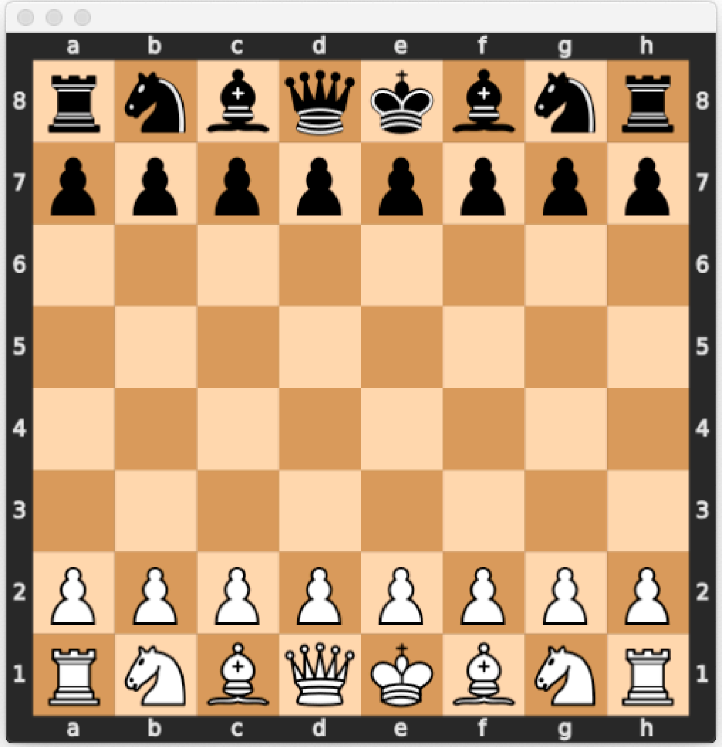

The environment is the world that the agent lives in

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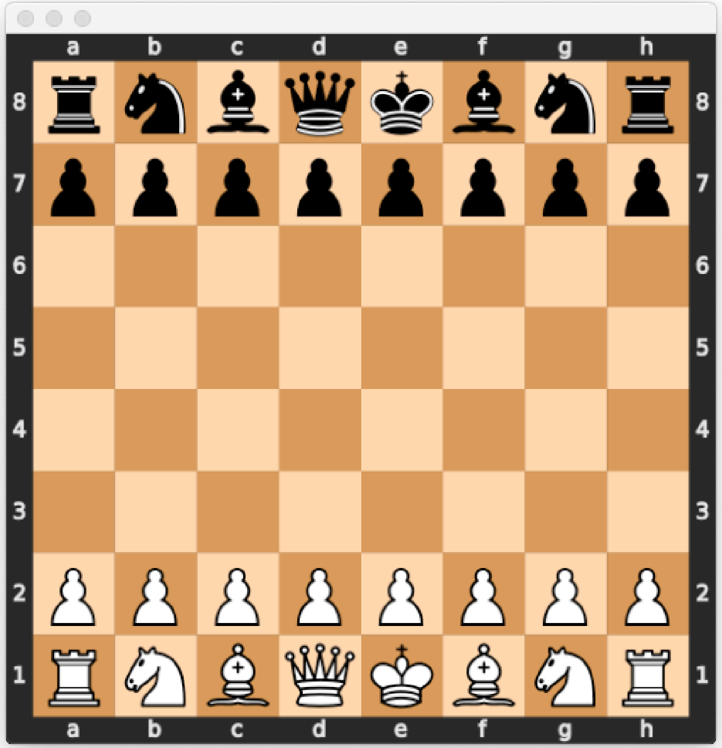

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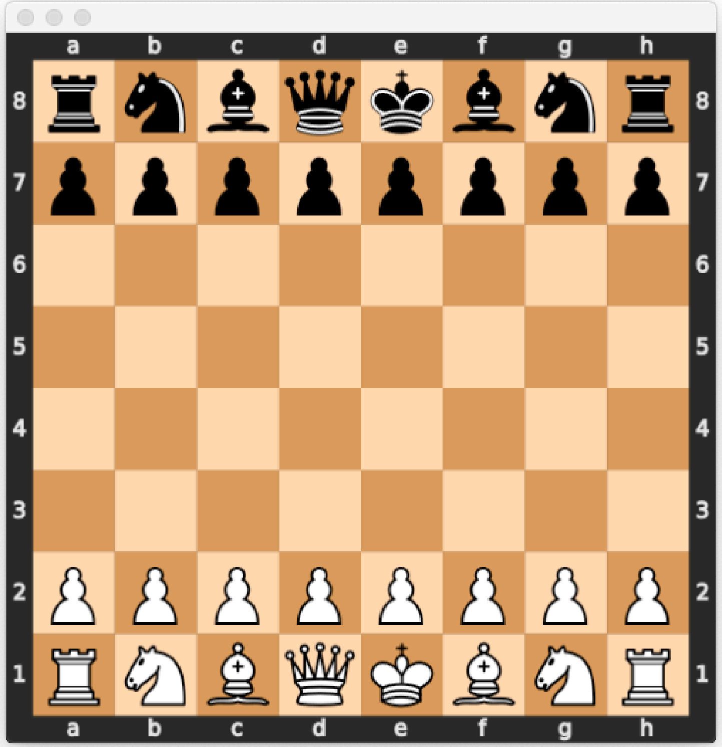

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The environment is a collection of rules

For example, each piece can only move in certain ways

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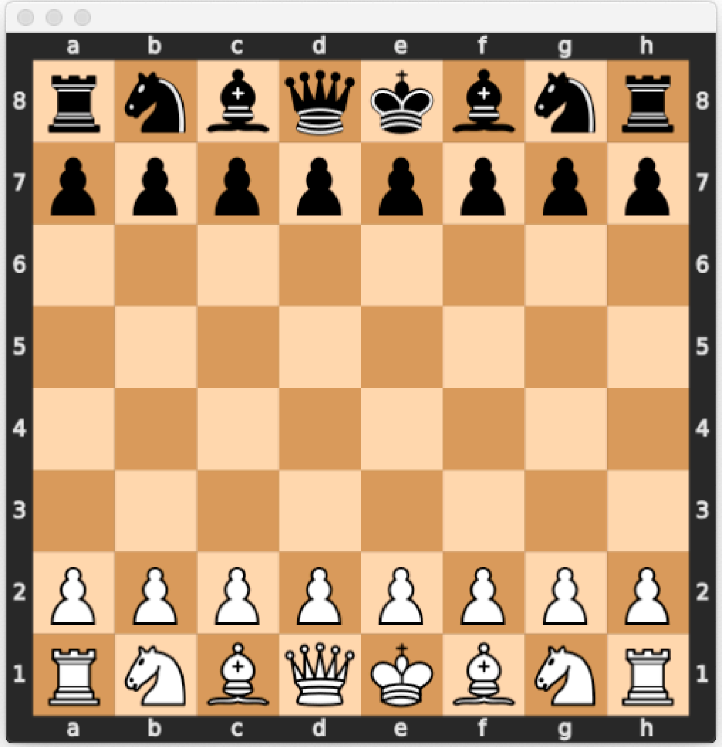

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The environment is a collection of rules

For example, each piece can only move in certain ways

If two pieces touch, then one piece dies

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The **state** describes the agent in the environment

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Now that you understand the agent, rewards, and environment, we will get more technical

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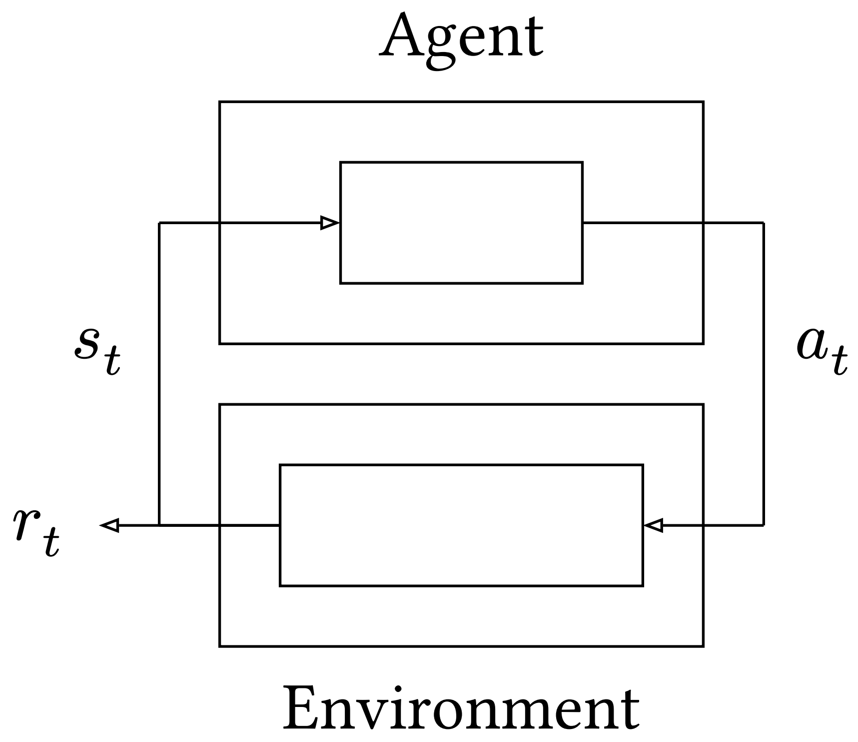

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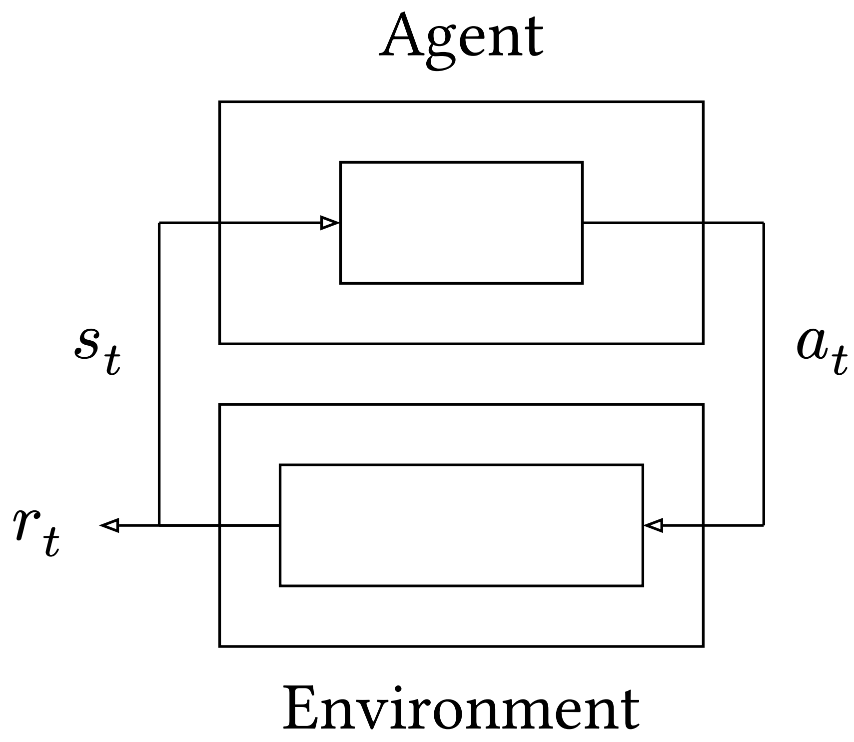

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- The agent takes **actions** in the environment

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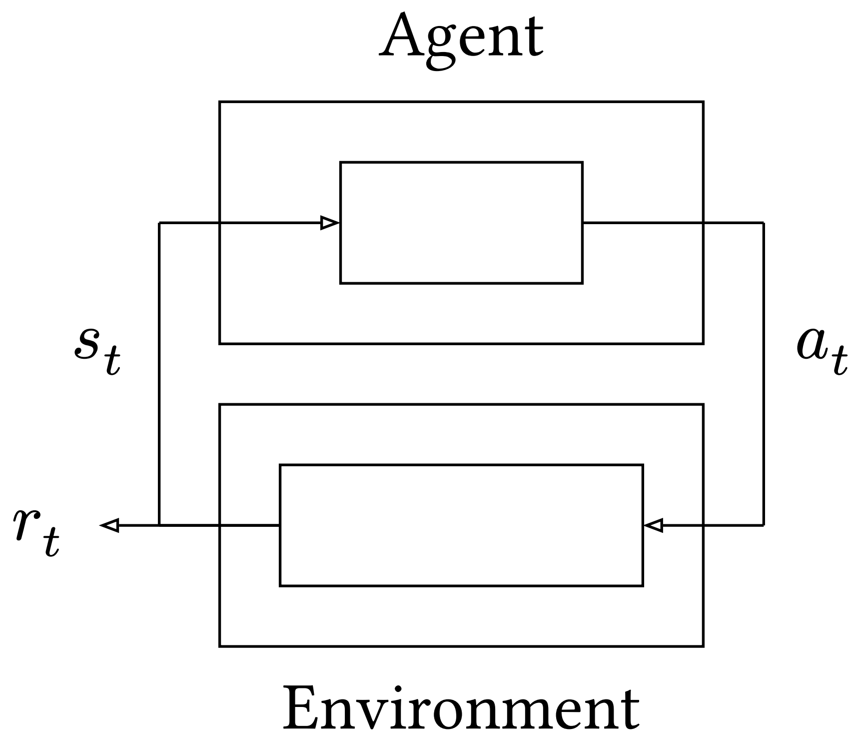

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- The agent takes **actions** in the environment
- Actions change the environment **state**, producing an new state and **reward**

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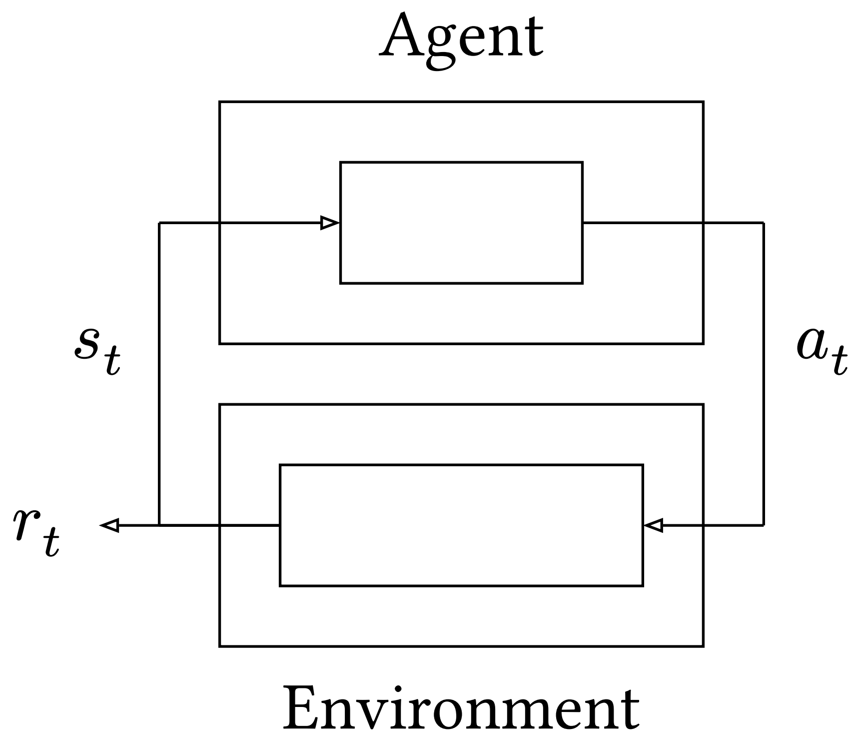

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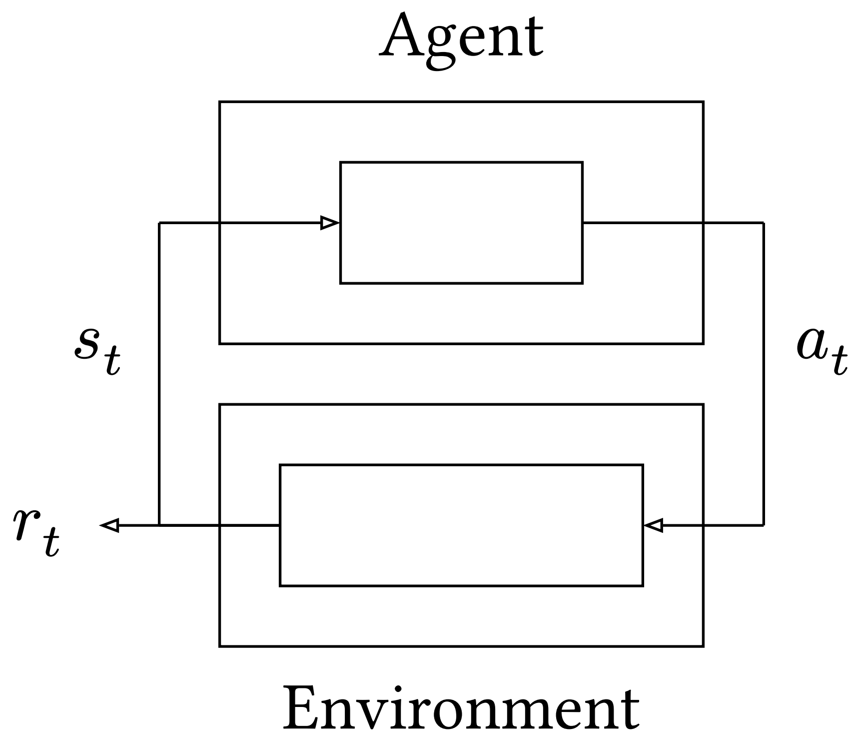

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- The agent takes **actions** in the environment
- Actions change the environment **state**, producing an new state and **reward**
- The cycle continues for $t = 0, 1, \ldots$
- Goal is to maximize the **cumulative reward**
  - Sum of rewards over **all** timestep

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By definition, RL solves **Markov Decision Processes (MDPs)**

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How you structure your problem is **critical** – more important than which algorithms you use, how much compute you have, etc.

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Let us formally introduce the MDP

Markov Decision Processes

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Let us briefly explain these terms.

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If the environment is a table, the state space might describe the positions of all objects on the table

$$\boldsymbol{s} = \begin{bmatrix} x_1 \\ y_1 \\ x_2 \\ y_2 \\ \vdots \end{bmatrix}$$

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A is the set of actions known as the **action space**

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What capabilities does the agent have?

Markov Decision Processes

A is the set of actions known as the **action space**

What capabilities does the agent have?

For the table example, I can apply a force to a specific object on the table

$$\boldsymbol{a} = \begin{bmatrix} F_x \\ F_y \\ i \end{bmatrix}$$

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Markov decision process because transition dynamics are **conditionally independent** of past states and actions

$$T(s_t, a_t \mid s_{t-1}, a_{t-1}, \ldots, s_0, a_0) = T(s_t, a_t)$$

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+100 for pushing objects onto the floor, or +100 for pushing objects to the centre

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? blocks give you mushrooms

Markov Decision Processes

Super Mario Bros. is a video game about Mario, an Italian plumber

Mario can move and jump

Touching a goomba kills Mario

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You collect coins and have a time limit and score

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Task: Define Super Mario MDP

Markov Decision Processes

State Space (S)?

Markov Decision Processes

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- Mario position/velocity $(\boldsymbol{r}, \dot{\boldsymbol{r}})$

Markov Decision Processes

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Markov Decision Processes

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- Number of coins collected

Markov Decision Processes

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- Which question blocks we opened

Markov Decision Processes

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- Goomba position/velocity and squished/not squished

Markov Decision Processes

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- Mario position/velocity $(\boldsymbol{r}, \dot{\boldsymbol{r}})$
- Score
- Number of coins collected
- The time remaining
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- Goomba position/velocity and squished/not squished

$$S = \{\mathbb{R}^4, \mathbb{Z}_+, \mathbb{Z}_+, \mathbb{Z}_+, \{0,1\}^m, \mathbb{R}^{4 \times k}, \{0,1\}^k\}$$

Markov Decision Processes

State Space (S)?

Markov Decision Processes

State Space (S)? $[0, 1]^{2 \times 256 \times 240 \times 3}$

Markov Decision Processes

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$$\begin{bmatrix} \begin{bmatrix} 255 \\ 0 \\ 0 \end{bmatrix} & \begin{bmatrix} 170 \\ 10 \\ 50 \end{bmatrix} & \dots \\ \begin{bmatrix} 10 \\ 100 \\ 235 \end{bmatrix} & \begin{bmatrix} 200 \\ 200 \\ 35 \end{bmatrix} & \dots \\ \vdots & & \ddots \end{bmatrix}, \begin{bmatrix} \begin{bmatrix} 255 \\ 0 \\ 0 \end{bmatrix} & \begin{bmatrix} 170 \\ 10 \\ 50 \end{bmatrix} & \dots \\ \begin{bmatrix} 10 \\ 100 \\ 235 \end{bmatrix} & \begin{bmatrix} 200 \\ 200 \\ 35 \end{bmatrix} & \dots \\ \vdots & & \ddots \end{bmatrix}$$

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Two images necessary to compute velocities!

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$$S = \mathbb{Z}_{<255}^{2 \times 256 \times 240 \times 3}$$

Markov Decision Processes

Action Space (A)?

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- Acceleration of Mario $\ddot{r}$

Markov Decision Processes

Action Space (A)?

- Acceleration of Mario $\ddot{r}$
  - But when playing Mario, we cannot explicitly set $\ddot{r}$

Markov Decision Processes

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- The Nintendo controller has $A, B, \uparrow, \downarrow, \leftarrow, \rightarrow$ buttons

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    - Cannot represent multiple buttons at once

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  - $A = \{A, B, \uparrow, \downarrow, \leftarrow, \rightarrow\}$
    - Cannot represent multiple buttons at once
  - $A = \{0, 1\}^6$
  - $$\left\{ \underbrace{\{0, 1, 2, 3, 4\}}_{\emptyset\text{,direction}} \times \underbrace{\{0, 1, 2, 3\}}_{\emptyset\text{,a,b,a+b}} \right\}$$

Markov Decision Processes

Transition Function (T)?

Markov Decision Processes

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- $T\left(\boldsymbol{s}_{\text{pixel}}, \rightarrow\right)$

Markov Decision Processes

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- $T\left(\boldsymbol{s}_{\text{pixel}}, \rightarrow\right)$
  - Move the Mario pixels right, unless a wall

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- $T\left(\boldsymbol{s}_{\text{pixel}}, \rightarrow\right)$
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  - Difficult to write down

Markov Decision Processes

Transition Function (T)?

- $T\left(\boldsymbol{s}_{\text{pixel}}, \rightarrow\right)$
  - Move the Mario pixels right, unless a wall
  - Difficult to write down
  - Deterministic

Markov Decision Processes

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Markov Decision Processes

Transition Function (T)?

- $T(s_r, \rightarrow)$

Markov Decision Processes

Transition Function (T)?

- $T(\boldsymbol{s}_r, \rightarrow)$
  - Changes Mario's $(\boldsymbol{r}, \dot{\boldsymbol{r}})$ in game memory

Markov Decision Processes

Transition Function (T)?

- $T(\boldsymbol{s}_r, \rightarrow)$
  - Changes Mario's $(\boldsymbol{r}, \dot{\boldsymbol{r}})$ in game memory
  - Human understandable, easier to implement for game developers

Markov Decision Processes

Question: In Mario, a single image frame is not a Markov state. How come?

Markov Decision Processes

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Answer: Cannot measure velocity.

Markov Decision Processes

Question: Why do we need velocity in the state?

Markov Decision Processes

Question: Why do we need velocity in the state?

Answer: If we don't have it, Markov property is violated

$T(s_t, a_t)$: Mario is moving $\uparrow, \downarrow, \leftarrow, \rightarrow$

$T(s_t, a_t \mid s_{t-1})$: Mario is moving $\rightarrow$ at 1 m/s

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Not conditionally independent! $T(s_t, a_t \mid s_{t-1}, a_{t-1}, ..., s_0, a_0) \neq T(s_t, a_t)$

Markov Decision Processes

Reward (R)?

Markov Decision Processes

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- 1 for beating the level and 0 otherwise

Markov Decision Processes

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- 1 for beating the level and 0 otherwise
- Total score

Markov Decision Processes

Reward (R)?

- 1 for beating the level and 0 otherwise
- Total score
- 1 for beating the level + $0.01 \cdot$ score

Markov Decision Processes

- S ✓

Markov Decision Processes

- $S \checkmark$
- $A \checkmark$

Markov Decision Processes

- S✓
- A✓
- T✓

Markov Decision Processes

- S ✓
- A ✓
- T ✓
- R ✓

Markov Decision Processes

- S✓
- A✓
- T✓
- R✓
- γ?

Markov Decision Processes

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The cumulative reward is called the **return (G)**

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Agent goal in RL is to maximize the **cumulative** reward

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Note that we care about all future rewards, not just the current reward!

Markov Decision Processes

Do humans maximize the return?

Markov Decision Processes

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Experiment: one cookie now, or two cookies in a year?

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$$G = \sum_{t=0}^{\infty} \gamma^t r_t = r_0 + \gamma r_1 + \gamma^2 r_2 + \dots$$

$$0 \leq \gamma \leq 1$$

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Where have we seen this before?

Markov Decision Processes

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With a reward of 1 at each timestep and $\gamma = 0.9$

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Markov Decision Processes

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We almost always choose to maximize the <u>discounted</u> return

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Exercise: Reinforcement learning also describes human and animal behaviors. How can you describe your behavior using reinforcement learning?

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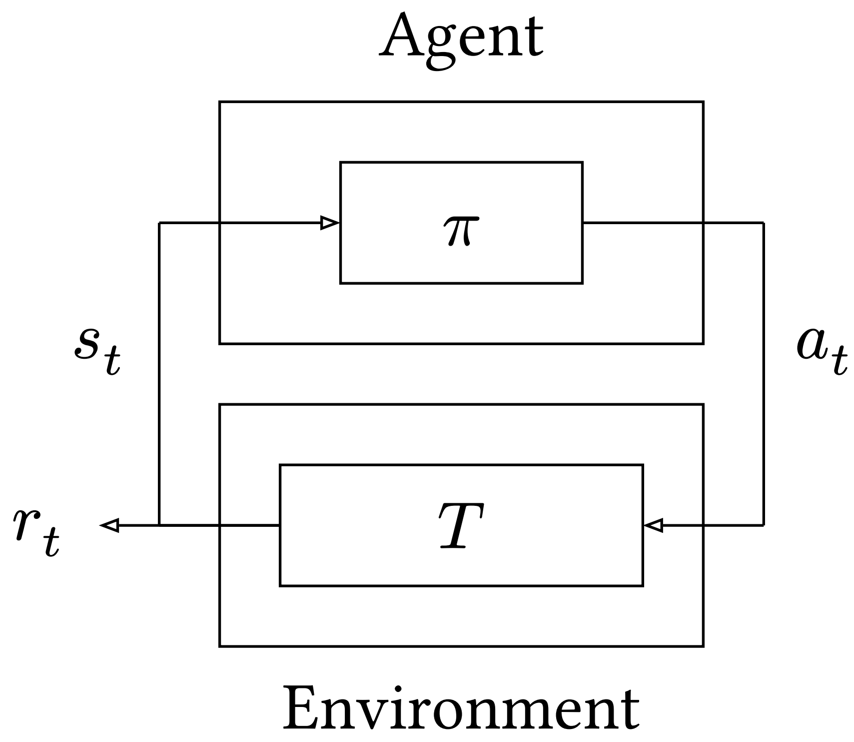
This happens internally when I decide to go to the pub after work

Agents and Policies

Agents and Policies



s_t: state, a_t: action,
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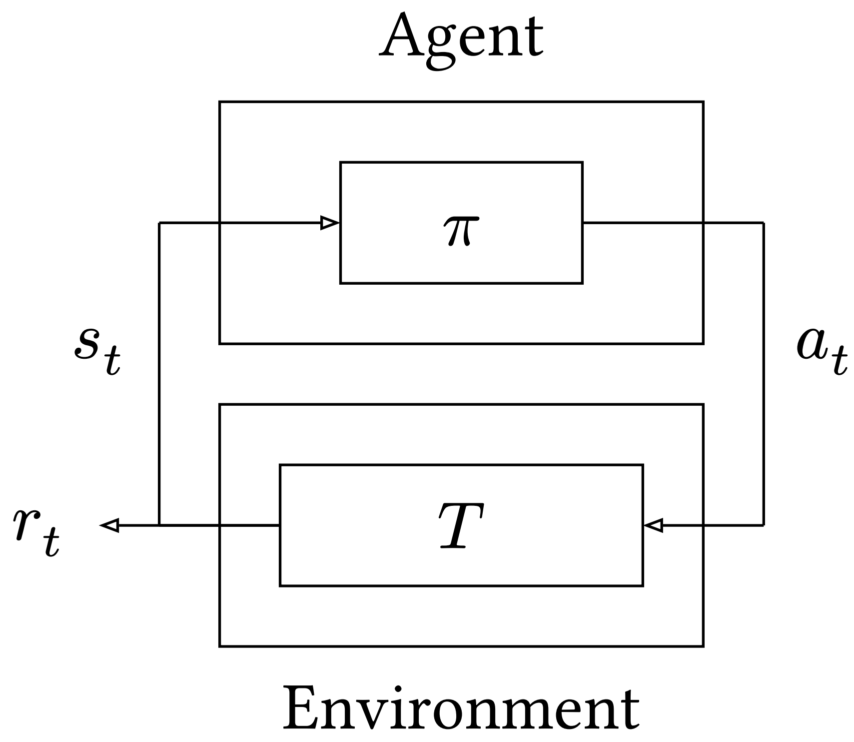
- We have defined the environment

Agents and Policies



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- We have defined the environment
- Now let us define the agent

Agents and Policies

The agent acts following a **policy** π.

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$$a_t \sim \pi(s_t)$$ Sample action from the policy

$$\pi(a_t \mid s_t)$$ Probability of taking each action

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In English: The optimal must consider the action distribution combined and state transition distribution to compute the reward/return

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We write the return as the expectation given our policy actions

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Now, our policy is truly optimal

Q Learning

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1. Derive the value function V
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The Plan:

1. Derive the value function V
2. Derive Q function from V
3. Derive an optimal policy from Q
4. Learn to train Q

Q Learning

Recall the discounted return for a specific policy π

$$G_\pi = \mathbb{E}\left[\sum_{t=0}^{\infty} \gamma^t r_t \middle| a_t \sim \pi(s_t) \right]$$

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In English: At each timestep, we take an action $a_t \sim \pi(s_t)$

follow the state transition function $s_{t+1} \sim T(s_t, a_t)$

Q Learning

Recall the discounted return for a specific policy π

$$G_\pi = \mathbb{E}\left[\sum_{t=0}^{\infty} \gamma^t r_t \;\middle|\; a_t \sim \pi(s_t)\right]$$

In English: At each timestep, we take an action $a_t \sim \pi(s_t)$

follow the state transition function $s_{t+1} \sim T(s_t, a_t)$

and get a reward $r_t = R(s_{t+1})$

Q Learning

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Question: Where does s_0 come from?

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Question: Where does s_0 come from?

Answer: Some higher power

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That is not a good answer

Q Learning

$$G_\pi = \mathbb{E}\left[\sum_{t=0}^{\infty} \gamma^t r_t \;\middle|\; a_t \sim \pi(s_t)\right]$$

What if we defined the return starting from a specific state s_0?

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Measures the **value** of a state (how good is it to be in this state?), for a given policy π

We call this the **Value Function (V_π)** $\quad V_\pi : S \to \mathbb{R}$

Q Learning

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Let's go one step further. What if we parameterize the value function with an initial action?

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When V depends on a specific action, we call it the **Q function**:

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a_0 affects your next state s_1, which affects the future

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Q function gives you a number denoting how much better your life will be for attending Cambridge (based on your behavior π). Takes into account reward (based on income, friend group, experiences, etc).

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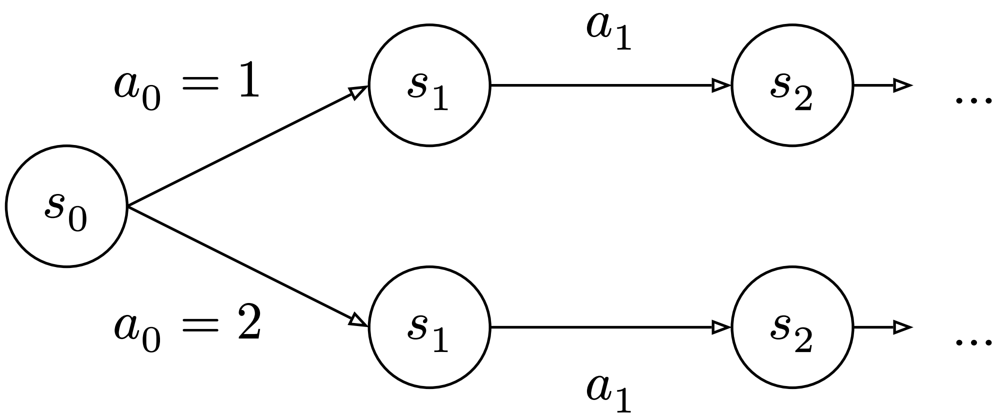

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We call this the **greedy policy**

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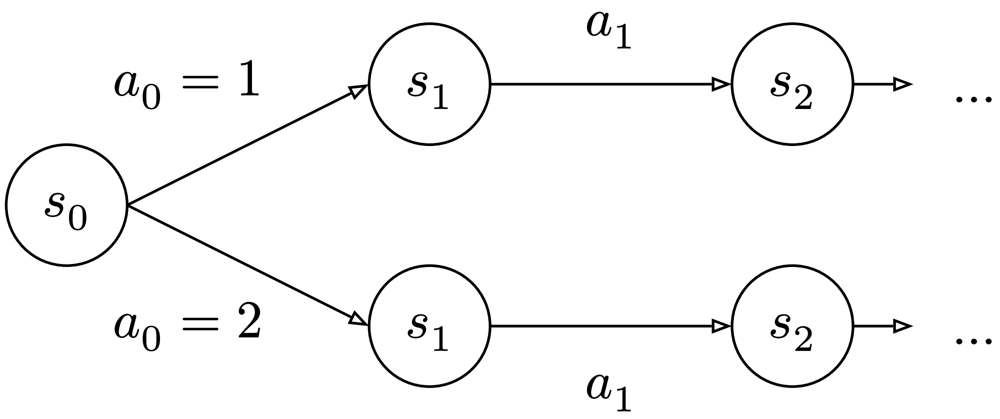

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Can we get rid of the infinite sum?

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Factor out γ

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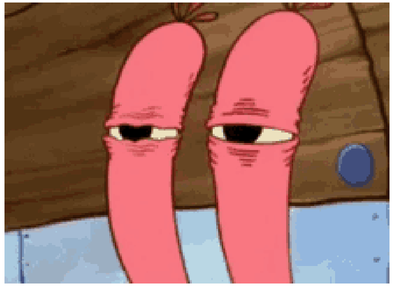

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The policy π_* takes the argmax over Q, which reduces to

$$Q_*(s_0, a_0) = \mathbb{E}[r_0 \mid a_0] + \gamma \cdot \max_{\{a \in A\}} Q_*(s_1, a)$$

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Literature often drops the expectation and time subscripts

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All we need is:

$$(s, a, r, \gamma, s')$$

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We can parameterize Q with parameters θ and try to approximate Q_*

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- Best driver

Q Learning

If we optimize this objective, we will find the optimal Q function

$$\lim_{\mathcal{L} \to 0} Q(s, a, \theta) = Q_*(s, a)$$

What does this mean?

Given enough time and data, we can learn the best possible policy

- Best chess player
- Best driver
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$$\min_{\theta} \left(Q(s, a, \theta) - \left(r + \gamma \cdot \max_{\{a' \in A\}} Q(s', a', \theta)\right)\right)^2$$

Q Learning

Q learning learns superhuman policies on many video games

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https://www.youtube.com/watch?v=O2QaSh4tNVw	SMB
https://youtu.be/VIwGxOdXGfw?si=A-CVLI6vEJHOxrvx&t=478	MK

Q Learning

How do we use RL with LLMs?

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LLMs get reward for helping humans

Q Learning

1. Reinforcement Learning, an Introduction (2018, Sutton and Barto)
   - Available for free online (legal)
   - All the RL theory you will ever need
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3.
4.
5.

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5. Special Topics in AI (Winter/Spring 2025)