Neural Networks

CISC 7026: Introduction to Deep Learning

University of Macau

Notation Change

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$$\boldsymbol{x} = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \end{bmatrix}, \quad \boldsymbol{X} = \begin{bmatrix} x_{1,1} & \cdots & x_{1,n} \\ \vdots & \ddots & \vdots \\ x_{m,1} & \cdots & x_{m,n} \end{bmatrix}$$

Agenda

1. Review
2. Multivariate linear regression
3. Limitations of linear regression
4. History of neural networks
5. Biological neurons
6. Artificial neurons
7. Wide neural networks
8. Deep neural networks
9. Practical considerations

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- *The causal effects of education on health outcomes in the UK Biobank.* Davies et al. *Nature Human Behaviour.*
- By staying in school, you are likely to live longer
- Being rich also helps, but education alone has a **causal** relationship with life expectancy

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$$f : X \times \Theta \mapsto Y$$

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$$f : X \times \Theta \mapsto Y$$

Approach: Learn the parameters θ such that

$$f(x, \theta) = y; \quad x \in X, y \in Y$$

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Started with a linear function f

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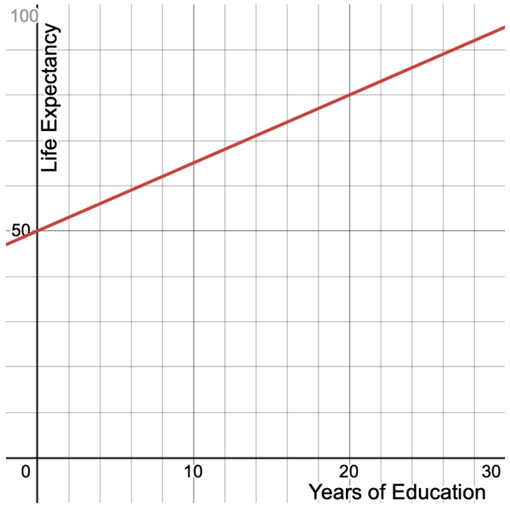
$$f(x, \boldsymbol{\theta}) = f\left(x, \begin{bmatrix} \theta_1 \\ \theta_0 \end{bmatrix}\right) = \theta_1 x + \theta_0$$



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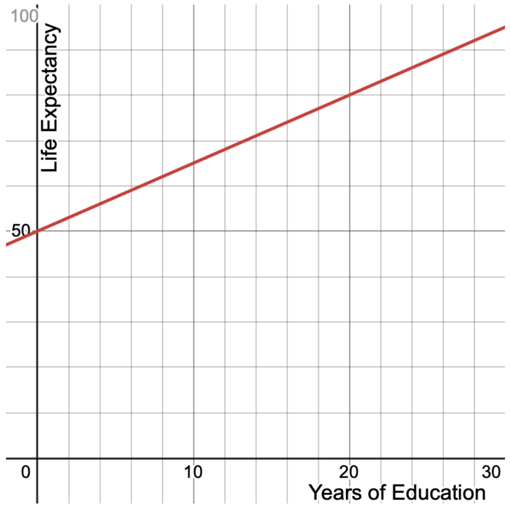

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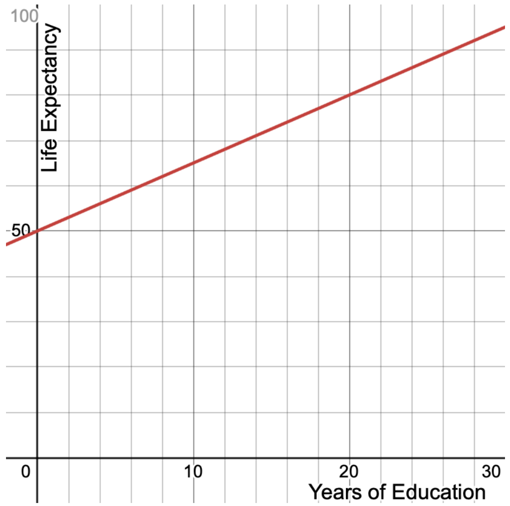

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$$\text{error}(f(x, \boldsymbol{\theta}), y) = (f(x, \boldsymbol{\theta}) - y)^2$$

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We wrote the loss function for a single datapoint $x_{[i]}, y_{[i]}$ using the square error

$$\mathcal{L}\left(x_{[i]}, y_{[i]}, \boldsymbol{\theta}\right) = \operatorname{error}\left(f\left(x_{[i]}, \boldsymbol{\theta}\right), y_{[i]}\right) = \left(f\left(x_{[i]}, \boldsymbol{\theta}\right) - y_{[i]}\right)^2$$

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$$\mathcal{L}(\boldsymbol{x}, \boldsymbol{y}, \boldsymbol{\theta}) = \sum_{i=1}^{n} \operatorname{error}\left(f\left(x_{[i]}, \boldsymbol{\theta}\right), y_{[i]}\right) = \sum_{i=1}^{n} \left(f\left(x_{[i]}, \boldsymbol{\theta}\right) - y_{[i]}\right)^2$$

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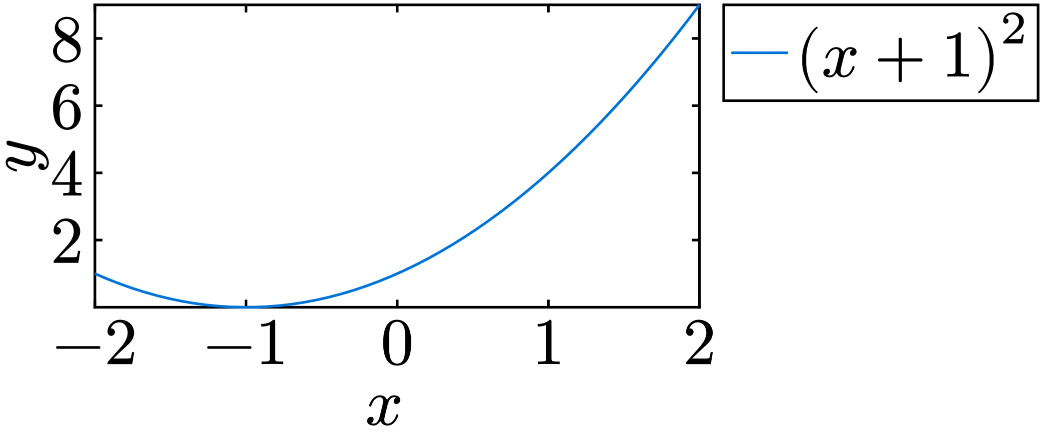

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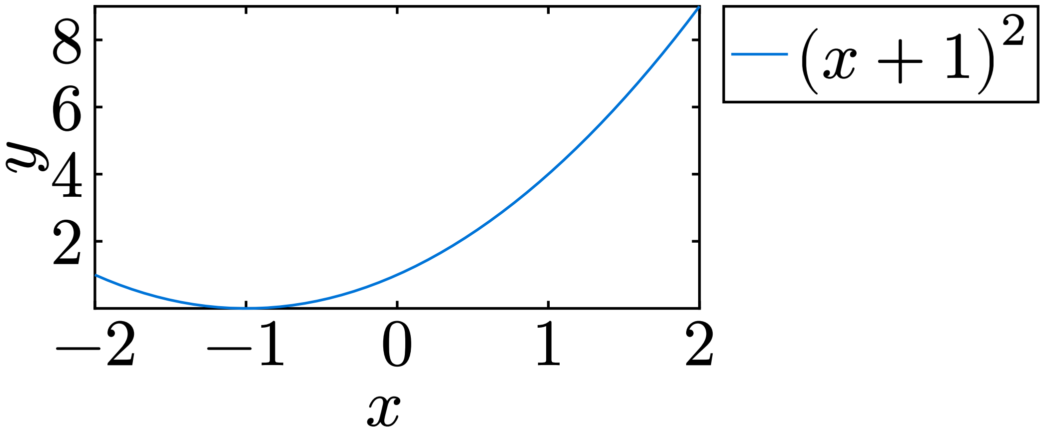

$$\arg\min_x f(x) = -1$$

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$$\arg\min_{\boldsymbol{\theta}} \mathcal{L}(\boldsymbol{x}, \boldsymbol{y}, \boldsymbol{\theta}) = \arg\min_{\boldsymbol{\theta}} \sum_{i=1}^{n} \text{error}\left(f\left(x_{[i]}, \boldsymbol{\theta}\right), y_{[i]}\right)$$

$$= \arg\min_{\boldsymbol{\theta}} \sum_{i=1}^{n} \left(f\left(x_{[i]}, \boldsymbol{\theta}\right) - y_{[i]}\right)^2$$

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$$\boldsymbol{\theta} = \left(\boldsymbol{X}_D^\top \boldsymbol{X}_D\right)^{-1} \boldsymbol{X}_D^\top \boldsymbol{y}$$

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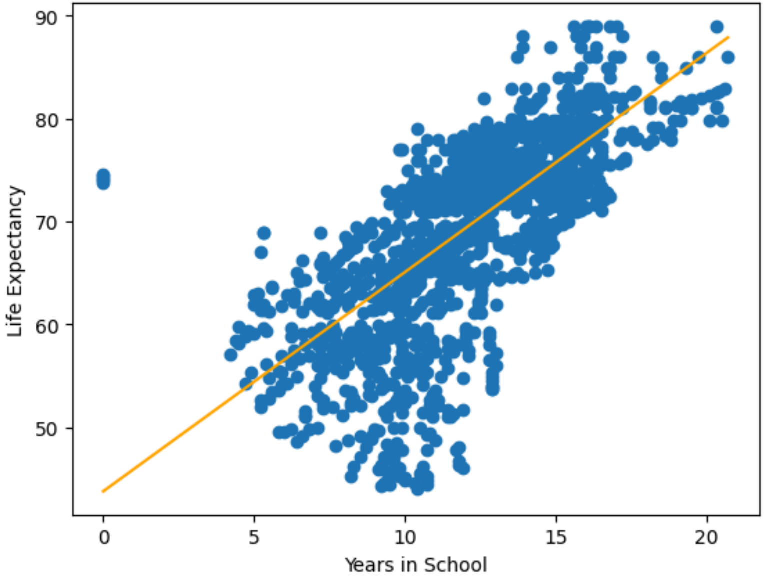

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Then, we used a trick to extend linear regression to nonlinear models

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$$\boldsymbol{X}_D = \begin{bmatrix} x_{[1]} & 1 \\ x_{[2]} & 1 \\ \vdots & \vdots \\ x_{[n]} & 1 \end{bmatrix} \Rightarrow \boldsymbol{X}_D = \begin{bmatrix} \log\left(1 + x_{[1]}\right) & 1 \\ \log\left(1 + x_{[2]}\right) & 1 \\ \vdots & \vdots \\ \log\left(1 + x_{[n]}\right) & 1 \end{bmatrix}$$

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$$\Theta \in \mathbb{R}^2 \Rightarrow \Theta \in \mathbb{R}^{m+1}$$

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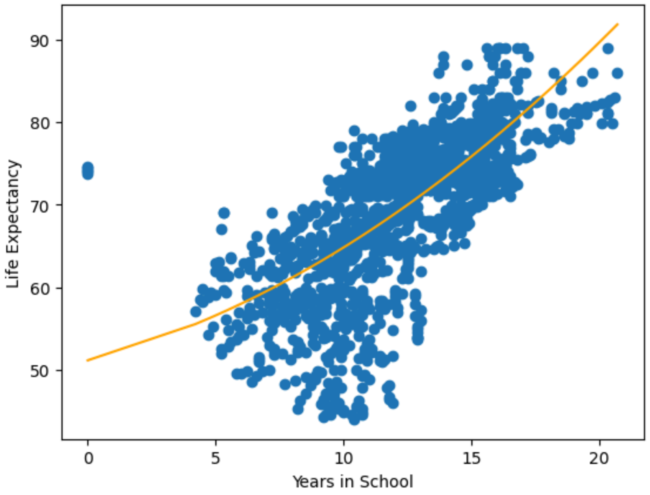
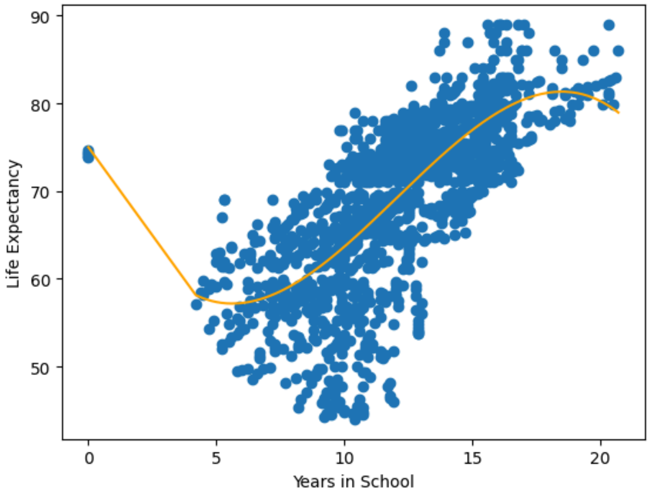
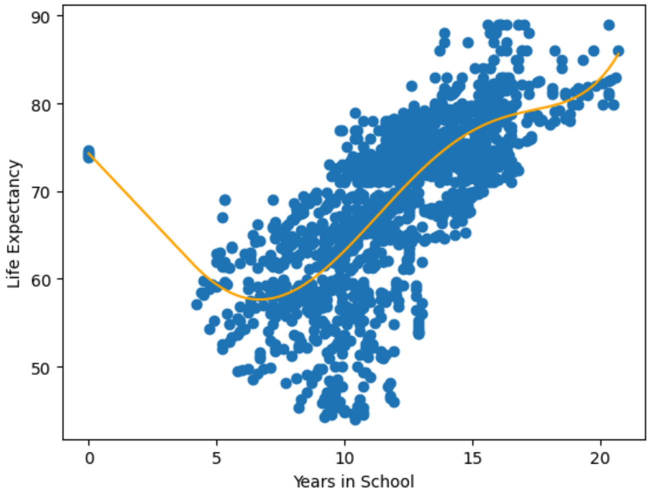


$m = 2$



$m = 3$



$m = 5$

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We care about **generalization** in machine learning

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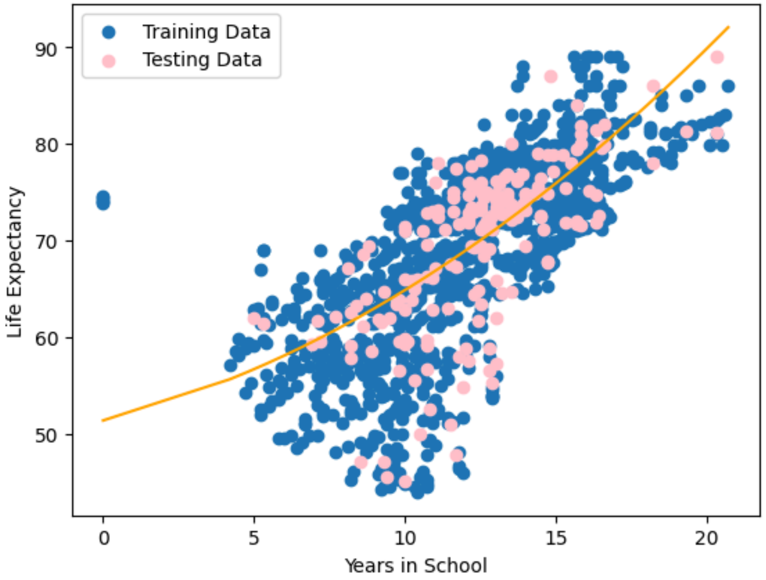

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We can solve these problems using linear regression too

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$x_{[i],1}$ refers to the first dimension of training data i

The design matrix for a **multivariate** linear system is

$$\boldsymbol{X}_D = \begin{bmatrix} x_{[1],d_x} & x_{[1],d_x-1} & \cdots & x_{[1],1} & 1 \\ x_{[2],d_x} & x_{[2],d_x-1} & \cdots & x_{[2],1} & 1 \\ \vdots & \vdots & \ddots & \vdots & \\ x_{[n],d_x} & x_{[n],d_x-1} & \cdots & x_{[n],1} & 1 \end{bmatrix}$$

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Remember $x_{[n],d_x}$ refers to dimension d_x of training data n

The solution is the same as before

$$\boldsymbol{\theta} = \left(\boldsymbol{X}_D^\top \boldsymbol{X}_D\right)^{-1} \boldsymbol{X}_D^\top \boldsymbol{y}$$

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Limitations of Linear Regression

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So far, we have seen:

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One-dimensional polynomial functions

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Combine them to create multi-dimensional polynomial functions

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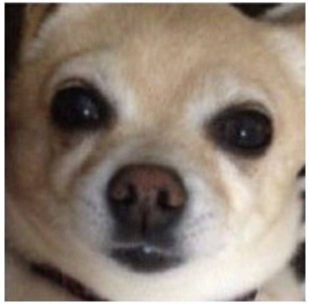

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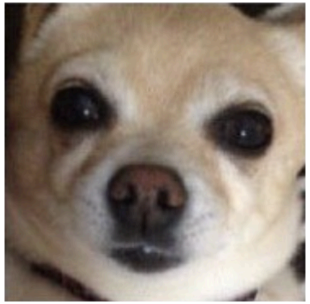

$$f : X \times \Theta \mapsto Y; \quad X : \text{Image}, \quad Y : \text{Number of } ♥$$

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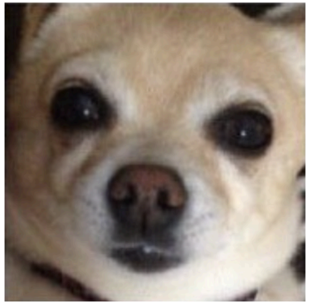

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$$X \in \mathbb{Z}_+^{256 \times 256} = \mathbb{Z}_+^{65536}; \quad Y \in \mathbb{Z}_+$$

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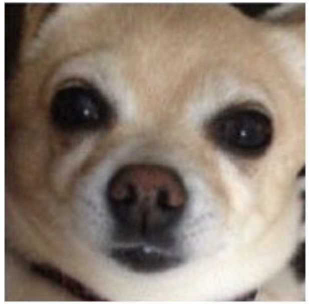
Let us do an example

Task: predict how many ♥ a photo gets on social media



$$f : X \times \Theta \mapsto Y; \quad X : \text{Image}, \quad Y : \text{Number of } ♥$$

$$X \in \mathbb{Z}_+^{256 \times 256} = \mathbb{Z}_+^{65536}; \quad Y \in \mathbb{Z}_+$$

Highly nonlinear task, use a polynomial with order $m = 20$

Limitations of Linear Regression

$$\boldsymbol{X}_D = \begin{bmatrix} \boldsymbol{x}_{D,[1]} & \cdots & \boldsymbol{x}_{D,[n]} \end{bmatrix}^\top$$

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$$\begin{bmatrix} \underbrace{x_{[i],d_x}^m x_{[i],d_x-1}^m \ldots x_{[i],1}^m}_{(d_x \Rightarrow 1, x^m)} & \underbrace{x_{[i],d_x}^m x_{[i],d_x-1}^m \ldots x_{[i],2}^m}_{(d_x \Rightarrow 2, x^m)} & \cdots & \underbrace{x_{[i],d_x}^{m-1} x_{[i],d_x-1}^{m-1} \ldots x_{[i],1}^m}_{(d_x \Rightarrow 1, x^{m-1})} & \cdots \end{bmatrix}$$

Question: How many columns in this matrix?

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Hint: $d_x = 2, m = 3$: $x^3 + y^3 + x^2 y + y^2 x + xy + x + y + 1$

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Answer: $(d_x)^m = 65536^{20} + 1 \approx 10^{96}$

Limitations of Linear Regression

How big is 10^{96}?

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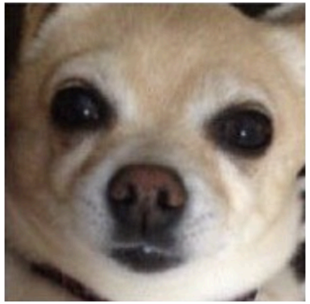

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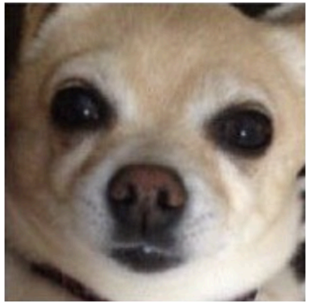

Polynomial regression does not scale to large inputs

Limitations of Linear Regression

Issues arise with other problems

1. **Poor scalability**
2. Polynomials do not generalize well

Limitations of Linear Regression

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$$\lim_{x \to \infty} \theta_m x^m + \theta_{m-1} x^{m-1} + \ldots$$

Equation of a polynomial

Limitations of Linear Regression

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$$\lim_{x \to \infty} x^m \left(\theta_m + \frac{\theta_{m-1}}{x} + \dots \right)$$

Factor out x^m

Limitations of Linear Regression

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$$\lim_{x \to \infty} x^m \cdot \lim_{x \to \infty} \left(\theta_m + \frac{\theta_{m-1}}{x} + \ldots \right)$$

Split the limit (limit of products)

Limitations of Linear Regression

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Rewrite

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$$\theta_m \lim_{x \to \infty} x^m = \infty$$

If $\theta_m > 0$

Limitations of Linear Regression

$\lim_{x \to \infty} x^m \cdot \lim_{x \to \infty} \left(\theta_m + \frac{\theta_{m-1}}{x} + \ldots \right)$	Split the limit (limit of products)
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$\theta_m \lim_{x \to \infty} x^m$	Rewrite
$\theta_m \lim_{x \to \infty} x^m = \infty$	If $\theta_m > 0$
$\theta_m \lim_{x \to \infty} x^m = -\infty$	If $\theta_m < 0$

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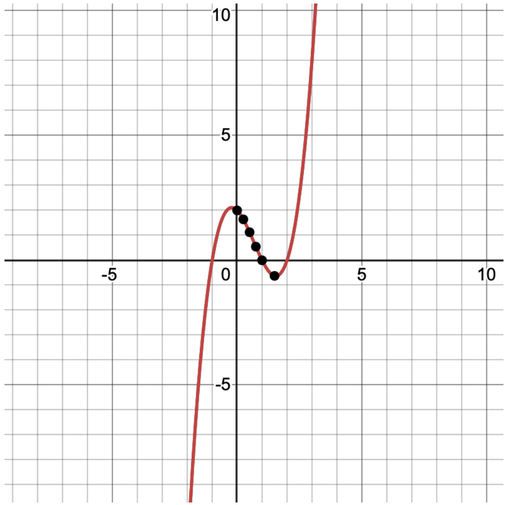

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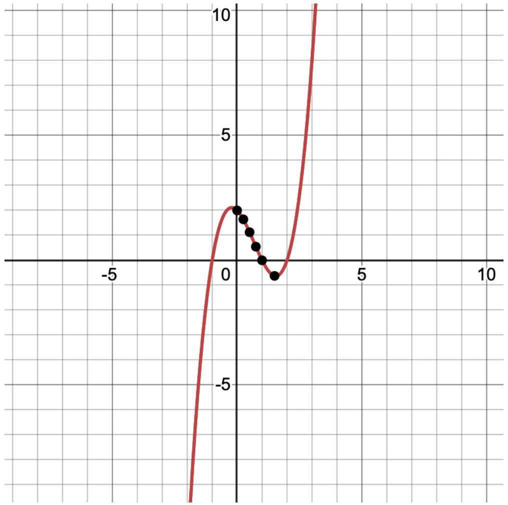

Remember, to predict new data we want our functions to generalize

Limitations of Linear Regression

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1. No analytical solution
2. High data requirement

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3. **Limitations of linear regression**
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5. Biological neurons
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History of Neural Networks

In 1939-1945, there was a World War

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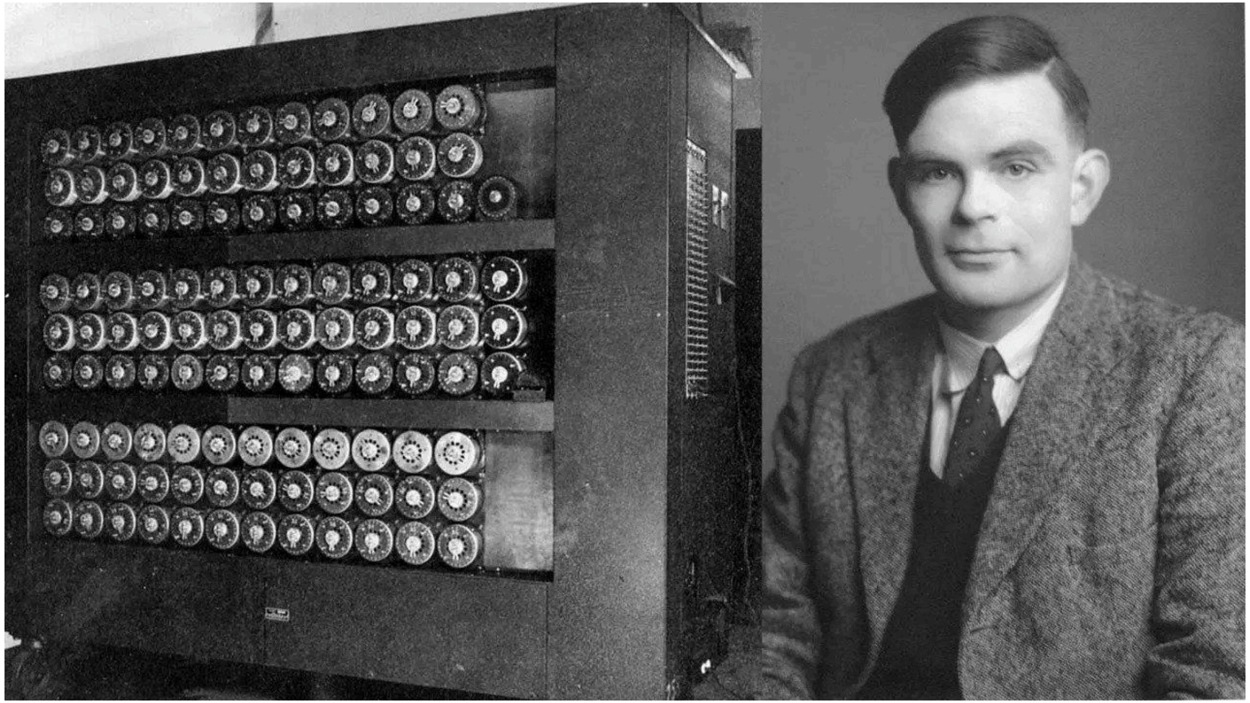
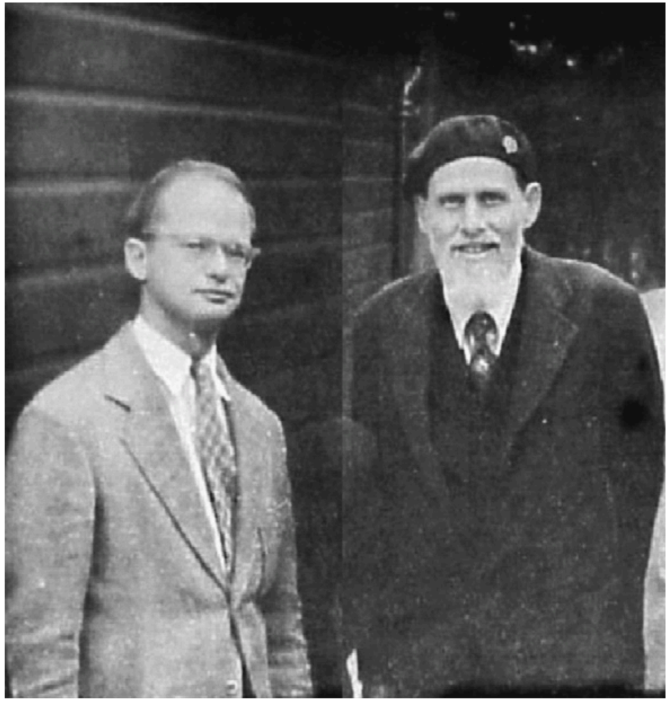


History of Neural Networks

Meanwhile, a neuroscientist and mathematician (McCullough and Pitts) were trying to understand the human brain



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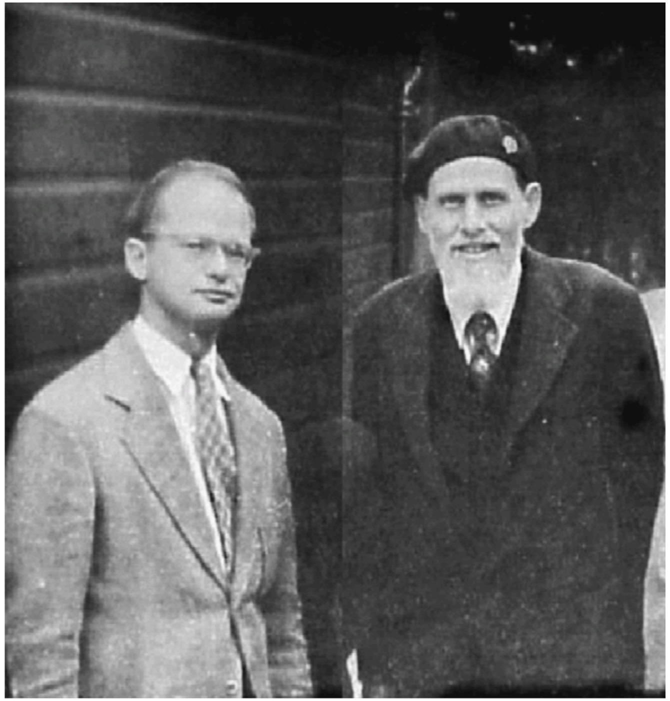

They designed the theory for the first neural network

History of Neural Networks

Rosenblatt implemented this neural network theory on a computer a few years later

History of Neural Networks

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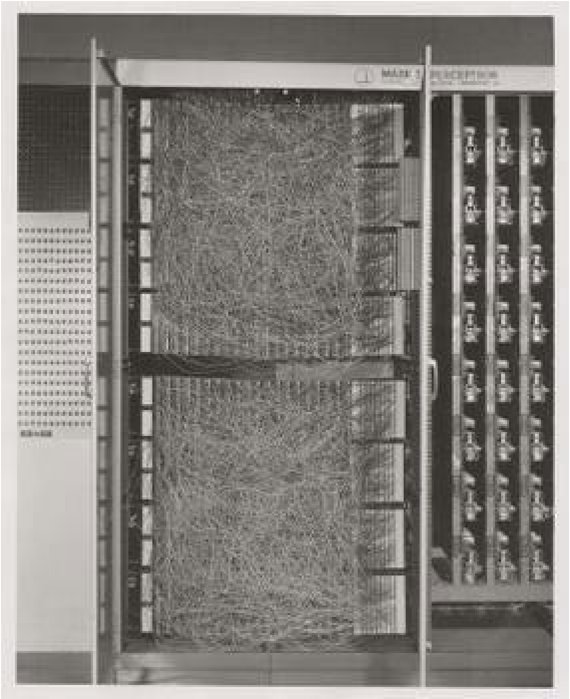
At the time, computers were very slow and expensive



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Through advances in theory and hardware, neural networks became slightly better

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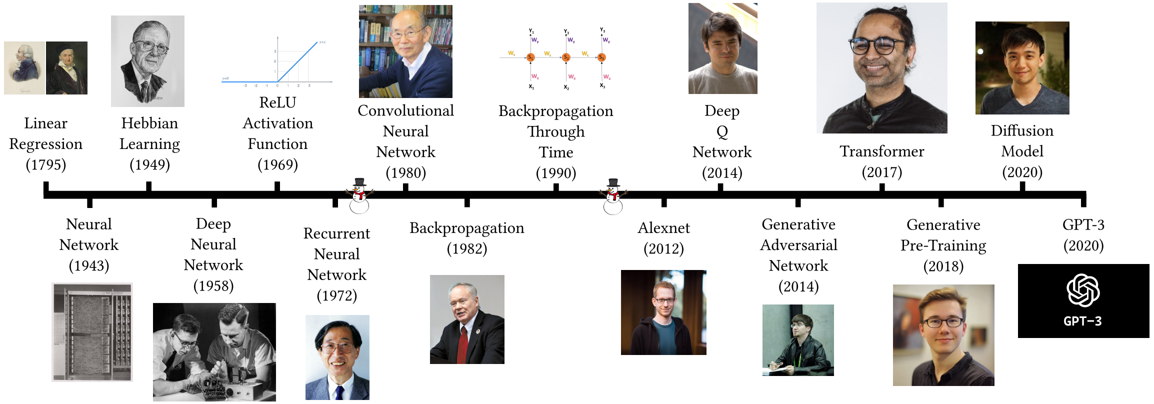

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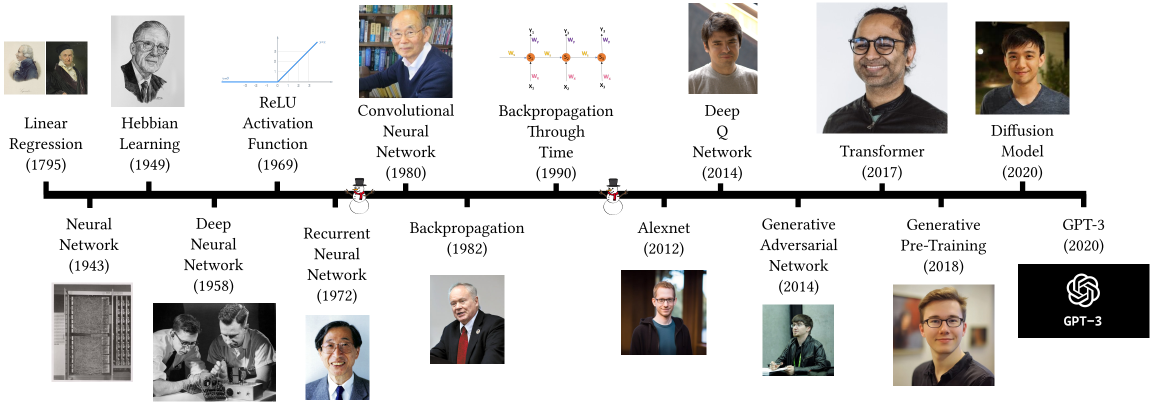

Around 2012, these improvements culminated in neural networks that perform like humans

History of Neural Networks

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It is a function, inspired by how the brain works

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$$f : X \times \Theta \mapsto Y$$

History of Neural Networks

Brains and neural networks rely on **neurons**

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First, let us review biological neurons

History of Neural Networks

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First, let us review biological neurons

Note: I am not a neuroscientist! I may make simplifications or errors with biology

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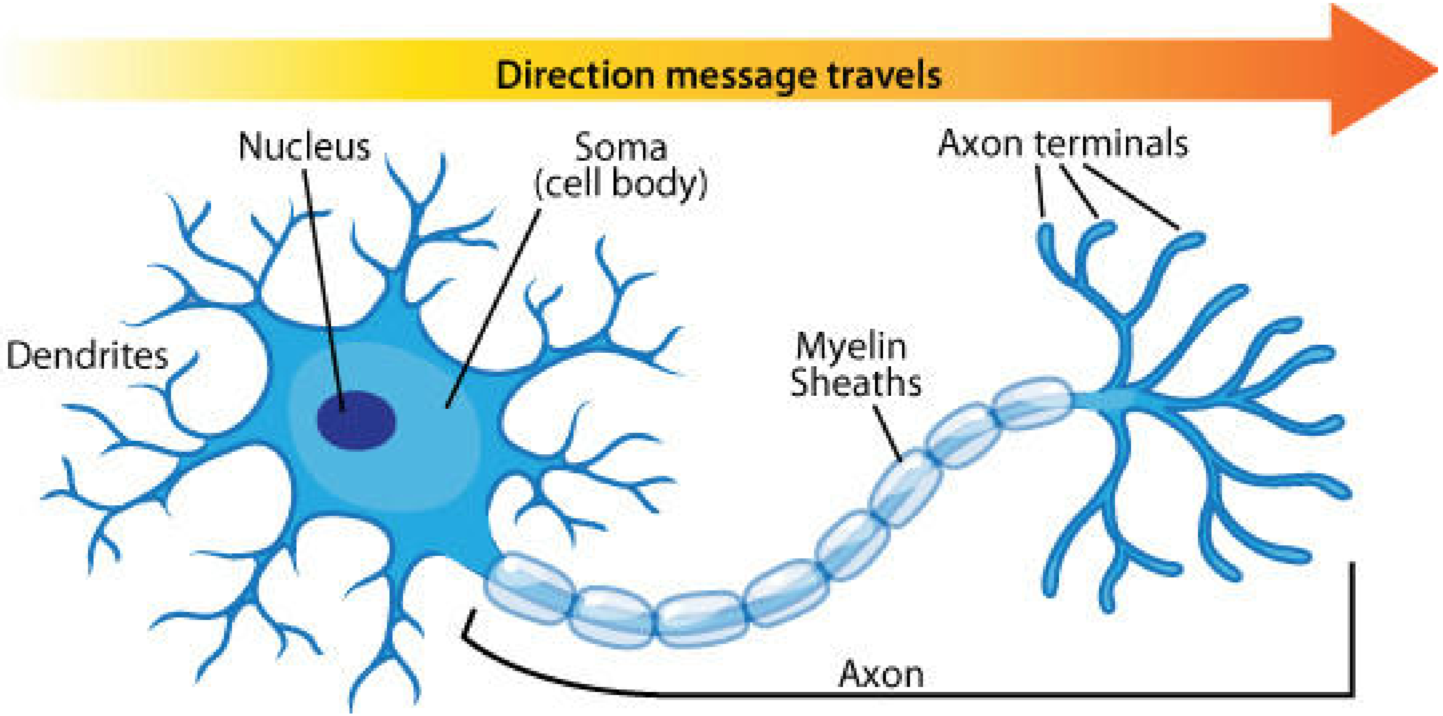
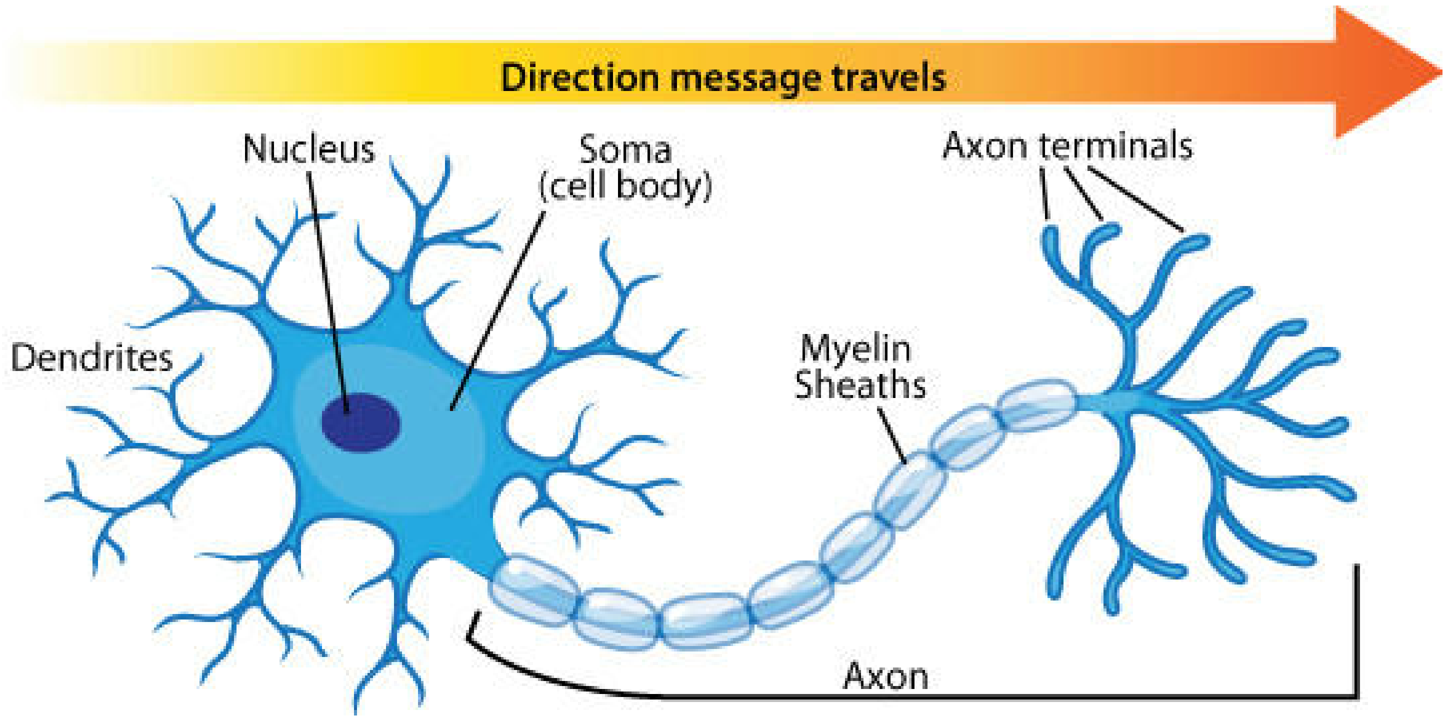
Biological Neurons



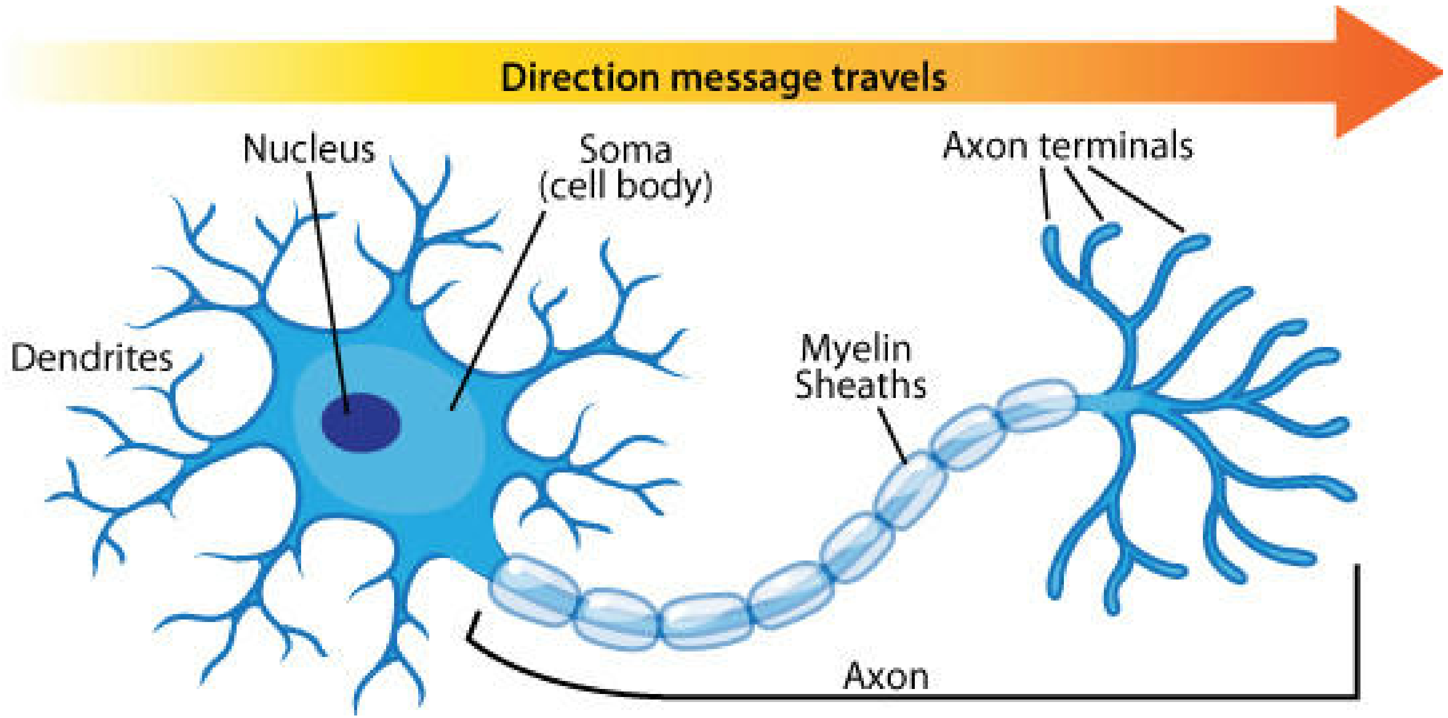
A simplified neuron consists of many parts

Biological Neurons



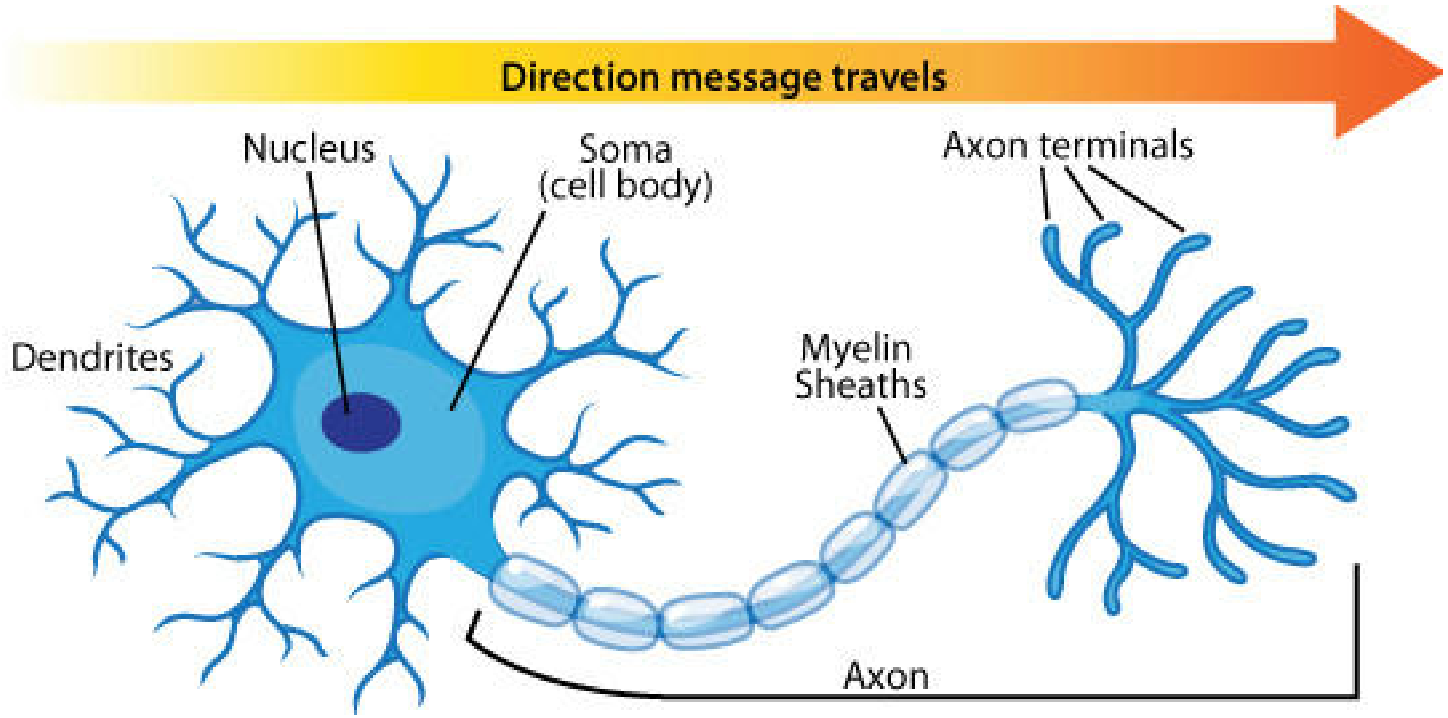
Neurons send messages based on messages received from other neurons

Biological Neurons



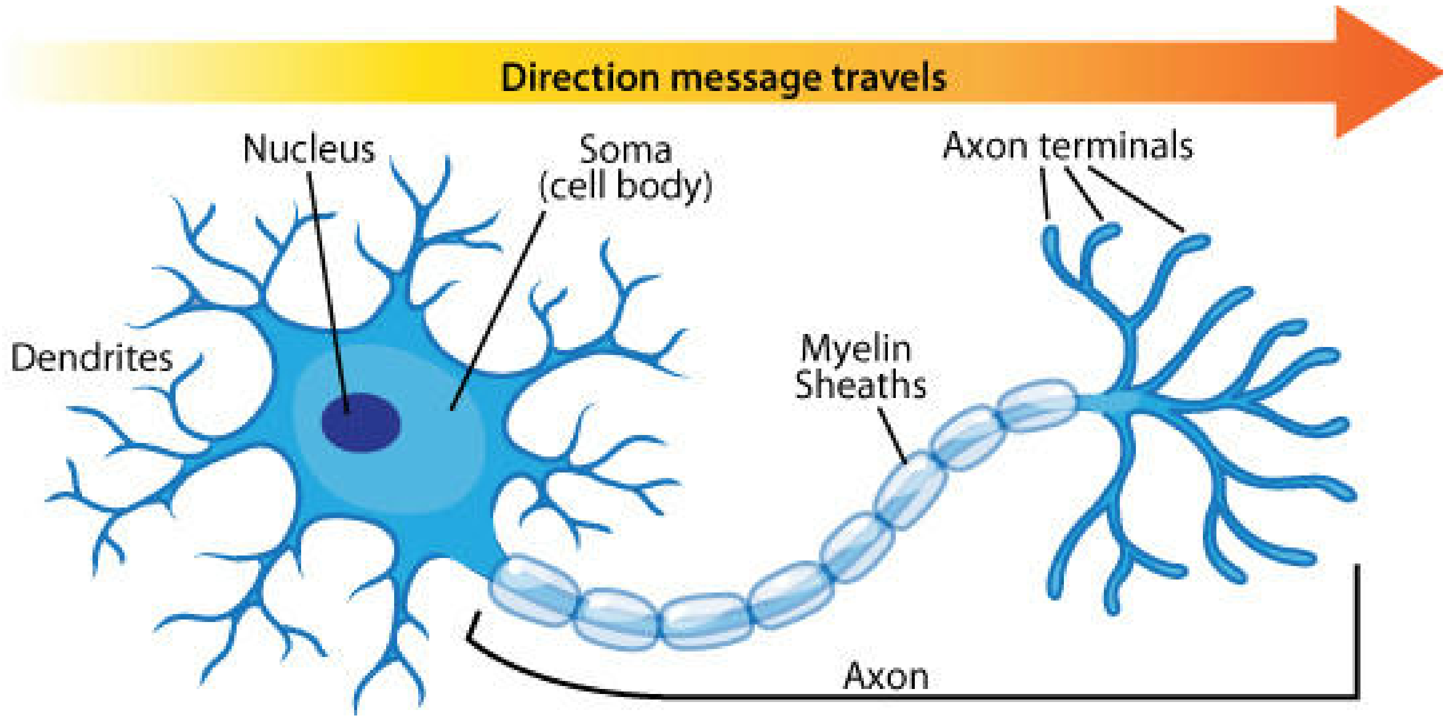
Incoming electrical signals travel along dendrites

Biological Neurons



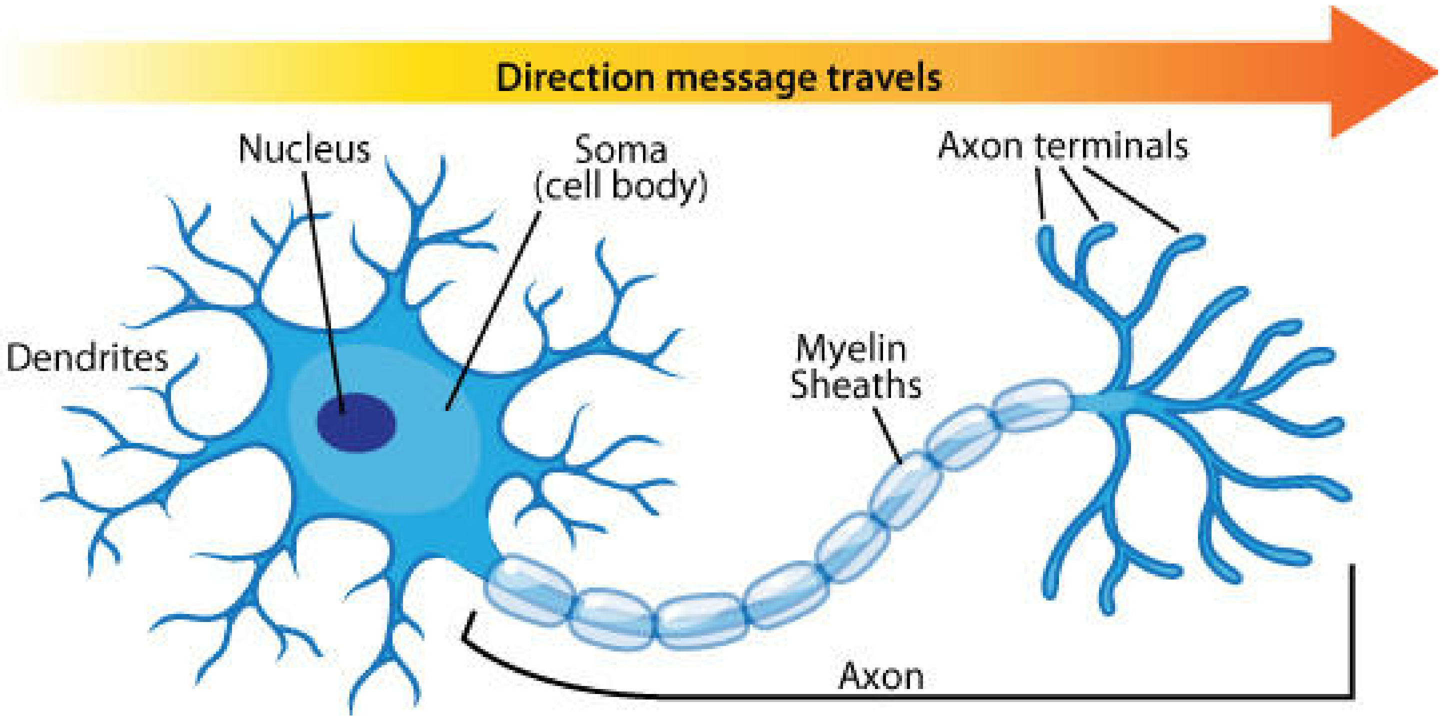
Dendrites are not all equal! Different dendrites have different diameters and structures

Biological Neurons



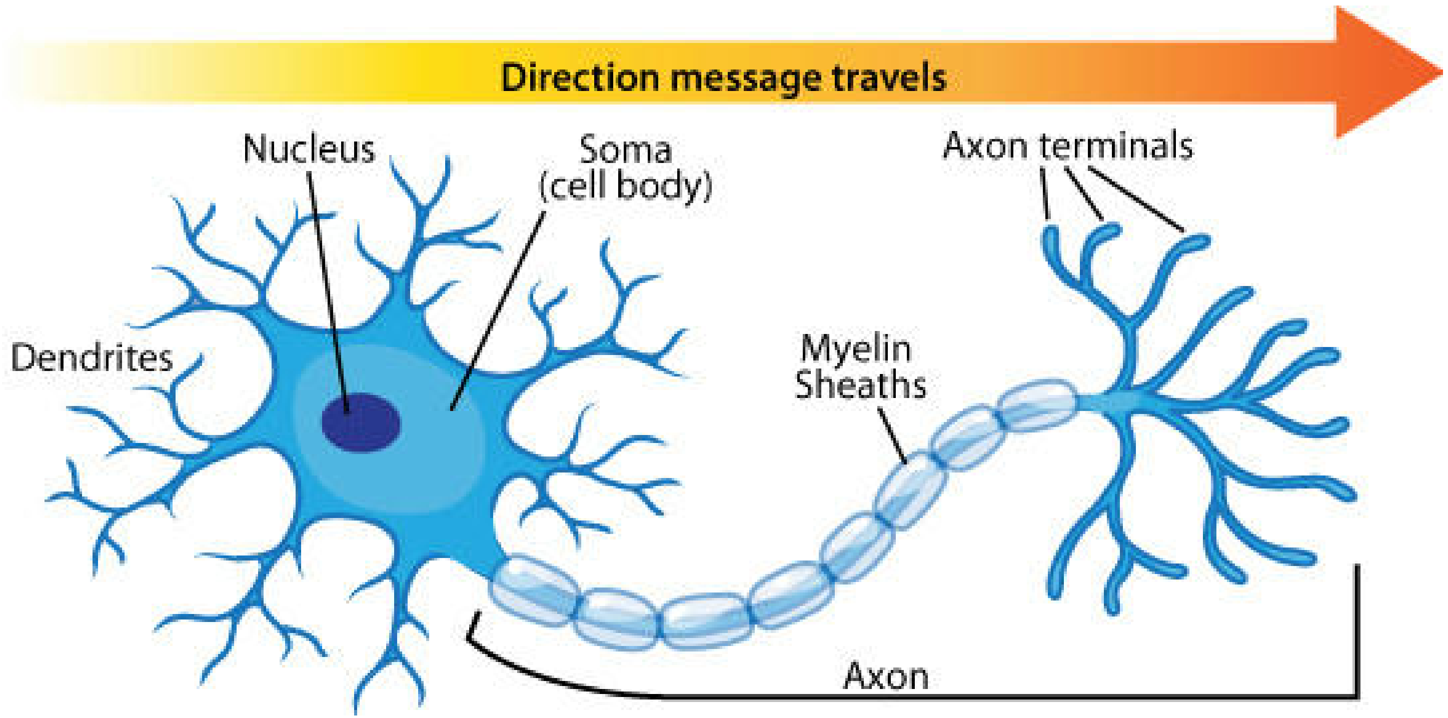
Electrical charges collect in the Soma (cell body)

Biological Neurons



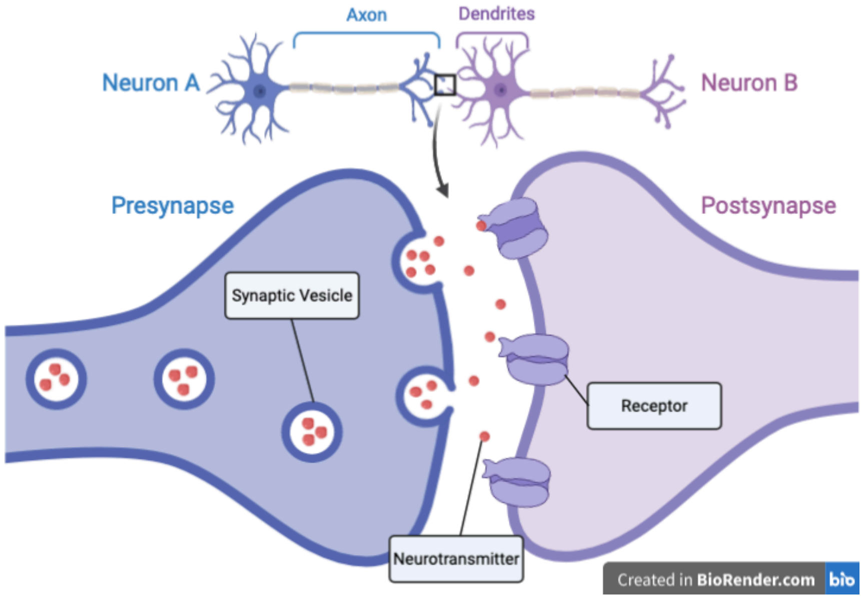
The axon outputs an electrical signal to other neurons

Biological Neurons



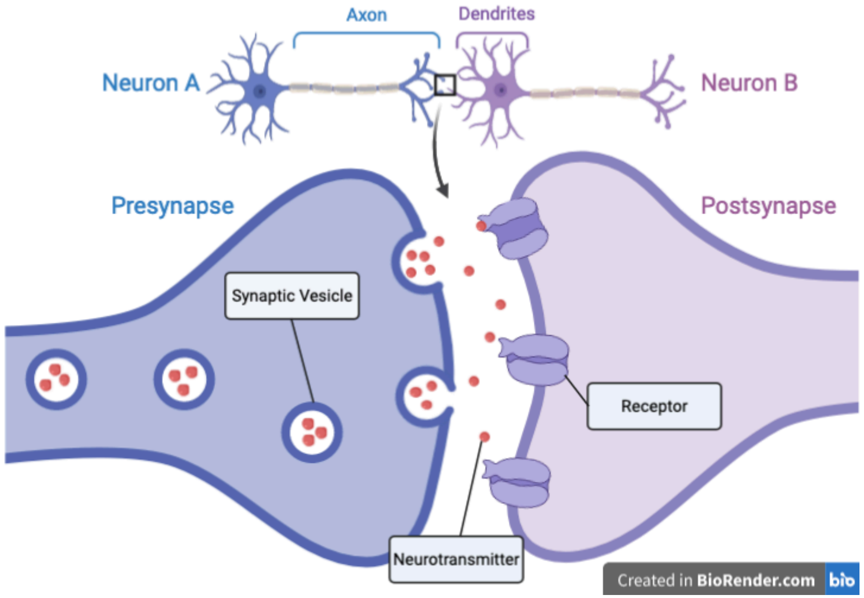
The axon terminals will connect to dendrites of other neurons through a synapse

Biological Neurons



The synapse converts electrical signal, to chemical signal, back to electrical signal

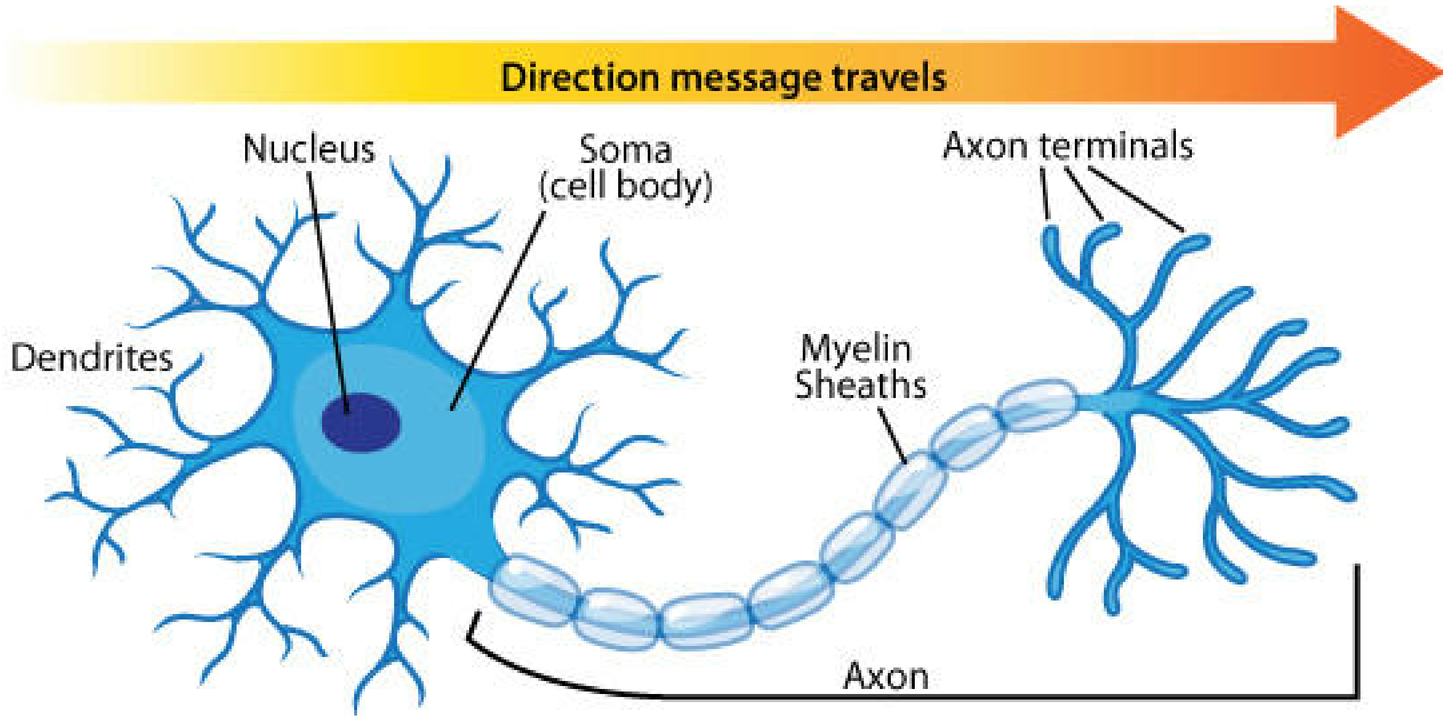
Biological Neurons



The synapse converts electrical signal, to chemical signal, back to electrical signal

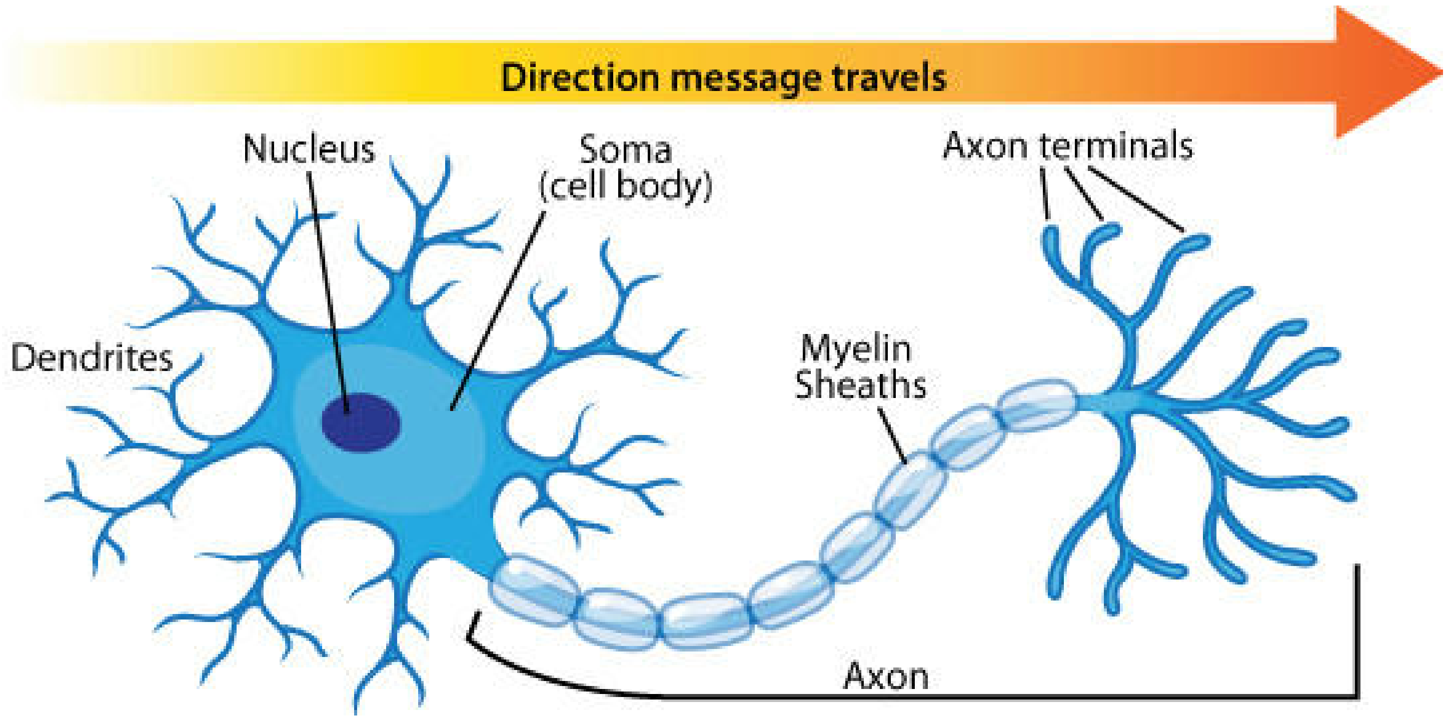
Synaptic weight determines how well a signal crosses the gap

Biological Neurons



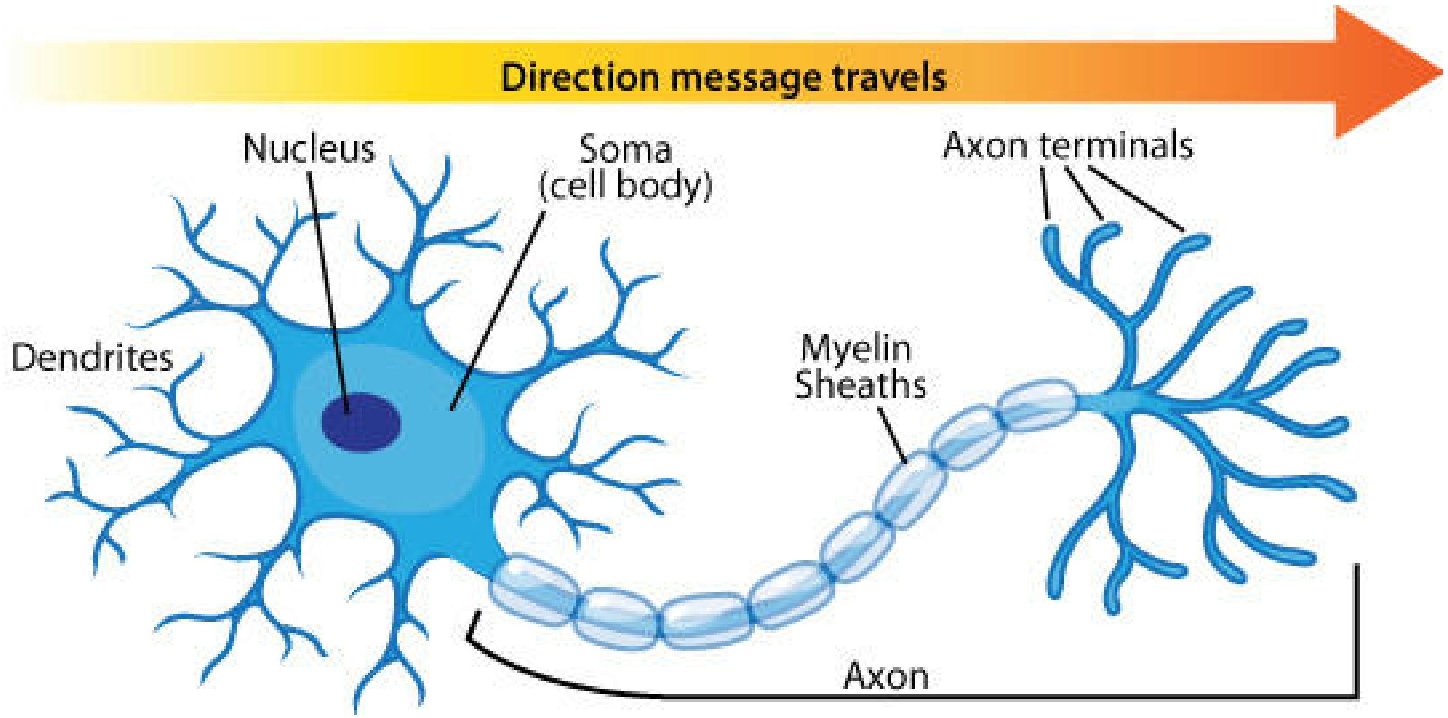
For our purposes, we can model the axon terminals, dendrites, and synapses to be one thing

Biological Neurons



The neuron takes many inputs, and produces a single output

Biological Neurons



The neuron will only output a signal down the axon ("fire") at certain times

Biological Neurons

How does a neuron decide to send an impulse ("fire")?

Biological Neurons

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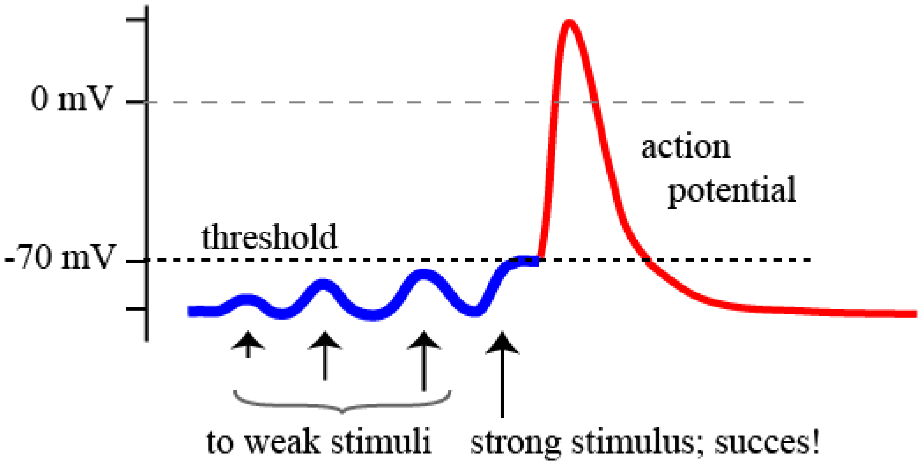
Incoming impulses (via dendrites) change the electric potential of the neuron



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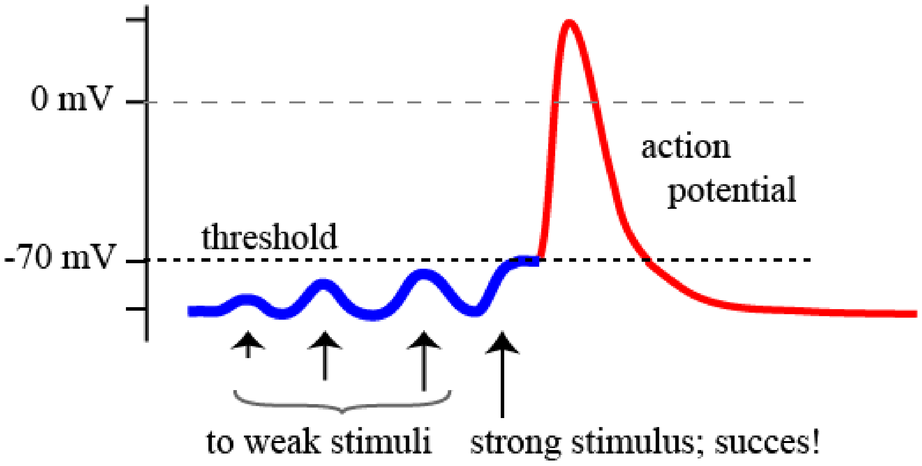

In a parallel circuit, we can sum voltages together

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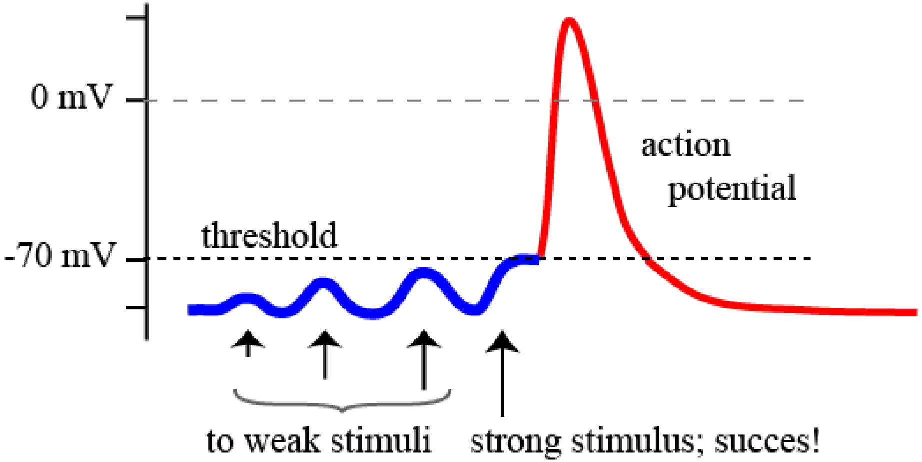
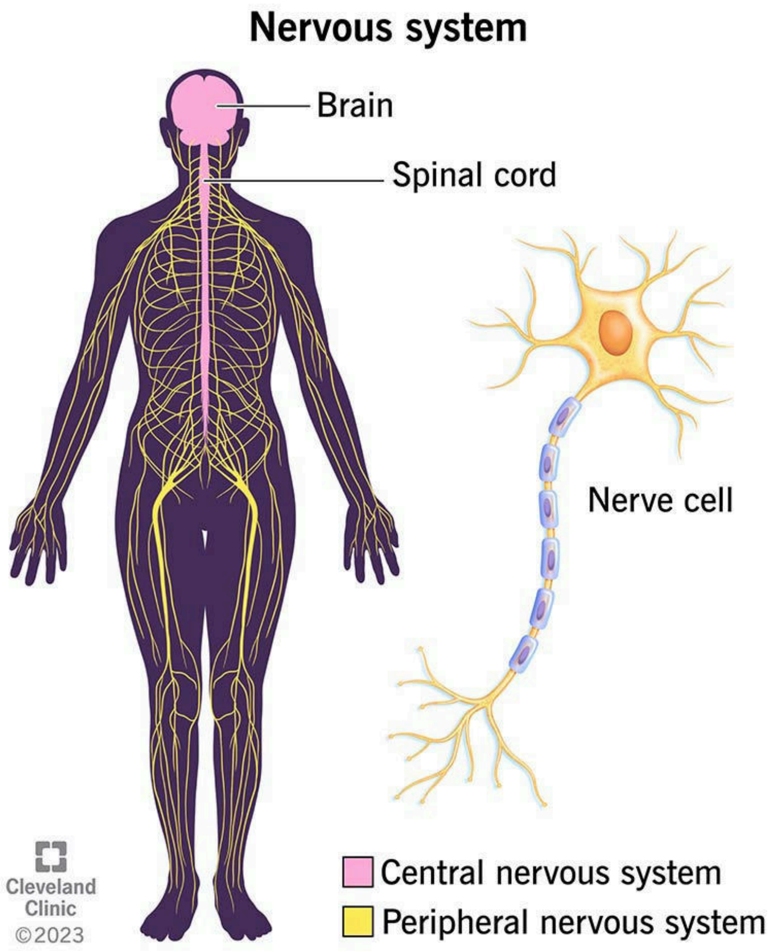


In a parallel circuit, we can sum voltages together

Many active dendrites will add together and trigger an impulse

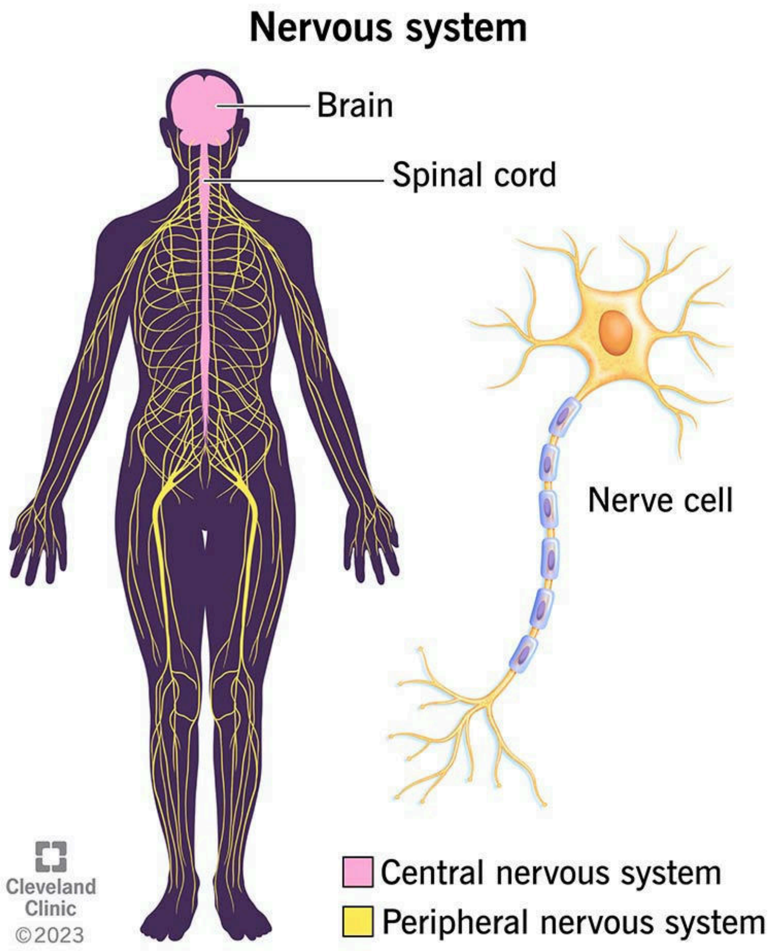
Biological Neurons

Pain triggers initial nerve impulse, starts a chain reaction into the brain



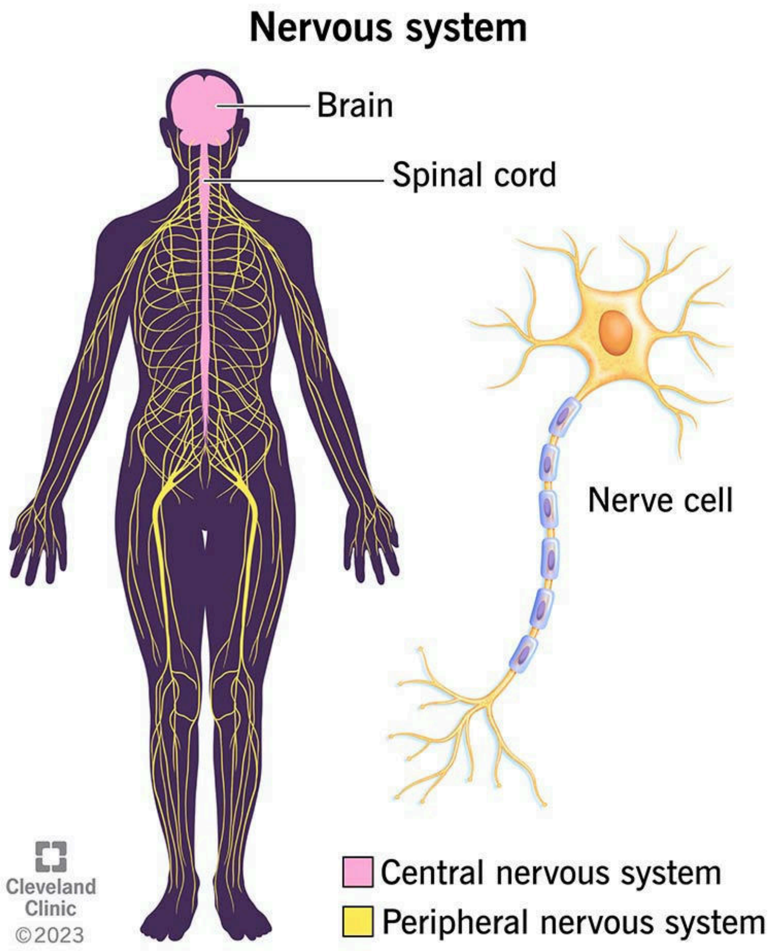
Biological Neurons

When the signal reaches the brain,
we will think



Biological Neurons

After thinking, we will take action



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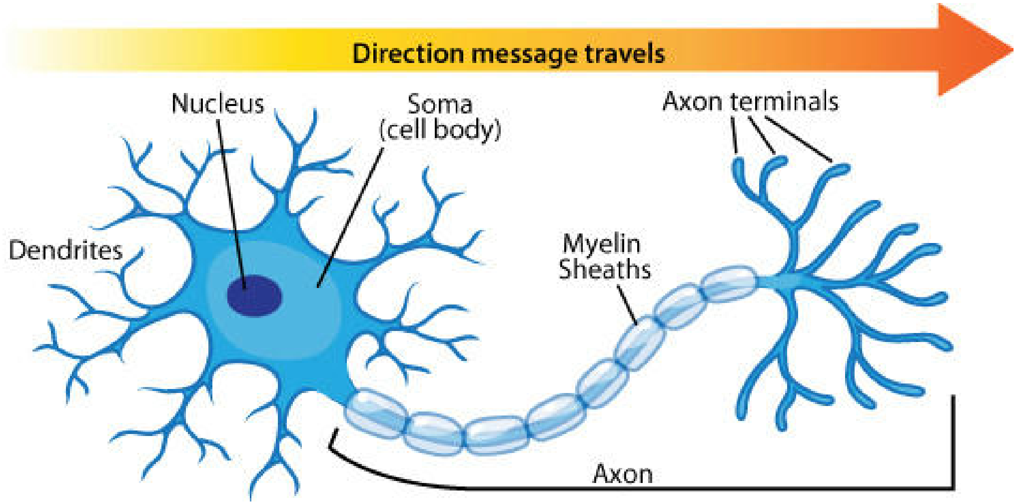
Artificial Neurons



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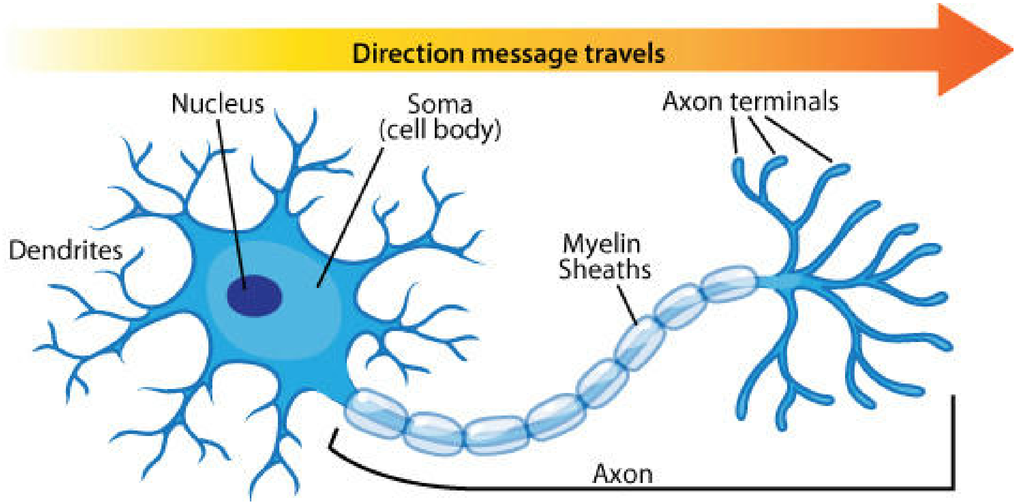
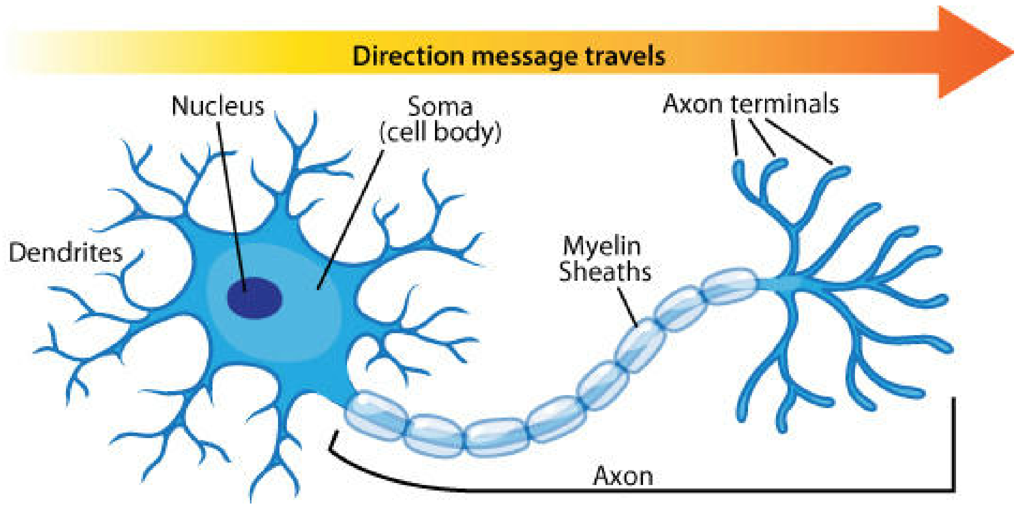


Question: How could we write a neuron as a function? $f : \_\_ \mapsto \_\_$

Artificial Neurons



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Answer:

$$f : \underbrace{\mathbb{R}^{d_x}}_{\text{Dendrite voltages}} \times \underbrace{\mathbb{R}^{d_x}}_{\text{Synaptic weight}} \mapsto \underbrace{\mathbb{R}}_{\text{Axon voltage}}$$

Artificial Neurons

Let us implement an artifical neuron as a function

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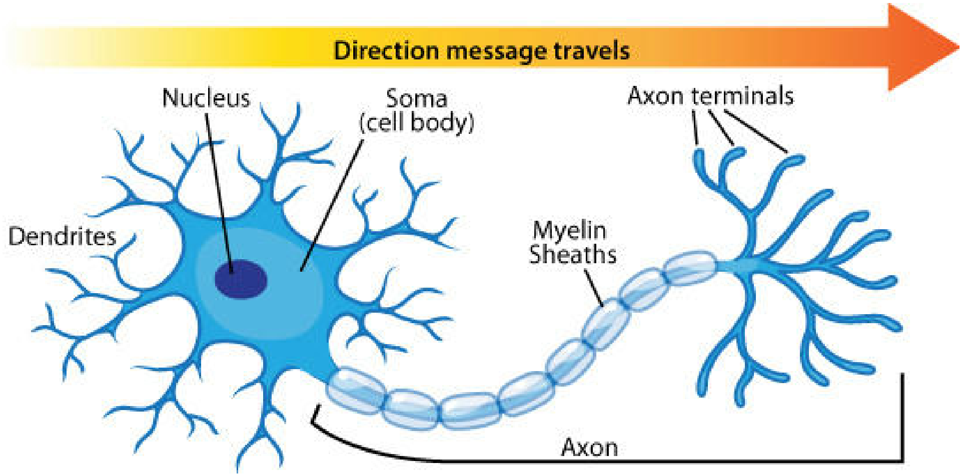

Neuron has a structure of dendrites with synaptic weights

Artificial Neurons

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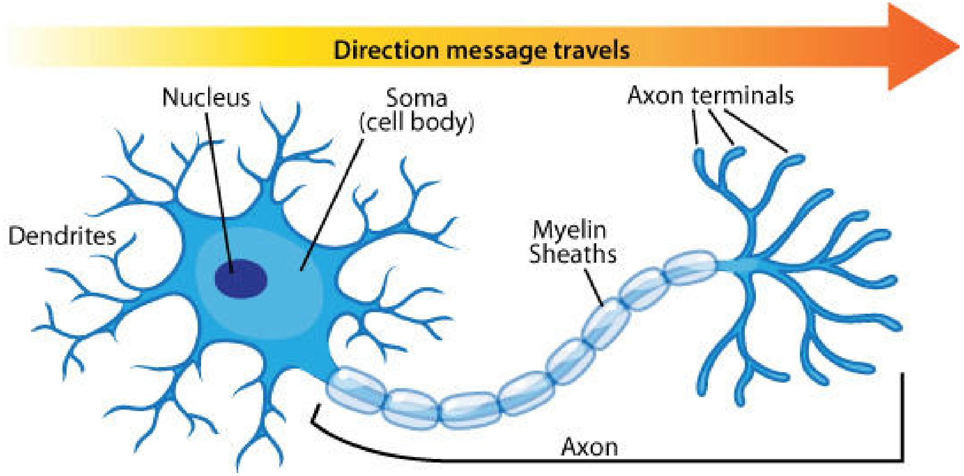


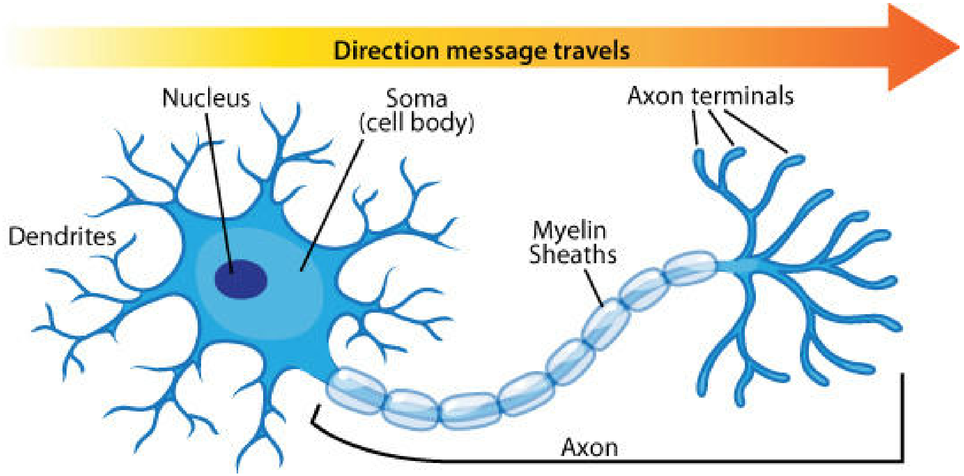
Neuron has a structure of dendrites with synaptic weights

$$f\left(\begin{bmatrix} \theta_1 \\ \theta_2 \\ \vdots \\ \theta_{d_x} \end{bmatrix}\right)$$

$$f(\boldsymbol{\theta})$$

Artificial Neurons

Let us implement an artifical neuron as a function



Each incoming dendrite has some voltage potential

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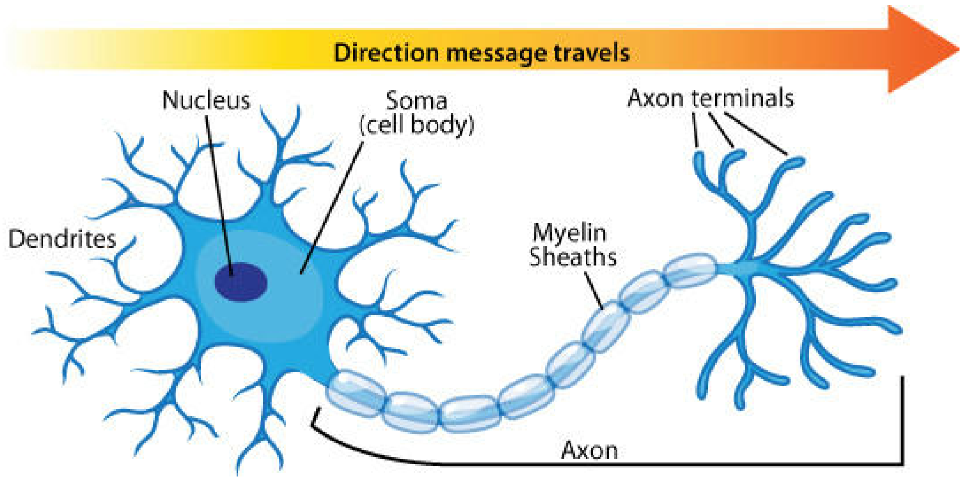

Each incoming dendrite has some voltage potential

$$f\left(\begin{bmatrix} x_1 \\ \vdots \\ x_{d_x} \end{bmatrix}, \begin{bmatrix} \theta_1 \\ \vdots \\ \theta_{d_x} \end{bmatrix}\right)$$

$$f(\boldsymbol{x}, \boldsymbol{\theta})$$

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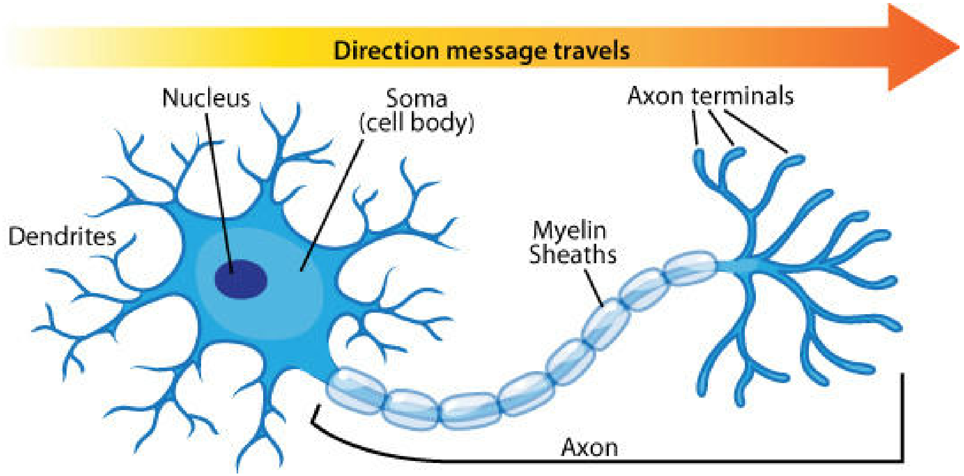

Voltage potentials sum together to give us the voltage in the cell body

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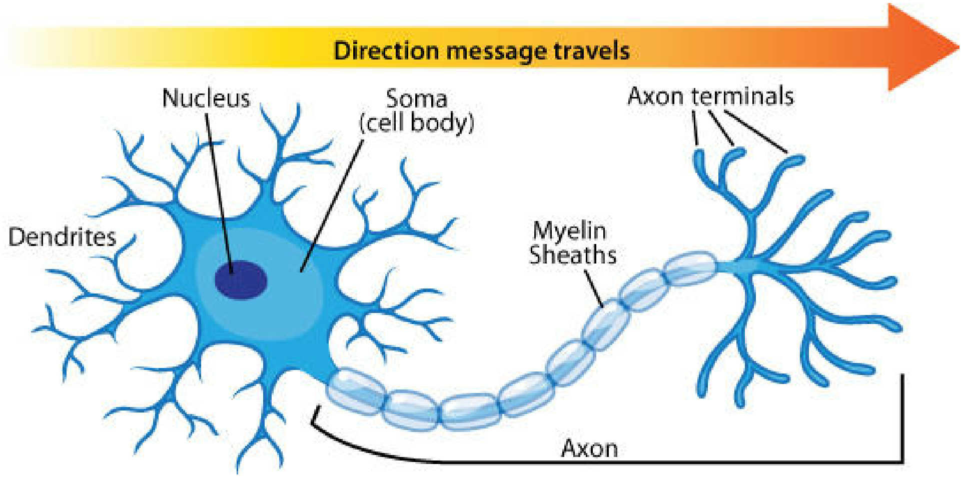

Voltage potentials sum together to give us the voltage in the cell body

$$f\left(\begin{bmatrix} x_1 \\ \vdots \\ x_{d_x} \end{bmatrix}, \begin{bmatrix} \theta_1 \\ \vdots \\ \theta_{d_x} \end{bmatrix}\right) = \sum_{j=1}^{d_x} \theta_j x_j$$

$$f(\boldsymbol{x}, \boldsymbol{\theta}) = \boldsymbol{\theta}^\top \boldsymbol{x}$$

Artificial Neurons

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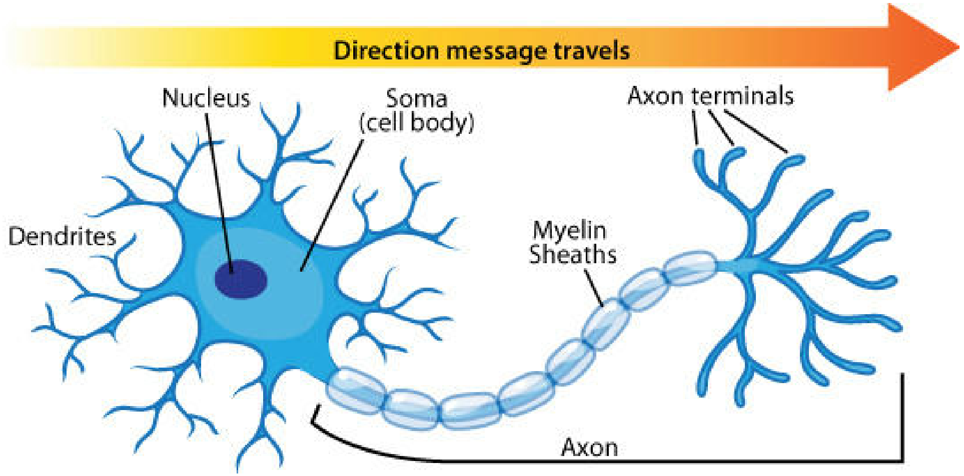

The axon fires only if the voltage is over a threshold

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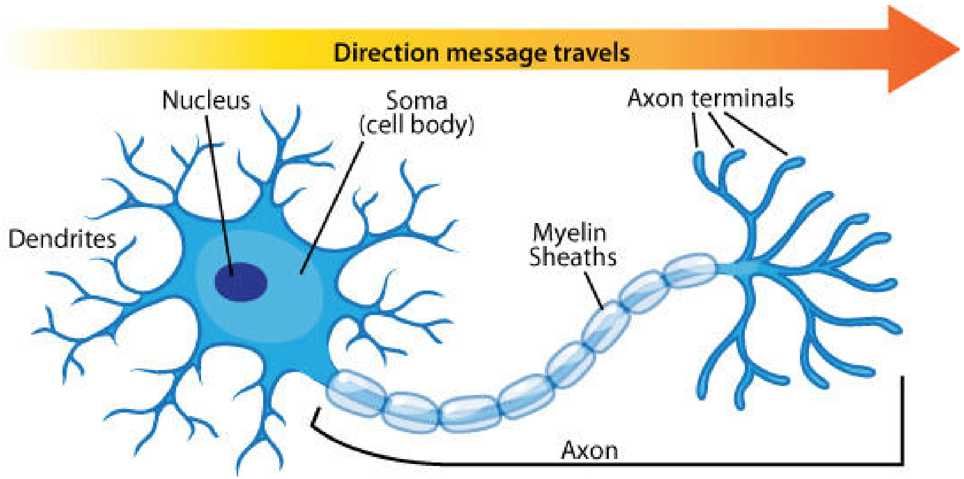

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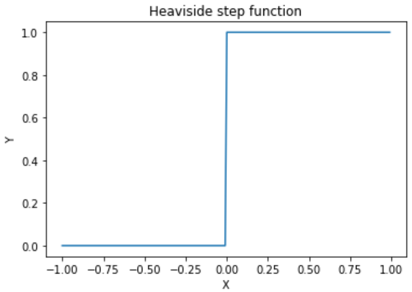
$$\sigma(x) = H(x) =$$



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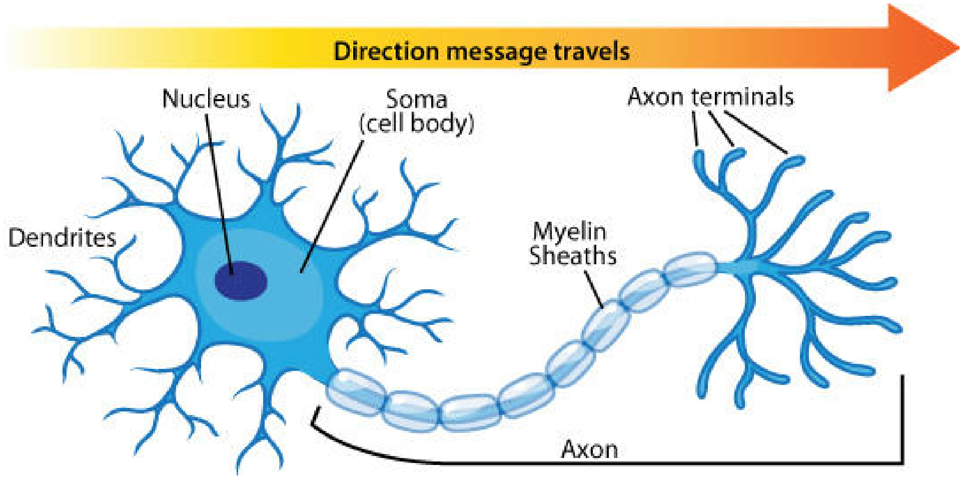

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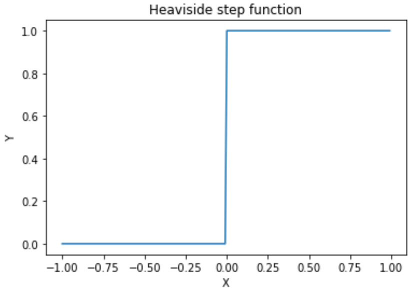
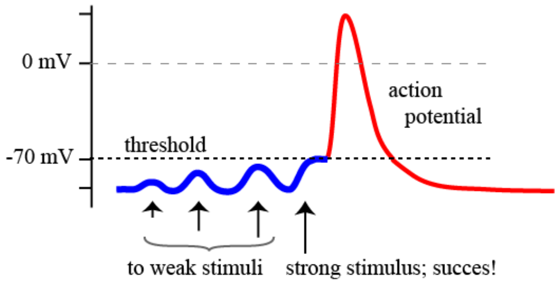
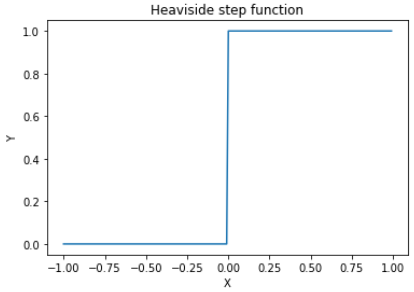


$$f\left(\begin{bmatrix} x_1 \\ \vdots \\ x_n \end{bmatrix}, \begin{bmatrix} \theta_1 \\ \vdots \\ \theta_n \end{bmatrix}\right) = \sigma\left(\sum_{j=1}^{d_x} \theta_j x_j\right)$$

Artificial Neurons

Maybe we want to vary the activation threshold





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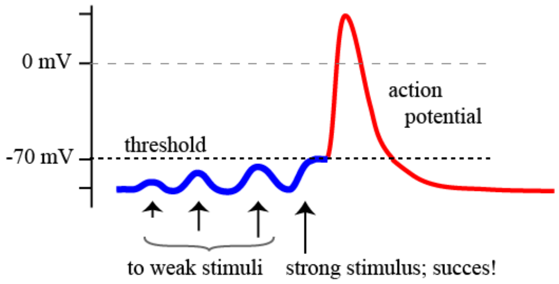
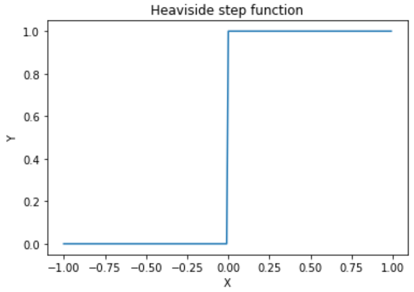
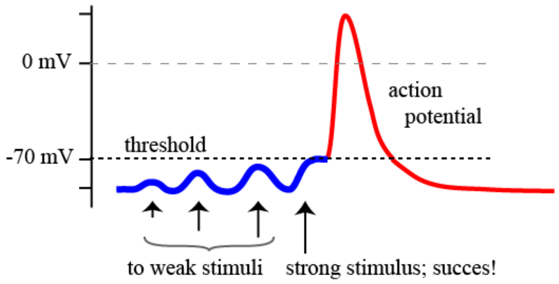


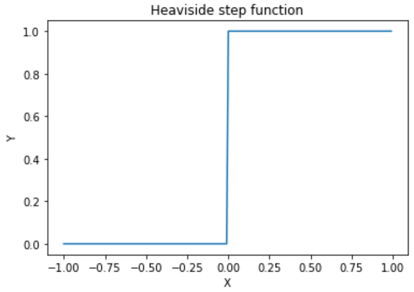


$$f\left(\begin{bmatrix} 1 \\ x_1 \\ \vdots \\ x_{d_x} \end{bmatrix}, \begin{bmatrix} \theta_0 \\ \theta_1 \\ \vdots \\ \theta_{d_x} \end{bmatrix}\right) = \sigma\left(\theta_0 + \sum_{j=1}^{d_x} \theta_j x_j\right) = \sigma\left(\sum_{j=0}^{d_x} \theta_j x_j\right)$$

Artificial Neurons

Maybe we want to vary the activation threshold





$$f\left(\begin{bmatrix} \color{red}{1} \\ x_1 \\ \vdots \\ x_{d_x} \end{bmatrix}, \begin{bmatrix} \color{red}{\theta_0} \\ \theta_1 \\ \vdots \\ \theta_{d_x} \end{bmatrix}\right) = \sigma\left(\color{red}{\theta_0} + \sum_{j=1}^{d_x} \theta_j x_j\right) = \sigma\left(\sum_{\color{red}{j=0}}^{d_x} \theta_j x_j\right)$$

$$\overline{\boldsymbol{x}} = \begin{bmatrix} 1 \\ \boldsymbol{x} \end{bmatrix}, \quad f(\boldsymbol{x}, \boldsymbol{\theta}) = \sigma(\boldsymbol{\theta}^\top \overline{\boldsymbol{x}})$$

Artificial Neurons

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Let us write out the full equation for a neuron

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$$f(\boldsymbol{x}, \boldsymbol{\theta}) = \sigma\left(\theta_0 1 + \theta_1 x_1 + \ldots + \theta_{d_x} x_{d_x}\right)$$

Artificial Neurons

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$$f(\boldsymbol{x}, \boldsymbol{\theta}) = \sigma\left(\theta_0 1 + \theta_1 x_1 + ... + \theta_{d_x} x_{d_x}\right)$$

Question: Does this look familiar to anyone?

Artificial Neurons

$$f(\boldsymbol{x}, \boldsymbol{\theta}) = \sigma(\boldsymbol{\theta}^\top \overline{\boldsymbol{x}})$$

This is the artificial neuron!

Let us write out the full equation for a neuron

$$f(\boldsymbol{x}, \boldsymbol{\theta}) = \sigma\left(\theta_0 1 + \theta_1 x_1 + \ldots + \theta_{d_x} x_{d_x}\right)$$

Question: Does this look familiar to anyone?

Answer: Inside σ is the multivariate linear model!

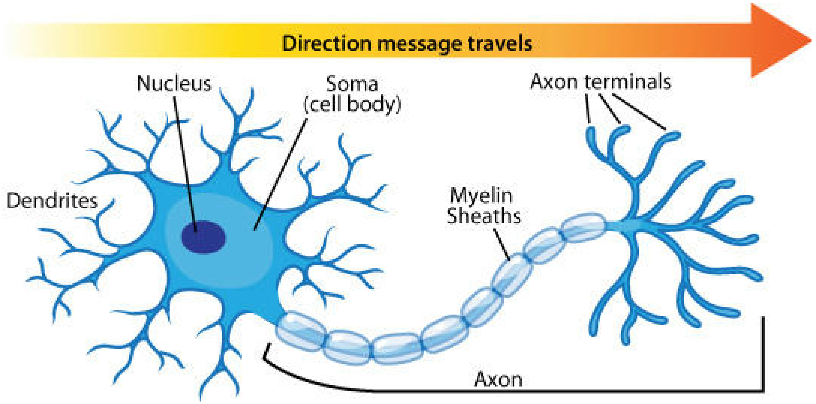
$$f(\boldsymbol{x}, \boldsymbol{\theta}) = \theta_{d_x} x_{d_x} + \theta_{d_x - 1} x_{d_x - 1} + \ldots + \theta_0 1$$

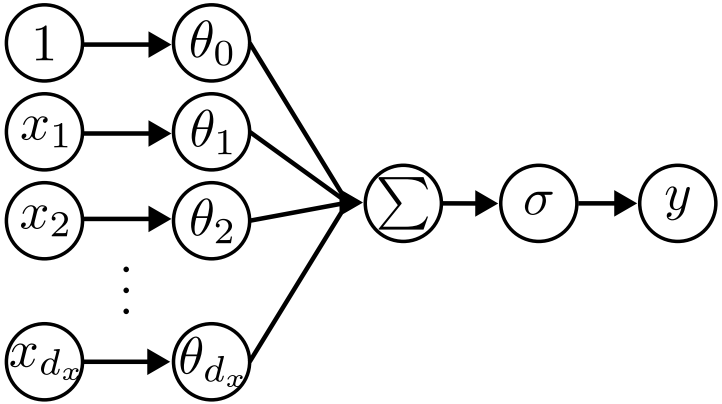
Artificial Neurons

We model a neuron using a linear model and activation function

Artificial Neurons

We model a neuron using a linear model and activation function





$$f(\boldsymbol{x}, \boldsymbol{\theta}) = \sigma(\boldsymbol{\theta}^\top \overline{\boldsymbol{x}})$$

Artificial Neurons

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Sometimes, we will write $\boldsymbol{\theta}$ as a bias and weight $b, \boldsymbol{w}$

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Sometimes, we will write $\boldsymbol{\theta}$ as a bias and weight $b, \boldsymbol{w}$

$$\boldsymbol{\theta} = \begin{bmatrix} b \\ \boldsymbol{w} \end{bmatrix}; \quad \begin{bmatrix} \theta_0 \\ \theta_1 \\ \vdots \\ \theta_{d_x} \end{bmatrix} = \begin{bmatrix} b \\ w_1 \\ \vdots \\ w_{d_x} \end{bmatrix}$$

Artificial Neurons

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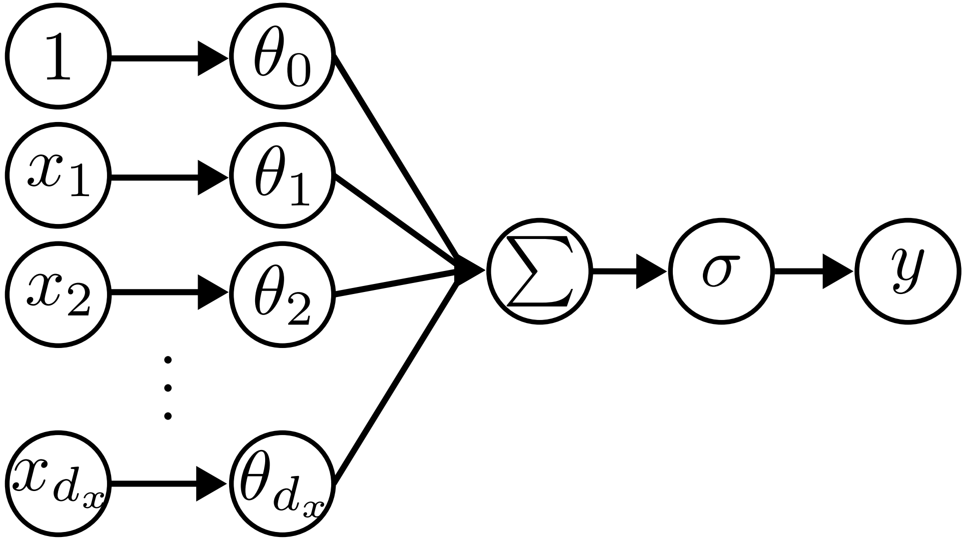
Sometimes, we will write $\boldsymbol{\theta}$ as a bias and weight $b, \boldsymbol{w}$

$$\boldsymbol{\theta} = \begin{bmatrix} b \\ \boldsymbol{w} \end{bmatrix}; \quad \begin{bmatrix} \theta_0 \\ \theta_1 \\ \vdots \\ \theta_{d_x} \end{bmatrix} = \begin{bmatrix} b \\ w_1 \\ \vdots \\ w_{d_x} \end{bmatrix}$$

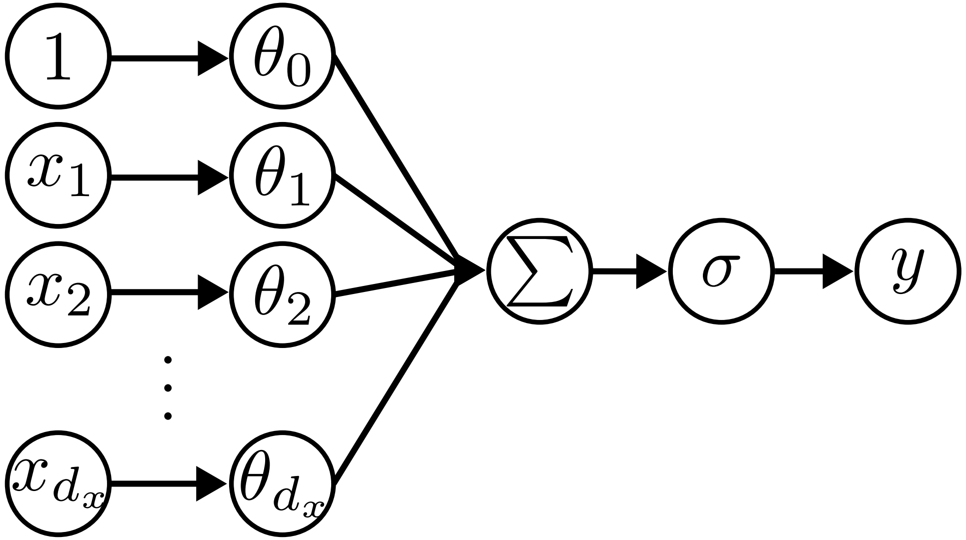
$$f\left(\boldsymbol{x}, \begin{bmatrix} b \\ \boldsymbol{w} \end{bmatrix}\right) = b + \boldsymbol{w}^\top \boldsymbol{x}$$

Relax

Artificial Neurons

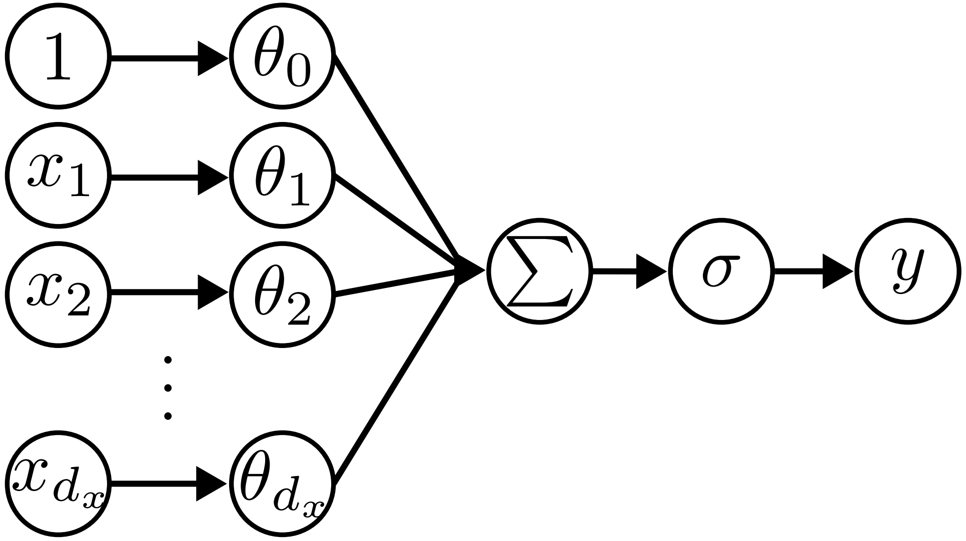


Artificial Neurons



In machine learning, we represent functions

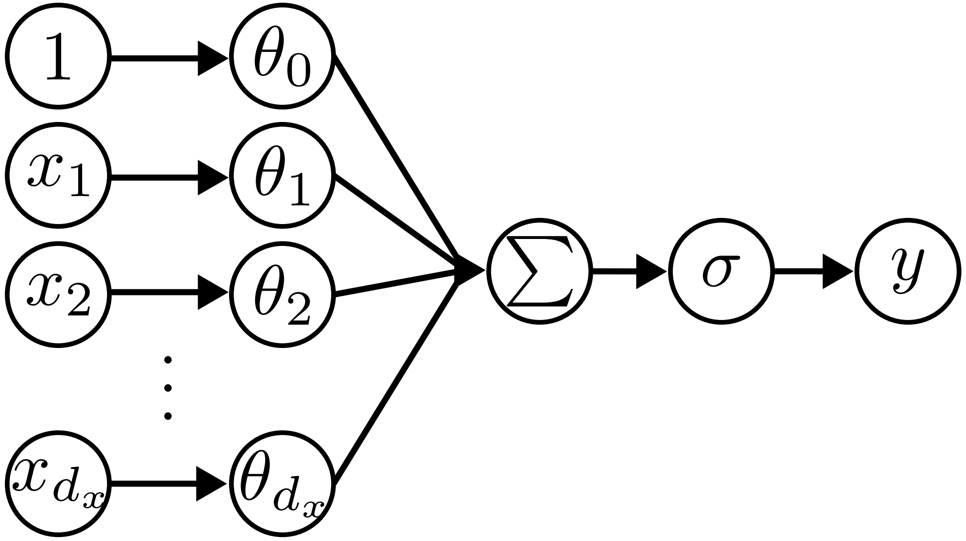
Artificial Neurons



In machine learning, we represent functions

What kinds of functions can our neuron represent?

Artificial Neurons



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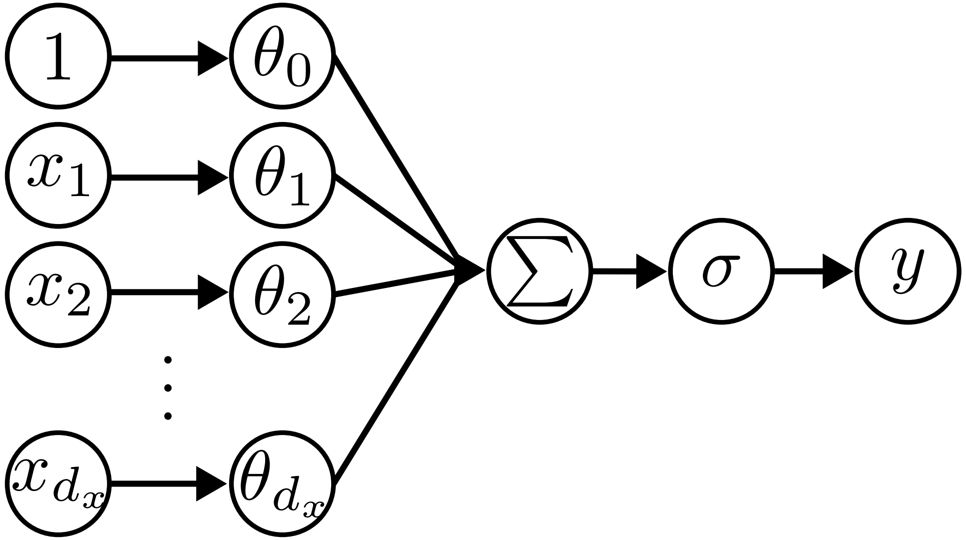
What kinds of functions can our neuron represent?

Let us consider some **boolean** functions

Artificial Neurons



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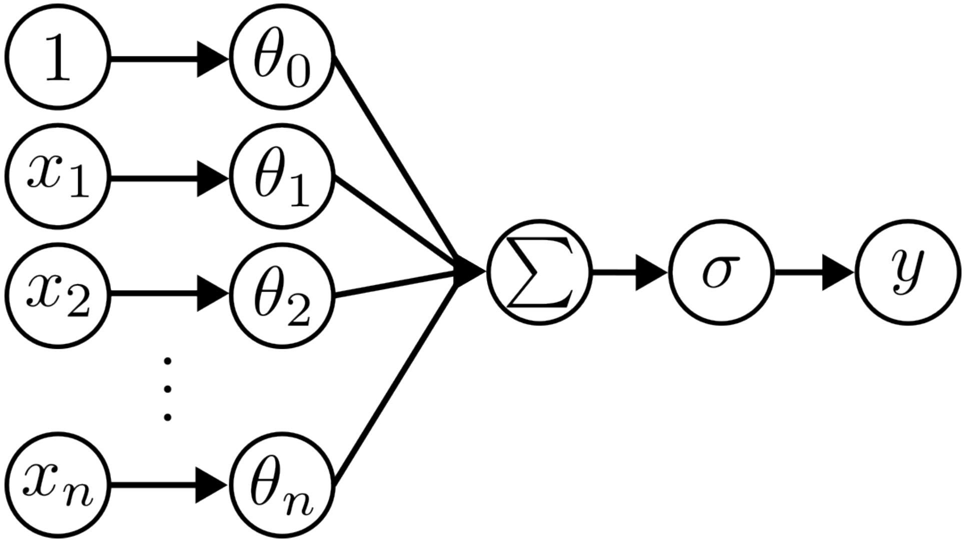
Let us start with a logical AND function

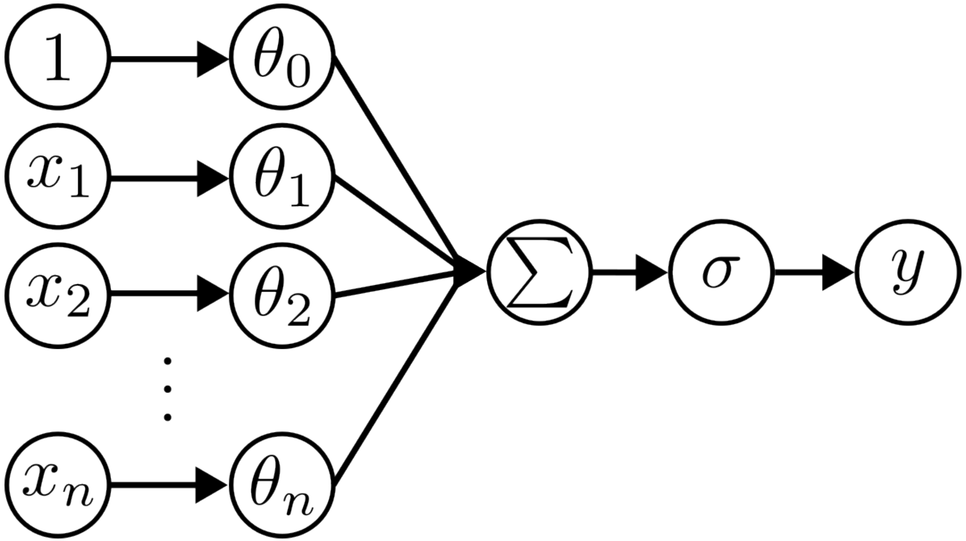
Artificial Neurons



Review: Activation function

(Heaviside step function)

Artificial Neurons



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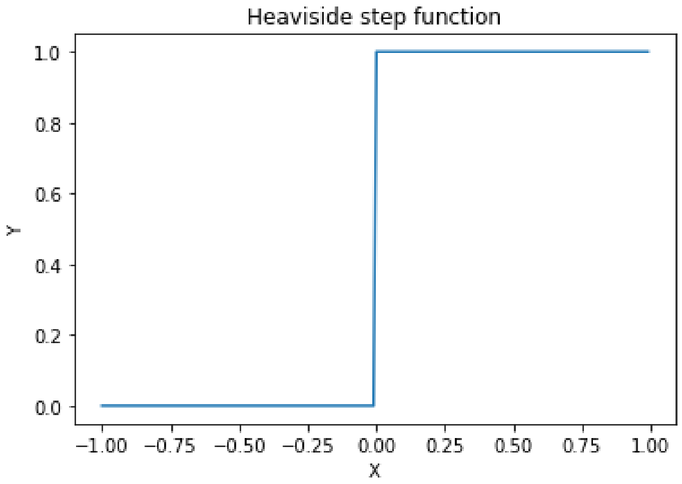

$$\sigma(x) = H(x) = \begin{cases} 1 \text{ if } x > 0 \\ 0 \text{ if } x \leq 0 \end{cases}$$

Artificial Neurons

Implement AND using an artificial neuron

Artificial Neurons

Implement AND using an artificial neuron

$$f\left(\begin{bmatrix} x_1 & x_2 \end{bmatrix}^\top, \begin{bmatrix} \theta_0 & \theta_1 & \theta_2 \end{bmatrix}^\top\right) = \sigma(\theta_0 1 + \theta_1 x_1 + \theta_2 x_2)$$

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Implement AND using an artificial neuron

$$f\left(\begin{bmatrix} x_1 & x_2 \end{bmatrix}^\top, \begin{bmatrix} \theta_0 & \theta_1 & \theta_2 \end{bmatrix}^\top\right) = \sigma(\theta_0 1 + \theta_1 x_1 + \theta_2 x_2)$$

$$\boldsymbol{\theta} = \begin{bmatrix} \theta_0 & \theta_1 & \theta_2 \end{bmatrix}^\top = \begin{bmatrix} -1 & 1 & 1 \end{bmatrix}^\top$$

Artificial Neurons

Implement AND using an artificial neuron

$$f\left([x_1 \;\; x_2]^\top, [\theta_0 \;\; \theta_1 \;\; \theta_2]^\top\right) = \sigma(\theta_0 1 + \theta_1 x_1 + \theta_2 x_2)$$

$$\boldsymbol{\theta} = [\theta_0 \;\; \theta_1 \;\; \theta_2]^\top = [-1 \;\; 1 \;\; 1]^\top$$

x_1	x_2	y	$f(x_1, x_2, \boldsymbol{\theta})$	$\hat{y}$
0	0	0	$\sigma(-1 \cdot 1 + 1 \cdot 0 + 1 \cdot 0) = \sigma(-1)$	0
0	1	0	$\sigma(-1 \cdot 1 + 1 \cdot 0 + 1 \cdot 1) = \sigma(0)$	0
1	0	0	$\sigma(-1 \cdot 1 + 1 \cdot 1 + 1 \cdot 0) = \sigma(0)$	0
1	1	1	$\sigma(-1 \cdot 1 + 1 \cdot 1 + 1 \cdot 1) = \sigma(1)$	1

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Implement OR using an artificial neuron

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x_1	x_2	y	$f(x_1, x_2, \boldsymbol{\theta})$	$\hat{y}$
0	0	0	$\sigma(1 \cdot 0 + 1 \cdot 0 + 1 \cdot 0) = \sigma(0)$	0
0	1	0	$\sigma(1 \cdot 0 + 1 \cdot 1 + 1 \cdot 0) = \sigma(1)$	1
1	0	1	$\sigma(1 \cdot 0 + 1 \cdot 0 + 1 \cdot 1) = \sigma(1)$	1
1	1	1	$\sigma(1 \cdot 0 + 1 \cdot 1 + 1 \cdot 1) = \sigma(2)$	1

Artificial Neurons

Implement XOR using an artificial neuron

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$$f\left(\begin{bmatrix} x_1 & x_2 \end{bmatrix}^\top, \begin{bmatrix} \theta_0 & \theta_1 & \theta_2 \end{bmatrix}^\top\right) = \sigma(\theta_0 1 + \theta_1 x_2 + \theta_2 x_2)$$

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$$\boldsymbol{\theta} = \begin{bmatrix} \theta_0 & \theta_1 & \theta_2 \end{bmatrix}^\top = \begin{bmatrix} ? & ? & ? \end{bmatrix}^\top$$

Artificial Neurons

Implement XOR using an artificial neuron

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$$\boldsymbol{\theta} = \begin{bmatrix} \theta_0 & \theta_1 & \theta_2 \end{bmatrix}^\top = \begin{bmatrix} ? & ? & ? \end{bmatrix}^\top$$

x_1	x_2	y	$f(x_1, x_2, \boldsymbol{\theta})$	$\hat{y}$
0	0	0	This is IMPOSSIBLE!	
0	1	1		
1	0	1		
1	1	0		

Artificial Neurons

Why can't we represent XOR using a neuron?

Artificial Neurons

Why can't we represent XOR using a neuron?

$$f\left([x_1 \;\; x_2]^\top, [\theta_0 \;\; \theta_1 \;\; \theta_2]^\top\right) = \sigma(1\theta_0 + x_1\theta_1 + x_2\theta_2)$$

Artificial Neurons

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We can only represent $\sigma(\text{linear function})$

Artificial Neurons

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XOR is not a linear combination of x_1, x_2!

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We want to represent any function, not just linear functions

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Let us think back to biology, maybe it has an answer

Artificial Neurons

Brain: Biological neurons $\rightarrow$ Biological neural network

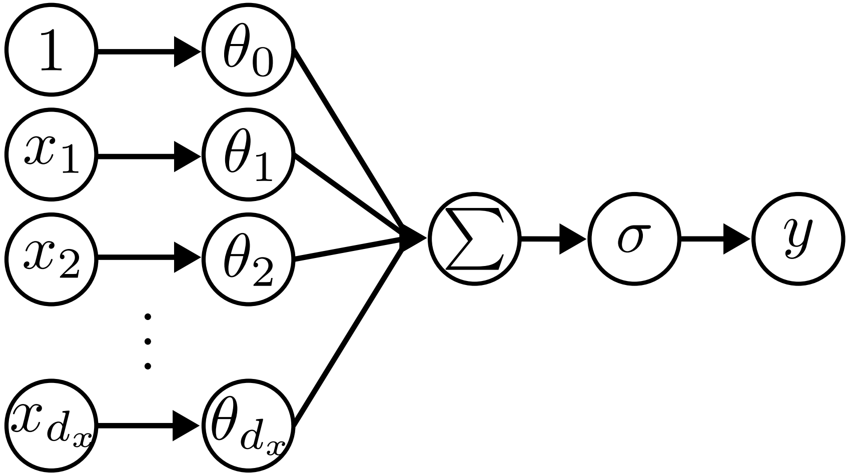
Artificial Neurons

Brain: Biological neurons $\rightarrow$ Biological neural network

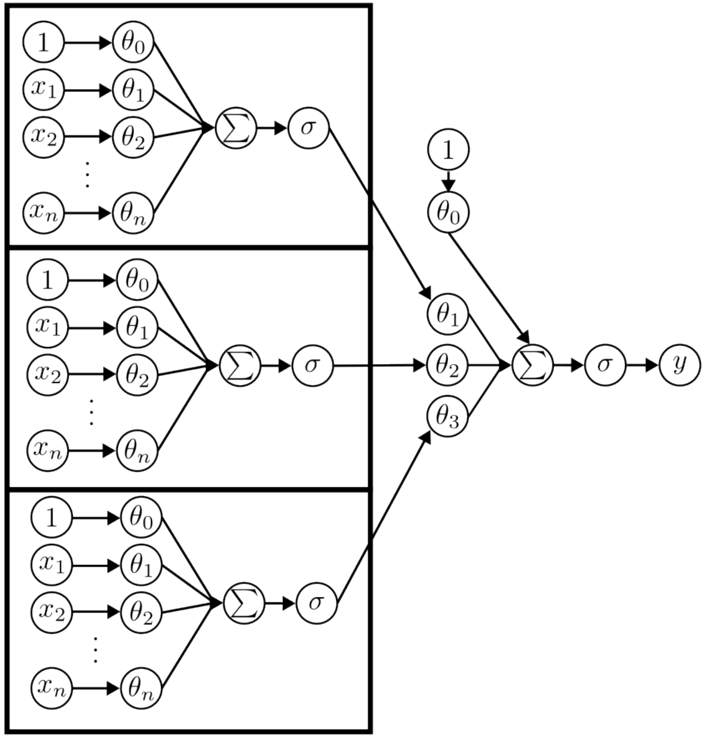
Computer: Artificial neurons $\rightarrow$ Artificial neural network

Artificial Neurons

Connect artificial neurons into a network

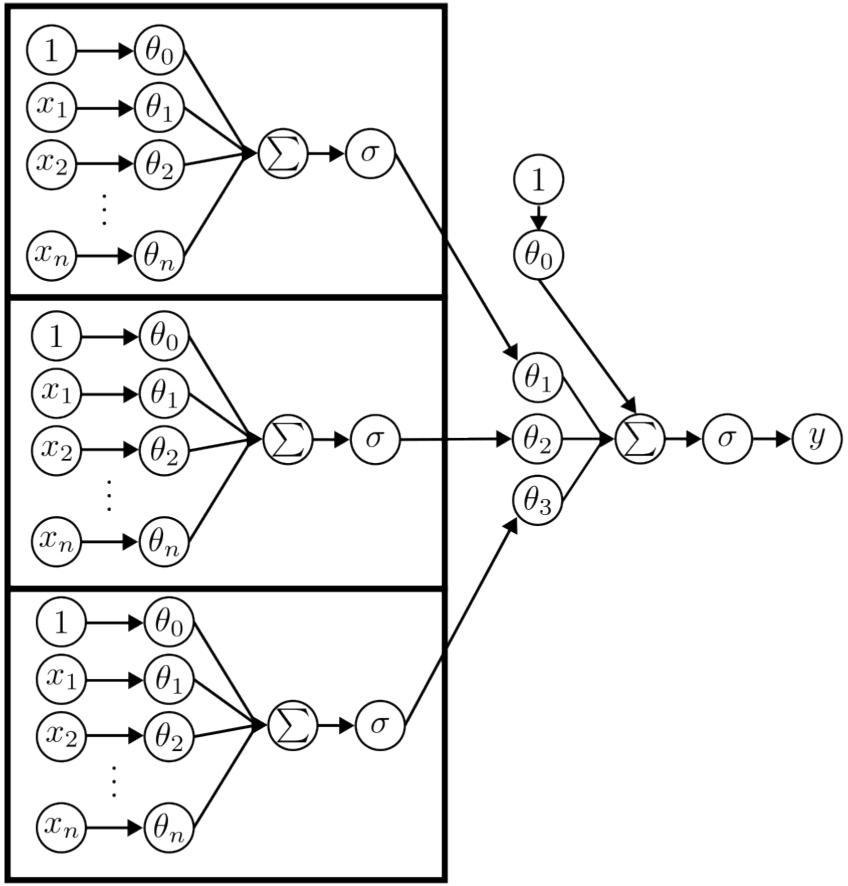


Neuron



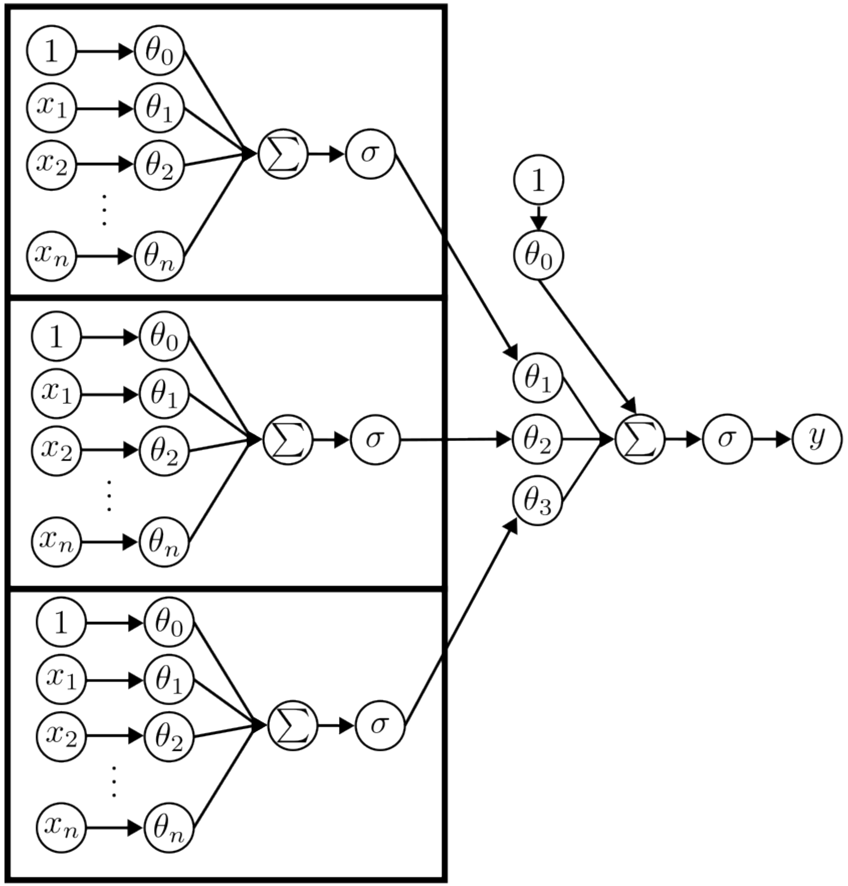
Neural Network

Artificial Neurons



Adding neurons in **parallel** creates a **wide** neural network

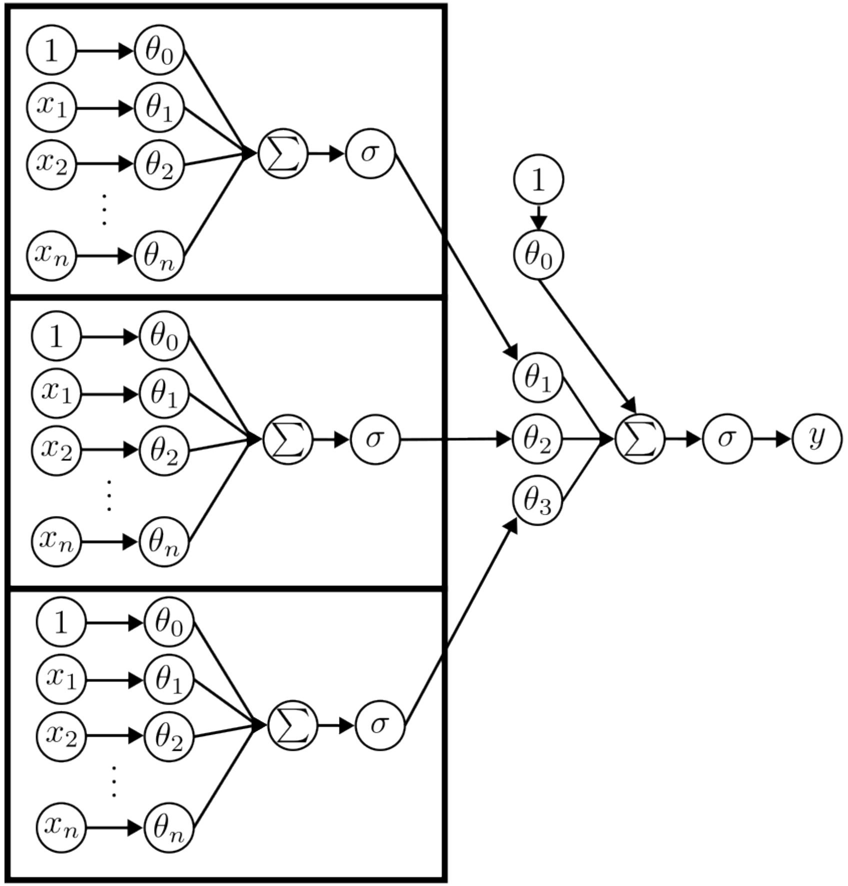
Artificial Neurons



Adding neurons in **parallel** creates a **wide** neural network

Adding neurons in **series** creates a **deep** neural network

Artificial Neurons

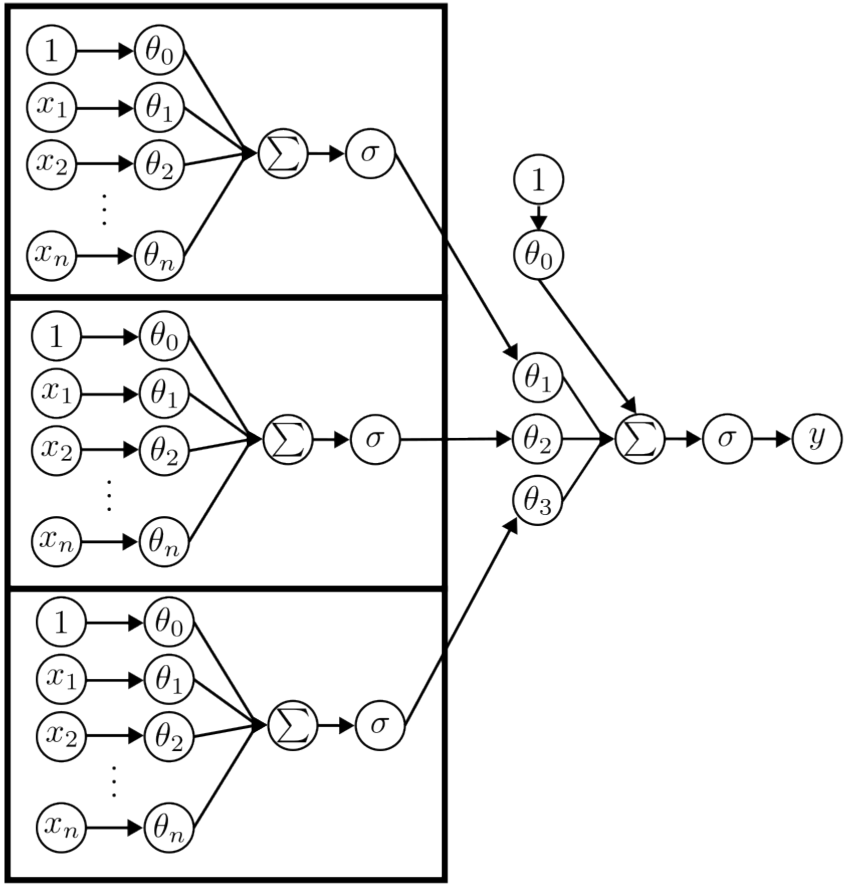


Adding neurons in **parallel** creates a **wide** neural network

Adding neurons in **series** creates a **deep** neural network

Today's powerful neural networks are both **wide** and **deep**

Artificial Neurons



Adding neurons in **parallel** creates a **wide** neural network

Adding neurons in **series** creates a **deep** neural network

Today's powerful neural networks are both **wide** and **deep**

Let us try to implement XOR using a wide and deep neural network

Agenda

1. Review
2. Multivariate linear regression
3. Limitations of linear regression
4. History of neural networks
5. Biological neurons
6. **Artificial neurons**
7. Wide neural networks
8. Deep neural networks
9. Practical considerations

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Wide Neural Networks

How do we express a **wide** neural network mathematically?

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d_y neurons (wide):

$$f : \mathbb{R}^{d_x} \times \Theta \mapsto \mathbb{R}^{d_y}$$

$$\Theta \in \mathbb{R}^{(d_x + 1) \times d_y}$$

Wide Neural Networks

For a single neuron:

$$f\left(\begin{bmatrix} x_1 \\ \vdots \\ x_{d_x} \end{bmatrix}, \begin{bmatrix} \theta_0 \\ \theta_1 \\ \vdots \\ \theta_{d_x} \end{bmatrix}\right) = \sigma\left(\sum_{i=0}^{d_x} \theta_i \overline{x}_i\right)$$

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$$f(\boldsymbol{x}, \boldsymbol{\theta}) = \sigma(b + \boldsymbol{w}^\top \boldsymbol{x})$$

For a wide network:

$$f\left(\begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_{d_x} \end{bmatrix}, \begin{bmatrix} \theta_{0,1} & \theta_{0,2} & \dots & \theta_{0,d_y} \\ \theta_{1,1} & \theta_{1,2} & \dots & \theta_{1,d_y} \\ \vdots & \vdots & \ddots & \vdots \\ \theta_{d_x,1} & \theta_{d_x,2} & \dots & \theta_{d_x,d_y} \end{bmatrix}\right) = \begin{bmatrix} \sigma\left(\sum_{i=0}^{d_x} \theta_{i,1} \overline{x}_i\right) \\ \sigma\left(\sum_{i=0}^{d_x} \theta_{i,2} \overline{x}_i\right) \\ \vdots \\ \sigma\left(\sum_{i=0}^{d_x} \theta_{i,d_y} \overline{x}_i\right) \end{bmatrix}$$

$$f(\boldsymbol{x}, \boldsymbol{\theta}) = \sigma(\boldsymbol{\theta}^\top \overline{\boldsymbol{x}}); \quad \boldsymbol{\theta}^\top \in \mathbb{R}^{d_y \times (d_x+1)}$$

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2. Multivariate linear regression
3. Limitations of linear regression
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6. Artificial neurons
7. **Wide neural networks**
8. Deep neural networks
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Deep Neural Networks

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$$\boldsymbol{\theta} = \begin{bmatrix} \boldsymbol{\theta}_1 & \boldsymbol{\theta}_2 & \ldots & \boldsymbol{\theta}_\ell \end{bmatrix}^\top = \begin{bmatrix} \boldsymbol{\varphi} & \boldsymbol{\psi} & \ldots & \boldsymbol{\xi} \end{bmatrix}^\top$$

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A deep network has many internal functions

$$f_1(\boldsymbol{x}, \boldsymbol{\varphi}) = \boldsymbol{\varphi}^\top \overline{\boldsymbol{x}} \quad f_2(\boldsymbol{x}, \boldsymbol{\psi}) = \boldsymbol{\psi}^\top \overline{\boldsymbol{x}} \quad \dots \quad f_\ell(\boldsymbol{x}, \boldsymbol{\xi}) = \boldsymbol{\xi}^\top \overline{\boldsymbol{x}}$$

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$$f(\boldsymbol{x}, \boldsymbol{\theta}) = f_\ell(\ldots f_2(f_1(\boldsymbol{x}, \boldsymbol{\varphi}), \boldsymbol{\psi}) \ldots \boldsymbol{\xi})$$

Deep Neural Networks

Written another way

$$z_1 = f_1(\boldsymbol{x}, \boldsymbol{\varphi}) = \boldsymbol{\varphi}^\top \overline{\boldsymbol{x}}$$

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$$\boldsymbol{z}_1 = f_1(\boldsymbol{x}, \boldsymbol{\varphi}) = \boldsymbol{\varphi}^\top \overline{\boldsymbol{x}}$$

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$$\boldsymbol{y} = f_\ell(\boldsymbol{x}, \boldsymbol{\xi}) = \boldsymbol{\xi}^\top \overline{\boldsymbol{z}}_{\ell-1}$$

We call each function a **layer**

Deep Neural Networks

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A deep neural network is made of many layers

Deep Neural Networks

What functions can we represent using a deep neural network?

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Proof Sketch: Approximate a continuous function $g : \mathbb{R} \mapsto \mathbb{R}$ using a linear combination of Heaviside functions

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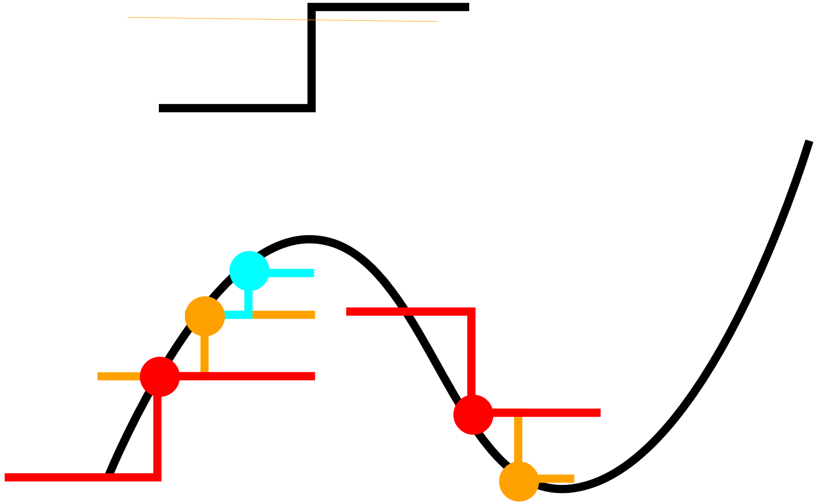

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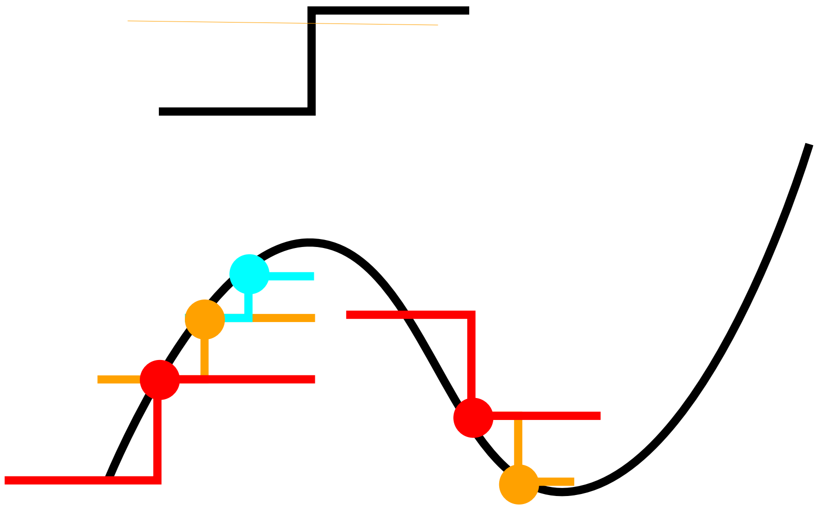

$$\exists \left(\boldsymbol{\theta} \in \mathbb{R}^{1 \times d_h}, \boldsymbol{\varphi} \in \mathbb{R}^{(d_h+1) \times d_1}\right) \text{ such that } \lim_{d_h \mapsto \infty} \left[\boldsymbol{\varphi}^\top \sigma\left(\overline{\boldsymbol{\theta}^\top \overline{x}}\right)\right]$$

Deep Neural Networks

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$$\mid g(\boldsymbol{x}) - f(\boldsymbol{x}, \boldsymbol{\theta}) \mid < \varepsilon$$

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Making the network deeper or wider decreases ε

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Very powerful finding! The basis of deep learning.

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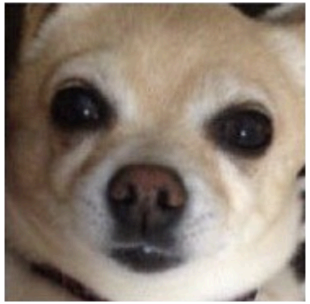
<u>Very powerful finding! The basis of deep learning.</u>

Task: predict how many ♥ a photo gets on social media



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Practical Considerations

We call wide neural networks **perceptrons**

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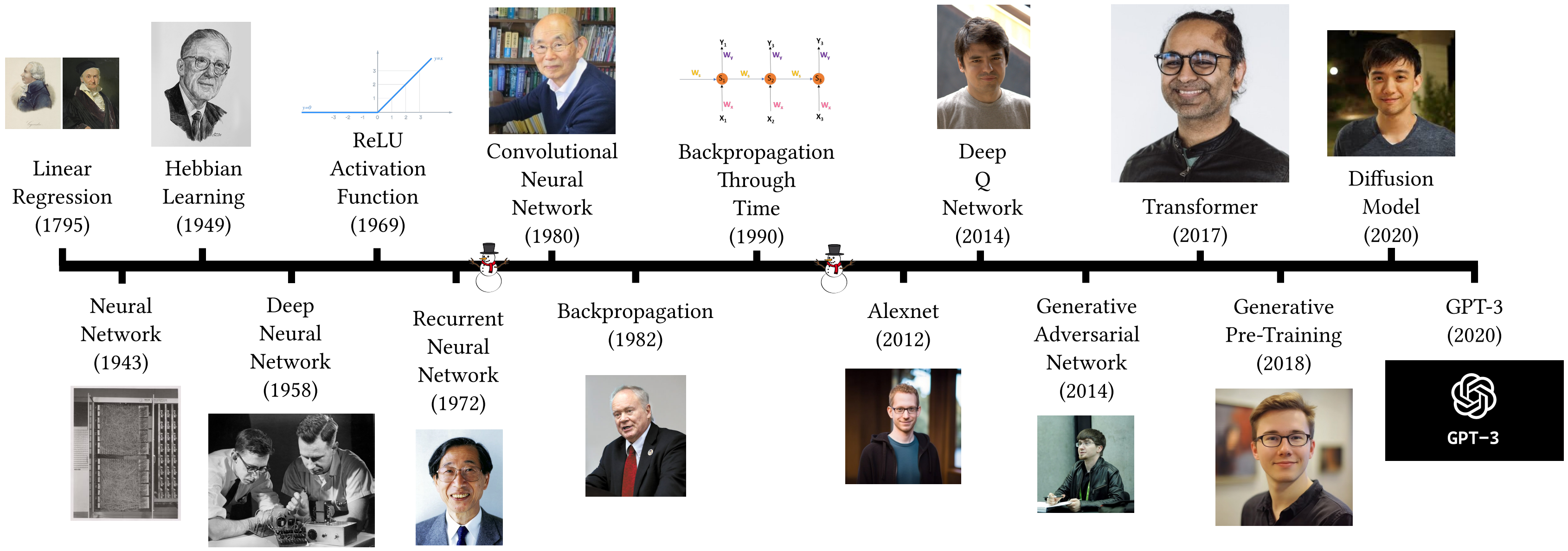

Practical Considerations

All the models we examine in this course will use MLPs

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- Recurrent neural networks

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It is very important to understand MLPs!

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It is very important to understand MLPs!

I will explain them again very simply

Practical Considerations

A **layer** is a linear operation and an activation function

$$f\left(\boldsymbol{x}, \begin{bmatrix} \boldsymbol{b} \\ \boldsymbol{W} \end{bmatrix}\right) = \sigma(\boldsymbol{b} + \boldsymbol{W}^\top \boldsymbol{x})$$

Many layers makes a deep neural network

$$\boldsymbol{z}_1 = f\left(\boldsymbol{x}, \begin{bmatrix} \boldsymbol{b}_1 \\ \boldsymbol{W}_1 \end{bmatrix}\right)$$

$$\boldsymbol{z}_2 = f\left(\boldsymbol{z}_1, \begin{bmatrix} \boldsymbol{b}_2 \\ \boldsymbol{W}_2 \end{bmatrix}\right)$$

$$\boldsymbol{y} = f\left(\boldsymbol{z}_2, \begin{bmatrix} \boldsymbol{b}_2 \\ \boldsymbol{W}_2 \end{bmatrix}\right)$$

Practical Considerations

Let us create a wide neural network in colab! https://colab.research.google.com/drive/1bLtf3QY-yROIif_EoQSU1WS7svd0q8j7?usp=sharing

Practical Considerations

Linear regression:

Practical Considerations

Linear regression:

$+$ Analytical solution

Practical Considerations

Linear regression:

+ Analytical solution

+ Low data requirement

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Linear regression:

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+ Low data requirement

– Poor scalability

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Neural networks:

– No analytical solution

– High data requirement

+ Scale to large inputs

Practical Considerations

Linear regression:

+ Analytical solution

+ Low data requirement

– Poor scalability

– Poor polynomials generalization

Neural networks:

– No analytical solution

– High data requirement

+ Scale to large inputs

+ Slightly better generalization

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