Transformers

CISC 7026 - Introduction to Deep Learning

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Review

Review

Last time, we derived various forms of **attention**

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We started with composite memory

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We started with composite memory

$$f(\boldsymbol{x}, \boldsymbol{\theta}) = \sum_{i=1}^{T} \boldsymbol{\theta}^\top \overline{\boldsymbol{x}}_i$$

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We started with composite memory

$$f(\boldsymbol{x}, \boldsymbol{\theta}) = \sum_{i=1}^{T} \boldsymbol{\theta}^\top \overline{\boldsymbol{x}}_i$$

Given large enough T, we will eventually run out of storage space

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Last time, we derived various forms of **attention**

We started with composite memory

$$f(\boldsymbol{x}, \boldsymbol{\theta}) = \sum_{i=1}^{T} \boldsymbol{\theta}^\top \overline{\boldsymbol{x}}_i$$

Given large enough T, we will eventually run out of storage space

The sum is a **lossy** operation that can store a limited amount of information

Review

So we introduced a forgetting term γ

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$$f(\boldsymbol{x}, \boldsymbol{\theta}) = \sum_{i=1}^{T} \gamma^{T-i} \cdot \boldsymbol{\theta}^\top \overline{\boldsymbol{x}}_i$$

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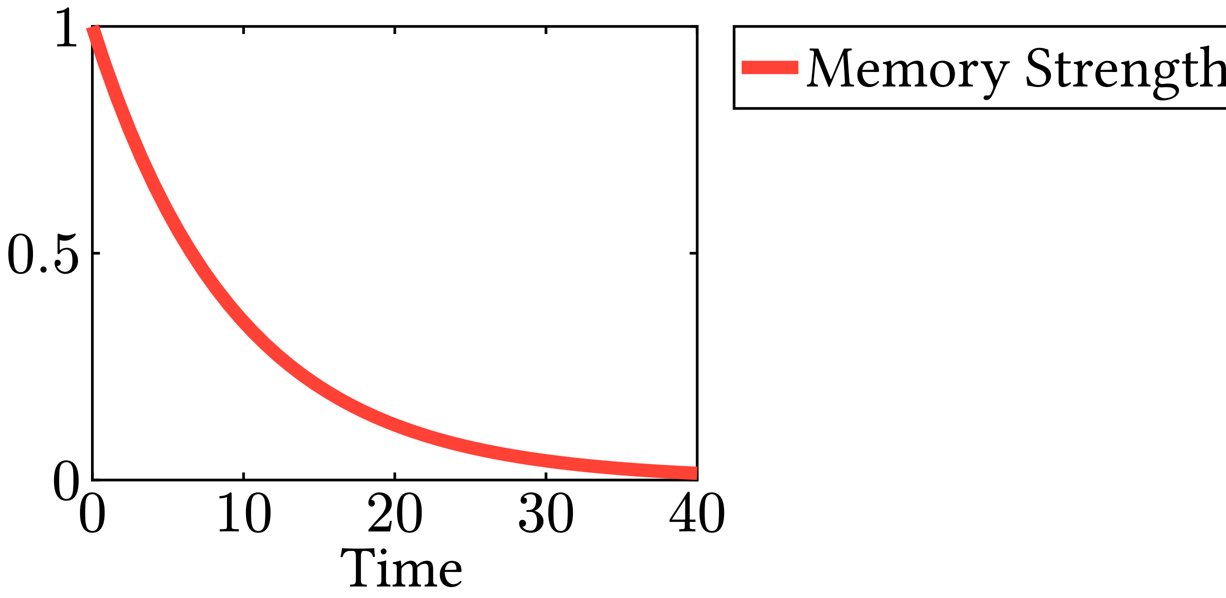
$$f(\boldsymbol{x}, \boldsymbol{\theta}) = \sum_{i=1}^{T} \gamma^{T-i} \cdot \boldsymbol{\theta}^\top \overline{\boldsymbol{x}}_i$$



Review

We went to a party and the forgetting seemed ok

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We went to a party and the forgetting seemed ok

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10 PM

Review

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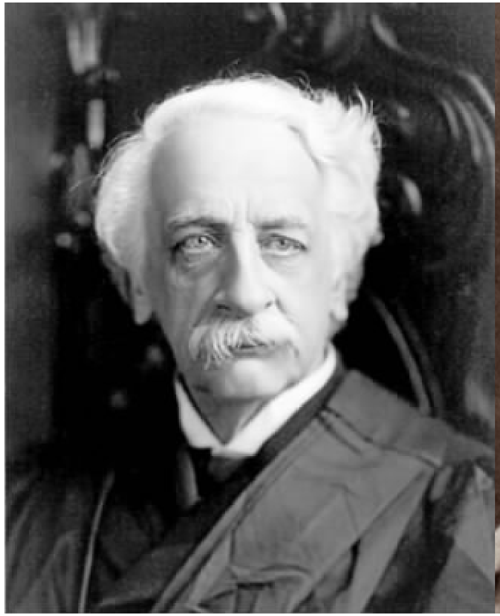   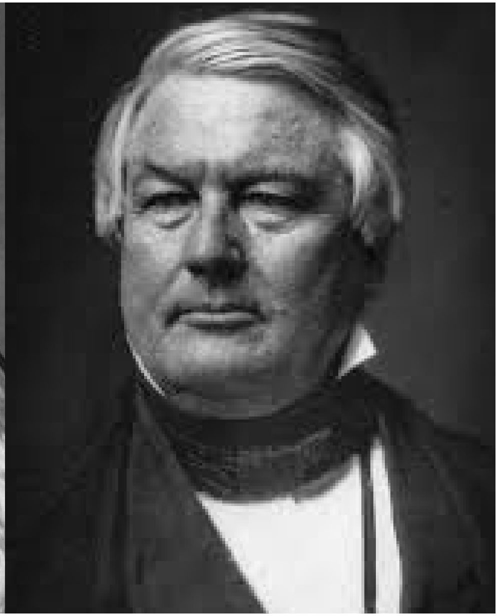

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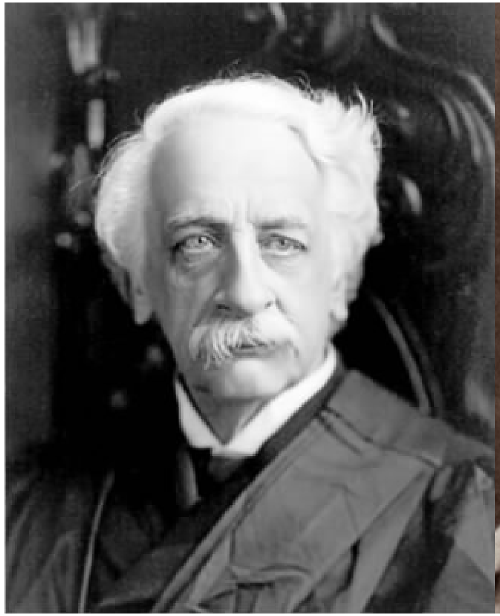

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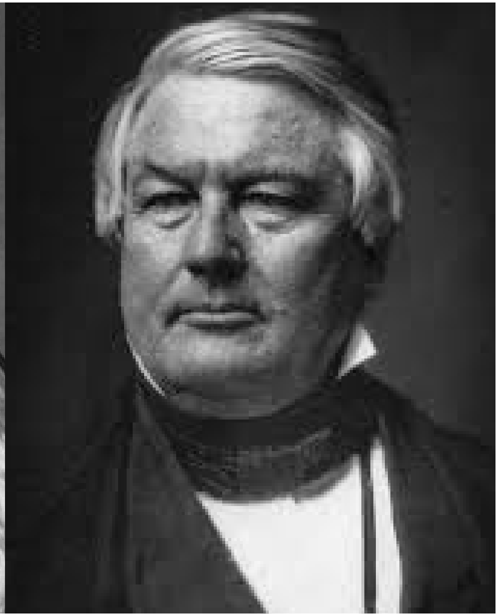

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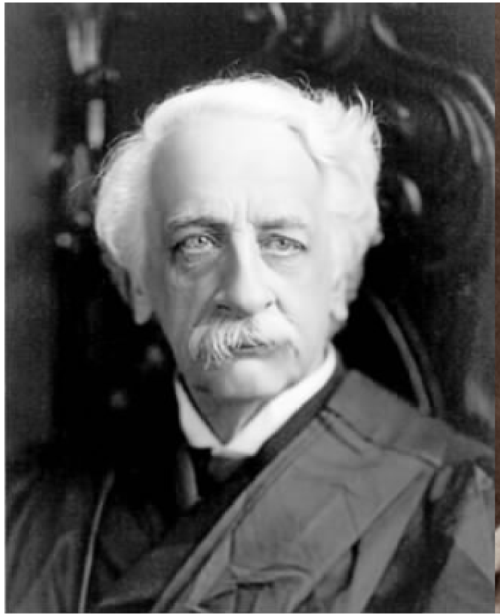

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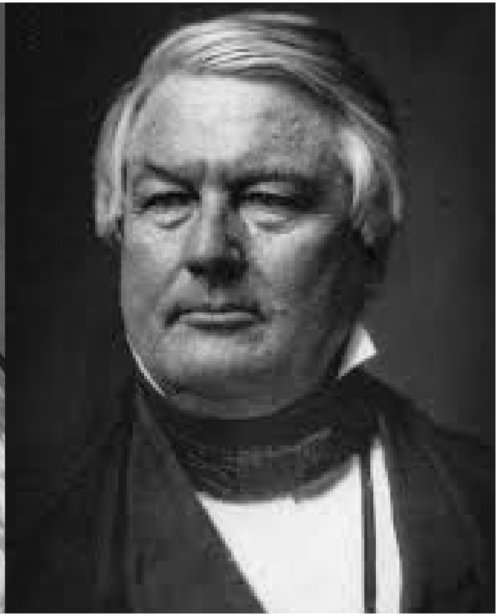


1 AM

Review

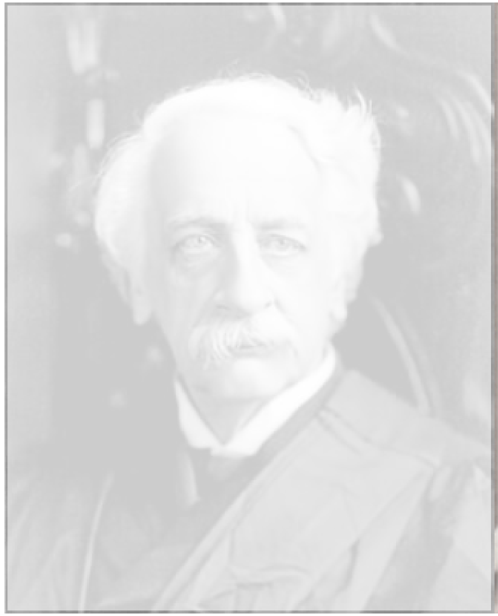
Review

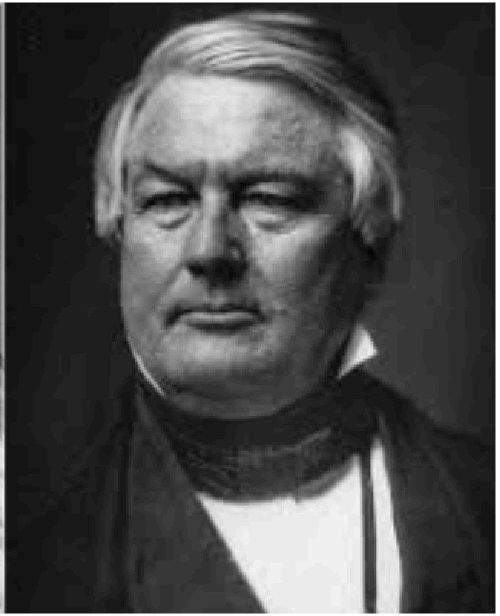
$$\gamma^3 \boldsymbol{\theta}^\top \overline{\boldsymbol{x}}_1$$

Review

$\gamma^3 \boldsymbol{\theta}^\top \overline{\boldsymbol{x}}_1$ $\gamma^2 \boldsymbol{\theta}^\top \overline{\boldsymbol{x}}_2$

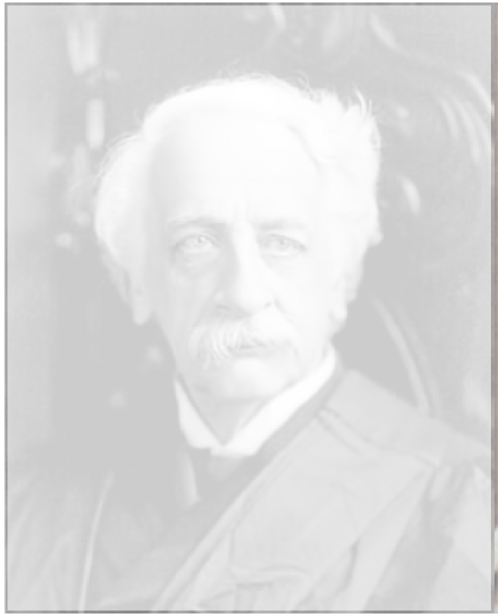
Review



$\gamma^3 \boldsymbol{\theta}^\top \overline{\boldsymbol{x}}_1$ $\gamma^2 \boldsymbol{\theta}^\top \overline{\boldsymbol{x}}_2$ $\gamma^1 \boldsymbol{\theta}^\top \overline{\boldsymbol{x}}_3$

Review



$$\gamma^3 \boldsymbol{\theta}^\top \overline{\boldsymbol{x}}_1$$

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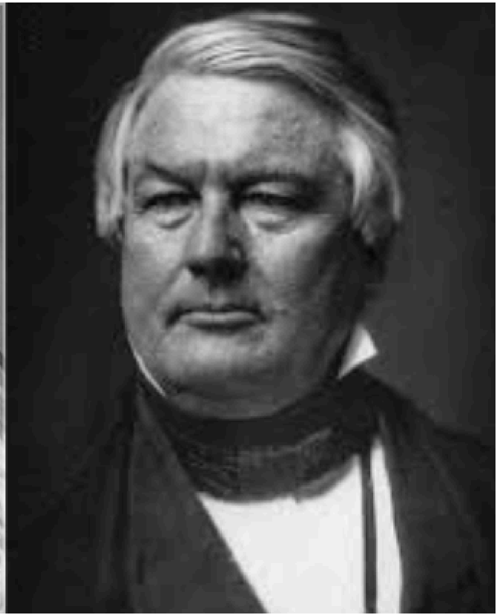
$$\gamma^1 \boldsymbol{\theta}^\top \overline{\boldsymbol{x}}_3$$



$$\gamma^0 \boldsymbol{\theta}^\top \overline{\boldsymbol{x}}_4$$

Review

But we encountered problems when Taylor Swift arrived at the party

Review

But we encountered problems when Taylor Swift arrived at the party

Review

But we encountered problems when Taylor Swift arrived at the party

$\gamma^4 \boldsymbol{\theta}^\top \overline{\boldsymbol{x}}_1$ $\quad$ $\gamma^3 \boldsymbol{\theta}^\top \overline{\boldsymbol{x}}_2$ $\quad$ $\gamma^2 \boldsymbol{\theta}^\top \overline{\boldsymbol{x}}_3$ $\quad$ $\gamma^1 \boldsymbol{\theta}^\top \overline{\boldsymbol{x}}_4$ $\quad$ $\gamma^0 \boldsymbol{\theta}^\top \overline{\boldsymbol{x}}_5$

Review

$\gamma^4 \boldsymbol{\theta}^\top \overline{\boldsymbol{x}}_1$ $\quad$ $\gamma^3 \boldsymbol{\theta}^\top \overline{\boldsymbol{x}}_2$ $\quad$ $\gamma^2 \boldsymbol{\theta}^\top \overline{\boldsymbol{x}}_3$ $\quad$ $\gamma^1 \boldsymbol{\theta}^\top \overline{\boldsymbol{x}}_4$ $\quad$ $\gamma^0 \boldsymbol{\theta}^\top \overline{\boldsymbol{x}}_5$

Review

$$\gamma^4 \boldsymbol{\theta}^\top \overline{\boldsymbol{x}}_1 \qquad \gamma^3 \boldsymbol{\theta}^\top \overline{\boldsymbol{x}}_2 \qquad \gamma^2 \boldsymbol{\theta}^\top \overline{\boldsymbol{x}}_3 \qquad \gamma^1 \boldsymbol{\theta}^\top \overline{\boldsymbol{x}}_4 \qquad \gamma^0 \boldsymbol{\theta}^\top \overline{\boldsymbol{x}}_5$$

With composite memory, we forget Taylor Swift!

Review

$\gamma^4 \boldsymbol{\theta}^\top \overline{\boldsymbol{x}}_1$ $\gamma^3 \boldsymbol{\theta}^\top \overline{\boldsymbol{x}}_2$ $\gamma^2 \boldsymbol{\theta}^\top \overline{\boldsymbol{x}}_3$ $\gamma^1 \boldsymbol{\theta}^\top \overline{\boldsymbol{x}}_4$ $\gamma^0 \boldsymbol{\theta}^\top \overline{\boldsymbol{x}}_5$

With composite memory, we forget Taylor Swift!

Our model of human memory was incomplete

Review

So we introduced **attention**

Review

So we introduced **attention**

The attention we pay to person i is

$$\lambda\left(\begin{bmatrix} \boldsymbol{x}_1 \\ \vdots \\ \boldsymbol{x}_T \end{bmatrix}, \boldsymbol{\theta}_\lambda\right)_i = \operatorname{softmax}\left(\begin{bmatrix} \boldsymbol{\theta}_\lambda^\top \overline{\boldsymbol{x}}_1 \\ \vdots \\ \boldsymbol{\theta}_\lambda^\top \overline{\boldsymbol{x}}_T \end{bmatrix}\right)_i = \frac{\exp\left(\boldsymbol{\theta}_\lambda^\top \overline{\boldsymbol{x}}_i\right)}{\sum_{j=1}^{T} \exp\left(\boldsymbol{\theta}_\lambda^\top \overline{\boldsymbol{x}}_j\right)}$$

Review

Review

$$\lambda\left(\begin{bmatrix} \boldsymbol{x}_1 \\ \vdots \\ \boldsymbol{x}_5 \end{bmatrix}, \boldsymbol{\theta}_\lambda\right)_1$$

$$\cdot\, \boldsymbol{\theta}^\top \overline{\boldsymbol{x}}_1$$

Review

$$\lambda\left(\begin{bmatrix} \boldsymbol{x}_1 \\ \vdots \\ \boldsymbol{x}_5 \end{bmatrix}, \boldsymbol{\theta}_\lambda\right)_1 \lambda\left(\begin{bmatrix} \boldsymbol{x}_1 \\ \vdots \\ \boldsymbol{x}_5 \end{bmatrix}, \boldsymbol{\theta}_\lambda\right)_2$$

$$\cdot\, \boldsymbol{\theta}^\top \overline{\boldsymbol{x}}_1 \qquad \cdot\, \boldsymbol{\theta}^\top \overline{\boldsymbol{x}}_2$$

Review

$$\lambda\left(\begin{bmatrix} \boldsymbol{x}_1 \\ \vdots \\ \boldsymbol{x}_5 \end{bmatrix}, \boldsymbol{\theta}_\lambda\right)_1 \lambda\left(\begin{bmatrix} \boldsymbol{x}_1 \\ \vdots \\ \boldsymbol{x}_5 \end{bmatrix}, \boldsymbol{\theta}_\lambda\right)_2 \lambda\left(\begin{bmatrix} \boldsymbol{x}_1 \\ \vdots \\ \boldsymbol{x}_5 \end{bmatrix}, \boldsymbol{\theta}_\lambda\right)_3$$

$$\cdot\, \boldsymbol{\theta}^\top \overline{\boldsymbol{x}}_1 \qquad \cdot\, \boldsymbol{\theta}^\top \overline{\boldsymbol{x}}_2 \qquad \cdot\, \boldsymbol{\theta}^\top \overline{\boldsymbol{x}}_3$$

Review

$$\lambda\left(\begin{bmatrix} \boldsymbol{x}_1 \\ \vdots \\ \boldsymbol{x}_5 \end{bmatrix}, \boldsymbol{\theta}_\lambda\right)_1 \lambda\left(\begin{bmatrix} \boldsymbol{x}_1 \\ \vdots \\ \boldsymbol{x}_5 \end{bmatrix}, \boldsymbol{\theta}_\lambda\right)_2 \lambda\left(\begin{bmatrix} \boldsymbol{x}_1 \\ \vdots \\ \boldsymbol{x}_5 \end{bmatrix}, \boldsymbol{\theta}_\lambda\right)_3 \lambda\left(\begin{bmatrix} \boldsymbol{x}_1 \\ \vdots \\ \boldsymbol{x}_5 \end{bmatrix}, \boldsymbol{\theta}_\lambda\right)_4$$

$$\cdot\, \boldsymbol{\theta}^\top \overline{\boldsymbol{x}}_1 \qquad \cdot\, \boldsymbol{\theta}^\top \overline{\boldsymbol{x}}_2 \qquad \cdot\, \boldsymbol{\theta}^\top \overline{\boldsymbol{x}}_3 \qquad \cdot\, \boldsymbol{\theta}^\top \overline{\boldsymbol{x}}_4$$

Review

$$\lambda\left(\begin{bmatrix} \boldsymbol{x}_1 \\ \vdots \\ \boldsymbol{x}_5 \end{bmatrix}, \boldsymbol{\theta}_\lambda\right)_1 \lambda\left(\begin{bmatrix} \boldsymbol{x}_1 \\ \vdots \\ \boldsymbol{x}_5 \end{bmatrix}, \boldsymbol{\theta}_\lambda\right)_2 \lambda\left(\begin{bmatrix} \boldsymbol{x}_1 \\ \vdots \\ \boldsymbol{x}_5 \end{bmatrix}, \boldsymbol{\theta}_\lambda\right)_3 \lambda\left(\begin{bmatrix} \boldsymbol{x}_1 \\ \vdots \\ \boldsymbol{x}_5 \end{bmatrix}, \boldsymbol{\theta}_\lambda\right)_4 \lambda\left(\begin{bmatrix} \boldsymbol{x}_1 \\ \vdots \\ \boldsymbol{x}_5 \end{bmatrix}, \boldsymbol{\theta}_\lambda\right)_5$$

$$\cdot\, \boldsymbol{\theta}^\top \overline{\boldsymbol{x}}_1 \qquad \cdot\, \boldsymbol{\theta}^\top \overline{\boldsymbol{x}}_2 \qquad \cdot\, \boldsymbol{\theta}^\top \overline{\boldsymbol{x}}_3 \qquad \cdot\, \boldsymbol{\theta}^\top \overline{\boldsymbol{x}}_4 \qquad \cdot\, \boldsymbol{\theta}^\top \overline{\boldsymbol{x}}_5$$

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$$0.70 \cdot \boldsymbol{\theta}^\top \overline{\boldsymbol{x}}_1$$

Review

$$0.70 \cdot \boldsymbol{\theta}^\top \overline{\boldsymbol{x}}_1 \qquad 0.04 \cdot \boldsymbol{\theta}^\top \overline{\boldsymbol{x}}_2$$

Review

$0.70 \cdot \boldsymbol{\theta}^\top \overline{\boldsymbol{x}}_1 \quad 0.04 \cdot \boldsymbol{\theta}^\top \overline{\boldsymbol{x}}_2 \quad 0.03 \cdot \boldsymbol{\theta}^\top \overline{\boldsymbol{x}}_3$

Review

$0.70 \cdot \boldsymbol{\theta}^\top \overline{\boldsymbol{x}}_1 \quad 0.04 \cdot \boldsymbol{\theta}^\top \overline{\boldsymbol{x}}_2 \quad 0.03 \cdot \boldsymbol{\theta}^\top \overline{\boldsymbol{x}}_3 \quad 0.20 \cdot \boldsymbol{\theta}^\top \overline{\boldsymbol{x}}_4$

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$$0.70 \cdot \boldsymbol{\theta}^\top \overline{\boldsymbol{x}}_1 \quad 0.04 \cdot \boldsymbol{\theta}^\top \overline{\boldsymbol{x}}_2 \quad 0.03 \cdot \boldsymbol{\theta}^\top \overline{\boldsymbol{x}}_3 \quad 0.20 \cdot \boldsymbol{\theta}^\top \overline{\boldsymbol{x}}_4 \quad 0.03 \cdot \boldsymbol{\theta}^\top \overline{\boldsymbol{x}}_5$$

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$$0.70 \cdot \boldsymbol{\theta}^\top \overline{\boldsymbol{x}}_1 \quad 0.04 \cdot \boldsymbol{\theta}^\top \overline{\boldsymbol{x}}_2 \quad 0.03 \cdot \boldsymbol{\theta}^\top \overline{\boldsymbol{x}}_3 \quad 0.20 \cdot \boldsymbol{\theta}^\top \overline{\boldsymbol{x}}_4 \quad 0.03 \cdot \boldsymbol{\theta}^\top \overline{\boldsymbol{x}}_5$$

$$0.70 + 0.04 + 0.03 + 0.20 + 0.03 = 1.0$$

Review

Then, we introduced **keys** and **queries**

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Query: Which person will help me on my exam?

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Query: Which person will help me on my exam?

Musician Lawyer Shopkeeper Chef Scientist

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$$\boldsymbol{K} = [\boldsymbol{k}_1 \ \ \boldsymbol{k}_2 \ \ \dots \ \ \boldsymbol{k}_T] = \left[\boldsymbol{\theta}_K^\top \boldsymbol{x}_1 \ \ \boldsymbol{\theta}_K^\top \boldsymbol{x}_2 \ \ \dots \ \ \boldsymbol{\theta}_K^\top \boldsymbol{x}_T\right], \quad \boldsymbol{K} \in \mathbb{R}^{d_h \times T}$$

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$$\boldsymbol{K} = [\boldsymbol{k}_1 \ \ \boldsymbol{k}_2 \ \ \dots \ \ \boldsymbol{k}_T] = [\boldsymbol{\theta}_K^\top \boldsymbol{x}_1 \ \ \boldsymbol{\theta}_K^\top \boldsymbol{x}_2 \ \ \dots \ \ \boldsymbol{\theta}_K^\top \boldsymbol{x}_T], \quad \boldsymbol{K} \in \mathbb{R}^{d_h \times T}$$

$$\boldsymbol{q} = \boldsymbol{\theta}_Q^\top \boldsymbol{x}_q, \quad \boldsymbol{\theta}_Q \in \mathbb{R}^{d_x \times d_h}, \quad \boldsymbol{q} \in \mathbb{R}^{d_h}$$

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$$\boldsymbol{K} = [\boldsymbol{k}_1 \ \ \boldsymbol{k}_2 \ \ \ldots \ \ \boldsymbol{k}_T] = \left[\boldsymbol{\theta}_K^\top \boldsymbol{x}_1 \ \ \boldsymbol{\theta}_K^\top \boldsymbol{x}_2 \ \ \ldots \ \ \boldsymbol{\theta}_K^\top \boldsymbol{x}_T\right], \quad \boldsymbol{K} \in \mathbb{R}^{d_h \times T}$$

$$\boldsymbol{q} = \boldsymbol{\theta}_Q^\top \boldsymbol{x}_q, \quad \boldsymbol{\theta}_Q \in \mathbb{R}^{d_x \times d_h}, \quad \boldsymbol{q} \in \mathbb{R}^{d_h}$$

$$\lambda\left(\begin{bmatrix} \boldsymbol{x}_1 \\ \vdots \\ \boldsymbol{x}_T \end{bmatrix}, \boldsymbol{\theta}\right) = \operatorname{softmax}(\boldsymbol{q}^\top \boldsymbol{K}) = \operatorname{softmax}(\boldsymbol{q}^\top [\boldsymbol{k}_1 \ \ \boldsymbol{k}_2 \ \ \ldots \ \ \boldsymbol{k}_T])$$

$$= \operatorname{softmax}\left(\left[\boldsymbol{q}^\top \boldsymbol{k}_1 \ \ \boldsymbol{q}^\top \boldsymbol{k}_2 \ \ \ldots \ \ \boldsymbol{q}^\top \boldsymbol{k}_T\right]\right)$$

Review

$$\boldsymbol{K} = [\boldsymbol{k}_1 \;\; \boldsymbol{k}_2 \;\; \dots \;\; \boldsymbol{k}_T] = \left[\boldsymbol{\theta}_K^\top \boldsymbol{x}_1 \;\; \boldsymbol{\theta}_K^\top \boldsymbol{x}_2 \;\; \dots \;\; \boldsymbol{\theta}_K^\top \boldsymbol{x}_T\right], \quad \boldsymbol{K} \in \mathbb{R}^{d_h \times T}$$

$$\boldsymbol{q} = \boldsymbol{\theta}_Q^\top \boldsymbol{x}_q, \quad \boldsymbol{\theta}_Q \in \mathbb{R}^{d_x \times d_h}, \quad \boldsymbol{q} \in \mathbb{R}^{d_h}$$

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$$= \mathrm{softmax}([\boldsymbol{q}^\top \boldsymbol{k}_1 \;\; \boldsymbol{q}^\top \boldsymbol{k}_2 \;\; \dots \;\; \boldsymbol{q}^\top \boldsymbol{k}_T])$$

We call this **dot-product attention**

Review

$$\boldsymbol{K} = [\boldsymbol{k}_1 \ \ \boldsymbol{k}_2 \ \ \ldots \ \ \boldsymbol{k}_T] = \left[\boldsymbol{\theta}_K^\top \boldsymbol{x}_1 \ \ \boldsymbol{\theta}_K^\top \boldsymbol{x}_2 \ \ \ldots \ \ \boldsymbol{\theta}_K^\top \boldsymbol{x}_T\right], \quad \boldsymbol{K} \in \mathbb{R}^{d_h \times T}$$

$$\boldsymbol{q} = \boldsymbol{\theta}_Q^\top \boldsymbol{x}_q, \quad \boldsymbol{\theta}_Q \in \mathbb{R}^{d_x \times d_h}, \quad \boldsymbol{q} \in \mathbb{R}^{d_h}$$

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$$= \operatorname{softmax}\left([\boldsymbol{q}^\top \boldsymbol{k}_1 \ \ \boldsymbol{q}^\top \boldsymbol{k}_2 \ \ \ldots \ \ \boldsymbol{q}^\top \boldsymbol{k}_T]\right)$$

We call this **dot-product attention**

Then we add attention back to the composite model

Review

$$f\left(\begin{bmatrix} \boldsymbol{x}_1 \\ \vdots \\ \boldsymbol{x}_T \end{bmatrix}, \boldsymbol{\theta}\right) = \sum_{i=1}^{T} \boldsymbol{\theta}^\top \boldsymbol{x}_i \cdot \lambda\left(\begin{bmatrix} \boldsymbol{x}_1 \\ \vdots \\ \boldsymbol{x}_T \end{bmatrix}, \boldsymbol{\theta}_\lambda\right)_i$$

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$$f\left(\begin{bmatrix} \boldsymbol{x}_1 \\ \vdots \\ \boldsymbol{x}_T \end{bmatrix}, \boldsymbol{\theta}\right) = \sum_{i=1}^{T} \boldsymbol{\theta}^\top \boldsymbol{x}_i \cdot \lambda\left(\begin{bmatrix} \boldsymbol{x}_1 \\ \vdots \\ \boldsymbol{x}_T \end{bmatrix}, \boldsymbol{\theta}_\lambda\right)_i$$

We relabel $\boldsymbol{\theta}$ to $\boldsymbol{\theta}_V$

$$f\left(\begin{bmatrix} \boldsymbol{x}_1 \\ \vdots \\ \boldsymbol{x}_T \end{bmatrix}, \boldsymbol{\theta}\right) = \sum_{i=1}^{T} \boldsymbol{\theta}_{V}^\top \boldsymbol{x}_i \cdot \lambda\left(\begin{bmatrix} \boldsymbol{x}_1 \\ \vdots \\ \boldsymbol{x}_T \end{bmatrix}, \boldsymbol{\theta}_\lambda\right)_i$$

Review

$$f\left(\begin{bmatrix} \boldsymbol{x}_1 \\ \vdots \\ \boldsymbol{x}_T \end{bmatrix}, \boldsymbol{\theta}\right) = \sum_{i=1}^{T} \boldsymbol{\theta}^\top \boldsymbol{x}_i \cdot \lambda\left(\begin{bmatrix} \boldsymbol{x}_1 \\ \vdots \\ \boldsymbol{x}_T \end{bmatrix}, \boldsymbol{\theta}_\lambda\right)_i$$

We relabel $\boldsymbol{\theta}$ to $\boldsymbol{\theta}_V$

$$f\left(\begin{bmatrix} \boldsymbol{x}_1 \\ \vdots \\ \boldsymbol{x}_T \end{bmatrix}, \boldsymbol{\theta}\right) = \sum_{i=1}^{T} \boldsymbol{\theta}_{V}^\top \boldsymbol{x}_i \cdot \lambda\left(\begin{bmatrix} \boldsymbol{x}_1 \\ \vdots \\ \boldsymbol{x}_T \end{bmatrix}, \boldsymbol{\theta}_\lambda\right)_i$$

In dot-product attention, we call $\boldsymbol{\theta}_V^\top \boldsymbol{x}_i$ the **value**

Review

In **dot product self attention** we create queries for all inputs

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In **dot product self attention** we create queries for all inputs

$$\boldsymbol{Q} = [\boldsymbol{q}_1 \ \ \boldsymbol{q}_2 \ \ \dots \ \ \boldsymbol{q}_T] = \left[\boldsymbol{\theta}_Q^\top \boldsymbol{x}_1 \ \ \boldsymbol{\theta}_Q^\top \boldsymbol{x}_2 \ \ \dots \ \ \boldsymbol{\theta}_Q^\top \boldsymbol{x}_T\right], \quad \boldsymbol{Q} \in \mathbb{R}^{T \times d_h}$$

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In **dot product self attention** we create queries for all inputs

$$\boldsymbol{Q} = [\boldsymbol{q}_1 \ \ \boldsymbol{q}_2 \ \ \dots \ \ \boldsymbol{q}_T] = \left[\boldsymbol{\theta}_Q^\top \boldsymbol{x}_1 \ \ \boldsymbol{\theta}_Q^\top \boldsymbol{x}_2 \ \ \dots \ \ \boldsymbol{\theta}_Q^\top \boldsymbol{x}_T\right], \quad \boldsymbol{Q} \in \mathbb{R}^{T \times d_h}$$

Review

Attention looks messy, but we can rewrite it in matrix form

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$$\boldsymbol{Q} = [\boldsymbol{q}_1 \ \ \boldsymbol{q}_2 \ \ \ldots \ \ \boldsymbol{q}_T] = \left[\boldsymbol{\theta}_Q^\top \boldsymbol{x}_1 \ \ \boldsymbol{\theta}_Q^\top \boldsymbol{x}_2 \ \ \ldots \ \ \boldsymbol{\theta}_Q^\top \boldsymbol{x}_T\right]$$

$$\boldsymbol{K} = [\boldsymbol{k}_1 \ \ \boldsymbol{k}_2 \ \ \ldots \ \ \boldsymbol{k}_T] = \left[\boldsymbol{\theta}_K^\top \boldsymbol{x}_1 \ \ \boldsymbol{\theta}_K^\top \boldsymbol{x}_2 \ \ \ldots \ \ \boldsymbol{\theta}_K^\top \boldsymbol{x}_T\right]$$

$$\boldsymbol{V} = [\boldsymbol{v}_1 \ \ \boldsymbol{v}_2 \ \ \ldots \ \ \boldsymbol{v}_T] = \left[\boldsymbol{\theta}_V^\top \boldsymbol{x}_1 \ \ \boldsymbol{\theta}_V^\top \boldsymbol{x}_2 \ \ \ldots \ \ \boldsymbol{\theta}_V^\top \boldsymbol{x}_T\right]$$

Review

Attention looks messy, but we can rewrite it in matrix form

$$\boldsymbol{Q} = [\boldsymbol{q}_1 \ \boldsymbol{q}_2 \ \dots \ \boldsymbol{q}_T] = [\boldsymbol{\theta}_Q^\top \boldsymbol{x}_1 \ \boldsymbol{\theta}_Q^\top \boldsymbol{x}_2 \ \dots \ \boldsymbol{\theta}_Q^\top \boldsymbol{x}_T]$$

$$\boldsymbol{K} = [\boldsymbol{k}_1 \ \boldsymbol{k}_2 \ \dots \ \boldsymbol{k}_T] = [\boldsymbol{\theta}_K^\top \boldsymbol{x}_1 \ \boldsymbol{\theta}_K^\top \boldsymbol{x}_2 \ \dots \ \boldsymbol{\theta}_K^\top \boldsymbol{x}_T]$$

$$\boldsymbol{V} = [\boldsymbol{v}_1 \ \boldsymbol{v}_2 \ \dots \ \boldsymbol{v}_T] = [\boldsymbol{\theta}_V^\top \boldsymbol{x}_1 \ \boldsymbol{\theta}_V^\top \boldsymbol{x}_2 \ \dots \ \boldsymbol{\theta}_V^\top \boldsymbol{x}_T]$$

$$\operatorname{attn}\left(\begin{bmatrix} \boldsymbol{x}_1 \\ \vdots \\ \boldsymbol{x}_T \end{bmatrix}, \boldsymbol{\theta}\right) = \operatorname{softmax}\left(\frac{\boldsymbol{Q}\boldsymbol{K}^\top}{\sqrt{d_h}}\right)\boldsymbol{V}$$

Review

Attention looks messy, but we can rewrite it in matrix form

$$\boldsymbol{Q} = [\boldsymbol{q}_1 \ \ \boldsymbol{q}_2 \ \ \dots \ \ \boldsymbol{q}_T] = [\boldsymbol{\theta}_Q^\top \boldsymbol{x}_1 \ \ \boldsymbol{\theta}_Q^\top \boldsymbol{x}_2 \ \ \dots \ \ \boldsymbol{\theta}_Q^\top \boldsymbol{x}_T]$$

$$\boldsymbol{K} = [\boldsymbol{k}_1 \ \ \boldsymbol{k}_2 \ \ \dots \ \ \boldsymbol{k}_T] = [\boldsymbol{\theta}_K^\top \boldsymbol{x}_1 \ \ \boldsymbol{\theta}_K^\top \boldsymbol{x}_2 \ \ \dots \ \ \boldsymbol{\theta}_K^\top \boldsymbol{x}_T]$$

$$\boldsymbol{V} = [\boldsymbol{v}_1 \ \ \boldsymbol{v}_2 \ \ \dots \ \ \boldsymbol{v}_T] = [\boldsymbol{\theta}_V^\top \boldsymbol{x}_1 \ \ \boldsymbol{\theta}_V^\top \boldsymbol{x}_2 \ \ \dots \ \ \boldsymbol{\theta}_V^\top \boldsymbol{x}_T]$$

$$\mathrm{attn}\left(\begin{bmatrix} \boldsymbol{x}_1 \\ \vdots \\ \boldsymbol{x}_T \end{bmatrix}, \boldsymbol{\theta} \right) = \mathrm{softmax}\left(\frac{\boldsymbol{Q}\boldsymbol{K}^\top}{\sqrt{d_h}} \right) \boldsymbol{V}$$

With attention, we can create the **transformer**

Review

Attention looks messy, but we can rewrite it in matrix form

$$\boldsymbol{Q} = [\boldsymbol{q}_1 \;\; \boldsymbol{q}_2 \;\; \ldots \;\; \boldsymbol{q}_T] = \left[\boldsymbol{\theta}_Q^\top \boldsymbol{x}_1 \;\; \boldsymbol{\theta}_Q^\top \boldsymbol{x}_2 \;\; \ldots \;\; \boldsymbol{\theta}_Q^\top \boldsymbol{x}_T\right]$$

$$\boldsymbol{K} = [\boldsymbol{k}_1 \;\; \boldsymbol{k}_2 \;\; \ldots \;\; \boldsymbol{k}_T] = \left[\boldsymbol{\theta}_K^\top \boldsymbol{x}_1 \;\; \boldsymbol{\theta}_K^\top \boldsymbol{x}_2 \;\; \ldots \;\; \boldsymbol{\theta}_K^\top \boldsymbol{x}_T\right]$$

$$\boldsymbol{V} = [\boldsymbol{v}_1 \;\; \boldsymbol{v}_2 \;\; \ldots \;\; \boldsymbol{v}_T] = \left[\boldsymbol{\theta}_V^\top \boldsymbol{x}_1 \;\; \boldsymbol{\theta}_V^\top \boldsymbol{x}_2 \;\; \ldots \;\; \boldsymbol{\theta}_V^\top \boldsymbol{x}_T\right]$$

$$\operatorname{attn}\left(\begin{bmatrix} \boldsymbol{x}_1 \\ \vdots \\ \boldsymbol{x}_T \end{bmatrix}, \boldsymbol{\theta}\right) = \operatorname{softmax}\left(\frac{\boldsymbol{Q}\boldsymbol{K}^\top}{\sqrt{d_h}}\right)\boldsymbol{V}$$

With attention, we can create the **transformer**

Review

Why should we care about the transformer?

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Why should we care about the transformer?

It is arguably the most powerful neural network architecture today

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- AlphaFold (Nobel prize)

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- AlphaFold (Nobel prize)
- ChatGPT, Qwen, LLaMA, etc

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- ChatGPT, Qwen, LLaMA, etc
- DinoV2

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Why should we care about the transformer?

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- AlphaFold (Nobel prize)
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Going Deeper

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Modern transformers can be very deep

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Modern transformers can be very deep

Very deep networks require two new training tricks

Going Deeper

Modern transformers can be very deep

Very deep networks require two new training tricks

We must understand these tricks before implementing the transformer

Going Deeper

Modern transformers can be very deep

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Trick 1: Residual connections

Going Deeper

Modern transformers can be very deep

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Trick 1: Residual connections

Trick 2: Layer normalization

Going Deeper

Modern transformers can be very deep

Very deep networks require two new training tricks

We must understand these tricks before implementing the transformer

Trick 1: Residual connections

Trick 2: Layer normalization

We will start with the **residual connection**

Going Deeper

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But there is a limit!

Going Deeper

Making the network too deep can hurt performance

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At each layer, we lose a little bit of information

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With enough layers, all the information in $\boldsymbol{x}$ is lost!

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Question: How can we prevent $\boldsymbol{x}$ from getting lost?

Going Deeper

We can feed $\boldsymbol{x}$ to each layer

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The first approach is called the **DenseNet** approach

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\boldsymbol{z}_1 &= f_1(\boldsymbol{x}, \boldsymbol{\theta}_1) \\
\boldsymbol{z}_2 &= f_2\left(\begin{bmatrix} \boldsymbol{x} \\ \boldsymbol{z}_1 \end{bmatrix}, \boldsymbol{\theta}_2\right) \\
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Question: Any issues with the DenseNet approach?

Answer: Very deep networks require too many parameters!

Going Deeper

The next method is called the **ResNet** approach

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\boldsymbol{z}_1 &= f_1(\boldsymbol{x}, \boldsymbol{\theta}_1) \\
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For very deep networks, we use ResNets over DenseNets

Going Deeper

ResNets use a **residual connection**

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Each layer is a small perturbation of $\boldsymbol{x}$

This prevents $\boldsymbol{x}$ from getting lost in very deep networks

Going Deeper

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$$f_1(\boldsymbol{x}, \boldsymbol{\theta}_1) = \sum_{i=1}^{d_x} \theta_{1,i} \cdot x_i$$

Question: If all $x_i = 1$, $\theta_{1,i} = 0.01$ and $d_x = 1000$, what is the output?

Going Deeper

$$f_1(\boldsymbol{x}, \boldsymbol{\theta}_1) = \sum_{i=1}^{d_x} \theta_{1,i} x_i$$

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Going Deeper

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What if we add another layer with the same d_x and θ?

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Going Deeper

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Question: What is the problem?

Going Deeper

Let us look at the gradient

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$$\nabla_{\boldsymbol{\theta}_1} f_2(f_1(\boldsymbol{x}, \boldsymbol{\theta}_1), \boldsymbol{\theta}_2) =$$

Going Deeper

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Can cause exploding or vanishing gradient

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Deeper network $\Rightarrow$ worse exploding/vanishing issues

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Can cause exploding or vanishing gradient

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Question: What can we do?

Answer: We can normalize the output of each layer

Going Deeper

Layer normalization normalizes the output of a layer

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First, layer normalization **centers** the output of the layer

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$$\mu = \frac{1}{d_y} \sum_{i=1}^{d_y} f(\boldsymbol{x}, \boldsymbol{\theta})_i$$

Going Deeper

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Question: What does this do?

Going Deeper

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$$f(\boldsymbol{x}, \boldsymbol{\theta}) - \mu$$

Question: What does this do?

Answer: Creates zero-mean output (both positive and negative values)

Going Deeper

$$\mu = d_y \sum_{i=1}^{d_y} f(\boldsymbol{x}, \boldsymbol{\theta})_i$$

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Then, layer normalization **rescales** the output by standard deviation

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Going Deeper

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Layer norm makes the output normally distributed, $y_i \sim \mathcal{N}(0, 1)$

Going Deeper

If the output is normally distrbuted:

Going Deeper

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- 99.7% of outputs $\in [-3, 3]$

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Going Deeper

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This helps prevent vanishing and exploding gradients

Going Deeper

Now, let's combine residual connections and layer norm and try our very deep network again

https://colab.research.google.com/drive/1qVIbQKpTuBYIa7FvC4IH-kJq-E0jmc0d#scrollTo=iQtXjGYiz5CD

Transformers

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Now we have everything we need to implement a transformer

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A transformer consists of many **transformer layers**

Transformers

```python
class TransformerLayer(nn.Module):
    def __init__(self):
        self.attn = Attention()
        self.mlp = Sequential(
            Linear(d_h, d_h), LeakyReLU(), Linear(d_h, d_h))
        self.norm = nn.LayerNorm(
            d_h, elementwise_affine=False)

    def forward(self, x):
        # Residual connection and layer norm
        x = self.norm(self.attn(x) + x)
        x = self.norm(self.mlp(x) + x)
        return x
```

Transformers

```python
class Transformer(nn.Module):
    def __init__(self):
        self.layer1 = TransformerLayer()
        self.layer2 = TransformerLayer()
        self.layer3 = TransformerLayer()

    def forward(self, x):
        x = self.layer1(x)
        x = self.layer2(x)
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Question: What are the input/output shapes of the transformer?

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Question: What are the input/output shapes of the transformer?

Answer: $f : \mathbb{R}^{T \times d_x} \times \Theta \mapsto \mathbb{R}^{T \times d_y}$

Positional Encoding

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Our transformer maps T inputs to T outputs

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- T pixels in an image

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Question: Do we care about the order of the T inputs?

Positional Encoding

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Question: Do we care about the order of the T inputs?

$$f\left(\begin{bmatrix} \boldsymbol{x}_1 \\ \boldsymbol{x}_2 \end{bmatrix}, \boldsymbol{\theta}\right) \stackrel{?}{=} f\left(\begin{bmatrix} \boldsymbol{x}_2 \\ \boldsymbol{x}_1 \end{bmatrix}, \boldsymbol{\theta}\right)$$

Positional Encoding

Our transformer maps T inputs to T outputs

$$f : \mathbb{R}^{T \times d_x} \times \Theta \mapsto \mathbb{R}^{T \times d_y}$$

- T words in a sentence
- T pixels in an image
- T amino acids in a protein

Question: Do we care about the order of the T inputs?

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Answer: It depends. In some tasks, yes. In others, no

Positional Encoding

Question: Detecting birds in an image of T pixels, does order matter?

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Answer: Yes!

Positional Encoding

Question: T robots searching for an object. Does order matter?

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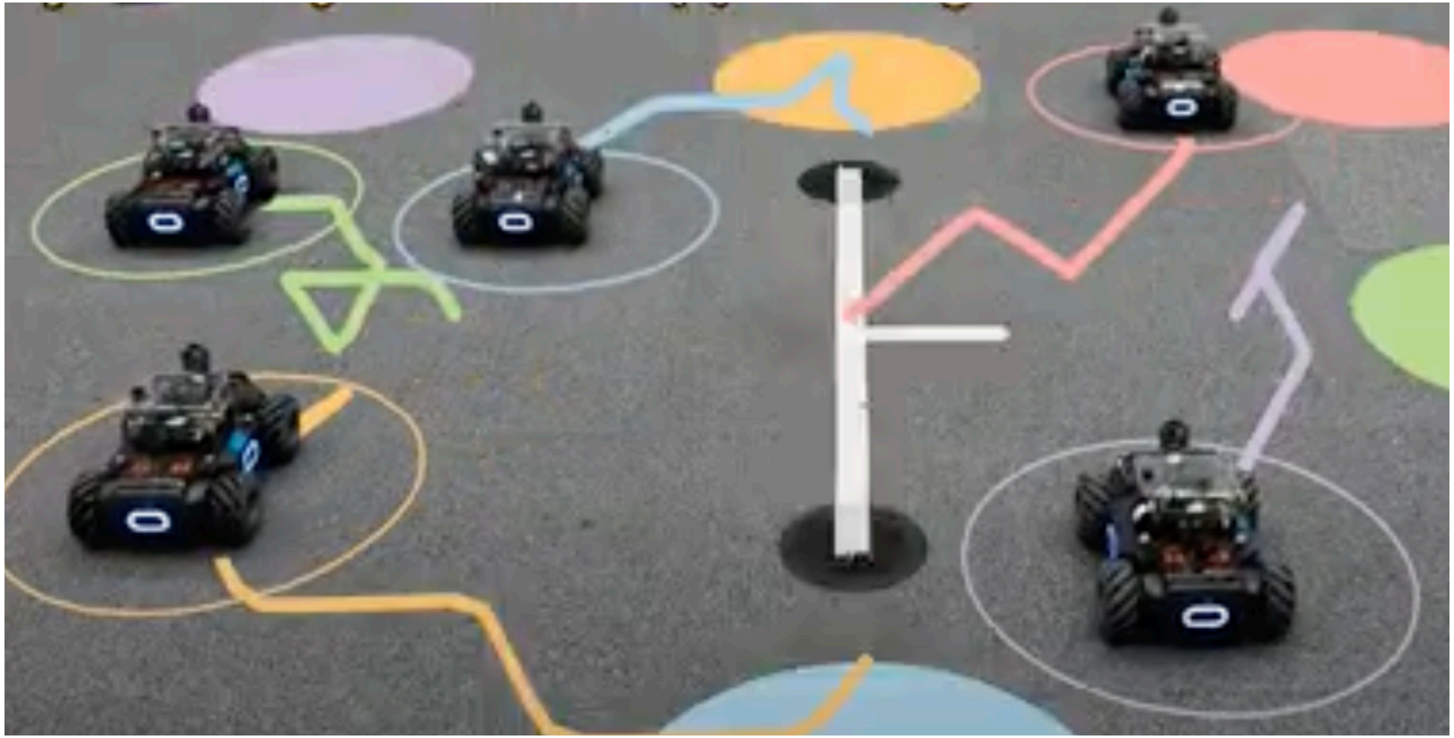

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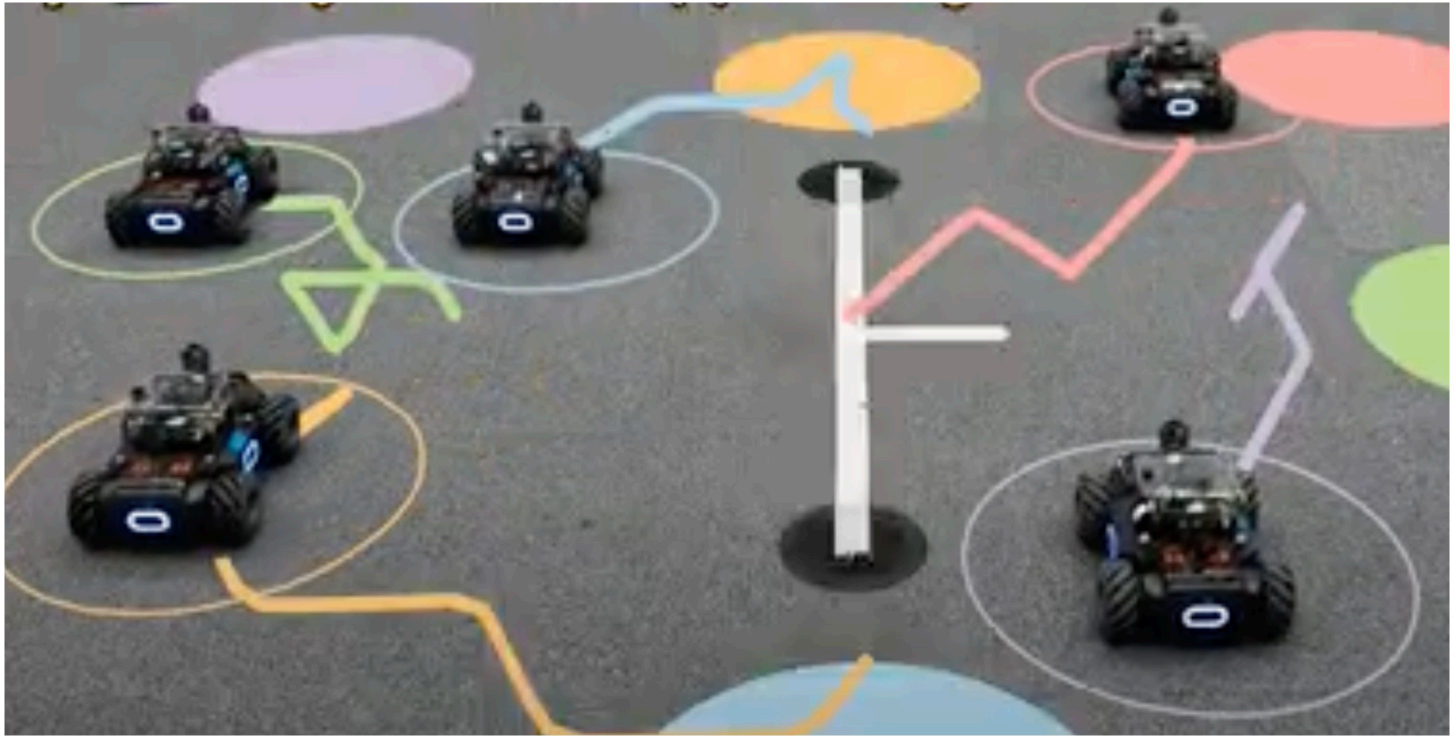

Answer: No!

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Question: We translate a sentence containing T words. Does order matter?

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Answer: Yes!

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For certain tasks, the order of inputs matters

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First, define a **permutation matrix** $\boldsymbol{P} \in \{0, 1\}^{T \times T}$ that reorders the inputs

Positional Encoding

Example 1:

$$\boldsymbol{P} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}; \quad \boldsymbol{a} = \begin{bmatrix} 3 \\ 4 \\ 5 \end{bmatrix};$$

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Positional Encoding

$$f\left(\boldsymbol{P}\begin{bmatrix}\boldsymbol{x}_1\\ \vdots\\ \boldsymbol{x}_n\end{bmatrix}\right) \neq \boldsymbol{P} f\left(\begin{bmatrix}\boldsymbol{x}_1\\ \vdots\\ \boldsymbol{x}_n\end{bmatrix}\right)$$

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Which is a transformer?

Positional Encoding

Recall dot product self attention

Positional Encoding

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$$\mathrm{attn}\left(\begin{bmatrix} \boldsymbol{x}_1 \\ \vdots \\ \boldsymbol{x}_T \end{bmatrix}, \boldsymbol{\theta} \right) = \mathrm{softmax}\left(\frac{\boldsymbol{Q}\boldsymbol{K}^\top}{\sqrt{d_h}} \right) \boldsymbol{V}$$

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Recall dot product self attention

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$$\boldsymbol{Q} = [\boldsymbol{q}_1 \;\; \boldsymbol{q}_2 \;\; \ldots \;\; \boldsymbol{q}_T] = \left[\boldsymbol{\theta}_Q^\top \boldsymbol{x}_1 \;\; \boldsymbol{\theta}_Q^\top \boldsymbol{x}_2 \;\; \ldots \;\; \boldsymbol{\theta}_Q^\top \boldsymbol{x}_T\right]$$

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Permuting the inputs reorders $\boldsymbol{Q}, \boldsymbol{K}, \boldsymbol{V}$

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$\boldsymbol{P}$ swaps row i and j

Positional Encoding

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Question: What does this mean?

Positional Encoding

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Question: What does this mean?

Answer: Attention/transformer **does not** understand order.

Equivariant, order **does not** matter to the transformer.

Positional Encoding

This makes sense, in our party attention example we never consider the order

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Transformer cannot determine order of inputs! **Equivariant** to ordering

Positional Encoding

The following sentences are the same to a transformer

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This is a problem! For some tasks, we care about input order

Can we make the transformer care about order?

Positional Encoding

Question: What are some ways we can introduce ordering?

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Answer 1: We can introduce forgetting (function of time)

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Positional Encoding

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RoPE: *Su, Jianlin, et al. "Roformer: Enhanced transformer with rotary position embedding." Neurocomputing.*

Positional Encoding

Question: What are some ways we can introduce ordering?

Answer 1: We can introduce forgetting (function of time)

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Answer 2: We can modify the inputs based on their ordering

We will focus on answer 2 because it is more common

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$$\text{attn}\left(\begin{bmatrix} \boldsymbol{x}_T \\ \boldsymbol{x}_1 \\ \vdots \\ \boldsymbol{x}_2 \end{bmatrix} + \begin{bmatrix} f_{\text{pos}}(T) \\ f_{\text{pos}}(1) \\ \vdots \\ f_{\text{pos}}(2) \end{bmatrix}, \boldsymbol{\theta}\right)$$

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In `torch`, this is called `nn.Embedding`

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Now, let us rewrite the transformer with the positional encoding

Positional Encoding

So, our final transformer is

```
class Transformer(nn.Module):
    def __init__(self):
        self.f_pos = nn.Embedding(d_x, T)
        self.layer1 = TransformerLayer()
        self.layer2 = TransformerLayer()

    def forward(self, x):
        x = x + f_pos(torch.arange(x.shape[0]))
        x = self.layer1(x)
        x = self.layer2(x)
        return x
```

Positional Encoding

Let us code up the transformer in colab

https://colab.research.google.com/drive/1qVIbQKpTuBYIa7FvC4IH-kJq-E0jmc0d#scrollTo=iQtXjGYiz5CD

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Let us see how to create inputs for our transformers

Text Transformers

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What if we represent a sentence like this

$$\underbrace{\begin{bmatrix} \text{John} \\ \text{likes} \\ \text{movies} \end{bmatrix}}_{d_x} \left.\vphantom{\begin{bmatrix} \text{John} \\ \text{likes} \\ \text{movies} \end{bmatrix}}\right\} T$$

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Step 3: Replace words with vector representations

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Now, let us write some pseudocode

Text Transformers

```
unique_words = set(sentence.split(" ") for sentence in xs)
word_index = {word: i for i, word in enumerate(unique_words)}
embeddings = nn.Embedding(len(tokens), d_x)
# Convert from words to parameters
xs = []
for sentence in sentences:
    xs.append([])
    for word in sentence:
        index = word_index[word]
        representation = embeddings[word_index]
        xs.append(representation)

print(xs)
>>> [[Tensor(...), Tensor(...), ...]]
```

Text Transformers

Now, feed our dataset to the transformer

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```
model = Transformer()
for sentence_representation in xs:
    # Convert list to tensor
    x = torch.stack(tokenized_sentence)
    y = model(x)
```

Image Transformers

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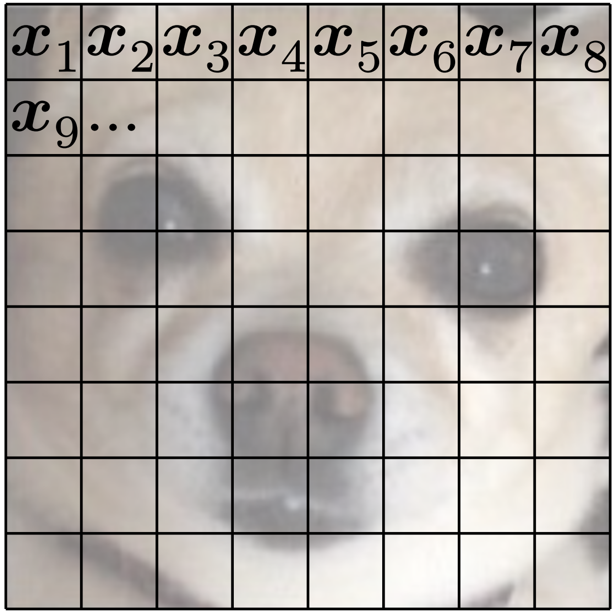

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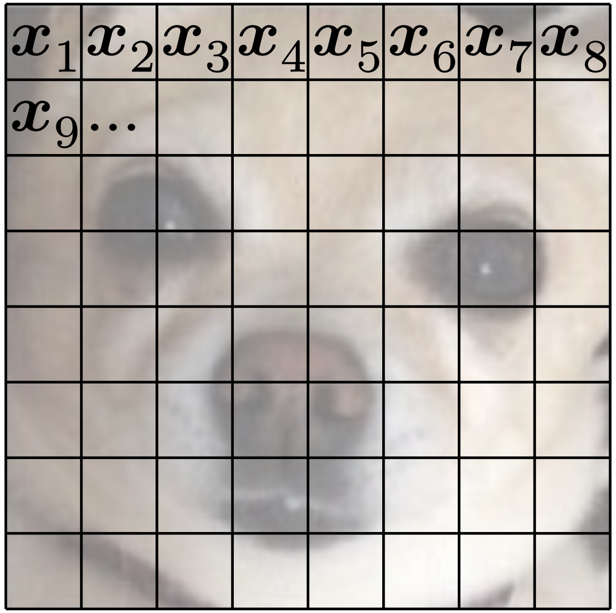

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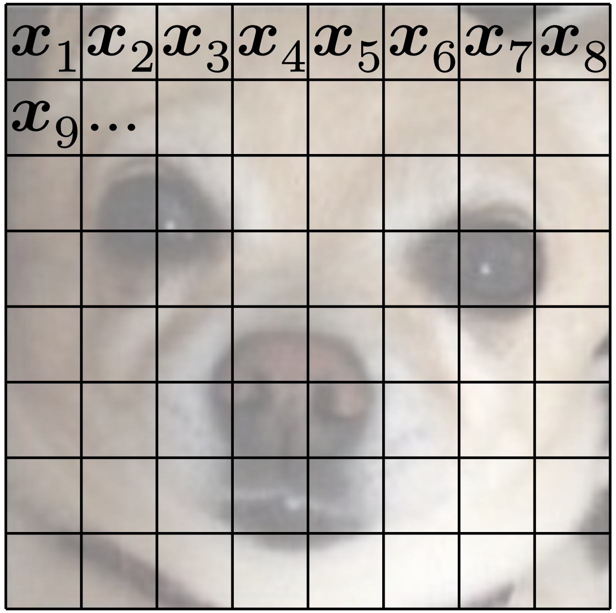

Then, feed a sequence of patches to the transformer

$$f\left(\begin{bmatrix} \boldsymbol{x}_1 \\ \vdots \\ \boldsymbol{x}_T \end{bmatrix}, \boldsymbol{\theta}\right)$$

Image Transformers

```
# Convert image into patches
patches = []
for i in range(0, x.shape[0] - k + 1, k):
    for j in range(0, x.shape[1] - k + 1, k):
        patches.append(
            x[i: i + k , j: j + k]
        )
patches = stack(patches, axis=0)
print(patches.shape)
>>> (T, k, k)

model = Transformer()
y = model(patches)
```

Generative Pre-Training

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Question: How do we train transformers?

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There are not enough graduate students to label training data!

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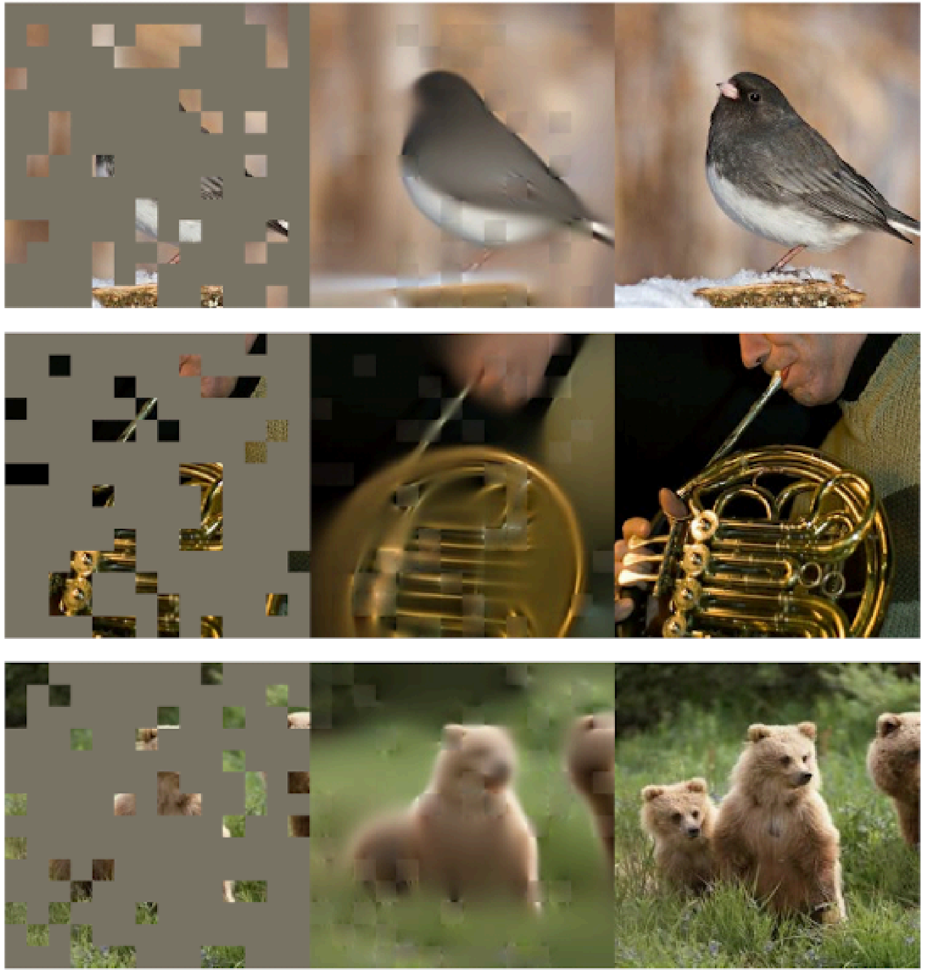
We **generate** the missing data

This method is **extremely** powerful

Generative Pre-Training



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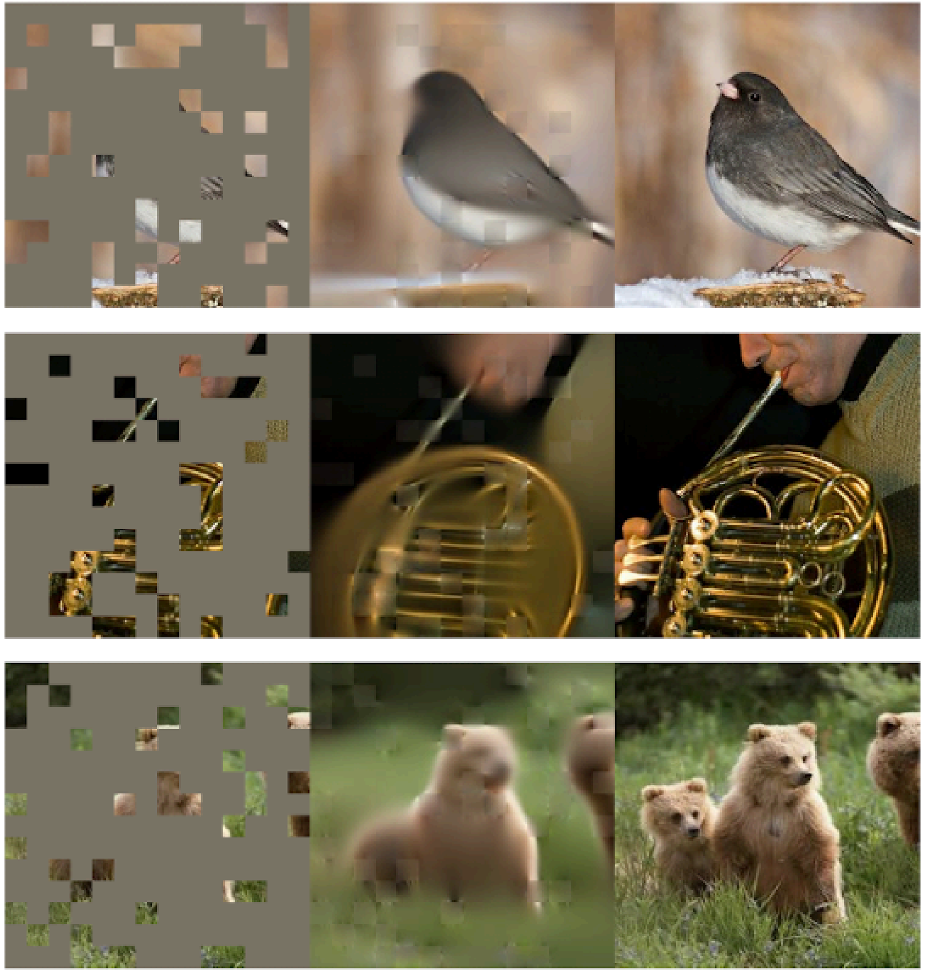

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$$P\left(\begin{bmatrix} \text{movies} \\ \text{Mary} \\ \text{dogs} \end{bmatrix} \middle| \text{John, likes}; \boldsymbol{\theta}\right) = \begin{bmatrix} 0.5 \\ 0.3 \\ 0.2 \end{bmatrix}$$

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$\boldsymbol{x}_T$ = "movies" $\quad\quad$ $P(\text{movies} \mid \text{John, likes}; \boldsymbol{\theta}) = 0.5$

Update $\boldsymbol{\theta}$ so that $P(\text{movies} \mid \text{John, likes}; \boldsymbol{\theta}) = 0.6$

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What about for images?

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$$= \arg\min_{\boldsymbol{\theta}} [-\log P(\text{pixel\_3} \mid \text{pixel\_1}, \text{pixel\_2}; \boldsymbol{\theta})]$$

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The square error represents a normal distribution over pixel values

$$= \arg\min_{\boldsymbol{\theta}} \left(f\left(\begin{bmatrix} \text{pixel\_1} \\ \text{pixel\_2} \end{bmatrix}, \boldsymbol{\theta}\right) - \text{pixel\_3}\right)^2$$

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Let us see what the models learn with a GPT loss

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Let us start with the image transformer

Generative Pre-Training

Anyone familiar with Da Vinci's painting *The Last Supper*?

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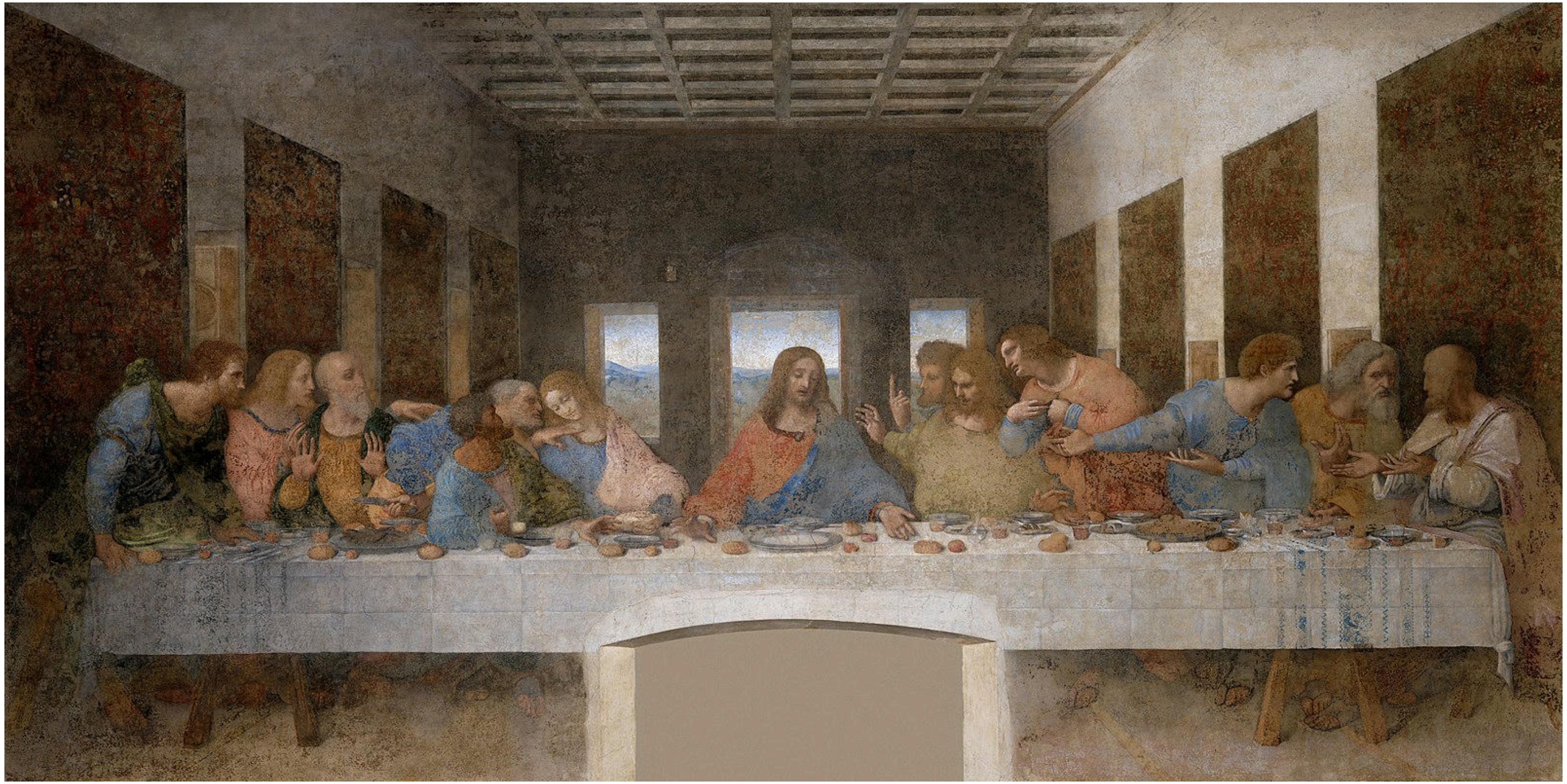
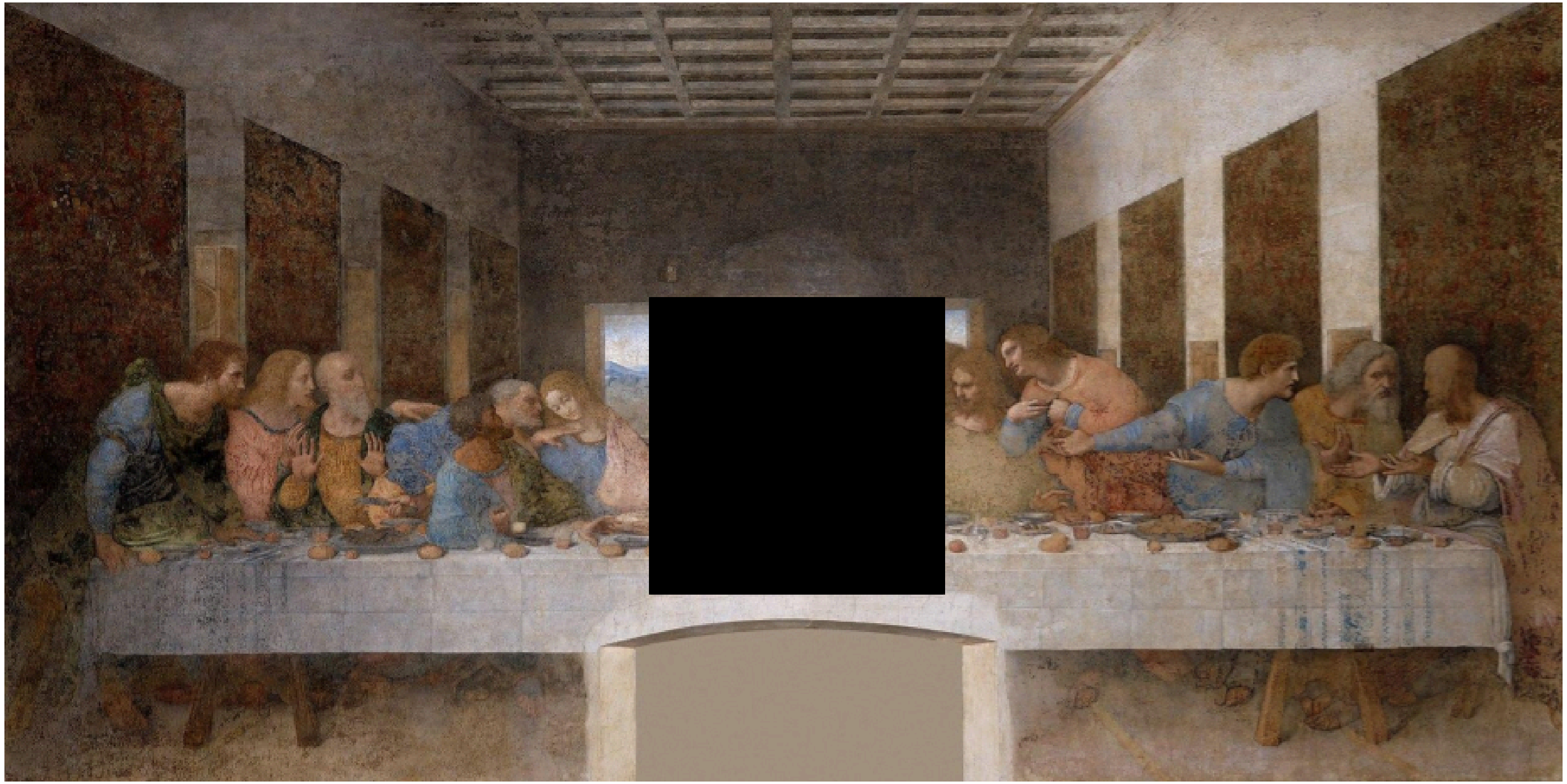
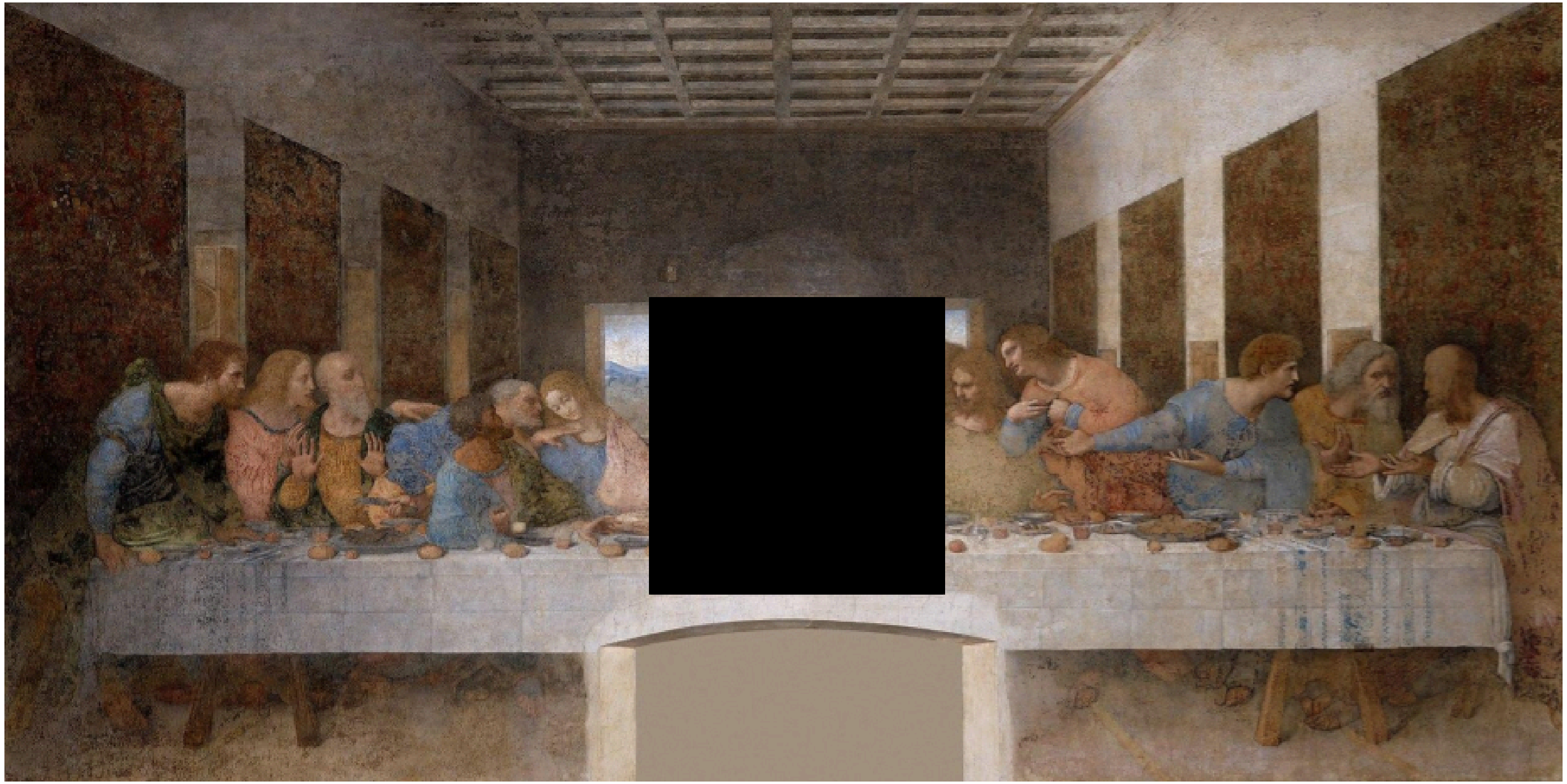


Generative Pre-Training



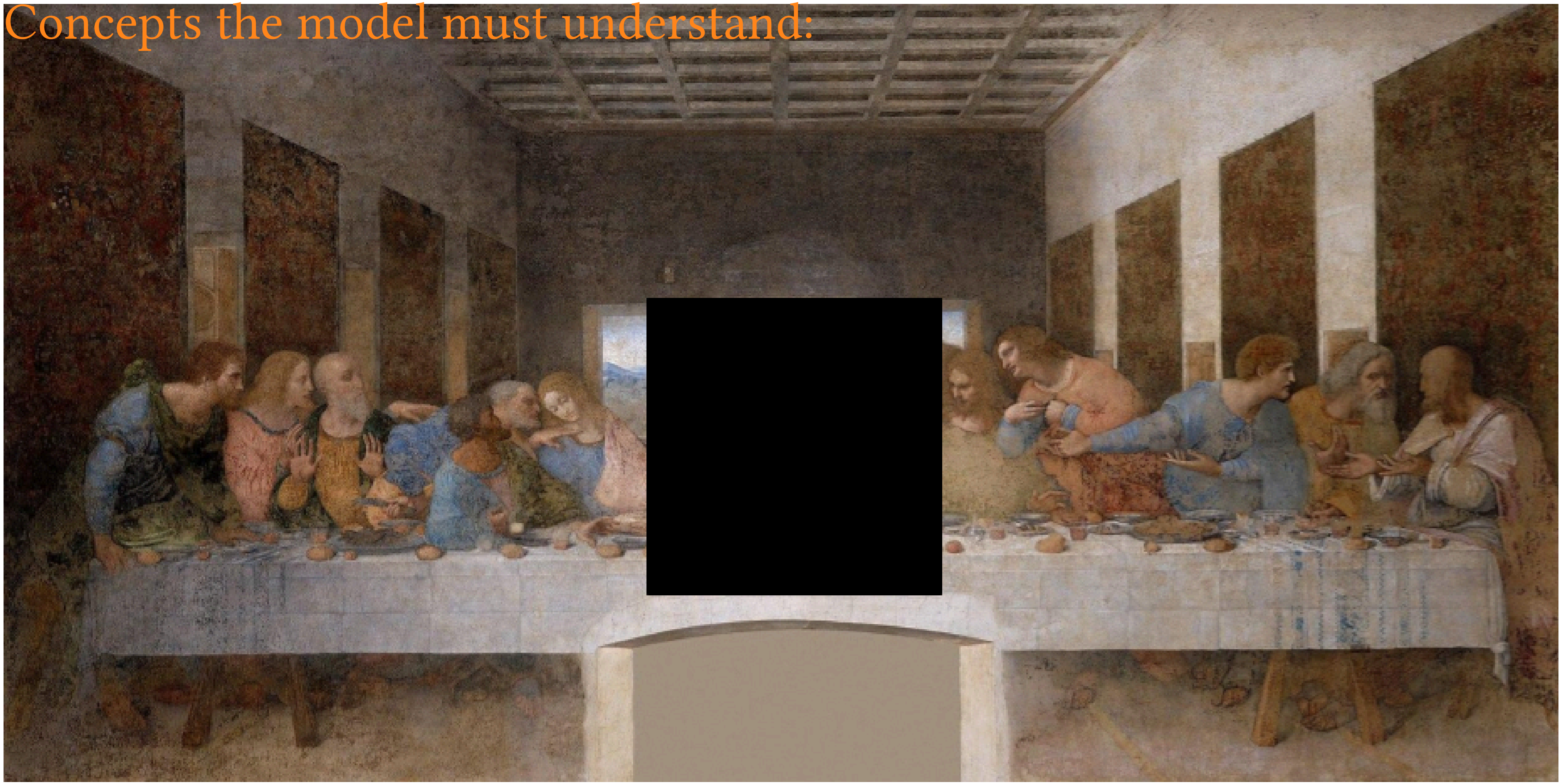
Generative Pre-Training



Question: What concepts does the vision transformer need to understand to predict the missing pixels?

Generative Pre-Training

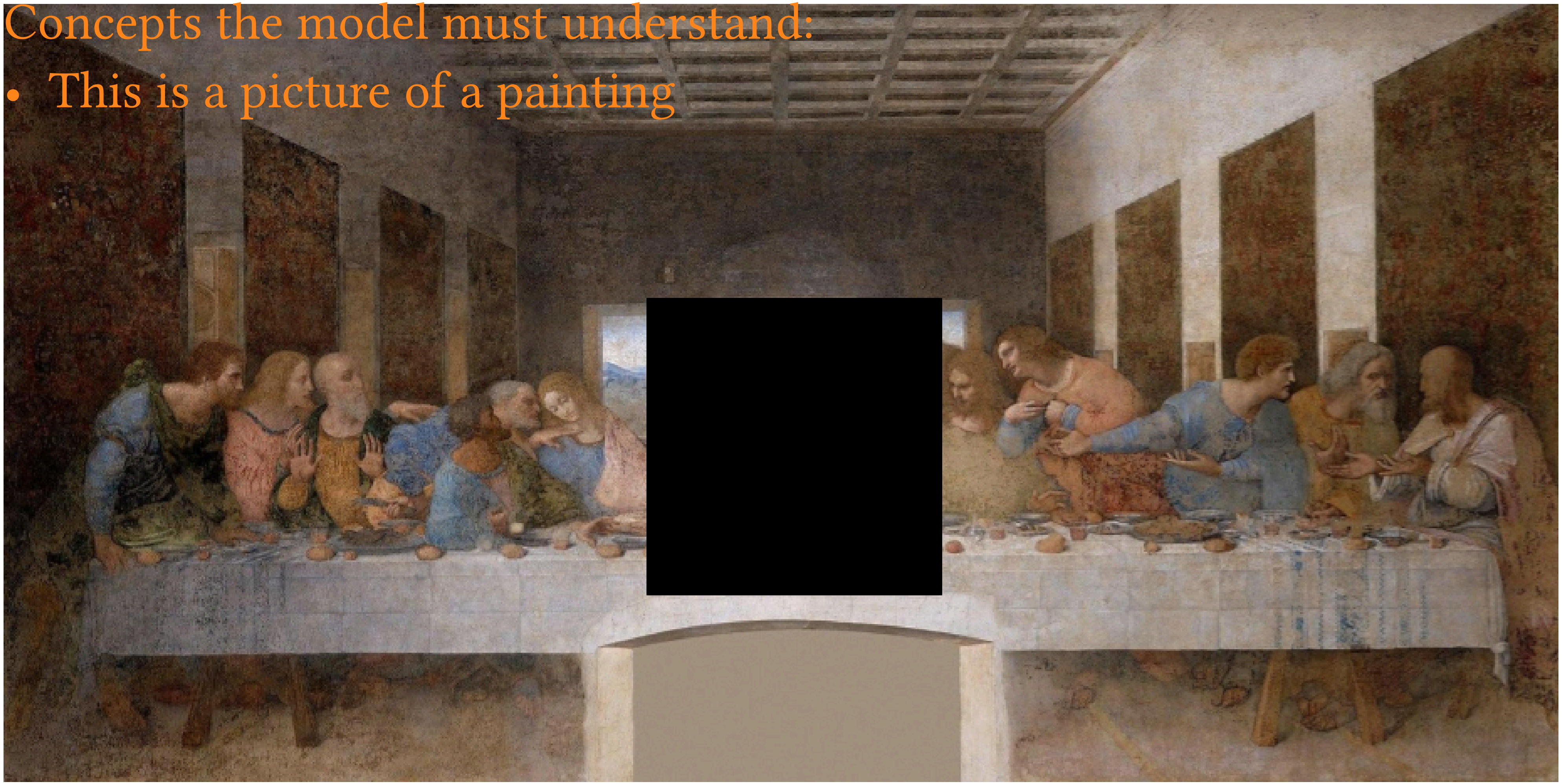
Concepts the model must understand:



Generative Pre-Training

Concepts the model must understand:

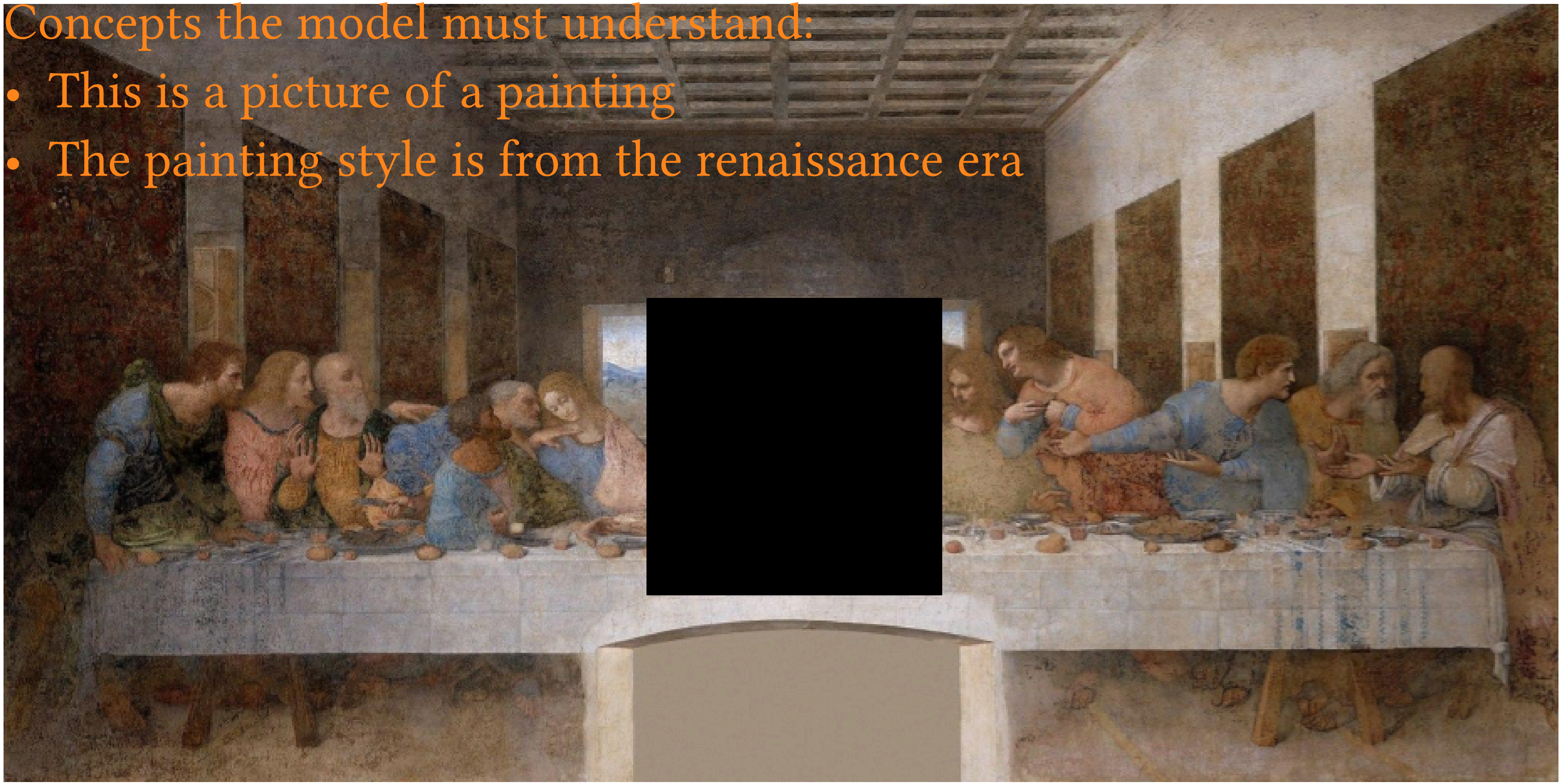
- This is a picture of a painting



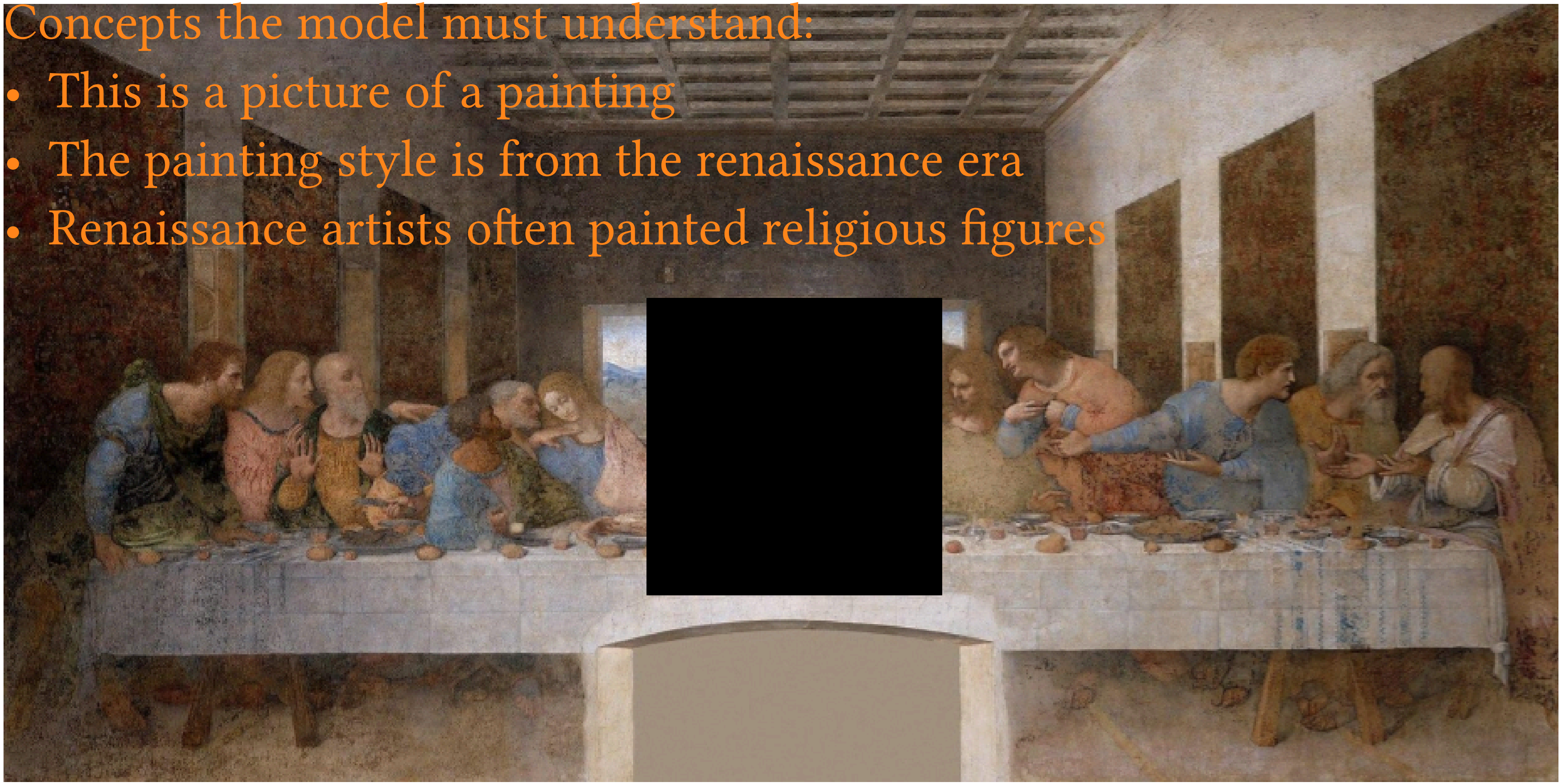
Generative Pre-Training

Concepts the model must understand:

- This is a picture of a painting
- The painting style is from the renaissance era

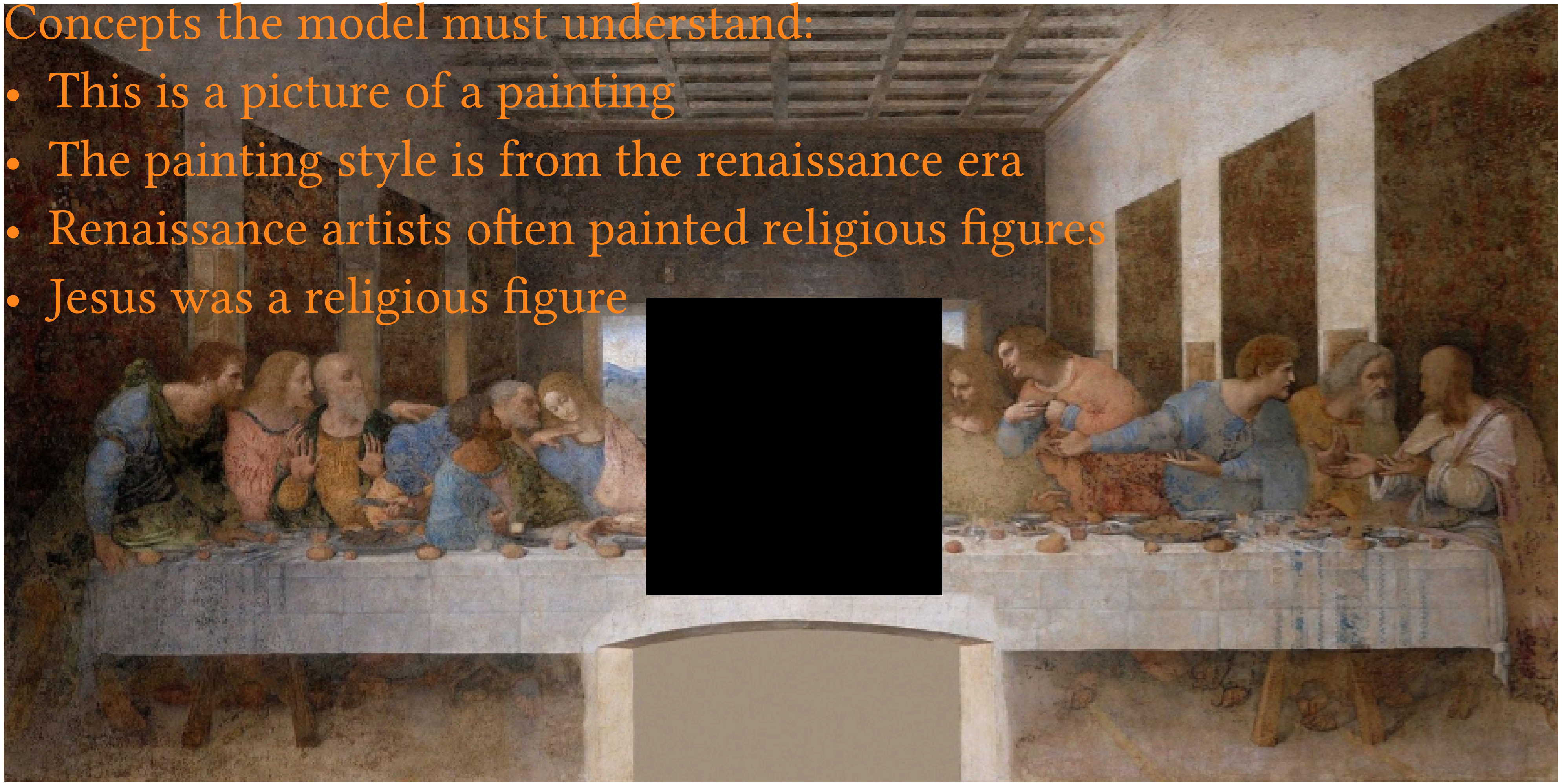


Generative Pre-Training



Concepts the model must understand:
- This is a picture of a painting
- The painting style is from the renaissance era
- Renaissance artists often painted religious figures

Generative Pre-Training



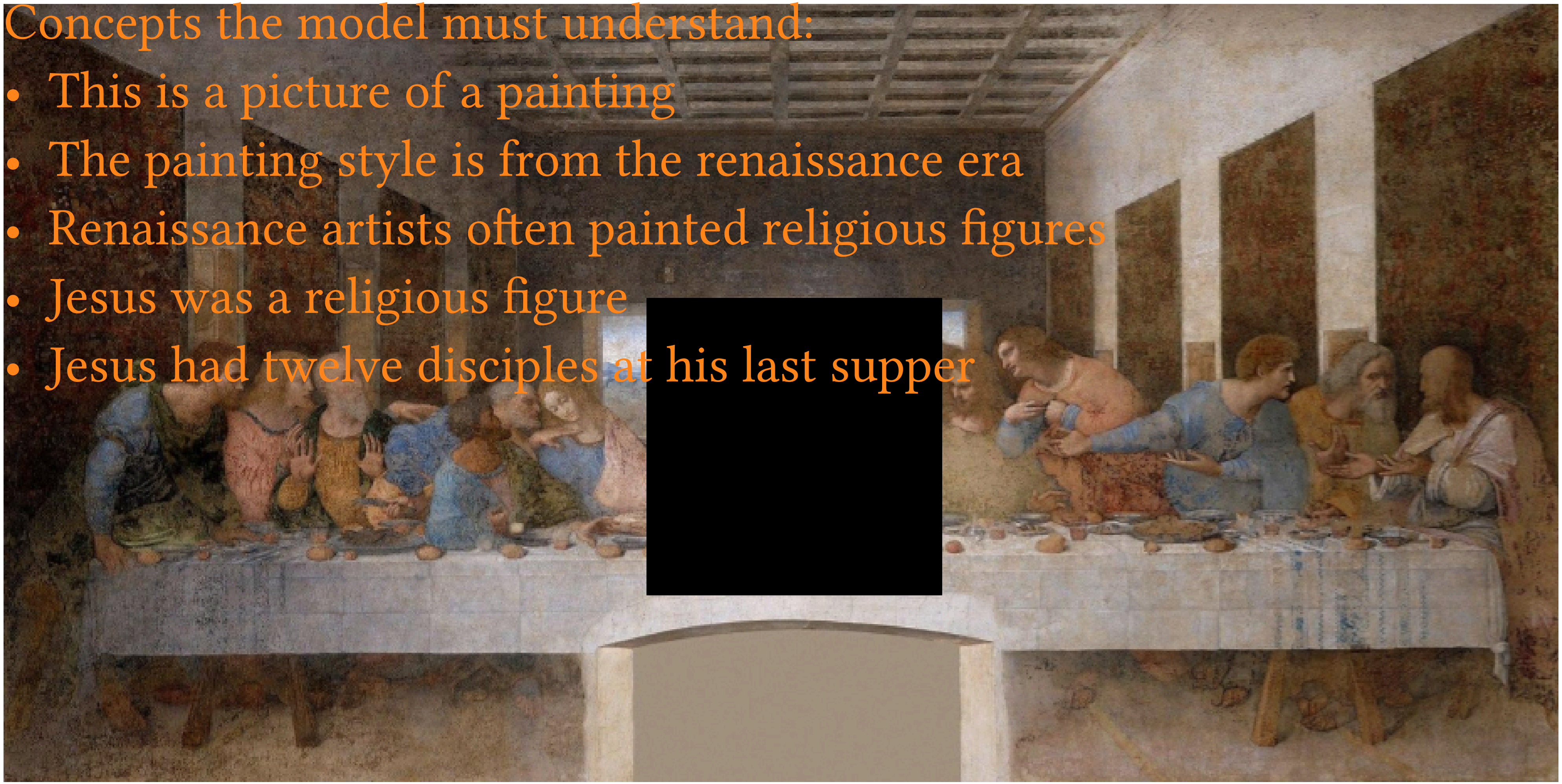
Concepts the model must understand:

- This is a picture of a painting
- The painting style is from the renaissance era
- Renaissance artists often painted religious figures
- Jesus was a religious figure

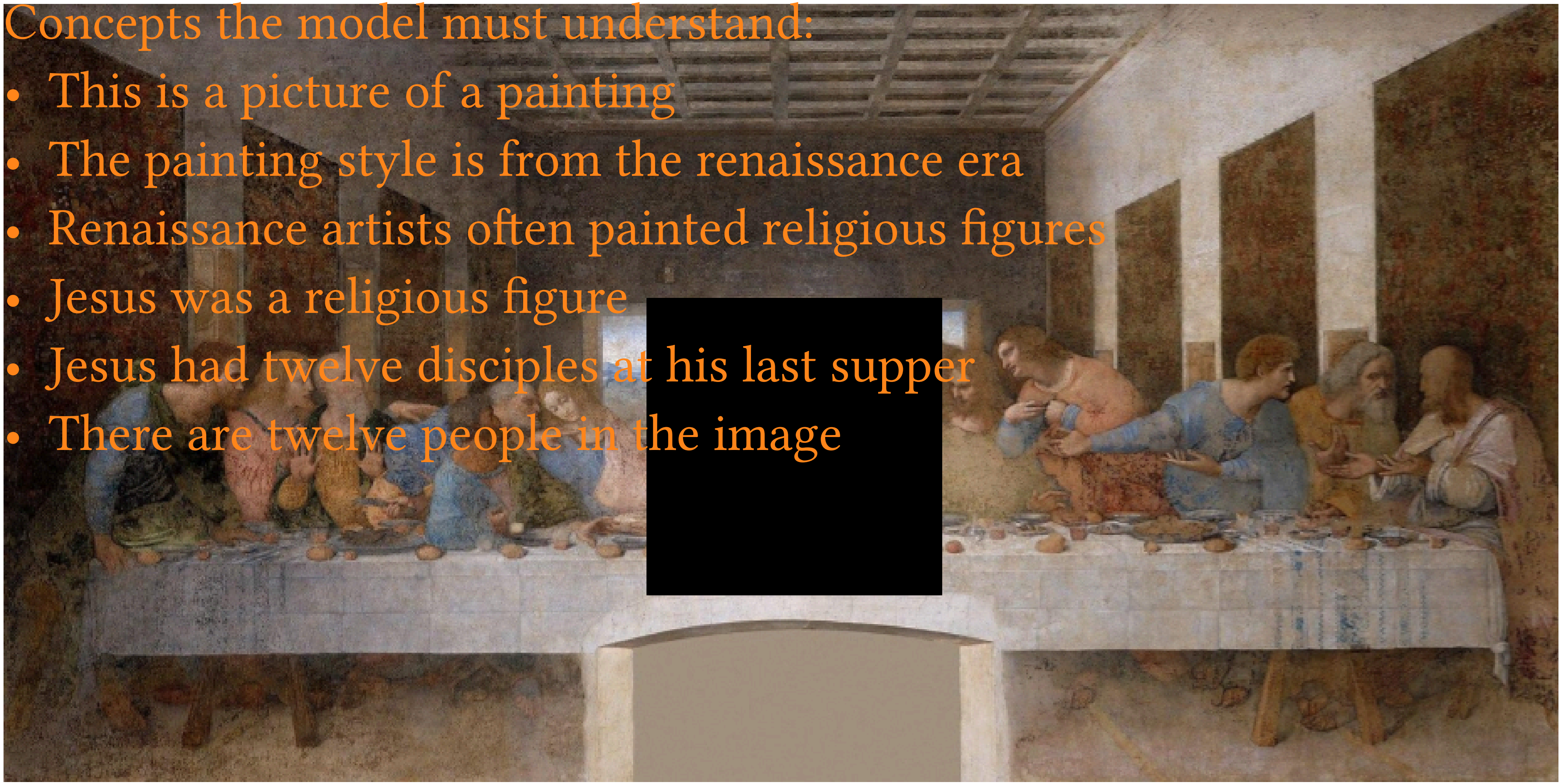
Generative Pre-Training

Concepts the model must understand:

- This is a picture of a painting
- The painting style is from the renaissance era
- Renaissance artists often painted religious figures
- Jesus was a religious figure
- Jesus had twelve disciples at his last supper



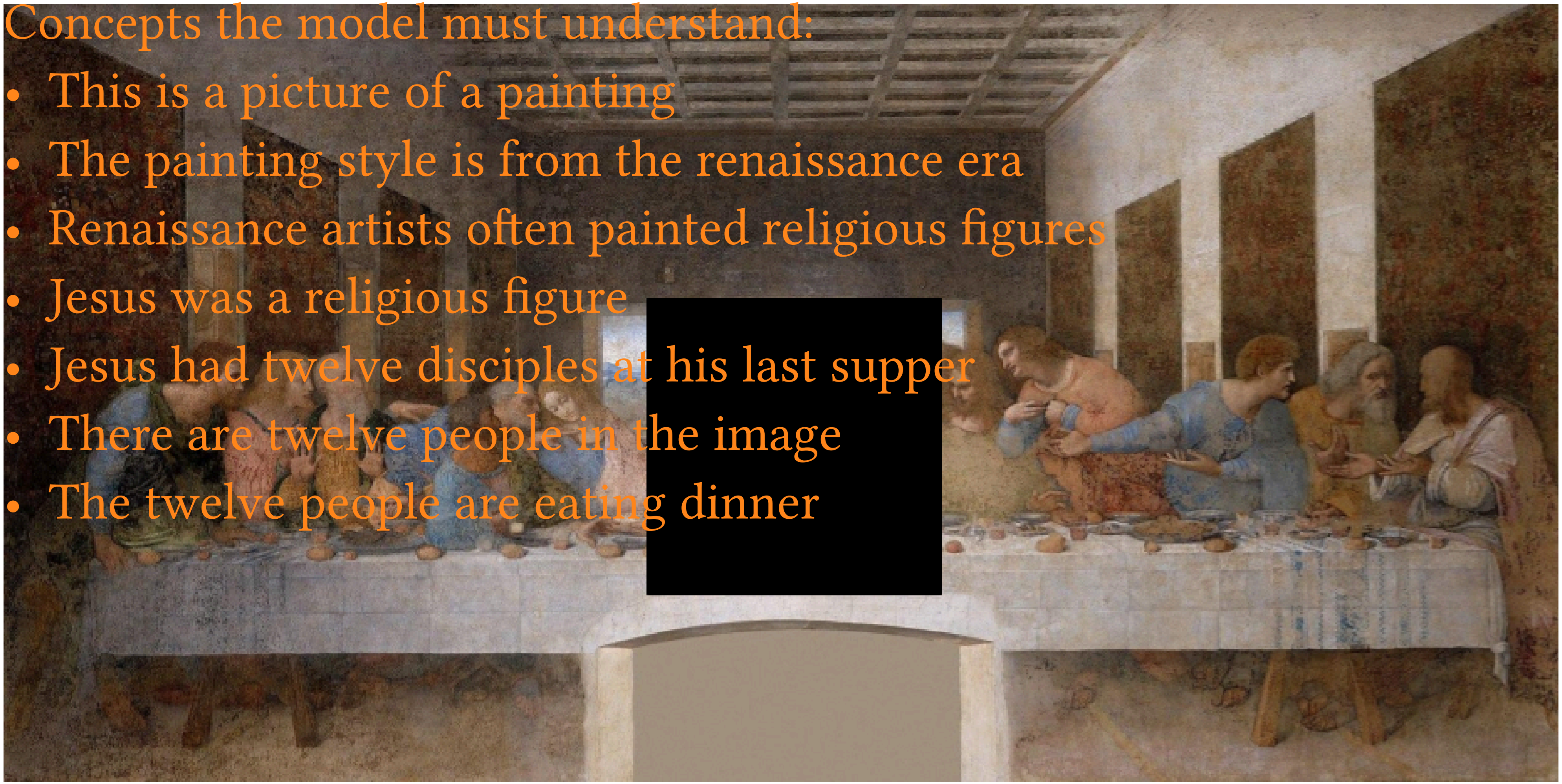
Generative Pre-Training



Concepts the model must understand:

- This is a picture of a painting
- The painting style is from the renaissance era
- Renaissance artists often painted religious figures
- Jesus was a religious figure
- Jesus had twelve disciples at his last supper
- There are twelve people in the image

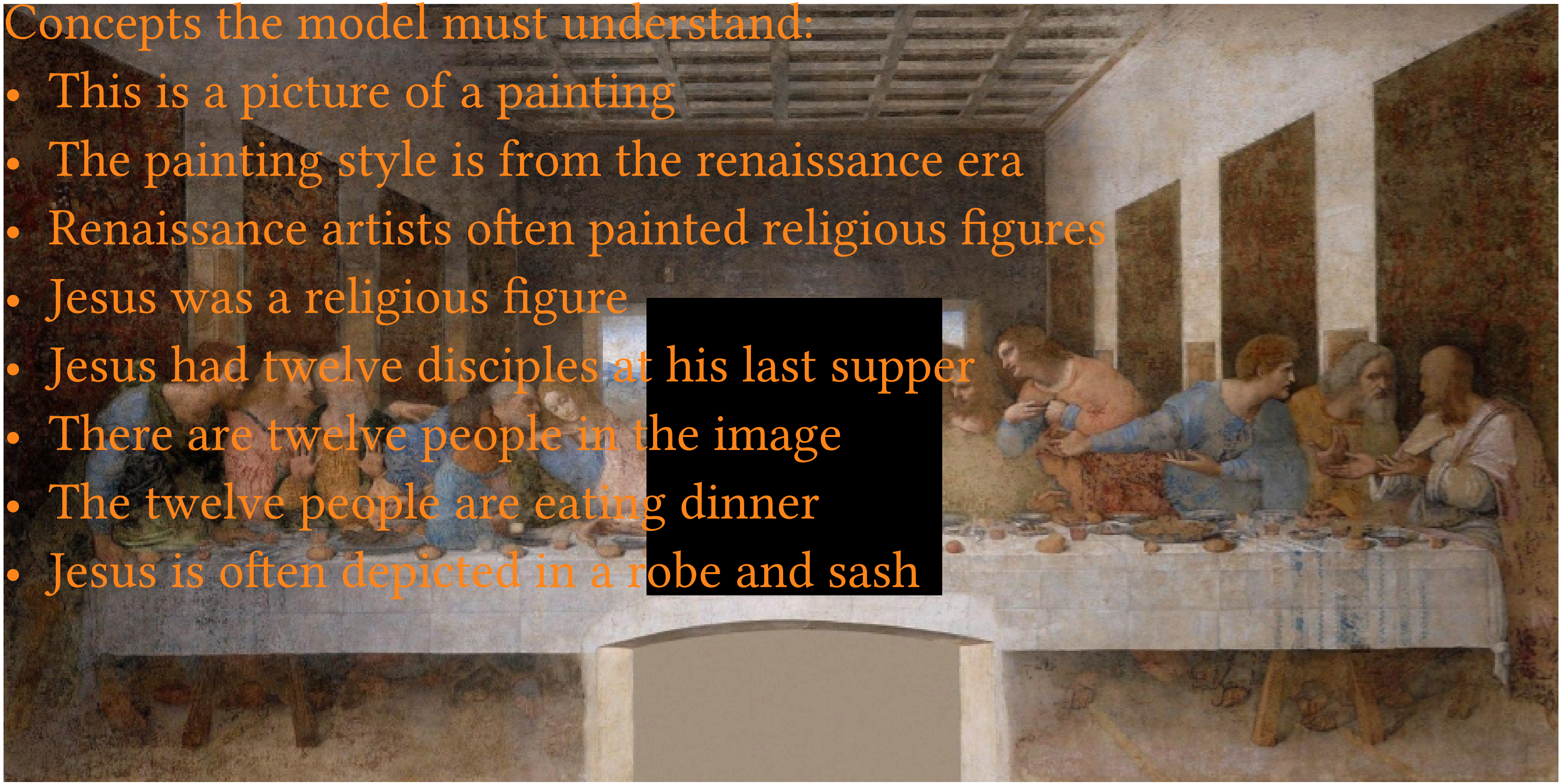
Generative Pre-Training



Concepts the model must understand:

- This is a picture of a painting
- The painting style is from the renaissance era
- Renaissance artists often painted religious figures
- Jesus was a religious figure
- Jesus had twelve disciples at his last supper
- There are twelve people in the image
- The twelve people are eating dinner

Generative Pre-Training



Concepts the model must understand:

- This is a picture of a painting
- The painting style is from the renaissance era
- Renaissance artists often painted religious figures
- Jesus was a religious figure
- Jesus had twelve disciples at his last supper
- There are twelve people in the image
- The twelve people are eating dinner
- Jesus is often depicted in a robe and sash

Generative Pre-Training

We train the model to fix the image

Generative Pre-Training

We train the model to fix the image

To fix the image, the model must understand so much of our world

Generative Pre-Training

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This is the power of generative pre-training

Generative Pre-Training

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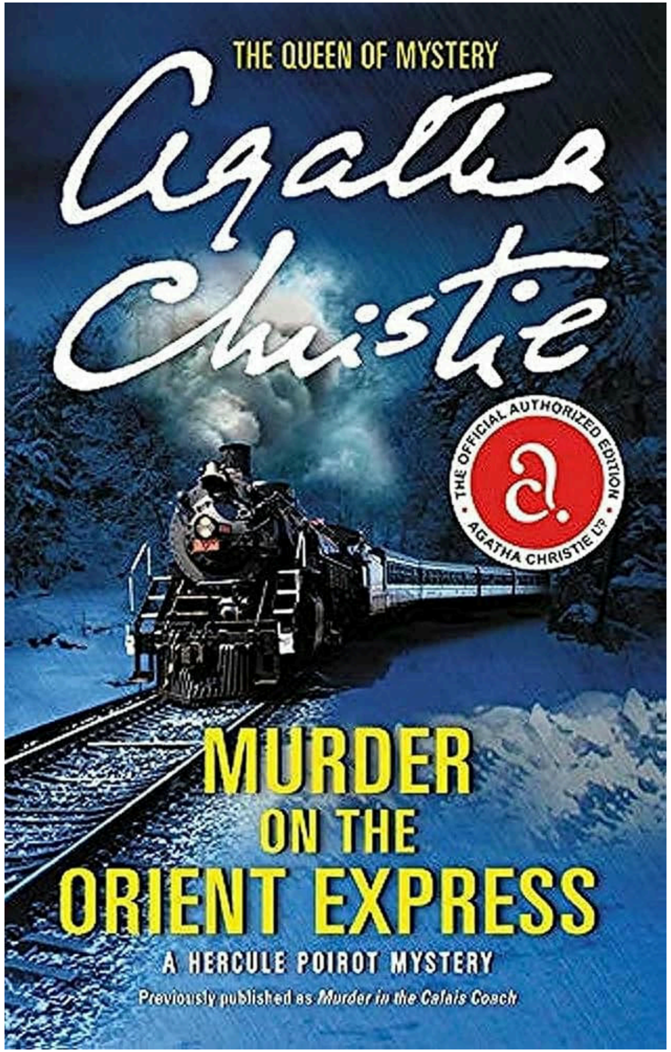
To fix the image, the model must understand so much of our world

This is the power of generative pre-training

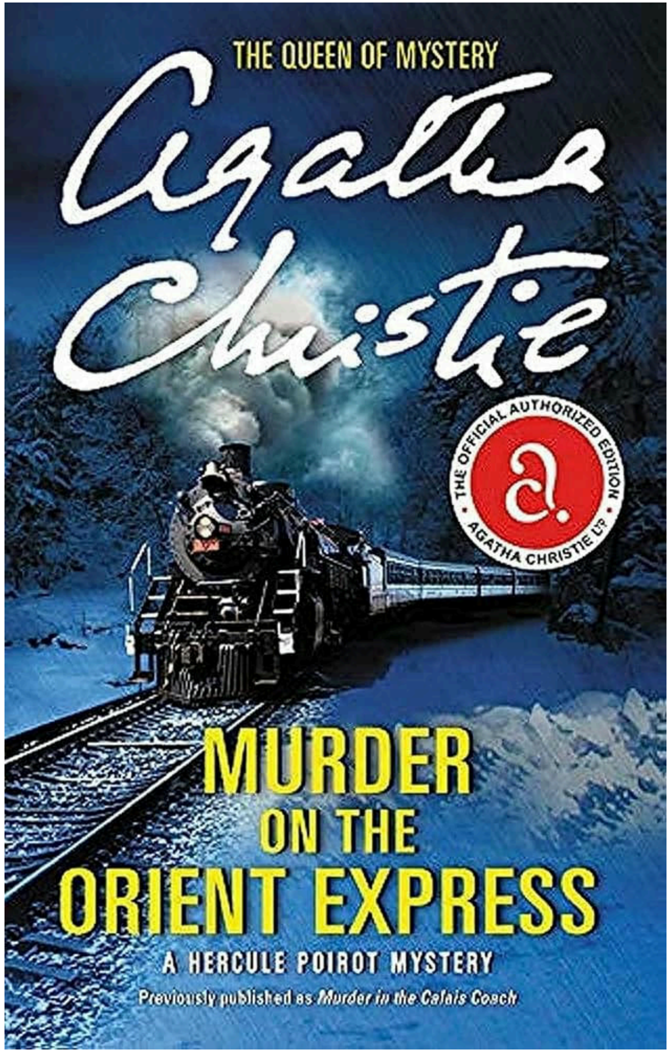
What about text transformers?

Generative Pre-Training



This is a mystery novel

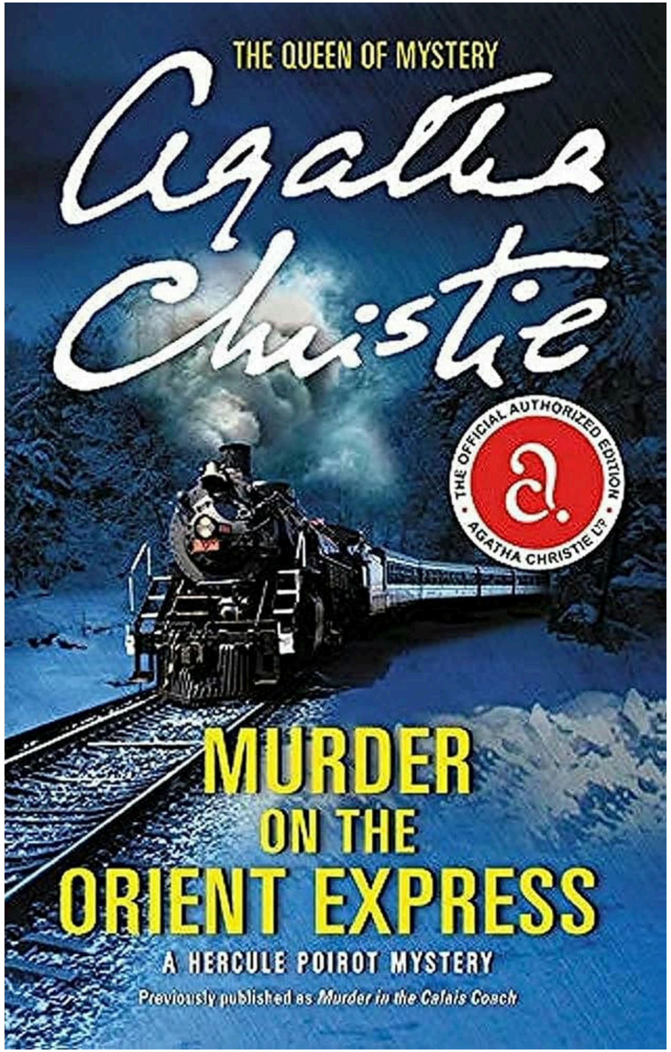
Generative Pre-Training



This is a mystery novel

Clues, intrigue, murder, etc

Generative Pre-Training

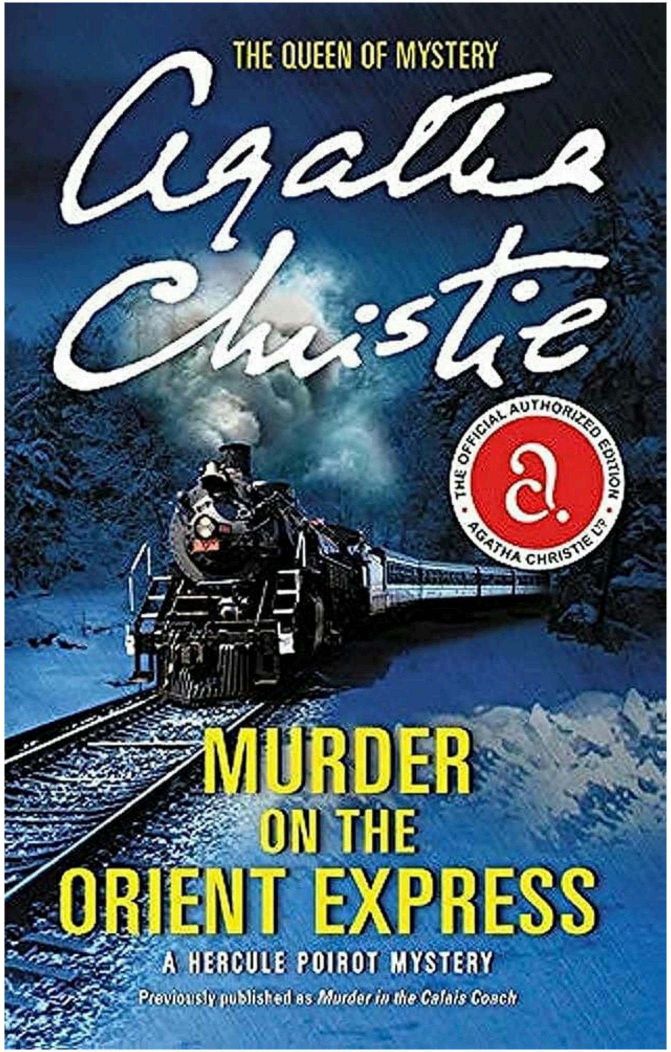


This is a mystery novel

Clues, intrigue, murder, etc

"Ah, said inspector Poirot, the murderer must be ________."

Generative Pre-Training



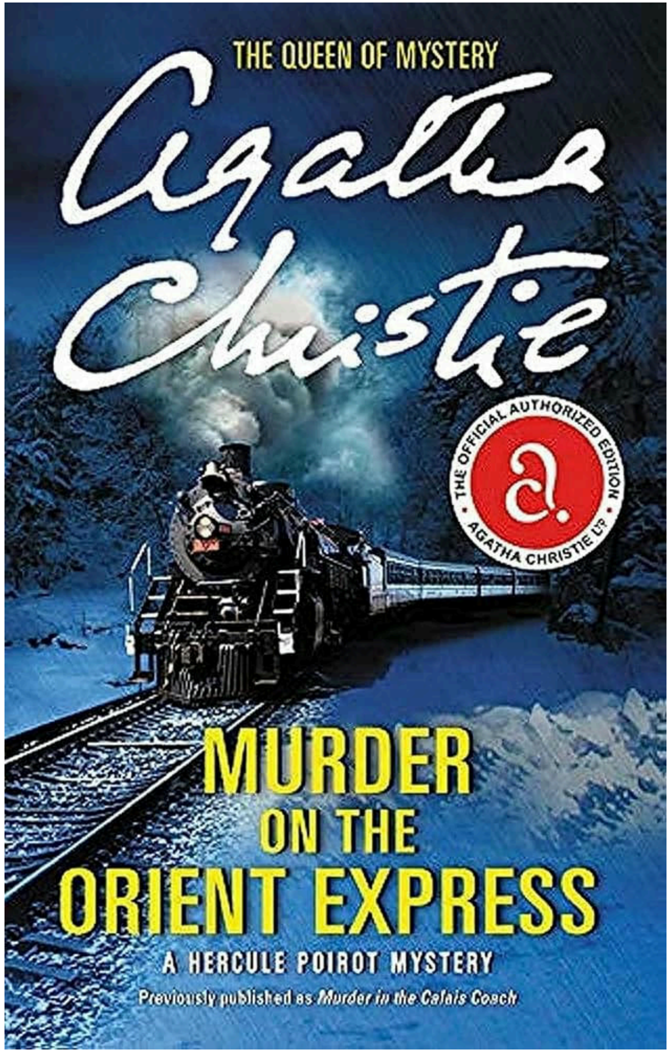
This is a mystery novel

Clues, intrigue, murder, etc

“Ah, said inspector Poirot, the murderer must be ________.”

To complete the sentence, the model must understand:

Generative Pre-Training



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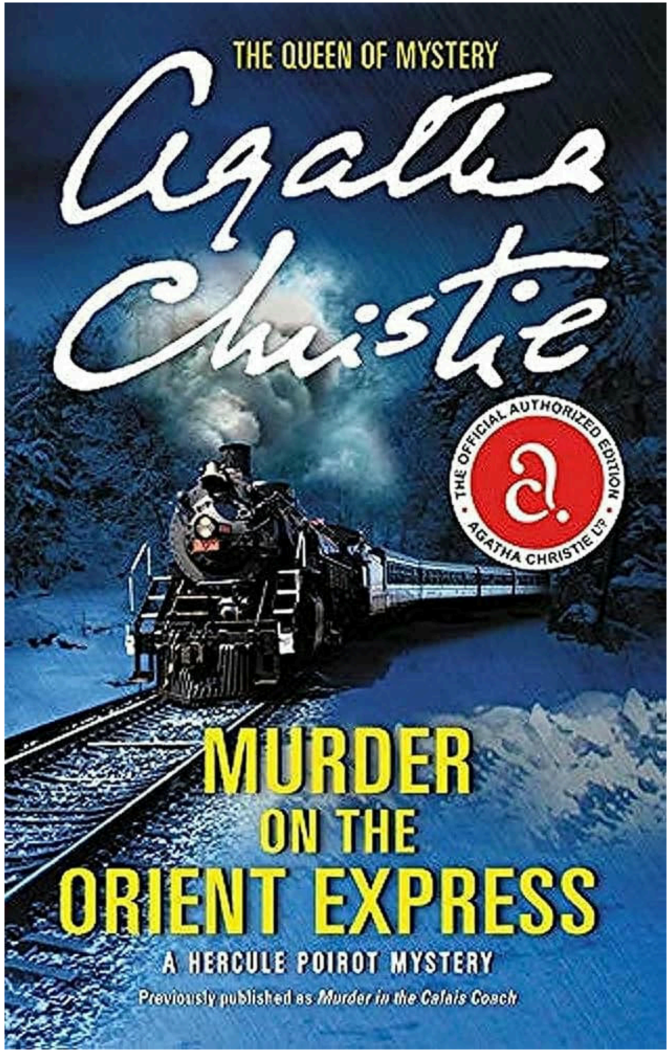
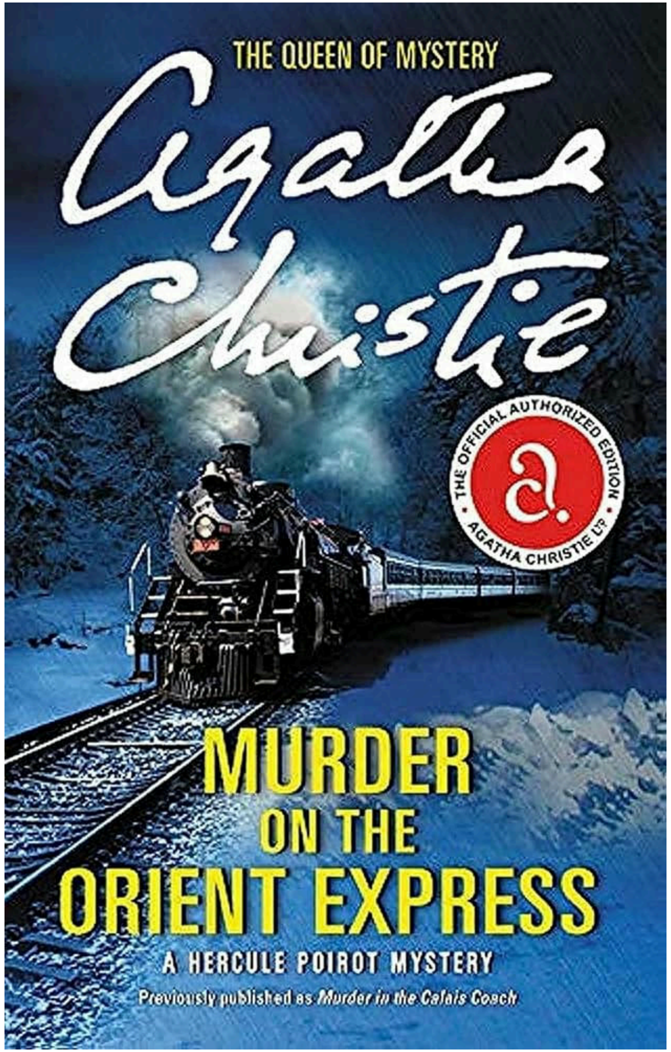


To complete the sentence, the model must understand:

- What a murder is

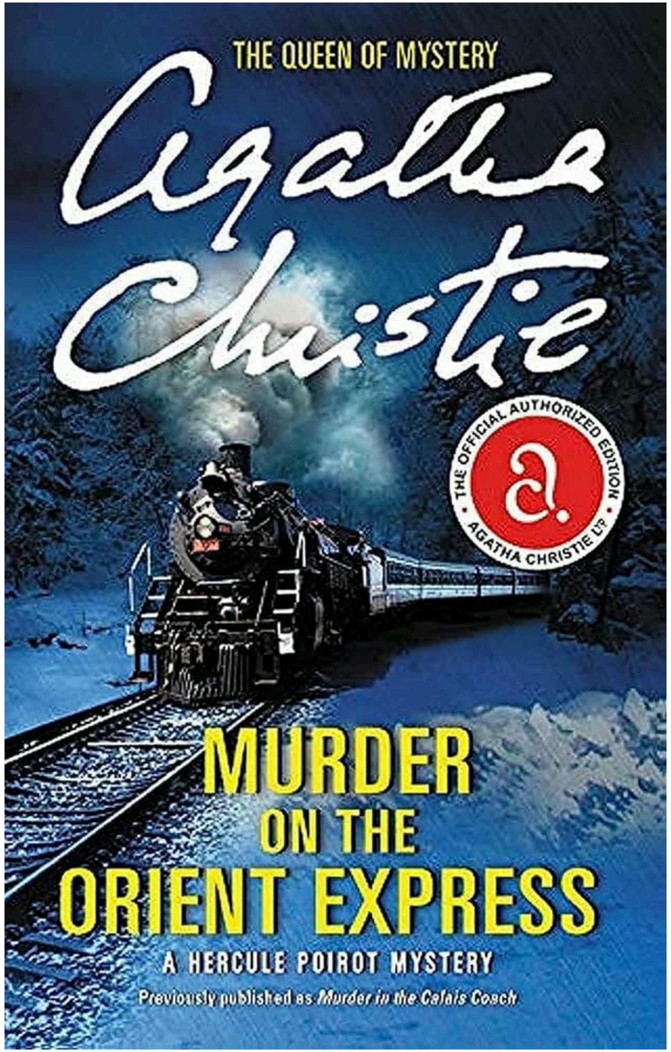
Generative Pre-Training



To complete the sentence, the model must understand:

- What a murder is
- What it means to be alive

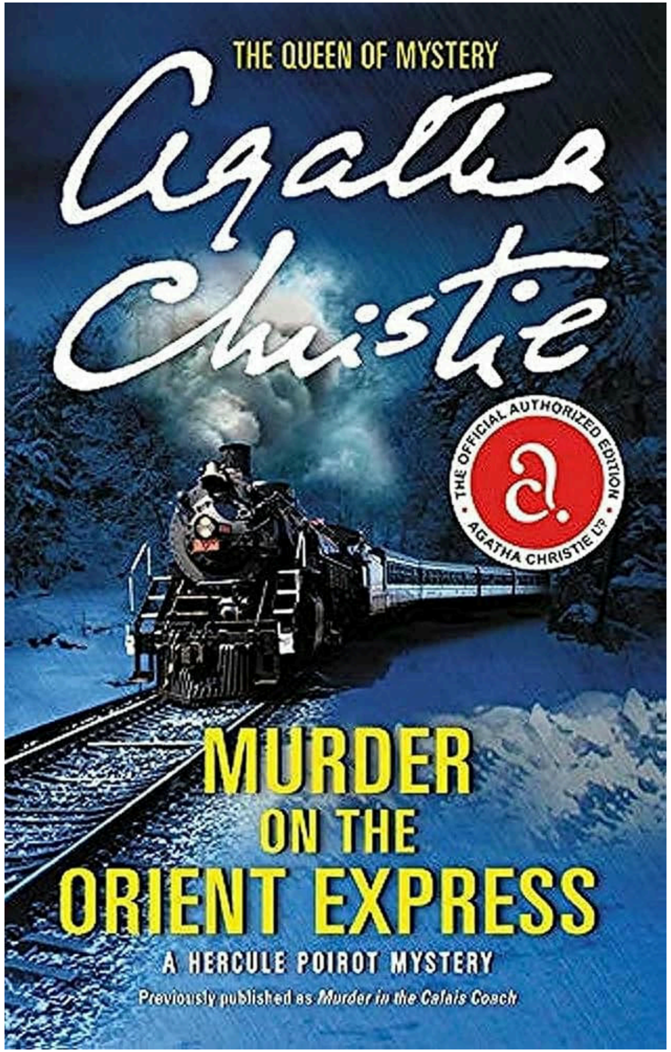
Generative Pre-Training



To complete the sentence, the model must understand:

- What a murder is
- What it means to be alive
- Emotions like anger, jealousy, betrayal, love

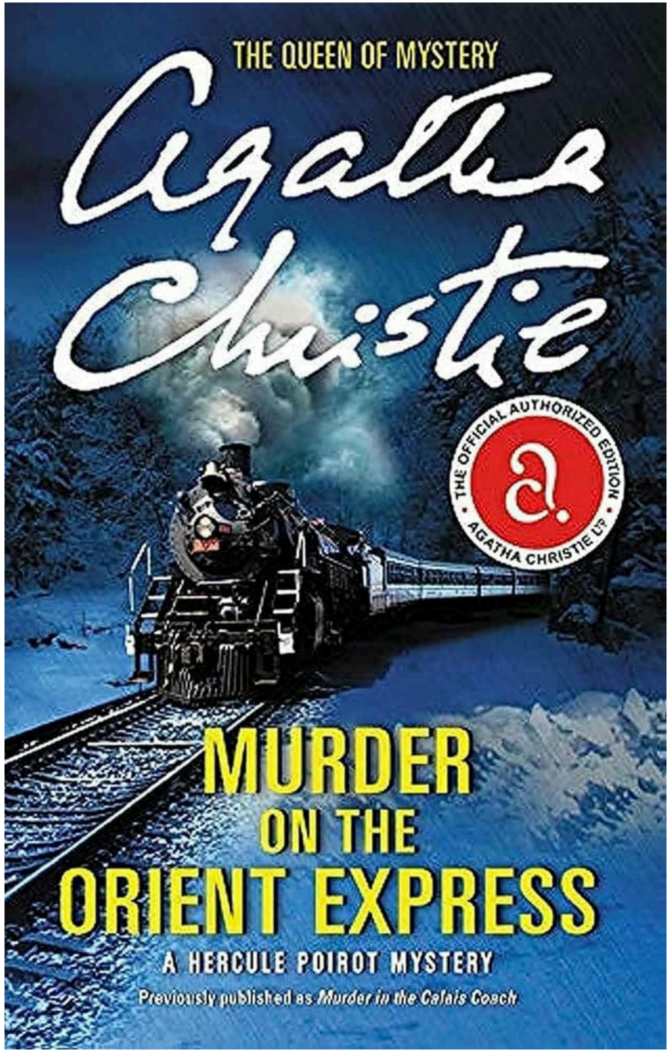
Generative Pre-Training



To complete the sentence, the model must understand:

- What a murder is
- What it means to be alive
- Emotions like anger, jealousy, betrayal, love
- Personalities of each character

Generative Pre-Training



To complete the sentence, the model must understand:

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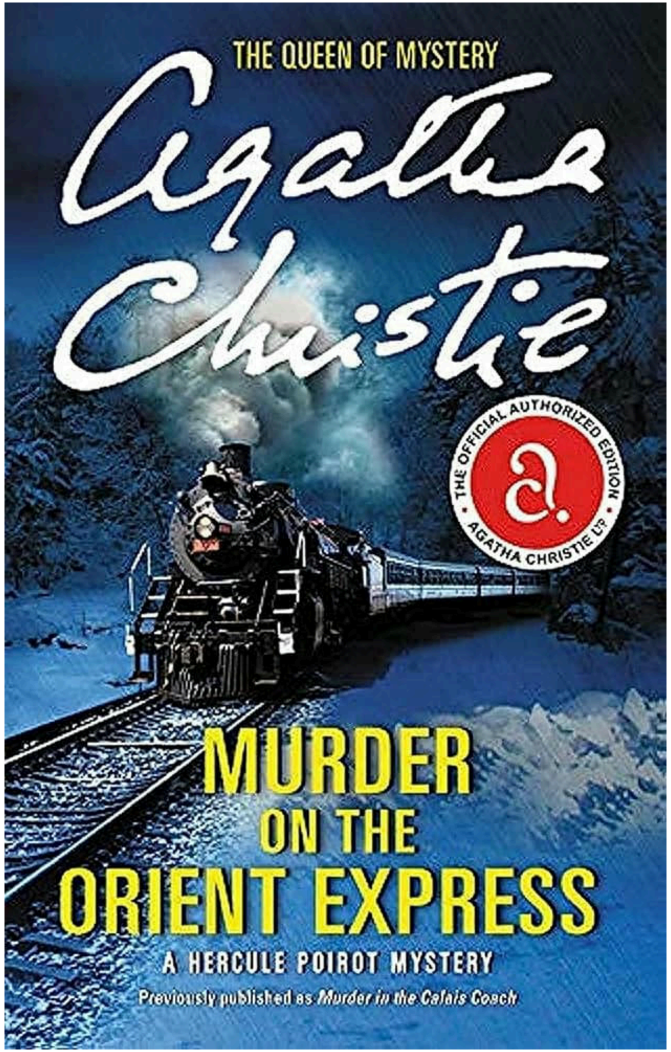
- What it means to be alive
- Emotions like anger, jealousy, betrayal, love
- Personalities of each character
- Why a human would murder another human

Generative Pre-Training



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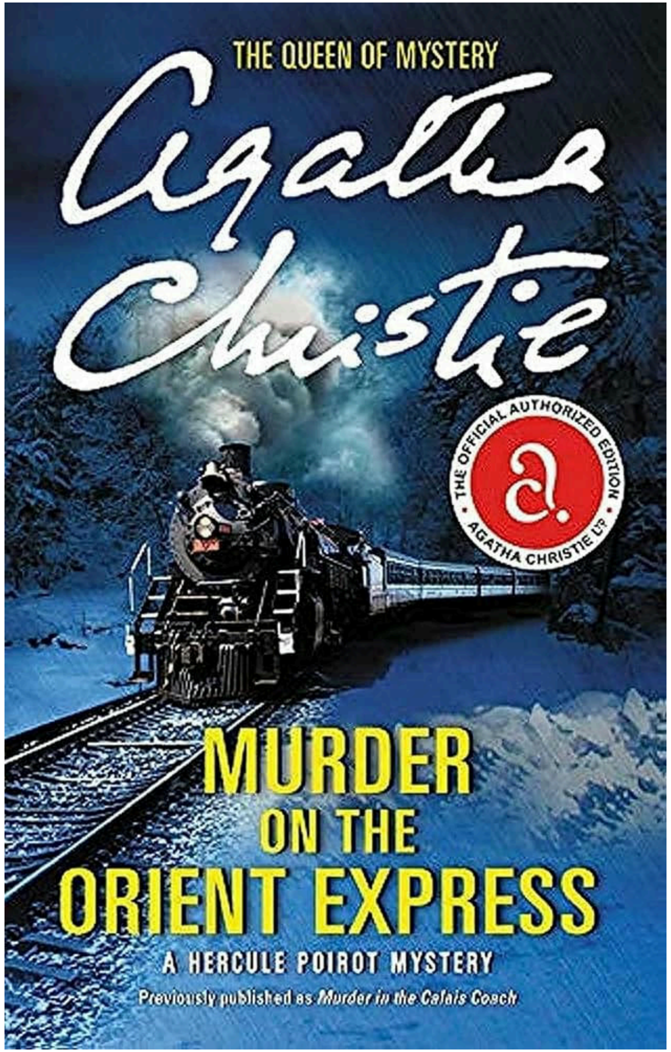
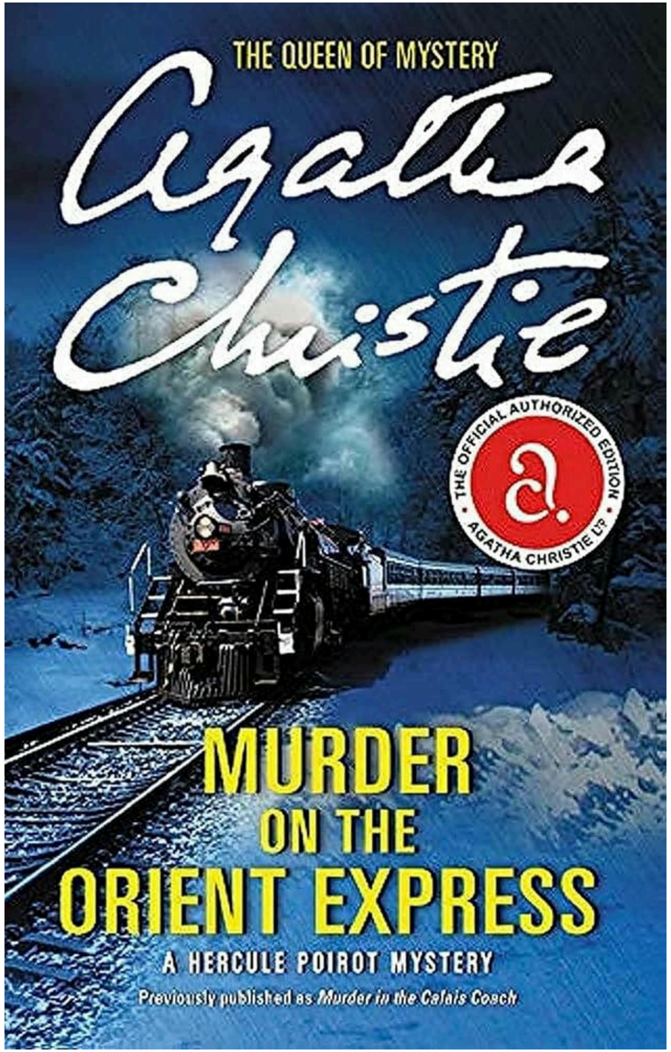


To complete the sentence, the model must understand:

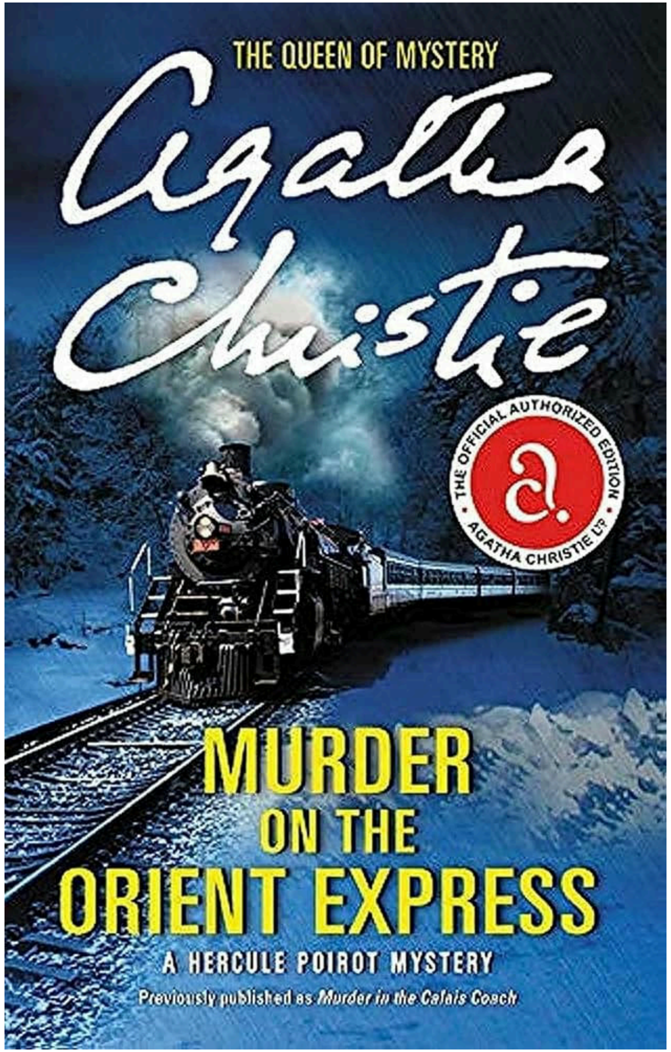
- What a murder is
- What it means to be alive
- Emotions like anger, jealousy, betrayal, love
- Personalities of each character
- Why a human would murder another human
- How humans react to emotions
- How to tell if someone lies

Generative Pre-Training



To predict the murderer, the model must understand so much about humans and our society

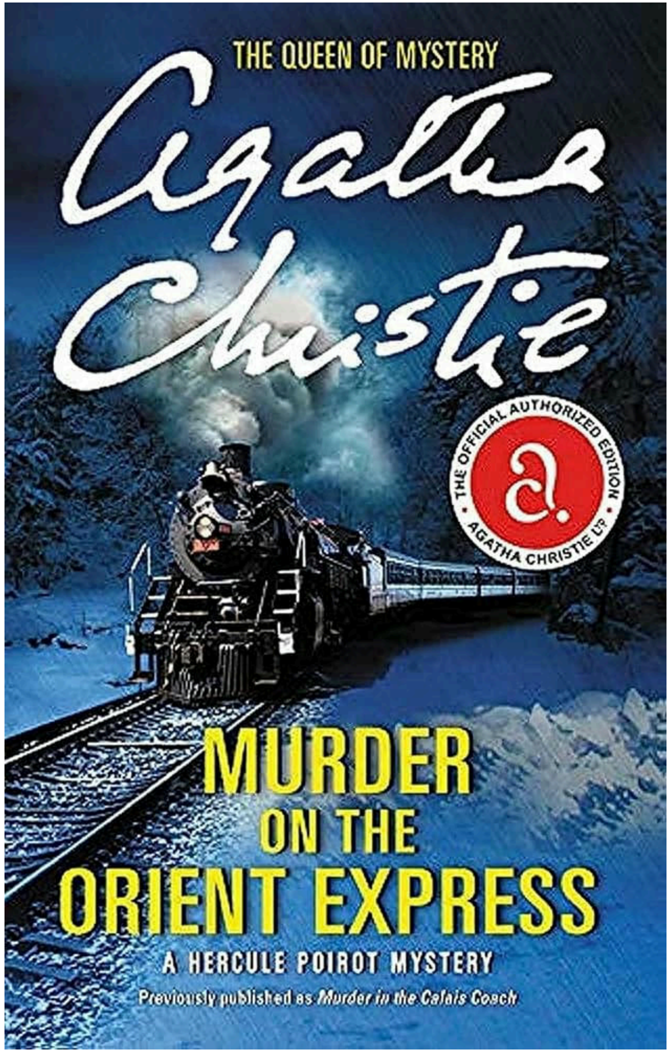
Generative Pre-Training



To predict the murderer, the model must understand so much about humans and our society

The Books3 dataset contains 200,000 books

Generative Pre-Training



To predict the murderer, the model must understand so much about humans and our society

The Books3 dataset contains 200,000 books

We train the model to predict the ending of all these books

Generative Pre-Training

We can apply this same concept to:

Generative Pre-Training

We can apply this same concept to:

- Predict missing base pairs in a strand of DNA

Generative Pre-Training

We can apply this same concept to:

- Predict missing base pairs in a strand of DNA
- Predict missing audio from a song

Generative Pre-Training

We can apply this same concept to:

- Predict missing base pairs in a strand of DNA
- Predict missing audio from a song
- Predict the outcome of particle collisions at the Large Hadron Supercollider

Generative Pre-Training

We can apply this same concept to:

- Predict missing base pairs in a strand of DNA
- Predict missing audio from a song
- Predict the outcome of particle collisions at the Large Hadron Supercollider

All we need is a large enough dataset!

Generative Pre-Training

What if we put the model in a robot?

Generative Pre-Training

What if we put the model in a robot?

Give the model what the robot sees and what the robot does

Generative Pre-Training

What if we put the model in a robot?

Give the model what the robot sees and what the robot does

Predict what the robot will see next

Generative Pre-Training

What if we put the model in a robot?

Give the model what the robot sees and what the robot does

Predict what the robot will see next

Call this a **world model** because it models the structure of our world

Generative Pre-Training

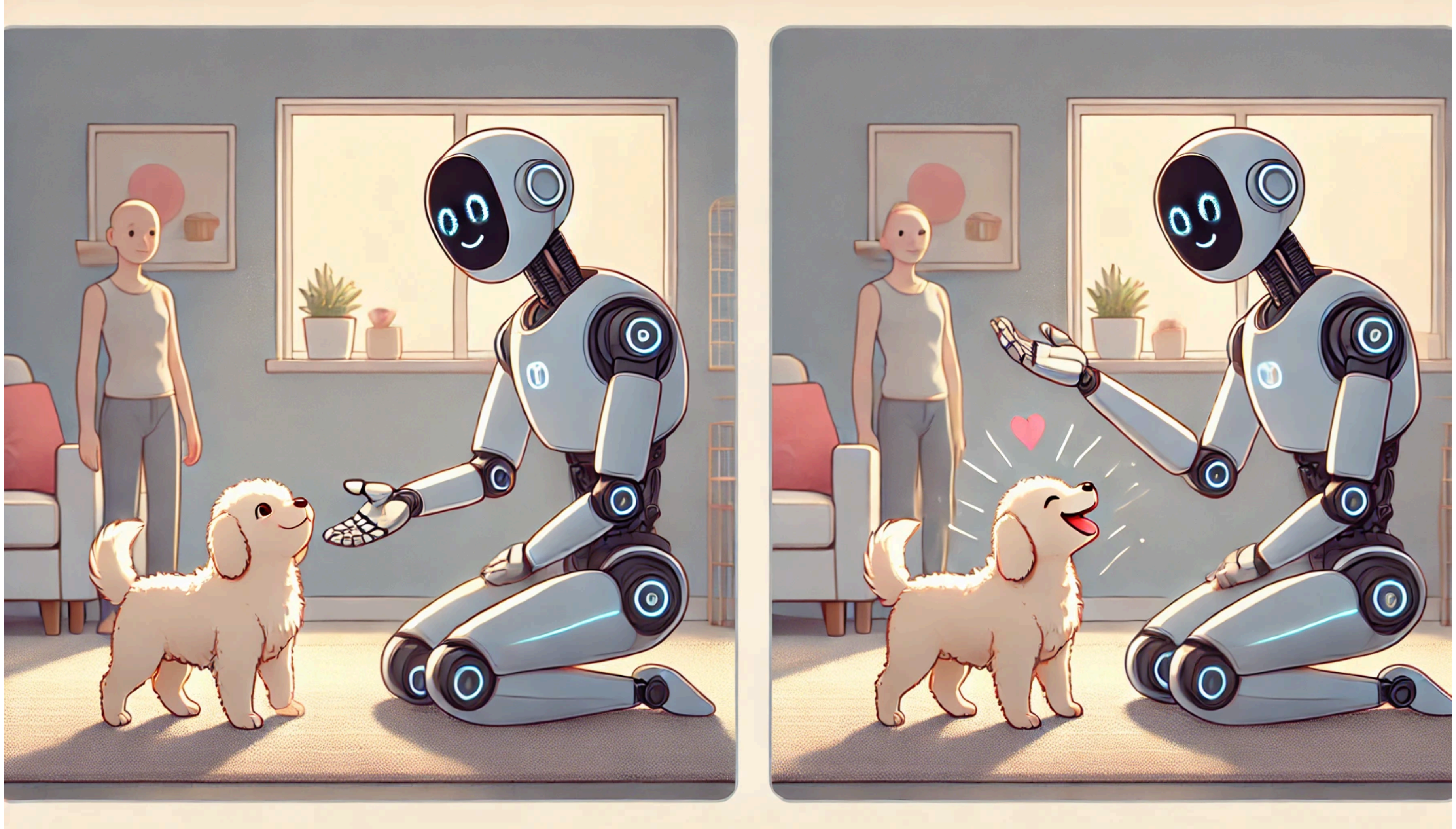
What if we put the model in a robot?

Give the model what the robot sees and what the robot does

Predict what the robot will see next

Call this a **world model** because it models the structure of our world

Generative Pre-Training



Generative Pre-Training

The world model must understand:

Generative Pre-Training

The world model must understand:

- How to control the robot

Generative Pre-Training

The world model must understand:

- How to control the robot
- How to pet a dog

Generative Pre-Training

The world model must understand:

- How to control the robot
- How to pet a dog
- Dogs have feelings

Generative Pre-Training

The world model must understand:

- How to control the robot
- How to pet a dog
- Dogs have feelings
- Petting makes dogs happy

Generative Pre-Training

The world model must understand:

- How to control the robot
- How to pet a dog
- Dogs have feelings
- Petting makes dogs happy
- Dogs smile when happy

Generative Pre-Training

Soon, I will apply for a grant to train a world model

Generative Pre-Training

Soon, I will apply for a grant to train a world model

If I win, I will need help creating a robot dataset

Generative Pre-Training

Soon, I will apply for a grant to train a world model

If I win, I will need help creating a robot dataset

I will need some humans to control our robots in the world

Generative Pre-Training

Soon, I will apply for a grant to train a world model

If I win, I will need help creating a robot dataset

I will need some humans to control our robots in the world

If you are interested, give me your email after class

Generative Pre-Training

These transformers learn and understand the structure of our world

Generative Pre-Training

These transformers learn and understand the structure of our world

But their understanding is trapped

Generative Pre-Training

These transformers learn and understand the structure of our world

But their understanding is trapped

They can only finish sentences or complete pictures

Generative Pre-Training

These transformers learn and understand the structure of our world

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How can we use this strong understanding to help humans?

Generative Pre-Training

These transformers learn and understand the structure of our world

But their understanding is trapped

They can only finish sentences or complete pictures

How can we use this strong understanding to help humans?

- Identify pictures of cancer

Generative Pre-Training

These transformers learn and understand the structure of our world

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They can only finish sentences or complete pictures

How can we use this strong understanding to help humans?

- Identify pictures of cancer
- Make scientific discoveries

Generative Pre-Training

These transformers learn and understand the structure of our world

But their understanding is trapped

They can only finish sentences or complete pictures

How can we use this strong understanding to help humans?
- Identify pictures of cancer
- Make scientific discoveries
- Minimize human suffering

Generative Pre-Training

These transformers learn and understand the structure of our world

But their understanding is trapped

They can only finish sentences or complete pictures

How can we use this strong understanding to help humans?

- Identify pictures of cancer
- Make scientific discoveries
- Minimize human suffering

Today, we use **reinforcement learning**

We will formally introduce reinforcement learning next lecture

Closing Remarks

Closing Remarks

This is the last in-person lecture

Closing Remarks

This is the last in-person lecture

I will record a video on reinforcement learning next week

Closing Remarks

This is the last in-person lecture

I will record a video on reinforcement learning next week

I will be here from 7:00PM on December 2 for questions/discussin on reinforcement learning

Closing Remarks

In this course, we started from Gauss in 1795

Closing Remarks

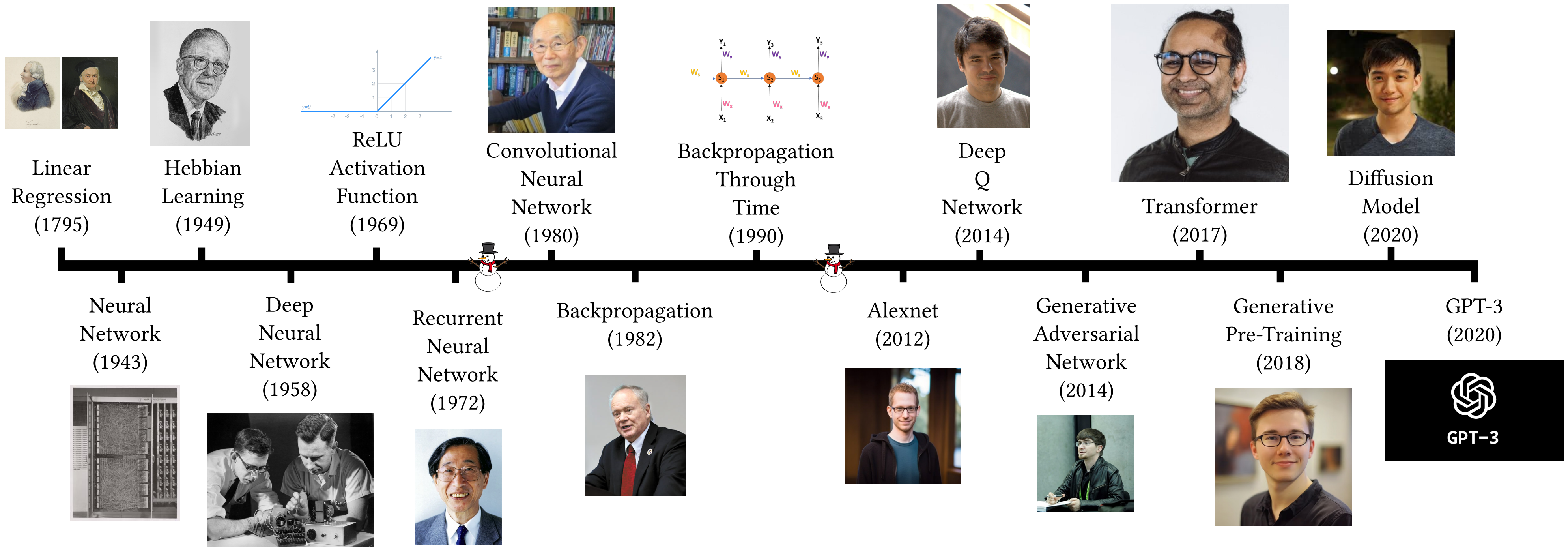
In this course, we started from Gauss in 1795

We built up concepts until we reached the modern age

Closing Remarks

In this course, we started from Gauss in 1795

We built up concepts until we reached the modern age



Closing Remarks

We learned about:

Closing Remarks

We learned about:

- Linear regression

Closing Remarks

We learned about:

- Linear regression
- Polynomial regression

Closing Remarks

We learned about:

- Linear regression
- Polynomial regression
- Biological neurons

Closing Remarks

We learned about:

- Linear regression
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- Artificial neurons

Closing Remarks

We learned about:

- Linear regression
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- Biological neurons
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- Perceptrons

Closing Remarks

We learned about:

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- Backpropagation

Closing Remarks

We learned about:

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- Biological neurons
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- Gradient descent

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We learned about:

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- Classification

Closing Remarks

We learned about:

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- Parameter initialization

Closing Remarks

We learned about:

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- Gradient descent
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- Parameter initialization
- Many activation functions

Closing Remarks

We learned about:

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- Classification
- Parameter initialization
- Many activation functions
- Stochastic gradient descent

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- Stochastic gradient descent
- RMSProp and Adam

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We learned about:

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- RMSProp and Adam
- Convolutional neural networks

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- Stochastic gradient descent
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- Convolutional neural networks
- Composite memory

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Closing Remarks

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- Composite memory
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- Autoencoders
- Variational autoencoders

Closing Remarks

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- Composite memory
- Recurrent neural networks
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- Graph neural networks

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- Variational autoencoders
- Graph neural networks
- Attention and transformers

Closing Remarks

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- Gradient descent
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- Many activation functions
- Stochastic gradient descent
- RMSProp and Adam
- Convolutional neural networks
- Composite memory
- Recurrent neural networks
- Autoencoders
- Variational autoencoders
- Graph neural networks
- Attention and transformers
- Generative pre-training

Closing Remarks

I hope you enjoyed the course!

Closing Remarks

I hope you enjoyed the course!

But there are many more topics to learn!

Closing Remarks

I hope you enjoyed the course!

But there are many more topics to learn!

Now, you have the tools to study deep learning on your own

Closing Remarks

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Now, you have the tools to study deep learning on your own

You have the tools to train neural networks for real problems

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You have the tools to train neural networks for real problems

Closing Remarks

In the first lecture, I asked everyone in this class for something

Closing Remarks

In the first lecture, I asked everyone in this class for something

Question: Do you remember what it was?

Closing Remarks

Deep learning is a powerful tool

Closing Remarks

Deep learning is a powerful tool

Like all powerful tools, deep learning can be used for good or evil

Closing Remarks

Deep learning is a powerful tool

Like all powerful tools, deep learning can be used for good or evil

- COVID-19 vaccine

Closing Remarks

Deep learning is a powerful tool

Like all powerful tools, deep learning can be used for good or evil

- COVID-19 vaccine
- Creating art

Closing Remarks

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- COVID-19 vaccine
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- Autonomous driving

Closing Remarks

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- COVID-19 vaccine
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- Autonomous driving

- Making DeepFakes

Closing Remarks

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- COVID-19 vaccine
- Creating art
- Autonomous driving

- Making DeepFakes
- Weapon guidance systems

Closing Remarks

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- COVID-19 vaccine
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- Autonomous driving

- Making DeepFakes
- Weapon guidance systems
- Discrimination

Closing Remarks

Deep learning is a powerful tool

Like all powerful tools, deep learning can be used for good or evil

- COVID-19 vaccine
- Creating art
- Autonomous driving

- Making DeepFakes
- Weapon guidance systems
- Discrimination

Before training a model, think about whether it is good or bad for the world

Course Evaluation

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Department instructed me to ask you for course feedback

Course Evaluation

Department instructed me to ask you for course feedback

We take this feedback seriously

Course Evaluation

Department instructed me to ask you for course feedback

We take this feedback seriously

Your feedback will impact future courses (and my job)

Course Evaluation

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Your feedback will impact future courses (and my job)

If you like the course, please say it!

Course Evaluation

Department instructed me to ask you for course feedback

We take this feedback seriously

Your feedback will impact future courses (and my job)

If you like the course, please say it!

Please be specific on what you like and do not like

Course Evaluation

Department instructed me to ask you for course feedback

We take this feedback seriously

Your feedback will impact future courses (and my job)

If you like the course, please say it!

Please be specific on what you like and do not like

Your likes/dislikes will change your future courses

Course Evaluation

I must leave the room to let you fill out this form

Course Evaluation

I must leave the room to let you fill out this form

Please scan the QR code and complete the survey

Course Evaluation

I must leave the room to let you fill out this form

Please scan the QR code and complete the survey

I will return in 10 minutes to see if everyone has finished

Course Evaluation

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https://isw.um.edu.mo/siaweb

Course Evaluation

If you participated in class come see me after class

Course Evaluation

If you participated in class come see me after class

- Answered a question

Course Evaluation

If you participated in class come see me after class

- Answered a question
- Asked a question