Recurrent Neural Networks

CISC 7026: Introduction to Deep Learning

University of Macau

Admin

~~Prof. Qingbiao Li GNN lecture 28 October~~

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November 4 holiday, no lecture

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Updated assignment 5 is on Moodle, due in 2 weeks

Agenda

1. Review
2. Sequence Modeling
3. Composite Memory
4. Linear Recurrence
5. Scans
6. Output Modeling
7. Recurrent Loss Functions
8. Backpropagation through Time
9. Recurrent Neural Networks
10. Coding

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Review

In perceptrons, each neuron in a layer is independent

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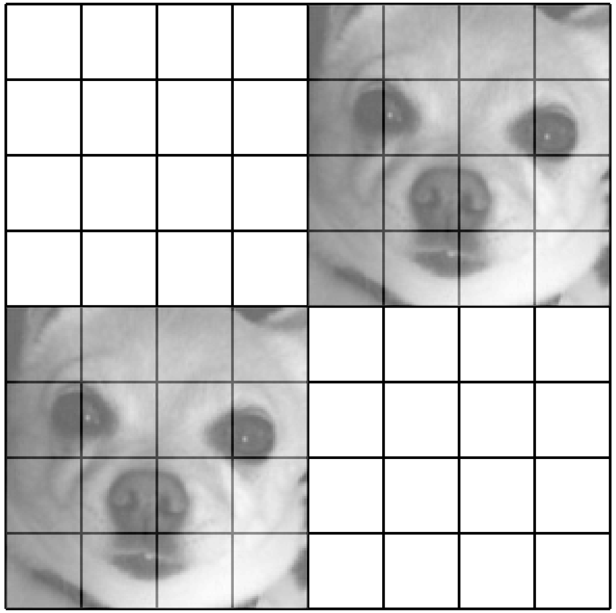

Review

These images are equivalent to a neural network

Review

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It is a miracle that our neural networks could classify clothing!

Review

A **signal** represents information as a function of time, space or some other variable

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$$x(t) = 2t + 1$$

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$$x(u, v) = \frac{u^2}{v} - 3$$

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$x(t), x(u, v)$ represent physical processes that we may or may not know

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In **signal processing**, we analyze the meaning of signals

Review

$$x(t) = \text{stock price}$$

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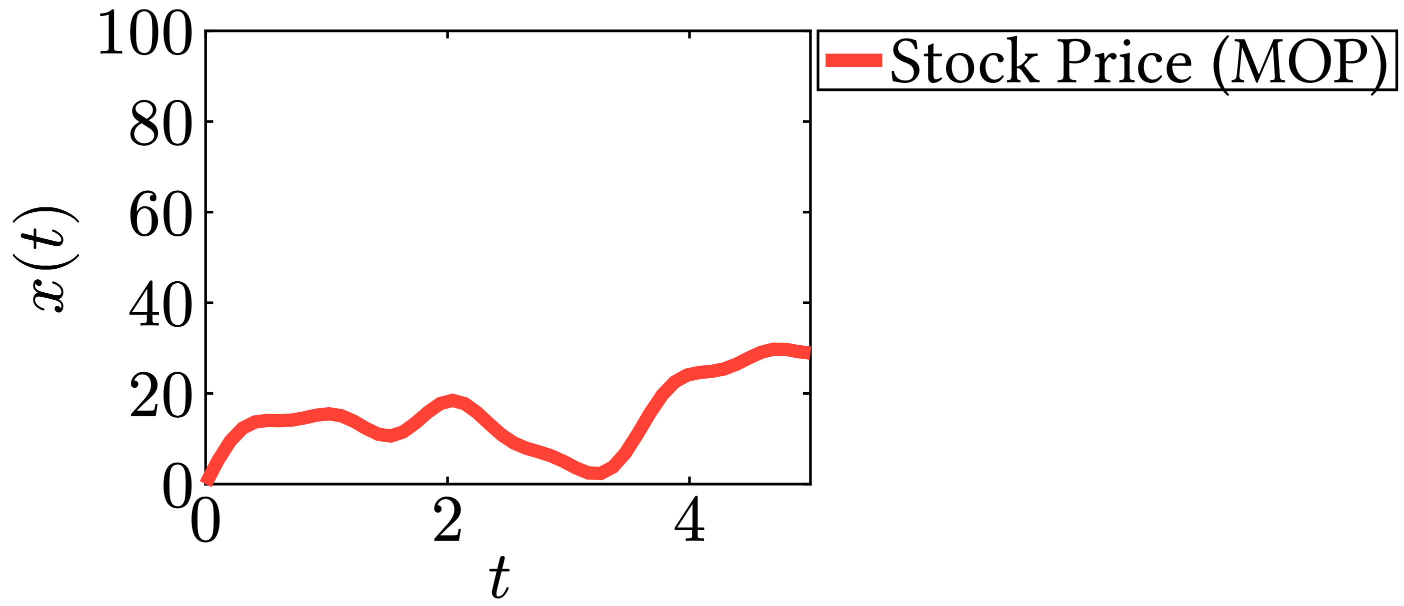

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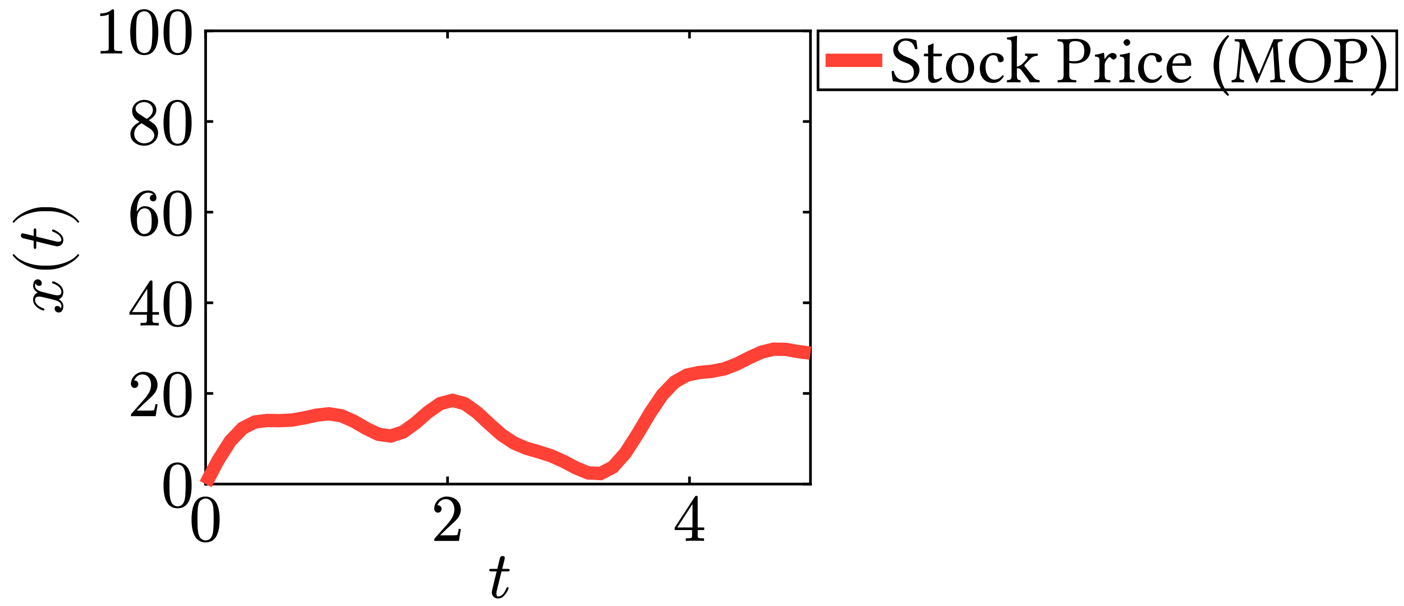

There is an underlying structure to $x(t)$

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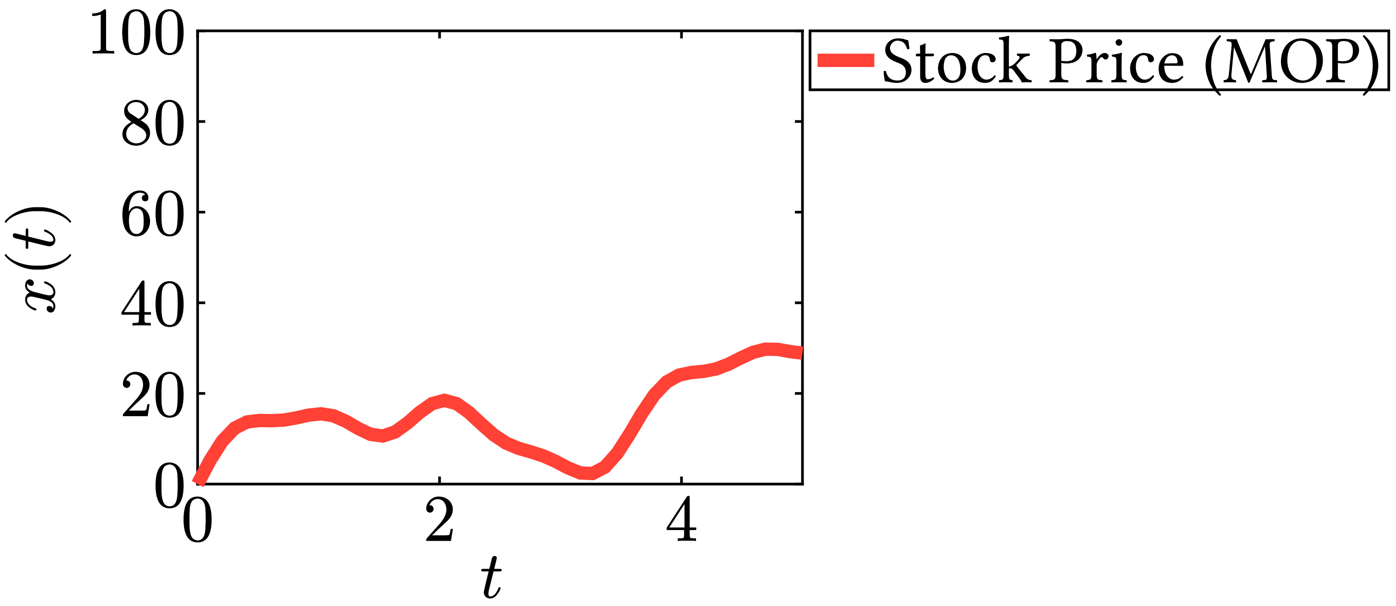

There is an underlying structure to $x(t)$

Structure: Tomorrow's stock price will be close to today's stock price

Review

$$x(t) = \text{audio}$$

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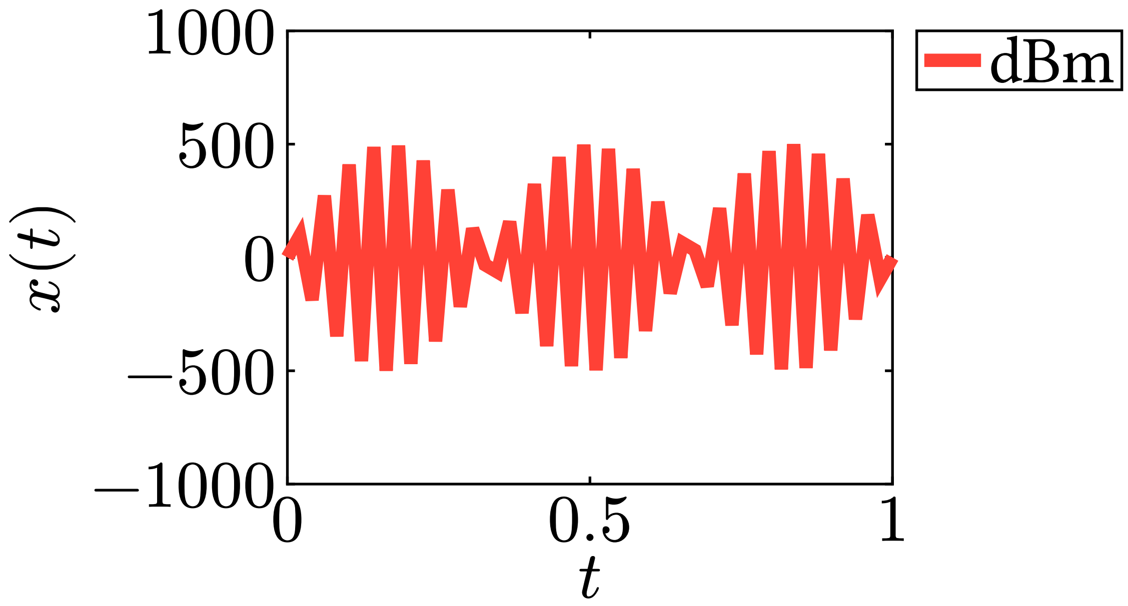

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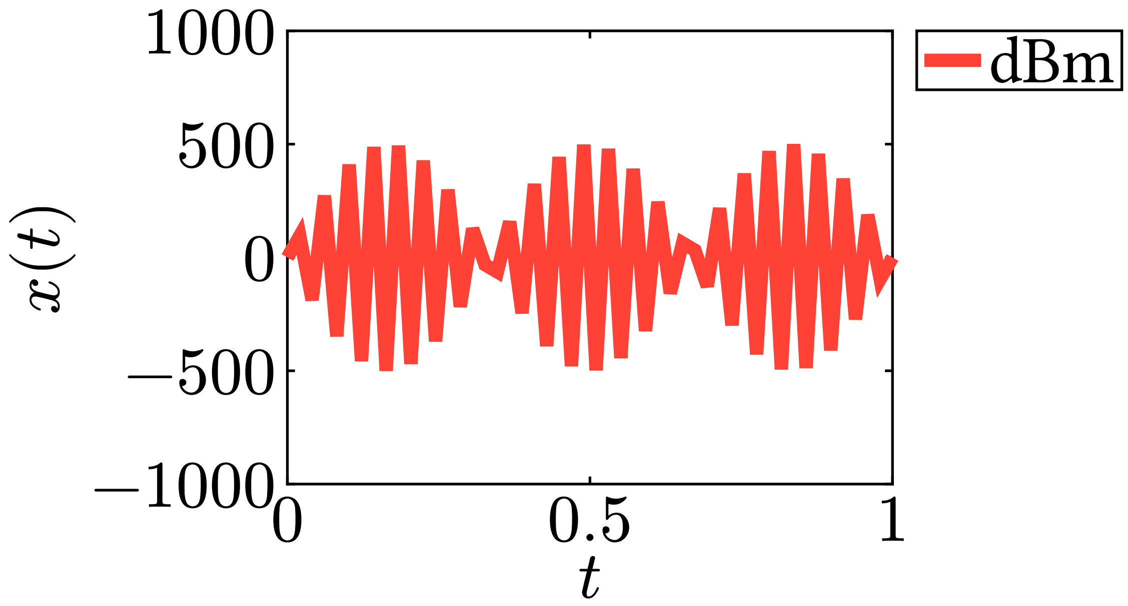

Structure: Nearby waves form syllables

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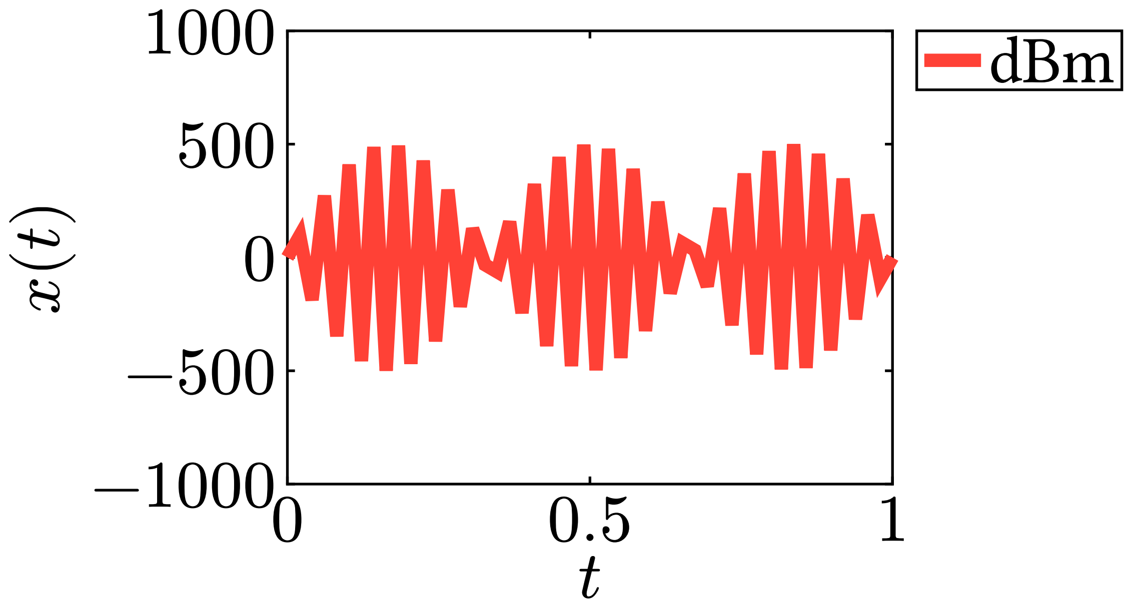

Structure: Nearby waves form syllables

Structure: Nearby syllables combine to create meaning

Review

$$x(u, v) = \text{image}$$

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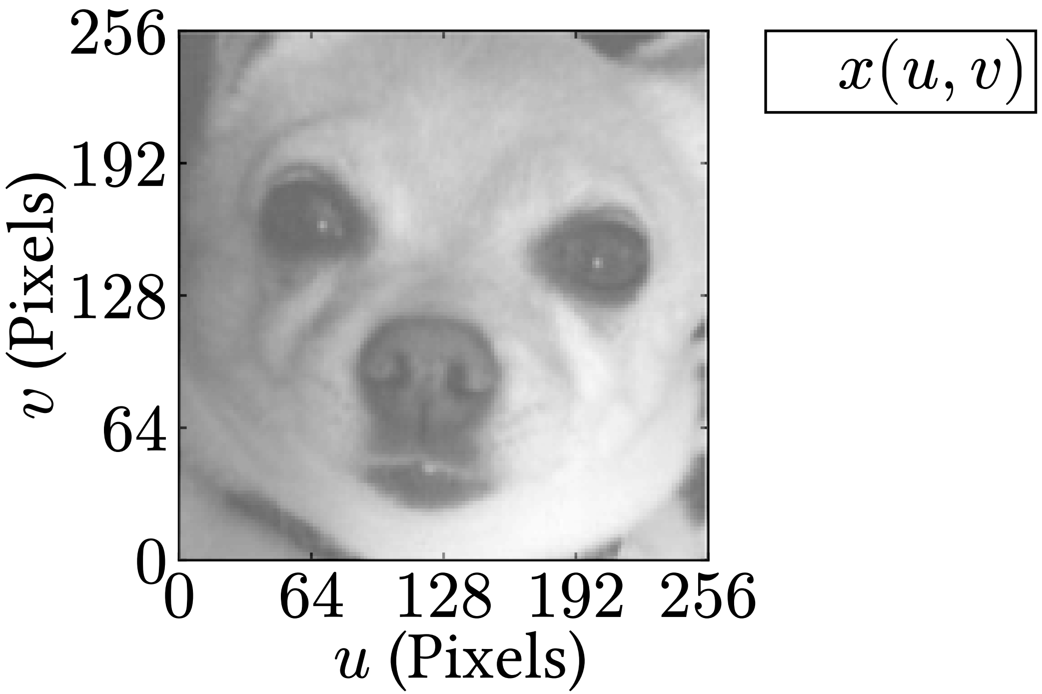

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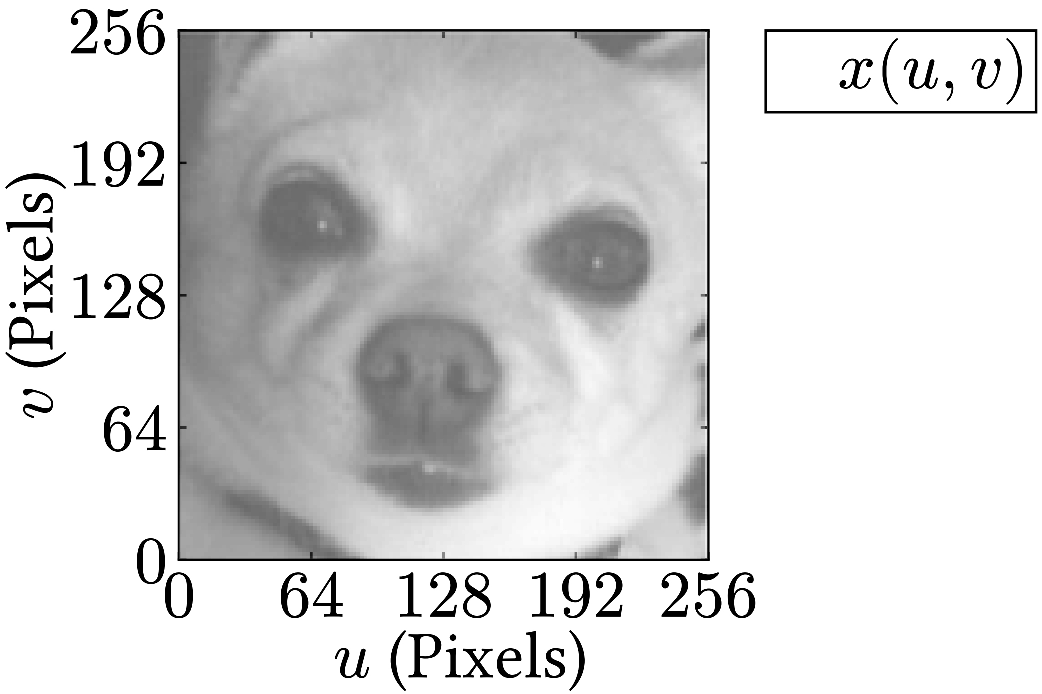


Structure: Repeated components (circles, symmetry, eyes, nostrils, etc)

Review

Two common properties of signals:

Review

Two common properties of signals:

Locality: Information concentrated over small regions of space/time

Review

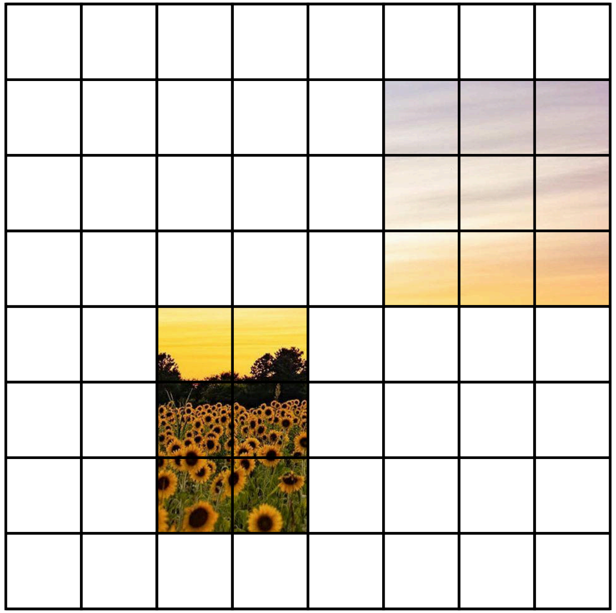
Two common properties of signals:

Locality: Information concentrated over small regions of space/time

Translation Equivariance: Shift in signal results in shift in output

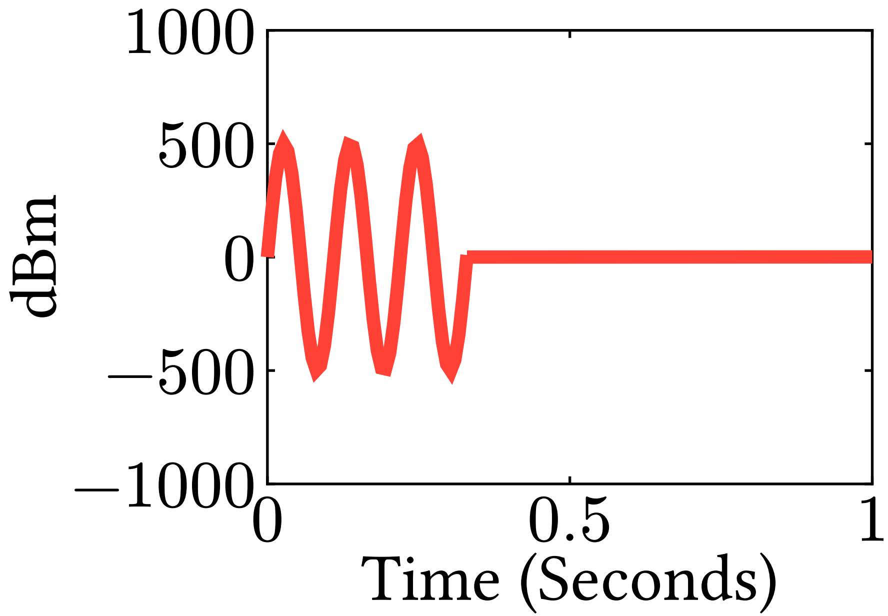
Review

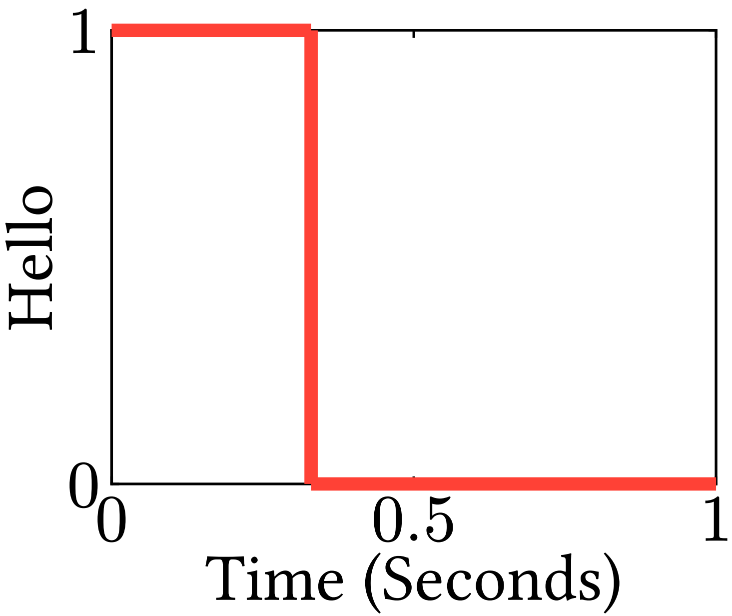
A more realistic scenario of locality and translation equivariance



Review

We use convolution to turn signals into useful signals





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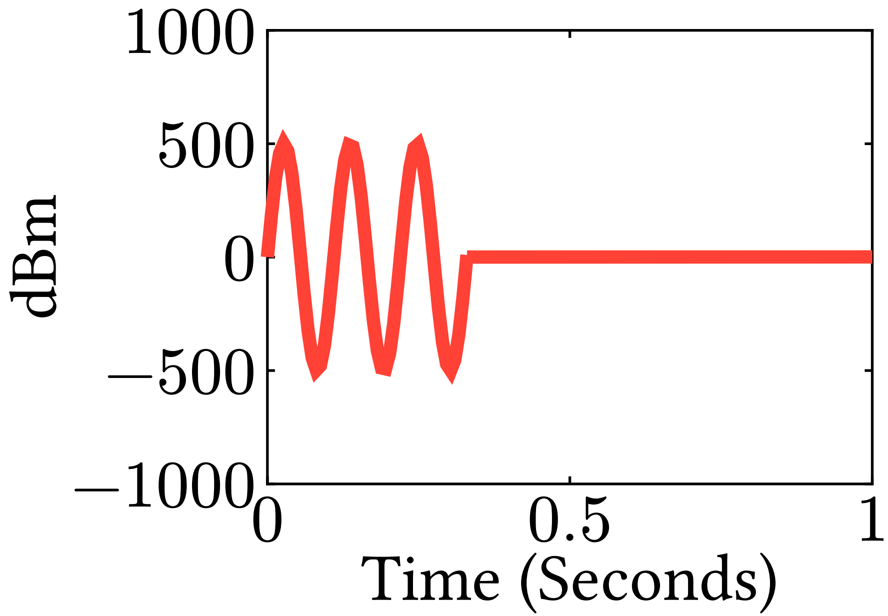

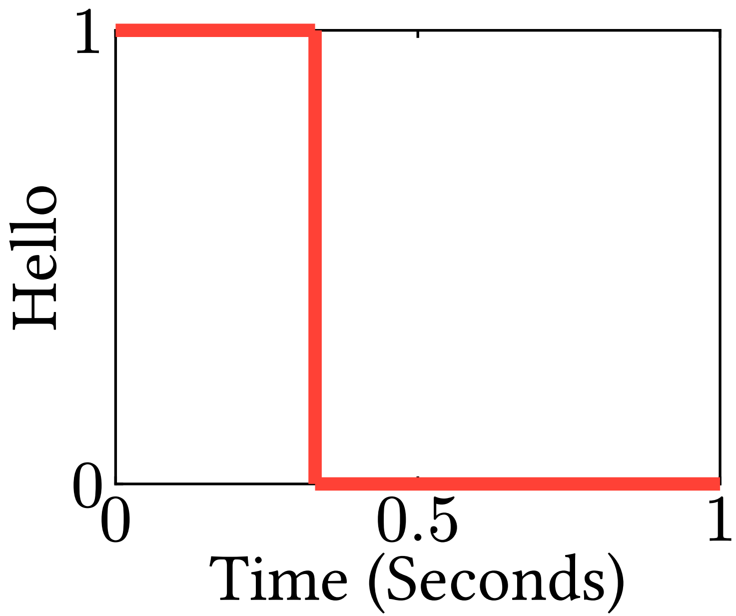

Convolution is translation equivariant and local

Review

Convolution is the sum of products of a signal $x(t)$ and a **filter** $g(t)$

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If the t is continuous in $x(t)$

$$x(t) * g(t) = \int_{-\infty}^{\infty} x(t - \tau) g(\tau) d\tau$$

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If the t is discrete in $x(t)$

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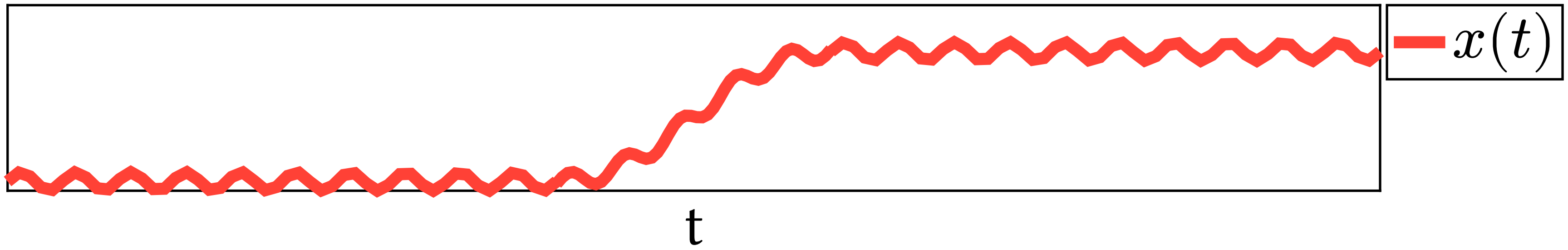
We slide the filter $g(t)$ across the signal $x(t)$

Review



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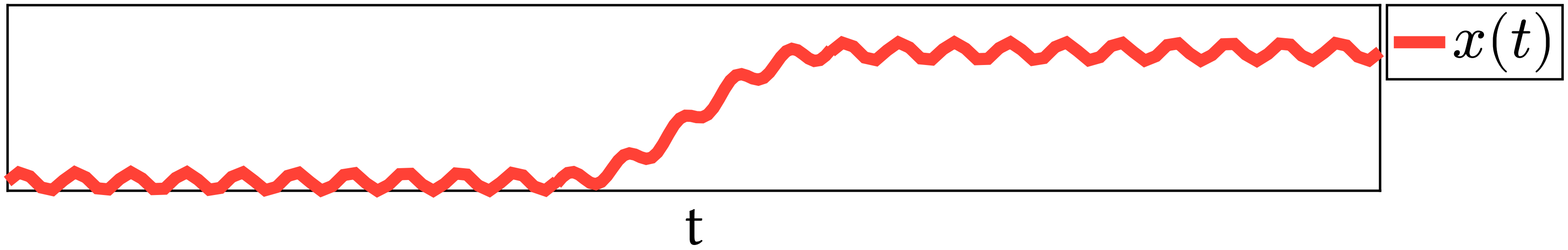

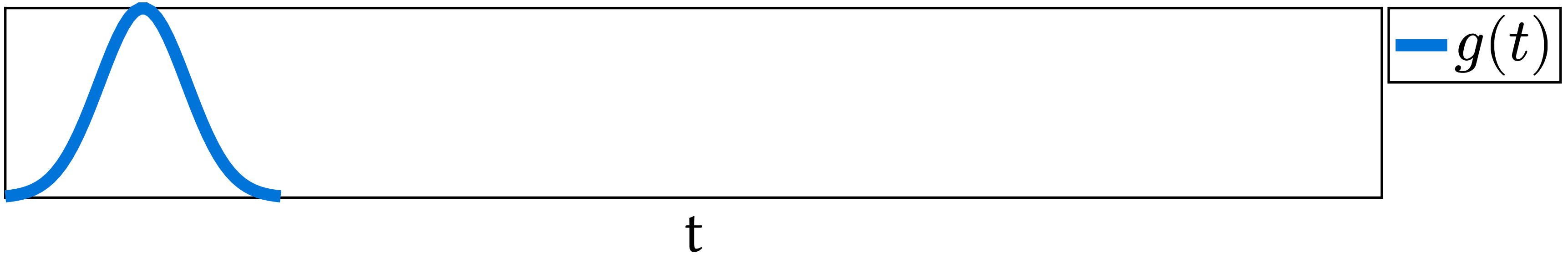

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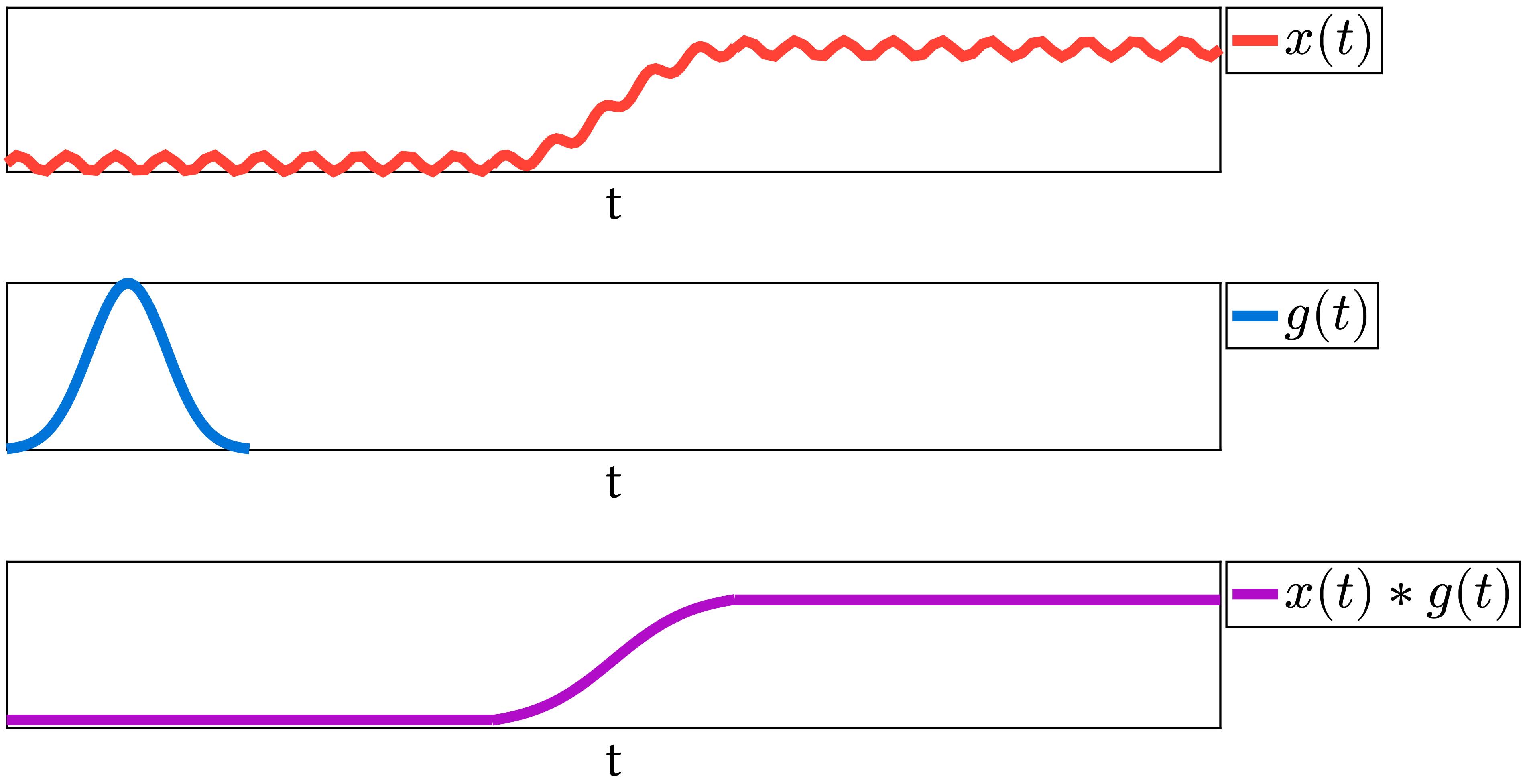

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$$\begin{bmatrix} x(t) \\ g(t) \\ y(t) \end{bmatrix} = \begin{bmatrix} 1 & 2 & 3 & 4 & 5 \\ 2 & 1 & & & \\ & & & & \end{bmatrix}$$

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$$\begin{bmatrix} x(t) \\ g(t) \\ y(t) \end{bmatrix} = \begin{bmatrix} 1 & {\color{red}2} & {\color{red}3} & 4 & 5 \\ & {\color{red}2} & {\color{red}1} & & \\ 4 & {\color{red}7} & & & \end{bmatrix}$$

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$$\begin{bmatrix} x(t) \\ g(t) \\ y(t) \end{bmatrix} = \begin{bmatrix} 1 & 2 & \color{red}{3} & \color{red}{4} & 5 \\ & & \color{red}{2} & \color{red}{1} & \\ 4 & 5 & \color{red}{10} & & \end{bmatrix}$$

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To make a convolution layer, we make the filter with trainable parameters

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Review

We can write both a perceptron and convolution in vector form

$$f(x(t), \boldsymbol{\theta}) = \sigma\left(\boldsymbol{\theta}^\top \begin{bmatrix} 1 \\ x(0.1) \\ x(0.2) \\ \vdots \end{bmatrix} \right)$$

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A convolution layer applies a "mini" perceptron to every few timesteps

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The output size depends on the signal length

Review

If we want a single output, we should **pool**

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$$z(t) = f(x(t), \boldsymbol{\theta}) = \begin{bmatrix} \sigma\left(\boldsymbol{\theta}^\top \begin{bmatrix} 1 \\ x(0.1) \\ x(0.2) \end{bmatrix}\right) & \sigma\left(\boldsymbol{\theta}^\top \begin{bmatrix} 1 \\ x(0.2) \\ x(0.3) \end{bmatrix}\right) & \dots \end{bmatrix}^\top$$

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$$\mathrm{SumPool}(z(t)) = \sigma\left(\boldsymbol{\theta}^\top \begin{bmatrix} 1 \\ x(0.1) \\ x(0.2) \end{bmatrix}\right) + \sigma\left(\boldsymbol{\theta}^\top \begin{bmatrix} 1 \\ x(0.2) \\ x(0.3) \end{bmatrix}\right) + \dots$$

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$$\mathrm{MeanPool}(z(t)) = \frac{1}{T - k + 1} \mathrm{SumPool}(z(t)); \quad \mathrm{MaxPool}(z(t)) = \max(z(t))$$

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Our examples considered:

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- 1 dimensional variable t

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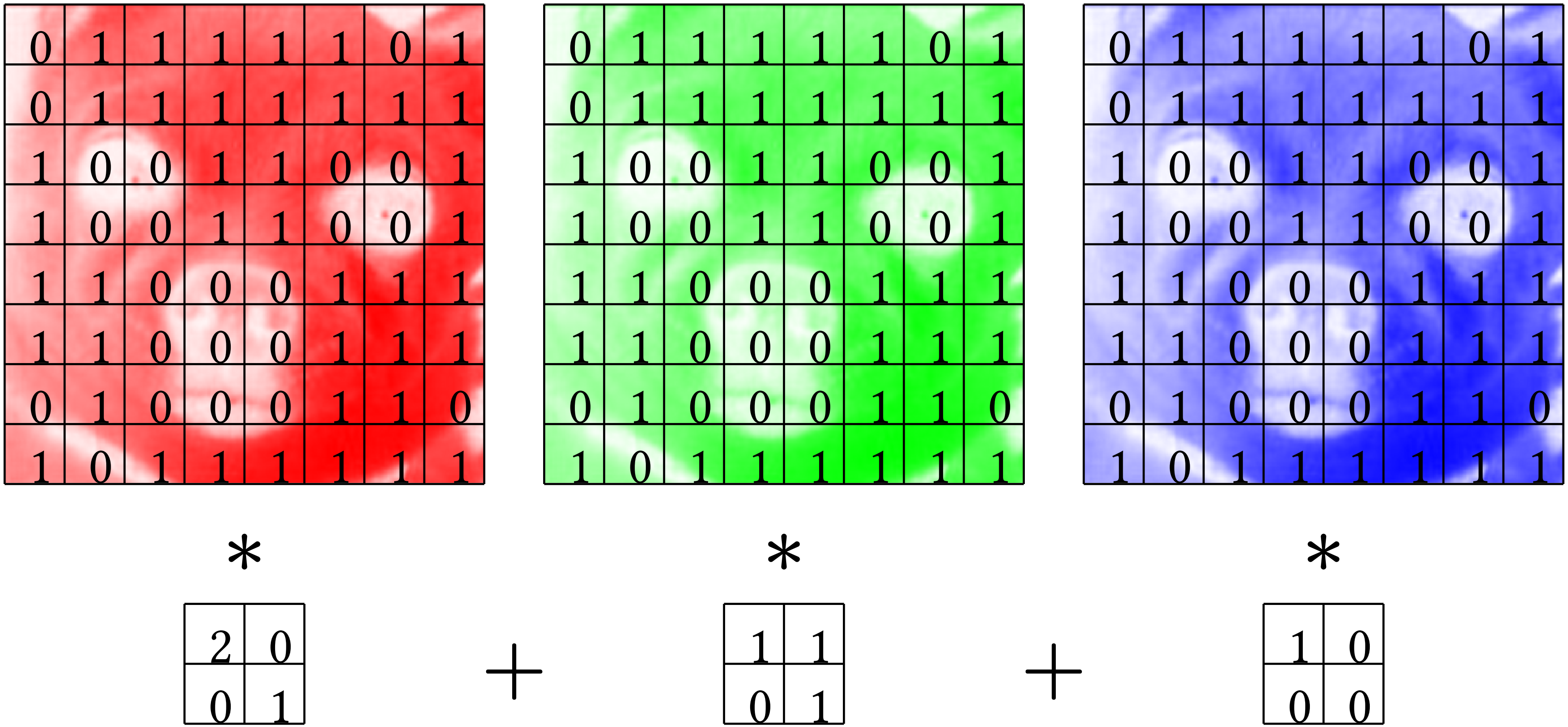
The idea is exactly the same

Review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0 1 1 1 1 1 1 1
1 0 0 1 1 0 0 1
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0 1 0 0 0 1 1 0
1 0 1 1 1 1 1 1
*
2 0
0 1
+
*
1 1
0 1
+
*
1 0
0 0

Review

One last thing, **stride** allows you to "skip" cells during convolution

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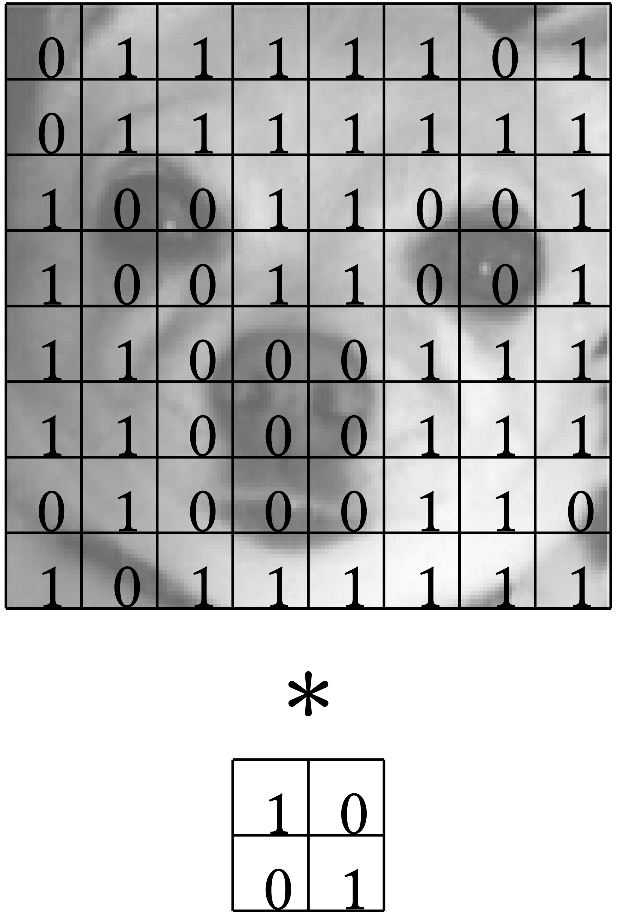
This can decrease the size of image without pooling

One last thing, **stride** allows you to “skip” cells during convolution

This can decrease the size of image without pooling

Padding adds zero pixels to the image to increase the output size



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Now, we will discuss a neuroscience approach to sequence modeling

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You can solve temporal tasks using either convolution or RNNs

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You can solve temporal tasks using either convolution or RNNs

So what is the difference between convolution and recurrent models?

Sequence Modeling

Convolution works over inputs of any variables (time, space, etc)

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Recurrent neural networks only work with time

Convolution makes use of locality and translation equivariance properties

Recurrent models do not assume locality or equivariance

Equivariance and locality make learning more efficient, but not all problems have this structure

Let us examine some real life signals, and see if these properties hold

Sequence Modeling

Example 1: You like dinosaurs as a child, you grow up and study dinosaurs for work

Sequence Modeling

Example 1: You like dinosaurs as a child, you grow up and study dinosaurs for work

Question: Is this local?

Sequence Modeling

Example 1: You like dinosaurs as a child, you grow up and study dinosaurs for work

Question: Is this local?

Answer: No, two related events separated by 20 years

Sequence Modeling

Example 2: Your parent changes your diaper

Sequence Modeling

Example 2: Your parent changes your diaper

Question: Translation equivariant?

Sequence Modeling

Example 2: Your parent changes your diaper

Question: Translation equivariant?

No! Ok if you are a baby, different meaning if you are an adult!

Sequence Modeling

Example 3: You hear a gunshot then see runners

Sequence Modeling

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Question: Translation equivariant?

Sequence Modeling

Example 3: You hear a gunshot then see runners

Question: Translation equivariant?

Answer: No! (1) gunshot, (2) see runners, enjoy the race. (1) see runners, (2) hear gunshot, you start running too!

Sequence Modeling

Example 3: You hear a gunshot then see runners

Question: Translation equivariant?

Answer: No! (1) gunshot, (2) see runners, enjoy the race. (1) see runners, (2) hear gunshot, you start running too!

Question: Any other examples?

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Problems without locality and translation equivariance are difficult to solve with convolution

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For these problems, we need something else!

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How do humans experience time and process temporal data?

Sequence Modeling

Problems without locality and translation equivariance are difficult to solve with convolution

For these problems, we need something else!

How do humans experience time and process temporal data?

Can we design a neural network based on human perceptions of time?

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Composite Memory

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We only perceive the present

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We only perceive the present

See dog $\rightarrow$ photoreceptors fire $\rightarrow$ neurons fire in the brain

Composite Memory

How do humans process temporal data?

We only perceive the present

See dog $\rightarrow$ photoreceptors fire $\rightarrow$ neurons fire in the brain

No dog $\rightarrow$ no photoreceptors fire $\rightarrow$ no neurons fire

Composite Memory

How do humans process temporal data?

We only perceive the present

See dog $\rightarrow$ photoreceptors fire $\rightarrow$ neurons fire in the brain

No dog $\rightarrow$ no photoreceptors fire $\rightarrow$ no neurons fire

We know there was a dog, even if we no longer see it

Composite Memory

How do humans process temporal data?

We only perceive the present

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We can reason over time by recording information as **memories**

Composite Memory

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Humans process temporal data by storing and recalling memories

Composite Memory

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Without the ability to reason over memories, we would only react to stimuli like bacteria

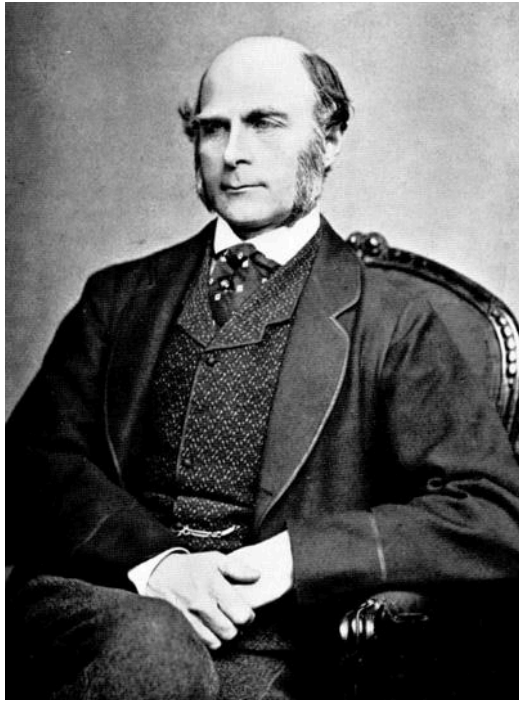
Composite Memory

So how do we model memory in humans?

Composite Memory

Francis Galton (1822-1911)

photo composite memory



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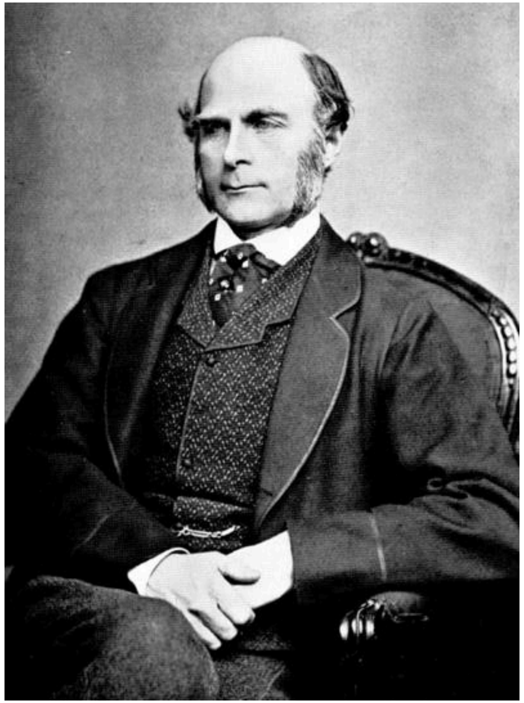
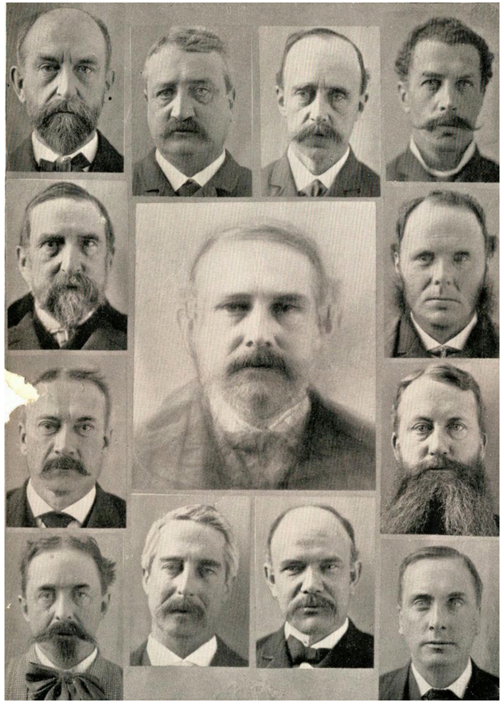


Composite photo of members of a party



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We repeat the same process for each new face

We can rewrite f as a **recurrent function**

Agenda

1. Review
2. Sequence Modeling
3. **Composite Memory**
4. Linear Recurrence
5. Scans
6. Output Modeling
7. Recurrent Loss Functions
8. Backpropagation through Time
9. Recurrent Neural Networks
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Linear Recurrence

Question: What is the meaning of $\boldsymbol{h}$ in humans?

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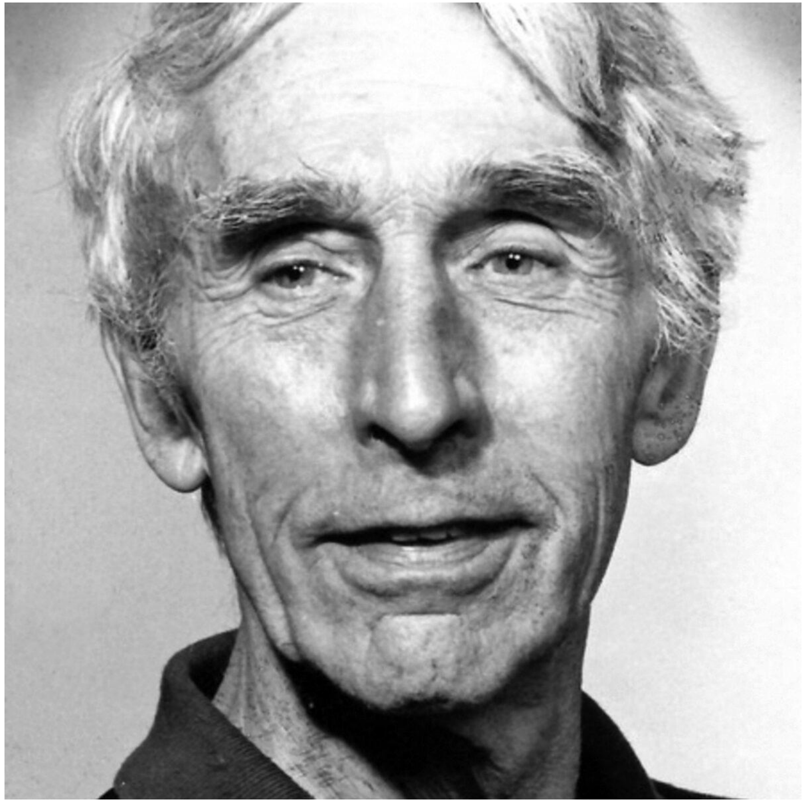

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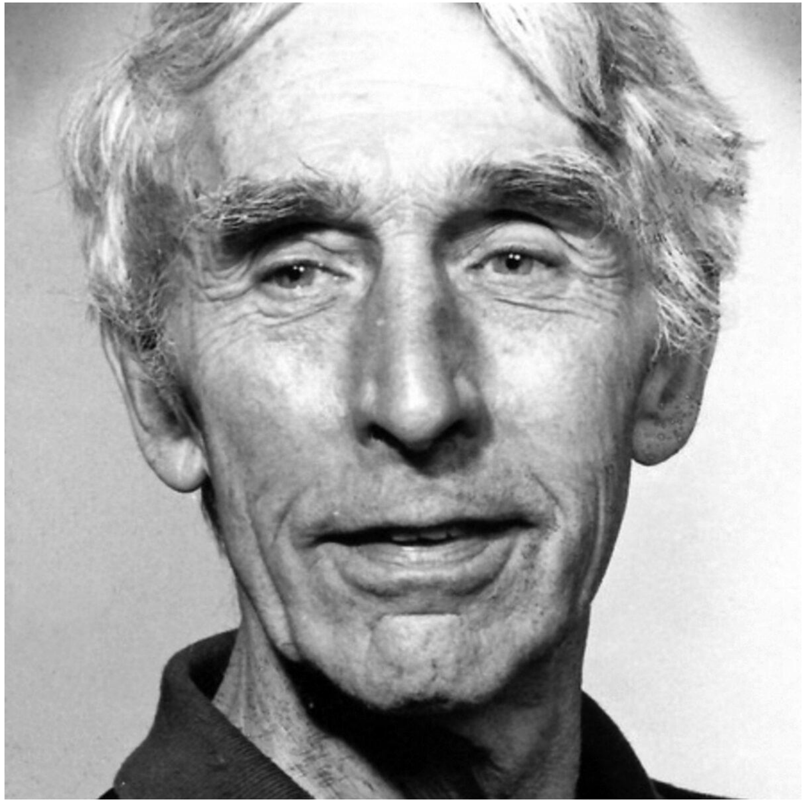

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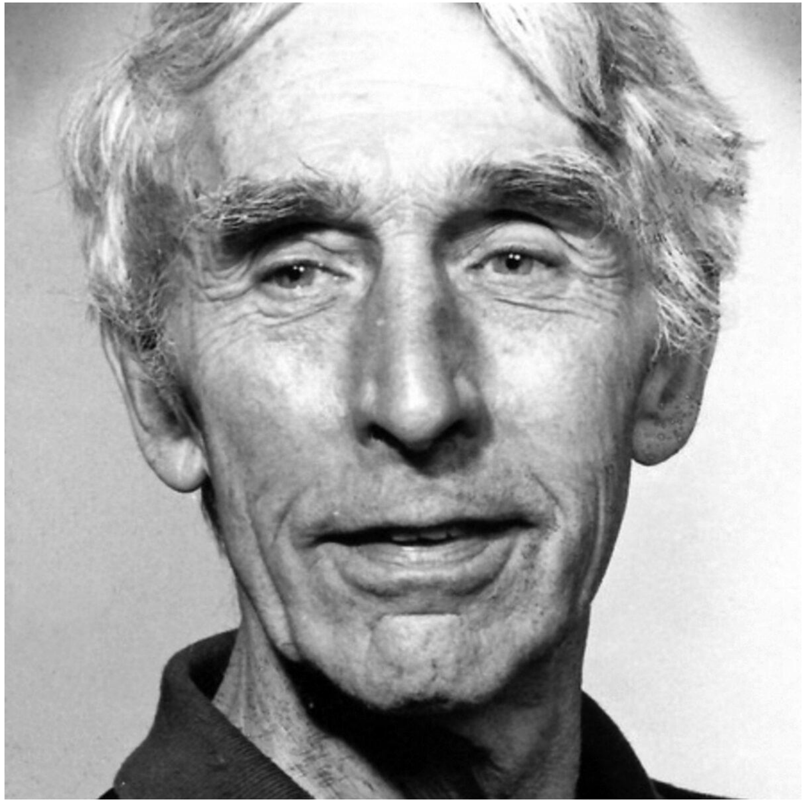

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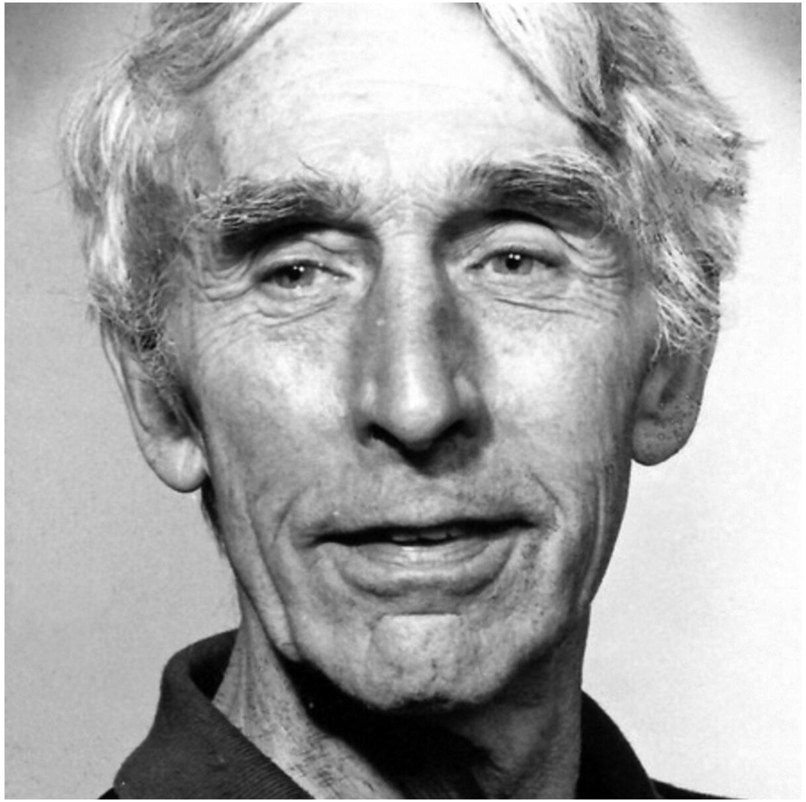

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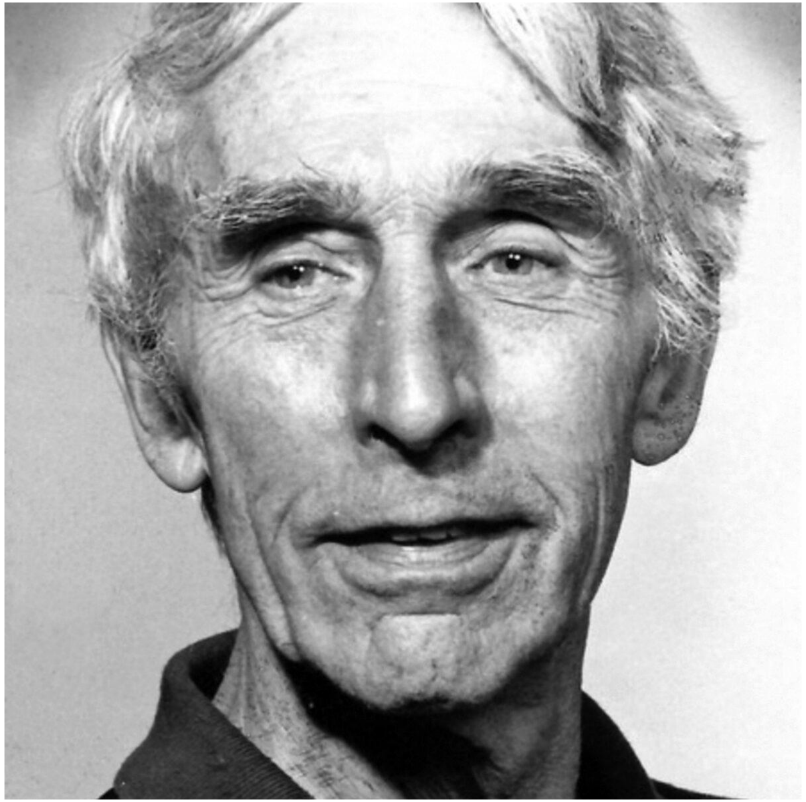

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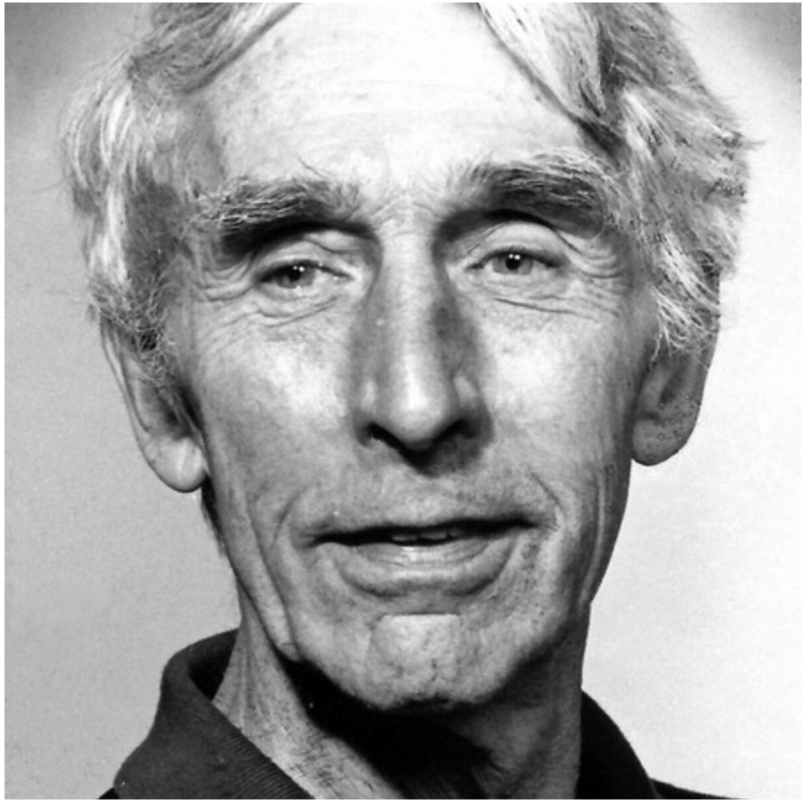

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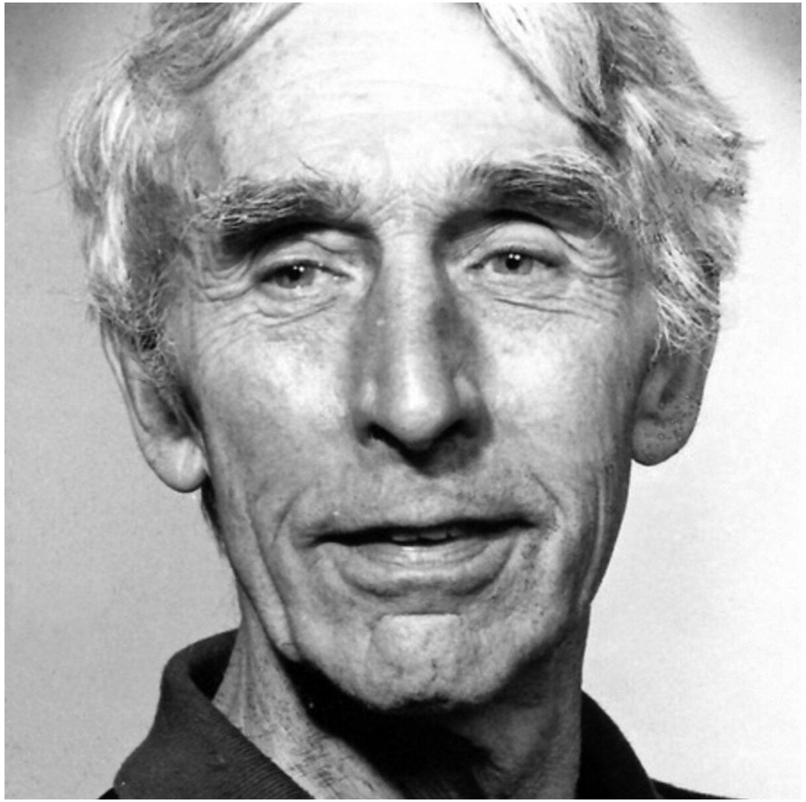

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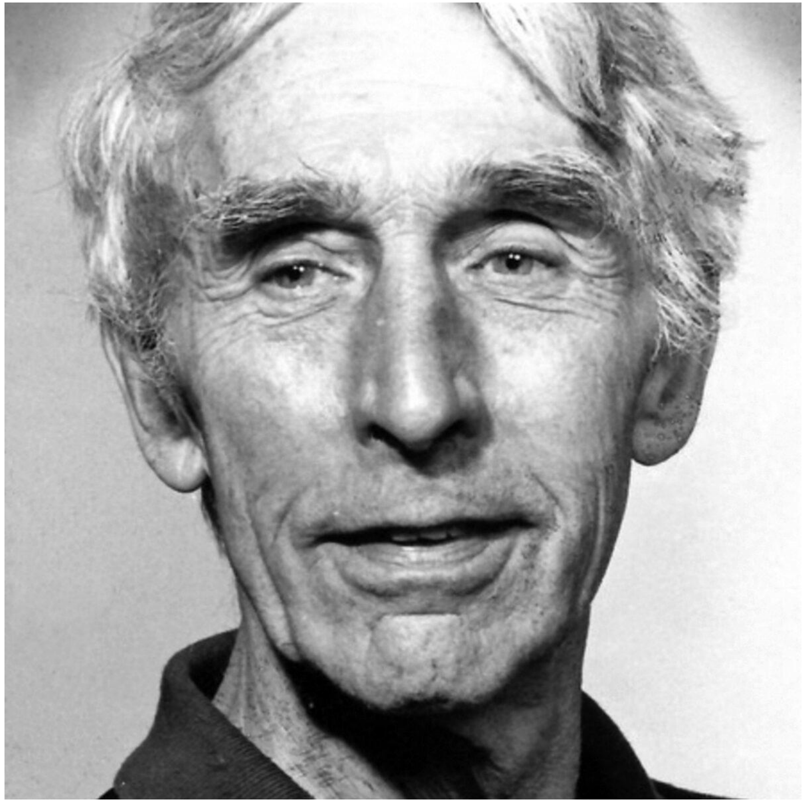

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Let us work out how forgetting works

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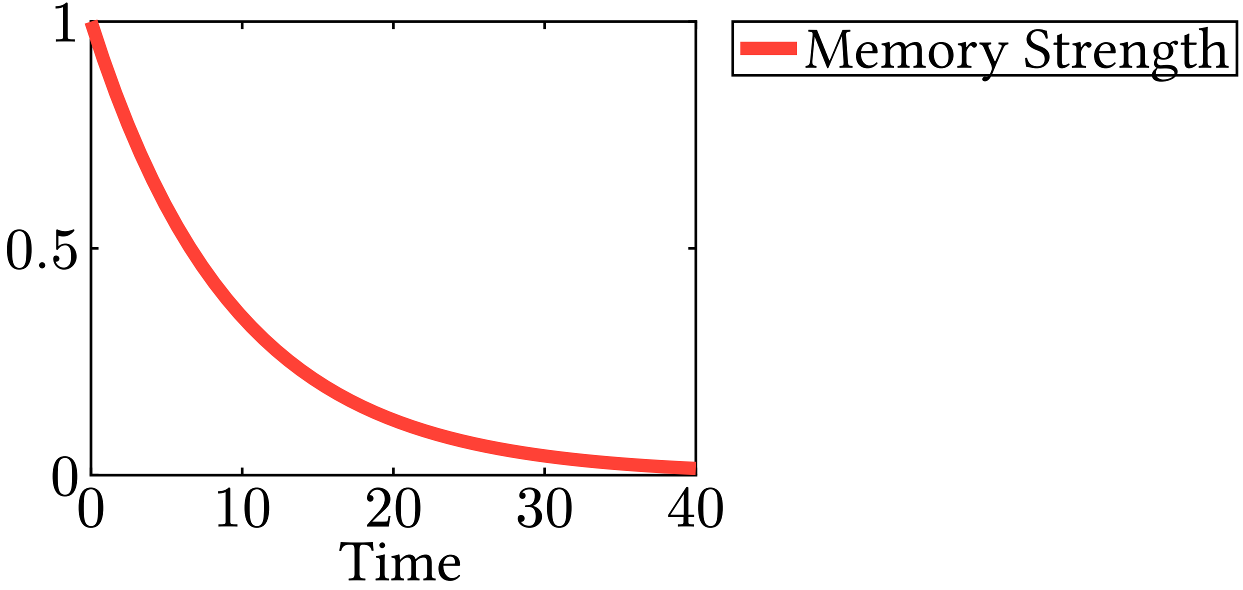

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The memory decay is smooth and differentiable

We can learn the parameters $\gamma, \boldsymbol{\theta}$ using gradient descent

Linear Recurrence

Morad et al., *Reinforcement Learning with Fast and Forgetful Memory.* Neural Information Processing Systems. (2024).

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Morad et al., *Reinforcement Learning with Fast and Forgetful Memory.* Neural Information Processing Systems. (2024).

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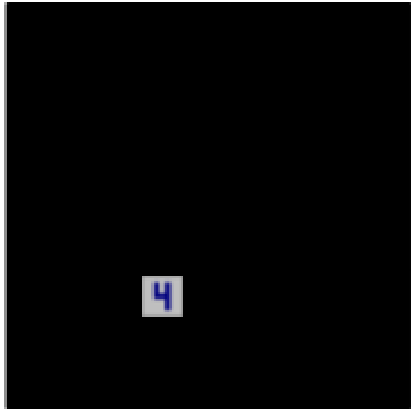
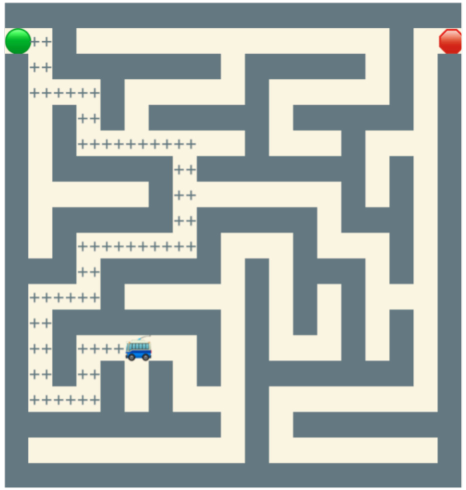
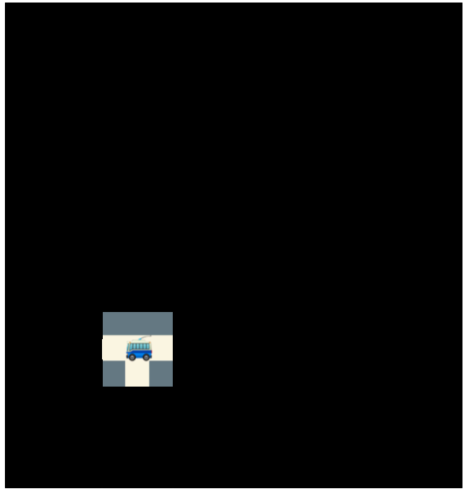

https://www.youtube.com/watch?v=0ey63XPB-4U&t=85s

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2. Sequence Modeling
3. Composite Memory
4. **Linear Recurrence**
5. Scans
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Scans

$$\boldsymbol{h}_1 = f(\boldsymbol{0}, \boldsymbol{x}_1, \boldsymbol{\theta}) = \gamma \boldsymbol{0} + \boldsymbol{\theta}^\top \overline{\boldsymbol{x}}_1$$

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How do we compute $\boldsymbol{h}_1, \boldsymbol{h}_2, ..., \boldsymbol{h}_T$ on a computer?

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How do we compute $\boldsymbol{h}_1, \boldsymbol{h}_2, ..., \boldsymbol{h}_T$ on a computer?

We use an algebraic operation called a **scan**

Scans

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Scans

```python
import jax
import jax.numpy as jnp

T, d_x, d_h = 10, 2, 4
xs, h0 = jnp.ones((T, d_x)), jnp.zeros((d_h,))
theta = [jnp.ones((d_h,)), jnp.ones((d_x, d_h))] # (b, W)

def f(h, x):
    b, W = theta
    result = h + (W.T @ x + b)
    return result, result # return one, return all

hT, hs = jax.lax.scan(f, init=h0, xs=xs) # Scan f over x
```

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```
def scan(f, h, xs):
    # h shape is (d_h,)
    # xs shape is (T, d_x)
    hs = []
    for x in xs:
        h = f(h, x, theta)
        hs.append(h)
    # output shape is (T, d_h)
    return torch.stack(hs)
```

Scans

```python
import torch
T, d_x, d_h = 10, 2, 4

xs, h0 = torch.ones((T, d_x)), torch.zeros((d_h,))
theta = (torch.ones((d_h,)), torch.ones((d_x, d_h)))

def f(h, x):
    b, W = theta
    result = h + (W.T @ x + b)
    return result # h

hs = scan(f, h0, xs)
```

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Answer: Yes, linear operations obey the associative property

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Example: I ask you your favorite ice cream flavor

You recall previous times you ate ice cream, but not your phone number

We will model this recall of memories using a function g

Output Modeling

Let g define our memory recall function

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$\boldsymbol{h}$: Everything you remember (hometown, birthday, etc)

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Now, we will combine f and g

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Step 1: Perform scan to find recurrent states

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Step 2: Perform recall on recurrent states

$$\begin{bmatrix} \boldsymbol{y}_1 \\ \vdots \\ \boldsymbol{y}_T \end{bmatrix} = \begin{bmatrix} g(\boldsymbol{h}_1, \boldsymbol{x}_1, \boldsymbol{\theta}_g) \\ \vdots \\ g(\boldsymbol{h}_T, \boldsymbol{x}_T, \boldsymbol{\theta}_g) \end{bmatrix}$$

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Questions? This is on the homework

Output Modeling

To summarize, we defined:

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- Recurrent function f

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- Scanned recurrence $\text{scan}(f)$

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To run our model:

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- Execute g over recurrent states to make outputs

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Relax

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Recurrent Loss Functions

Let us examine some example tasks:

Recurrent Loss Functions

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- Clock

Recurrent Loss Functions

Let us examine some example tasks:

- Clock
- Explaining a video

Recurrent Loss Functions

Task: Clock – keep track of time

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Every minute, the minute hand ticks

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Count/remember the ticks to know the time

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$$X \in \{0, 1\}, \quad Y \in \mathbb{R}^2$$

Recurrent Loss Functions

Example input sequence: $\begin{bmatrix} 1 \\ 1 \\ \vdots \\ 1 \end{bmatrix}$

Recurrent Loss Functions

Example input sequence: $\begin{bmatrix} 1 \\ 1 \\ \vdots \\ 1 \end{bmatrix}$

Desired output sequence $\begin{bmatrix} 0 & 1 \\ 0 & 2 \\ \vdots & \vdots \\ 2 & 13 \end{bmatrix}$

We have a corresponding label $\boldsymbol{y}$ for each input x

Recurrent Loss Functions

Can use square error

Recurrent Loss Functions

Can use square error

First, scan f over the inputs to find h

$$\boldsymbol{h}_{[i],j} = \text{scan}(f)\left(\boldsymbol{h}_0, \begin{bmatrix} \boldsymbol{x}_{[i],1} \\ \vdots \\ \boldsymbol{x}_{[i],T} \end{bmatrix}, \boldsymbol{\theta}_f \right)$$

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$$\mathcal{L}(\boldsymbol{X}, \boldsymbol{Y}, \boldsymbol{\theta}) = \sum_{i=1}^{n} \sum_{j=1}^{T} \left[g\left(\boldsymbol{h}_{[i],j}, \boldsymbol{x}_{[i],j}, \boldsymbol{\theta}_g\right) - \boldsymbol{y}_{[i],j}\right]^2$$

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Onto the next task

Recurrent Loss Functions

Task: Watch a video, then explain it

$$X \in \mathbb{Z}^{3 \times 32 \times 32}, \quad Y \in \{\text{comedy show}, \text{action movie}, ...\}$$

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Example input sequence: $\begin{bmatrix} I_1 \\ I_2 \\ \vdots \\ I_T \end{bmatrix}$

Example output: "dancing dog"

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Unlike before, we have many inputs but just one output!

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$$\mathcal{L}(\boldsymbol{X}, \boldsymbol{Y}, \boldsymbol{\theta}) = \sum_{i=1}^{n} \sum_{j=1}^{d_y} y_{[i],j} \log g\left(\boldsymbol{h}_{[i],T}, \boldsymbol{x}_{[i],T}, \boldsymbol{\theta}_g\right)_j$$

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We only care about the $\boldsymbol{h}_T$

Recurrent Loss Functions

To summarize, we use standard losses for recurrent loss functions

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Just be careful – we often sum over an additional axis

$$\sum_{i=1}^{n} \sum_{j=1}^{T} \ldots$$

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Backpropagation through Time

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Step 2: Update parameters using gradient

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Just like all other models, we train recurrent models using gradient descent

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How do we compute gradients for recurrent functions?

Backpropagation through Time

First, compute gradient of f

$$f(\boldsymbol{h}, \boldsymbol{x}, \boldsymbol{\theta}) = \gamma \boldsymbol{h} + \boldsymbol{\theta}^\top \overline{\boldsymbol{x}}$$

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Question: What is $\nabla_{\boldsymbol{\theta}} f$?

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Too easy, now let us find the gradient of scan(f)

Backpropagation through Time

$$\text{scan}(f)\left(\boldsymbol{h}_0, \begin{bmatrix} \boldsymbol{x}_1 \\ \vdots \\ \boldsymbol{x}_T \end{bmatrix}, \boldsymbol{\theta}\right) = \begin{bmatrix} f(h_0, x_1, \boldsymbol{\theta}) \\ f(h_1, x_2, \boldsymbol{\theta}) \\ \vdots \\ f(h_{T-1}, x_T, \boldsymbol{\theta}) \end{bmatrix} = \begin{bmatrix} f(h_0, x_1) \\ f(f(h_0, x_1), x_2) \\ \vdots \\ f(...f(h_0, x_1)..., x_T) \end{bmatrix}$$

Question: What is $\nabla_{\boldsymbol{\theta}} \text{scan}(f)$?

Hint: Chain rule

Backpropagation through Time

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Question: What is $\nabla_{\boldsymbol{\theta}} \text{scan}(f)$?

Hint: Chain rule

$$\nabla_{\boldsymbol{\theta}} \text{scan}(f)\left(\boldsymbol{h}_0, \begin{bmatrix} \boldsymbol{x}_1 \\ \vdots \\ \boldsymbol{x}_T \end{bmatrix}, \boldsymbol{\theta}\right) = \begin{bmatrix} \nabla_{\boldsymbol{\theta}}[f](h_0, x_1) \\ \nabla_{\boldsymbol{\theta}}[f](f(h_0, x_1), x_2) \nabla_{\boldsymbol{\theta}}[f](h_0, x_1) \\ \vdots \\ \nabla_{\boldsymbol{\theta}}[f](\ldots f(h_0, x_1) \ldots, x_T) \ldots \cdot \nabla_{\boldsymbol{\theta}}[f](h_0, x_1) \end{bmatrix}$$

Backpropagation through Time

$$\nabla_{\boldsymbol{\theta}} \operatorname{scan}(f) \left(\boldsymbol{h}_0, \begin{bmatrix} \boldsymbol{x}_1 \\ \vdots \\ \boldsymbol{x}_T \end{bmatrix}, \boldsymbol{\theta} \right) = \begin{bmatrix} \nabla_{\boldsymbol{\theta}}[f] \\ \nabla_{\boldsymbol{\theta}}[f] \nabla_{\boldsymbol{\theta}}[f] \\ \nabla_{\boldsymbol{\theta}}[f] \nabla_{\boldsymbol{\theta}}[f] \nabla_{\boldsymbol{\theta}}[f] \\ \vdots \end{bmatrix}$$

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Question: Any issues with this?

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Question: Any issues with this?

What if $\nabla_{\boldsymbol{\theta}} f$ is $\ll 1$ or $\gg 1$?

Agenda

1. Review
2. Sequence Modeling
3. Composite Memory
4. Linear Recurrence
5. Scans
6. Output Modeling
7. Recurrent Loss Functions
8. **Backpropagation through Time**
9. Recurrent Neural Networks
10. Coding

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Recurrent Neural Networks

Until now, f was a linear function

Recurrent Neural Networks

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If we make f a neural network, then we have a **recurrent neural network** (RNN)

Recurrent Neural Networks

The simplest recurrent neural network is the **Elman Network**

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Answer: Forgetting!

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Add forgetting

$$f(\boldsymbol{h}, \boldsymbol{x}, \boldsymbol{\theta}) = \sigma\left(\boldsymbol{\theta}_1^\top \boldsymbol{h} \odot f_{\text{forget}}(\boldsymbol{h}, \boldsymbol{x}, \boldsymbol{\theta}) + \boldsymbol{\theta}_2^\top \overline{\boldsymbol{x}}\right)$$

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Through gradient descent, neural network learns which memories to forget

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Minimal gated unit (MGU) is a modern RNN

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MGU defines two helper functions

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Left term forgets old, right term replaces forgotten memories

Recurrent Neural Networks

There are even more complicated models

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- Long Short-Term Memory (LSTM)

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LSTM has 6 different functions! Too complicated to review

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MGU is simpler and performs similarly to LSTM and GRU

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Recall the gradient for the linear recurrence

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Question: What is the gradient for $\operatorname{scan}(f)$ of Elman network?

Recurrent Neural Networks

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$$\nabla[\sigma] \cdot \nabla[\sigma] \cdot \ldots = 0$$

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All RNNs suffer from either exploding gradient (ReLU) or vanishing gradient (sigmoid). Active area of research!

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Coding

Jax RNN https://colab.research.google.com/drive/147z7FNGyERV8oQ_4gZmxDVdeoNt0hKta#scrollTo=TUMonlJ1u8Va

Homework https://colab.research.google.com/drive/1CNaDxx1yJ4-phyMvgbxECL8ydZYBGQGt?usp=sharing