Transformers

CISC 7026 - Introduction to Deep Learning

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Review

Review

Last time, we derived various forms of **attention**

We started with composite memory

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$$f(\boldsymbol{x}, \boldsymbol{\theta}) = \sum_{i=1}^{T} \boldsymbol{\theta}^\top \overline{\boldsymbol{x}}_i$$

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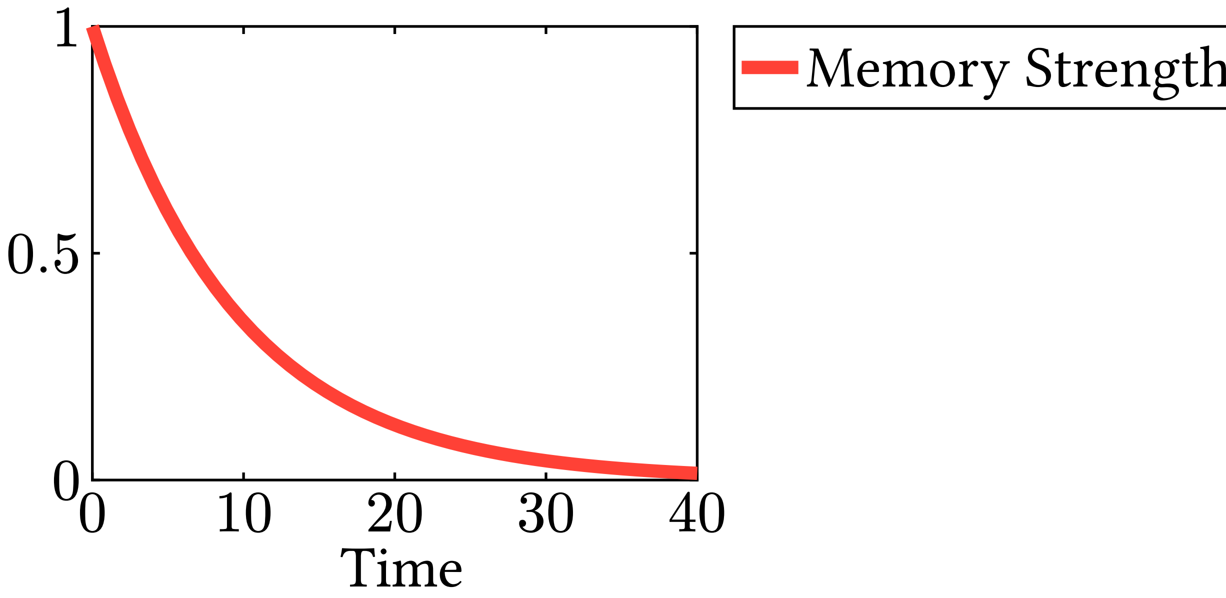

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We went to a party and the forgetting seemed ok

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10 PM

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10 PM 11 PM

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10 PM 11 PM 12 AM

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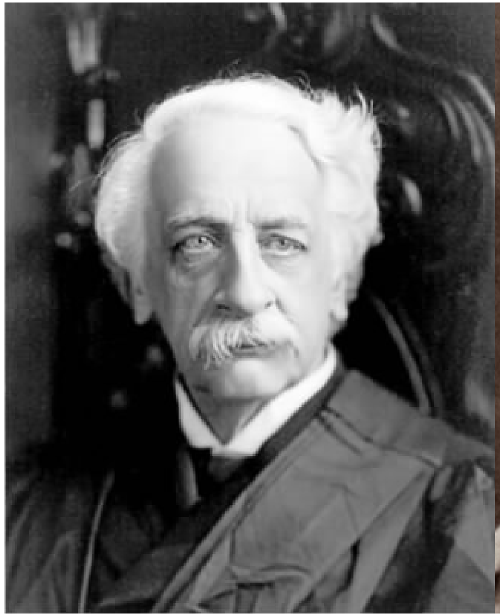

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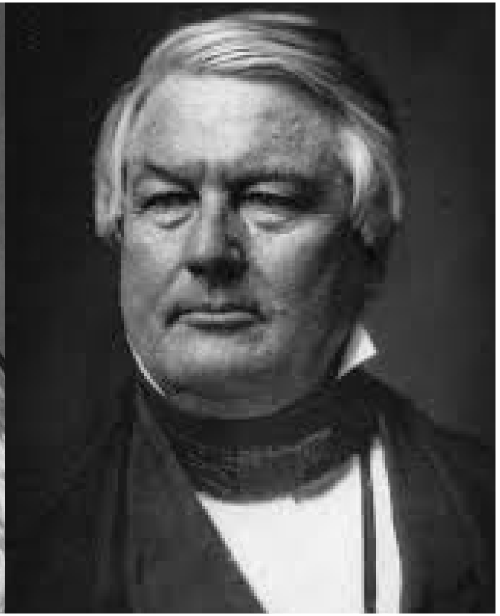

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Review

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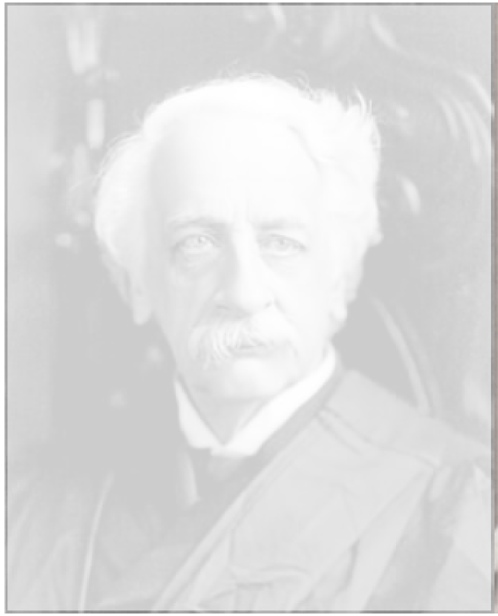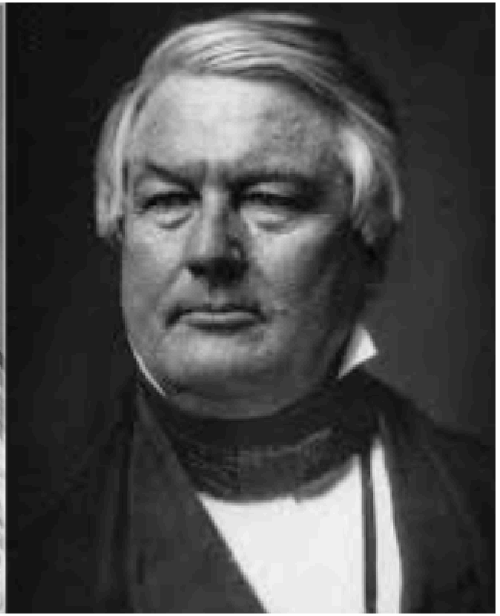

$$\gamma^3 \boldsymbol{\theta}^\top \overline{\boldsymbol{x}}_1$$

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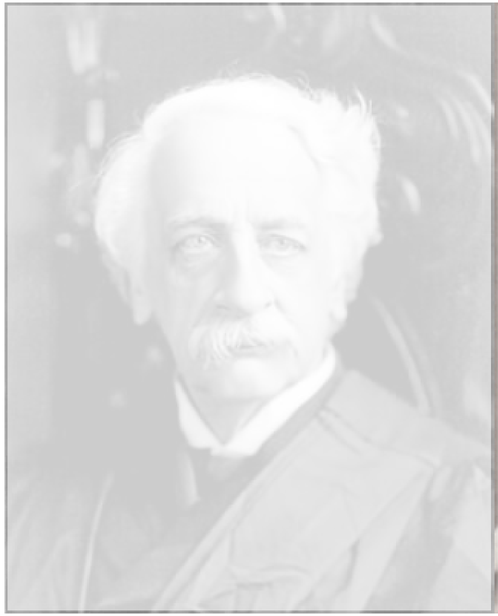


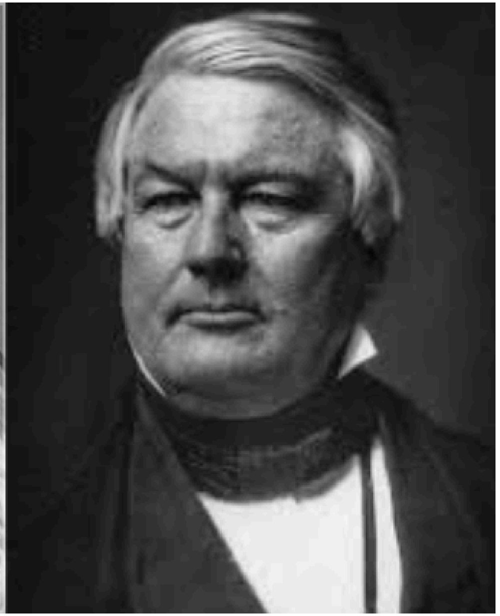

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With our current model, we forget Taylor Swift!

Our model of human memory is incomplete

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Last time we studied attention

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Overview of transformer application and domains

Going Deeper

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We previously reviewed training tricks

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- Deeper networks

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- Parameter initialization

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- Weight decay

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- Parameter initialization
- Stochastic gradient descent
- Adaptive optimization
- Weight decay

These methods empirically improve performance, but we do not always understand why

Going Deeper

Modern transformers can be very deep

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For this reason, they use two new training tricks to enable very deep models

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We will start with the **residual connection**

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Remember that a two-layer MLP is a universal function approximator

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Making the network too deep can hurt performance

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The theory is that the input information is **lost** somewhere in the network

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$$\boldsymbol{y} = f_k(...f_2(f_1(\boldsymbol{x}, \boldsymbol{\theta}_1), \boldsymbol{\theta_2}), ..., \boldsymbol{\theta}_k)$$

Going Deeper

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Claim: If the input information is present throughout the network, then we should be able to learn the identity function $f(x) = x$

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Question: We have seen a similar model, what was it?

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https://colab.research.google.com/drive/1qVIbQKpTuBYIa7FvC4IH-kJq-E0jmc0d#scrollTo=bg74S-AvbmJz

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Very deep networks struggle to learn the identity function

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Very deep networks struggle to learn the identity function

If the input information is available, then learning the identity function should be very easy!

Question: How can we prevent the input from getting lost?

Going Deeper

We can feed the input to each layer

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The first approach is called the **DenseNet** approach

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$$\boldsymbol{z}_1 = f_1(\boldsymbol{x}, \boldsymbol{\theta}_1)$$

$$\boldsymbol{z}_2 = f_2\left(\begin{bmatrix} \boldsymbol{x} \\ \boldsymbol{z}_1 \end{bmatrix}, \boldsymbol{\theta}_2\right)$$

$$\vdots$$

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Question: Any issues with the DenseNet approach?

Answer: Very deep networks require too many parameters!

Going Deeper

The next method is called the **ResNet** approach

$$\begin{aligned}
\boldsymbol{z}_1 &= f_1(\boldsymbol{x}, \boldsymbol{\theta}_1) \\
\boldsymbol{z}_2 &= f_2(\boldsymbol{x}, \boldsymbol{\theta}_2) + \boldsymbol{x} \\
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$$f(\boldsymbol{x}, \boldsymbol{\theta}) = 0; \quad f(\boldsymbol{x}, \boldsymbol{\theta}) + x = x$$

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This helps prevent information from getting lost in very deep networks

Going Deeper

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$$f_1(\boldsymbol{x}, \boldsymbol{\theta}_1) = \sum_{i=1}^{d_x} \theta_{1,i} x_i$$

Question: If all $x_i = 1, \theta_{1,i} = 0.01$ and $d_x = 1000$, what is the output?

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What if we add another layer with the same d_x and θ?

$$f_2(\boldsymbol{z}, \boldsymbol{\theta}_2) = \sum_{i=1}^{1000} 0.01 \cdot 10 = 100$$

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Question: What is the problem?

Going Deeper

Let us look at the gradient

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$$\nabla_{\boldsymbol{\theta}_1} f_2(f_1(\boldsymbol{x}, \boldsymbol{\theta}_1), \boldsymbol{\theta}_2) =$$

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$$\nabla_{\boldsymbol{\theta}_1} f_2(f_1(\boldsymbol{x}, \boldsymbol{\theta}_1), \boldsymbol{\theta}_2) = \nabla_{\boldsymbol{\theta}_1}[f_2](f_1(\boldsymbol{x}, \boldsymbol{\theta}_1)) \cdot \nabla_{\boldsymbol{\theta}_1}[f_1](\boldsymbol{x}, \boldsymbol{\theta}_1)$$

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$$\approx 100 \cdot 10$$

Can cause exploding or vanishing gradient

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Deeper network $\Rightarrow$ worse exploding/vanishing issues

Question: What can we do?

Going Deeper

Let us look at the gradient

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$$\mu = d_y \sum_{i=1}^{d_y} f(\boldsymbol{x}, \boldsymbol{\theta})_i$$

$$f(\boldsymbol{x}, \boldsymbol{\theta}) - \mu$$

Question: What does this do?

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Question: What does this do?

Answer: Makes output have zero mean (both positive and negative values)

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Going Deeper

$$\mu = d_y \sum_{i=1}^{d_y} f(\boldsymbol{x}, \boldsymbol{\theta})_i \qquad\qquad f(\boldsymbol{x}, \boldsymbol{\theta}) - \mu$$

Then, layer normalization **rescales** the outputs

$$\sigma = \frac{\sqrt{\sum_{i=1}^{d_y} f(\boldsymbol{x}, \boldsymbol{\theta}_i - \mu)^2}}{d_y}$$

$$\mathrm{LN}(f(\boldsymbol{x}, \boldsymbol{\theta})) = \frac{f(\boldsymbol{x}, \boldsymbol{\theta}) - \mu}{\sigma}$$

Now, the output of the layer is normally distributed

Going Deeper

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- 99.7% of outputs $\in [-3, 3]$

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- 99.99% of outputs $\in [-4, 4]$
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This helps prevent vanishing and exploding gradients

Going Deeper

Now, let's combine residual connections and layer norm and try our very deep network again

TODO COLAB

Going Deeper

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Going Deeper

Layer Norm

Initialization and L2 normalization keep the weights small

But the outputs of each layer can still be large or small

Chain rule example

Transformers

Transformers

Now we have everything we need to implement a transformer

Positional Encoding

Text Transformers

Image Transformers

Unsupervised Training

Unsupervised Training

Predict the future

World Models

World Models

What if transformers could interact with the world?

Course Evaluation

Course Evaluation

Department instructed me to ask you for course feedback

Course Evaluation

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If you are not comfortable writing English, write Chinese

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I must leave the room to let you fill out this form

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I must leave the room to let you fill out this form

Please scan the QR code and complete the survey

Course Evaluation

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Department has suggested 10 minutes

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https://isw.um.edu.mo/siaweb

Course Evaluation

Research data labeling and collection

If you participated, come up