Graph Neural Networks

CISC 7026: Introduction to Deep Learning

University of Macau

Agenda

1. Review
2. Quiz
3. Introduction to Prof. Li
4. Graph Neural Networks

GNNs

$$f(\boldsymbol{x}, \boldsymbol{\theta})_j = \boldsymbol{\theta}^\top \overline{\boldsymbol{x}}$$

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$$f : X^t \mapsto Z$$

$$f^{-1} : Z \mapsto X^t$$

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You CPU has a H.264 decoder built in to make this fast

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Task: Compress images for your clothing website to save on costs

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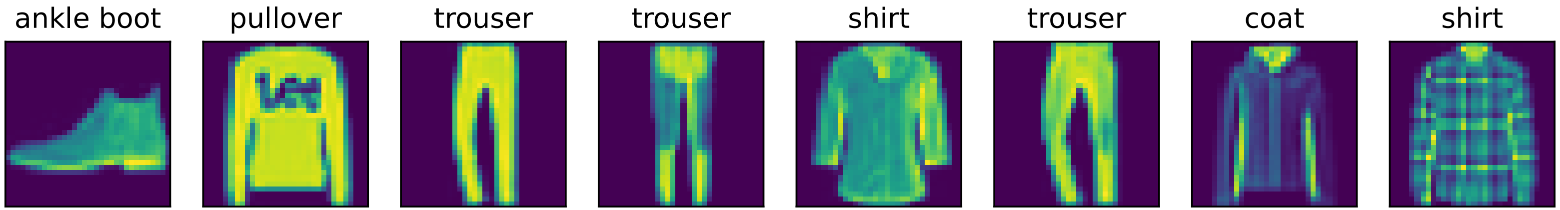

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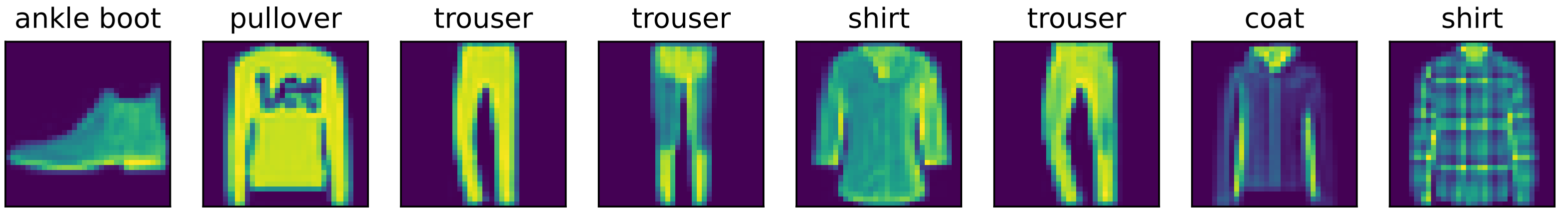

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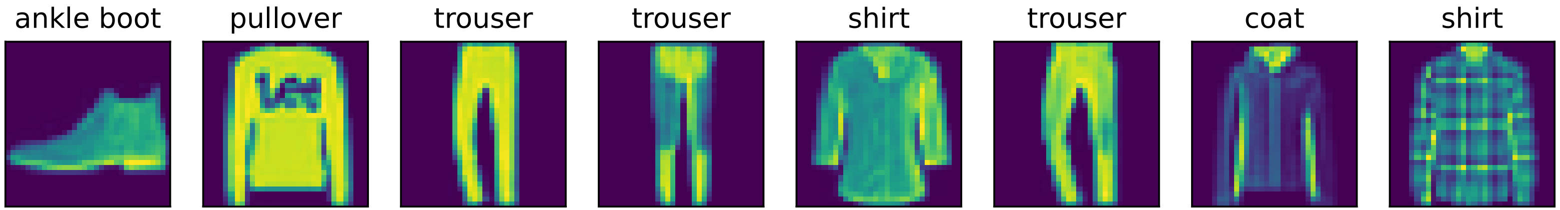

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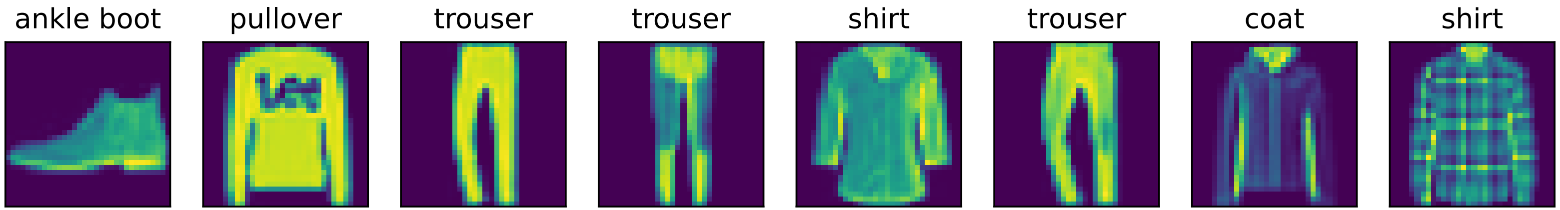

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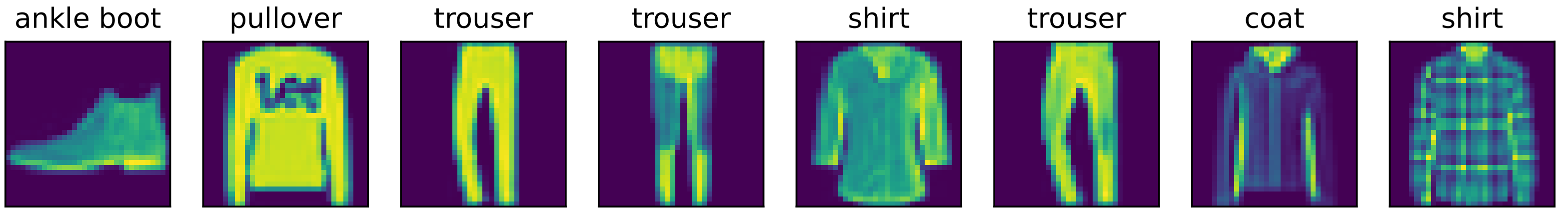

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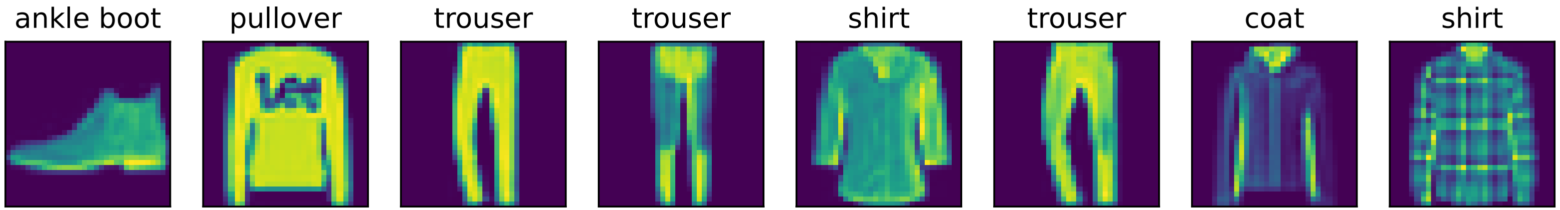

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How do we find $\boldsymbol{\theta}$?

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What if we plug $\boldsymbol{z}$ into the second equation?

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Let us try another way

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Turn this into a loss function using the square error

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Forces the networks to compress and reconstruct $\boldsymbol{x}$

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Define over the entire dataset

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We call this the **reconstruction loss**

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We call this the **reconstruction loss**

It is an unsupervised loss because we only provide $\boldsymbol{X}$ and not $\boldsymbol{Y}$!

Review

We can use autoencoders for more than compression

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We can make **denoising autoencoders** that remove noise

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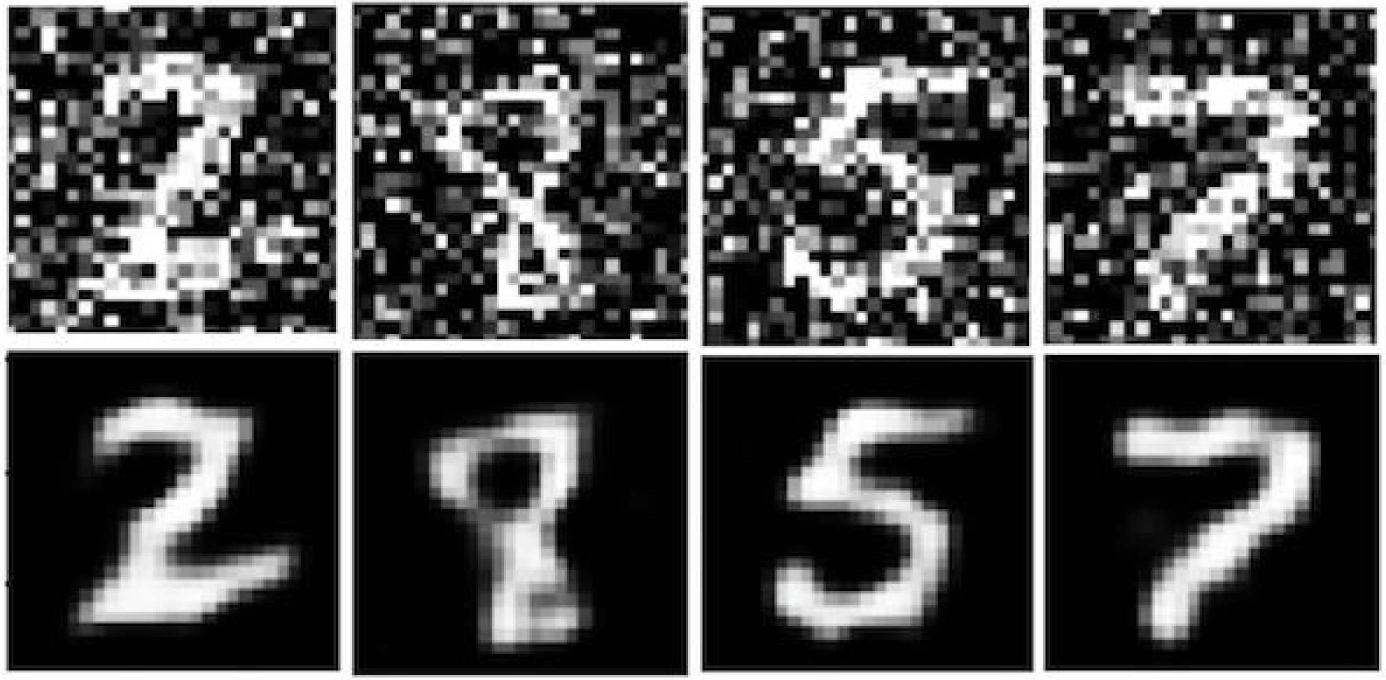

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Generate some noise $\quad \varepsilon \sim \mathcal{N}(\boldsymbol{\mu}, \boldsymbol{\sigma})$

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Original loss $\mathcal{L}(\boldsymbol{X}, \boldsymbol{\theta}) = \sum_{i=1}^{n} \sum_{j=1}^{d_x} \left(x_{[i],j} - f^{-1}\left(f\left(\boldsymbol{x}_{[i]}, \boldsymbol{\theta}_e\right), \boldsymbol{\theta}_d\right)_j \right)^2$

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Denoising loss $$\mathcal{L}(\boldsymbol{X}, \boldsymbol{\theta}) = \sum_{i=1}^{n} \sum_{j=1}^{d_x} \left(x_{[i],j} - f^{-1}\left(f\left(\boldsymbol{x}_{[i]} + \boldsymbol{\varepsilon}, \boldsymbol{\theta}_e \right), \boldsymbol{\theta}_d \right)_j \right)^2$$

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Autoencoder will learn to remove noise when reconstructing image

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Then, we discussed variational autoencoders

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However, to save time we will review these and write code next time

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Quiz

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No cheating, you will get 0 and I tell Dean

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You will have 20 minutes

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Good luck!

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Today, he will teach you about **Graph Neural Networks**