Reinforcement Learning

CISC 7026 - Introduction to Deep Learning

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University of Macau

Lecture Goal: Provide a proper understanding of the theoretical foundations of reinforcement learning

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Lecture Goal: Give you enough information to begin learning RL on your own

What is RL?

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What does this mean?

What is RL?

Example: You train a model f to play chess

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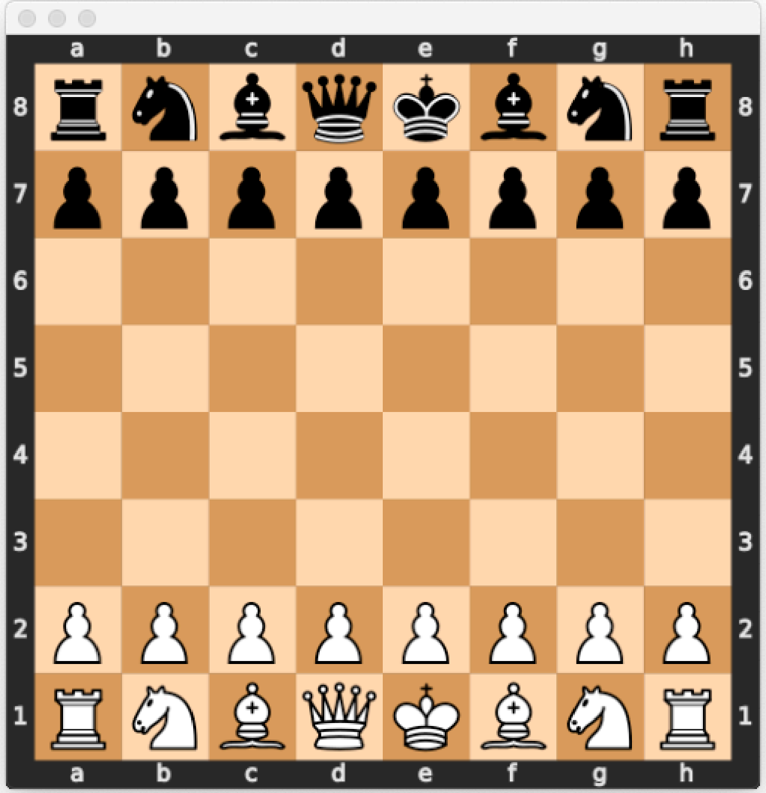

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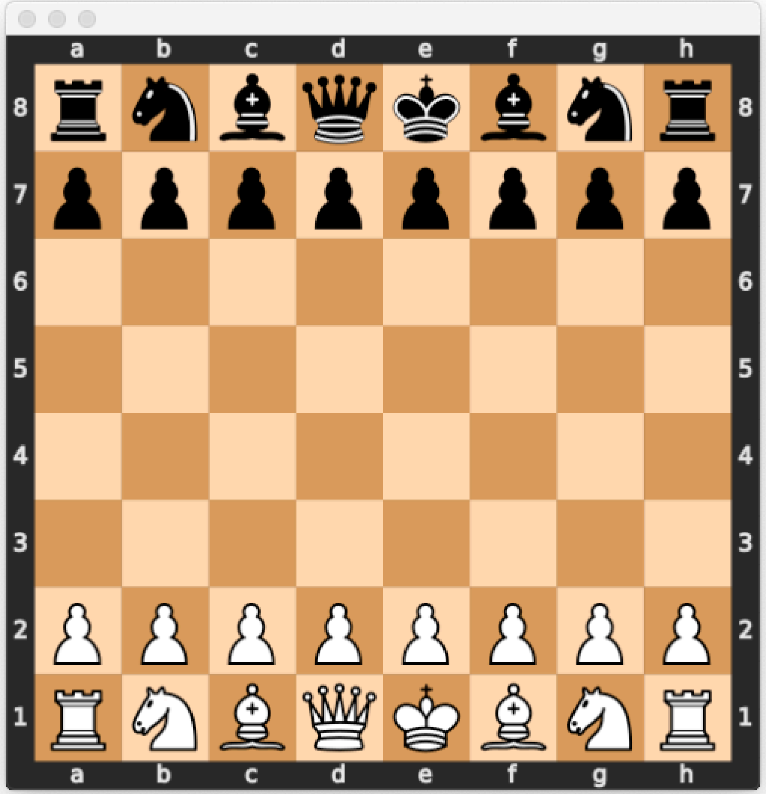

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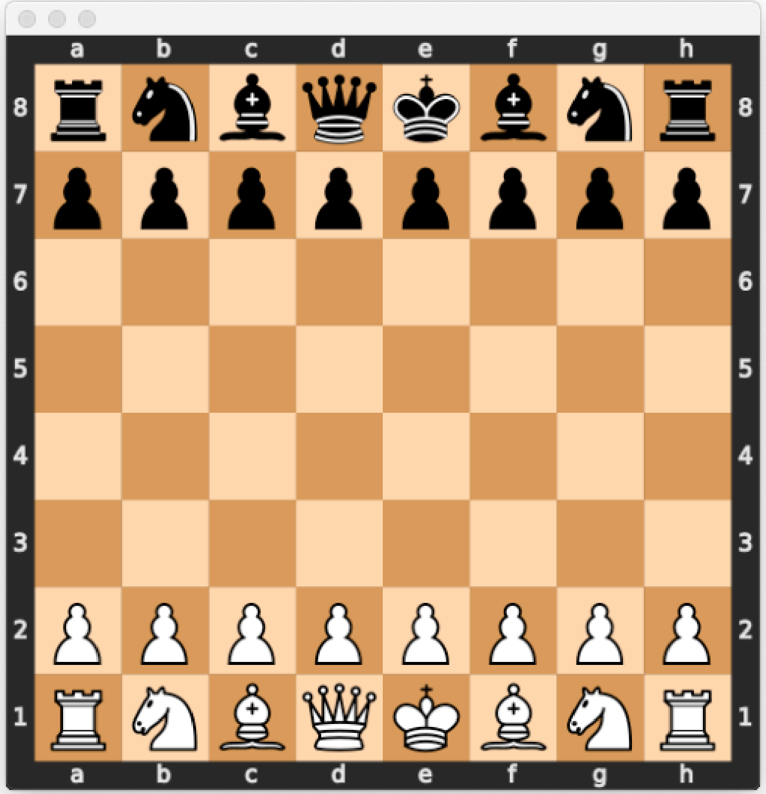

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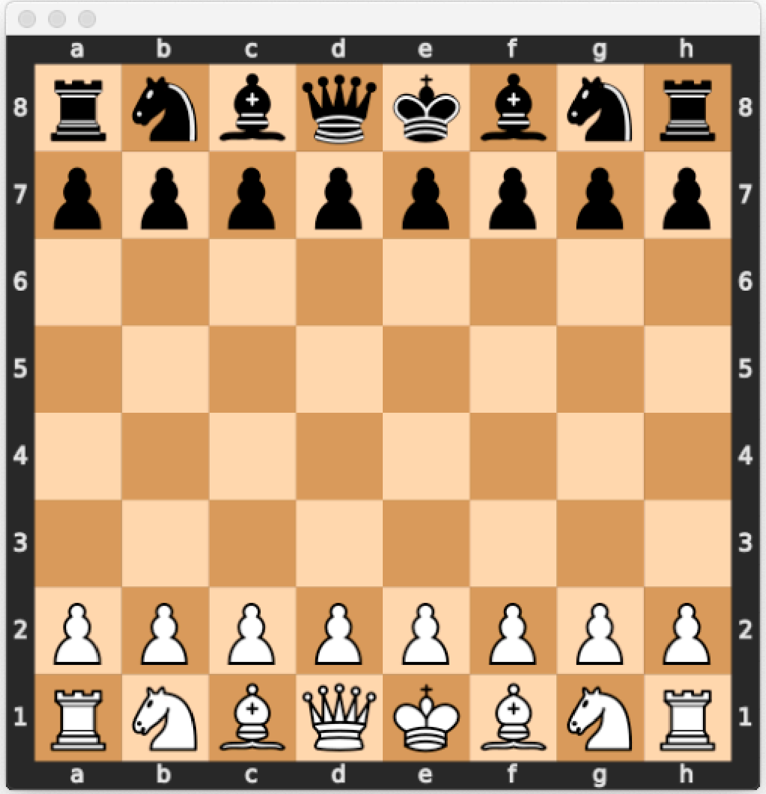

We do not know the answer

But RL can tell us!

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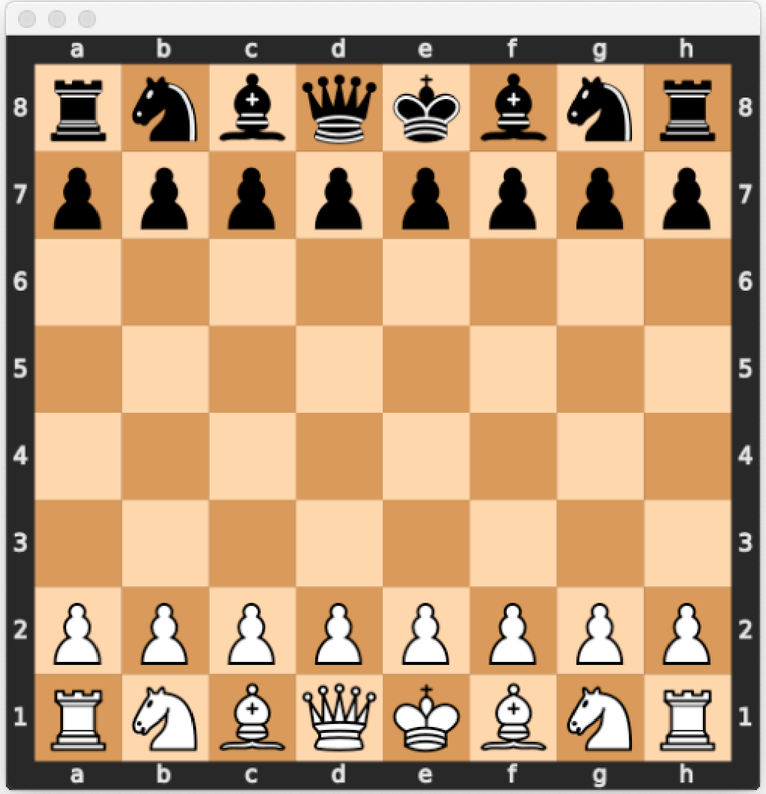

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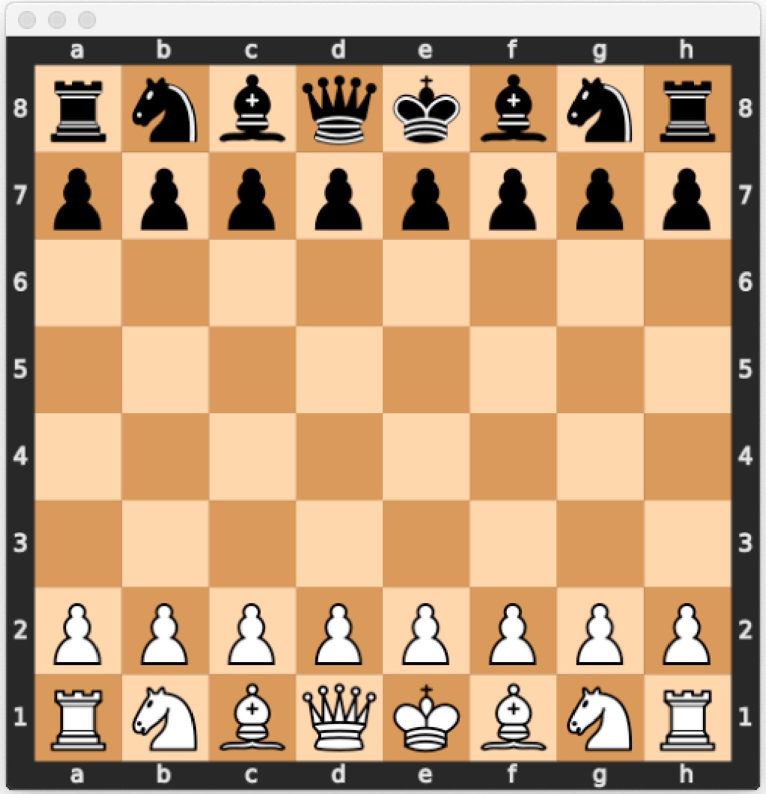

An answer gives us just one move

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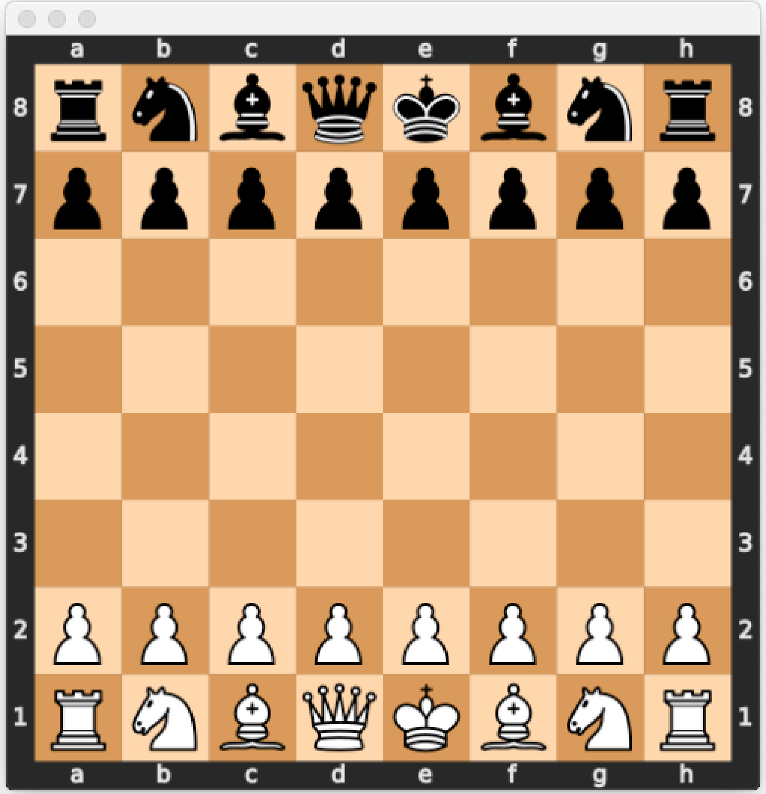

An answer gives us just one move

We need many moves to win

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RL gives us the best sequence of moves to achieve a result

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- Win a game of chess

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- Win a game of chess
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- Reduce human suffering

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RL gives us the best sequence of moves to achieve a result

- Win a game of chess
- Drive a customer to the store
- Cook a tasty meal
- Surgically remove cancer from the patient
- Prevent climate change
- Reduce human suffering
- Find your own purpose (achieve conciousness)

What is RL?

https://www.youtube.com/watch?v=Zeyv1bN9v4A	GT
https://www.youtube.com/watch?v=kopoLzvh5jY&t=1s	H&S
https://www.youtube.com/watch?v=eHipy_j29Xw	DoTA

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Real applications of RL:

- Autonomous vehicles
- Video game NPCs
- Behavior modeling in psychology/ecology/biology
- Material and drug design
- Finance
- Alignment in large language models
  - Artificial General Intelligence?
- Anywhere with cause and effect
  - Where you **change** the world by **interacting** with it

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RL is more complicated than supervised learning

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Instead of a model and dataset, we have an **agent** and **environment**

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Agent

Environment

What is RL?

The agent receives a positive reward for doing good

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The agent receives a positive reward for doing good

And a negative reward for doing bad

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The agent receives a positive reward for doing good

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Eventually, the agent only does good behaviors

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Humans learn by reinforcement learning too

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Enough about the agent, let us talk about the environment

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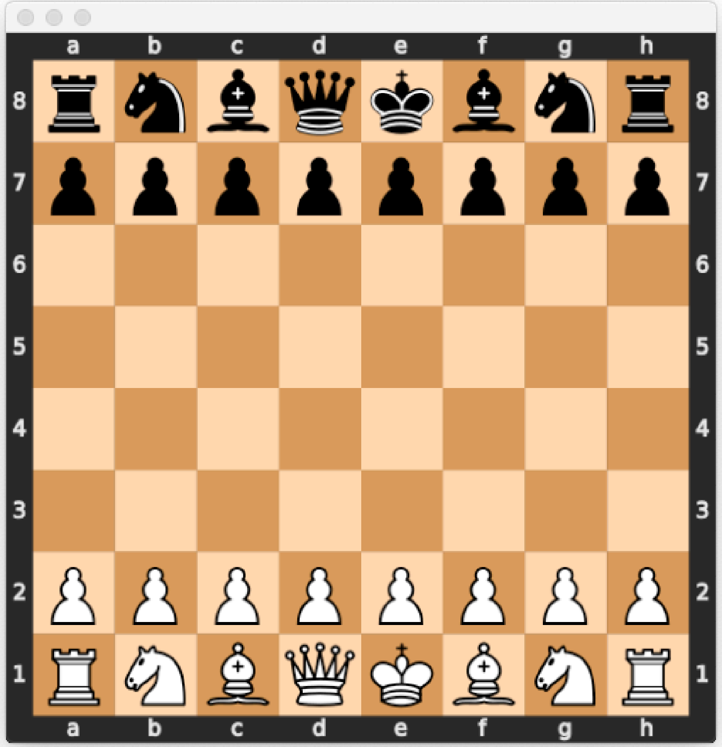

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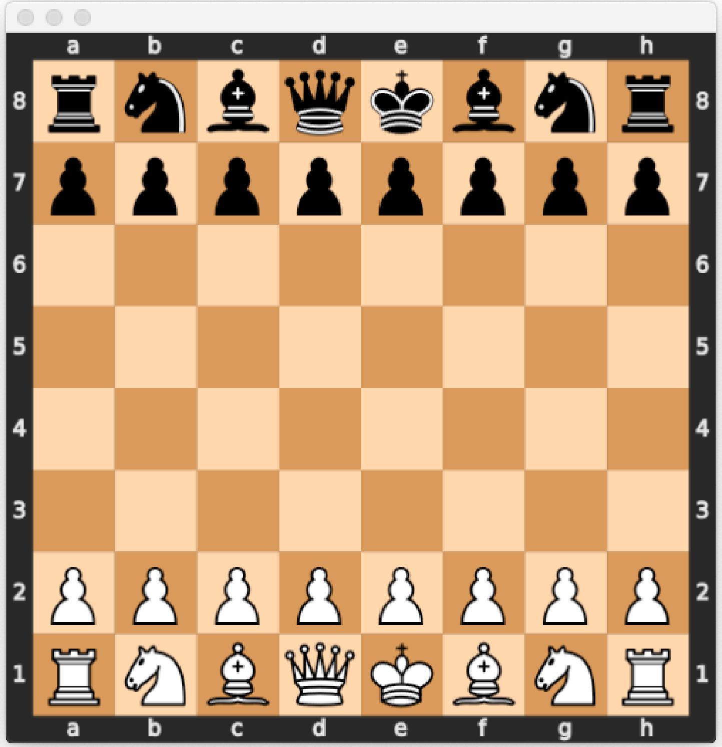

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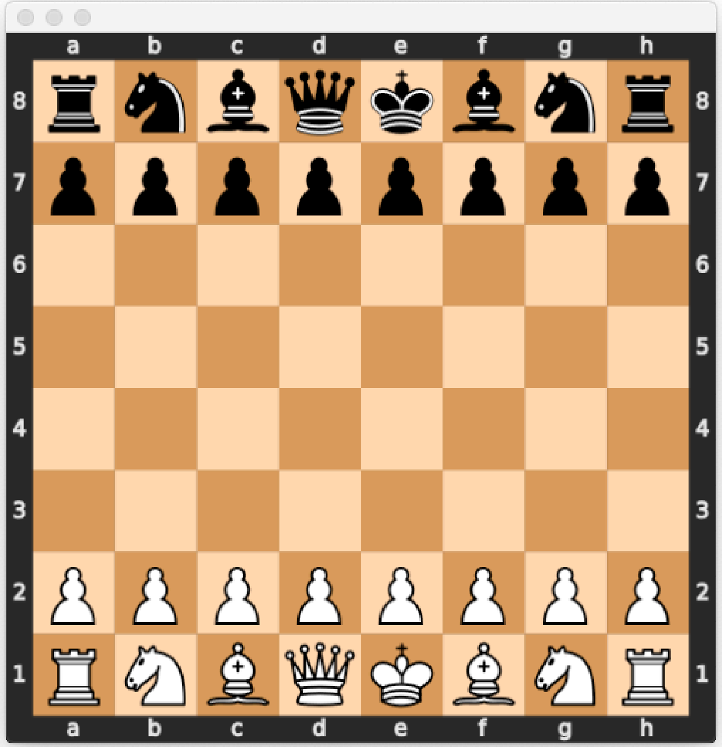

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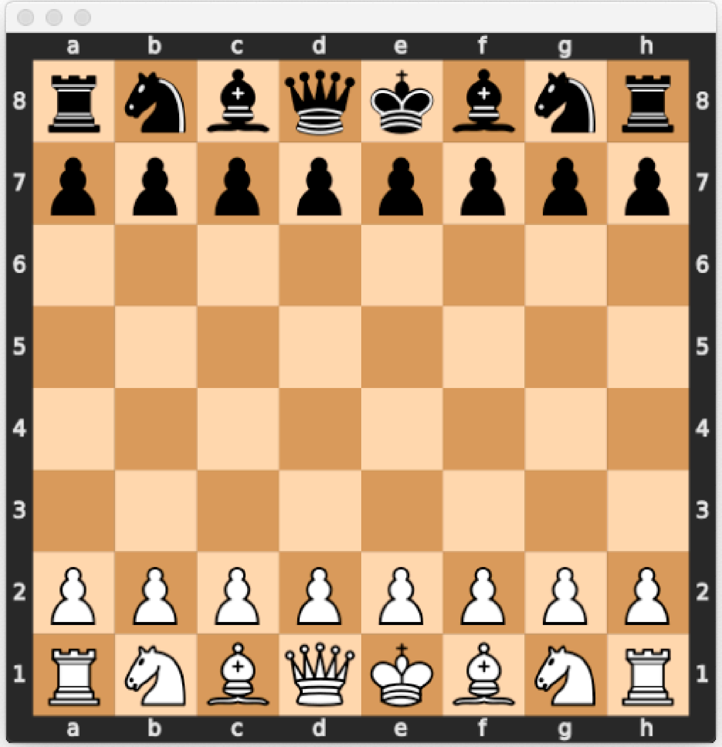

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For example, each piece can only move in certain ways

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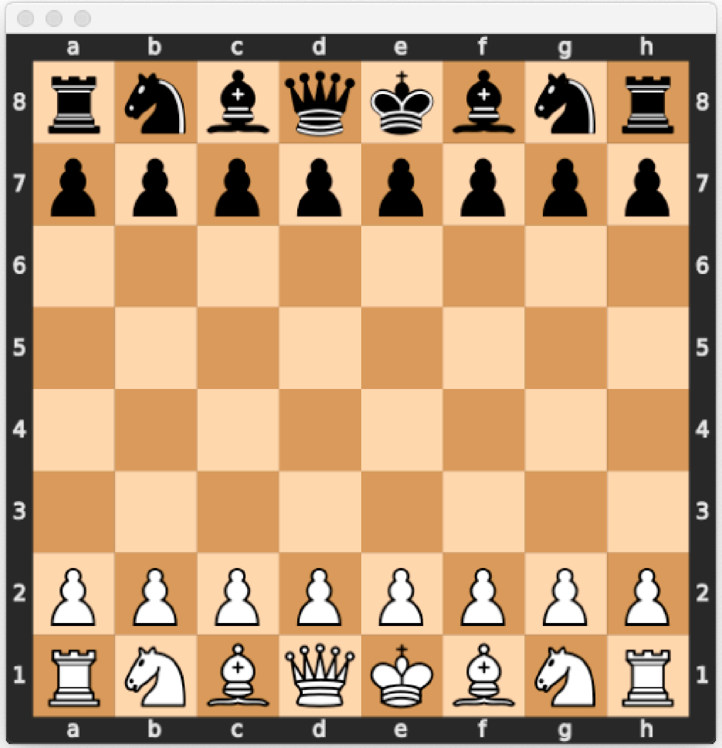

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For example, each piece can only move in certain ways

If two pieces touch, then one piece dies

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Now that you understand the agent, rewards, and environment, we will get more technical

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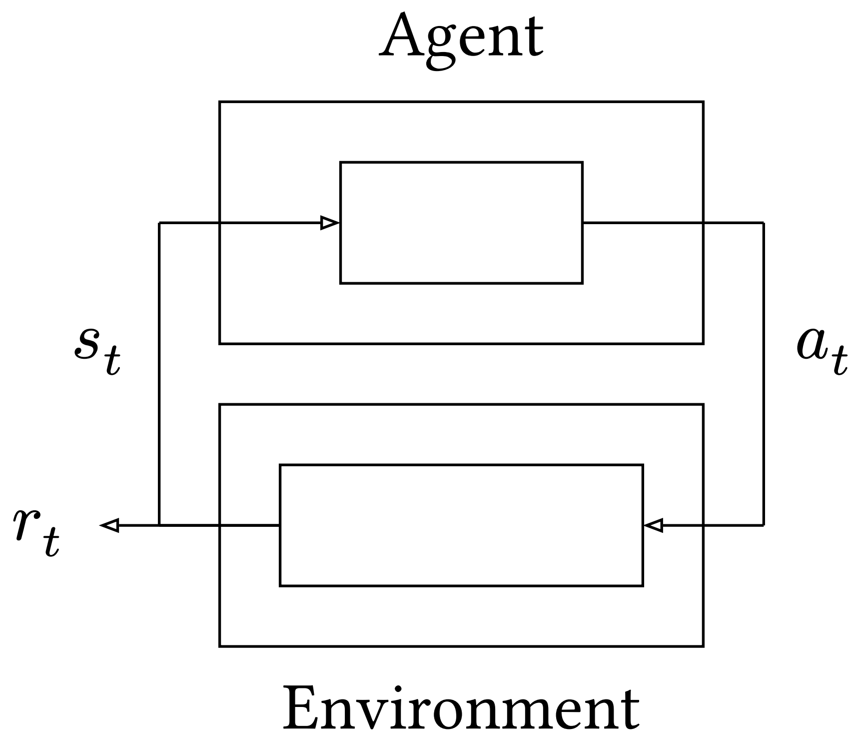

s_t: state, a_t: action, r_t: reward

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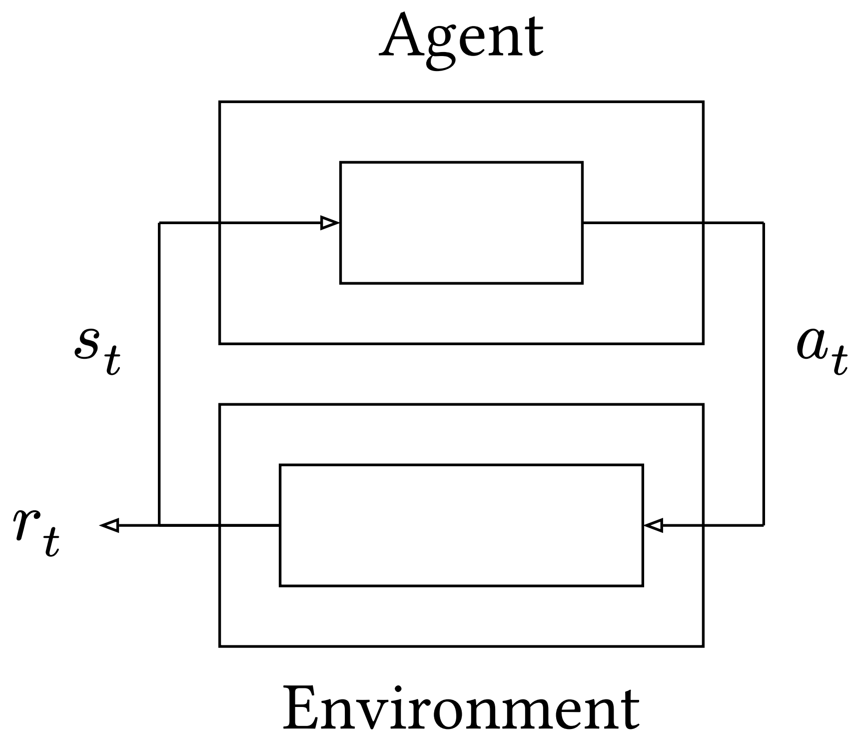

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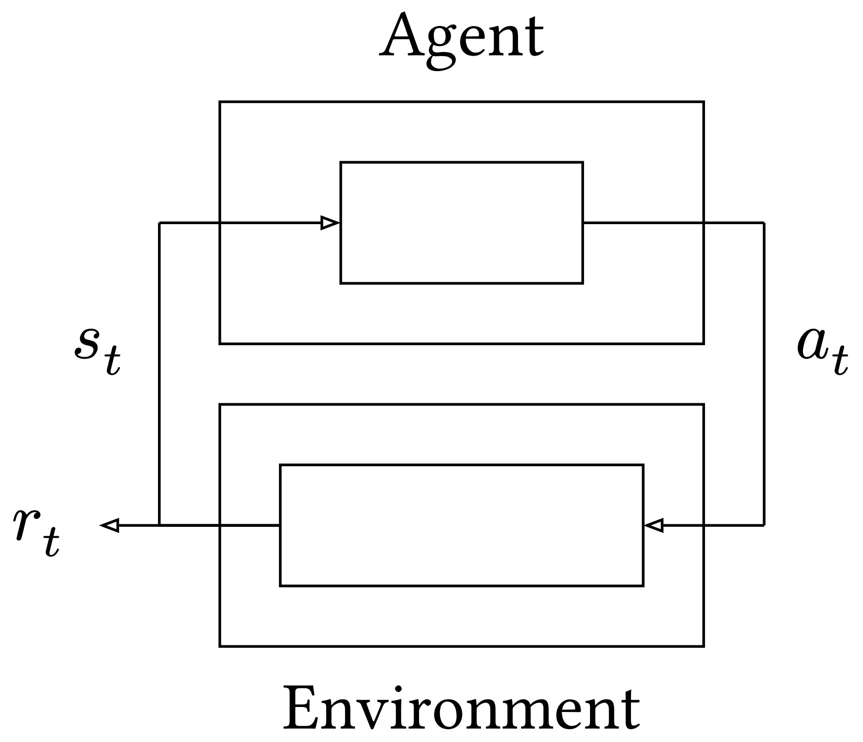

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- The agent takes **actions** in the environment
- Actions change the environment **state**, producing an new state and **reward**

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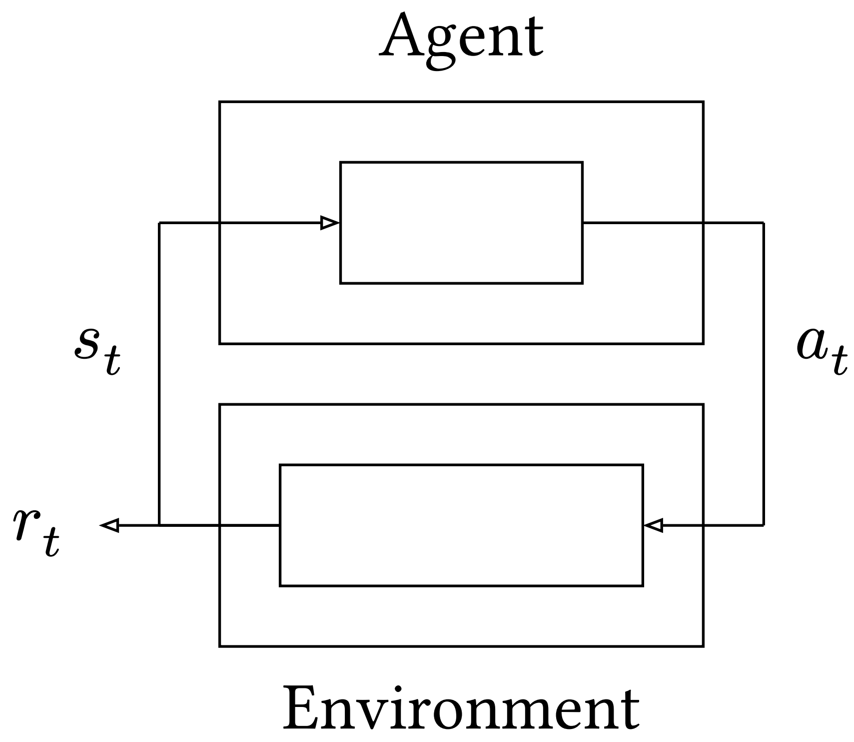

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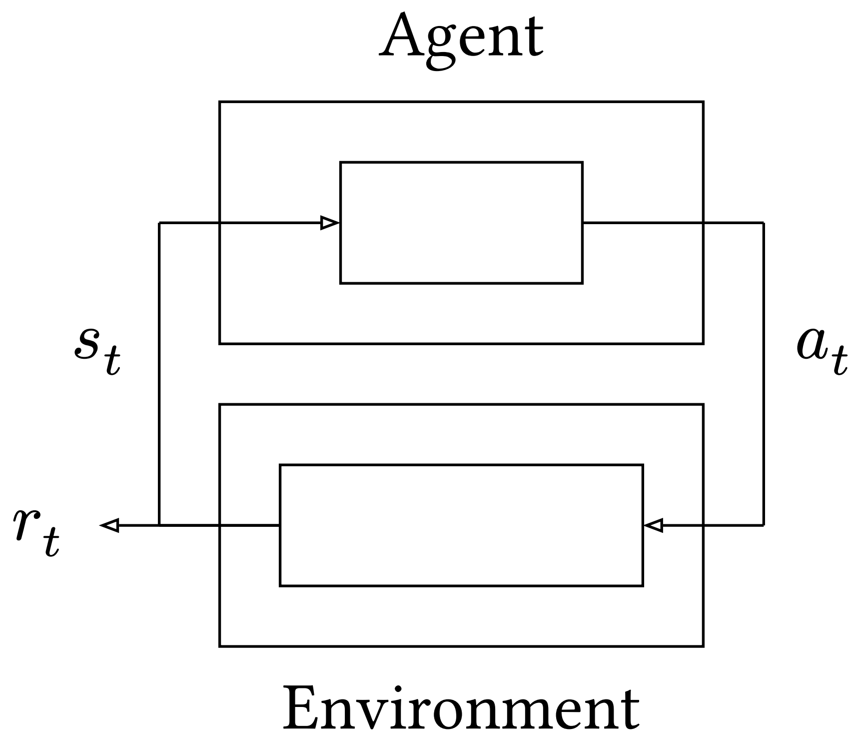

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- Goal is to maximize the **cumulative reward**

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By definition, RL solves **Markov Decision Processes (MDPs)**.

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To solve a problem using RL, we first must convert the problem into an MDP. We call this converted problem the **environment**.

How you structure your problem is **critical** – more important than which algorithms you use, how much compute you have, etc.

Markov Decision Processes

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Let us briefly explain these terms.

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We need a way to describe what state the environment is in

If the environment is a table, the state space might describe the positions of all objects on the table

$$\boldsymbol{s} = \begin{bmatrix} x_1 \\ y_1 \\ x_2 \\ y_2 \\ \vdots \end{bmatrix}$$

Markov Decision Processes

A is the set of actions known as the **action space**

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What capabilities does the agent have?

Markov Decision Processes

A is the set of actions known as the **action space**

What capabilities does the agent have?

For the table example, I can apply a force to a specific object on the table

$$\boldsymbol{a} = \begin{bmatrix} F_x \\ F_y \\ i \end{bmatrix}$$

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$$T\left(\underbrace{\begin{bmatrix} x_1 \\ y_1 \\ x_2 \\ y_2 \\ \vdots \end{bmatrix}}_{\text{state}}, \underbrace{\begin{bmatrix} F_x \\ F_y \\ i \end{bmatrix}}_{\text{action}}\right) = \underbrace{\Delta \begin{bmatrix} x_1 \\ y_1 \\ x_2 \\ y_2 \\ \vdots \end{bmatrix}}_{\text{next state dist.}}$$

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This is a **Markov** decision process because transition dynamics are **conditionally independent** of past states and actions

$$T(s_t, a_t \mid s_{t-1}, a_{t-1}, ..., s_0, a_0) = T(s_t, a_t)$$

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+100 for pushing objects onto the floor, or +100 for pushing objects to the centre

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You collect coins and have a time limit and score

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Mario can move and jump

Touching a goomba kills Mario

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? blocks give you mushrooms

You collect coins and have a time limit and score

Task: Define Super Mario MDP

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State Space (S)?

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- Mario position/velocity $(x, y, \dot{x}, \dot{y})$

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- Mario position/velocity $(x, y, \dot{x}, \dot{y})$
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- Number of coins collected

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- The time remaining

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- Goomba position/velocity and squished/not squished

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- Mario position/velocity $(x, y, \dot{x}, \dot{y})$
- Score
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- Which question blocks we opened
- Goomba position/velocity and squished/not squished

$$S = \{\mathbb{R}^4, \mathbb{Z}_+, \mathbb{Z}_+, \mathbb{Z}_+, \{0, 1\}^m, \mathbb{R}^{4 \times k}, \{0, 1\}^k\}$$

Markov Decision Processes

State Space (S)?

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State Space (S)? $[0, 1]^{2 \times 256 \times 240 \times 3}$

Markov Decision Processes

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$$
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Markov Decision Processes

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Markov Decision Processes

Action Space (A)?

Markov Decision Processes

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- Acceleration of Mario $\ddot{x}$

Markov Decision Processes

Action Space (A)?

- Acceleration of Mario $\ddot{x}$
  - But when playing Mario, we cannot explicitly set $\ddot{x}$

Markov Decision Processes

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  - $A = \{0, 1\}^6$
  - $$\left\{ \underbrace{\{0, 1, 2, 3, 4\}}_{\emptyset,\text{direction}} \times \underbrace{\{0, 1, 2, 3\}}_{\emptyset,\text{a,b,a+b}} \right\}$$

Markov Decision Processes

Transition Function (T)?

Markov Decision Processes

Transition Function (T)?

- $T(\text{pixel\_state}, \text{right})$

Markov Decision Processes

Transition Function (T)?

- $T(\text{pixel\_state}, \text{right})$
  - Move the Mario pixels right, unless a wall
  - Difficult to write down
  - Deterministic

Markov Decision Processes

Transition Function (T)?

Markov Decision Processes

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- $T(\text{pos\_vel\_state}, \text{right})$

Markov Decision Processes

Transition Function (T)?

- $T(\text{pos\_vel\_state}, \text{right})$
  - Changes Mario's $(x, y, \dot{x}, \dot{y})$ in game memory
  - Human understandable, easier to implement for game developers

Markov Decision Processes

Question: In Mario, a single image frame is not a Markov state. How come?

Markov Decision Processes

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Answer: Cannot measure velocity.

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Markov Decision Processes

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Answer: If we don't have it, Markov property is violated

$T(s_t, a_t)$: Mario is going up, down, left or right

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Not conditionally independent!
$T(s_t, a_t \mid s_{t-1}, a_{t-1}, \ldots, s_0, a_0) \neq T(s_t, a_t)$

Markov Decision Processes

Reward (R)?

Markov Decision Processes

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Markov Decision Processes

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- 1 for beating the level and 0 otherwise
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Markov Decision Processes

Reward (R)?

- 1 for beating the level and 0 otherwise
- Total score
- 1 for beating the level + $0.01 \cdot$ score

Markov Decision Processes

- S✓
- A✓
- T✓
- R✓
- γ?

Markov Decision Processes

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Note that we care about all future rewards, not just the current reward!

Markov Decision Processes

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Markov Decision Processes

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Experiment: one cookie now, or two cookies in a year?

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We almost always choose to maximize the <u>discounted</u> return

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Exercise: Reinforcement learning also describes human and animal behaviors. How can you describe your behavior using reinforcement learning?

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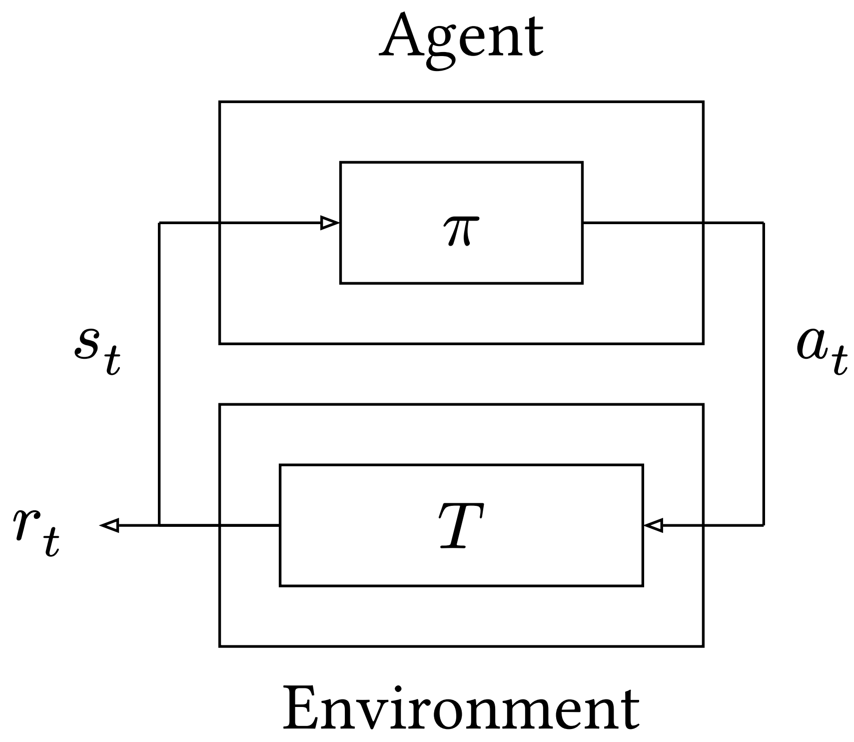
This happens internally when I decide to go to the pub after work

Agents and Policies

Agents and Policies



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- We have defined the environment

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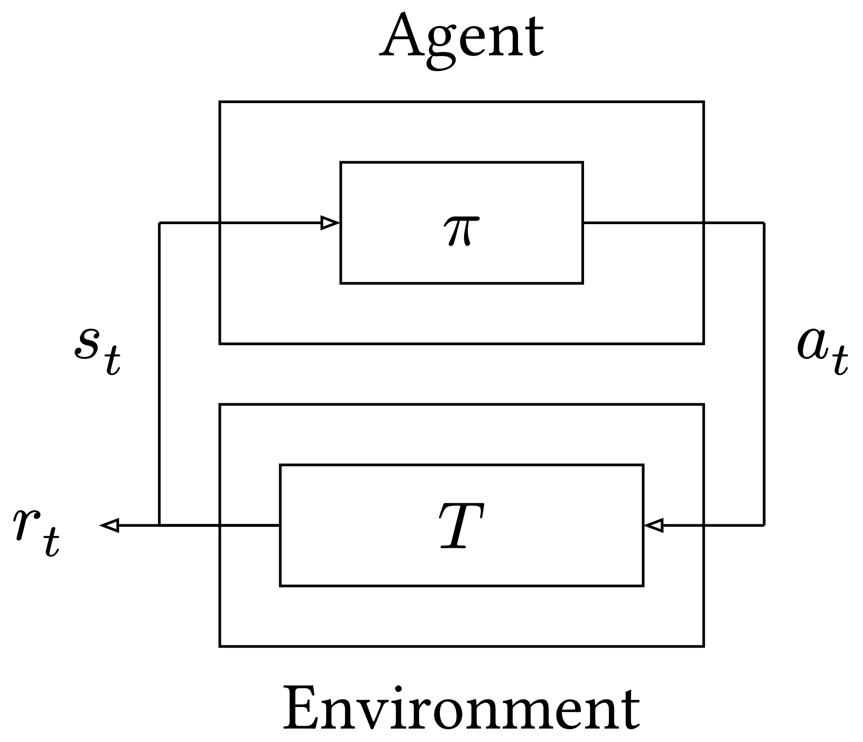

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- We have defined the environment
- Now let us define the agent

Agents and Policies

The agent acts following a **policy** π.

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$$\pi(a_t \mid s_t)$$

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Now, our policy is truly optimal

Q Learning

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We use **algorithms** to search for the optimal policy π_*

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1. Derive the value function V
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Recall the discounted return for a specific policy π

$$G_\pi = \mathbb{E}\left[\sum_{t=0}^{\infty} \gamma^t r_t \mid a_t \sim \pi(s_t)\right]$$

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and get a reward $r_t = R(s_{t+1})$

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That is not a good answer

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Measures the value of a state (how good is it to be in this state?), for a given policy π

We call this the **Value Function (V_π)** $\quad V_\pi : S \to \mathbb{R}$

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The Plan:

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When V depends on a specific action, we call it the **Q function**:

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a_0 affects your next state s_1, which affects the future

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Example: You have summer school offers from Cambridge and Oxford

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Q function gives you a number denoting how much better your life will be for attending Cambridge (based on your behavior π). Takes into account reward (based on income, friend group, experiences, etc).

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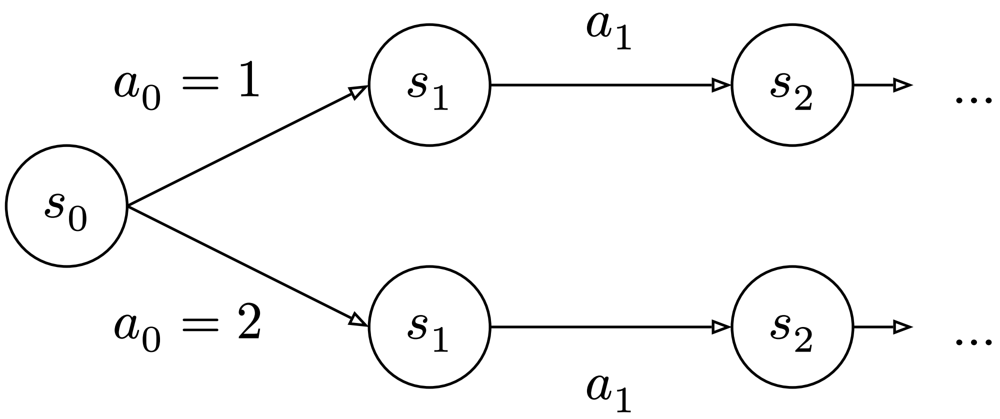

Q Learning

The Plan:

1. Derive the value function V
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3. **Derive an optimal policy from Q**
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Write this more formally

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We call this the **greedy policy**

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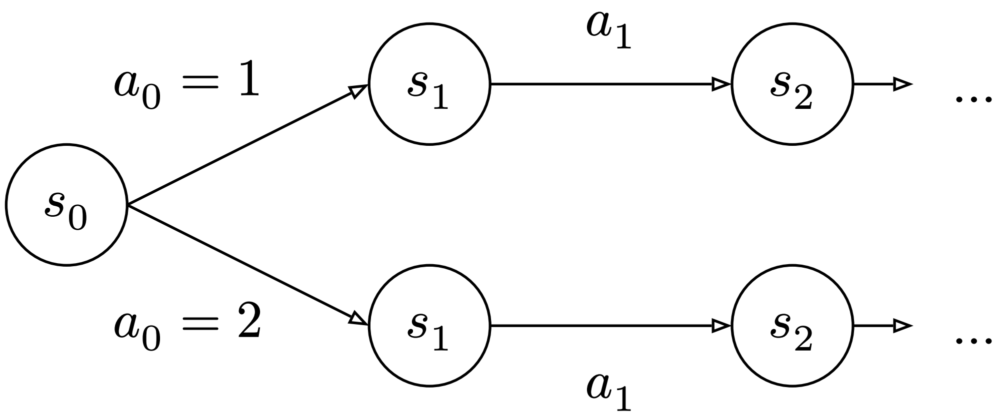

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Can we get rid of the infinite sum?

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Factor out γ

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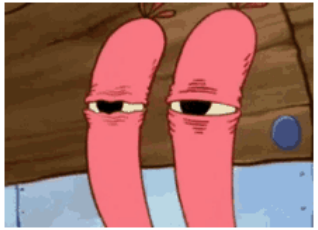

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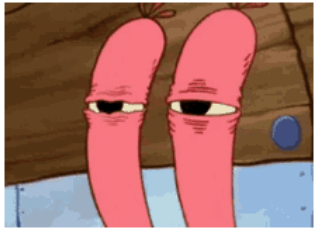

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The policy π_* takes the argmax over Q, which reduces to

$$Q_*(s_0, a_0) = \mathbb{E}[r_0 \mid a_0] + \gamma \cdot \max_{\{a \in A\}} Q_*(s_1, a)$$

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All we need is:

$$(s, a, r, \gamma, s')$$

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We can parameterize Q with parameters θ and try to approximate Q_*

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