Convolution

CISC 7026: Introduction to Deep Learning

University of Macau

Agenda

1. Review
2. Signal Processing
3. Convolution
4. Convolutional Neural Networks
5. Additional Dimensions
6. Coding

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Review

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To do so, we must think of the world as a collection of signals

Signal Processing

A **signal** represents information as a function of time, space or some other variable

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$$x(u, v) = \ldots$$

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Signal processing is a field of research that focuses on analyzing the meaning of signals

Knowing the meaning of signals is very useful

Signal Processing

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$$x(t) = \text{stock price}$$

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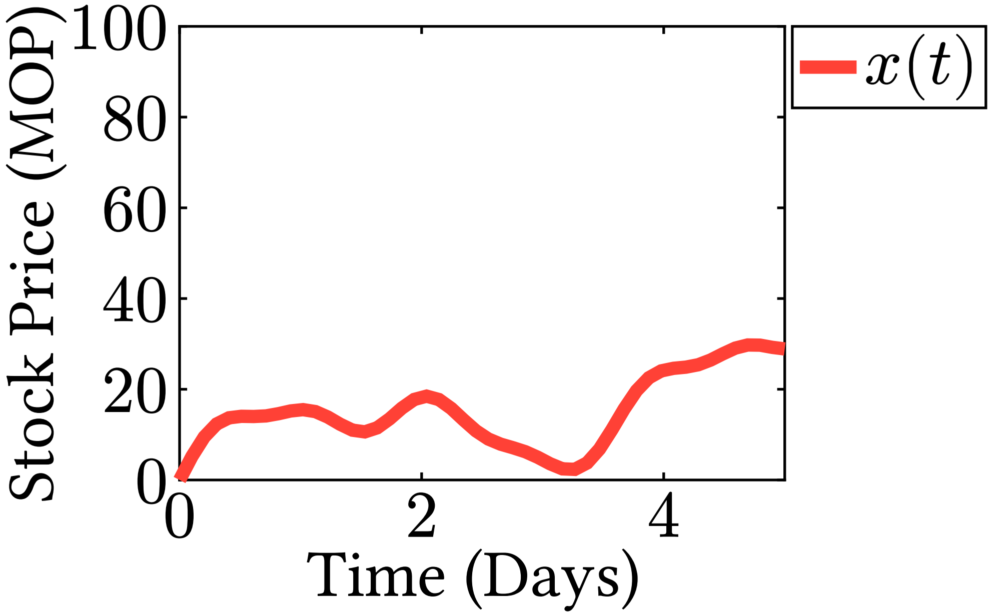

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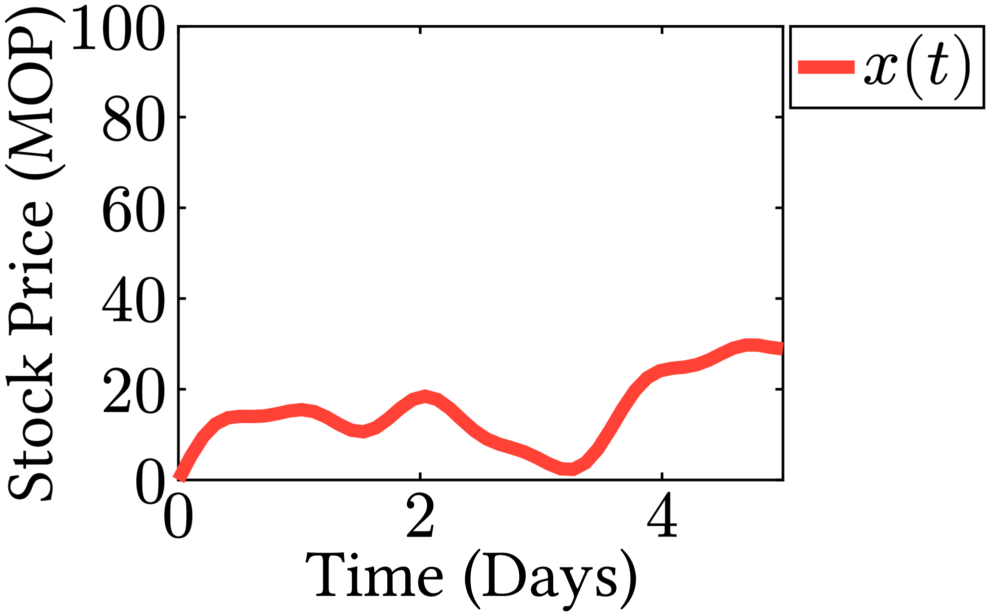

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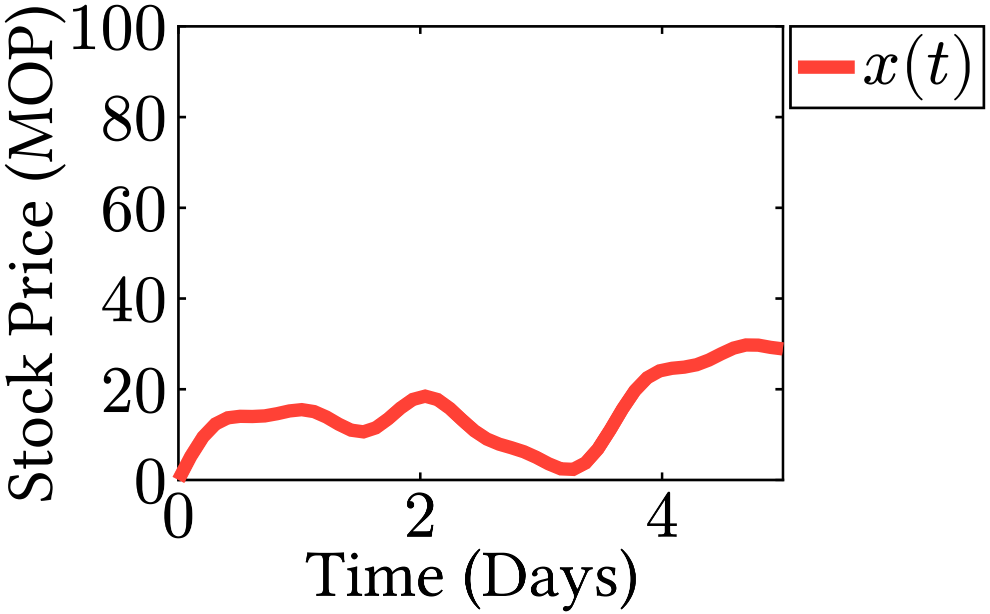

There is an underlying structure that we do not fully understand

Structure: Tomorrow's stock price will be close to today's stock price

Signal Processing

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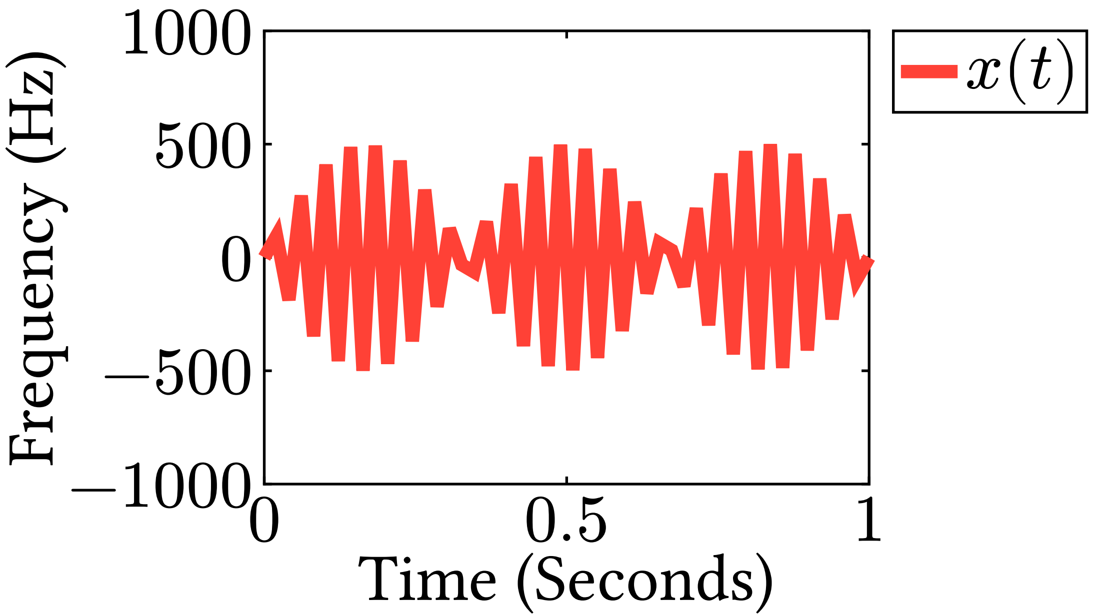

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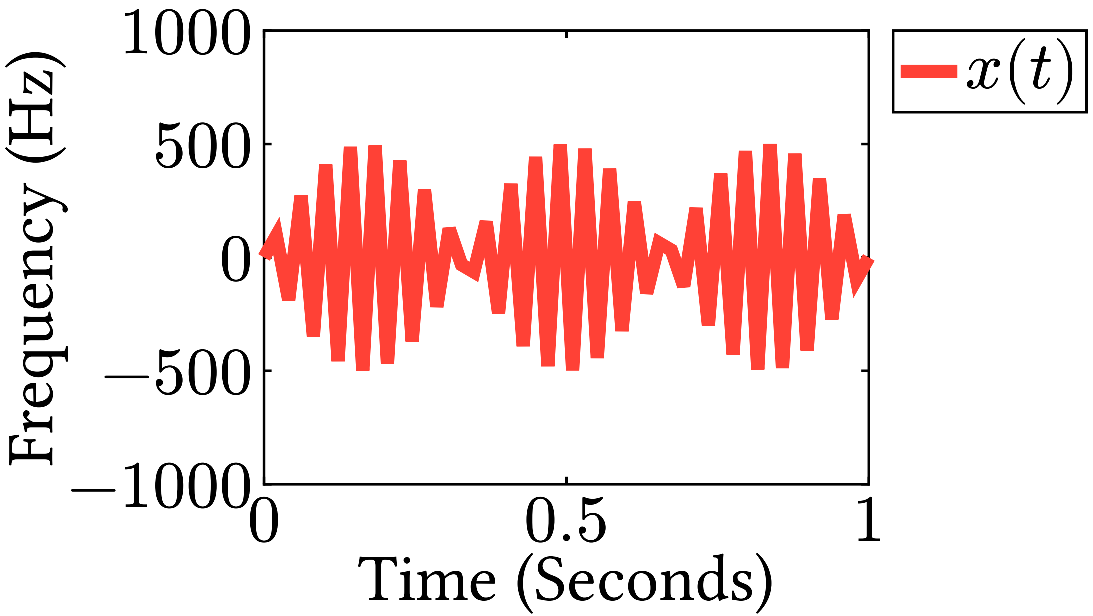

Structure: Nearby waves form syllables

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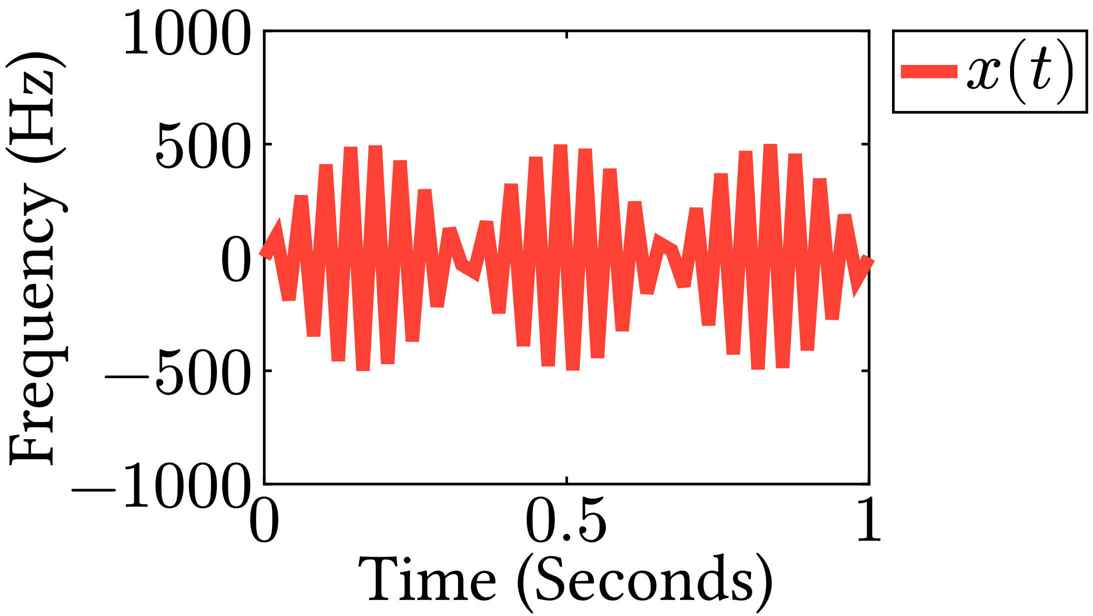

Structure: Nearby waves form syllables

Structure: Nearby syllables combine to create meaning

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$$x(u, v) = \text{image}$$

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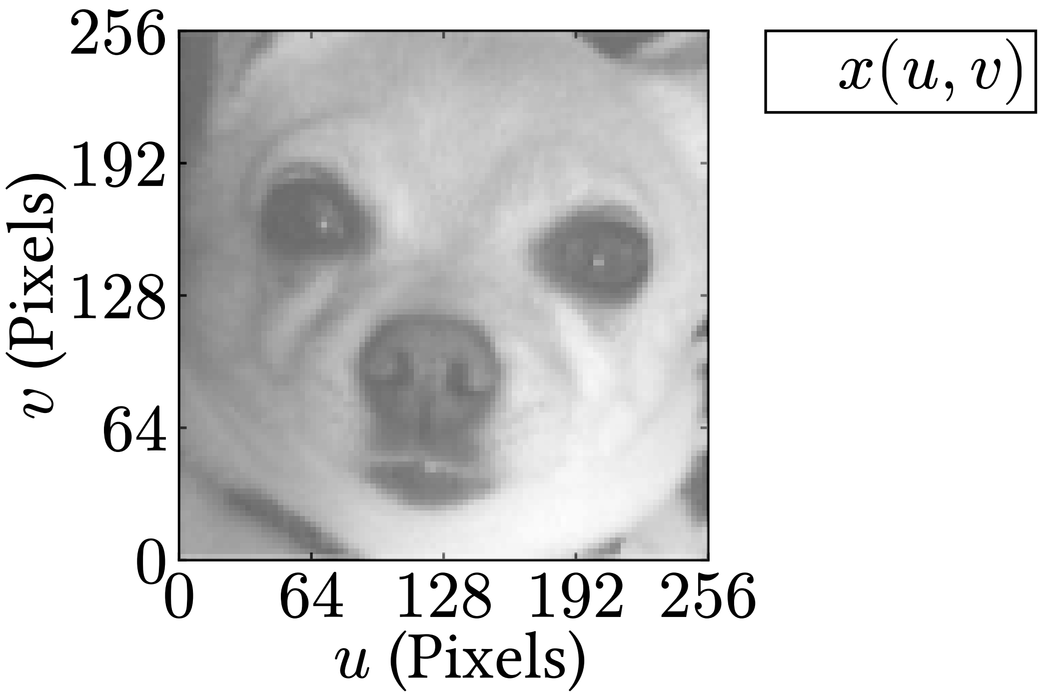

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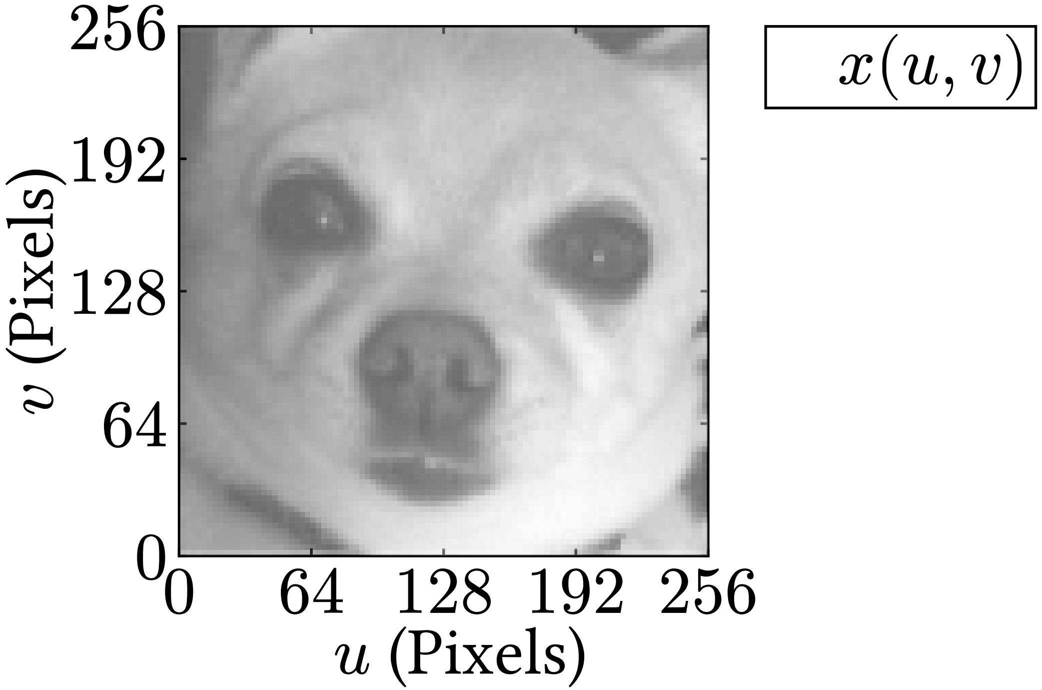

Structure: Repeated components (circles, symmetry, eyes, nostrils, etc)

Signal Processing

In signal processing, we often consider:

Signal Processing

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- Locality

Signal Processing

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- Translation equivariance

Signal Processing

Locality: Information concentrated over small regions of space/time

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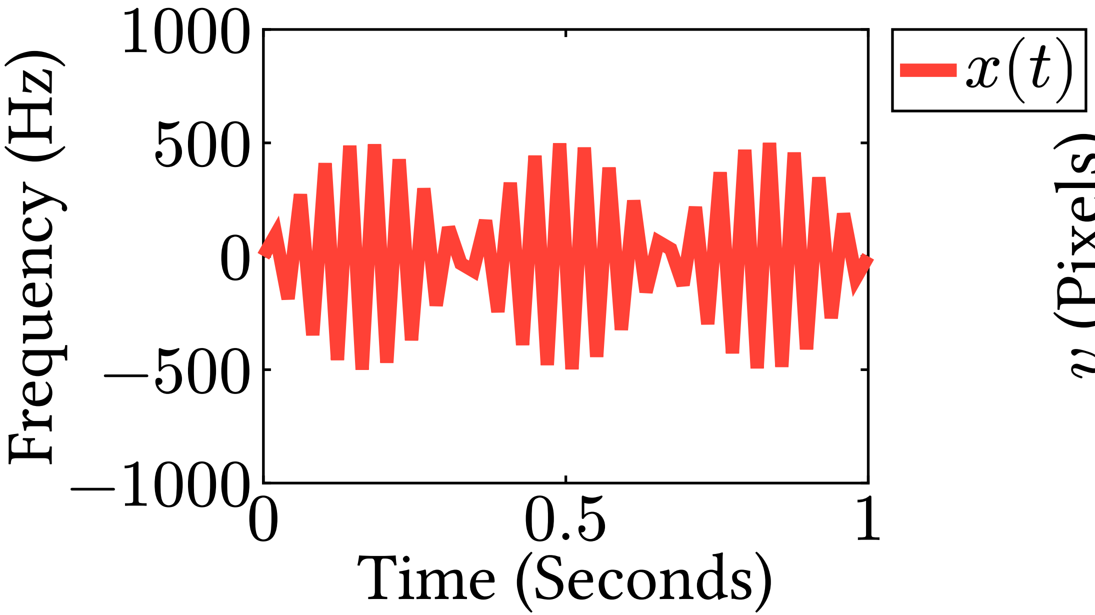

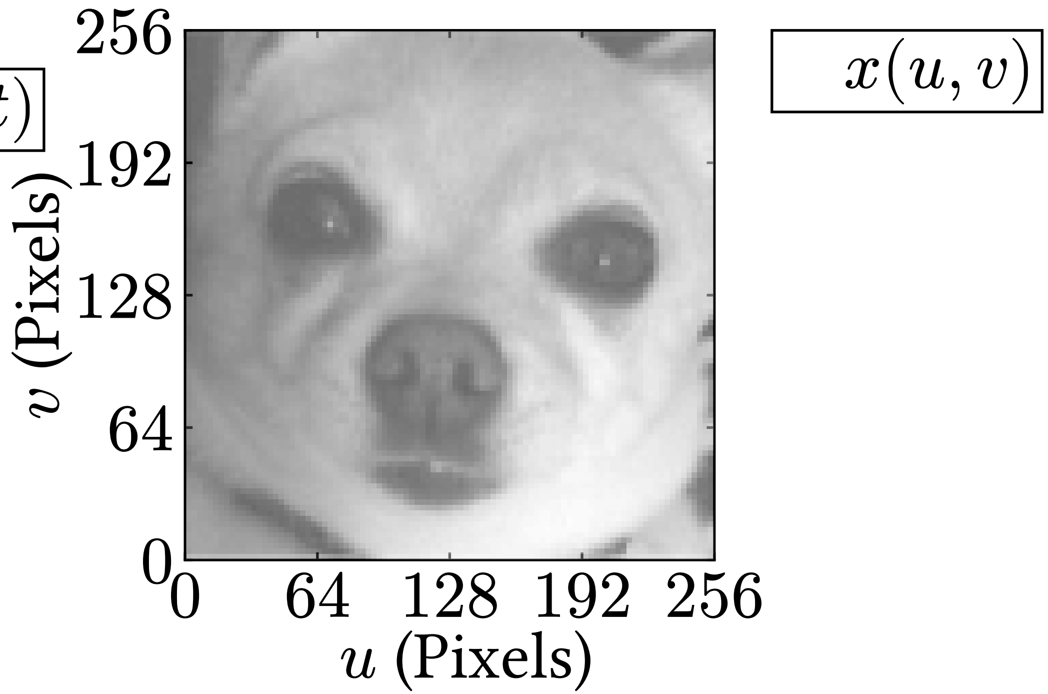

Signal Processing

Translation Equivariance: Shift in signal results in shift in output

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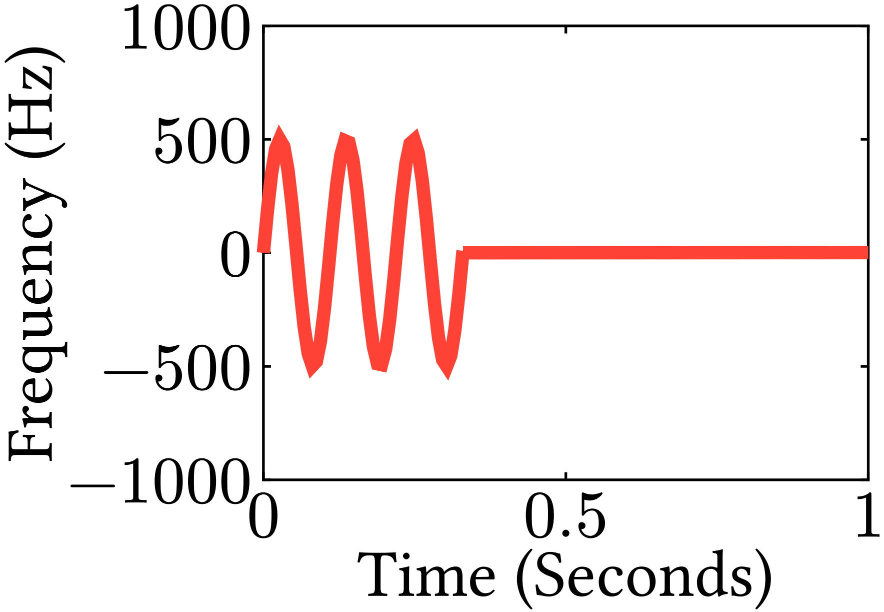

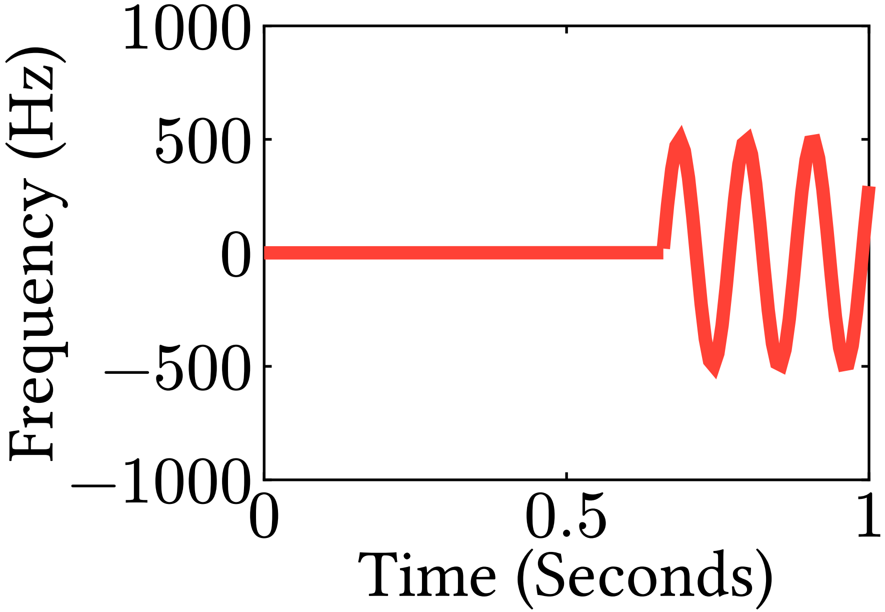

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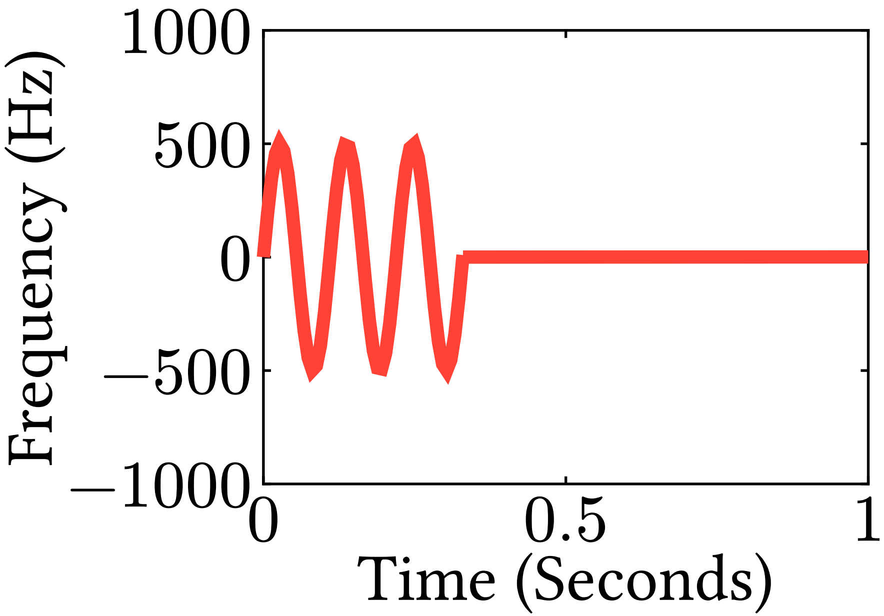

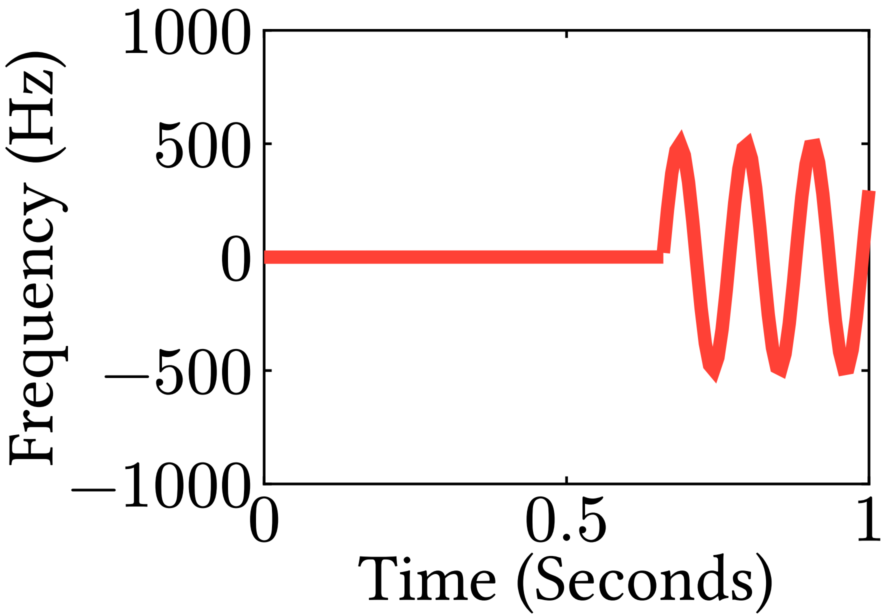

Both say “hello”

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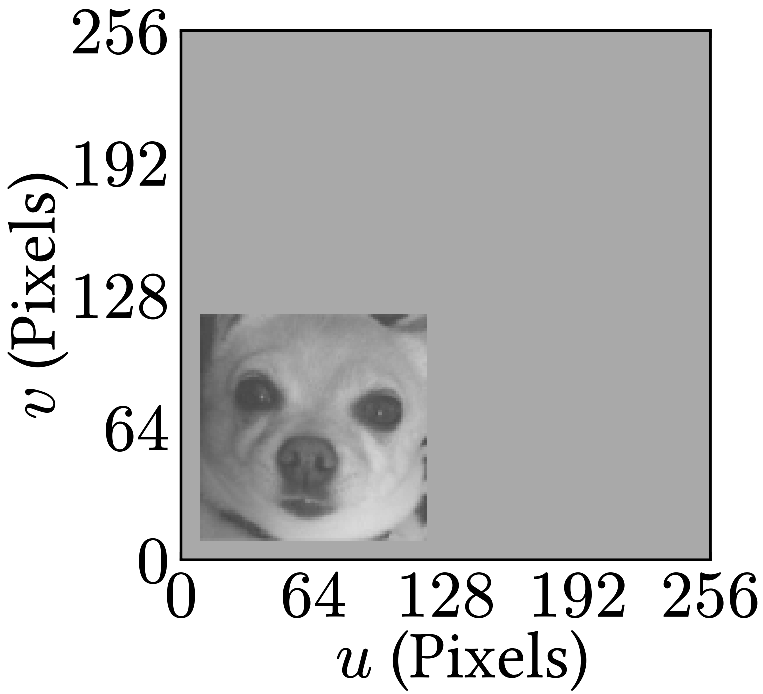

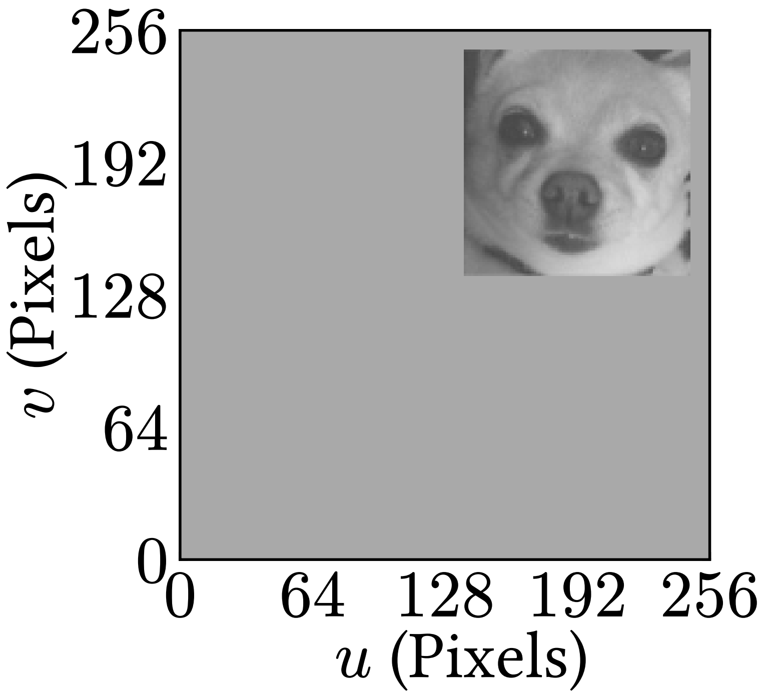

Both contain a dog

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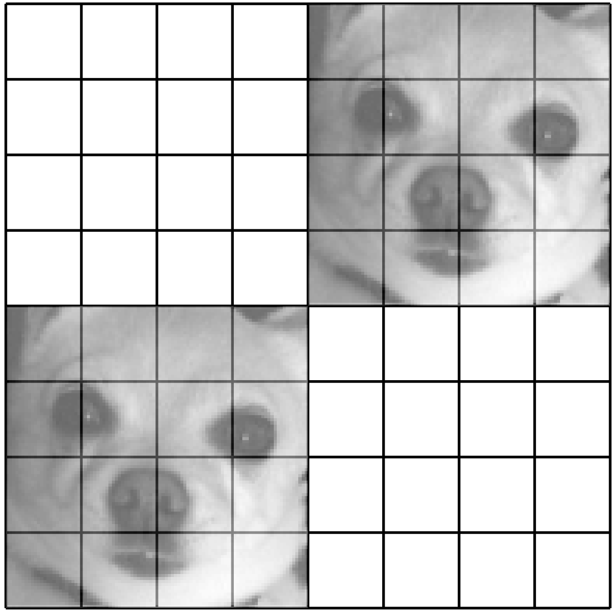

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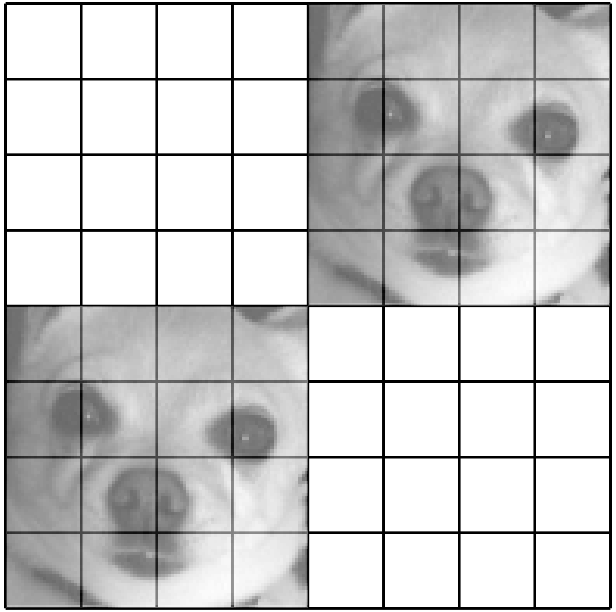

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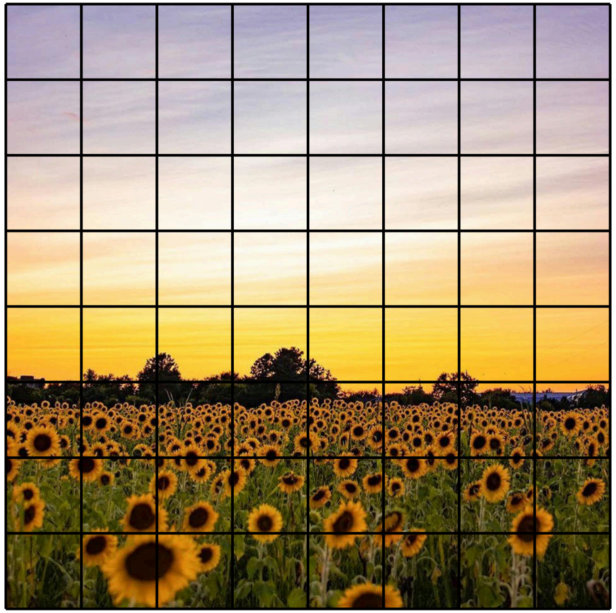

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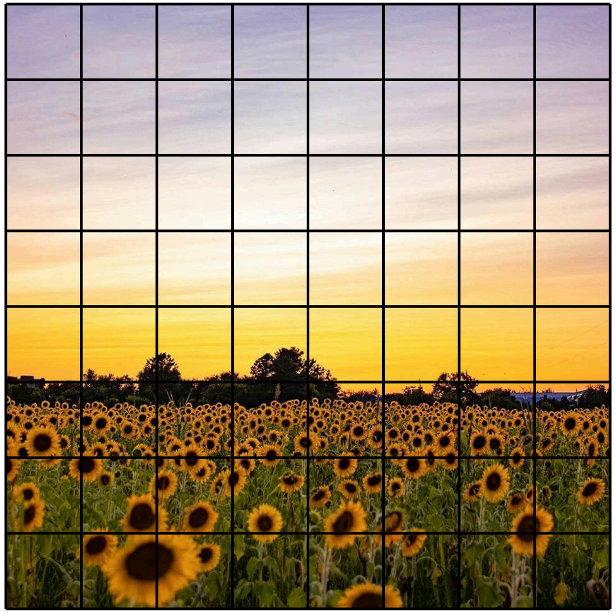

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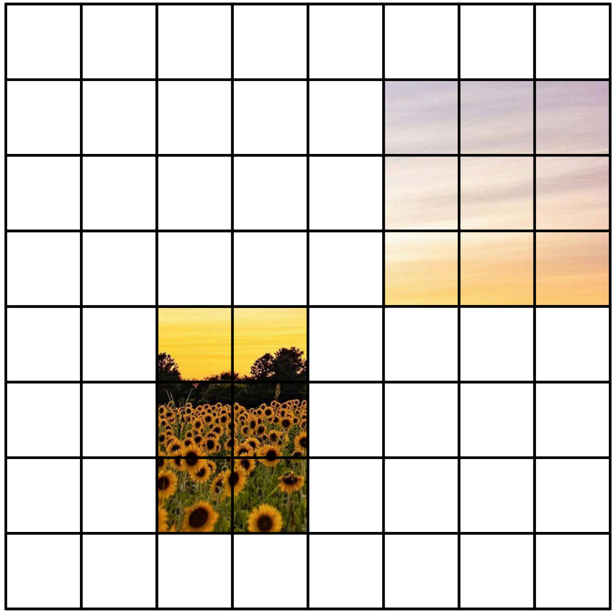

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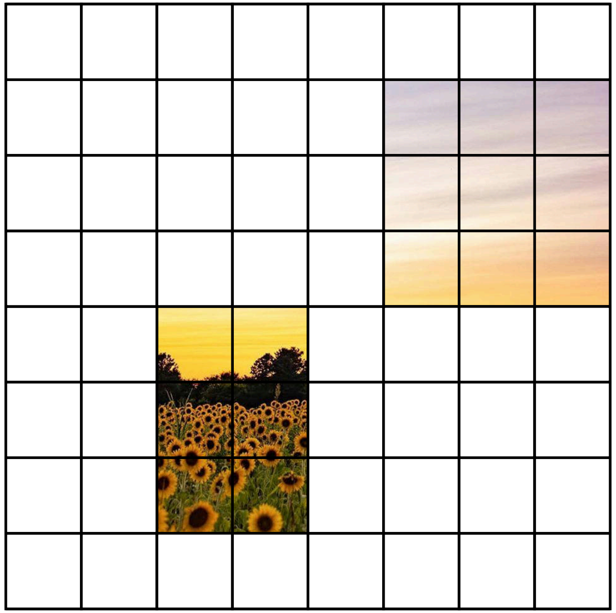

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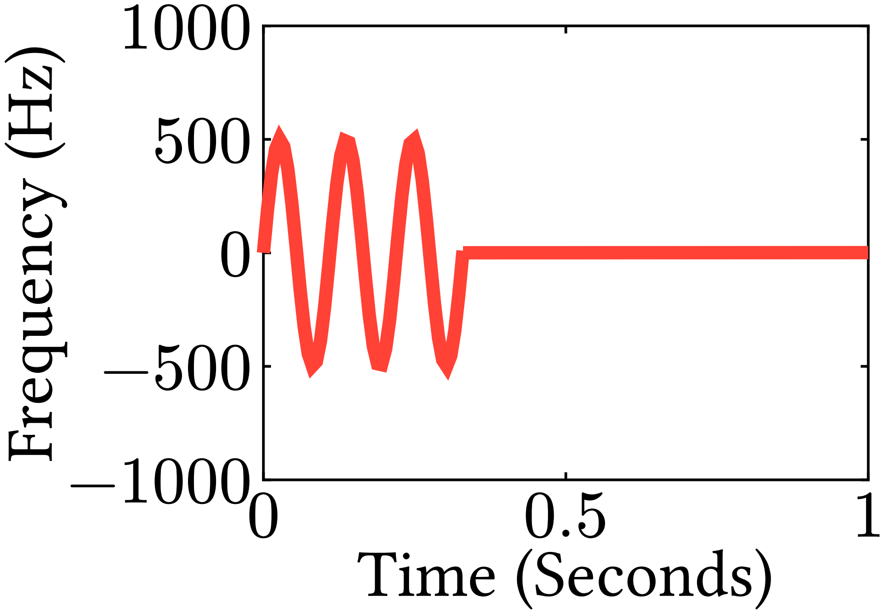

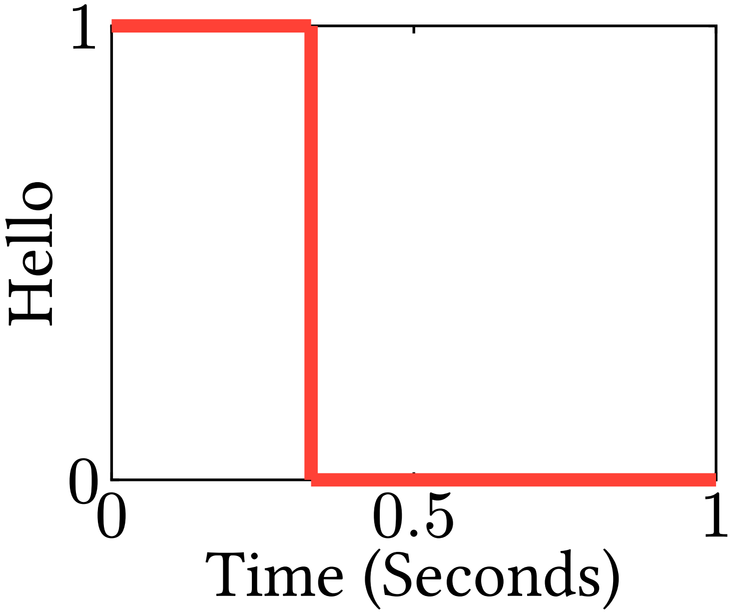

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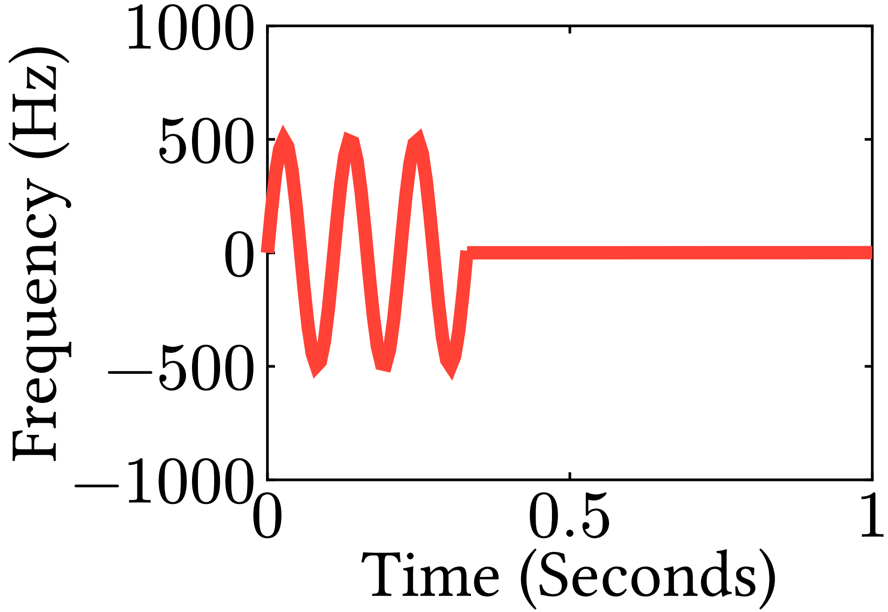

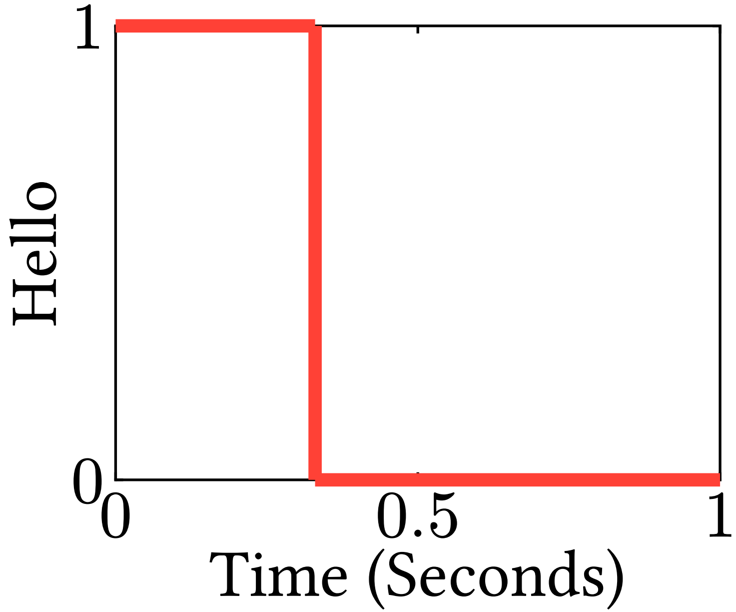

A standard way to transform signals is **convolution**

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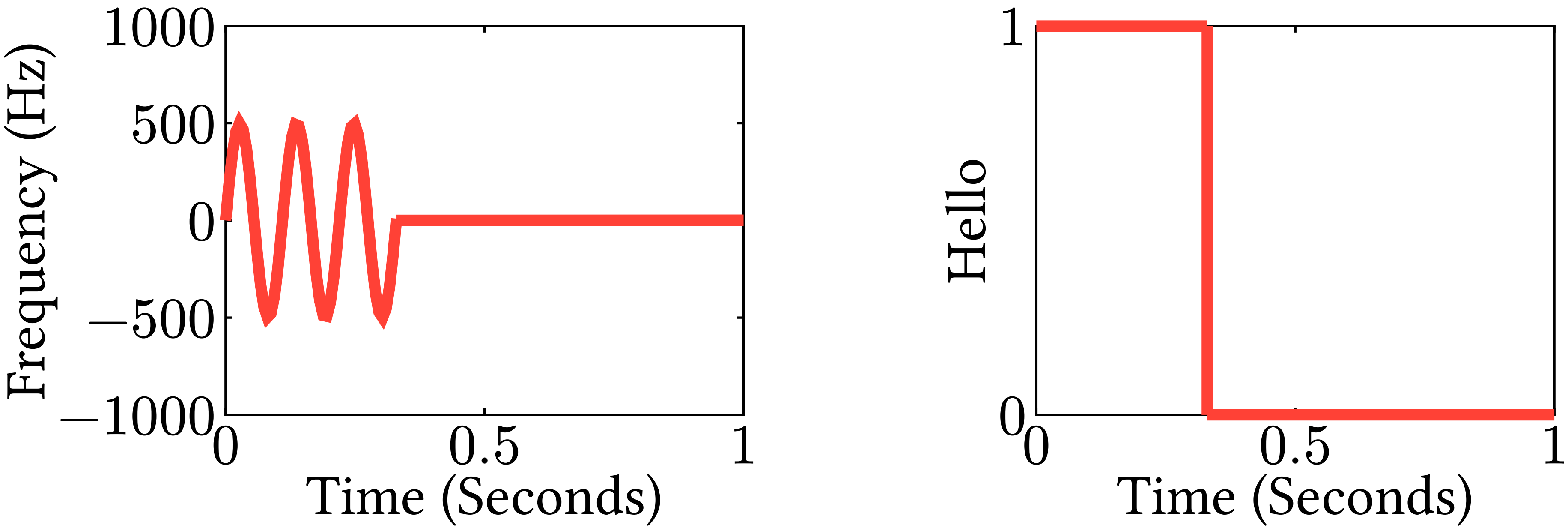

A standard way to transform signals is **convolution**

Convolution is translation equivariant and can be local

Convolution

Convolution is the sum of products of a signal $x(t)$ and a **filter** $g(t)$

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If the t is continuous in $x(t)$

$$x(t) * g(t) = \int_{-\infty}^{\infty} x(t - \tau) g(\tau) d\tau$$

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If the t is discrete in $x(t)$

$$x(t) * g(t) = \sum_{\tau = -\infty}^{\infty} x(t - \tau) g(\tau)$$

We slide the filter $g(t)$ across the signal $x(t)$

Convolution

Example: Let us examine a low-pass filter

Convolution

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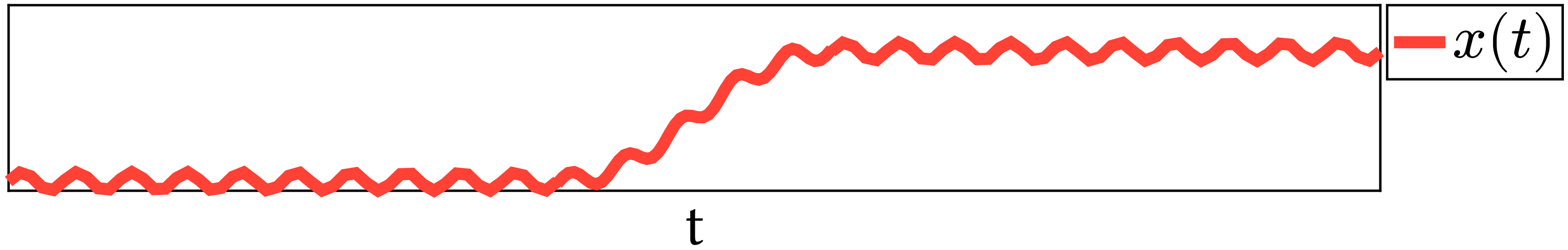
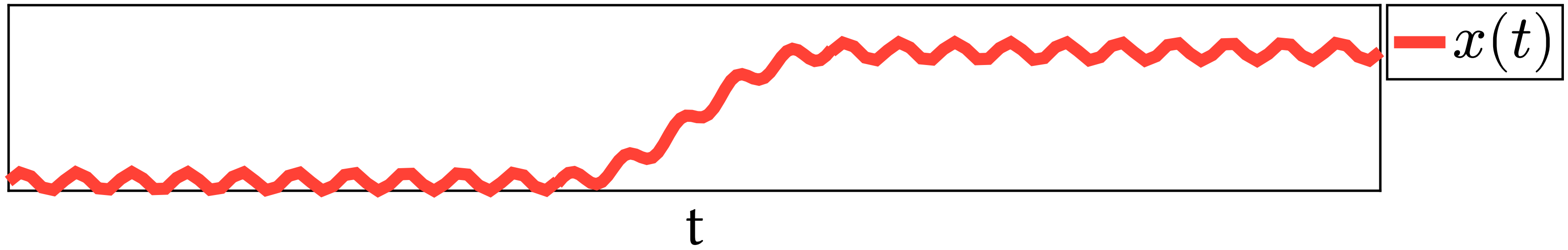
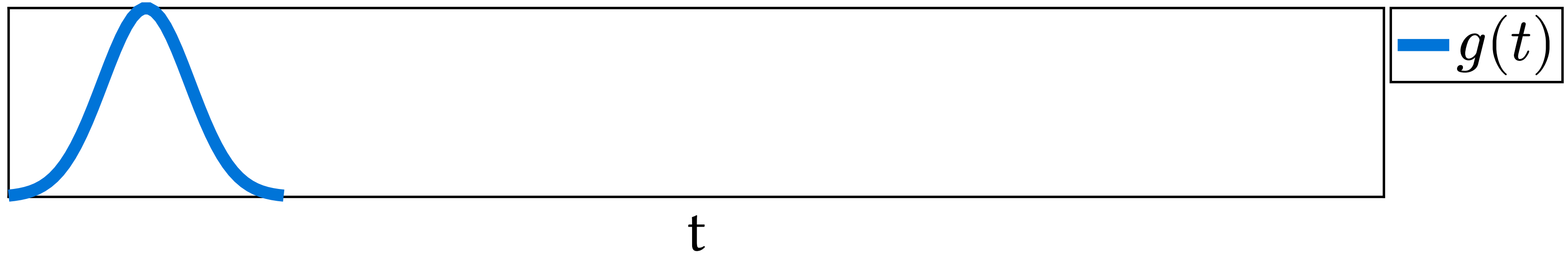
The filter will take a signal and remove noise, producing a cleaner signal

Convolution



Convolution





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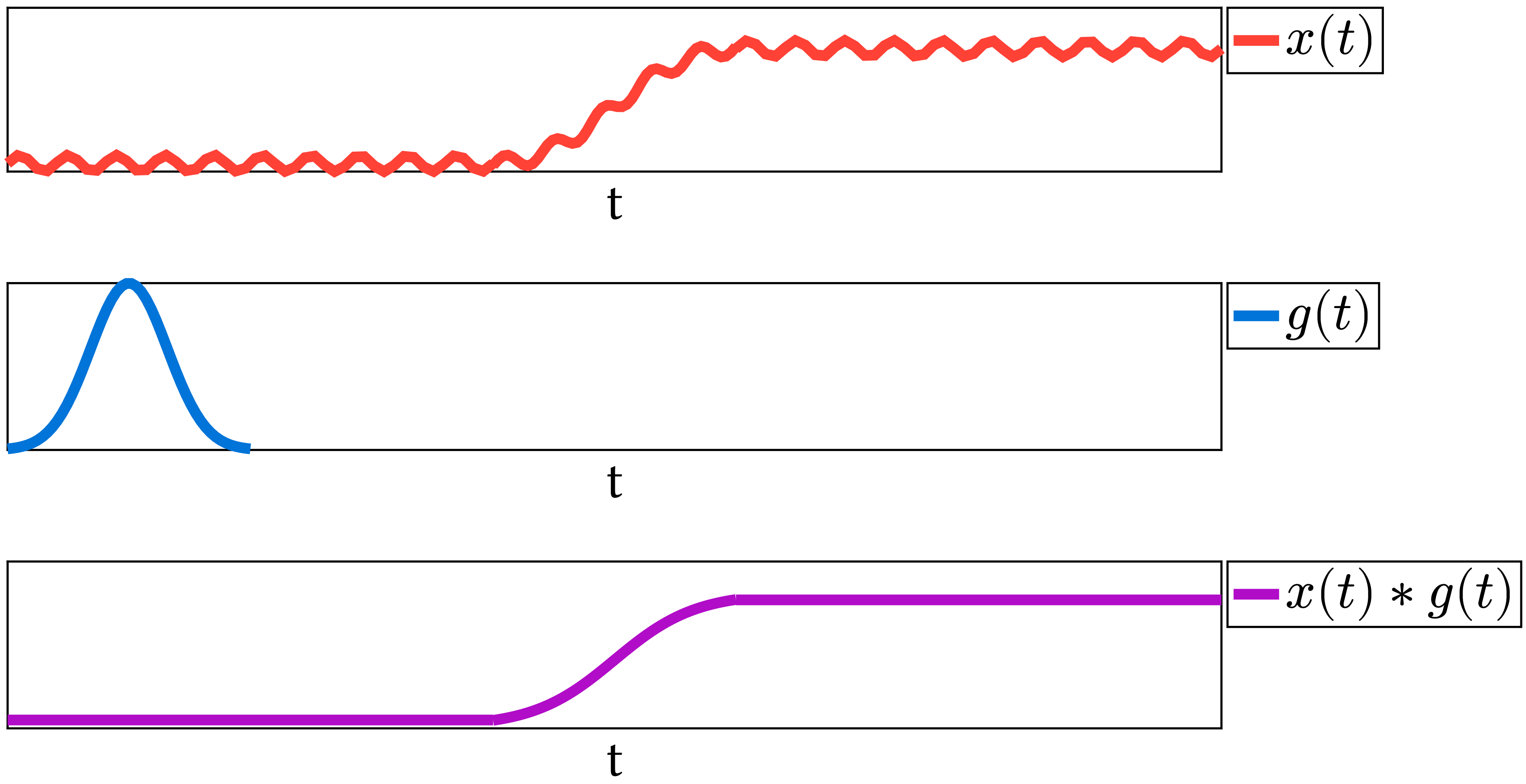

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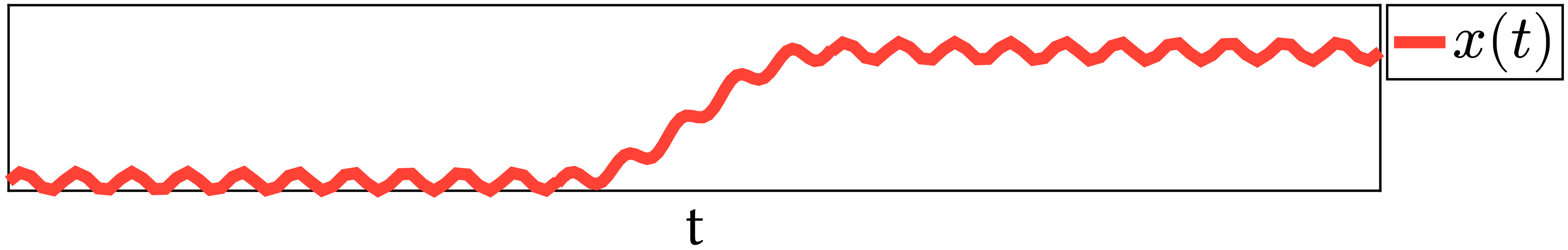
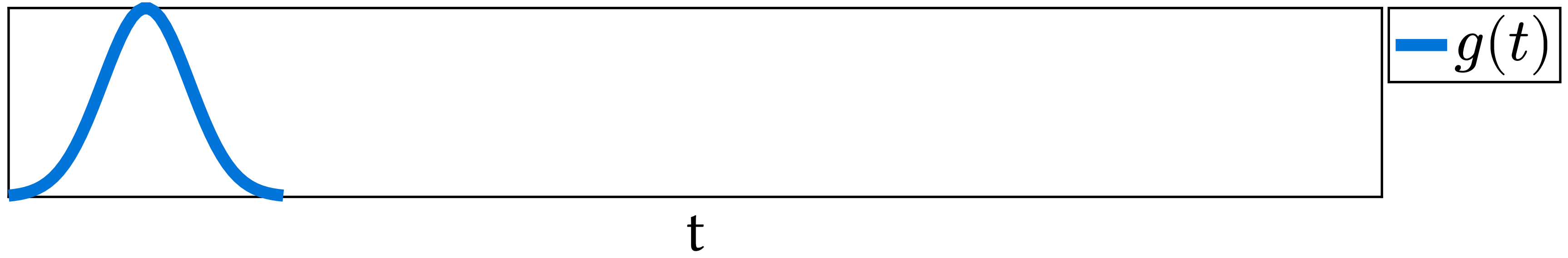
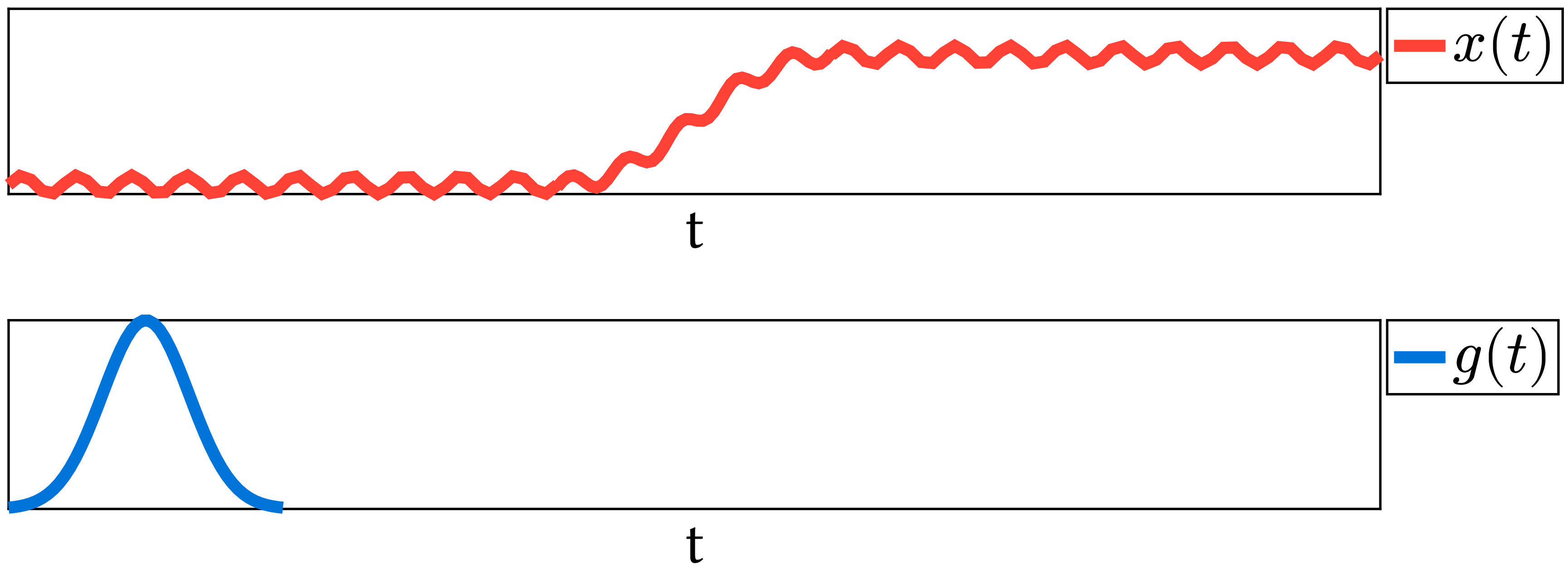




Convolution is **local** to the filter $g(t)$

Convolution



Convolution is **local** to the filter $g(t)$

Convolution is also **equivariant** to time/space shifts

Convolution

Often, we use continuous time/space convolution for analog signals

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- Anything stored as bits instead of a function

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- Images
- Quantized audio
- Anything stored as bits instead of a function

But it is good to know both! Continuous variables for theory. Discrete variables for software

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$$\begin{bmatrix} g(t) \\ x(t) \\ y(t) \end{bmatrix} = \begin{bmatrix} 2 & 1 & & & \\ 1 & 2 & 3 & 4 & 5 \\ & & & & \end{bmatrix}$$

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$$\begin{bmatrix} g(t) \\ x(t) \\ y(t) \end{bmatrix} = \begin{bmatrix} & \color{red}{2} & \color{red}{1} & & \\ 1 & \color{red}{2} & \color{red}{3} & 4 & 5 \\ 4 & \color{red}{5} & & & \end{bmatrix}$$

Convolution

$$\begin{bmatrix} g(t) \\ x(t) \\ y(t) \end{bmatrix} = \begin{bmatrix} & & 2 & 1 & \\ 1 & 2 & 3 & 4 & 5 \\ 4 & 5 & 10 & & \end{bmatrix}$$

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Question: Does anybody see a connection to neural networks?

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Hint: What if I rewrite the filter?

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$$\begin{bmatrix} g(t) \\ x(t) \\ y(t) \end{bmatrix} = \begin{bmatrix} \theta_2 & \theta_1 & & & \\ 1 & 2 & 3 & 4 & 5 \\ 4 & 5 & 10 & 13 & \end{bmatrix}$$

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Just like neural networks, convolution is a linear operation

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It is a weighted sum of the inputs, just like a neuron

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Question: How does convolution differ from a neuron?

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Question: How does convolution differ from a neuron?

Answer: In a neuron, each input x_i has a different parameter θ_i. In convolution, we reuse (slide) θ_i over $x_1, x_2, ...$

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Efficiently expands neural networks to images, videos, sounds, etc

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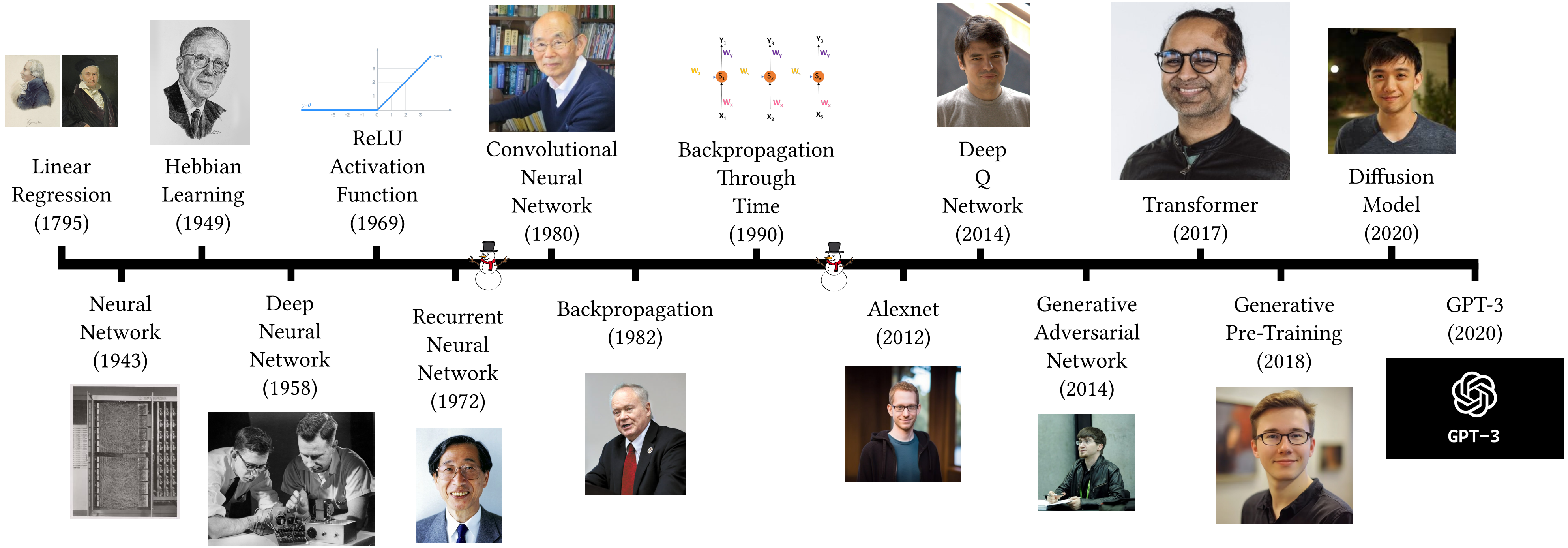
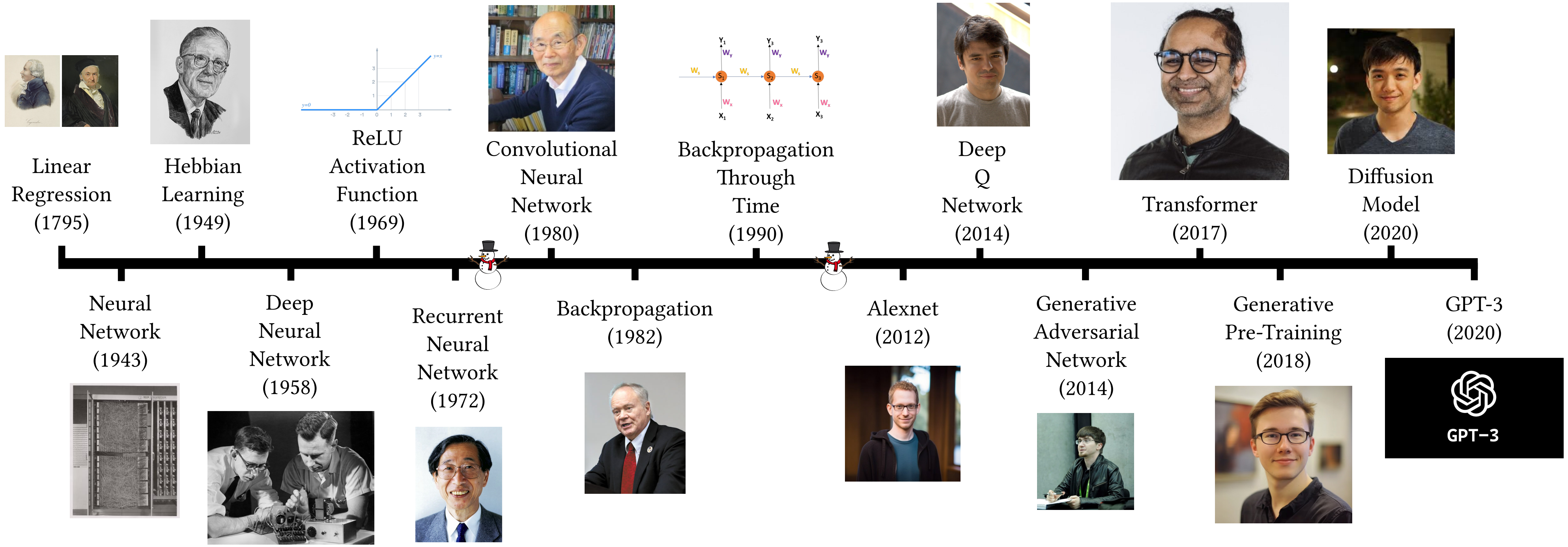


Convolutional Neural Networks

CNNs have been around since the 1970's



2012: GPU and CNN efficiency resulted in breakthroughs

Convolutional Neural Networks

So how does a convolutional neural network work?

Convolutional Neural Networks

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Like before, we will start with linear functions and derive a convolutional layer

Convolutional Neural Networks

Recall the neuron

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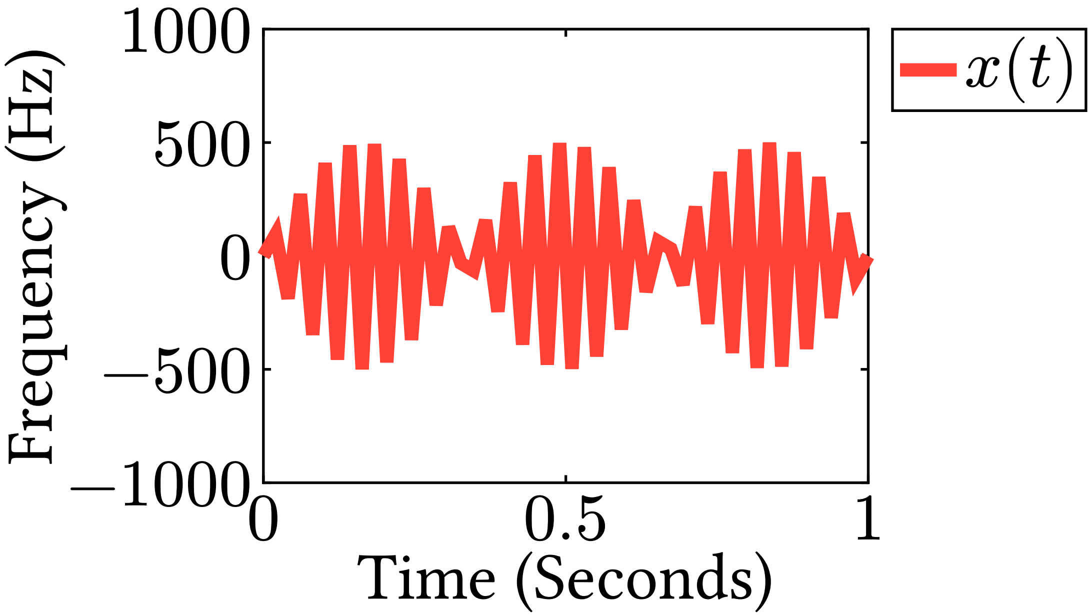

Convolutional Neural Networks

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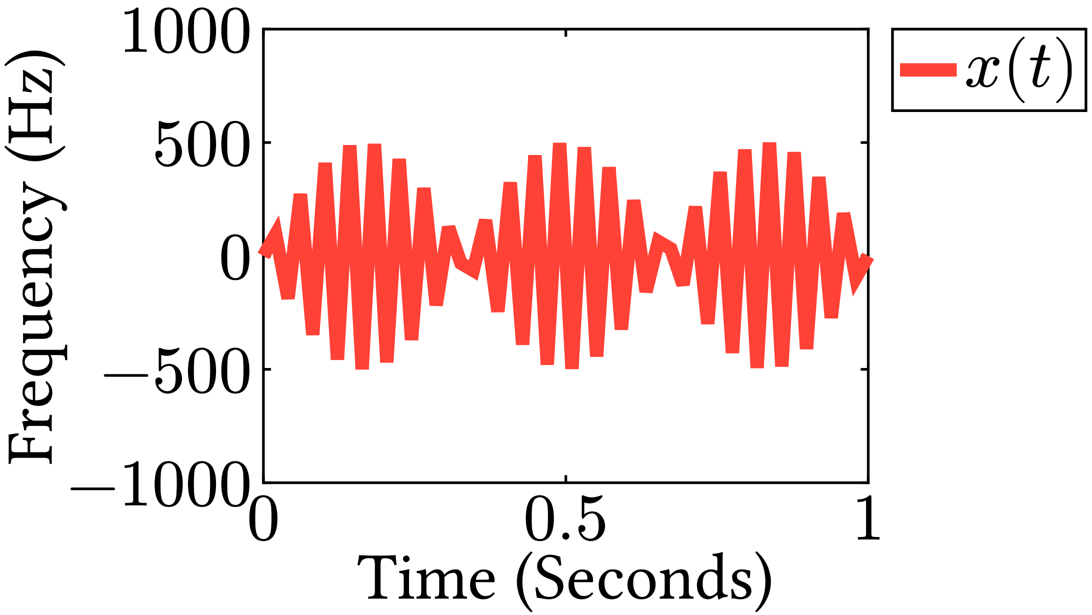
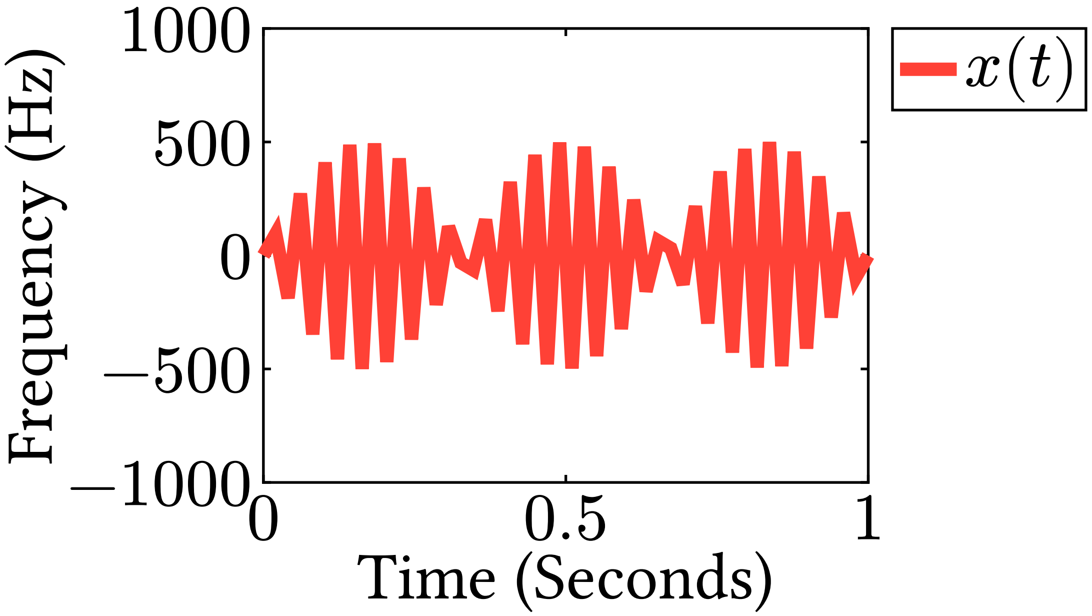


$$f\left(\begin{bmatrix} x(0.1) \\ x(0.2) \\ \vdots \end{bmatrix}, \boldsymbol{\theta}\right) =$$

$$\sigma(\theta_0 + \theta_1 x(0.1) + \theta_2 x(0.2) + \ldots)$$

Convolutional Neural Networks



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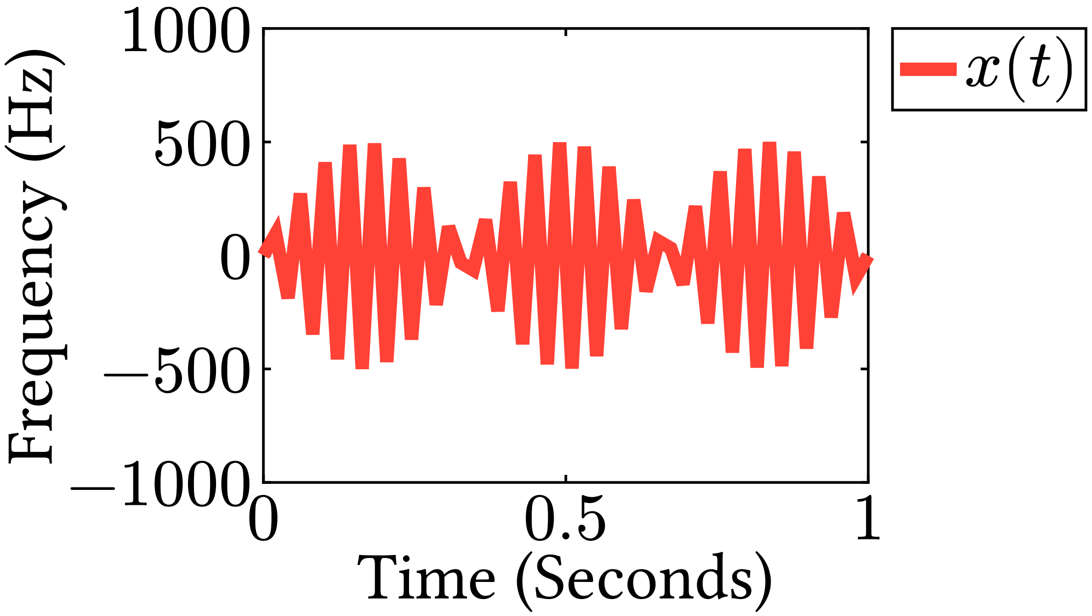

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Question: Any problems besides locality/equivariance?

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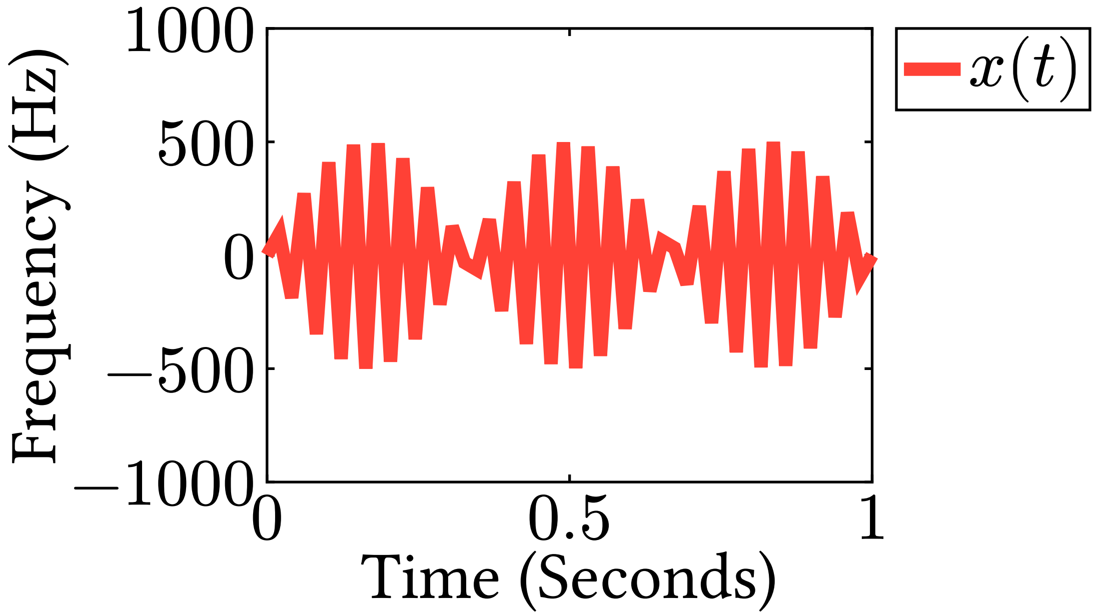

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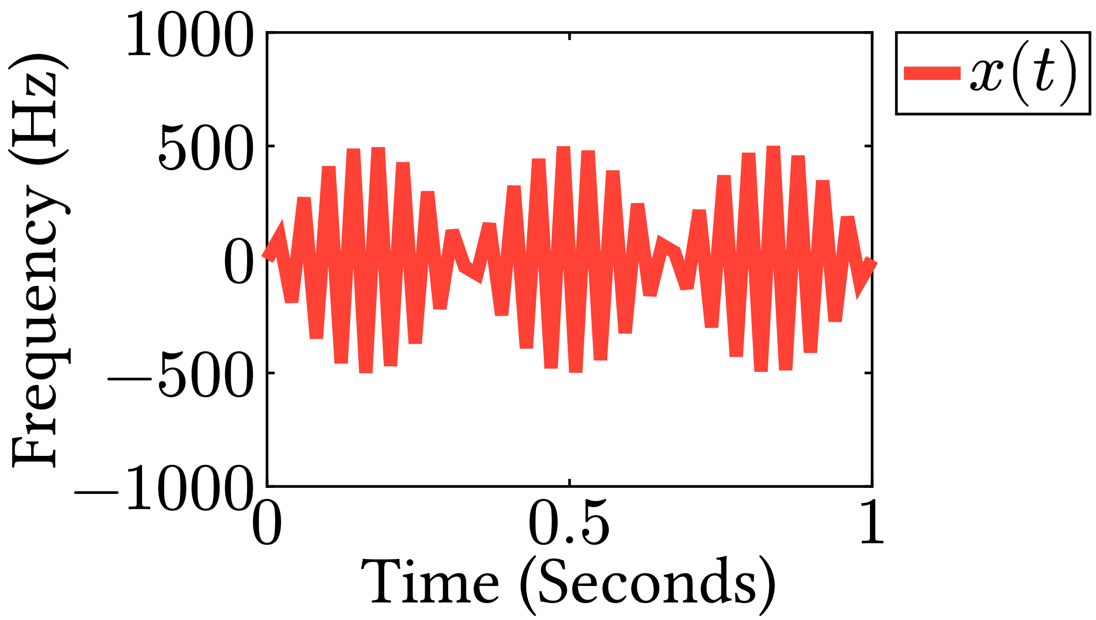
Convolutional Neural Networks



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Answer 1: Parameters scale with sequence length

Answer 2: Parameters only for exactly 1 second waveforms

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To fix problems, each timestep cannot use different parameters

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$$f\left(\begin{bmatrix} x(0.1) \\ x(0.2) \\ \vdots \end{bmatrix}, \begin{bmatrix} \theta_0 \\ \theta_1 \\ \vdots \end{bmatrix} \right)$$

$$= \sigma(\theta_0 + \theta_1 x(0.1) + \theta_2 x(0.2) + \theta_3 x(0.3) + \theta_4 x(0.4) + \theta_5 x(0.5) + ...)$$

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$$f\left(\begin{bmatrix} x(0.1) \\ x(0.2) \\ \vdots \end{bmatrix}, \begin{bmatrix} \theta_0 \\ \theta_1 \\ \theta_2 \end{bmatrix} \right) = \begin{bmatrix} \sigma(\theta_0 + \theta_1 x(0.1) + \theta_2 x(0.2)) \\ \sigma(\theta_0 + \theta_1 x(0.2) + \theta_2 x(0.3)) \\ \sigma(\theta_0 + \theta_1 x(0.3) + \theta_2 x(0.4)) \\ \vdots \end{bmatrix}$$

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We can write both the neuron and convolution in vector form

$$f(x(t), \boldsymbol{\theta}) = \sigma\left(\boldsymbol{\theta}^\top \begin{bmatrix} 1 \\ x(0.1) \\ x(0.2) \\ \vdots \end{bmatrix} \right)$$

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$$f(x(t), \boldsymbol{\theta}) = \sigma\left(\boldsymbol{\theta}^\top \begin{bmatrix} 1 \\ x(0.1) \\ x(0.2) \\ \vdots \end{bmatrix}\right) \qquad f(x(t), \boldsymbol{\theta}) = \begin{bmatrix} \sigma\left(\boldsymbol{\theta}^\top \begin{bmatrix} 1 \\ x(0.1) \\ x(0.2) \end{bmatrix}\right) \\ \sigma\left(\boldsymbol{\theta}^\top \begin{bmatrix} 1 \\ x(0.2) \\ x(0.3) \end{bmatrix}\right) \\ \vdots \end{bmatrix}$$

A convolution layer applies a "mini" perceptron to every few timesteps

Convolutional Neural Networks

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Question: What is the shape of the results?

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Answer 2: $T - k$, where T is sequence length and k filter length

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$$\text{SumPool}(z(t)) = \sigma\left(\boldsymbol{\theta}^\top \begin{bmatrix} 1 \\ x(0.1) \\ x(0.2) \end{bmatrix} \right) + \sigma\left(\boldsymbol{\theta}^\top \begin{bmatrix} 1 \\ x(0.2) \\ x(0.3) \end{bmatrix} \right) + \dots$$

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$$z(t) = f(x(t), \boldsymbol{\theta}) = \left[\sigma\left(\boldsymbol{\theta}^\top \begin{bmatrix} 1 \\ x(0.1) \\ x(0.2) \end{bmatrix} \right) \ \sigma\left(\boldsymbol{\theta}^\top \begin{bmatrix} 1 \\ x(0.2) \\ x(0.3) \end{bmatrix} \right) \ \ldots \right]^\top$$

$$\mathrm{SumPool}(z(t)) = \sigma\left(\boldsymbol{\theta}^\top \begin{bmatrix} 1 \\ x(0.1) \\ x(0.2) \end{bmatrix} \right) + \sigma\left(\boldsymbol{\theta}^\top \begin{bmatrix} 1 \\ x(0.2) \\ x(0.3) \end{bmatrix} \right) + \ldots$$

$$\mathrm{MeanPool}(z(t)) = \frac{1}{T} \, \mathrm{SumPool}(z(t))$$

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Things will get more complicated, but the core idea is exactly the same

Agenda

1. Review
2. Signal Processing
3. Convolution
4. **Convolutional Neural Networks**
5. Additional Dimensions
6. Coding

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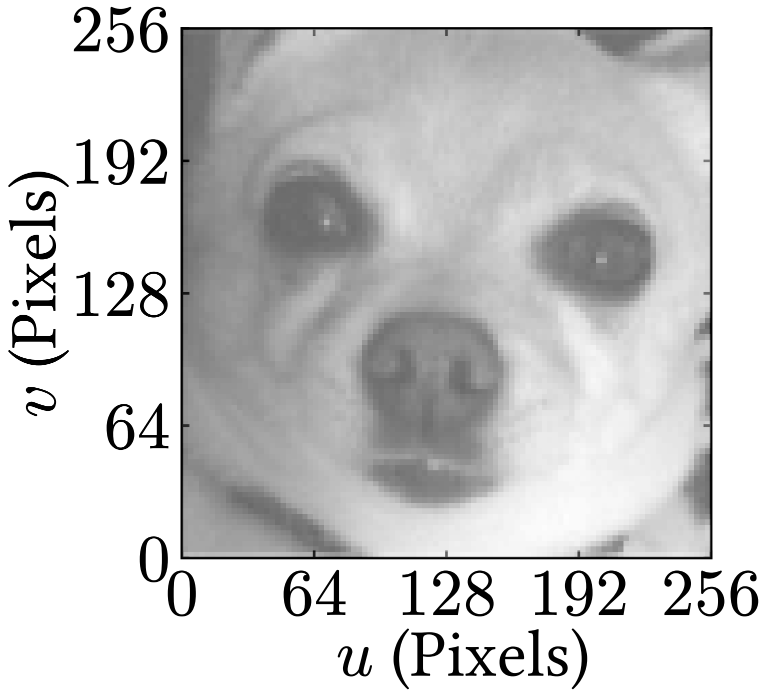
Additional Dimensions



$$\boxed{x(u, v)}$$

Question: How many input dimensions for x?

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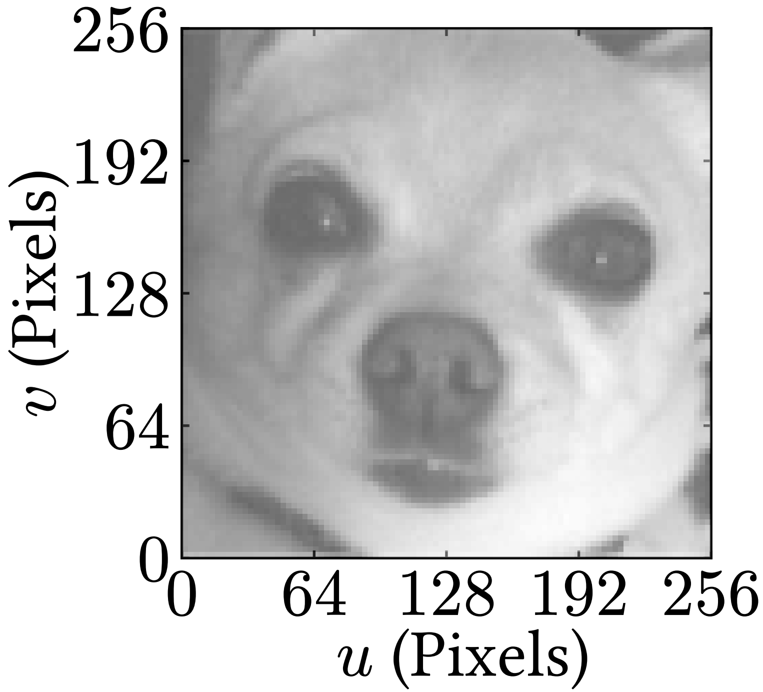

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Answer: 2, u, v

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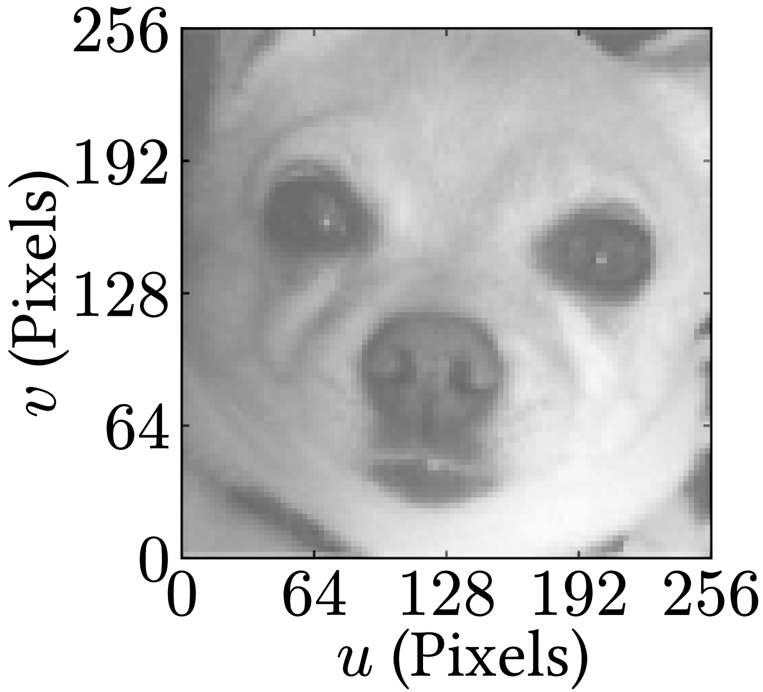

$\boxed{x(u, v)}$

Question: How many input dimensions for x?

Answer: 2, u, v

Question: How many output dimensions for x?

Answer: 1, black/white value

$$x : \underbrace{\mathbb{Z}_{0,255}}_{\text{width}} \times \underbrace{\mathbb{Z}_{0,255}}_{\text{height}} \mapsto \underbrace{\mathbb{Z}_{0,255}}_{\text{Color values}}$$

Additional Dimensions

$$\begin{bmatrix} 0 & 1 & 1 & 1 & 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 0 & 0 & 1 & 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 & 1 & 0 & 0 & 1 \\ 1 & 1 & 0 & 0 & 0 & 1 & 1 & 1 \\ 1 & 1 & 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 & 0 & 1 & 1 & 0 \\ 1 & 0 & 1 & 1 & 1 & 1 & 1 & 1 \end{bmatrix}$$

$$*$$

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

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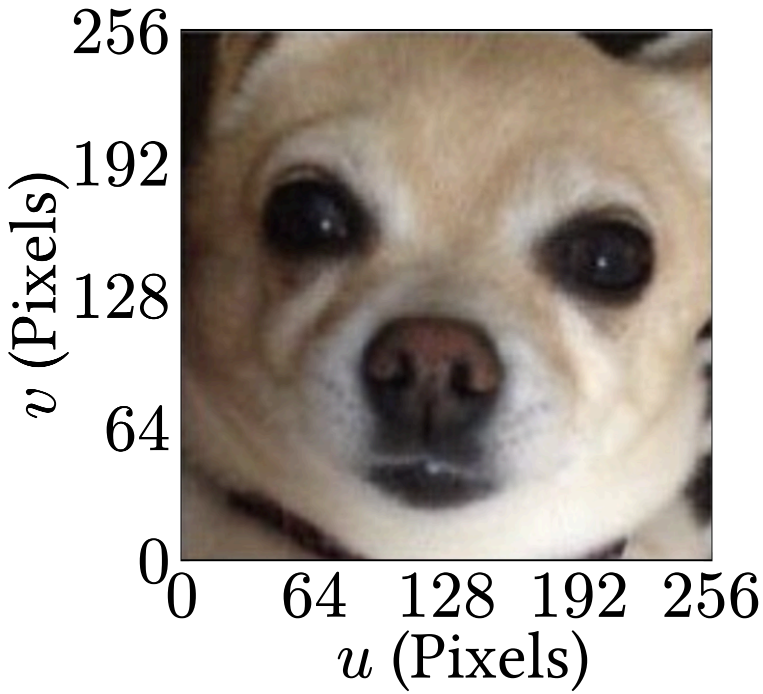

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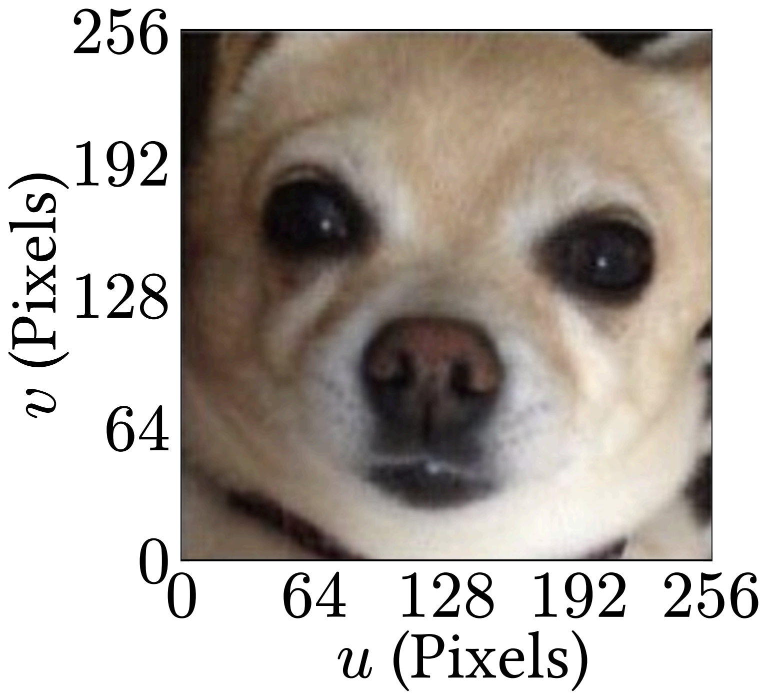

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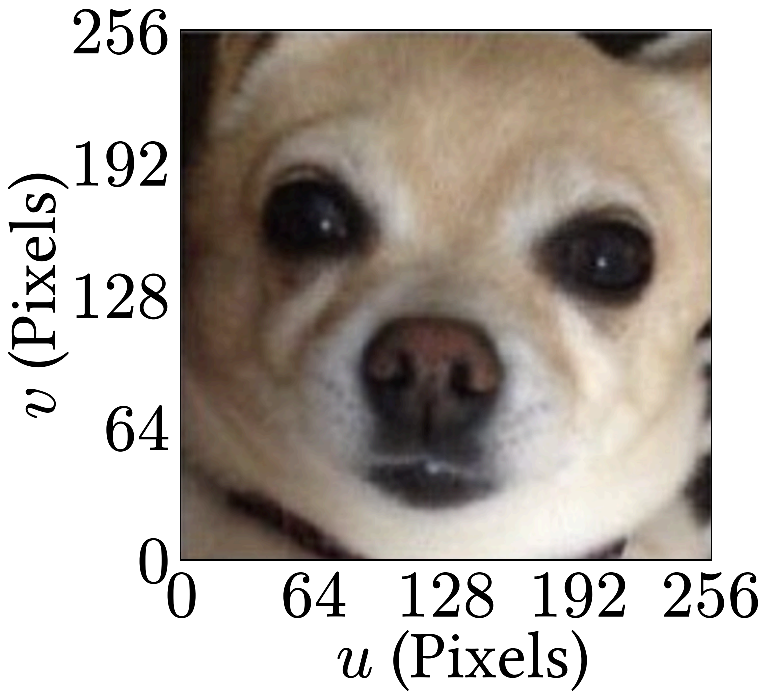

$\boxed{x(u, v)}$

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Question: How many output dimensions for x?

Answer: 3 – red, green, and blue channels

$$x : \underbrace{\mathbb{Z}_{0,255}}_{\text{width}} \times \underbrace{\mathbb{Z}_{0,255}}_{\text{height}} \mapsto \underbrace{[0, 1]^3}_{\text{Color values}}$$

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$$\begin{bmatrix} \frac{R}{255} & \frac{G}{255} & \frac{B}{255} \end{bmatrix}$$

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Each pixel contains 3 colors (channels)

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And the pixels extend in 2 directions (variable)

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$$\boldsymbol{x}(u, v) = \begin{bmatrix} \underbrace{\begin{bmatrix} 130 & 140 & 120 & 103 \\ 80 & 140 & 120 & 105 \\ 130 & 140 & 75 & 165 \\ 210 & 140 & 90 & 150 \end{bmatrix}}_{\text{red}} & \underbrace{\begin{bmatrix} 130 & 140 & 75 & 165 \\ 210 & 140 & 90 & 150 \\ 130 & 140 & 120 & 103 \\ 80 & 140 & 120 & 105 \end{bmatrix}}_{\text{green}} & \underbrace{\begin{bmatrix} 210 & 140 & 90 & 150 \\ 130 & 140 & 75 & 165 \\ 110 & 140 & 120 & 103 \\ 80 & 140 & 120 & 105 \end{bmatrix}}_{\text{blue}} \end{bmatrix}^\top$$

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This form is called CHW (channel, height, width) format

Convolutional filter must process this data!

Additional Dimensions

$$
\begin{bmatrix}
0&1&1&1&1&1&0&1\\
0&1&1&1&1&1&1&1\\
1&0&0&1&1&0&0&1\\
1&0&0&1&1&0&0&1\\
1&1&0&0&0&1&1&1\\
1&1&0&0&0&1&1&1\\
0&1&0&0&0&1&1&0\\
1&0&1&1&1&1&1&1
\end{bmatrix}
*
\begin{bmatrix}1&0\\0&1\end{bmatrix}
+
\begin{bmatrix}
0&1&1&1&1&1&0&1\\
0&1&1&1&1&1&1&1\\
1&0&0&1&1&0&0&1\\
1&0&0&1&1&0&0&1\\
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$$

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I will not bore you with the full equations

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Question: What is the shape of $\boldsymbol{\theta}$ for a single layer?

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Question: What is the shape of $\boldsymbol{\theta}$ for a single layer?

Answer:

$$\boldsymbol{\theta} \in \mathbb{R}^{c_x \times c_y \times (k+1) \times k}$$

- Input channels: c_x
- Output channels: c_y
- Filter u (height): $k + 1$
- Filter v (width): k

Additional Dimensions

```
import torch
c_x = 3 # Number of colors
c_y = 32
k = 2 # Filter size
h, w = 128, 128 # Image size

conv1 = torch.nn.Conv2d(
  in_channels=c_x,
  out_channels=c_y,
  kernel_size=2
)
image = torch.rand((1, c_x, h, w)) # Torch requires BCHW
out = conv1(image) # Shape(1, c_y, h - k, w - k)
```

Additional Dimensions

One last thing, stride allows you to "skip" cells during convolution

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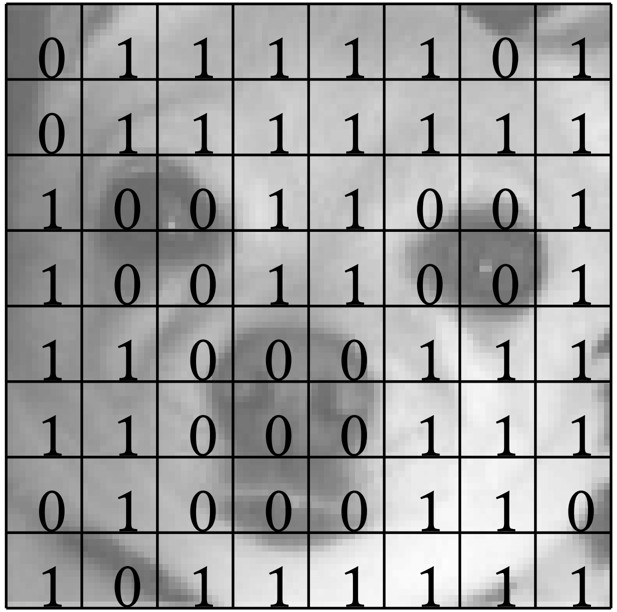

$$*$$

1	0
0	1

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Coding

```
import jax, equinox
c_x = 3 # Number of colors
c_y = 32
k = 2 # Filter size
h, w = 128, 128 # Image size
conv1 = equinox.nn.Conv2d(
  in_channels=c_x,
  out_channels=c_y,
  kernel_size=2,
  key=jax.random.key(0)
)
image = jax.random.uniform(jax.random.key(1), (c_x, h, w))
out = conv1(image) # Shape(c_y, h - k, w - k)
```

Coding

```
import torch
conv1 = torch.nn.Conv2d(3, c_h, 2)
pool1 = torch.nn.AdaptivePool2d((a, a))
conv2 = torch.nn.Conv2d(c_h, c_y, 2)
pool2 = torch.nn.AdaptivePool2d((b, b))
linear = torch.nn.Linear(c_y * b * b)
z_1 = conv1(image)
z_1 = torch.nn.functional.leaky_relu(z_1)
z_1 = pool(z_1) # Shape(1, c_h, a, a)
z_2 = conv1(z1)
z_2 = torch.nn.functional.leaky_relu(z_2)
z_2 = pool(z_2) # Shape(1, c_y, b, b)
z_3 = linear(z_2.flatten())
```

Coding

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conv2 = equinox.nn.Conv2d(c_h, c_y, 2)
pool2 = equinox.nn.AdaptivePool2d((b, b))
linear = equinox.nn.Linear(c_y * b * b)
z_1 = conv1(image,(3, h, w))
z_1 = jax.nn.leaky_relu(z_1)
z_1 = pool(z_1) # Shape(c_h, a, a)
z_2 = conv1(z1)
z_2 = jax.nn.leaky_relu(z_2)
z_2 = pool(z_2) # Shape(c_y, b, b)
z_3 = linear(z_2.flatten())
```

Coding

Single channel, single filter, single variable, $\boldsymbol{\theta} \in \mathbb{R}^{k+1}, k = 2$

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$$f(x(t), \boldsymbol{\theta}) = \begin{bmatrix} \sigma\left(\boldsymbol{\theta}^\top \begin{bmatrix} 1 \\ x(1) \\ x(2) \end{bmatrix}\right) & \sigma\left(\boldsymbol{\theta}^\top \begin{bmatrix} 1 \\ x(2) \\ x(3) \end{bmatrix}\right) & \dots \end{bmatrix}^\top$$

Single channel, single filter, **two variables**, $\boldsymbol{\theta} \in \mathbb{R}^{2k+1}, k = 2$

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Single channel, single filter, **two variables**, $\boldsymbol{\theta} \in \mathbb{R}^{2k+1}, k = 2$

$$f(x(t), \boldsymbol{\theta}) = \begin{bmatrix} \sigma\left(\boldsymbol{\theta}^\top \begin{bmatrix} 1 & 0 & 0 \\ x(1,1) & x(1,2) & x(1,3) \\ x(2,1) & x(2,2) & x(2,3) \end{bmatrix}\right) & \sigma\left(\boldsymbol{\theta}^\top \begin{bmatrix} 1 & 0 & 0 \\ x(2,1) & x(2,2) & x(2,3) \\ x(3,1) & x(3,2) & x(3,3) \end{bmatrix}\right) \end{bmatrix}^\top$$

Coding

Three channels, single filter, two variables, $\boldsymbol{\theta} \in \mathbb{R}^{2k+1}, k = 2$

$$f_r(x(t), \boldsymbol{\theta}) = \left[\sigma\left(\boldsymbol{\theta}_r^\top \begin{bmatrix} 1 & 0 & 0 \\ x(1,1) & x(1,2) & x(1,3) \\ x(2,1) & x(2,2) & x(2,3) \end{bmatrix} \right) \; \sigma\left(\boldsymbol{\theta}_r^\top \begin{bmatrix} 1 & 0 & 0 \\ x(2,1) & x(2,2) & x(2,3) \\ x(3,1) & x(3,2) & x(3,3) \end{bmatrix} \right) \right]^\top$$

Coding

Three channels, single filter, two variables, $\boldsymbol{\theta} \in \mathbb{R}^{2k+1}, k = 2$

$$f_r(x(t), \boldsymbol{\theta}) = \left[\sigma\left(\boldsymbol{\theta}_r^\top \begin{bmatrix} 1 & 0 & 0 \\ x(1,1) & x(1,2) & x(1,3) \\ x(2,1) & x(2,2) & x(2,3) \end{bmatrix} \right) \sigma\left(\boldsymbol{\theta}_r^\top \begin{bmatrix} 1 & 0 & 0 \\ x(2,1) & x(2,2) & x(2,3) \\ x(3,1) & x(3,2) & x(3,3) \end{bmatrix} \right) \right]^\top$$

Coding

We only considered one filter

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In a perceptron, many parallel neurons makes a wide neural network

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In a CNN, many parallel filters makes a wide CNN

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In a CNN, many parallel filters makes a wide CNN

$$\boldsymbol{z}(t) = f(x(t), \boldsymbol{\theta}) = \begin{bmatrix} \sigma\left(\boldsymbol{\theta}^\top \begin{bmatrix} 1 \\ x(0.1) \\ x(0.2) \end{bmatrix}\right) & \sigma\left(\boldsymbol{\theta}^\top \begin{bmatrix} 1 \\ x(0.2) \\ x(0.3) \end{bmatrix}\right) & \dots \end{bmatrix}^\top$$

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Convolution

Neuron:

$$\sigma(\boldsymbol{\theta}_1^\top \overline{\boldsymbol{x}}(1) + \boldsymbol{\theta}_2^\top \overline{\boldsymbol{x}}(2) + \ldots) = \sigma\left(\sum_{i=0}^{d_x} \theta_{1,i} \overline{x}_i + \sum_{i=0}^{d_x} \theta_{2,i} \overline{x}_i + \ldots\right)$$

Convolution

Convolution

Neuron:

$$\sigma(\boldsymbol{\theta}_1^\top \overline{\boldsymbol{x}}(1) + \boldsymbol{\theta}_2^\top \overline{\boldsymbol{x}}(2) + \ldots) = \sigma\left(\sum_{i=0}^{d_x} \theta_{1,i} \overline{x}_i + \sum_{i=0}^{d_x} \theta_{2,i} \overline{x}_i + \ldots\right)$$

Convolution:

$$\boldsymbol{\theta}_1^\top \overline{\boldsymbol{x}}(t) + \boldsymbol{\theta}_2^\top \overline{\boldsymbol{x}}(t+1) = \left(\sum_{i=0}^{d_x} \theta_{1,i} \overline{x}_i(t)\right) + \left(\sum_{i=0}^{d_x} \theta_{2,i} \overline{x}_i(t+1)\right)$$

Convolution

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$$\sigma(\boldsymbol{\theta}_1^\top \overline{\boldsymbol{x}}(1) + \boldsymbol{\theta}_2^\top \overline{\boldsymbol{x}}(2) + \ldots) = \sigma\left(\sum_{i=0}^{d_x} \theta_{1,i} \overline{x}_i + \sum_{i=0}^{d_x} \theta_{2,i} \overline{x}_i + \ldots \right)$$

Convolution:

$$\boldsymbol{\theta}_1^\top \overline{\boldsymbol{x}}(t) + \boldsymbol{\theta}_2^\top \overline{\boldsymbol{x}}(t+1) = \left(\sum_{i=0}^{d_x} \theta_{1,i} \overline{x}_i(t) \right) + \left(\sum_{i=0}^{d_x} \theta_{2,i} \overline{x}_i(t+1) \right)$$

Convolution

$$\begin{bmatrix} \boldsymbol{\theta_1} \\ \boldsymbol{\theta_2} \end{bmatrix} * \overline{\boldsymbol{x}}(t) = \begin{bmatrix} \boldsymbol{\theta}_1^\top \overline{\boldsymbol{x}}(0) + \boldsymbol{\theta}_2^\top \overline{\boldsymbol{x}}(1) & \theta_1^\top \overline{\boldsymbol{x}}(1) + \boldsymbol{\theta}_2^\top \overline{\boldsymbol{x}}(2) & \dots \end{bmatrix}$$

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We call this a **convolutional layer**

Convolution

$$\begin{bmatrix} \boldsymbol{\theta}_1 \\ \boldsymbol{\theta}_2 \end{bmatrix} * \overline{\boldsymbol{x}}(t) = [\boldsymbol{\theta}_1^\top \overline{\boldsymbol{x}}(0) + \boldsymbol{\theta}_2^\top \overline{\boldsymbol{x}}(1) \quad \theta_1^\top \overline{\boldsymbol{x}}(1) + \boldsymbol{\theta}_2^\top \overline{\boldsymbol{x}}(2) \quad ...]$$

We call this a **convolutional layer**

Question: Anything missing?

Convolution

$$\begin{bmatrix} \boldsymbol{\theta}_1 \\ \boldsymbol{\theta}_2 \end{bmatrix} * \overline{\boldsymbol{x}}(t) = \begin{bmatrix} \boldsymbol{\theta}_1^\top \overline{\boldsymbol{x}}(0) + \boldsymbol{\theta}_2^\top \overline{\boldsymbol{x}}(1) & \theta_1^\top \overline{\boldsymbol{x}}(1) + \boldsymbol{\theta}_2^\top \overline{\boldsymbol{x}}(2) & \dots \end{bmatrix}$$

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Answer: Activation function!

Convolution

$$\begin{bmatrix} \boldsymbol{\theta}_1 \\ \boldsymbol{\theta}_2 \end{bmatrix} * \overline{\boldsymbol{x}}(t) = \begin{bmatrix} \boldsymbol{\theta}_1^\top \overline{\boldsymbol{x}}(0) + \boldsymbol{\theta}_2^\top \overline{\boldsymbol{x}}(1) & \theta_1^\top \overline{\boldsymbol{x}}(1) + \boldsymbol{\theta}_2^\top \overline{\boldsymbol{x}}(2) & \dots \end{bmatrix}$$

We call this a **convolutional layer**

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$$\begin{bmatrix} \sigma(\boldsymbol{\theta}_1^\top \overline{\boldsymbol{x}}(0) + \boldsymbol{\theta}_2^\top \overline{\boldsymbol{x}}(1)) & \sigma(\theta_1^\top \overline{\boldsymbol{x}}(1) + \boldsymbol{\theta}_2^\top \overline{\boldsymbol{x}}(2)) & \dots \end{bmatrix}$$

Convolution

$$\begin{bmatrix} \boldsymbol{\theta}_1 \\ \boldsymbol{\theta}_2 \end{bmatrix} * \overline{\boldsymbol{x}}(t) = \begin{bmatrix} \boldsymbol{\theta}_1^\top \overline{\boldsymbol{x}}(0) + \boldsymbol{\theta}_2^\top \overline{\boldsymbol{x}}(1) & \theta_1^\top \overline{\boldsymbol{x}}(1) + \boldsymbol{\theta}_2^\top \overline{\boldsymbol{x}}(2) & \ldots \end{bmatrix}$$

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Much better

Convolution

Convolution is **local**, in this example, we only consider two consecutive timesteps

Convolution

Convolution is **local**, in this example, we only consider two consecutive timesteps

Convolution is **shift equivariant**, if $\boldsymbol{\theta}_1, \boldsymbol{\theta}_2$ detect "hello", it does not matter whether "hello" occurs at $x(0), x(1)$ or $x(100), x(101)$

Convolution

Convolution

```python
import jax, equinox
# Assume a sequence of length m
# Each timestep has dimension d_x
x = stock_data # Shape (d_x, time)
conv_layer = equinox.nn.Conv1d(
  in_channels=d_x,
  out_channels=d_y,
  kernel_size=k # Size of filter in timesteps/parameters,
  key=jax.random.key(0)
)

z = jax.nn.leaky_relu(conv_layer(x))
```

Convolution

```
import torch
# Assume a sequence of length m
# Each timestep has dimension d_x
# Torch requires 3 dims! Be careful!
x = stock_data # Shape (batch, d_x, time)
conv_layer = torch.nn.Conv1d(
  in_channels=d_x,
  out_channels=d_y,
  kernel_size=k # Size of filter in timesteps/parameters,
)

z = jax.nn.leaky_relu(conv_layer(x))
```

Agenda

1. Review
2. Signal Processing
3. **Convolution**
4. Convolutional Neural Networks
5. Additional Dimensions
6. Coding

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2D Convolution

We defined convolution over one variable t

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For images, we often have two variables denoting width and height u, v

$$x(u, v)$$

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We can also do convolutions over two dimensions

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$$x(u, v)$$

We can also do convolutions over two dimensions

Most image-based neural networks use convolutions