Transformers

CISC 7026 - Introduction to Deep Learning

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Review

Review

Last time, we derived various forms of **attention**

We started with composite memory

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$$f(\boldsymbol{x}, \boldsymbol{\theta}) = \sum_{i=1}^{T} \boldsymbol{\theta}^\top \overline{\boldsymbol{x}}_i$$

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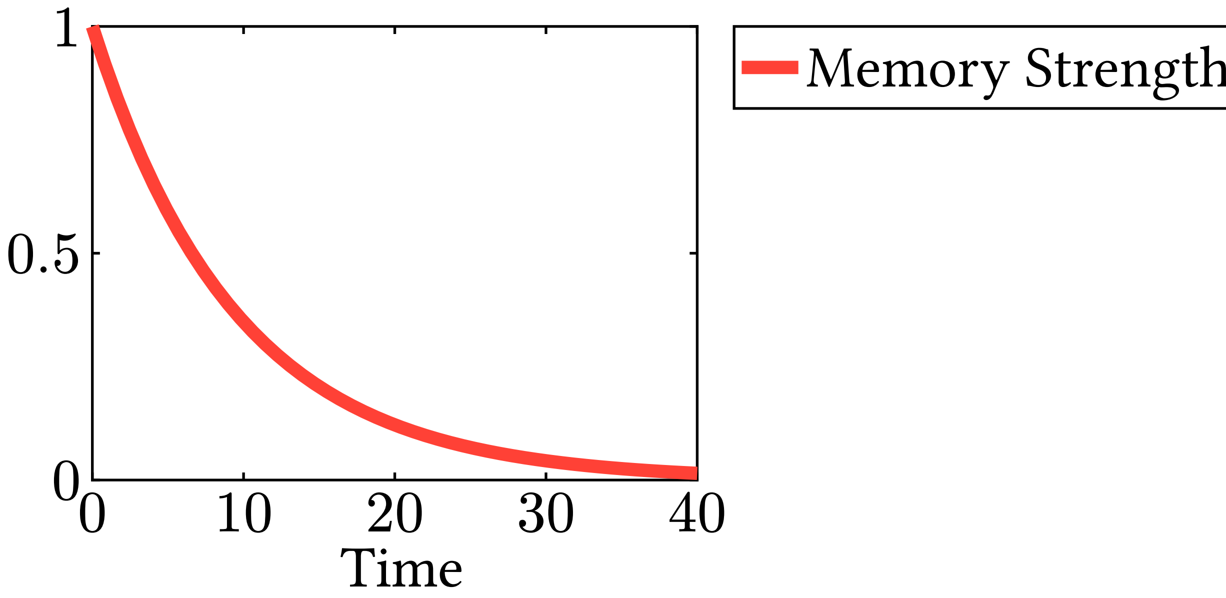

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10 PM

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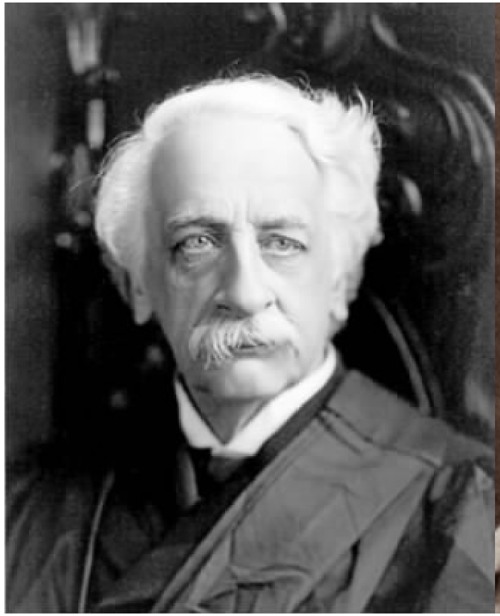


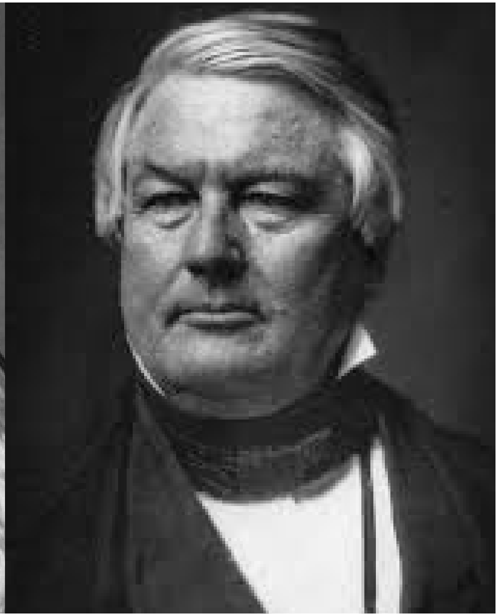

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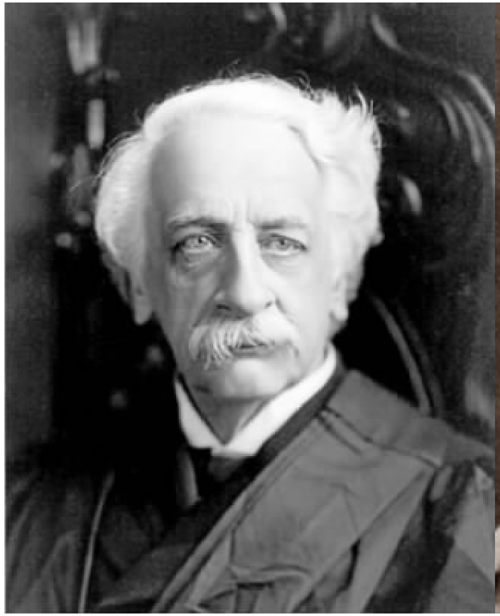

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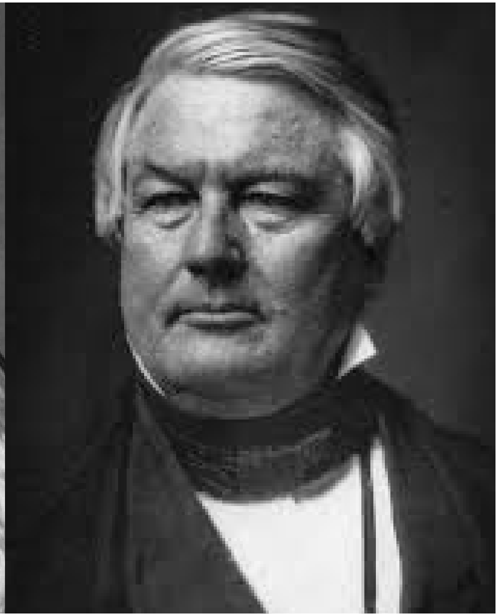

1 AM

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$$\gamma^3 \boldsymbol{\theta}^\top \overline{\boldsymbol{x}}_1$$

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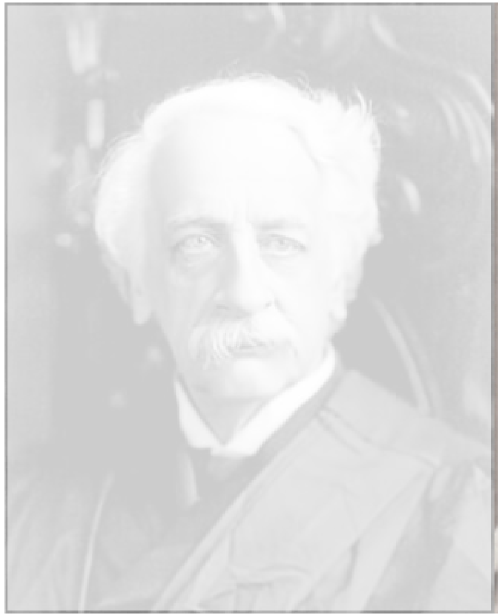

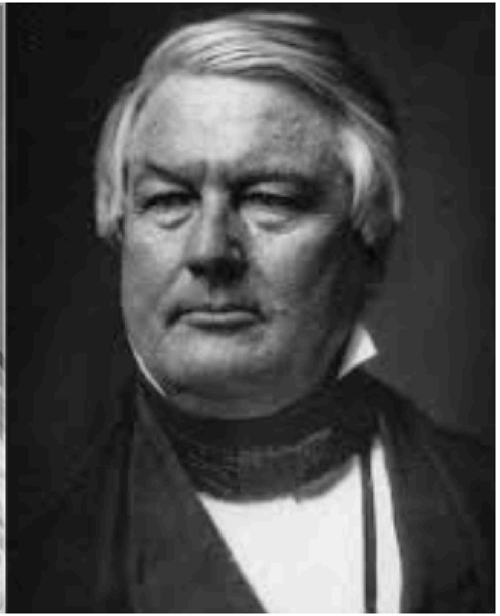

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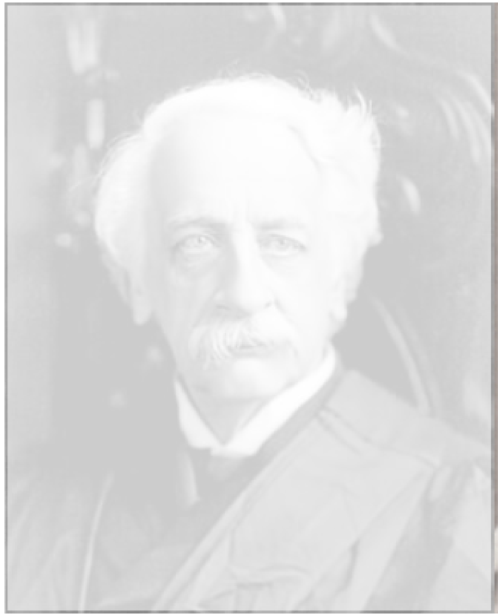

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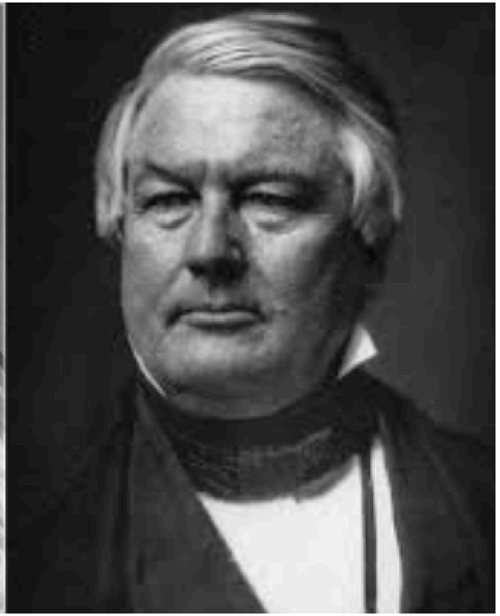

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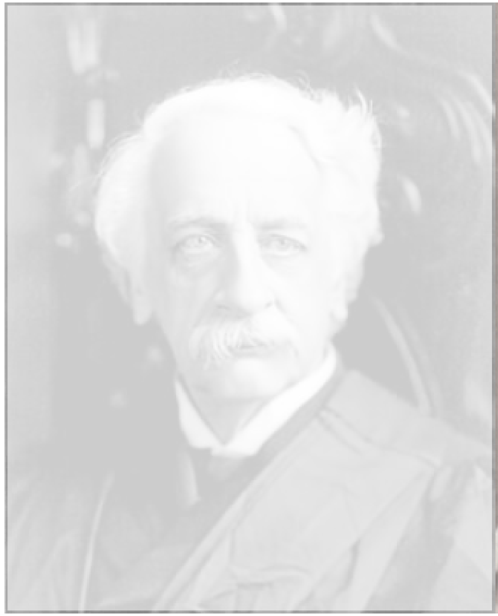


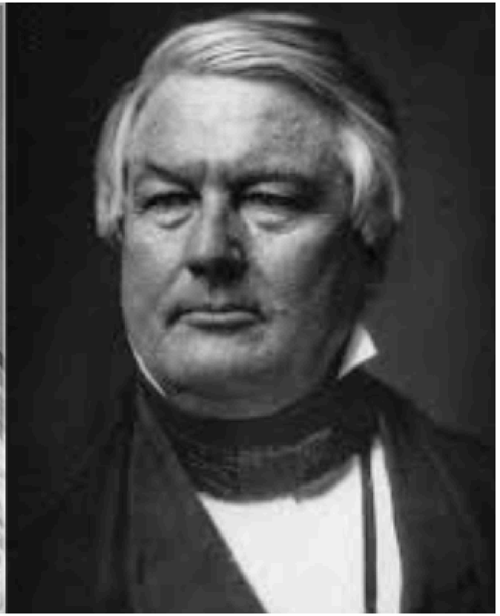

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With our current model, we forget Taylor Swift!

Our model of human memory is incomplete

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Overview of transformer application and domains

Going Deeper

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We previously reviewed training tricks

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- Deeper networks

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- Parameter initialization

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These methods empirically improve performance, but we do not always understand why

Going Deeper

Modern transformers can be very deep

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For this reason, they use two new training tricks to enable very deep models

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We will start with the **residual connection**

Going Deeper

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Making the network too deep can hurt performance

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$$\boldsymbol{y} = f_k(...f_2(f_1(\boldsymbol{x}, \boldsymbol{\theta}_1), \boldsymbol{\theta}_{\mathbf{2}}), ..., \boldsymbol{\theta}_k)$$

Going Deeper

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Claim: If the input information is present throughout the network, then we should be able to learn the identity function $f(x) = x$

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Question: We have seen a similar model, what was it?

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Question: Do you agree or disagree with the claim?

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https://colab.research.google.com/drive/1qVIbQKpTuBYIa7FvC4IH-kJq-E0jmc0d#scrollTo=bg74S-AvbmJz

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Very deep networks struggle to learn the identity function

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Very deep networks struggle to learn the identity function

If the input information is available, then learning the identity function should be very easy!

Question: How can we prevent the input from getting lost?

Going Deeper

We can feed the input to each layer

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The first approach is called the **DenseNet** approach

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\begin{aligned}
\boldsymbol{z}_1 &= f_1(\boldsymbol{x}, \boldsymbol{\theta}_1) \\
\boldsymbol{z}_2 &= f_2\left(\begin{bmatrix} \boldsymbol{x} \\ \boldsymbol{z}_1 \end{bmatrix}, \boldsymbol{\theta}_2\right) \\
&\vdots \\
\boldsymbol{z}_k &= f_k\left(\begin{bmatrix} \boldsymbol{x} \\ \boldsymbol{z}_1 \\ \vdots \\ \boldsymbol{z}_{k-1} \end{bmatrix}, \boldsymbol{\theta}_k\right)
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Question: Any issues with the DenseNet approach?

Answer: Very deep networks require too many parameters!

Going Deeper

The next method is called the **ResNet** approach

$$
\begin{aligned}
\boldsymbol{z}_1 &= f_1(\boldsymbol{x}, \boldsymbol{\theta}_1) \\
\boldsymbol{z}_2 &= f_2(\boldsymbol{x}, \boldsymbol{\theta}_2) + \boldsymbol{x} \\
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This helps prevent information from getting lost in very deep networks

Going Deeper

The second trick is called **layer normalization**

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$$f_1(\boldsymbol{x}, \boldsymbol{\theta}_1) = \sum_{i=1}^{d_x} \theta_{1,i} x_i$$

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$$f_1(\boldsymbol{x}, \boldsymbol{\theta}_1) = \sum_{i=1}^{d_x} \theta_{1,i} x_i$$

Question: If all $x_i = 1, \theta_{1,i} = 0.01$ and $d_x = 1000$, what is the output?

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What if we add another layer with the same d_x and θ?

$$f_2(\boldsymbol{z}, \boldsymbol{\theta}_2) = \sum_{i=1}^{1000} 0.01 \cdot 10 = 100$$

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Question: What is the problem?

Going Deeper

Let us look at the gradient

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$$\nabla_{\boldsymbol{\theta}_1} f_2(f_1(\boldsymbol{x}, \boldsymbol{\theta}_1), \boldsymbol{\theta}_2) =$$

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$$\nabla_{\boldsymbol{\theta}_1} f_2(f_1(\boldsymbol{x}, \boldsymbol{\theta}_1), \boldsymbol{\theta}_2) = \nabla_{\boldsymbol{\theta}_1}[f_2](f_1(\boldsymbol{x}, \boldsymbol{\theta}_1)) \cdot \nabla_{\boldsymbol{\theta}_1}[f_1](\boldsymbol{x}, \boldsymbol{\theta}_1)$$

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$$\approx 100 \cdot 10$$

Can cause exploding or vanishing gradient

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Deeper network $\Rightarrow$ worse exploding/vanishing issues

Question: What can we do?

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$$\mu = d_y \sum_{i=1}^{d_y} f(\boldsymbol{x}, \boldsymbol{\theta})_i$$

$$f(\boldsymbol{x}, \boldsymbol{\theta}) - \mu$$

Question: What does this do?

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Question: What does this do?

Answer: Makes output have zero mean (both positive and negative values)

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$$\mu = d_y \sum_{i=1}^{d_y} f(\boldsymbol{x}, \boldsymbol{\theta})_i \qquad\qquad f(\boldsymbol{x}, \boldsymbol{\theta}) - \mu$$

Then, layer normalization **rescales** the outputs

$$\sigma = \frac{\sqrt{\sum_{i=1}^{d_y} f(\boldsymbol{x}, \boldsymbol{\theta}_i - \mu)^2}}{d_y}$$

$$\mathrm{LN}(f(\boldsymbol{x}, \boldsymbol{\theta})) = \frac{f(\boldsymbol{x}, \boldsymbol{\theta}) - \mu}{\sigma}$$

Now, the output of the layer is normally distributed

Going Deeper

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- 99.99% of outputs $\in [-4, 4]$
- 99.9999% of outputs $\in [-5, 5]$

This helps prevent vanishing and exploding gradients

Going Deeper

Now, let's combine residual connections and layer norm and try our very deep network again

TODO COLAB

Transformers

Transformers

Now we have everything we need to implement a transformer

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- Attention

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A deep neural network consists of many layers

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A deep neural network consists of many layers

A transformer consists of many **transformer blocks**

Transformers

```
class TransformerBlock(nn.Module):
    def __init__(self):
        self.attn = Attention()
        self.mlp = Sequential(
            Linear(d_h, d_h), LeakyReLU(), Linear(d_h, d_h))
        self.norm1 = nn.LayerNorm(d_h)
        self.norm2 = nn.LayerNorm(d_h)

    def forward(self, x):
        x = self.norm1(self.attn(x) + x)
        x = self.norm2(self.mlp(x) + x)
        return x
```

Transformers

```python
class Transformer(nn.Module):
    def __init__(self):
        self.block1 = TransformerBlock()
        self.block2 = TransformerBlock()
        self.block3 = TransformerBlock()

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Question: What are the input/output shapes of the transformer?

Answer: $f : \mathbb{R}^{T \times d_x} \mapsto \mathbb{R}^{T \times d_h}$

Positional Encoding

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Our transformer maps T inputs to T outputs

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Question: Do we care about the order of the T inputs?

Answer: In some tasks, yes! In others, no

Positional Encoding

Question: Detecting birds in an image of T pixels, does order matter?

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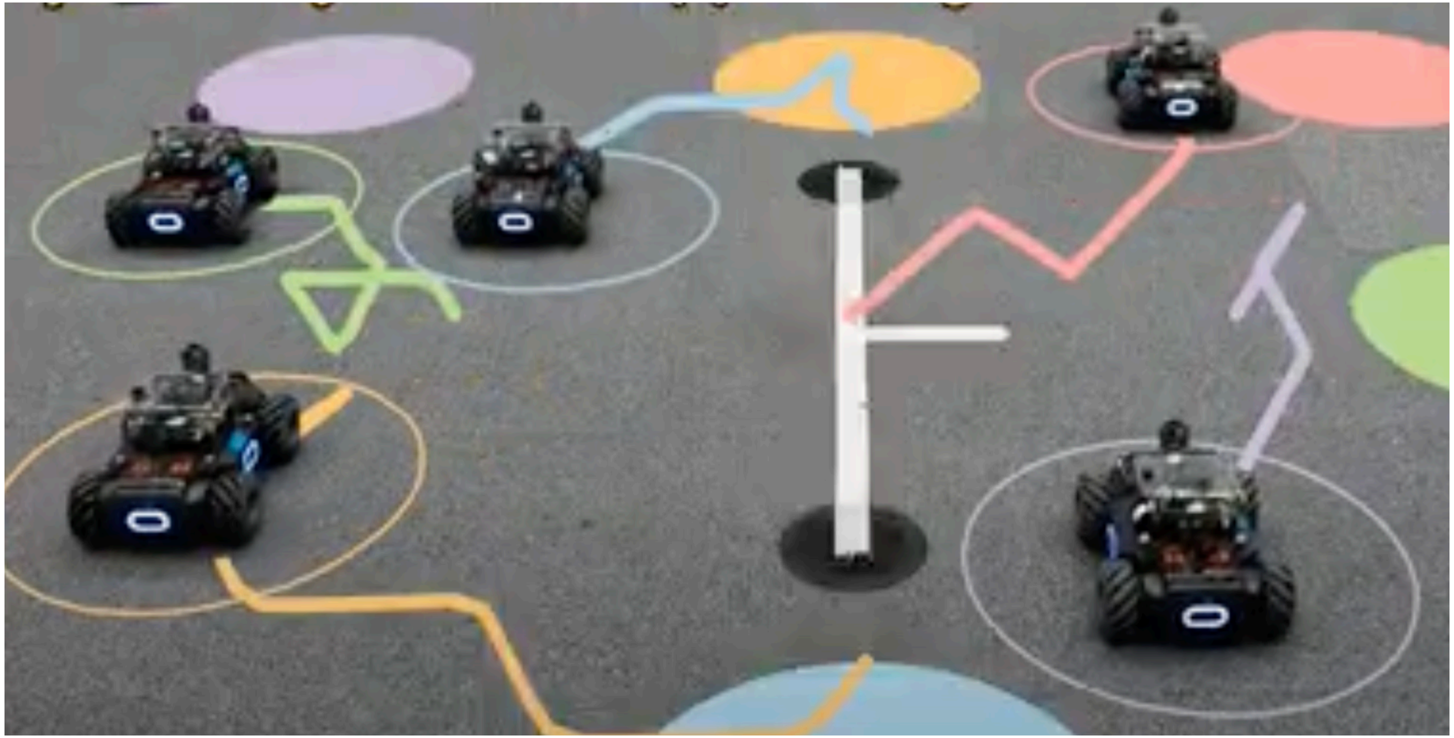

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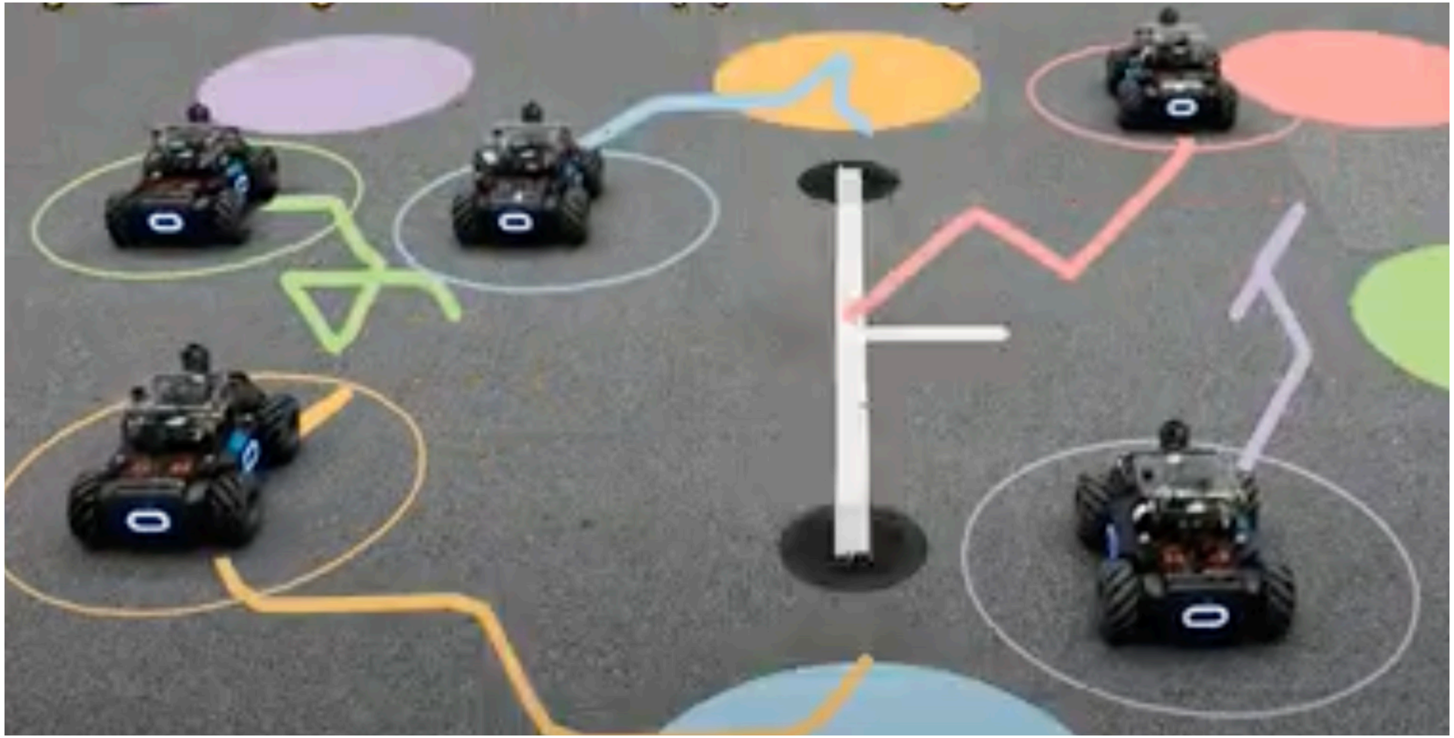

Answer: No!

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Question: We translate a sentence containing T words. Does order matter?

$$\begin{bmatrix} \boldsymbol{x}_1 \\ \boldsymbol{x}_2 \\ \boldsymbol{x}_3 \\ \boldsymbol{x}_4 \\ \boldsymbol{x}_5 \end{bmatrix} = \begin{bmatrix} \text{The} \\ \text{dog} \\ \text{licks} \\ \text{the} \\ \text{owner} \end{bmatrix}$$

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Example 1:

$$P = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}; \quad a = \begin{bmatrix} 3 \\ 4 \\ 5 \end{bmatrix};$$

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Recall dot product self attention

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$$f\left(\boldsymbol{P}\begin{bmatrix}\boldsymbol{x}_1\\ \vdots \\ \boldsymbol{x}_n\end{bmatrix}\right) = \boldsymbol{P}f\left(\begin{bmatrix}\boldsymbol{x}_1\\ \vdots \\ \boldsymbol{x}_n\end{bmatrix}\right)$$

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Question: What does this mean?

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Question: What does this mean?

Answer: Attention is equivariant, order **does not** matter

Positional Encoding

This makes sense, in our party example we never consider the order

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Transformer cannot determine order of inputs! **Equivariant** to ordering

Positional Encoding

The following sentences are the same to a transformer

$$\begin{bmatrix} \boldsymbol{x}_1 \\ \boldsymbol{x}_2 \\ \boldsymbol{x}_3 \\ \boldsymbol{x}_4 \\ \boldsymbol{x}_5 \end{bmatrix} = \begin{bmatrix} \text{The} \\ \text{dog} \\ \text{licks} \\ \text{the} \\ \text{owner} \end{bmatrix}$$

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This is a problem! For some tasks, we care about input order

Positional Encoding

Question: What are some ways we can introduce ordering?

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Answer 2: We can modify the inputs based on their ordering

We will focus on answer 2 because it is more common

Positional Encoding

$$\operatorname{attn}\left(\begin{bmatrix} \boldsymbol{x}_1 \\ \boldsymbol{x}_2 \\ \vdots \\ \boldsymbol{x}_T \end{bmatrix}, \boldsymbol{\theta} \right)$$

Positional Encoding

$$\mathrm{attn}\left(\begin{bmatrix} \boldsymbol{x}_1 \\ \boldsymbol{x}_2 \\ \vdots \\ \boldsymbol{x}_T \end{bmatrix}, \boldsymbol{\theta} \right) \qquad \mathrm{attn}\left(\begin{bmatrix} \boldsymbol{x}_1 \\ \boldsymbol{x}_2 \\ \vdots \\ \boldsymbol{x}_T \end{bmatrix} + \begin{bmatrix} f_{\mathrm{pos}}(1) \\ f_{\mathrm{pos}}(2) \\ \vdots \\ f_{\mathrm{pos}}(3) \end{bmatrix}, \boldsymbol{\theta} \right)$$

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$$f(i) = \begin{bmatrix} \boldsymbol{\theta}_1 \\ \boldsymbol{\theta}_2 \\ \vdots \\ \boldsymbol{\theta}_T \end{bmatrix}_i$$

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In `torch`, this is called `nn.Embedding`

Positional Encoding

So, our final transformer is

```python
class Transformer(nn.Module):
    def __init__(self):
        self.position_embedding = nn.Embedding(d_x)
        self.block1 = TransformerBlock()
        self.block2 = TransformerBlock()

    def forward(self, x):
        x = x + embedding(torch.arange(x.shape[0]))
        x = self.block1(x)
        x = self.block2(x)
        return x
```

Positional Encoding

Let us code up the transformer in colab

https://colab.research.google.com/drive/1qVIbQKpTuBYIa7FvC4IH-kJq-E0jmc0d#scrollTo=iQtXjGYiz5CD

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Now that we understand the transformer, how do we use it?

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$$\text{unique}\left(\begin{bmatrix}\text{John likes movies} \\ \text{Mary likes movies} \\ \text{I like dogs}\end{bmatrix}\right) = [\text{John} \quad \text{likes} \quad \text{movies} \quad \text{Mary} \quad I \quad \text{dogs}]$$

Text Transformers

Then assign a parameter to each word

$$\begin{bmatrix} \text{John} \\ \text{likes} \\ \text{movies} \\ \text{Mary} \\ I \\ \text{dogs} \end{bmatrix} = \begin{bmatrix} \boldsymbol{\theta}_1 \\ \boldsymbol{\theta}_2 \\ \boldsymbol{\theta}_3 \\ \boldsymbol{\theta}_4 \\ \boldsymbol{\theta}_5 \\ \boldsymbol{\theta}_6 \end{bmatrix}$$

$$\begin{bmatrix} \boldsymbol{\theta}_1 & \boldsymbol{\theta}_2 & \boldsymbol{\theta}_3 \end{bmatrix} =$$

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$$[\boldsymbol{\theta}_1 \;\; \boldsymbol{\theta}_2 \;\; \boldsymbol{\theta}_3] = \text{John likes movies}$$

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$$[\boldsymbol{\theta}_1 \ \ \boldsymbol{\theta}_2 \ \ \boldsymbol{\theta}_3] = \text{John likes movies}$$

$$[\boldsymbol{\theta}_4 \ \ \boldsymbol{\theta}_2 \ \ \boldsymbol{\theta}_1] = \text{Mary likes John}$$

Text Transformers

```python
unique_words = set(sentence.split(" ") for sentence in xs)
tokens = {word: i for i, word in enumerate(unique_words)}
embeddings = nn.Embedding(len(tokens), d_x)
# Convert from words to parameters
xs = []
for sentence in sentences:
    xs.append([])
    for word in sentence:
        index = embeddings[word]
        token = tokens[index]
        xs.append(token)

print(xs)
>>> [[Tensor(...), Tensor(...), ...]]
```

Text Transformers

```
model = Transformer()
for tokenized_sentence in xs:
    # Convert list to tensor
    x = torch.stack(tokenized_sentence)
    y = model(x)
```

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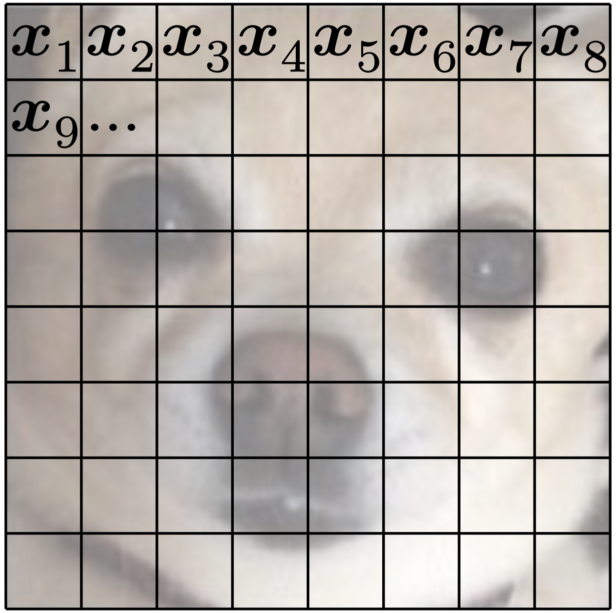

Image Transformers

```python
# Convert image into patches
patches = []
for i in range(0, x.shape[0] - k + 1, k):
    for j in range(0, x.shape[1] - k + 1, k):
        patches.append(
            x[i: i + k , j: j + k]
        )
patches = stack(patches, axis=0)
print(patches.shape)
>>> (T, k, k)

model = Transformer()
y = model(patches)
```

Unsupervised Training

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In practice, transformers require lots of training data

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Transformers are limited by the size of datasets today

There are not enough students to label training data!

Unsupervised Training

Question: How can we train transformers if we cannot create big enough datasets?

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This method is **extremely** powerful

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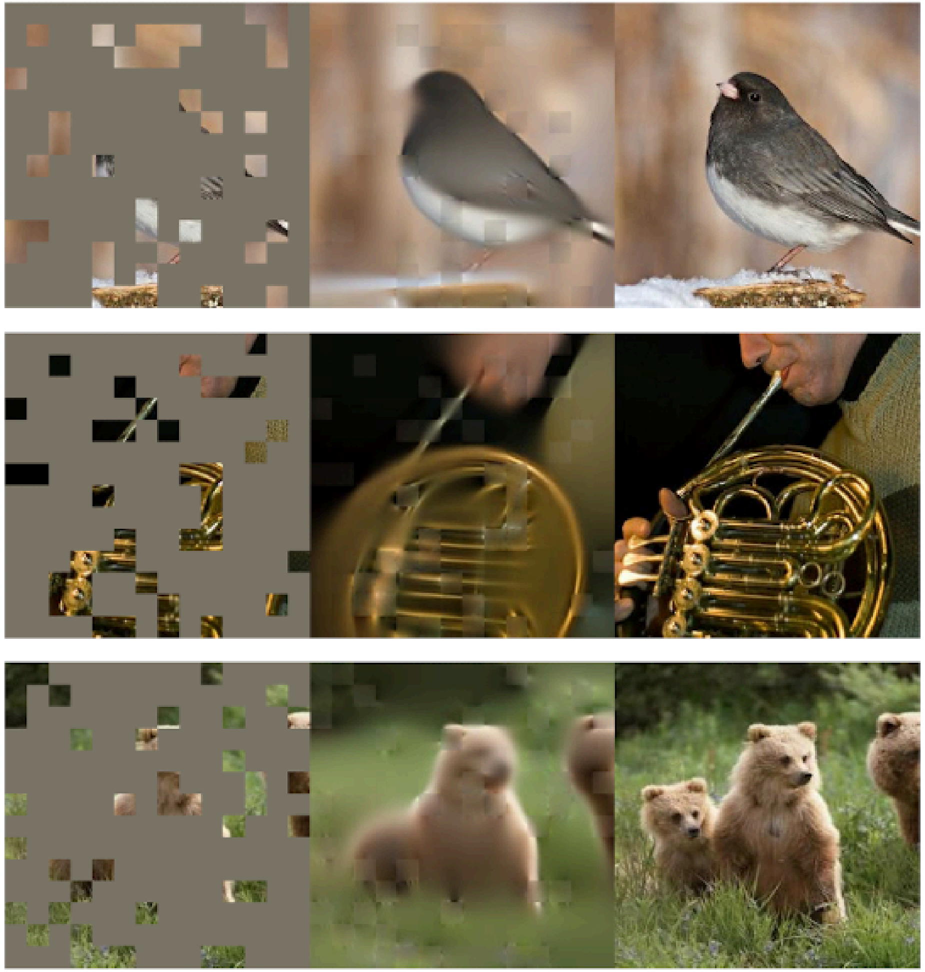

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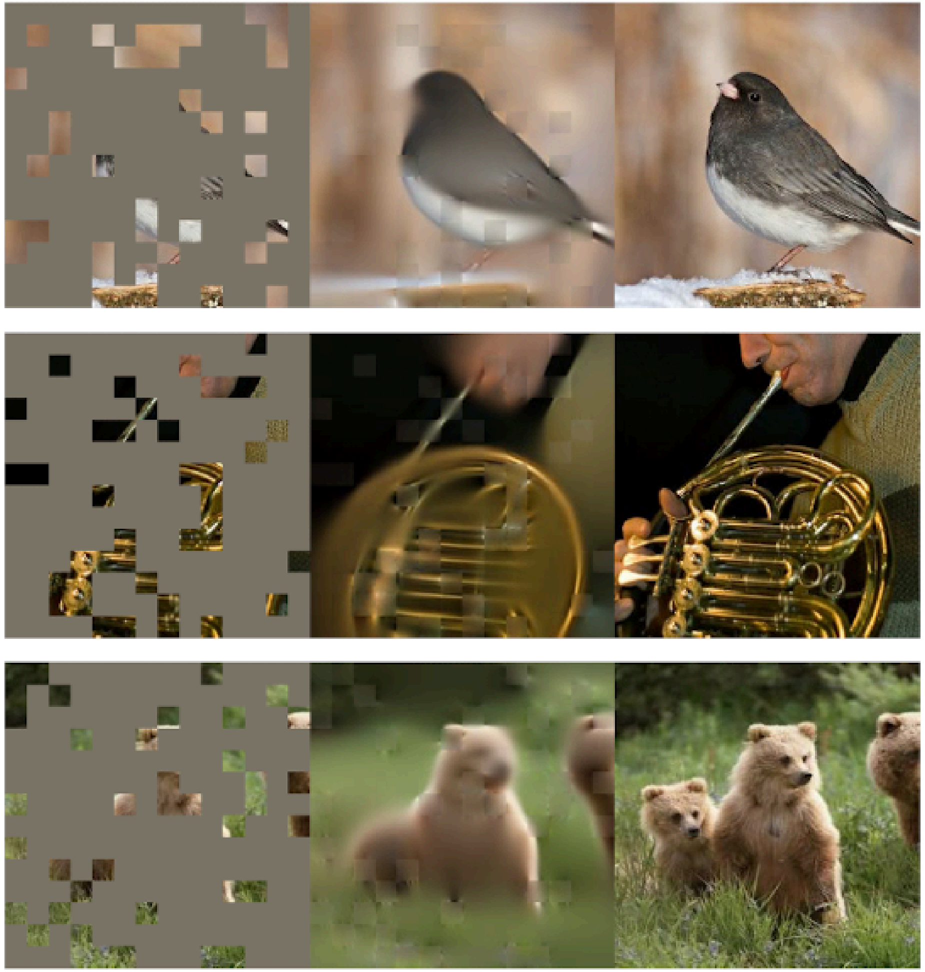

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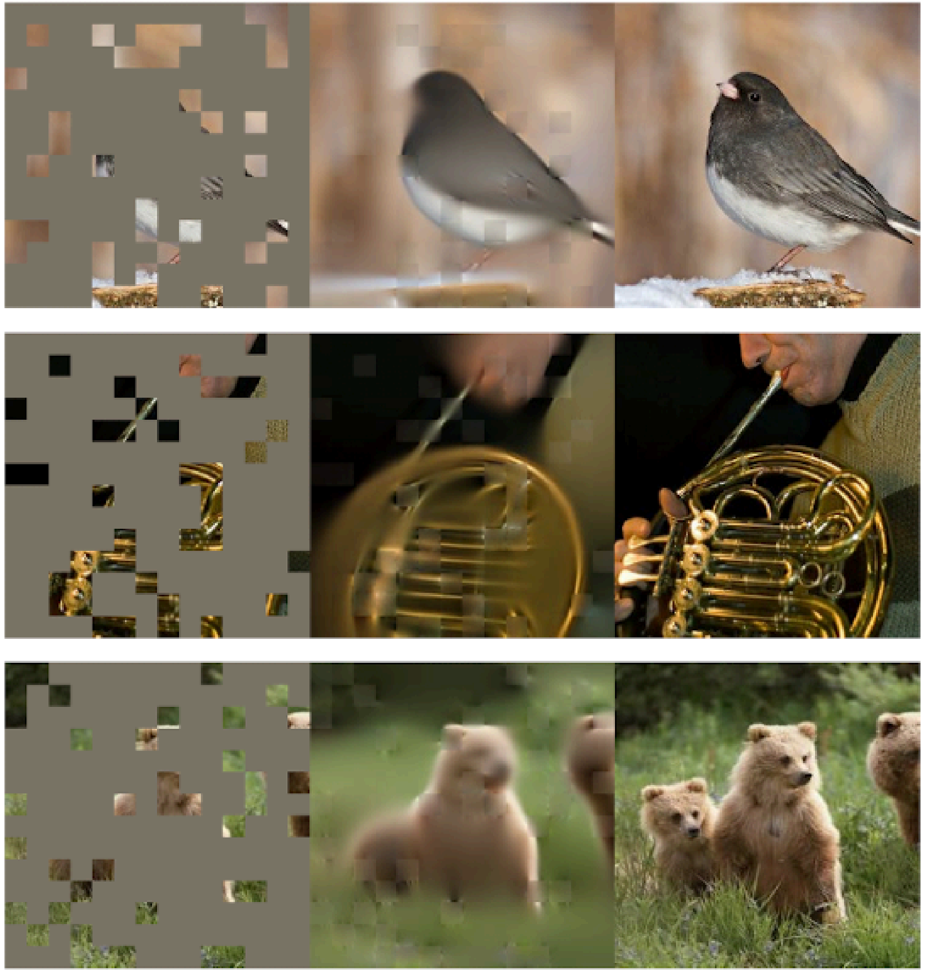

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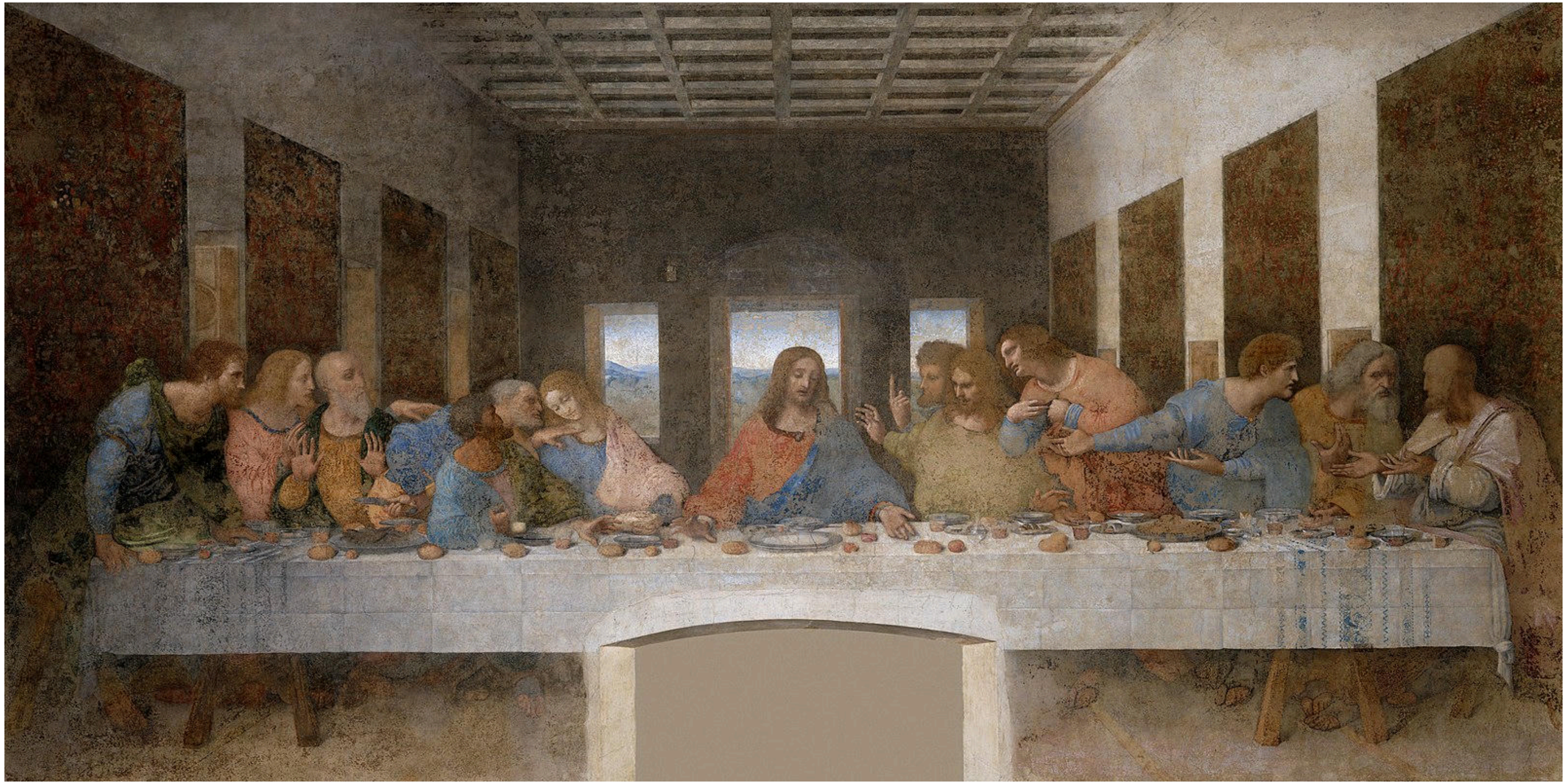

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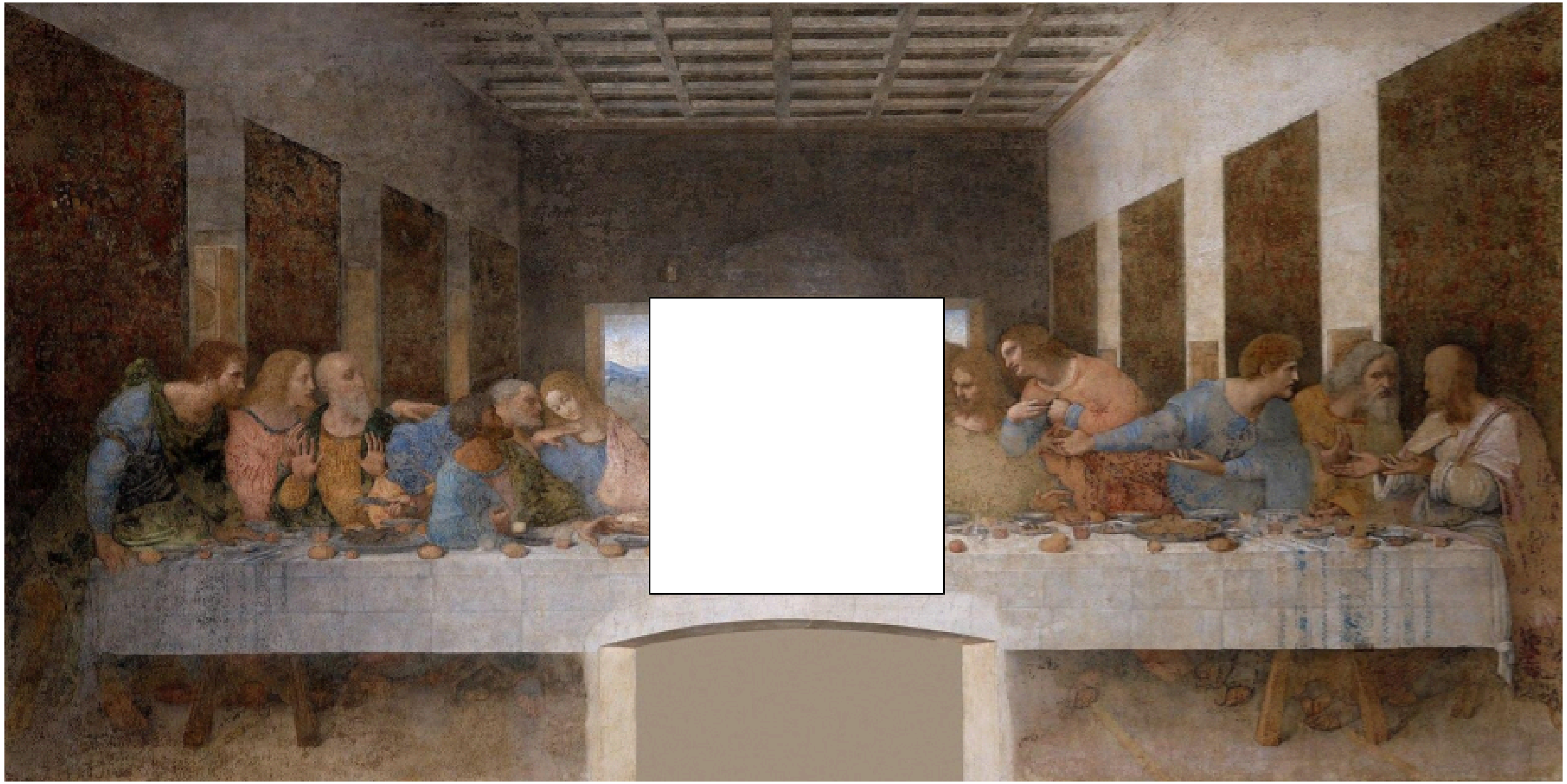

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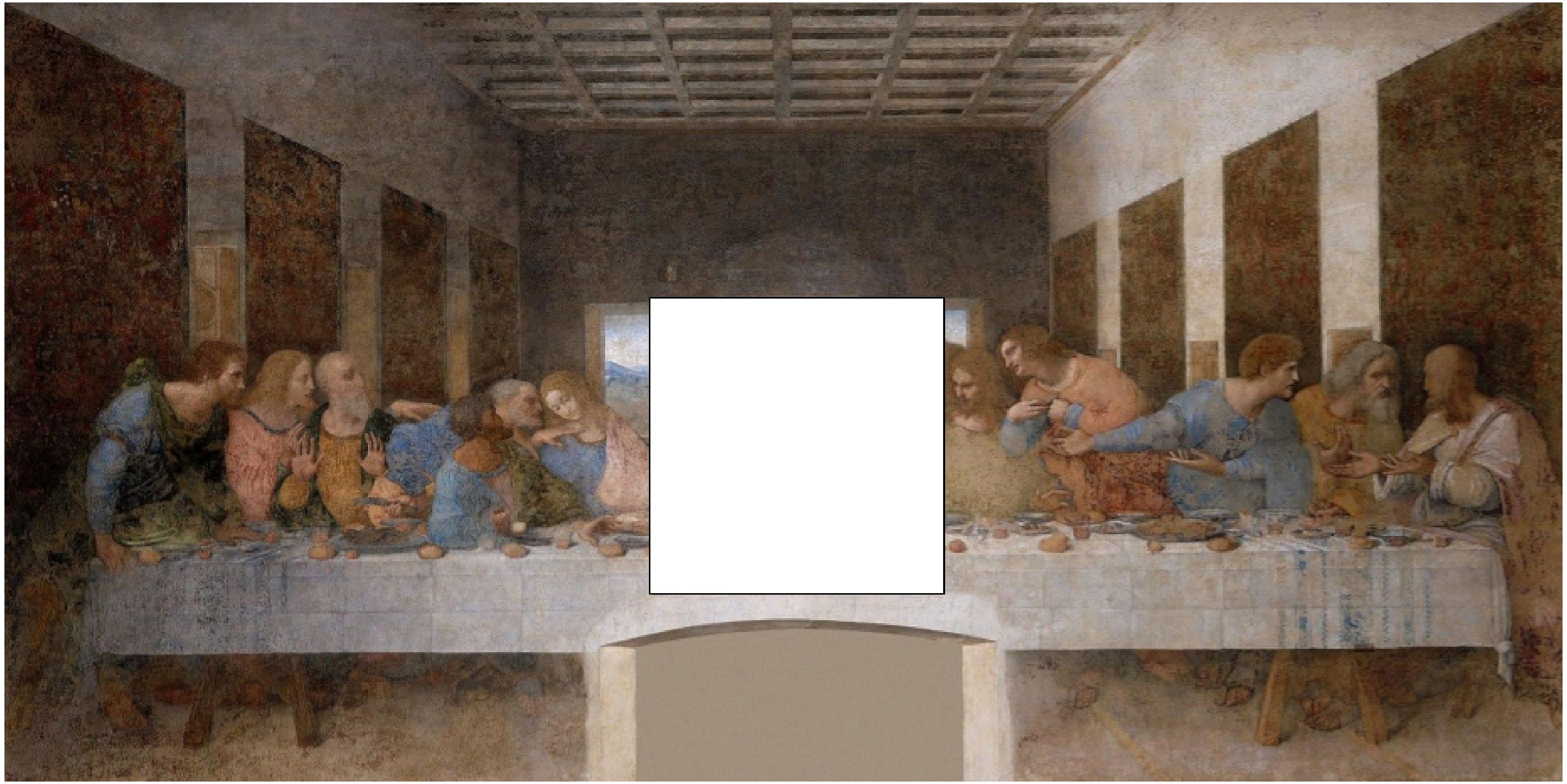
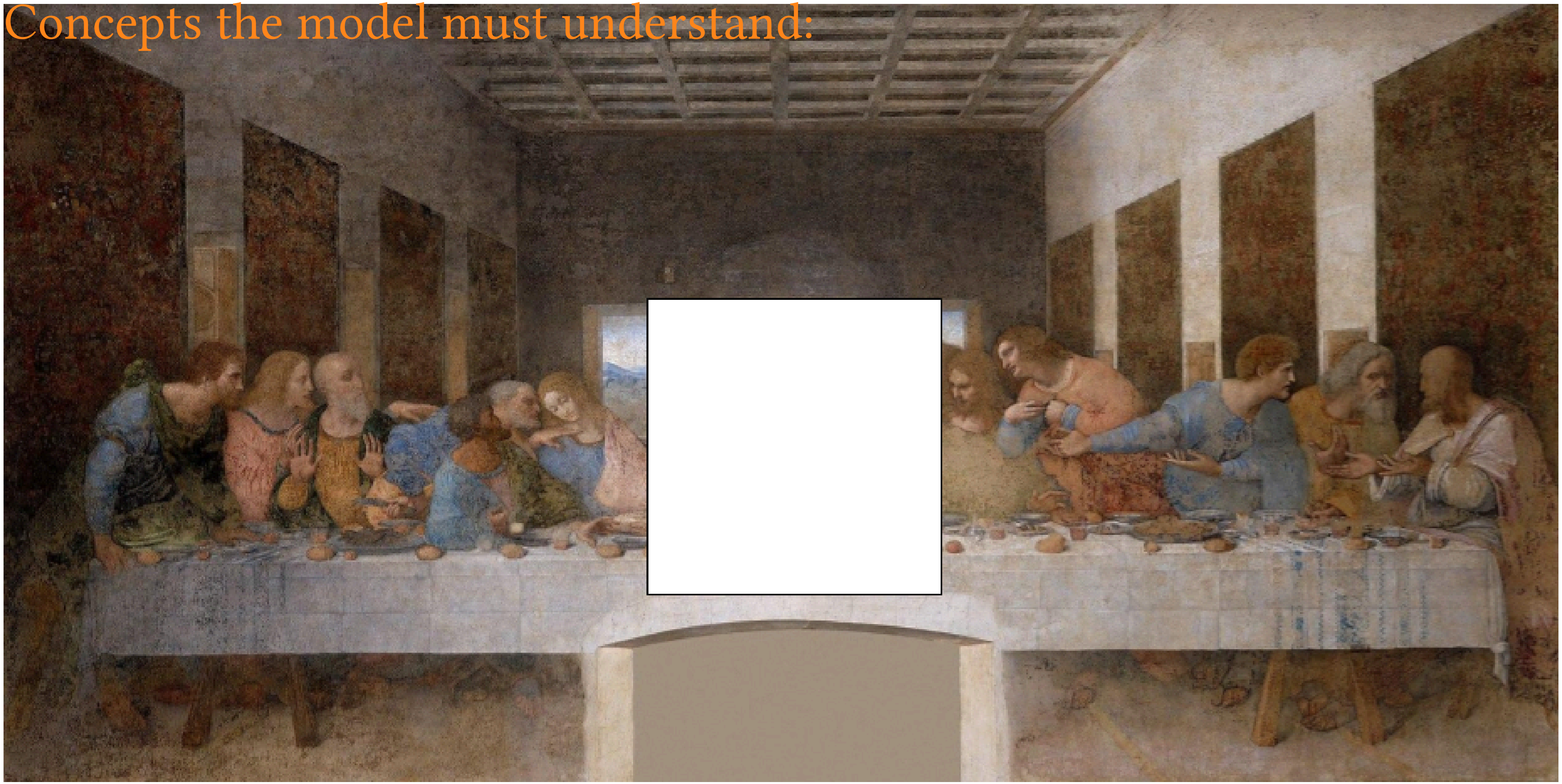


Question: What concepts does the vision transformer need to understand to predict the missing pixels?

Unsupervised Training

Concepts the model must understand:



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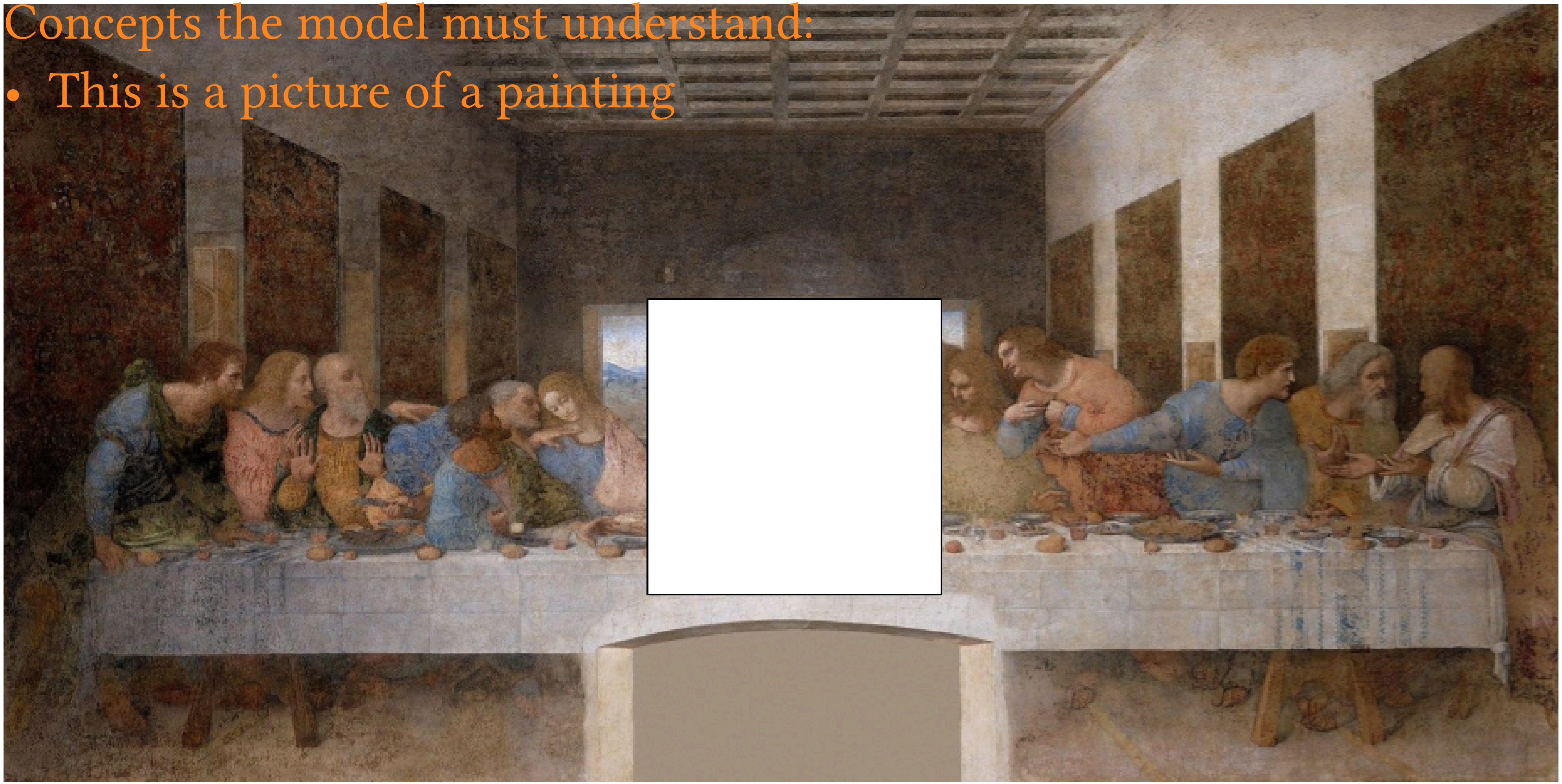
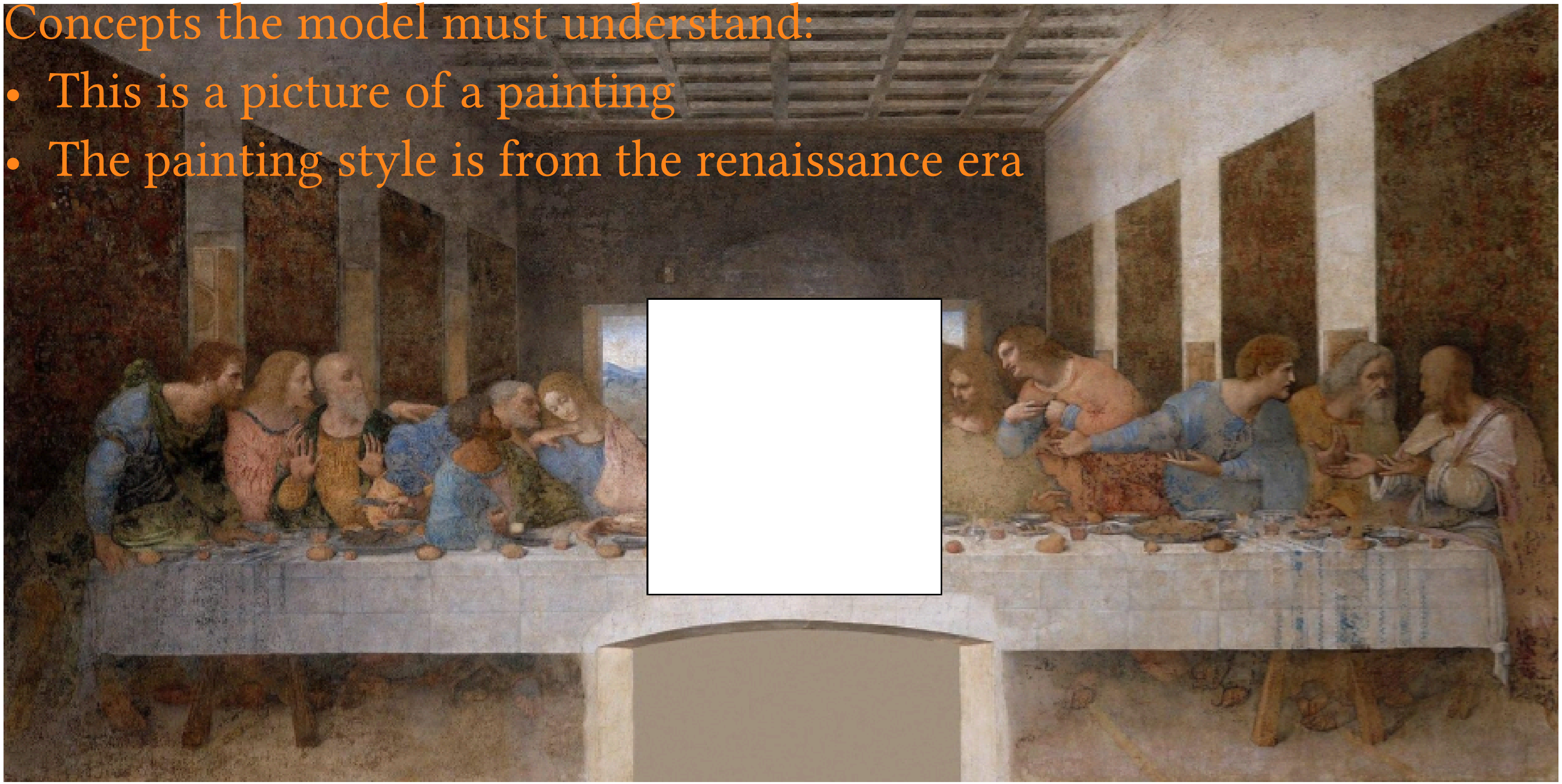


Unsupervised Training

Concepts the model must understand:

- This is a picture of a painting
- The painting style is from the renaissance era



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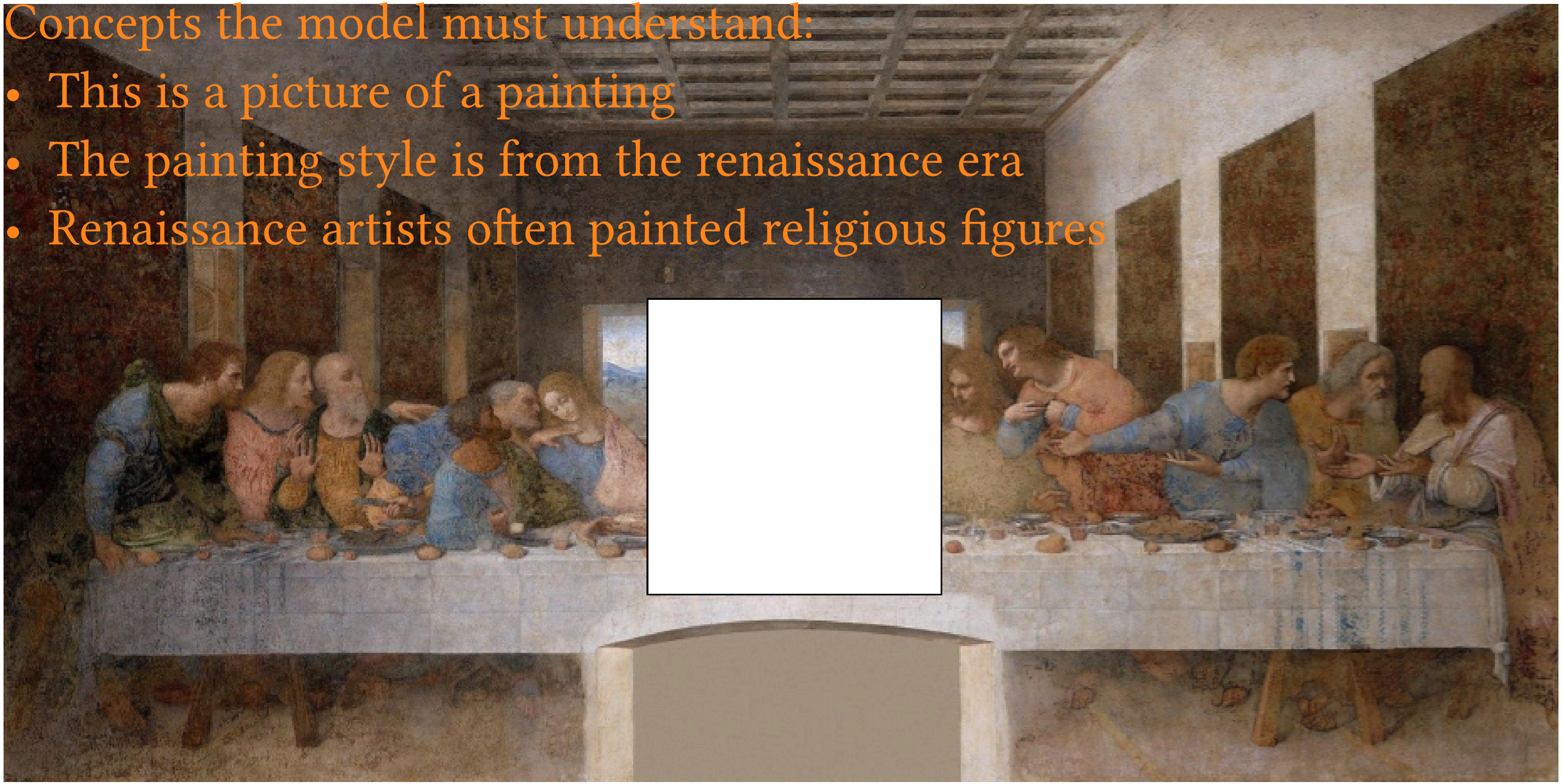

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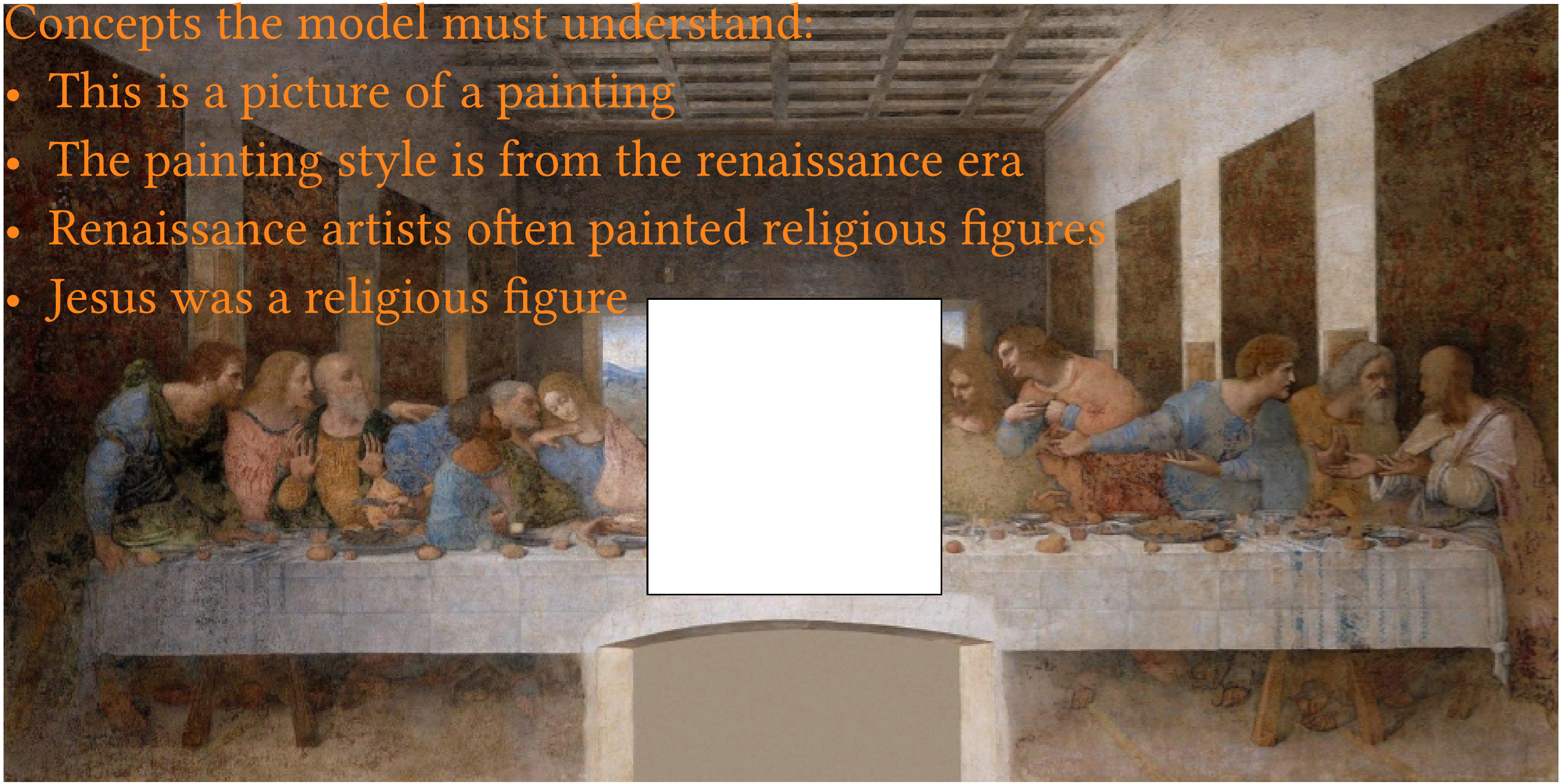

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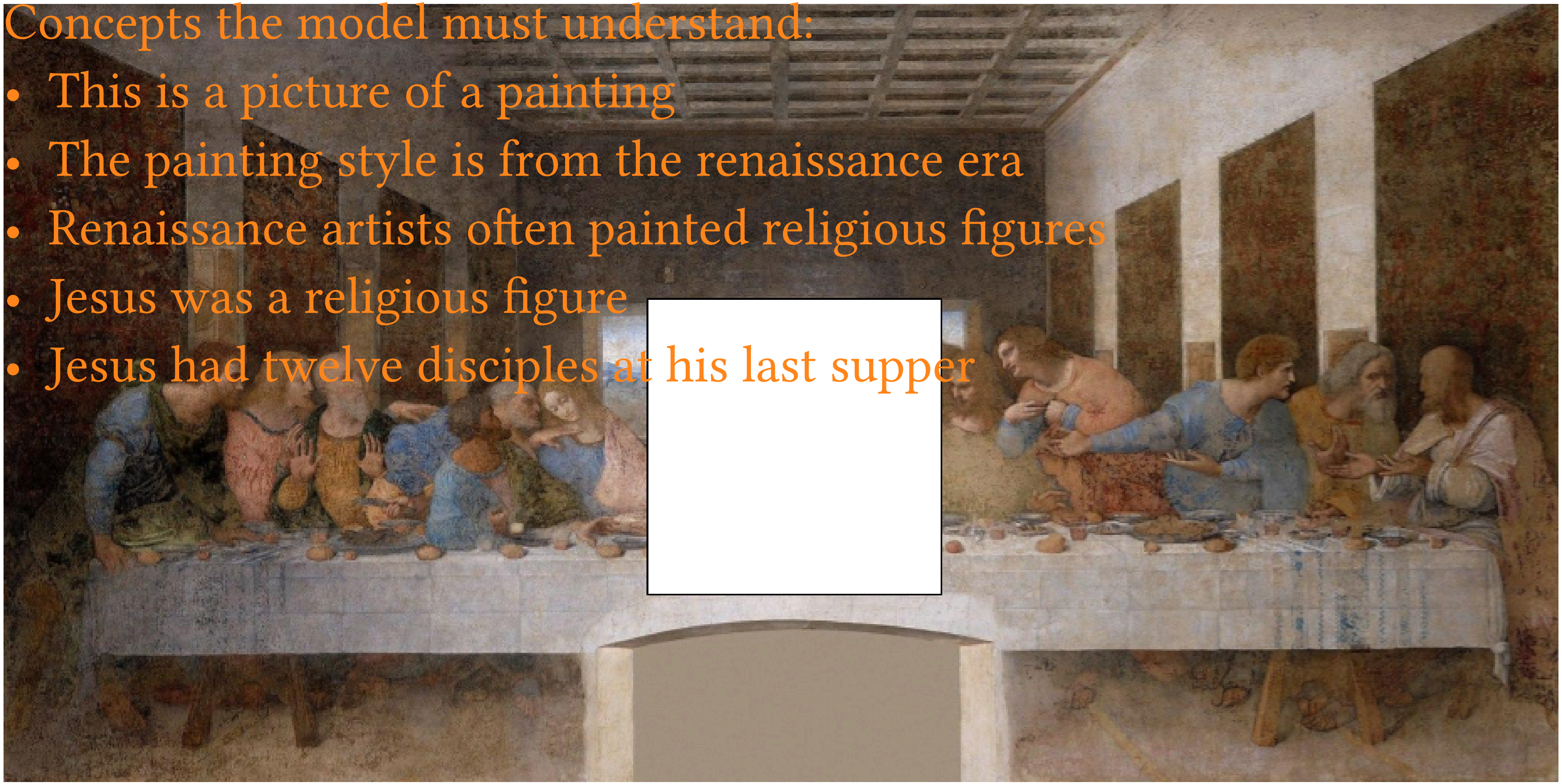

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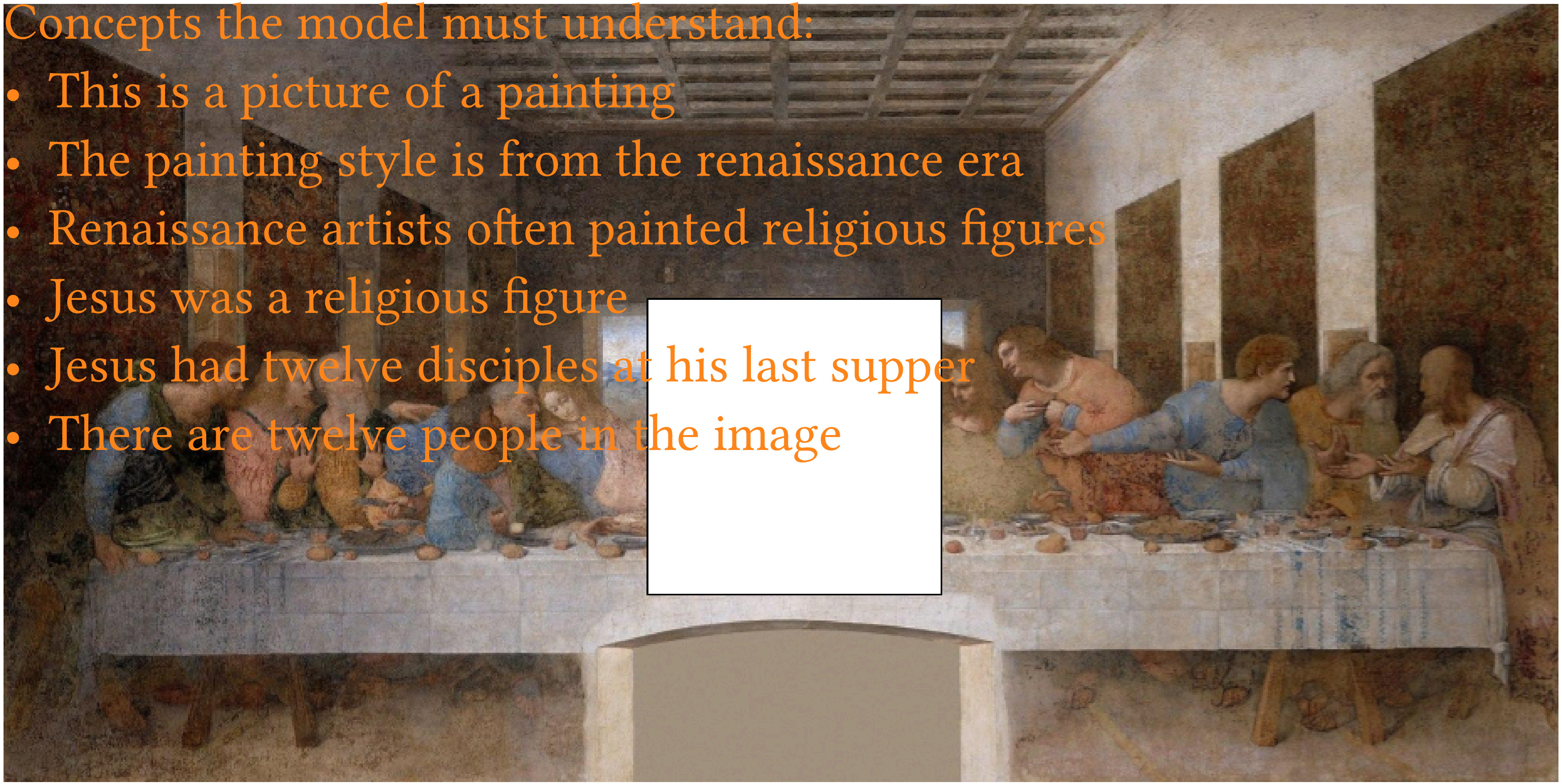

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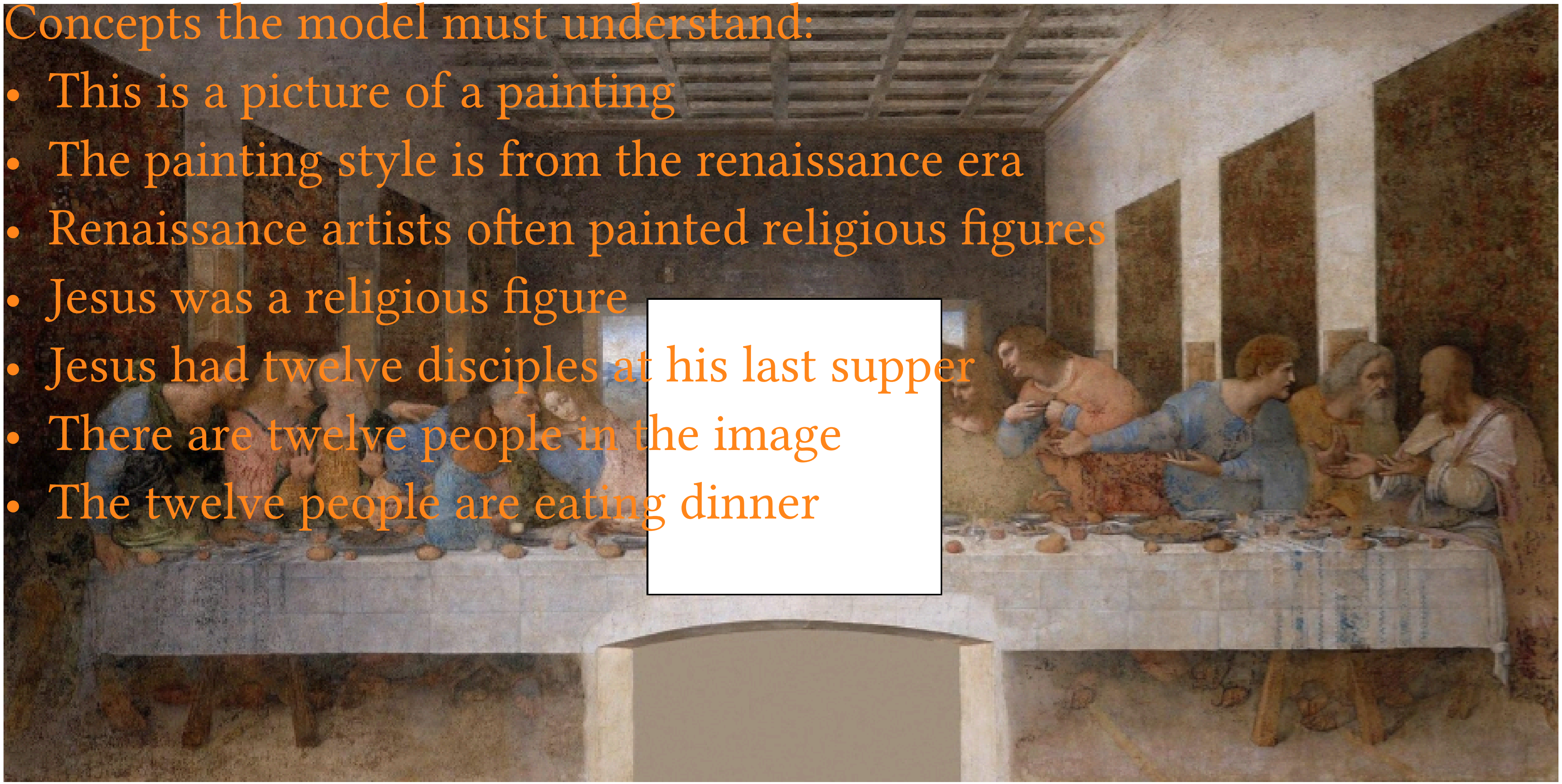

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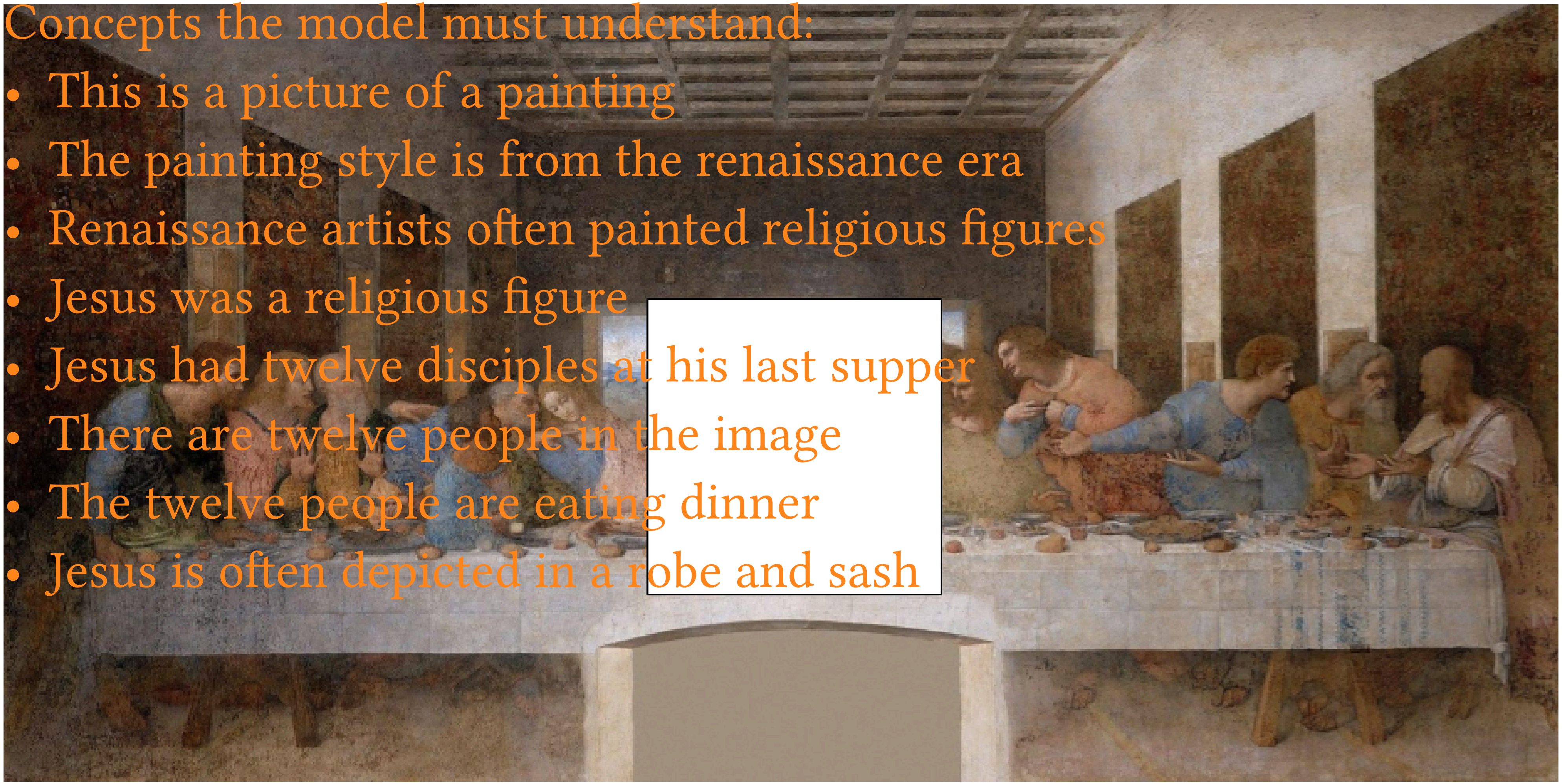

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- There are twelve people in the image
- The twelve people are eating dinner
- Jesus is often depicted in a robe and sash

Unsupervised Training

We train the model to fix the image

Unsupervised Training

We train the model to fix the image

To fix the image, the model must understand so much of our world

Unsupervised Training

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This is the power of generative pre-training

Unsupervised Training

We train the model to fix the image

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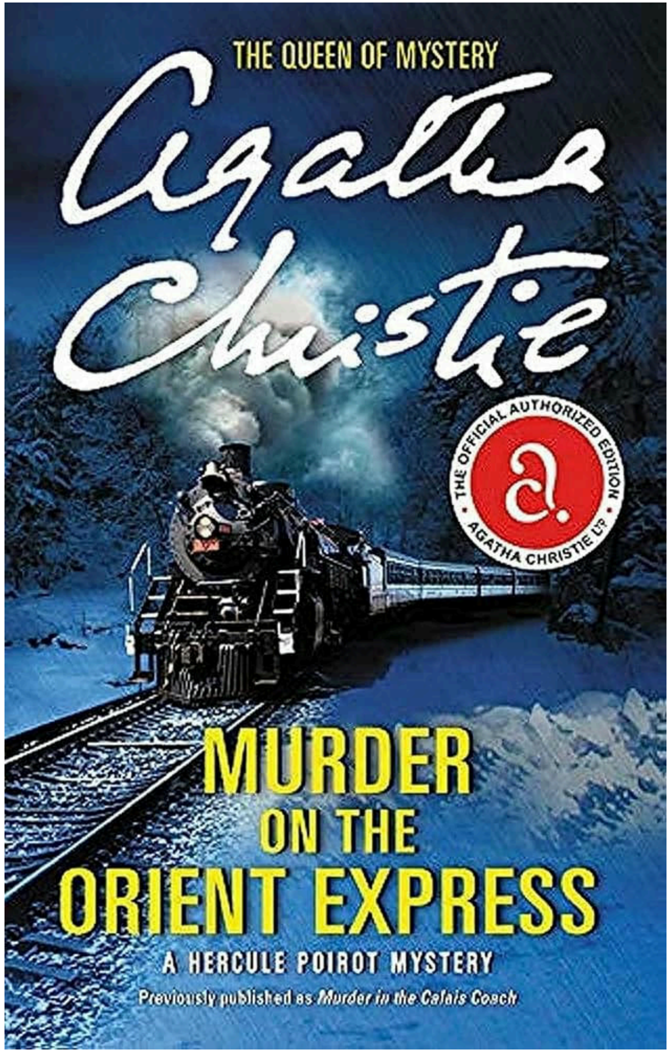
This is the power of generative pre-training

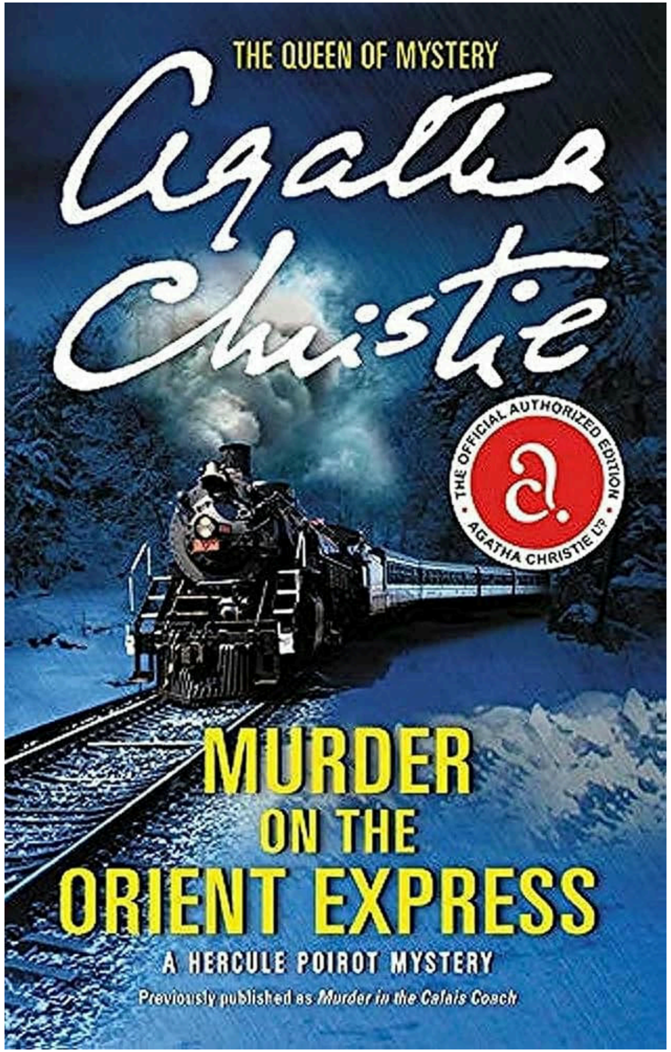
What about text transformers?

Unsupervised Training



This is a mystery novel

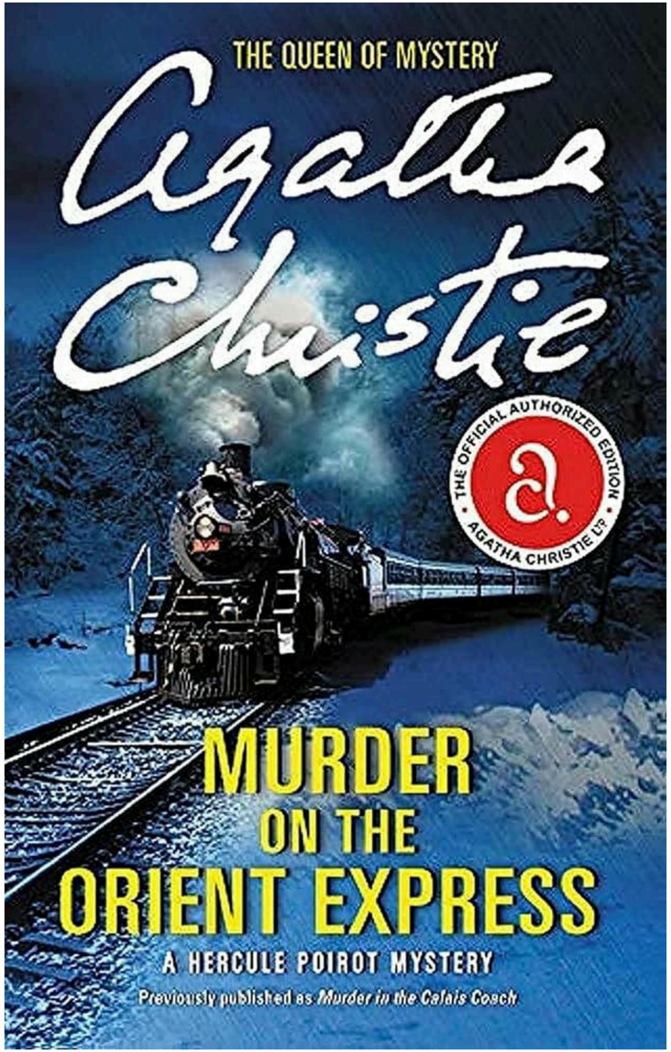
Unsupervised Training



This is a mystery novel

Clues, intrigue, murder, etc

Unsupervised Training

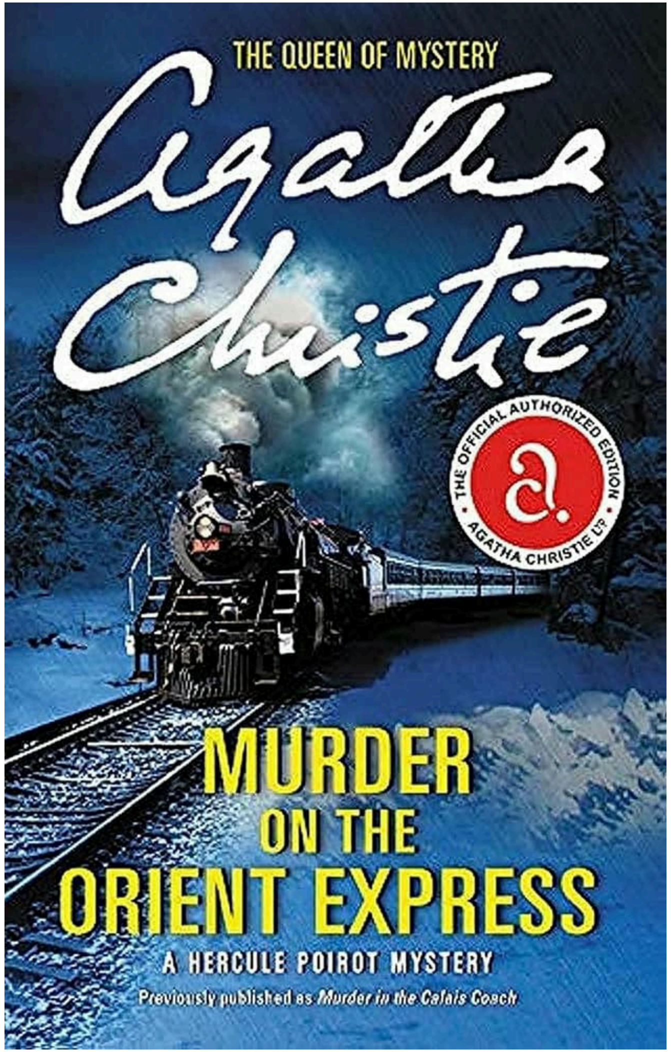


This is a mystery novel

Clues, intrigue, murder, etc

“Ah, said inspector Poirot, the murderer must be _________.”

Unsupervised Training



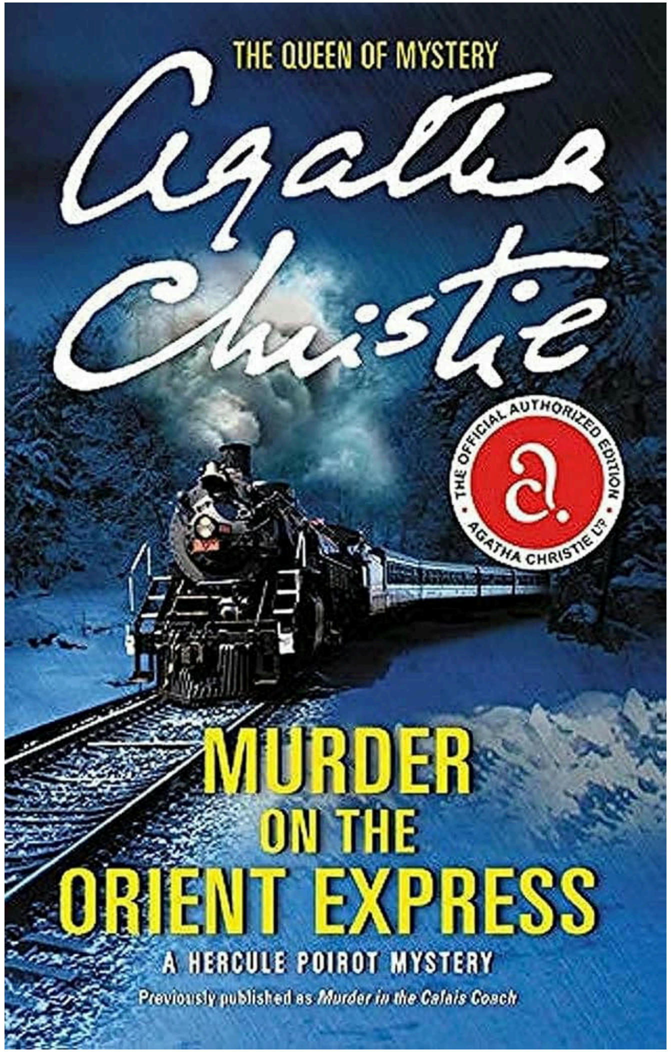
This is a mystery novel

Clues, intrigue, murder, etc

"Ah, said inspector Poirot, the murderer must be ________."

To complete the sentence, the model must understand:

Unsupervised Training



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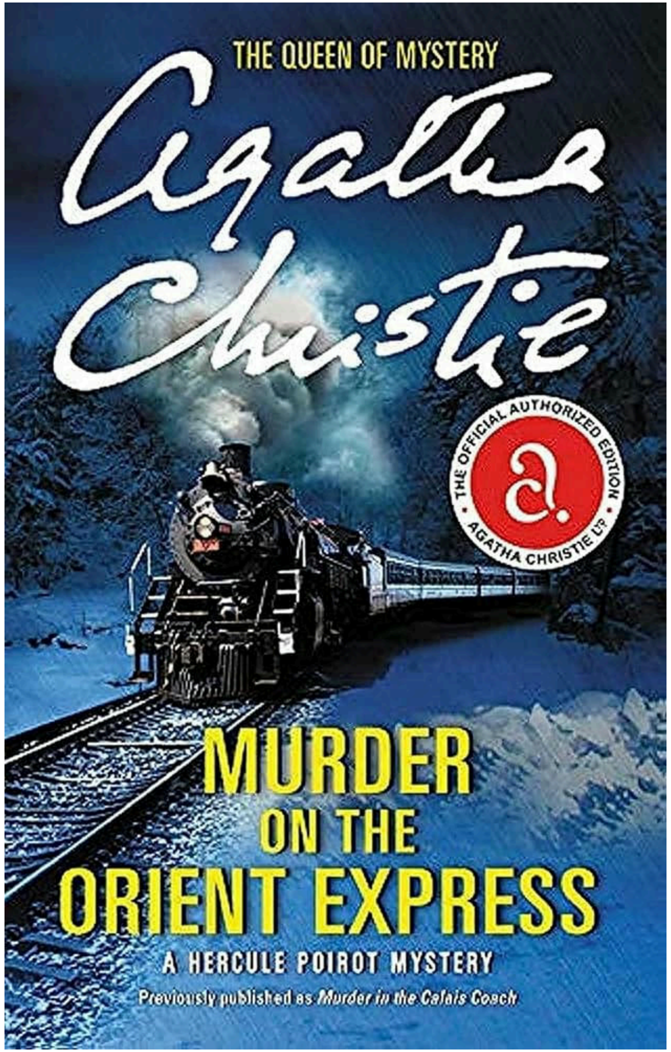

To complete the sentence, the model must understand:

- What a murder is

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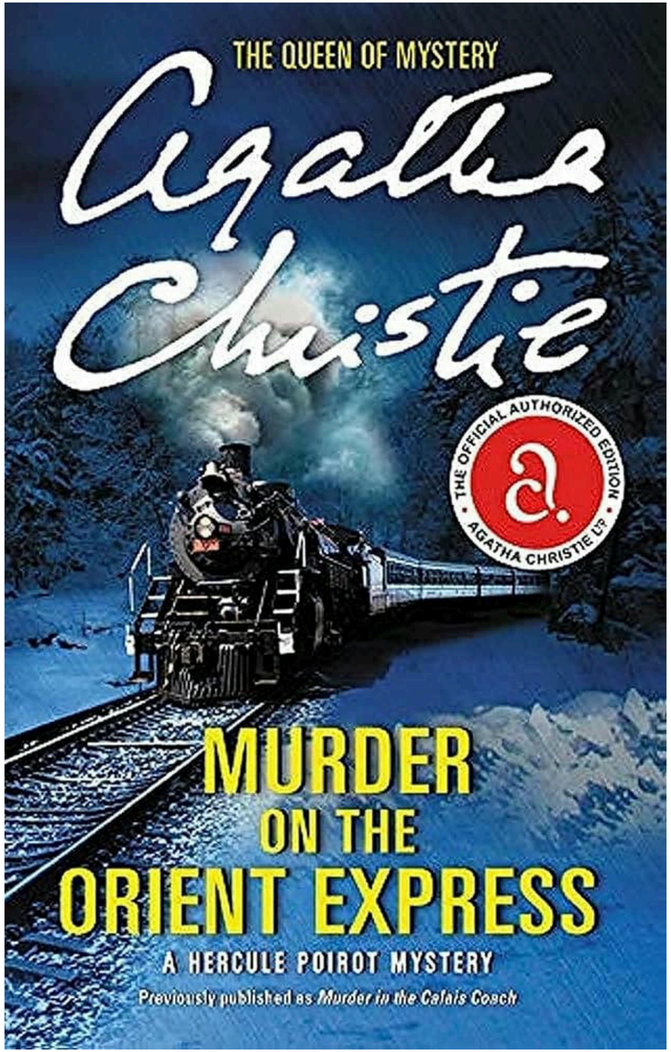
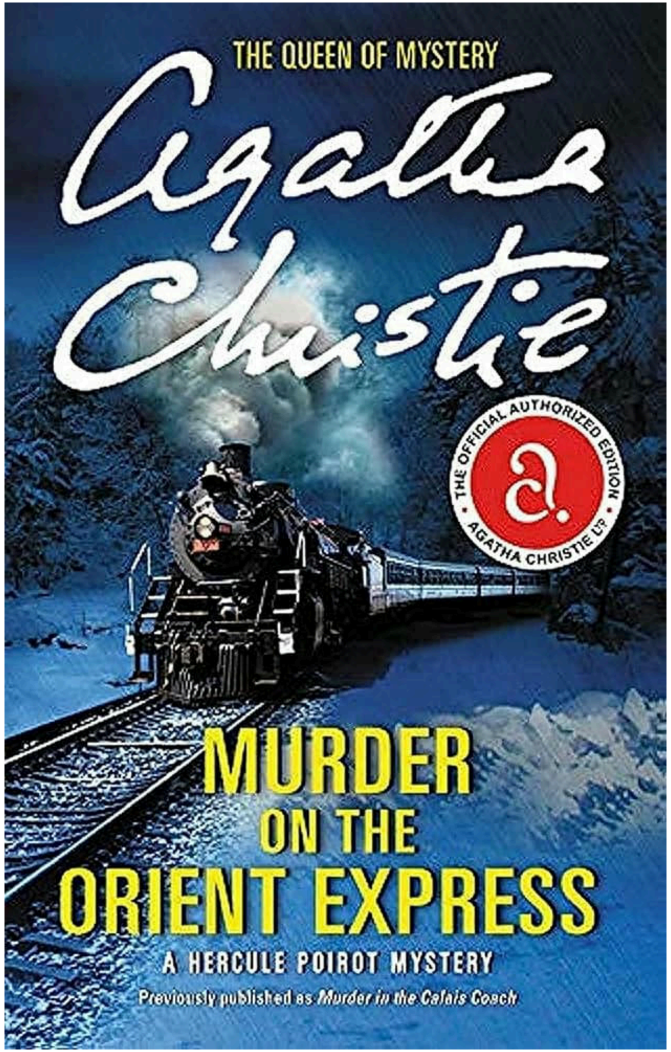


To complete the sentence, the model must understand:

- What a murder is
- What it means to be alive

Unsupervised Training



To complete the sentence, the model must understand:

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- Emotions like anger, jealousy, betrayal, love

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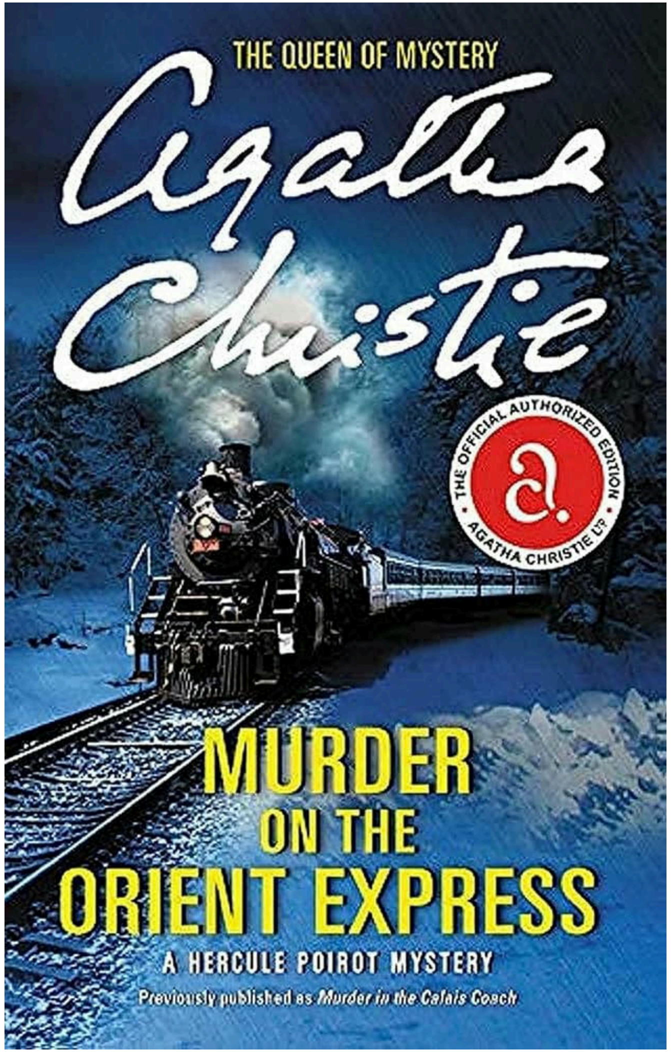
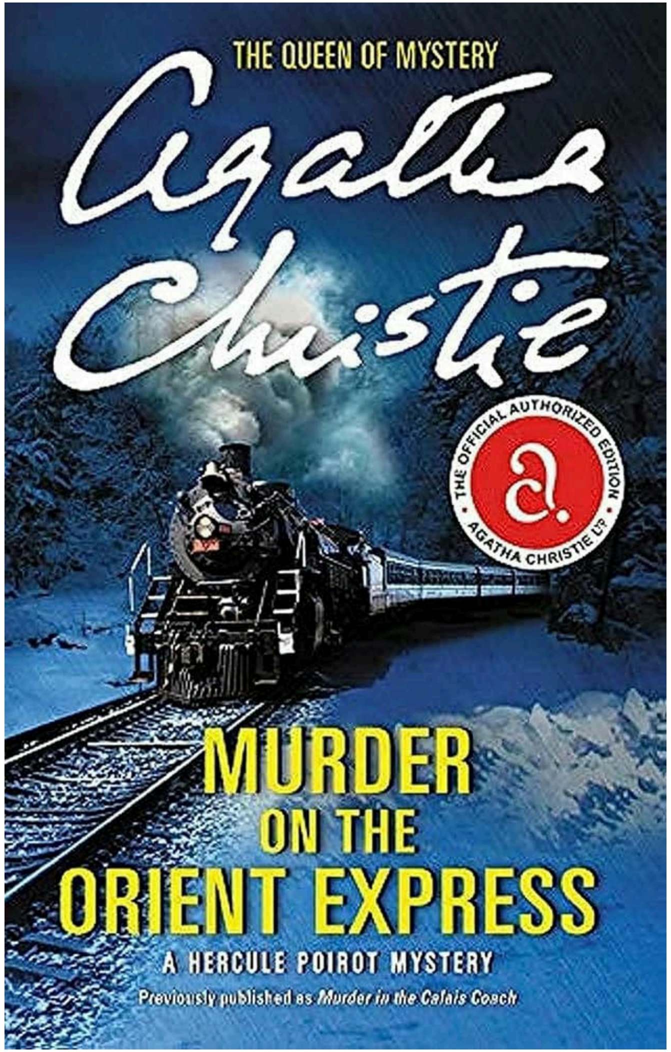


To complete the sentence, the model must understand:

- What a murder is
- What it means to be alive
- Emotions like anger, jealousy, betrayal, love
- Personalities of each character

Unsupervised Training



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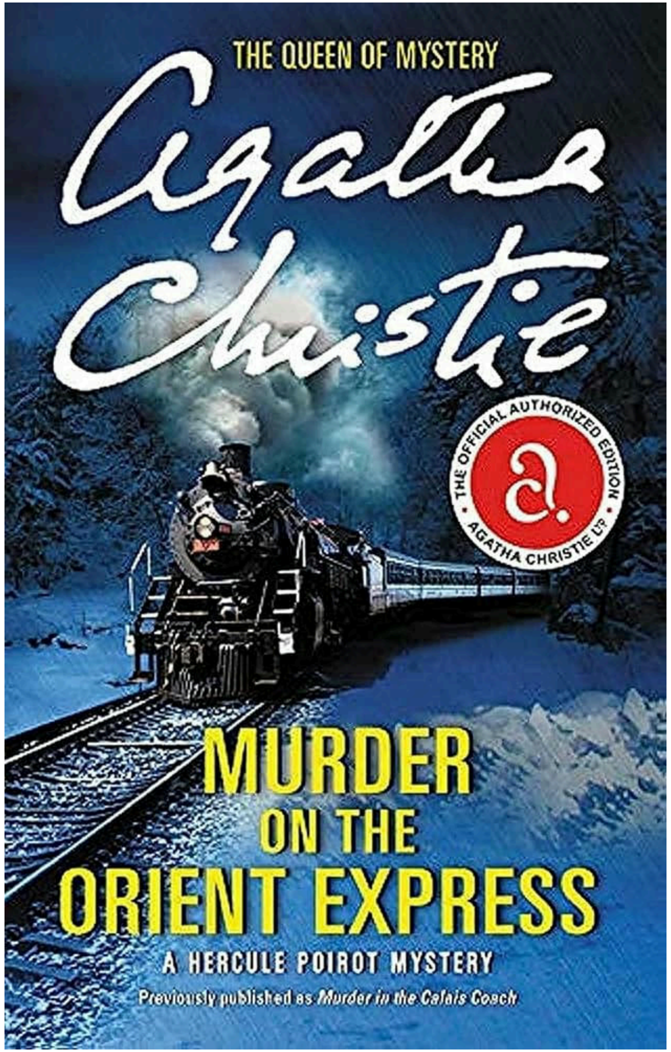
- Why a human would murder another human

Unsupervised Training



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- Why a human would murder another human
- How humans react to emotions

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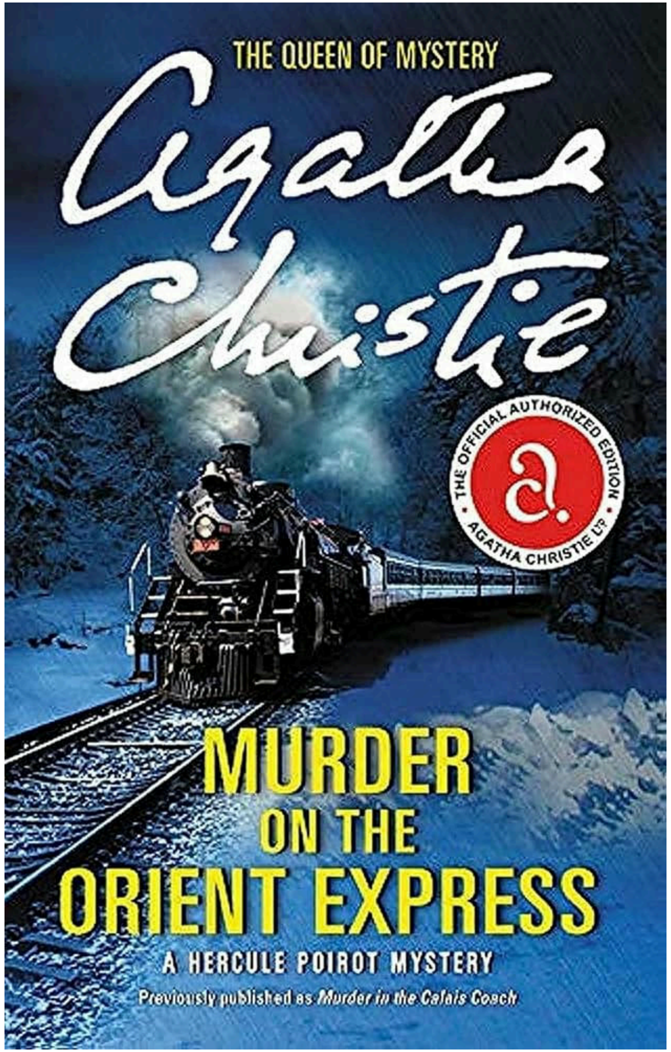
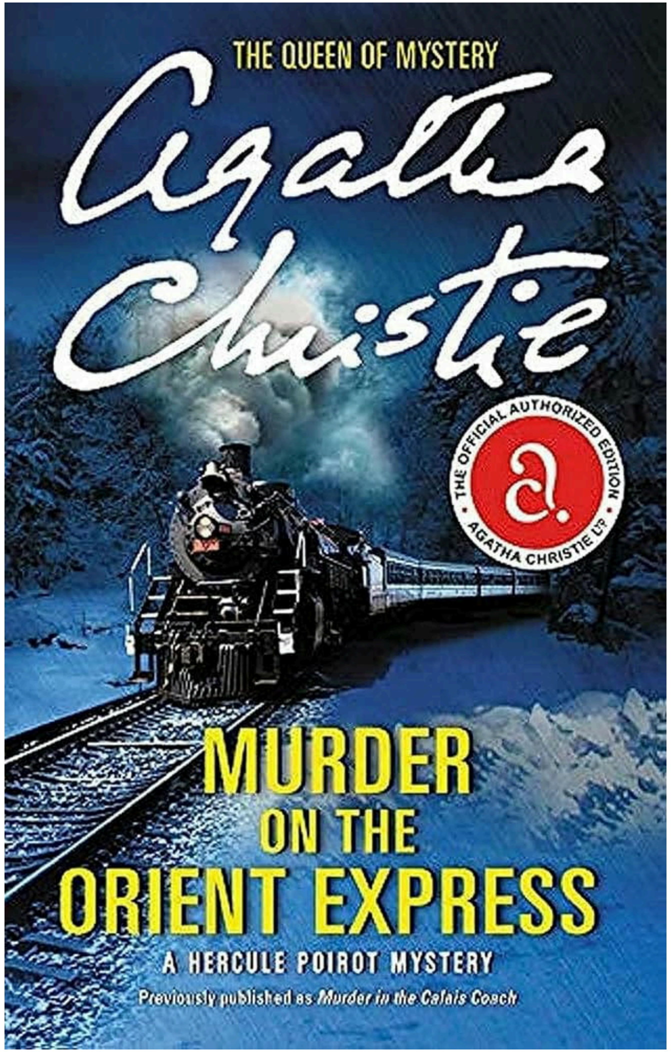


To complete the sentence, the model must understand:

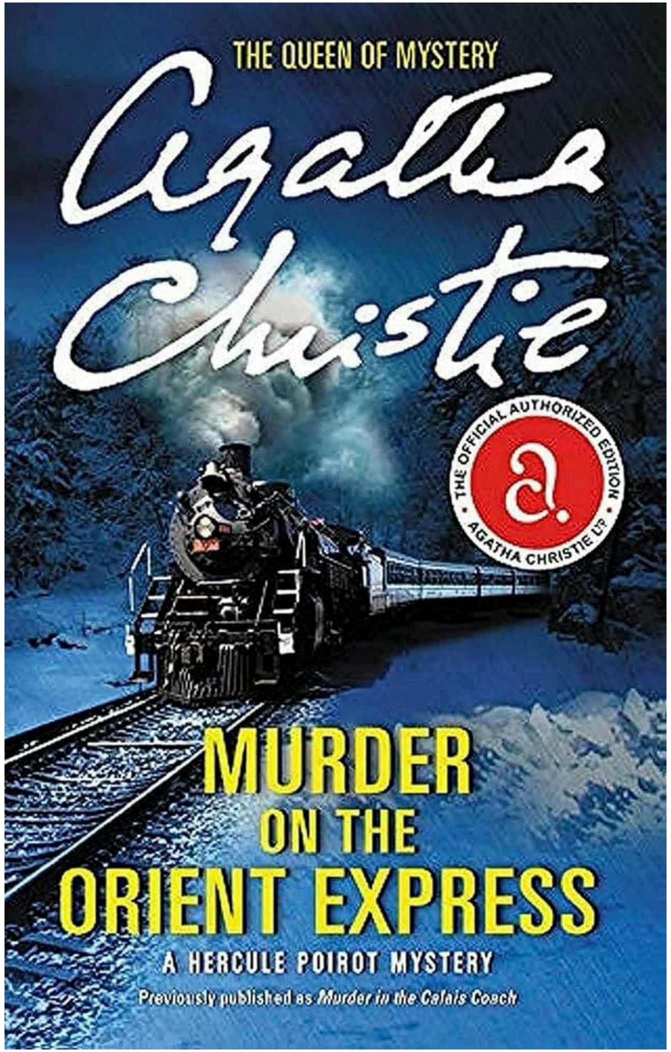
- What a murder is
- What it means to be alive
- Emotions like anger, jealousy, betrayal, love
- Personalities of each character
- Why a human would murder another human
- How humans react to emotions
- How to tell if someone lies

Unsupervised Training



To predict the murderer, the model must understand so much about humans and our society

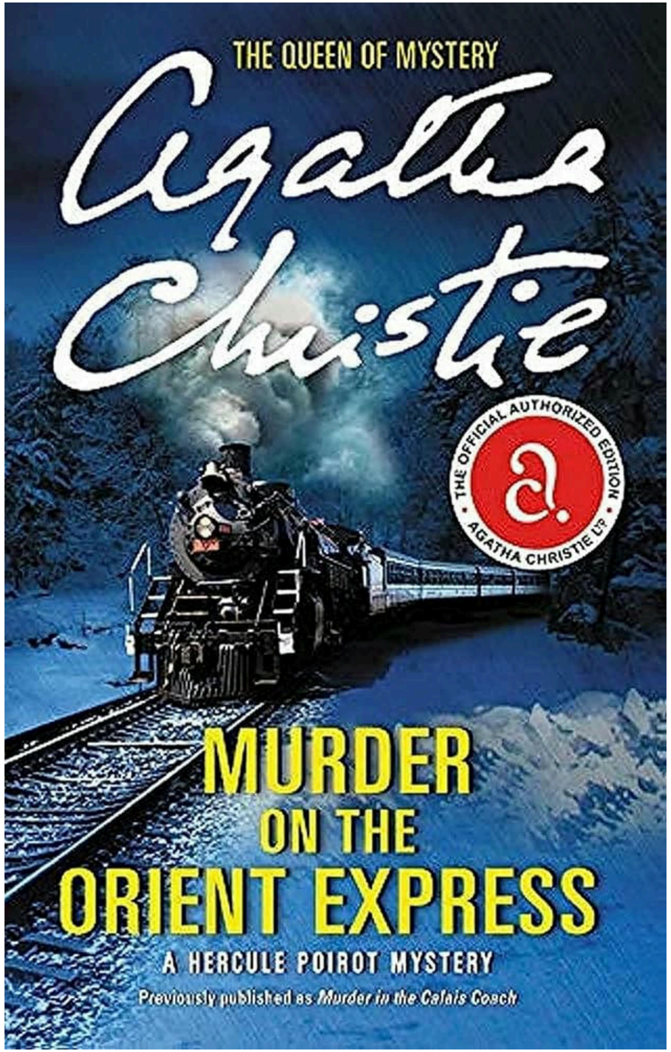
Unsupervised Training



To predict the murderer, the model must understand so much about humans and our society

The Books3 dataset contains 200,000 books

Unsupervised Training



To predict the murderer, the model must understand so much about humans and our society

The Books3 dataset contains 200,000 books

We train the model to predict the ending of all these books

Unsupervised Training

We can apply this same concept to:

Unsupervised Training

We can apply this same concept to:

- Predict missing base pairs in a strand of DNA

Unsupervised Training

We can apply this same concept to:

- Predict missing base pairs in a strand of DNA
- Predict missing audio from a song

Unsupervised Training

We can apply this same concept to:

- Predict missing base pairs in a strand of DNA
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- Predict the outcome of particle collisions at the Large Hadron Supercollider

Unsupervised Training

We can apply this same concept to:

- Predict missing base pairs in a strand of DNA
- Predict missing audio from a song
- Predict the outcome of particle collisions at the Large Hadron Supercollider

All we need is a large enough dataset!

Unsupervised Training

What if we put the model in a robot?

Unsupervised Training

What if we put the model in a robot?

Give the model what the robot sees and what the robot does

Unsupervised Training

What if we put the model in a robot?

Give the model what the robot sees and what the robot does

Predict what the robot will see next

Unsupervised Training

What if we put the model in a robot?

Give the model what the robot sees and what the robot does

Predict what the robot will see next

We call this a **world model**

Unsupervised Training

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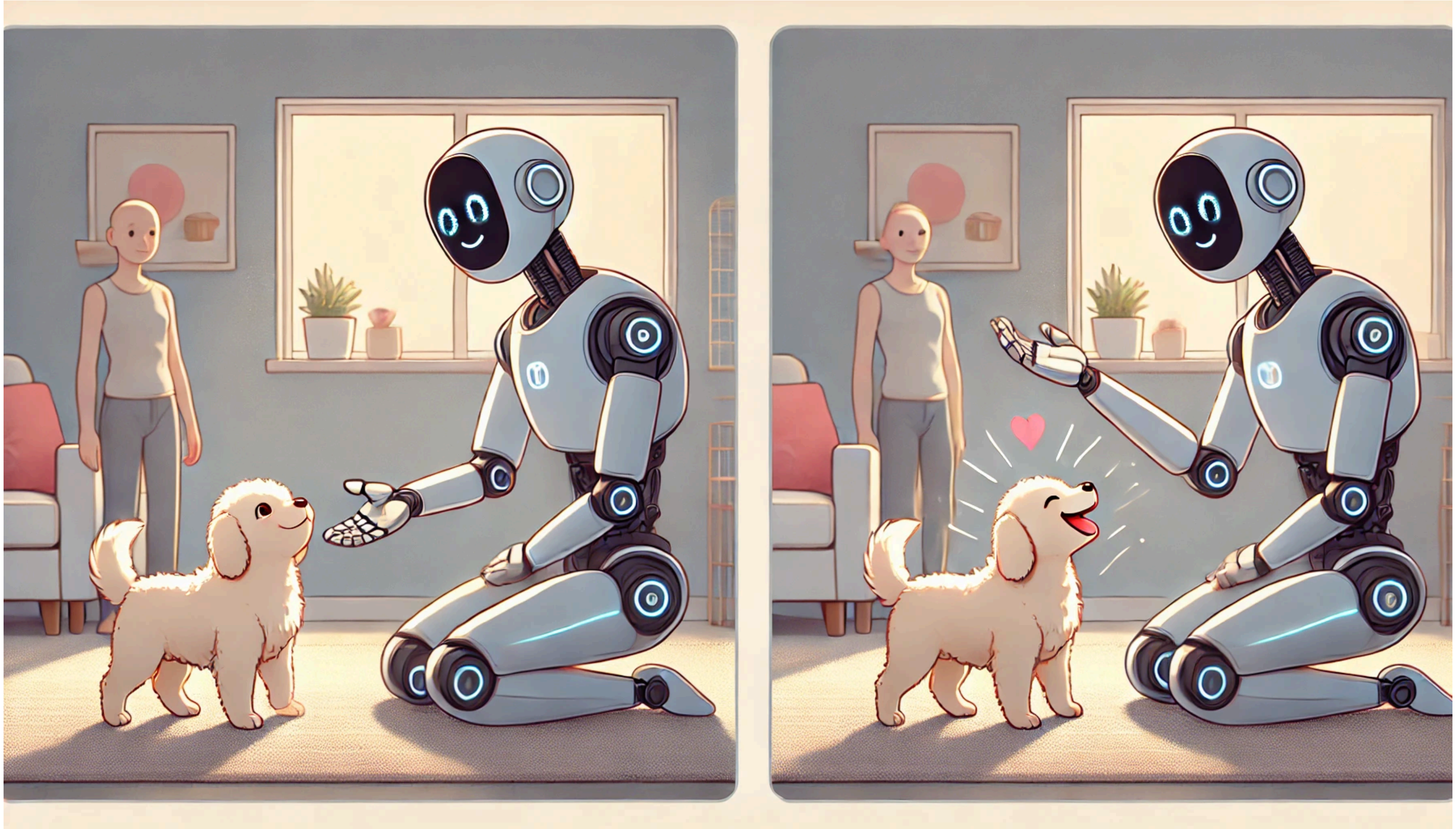

Unsupervised Training

Soon, I will apply for a grant to train a world model

Unsupervised Training

Soon, I will apply for a grant to train a world model

If I win, I will need help creating a robot dataset

Unsupervised Training

Soon, I will apply for a grant to train a world model

If I win, I will need help creating a robot dataset

I will need some humans to control our robots in the world

Unsupervised Training

Soon, I will apply for a grant to train a world model

If I win, I will need help creating a robot dataset

I will need some humans to control our robots in the world

If you are interested, give me your email after class

Unsupervised Training

These transformers learn and understand the structure of our world

Unsupervised Training

These transformers learn and understand the structure of our world

But their understanding is trapped

Unsupervised Training

These transformers learn and understand the structure of our world

But their understanding is trapped

They can only finish sentences or complete pictures

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How can we use this strong understanding to help humans?

Unsupervised Training

These transformers learn and understand the structure of our world

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How can we use this strong understanding to help humans?

- Identify pictures of cancer

Unsupervised Training

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How can we use this strong understanding to help humans?
- Identify pictures of cancer
- Make scientific discoveries

Unsupervised Training

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How can we use this strong understanding to help humans?

- Identify pictures of cancer
- Make scientific discoveries
- Minimize human suffering

Unsupervised Training

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But their understanding is trapped

They can only finish sentences or complete pictures

How can we use this strong understanding to help humans?

- Identify pictures of cancer
- Make scientific discoveries
- Minimize human suffering

Today, we use **reinforcement learning**

We will formally introduce reinforcement learning next lecture

Closing Remarks

Closing Remarks

This is the last in-person lecture

Closing Remarks

This is the last in-person lecture

I will record a video on reinforcement learning next week

Closing Remarks

This is the last in-person lecture

I will record a video on reinforcement learning next week

I will be here from 7:00PM on December 2 for questions/discussin on reinforcement learning

Closing Remarks

In this course, we started from Gauss in 1795

Closing Remarks

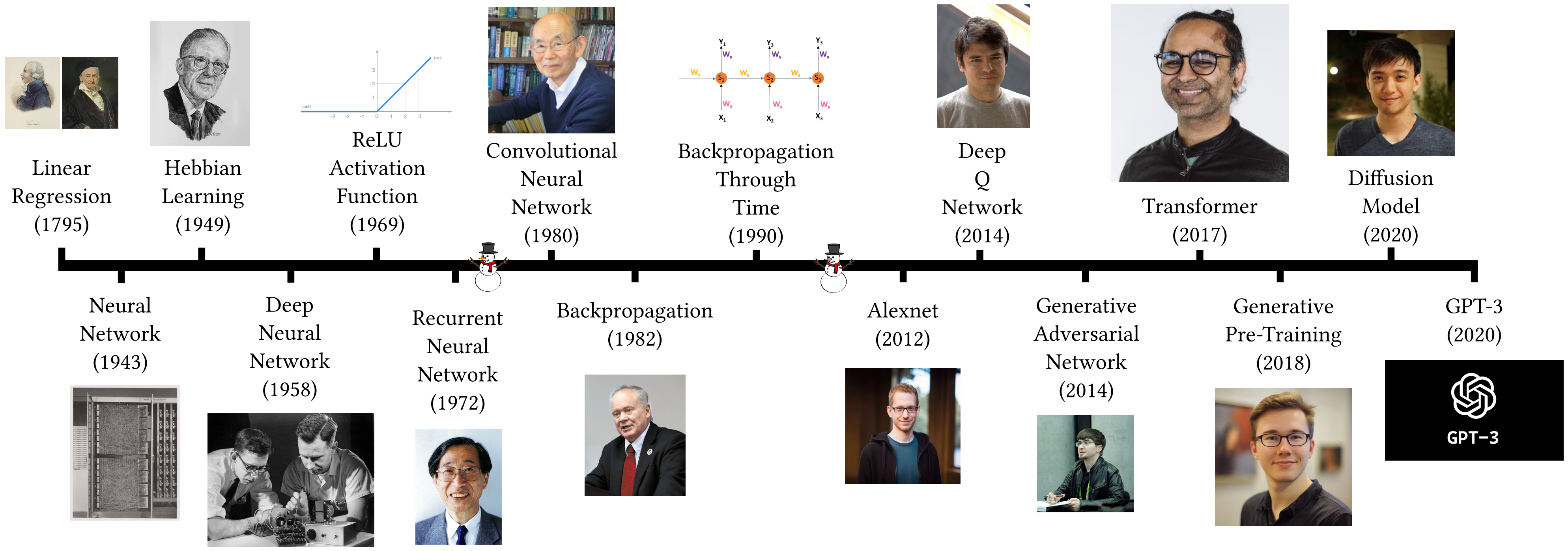
In this course, we started from Gauss in 1795

We built up concepts until we reached the modern age

Closing Remarks

In this course, we started from Gauss in 1795

We built up concepts until we reached the modern age



Closing Remarks

We learned about:

Closing Remarks

We learned about:

- Linear regression

Closing Remarks

We learned about:

- Linear regression
- Polynomial regression

Closing Remarks

We learned about:

- Linear regression
- Polynomial regression
- Biological neurons

Closing Remarks

We learned about:

- Linear regression
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- Artificial neurons

Closing Remarks

We learned about:

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- Polynomial regression
- Biological neurons
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- Perceptrons

Closing Remarks

We learned about:

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- Backpropagation

Closing Remarks

We learned about:

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- Gradient descent

Closing Remarks

We learned about:

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- Classification

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We learned about:

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- Classification
- Parameter initialization

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We learned about:

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- Many activation functions

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- Stochastic gradient descent

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- Variational autoencoders

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- Graph neural networks
- Attention and transformers

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- Autoencoders
- Variational autoencoders
- Graph neural networks
- Attention and transformers
- Generative pre-training

Closing Remarks

There are many more topics to cover!

Closing Remarks

There are many more topics to cover!

Now, you have the tools to study deep learning

Closing Remarks

There are many more topics to cover!

Now, you have the tools to study deep learning

You have the tools to train neural networks for real problems

Closing Remarks

In the first lecture, I asked everyone in this class for something

Closing Remarks

In the first lecture, I asked everyone in this class for something

Question: Do you remember what it was?

Closing Remarks

Deep learning is a powerful tool

Closing Remarks

Deep learning is a powerful tool

Like all powerful tools, deep learning can be used for good or evil

Closing Remarks

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Like all powerful tools, deep learning can be used for good or evil

- COVID-19 vaccine

Closing Remarks

Deep learning is a powerful tool

Like all powerful tools, deep learning can be used for good or evil

- COVID-19 vaccine
- Creating art

Closing Remarks

Deep learning is a powerful tool

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- COVID-19 vaccine
- Creating art
- Autonomous driving

Closing Remarks

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Like all powerful tools, deep learning can be used for good or evil

- COVID-19 vaccine
- Creating art
- Autonomous driving

- Making DeepFakes

Closing Remarks

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- COVID-19 vaccine
- Creating art
- Autonomous driving

- Making DeepFakes
- Weapon guidance systems

Closing Remarks

Deep learning is a powerful tool

Like all powerful tools, deep learning can be used for good or evil

- COVID-19 vaccine
- Creating art
- Autonomous driving

- Making DeepFakes
- Weapon guidance systems
- Discrimination

Closing Remarks

Deep learning is a powerful tool

Like all powerful tools, deep learning can be used for good or evil

- COVID-19 vaccine
- Creating art
- Autonomous driving

- Making DeepFakes
- Weapon guidance systems
- Discrimination

Before training a model, think about whether it is good or bad for the world

Course Evaluation

Course Evaluation

Department instructed me to ask you for course feedback

Course Evaluation

Department instructed me to ask you for course feedback

We take this feedback seriously

Course Evaluation

Department instructed me to ask you for course feedback

We take this feedback seriously

Your feedback will impact future courses (and my job)

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If you like the course, please say it!

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If you like the course, please say it!

Please be specific on what you like and do not like

Course Evaluation

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We take this feedback seriously

Your feedback will impact future courses (and my job)

If you like the course, please say it!

Please be specific on what you like and do not like

Your likes/dislikes will change your future courses

Course Evaluation

I must leave the room to let you fill out this form

Course Evaluation

I must leave the room to let you fill out this form

Please scan the QR code and complete the survey

Course Evaluation

I must leave the room to let you fill out this form

Please scan the QR code and complete the survey

Department has suggested 10 minutes

Course Evaluation

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Department has suggested 10 minutes

I will return in 10 minutes

Course Evaluation

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I will return in 10 minutes

https://isw.um.edu.mo/siaweb

Course Evaluation

If you participated in class come see me after class

Course Evaluation

If you participated in class come see me after class

- Answered a question

Course Evaluation

If you participated in class come see me after class

- Answered a question
- Asked a question