Offline RL

CISC 7404 - Decision Making

Steven Morad

University of Macau

Admin

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Final quiz on 24 April (2 weeks), format subject to change

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- 1 question fundamental RL (V/Q/PG)

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Homework 2 grading deadline next Wednesday

Review

Offline RL

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Review: In on-policy RL, each iteration we collect a **new** dataset using our policy

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In off-policy RL, each iteration we **update** our dataset using our policy

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In off-policy RL, each iteration we **update** our dataset using our policy

In imitation learning, we are given a fixed **expert** dataset

Question: Did you find imitation learning interesting? Why?

Offline RL

Imitation learning from a fixed dataset is attractive for many reasons

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Question: Can we learn policies from fixed datasets that do better than the experts?

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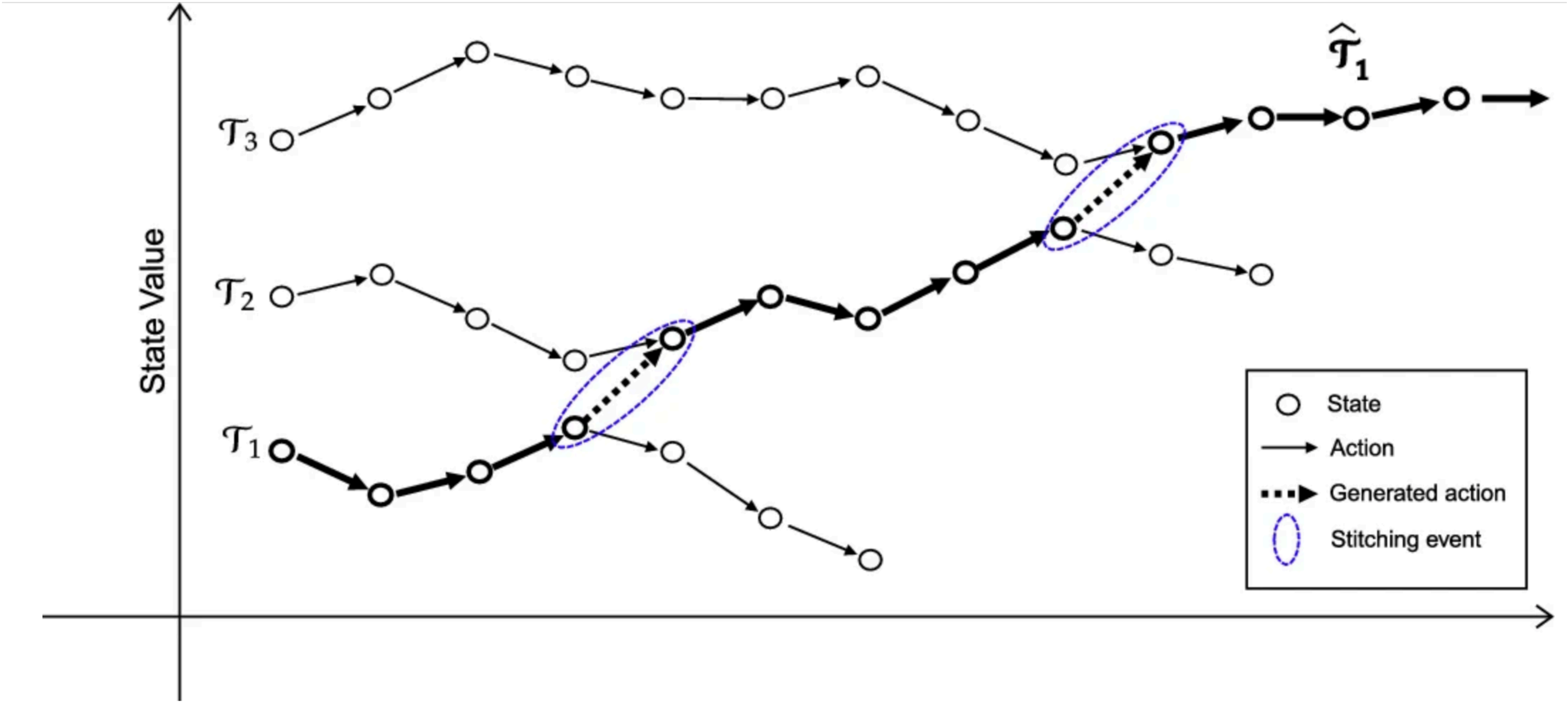
- How is this possible without exploration?

In imitation learning, we learn to imitate dataset trajectories

In offline RL, we learn to **stitch** together subtrajectories into optimal trajectories

Offline RL



State Value
$\mathcal{T}_3$
$\mathcal{T}_2$
$\mathcal{T}_1$
$\hat{\mathcal{T}}_1$
State
Action
Generated action
Stitching event

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I will also present my work on offline RL at ICRA next month

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https://sites.google.com/view/llm-marl/home

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$$(S, A, \mathcal{R}, \mathrm{Tr}, \gamma, \boldsymbol{X})$$

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$$\mathcal{R}\left(s_{t+1}\right) =?$$

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- Improve behavior cloning with rewards

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Let us begin with behavior cloning first

Behavioral Cloning with Rewards

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Recall the behavior cloning objective

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$$\arg\min_{\theta_\pi} \sum_{s \in \boldsymbol{X}} \mathrm{KL}\big(\pi\big(a \mid s; \theta_\beta\big), \pi\big(a \mid s; \theta_\pi\big)\big)$$

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$$\arg\min_{\theta_\pi} \sum_{s \in \boldsymbol{X}} \sum_{a \in A} -\pi\left(a \mid s; \theta_\beta\right) \log \pi\left(a \mid s; \theta_\pi\right)$$

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- Two possible actions $A = \{a_+, a_-\}$

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Expert must behave better in one state than the other!

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Behavioral Cloning with Rewards

$$\pi(a_+ \mid s_0; \theta_\beta) = 0.5$$

$$\pi(a_- \mid s_0; \theta_\beta) = 0.5$$

Behavioral Cloning with Rewards

$$\pi\left(a_{+} \mid s_0; \theta_\beta\right) = 0.5 \qquad \pi\left(a_{-} \mid s_0; \theta_\beta\right) = 0.5$$

Question: Which action is better behavior?

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Answer: Randomly choose $a \in \{a_+, a_-\}$

Question: Is this a good idea?

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Question: We know which action is better, how can we measure this?

Answer: Reward! In BC, no reward. In offline RL, we have reward!

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Expert has equal probability for both good and bad actions

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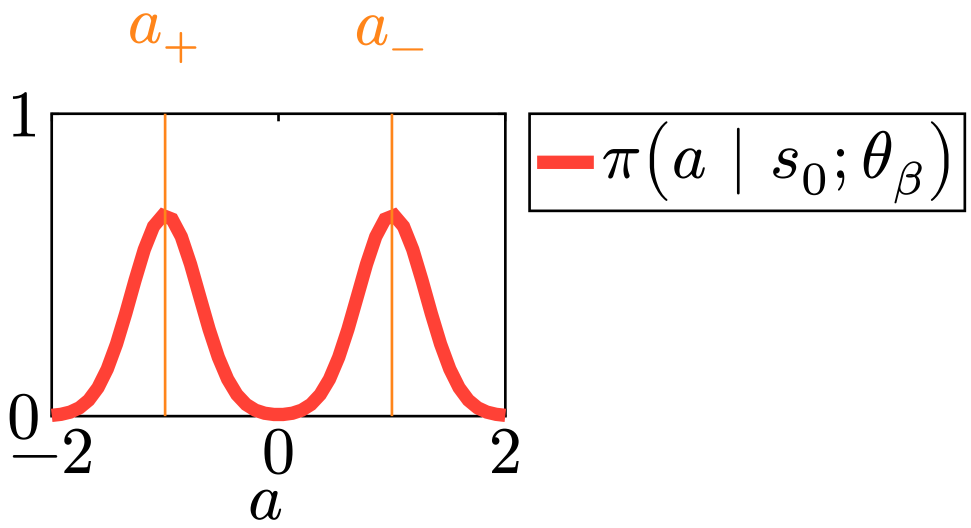

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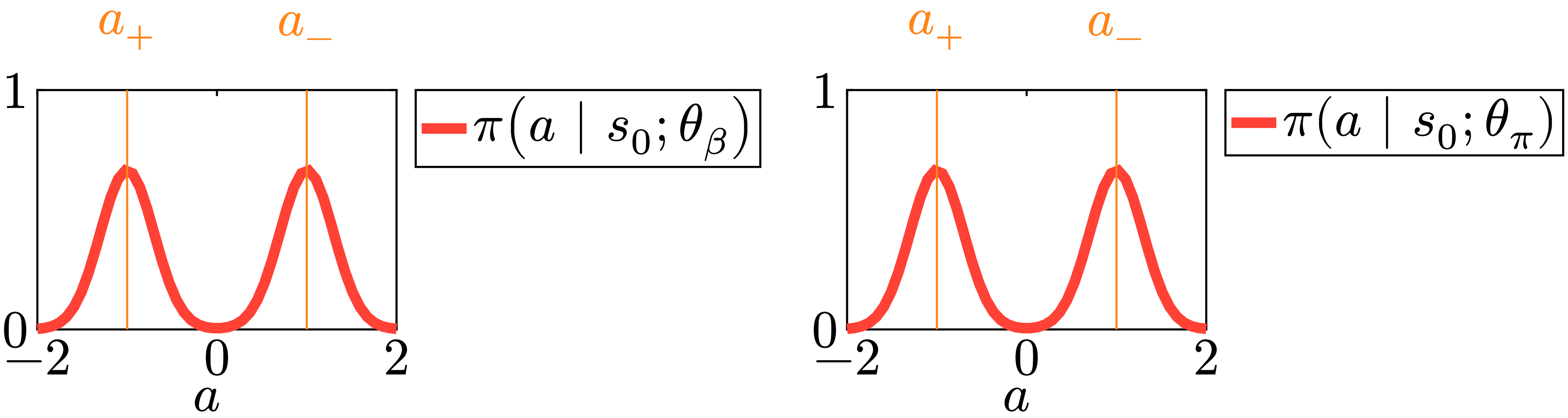

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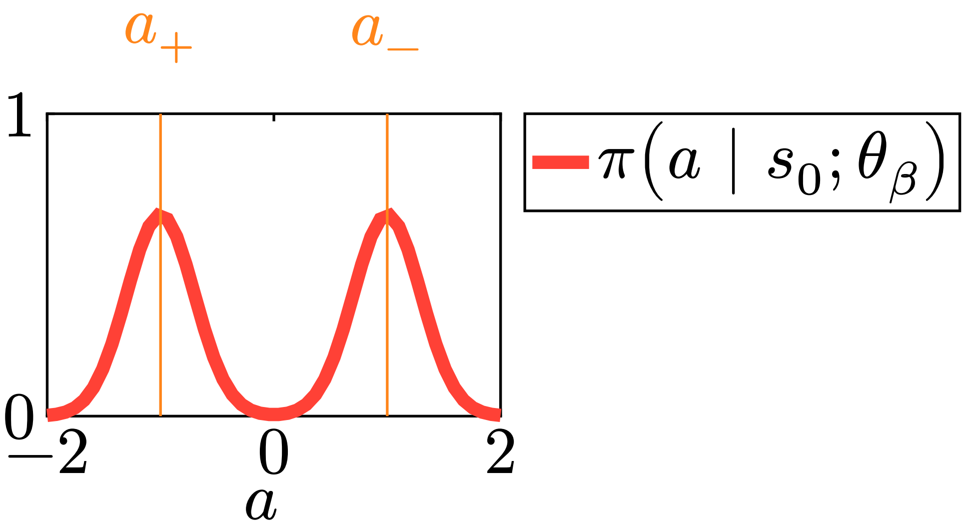

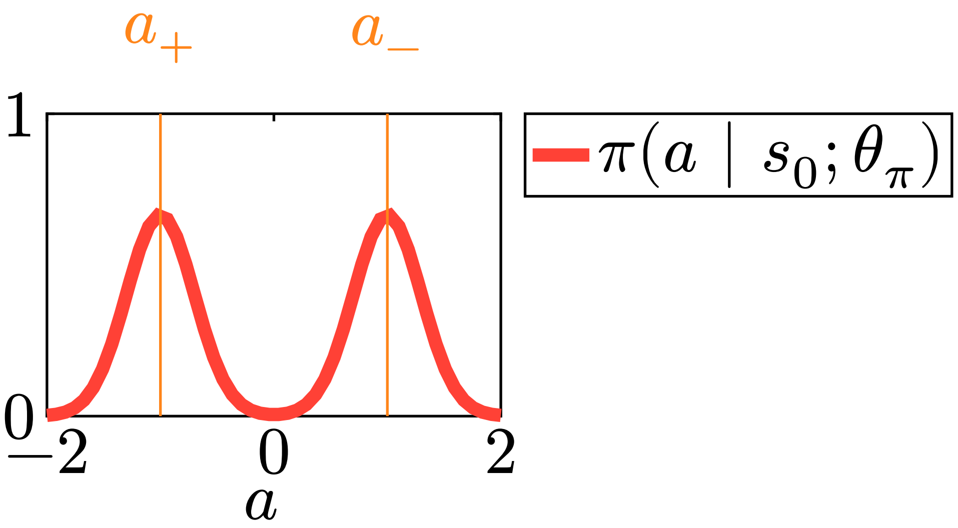

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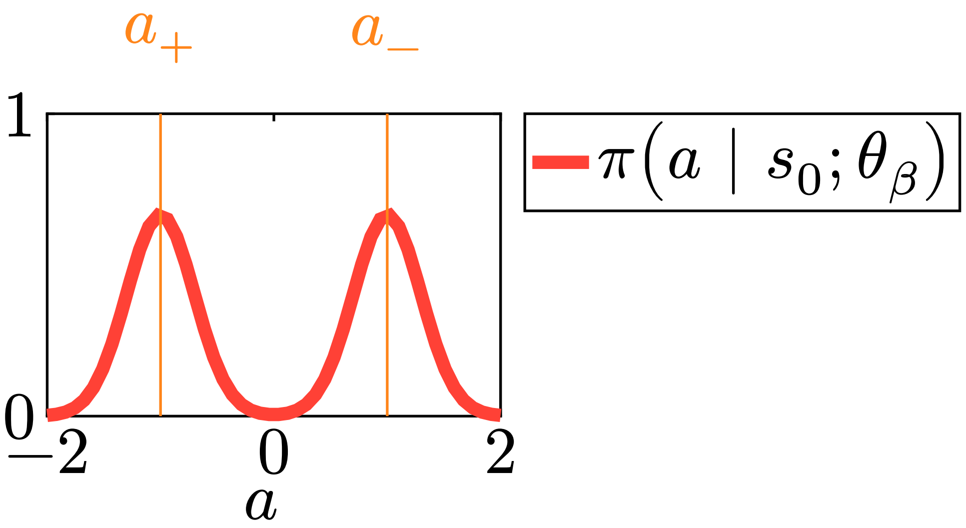

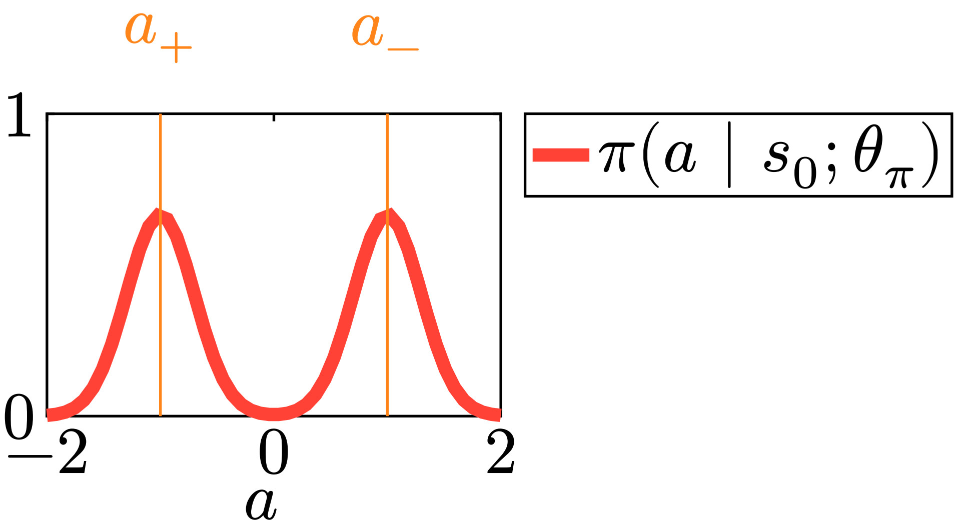

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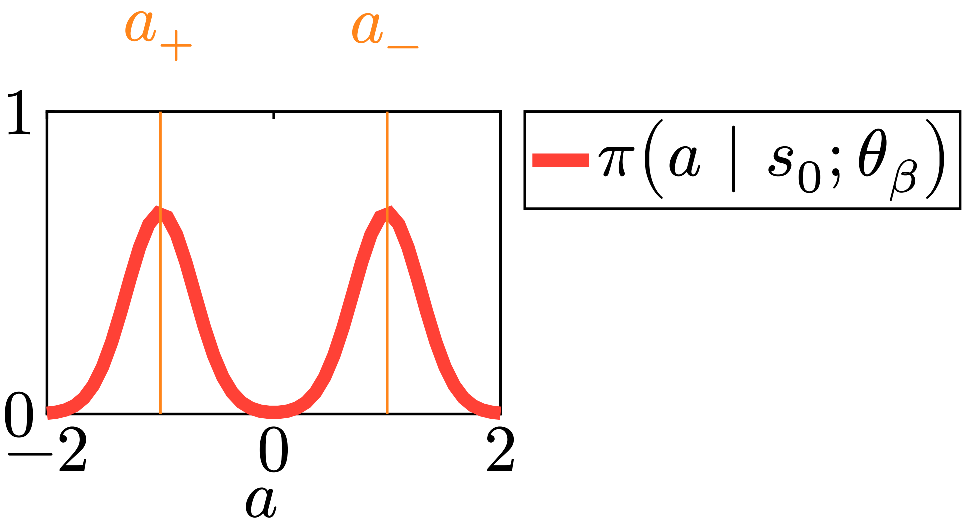
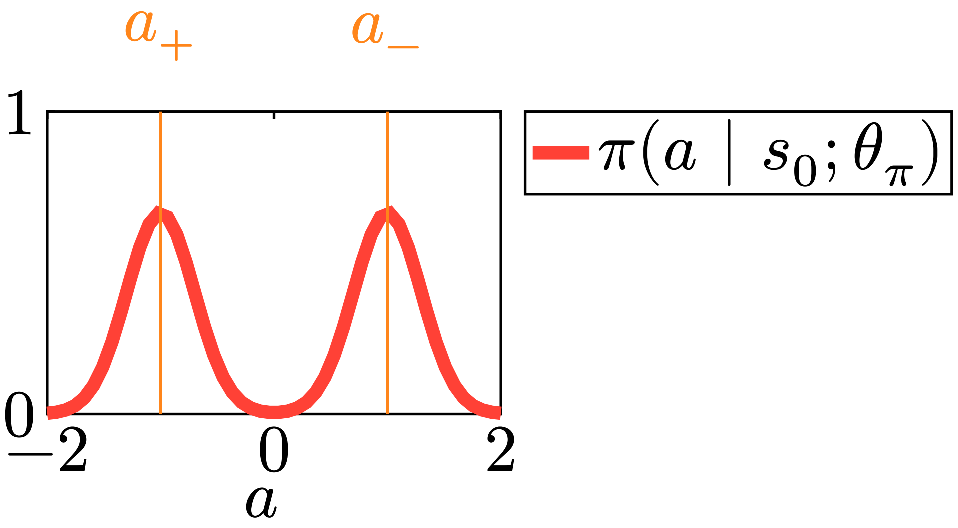
With offline RL, we have empirical rewards/returns, we can do better!

$$\hat{\mathbb{E}}[\mathcal{R}(s_1) \mid s_0, a_0] \qquad \hat{\mathbb{E}}\left[\mathcal{G}(\boldsymbol{\tau}) \mid s_0; \theta_\beta\right]$$

Behavioral Cloning with Rewards





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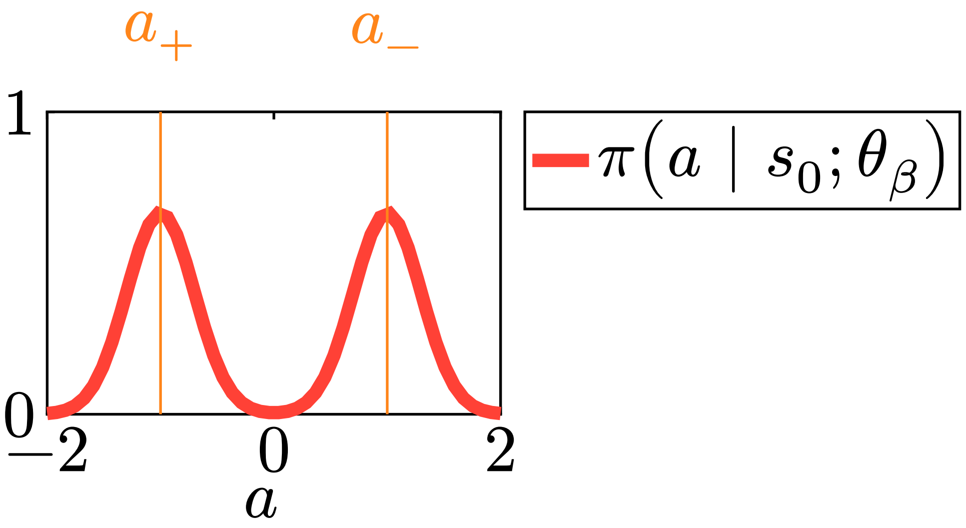

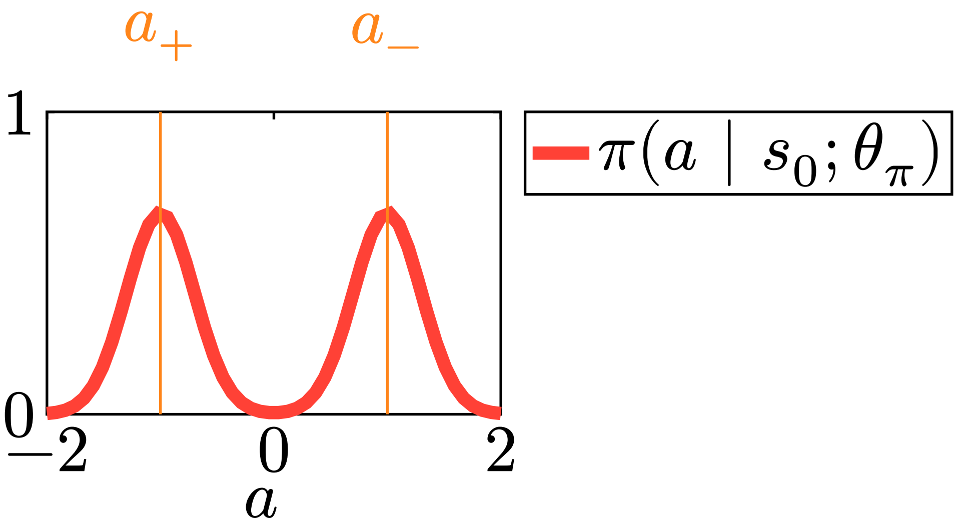

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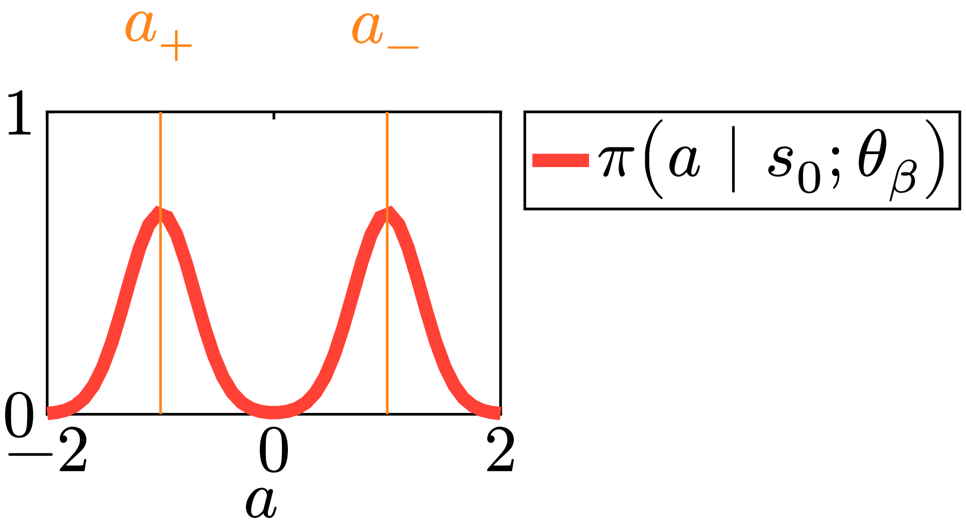
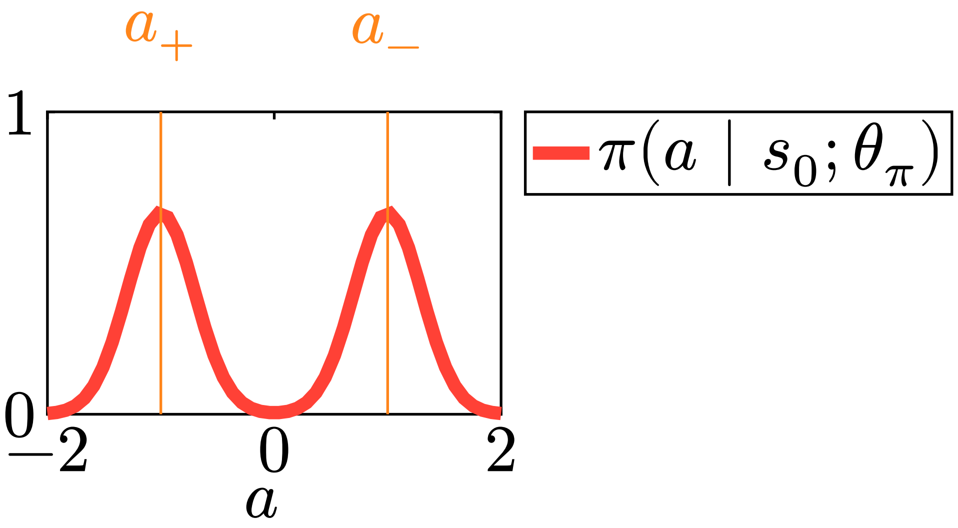


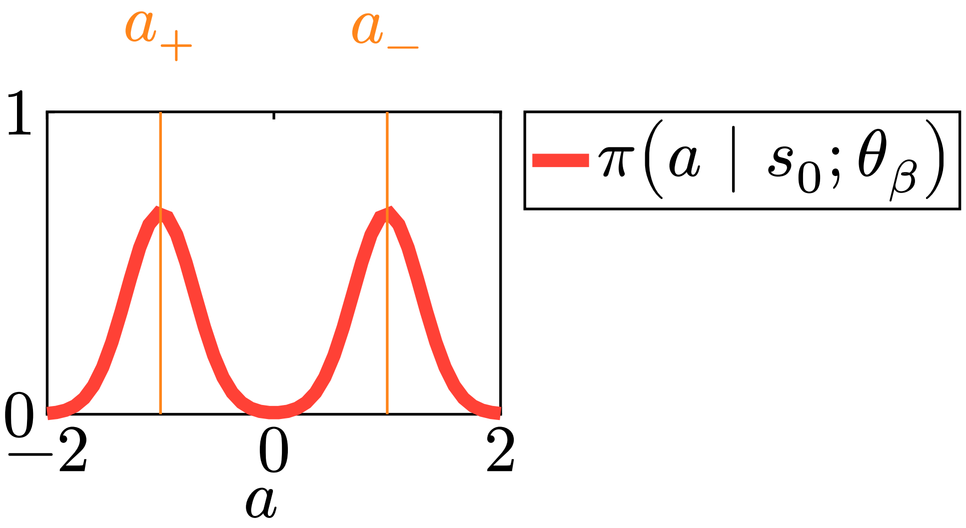


$\hat{\mathbb{E}}[\mathcal{R}(s_1) \mid s_0; \theta_\beta]$ $\quad\quad$ $\hat{\mathbb{E}}[\mathcal{G}(\boldsymbol{\tau}) \mid s_0; \theta_\beta]$

Question: How should we change $\pi(a_+ \mid s_0; \theta_\pi)$ and $\pi(a_- \mid s_0; \theta_\pi)$?

Answer: Increase probability of a_+, decrease probability of a_-

Behavioral Cloning with Rewards



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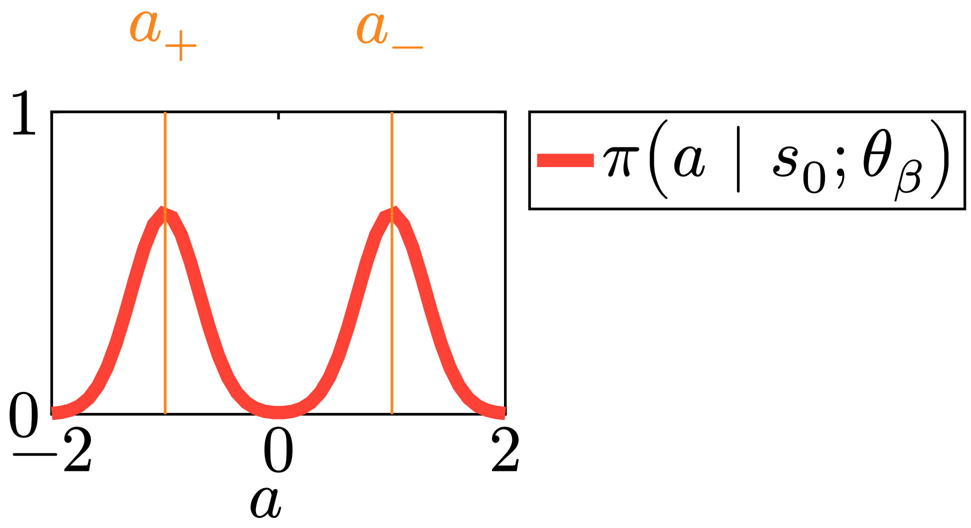

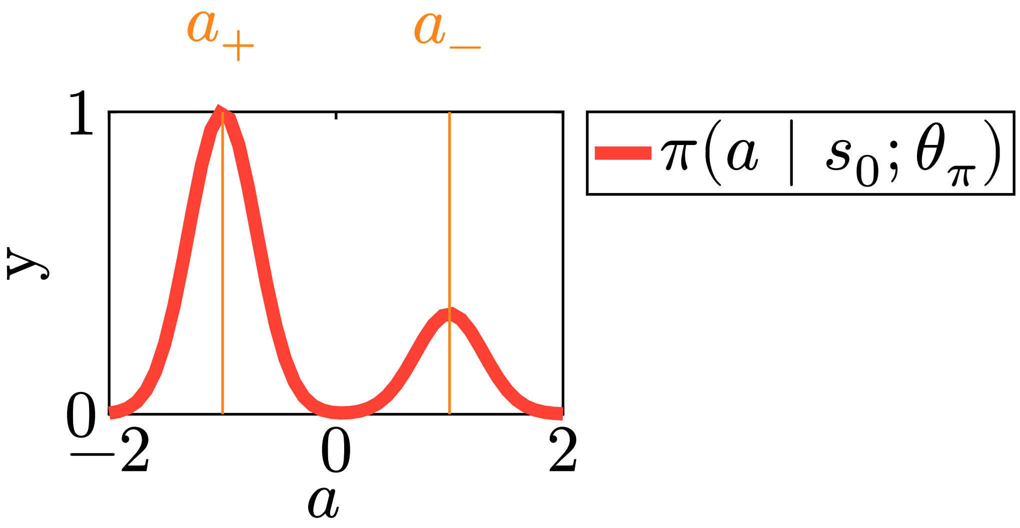

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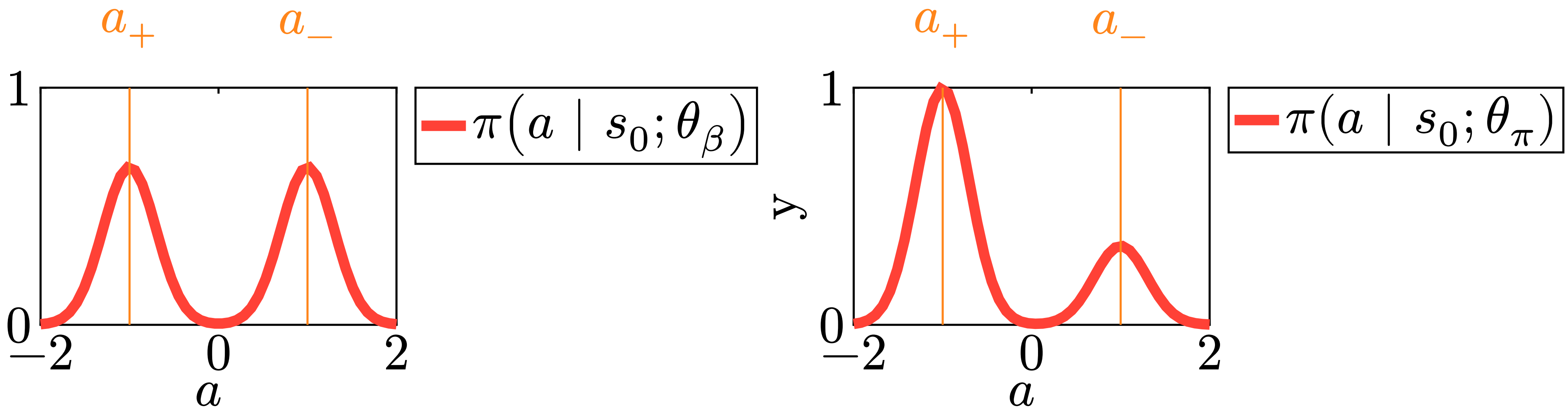

We want to reweight action probabilities based on reward or return

Behavioral Cloning with Rewards

$$\arg\min_{\theta_\pi} \sum_{a \in \{a_+, a_-\}} -\pi(a \mid s_0; \theta_\beta) \log \pi(a \mid s_0; \theta_\pi)$$

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$$r_+ > r_- \Rightarrow f(r_+) > f(r_-)$$

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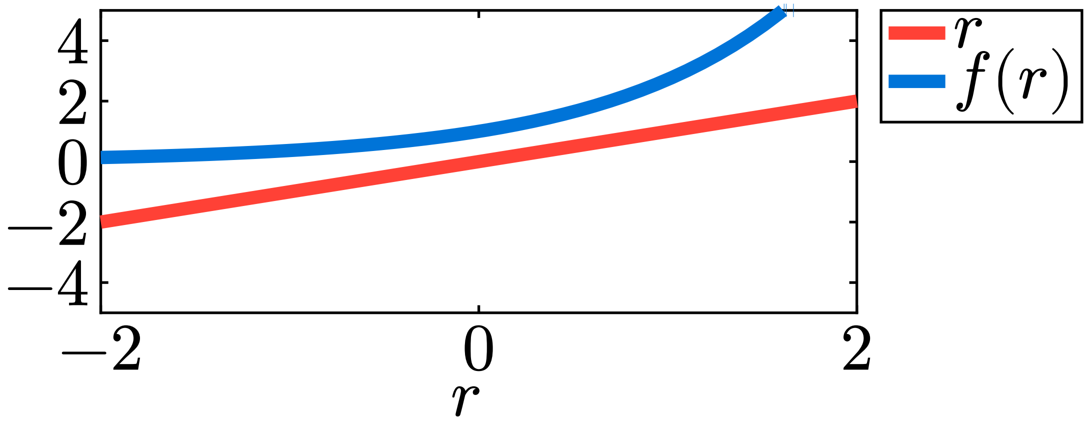

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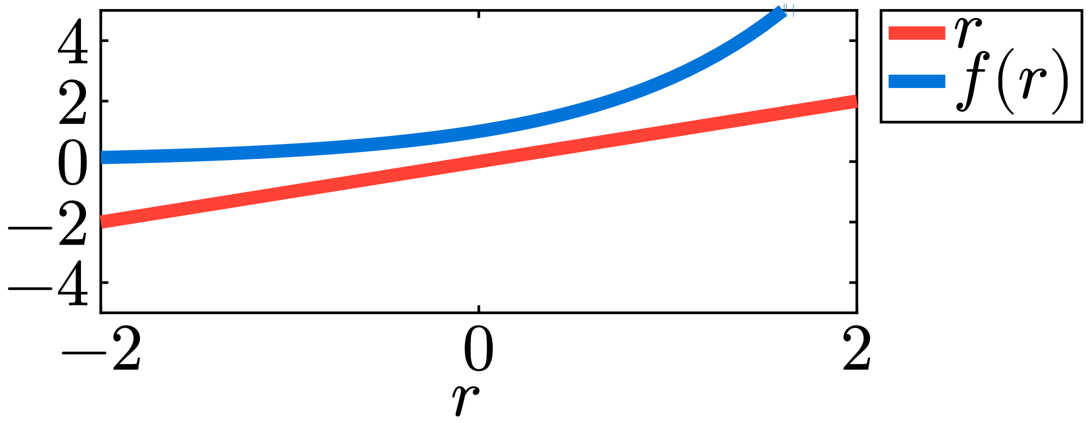

$$f(r) = e^r$$

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- Introduce TD value function (remove infinite sum)
- Introduce advantage (normalize return)

Behavioral Cloning with Rewards

$$\theta_\pi = \arg\min_{\theta_\pi} \sum_{s_0 \in \boldsymbol{X}} \sum_{a \in A} -\pi(a \mid s_0; \theta_\beta) \log \pi(a \mid s_0; \theta_\pi) \cdot \exp\left(\hat{\mathbb{E}}\left[\mathcal{G}(\boldsymbol{\tau}) \mid s_0; \theta_\beta\right]\right)$$

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Behavioral Cloning with Rewards

Definition: Monotonic Advantage Re-Weighted Imitation Learning (MARWIL) reweights the behavior cloning objective based on the advantage

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Answer: Yes, because we learn V for θ_β not θ_π

Behavioral Cloning with Rewards

Add improvements to MARWIL to derive other offline RL algorithms

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- Implicit Q learning (IQL)
- Maximum a Posteriori Optimization (MPO)

The Deadly Triad

The Deadly Triad

There are two standard approaches to offline RL

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1. Reweight the BC loss using rewards/returns
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Finished first, now let us visit the second

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- $\theta_\beta \neq \theta_\pi$, cannot use on-policy method

Off-policy can learn from any trajectories

- Trajectory collected following θ_β
- Can use θ_β trajectory to update θ_π

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Question: Temporal Difference or Monte Carlo Q learning?

- MC is on-policy
- Only TD Q learning is off-policy

The Deadly Triad

Recall the standard Q learning algorithm

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Recall the standard Q learning algorithm

```
while not terminated:
    transition = env.step(action)
    buffer.append(transition)
    train_data = buffer.sample()
    J = grad(td_loss)(theta_Q, theta_T, Q, train_data)
    theta_Q = opt.update(theta_Q, J)
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Question: What can we do to make this offline? Without exploration?

Answer:

- Put dataset into replay buffer
- Get rid of env code

The Deadly Triad

```python
for x in X:
    buffer.append(x) # Add dataset to replay buffer
# Comment out exploration code
# while not terminated:
    # transition = env.step(action)
    # buffer.append(transition)
for epoch in num_epochs:
    train_data = buffer.sample()
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Question: Will this work?

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Question: Will this work?

Answer: It can! But only for very simple problems

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Let us investigate why

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Let us further investigate the deadly triad

The Deadly Triad

Imagine a toy MDP

The Deadly Triad

Imagine a toy MDP

$$S = \{s\}$$

The Deadly Triad

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$$S = \{s\} \quad A = \{1, 2\}$$

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Imagine a toy MDP

$$S = \{s\} \quad A = \{1, 2\} \quad Q(s, a, \theta_Q) = \theta_Q \cdot a \quad \mathcal{R}(s) = 0 \quad \gamma = 1$$

Can update θ_Q using simple TD update

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Let us perform some updates to θ_Q and see what happens

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Initialize $\theta_Q = 1$, update for $a = 1$

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$$\theta_Q = 1 - [1 - 2]$$

$$\theta_Q = 2$$

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Repeat with $\theta_Q = 2$, update for $a = 1$

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Offline RL guarantees case 3, because we will never explore

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3. Visit all possible states/actions	Not possible with fixed dataset

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Question: What if the state space is continuous? Will this work?

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We can learn θ_β using behavioral cloning

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- Too much extrapolation leads to deadly triad!

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We need to balance the second term a little better

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We can subtract Q for the action we take in the dataset!

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This balances the second term to be closer to zero

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Constraining Q

Definition: Conservative Q Learning (CQL) learns a Q function that minimizes Q for out of distribution actions

$$\theta_{Q,i+1} = \arg\min_{\theta_{Q,i}} \underbrace{\left(Q\left(s_0, a_0, \theta_\pi, \theta_{Q,i}\right) - y\right)^2}_{\text{TD error}} + z^2$$

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$$z = \underbrace{\left(\log \sum_{a \in A} \exp\left(Q\left(s_1, a, \theta_\pi, \theta_{Q,i}\right)\right)\right)}_{\text{Push down Q for all } a} - \underbrace{Q\left(s_1, a_1, \theta_\pi, \theta_Q\right)}_{\substack{\text{Push up Q for} \\ \text{in-distribution } a}}$$

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- Like IL, but learns optimal policy instead of expert policy

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- Breaking the deadly triad with Q learning