Policy Gradient

CISC 7404 - Decision Making

Steven Morad

University of Macau

Admin

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Homework 1 was due yesterday 23:59

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We are about 50% finished with the course

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Any opinions/feedback on the course?

- Can also talk after class
- Or email smorad at um.edu.mo

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If you want full participation marks, you must participate in lecture

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Right now, the following students have full participation marks:

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Right now, the following students have full participation marks:

- LIU KEJIA
- LIU HUANRONG
- HOI HOU HONG
- CHEN ZELAI
- WANG ZEKANG
- HE ZHE
- WANG MENGQI
- ZHANG BORONG
- HE ENHAO
- QIAO YULIN
- ZHUANG HANQIN
- KAM KA HOU

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A cute video of trajectory optimization

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https://www.youtube.com/watch?v=tudxHzZ5_ls

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http://www.incompleteideas.net/IncIdeas/BitterLesson.html

Review

Parameterized Policies

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There are two types of algorithms

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- Value based methods (Q learning)

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Policy gradient can change pretrained model parameters

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$$\pi(a_t \mid s_t; \theta_\pi) = \begin{cases} 1 \text{ if } a_t = \text{arg max}_{a \in A} Q(s_t, a, \theta_\pi) \\ 0 \text{ otherwise} \end{cases}$$

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So why do we need a new algorithm?

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Example: Consider a Unitree BenBen, with 12 joints

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To learn to motion, we must learn actions for all joints $A \in [0, 2\pi]^{12}$

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Answer: No, $\arg\max_{a \in A}$, but A is infinite. How can we take arg max over an infinite set?

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Does that sound impossible?

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We can improve the action distribution over time

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Question: How should we change θ_π?

Answer: Change θ_π so we reach good $s \in S$, making the return larger

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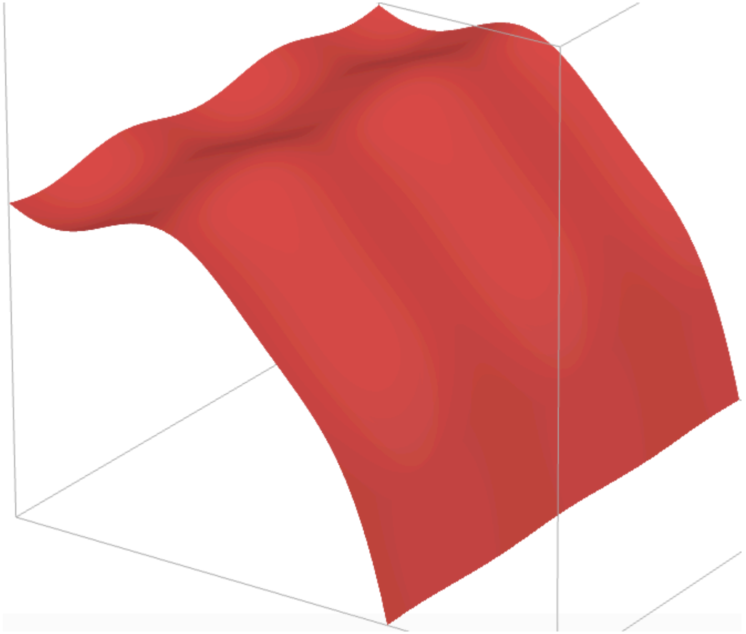

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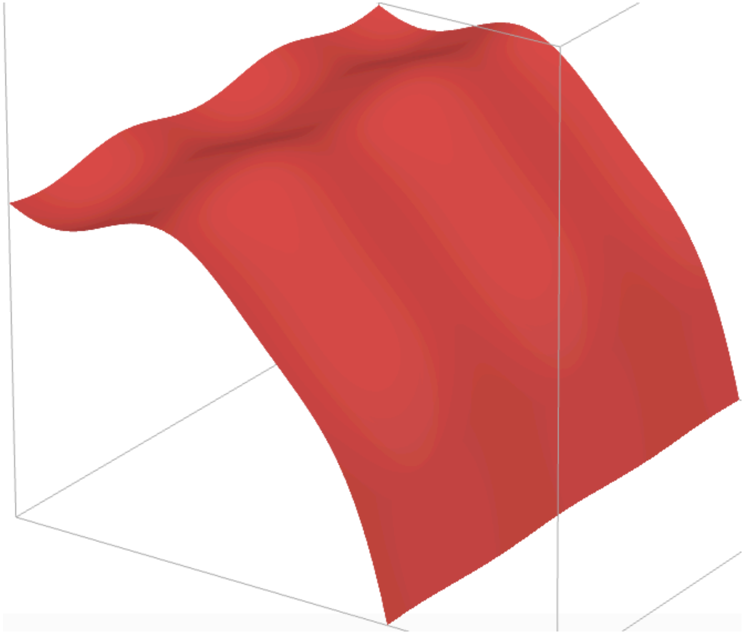

$$\theta_{\pi, i+1} = \theta_{\pi, i} + \alpha \cdot \nabla_{\theta_{\pi, i}} \mathbb{E}\left[\mathcal{G}(\boldsymbol{\tau}) \mid s_0; \theta_{\pi, i}\right]$$

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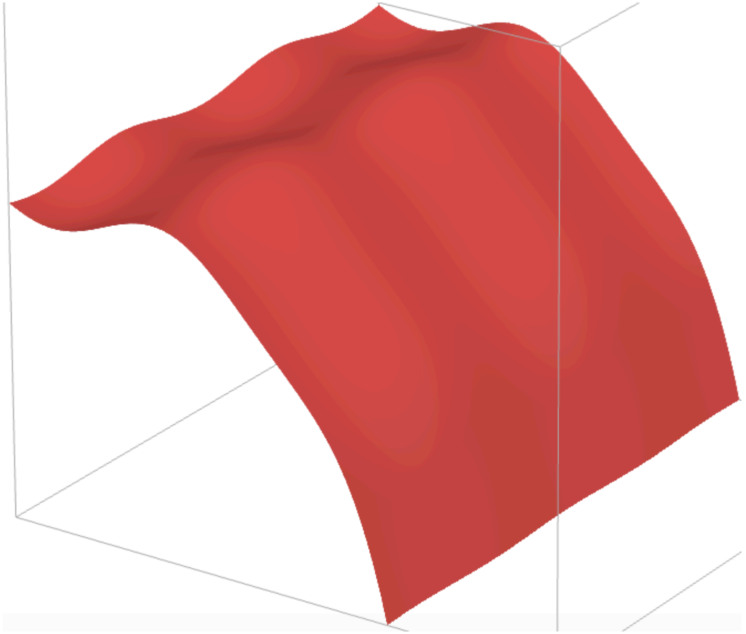

Current policy

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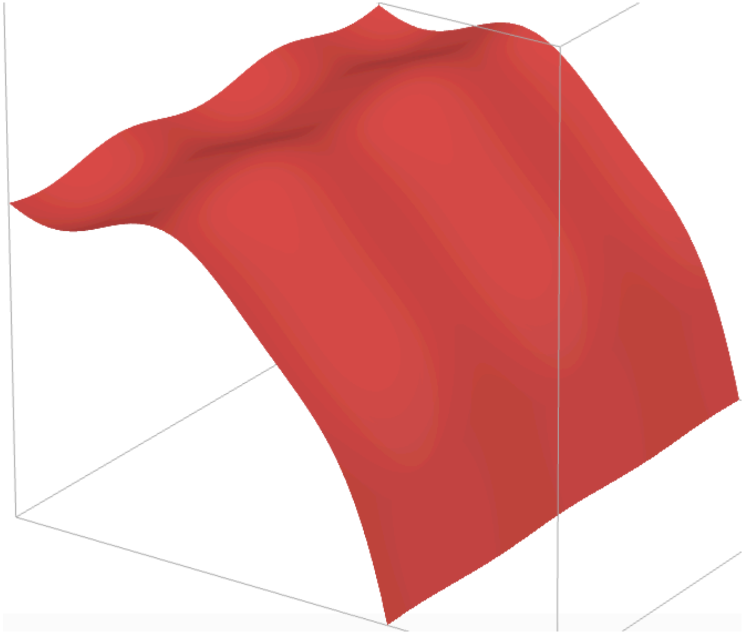

$$\theta_{\pi, i+1} = \underbrace{\theta_{\pi, i}}_{\text{Current policy}} + \alpha \cdot \underbrace{\nabla_{\theta_{\pi, i}} \mathbb{E}[\mathcal{G}(\boldsymbol{\tau}) \mid s_0; \theta_{\pi, i}]}_{\theta \text{ direction that maximizes return}}$$

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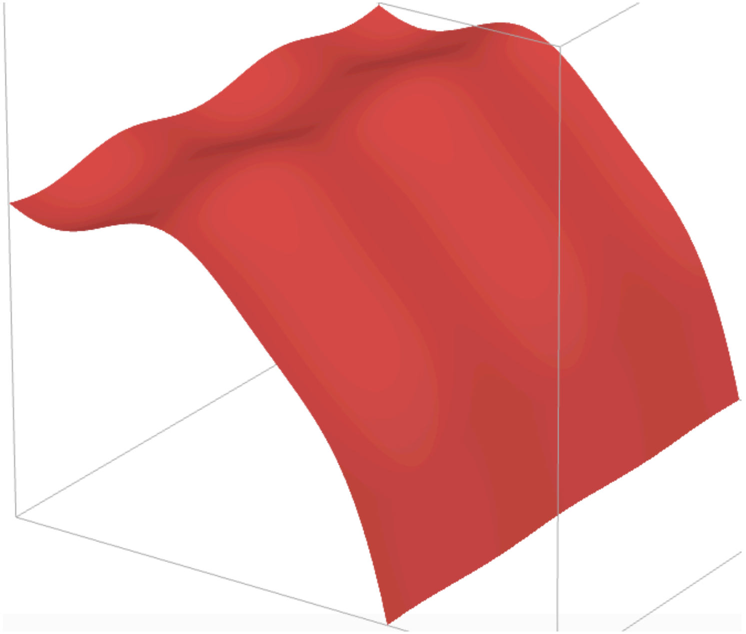

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If find $\nabla_{\theta_\pi} \mathbb{E}[\mathcal{G}(\boldsymbol{\tau}) \mid s_0; \theta_\pi]$, we can improve the policy and return

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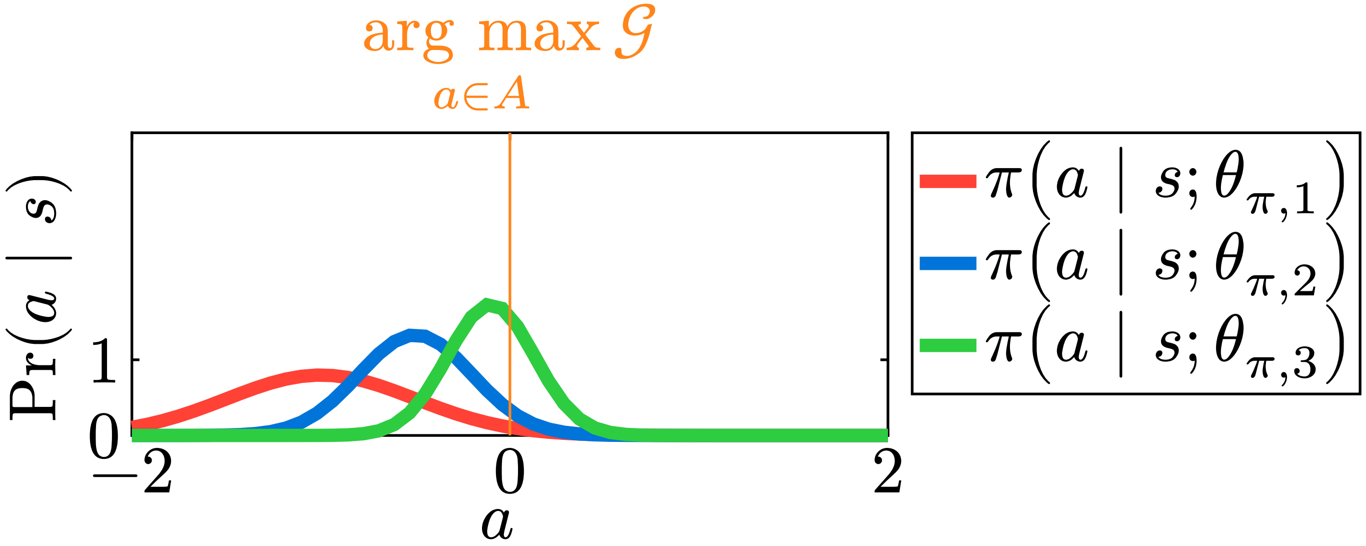

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First, combine top two equations so we have more space

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Plug line 2 into line 1

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$$\left(\sum_{s_1, \ldots, s_n \in S} \prod_{t=0}^{n} \left(\sum_{a_t \in A} \mathrm{Tr}(s_{t+1} \mid s_t, a_t) \cdot \pi(a_t \mid s_t; \theta_\pi) \right) \right)$$

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Take the gradient with respect to θ_π of both sides

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Take the gradient with respect to θ_π of both sides

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Move gradient inside sums

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Rewrite $\Pr(s_{n+1} \mid s_0; \theta_\pi)$ by pulling action sum outside

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First term is a constant factor, pull out constant

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Can use chain and product rule, but will create a mess of terms

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$$\nabla_{\theta_\pi} [\pi(a_n \mid s_n; \theta_\pi) \dots \cdot \pi(a_0 \mid s_0; \theta_\pi)] =$$

$$\nabla_{\theta_\pi} [\pi(a_n \mid s_n; \theta_\pi)] \cdot \pi(a_{n-1} \mid s_{n-1}; \theta_\pi) \dots$$

It will be very expensive/intractable to compute all terms!

Policy Gradient

Log-derivative trick:

Policy Gradient

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Question: What is

$$\nabla_x \log(f(x))$$

Policy Gradient

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Apply log-derivative trick to $\nabla \prod$

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The log of products is the sum of logs: $\log(ab) = \log(a) + \log(b)$

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Gradient of sum is sum of gradients

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Last step, recombine product

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$$\nabla_{\theta_\pi} \mathbb{E}[\mathcal{G}(\boldsymbol{\tau}) \mid s_0; \theta_\pi] = \sum_{n=0}^{\infty} \gamma^n \sum_{s_{n+1} \in S} \mathcal{R}(s_{n+1}) \cdot \sum_{s_1, \dots, s_n \in S} \sum_{a_0, \dots, a_n \in A}$$

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But this is a bit messy, and we don't know how to train it yet!

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Can we write it in a simpler form so we can approximate it?

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Question: Is this familiar?

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Careful, π relies on n – cannot write this way

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Answer: On-policy, empirical return based on θ_π

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$$\nabla_{\theta_\pi} \mathbb{E}[\mathcal{G}(\boldsymbol{\tau}) \mid s; \theta_\pi] \approx \sum_{t=0}^{\infty} \hat{\mathbb{E}}[\mathcal{G}(\boldsymbol{\tau}) \mid s_t; \theta_\pi] \cdot \nabla_{\theta_\pi} \log \pi(a_t \mid s_t; \theta_\pi)$$

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$$\nabla_{\theta_\pi} \mathbb{E}[\mathcal{G}(\boldsymbol{\tau}) \mid s_0; \theta_\pi] = V(s_0, \theta_\pi) \cdot \nabla_{\theta_\pi} \log \pi(a_0 \mid s_0; \theta_\pi)$$

$$\nabla_{\theta_\pi} \mathbb{E}[\mathcal{G}(\boldsymbol{\tau}) \mid s_0; \theta_\pi] = Q(s_0, a_0, \theta_\pi) \cdot \nabla_{\theta_\pi} \log \pi(a_0 \mid s_0; \theta_\pi)$$

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We call this **actor-critic**, more discussion next time

Coding

Coding

We can implement our policy using all sorts of action distributions

[1] "Improving stochastic policy gradients in continuous control with deep reinforcement learning using the beta distribution." International conference on machine learning. PMLR, 2017.

Coding

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For discrete tasks, we often use **categorical** distributions

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For discrete tasks, we often use **categorical** distributions

For continuous tasks, we usually use **normal** distributions

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Coding

We can implement our policy using all sorts of action distributions

For discrete tasks, we often use **categorical** distributions

For continuous tasks, we usually use **normal** distributions

However, some people say beta distributions work better![1]

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Coding

The coding looks a little bit different than the math

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We often separate the model, policy distribution, and sampled action

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We often separate the model, policy distribution, and sampled action

```
# Compute "logits", unnormalized probabilities
z = model(x)
# Create distribution pi(a | s; theta_pi)
p_a = dist(z) # For loss function
# Sample action that we use in environment
a = sample(p_a) # For env step
```

Coding

Create a model for discrete action spaces

Coding

Create a model for discrete action spaces

```
discrete_action_model = nn.Sequential([
    nn.Linear(state_size, hidden_size),
    nn.Lambda(leaky_relu),
    nn.Linear(hidden_size, hidden_size),
    nn.Lambda(leaky_relu)
    # Output logits for possible actions
    nn.Linear(hidden_size, action_size),
])
```

Coding

Need a function to get actions for our environment

```
def sample_action(model, state, key):
    z = model(state)
    # BE VERY CAREFUL, always read documentation
    # Sometimes takes UNNORMALIZED logits, sometimes probs
    action_probs = softmax(model, state)
    a = categorical(key, action_probs)
    a = categorical(key, z) # Does not even use pi
    return a
```

Coding

```python
def REINFORCE_loss(model, episode):
    """REINFORCE for discrete actions"""
    G = compute_return(rewards) # empirical return
    # We need log(pi(a | s)), softmax => probs
    # log_softmax more stable than log(softmax(x))
    log_probs = log_softmax(model(episode.states))
    # We only update the policy for the actions we took
    # Discrete/categorical actions
    # Can use sum or mean
    policy_gradient = mean(G * log_probs[episode.actions])
    # Want gradient ascent, most library do gradient descent
    return -policy_gradient
```

Coding

What about continuous action spaces?

Coding

What about continuous action spaces?

```
continuous_action_model = nn.Sequential([
    nn.Linear(state_size, hidden_size),
    nn.Lambda(leaky_relu),
    nn.Linear(hidden_size, hidden_size),
    nn.Lambda(leaky_relu)
    nn.Linear(hidden_size, 2 * action_size),
    # Like to use a diagonal multivariate Gaussian
    # Assumes independence between actions (approximation)
    nn.Lambda(lambda x: split(x, 2))
])
```

Coding

```python
def sample_action(model, state, key):
    # Log(sigma) because neural network outputs +/-
    # sigma only + but log_sigma can be +/-
    mu, log_sigma = model(state)
    a = normal(key, mu, exp(sigma))
    return a
```

Coding

```python
def REINFORCE_loss(model, episode):
    """REINFORCE for continuous actions using Gaussian pi"""
    G = compute_return(rewards) # empirical return
    # Policy outputs mean and log(std dev)
    mus, log_sigmas = model(episode.states)
    # Log probability of action we took under action dist
    # log pi(a = episode.a | s)
    log_probs = -(
        (episode.actions - mus) ** 2
        / (2 * exp(log_sigmas) ** 2) + log_sigmas
    )
    policy_gradient = mean(G * log_probs)
    # Want gradient ascent, library does gradient descent
    return -policy_gradient
```

Homework

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Homework 2 is the final homework, and it is a little special

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Choose only one assignment

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- Policy gradient (easier, max 80/100)

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You can try and estimate the return for completing each assignment

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If you solve policy gradient early, then try deep Q learning

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If you solve policy gradient early, then try deep Q learning

Deep Q learning requires more hyperparameter tuning

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If you wait until the day before, you will not succeed

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Due in 3 weeks (04.09)

Homework

https://colab.research.google.com/drive/1JWfMYviwt7tgU08QDeIZV82MuzVQZbX1?usp=sharing

https://colab.research.google.com/drive/1qKXsaOpT27paCmPA-Hbh_-PtbQnrrkla?usp=sharing