Decision Processes

CISC 7404 - Decision Making

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Review

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Last time, we reviewed probability and bandits

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Markov Processes

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Decisions must make some change in the world

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- DNA sequences
- Cryptography
- History

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Let us do an example to understand this

Markov Processes

Problem: Predict the weather

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$$= \begin{bmatrix} 0.4 & 0.3 & 0.3 \\ 0.5 & 0.3 & 0.2 \\ 0.5 & 0.1 & 0.4 \end{bmatrix}$$

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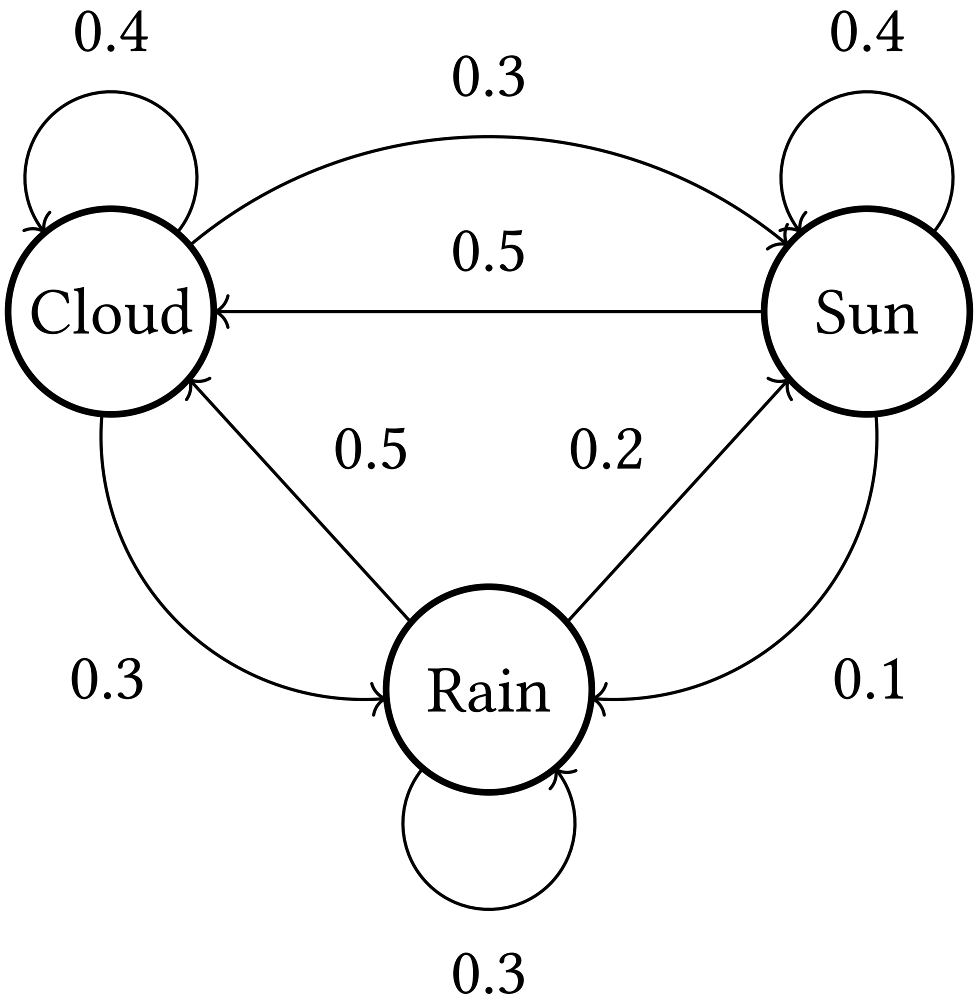

Markov Processes

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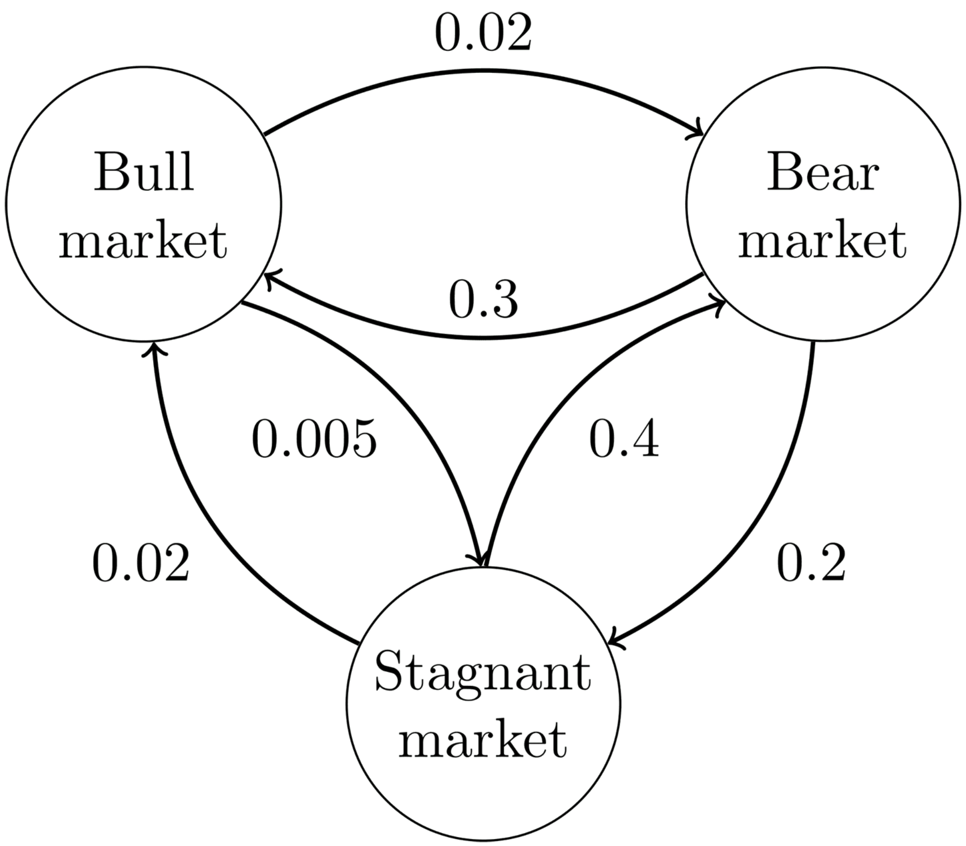

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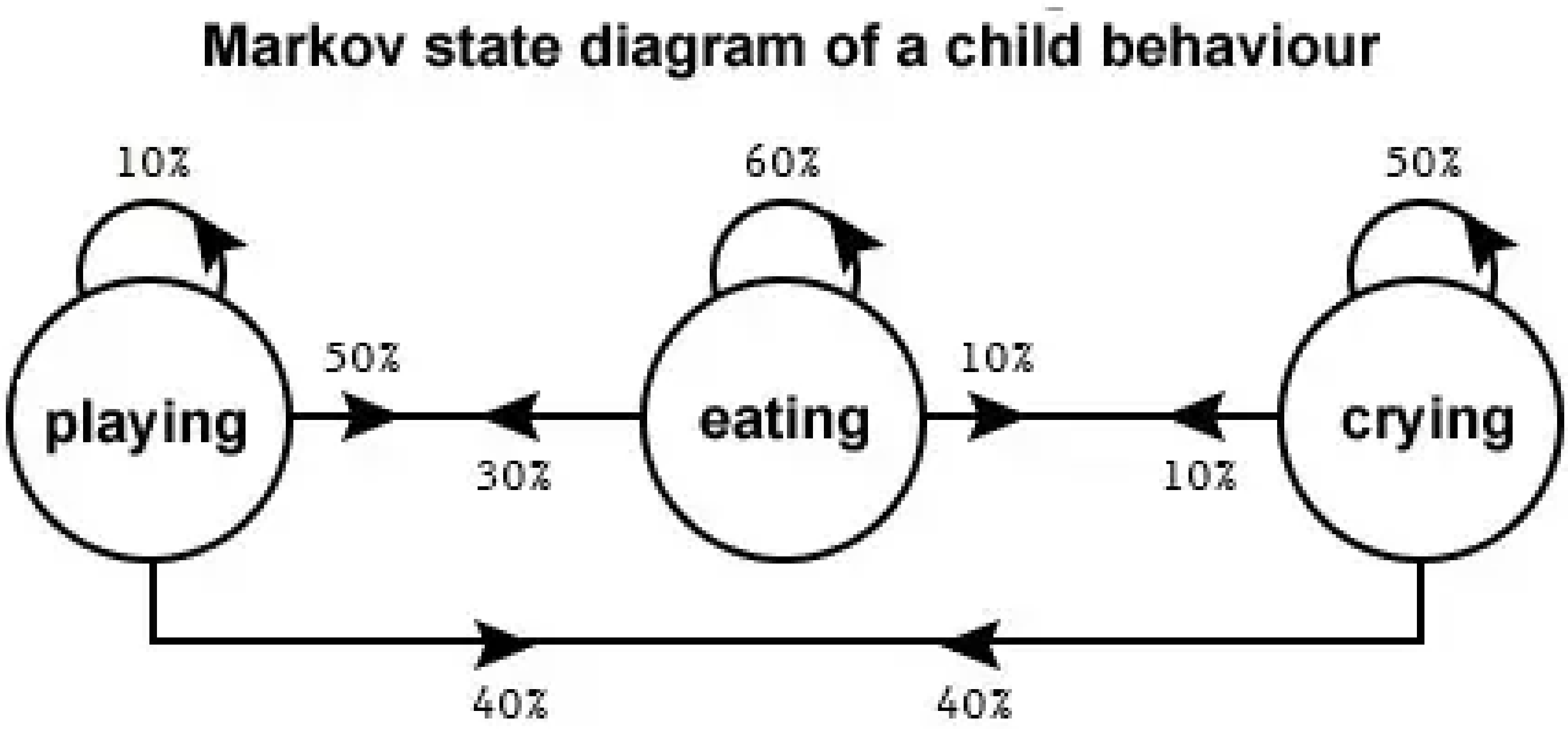

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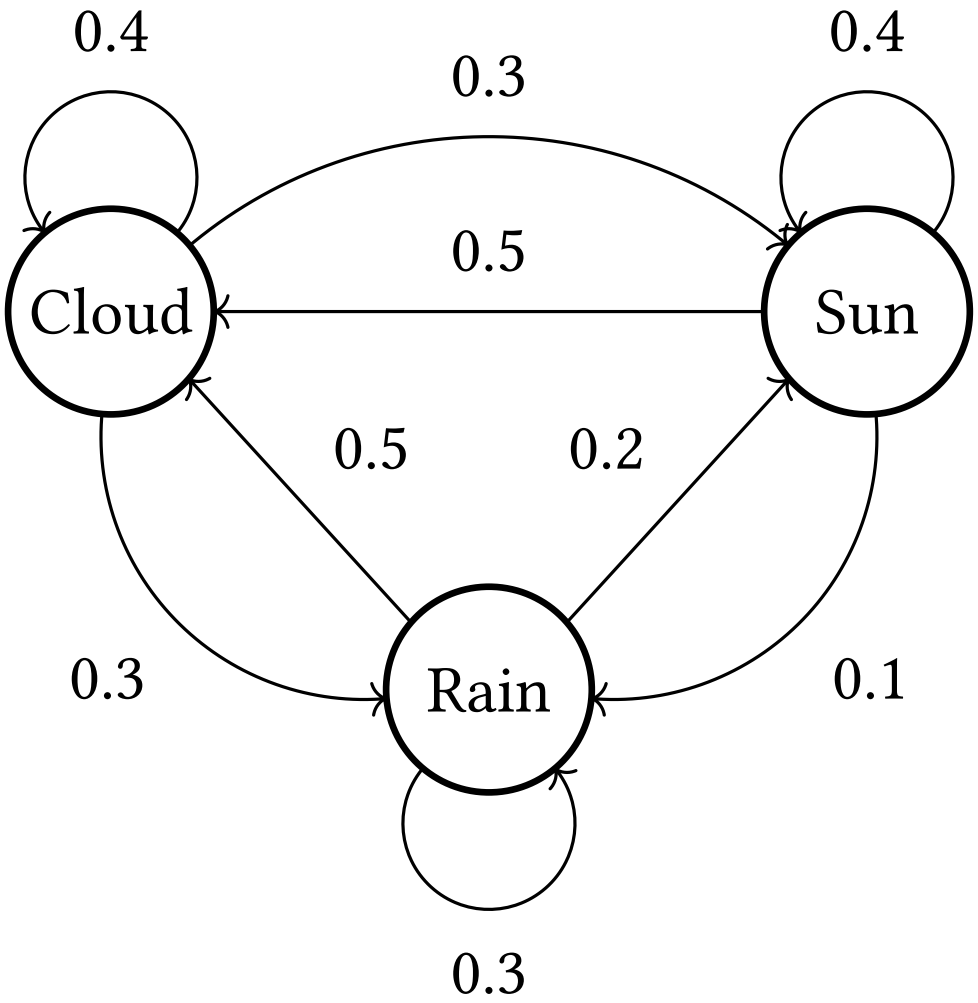
To compute the next node, we only look at the current node



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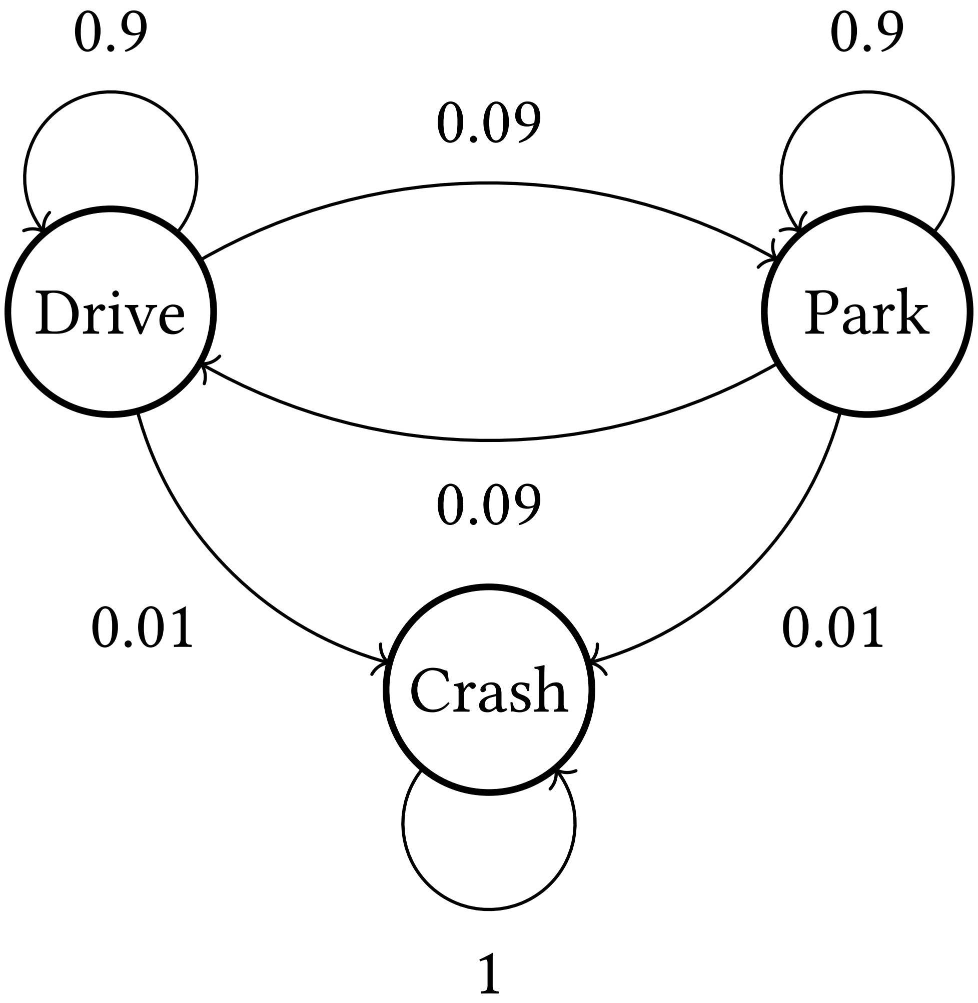
Question: How can we model this?

Answer: We create a **terminal state** that we cannot leave

Markov Processes



Upon reaching a terminal state, we get stuck

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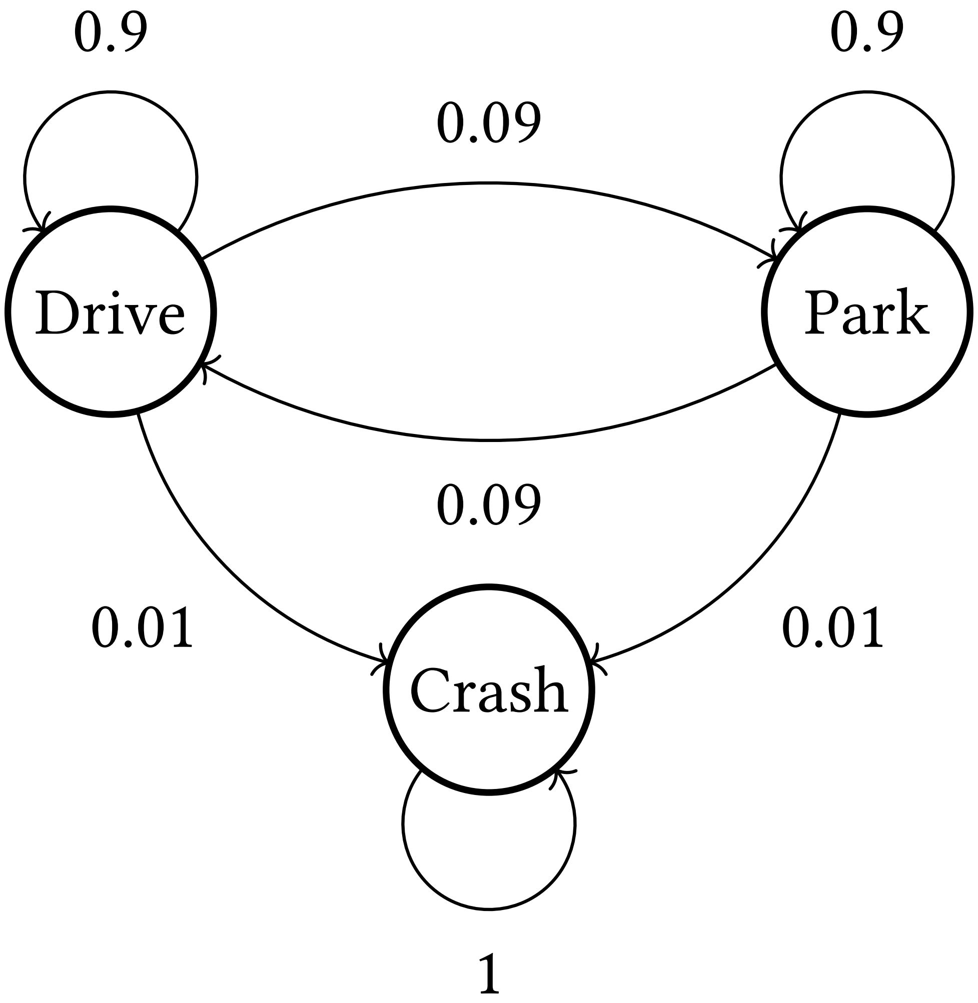

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Once we crash our car, we cannot drive or park any more

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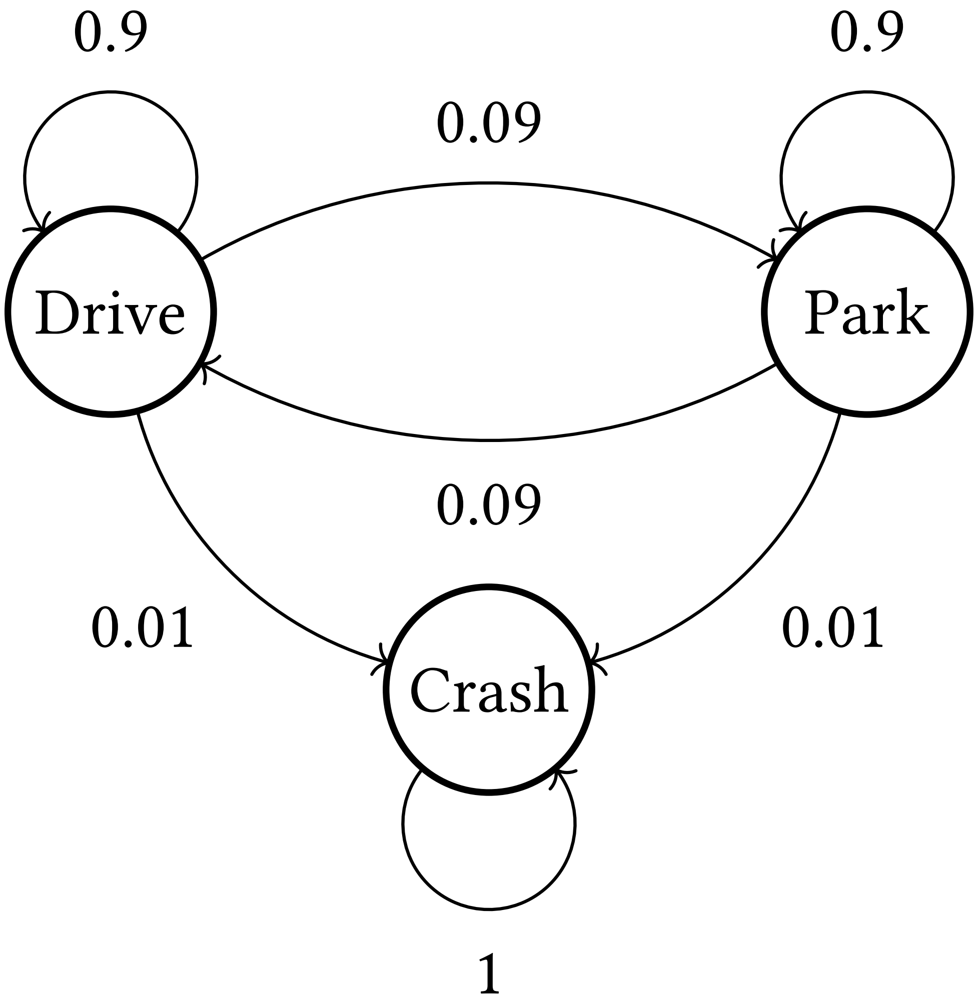

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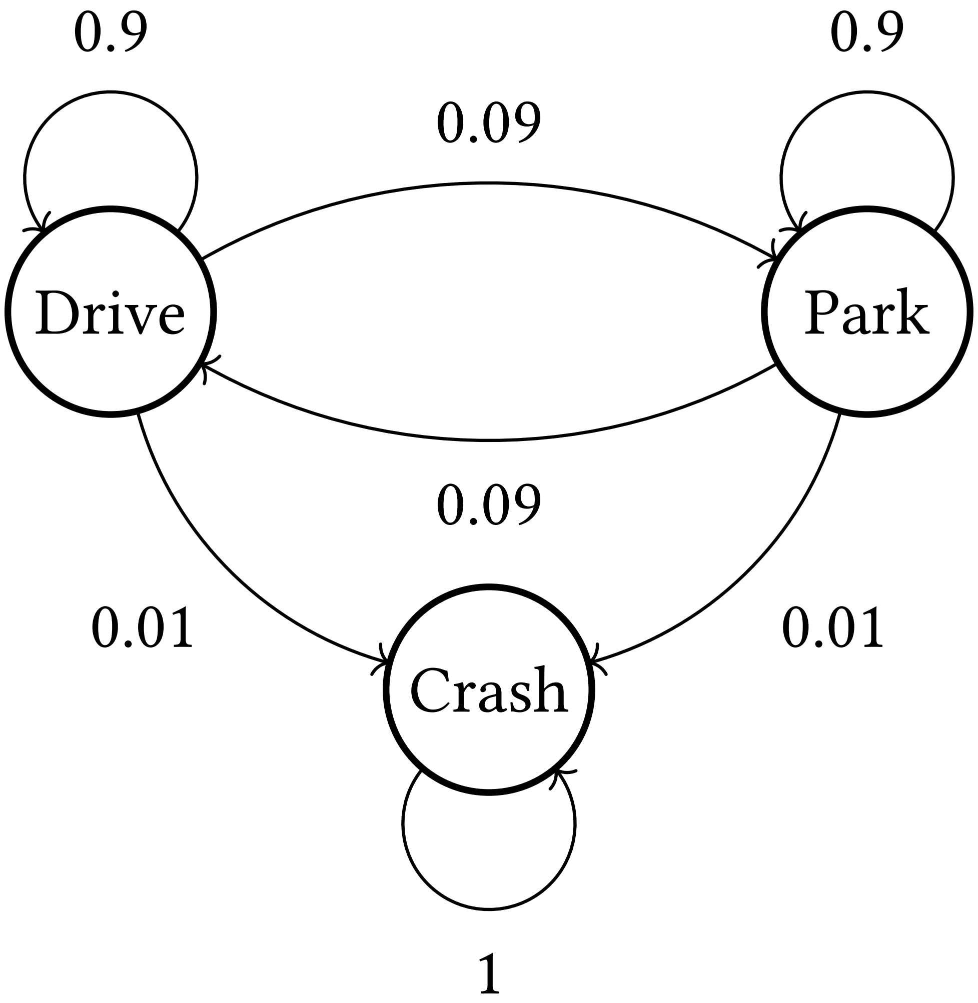
Once we crash our car, we cannot drive or park any more

The only transition from a terminal state is back to itself

$$\Pr(s' = s_{\text{terminal}} \mid s = s_{\text{terminal}}) = 1.0$$

Markov Processes



We call the sequence of states until the terminal state an **episode**

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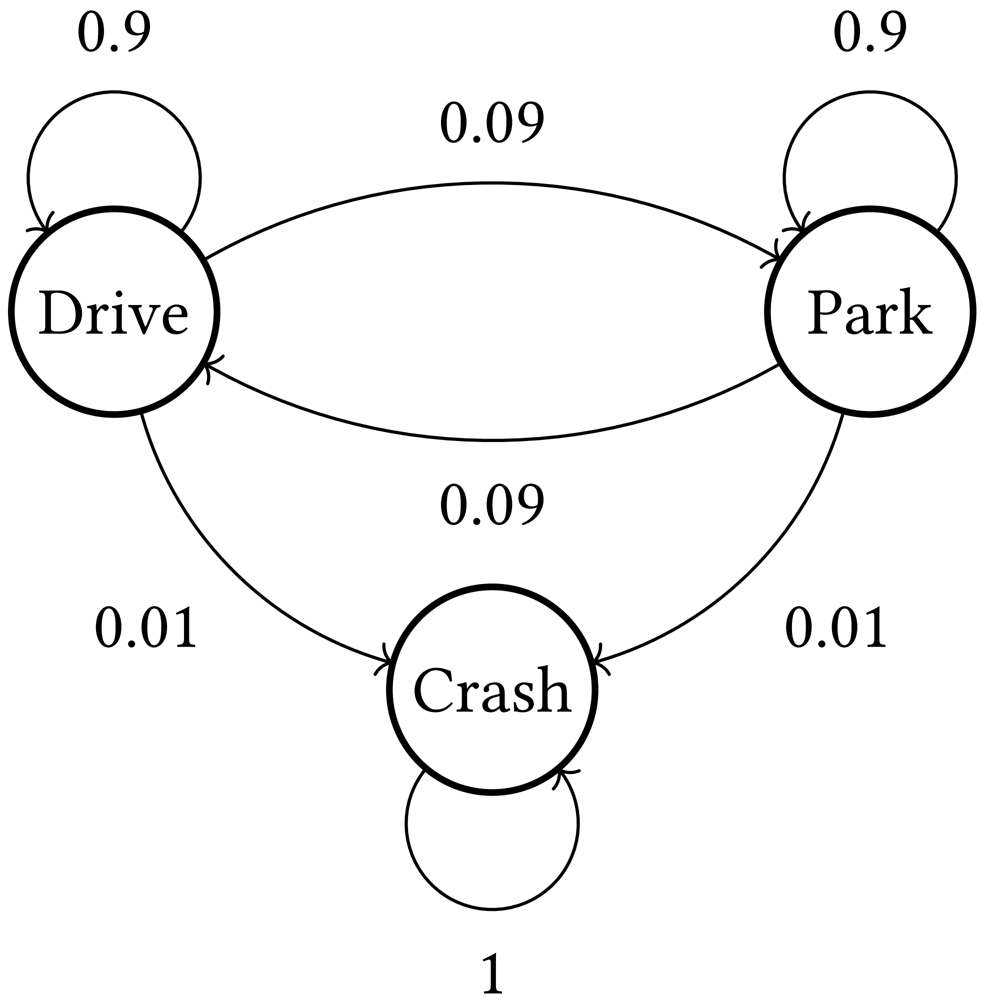

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$$\begin{bmatrix} s_0 \\ s_1 \\ s_2 \\ \vdots \\ s_n \end{bmatrix}$$

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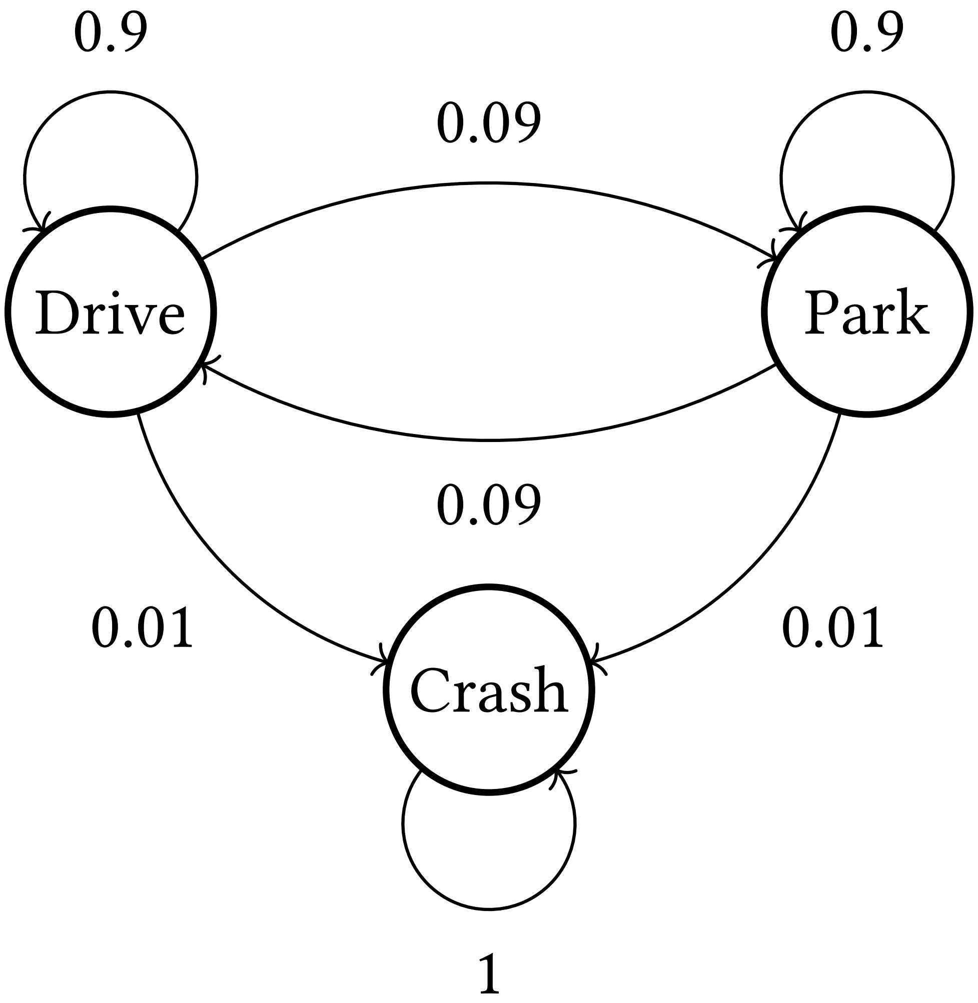

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$$\begin{bmatrix} s_0 \\ s_1 \\ s_2 \\ \vdots \\ s_n \end{bmatrix} = \begin{bmatrix} \text{Drive} \\ \text{Drive} \\ \text{Park} \\ \vdots \\ \text{Crash} \end{bmatrix}$$

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Design an MDP about a problem you care about

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- Create a terminal state

Markov Control Processes

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We will modify the Markov process for decision making

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We introduce the **agent** to make decisions that change the environment

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The agent takes **actions** $a \in A$ that change the environment

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The action space A defines what our agent can do

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$$(S, T)$$

$$T : S \mapsto \Delta S$$

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Markov process	Markov control process
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Let us see an example

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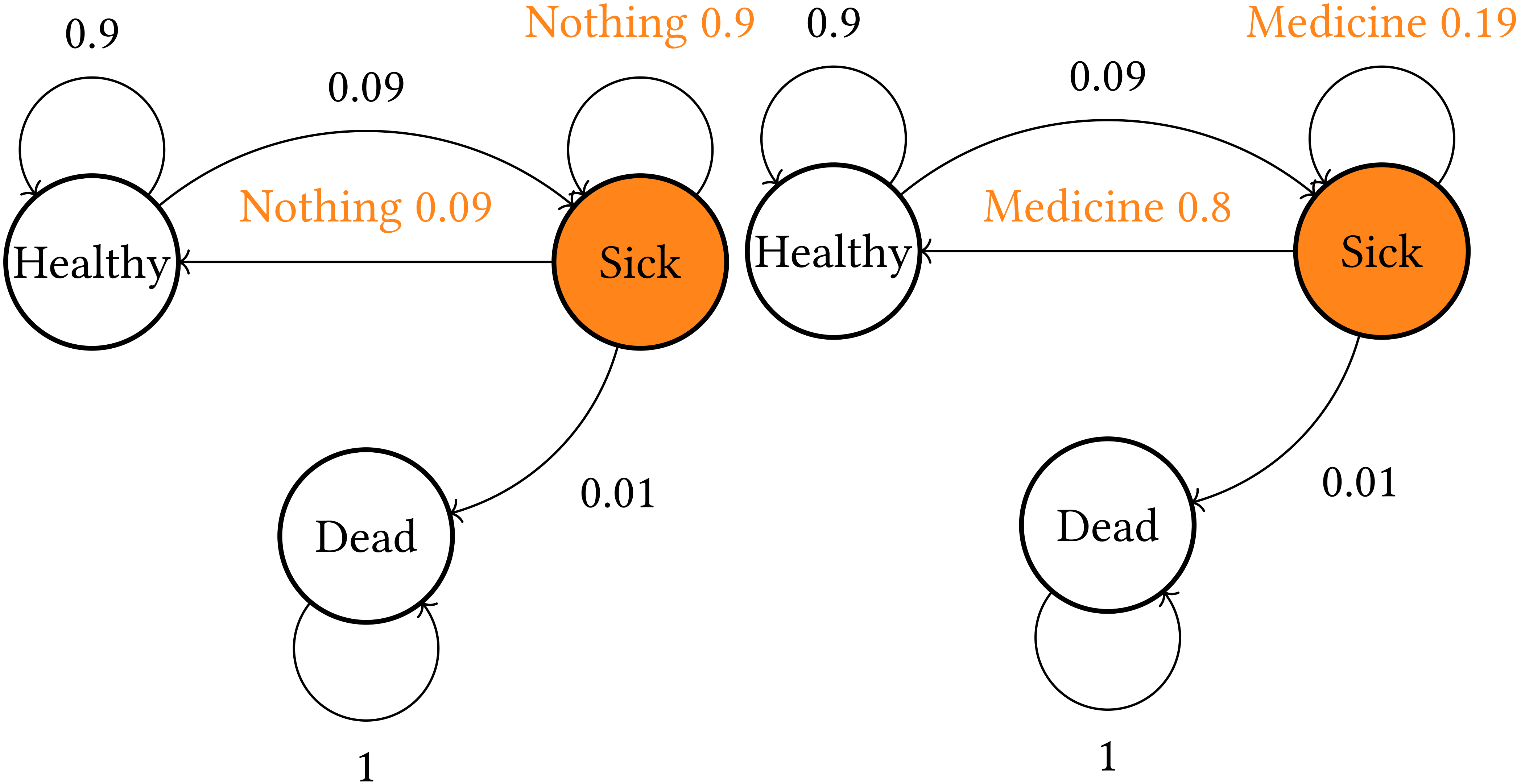

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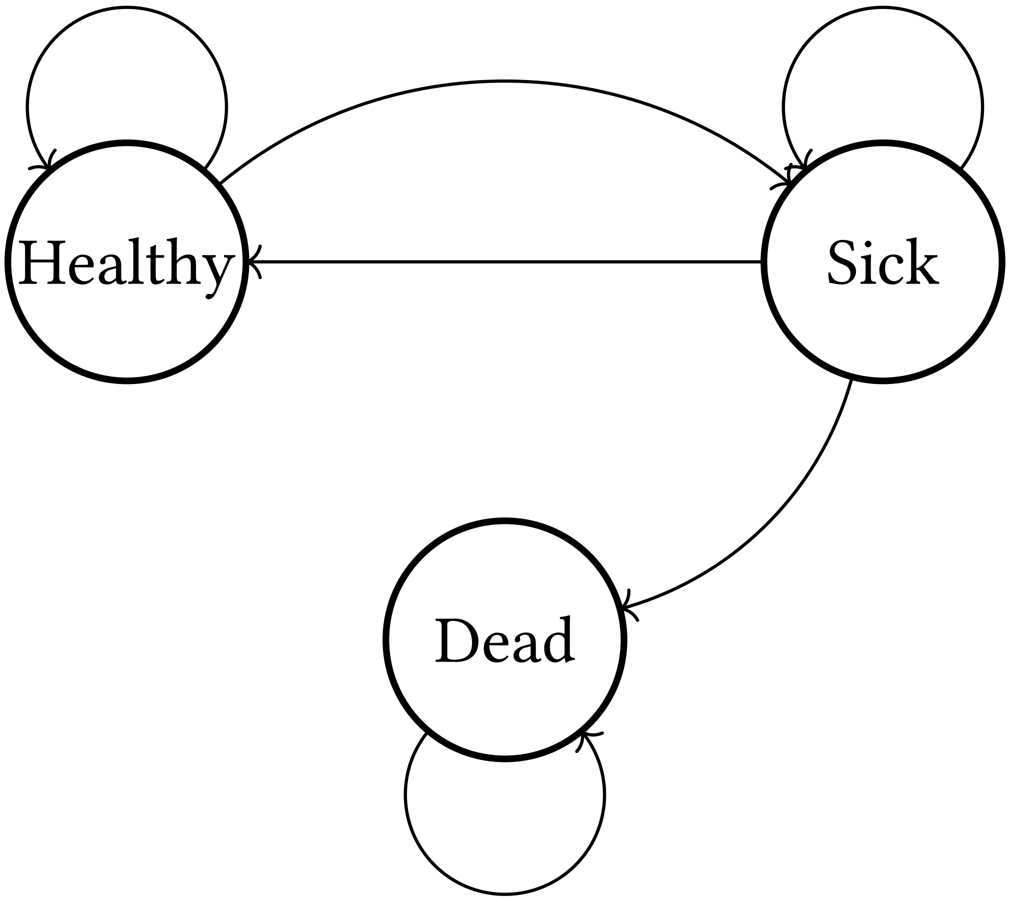

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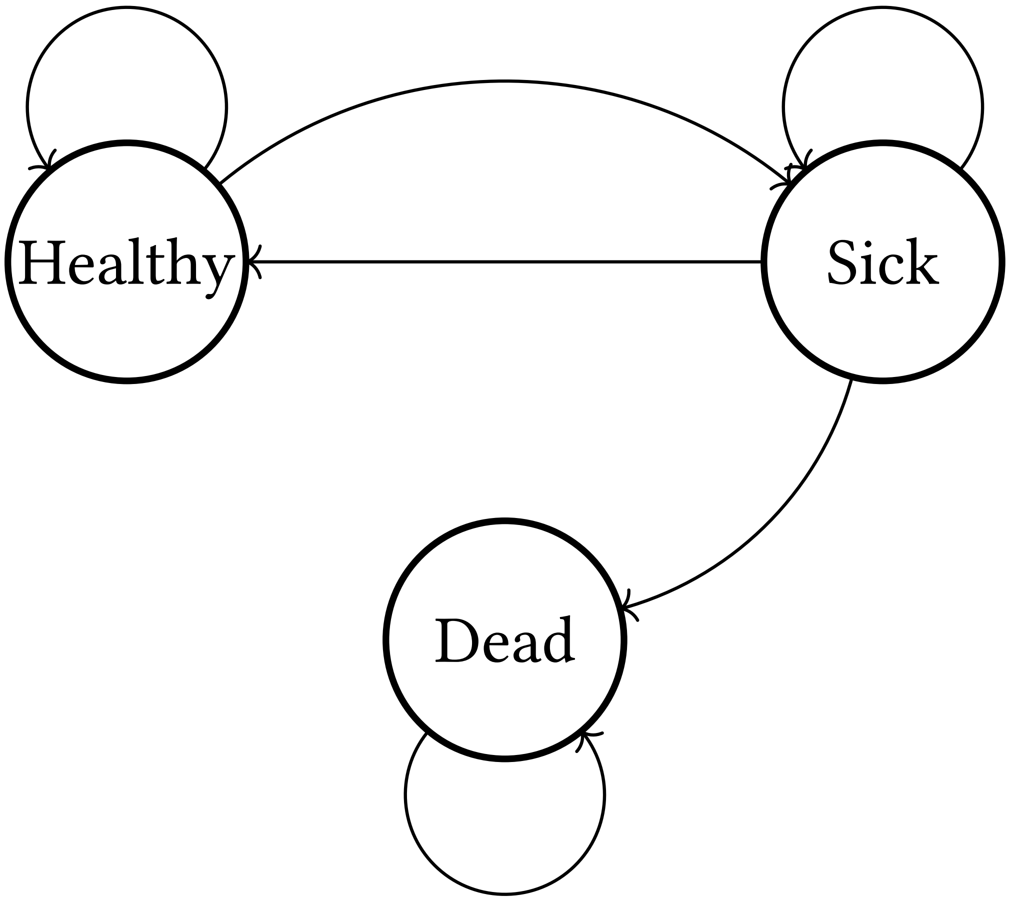

The **trajectory** contains the states and actions until a terminal state

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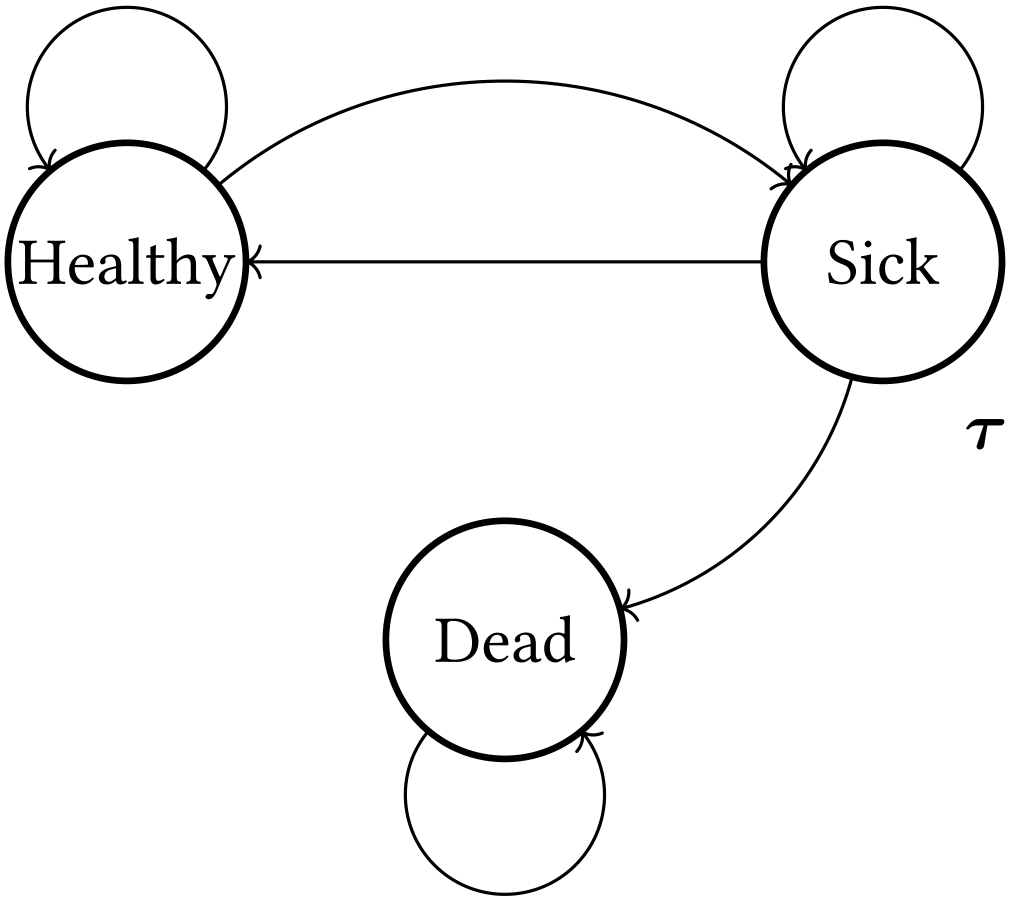
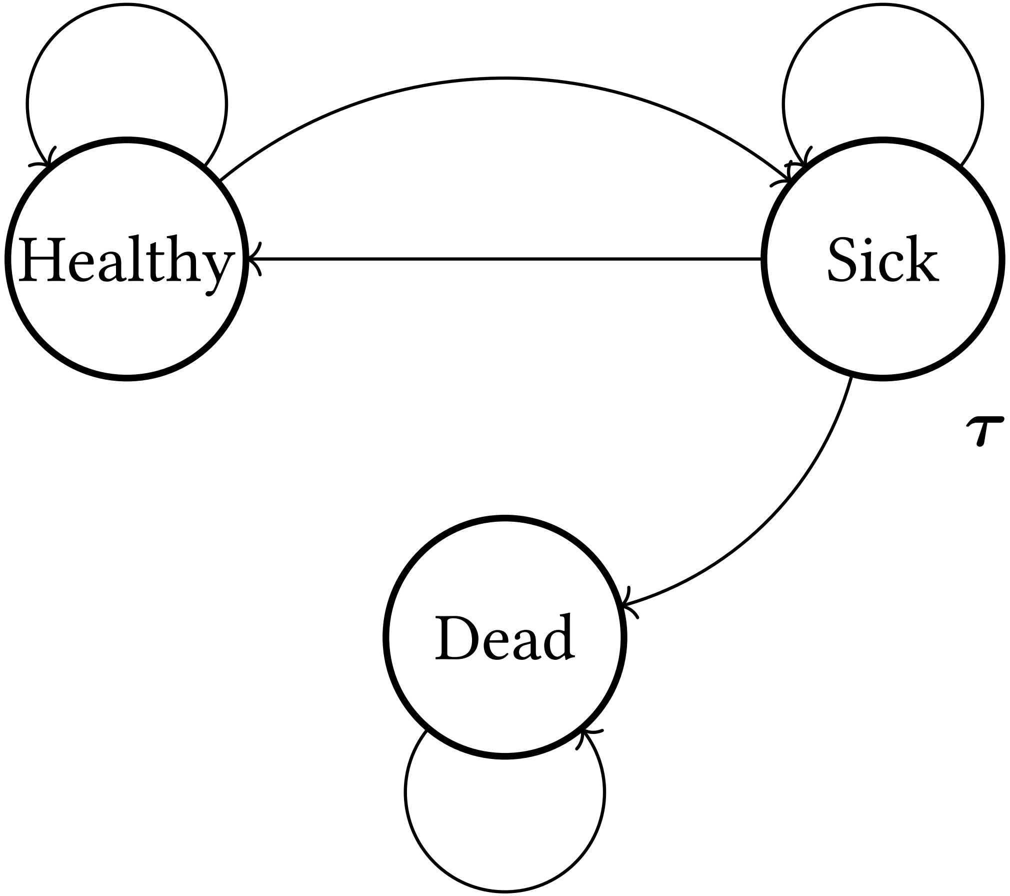


The **trajectory** contains the states and actions until a terminal state

$$\boldsymbol{\tau} = \begin{bmatrix} s_0 & a_0 \\ s_1 & a_1 \\ s_2 & a_2 \\ \vdots & \vdots \\ s_n & \emptyset \end{bmatrix}$$

Markov Control Processes



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$$\boldsymbol{\tau} = \begin{bmatrix} s_0 & a_0 \\ s_1 & a_1 \\ s_2 & a_2 \\ \vdots & \vdots \\ s_n & \emptyset \end{bmatrix} = \begin{bmatrix} \text{Healthy} & \text{Nothing} \\ \text{Sick} & \text{Nothing} \\ \text{Sick} & \text{Medicine} \\ \vdots & \vdots \\ \text{Dead} & \emptyset \end{bmatrix}$$

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How can we make optimal decisions if we cannot tell how good a decision is?

We need something to tell us how good it is to be in a state!

Markov Decision Processes

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$$R : S \times A \mapsto \mathbb{R}$$

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Sutton and Barto:	This course:
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Markov decision process

$$(S, A, T, R, \gamma)$$

$$T : S \times A \mapsto \Delta S$$

$$R : S \mapsto \mathbb{R}$$

Markov Decision Processes

The **history** contains the states, actions, and rewards until termination

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$$\boldsymbol{H} = \begin{bmatrix} s_0 & a_0 & r_0 \\ s_1 & a_1 & r_1 \\ \vdots & \vdots & \vdots \\ s_n & \emptyset & r_n \end{bmatrix}$$

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$$R(s_d) = 10 \qquad R(s_n) = 15$$

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We can write this mathematically as

$$\arg\max_{s \in S} R(s)$$

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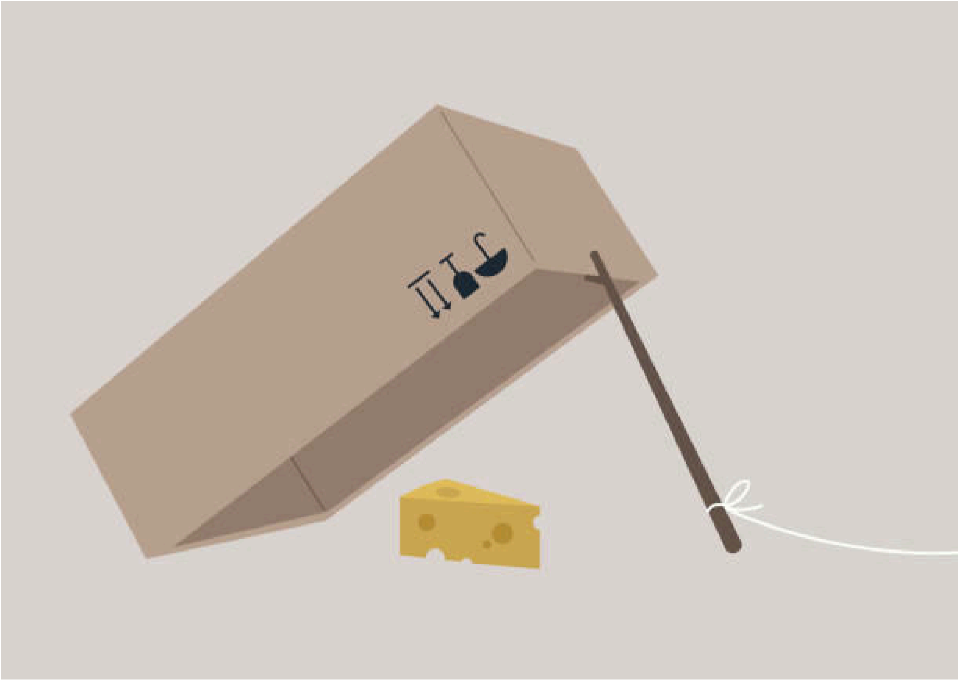

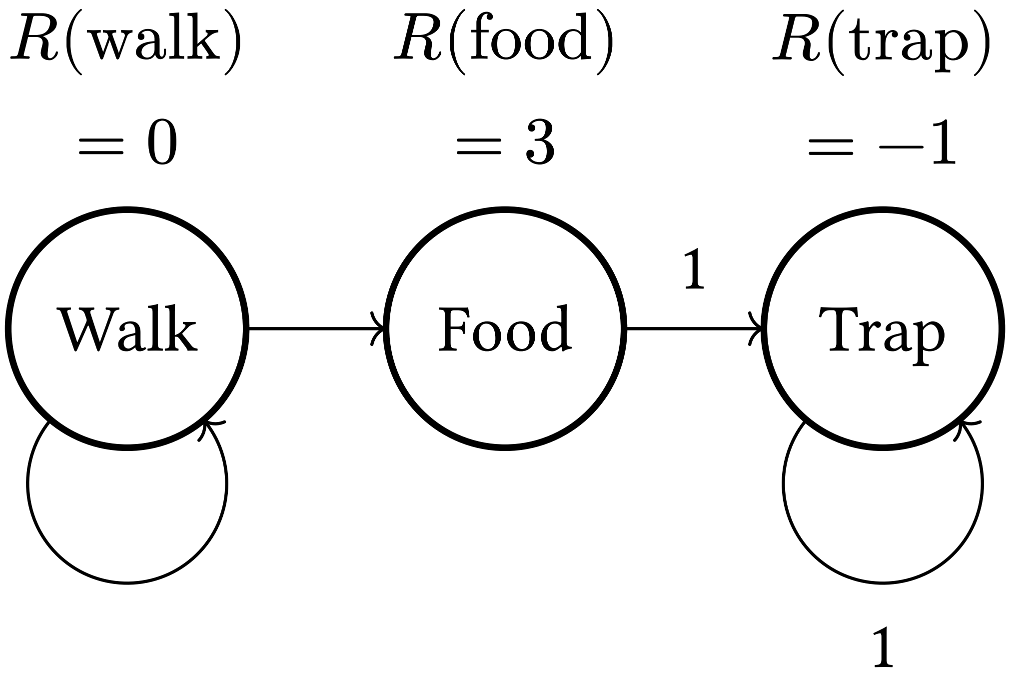

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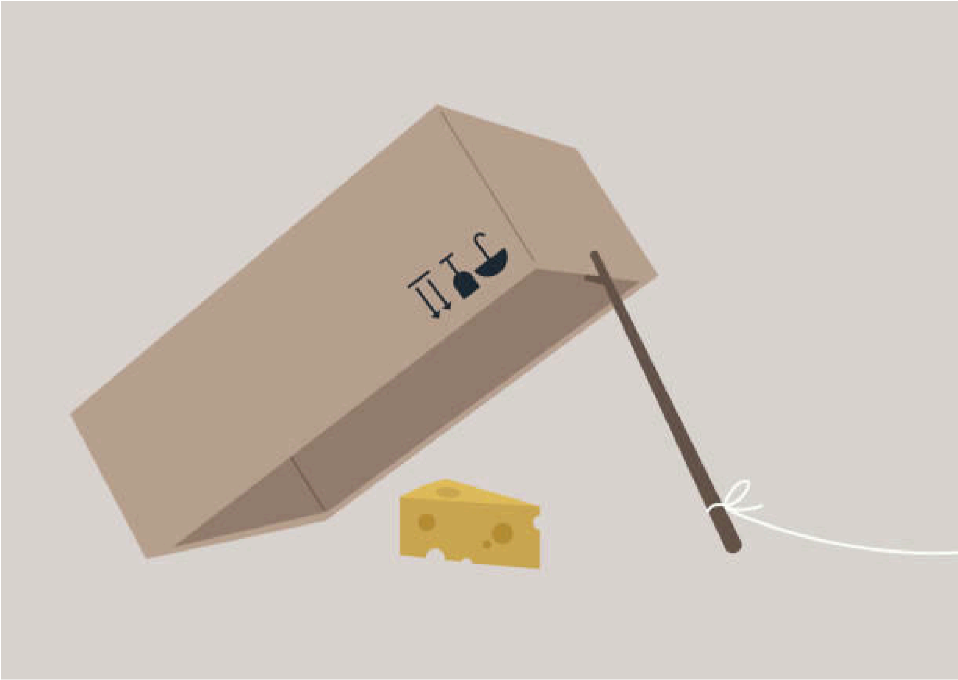

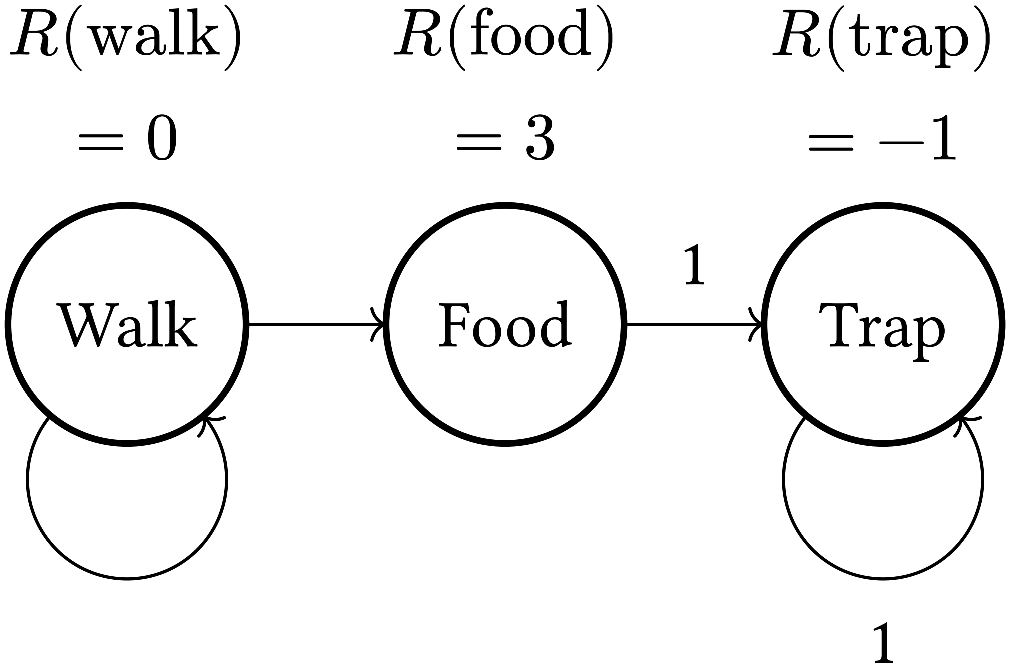

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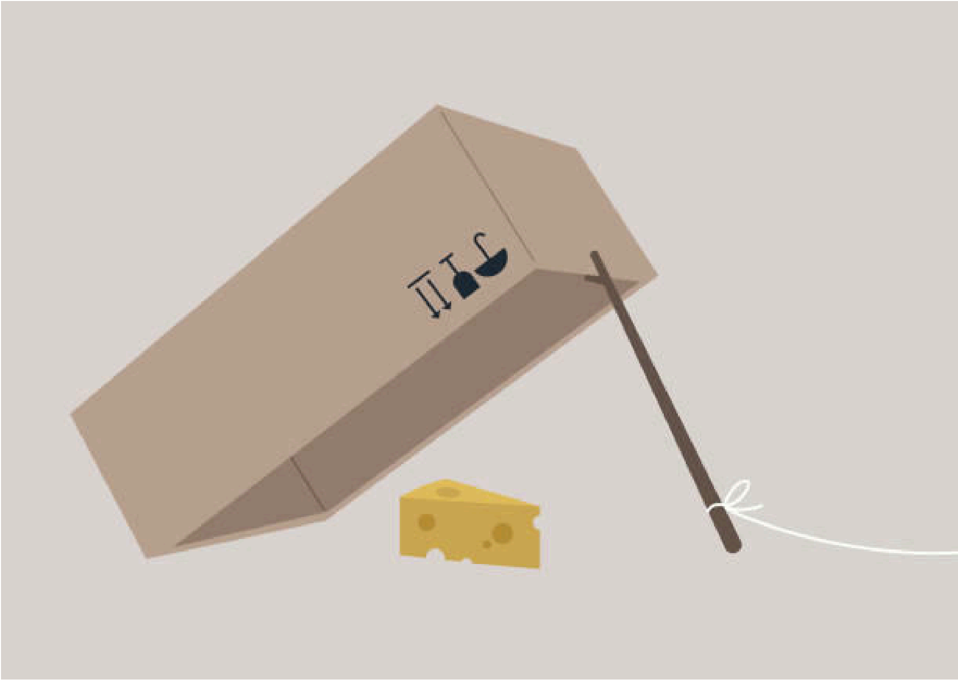
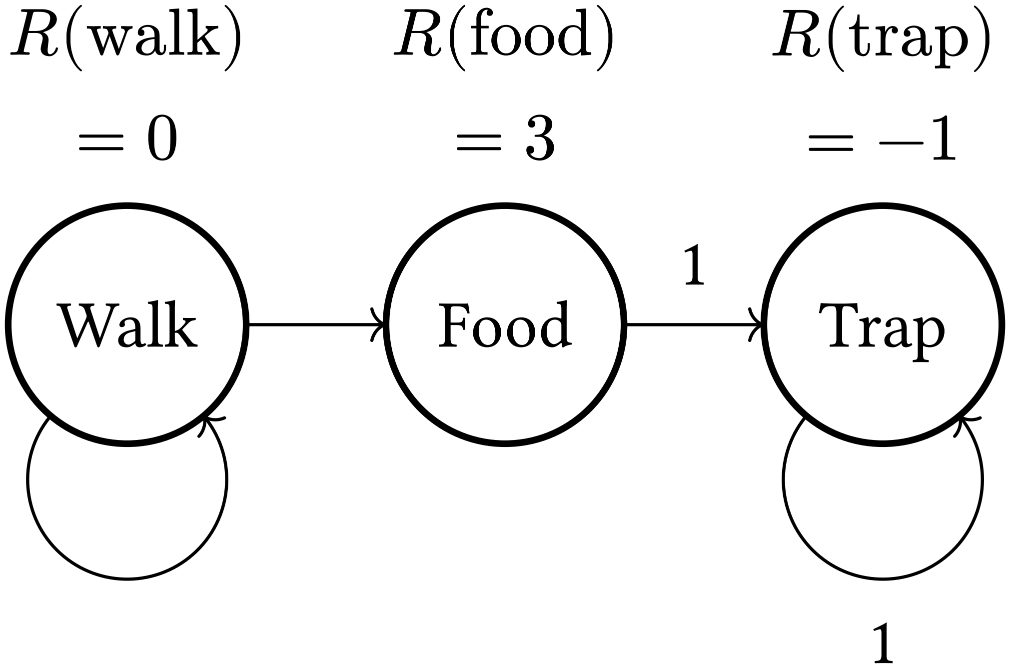
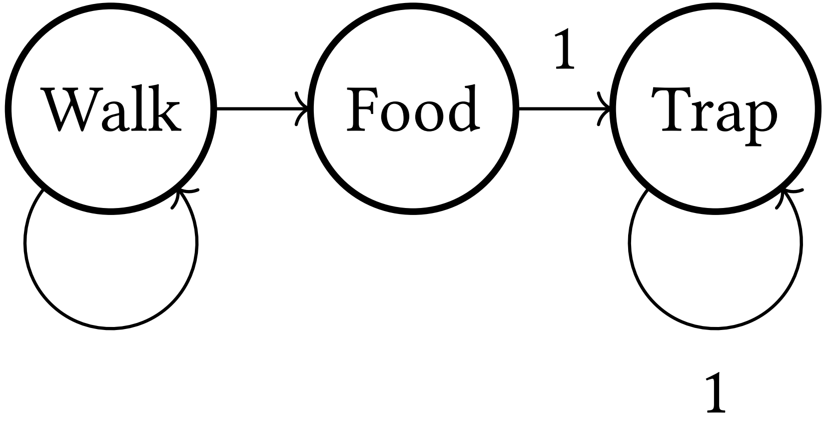




$$\arg\max_{s \in S} R(s) = \text{food}$$

Markov Decision Processes

$R(\text{walk}) = 0 \quad R(\text{food}) = 3 \quad R(\text{trap}) = -1$



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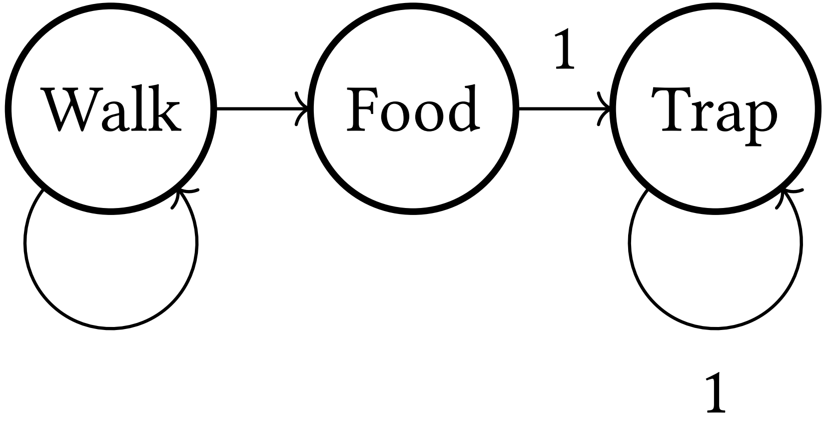

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$$G = \sum_{t=0}^{\infty} R(s_{t+1})$$

Markov Decision Processes

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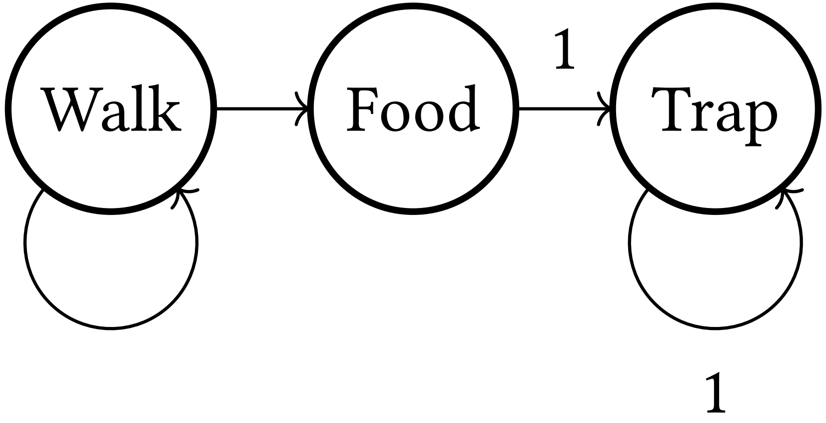

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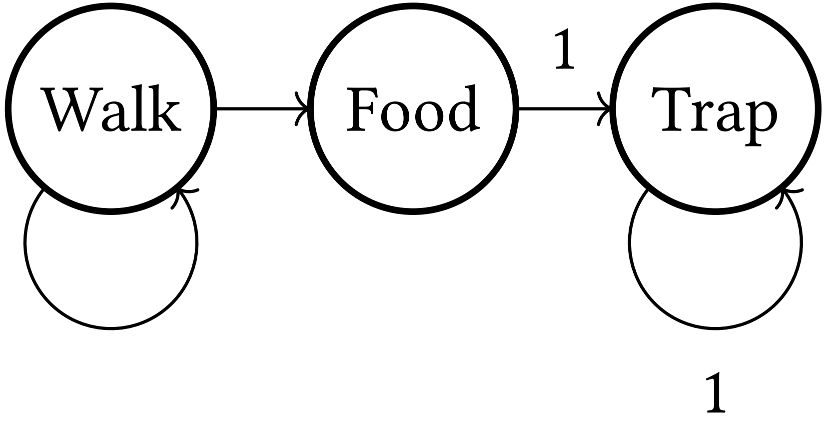

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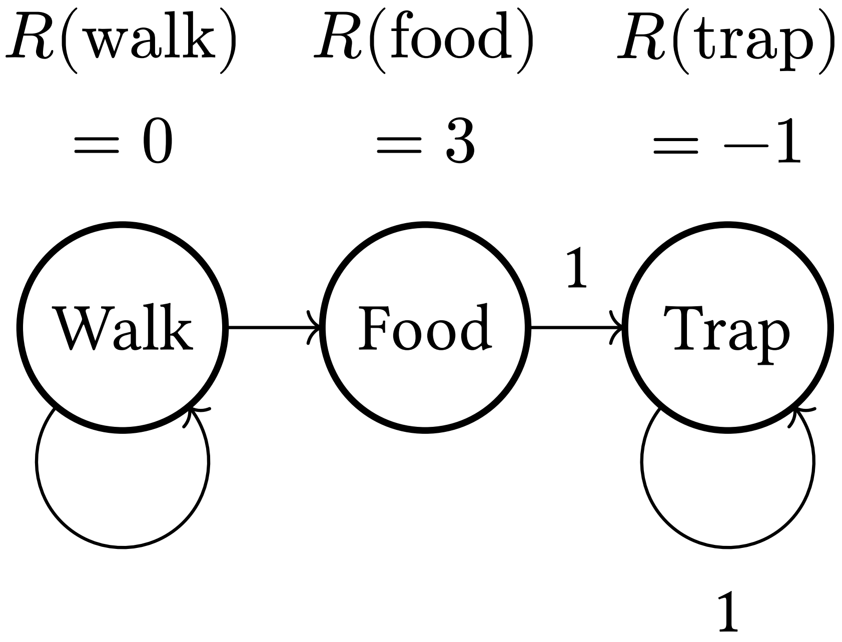

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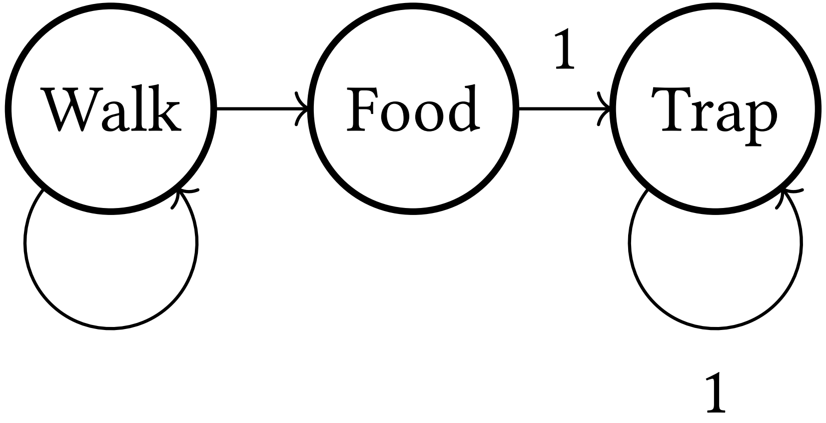

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Now, we make better decisions!

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Consider one more example

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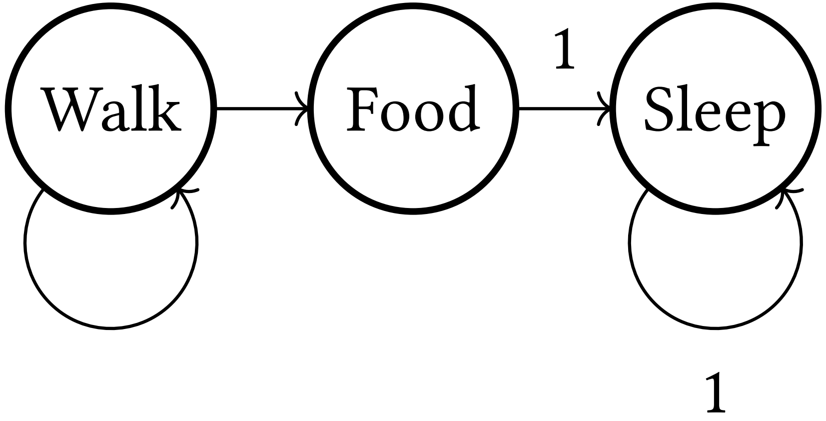

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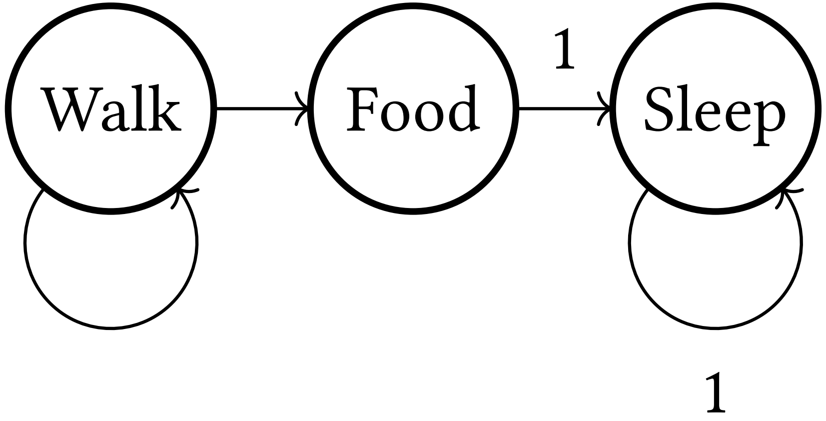

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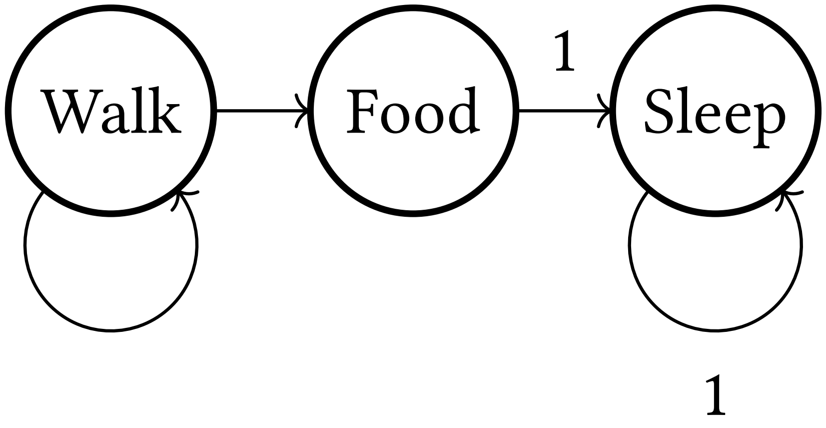

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Question: How?

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We can introduce a **discount** term $\gamma \in [0, 1]$ to the return

With $\gamma = 1$

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Markov Decision Processes

Without γ

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Thus, our objective is

$$\arg \max_{s \in S} G = \arg \max_{s \in S} \sum_{t=0}^{\infty} \gamma^t R(s_{t+1})$$

Markov Decision Processes

Let us review

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For the rest of the course, we will solve MDPs

Markov Decision Processes

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Markov Decision Processes

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Maximize the discounted return

$$\underset{s \in S}{\arg\max}\, G = \underset{s \in S}{\arg\max} \sum_{t=0}^{\infty} \gamma^t R(s_{t+1})$$

You must understand the discounted return!

Markov Decision Processes

Understanding MDPs is the **most important part** of RL

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Make sure you understand MDPs!

Exercise

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Model Super Mario Bros as an MDP

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Model Super Mario Bros as an MDP Design the:

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- State space S

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Compute discounted return for:

- Eat mushroom at $t = 10$
- Collect coins at $t = 11, 12$
- Die to bowser at $t = 20$

Coding

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In this course, we will implemented MDPs using **gymnasium**

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https://gymnasium.farama.org/api/env/

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Gymnasium uses **observations** instead of **states**

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If the Markov property is broken, $s_t \in S$ is not a Markov state

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Then, we change $s_t \in S$ to an **observation** $o_t \in O$ (more later)

Coding

```
import gymnasium as gym

MyMDP(gym.Env):
  def __init__(self):
    self.action_space = gym.spaces.Discrete(3) # A
    self.observation_space = gym.spaces.Discrete(5) # S

  def reset(self, seed=None) -> Tuple[Observation, Dict]

  def step(self, action) -> Tuple[
    Observation, Reward, Terminated, Truncated, Dict
  ]
```

Coding

https://colab.research.google.com/drive/1rDNik5oRl27si8wdtMLE7Y41U5J2bx-I#scrollTo=9pOLI5OgKvoE

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Too many A's last term, exam will be **difficult**