



Actor Critic II

CISC 7404 - Decision Making

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Quiz	2
Admin	6
Review	15
Actor Critic	16
Deterministic Policy Gradient	20
Deep Deterministic Policy Gradient	38
Coding	51
Max Entropy RL	60
Thoughts on Model-Free RL	71
Final Project Tips	75

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- After my explanation, you will have 75 minutes to complete the exam
- After you are done, give me your exam and go relax outside, we resume class at 8:30

Quiz

- There may or may not be different versions of the exam

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- Instructions are in both english and chinese, english instructions take precedence
- Good luck!

Quiz

- 在所有学生收起电脑/笔记/手机后,我会分发试卷。
- 如果在此之后仍有电脑/笔记/手机未收,将视为作弊。
- 试卷会背面朝下发下,在我宣布开始前请勿翻面。
- 试卷翻面后,我会简要说明每道题的注意事项。
- 说明结束后,你们有 75 分钟完成考试。
- 交卷后请到教室外休息,8:30 恢复上课。
- 试卷可能存在不同版本,细节略有差异。
- 若你的试卷上出现其他版本的答案,将被判定为作弊。
- 试卷说明为中英双语,若内容冲突以英文为准。
- 祝各位考试顺利!

Admin

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You should be able to understand research papers on:

- MDPs, bandits, RL algorithms, even some control problems!

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- Imitation Learning
- Offline RL
- Memory and POMDPs
- Large Language Models

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Question: Should we replace a topic with something else?

Admin

- Imitation learning
 - Sometimes, designing a reward function is hard
 - It is easier to demonstrate desired behavior to agents
 - Instead of reward for surgery, do what human does
 - Instead of reward for self driving, do what human does
 - With imitation learning, can learn behaviors without rewards
 - Closer to supervised learning, easier to train
 - Policies are not better than dataset/humans

Admin

- Offline RL
 - RL without exploration
 - How can we learn policies from a fixed dataset?
 - Learn surgery from surgical videos (no need to kill patients)
 - Learn driving from Xiaomi driving dataset (no need to crash cars)
 - Unlike imitation learning, can do **better** than dataset
 - Very new topic (2-3 years old)
 - Does not work very well (yet)

Admin

- Memory and POMDPs (my research focus)
 - So far, we focused on video games
 - MDP
 - Many interesting problems are not Markov
 - Think of robot with camera, not Markov
 - Almost every task has sensor noise, not Markov
 - Can we extend RL to work for virtually any problem?
 - Yes, requires long-term memory
 - LSTM, transformer, etc
 - May also have time to introduce world models
 - Dreamer, TD-MPC, etc

Admin

- Large Language Models
 - Can train LLMs using unsupervised learning
 - They only learn to predict next word
 - We use RL to teach them to interact with humans
 - Apply policy gradient to textual MDP
 - DeepSeek math/GRPO
 - RL-adjacent methods (DPO)

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Also a possibility to split lecture:

- E.g., 1 hour imitation learning, 1 hour something new

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Alternative topics:

- Multi-agent RL
- Model-based RL and world-models
- Evolutionary algorithms

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Homework 2 progress

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If you did not already start, you might be in trouble

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Experiments take a long time, start as soon as possible

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Harder and requires more debugging than FrozenLake assignment

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Those using Tencent AI Arena (Honor of Kings):

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 - Instead, consider DDPG, SAC, TRPO, DQN variant, etc
- Cannot install new python libraries (Tencent security issue)
 - No jax, must use torch
 - You must learn Tencent's strange callback system
 - Prevents copy/pasting, so torch is ok

Review

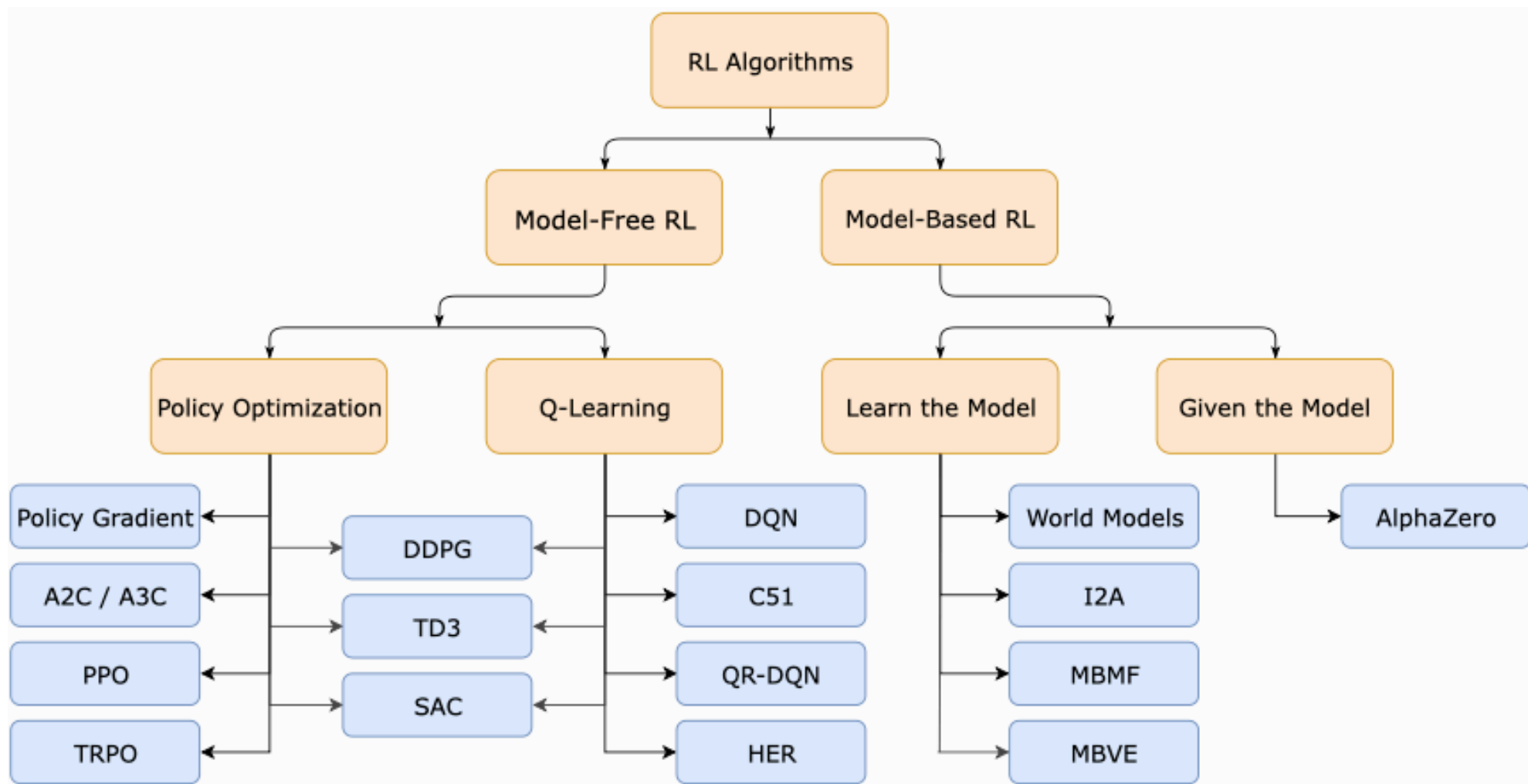
Actor Critic

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Alternative descriptions of actor critic algorithms

<https://lilianweng.github.io/posts/2018-04-08-policy-gradient/>

Actor Critic



Actor Critic

There are two approaches to actor critic

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1. **Policy gradient based:**

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PG with V instead of MC

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2. **Q learning based:**

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Learn Q for a specific policy

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Deterministic Policy Gradient

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Question: Why did we introduce policy gradient methods?

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Deterministic Policy Gradient



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Question: Why did Q learning fail BenBen?

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$$A = [0, 2\pi]^{12}$$

Deterministic Policy Gradient



Question: Why did Q learning fail BenBen?

$$A = [0, 2\pi]^{12}$$

$$\pi(a_t \mid s_t; \theta_\pi) = \begin{cases} 1 & \text{if } a_t = \arg \max_{a_t \in A} Q(s_t, a_t, \theta_\pi) \\ 0 & \text{otherwise} \end{cases}$$

Deterministic Policy Gradient



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Infinitely many a_t – compute Q for each and take arg max over all

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Let us quickly review the Q function and value function

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Start with general form of Temporal Difference Q function

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$$Q(s_0, a_0, \theta_\pi) = \underbrace{\mathbb{E}[\mathcal{R}(s_{t+1}) \mid s_0, a_0]}_{\text{Reward for taking } a_0} + \gamma \underbrace{V(s_1, \theta_\pi)}_{\mathcal{G} \text{ following } \theta_\pi}$$

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For the greedy policy, we can reduce Q further

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Cannot use the max Q function with BenBen

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Question: Can we use continuous a with Q ?

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Can we learn a different deterministic policy for continuous actions?

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Question: What method can we use to learn θ_π ?

Deterministic Policy Gradient

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Let me explain what I mean

Deterministic Policy Gradient

With deterministic policy, μ inside Tr means chain rule

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With stochastic policy, π **not inside** Tr (product rule)

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We use stochastic policies in RL because of this

Deep Deterministic Policy Gradient

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$$\nabla_{\theta_{\mu}} \mathbb{E}[\mathcal{G}(\tau) \mid s_0; \theta_{\mu}]$$

Deep Deterministic Policy Gradient

$$\nabla_{\theta_{\mu}} \mathbb{E}[\mathcal{G}(\tau) \mid s_0; \theta_{\mu}] \qquad \theta_{\pi, i+1} = \theta_{\pi, i} + \alpha \cdot \nabla_{\theta_{\mu}} \mathbb{E}[\mathcal{G}(\tau) \mid s_0; \theta_{\mu}]$$

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Let us try to derive deterministic policy gradient again

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This time, take gradient of V instead of gradient of $\mathbb{E}[\mathcal{G}(\tau) \mid s_0; \theta_\mu]$

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$$V(s_0, \theta_\mu) = \mathbb{E}[\mathcal{R}(s_1) \mid s_0; \theta_\mu] + \gamma V(s_0, \theta_\mu)$$

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$$\nabla_{\theta_\mu} V(s_0, \theta_\mu) = \nabla_{\theta_\mu} Q(s_0, \mu(s_0, \theta_\mu), \theta_\mu)$$

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Let us inspect these terms more closely

Deep Deterministic Policy Gradient

How θ_μ changes a

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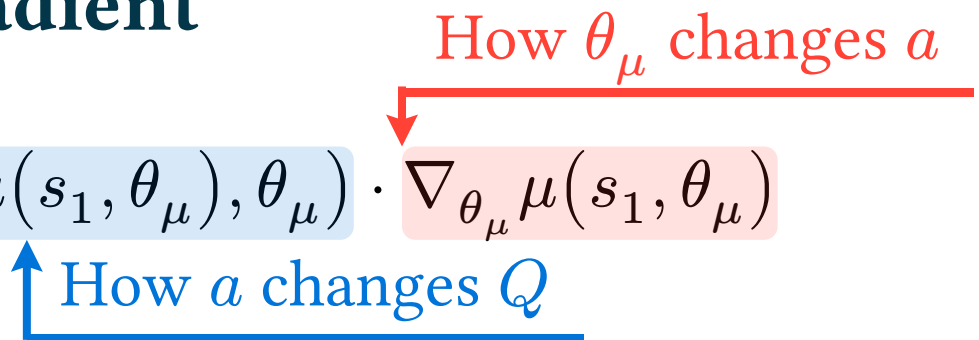
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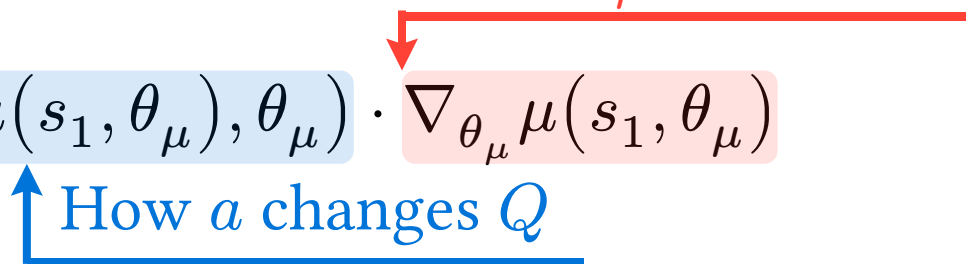
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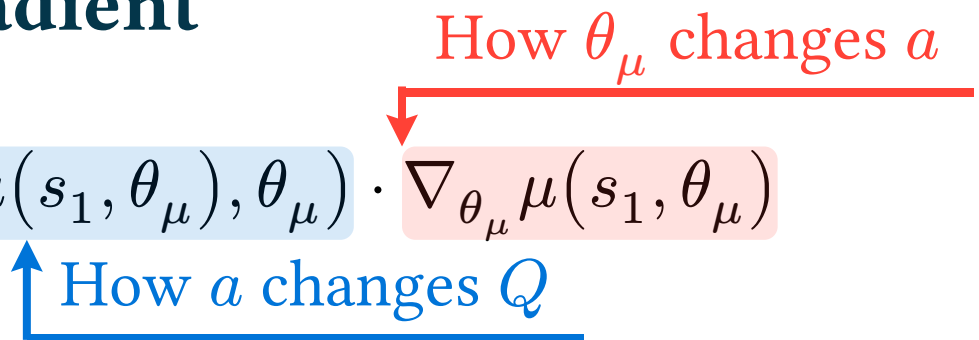
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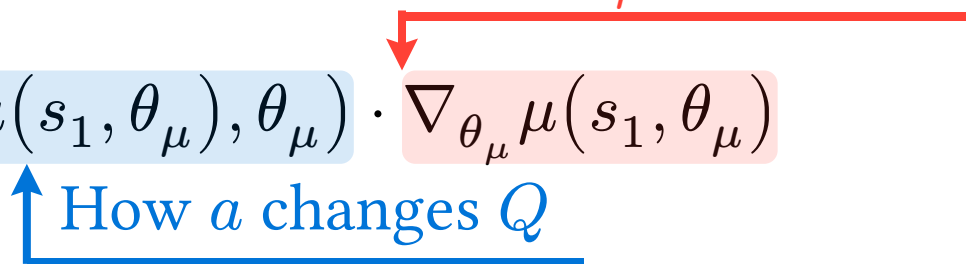
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We can backpropagate through Q without worrying about recursion

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Writing the code makes it look easy

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Writing the code makes it look easy

```
def V(s, Q_nn, mu_nn):  
    a = mu_nn(s)  
    return Q_nn(s, a) # Return value
```

```
# Learn the policy that maximizes V  
# Make sure to differentiate w.r.t policy parameters!  
J = grad(V, argnums=2)(states, Q_nn, mu_nn)  
mu_nn = optimizer.update(mu_nn, J)
```

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$$Q(s_0, a_0, \theta_\mu, \theta_Q) = \mathbb{E}[\mathcal{R}(s_1) \mid s_0, a_0] + \gamma Q(s_1, \mu(s_1, \theta_\mu), \theta_\mu, \theta_Q)$$

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```
def V(s, Q_nn, mu_nn):  
    a = mu_nn(s)  
    return Q_nn(s, a) # return value  
def TD_loss(s, a, r, next_s, Q_nn, mu_nn):  
    return (V(s, Q_nn, mu_nn) - td_label) ** 2  
# Now, we learn params of Q following policy (argnums=2)  
J = grad(TD_loss, argnums=1)(s, a, r, next_s, Q_nn, mu_nn)  
Q_nn = optimizer.update(Q_nn, J)
```

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Definition: Deep Deterministic Policy Gradient (DDPG) decomposes V into a deterministic policy μ and Q , learning them jointly

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$$\theta_{Q,i+1} = \arg \min_{\theta_{Q,i}}$$

$$\left(Q(s_0, a_0, \theta_{\mu,i}, \theta_{Q,i}) - \left(\hat{\mathbb{E}}[\mathcal{R}(s_1) \mid s_0, a_0] + \gamma Q(s_1, \mu(s_1, \theta_{\mu,i}), \theta_{\mu,i}, \theta_{Q,i}) \right) \right)$$

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$$\theta_{\mu,i+1} = \theta_{\mu,i} + \alpha \cdot Q(s_0, \mu(s_0, \theta_\mu), \theta_{\mu,i}, \theta_{Q,i+1})$$

Repeat until convergence, $\theta_{\mu,i+1} = \theta_{\mu,i}$, $\theta_{Q,i+1} = \theta_{Q,i}$

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Question: Is DDPG on-policy or off-policy?

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Almost **all** good off-policy actor-critic algorithms are based on DDPG

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Because μ is a neural network, it can generalize to continuous s, a

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- More recent papers find using normal noise produces similar results

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mu = Sequential([  
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BenBen: $A = [0, 2\pi]^{12}$, so `action_dims=12`

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```
def bound_action(action, lower, upper):  
    return 0.5 * (upper + lower) + 0.5 * (upper - lower)  
        * tanh(action)
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def bound_action(action, lower, upper):  
    return 0.5 * (upper + lower) + 0.5 * (upper - lower)  
        * tanh(action)  
  
def sample_action(mu, state, A_bounds, std):  
    action = mu(state)  
    noisy_action = action + normal(0, std) # Explore  
    return bound_action(noisy_action, *A_bounds)
```

Coding

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Coding

Now construct Q

```
Q = Sequential([
    # Different from DQN network
    # Input action and state together
    Lambda(lambda s, a: concatenate(s, a)),
    Linear(state_size + action_dims, hidden_size),
    Lambda(leaky_relu),
    Linear(hidden_size, hidden_size),
    Lambda(leaky_relu),
    Linear(hidden_size, 1), # Single value for Q(s, a)
])
```

Coding

```
while not terminated:
    # Exploration: make sure actions within action space!
    action = sample_action(mu, state, bounds, std)
    transition = env.step(action)
    replay_buffer.append(transition)
    data = replay_buffer.sample()
    # Theta_pi params are in mu neural network
    # Argnums tells us differentiation variable
    J_Q = grad(Q_loss, argnums=0)(theta_Q, theta_T, mu, data)
    theta_Q = apply_updates(J_Q, ...)
    J_mu = grad(mu_loss, argnums=0)(mu, theta_Q, data)
    mu = apply_updates(J_mu, ...)
    if step % 200 == 0: # Target network necessary
        theta_T = theta_Q
```

Coding

```
def Q_loss(theta_Q, theta_T, theta_pi, data):  
    Qnet = combine(Q, theta_Q)  
    Tnet = combine(Q, theta_T) # Target network  
    # Predict Q values for action we took  
    prediction = vmap(Qnet)(data.state, data.action)  
    # Now compute labels using TD error  
    next_action = vmap(mu)(data.next_state)  
    # NOTE: No max! Mu approximates argmax  
    next_Q = vmap(Tnet)(data.next_state, next_action)  
    label = data.reward + gamma * data.done * next_Q  
    return (prediction - label) ** 2
```

Coding

```
def mu_loss(mu, theta_Q, data):  
    # Find the action that maximizes the Q function  
    Qnet = combine(Q, theta_Q)  
    # Instead of multiply, chain rule -- plug action into Q  
    action = vmap(mu)(data.state)  
    q_value = vmap(Qnet)(data.state, action)  
    # Update the policy parameters to maximize the Q value  
    # Gradient ascent but we min loss, use negative  
    return -q_value
```

Coding

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- Add noise to target

$$Q(s, \mu(s, \theta_\mu) + \eta, \theta_\mu, \theta_T)$$

- Learns two Q functions, use the minimum as target

$$\min_{i \in 1,2} Q(s, \mu(s, \theta_\mu) + \eta, \theta_\mu, \theta_{T,i})$$

- Update Q functions more often than policy

Max Entropy RL

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Many algorithms add improvements to DDPG

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We will very briefly cover max-entropy RL

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We will very briefly cover max-entropy RL

First, let us introduce entropy

Max Entropy RL

Entropy measures the uncertainty of a distribution

Max Entropy RL

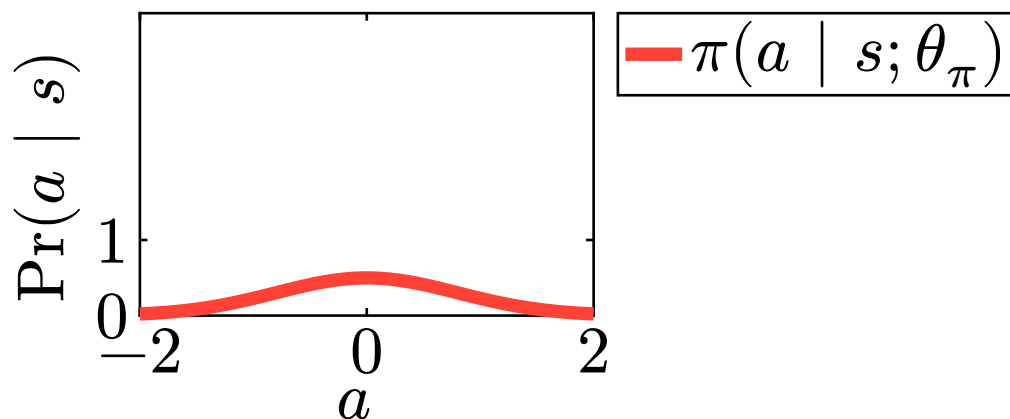
Entropy measures the uncertainty of a distribution

$$H(\pi(a \mid s; \theta_\pi))$$

Max Entropy RL

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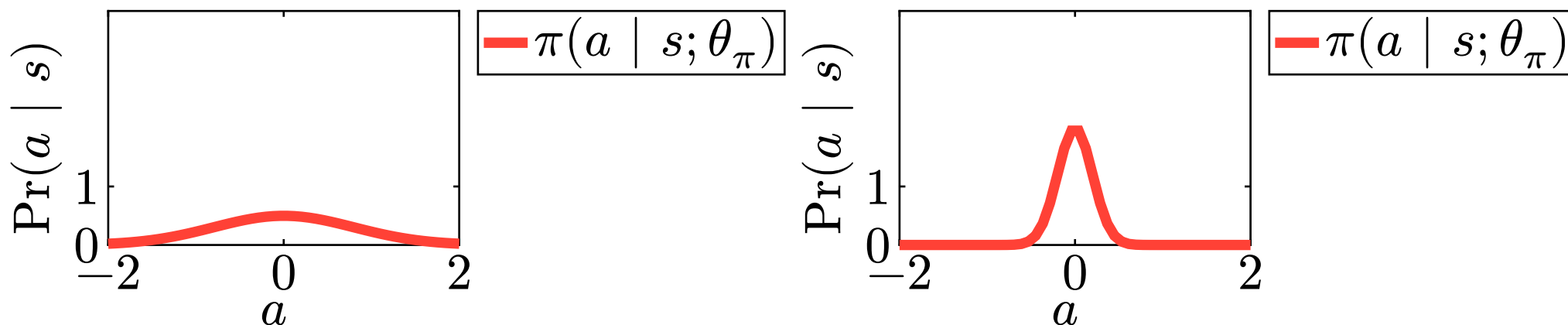
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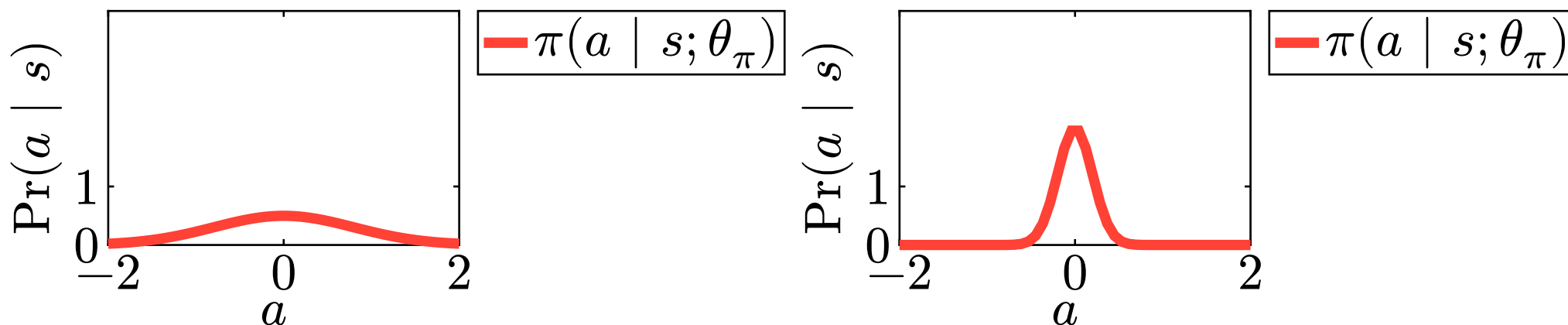
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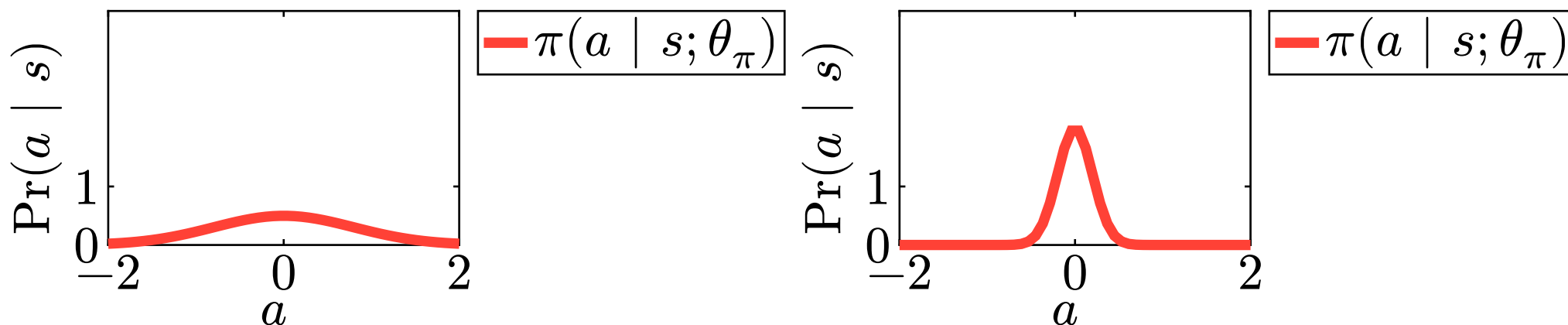


Question: Which policy has higher entropy?

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Left policy, more uncertain/random

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We learn a policy that maximizes the return and **behaves randomly**

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 - Further discussion in offline RL lecture

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↑ Noise terms

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We get SAC!

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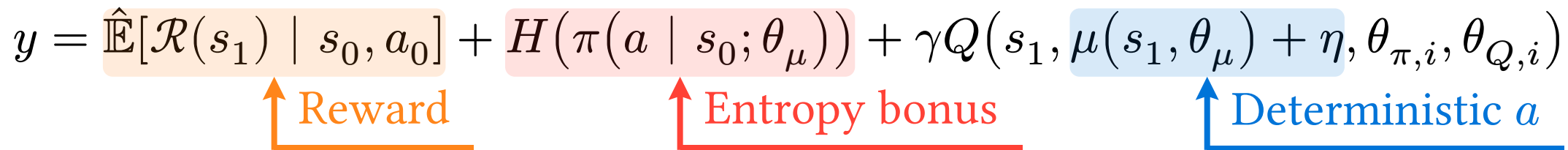
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Reward Entropy bonus Deterministic a

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Repeat until convergence, $\theta_{\mu,i+1} = \theta_{\mu,i}$, $\theta_{Q,i+1} = \theta_{Q,i}$

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Like PPO, there are many variants of SAC

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Coding SAC could take an entire lecture, read CleanRL

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Duality between policy gradient actor critic and Q learning actor critic

Introduces the concept, simple

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- Fewer hyperparameters
- Tuned DDPG can likely outperform untuned SAC

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 - Think about why it learned to do this (exploiting bugs in MDP)

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You must use your brain to be successful!