Bandits

CISC 7404 - Decision Making

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Review

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In this course, we will focus primarily on reinforcement learning

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But reinforcement learning is a method, not a problem

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But reinforcement learning is a method, not a problem

The problem is **decision making**

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In this course, we focus on **optimal** decision making

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With an optimal decision making machine, you can create:
- Best possible doctor (which medicine to give?)
- Best possible lawyer (what to argue?)
- Best possible scientist (what to research?)

If the machine understands **why** it makes decisions, it is conscious

Set Notation

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Let us review some notation I will use in the course

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If you ever get confused, come back to these slides

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Vectors

$$\boldsymbol{x} = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}$$

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$$\boldsymbol{x} = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}$$

Matrices

$$\boldsymbol{X} = \begin{bmatrix} x_{1,1} & x_{1,2} & \dots & x_{1,n} \\ x_{2,1} & x_{2,2} & \dots & x_{2,n} \\ \vdots & \vdots & \ddots & \vdots \\ x_{m,1} & x_{m,2} & \dots & x_{m,n} \end{bmatrix}$$

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We will represent **tensors** as nested vectors or matrices

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$$\boldsymbol{x} = \begin{bmatrix} \boldsymbol{x}_1 \\ \boldsymbol{x}_2 \\ \vdots \\ \boldsymbol{x}_n \end{bmatrix}$$

Each $\boldsymbol{x}_i$ is a vector

Set Notation

Same for matrices

Tensor of matrices

$$\boldsymbol{X} = \begin{bmatrix} \boldsymbol{x}_{1,1} & \boldsymbol{x}_{1,2} & \dots & \boldsymbol{x}_{1,n} \\ \boldsymbol{x}_{2,1} & \boldsymbol{x}_{2,2} & \dots & \boldsymbol{x}_{2,n} \\ \vdots & \vdots & \ddots & \vdots \\ \boldsymbol{x}_{m,1} & \boldsymbol{x}_{m,2} & \dots & \boldsymbol{x}_{m,n} \end{bmatrix}$$

Set Notation

Question: What is the difference between the following?

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$\mathbb{R}$	Set of all real numbers $\{1, 2.03, \pi, ...\}$

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$\mathbb{Z}$	Set of all integers $\{-2, -1, 0, 1, 2, ...\}$

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$\mathbb{Z}_+$	Set of all **positive** integers $\{1, 2, ...\}$

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$$[0, 1]$$

Closed interval

$0.0, 0.01, 0.00...1, 0.99, 1.0$

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$\{0, 1\}^{k \times k}$	A matrix of boolean values of shape k by k

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You can think of this as a for loop

```
output = {} # Set
for x in Z:
  output.insert(x + 1)
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output = {x + 1 for x in Z}
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$$f \quad : \quad \mathbb{R} \quad \mapsto \quad \mathbb{Z}$$

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Question: What functions could f be?

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$$X \in \mathbb{R}^n; \Theta \in \mathbb{R}^{m \times n}; Y \in [0, 1]^{n \times m}$$

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We want to make optimal decisions, so we will often take the minimum or maximum of functions

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$$f(x) = -(x+1)^2$$

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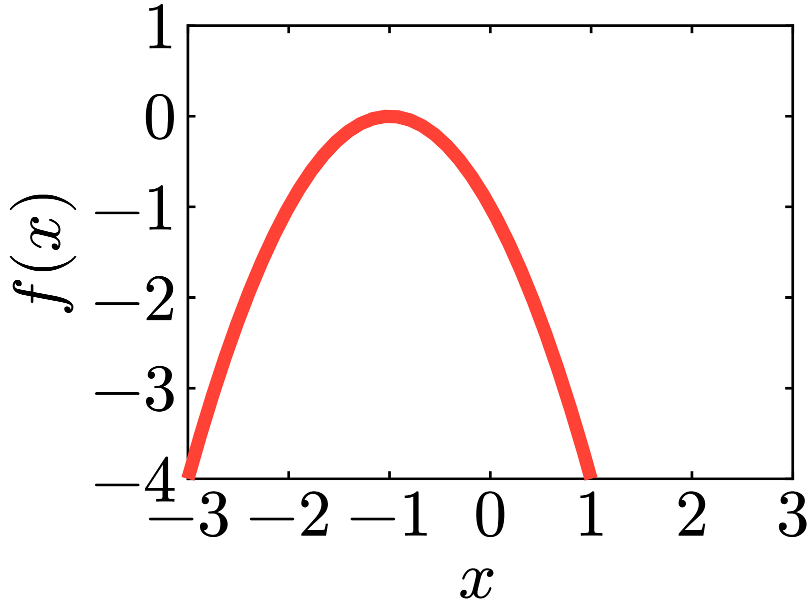

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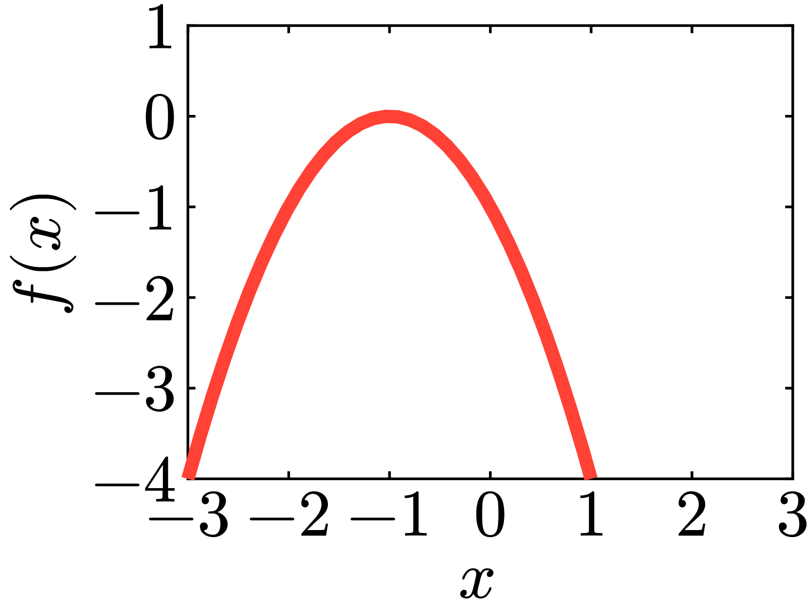

$$\max_{x \in \mathbb{R}} f(x)?$$

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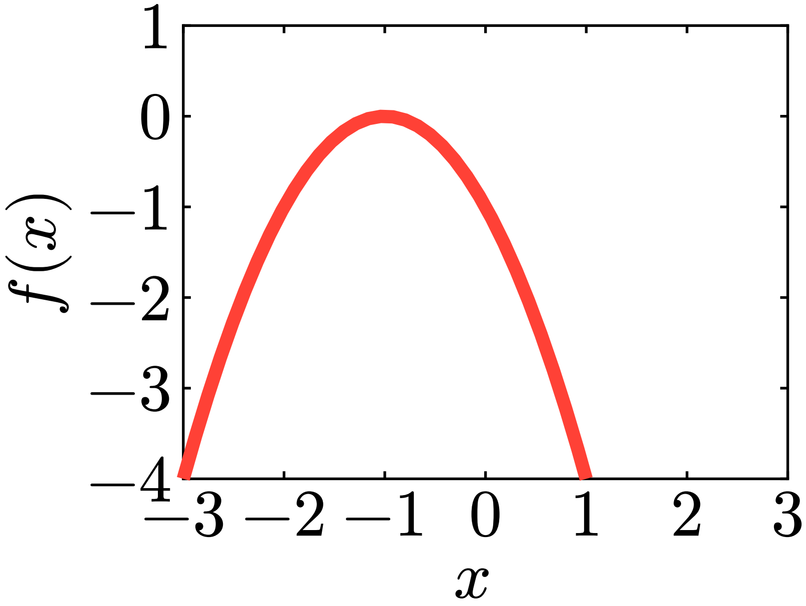

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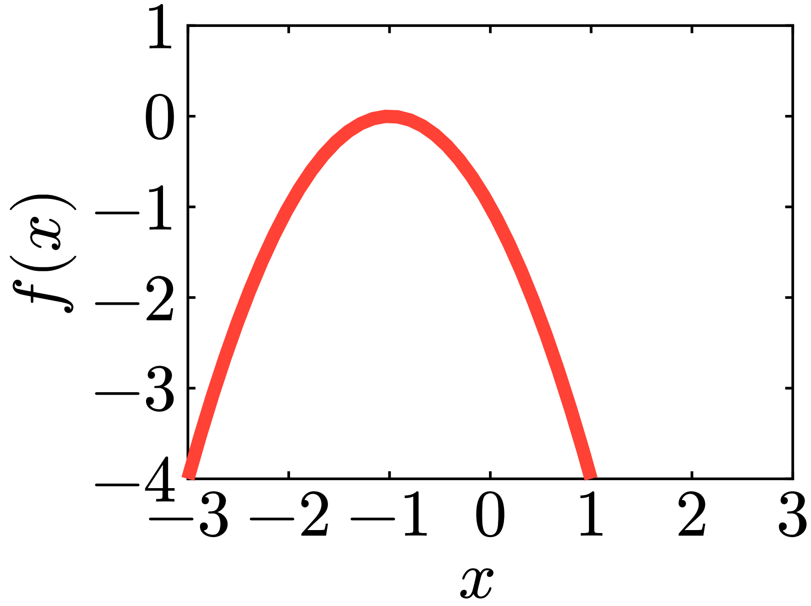

$$\max_{x \in \mathbb{R}} f(x)? \qquad \arg\max_{x \in \mathbb{R}} f(x)? \qquad \arg\max_{x \in \mathbb{Z}_+} f(x)?$$

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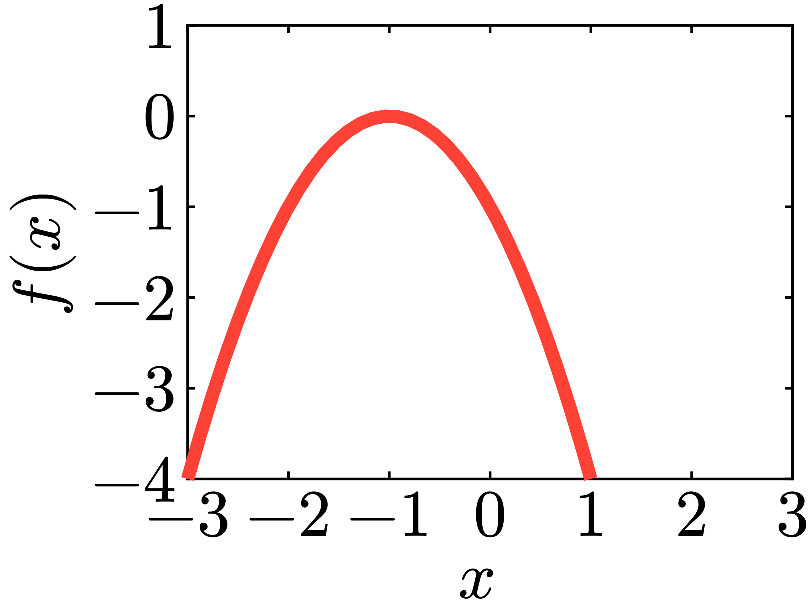

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$$0$$

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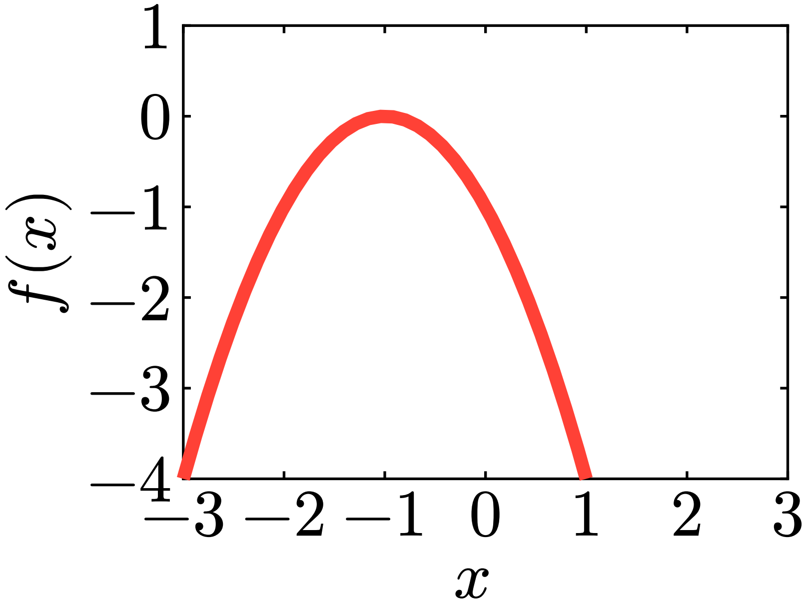

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$$-1$$

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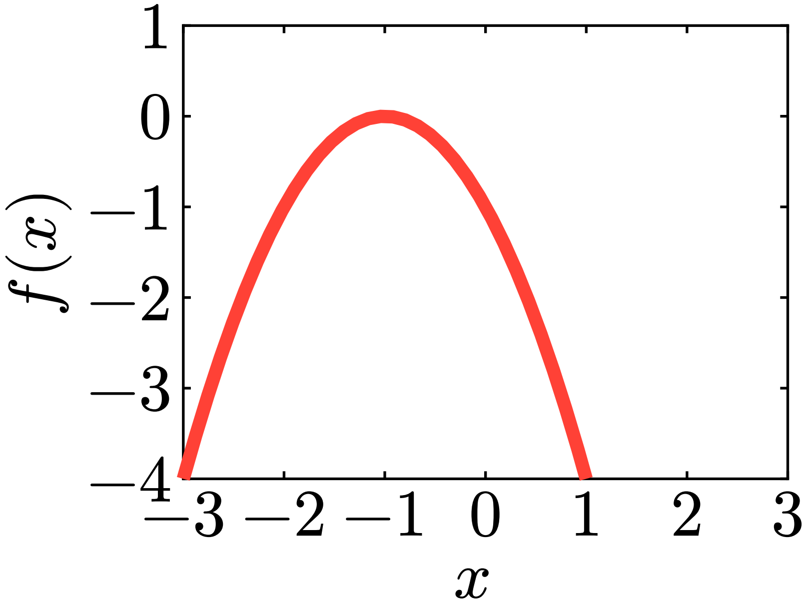

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$$0 \qquad -1 \qquad 1$$

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$$\left\{ x^{\frac{1}{2}} \mid x \in \mathbb{R}_+ \right\}$$

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Question: What is this?

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Answer:

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Question: What is this?

Answer:

- An infinitely large set of all real numbers greater than zero

Exercises

$$\left\{x^{\frac{1}{2}} \mid x \in \mathbb{R}_{+}\right\}$$

Question: What is this?

Answer:

- An infinitely large set of all real numbers greater than zero
- The results of evaluating $f(x) = \sqrt{x}$ for all positive real numbers

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Today’s lecture will be difficult

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If you can understand it, then reinforcement learning will be easy

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Bandits are the simplest decision making problem

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Question: What is a bandit?

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A bandit steals your money

Bandits

Here is the bandit we will focus on in this course

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This is a **one-armed** bandit

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Question: How does a one-armed bandit steal your money?

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Answer: You win less money than you put in

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Your chance of winning is $\frac{1}{200}$

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Let us see if we can make money playing this game

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For our bandit, we have two possible outcomes

$$\Omega \in \{\text{win}, \text{lose}\}$$

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An **event** is a set of outcomes

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$$E_{\text{win}} = \{\text{win}\}; \quad E_{\text{lose}} = \{\text{lose}\}; \quad E_{\text{any}} = \{\text{win}, \text{lose}\}$$

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We define the probabilites over the outcome and event spaces

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Event probabilities do not always sum to one

$$E_{\text{win}} = \{\text{win}\} \qquad \sum_{\varepsilon \in E} \Pr(\varepsilon) \leq 1$$

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Question: What is the random variable for the bandit?

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Question: What is the random variable for the bandit?

$$\mathcal{X} : \{\text{lose}, \text{win}\} \mapsto \{-10, 1000\}$$

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$$\frac{199}{200} \cdot -10 + \frac{1}{200} \cdot 1000 = -4.95$$

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Negative reward means we lose money

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If play the game more, the mean reward converges to the expectation

$$\lim_{n \to \infty} \sum_{t=1}^{n} r_t = n \cdot \mathbb{E}[\mathcal{X}] = -4.95n$$

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The more you play, the closer you get to $n \cdot \mathbb{E}[\mathcal{X}]$

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Question: Could a gambler find out $\mathbb{E}[\mathcal{X}]$?

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Gambler only has access to the rewards

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After playing enough, the gambler can approximate the expectation!

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- Near zero: The gambler wins sometimes and will continue to play

Multiarmed Bandits

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The bandit problem is useful for casino owners and gamblers

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We will make the problem more interesting

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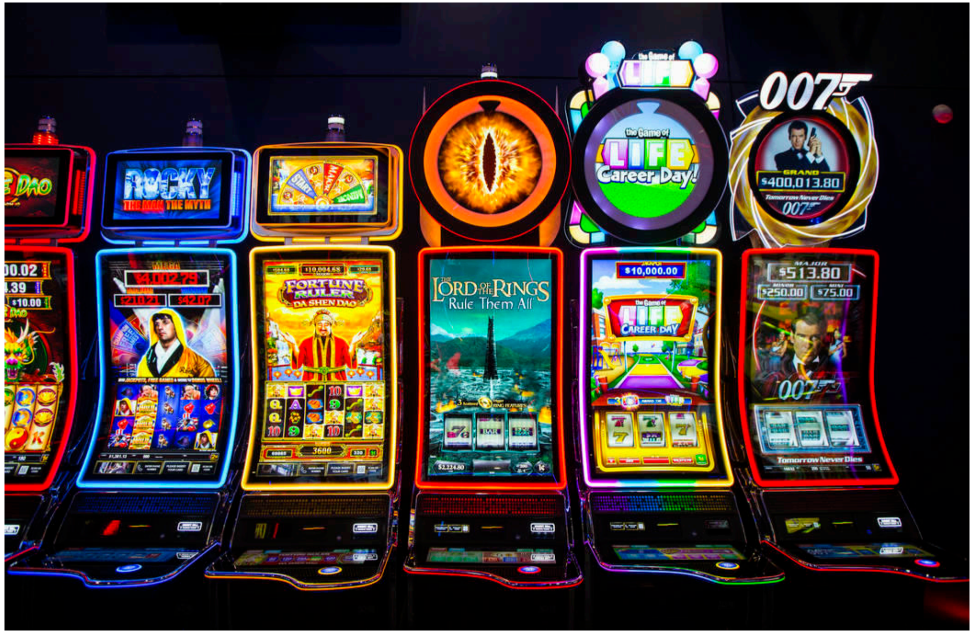

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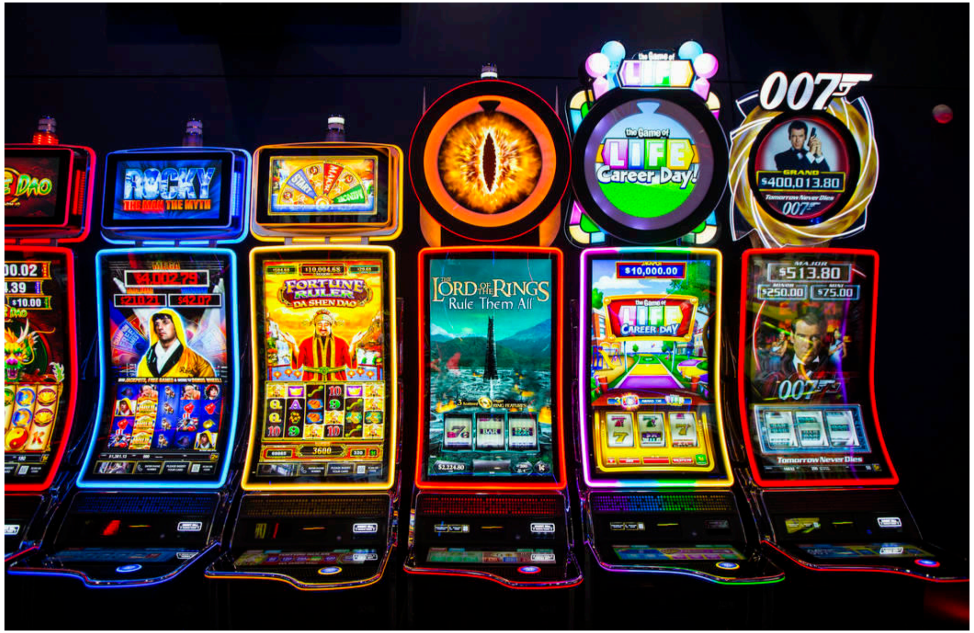

Question: Which machine do you play?

Multiarmed Bandits

We call this the **multi-armed bandit** problem

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You don't know the expected value of each arm. Which should you pull?

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We can find the best medicine while healing the most people

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TikTok select videos to maximize your $\mathbb{E}$[❤️]

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Which actions should you take to make the most money?

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- Pick a to estimate bandits

$$\mathbb{E}[\mathcal{X}_a \mid a \in 1...k]$$

Multiarmed Bandits

This is a hard problem!

We need to estimate $\mathbb{E}[\mathcal{X}_1], \mathbb{E}[\mathcal{X}_2], ..., \mathbb{E}[\mathcal{X}_k]$ to find the best $\mathcal{X}$

But it takes ∞ money to find $\mathbb{E}[\mathcal{X}]$!

$$\mathbb{E}[\mathcal{X}] = \lim_{n \to \infty} \frac{1}{n} \sum_{t=1}^{n} r_t$$

Which action $a \in \{1...k\}$ do we choose? Which bandit do we play?

We want to:

- Pick a to estimate bandits $\mathbb{E}[\mathcal{X}_a \mid a \in 1...k]$
- Pick a to make the most money $\arg\max_{a \in \{1...k\}} \mathbb{E}[\mathcal{X}_a]$

Multiarmed Bandits

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Exploration:

$$\mathbb{E}[\mathcal{X}_a \mid a \in \{1...k\}]$$

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It is important to understand the difference between exploration and exploitation! Any questions?

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$$a \sim \text{uniform}(\{1...k\})$$

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Question: How can we achieve both goals at once?

Answer: Sometimes choose a to explore, sometimes choose a to exploit

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$$u \sim \mathrm{uniform}([0, 1])$$

$$\text{if } u < 0.5 \text{ then } a \sim \mathrm{uniform}(\{1...k\})$$

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$\varepsilon \approx 1$ when we trust our estimates of $\mathbb{E}[\mathcal{X}]$

$\varepsilon \approx 0$ when we do not trust our estimates of $\mathbb{E}[\mathcal{X}]$

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- Sometimes it suggests study videos, to understand if you like study videos more

Questions?

Coding

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Let us code some multiarmed bandits!

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https://colab.research.google.com/drive/1cyNLRa-J8oe7pgy_gs2mcypZPqqaquoa