



Actor Critic II

CISC 7404 - Decision Making

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- After my explanation, you will have 75 minutes to complete the exam

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- After my explanation, you will have 75 minutes to complete the exam
- After you are done, give me your exam and go relax outside, we resume class at 8:30

Quiz

- There may or may not be different versions of the exam

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- Instructions are in both english and chinese, english instructions take precedence
- Good luck!

Quiz

- 在所有学生收起电脑/笔记/手机后,我会分发试卷。
- 如果在此之后仍有电脑/笔记/手机未收,将视为作弊。
- 试卷会背面朝下发下,在我宣布开始前请勿翻面。
- 试卷翻面后,我会简要说明每道题的注意事项。
- 说明结束后,你们有 75 分钟完成考试。
- 交卷后请到教室外休息,8:30 恢复上课。
- 试卷可能存在不同版本,细节略有差异。
- 若你的试卷上出现其他版本的答案,将被判定为作弊。
- 试卷说明为中英双语,若内容冲突以英文为准。
- 祝各位考试顺利!

Admin

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You should be able to understand research papers on:

- MDPs, bandits, RL algorithms, even some control problems!

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- Imitation Learning
- Offline RL
- Memory and POMDPs
- Large Language Models

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Question: Should we replace a topic with something else?

Admin

- Imitation learning
 - Sometimes, designing a reward function is hard
 - It is easier to demonstrate desired behavior to agents
 - Instead of reward for surgery, do what human does
 - Instead of reward for self driving, do what human does
 - With imitation learning, can learn behaviors without rewards
 - Closer to supervised learning, easier to train
 - Policies are not better than dataset/humans

Admin

- Offline RL
 - RL without exploration
 - How can we learn policies from a fixed dataset?
 - Learn surgery from surgical videos (no need to kill patients)
 - Learn driving from Xiaomi driving dataset (no need to crash cars)
 - Unlike imitation learning, can do **better** than dataset
 - Very new topic (2-3 years old)
 - Does not work very well (yet)

Admin

- Memory and POMDPs (my research focus)
 - So far, we focused on video games
 - MDP
 - Many interesting problems are not Markov
 - Think of robot with camera, not Markov
 - Almost every task has sensor noise, not Markov
 - Can we extend RL to work for virtually any problem?
 - Yes, requires long-term memory
 - LSTM, transformer, etc
 - May also have time to introduce world models
 - Dreamer, TD-MPC, etc

Admin

- Large Language Models
 - Can train LLMs using unsupervised learning
 - They only learn to predict next word
 - We use RL to teach them to interact with humans
 - Apply policy gradient to textual MDP
 - DeepSeek math/GRPO
 - RL-adjacent methods (DPO)

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Also a possibility to split lecture:

- E.g., 1 hour imitation learning, 1 hour something new

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Alternative topics:

- Multi-agent RL
- Model-based RL and world-models
- Evolutionary algorithms

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Homework 2 progress

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If you did not already start, you might be in trouble

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Experiments take a long time, start as soon as possible

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Experiments take a long time, start as soon as possible

Harder and requires more debugging than FrozenLake assignment

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Those using Tencent AI Arena (Honor of Kings):

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 - Instead, consider DDPG, SAC, TRPO, DQN variant, etc
- Cannot install new python libraries (Tencent security issue)
 - No jax, must use torch
 - You must learn Tencent's strange callback system
 - Prevents copy/pasting, so torch is ok

Review

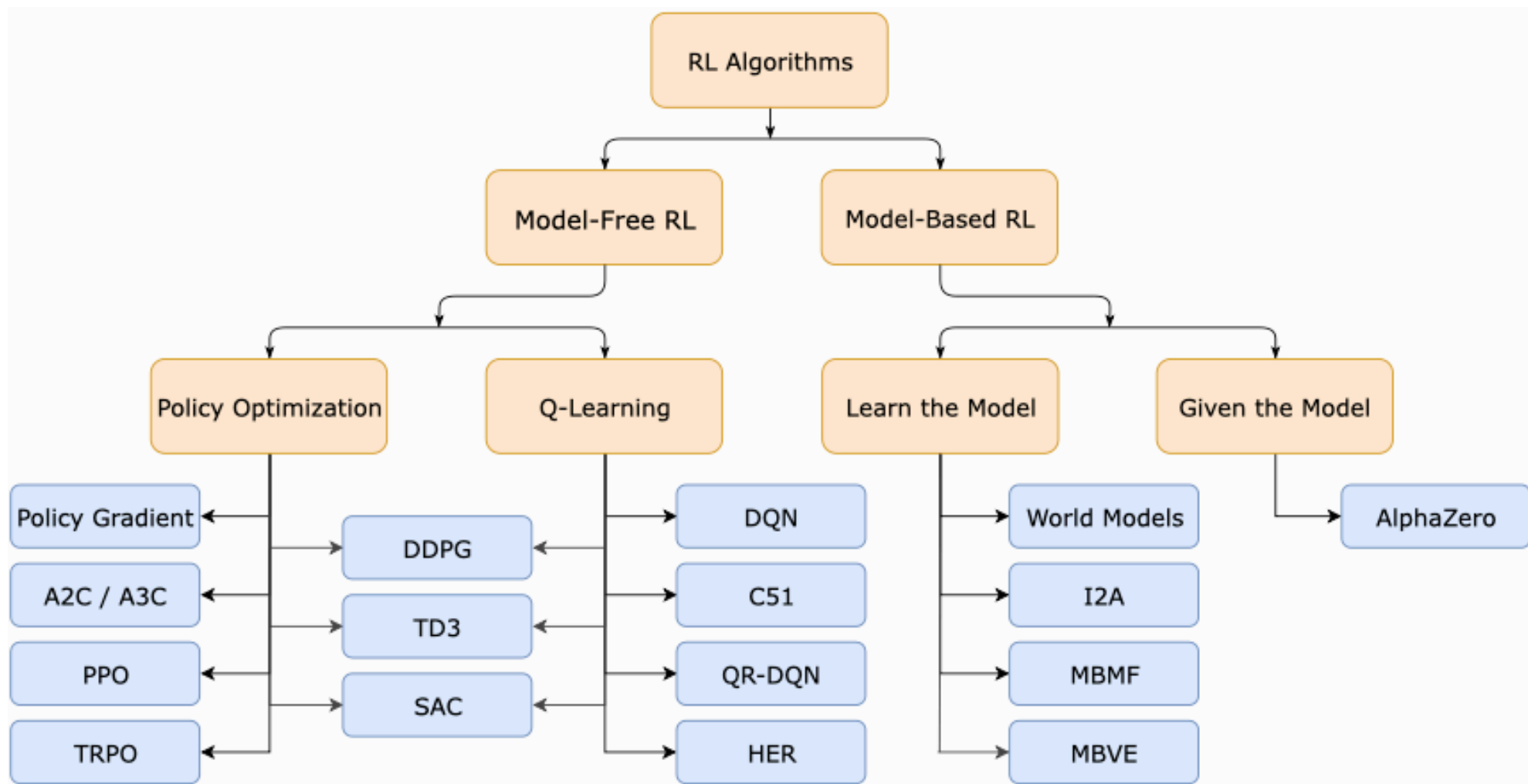
Actor Critic

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Alternative descriptions of actor critic algorithms

<https://lilianweng.github.io/posts/2018-04-08-policy-gradient/>

Actor Critic



Actor Critic

There are two approaches to actor critic

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1. **Policy gradient based:**

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PG with V instead of MC

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2. **Q learning based:**

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Learn Q for a specific policy

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Deterministic Policy Gradient

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Question: Why did we introduce policy gradient methods?

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Deterministic Policy Gradient



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Question: Why did Q learning fail BenBen?

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$$A = [0, 2\pi]^{12}$$

Deterministic Policy Gradient



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$$A = [0, 2\pi]^{12}$$

$$\pi(a_t \mid s_t; \theta_\pi) = \begin{cases} 1 & \text{if } a_t = \arg \max_{a_t \in A} Q(s_t, a_t, \theta_\pi) \\ 0 & \text{otherwise} \end{cases}$$

Deterministic Policy Gradient



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Infinitely many a_t – compute Q for each and take arg max over all

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Let us quickly review the Q function and value function

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Start with general form of Temporal Difference Q function

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$$Q(s_0, a_0, \theta_\pi) = \underbrace{\mathbb{E}[\mathcal{R}(s_1) \mid s_0, a_0]}_{\text{Reward for taking } a_0} + \gamma \underbrace{V(s_1, \theta_\pi)}_{\mathcal{G} \text{ following } \theta_\pi}$$

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Cannot use the max Q function with BenBen

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Question: Can we use continuous a with Q ?

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Can we learn a different deterministic policy for continuous actions?

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Question: What method can we use to learn θ_π ?

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Question: What method can we use to learn θ_π ?

Answer: Policy gradient

Deterministic Policy Gradient

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Question: What method can we use to learn θ_π ?

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$$\nabla_x f(g(x)) = \nabla_g[f(g(x))] \nabla_x g(x)$$

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Let me explain what I mean

Deterministic Policy Gradient

With deterministic policy, μ inside Tr means chain rule

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We use stochastic policies in RL because of this

Deep Deterministic Policy Gradient

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$$\nabla_{\theta_{\mu}} \mathbb{E}[\mathcal{G}(\tau) \mid s_0; \theta_{\mu}]$$

Deep Deterministic Policy Gradient

$$\nabla_{\theta_{\mu}} \mathbb{E}[\mathcal{G}(\tau) \mid s_0; \theta_{\mu}] \qquad \theta_{\pi, i+1} = \theta_{\pi, i} + \alpha \cdot \nabla_{\theta_{\mu}} \mathbb{E}[\mathcal{G}(\tau) \mid s_0; \theta_{\mu}]$$

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Let us try to derive deterministic policy gradient again

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This time, take gradient of V instead of gradient of $\mathbb{E}[\mathcal{G}(\tau) \mid s_0; \theta_\mu]$

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Deep Deterministic Policy Gradient

$$\nabla_{\theta_\mu} V(s_0, \theta_\mu) = \nabla_{\theta_\mu} Q(s_0, \mu(s_0, \theta_\mu), \theta_\mu)$$

Use the chain rule

$$\nabla_x f(g(x)) = \nabla_g[f(g(x))] \nabla_x g(x)$$

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Let us inspect these terms more closely

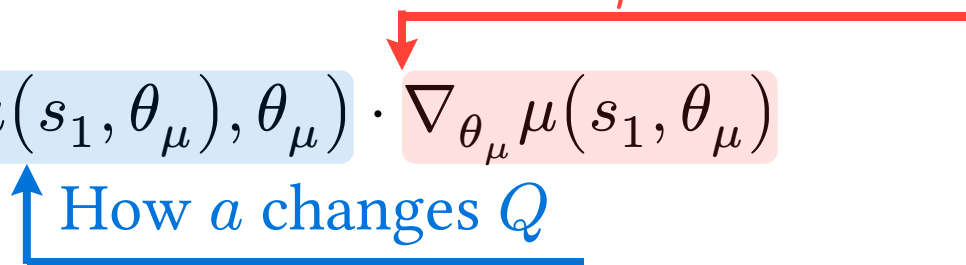
Deep Deterministic Policy Gradient

How θ_μ changes a

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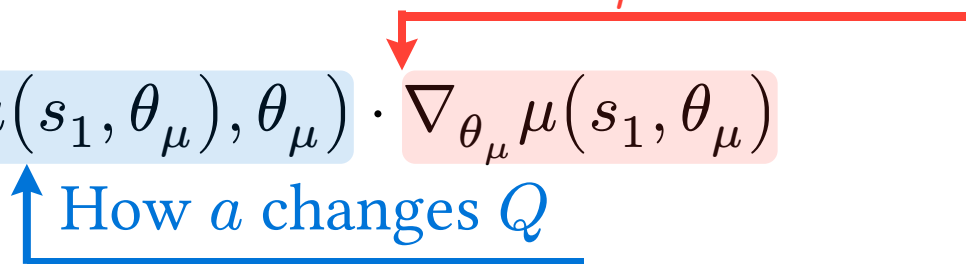
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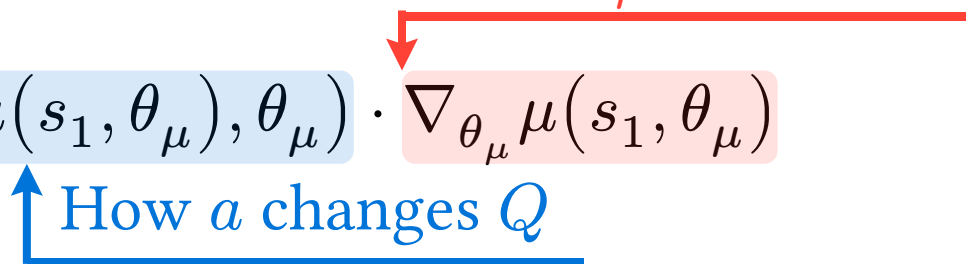
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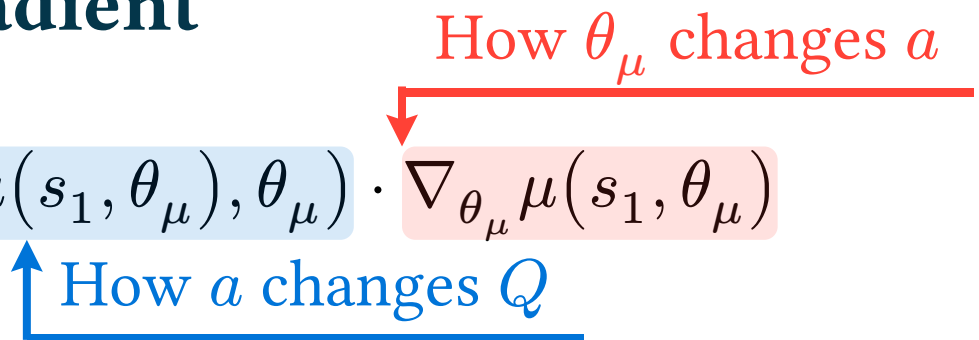
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Analytical solution for the first term is difficult (recursive definition)

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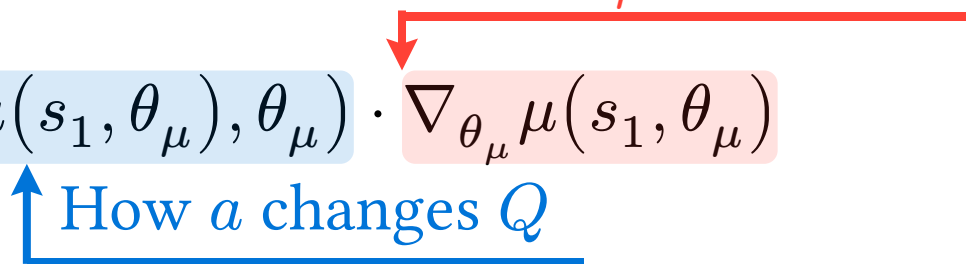
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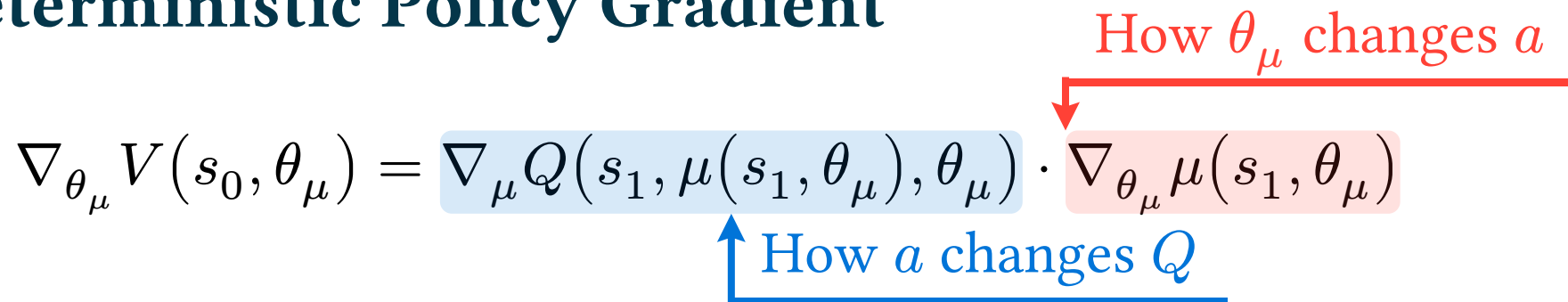
We can backpropagate through Q without worrying about recursion

Deep Deterministic Policy Gradient

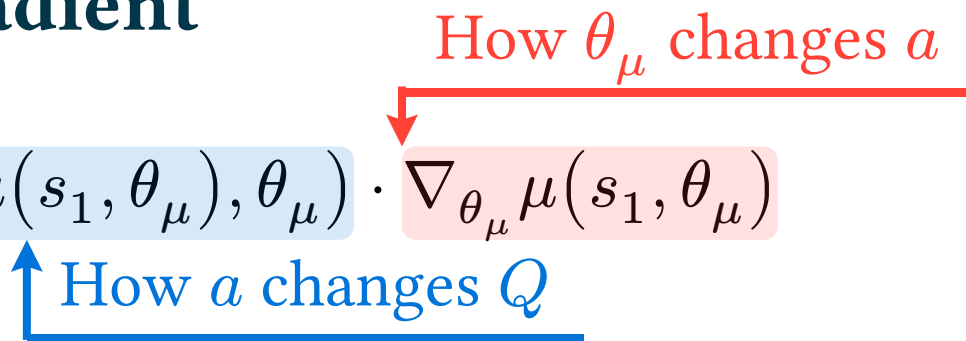
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Writing the code makes it look easy

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How a changes Q

Writing the code makes it look easy

```
def V(s, Q_nn, mu_nn):  
    a = mu_nn(s)  
    return Q_nn(s, a) # Return value  
  
# Learn the policy that maximizes V  
# Make sure to differentiate w.r.t policy parameters!  
J = grad(V, argnums=2)(states, Q_nn, mu_nn)  
mu_nn = optimizer.update(mu_nn, J) # Grad ascent
```

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$$Q(s_0, a_0, \theta_\mu, \theta_Q) = \mathbb{E}[\mathcal{R}(s_1) \mid s_0, a_0] + \gamma Q(s_1, \mu(s_1, \theta_\mu), \theta_\mu, \theta_Q)$$

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```
def V(s, Q_nn, mu_nn):  
    a = mu_nn(s)  
    return Q_nn(s, a) # return value  
def TD_loss(s, a, r, next_s, Q_nn, mu_nn):  
    return (V(s, Q_nn, mu_nn) - td_label) ** 2  
# Now, we learn params of Q following policy  
J = grad(TD_loss, argnums=4)(s, a, r, next_s, Q_nn, mu_nn)  
Q_nn = optimizer.update(Q_nn, J) # Grad descent
```

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Definition: Deep Deterministic Policy Gradient (DDPG) decomposes V into a deterministic policy μ and Q , learning them jointly

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$$\theta_{Q,i+1} = \arg \min_{\theta_{Q,i}}$$

$$\left(Q(s_0, a_0, \theta_{\mu,i}, \theta_{Q,i}) - \left(\hat{\mathbb{E}}[\mathcal{R}(s_1) \mid s_0, a_0] + \gamma Q(s_1, \mu(s_1, \theta_{\mu,i}), \theta_{\mu,i}, \theta_{Q,i}) \right) \right)$$

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Repeat until convergence, $\theta_{\mu,i+1} = \theta_{\mu,i}$, $\theta_{Q,i+1} = \theta_{Q,i}$

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$$\left(Q(s_0, a_0, \theta_{\mu,i}, \theta_{Q,i}) - \left(\hat{\mathbb{E}}[\mathcal{R}(s_1) \mid s_0, a_0] + \gamma Q(s_1, \mu(s_1, \theta_{\mu,i}), \theta_{\mu,i}, \theta_{Q,i}) \right) \right)$$

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Question: Is DDPG on-policy or off-policy?

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Almost **all** good off-policy actor-critic algorithms are based on DDPG

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Because μ is a neural network, it can generalize to continuous s, a

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- More recent papers find using normal noise produces similar results

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mu = Sequential([  
    Linear(state_size, hidden_size),  
    Lambda(leaky_relu),  
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BenBen: $A = [0, 2\pi]^{12}$, so `action_dims=12`

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```
def bound_action(action, lower, upper):  
    return 0.5 * (upper + lower) + 0.5 * (upper - lower)  
        * tanh(action)
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```
def sample_action(mu, state, A_bounds, std):  
    action = mu(state)  
    noisy_action = action + normal(0, std) # Explore  
    return bound_action(noisy_action, *A_bounds)
```

Coding

Now construct Q

Coding

Now construct Q

```
Q = Sequential([
    # Different from DQN network
    # Input action and state together
    Lambda(lambda s, a: concatenate(s, a)),
    Linear(state_size + action_dims, hidden_size),
    Lambda(leaky_relu),
    Linear(hidden_size, hidden_size),
    Lambda(leaky_relu),
    Linear(hidden_size, 1), # Single value for Q(s, a)
])
```

Coding

```
while not terminated:
    # Exploration: make sure actions within action space!
    action = sample_action(mu, state, bounds, std)
    transition = env.step(action)
    replay_buffer.append(transition)
    data = replay_buffer.sample()
    # Theta_pi params are in mu neural network
    # Argnums tells us differentiation variable
    J_Q = grad(Q_loss, argnums=0)(theta_Q, theta_T, mu, data)
    theta_Q = apply_updates(J_Q, ...)
    J_mu = grad(mu_loss, argnums=0)(mu, theta_Q, data)
    mu = apply_updates(J_mu, ...)
    if step % 200 == 0: # Target network necessary
        theta_T = theta_Q
```

Coding

```
def Q_loss(theta_Q, theta_T, theta_pi, data):  
    Qnet = combine(Q, theta_Q)  
    Tnet = combine(Q, theta_T) # Target network  
    # Predict Q values for action we took  
    prediction = vmap(Qnet)(data.state, data.action)  
    # Now compute labels using TD error  
    next_action = vmap(mu)(data.next_state)  
    # NOTE: No max! Mu approximates argmax  
    next_Q = vmap(Tnet)(data.next_state, next_action)  
    label = data.reward + gamma * data.done * next_Q  
    return (prediction - label) ** 2
```

Coding

```
def mu_loss(mu, theta_Q, data):  
    # Find the action that maximizes the Q function  
    Qnet = combine(Q, theta_Q)  
    # Instead of multiply, chain rule -- plug action into Q  
    action = vmap(mu)(data.state)  
    q_value = vmap(Qnet)(data.state, action)  
    # Update the policy parameters to maximize the Q value  
    # Gradient ascent but we min loss, use negative  
    return -q_value
```

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You can also read about Twin Delayed DDPG (TD3)

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- Add noise to target

$$Q(s, \mu(s, \theta_\mu) + \eta, \theta_\mu, \theta_T)$$

- Learns two Q functions, use the minimum as target

$$\min_{i \in 1,2} Q(s, \mu(s, \theta_\mu) + \eta, \theta_\mu, \theta_{T,i})$$

- Update Q functions more often than policy

Max Entropy RL

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Many algorithms add improvements to DDPG

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We will very briefly cover max-entropy RL

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We will very briefly cover max-entropy RL

First, let us introduce entropy

Max Entropy RL

Entropy measures the uncertainty of a distribution

Max Entropy RL

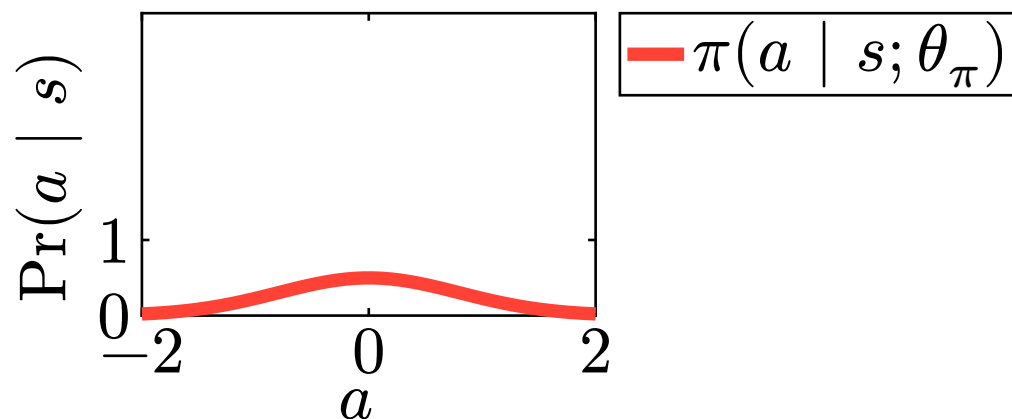
Entropy measures the uncertainty of a distribution

$$H(\pi(a \mid s; \theta_\pi))$$

Max Entropy RL

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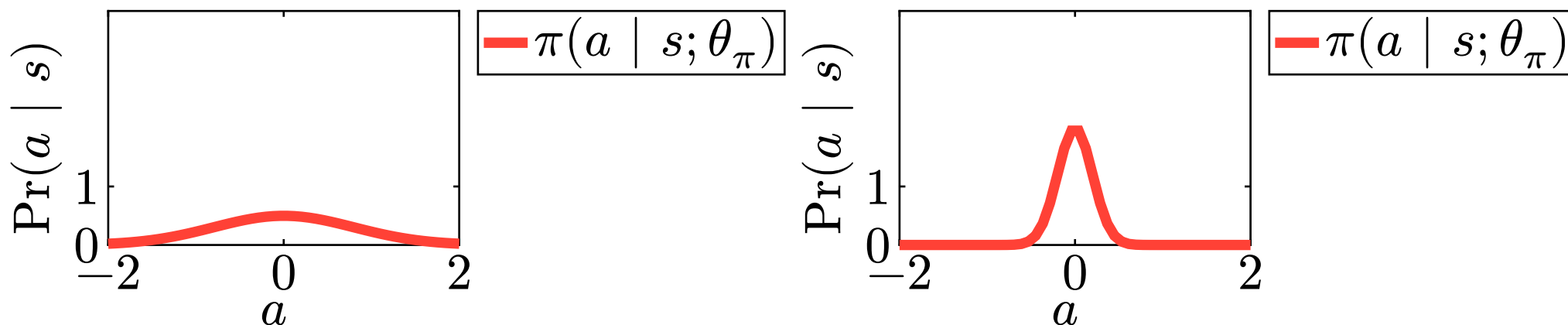
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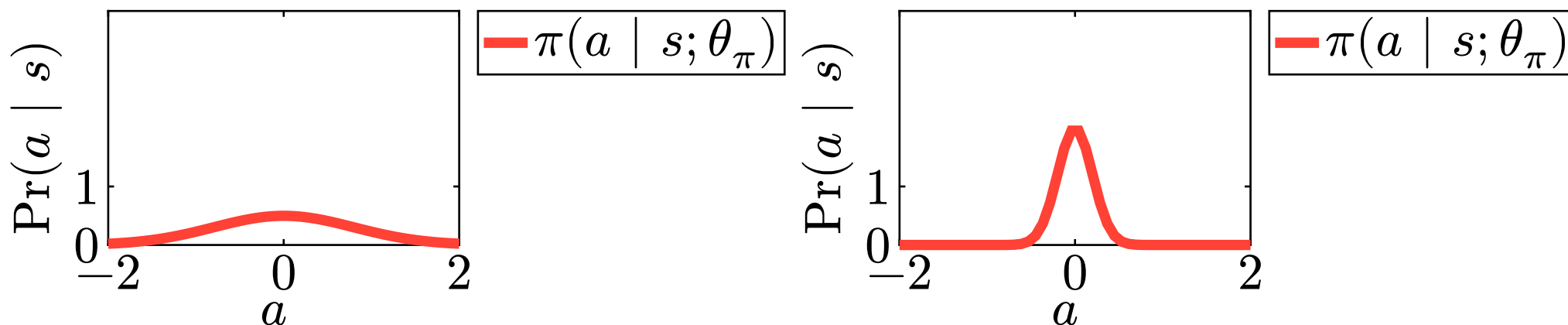
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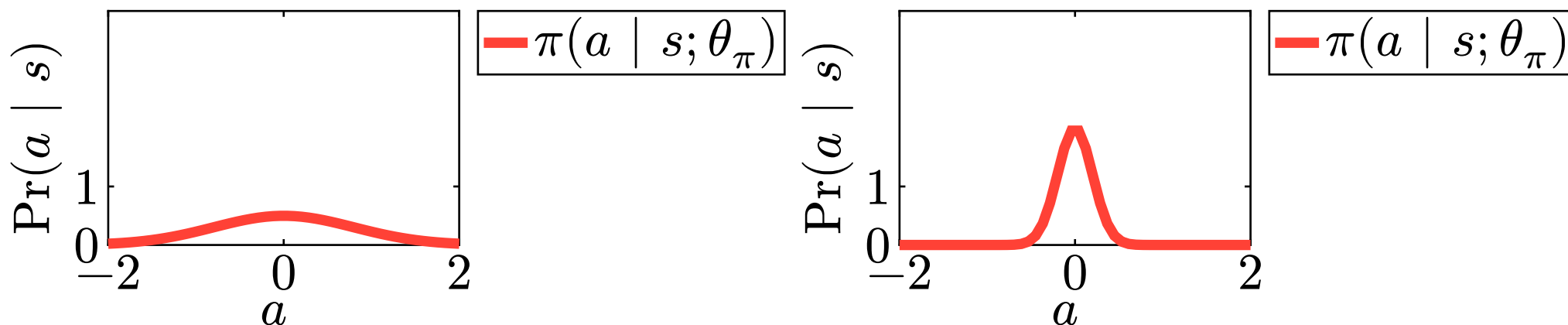


Question: Which policy has higher entropy?

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Left policy, more uncertain/random

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We learn a policy that maximizes the return and **behaves randomly**

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- There are theoretical reasons, but they are not very good

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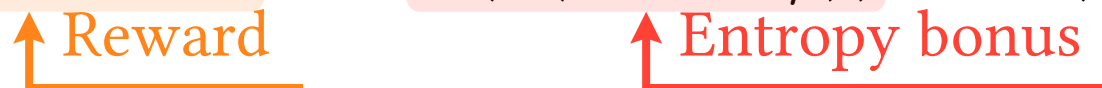
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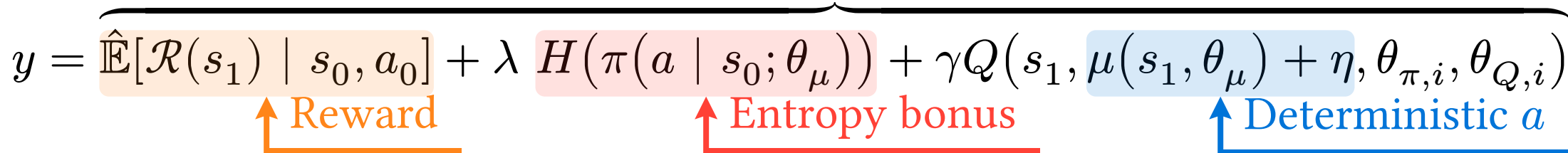
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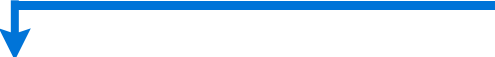

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Repeat until convergence, $\theta_{\mu, i+1} = \theta_{\mu, i}$, $\theta_{Q, i+1} = \theta_{Q, i}$

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Coding SAC could take an entire lecture, read CleanRL

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Tuned DDPG/A2C perform 95% as good as tuned SAC/PPO

Thoughts on Model-Free RL

What algorithm is best in 2025?

For discrete actions (Atari), DQN variants still perform best

For continuous actions (MuJoCo), SAC performs best

PPO is less sensitive to hyperparameters than SAC/DQN

- Noobs like PPO, they cannot tune hyperparameters
- Often performs worse than tuned DQN/SAC

Tuned DDPG/A2C perform 95% as good as tuned SAC/PPO

- Much easier to debug

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- Record some episodes (videos/frames/etc)
- Watch the policy, what did it learn to do?
 - Think about why it learned to do this (exploiting bugs in MDP)

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