



Deep Value

CISC 7404 - Decision Making

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Admin

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I noticed the last 1 hour of class everyone looks tired and sad

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Three hours is a long time to pay attention, especially at night

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Would you prefer:

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Would you prefer:

1. To have a long break ($1.5h + 0.5h + 1h$) in the middle?
- 2.
- 3.

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Would you prefer:

1. To have a long break ($1.5h + 0.5h + 1h$) in the middle?
2. No breaks, end lecture early after 2 or 2.25 hours
- 3.

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Would you prefer:

1. To have a long break ($1.5\text{h} + 0.5\text{h} + 1\text{h}$) in the middle?
2. No breaks, end lecture early after 2 or 2.25 hours
3. Keep as-is (approximately 3 hours + 10 min break)

Admin

HW1 bug:

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There was a bug in the `update_Q_TD0` starter code, thanks He Zhe!

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Before:

```
terminateds = jnp.concatenate([jnp.zeros(states.shape[0],  
dtype=bool), jnp.array([1], dtype=bool)])
```

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There was a bug in the update_Q_TD0 starter code, thanks He Zhe!

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terminateds = jnp.concatenate([jnp.zeros(states.shape[0],  
dtype=bool), jnp.array([1], dtype=bool)])
```

After:

```
terminateds = jnp.concatenate([jnp.zeros(states.shape[0] - 1,  
dtype=bool), jnp.array([1], dtype=bool)])
```

$$Q_{i+1}(s_0, a_0, \theta_\pi) = Q_i(s_0, a_0, \theta_\pi) - \alpha \cdot \eta$$


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$$\eta = Q_i(s_0, a_0, \theta_\pi) - \left(\hat{\mathbb{E}}[\mathcal{R}(s_1) \mid s_0, a_0] + \neg d_0 \gamma \max_{a \in A} Q_i(s_1, a, \theta_\pi) \right)$$

Admin

$$Q_{i+1}(s_0, a_0, \theta_\pi) = Q_i(s_0, a_0, \theta_\pi) - \alpha \cdot \eta$$

Predicted value


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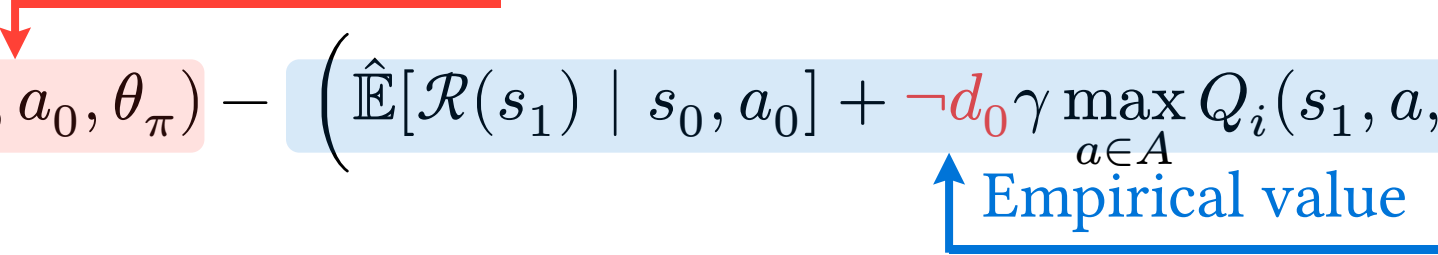
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Empirical value



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Empirical value

小心! If s_1 is a terminal state, future value is 0 ($\neg d_0 = \text{not terminated}$)

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Empirical value

小心! If s_1 is a terminal state, future value is 0 ($\neg d_0$ = not terminated)

Without the $\neg d$ term, takes longer to train!

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Instead, **you** will implement deep Q learning for your second homework

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Homework 2:

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Homework 2:

- Deep Q learning

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Homework 2:

- Deep Q learning
- Deep policy gradient

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Homework 2:

- Deep Q learning
- Deep policy gradient

Will release after homework 1 due date

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Next quiz in 2-3 weeks

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As before, I will announce the quiz one week before

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Focus on

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- Returns

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Review

Deep Learning Review

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We model neural networks as parameterized functions

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$$f : X \times \Theta \mapsto Y$$

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Map an input $x \in X$ and parameters $\theta \in \Theta$ to output space Y

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Map an input $x \in X$ and parameters $\theta \in \Theta$ to output space Y

$$f(x, \theta)$$

Deep Learning Review

Neural networks consist of **artificial neurons**

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$$\boldsymbol{x} \in \mathbb{R}^{d_x}, \boldsymbol{\theta} \in \mathbb{R}^{d_x+1}$$

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$$\boldsymbol{x} \in \mathbb{R}^{d_x}, \boldsymbol{\theta} \in \mathbb{R}^{d_x+1}$$

$$\overline{\boldsymbol{x}} = \begin{bmatrix} 1 & x_1 & x_2 & \dots & x_{d_x} \end{bmatrix}^\top \in \mathbb{R}^{d_x+1}$$

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$$\sigma(x) = \frac{1}{1 + e^{-x}}$$

Or ReLU

$$\sigma(x) = \max(0, x)$$

Deep Learning Review

We combine individual neurons into a **layer**

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$$f : \mathbb{R}^{d_x} \times \Theta \mapsto \mathbb{R}$$

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Layer of d_y neurons:

Deep Learning Review

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$$\Theta = \mathbb{R}^{(d_x+1) \times d_y}$$

Deep Learning Review

For a single neuron

Deep Learning Review

For a single neuron

$$f\left(\begin{bmatrix} x_1 \\ \vdots \\ x_{d_x} \end{bmatrix}, \begin{bmatrix} \theta_0 \\ \theta_1 \\ \vdots \\ \theta_{d_x} \end{bmatrix}\right) = \sigma\left(\sum_{i=0}^{d_x} \theta_i \bar{x}_i\right)$$

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For a wide network

Deep Learning Review

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Deep Learning Review

We can combine layers to create a **deep** neural network

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A wide network:

$$f(x, \theta) = \sigma(\theta^\top \bar{x})$$

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$$f_1(x, \varphi) = \sigma(\varphi^\top \bar{x})$$

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$$f(x, \theta) = f_\ell(\dots f_2(f_1(x, \varphi), \psi) \dots \xi)$$

Deep Learning Review

Written another way

Deep Learning Review

Written another way

$$z_1 = f_1(x, \varphi) = \sigma(\varphi^\top \bar{x})$$

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$$z_1 = f_1(x, \varphi) = \sigma(\varphi^\top \bar{x})$$

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$$z_1 = f_1(x, \varphi) = \sigma(\varphi^\top \bar{x})$$

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\vdots

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$$y = f_\ell(z_{\ell-1}, \xi) = \sigma(\xi^\top \bar{z}_{\ell-1})$$

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We call each function a **layer**

Deep Learning Review

Written another way

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\vdots

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We call each function a **layer**

A deep neural network is made of many layers

Deep Learning Review

We can create different models for different tasks

Deep Learning Review

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Standard tasks:

Deep Learning Review

We can create different models for different tasks

Standard tasks: Multi-layer perceptron (MLP)

Deep Learning Review

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Standard tasks: Multi-layer perceptron (MLP)

Image tasks:

Deep Learning Review

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Standard tasks: Multi-layer perceptron (MLP)

Image tasks: Convolutional neural network (CNN)

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Temporal tasks:

Deep Learning Review

We can create different models for different tasks

Standard tasks: Multi-layer perceptron (MLP)

Image tasks: Convolutional neural network (CNN)

Temporal tasks: Recurrent neural network (RNN)

Deep Learning Review

We can create different models for different tasks

Standard tasks: Multi-layer perceptron (MLP)

Image tasks: Convolutional neural network (CNN)

Temporal tasks: Recurrent neural network (RNN)

Image, temporal tasks: Transformer

Deep Learning Review

What functions can we represent using deep neural networks?

Deep Learning Review

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A deep neural network is a **universal function approximator**

Deep Learning Review

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It can approximate **any** continuous function $g(x)$ to precision η

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$$| g(x) - f(x, \theta) | < \eta$$

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Making the network deeper or wider decreases η

Deep Learning Review

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Very powerful finding! The basis of deep learning.

Deep Learning Review

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$$| g(x) - f(x, \theta) | < \eta$$

Making the network deeper or wider decreases η

Very powerful finding! The basis of deep learning.

Although such θ exists, it can be hard to find

Deep Learning Review

Finding θ is an optimization problem

Deep Learning Review

Finding θ is an optimization problem

In particular, we optimize a **loss function**

Deep Learning Review

Finding θ is an optimization problem

In particular, we optimize a **loss function**

$$\mathcal{L} : X \times Y \times \Theta \mapsto \mathbb{R}$$

Deep Learning Review

Finding θ is an optimization problem

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$$\arg \min_{\theta} \mathcal{L}(x, y, \theta)$$

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Finding θ is an optimization problem

In particular, we optimize a **loss function**

$$\mathcal{L} : X \times Y \times \Theta \mapsto \mathbb{R}$$

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Loss function measures the error between $f(x, \theta)$ and desired $g(x) = y$

Deep Learning Review

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In particular, we optimize a **loss function**

$$\mathcal{L} : X \times Y \times \Theta \mapsto \mathbb{R}$$

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Loss function measures the error between $f(x, \theta)$ and desired $g(x) = y$

In this class, we will build loss functions from two error functions

Deep Learning Review

Square error: The squared distance over a dataset of size n

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Square error: The squared distance over a dataset of size n

$$\sum_{i=1}^n \sum_{j=1}^{d_y} \left(f(\mathbf{x}_{[i]}, \boldsymbol{\theta})_j - g(\mathbf{x})_j \right)^2 = \sum_{i=1}^n \sum_{j=1}^{d_y} \left(f(\mathbf{x}_{[i]}, \boldsymbol{\theta})_j - y_{[i],j} \right)^2$$

Deep Learning Review

Square error: The squared distance over a dataset of size n

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Cross entropy error: The categorical error over a dataset of size n

Deep Learning Review

Square error: The squared distance over a dataset of size n

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Cross entropy error: The categorical error over a dataset of size n

$$-\sum_{i=1}^n \sum_{j=1}^{d_y} P\left(g(\mathbf{x}_{[i]})_j \mid \mathbf{x}_{[i]}\right) \log f(\mathbf{x}_{[i]}, \boldsymbol{\theta})_j = -\sum_{i=1}^n \sum_{j=1}^{d_y} P(y_{[i],j} \mid \mathbf{x}_{[i]}) \log f(\mathbf{x}_{[i]}, \boldsymbol{\theta})_j$$

Deep Learning Review

Square error:

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Question: Which one will we use for Q learning?

Deep Learning Review

Square error:

$$\sum_{i=1}^n \sum_{j=1}^{d_y} \left(f(\mathbf{x}_{[i]}, \boldsymbol{\theta})_j - y_{[i],j} \right)^2$$

Cross entropy error:

$$- \sum_{i=1}^n \sum_{j=1}^{d_y} P(y_{[i],j} \mid \mathbf{x}_{[i]}) \log f(\mathbf{x}_{[i]}, \boldsymbol{\theta})_j$$

Question: Which one will we use for Q learning?

Answer: Predict a scalar (expected return), so square error (regression)

Deep Learning Review

We can use both errors in a loss function

Deep Learning Review

We can use both errors in a loss function

$$\mathcal{L}(\mathbf{X}, \mathbf{Y}, \boldsymbol{\theta}) = \sum_{i=1}^n \sum_{j=1}^{d_y} \left(f(\mathbf{x}_{[i]}, \boldsymbol{\theta})_j - y_{[i],j} \right)^2$$

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Deep Learning Review

Question: Which search method do we use?

Deep Learning Review

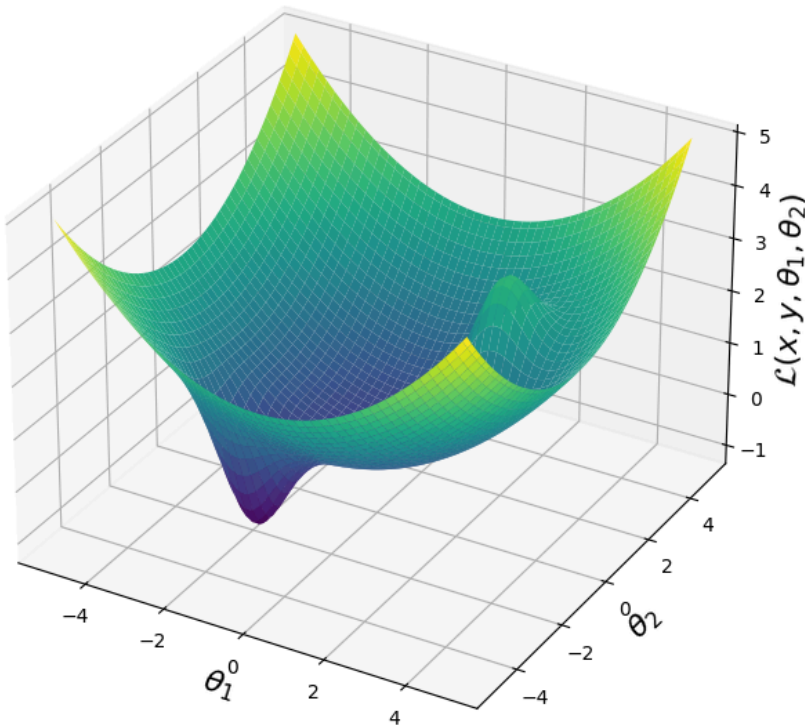
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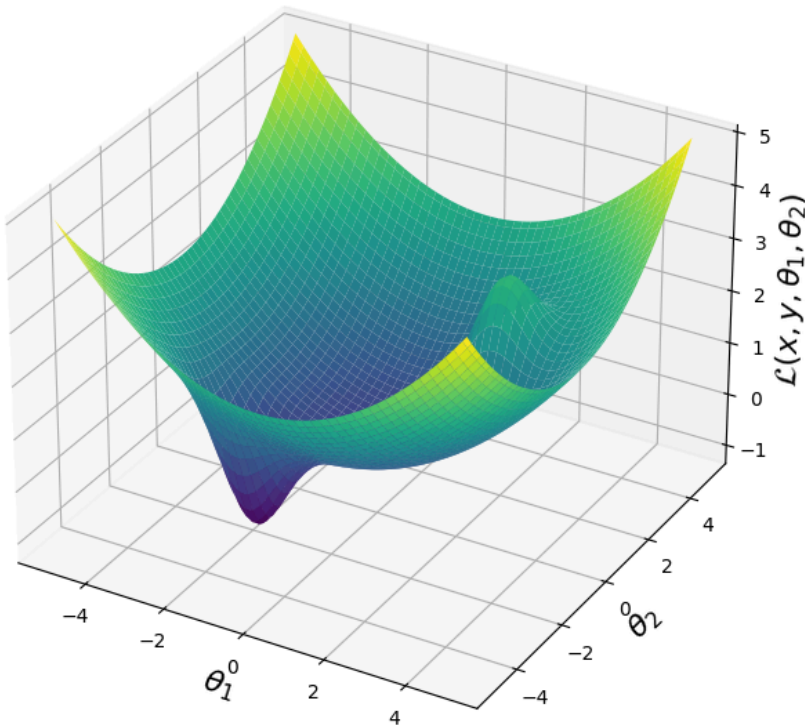
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Deep Learning Review

```
1: function GRADIENT DESCENT( $\mathbf{X}$ ,  $\mathbf{Y}$ ,  $\mathcal{L}$ ,  $t$ ,  $\alpha$ )
2:     ▷ Randomly initialize parameters
3:      $\boldsymbol{\theta} \leftarrow \text{Glorot}()$ 
4:     for  $i \in 1 \dots t$  do
5:         ▷ Compute the gradient of the loss
6:          $\mathbf{J} \leftarrow \nabla_{\boldsymbol{\theta}} \mathcal{L}(\mathbf{X}, \mathbf{Y}, \boldsymbol{\theta})$ 
7:         ▷ Update the parameters using the negative gradient
8:          $\boldsymbol{\theta} \leftarrow \boldsymbol{\theta} - \alpha \cdot \mathbf{J}$ 
9:     return  $\boldsymbol{\theta}$ 
```

Deep Learning Review

We can put it all together in jax and equinox

```
from jax import random
from equinox import nn
```

```
seed = random.key(0)
key, *net_keys= random.split(seed, 4)
net = nn.Sequential([
    nn.Linear(d_x, d_h, key=net_keys[0]),
    nn.Lambda(jax.nn.leaky_relu),
    nn.Linear(d_h, d_h, key=net_keys[1]),
    nn.Lambda(jax.nn.leaky_relu),
    nn.Linear(d_h, d_y, key=net_keys[2]),
])
```

Deep Learning Review

We can extract parameters using `eqx.partition`

```
import equinox as eqx
# Get all arrays (parameters) in the network
theta, f = eqx.partition(net, eqx.is_array)
# Add one to every parameter
theta = jax.tree.map(theta, lambda x: x + 1)
# Put the new parameters back into network
net = eqx.combine(theta, f)
```

Deep Learning Review

```
import jax.numpy as jnp
import equinox as eqx

def L_square(net, x, y):
    # vmap applies network to batch of data
    prediction = eqx.filter_vmap(net)(x)
    return ((prediction - y) ** 2).mean()

def L_cross_entropy(net, x, y):
    # Net outputs probabilities
    # And y is one-hot, e.g. [0, 0, 1, 0]
    prediction = eqx.filter_vmap(net)(x)
    return -(y * jnp.log(prediction)).sum(-1).mean()
```

Deep Learning Review

```
import optax
import equinox as eqx
opt = optax.adam(learning_rate=3e-4)
# Adam needs to track momentum and variance
opt_state = opt.init(eqx.filter(net, eqx.is_array))
# Gradient of loss function is a function
grad_L = eqx.filter_grad(L_square)
# Evaluate grad_L at x, y, theta to find J
J = grad_L(net, x, y)
# Compute parameter update using J (adam)
updates, opt_state = opt.update(
    grads, opt_state, params=eqx.filter(net, eqx.is_array)
)
net = eqx.apply_updates(net, updates) # Update params
```

Deep Learning Review

```
def train_one_batch(net, batch, opt_state):  
    x, y = batch  
    grads = eqx.filter_grad(L_square)(net, x, y)  
    updates, opt_state = opt.update(  
        grads, opt_state, params=eqx.filter(net, eqx.is_array)  
    )  
    net = eqx.apply_updates(net, updates) # Update params  
    return net, opt_state  
  
for epoch in range(num_epochs):  
    for batch in dataset:  
        # Can use eqx.filter_jit(f) for speedup  
        net, opt_state = train_one_batch(net, batch, opt_state)
```

Deep Learning Review

Dirty secret of deep learning:

Deep Learning Review

Dirty secret of deep learning: We do not understand deep learning

Biological inspiration, theoretical bounds, and mathematical guarantees

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This applies even more to **deep reinforcement learning**

Motivation

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In the homework, you trained a Q function to solve a task

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Let me demonstrate this with an example problem

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Example: Learn a policy to pick up trash and put it in the bin

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This would be a large matrix!

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Discretize to 128×128 grid squares

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Very large but not infinite

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With TD updates, updating one cell means we must update all cells

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With TD updates, updating one cell means we must update all cells

It can take many states and actions for Q converge (HW up to 100k)

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There is a lower sample complexity bound on convergence¹

¹Li, Gen, et al. “Is Q-Learning Minimax Optimal? A Tight Sample Complexity Analysis.” Oper. Res. (2024).

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$$\frac{|S| |A|}{(1 - \gamma)^5 \cdot \eta^2}$$

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$64 \times A$ petabytes of rewards to learn Q

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We can say the same for deep RL

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Let us try and learn the Q function using a deep neural network

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Before:

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$$Q(s, a, \theta_{\pi}, \theta_Q)$$

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The Q function estimates the policy-conditioned discounted return

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$$Q(s_0, a_0, \theta_\pi, \theta_Q) = \mathbb{E}[\mathcal{G}(\tau) \mid s_0, a_0; \theta_\pi]$$

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Make this an optimization objective, so we can train a network

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We must have an f , θ , and y

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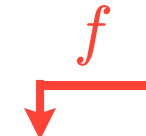
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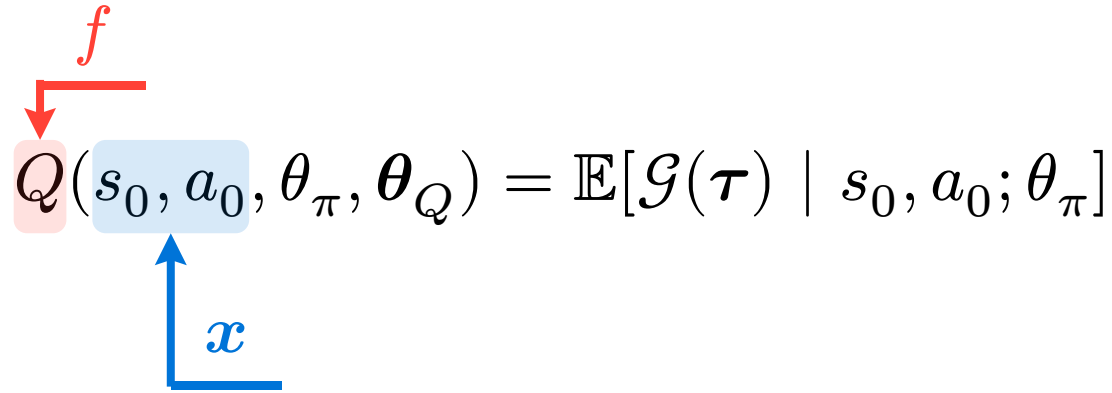
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The diagram shows the equation $Q(s_0, a_0, \theta_\pi, \theta_Q) = \mathbb{E}[\mathcal{G}(\tau) \mid s_0, a_0; \theta_\pi]$. A red arrow labeled f points to the Q function. A blue arrow labeled x points to the state-action pair (s_0, a_0) .

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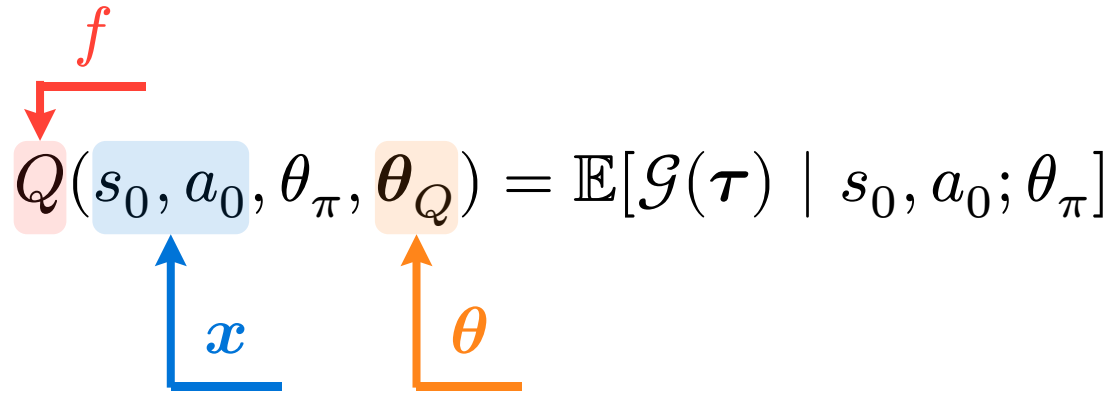
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$$Q(s_0, a_0, \theta_\pi, \theta_Q) = \mathbb{E}[\mathcal{G}(\tau) \mid s_0, a_0; \theta_\pi]$$

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Diagram illustrating the Q function equation: $Q(s_0, a_0, \theta_\pi, \theta_Q) = \mathbb{E}[\mathcal{G}(\tau) \mid s_0, a_0; \theta_\pi]$. The components are color-coded and labeled with arrows:

- Q (red box) is pointed to by a red arrow labeled f .
- s_0 (blue box) is pointed to by a blue arrow labeled x .
- a_0 (light blue box) is pointed to by a blue arrow labeled x .
- θ_π (purple box) is pointed to by a purple arrow labeled "Policy, function of θ_Q ".
- θ_Q (orange box) is pointed to by an orange arrow labeled θ .

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Use a distance measure we can minimize, choose square error

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$$Q(s_0, a_0, \theta_\pi, \theta_Q) = \mathbb{E}[\mathcal{G}(\tau) \mid s_0, a_0; \theta_\pi]$$

Move everything to left

$$Q(s_0, a_0, \theta_\pi, \theta_Q) - \mathbb{E}[\mathcal{G}(\tau) \mid s_0, a_0; \theta_\pi] = 0$$

Use a distance measure we can minimize, choose square error

$$\left(Q(s_0, a_0, \theta_\pi, \theta_Q) - \mathbb{E}[\mathcal{G}(\tau) \mid s_0, a_0; \theta_\pi] \right)^2 = 0$$

Deep Q Learning

$$\left(Q(s_0, a_0, \theta_\pi, \theta_Q) - \mathbb{E}[\mathcal{G}(\tau) \mid s_0, a_0; \theta_\pi]\right)^2 = 0$$

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Question: Missing anything?

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Question: Missing anything? Hint: Other loss functions have sums

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Answer: Minimize over all possible states and actions

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Answer: Monte Carlo and Temporal Difference

Deep Q Learning

Monte Carlo Objective:

Deep Q Learning

Monte Carlo Objective:

$$\arg \min_{\boldsymbol{\theta}_Q} \left[\sum_{s_0 \in S} \sum_{a_0 \in A} \left(Q(s_0, a_0, \theta_\pi, \boldsymbol{\theta}_Q) - \sum_{t=0}^{\infty} \gamma^t \mathbb{E}[\mathcal{R}(s_{t+1}) \mid s_0, a_0; \theta_\pi] \right)^2 \right]$$

Deep Q Learning

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Temporal Difference Objective:

Deep Q Learning

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We approximate the expected reward empirically

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Question: Call this Monte Carlo return because of this objective. Why?

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Monte Carlo is a famous casino. We approximate the expected return by “gambling” over the episode

Deep Q Learning

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Can train over batch/dataset \mathbf{X} containing many episodes \mathbf{x}

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Now, lets do the TD loss function

Deep Q Learning

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Rewrite over the episode x

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Empirically compute expected reward

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Empirically compute expected reward

$$\arg \min_{\theta_Q} \mathcal{L}(x, \theta_Q) = \arg \min_{\theta_Q}$$

$$\sum_{\substack{s_i, a_i, r_i, d_i, s_{i+1} \\ \in x}} \left(Q(s_i, a_i, \theta_\pi, \theta_Q) - \left(r_i + \neg d_0 \gamma \arg \max_{a \in A} Q(s_i, a, \theta_\pi, \theta_Q) \right) \right)^2$$

Deep Q Learning

$$\arg \min_{\theta_Q} \mathcal{L}(x, \theta_Q) = \arg \min_{\theta_Q}$$

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Do it over a batch

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Do it over a batch

$$\arg \min_{\theta_Q} \mathcal{L}(\mathbf{x}, \theta_Q) = \arg \min_{\theta_Q} \sum_{\mathbf{x}_{[j]} \in \mathbf{X}}$$

$$\sum_{\substack{s_i, a_i, r_i, d_i, s_{i+1} \\ \in \mathbf{x}}} \left(Q(s_i, a_i, \theta_\pi, \theta_Q) - \left(r_i + \gamma \arg \max_{a \in A} Q(s_i, a, \theta_\pi, \theta_Q) \right) \right)^2$$

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To summarize, our two loss functions:

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Can optimize both loss functions using gradient descent

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RL optimization is more difficult than supervised learning

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Supervised Learning:

Deep Q Learning

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RL optimization is more difficult than supervised learning

Supervised Learning:

- Static inputs

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Supervised Learning:

- Static inputs
- Static labels

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Supervised Learning:

- Static inputs
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- Limited dataset

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Supervised Learning:

- Static inputs
- Static labels
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 - Human can clean
 - Bad to overfit

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Reinforcement Learning:

Deep Q Learning

Can optimize both loss functions using gradient descent

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Supervised Learning:

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Reinforcement Learning:

- Inputs change as θ_π changes

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Reinforcement Learning:

- Inputs change as θ_π changes
 - Visit new/different states
- Labels change as θ_π changes
 - $\mathbb{E}[\mathcal{G}(\tau) \mid \theta_\pi]$
- Infinite dataset
 - Can always collect from env
 - Bad θ_π means bad dataset
 - Overfitting no problem

Experience Replay

Experience Replay

Optimization is difficult in RL

Experience Replay

Optimization is difficult in RL

Most RL papers train for 10M-10B environment steps

Experience Replay

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It takes a long time to train a deep Q function

Experience Replay

Optimization is difficult in RL

Most RL papers train for 10M-10B environment steps

It takes a long time to train a deep Q function

Let us see if we can improve training speed

Experience Replay

```
for epoch in range(num_epochs):
    terminated = False
    s = env.reset()
    episode = []
    # Step between 1 and infinity times to get one episode
    while not terminated:
        a = policy(s, theta_Q)
        next_s, r, d = env.step(action)
        episode.append([s, a, r, d, next_s])
    # Compute gradient over episode
    J = grad(L)(theta_Q, episode)
    theta_Q = update(theta_Q, grad)
```

Experience Replay

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for epoch in range(num_epochs):  
    terminated = False  
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Question: Which part is slowest?

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    # Compute gradient over episode  
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    theta_Q = update(theta_Q, grad)
```

Question: Which part is slowest? **Answer:** Collecting episodes

Experience Replay

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    # Compute gradient over episode
    J = grad(L)(theta_Q, episode)
    theta_Q = update(theta_Q, grad)
```

Collect episode, train, throw away episode, start again

Experience Replay

What if we reuse episodes?

Experience Replay

What if we reuse episodes?

```
episodes = []
for epoch in range(num_epochs):
    terminated = False
    s = env.reset()
    episode = []
    while not terminated:
        a = policy(s, theta_Q)
        next_s, r, d = env.step(action)
        episode.append([s, a, r, d, next_s])
    episodes.append(episode)
J = grad(L)(theta_Q, episodes) # Train over ALL episodes
theta_Q = update(theta_Q, grad)
```

Experience Replay

When we reuse episodes, we call it **experience replay**

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Store episodes in a **replay buffer**

(list)

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(list)

$$B_t = \begin{bmatrix} s_1 & a_1 & r_1 & d_1 \\ \vdots & \vdots & \vdots & \vdots \\ s_t & a_t & r_t & d_t \end{bmatrix}$$

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Create a dataset from the buffer

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$$X_t = \begin{bmatrix} s_{31} & a_{31} & r_{31} & d_{31} \\ \vdots & \vdots & \vdots & \vdots \\ s_4 & a_4 & r_4 & d_4 \end{bmatrix}$$

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Train on the dataset

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Train on the dataset

$$\arg \min_{\theta_Q} \mathcal{L}(X_t, \theta_Q)$$

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Humans do experience replay when they dream!

Experience Replay

On-policy algorithms must throw away episodes after training

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Must collect data using the current policy, cannot use experience replay

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Off-policy algorithms can reuse old episodes and use experience replay

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Question: Which is Q learning?

Experience Replay

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Question: Which is Q learning?

Let us find out!

Experience Replay

Start with the Monte Carlo return

Experience Replay

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$$\arg \min_{\theta_Q} \left[\sum_{s_0 \in S} \sum_{a_0 \in A} \left(Q(s_0, a_0, \theta_\pi, \theta_Q) - \sum_{t=0}^{\infty} \gamma^t \hat{\mathbb{E}}[\mathcal{R}(s_{t+1}) \mid s_0, a_0; \theta_\pi] \right)^2 \right]$$

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Question: On-policy or off-policy? **Answer:** On-policy. Why?

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Our return is conditioned on the policy

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Our return is conditioned on the policy

If the policy changes, the return $r_0 + \gamma r_1 + \gamma^2 r_2 + \dots$ is not valid!

Experience Replay

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We need $\hat{\mathbb{E}}[\mathcal{R}(s_{t+1}) \mid s_0, a_0; \theta_\pi]$

Experience Replay

What about TD return?

Experience Replay

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$$\arg \min_{\boldsymbol{\theta}_Q} \left[\sum_{s_0 \in S} \sum_{a_0 \in A} \left(Q(s_0, a_0, \theta_\pi, \boldsymbol{\theta}_Q) - \left(\hat{\mathbb{E}}[\mathcal{R}(s_1) \mid s_0, a_0] + \gamma \max_{a \in A} Q(s_1, a, \theta_\pi, \boldsymbol{\theta}_Q) \right) \right)^2 \right]$$

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Q function depends on θ_π , but reward does not!

Do we know $\arg \max_{a \in A} Q(s_1, a, \theta_\pi, \theta_Q)$?

Experience Replay

What about TD return?

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Question: On-policy or off-policy? **Answer:** Off-policy. Why?

Q function depends on θ_π , but reward does not!

Do we know $\arg \max_{a \in A} Q(s_1, a, \theta_\pi, \boldsymbol{\theta}_Q)$? Yes! Just plug in s_1

Experience Replay

To summarize:

Experience Replay

To summarize:

Monte Carlo Q learning is on-policy

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Cannot reuse data, takes a long time to train

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To summarize:

Monte Carlo Q learning is on-policy

Cannot reuse data, takes a long time to train

Temporal Difference Q learning is special!

It is off-policy, can reuse data and train faster

TD is not always better than MC

MC needs more training data, but TD has harder optimization

Target Networks

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If you train a deep Q network using TD, you will find

$$Q(s_0, a_0, \theta_\pi, \theta_Q) = \infty$$

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Question: Can you see why?

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Question: Can you see why? Hint: What if $s_0 \approx s_1$?

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Question: Can you see why? Hint: What if $s_0 \approx s_1$?

$$Q(s_0, a_0, \theta_\pi, \theta_Q) = r_0 + \max_{a \in A} Q(s_0, a_0, \theta_\pi, \theta_Q)$$

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Target Networks

Usually, the target parameters are older parameters

```
theta_Q = ... # Initialize parameters
theta_T = theta_Q.copy()

for epoch in range(num_epochs):
    grad = grad(L)(theta_Q, theta_T, X)
    theta_Q = optimizer.update(theta_Q, grad)
    if epoch % 200 == 0:
        # Update target parameters
        theta_T = theta_Q.copy()
```

Deep Q Networks

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¹Human-level control through deep reinforcement learning. *Nature*. 2014.

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You have all the tools you need to implement DQN, except for one

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$$a = \arg \max_i \begin{bmatrix} Q(s, a = 1, \theta_{\pi}, \theta_Q) \\ Q(s, a = 2, \theta_{\pi}, \theta_Q) \\ \vdots \\ Q(s, a = i, \theta_{\pi}, \theta_Q) \end{bmatrix}$$

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For each action, we must execute Q network $|A|$ times. Not efficient!

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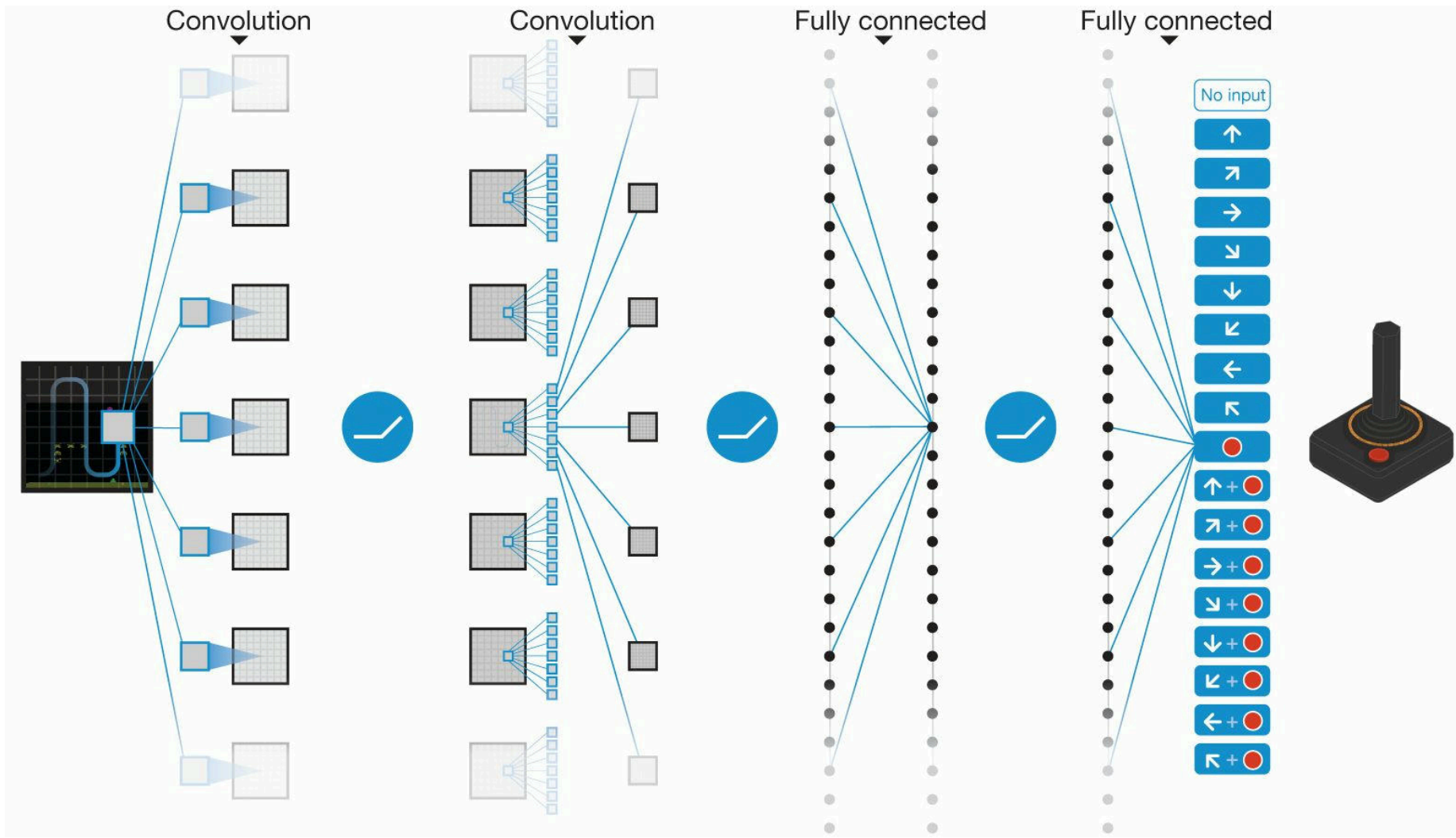
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The neural network outputs $|A|$ values – one for each action

$$a = \arg \max_i Q(s, \theta_\pi, \theta_Q)_i$$

This is $|A|$ times faster!

Deep Q Networks



Deep Q Networks

```
Q = nn.Sequential([...])
theta_T = partition(Q, is_array)[0]
replay_buffer = deque(maxsize=50_000)
for epoch in range(num_epochs):
    while not terminated:
        a = random_action if epoch < k else epsilon_greedy(Q)
        s, r, d, next_s = env.step(a)
        replay_buffer.insert((s, a, r, d, next_s))
        X = random.sample(replay_buffer, batch_size)
        theta_Q, model = eqx.partition(Q, is_array)
        theta_Q = td_update(theta_Q, theta_T, Q, X)
        theta_T = copy(theta_Q) if epoch % j == 0 else theta_T
        Q = eqx.combine(theta_Q, model)
```

Deep Q Networks

Finally, let us look at some successes of deep Q learning

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Pokemon <https://youtu.be/DcYLT37ImBY?si=AeR2WkQg4X-tWa5v>