Actor Critic II

CISC 7404 - Decision Making

Steven Morad

University of Macau

Quiz

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- If your exam has the answer for another version, it is cheating
- Instructions are in both english and chinese, english instructions take precedence
- Good luck!

Quiz

- 在所有学生收起电脑/笔记/手机后,我会分发试卷。
- 如果在此之后仍有电脑/笔记/手机未收,将视为作弊。
- 试卷会背面朝下发下,在我宣布开始前请勿翻面。
- 试卷翻面后,我会简要说明每道题的注意事项。
- 说明结束后,你们有 75 分钟完成考试。
- 交卷后请到教室外休息,8:30 恢复上课。
- 试卷可能存在不同版本,细节略有差异。
- 若你的试卷上出现其他版本的答案,将被判定为作弊。
- 试卷说明为中英双语,若内容冲突以英文为准。
- 祝各位考试顺利!

Admin

Admin

After today, we finish the foundational course material

Next week, we begin to investigate other parts of decision making

Right now, my plan is:

- Offline RL
- Memory and POMDPs
- Imitation Learning
- Large Language Models

Question: Should we replace a topic with something else?

Admin

- Offline RL
  - RL without exploration
  - How can we learn policies from a fixed dataset?
    - Learn surgery from surgical videos (no need to kill patients)
    - Learn driving from Xiaomi driving dataset (no need to crash cars)
  - Very new topic (2-3 years old)
  - Does not work very well (yet)

Admin

- Memory and POMDPs (my research focus)
  - So far, we always assume MDPs
  - Many interesting problems are not Markov
  - Can we extend RL to work for virtually any problem?
    - Yes, requires long-term memory
    - LSTM, transformer, etc
  - May also have time to introduce world models
    - Dreamer, TD-MPC, etc

Admin

- Imitation learning
  - Sometimes, designing a reward function is hard
  - It is easier to demonstrate desired behavior to agents
  - Agents can copy your behaviors without rewards
  - Closer to supervised learning, easier to train
    - Policies are not better than humans

Admin

- Large Language Models
  - Can train LLMs using unsupervised learning
    - They only learn to predict next word
  - We use RL to teach them to interact with humans
    - Apply policy gradient to language
    - GRPO
    - RL-adjacent methods (DPO)

Admin

Also a possibility to split lecture:

- E.g., 1 hour imitation learning, 1 hour something new

Question: Any topic sound boring?

Question: Any suggestions for other topics?

- Maybe just focus on a specific paper?

Alternative topics:

- Multi-agent RL
- Model-based RL and world-models
- Evolutionary algorithms

Admin

Homework 2 progress

If you did not already start, you might be in trouble

Experiments take a long time, start as soon as possible

Harder and requires more debugging than `FrozenLake` assignment

Admin

Those using Tencent AI Arena (Honor of Kings):

- Backup project should not rely on Honor of Kings
- **Make sure to save your code regularly**
  - Everything stored on Tencent VMs, not sure how safe code is
- There is already a PPO baseline, cannot use PPO
  - Instead, consider DDPG, SAC, TRPO, etc
- Cannot install new python libraries (Tencent security issue)
  - No `jax`, must use `torch`
  - You must learn Tencent's strange callback system
    - Prevents copy/pasting, so `torch` is ok

Review

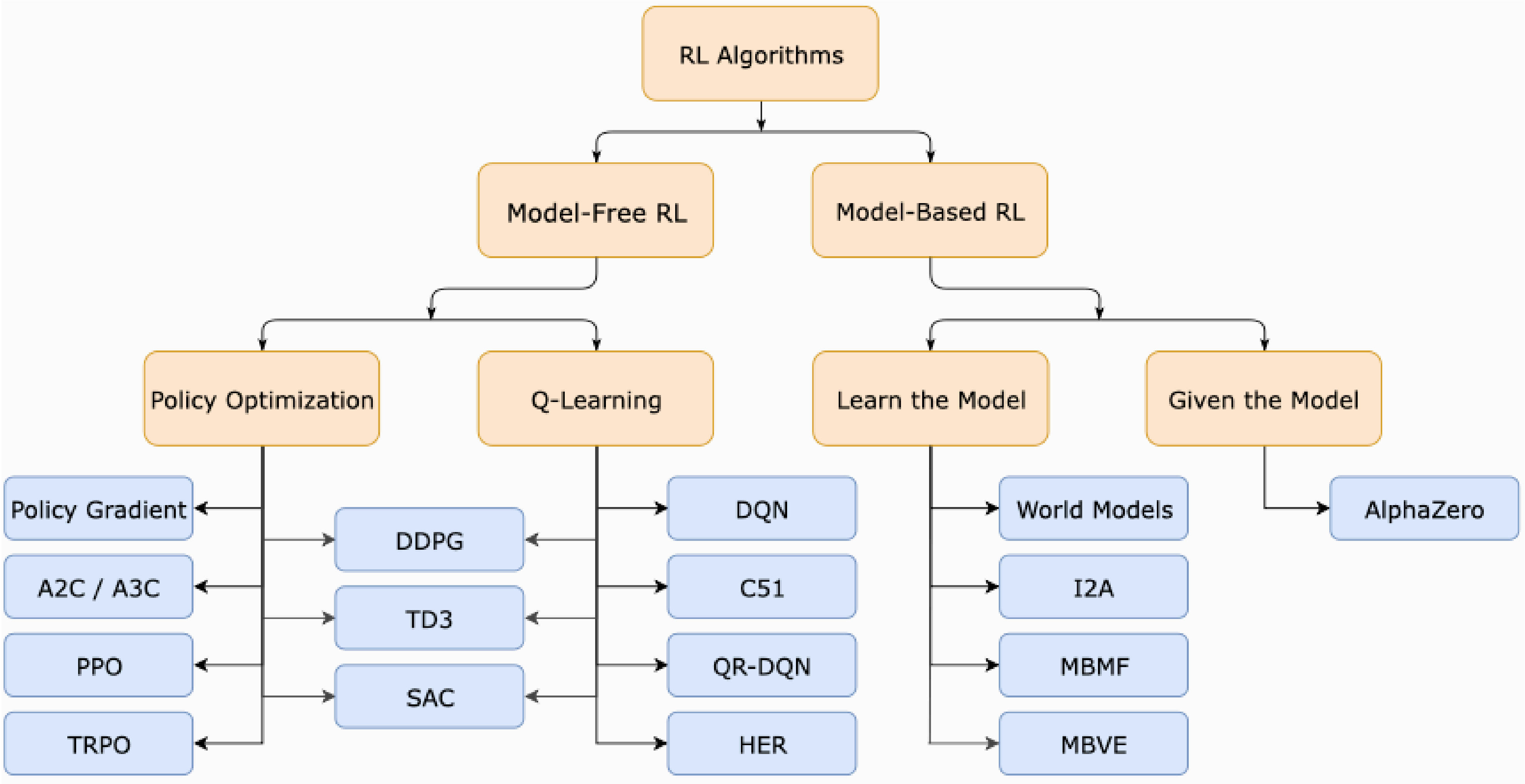
Actor Critic

Actor Critic

Alternative descriptions of actor critic algorithms

https://lilianweng.github.io/posts/2018-04-08-policy-gradient/

Actor Critic



Actor Critic

There are two approaches to actor critic

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1. **Policy gradient based:**

Actor Critic

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PG with V instead of MC

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Learn policy to maximize Q

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Actor Critic

Deterministic Policy Gradient

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Question: Why did we introduce policy gradient methods?

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Question: Why did Q learning fail BenBen?

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$$A = [0, 2\pi]^{12}$$

Deterministic Policy Gradient

Question: Why did Q learning fail BenBen?

$$A = [0, 2\pi]^{12}$$

$$\pi(a_t \mid s_t; \theta_\pi) = \begin{cases} 1 \text{ if } a_t = \arg\max_{a_t \in A} Q(s_t, a_t, \theta_\pi) \\ 0 \text{ otherwise} \end{cases}$$

Deterministic Policy Gradient

Question: Why did Q learning fail BenBen?

$$A = [0, 2\pi]^{12}$$

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Infinitely many a_t – compute Q for each and take arg max over all

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Let us quickly review the Q function and value function

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The most general form of Q

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The reward for taking a_0 and following θ_π afterward

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Question: Can we learn a continuous policy using Q?

Deterministic Policy Gradient

What if we use Q to learn a policy?

Deterministic Policy Gradient

What if we use Q to learn a policy?

Question: What method can we use for policy learning?

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$$\mu : S \times \Theta \mapsto A$$

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Recall policy gradient

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We find θ_π using gradient ascent

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$$\theta_{i+1} = \theta_i + \alpha \cdot \nabla_{\theta_\pi} \mathbb{E}[\mathcal{G}(\boldsymbol{\tau}) \mid s_0; \theta_\pi]$$

Deterministic Policy Gradient

Recall policy gradient

We find θ_π using gradient ascent

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We need to know $\nabla_{\theta_\pi} \mathbb{E}[\mathcal{G}(\boldsymbol{\tau}) \mid s_0; \theta_\pi]$

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We found this for stochastic policy gradient

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How does a deterministic policy change $\nabla_{\theta_\mu} \mathbb{E}\left[\mathcal{G}(\boldsymbol{\tau}) \mid s_0; \theta_\mu\right]$?

Deterministic Policy Gradient

The expected return with a **stochastic** policy

$$\mathbb{E}[\mathcal{G}(\boldsymbol{\tau}) \mid s_0; \theta_\pi] = \sum_{n=0}^{\infty} \gamma^n \sum_{s_{n+1} \in S} \mathcal{R}(s_{n+1}) \cdot \Pr(s_{n+1} \mid s_0; \theta_\pi)$$

$$\Pr(s_{n+1} \mid s_0; \theta_\pi) = \sum_{s_1, \dots, s_n \in S} \prod_{t=0}^{n} \left(\sum_{a_t \in A} \mathrm{Tr}(s_{t+1} \mid s_t, a_t) \cdot \pi(a_t \mid s_t; \theta_\pi) \right)$$

The state distribution with a **deterministic** policy

$$\Pr(s_{n+1} \mid s_0; \theta_\mu) = \sum_{s_1, \dots, s_n \in S} \prod_{t=0}^{n} \mathrm{Tr}(s_{t+1} \mid s_t, \mu(s_t, \theta_\mu))$$

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Let us continue to derive policy gradient with a **deterministic** policy μ

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Plug state distribution into expected return

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$$\Pr(s_{n+1} \mid s_0; \theta_\mu) = \sum_{s_1, \dots, s_n \in S} \prod_{t=0}^{n} \mathrm{Tr}(s_{t+1} \mid s_t, \mu(s_t, \theta_\mu))$$

Plug state distribution into expected return

$$\mathbb{E}[\mathcal{G}(\boldsymbol{\tau}) \mid s_0; \theta_\mu] = \sum_{n=0}^{\infty} \gamma^n \sum_{s_{n+1} \in S} \mathcal{R}(s_{n+1}) \sum_{s_1, \dots, s_n \in S} \prod_{t=0}^{n} \mathrm{Tr}(s_{t+1} \mid s_t, \mu(s_t, \theta_\mu))$$

Deterministic Policy Gradient

$$\nabla_{\theta_\mu} \left[\mathbb{E}[\mathcal{G}(\boldsymbol{\tau})] \mid s_0; \theta_\mu \right]$$

$$= \nabla_{\theta_\mu} \left[\sum_{n=0}^{\infty} \gamma^n \sum_{s_{n+1} \in S} \mathcal{R}(s_{n+1}) \sum_{s_1, \dots, s_n \in S} \prod_{t=0}^{n} \mathrm{Tr}\left(s_{t+1} \mid s_t, \mu(s_t, \theta_\mu)\right) \right]$$

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Gradient of sum is sum of gradient, move the gradient inside the sums

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Recall the log trick

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Recall the log trick $\nabla_x f(x) = f(x) \nabla_x \log f(x)$

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Log of products is sum of logs

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Move the gradient inside sum

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Use the chain rule $\qquad \nabla_x f(g(x)) = \nabla_g [f(g(x))] \nabla_x g(x)$

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We know must know ∇ Tr to find the deterministic policy gradient

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Let me explain what I mean

Deterministic Policy Gradient

With deterministic policy, μ inside Tr means chain rule

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With stochastic policy, we multiply Tr by π (product rule)

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Tr comes out of the sum and disappears into the expected return

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This is one reason why we always consider stochastic π

Deep Deterministic Policy Gradient

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Deep Deterministic Policy Gradient

$$\nabla_{\theta_\mu} \mathbb{E}[\mathcal{G}(\boldsymbol{\tau}) \mid s_0; \theta_\mu] \qquad \theta_{\pi, i+1} = \theta_{\pi, i} + \alpha \cdot \nabla_{\theta_\mu} \mathbb{E}[\mathcal{G}(\boldsymbol{\tau}) \mid s_0; \theta_\mu]$$

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We failed – a deterministic policy gradient does not seem to work

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$$\nabla_{\theta_\mu} Q(s_0, a_0, \theta_\mu)$$

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In English: Find the policy parameters θ_μ that maximize the value

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Let us figure out the gradient of Q

Deep Deterministic Policy Gradient

$$Q(s_0, a_0, \theta_\mu) = \mathbb{E}[\mathcal{R}(s_1) \mid s_0, a_0] + \gamma Q(s_1, a, \theta_\mu); \quad a = \mu(s_1, \theta_\mu)$$

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Take the gradient of both sides

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$$\nabla_{\theta_\mu} Q(s_0, a_0, \theta_\mu) = \nabla_{\theta_\mu} \left[\mathbb{E}[\mathcal{R}(s_1) \mid s_0, a_0] + \gamma Q(s_1, \mu(s_1, \theta_\mu), \theta_\mu)\right]$$

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Initial reward only depends on action, not θ_μ – gradient is zero

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Pull out γ

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Deep Deterministic Policy Gradient

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Let us inspect these terms more closely

Deep Deterministic Policy Gradient

How θ_μ changes a

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But what if Q is a neural network?

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Writing the code makes it look easy

```
def Q(s, a, Q_nn, mu_nn):
    a = mu_nn(s)
    return Q_nn(s, a)

# Optimize policy to maximize Q
# Make sure to differentiate w.r.t mu parameters!
J = grad(Q, argnums=3)(states, actions, Q_nn, mu_nn)
mu_nn = optimizer.update(mu_nn, J)
```

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```
def Q(s, a, Q_nn, mu_nn):
    a = mu_nn(s)
    return Q_nn(s, a)
# Before, we learned policy params to maximize Q
# Now, we learn params of Q following policy (argnums=2)
J = grad(Q, argnums=2)(states, actions, Q_nn, mu_nn)
Q_nn = optimizer.update(Q_nn, J)
```

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Definition: Deep Deterministic Policy Gradient (DDPG) jointly learns a Q function for deterministic policy μ, and the policy parameters θ_μ

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$$\theta_{Q,i+1} = \arg\min_{\theta_{Q,i}}$$

$$\left(Q\left(s_0, a_0, \theta_{\mu,i}, \theta_{Q,i}\right) - \left(\hat{\mathbb{E}}[\mathcal{R}(s_1) \mid s_0, a_0] + \gamma Q\left(s_1, \mu\left(s_1, \theta_{\mu,i}\right), \theta_{\mu,i}, \theta_{Q,i}\right)\right)\right)$$

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Step 2: Learn θ_μ that maximizes Q

$$\theta_{\mu,i+1} = \theta_{\mu,i} + \alpha \cdot Q\left(s_0, \mu\left(s_0, \theta_\mu\right), \theta_{\mu,i}, \theta_{Q,i+1}\right)$$

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$$\theta_{\mu,i+1} = \theta_{\mu,i} + \alpha \cdot Q(s_0, \mu(s_0, \theta_\mu), \theta_{\mu,i}, \theta_{Q,i+1})$$

Repeat until convergence, $\theta_{\mu,i+1} = \theta_{\mu,i}, \quad \theta_{Q,i+1} = \theta_{Q,i}$

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Almost **all** good off-policy actor-critic algorithms are based on DDPG

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- μ neural network approximation of $\arg\max_{a \in A} Q(s, a)$
- Policy learning is learning arg max over infinite action space

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Like policy gradient, the math and code is different

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BenBen: $A = [0, 2\pi]^{12}$, so `action_dims=12`

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```
def bound_action(action, lower, upper):
    return 0.5 * (upper + lower) + 0.5 * (upper - lower)
        * tanh(action)
```

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```
def bound_action(action, lower, upper):
    return 0.5 * (upper + lower) + 0.5 * (upper - lower)
        * tanh(action)

def sample_action(mu, state, A_bounds, std):
    action = mu(state)
    noisy_action = action + normal(0, std) # Explore
    return bound_action(noisy_action, *A_bounds)
```

Coding

Now construct Q

Coding

Now construct Q

```
Q = Sequential([
    # Different from DQN network
    # Input action and state together
    Lambda(lambda s, a: concatenate(s, a)),
    Linear(state_size + action_dims, hidden_size),
    Lambda(leaky_relu),
    Linear(hidden_size, hidden_size),
    Lambda(leaky_relu),
    Linear(hidden_size, 1), # Single value for Q(s, a)
])
```

Coding

```
while not terminated:
    # Exploration: make sure actions within action space!
    action = sample_action(mu, state, bounds, std)
    transition = env.step(action)
    replay_buffer.append(transition)
    data = replay_buffer.sample()
    # Theta_pi params are in mu neural network
    # Argnums tells us differentiation variable
    J_Q = grad(Q_loss, argnums=0)(theta_Q, theta_T, mu, data)
    J_mu = grad(mu_loss, argnums=0)(mu, theta_Q, data)
    theta_Q, mu = apply_updates(J_Q, J_mu, ...)
    if step % 200 == 0: # Target network necessary
        theta_T = theta_Q
```

Coding

```
def Q_loss(theta_Q, theta_T, theta_pi, data):
    Qnet = combine(Q, theta_Q)
    Tnet = combine(Q, theta_T) # Target network
    # Predict Q values for action we took
    prediction = vmap(Qnet)(data.state, data.action)
    # Now compute labels
    next_action = vmap(mu)(data.next_state)
    # NOTE: No argmax! Mu approximates argmax
    next_Q = vmap(Tnet)(data.next_state, next_action)
    label = reward + gamma * data.done * next_Q
    return (prediction - label) ** 2
```

Coding

```
def mu_loss(mu, theta_Q, data):
    # Find the action that maximizes the Q function
    Qnet = combine(Q, theta_Q)
    # Instead of multiply, chain rule -- plug action into Q
    action = vmap(mu)(data.state)
    q_value = vmap(Qnet)(data.state, action)
    # Update the policy parameters to maximize the Q value
    # Gradient ascent but we min loss, use negative
    return -q_value
```

Max Entropy RL

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Many algorithms add improvements to DDPG

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We will very briefly cover max-entropy RL

First, let us introduce entropy

Max Entropy RL

Entropy measures the uncertainty of a distribution

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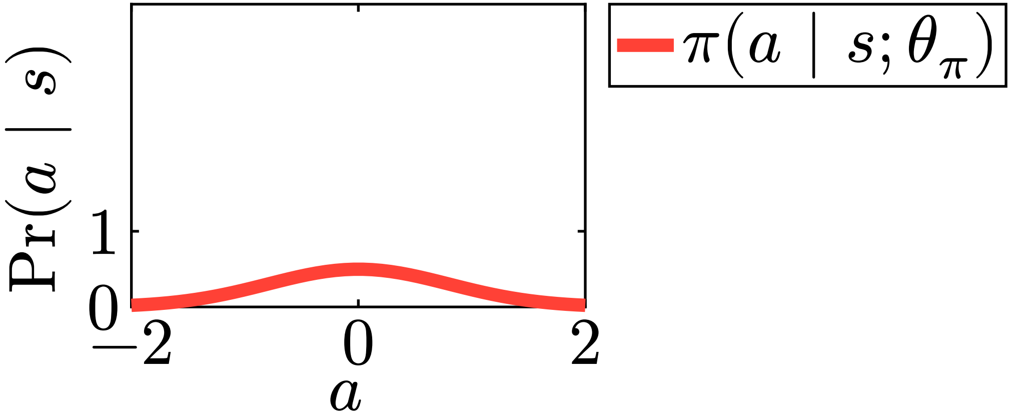

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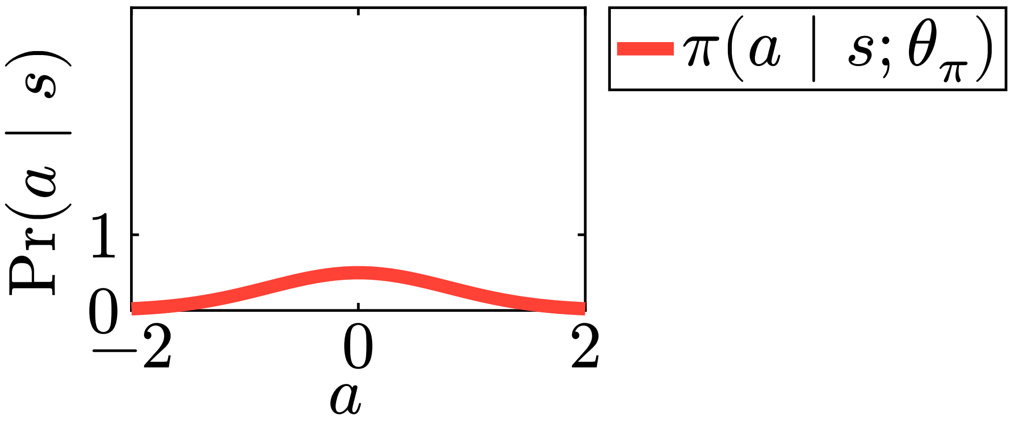

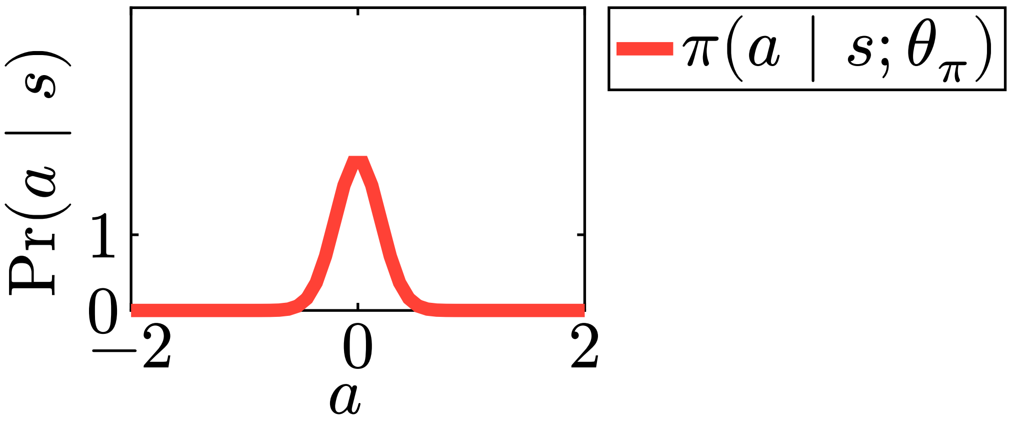

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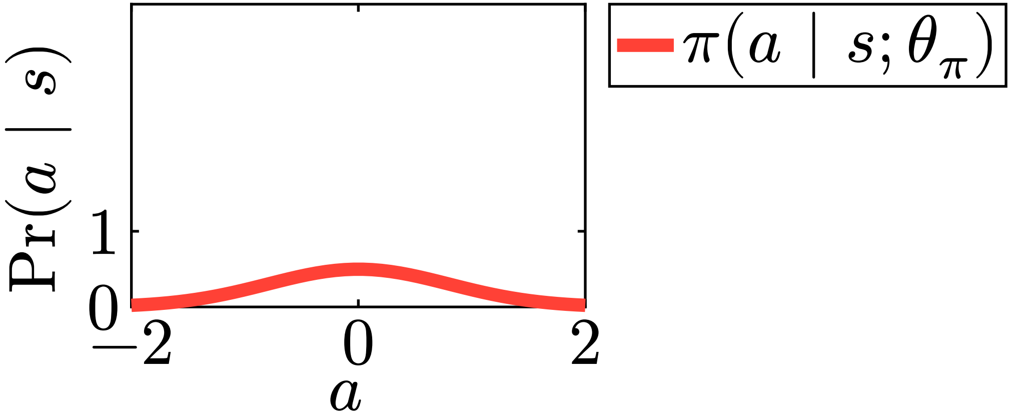

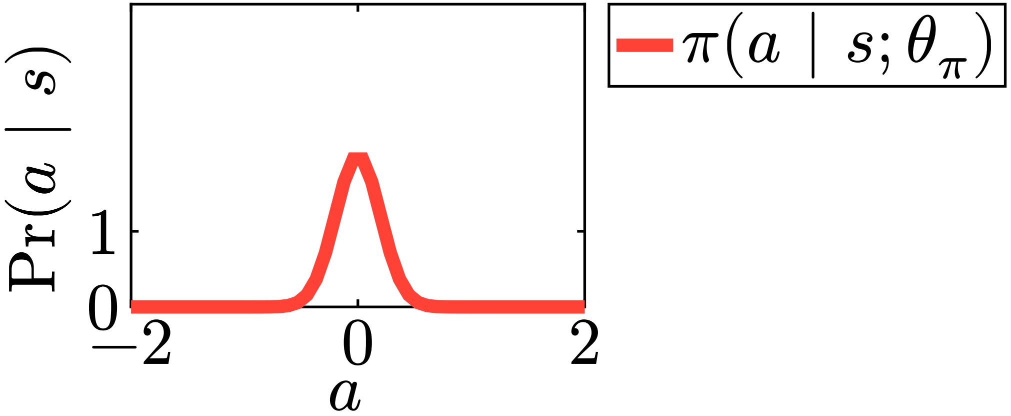

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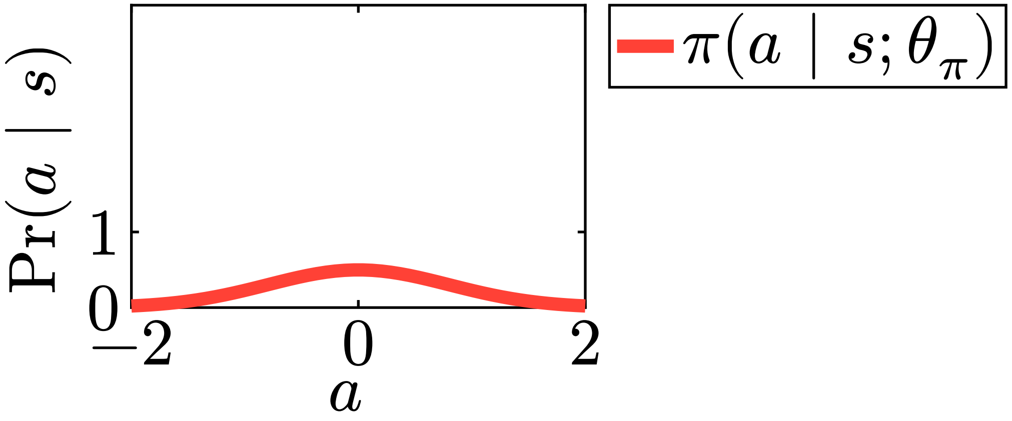

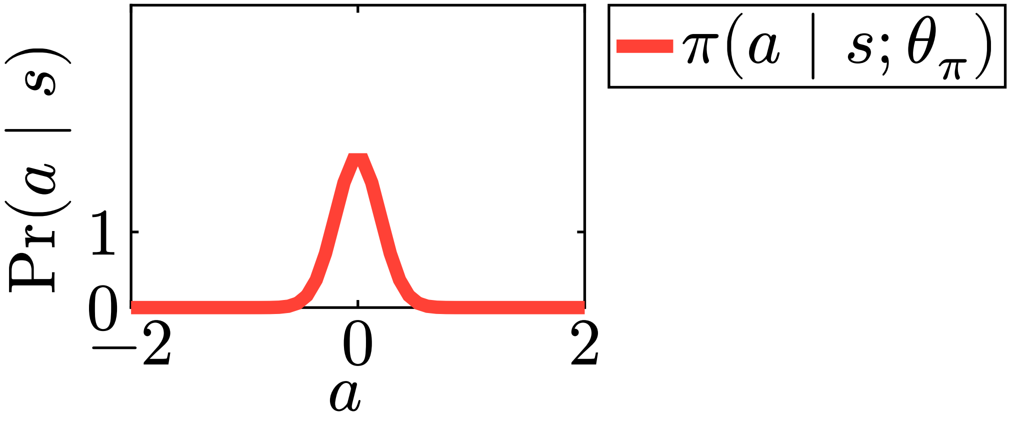

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Left policy, more uncertain/random

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We want a policy that is both random and maximizes the return

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- Better exploration during training
- More stable training (explain further in offline RL)

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We get SAC!

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Repeat until convergence, $\theta_{\mu, i+1} = \theta_{\mu, i}, \quad \theta_{Q, i+1} = \theta_{Q, i}$

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Like PPO, SAC is complicated – uses many “implementation tricks”

- Often not documented
- CleanRL describes modern SAC, using tricks from 5+ papers
- https://docs.cleanrl.dev/rl-algorithms/sac/#implementation-details_1

Coding SAC could take an entire lecture, read CleanRL

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  - Think about why it learned to do this (exploiting bugs in MDP)

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You must use your brain to be successful!