Bandits

CISC 7404 - Decision Making

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Notation

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Let us review some notation I will use in the course

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If you ever get confused, come back to these slides

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Vectors

$$\boldsymbol{x} = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}$$

Matrix

$$\boldsymbol{X} = \begin{bmatrix} x_{1,1} & x_{1,2} & \dots & x_{1,n} \\ x_{2,1} & x_{2,2} & \dots & x_{2,n} \\ \vdots & \vdots & \ddots & \vdots \\ x_{m,1} & x_{m,2} & \dots & x_{m,n} \end{bmatrix}$$

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We will represent vectors or matrices of **tensors**

Vector of tensors

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Each $\boldsymbol{x}_i$ could be a vector, matrix, 3x3 tensor, etc

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Same for matrices

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Question: What is the difference between the following?

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$$X = \{1, 2, 3, 4\}$$

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We will represent important sets with blackboard font

$\mathbb{R}$	Set of all real numbers $\{1, 2.03, \pi, ...\}$
$\mathbb{Z}$	Set of all integers $\{-2, -1, 0, 1, 2, ...\}$
$\mathbb{Z}_+$	Set of all **positive** integers $\{1, 2, ...\}$

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The max operator returns the maximum of a function over its domain

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$$\max_x f(x)$$

The arg max operator returns the input that maximizes a function

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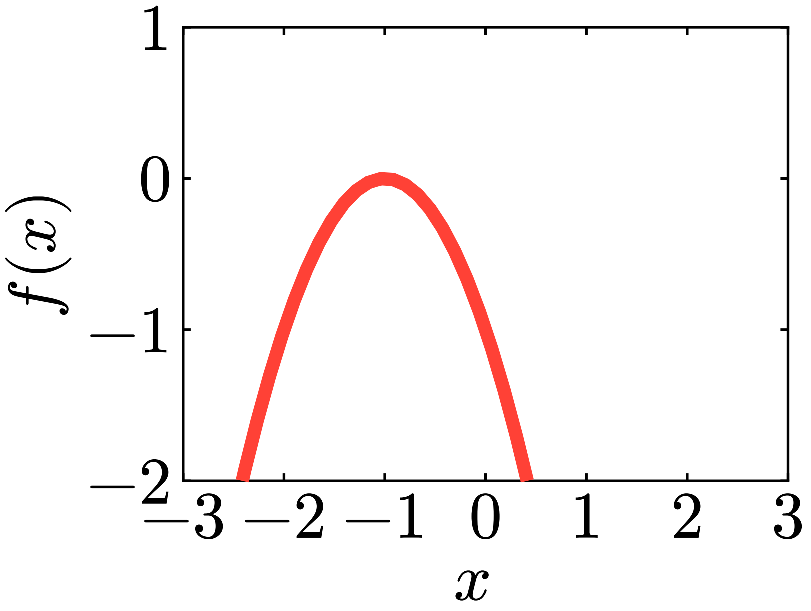

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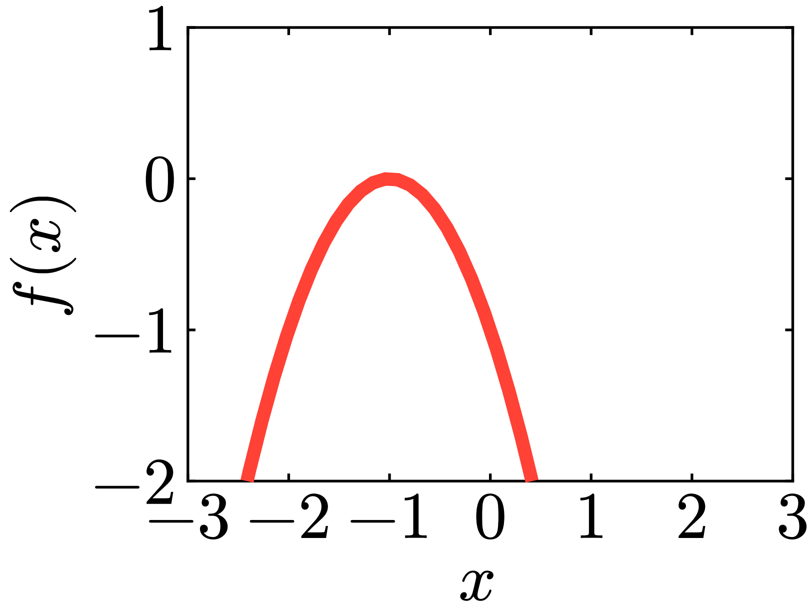

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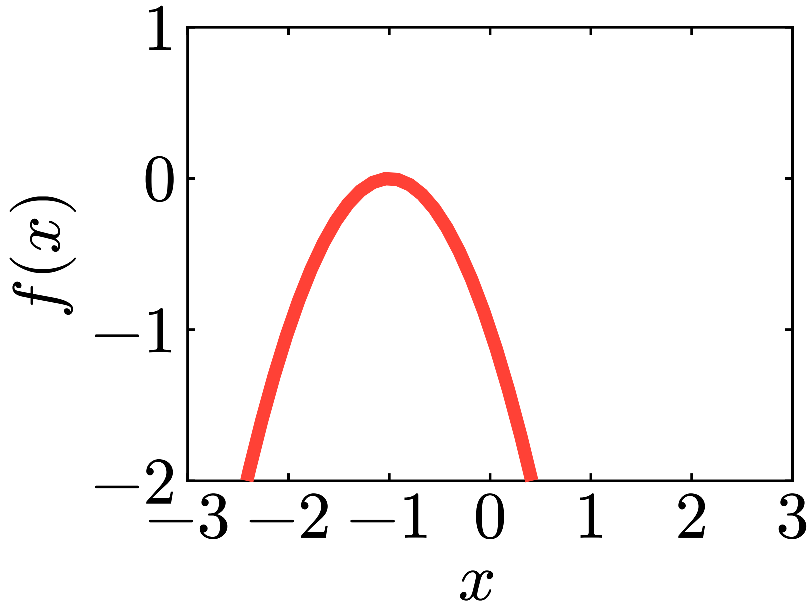

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$\{0, 1\}^n$	Set of all boolean vectors of length n

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We define **functions** or **maps** between sets

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Question: What does this function do?

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A bandit steals your money

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This is a **one-armed** bandit

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Let us see if we can make money playing this game

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$$\Pr(\mathcal{X} = x) = \left\{ \underbrace{\omega}_{\text{Outcome}} \in \underbrace{\Omega}_{\text{Outcomes}} \;\middle|\; \underbrace{\mathcal{X}(\omega)}_{\text{Outcome to real}} = \underbrace{x}_{\text{Real}} \right\}$$

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But we can combine them to find out

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$$-10 \cdot \frac{199}{200} + 1000 \cdot \frac{1}{200} = -4.95$$

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$$r_1 = -10$$
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As we play the game more and more, we converge to the expectation

$$\lim_{n \to \infty} \sum_{t=1}^{n} r_t = -4.95n = n\mathbb{E}[\mathcal{X}]$$

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Question: What is the best way to make money with the bandit?

Answer: Do not play! If you must, play as little as possible

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After playing enough, the gambler can approximate the expectation!

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- The expected value $\mathbb{E}[\mathcal{X}]$
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Make sure the expected value is **negative but near zero**:

Bandits

Exercise: You start a new casino in Macau. Create a bandit with the following outcomes $\Omega \in \{\text{Win Lemon}, \text{Win Cherry}, \text{Win BAR}, \text{Lose}\}$

Write down:

- Probability for each outcome $P(\omega); \quad \forall \omega \in \Omega$
- The random variable for each outcome $\mathcal{X}(\omega); \quad \forall \omega \in \Omega$
- The expected value $\mathbb{E}[\mathcal{X}]$
- How much money the gambler loses after 1000 plays

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Multiarmed Bandits

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If $\mathbb{E}[\mathcal{X}] > 0$ you should gamble

If $\mathbb{E}[\mathcal{X}] < 0$ you should not gamble

We will consider a more interesting problem

Multiarmed Bandits

You arrive at the Londoner with 1000 MOP and want to win money

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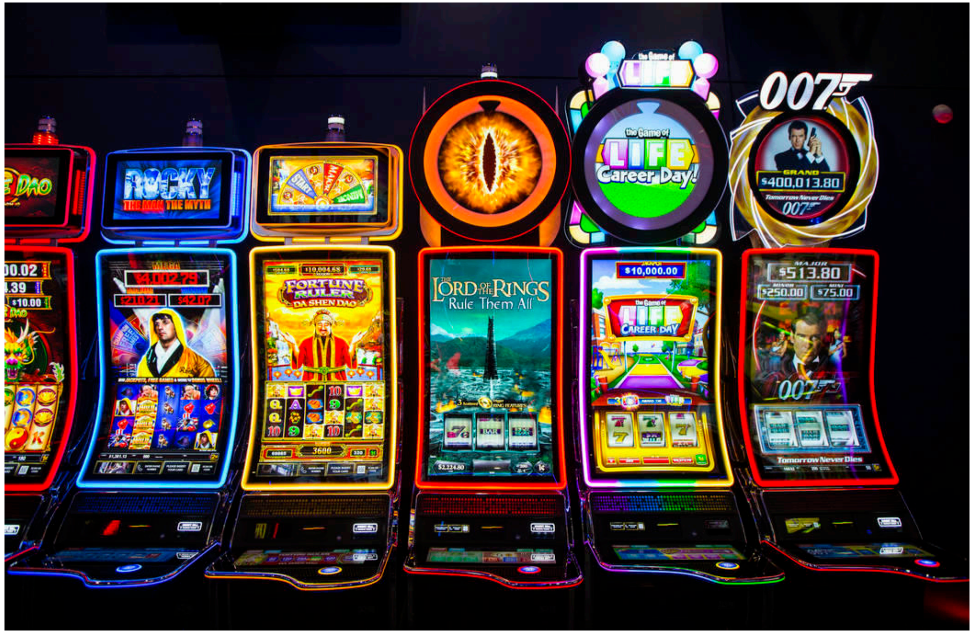

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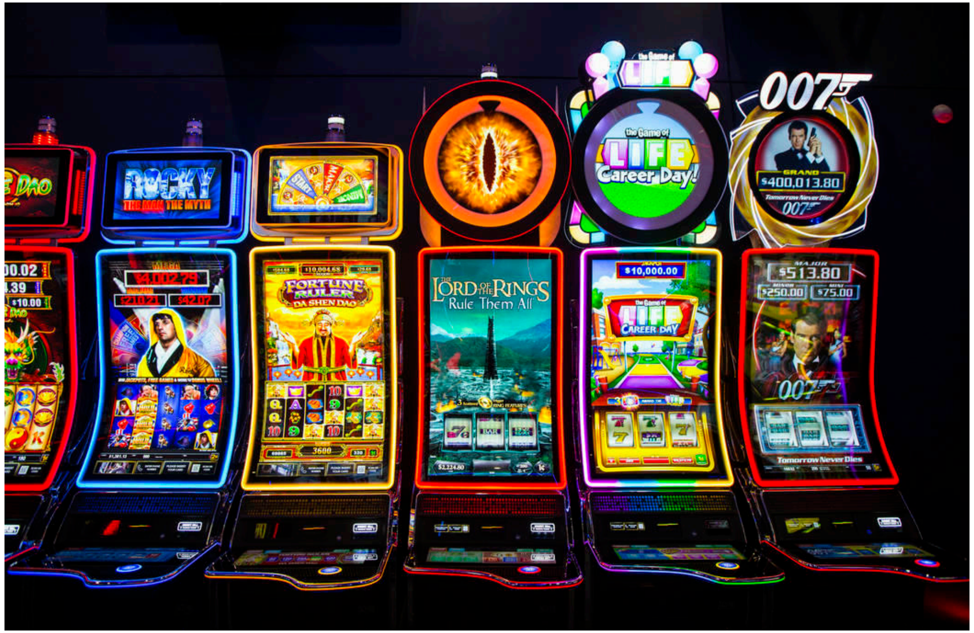

Question: Which machine do you play?

Multiarmed Bandits

We call this the **multi-armed bandit** problem

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You don't know the expected value of each arm. Which should you pull?

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Medicine C

We can find the best medicine while healing the most people

Multiarmed Bandits

YouTube, Youku, BiliBili, TikTok, Netflix use bandits to suggest videos

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YouTube tries to find your favorite video category

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Problem: We have k bandits, and each bandit is a random variable

$$\mathcal{X}_1, \mathcal{X}_2, ..., \mathcal{X}_k$$

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Question: How should we approach this problem?

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We want to:

- Pick a to approximate bandits $\mathbb{E}[\mathcal{X}_a \mid a \in 1...k]$
- Pick a to make the most money $\underset{a \in 1...k}{\arg\max}\, \mathbb{E}[\mathcal{X}_a]$

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We have names for each goal

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Exploration:

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It is important you understand this! Any questions?

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Question: How can we achieve both goals at once?

Answer: Sometimes choose a to explore, sometimes choose a to exploit

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Answer:

- $\varepsilon \approx 1$ when we trust our estimates $\mathbb{E}[\mathcal{X}]$
- $\varepsilon \approx 0$ when we do not trust our estimates

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