Bandits

CISC 7404 - Decision Making

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University of Macau

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Notation

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Let us review some notation I will use in the course

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If you ever get confused, come back to these slides

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Vectors

$$\boldsymbol{x} = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}$$

Matrix

$$\boldsymbol{X} = \begin{bmatrix} x_{1,1} & x_{1,2} & \cdots & x_{1,n} \\ x_{2,1} & x_{2,2} & \cdots & x_{2,n} \\ \vdots & \vdots & \ddots & \vdots \\ x_{m,1} & x_{m,2} & \cdots & x_{m,n} \end{bmatrix}$$

Notation

We will represent vectors or matrices of **tensors**

Vector of tensors

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Each $\boldsymbol{x}_i$ could be a vector, matrix, 3x3 tensor, etc

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Same for matrices

Matrix of tensors

$$\boldsymbol{X} = \begin{bmatrix} \boldsymbol{x}_{1,1} & \boldsymbol{x}_{1,2} & \dots & \boldsymbol{x}_{1,n} \\ \boldsymbol{x}_{2,1} & \boldsymbol{x}_{2,2} & \dots & \boldsymbol{x}_{2,n} \\ \vdots & \vdots & \ddots & \vdots \\ \boldsymbol{x}_{m,1} & \boldsymbol{x}_{m,2} & \dots & \boldsymbol{x}_{m,n} \end{bmatrix}$$

Notation

Question: What is the difference between the following?

$$\boldsymbol{X} = \begin{bmatrix} x_{1,1} & x_{1,2} & \cdots & x_{1,n} \\ x_{2,1} & x_{2,2} & \cdots & x_{2,n} \\ \vdots & \vdots & \ddots & \vdots \\ x_{m,1} & x_{m,2} & \cdots & x_{m,n} \end{bmatrix}$$

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Capital letters will often refer to **sets**

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$$X = \{1, 2, 3, 4\}$$

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We will represent important sets with blackboard font

$\mathbb{R}$	Set of all real numbers $\{1, 2.03, \pi, ...\}$
$\mathbb{Z}$	Set of all integers $\{-2, -1, 0, 1, 2, ...\}$
$\mathbb{Z}_+$	Set of all **positive** integers $\{1, 2, ...\}$

Notation

The max operator returns the maximum of a function over its domain

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$$\max_x f(x)$$

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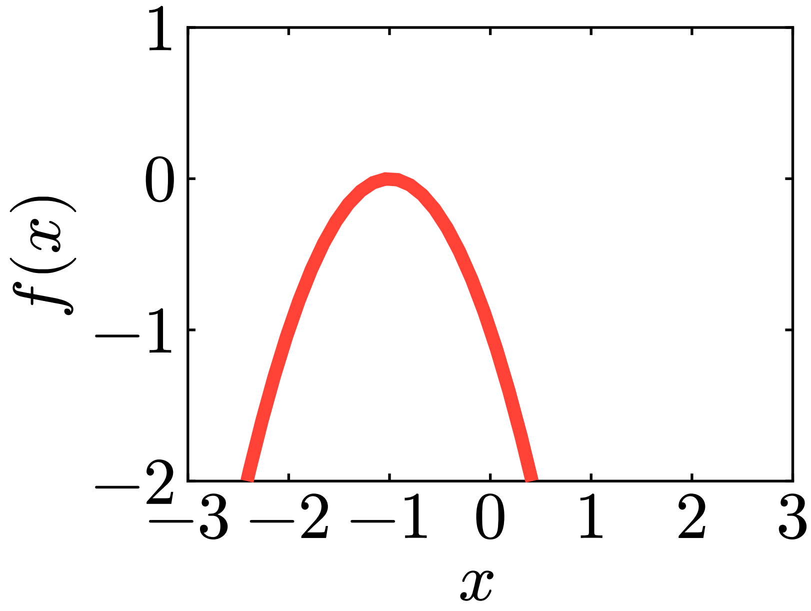

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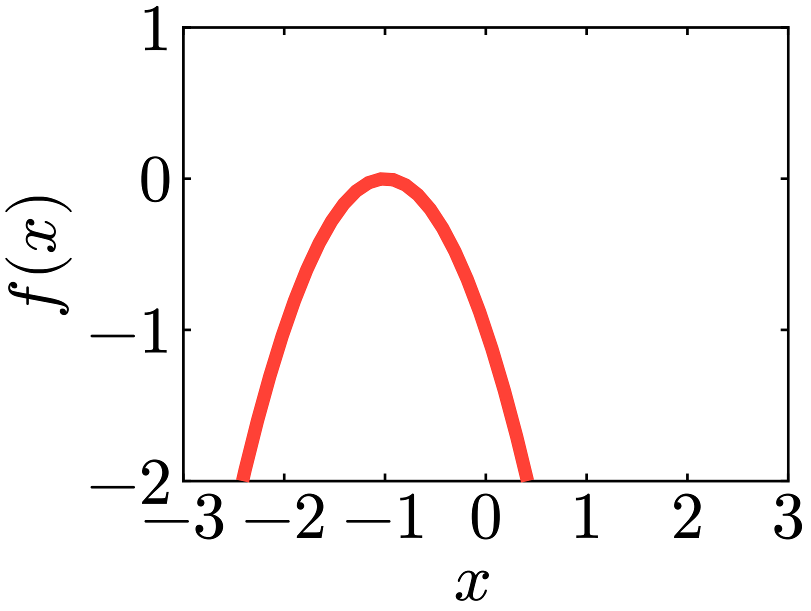

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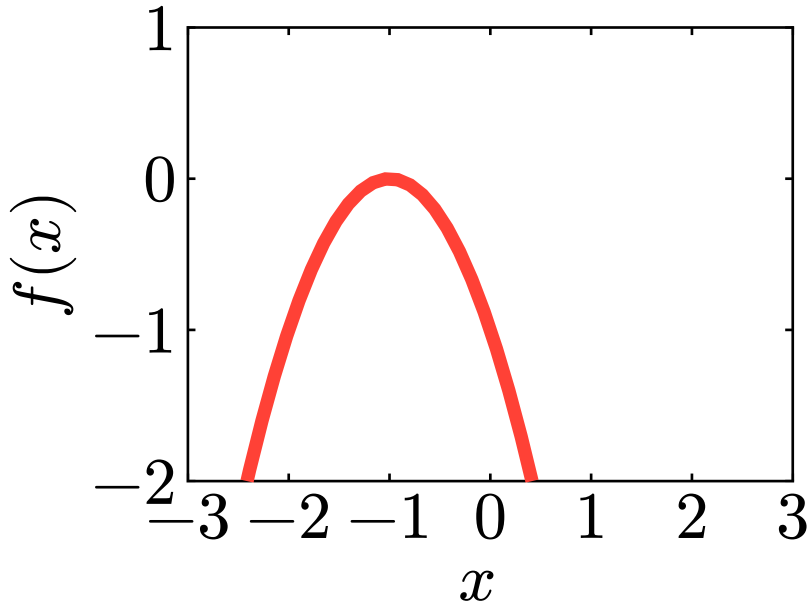

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$\{0, 1\}^n$	Set of all boolean vectors of length n

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We define **functions** or **maps** between sets

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Question: What does this function do?

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Today's lecture will be difficult

But if you can understand it, then reinforcement learning will be easy for you

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The simplest form of decision making problem is the **bandit**

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A bandit steals your money

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This is a **one-armed** bandit

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Question: How does a one-armed bandit steal your money?

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Let us see if we can make money playing this game

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$$\sum_{\omega \in \Omega} \Pr(\omega) = 1$$

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$$\Pr(\mathcal{X} = x) = \left\{ \Pr\left(\underbrace{\mathcal{X}(\omega)}_{\text{Outcome to real}} = \underbrace{x}_{\text{Real}} \right) \middle| \underbrace{\omega}_{\text{Outcome}} \in \underbrace{\Omega}_{\text{Outcomes}} \right\}$$

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But we can combine them to find out

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$$\mathbb{E}[\mathcal{X}] = \sum_{\omega \in \Omega} \mathcal{X}(\omega) \cdot \Pr(\omega)$$

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$$\Pr(\text{lose}) \cdot \mathcal{X}(\text{lose}) + \Pr(\text{win}) \cdot \mathcal{X}(\text{win})$$

$$\frac{199}{200} \cdot -10 + \frac{1}{200} \cdot 1000 = -4.95$$

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Question: What does $\mathbb{E}[\mathcal{X}] = -4.95$ mean?

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\begin{gathered}
r_1 = -10 \\
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Negative reward means we lose money

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As we play the game more and more, we converge to the expectation

$$\lim_{n \to \infty} \sum_{t=1}^{n} r_t = -4.95n = n\mathbb{E}[\mathcal{X}]$$

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Answer: Do not play! If you must, play as little as possible

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If you know $\mathbb{E}[\mathcal{X}]$, you know the result of gambling

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After playing enough, the gambler can approximate the expectation!

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- Probability for each outcome $P(\omega); \quad \forall \omega \in \Omega$
- The random variable for each outcome $\mathcal{X}(\omega); \quad \forall \omega \in \Omega$

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- The random variable for each outcome $\mathcal{X}(\omega); \quad \forall \omega \in \Omega$
- The expected value $\mathbb{E}[\mathcal{X}]$

Bandits

Exercise: You start a new casino in Macau. Create a bandit with the following outcomes $\Omega \in \{\text{Win Lemon}, \text{Win Cherry}, \text{Win BAR}, \text{Lose}\}$

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Make sure the expected value is **negative but near zero**:

- Negative: The gambler loses money and you win money
- Near zero: The gambler wins sometimes and will continue to play

Multiarmed Bandits

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The bandit problem is useful for casino owners and gamblers

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If $\mathbb{E}[\mathcal{X}] > 0$ you should gamble

If $\mathbb{E}[\mathcal{X}] < 0$ you should not gamble

We will consider a more interesting problem

Multiarmed Bandits

You arrive at the Londoner with 1000 MOP and want to win money

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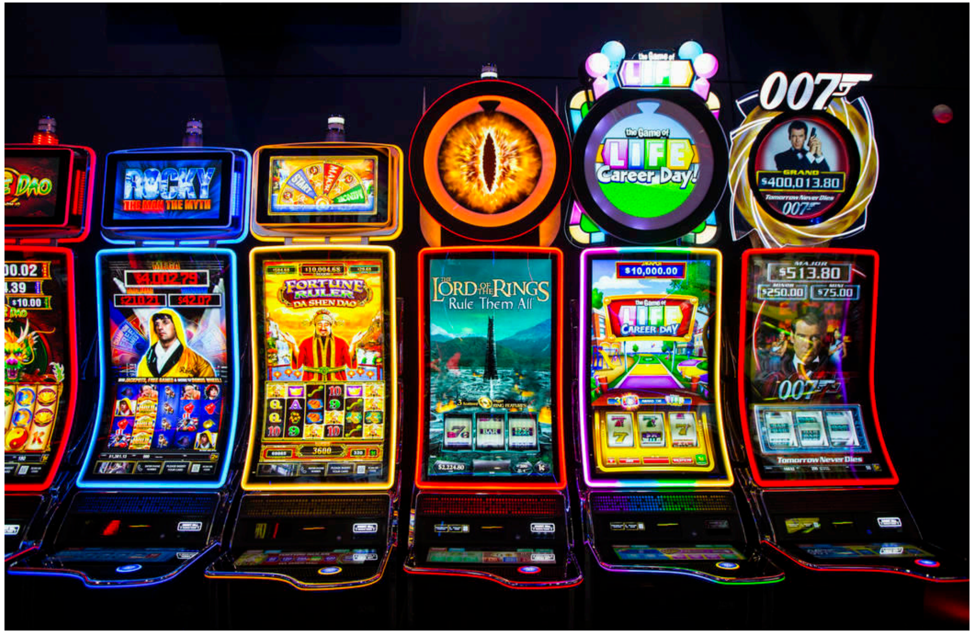

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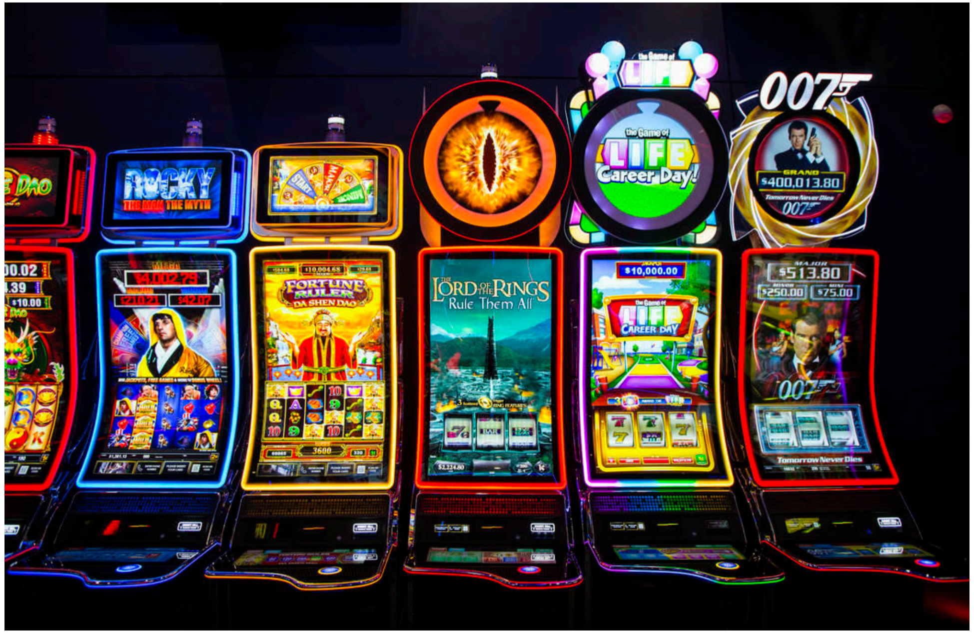

Question: Which machine do you play?

Multiarmed Bandits

We call this the **multi-armed bandit** problem

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You don't know the expected value of each arm. Which should you pull?

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We can find the best medicine while healing the most people

Multiarmed Bandits

YouTube, Youku, BiliBili, TikTok, Netflix use bandits to suggest videos

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TikTok tries to find your favorite video category

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Problem: We have k bandits, and each bandit is a random variable

$$\mathcal{X}_1, \mathcal{X}_2, ..., \mathcal{X}_k$$

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Question: How should we approach this problem?

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We want to:

- Pick a to approximate bandits $\quad \mathbb{E}[\mathcal{X}_a \mid a \in 1...k]$
- Pick a to make the most money $\quad \arg\max_{a \in 1...k} \mathbb{E}[\mathcal{X}_a]$

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We have names for each goal

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Exploration:

$$\mathbb{E}[\mathcal{X}_a \mid a \in 1...k]$$

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It is important you understand this! Any questions?

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Question: How can we achieve both goals at once?

Answer: Sometimes choose a to explore, sometimes choose a to exploit

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$$u \sim \text{uniform}([0, 1])$$

$$\text{if } u < 0.5 \text{ then } a \sim \text{uniform}(\{1 ... k\})$$

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Question: When should $\varepsilon \approx 1$? When should $\varepsilon \approx 0$?

Answer:

- $\varepsilon \approx 1$ when we trust our estimates $\mathbb{E}[\mathcal{X}]$
- $\varepsilon \approx 0$ when we do not trust our estimates

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- Sometimes it suggests study videos

Coding

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https://colab.research.google.com/drive/1cyNLRa-J8oe7pgy_gs2mcypZPqqaquoa