Decision Processes

CISC 7404 - Decision Making

Steven Morad

University of Macau

Markov chains

How should we structure decision making problems?

Recall MAB bandit

Agent/environment interface

Chess bot example

Why is chess not MAB bandits?

Sequential decision making

Agent makes decisions/moves pieces

Env is set of rules

Input/output function

Gymnasium

Review

Review

RL and decision making designed to solve only MDPs

Markov Processes

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Decisions must make some change in the world

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- Cryptography
- History

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Let us do an example to understand this

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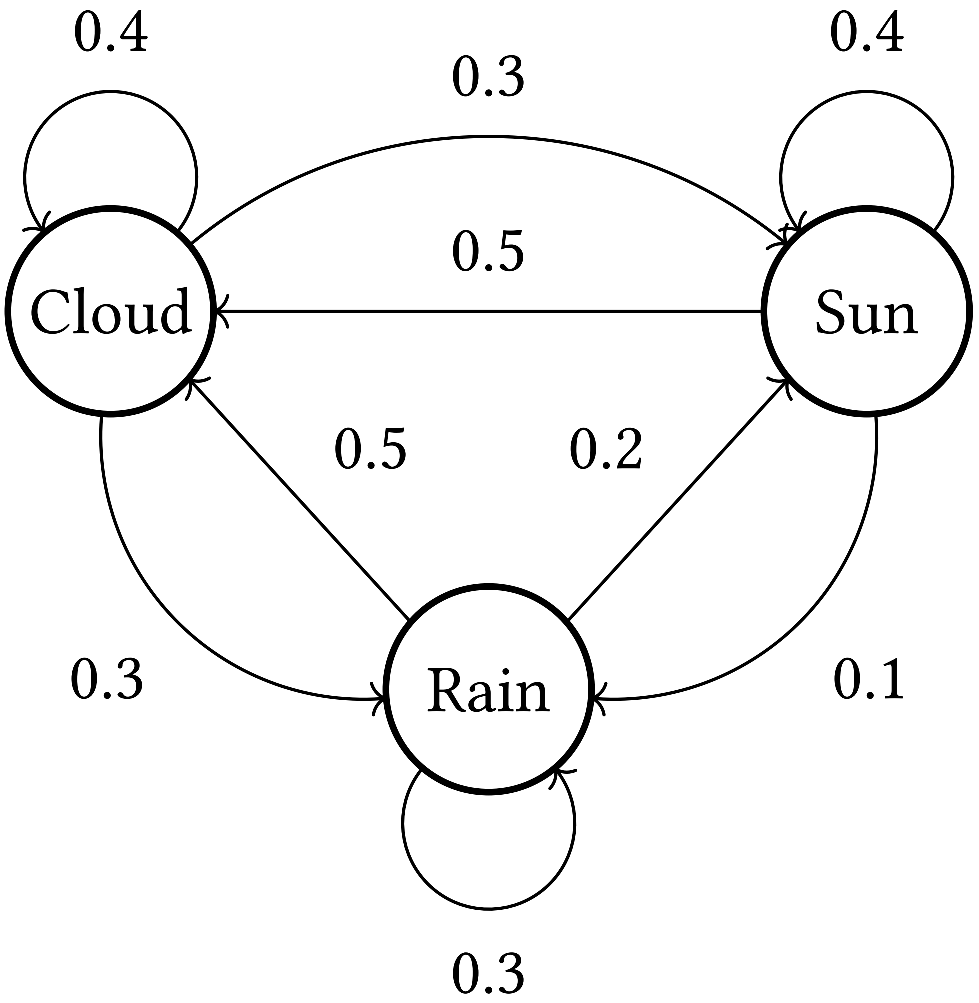

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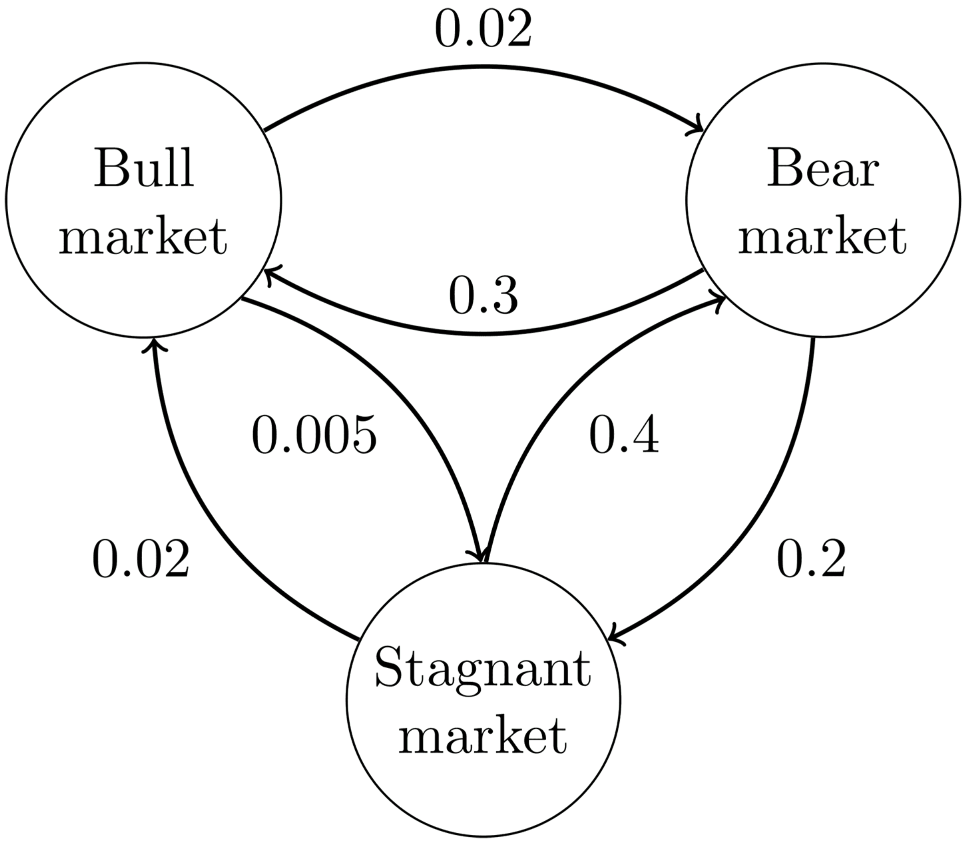

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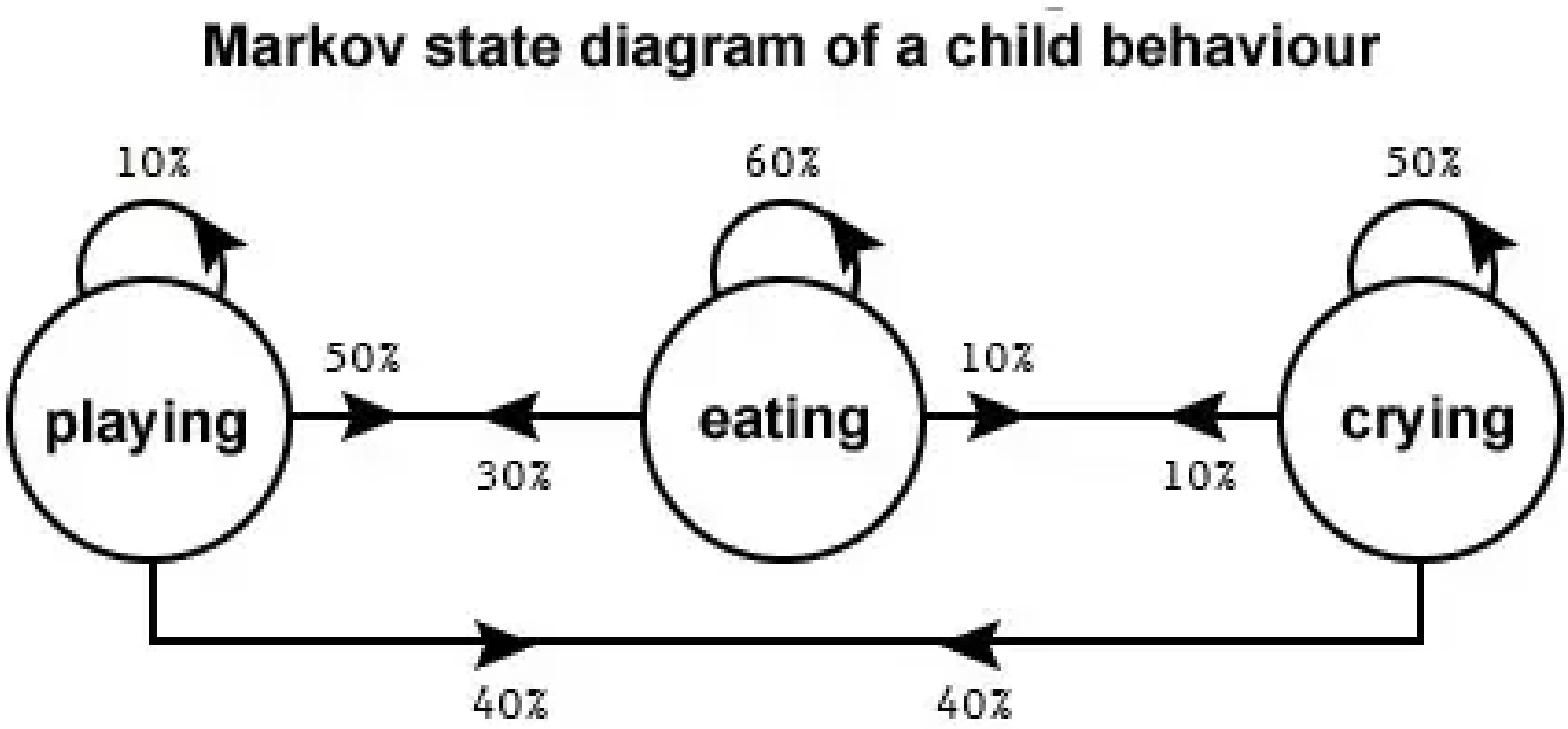

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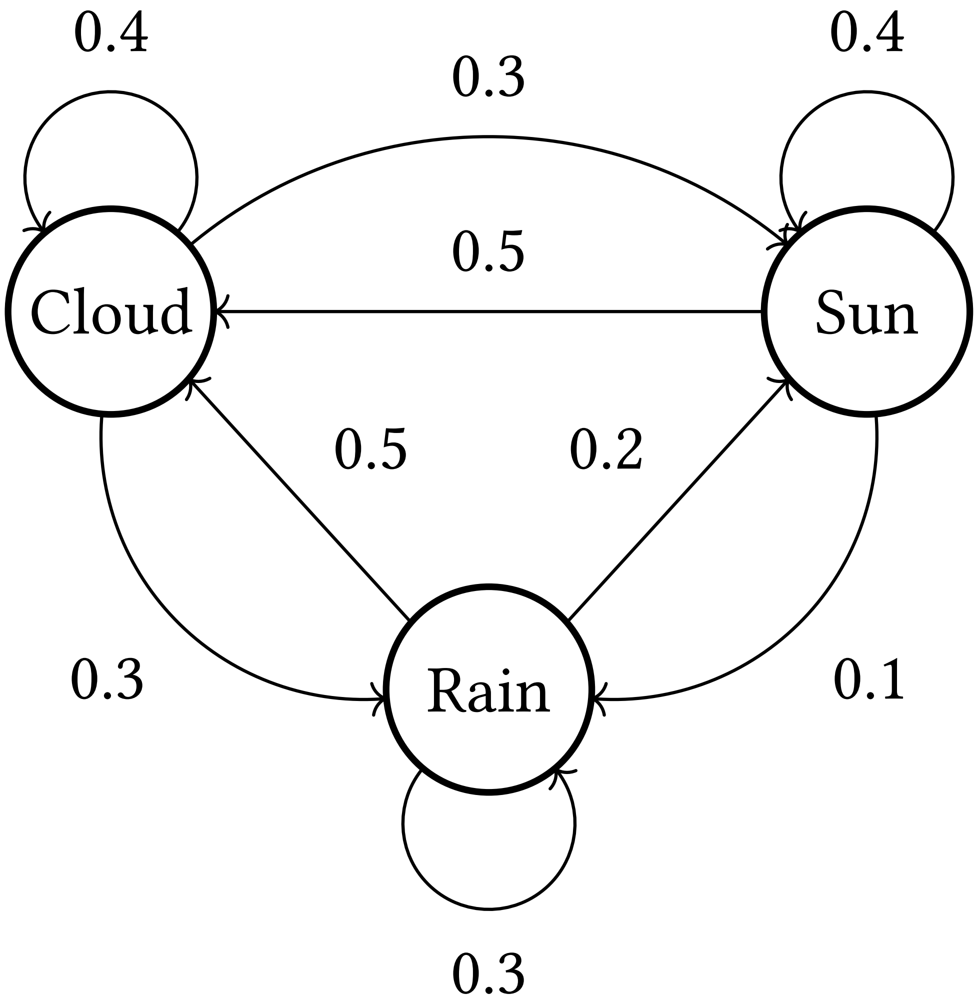
To compute the next node, we only look at the current node



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Question: How can we model this?

Answer: We create a **terminal state** that we cannot leave

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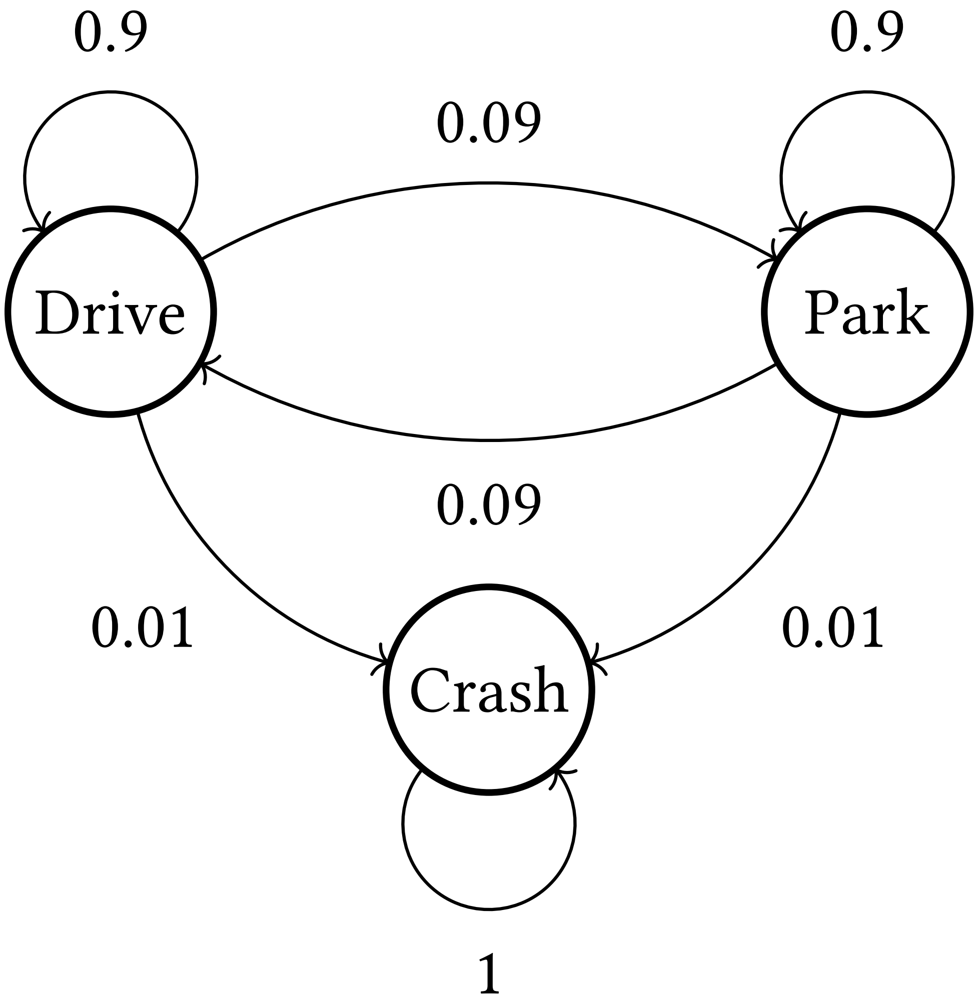

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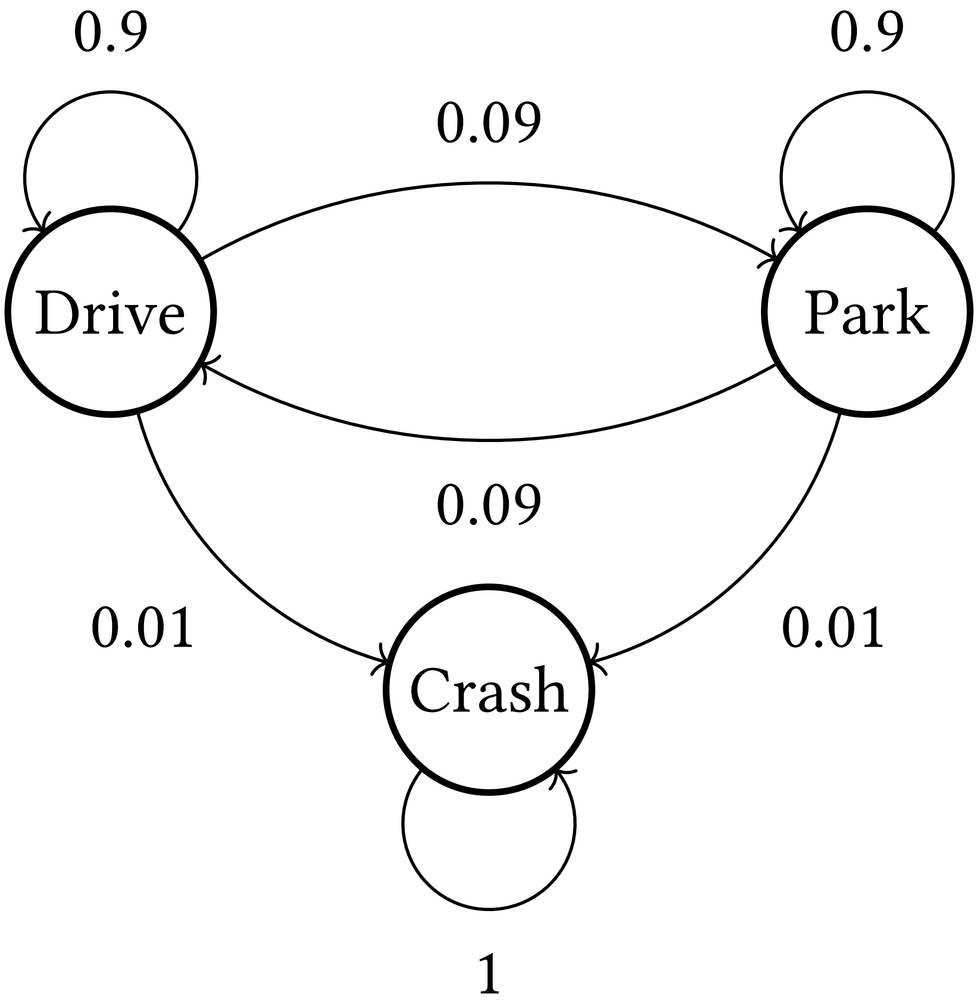

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Once we crash our car, we cannot drive or park any more

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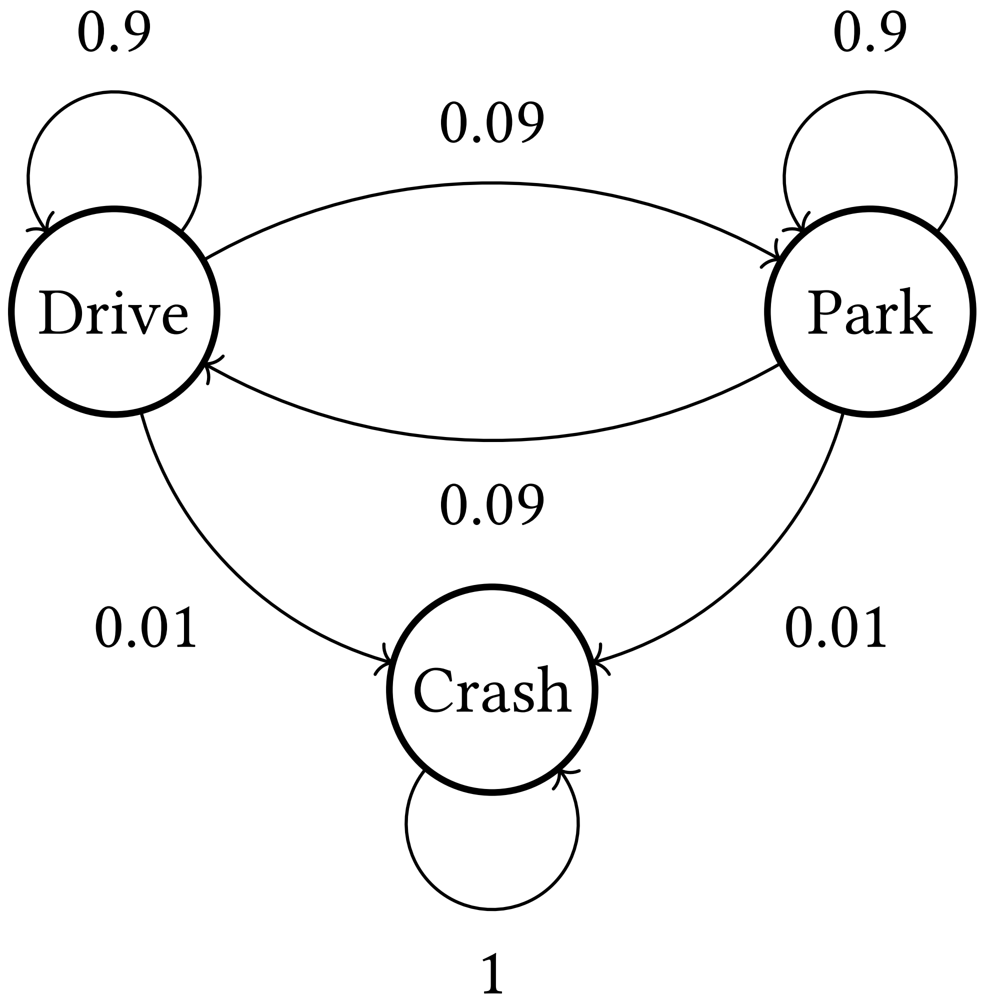

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The only transition from a terminal state is back to itself

$$\Pr(s' = s_{\text{terminal}} \mid s = s_{\text{terminal}}) = 1.0$$

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- Create a terminal state

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We will modify the Markov process for decision making

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We introduce the **agent** to make decisions that change the environment

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Let us see an example

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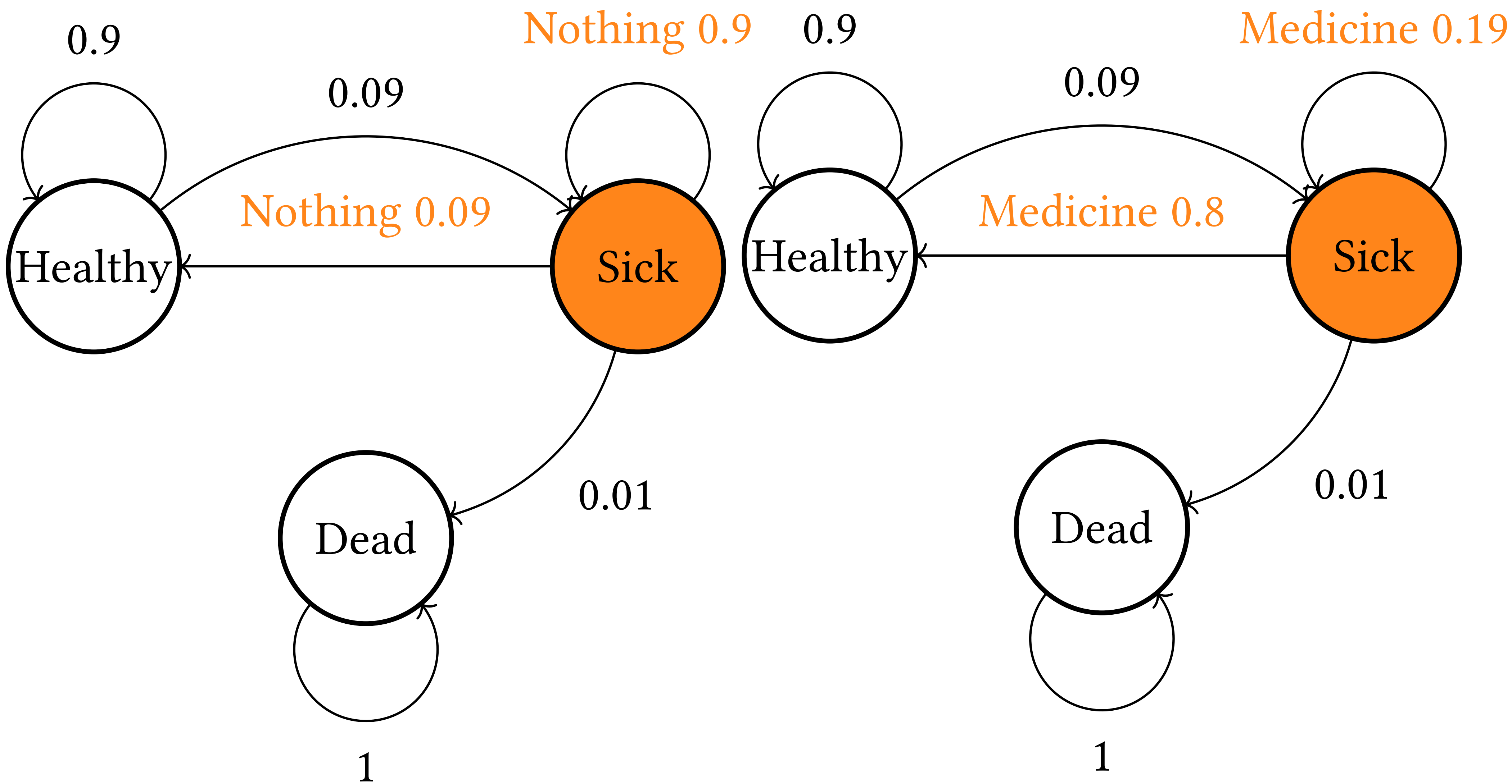

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How can we make optimal decisions if we cannot tell how good a decision is?

We need something to tell us how good it is to be in a state!

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Markov decision process

$$(S, A, T, R, \gamma)$$

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We can write this mathematically as

$$\arg\max_{s \in S} R(s)$$

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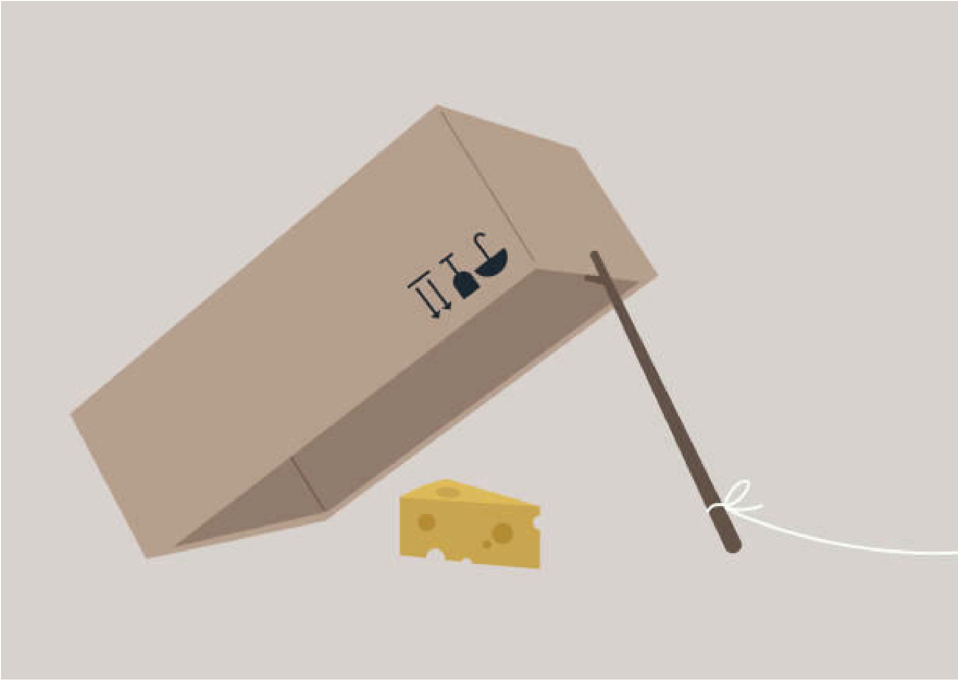

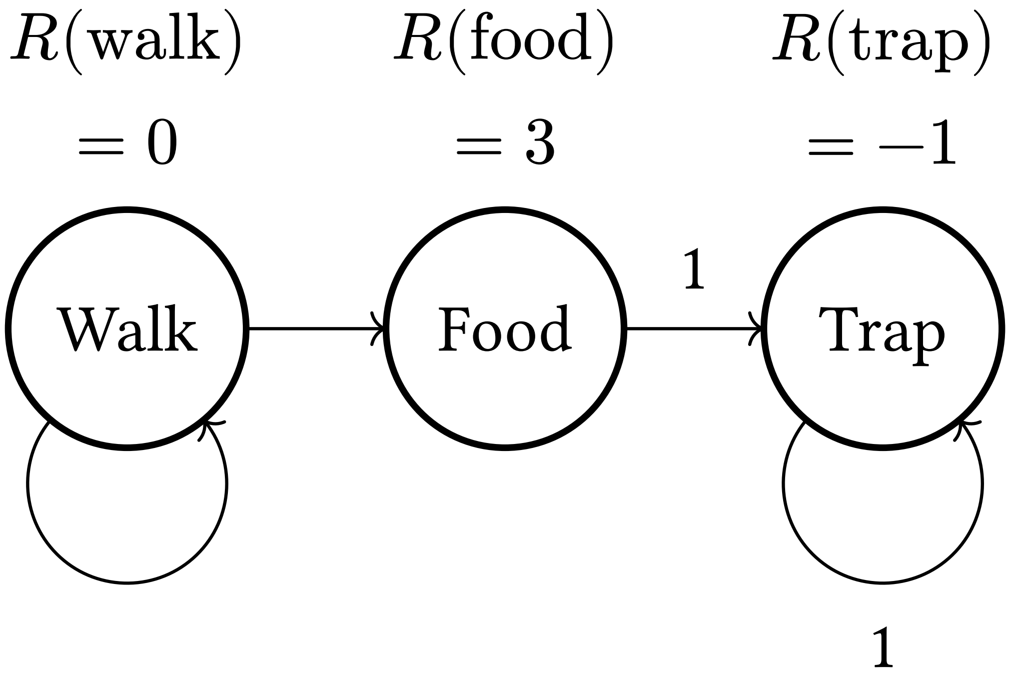

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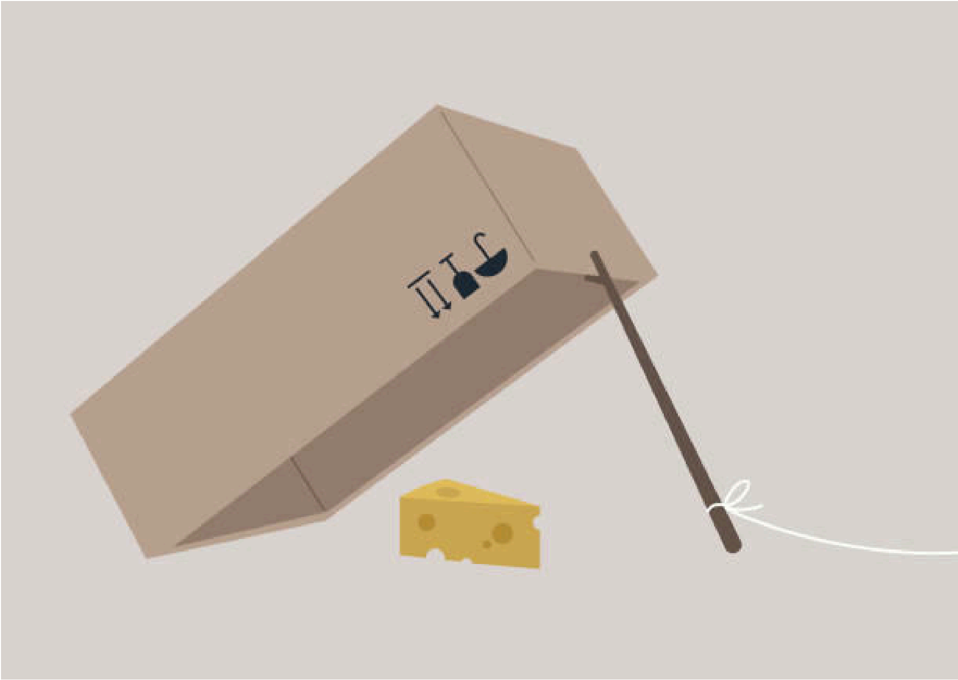

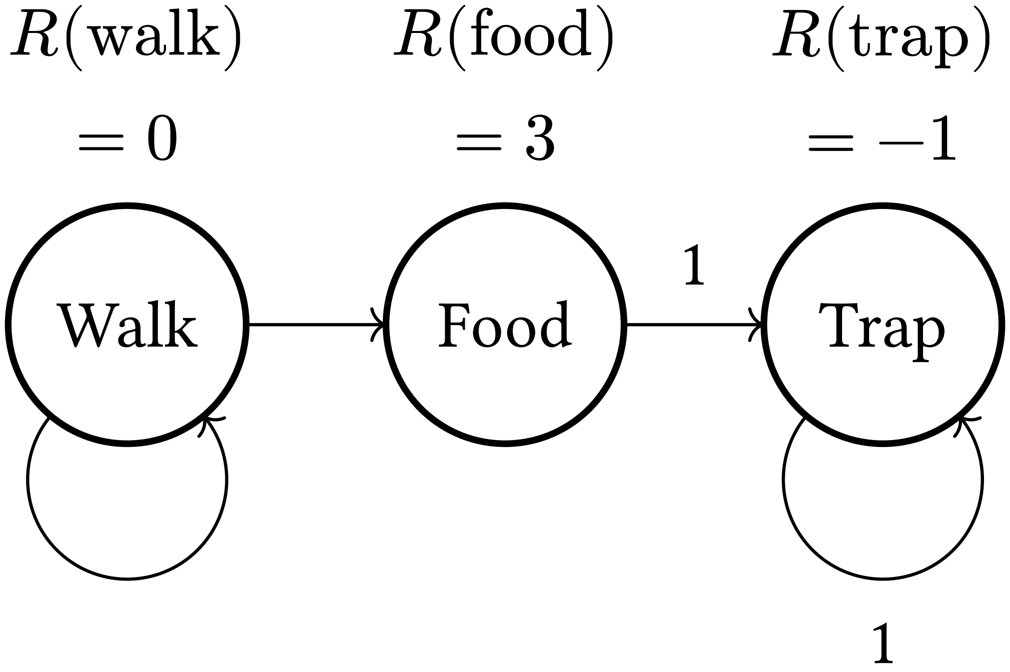

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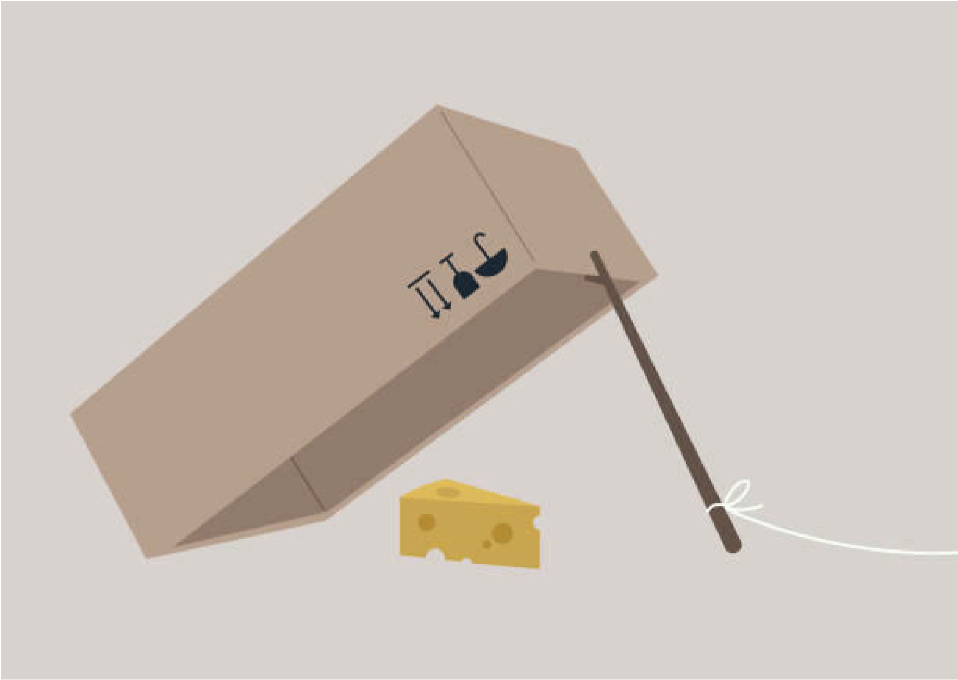

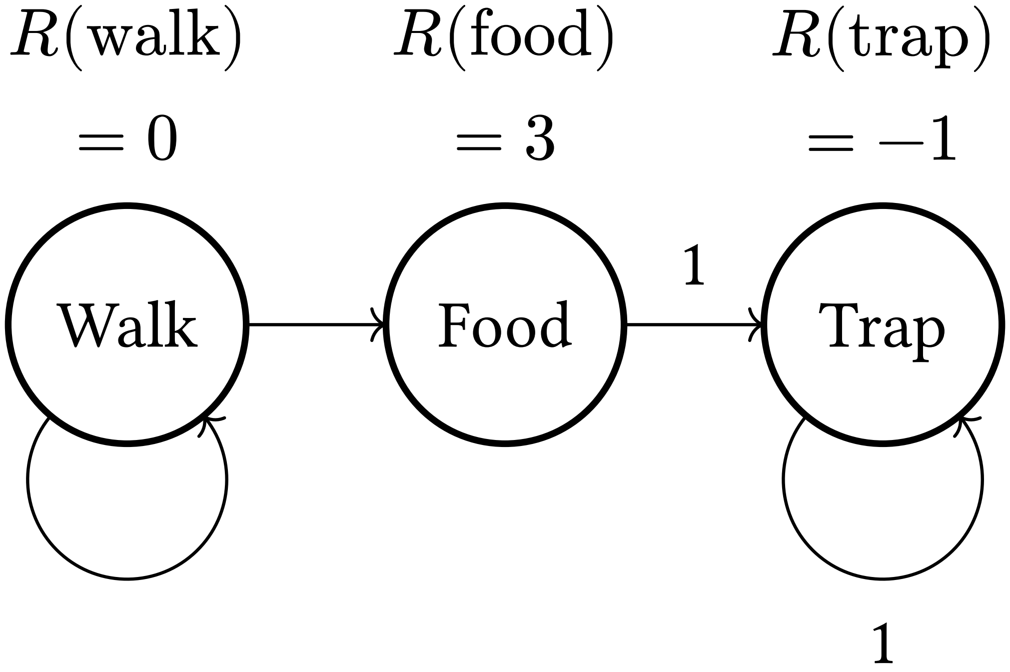

$$\arg\max_{s \in S} R(s) = \text{food}$$

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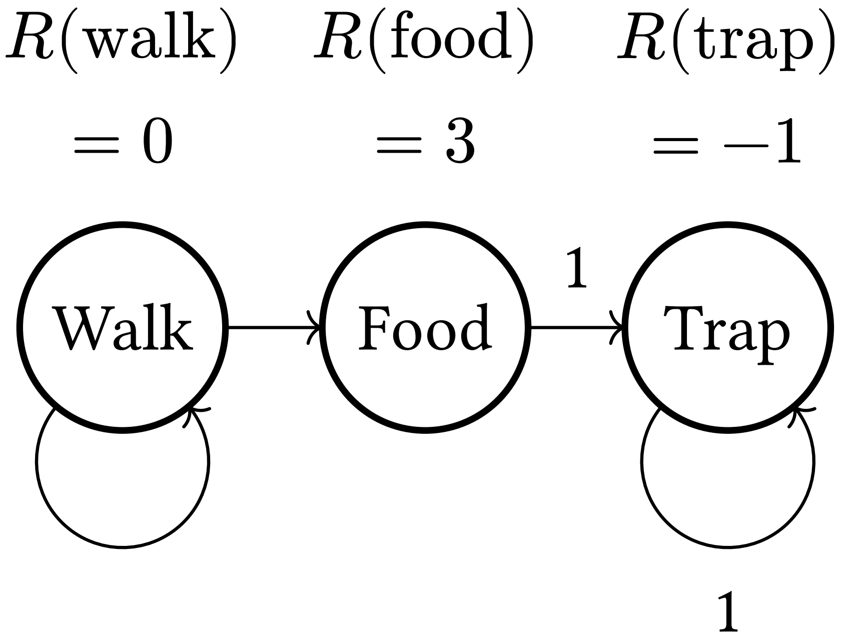

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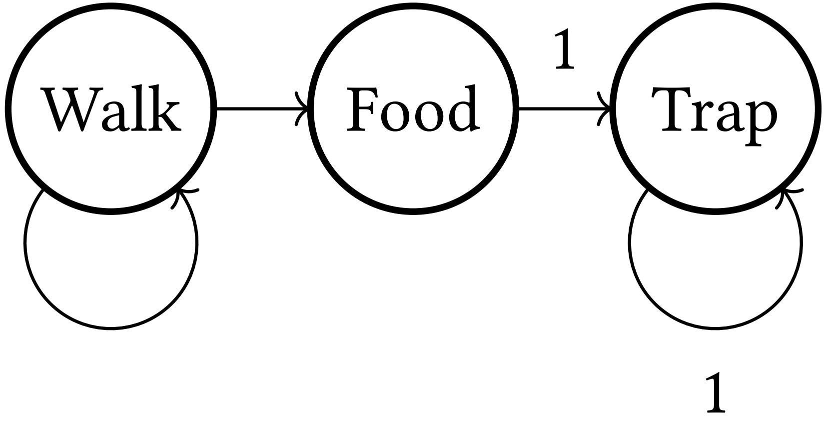
Markov Decision Processes

$R(\text{walk}) = 0$ $\quad R(\text{food}) = 3$ $\quad R(\text{trap}) = -1$



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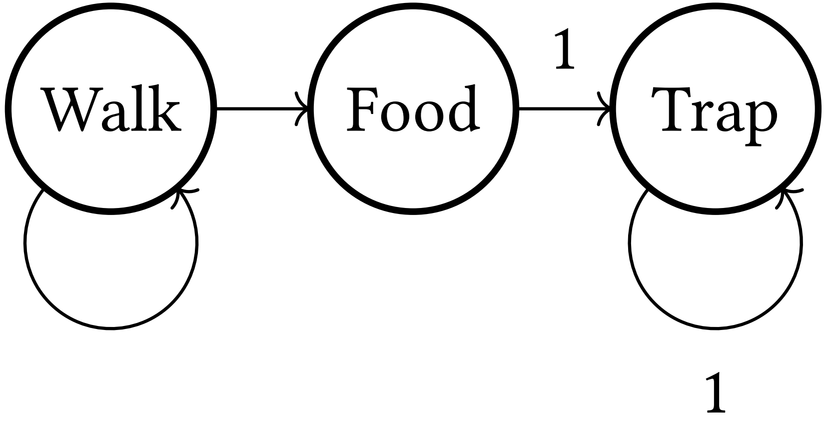

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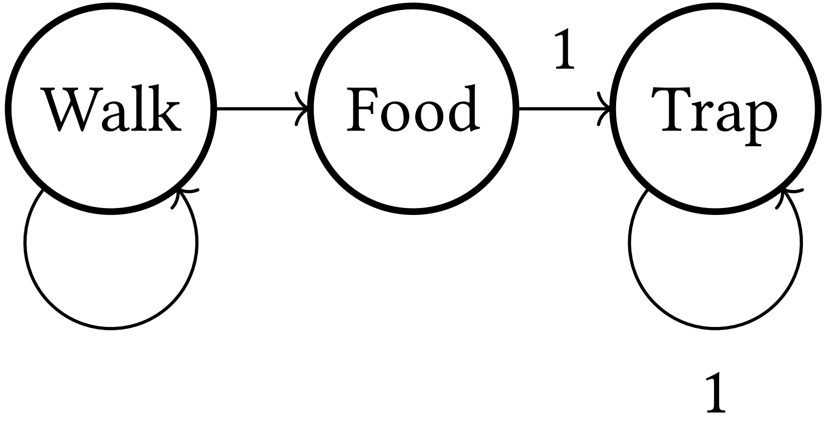

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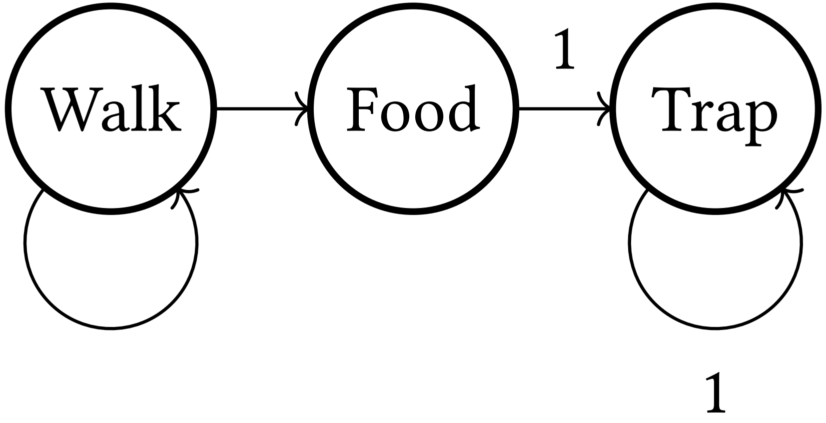

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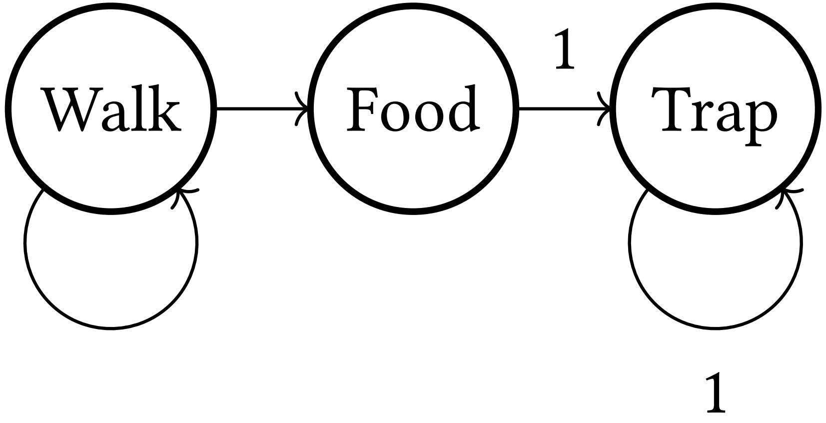

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Now, we make better decisions!

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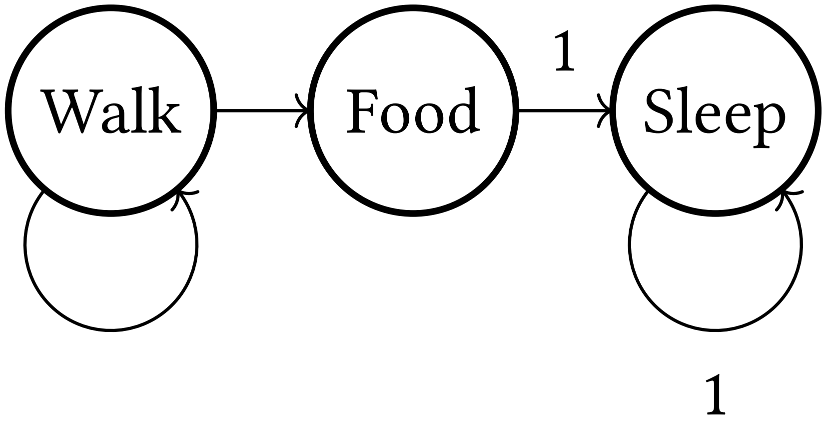

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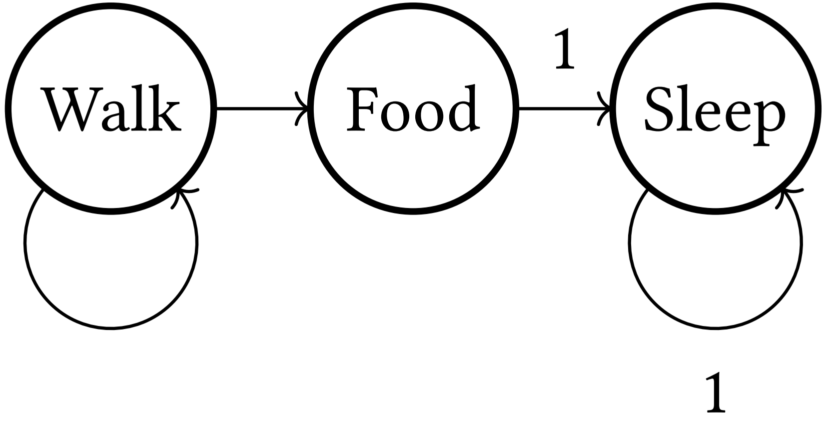
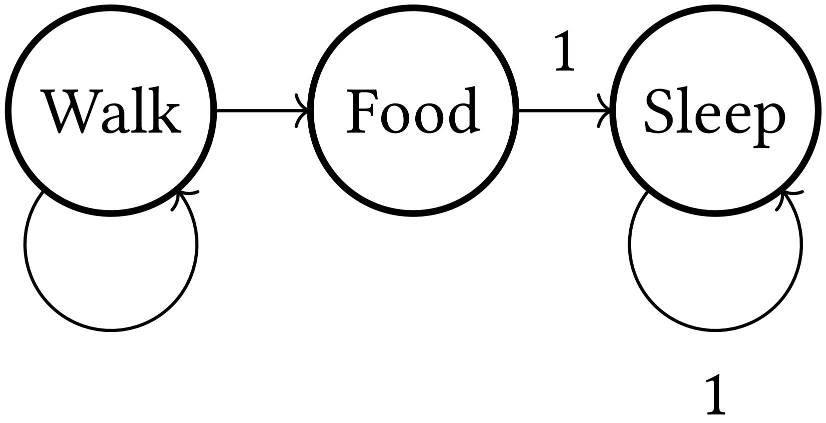


Question: What is the optimal sequence of states?

Markov Decision Processes

Consider one more example

$R(\text{walk}) = 0$ $R(\text{food}) = 3$ $R(\text{sleep}) = 0$



Question: What is the optimal sequence of states?

$$\text{Walk} + \text{Food} + \text{Sleep} + ... = 0 + 3 + 0 + ... = 3$$
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The return is an infinite sum

$$G = \sum_{t=0}^{\infty} R(s_t)$$

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Question: How?

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We can introduce a **discount** term $\gamma \in [0, 1]$ to the return

With $\gamma = 1$

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Markov Decision Processes

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Markov Decision Processes

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Thus, our objective is

$$\arg\max_{s \in S} G = \arg\max_{s \in S} \sum_{t=0}^{\infty} \gamma^t R(s_t)$$

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For the rest of the course, we will solve MDPs

Exercise

Reinforcement Learning

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Let us put everything together

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At each timestep we:

Reinforcement Learning

Let us put everything together

At each timestep we:

- Take an action a

Reinforcement Learning

Let us put everything together

At each timestep we:

- Take an action a
- Change states: $\Pr(s' \mid s, a)$

Reinforcement Learning

Understanding MDPs is the **most important part** of RL

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Existing software can train RL agents on your MDP

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Make sure you understand MDPs!

Reinforcement Learning

Markov decision process

$$(S, A, T, R, \gamma)$$

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