Value

CISC 7404 - Decision Making

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Review

Policy-Conditioned Returns

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Trajectory optimization is model-based algorithm

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Guaranteed optimal policy, given infinite compute

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Expensive to train, but very cheap to use

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Recall the return from trajectory optimization

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$$[\mathcal{G}(\boldsymbol{\tau}) \mid s_0, a_0, a_1, ...] = \sum_{t=0}^{\infty} \gamma^t \mathbb{E}[\mathcal{R}(s_{t+1}) \mid s_0, a_0, a_1, ...]$$

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- Random
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- Maximize $\mathcal{G}$

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$$\pi : S \times \Theta \mapsto \Delta A$$

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Example policy, greedy policy

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$$\pi(a_t \mid s_t; \theta_\pi) = \begin{cases} 1 \text{ if } a_t = \arg\max_{a_t \in A} \mathbb{E}[\mathcal{G}(\boldsymbol{\tau}) \mid s_0, a_0, a_1, ...] \\ 0 \text{ otherwise} \end{cases}$$

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Must construct and evaluate decision tree at each timestep!

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Conditioning the return on actions is annoying

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Must compute infinitely many actions and outcomes for the return

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What if we condition on a policy, instead of specific actions?

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Condition on distribution parameterized by θ_π instead of many actions

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Remember, $\pi(a \mid s; \theta_\pi)$ provides a distribution over the action space

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How does $\mathbb{E}[\mathcal{R}(s_{t+1})]$ change when we condition on θ_π?

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Question: What changes when we condition on θ_π?

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Answer: State transition function

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Combine the policy distribution with next state distribution

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Write out the first few timesteps

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Write out the first few timesteps

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Derive a general form for $\Pr(s_{n+1} \mid s_0; \theta_\pi)$

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$$\Pr(s_{n+1} \mid s_0; \theta_\pi) = \sum_{s_1, \dots, s_n \in S} \prod_{t=0}^{n} \left(\sum_{a_t \in A} \mathrm{Tr}(s_{t+1} \mid s_t, a_t) \cdot \pi(a_t \mid s_t; \theta) \right)$$

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Plug back into our expected reward

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$$\mathbb{E}[\mathcal{R}(s_{t+1}) \mid s_0; \theta_\pi] = \sum_{s_{t+1} \in S} \mathcal{R}(s_{t+1}) \cdot \Pr(s_{t+1} \mid s_0; \theta_\pi)$$

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Need to plug expected reward back into expected discounted return

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- DQN
- DDPG/SAC
- A3C/PPO/GRPO

Goal: find the θ_π (policy parameters) to maximize the expected return

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It is a critical part of decision making

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We can compute

$$V(s_0 = S_a, \theta_\pi), V(s_0 = S_b, \theta_\pi), V(s_0 = S_c, \theta_\pi)$$

To find the value of any state $S_a, S_b, S_c, ...$

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$$V(s_0, \theta_\pi) = \sum_{s_g \in \{S_x, S_y, S_z\}} \gamma^{t_g} \mathcal{R}(s_g) \cdot \Pr(s_g \mid s_0; \theta_\pi)$$

Where should you live?

TD Value Functions

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Replace infinite sum with value function

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Compute the return with a single transition $s_0 \to s_1$

Evaluate infinite-depth decision tree with one function

TD Value Functions

To summarize, we can represent the value function in two ways:

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They produce the same result, but with different computation

Q Functions

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We call the modified value function, a Q function

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Consider the Temporal Difference value function

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What if we wanted a mix of both?

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We can derive the Q function from the value function

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Q Functions

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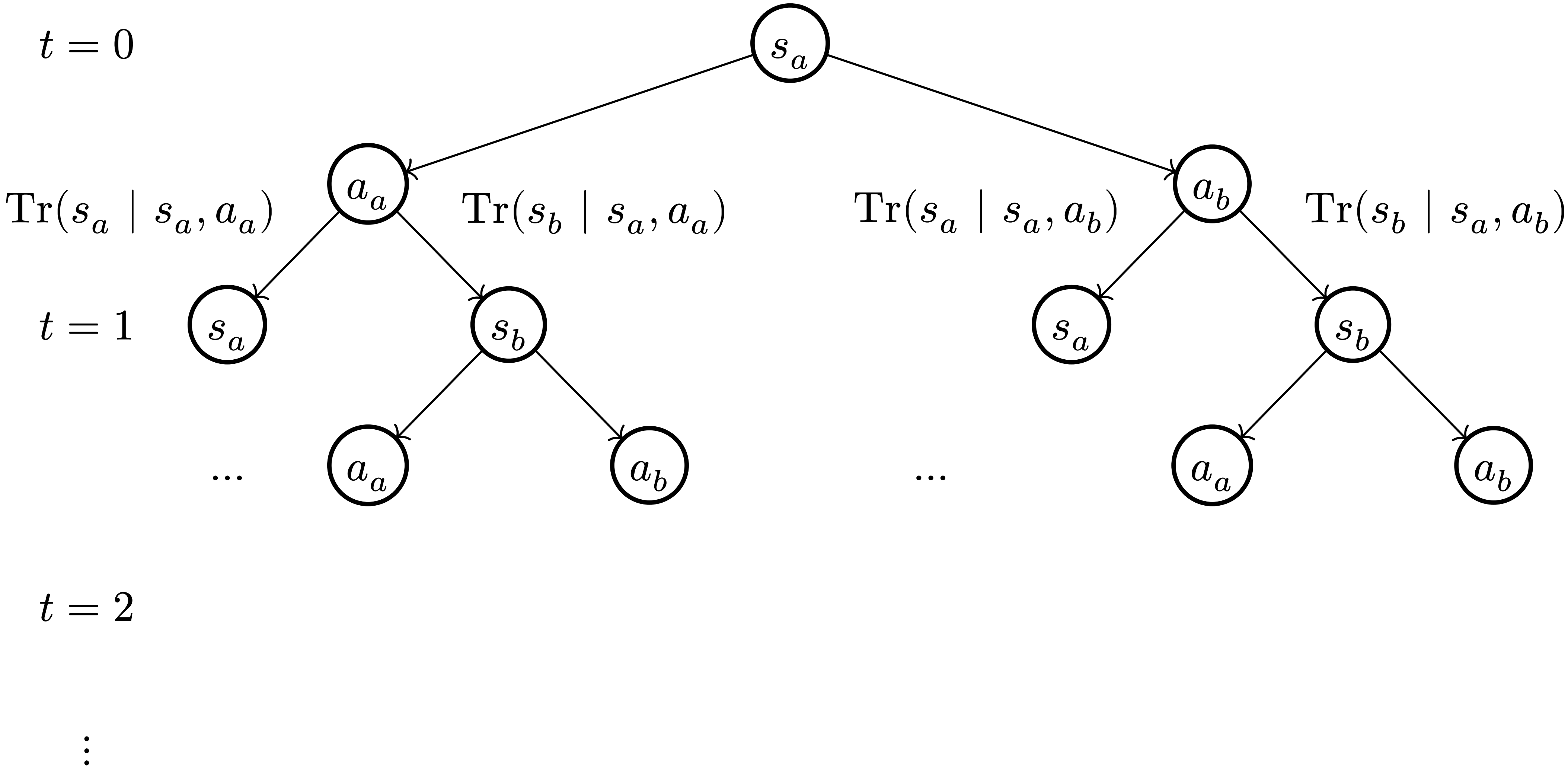
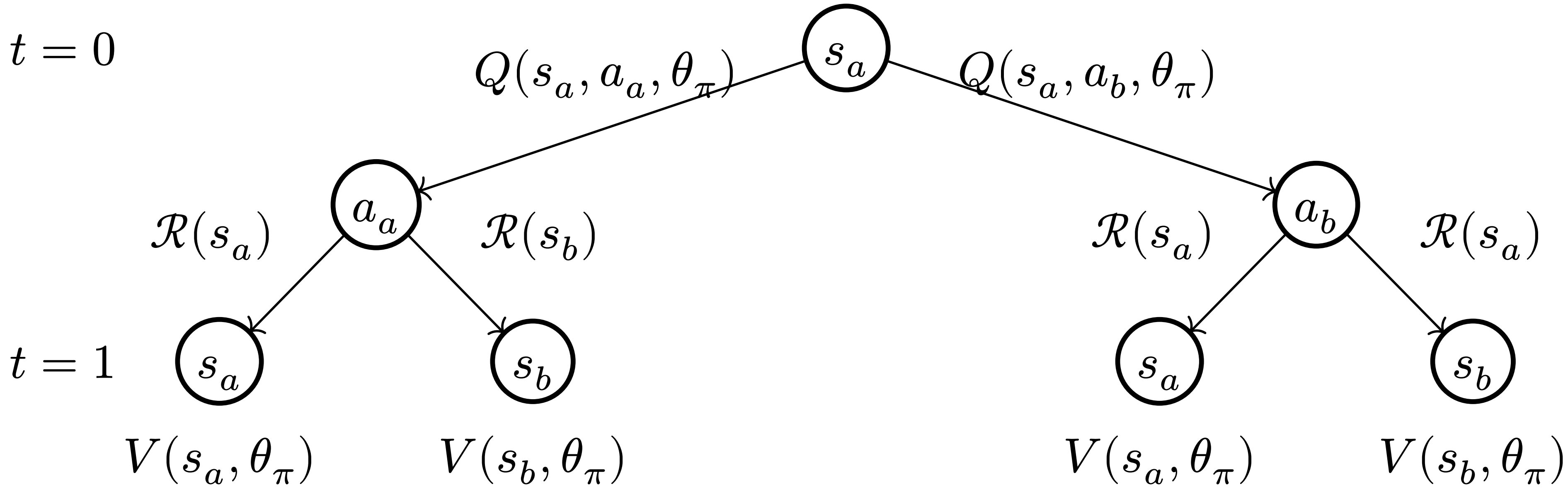
We collapsed the infinite decision tree into a single level

Q Functions


$t = 0$
s_a
a_a
a_b
$\mathrm{Tr}(s_a \mid s_a, a_a)$
$\mathrm{Tr}(s_b \mid s_a, a_a)$
$\mathrm{Tr}(s_a \mid s_a, a_b)$
$\mathrm{Tr}(s_b \mid s_a, a_b)$
$t = 1$
s_a
s_b
s_a
s_b
...
a_a
a_b
...
a_a
a_b
$t = 2$
$\vdots$

Q Functions



$t = 0$
s_a
$Q(s_a, a_a, \theta_\pi)$
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a_a
a_b
$\mathcal{R}(s_a)$
$\mathcal{R}(s_b)$
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We now have all the information we need to implement Q learning

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Let us find out

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$$Q(s_0, a_0, \theta_\pi) = \mathbb{E}[\mathcal{R}(s_1) \mid s_0, a_0] + \underbrace{\boxed{\sum_{t=1}^{\infty} \gamma^t \mathbb{E}[\mathcal{R}(s_{t+1}) \mid s_1; \theta_\pi]}}_{\text{Return following } \pi}$$

Q Learning

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Can run policy in environment to create episodes

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Can run policy in environment to create episodes

```
states, next_states, rewards, terminateds = [], [], [], []
state = environment.reset()
while not terminated:
    action = policy.sample(state)
    next_state, reward, terminated = environment.step(action)

    states.append(state), next_states.append(next_state), ...
    state = next_state

episode = (states, next_states, rewards, terminateds)
```

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What policy do we sample actions from?

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In the limit, we sample all possible actions in all states

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Navigation example, reward of 1 for reaching center tile

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https://mohitmayank.com/interactive_q_learning/q_learning.html

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- Defined training objective (TD and MC updates)

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Q Learning

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- Defined training objective (TD and MC updates)
- Defined dataset (episodes)
- Need to define model (Q function)!

Q Learning

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Today and for homework, use a simple matrix

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$$\begin{bmatrix} Q(S_1, A_1) & Q(S_1, A_2) & \dots \\ Q(S_2, A_1) & Q(S_2, A_2) & \dots \\ \vdots & \vdots & \ddots \end{bmatrix}$$

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$Q_{i,j}$ gives Q value for state $s = S_i$ and action $a = A_j$

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