# Decision Processes

## CISC 7404 - Decision Making

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# Review

# Markov Processes

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- History

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$$s_{t+1} \sim \mathrm{Tr}(\cdot \mid s_t)$$

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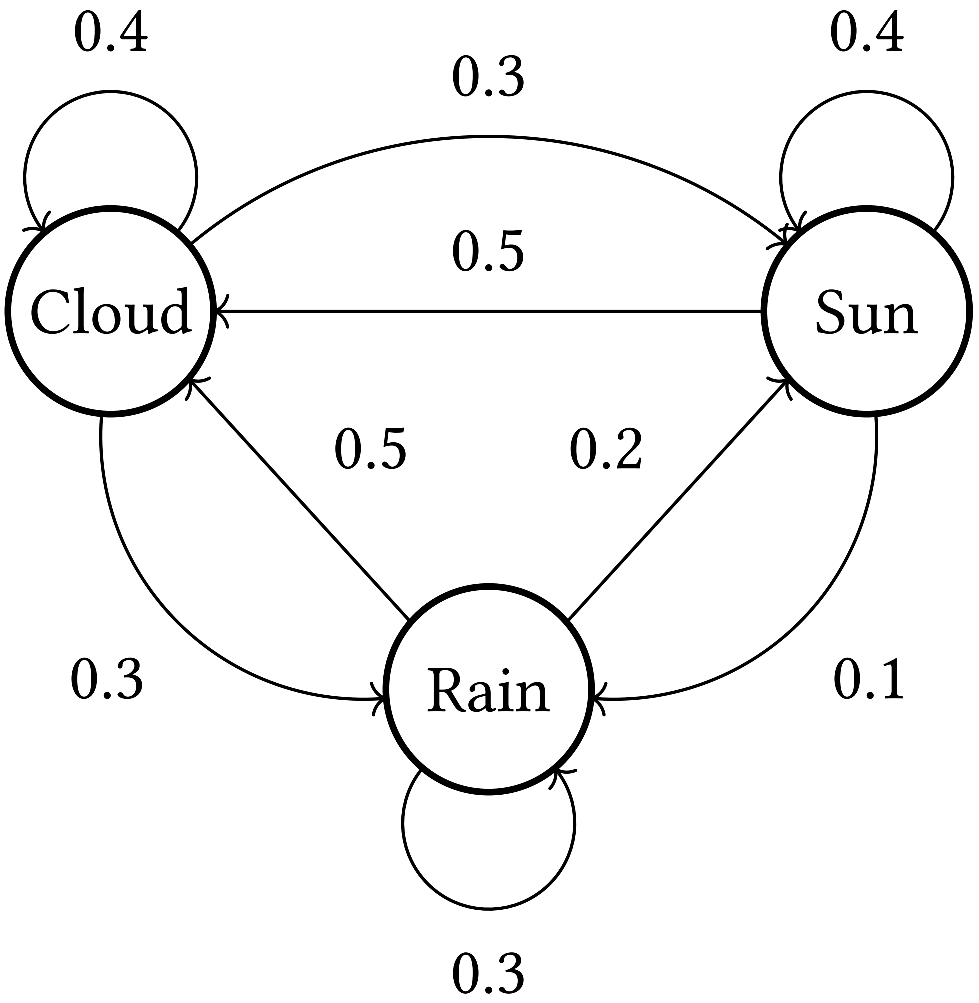

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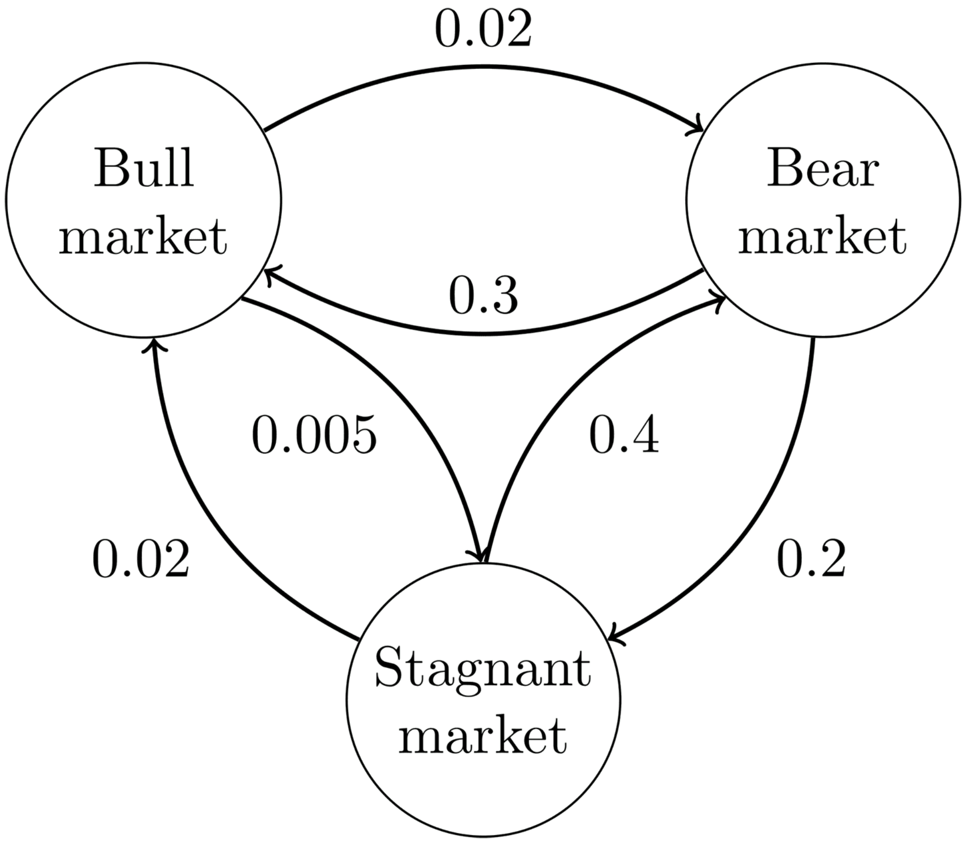

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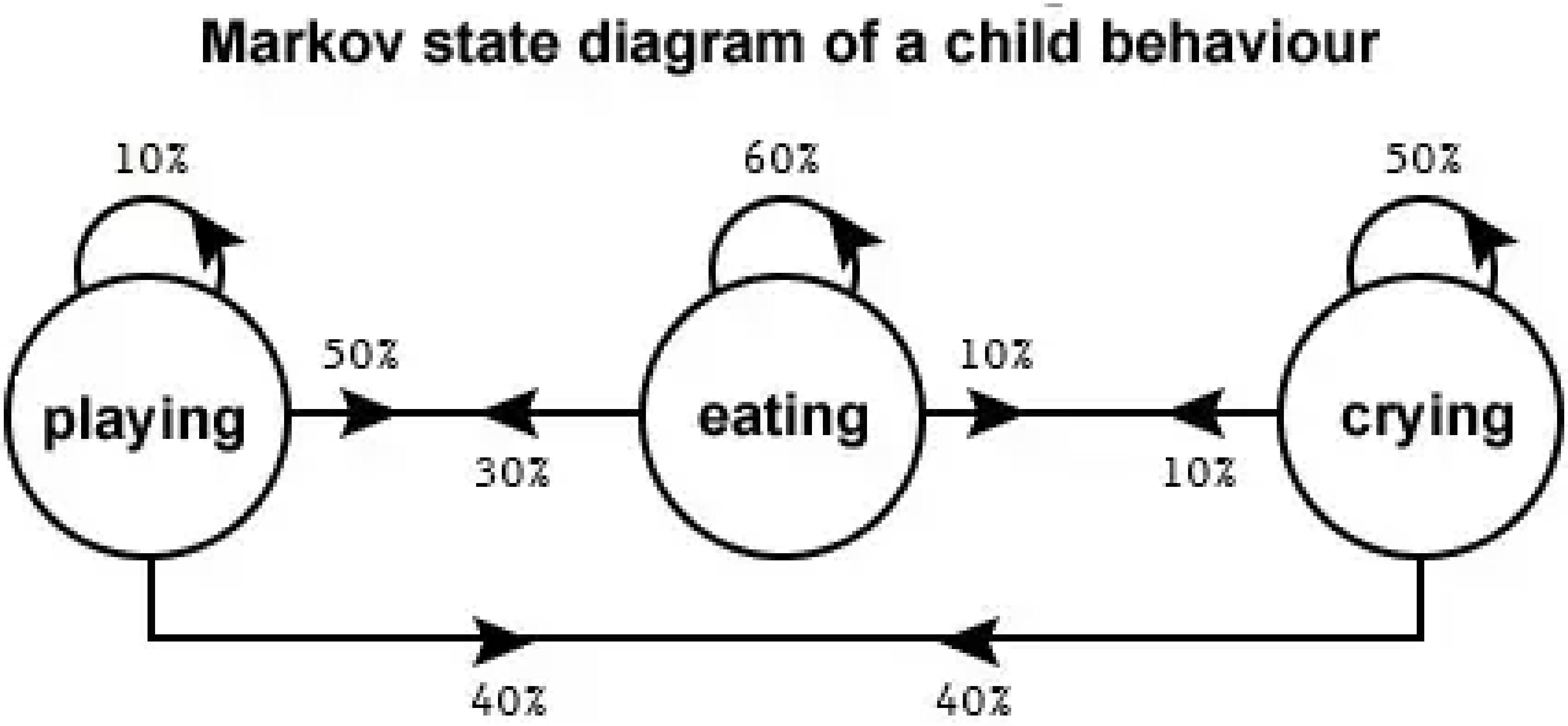

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$0.3 \neq 0.4, \Pr(s_{t+1} \mid s_t) \neq \Pr, (s_{t+1} \mid s_t, s_{t-1}, ..., s_0)$, **not** Markov

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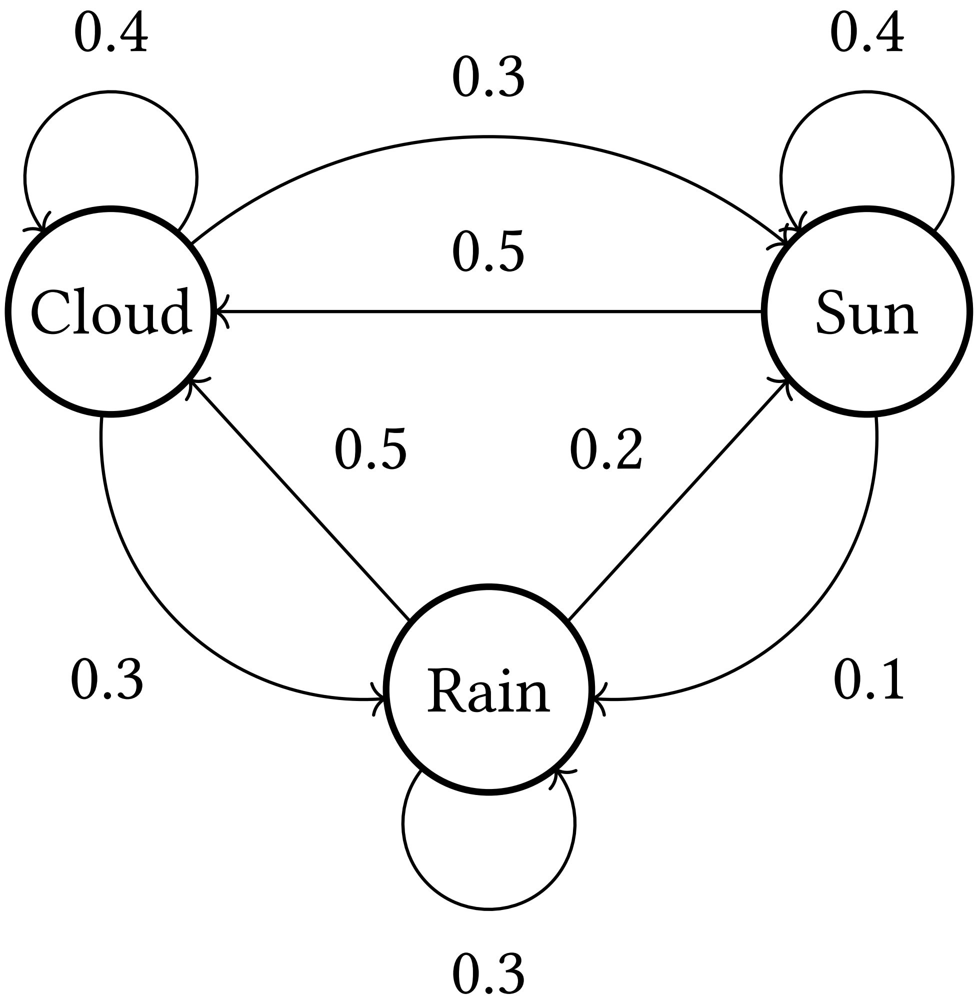

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Can we derive a general form for $P(s_n \mid s_0)$?

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Generalize to any timestep $n$

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If $s$ represents someone's mind, you can predict their future thoughts

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**Question:** How can we model a Markov process that ends?

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**Question:** How can we model a Markov process that ends?

**Answer:** We create a **terminal state** that we can enter but cannot leave

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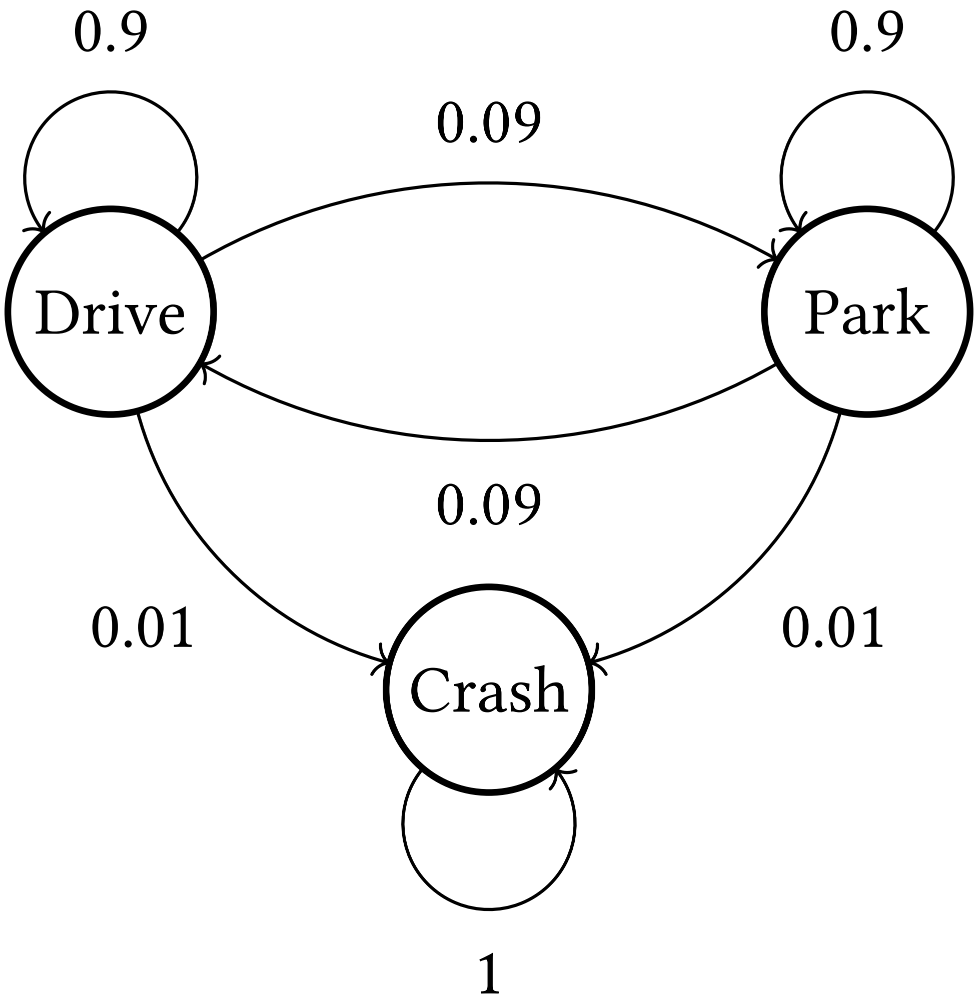

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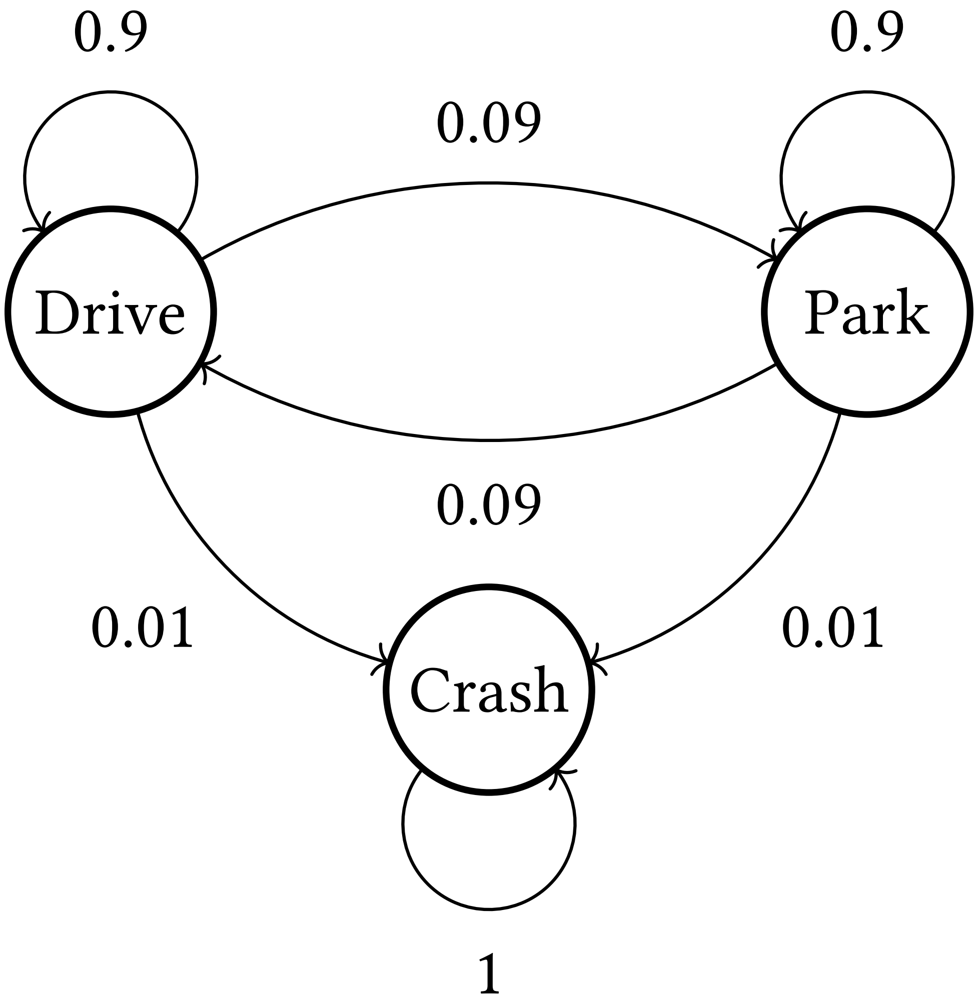


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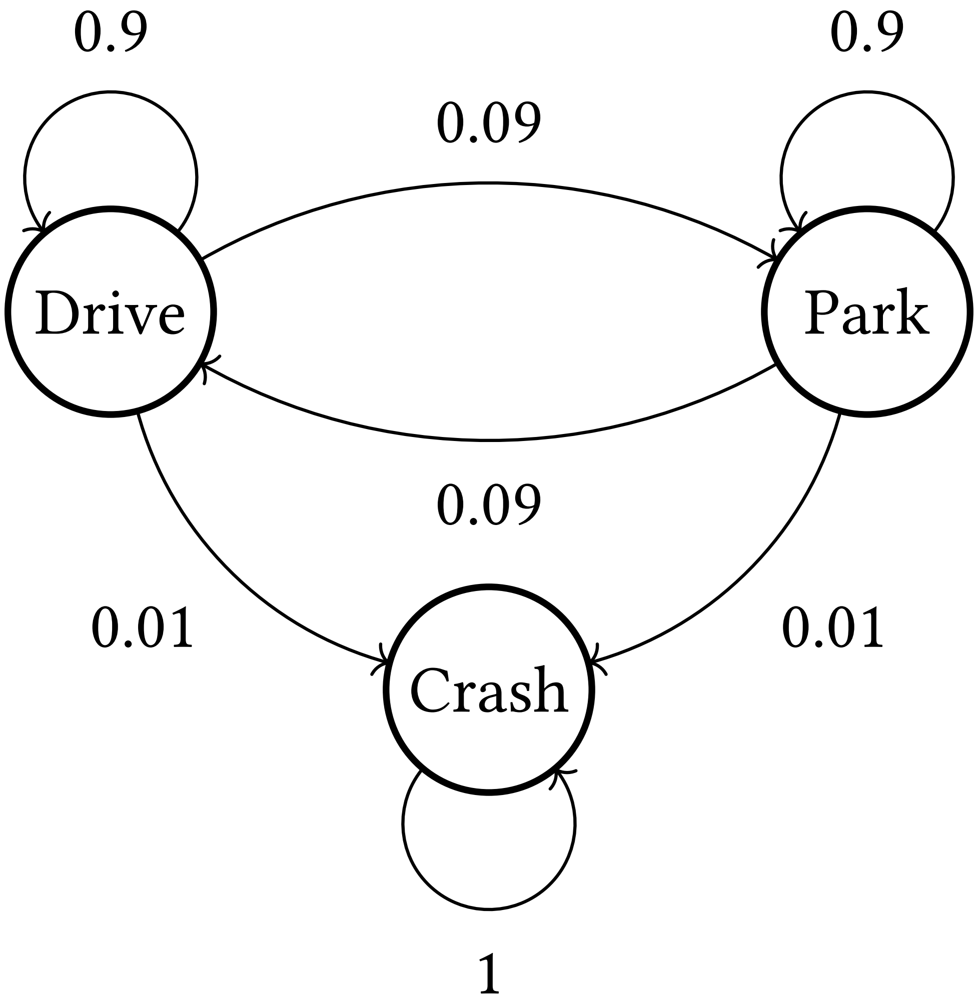


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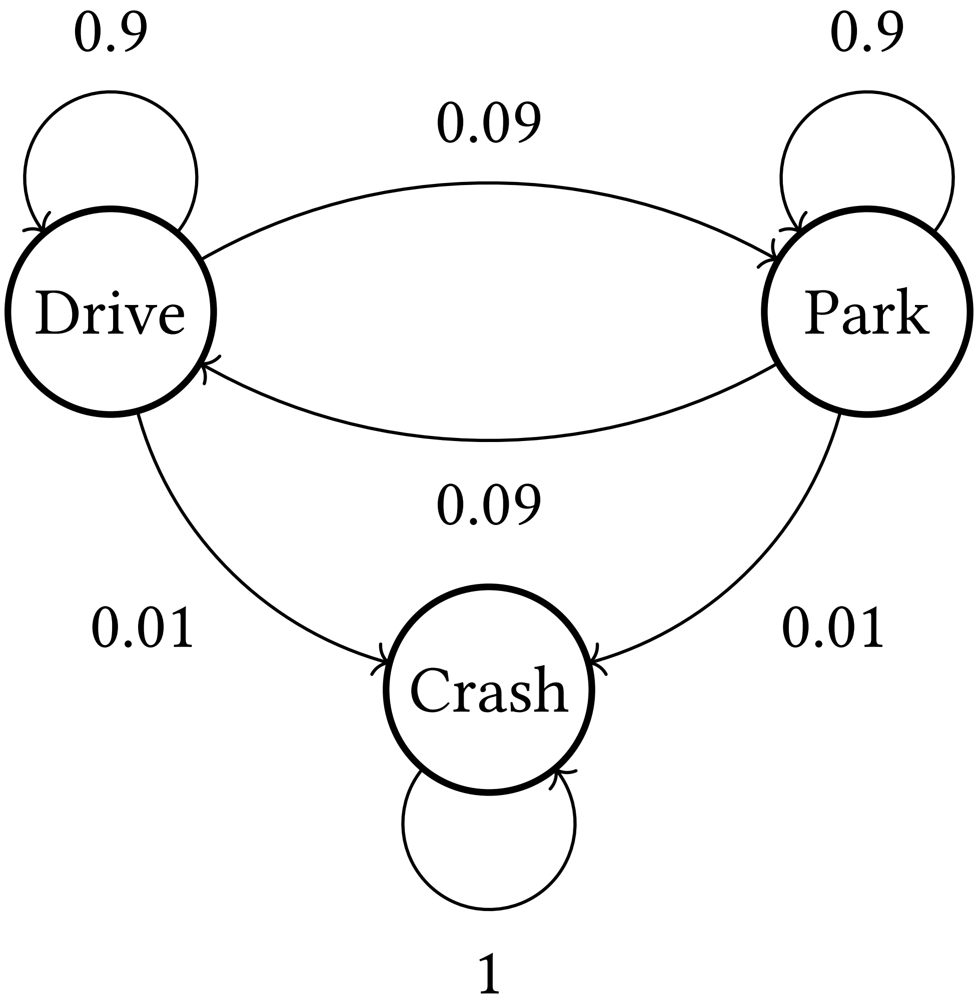


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The only transition from a terminal state is back to itself

$$\Pr(s_{t+1} = \text{term} \mid s_t = \text{term}) = 1$$
$$\Pr(s_{t+1} = \text{not term} \mid s_t = \text{term}) = 0$$

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The point of decision making is to choose our fate

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The agent changes the environment with its decisions

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In a Markov process, the future follows a predefined evolution

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The agent takes **actions** $a \in A$ that change the environment

The action space $A$ defines what our agent can do rrr

| Markov process | Markov control process |
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Let us see an example

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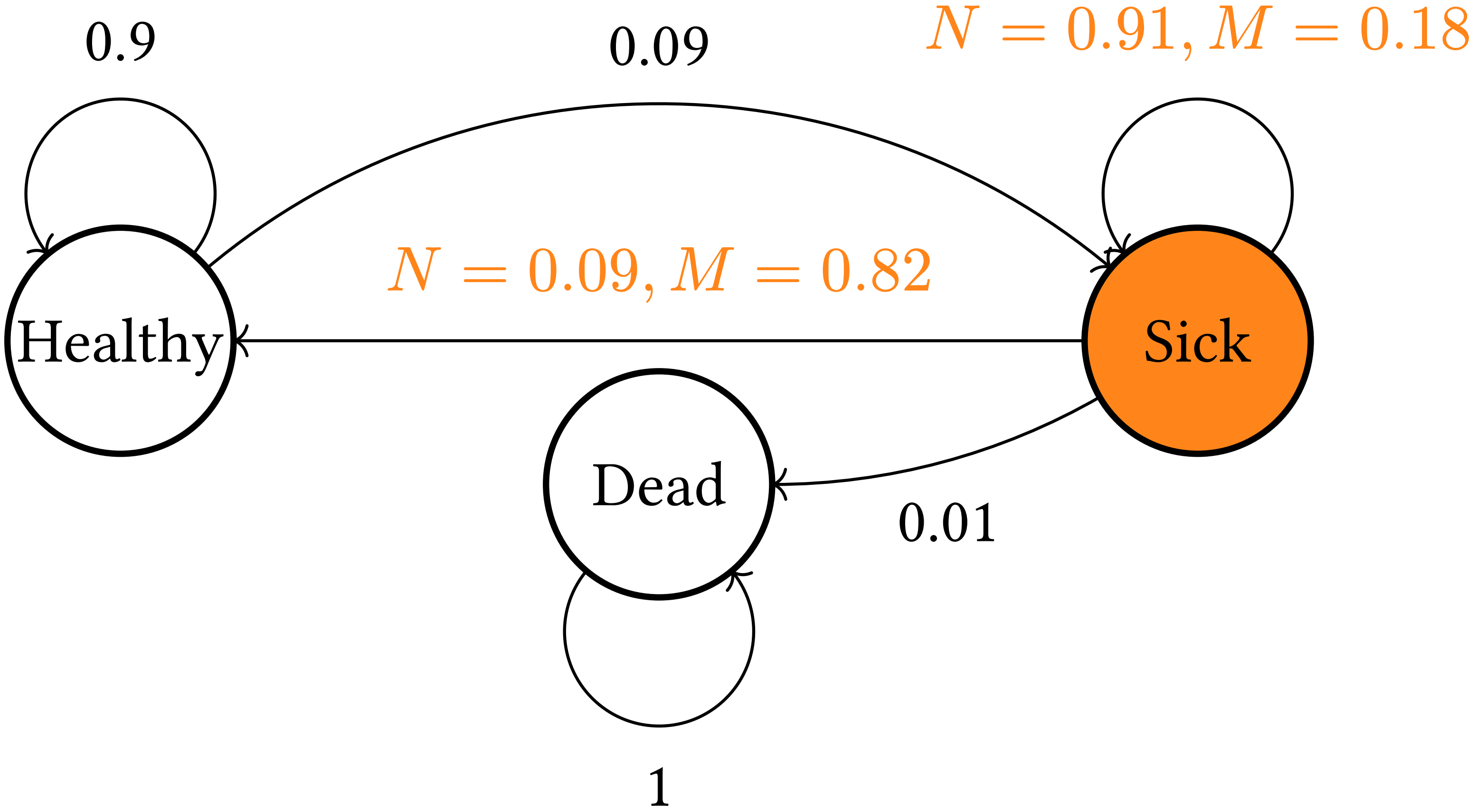

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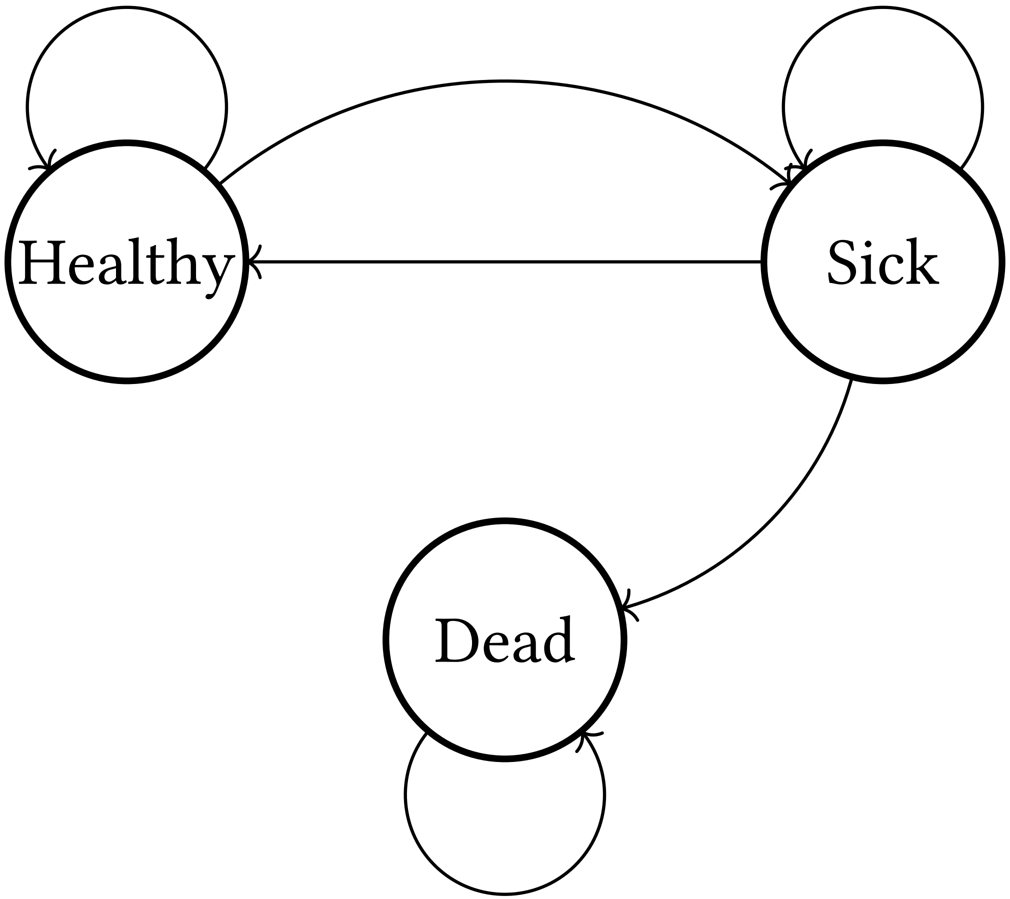

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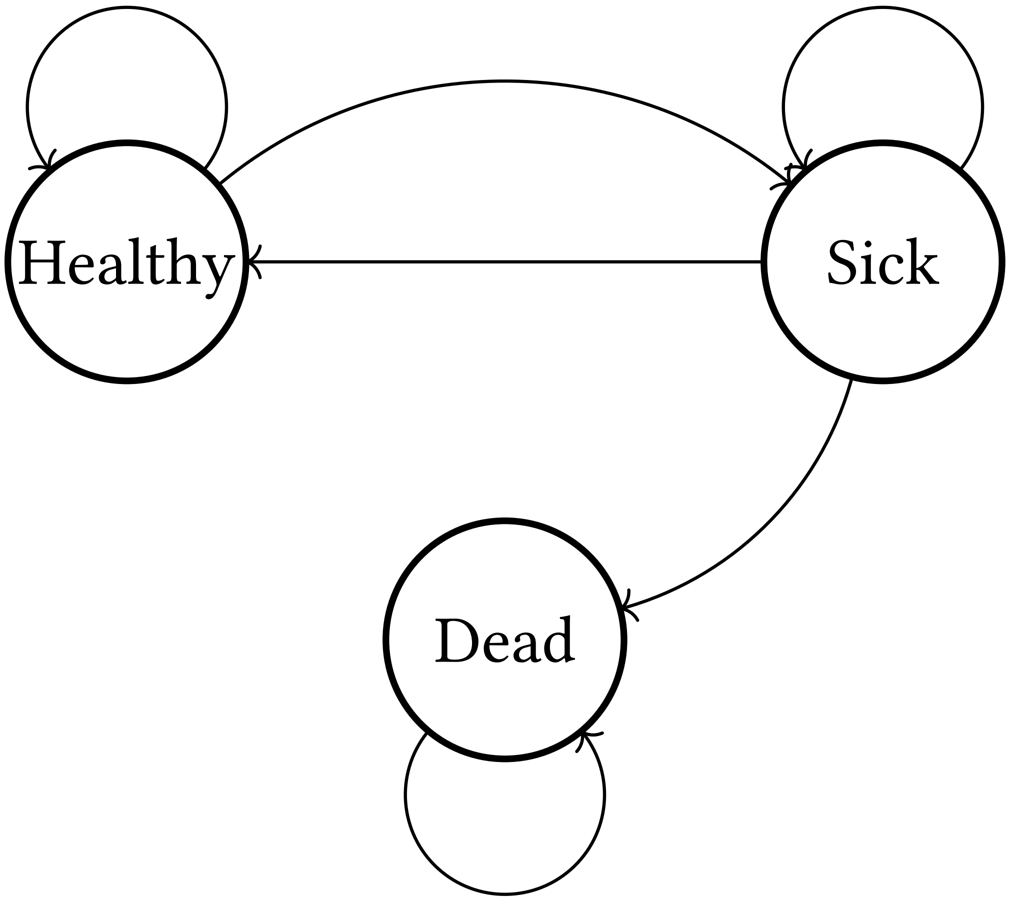


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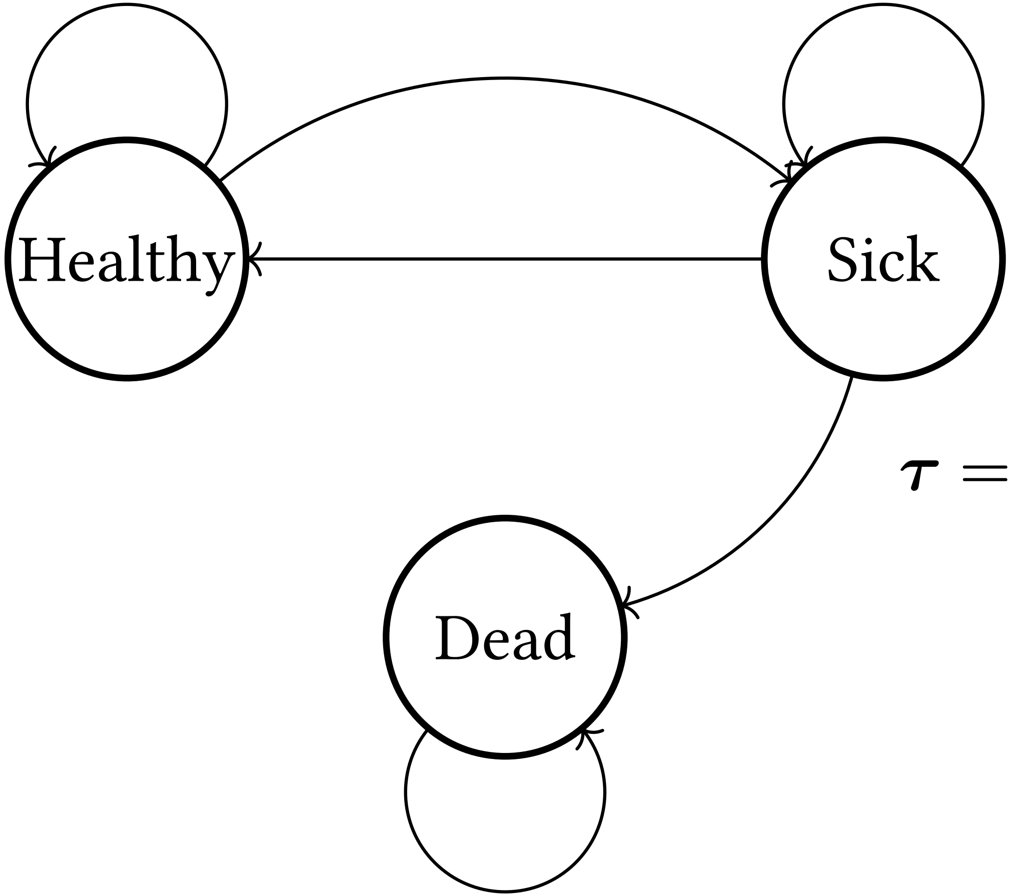


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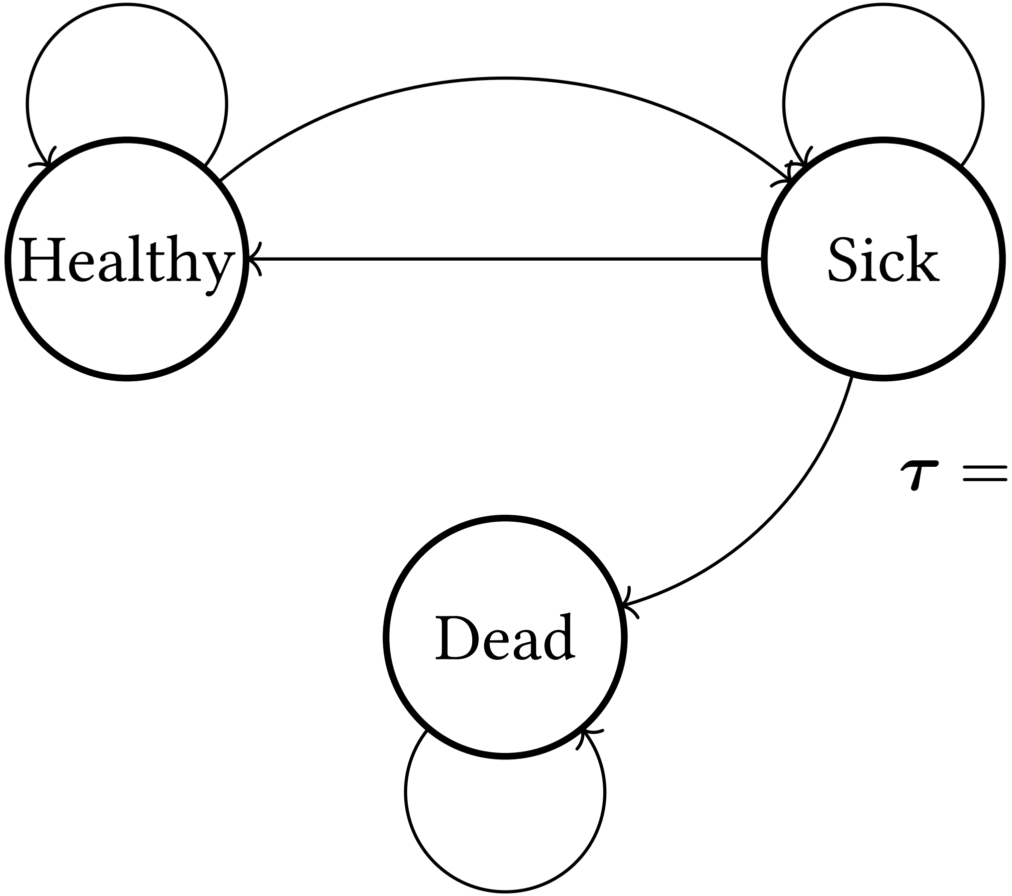


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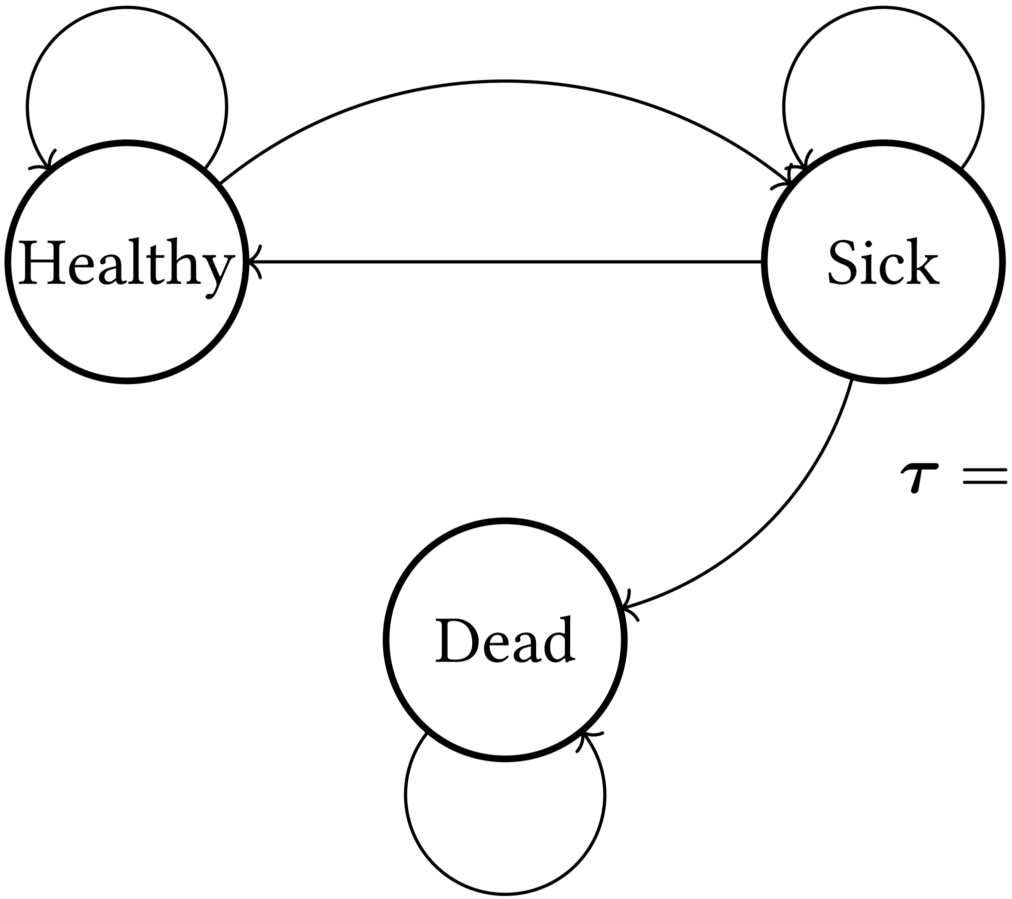


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If there is no terminal state, the trajectory can be infinitely long!

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We need a way to determine “good” and “bad” decisions

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You can always make these equivalent by modifying the MDP

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$$\boldsymbol{E} = \begin{bmatrix} s_0 & a_0 & r_0 \\ s_1 & a_1 & r_1 \\ \vdots & \vdots & \vdots \\ s_{n-1} & a_{n-1} & r_{n-1} \\ s_n & \emptyset & \emptyset \end{bmatrix} = [\boldsymbol{\tau} \;\; \boldsymbol{r}]$$

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We can write this mathematically as

$$\arg\max_{s \in S} R(s)$$

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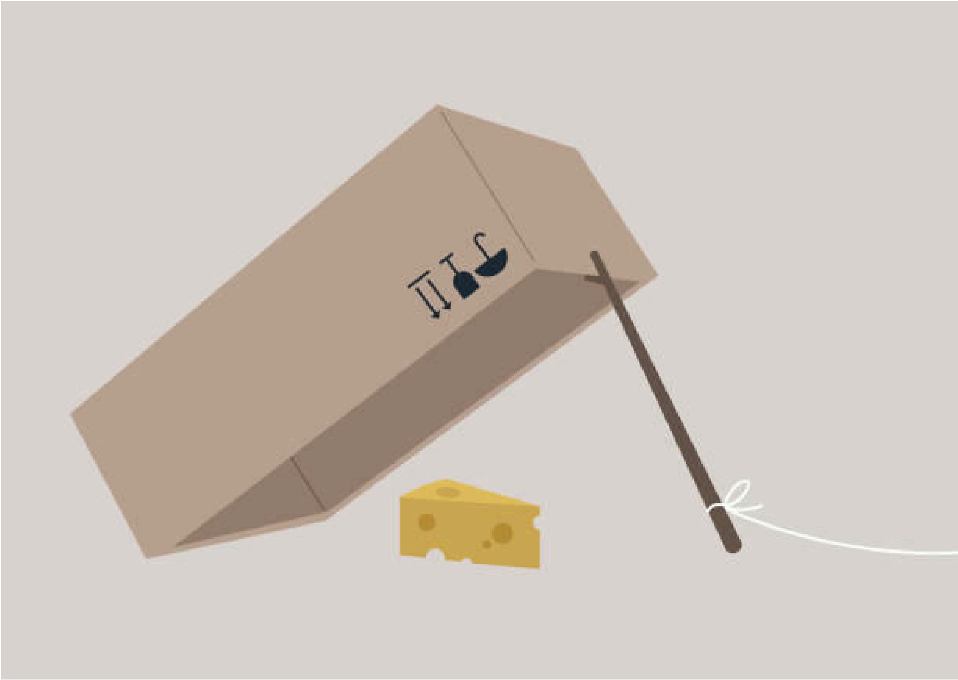

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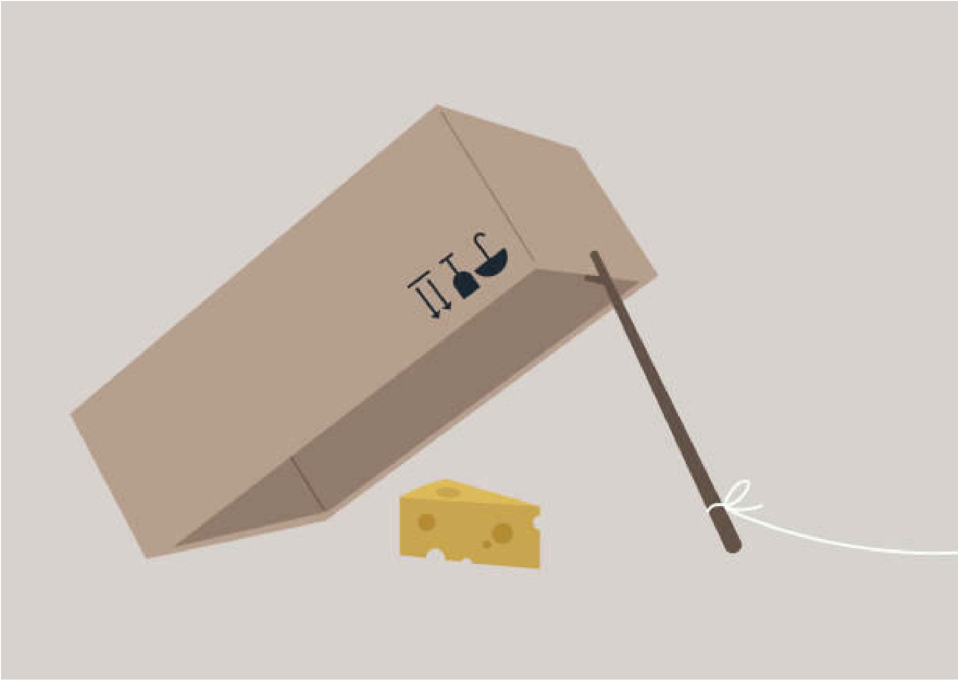

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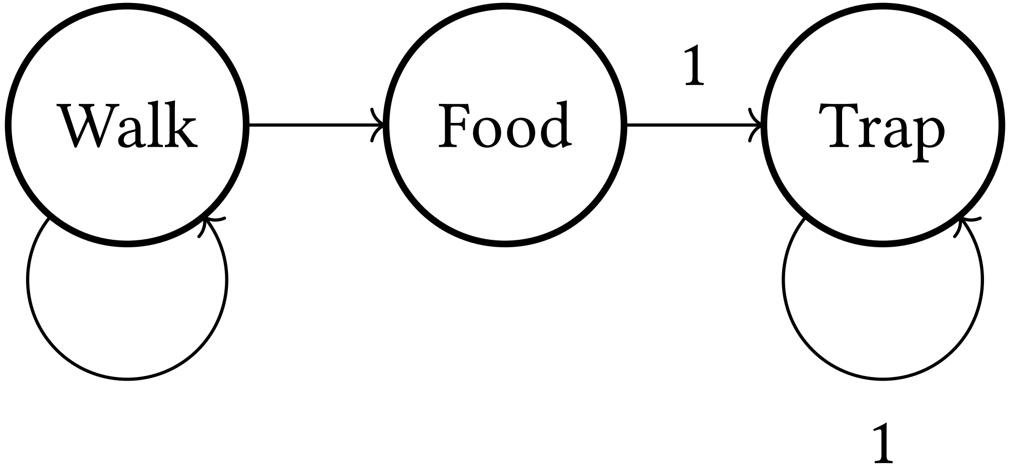

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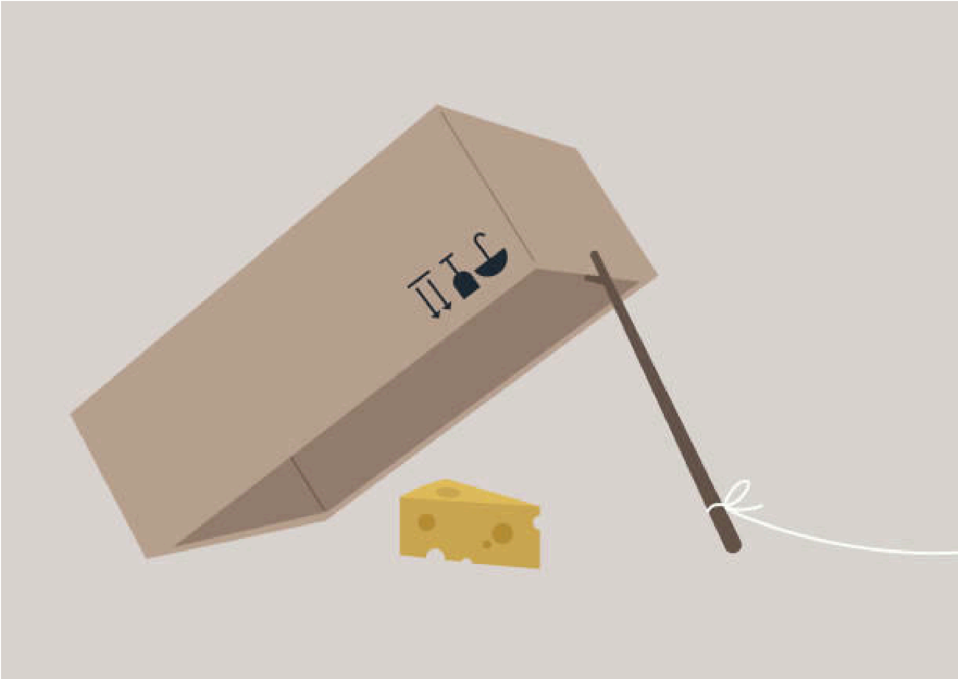

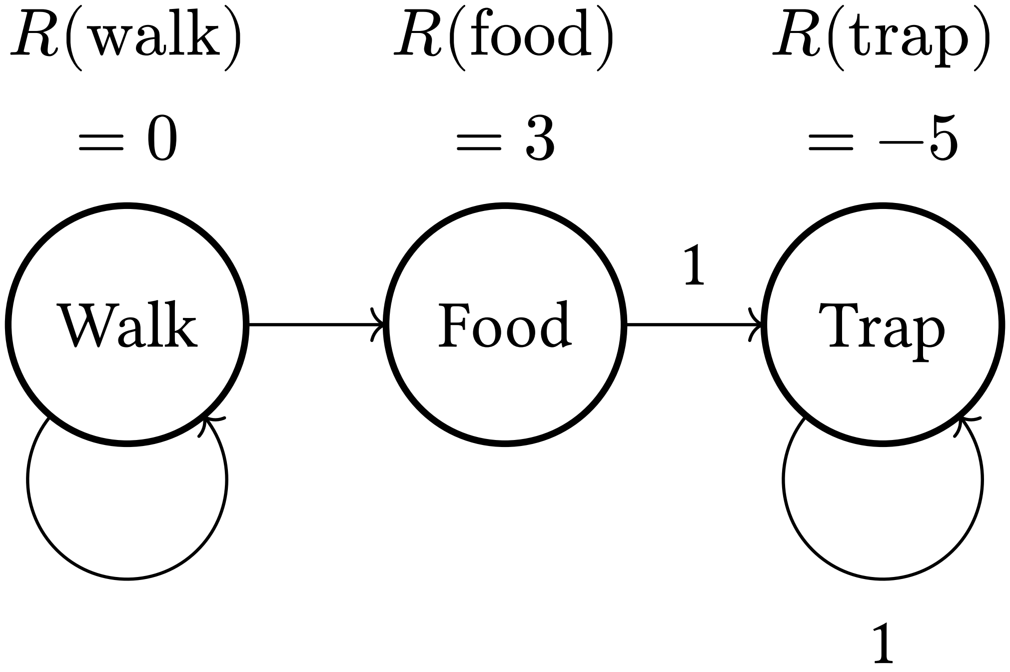


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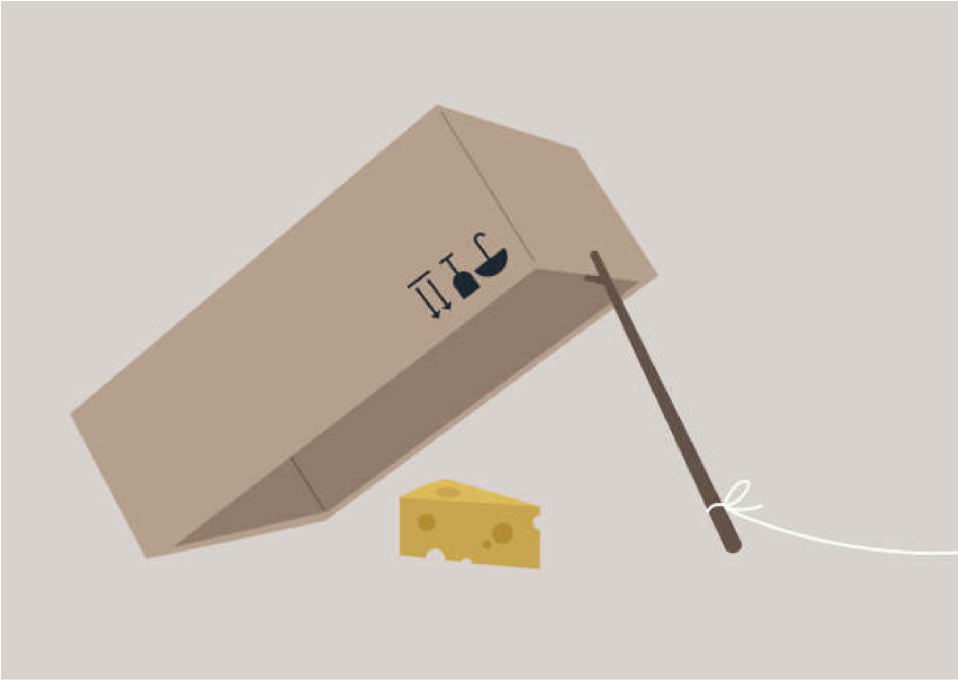

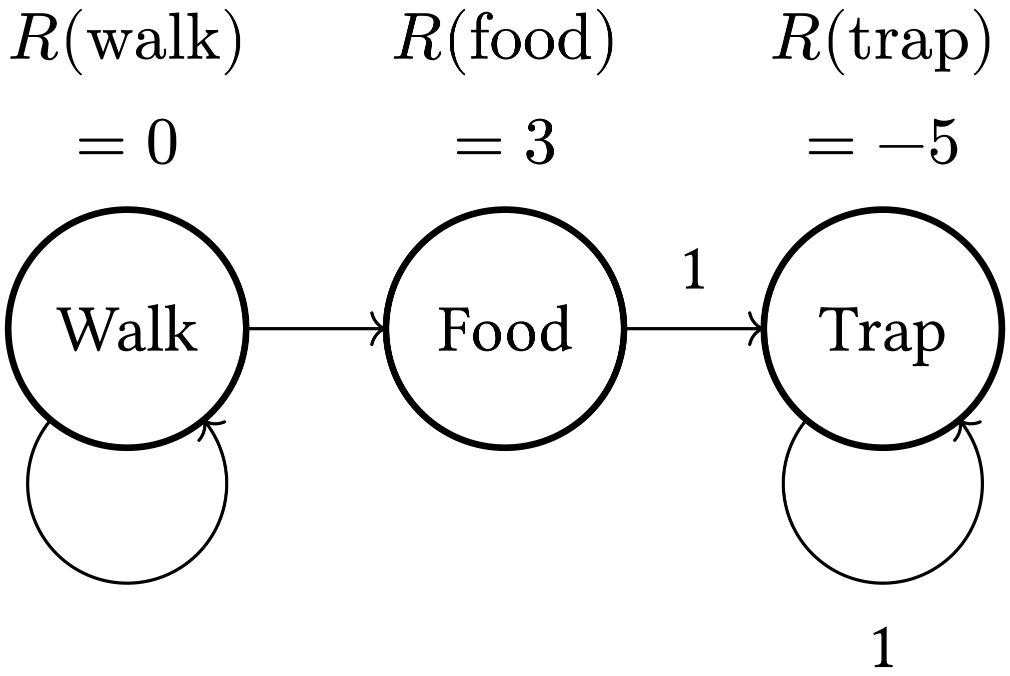


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If we maximize the reward, we are **too greedy**

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$$\begin{aligned} G(\boldsymbol{\tau}) &= G(s_0, a_0, s_1, a_1, ...) \\ &= \sum_{t=0}^{\infty} R(s_{t+1}) \end{aligned}$$

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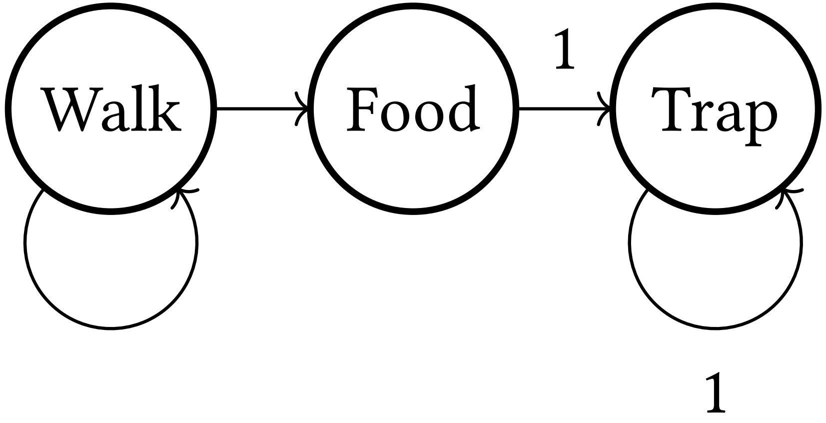


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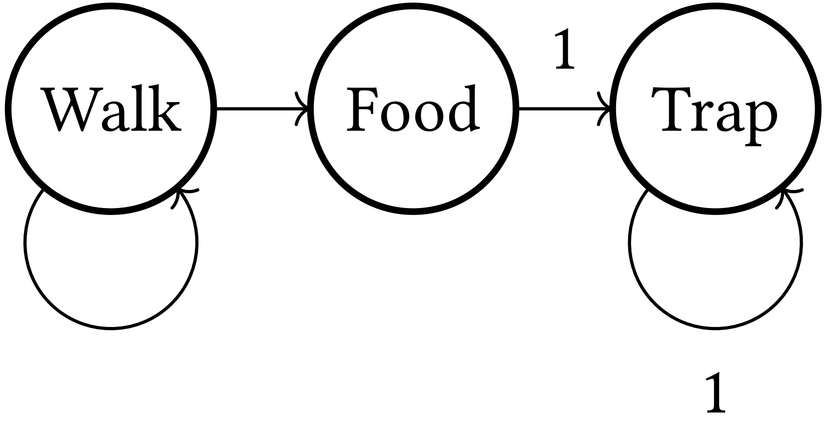


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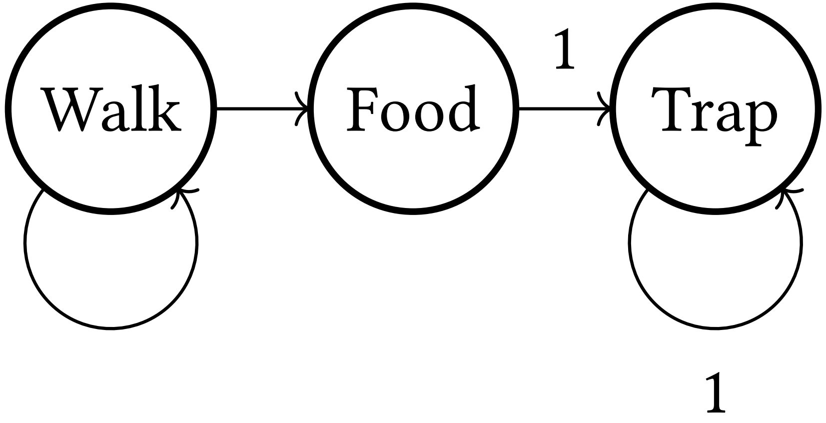


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By considering the future rewards, we can make optimal decisions

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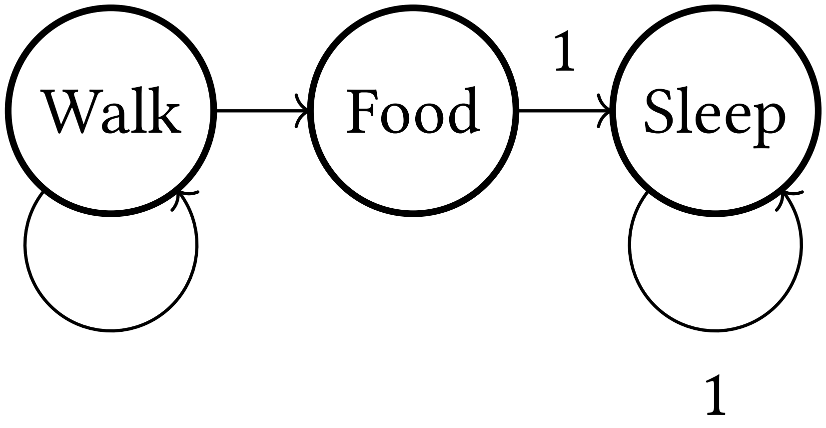

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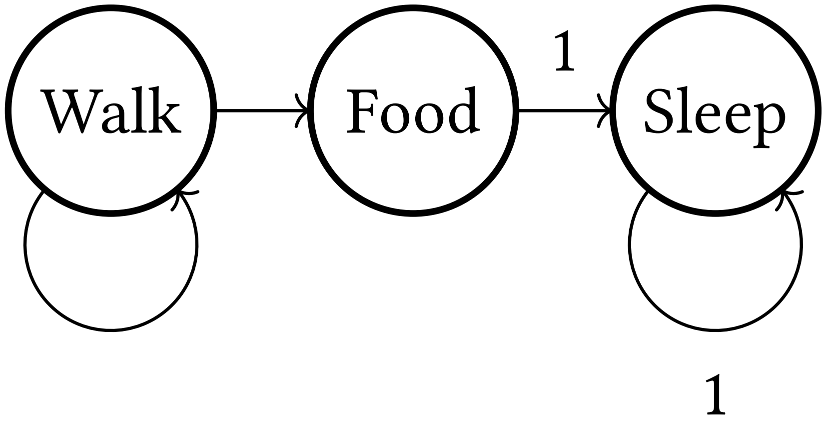


**Question:** What is the optimal sequence of states?

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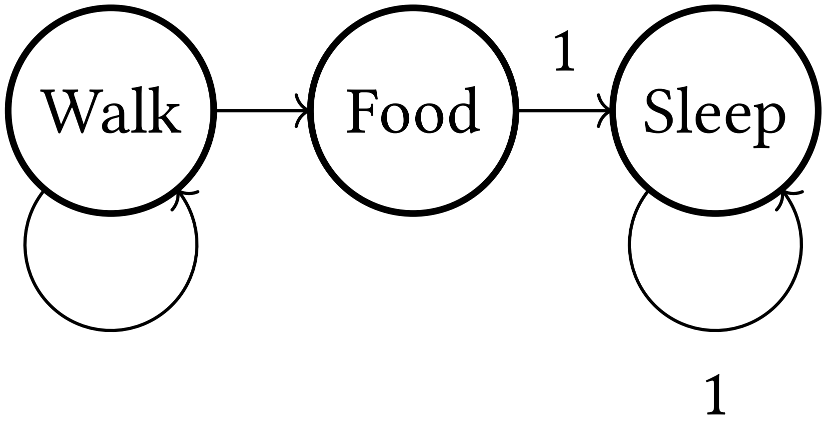


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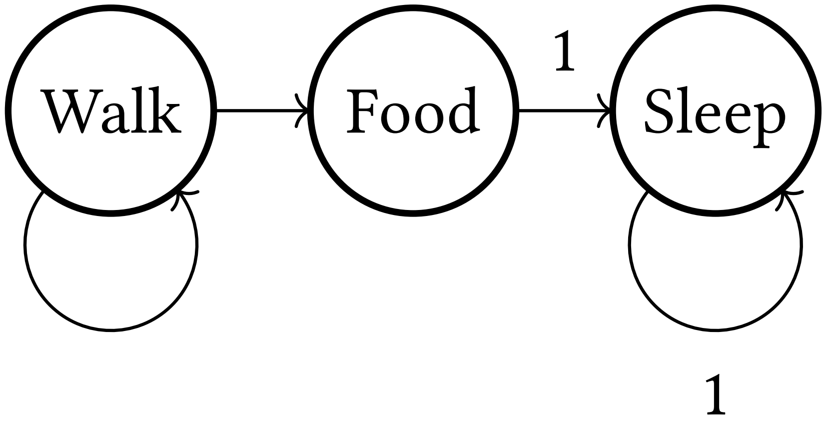


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The discounted return lets makes us prefer rewards sooner, like humans

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For the rest of the course, we maximize the discounted return

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If our agent maximizes the discounted return, then it is **optimal**

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You must understand the discounted return!

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Make sure you understand MDPs!

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Design a Super Mario Bros MDP

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Compute discounted return for:

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- Die to bowser at $t = 20$
- Game over screen at $t = 21...\infty$

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https://gymnasium.farama.org/api/env/

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Gymnasium uses observations, but for MDPs we treat them as states

# Coding

```
import gymnasium as gym

MyMDP(gym.Env):
  def __init__(self):
    self.action_space = gym.spaces.Discrete(3) # A
    self.observation_space = gym.spaces.Discrete(5) # S

  def reset(self, seed=None) -> Tuple[Observation, Dict]

  def step(self, action) -> Tuple[
    Observation, Reward, Terminated, Truncated, Dict
  ]
```

# Coding

https://colab.research.google.com/drive/1rDNik5oRl27si8wdtMLE7Y41U5J2bx-I#scrollTo=9pOLI5OgKvoE

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Too many A’s last term, exam will be **difficult**