Policy Gradient

CISC 7404 - Decision Making

Steven Morad

University of Macau

Admin

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Homework 1 was due yesterday 23:59

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We are about 50% finished with the course

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Any opinions/feedback on the course?
- Can also talk after class
- Or email smorad at um.edu.mo

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If you want full participation marks, you must participate in lecture

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Right now, the following students have full participation marks:

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Right now, the following students have full participation marks:

- LIU KEJIA
- LIU HUANRONG
- HOI HOU HONG
- CHEN ZELAI
- WANG ZEKANG
- HE ZHE
- WANG MENGQI
- ZHANG BORONG
- HE ENHAO
- QIAO YULIN
- YAO CHENYU
- KAM KA HOU

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Ask questions at office hours

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A cute video of trajectory optimization

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https://www.youtube.com/watch?v=tudxHzZ5_ls

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http://www.incompleteideas.net/IncIdeas/BitterLesson.html

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Policy gradient can change pretrained model parameters

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$$\pi(a_t \mid s_t; \theta_\pi) = \begin{cases} 1 \text{ if } a_t = \arg\max_{a \in A} Q(s_t, a, \theta_\pi) \\ 0 \text{ otherwise} \end{cases}$$

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So why do we need a new algorithm?

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Example: Consider a Unitree BenBen, with 12 joints

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To learn to motion, we must learn actions for all joints $A \in [0, 2\pi]^{12}$

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Answer: No, $\arg\max_{a \in A}$, but A is infinite. How can we take $\arg\max$ over an infinite set?

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Does that sound impossible?

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We can improve the action distribution over time

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Question: How should we change θ_π?

Answer: Change θ_π so we reach good $s \in S$, making the return larger

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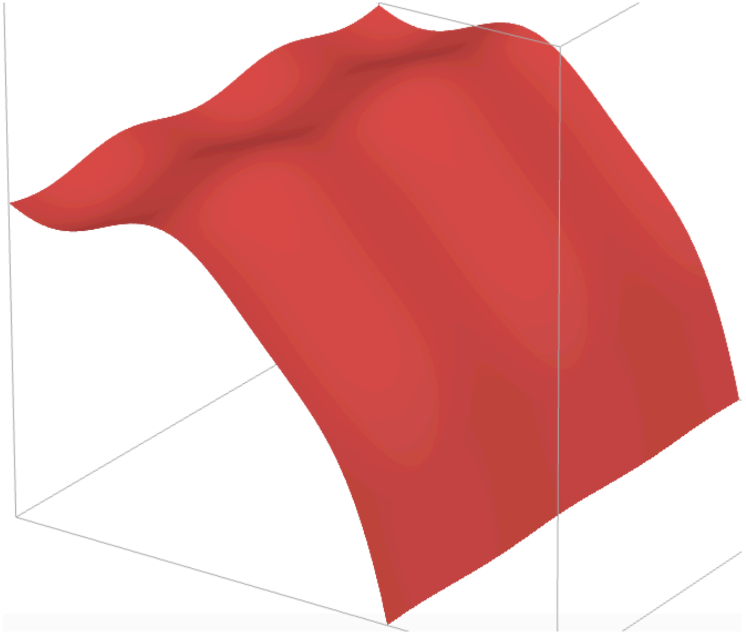

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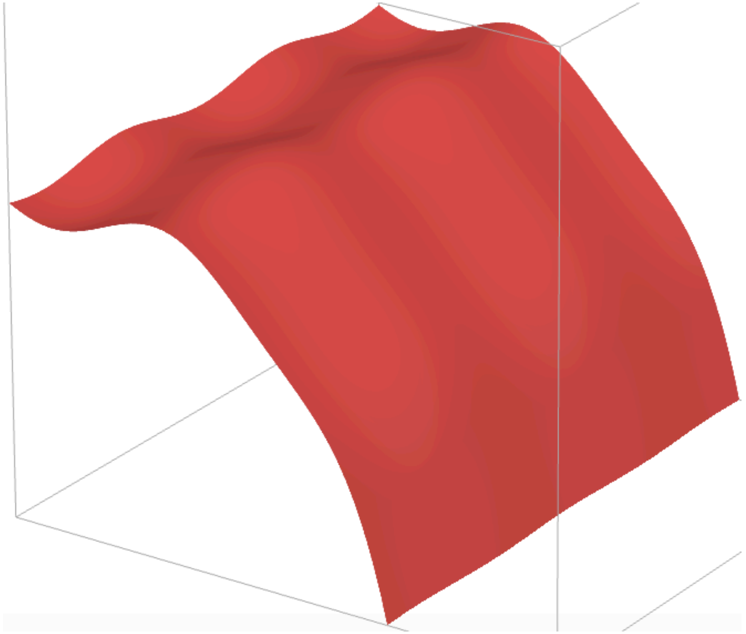

$$\theta_{\pi, i+1} = \theta_{\pi, i} + \alpha \cdot \nabla_{\theta_{\pi, i}} \mathbb{E}\left[\mathcal{G}(\boldsymbol{\tau}) \mid s_0; \theta_{\pi, i}\right]$$

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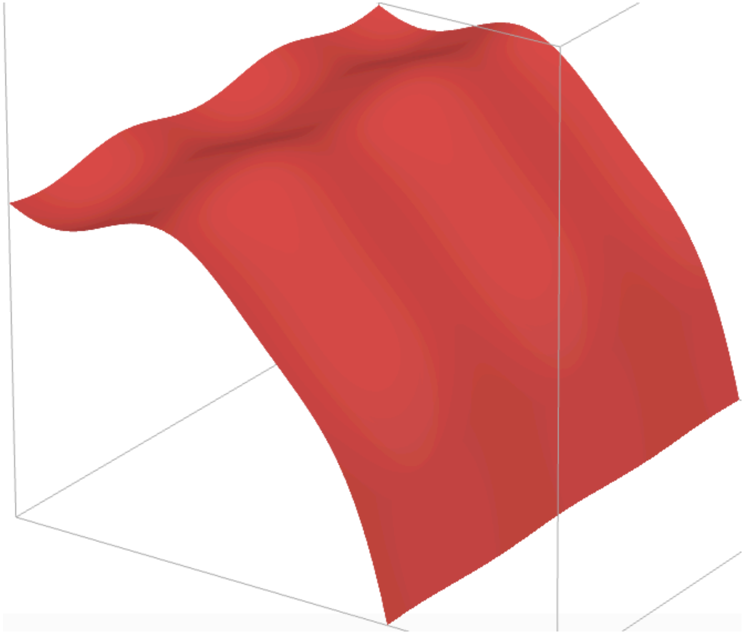

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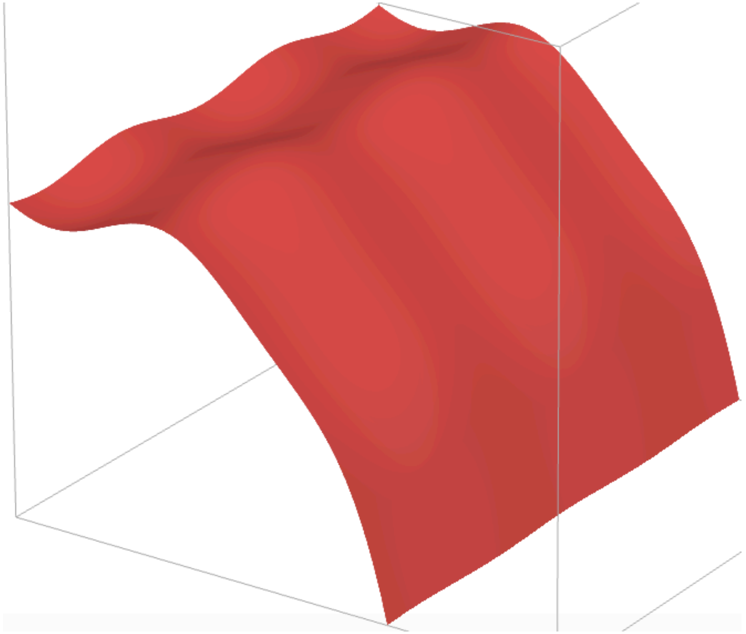

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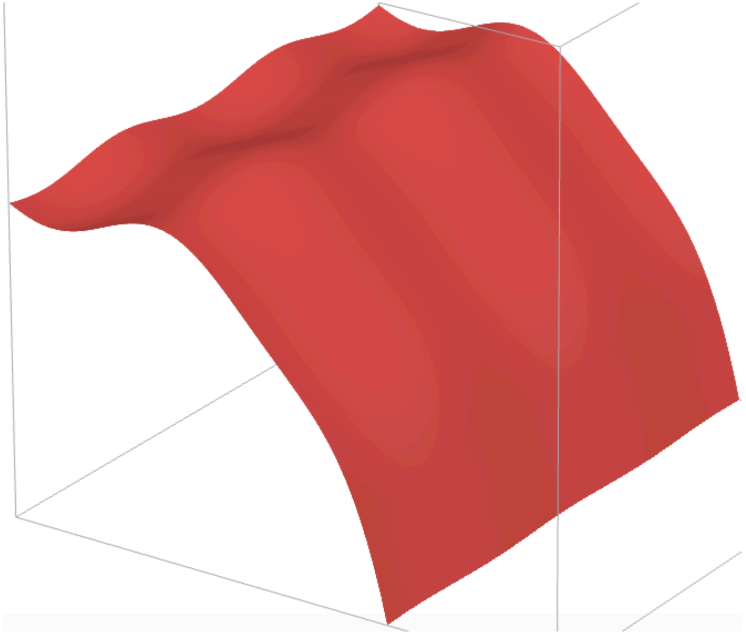

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If find $\nabla_{\theta_\pi} \mathbb{E}[\mathcal{G}(\boldsymbol{\tau}) \mid s_0; \theta_\pi]$, we can improve the policy and return

Policy Gradient

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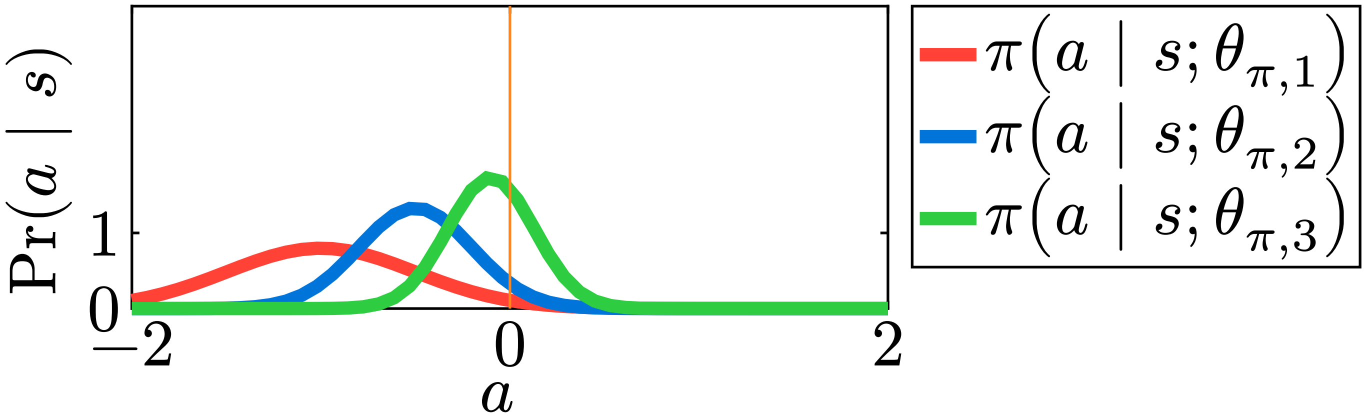
$\underset{a \in A}{\arg\max}\, \mathcal{G}$



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First, combine top two equations so we have more space

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$$\left(\sum_{s_1, \ldots, s_n \in S} \prod_{t=0}^{n} \left(\sum_{a_t \in A} \mathrm{Tr}(s_{t+1} \mid s_t, a_t) \cdot \pi(a_t \mid s_t; \theta_\pi) \right) \right)$$

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Take the gradient with respect to θ_π of both sides

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Take the gradient with respect to θ_π of both sides

$$\nabla_{\theta_\pi} \mathbb{E}[\mathcal{G}(\boldsymbol{\tau}) \mid s_0; \theta_\pi] = \nabla_{\theta_\pi} \left[\sum_{n=0}^{\infty} \gamma^n \sum_{s_{n+1} \in S} \mathcal{R}(s_{n+1}) \cdot \right.$$

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Move gradient inside sums

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Rewrite $\Pr(s_{n+1} \mid s_0; \theta_\pi)$ by pulling action sum outside

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Move the gradient operator further inside the sum

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Can use chain and product rule, but will create a mess of terms

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Can use chain and product rule, but will create a mess of terms

Have to backpropagate through n^2 products, which is intractable

Policy Gradient

Log-derivative trick:

Policy Gradient

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Policy Gradient

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Apply log-derivative trick to $\nabla \prod$

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The log of products is the sum of logs: $\log(ab) = \log(a) + \log(b)$

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Gradient with respect to θ, no θ in Tr, Tr is constant that disappears

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$$\nabla_{\theta_\pi} \mathbb{E}[\mathcal{G}(\boldsymbol{\tau}) \mid s_0; \theta_\pi] = \sum_{n=0}^{\infty} \gamma^n \sum_{s_{n+1} \in S} \mathcal{R}(s_{n+1}) \cdot \sum_{s_1, \ldots, s_n \in S} \sum_{a_0, \ldots, a_n \in A}$$

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This is the **policy gradient**

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This is the **policy gradient**

Rewrote the gradient of the return in terms of the gradient of policy

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$$\nabla_{\theta_\pi} \mathbb{E}[\mathcal{G}(\boldsymbol{\tau}) \mid s_0; \theta_\pi] = \sum_{n=0}^{\infty} \gamma^n \sum_{s_{n+1} \in S} \mathcal{R}(s_{n+1})$$

$$\cdot \sum_{s_1, \ldots, s_n \in S} \sum_{a_0, \ldots, a_n \in A} \left(\prod_{t=0}^{n} \mathrm{Tr}(s_{t+1} \mid s_t, a_t) \cdot \pi(a_t \mid s_t; \theta_\pi) \right) \left[\sum_{t=0}^{n} \nabla_{\theta_\pi} \log \pi(a_t \mid s_t; \theta_\pi) \right]$$

Policy Gradient

$$\nabla_{\theta_\pi} \mathbb{E}[\mathcal{G}(\boldsymbol{\tau}) \mid s_0; \theta_\pi] = \sum_{n=0}^{\infty} \gamma^n \sum_{s_{n+1} \in S} \mathcal{R}(s_{n+1})$$

$$\cdot \sum_{s_1, \ldots, s_n \in S} \sum_{a_0, \ldots, a_n \in A} \left(\prod_{t=0}^{n} \mathrm{Tr}(s_{t+1} \mid s_t, a_t) \cdot \pi(a_t \mid s_t; \theta_\pi) \right) \left[\sum_{t=0}^{n} \nabla_{\theta_\pi} \log \pi(a_t \mid s_t; \theta_\pi) \right]$$

Question: Is this familiar?

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Careful rewriting as return because π relies on n

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We can write the gradient of the return in terms of the policy gradient

Policy Gradient

Definition: The policy gradient family of algorithms

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Update the parameters iteratively until convergence

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Using the policy gradient

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Question: How to find this?

Answer: Estimate expectation empirically

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Answer: Estimate expectation empirically

$$\nabla_{\theta_\pi} \mathbb{E}[\mathcal{G}(\boldsymbol{\tau}) \mid s_0; \theta_\pi] \approx \hat{\mathbb{E}}[\mathcal{G}(\boldsymbol{\tau}) \mid s_0; \theta_\pi] \cdot \sum_{s, a \in \boldsymbol{\tau}} \nabla_{\theta_\pi} \log \pi(a \mid s; \theta_\pi)$$

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HINT:

- On-policy algorithms require data collected with θ_π
- Off-policy algorithms can use data collected with any policy

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- Off-policy algorithms can use data collected with any policy

Answer: On-policy, empirical return based on θ_π

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Answer: Value function or Q function

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$$\nabla_{\theta_\pi} \mathbb{E}[\mathcal{G}(\boldsymbol{\tau}) \mid s_0; \theta_\pi] = V(s_0; \theta_\pi, \theta_V) \cdot \sum_{s,a \in \boldsymbol{\tau}} \log \pi(a \mid s; \theta_\pi)$$

Policy Gradient

$$\nabla_{\theta_\pi} \mathbb{E}[\mathcal{G}(\boldsymbol{\tau}) \mid s_0; \theta_\pi] = \mathbb{E}[\mathcal{G}(\boldsymbol{\tau}) \mid s_0; \theta_\pi] \cdot \sum_{s,a \in \boldsymbol{\tau}} \log \pi(a \mid s; \theta_\pi)$$

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We call this **actor-critic**, more discussion next time

Coding

Coding

We can implement our policy using all sorts of action distributions

[1] "Improving stochastic policy gradients in continuous control with deep reinforcement learning using the beta distribution." International conference on machine learning. PMLR, 2017.

Coding

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For discrete tasks, we often use **categorical** distributions

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For discrete tasks, we often use **categorical** distributions

For continuous tasks, we usually use **normal** distributions

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Coding

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For discrete tasks, we often use **categorical** distributions

For continuous tasks, we usually use **normal** distributions

However, some people say Beta distributions work better![1]

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Coding

Create a policy for discrete action spaces

Coding

Create a policy for discrete action spaces

```
discrete_action_policy = nn.Sequential([
    nn.Linear(state_size, hidden_size),
    nn.Lambda(leaky_relu),
    nn.Linear(hidden_size, hidden_size),
    nn.Lambda(leaky_relu)
    nn.Linear(hidden_size, action_size),
    # Probability over possible actions
    nn.Lambda(jax.nn.log_softmax) # log(softmax(x))
])
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We use `log_softmax` for numerically stable gradients

Coding

```python
def REINFORCE_loss(theta_pi, episode):
    """REINFORCE for discrete actions"""
    G = compute_return(rewards) # empirical return
    # Policy already outputs log(softmax(x))
    log_probs = pi(episode.states, theta_pi)
    # We only update the policy for the actions we took
    # Discrete/categorical actions
    # Can use sum or mean
    policy_gradient = mean(G * log_probs[episode.actions])
    # Want gradient ascent, most library do gradient descent
    return -policy_gradient
```

Coding

What about continuous action spaces?

Coding

What about continuous action spaces?

```
continuous_action_policy = nn.Sequential([
    nn.Linear(state_size, hidden_size),
    nn.Lambda(leaky_relu),
    nn.Linear(hidden_size, hidden_size),
    nn.Lambda(leaky_relu)
    nn.Linear(hidden_size, 2 * action_size),
    # Like to use a diagonal multivariate Gaussian
    # Assumes independence between actions (approximation)
    # Produce mu and log_sigma for each action dim
    nn.Lambda(lambda x: split(x, 2))
])
```

Coding

```
def REINFORCE_loss(theta_pi, episode):
    """REINFORCE for continuous actions using Gaussian pi"""
    G = compute_return(rewards) # empirical return
    # Policy outputs mean and log(std dev)
    mus, log_sigmas = pi(episode.states, theta_pi)
    # Log probability from equation of Gaussian
    log_probs = -(
        (episode.actions - mus) ** 2
        / (2 * exp(log_sigmas) ** 2)
        + log_sigmas
    )
    policy_gradient = mean(G * log_probs)
    # Want gradient ascent, most library do gradient descent
    return -policy_gradient
```

Homework

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Homework 2 is the final homework, and it is a little special

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My suggestion: start with policy gradient

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If you solve policy gradient early, then try deep Q learning

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Deep Q learning requires more hyperparameter tuning

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Start early, one training run can take up to an hour

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If you have bugs, you will need to restart training

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Due in 3 weeks (04.09)