Actor Critic II

CISC 7404 - Decision Making

Steven Morad

University of Macau

Quiz

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- After you are done, give me your exam and go relax outside, we resume class at 8:30

Quiz

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- Instructions are in both english and chinese, english instructions take precedence
- Good luck!

Quiz

- 在所有学生收起电脑/笔记/手机后,我会分发试卷。
- 如果在此之后仍有电脑/笔记/手机未收,将视为作弊。
- 试卷会背面朝下发下,在我宣布开始前请勿翻面。
- 试卷翻面后,我会简要说明每道题的注意事项。
- 说明结束后,你们有 75 分钟完成考试。
- 交卷后请到教室外休息,8:30 恢复上课。
- 试卷可能存在不同版本,细节略有差异。
- 若你的试卷上出现其他版本的答案,将被判定为作弊。
- 试卷说明为中英双语,若内容冲突以英文为准。
- 祝各位考试顺利!

Admin

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You should be able to understand research papers on:

- MDPs, bandits, RL algorithms, even some control problems!

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- Imitation Learning
- Offline RL
- Memory and POMDPs
- Large Language Models

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Question: Should we replace a topic with something else?

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- Imitation learning
  - Sometimes, designing a reward function is hard
  - It is easier to demonstrate desired behavior to agents
    - Instead of reward for surgery, do what human does
    - Instead of reward for self driving, do what human does
  - With imitation learning, can learn behaviors without rewards
  - Closer to supervised learning, easier to train
  - Policies are not better than dataset/humans

Admin

- Offline RL
  - RL without exploration
  - How can we learn policies from a fixed dataset?
    - Learn surgery from surgical videos (no need to kill patients)
    - Learn driving from Xiaomi driving dataset (no need to crash cars)
  - Unlike imitation learning, can do **better** than dataset
  - Very new topic (2-3 years old)
    - Does not work very well (yet)

Admin

- Memory and POMDPs (my research focus)
  - So far, we focused on video games
    - MDP
  - Many interesting problems are not Markov
    - Think of robot with camera, not Markov
    - Almost every task has sensor noise, not Markov
  - Can we extend RL to work for virtually any problem?
    - Yes, requires long-term memory
    - LSTM, transformer, etc
  - May also have time to introduce world models
    - Dreamer, TD-MPC, etc

Admin

- Large Language Models
  - Can train LLMs using unsupervised learning
    - They only learn to predict next word
  - We use RL to teach them to interact with humans
    - Apply policy gradient to textual MDP
    - DeepSeek math/GRPO
    - RL-adjacent methods (DPO)

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Also a possibility to split lecture:

- E.g., 1 hour imitation learning, 1 hour something new

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Alternative topics:

- Multi-agent RL
- Model-based RL and world-models
- Evolutionary algorithms

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Homework 2 progress

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If you did not already start, you might be in trouble

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Experiments take a long time, start as soon as possible

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Harder and requires more debugging than `FrozenLake` assignment

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Those using Tencent AI Arena (Honor of Kings):

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- Cannot install new python libraries (Tencent security issue)
  - No `jax`, must use `torch`
  - You must learn Tencent's strange callback system
    - Prevents copy/pasting, so `torch` is ok

Review

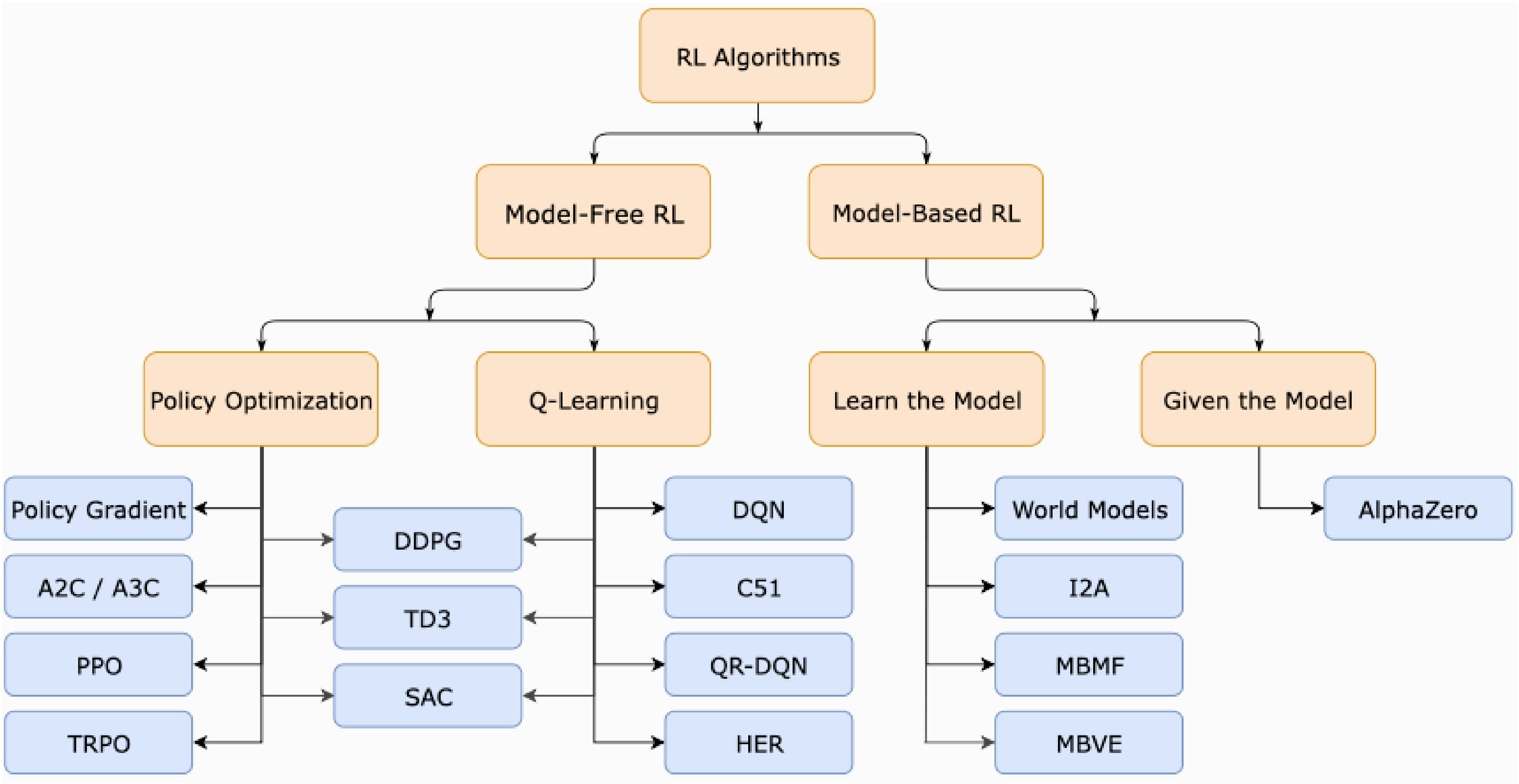
Actor Critic

Actor Critic

Alternative descriptions of actor critic algorithms

https://lilianweng.github.io/posts/2018-04-08-policy-gradient/

Actor Critic



Actor Critic

There are two approaches to actor critic

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1. **Policy gradient based:**

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Learn Q for a specific policy

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Deterministic Policy Gradient

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Question: Why did we introduce policy gradient methods?

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$$A = [0, 2\pi]^{12}$$

Deterministic Policy Gradient

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$$\pi(a_t \mid s_t; \theta_\pi) = \begin{cases} 1 \text{ if } a_t = \arg\max_{a_t \in A} Q(s_t, a_t, \theta_\pi) \\ 0 \text{ otherwise} \end{cases}$$

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Infinitely many a_t – compute Q for each and take arg max over all

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Let us quickly review the Q function and value function

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Start with general form of Temporal Difference Q function

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$$Q(s_0, a_0, \theta_\pi) = \underbrace{\mathbb{E}[\mathcal{R}(s_{t+1}) \mid s_0, a_0]}_{\text{Reward for taking } a_0} + \gamma \underbrace{V(s_1, \theta_\pi)}_{\mathcal{G} \text{ following } \theta_\pi}$$

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Q function $\Rightarrow$ no problem, policy $\Rightarrow$ problem $\quad \operatorname{arg\,max}_{a \in A} Q(s, a, \theta_\pi)$

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Can we learn a different deterministic policy for continuous actions?

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Question: What method can we use to learn θ_π?

Deterministic Policy Gradient

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$$= \sum_{n=0}^{\infty} \gamma^n \sum_{s_{n+1} \in S} \mathcal{R}(s_{n+1}) \cdot \sum_{s_1, \ldots, s_n \in S} \prod_{t=0}^{n} \mathrm{Tr}(s_{t+1} \mid s_t, \mu(s_t, \theta_\mu))$$

$$\sum_{t=0}^{n} \nabla_{\mu} [\log \mathrm{Tr}(s_{t+1} \mid s_t, \mu(s_t, \theta_\mu))] \cdot \nabla_{\theta_\mu} \mu(s_t, \theta_\mu)$$

Deterministic Policy Gradient

$$= \sum_{n=0}^{\infty} \gamma^n \sum_{s_{n+1} \in S} \mathcal{R}(s_{n+1}) \cdot \sum_{s_1, \ldots, s_n \in S} \prod_{t=0}^{n} \mathrm{Tr}(s_{t+1} \mid s_t, \mu(s_t, \theta_\mu))$$

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Question: Any problems?

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We must know ∇ Tr to find the deterministic policy gradient

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In model-free RL, we do not know $\nabla \mathrm{Tr}$ – it might not be differentiable!

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Let me explain what I mean

Deterministic Policy Gradient

With deterministic policy, μ inside Tr means chain rule

Deterministic Policy Gradient

With deterministic policy, μ inside Tr means chain rule

$$\sum_{t=0}^{n} \nabla_{\theta_\mu} \log \mathrm{Tr}\left(s_{t+1} \mid s_t, \mu\left(s_t, \theta_\mu\right)\right)$$

Deterministic Policy Gradient

With deterministic policy, μ inside Tr means chain rule

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With stochastic policy, π **not inside** Tr (product rule)

Deterministic Policy Gradient

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With stochastic policy, π **not inside** Tr (product rule)

$$\sum_{t=0}^{n} \sum_{a_t \in A} \nabla_{\theta_\pi} \left[\mathrm{Tr}(s_{t+1} \mid s_t, a_t) \cdot \log \pi(a_t \mid s_t; \theta_\pi)\right]$$

Deterministic Policy Gradient

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We use stochastic policies in RL because of this

Deep Deterministic Policy Gradient

Deep Deterministic Policy Gradient

$$\nabla_{\theta_\mu} \mathbb{E}\left[\mathcal{G}(\boldsymbol{\tau}) \mid s_0; \theta_\mu\right]$$

Deep Deterministic Policy Gradient

$$\nabla_{\theta_\mu} \mathbb{E}[\mathcal{G}(\boldsymbol{\tau}) \mid s_0; \theta_\mu] \qquad\qquad \theta_{\pi, i+1} = \theta_{\pi, i} + \alpha \cdot \nabla_{\theta_\mu} \mathbb{E}[\mathcal{G}(\boldsymbol{\tau}) \mid s_0; \theta_\mu]$$

Deep Deterministic Policy Gradient

$$\nabla_{\theta_\mu} \mathbb{E}[\mathcal{G}(\boldsymbol{\tau}) \mid s_0; \theta_\mu] \qquad \theta_{\pi, i+1} = \theta_{\pi, i} + \alpha \cdot \nabla_{\theta_\mu} \mathbb{E}[\mathcal{G}(\boldsymbol{\tau}) \mid s_0; \theta_\mu]$$

We failed – a deterministic policy gradient does not seem to work

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Can we try and optimize something else?

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Question: Could we replace $\nabla_{\theta_\mu} \mathbb{E}[\mathcal{G}(\boldsymbol{\tau}) \mid s_0; \theta_\mu]$ with something else?

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$$\nabla_{\theta_\mu} V\left(s_0, \theta_\mu\right)$$

Deep Deterministic Policy Gradient

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Let us try to derive deterministic policy gradient again

Deep Deterministic Policy Gradient

$$\nabla_{\theta_\mu} \mathbb{E}[\mathcal{G}(\boldsymbol{\tau}) \mid s_0; \theta_\mu] \qquad \theta_{\pi,i+1} = \theta_{\pi,i} + \alpha \cdot \nabla_{\theta_\mu} \mathbb{E}[\mathcal{G}(\boldsymbol{\tau}) \mid s_0; \theta_\mu]$$

We failed – a deterministic policy gradient does not seem to work

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$$\nabla_{\theta_\mu} V(s_0, \theta_\mu) \qquad \nabla_{\theta_\mu} Q(s_0, a_0, \theta_\mu)$$

Let us try to derive deterministic policy gradient again

This time, take gradient of Q instead of gradient of $\mathbb{E}[\mathcal{G}(\boldsymbol{\tau}) \mid s_0; \theta_\mu]$

Deep Deterministic Policy Gradient

$$Q(s_0, a_0, \theta_\mu) = \mathbb{E}[\mathcal{R}(s_1) \mid s_0, a_0] + \gamma Q(s_1, a, \theta_\mu); \quad a = \mu(s_1, \theta_\mu)$$

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Plug in μ for a

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Take the gradient of both sides

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Take the gradient of both sides

$$\nabla_{\theta_\mu} Q(s_0, a_0, \theta_\mu) = \nabla_{\theta_\mu} \left[\mathbb{E}[\mathcal{R}(s_1) \mid s_0, a_0] + \gamma Q(s_1, \mu(s_1, \theta_\mu), \theta_\mu)\right]$$

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Gradient of sum is sum of gradients

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Initial reward only depends on action, not θ_μ – gradient is zero

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Pull out γ

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Gradient ascent on $\nabla_{\theta_\mu} Q(s_0, a_0, \theta_\mu)$, can ignore constant γ

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Can rewrite Q as V because we don't use a_0 anymore

Deep Deterministic Policy Gradient

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Let us inspect these terms more closely

Deep Deterministic Policy Gradient

How θ_μ changes a

$$\nabla_{\theta_\mu} V(s_0, \theta_\mu) = \nabla_\mu Q(s_1, \mu(s_1, \theta_\mu), \theta_\mu) \cdot \nabla_{\theta_\mu} \mu(s_1, \theta_\mu)$$

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Let us inspect these terms more closely

The second term is easy to compute, gradient of deterministic policy

Deep Deterministic Policy Gradient

How θ_μ changes a

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How a changes Q

Let us inspect these terms more closely

The second term is easy to compute, gradient of deterministic policy

Analytical solution for the first term is difficult (recursive definition)

Deep Deterministic Policy Gradient

$$\nabla_{\theta_\mu} V(s_0, \theta_\mu) = \underbrace{\nabla_\mu Q(s_1, \mu(s_1, \theta_\mu), \theta_\mu)}_{\text{How } a \text{ changes } Q} \cdot \overbrace{\nabla_{\theta_\mu} \mu(s_1, \theta_\mu)}^{\text{How } \theta_\mu \text{ changes } a}$$

Let us inspect these terms more closely

The second term is easy to compute, gradient of deterministic policy

Analytical solution for the first term is difficult (recursive definition)

But what if Q is a neural network?

Deep Deterministic Policy Gradient

How θ_μ changes a

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How a changes Q

Let us inspect these terms more closely

The second term is easy to compute, gradient of deterministic policy

Analytical solution for the first term is difficult (recursive definition)

But what if Q is a neural network?

We can backpropagate through Q without worrying about recursion

Deep Deterministic Policy Gradient

How θ_μ changes a

$$\nabla_{\theta_\mu} V(s_0, \theta_\mu) = \nabla_\mu Q(s_1, \mu(s_1, \theta_\mu), \theta_\mu) \cdot \nabla_{\theta_\mu} \mu(s_1, \theta_\mu)$$

How a changes Q

Deep Deterministic Policy Gradient

$$\nabla_{\theta_\mu} V(s_0, \theta_\mu) = \underbrace{\nabla_\mu Q(s_1, \mu(s_1, \theta_\mu), \theta_\mu)}_{\text{How } a \text{ changes } Q} \cdot \overbrace{\nabla_{\theta_\mu} \mu(s_1, \theta_\mu)}^{\text{How } \theta_\mu \text{ changes } a}$$

Writing the code makes it look easy

Deep Deterministic Policy Gradient

How θ_μ changes a

$$\nabla_{\theta_\mu} V(s_0, \theta_\mu) = \nabla_\mu Q(s_1, \mu(s_1, \theta_\mu), \theta_\mu) \cdot \nabla_{\theta_\mu} \mu(s_1, \theta_\mu)$$

How a changes Q

Writing the code makes it look easy

```
def V(s, Q_nn, mu_nn):
    a = mu_nn(s)
    return Q_nn(s, a)

# Learn the policy that maximizes V
# Make sure to differentiate w.r.t policy parameters!
J = grad(V, argnums=2)(states, Q_nn, mu_nn)
mu_nn = optimizer.update(mu_nn, J)
```

Deep Deterministic Policy Gradient

This assumes we know the Q function for θ_μ

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$$Q(s_0, a_0, \theta_\mu, \theta_Q) = \mathbb{E}[\mathcal{R}(s_1) \mid s_0, a_0] + \gamma Q(s_1, \mu(s_1, \theta_\mu), \theta_\mu, \theta_Q)$$

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```
def V(s, Q_nn, mu_nn):
    a = mu_nn(s)
    return Q_nn(s, a)
# Before, we learned policy params to maximize Q
# Now, we learn params of Q following policy (argnums=2)
J = grad(V, argnums=1)(states, Q_nn, mu_nn)
Q_nn = optimizer.update(Q_nn, J)
```

Deep Deterministic Policy Gradient

Definition: Deep Deterministic Policy Gradient (DDPG) jointly learns a Q function for deterministic policy θ_{μ}, and the policy parameters θ_{μ}

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$$\theta_{Q,i+1} = \arg\min_{\theta_{Q,i}}$$

$$\left(Q\left(s_0, a_0, \theta_{\mu,i}, \theta_{Q,i}\right) - \left(\hat{\mathbb{E}}[\mathcal{R}(s_1) \mid s_0, a_0] + \gamma Q\left(s_1, \mu\left(s_1, \theta_{\mu,i}\right), \theta_{\mu,i}, \theta_{Q,i}\right)\right)\right)$$

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Step 2: Learn θ_μ that maximizes Q

$$\theta_{\mu,i+1} = \theta_{\mu,i} + \alpha \cdot Q\left(s_0, \mu\left(s_0, \theta_\mu\right), \theta_{\mu,i}, \theta_{Q,i+1}\right)$$

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Repeat until convergence, $\theta_{\mu,i+1} = \theta_{\mu,i}, \quad \theta_{Q,i+1} = \theta_{Q,i}$

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Question: Is DDPG on-policy or off-policy? Why?

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Almost **all** good off-policy actor-critic algorithms are based on DDPG

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- μ neural network approximation of $\arg\max_{a \in A} Q(s, a)$
- Policy learning is learning arg max over infinite action space

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$$\pi(a \mid s; \theta_\pi) = \begin{cases} 1 - \varepsilon : a = \arg\max_{a \in A} Q(s, a, \theta_\pi) \\ \varepsilon : \text{uniform}(A) \end{cases}$$

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Add some random noise to the action $a = \mu(s, \mu_\pi) + \eta$

$$\pi(a \mid s; \mu_\pi) = \text{Normal}(\mu(s, \mu_\pi), \sigma)$$

Coding

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First, construct deterministic policy

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mu = Sequential([
    Linear(state_size, hidden_size),
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BenBen: $A = [0, 2\pi]^{12}$, so `action_dims=12`

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- BenBen: $A \in [0, 2\pi]^{12}$, `lower=[0, 0, ...]`, `upper=[2pi, 2pi, ...]`

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```
def bound_action(action, lower, upper):
    return 0.5 * (upper + lower) + 0.5 * (upper - lower)
        * tanh(action)
```

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```
def bound_action(action, lower, upper):
    return 0.5 * (upper + lower) + 0.5 * (upper - lower)
        * tanh(action)

def sample_action(mu, state, A_bounds, std):
    action = mu(state)
    noisy_action = action + normal(0, std) # Explore
    return bound_action(noisy_action, *A_bounds)
```

Coding

Now construct Q

Coding

Now construct Q

```
Q = Sequential([
    # Different from DQN network
    # Input action and state together
    Lambda(lambda s, a: concatenate(s, a)),
    Linear(state_size + action_dims, hidden_size),
    Lambda(leaky_relu),
    Linear(hidden_size, hidden_size),
    Lambda(leaky_relu),
    Linear(hidden_size, 1), # Single value for Q(s, a)
])
```

Coding

```
while not terminated:
    # Exploration: make sure actions within action space!
    action = sample_action(mu, state, bounds, std)
    transition = env.step(action)
    replay_buffer.append(transition)
    data = replay_buffer.sample()
    # Theta_pi params are in mu neural network
    # Argnums tells us differentiation variable
    J_Q = grad(Q_loss, argnums=0)(theta_Q, theta_T, mu, data)
    J_mu = grad(mu_loss, argnums=0)(mu, theta_Q, data)
    theta_Q, mu = apply_updates(J_Q, J_mu, ...)
    if step % 200 == 0: # Target network necessary
        theta_T = theta_Q
```

Coding

```python
def Q_loss(theta_Q, theta_T, theta_pi, data):
    Qnet = combine(Q, theta_Q)
    Tnet = combine(Q, theta_T) # Target network
    # Predict Q values for action we took
    prediction = vmap(Qnet)(data.state, data.action)
    # Now compute labels
    next_action = vmap(mu)(data.next_state)
    # NOTE: No argmax! Mu approximates argmax
    next_Q = vmap(Tnet)(data.next_state, next_action)
    label = reward + gamma * data.done * next_Q
    return (prediction - label) ** 2
```

Coding

```
def mu_loss(mu, theta_Q, data):
    # Find the action that maximizes the Q function
    Qnet = combine(Q, theta_Q)
    # Instead of multiply, chain rule -- plug action into Q
    action = vmap(mu)(data.state)
    q_value = vmap(Qnet)(data.state, action)
    # Update the policy parameters to maximize the Q value
    # Gradient ascent but we min loss, use negative
    return -q_value
```

Max Entropy RL

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Many algorithms add improvements to DDPG

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We will very briefly cover max-entropy RL

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We will very briefly cover max-entropy RL

First, let us introduce entropy

Max Entropy RL

Entropy measures the uncertainty of a distribution

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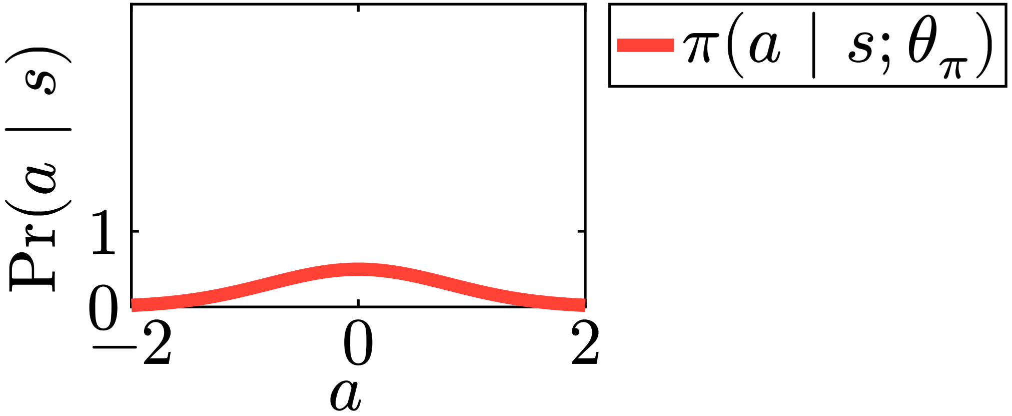

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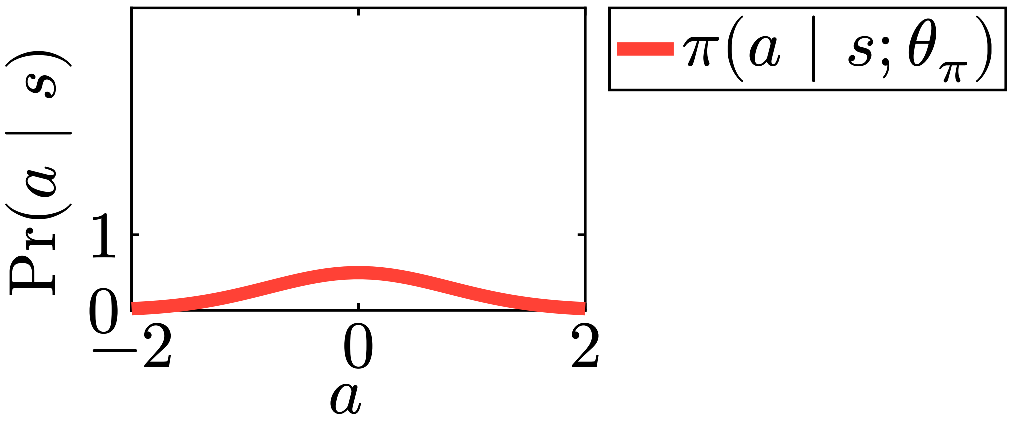

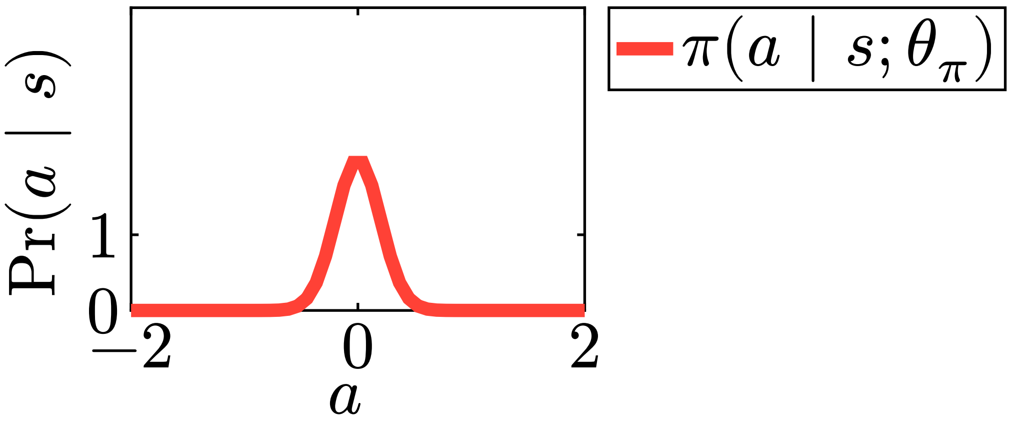

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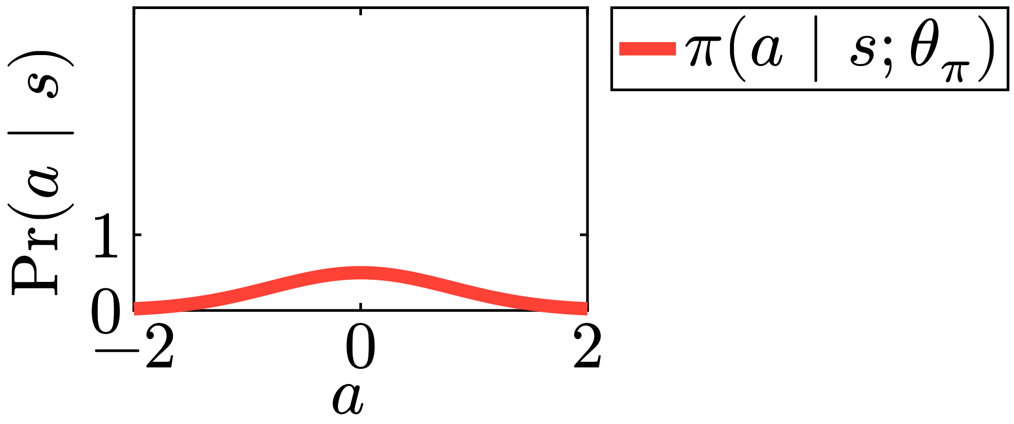

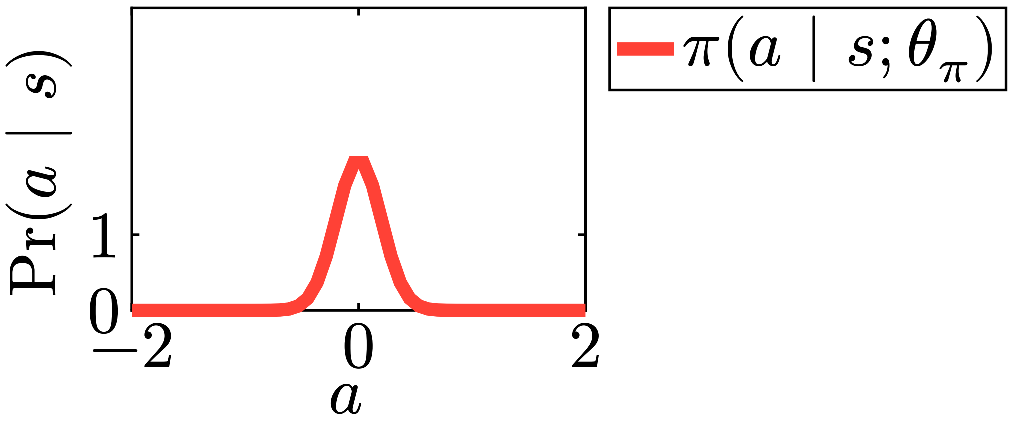

Question: Which policy has higher entropy?

Max Entropy RL

Entropy measures the uncertainty of a distribution

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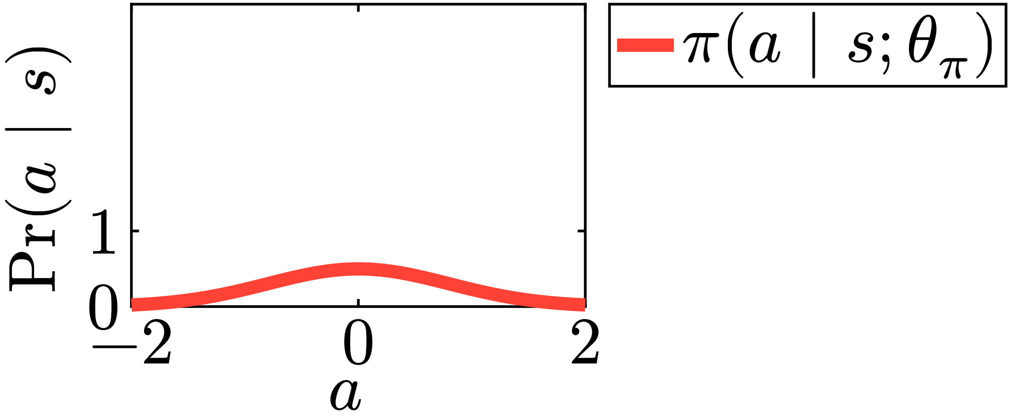

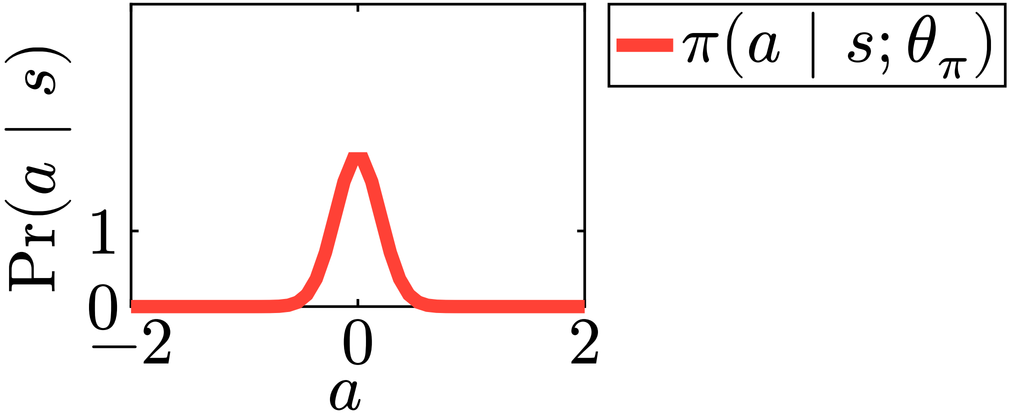

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Left policy, more uncertain/random

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We want a policy that is both random and maximizes the return

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- More stable training (explain further in offline RL)

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But f is deterministic when we know η

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We get SAC!

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Repeat until convergence, $\theta_{\mu, i+1} = \theta_{\mu, i}, \quad \theta_{Q, i+1} = \theta_{Q, i}$

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- Often not documented
- CleanRL describes modern SAC, using tricks from 5+ papers
- https://docs.cleanrl.dev/rl-algorithms/sac/#implementation-details_1

Coding SAC could take an entire lecture, read CleanRL

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  - Think about why it learned to do this (exploiting bugs in MDP)

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Theory is absolutely necessary to understand **why** your policy fails, and **how** to fix it

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In RL, your code never works on the first try

Even if it is correct, you need to play with hyperparameters

Theory is absolutely necessary to understand **why** your policy fails, and **how** to fix it

You must use your brain to be successful!