Decision Processes

CISC 7404 - Decision Making

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Review

Markov Processes

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Decisions must make some change in the world

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- Cryptography
- History

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$$s_{t+1} \sim \mathrm{Tr}(\cdot \mid s_t)$$

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$$= \begin{bmatrix} \Pr(C \mid C) & \Pr(R \mid C) & \Pr(S \mid C) \\ \Pr(C \mid R) & \Pr(R \mid R) & \Pr(S \mid R) \\ \Pr(C \mid S) & \Pr(R \mid S) & \Pr(S \mid S) \end{bmatrix}$$

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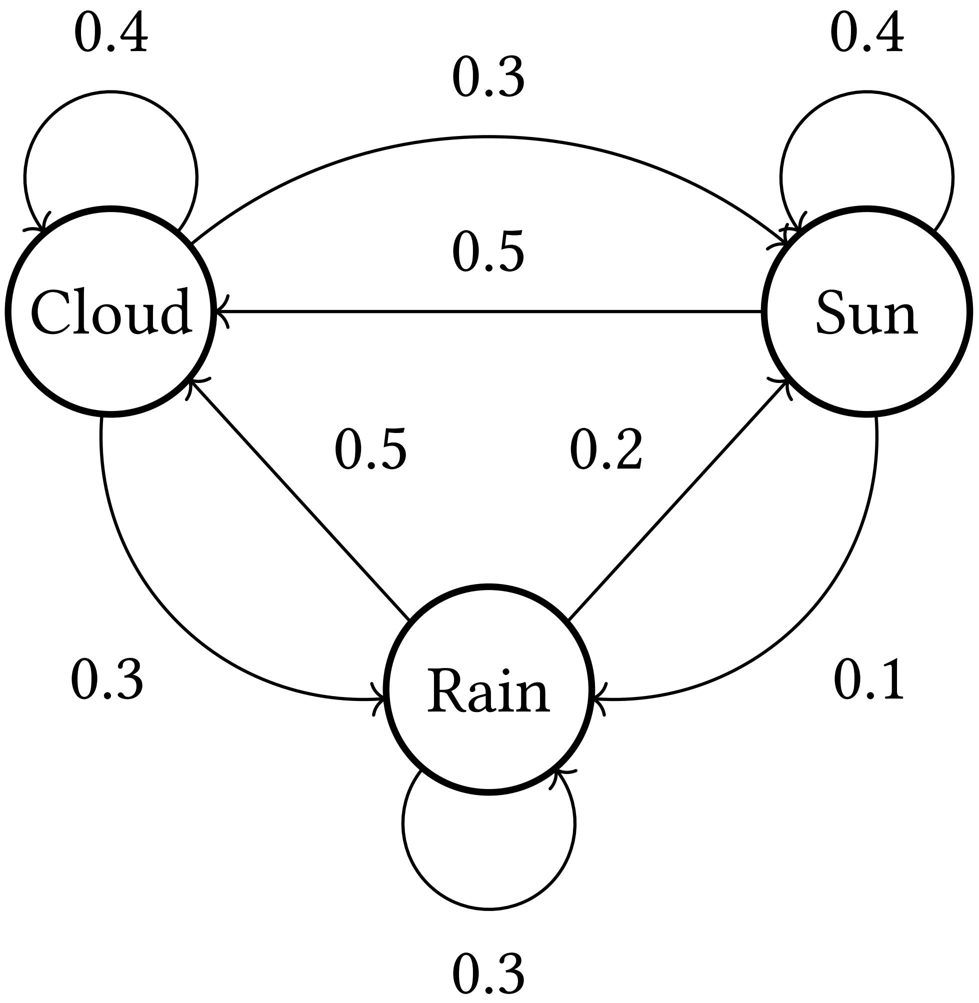

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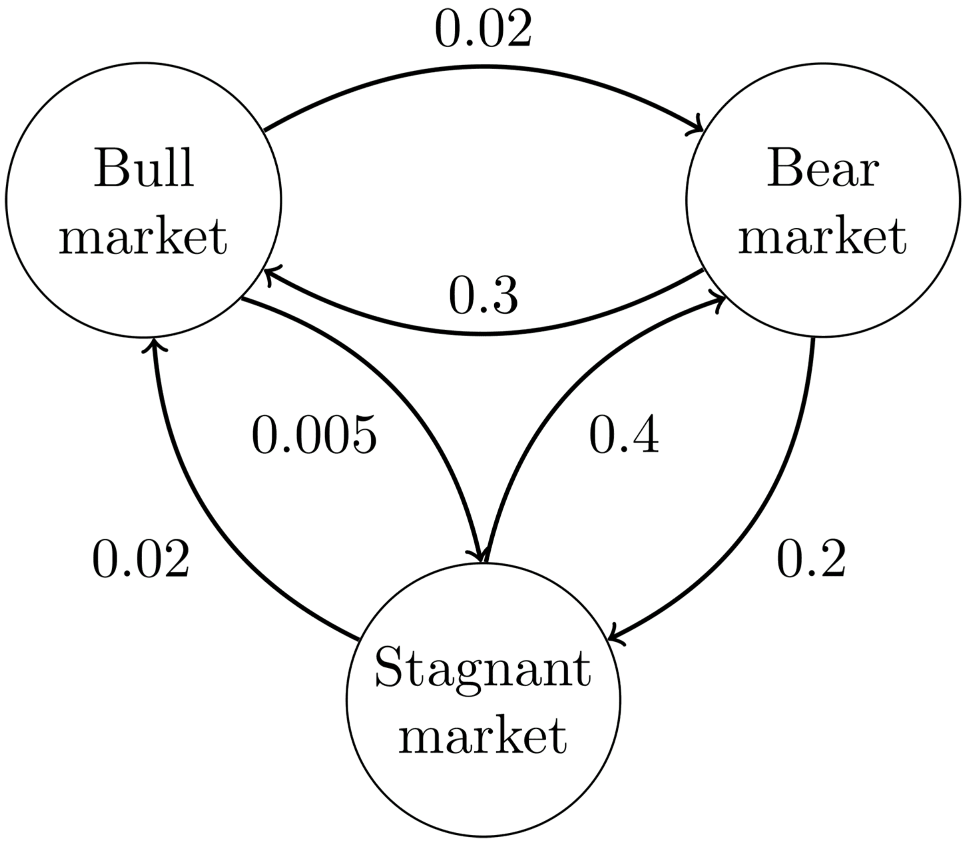

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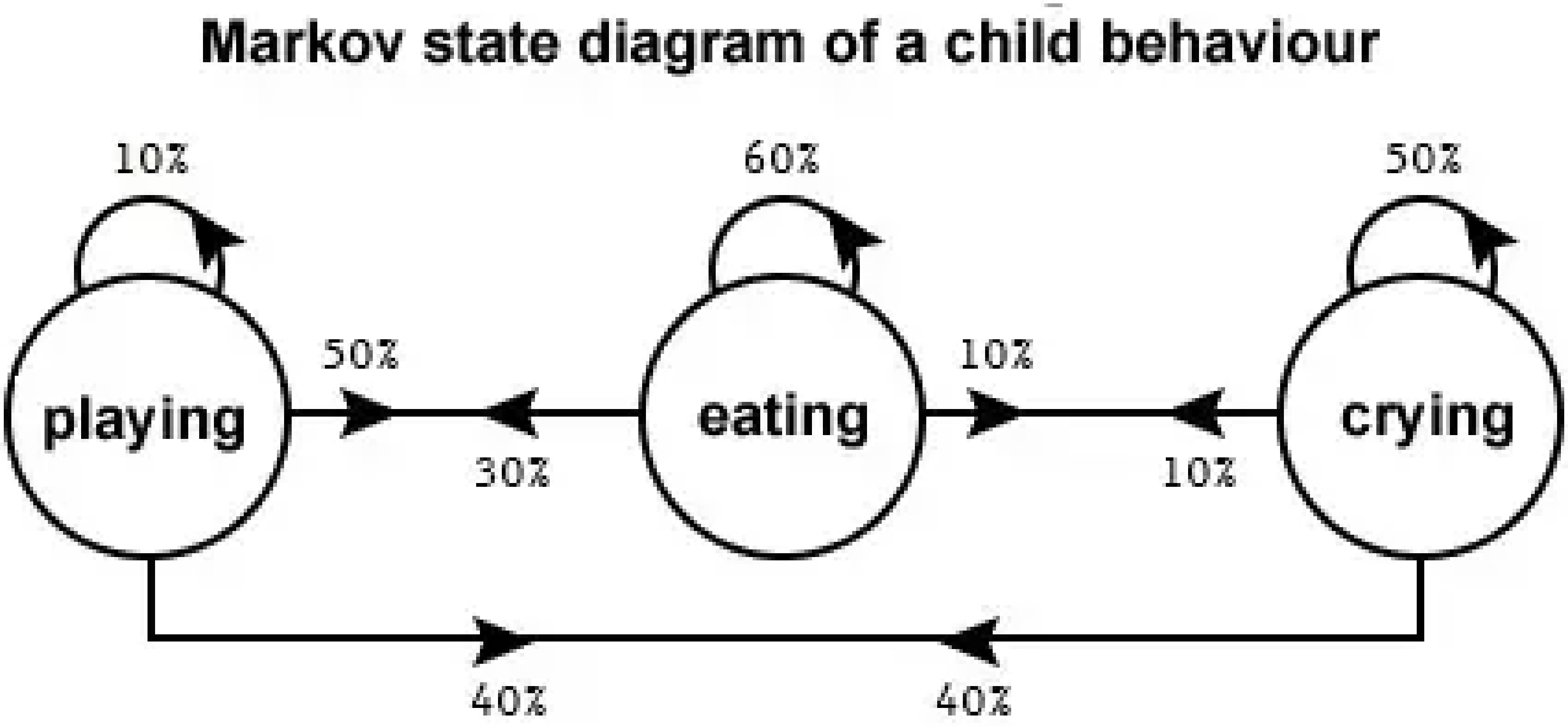

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$0.3 \neq 0.4, \Pr(s_{t+1} \mid s_t) \neq \Pr, (s_{t+1} \mid s_t, s_{t-1}, ..., s_0)$, **not** Markov

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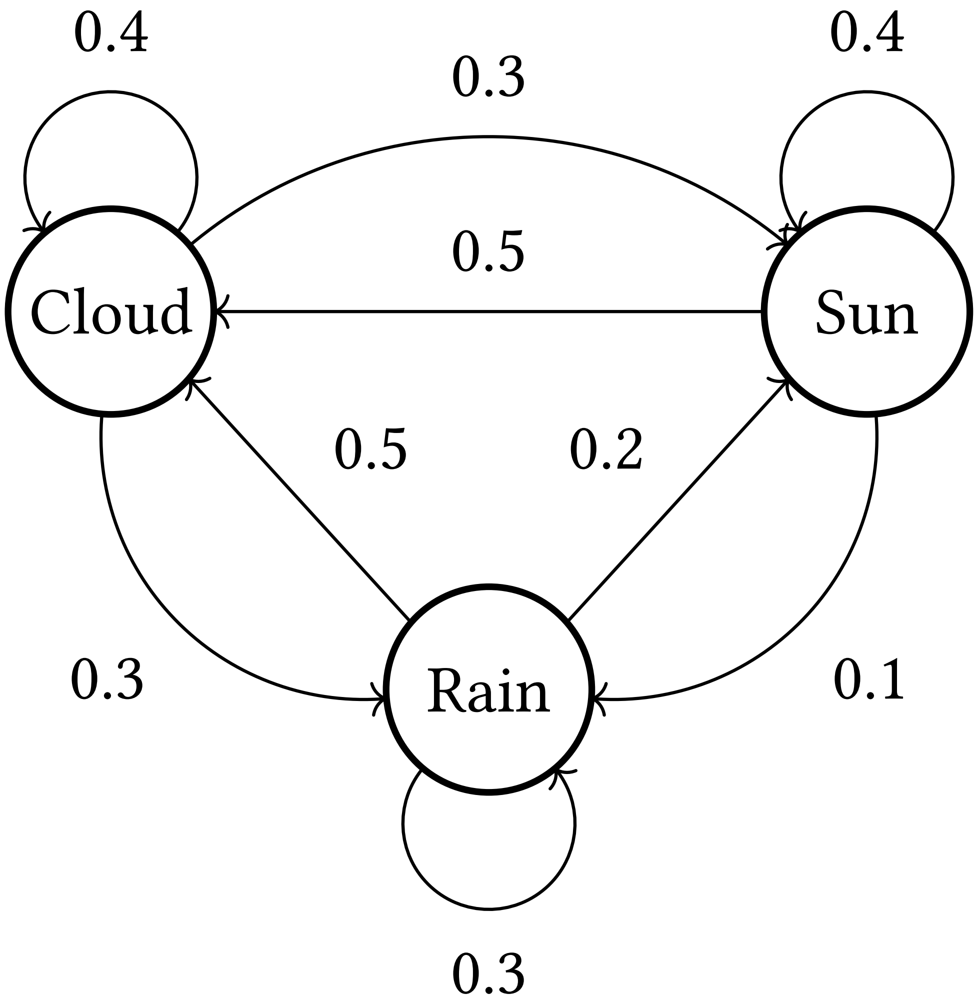

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Paths from all possible s_1 to s_2

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$$\Pr(s_2 \mid s_0 = s) = \underbrace{\sum_{s_1 \in S}}_{\text{Paths from all possible } s_1 \text{ to } s_2} \Pr(s_2 \mid s_1) \Pr(s_1 \mid s_0 = s)$$

$$\Pr(s_3 \mid s_0 = s) = \sum_{s_2 \in S} \Pr(s_3 \mid s_2) \sum_{s_1 \in S} \Pr(s_2 \mid s_1) \Pr(s_1 \mid s_0 = s)$$

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Can we derive a general form for $P(s_n \mid s_0)$?

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Product of sum is the sum of products

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Combine the sums

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Generalize to any timestep n

$$\Pr(s_n \mid s_0) = \sum_{s_1, s_2, \ldots s_{n-1} \in S} \prod_{t=0}^{n-1} \Pr(s_{t+1} \mid s_t)$$

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This expression tells us how the Markov process evolves over time

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We can predict the future, n timesteps from now

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If s represents someone's mind, you can predict their future thoughts

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Question: How can we model a Markov process that ends?

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Question: How can we model a Markov process that ends?

Answer: We create a **terminal state** that we can enter but cannot leave

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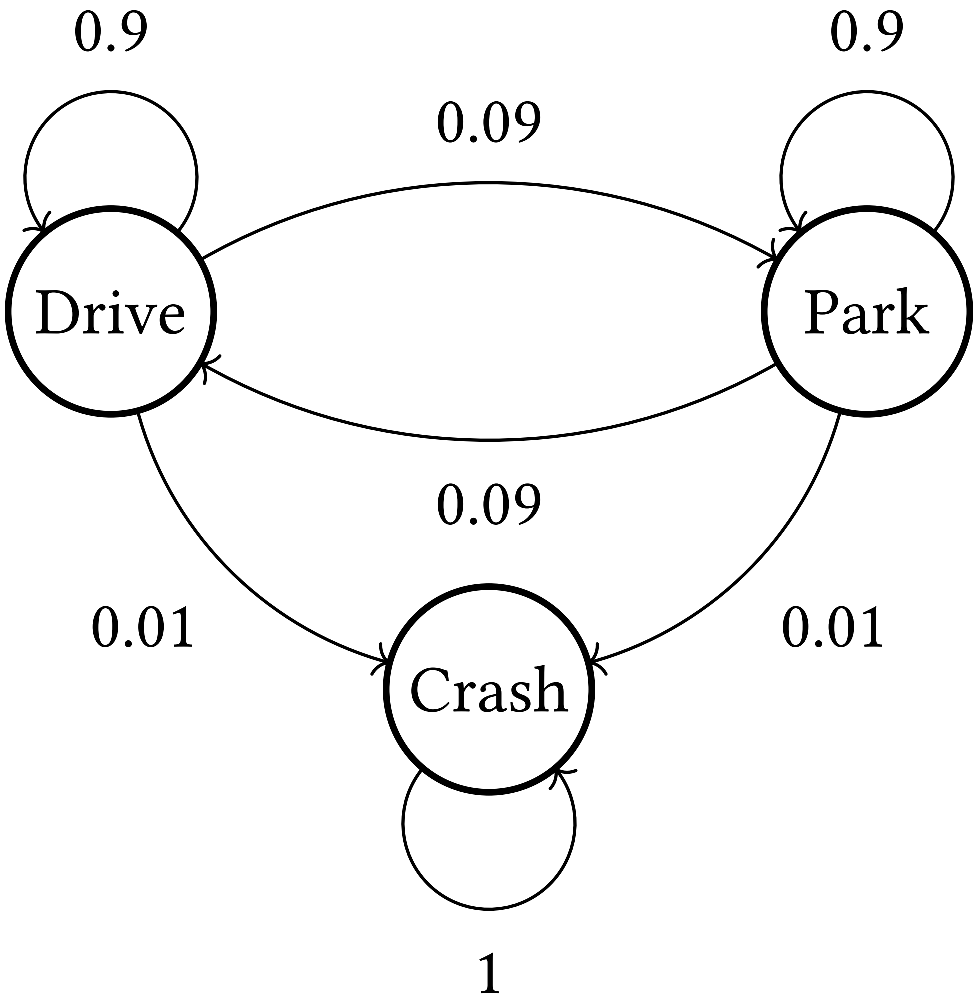

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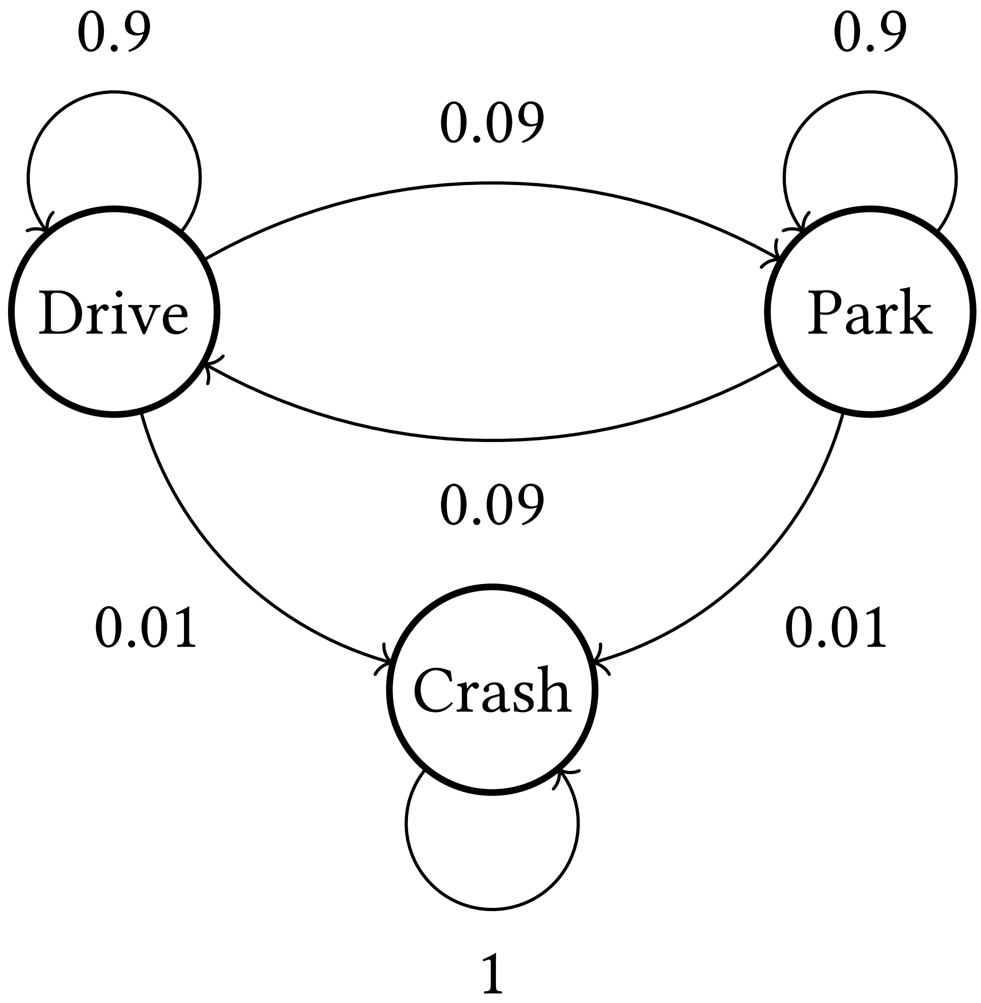

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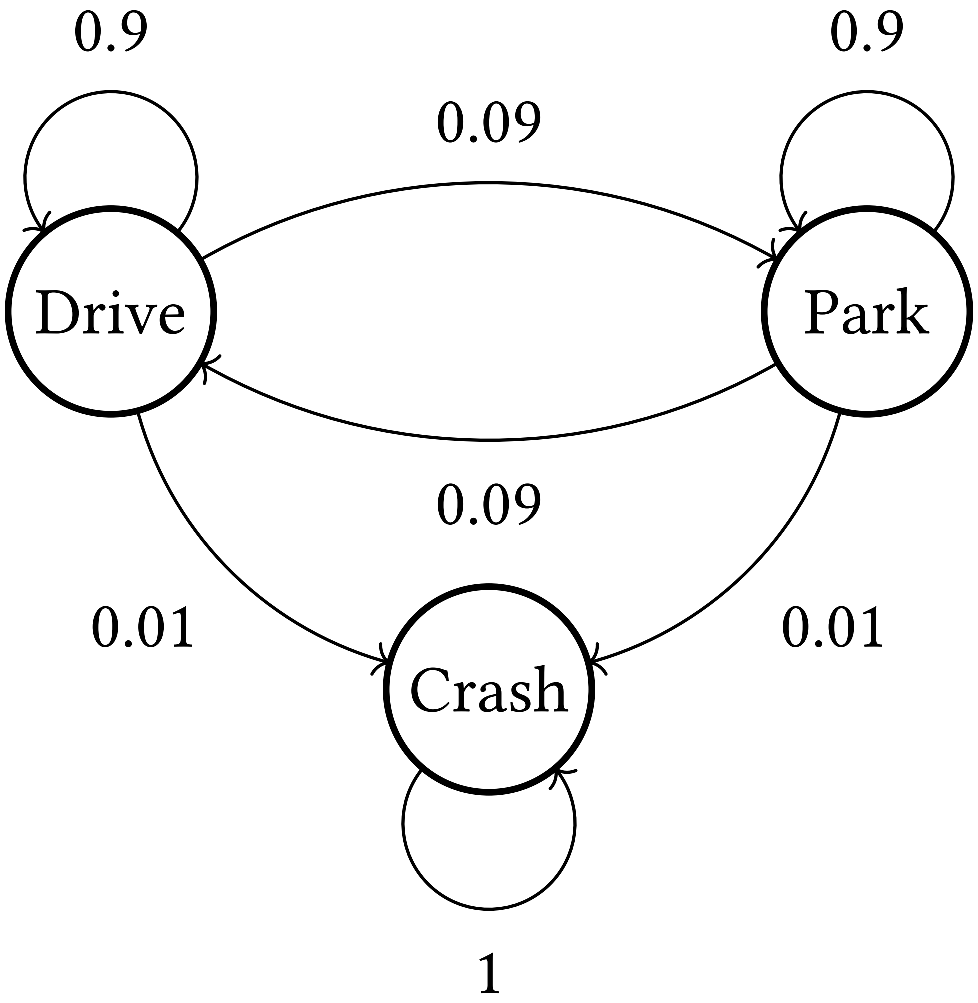

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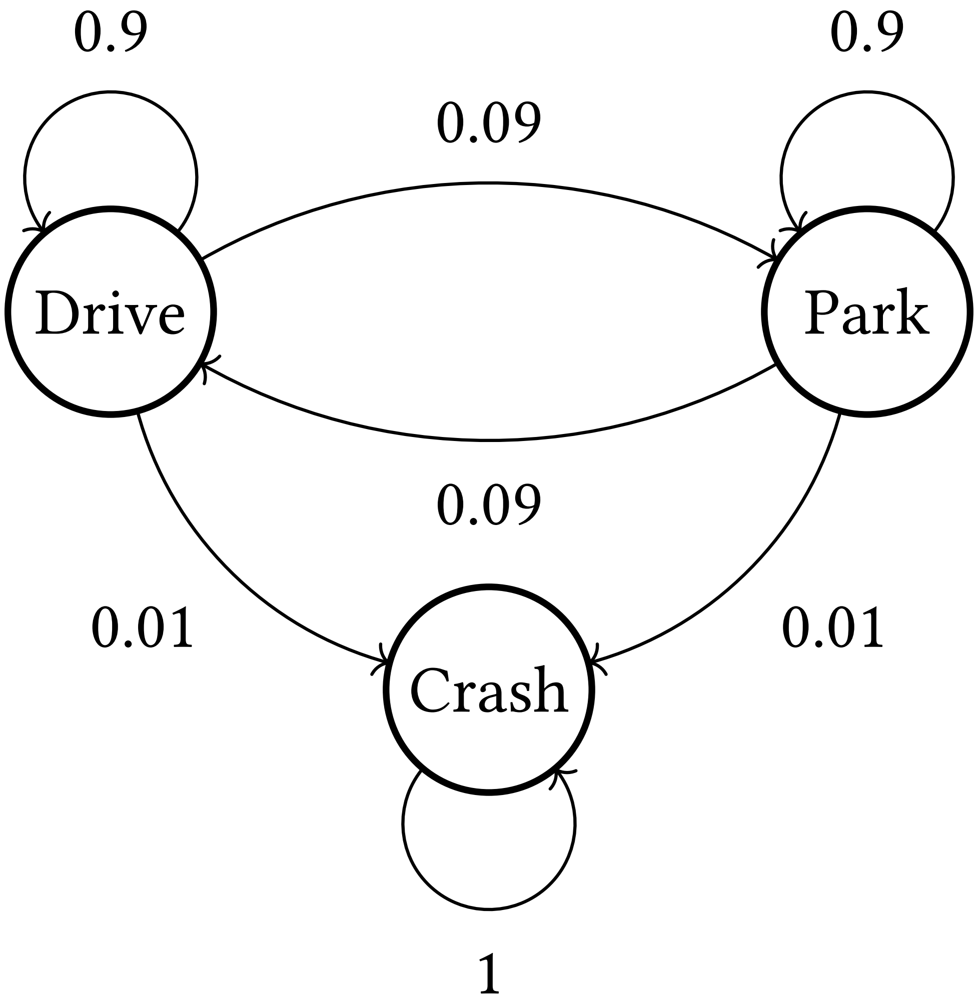

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The only transition from a terminal state is back to itself

$$\Pr(s_{t+1} = \text{term} \mid s_t = \text{term}) = 1$$

$$\Pr(s_{t+1} = \text{not term} \mid s_t = \text{term}) = 0$$

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The point of decision making is to choose our fate

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The agent changes the environment with its decisions

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Markov process

$$(S, \mathrm{Tr})$$

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In a Markov control process, we can control the evolution!

Markov Control Processes

The agent takes **actions** $a \in A$ that change the environment

The action space A defines what our agent can do rrr

Markov process	Markov control process
(S, Tr)	(S, A, Tr)
$\mathrm{Tr} : S \mapsto \Delta S$	$\mathrm{Tr} : S \times A \mapsto \Delta S$

In a Markov process, the future follows a predefined evolution

In a Markov control process, we can control the evolution!

Let us see an example

Markov Control Processes

$$S = \{\text{Healthy}, \text{Sick}, \text{Dead}\}$$

Markov Control Processes

$$S = \{\text{Healthy}, \text{Sick}, \text{Dead}\} \qquad A = \{\text{Nothing}, \text{Medicine}\} = \{N, M\}$$

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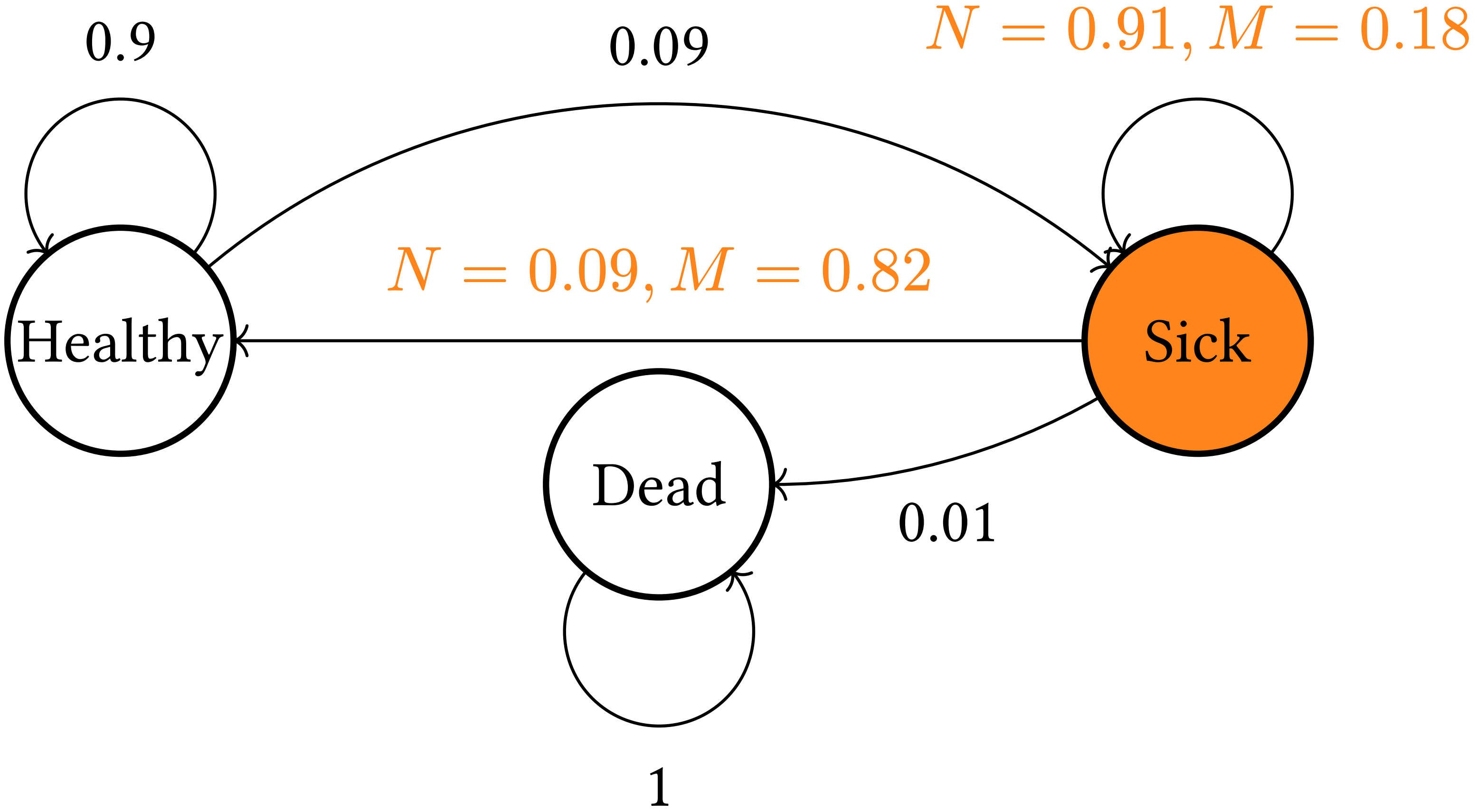
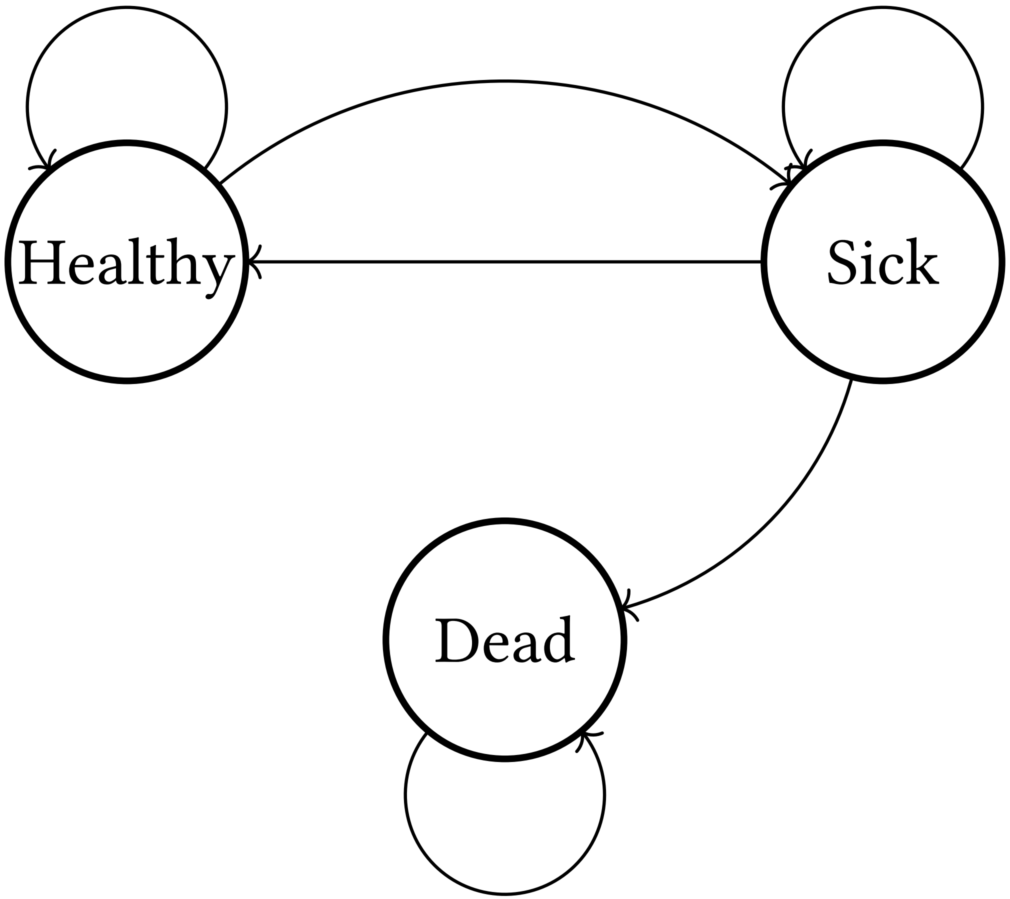


Markov Control Processes



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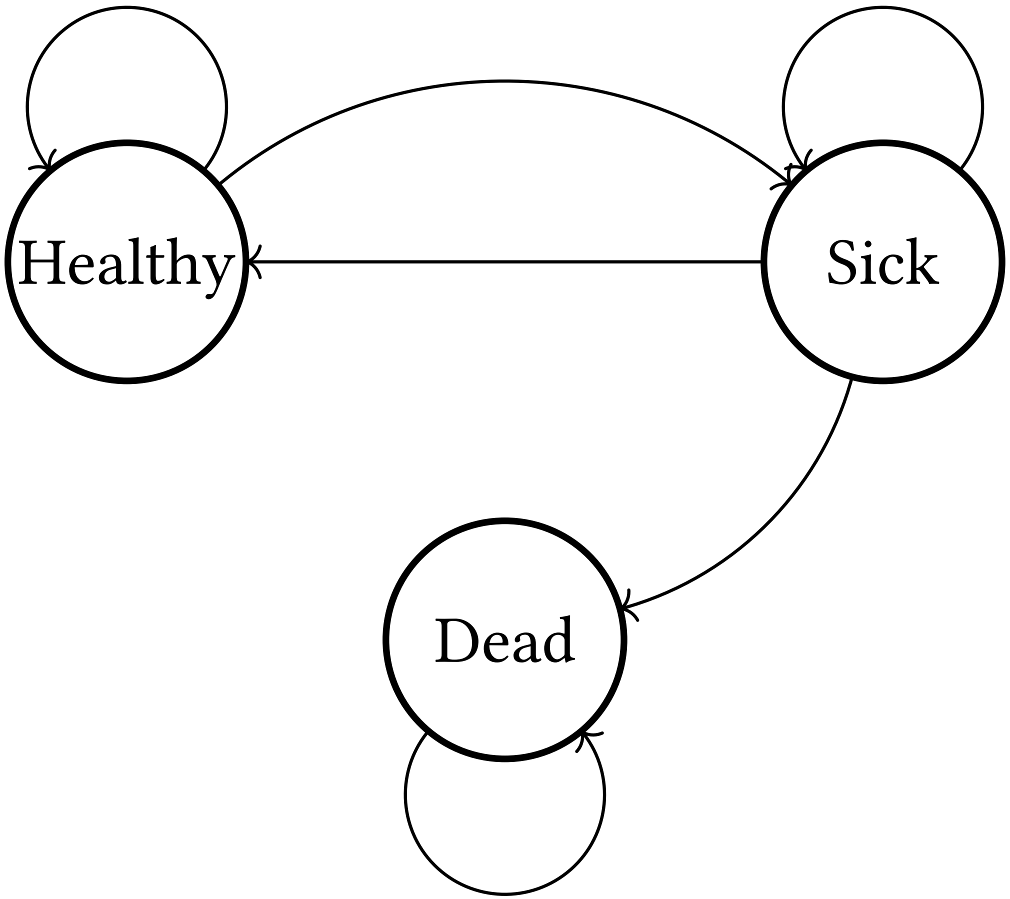
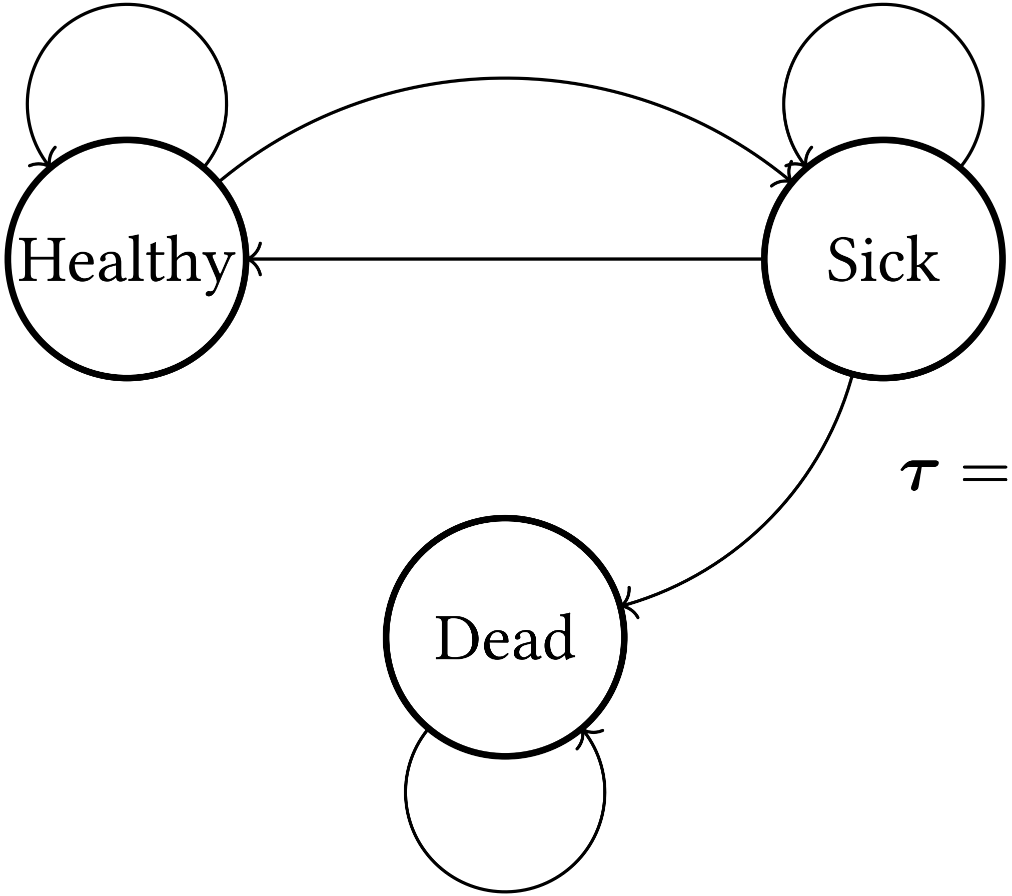


The **trajectory** contains the states and actions until a terminal state

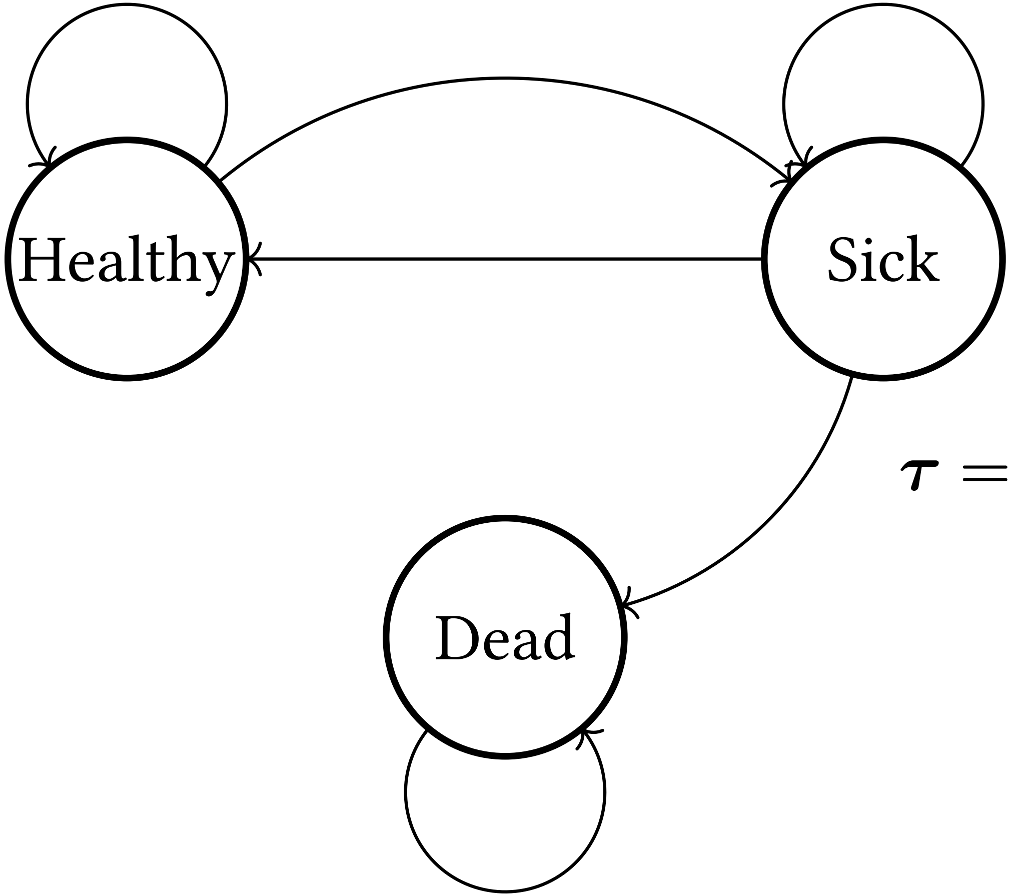
Markov Control Processes



The **trajectory** contains the states and actions until a terminal state

$$\boldsymbol{\tau} = \begin{bmatrix} s_0 & a_0 \\ s_1 & a_1 \\ s_2 & a_2 \\ \vdots & \vdots \\ s_{n-1} & a_{n-1} \\ s_n & \emptyset \end{bmatrix}$$

Markov Control Processes



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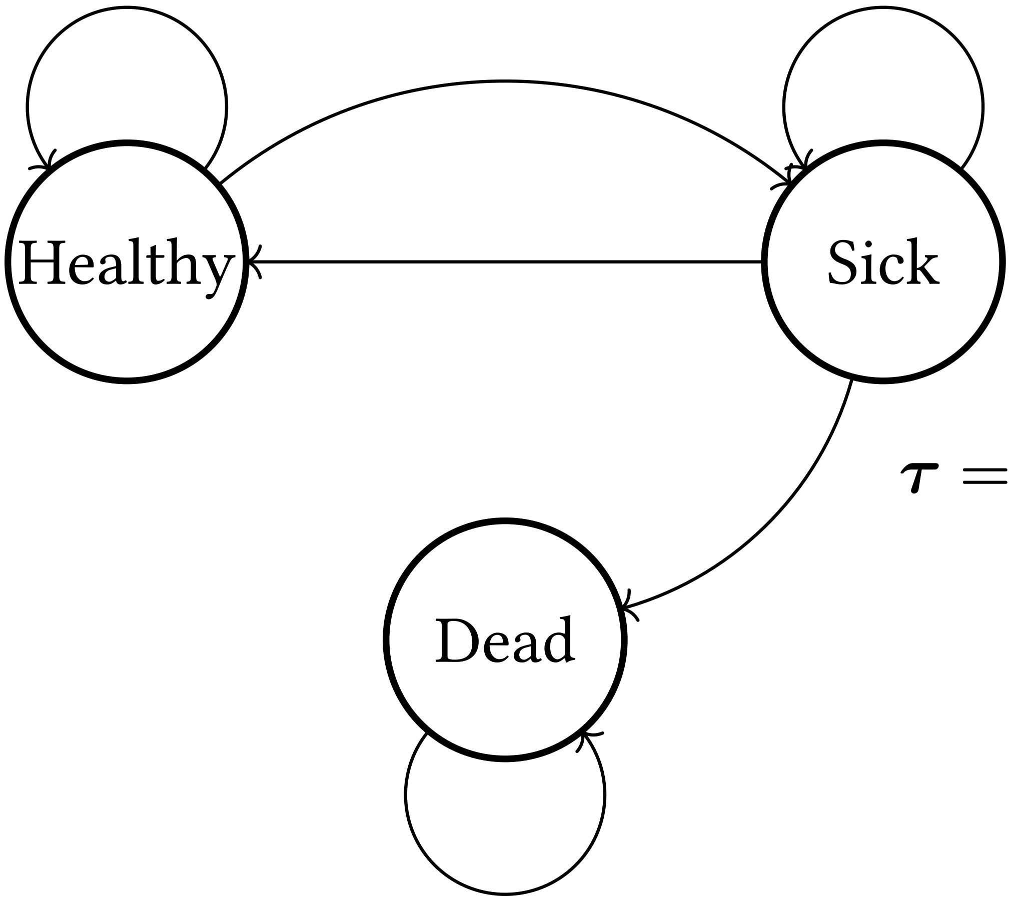

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If there is no terminal state, the trajectory can be infinitely long!

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They do not tell us which states are good to visit, or provide a goal

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How can we make optimal decisions if we do not have a goal or objective?

We need a way to determine “good” and “bad” decisions

Markov Decision Processes

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Markov decision processes (MDPs) add a measure of "goodness" to Markov control processes

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$$R : S \times A \mapsto \mathbb{R}$$

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You can always make these equivalent by modifying the MDP

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$$(S, A, \mathrm{Tr}, R, \gamma)$$

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In an MDP, an **episode** contains the trajectory and also the rewards

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$$\boldsymbol{E} = \begin{bmatrix} s_0 & a_0 & r_0 \\ s_1 & a_1 & r_1 \\ \vdots & \vdots & \vdots \\ s_{n-1} & a_{n-1} & r_{n-1} \\ s_n & \emptyset & \emptyset \end{bmatrix} = [\boldsymbol{\tau} \;\; \boldsymbol{r}]$$

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We can write this mathematically as

$$\arg\max_{s \in S} R(s)$$

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However, maximizing the reward is not always ideal

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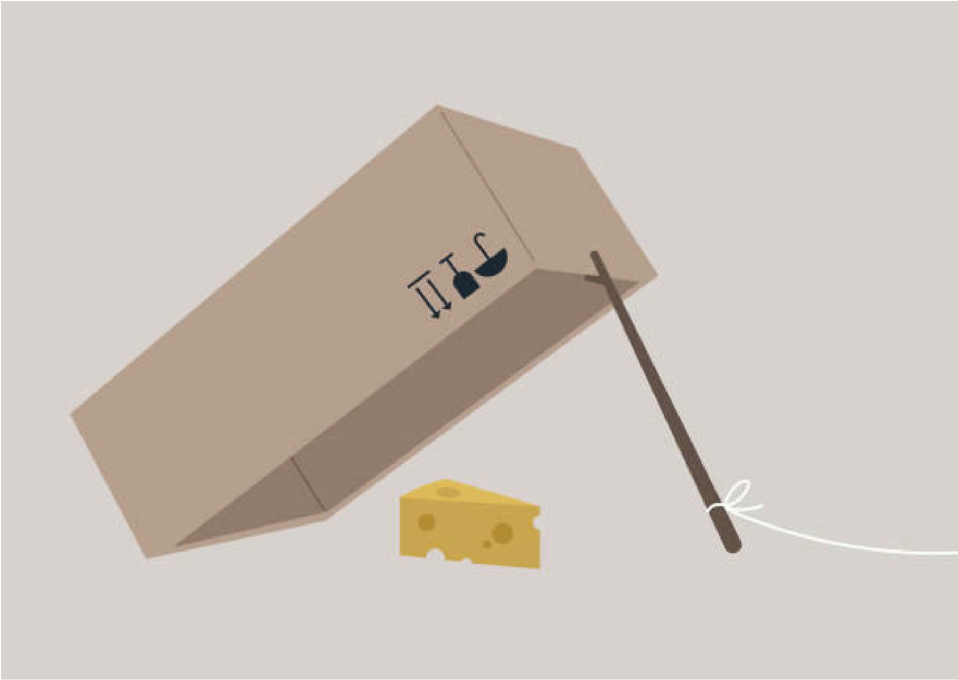

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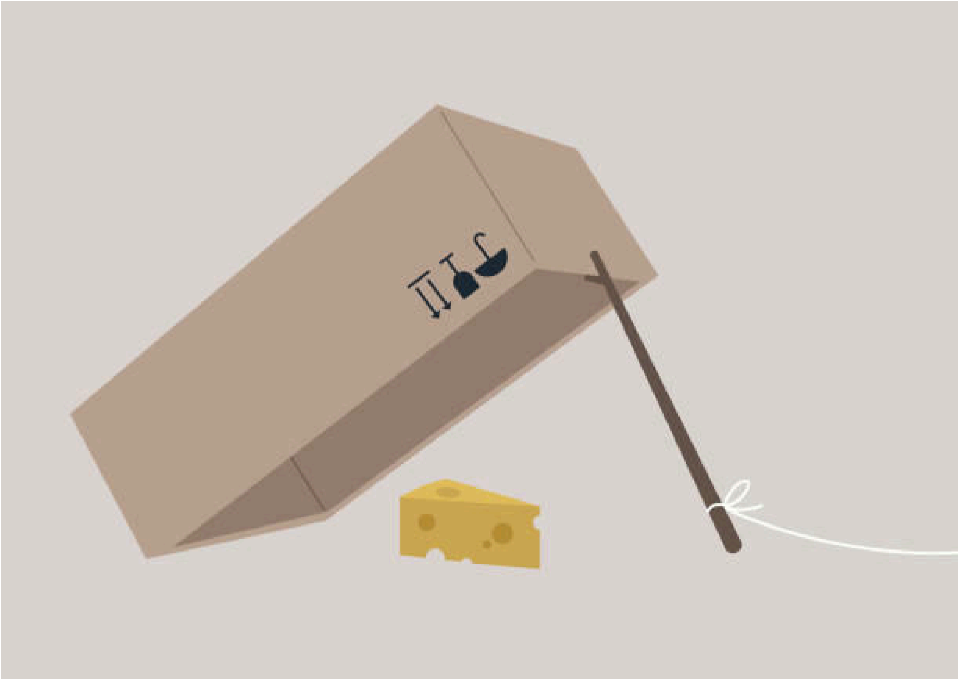

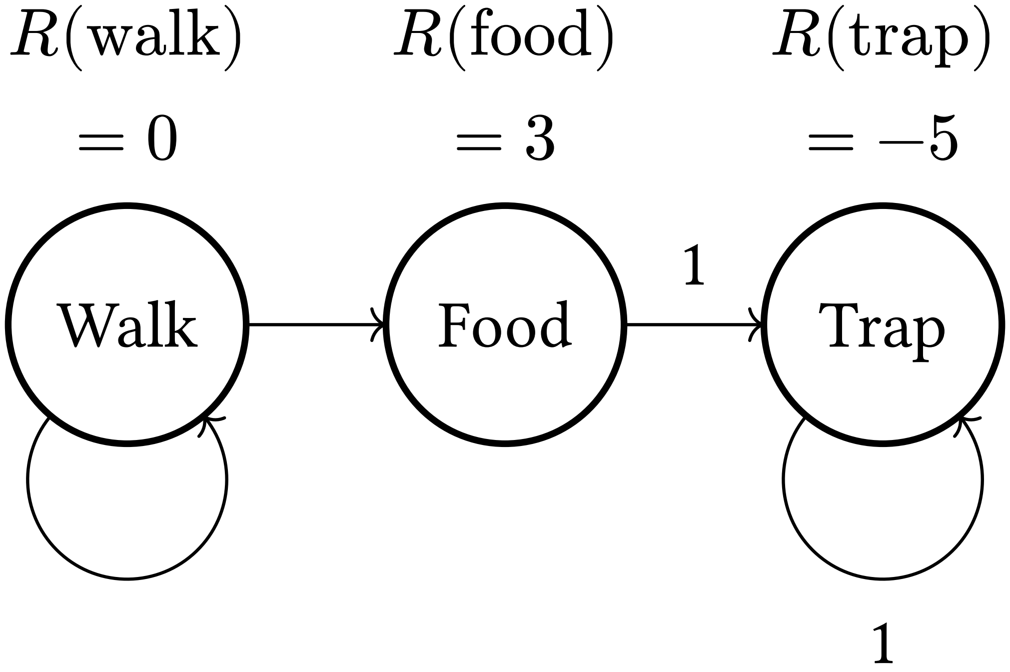

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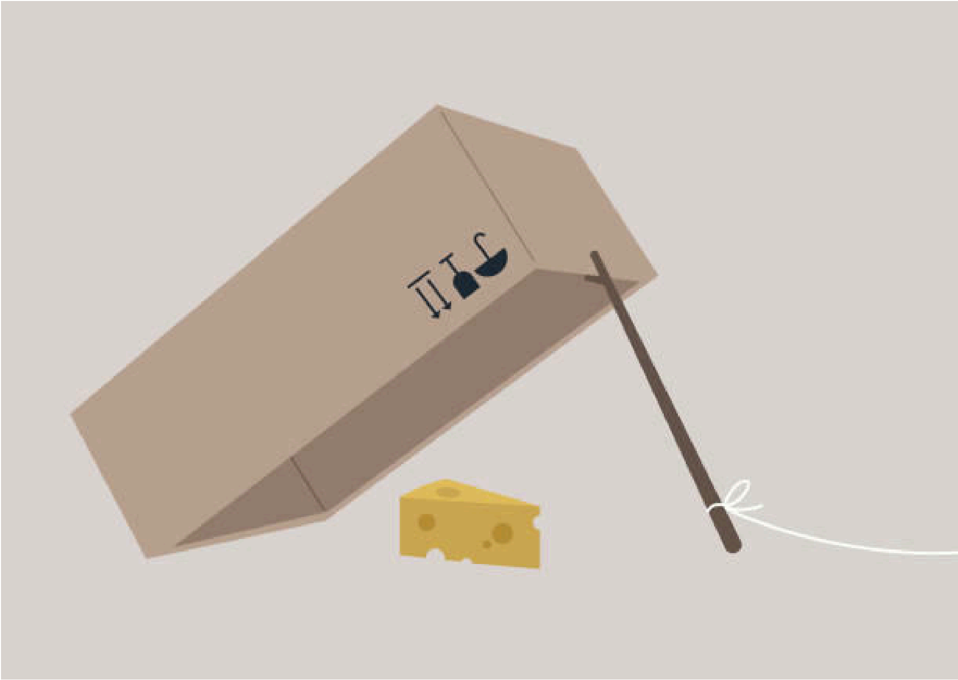

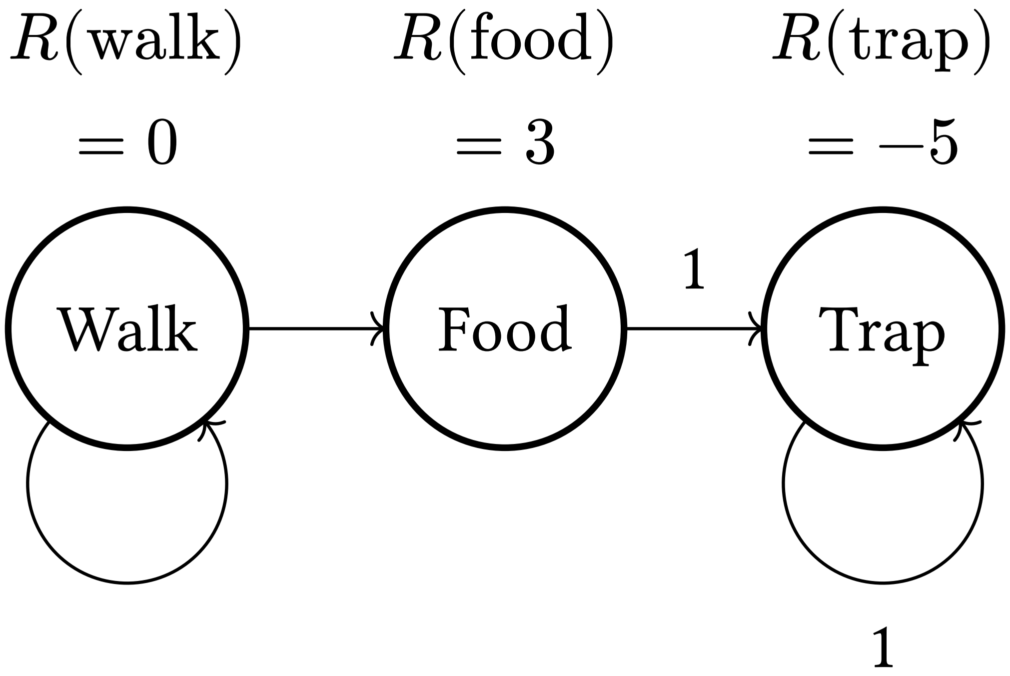

$$\arg\max_{a \in A} R(s) = \text{take the food}$$

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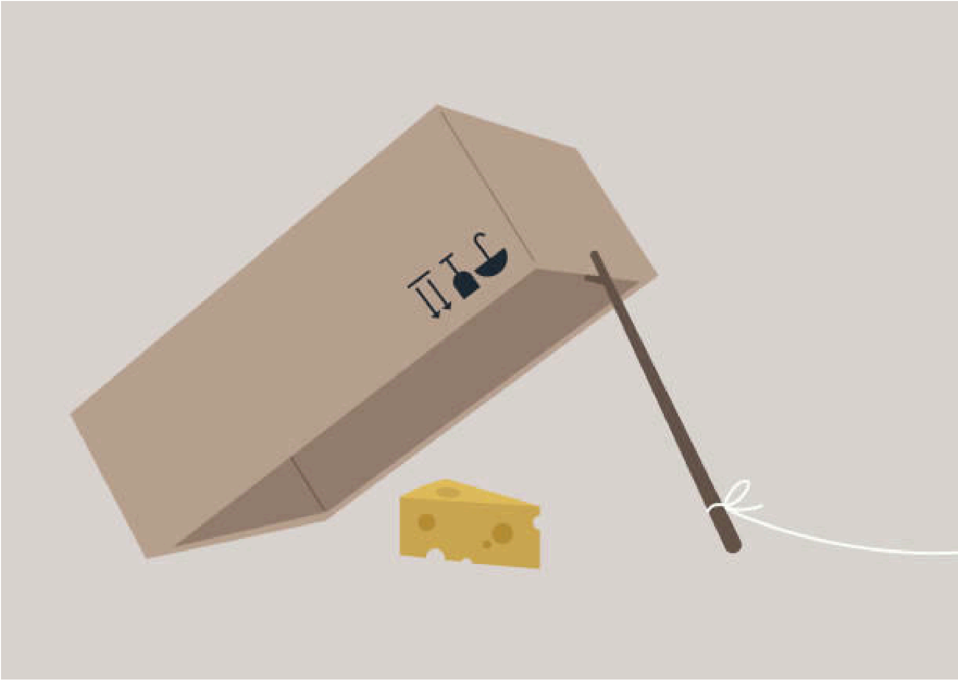

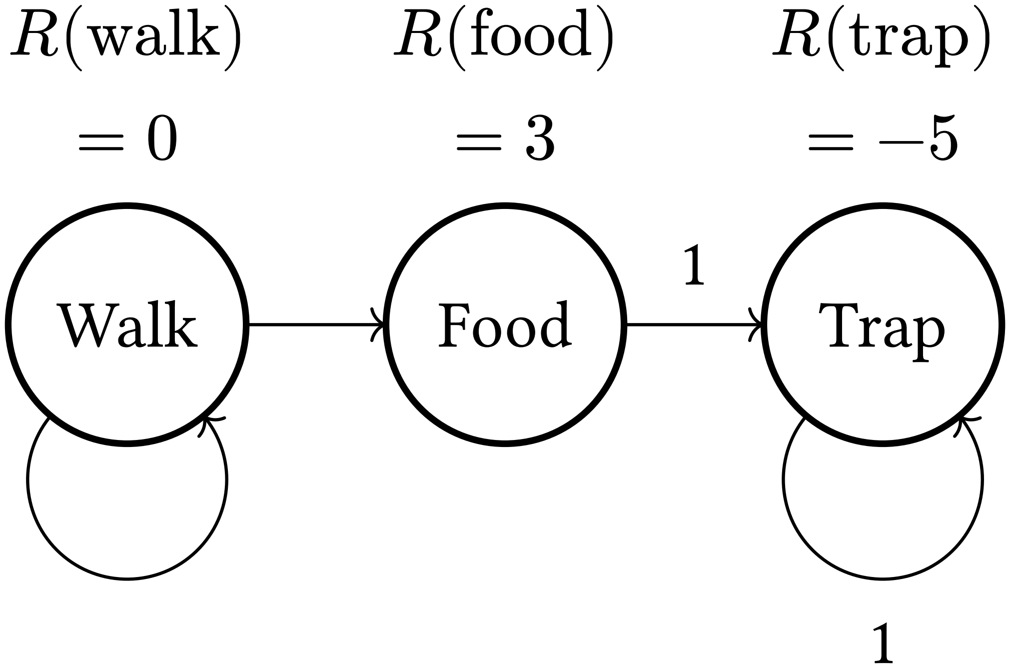

$$\arg\max_{a \in A} R(s) = \text{take the food}$$

If we maximize the reward, we are **too greedy**

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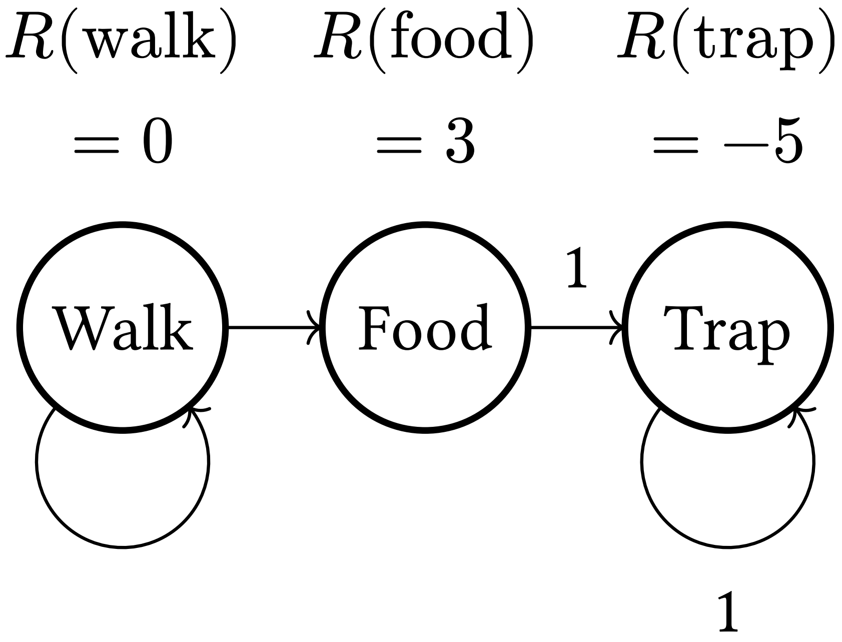
$$\begin{aligned} G(\boldsymbol{\tau}) &= G(s_0, a_0, s_1, a_1, ...) \\ &= \sum_{t=0}^{\infty} R(s_{t+1}) \end{aligned}$$

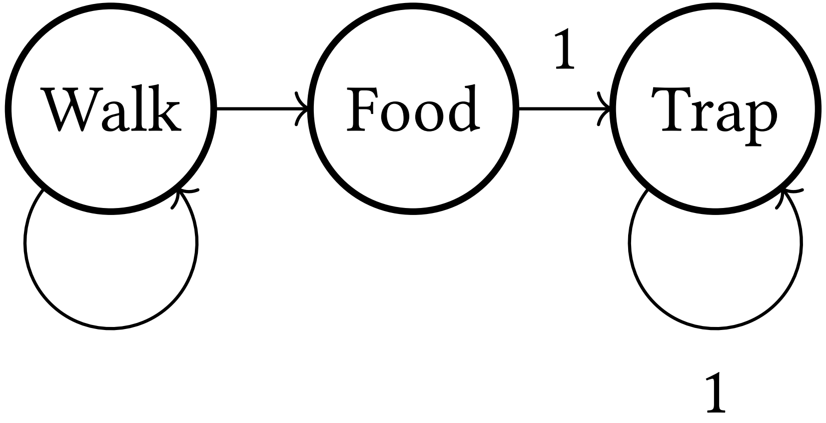
Markov Decision Processes



$$G(\boldsymbol{\tau}_{\text{greedy}}) = R(\text{food}) + R(\text{trap}) + R(\text{trap}) + \ldots = 3 - 5 - 5 - \ldots = -\infty$$

Markov Decision Processes

$R(\text{walk}) = 0$ $\quad R(\text{food}) = 3$ $\quad R(\text{trap}) = -5$



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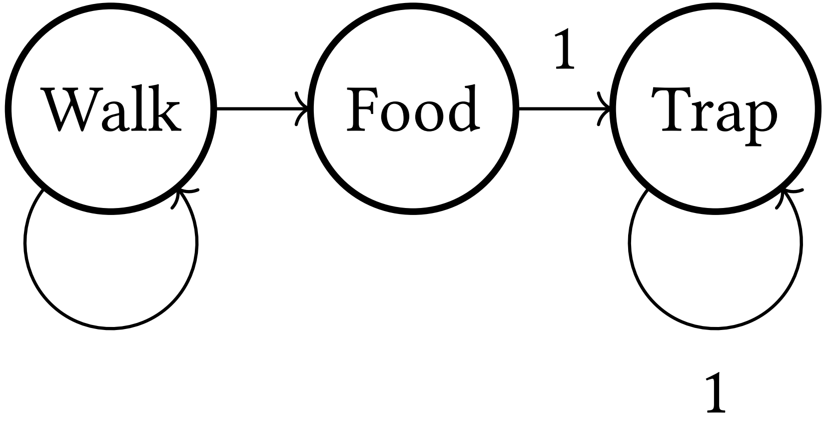

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By considering the future rewards, we can make optimal decisions

Markov Decision Processes

Consider one more example

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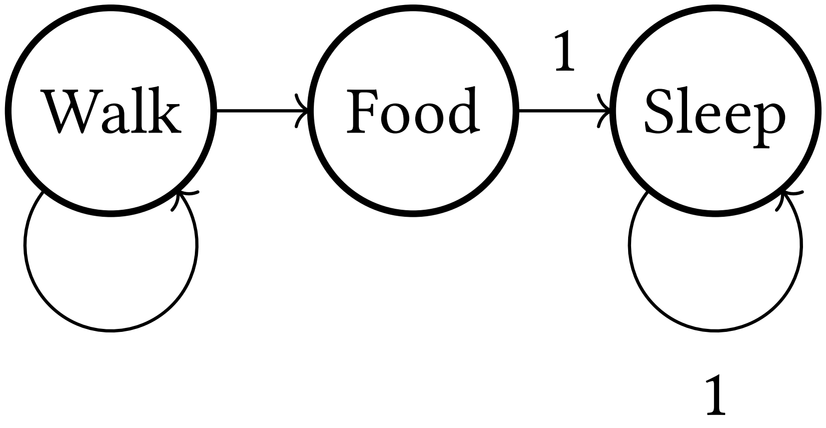
$R(\text{walk}) = 0$ $\quad R(\text{food}) = 3$ $\quad R(\text{sleep}) = 0$



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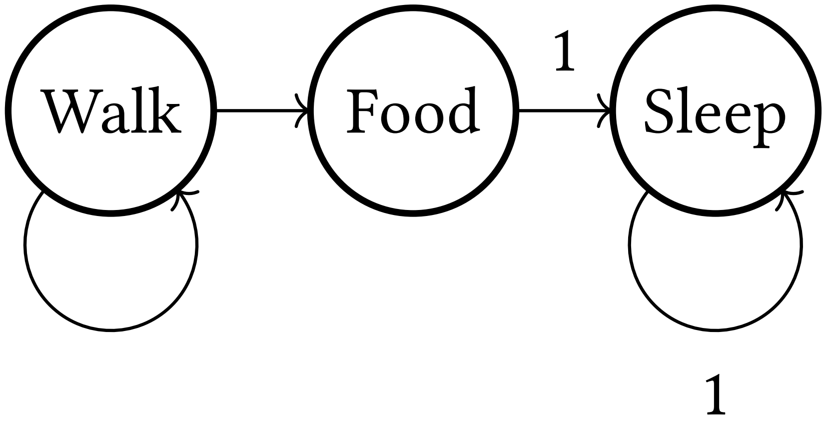

Question: What is the optimal sequence of states?

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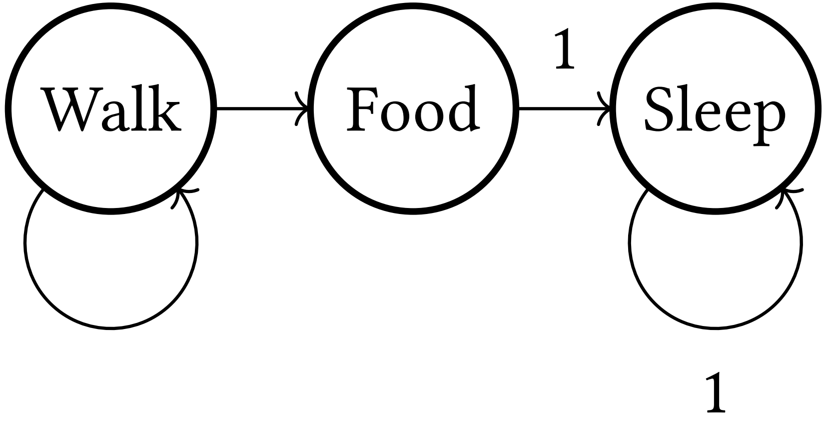

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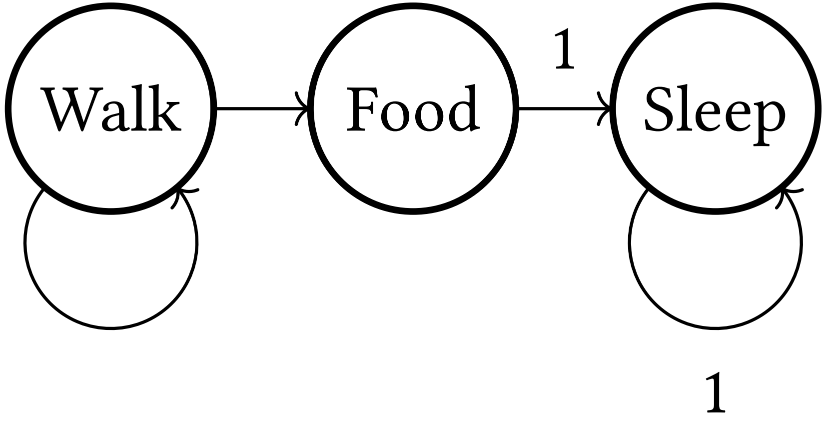

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$$G(\boldsymbol{\tau}) = \quad ? \quad = 1 + 0.9 + 0.8 + \dots$$

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With $\gamma = 0.9$

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$$G(\boldsymbol{\tau}) = \sum_{t=0}^{\infty} \gamma^t R(s_{t+1})$$

With $\gamma = 0$

$$G(\boldsymbol{\tau}) = 1 + 1 + 1 + \ldots$$

With $\gamma = 0.9$

$$G(\boldsymbol{\tau}) = (0.9^0 \cdot 1) + (0.9^1 \cdot 1) + (0.9^2 \cdot 1) +$$

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We call this the **discounted return**

The discounted return lets makes us prefer rewards sooner, like humans

Markov Decision Processes

For the rest of the course, we maximize the discounted return

$$\arg\max_{\boldsymbol{\tau}} G(\boldsymbol{\tau}) = \arg\max_{s \in S} \sum_{t=0}^{\infty} \gamma^t R(s_{t+1})$$

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If our agent maximizes the discounted return, then it is **optimal**

Markov Decision Processes

Let us review

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Definition: A Markov decision process (MDP) is a tuple (S, A, T, R, γ)

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Maximize the discounted return of the MDP

$$\arg\max_{\boldsymbol{\tau}} G(\boldsymbol{\tau}) = \arg\max_{s \in S} \sum_{t=0}^{\infty} \gamma^t R(s_{t+1})$$

You must understand the discounted return!

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Make sure you understand MDPs!

Exercise

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Design a Super Mario Bros MDP

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Exercise

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- Reward function R
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Your states are: eat mushroom, collect coins, die, game over

Compute discounted return for:

- Eat mushroom at $t = 10$
- Collect coins at $t = 11, 12$
- Die to bowser at $t = 20$
- Game over screen at $t = 21...\infty$
- $r = 0$ for other timesteps

Coding

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In this course, we will implemented MDPs using **gymnasium**

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https://gymnasium.farama.org/api/env/

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Gymnasium uses **observations** instead of **states**

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Gymnasium uses observations, but for MDPs we treat them as states

Coding

```
import gymnasium as gym

MyMDP(gym.Env):
  def __init__(self):
    self.action_space = gym.spaces.Discrete(3) # A
    self.observation_space = gym.spaces.Discrete(5) # S

  def reset(self, seed=None) -> Tuple[Observation, Dict]

  def step(self, action) -> Tuple[
    Observation, Reward, Terminated, Truncated, Dict
  ]
```

Coding

https://colab.research.google.com/drive/1rDNik5oRl27si8wdtMLE7Y41U5J2bx-I#scrollTo=9pOLI5OgKvoE

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Too many A’s last term, exam will be **difficult**