Deep Value

CISC 7404 - Decision Making

Steven Morad

University of Macau

Admin

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I noticed the last 1 hour of class everyone looks tired and sad

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Three hours is a long time to pay attention, especially at night

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Would you prefer:

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Would you prefer:

1. To have a long break (1.5h + 0.5h + 1h) in the middle?
2.
3.

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Would you prefer:

1. To have a long break (1.5h + 0.5h + 1h) in the middle?
2. No breaks, end lecture early after 2 or 2.25 hours
3.

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Would you prefer:

1. To have a long break (1.5h + 0.5h + 1h) in the middle?
2. No breaks, end lecture early after 2 or 2.25 hours
3. Keep as-is (approximately 3 hours + 10 min break)

Admin

HW1 bug:

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There was a bug in the `update_Q_TD0` starter code, thanks He Zhe!

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Before:

```
terminateds = jnp.concatenate([jnp.zeros(states.shape[0],
dtype=bool), jnp.array([1], dtype=bool)])
```

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There was a bug in the `update_Q_TD0` starter code, thanks He Zhe!

Before:

```
terminateds = jnp.concatenate([jnp.zeros(states.shape[0],
dtype=bool), jnp.array([1], dtype=bool)])
```

After:

```
terminateds = jnp.concatenate([jnp.zeros(states.shape[0] - 1,
dtype=bool), jnp.array([1], dtype=bool)])
```

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$$Q_{i+1}(s_0, a_0, \theta_\pi) = Q_i(s_0, a_0, \theta_\pi) - \alpha \cdot \eta$$

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$$\eta = Q_i(s_0, a_0, \theta_\pi) - \left(\hat{\mathbb{E}}[\mathcal{R}(s_1) \mid s_0, a_0] + \neg d_0 \gamma \max_{a \in A} Q_i(s_1, a, \theta_\pi) \right)$$

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$$Q_{i+1}(s_0, a_0, \theta_\pi) = Q_i(s_0, a_0, \theta_\pi) - \alpha \cdot \eta$$

Predicted value

$$\eta = \boxed{Q_i(s_0, a_0, \theta_\pi)} - \left(\hat{\mathbb{E}}[\mathcal{R}(s_1) \mid s_0, a_0] + \neg d_0 \gamma \max_{a \in A} Q_i(s_1, a, \theta_\pi) \right)$$

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Empirical value

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$$\eta = \overbrace{Q_i(s_0, a_0, \theta_\pi)}^{\text{Predicted value}} - \underbrace{\left(\hat{\mathbb{E}}[\mathcal{R}(s_1) \mid s_0, a_0] + \neg d_0 \gamma \max_{a \in A} Q_i(s_1, a, \theta_\pi) \right)}_{\text{Empirical value}}$$

小心! If s_1 is a terminal state, future value is 0 ($\neg d_0 =$ not terminated)

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Predicted value

$$\eta = Q_i(s_0, a_0, \theta_\pi) - \left(\hat{\mathbb{E}}[\mathcal{R}(s_1) \mid s_0, a_0] + \neg d_0 \gamma \max_{a \in A} Q_i(s_1, a, \theta_\pi) \right)$$

Empirical value

小心! If s_1 is a terminal state, future value is 0 ($\neg d_0 = \text{not terminated}$)

Without the $\neg d$ term, takes longer to train!

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Homework 2:

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Homework 2:

- Deep Q learning

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Homework 2:

- Deep Q learning
- Deep policy gradient

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Homework 2:

- Deep Q learning
- Deep policy gradient

Will release after homework 1 due date

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Next quiz in 2-3 weeks

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- Policy gradient

Review

Deep Learning Review

Deep Learning Review

We model neural networks as parameterized functions

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$$f : X \times \Theta \mapsto Y$$

Deep Learning Review

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Map an input $\boldsymbol{x} \in X$ and parameters $\boldsymbol{\theta} \in \Theta$ to output space Y

Deep Learning Review

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Map an input $\boldsymbol{x} \in X$ and parameters $\boldsymbol{\theta} \in \Theta$ to output space Y

$$f(\boldsymbol{x}, \boldsymbol{\theta})$$

Deep Learning Review

Neural networks consist of **artificial neurons**

Deep Learning Review

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$$\boldsymbol{x} \in \mathbb{R}^{d_x}, \boldsymbol{\theta} \in \mathbb{R}^{d_x + 1}$$

Deep Learning Review

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$$\boldsymbol{x} \in \mathbb{R}^{d_x}, \boldsymbol{\theta} \in \mathbb{R}^{d_x+1}$$

$$\overline{\boldsymbol{x}} = \begin{bmatrix} 1 & x_1 & x_2 & \ldots & x_{d_x} \end{bmatrix}^\top \in \mathbb{R}^{d_x+1}$$

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$$\sigma(x) = \frac{1}{1 + e^{-x}}$$

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Or ReLU

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$$\sigma(x) = \frac{1}{1 + e^{-x}}$$

Or ReLU

$$\sigma(x) = \max(0, x)$$

Deep Learning Review

We combine individual neurons into a **layer**

Deep Learning Review

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$$f : \mathbb{R}^{d_x} \times \Theta \mapsto \mathbb{R}$$

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Layer of d_y neurons:

Deep Learning Review

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Layer of d_y neurons:

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$$\Theta = \mathbb{R}^{(d_x + 1) \times d_y}$$

Deep Learning Review

For a single neuron

Deep Learning Review

For a single neuron

$$f\left(\begin{bmatrix} x_1 \\ \vdots \\ x_{d_x} \end{bmatrix}, \begin{bmatrix} \theta_0 \\ \theta_1 \\ \vdots \\ \theta_{d_x} \end{bmatrix}\right) = \sigma\left(\sum_{i=0}^{d_x} \theta_i \overline{x}_i\right)$$

Deep Learning Review

For a single neuron

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For a wide network

Deep Learning Review

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For a wide network

$$f\left(\begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_{d_x} \end{bmatrix}, \begin{bmatrix} \theta_{0,1} & \theta_{0,2} & \cdots & \theta_{0,d_y} \\ \theta_{1,1} & \theta_{1,2} & \cdots & \theta_{1,d_y} \\ \vdots & \vdots & \ddots & \vdots \\ \theta_{d_x,1} & \theta_{d_x,2} & \cdots & \theta_{d_x,d_y} \end{bmatrix}\right) = \begin{bmatrix} \sigma\left(\sum_{i=0}^{d_x} \theta_{i,1} \overline{x}_i\right) \\ \sigma\left(\sum_{i=0}^{d_x} \theta_{i,2} \overline{x}_i\right) \\ \vdots \\ \sigma\left(\sum_{i=0}^{d_x} \theta_{i,d_y} \overline{x}_i\right) \end{bmatrix}$$

Deep Learning Review

We can combine layers to create a **deep** neural network

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A wide network:

$$f(\boldsymbol{x}, \boldsymbol{\theta}) = \sigma(\boldsymbol{\theta}^\top \overline{\boldsymbol{x}})$$

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$$f_1(\boldsymbol{x}, \boldsymbol{\varphi}) = \sigma(\boldsymbol{\varphi}^\top \overline{\boldsymbol{x}})$$

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$$f_1(\boldsymbol{x}, \boldsymbol{\varphi}) = \sigma(\boldsymbol{\varphi}^\top \overline{\boldsymbol{x}}) \quad f_2(\boldsymbol{x}, \boldsymbol{\psi}) = \sigma(\boldsymbol{\psi}^\top \overline{\boldsymbol{x}})$$

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$$f(\boldsymbol{x}, \boldsymbol{\theta}) = f_\ell(...f_2(f_1(\boldsymbol{x}, \boldsymbol{\varphi}), \boldsymbol{\psi})...\boldsymbol{\xi})$$

Deep Learning Review

Written another way

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$$\boldsymbol{z}_1 = f_1(\boldsymbol{x}, \boldsymbol{\varphi}) = \sigma(\boldsymbol{\varphi}^\top \overline{\boldsymbol{x}})$$

Deep Learning Review

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$$\boldsymbol{z}_1 = f_1(\boldsymbol{x}, \boldsymbol{\varphi}) = \sigma(\boldsymbol{\varphi}^\top \overline{\boldsymbol{x}})$$

$$\boldsymbol{z}_2 = f_2(\boldsymbol{z}_{\mathbf{1}}, \boldsymbol{\psi}) = \sigma(\boldsymbol{\psi}^\top \overline{\boldsymbol{z}}_1)$$

Deep Learning Review

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$$\boldsymbol{y} = f_\ell(\boldsymbol{z}_{\ell-1}, \boldsymbol{\xi}) = \sigma(\boldsymbol{\xi}^\top \overline{\boldsymbol{z}}_{\ell-1})$$

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We call each function a **layer**

Deep Learning Review

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We call each function a **layer**

A deep neural network is made of many layers

Deep Learning Review

We can create different models for different tasks

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Standard tasks:

Deep Learning Review

We can create different models for different tasks

Standard tasks: Multi-layer perceptron (MLP)

Deep Learning Review

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Image tasks:

Deep Learning Review

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Standard tasks: Multi-layer perceptron (MLP)

Image tasks: Convolutional neural network (CNN)

Deep Learning Review

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Temporal tasks:

Deep Learning Review

We can create different models for different tasks

Standard tasks: Multi-layer perceptron (MLP)

Image tasks: Convolutional neural network (CNN)

Temporal tasks: Recurrent neural network (RNN)

Deep Learning Review

We can create different models for different tasks

Standard tasks: Multi-layer perceptron (MLP)

Image tasks: Convolutional neural network (CNN)

Temporal tasks: Recurrent neural network (RNN)

Image, temporal tasks: Transformer

Deep Learning Review

What functions can we represent using deep neural networks?

Deep Learning Review

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A deep neural network is a **universal function approximator**

Deep Learning Review

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It can approximate **any** continuous function $g(x)$ to precision η

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$$| \ g(\boldsymbol{x}) - f(\boldsymbol{x}, \boldsymbol{\theta}) \ | < \eta$$

Deep Learning Review

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A deep neural network is a **universal function approximator**

It can approximate **any** continuous function $g(x)$ to precision η

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Making the network deeper or wider decreases η

Deep Learning Review

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Very powerful finding! The basis of deep learning.

Deep Learning Review

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<u>Very powerful finding! The basis of deep learning.</u>

Although such $\boldsymbol{\theta}$ exists, it can be hard to find

Deep Learning Review

Finding $\boldsymbol{\theta}$ is an optimization problem

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In particular, we optimize a **loss function**

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$$\mathcal{L} : X \times Y \times \Theta \mapsto \mathbb{R}$$

Deep Learning Review

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In particular, we optimize a **loss function**

$$\mathcal{L} : X \times Y \times \Theta \mapsto \mathbb{R}$$

$$\arg\min_{\boldsymbol{\theta}} \mathcal{L}(\boldsymbol{x}, \boldsymbol{y}, \boldsymbol{\theta})$$

Deep Learning Review

Finding $\boldsymbol{\theta}$ is an optimization problem

In particular, we optimize a **loss function**

$$\mathcal{L} : X \times Y \times \Theta \mapsto \mathbb{R}$$

$$\arg \min_{\boldsymbol{\theta}} \mathcal{L}(\boldsymbol{x}, \boldsymbol{y}, \boldsymbol{\theta})$$

Loss function measures the error between $f(\boldsymbol{x}, \boldsymbol{\theta})$ and desired $g(\boldsymbol{x}) = \boldsymbol{y}$

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In this class, we will build loss functions from two error functions

Deep Learning Review

Square error: The squared distance over a dataset of size n

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Square error: The squared distance over a dataset of size n

$$\sum_{i=1}^{n} \sum_{j=1}^{d_y} \left(f\left(\boldsymbol{x}_{[i]}, \boldsymbol{\theta}\right)_j - g(\boldsymbol{x})_j \right)^2 = \sum_{i=1}^{n} \sum_{j=1}^{d_y} \left(f\left(\boldsymbol{x}_{[i]}, \boldsymbol{\theta}\right)_j - y_{[i],j} \right)^2$$

Deep Learning Review

Square error: The squared distance over a dataset of size n

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Cross entropy error: The categorical error over a dataset of size n

Deep Learning Review

Square error: The squared distance over a dataset of size n

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Cross entropy error: The categorical error over a dataset of size n

$$- \sum_{i=1}^{n} \sum_{j=1}^{d_y} P\left(g\left(\boldsymbol{x}_{[i]}\right)_j \mid \boldsymbol{x}_{[i]} \right) \log f\left(\boldsymbol{x}_{[i]}, \boldsymbol{\theta}\right)_j = - \sum_{i=1}^{n} \sum_{j=1}^{d_y} P\left(y_{[i],j} \mid \boldsymbol{x}_{[i]} \right) \log f\left(\boldsymbol{x}_{[i]}, \boldsymbol{\theta}\right)_j$$

Deep Learning Review

Square error:

$$\sum_{i=1}^{n} \sum_{j=1}^{d_y} \left(f\left(\boldsymbol{x}_{[i]}, \boldsymbol{\theta}\right)_j - y_{[i],j} \right)^2$$

Deep Learning Review

Square error:

$$\sum_{i=1}^{n} \sum_{j=1}^{d_y} \left(f\left(\boldsymbol{x}_{[i]}, \boldsymbol{\theta}\right)_j - y_{[i],j} \right)^2$$

Cross entropy error:

$$- \sum_{i=1}^{n} \sum_{j=1}^{d_y} P\left(y_{[i],j} \mid \boldsymbol{x}_{[i]}\right) \log f\left(\boldsymbol{x}_{[i]}, \boldsymbol{\theta}\right)_j$$

Deep Learning Review

Square error:

$$\sum_{i=1}^{n} \sum_{j=1}^{d_y} \left(f\left(\boldsymbol{x}_{[i]}, \boldsymbol{\theta}\right)_j - y_{[i],j} \right)^2$$

Cross entropy error:

$$- \sum_{i=1}^{n} \sum_{j=1}^{d_y} P\left(y_{[i],j} \mid \boldsymbol{x}_{[i]}\right) \log f\left(\boldsymbol{x}_{[i]}, \boldsymbol{\theta}\right)_j$$

Question: Which one will we use for Q learning?

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Question: Which one will we use for Q learning?

Answer: Predict a scalar (expected return), so square error (regression)

Deep Learning Review

We can use both errors in a loss function

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$$\mathcal{L}(\boldsymbol{X}, \boldsymbol{Y}, \boldsymbol{\theta}) = \sum_{i=1}^{n} \sum_{j=1}^{d_y} \left(f\left(\boldsymbol{x}_{[i]}, \boldsymbol{\theta}\right)_j - y_{[i],j} \right)^2$$

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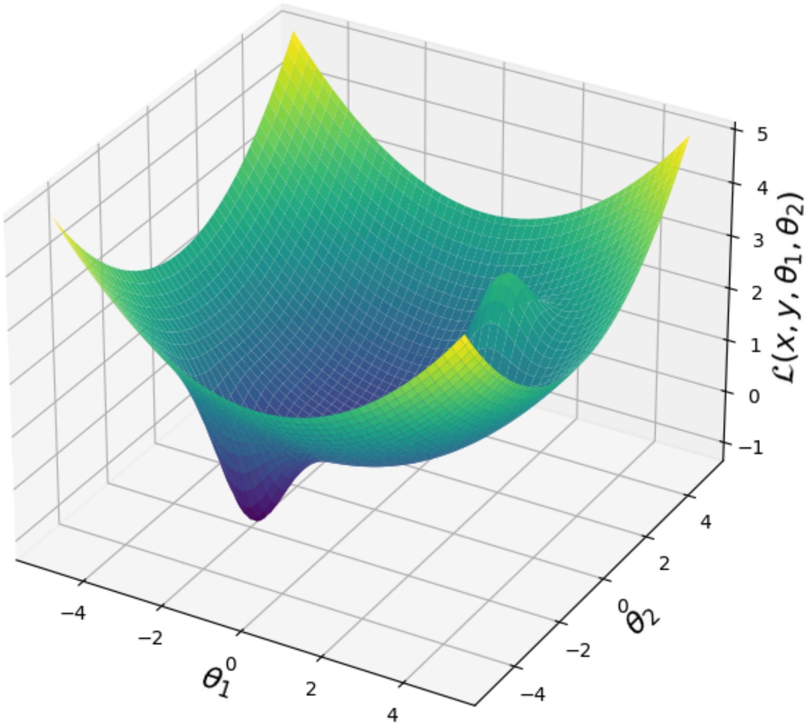

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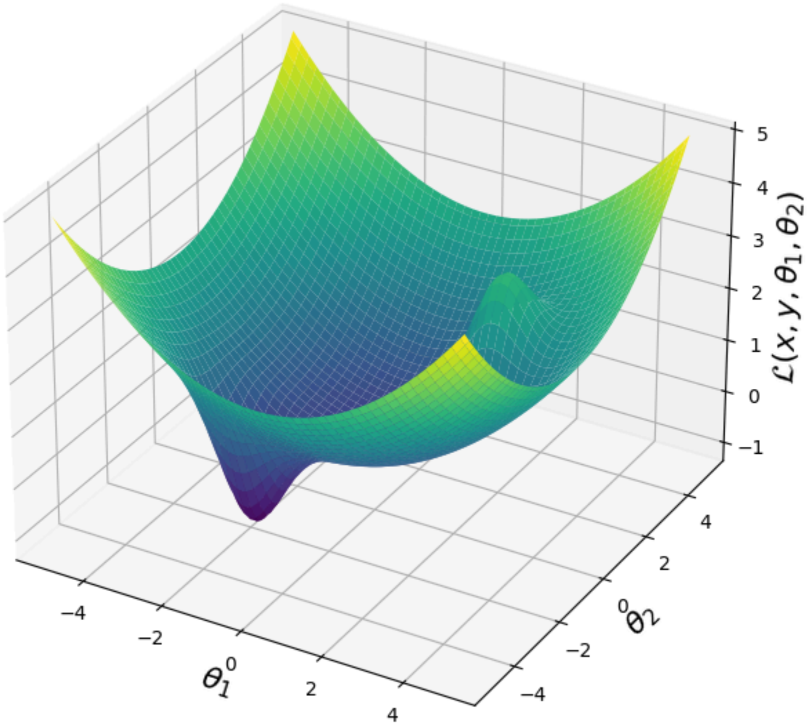

Deep Learning Review

function GRADIENT DESCENT($\boldsymbol{X}$, $\boldsymbol{Y}$, $\mathcal{L}$, t, α)

    ▷ Randomly initialize parameters

    $\boldsymbol{\theta} \leftarrow \text{Glorot}()$

    for $i \in 1 ... t$ **do**

        ▷ Compute the gradient of the loss

        $\boldsymbol{J} \leftarrow \nabla_{\boldsymbol{\theta}} \mathcal{L}(\boldsymbol{X}, \boldsymbol{Y}, \boldsymbol{\theta})$

        ▷ Update the parameters using the negative gradient

        $\boldsymbol{\theta} \leftarrow \boldsymbol{\theta} - \alpha \cdot \boldsymbol{J}$

    return $\boldsymbol{\theta}$

Deep Learning Review

We can put it all together in jax and equinox

```
from jax import random
from equinox import nn

seed = random.key(0)
key, *net_keys= random.split(seed, 4)
net = nn.Sequential([
  nn.Linear(d_x, d_h, key=net_keys[0]),
  nn.Lambda(jax.nn.leaky_relu),
  nn.Linear(d_h, d_h, key=net_keys[1]),
  nn.Lambda(jax.nn.leaky_relu),
  nn.Linear(d_h, d_y, key=net_keys[2]),
])
```

Deep Learning Review

We can extract parameters using `eqx.partition`

```
import equinox as eqx
# Get all arrays (parameters) in the network
theta, f = eqx.partition(net, eqx.is_array)
# Add one to every parameter
theta = jax.tree.map(theta, lambda x: x + 1)
# Put the new parameters back into network
net = eqx.combine(theta, f)
```

Deep Learning Review

```python
import jax.numpy as jnp
import equinox as eqx

def L_square(net, x, y):
  # vmap applies network to batch of data
  prediction = eqx.filter_vmap(net)(x)
  return ((prediction - y) ** 2).mean()

def L_cross_entropy(net, x, y):
  # Net outputs probabilities
  # And y is one-hot, e.g. [0, 0, 1, 0]
  prediction = eqx.filter_vmap(net)(x)
  return -(y * jnp.log(prediction)).sum(-1).mean()
```

Deep Learning Review

```python
import optax
import equinox as eqx
opt = optax.adam(learning_rate=3e-4)
# Adam needs to track momentum and variance
opt_state = opt.init(eqx.filter(net, eqx.is_array))
# Gradient of loss function is a function
grad_L = eqx.filter_grad(L_square)
# Evaluate grad_L at x, y, theta to find J
J = grad_L(net, x, y)
# Compute parameter update using J (adam)
updates, opt_state = opt.update(
    grads, opt_state, params=eqx.filter(net, eqx.is_array)
)
net = eqx.apply_updates(net, updates) # Update params
```

Deep Learning Review

```python
def train_one_batch(net, batch, opt_state):
  x, y = batch
  grads = eqx.filter_grad(L_square)(net, x, y)
  updates, opt_state = opt.update(
    grads, opt_state, params=eqx.filter(net, eqx.is_array)
  )
  net = eqx.apply_updates(net, updates) # Update params
  return net, opt_state

for epoch in range(num_epochs):
  for batch in dataset:
    # Can use eqx.filter_jit(f) for speedup
    net, opt_state = train_one_batch(net, batch, opt_state)
```

Deep Learning Review

Dirty secret of deep learning:

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Dirty secret of deep learning: We do not understand deep learning

Biological inspiration, theoretical bounds, and mathematical guarantees

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This applies even more to **deep reinforcement learning**

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Let me demonstrate this with an example problem

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This would be a large matrix!

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Discretize to 128×128 grid squares

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Very large but not infinite

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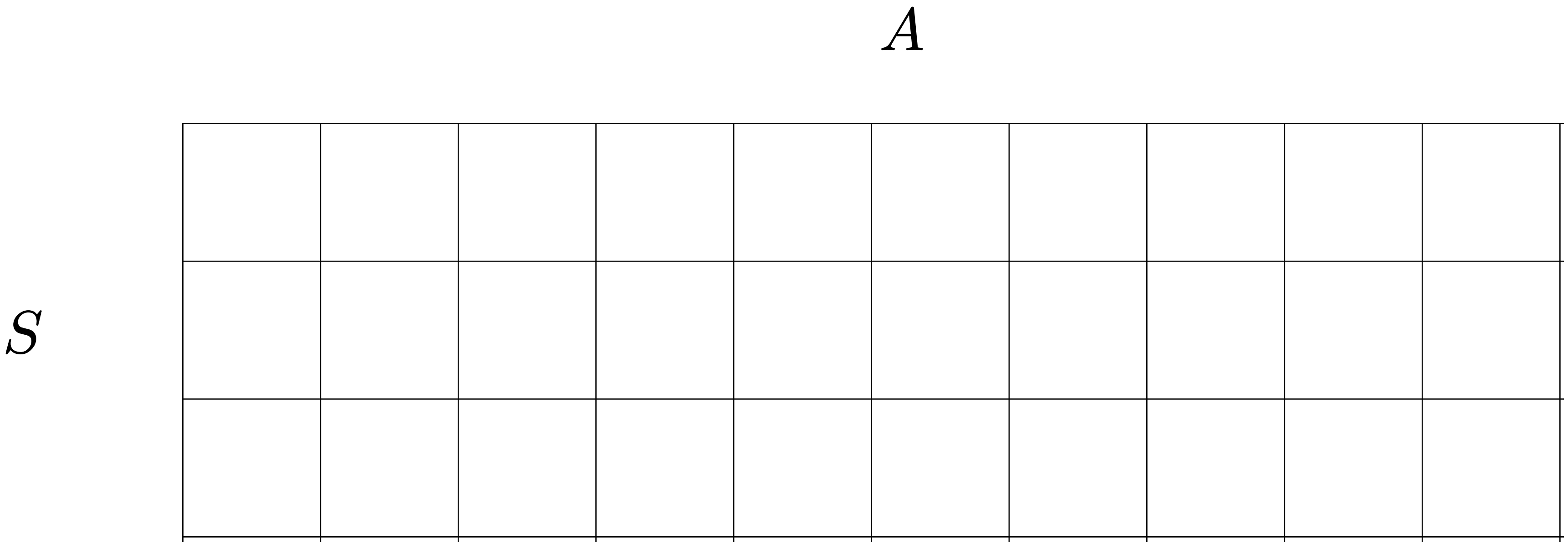

We must update Q for each s, a separately

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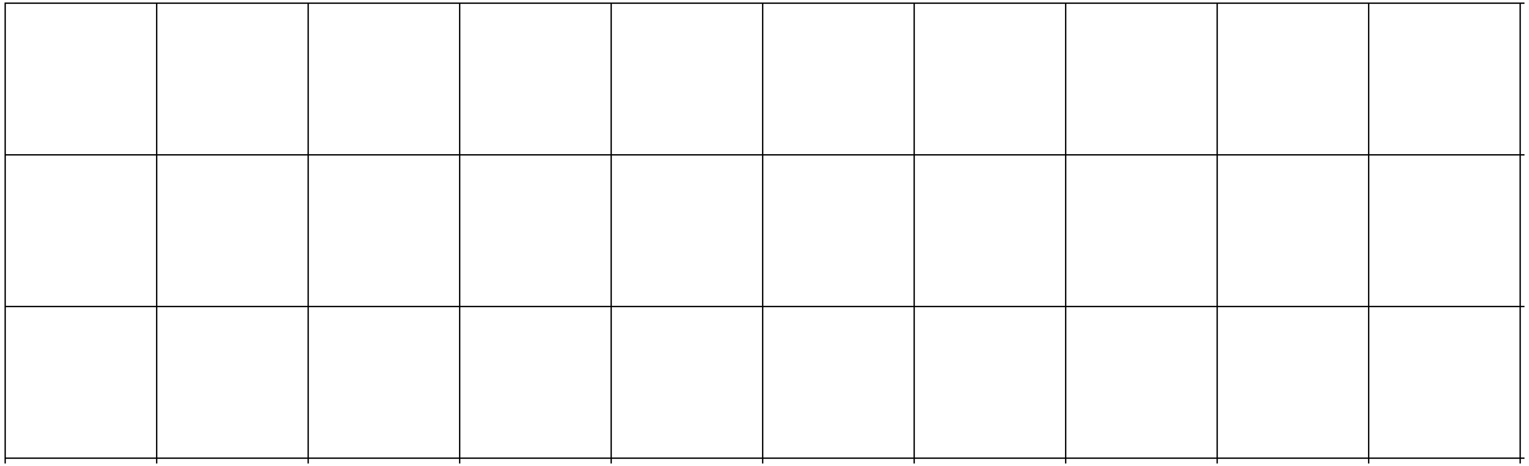

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With TD updates, updating one cell means we must update all cells

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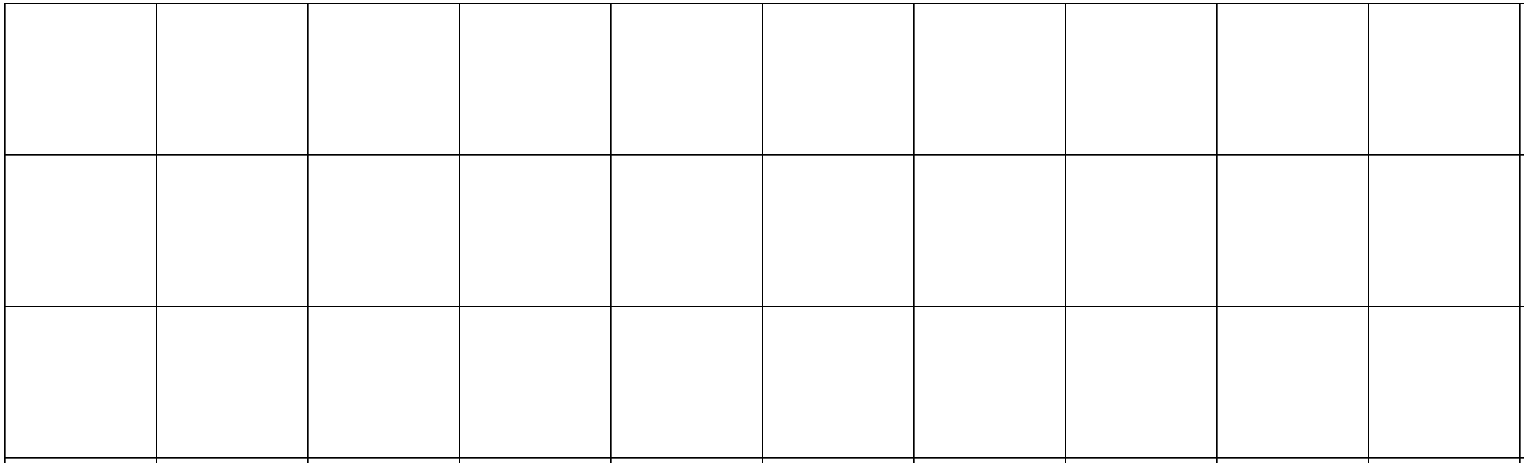

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It can take many states and actions for Q converge (HW up to 100k)

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There is a lower sample complexity bound on convergence[1]

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$64 \times A$ petabytes of rewards to learn Q

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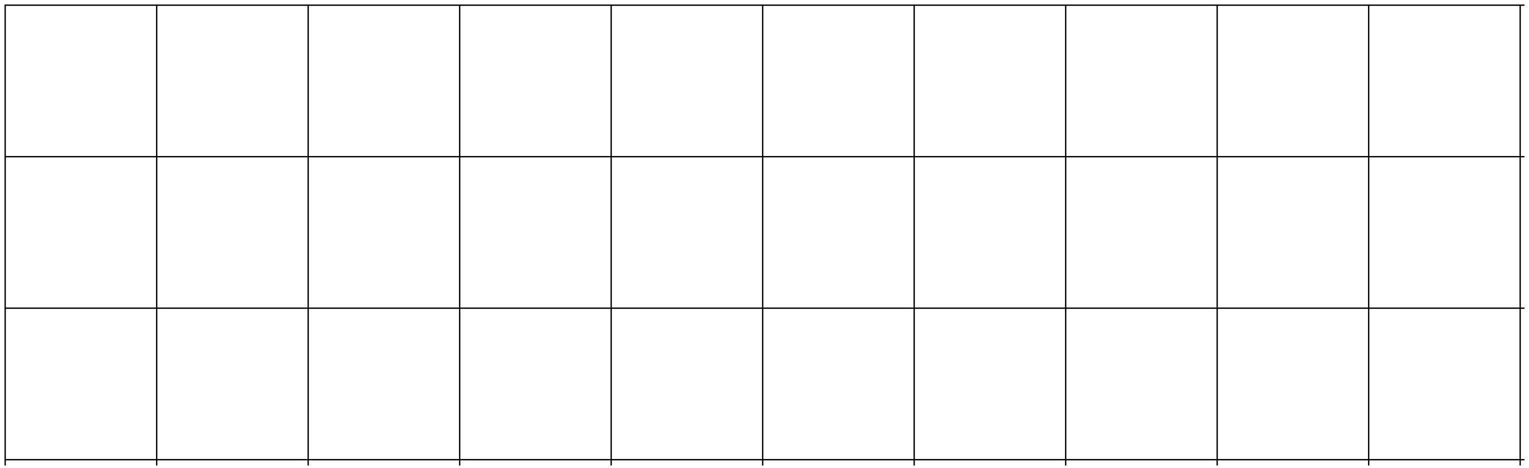

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We can say the same for deep RL

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Answer: Monte Carlo and Temporal Difference

Deep Q Learning

Monte Carlo Objective:

Deep Q Learning

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Deep Q Learning

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Deep Q Learning

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Question: Can we replace $\mathbb{E}$ with something we know? Hint: Gambling

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Monte Carlo is a famous casino. We approximate the expected return by "gambling" over the episode

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Can train over batch/dataset $\boldsymbol{X}$ containing many episodes $\boldsymbol{x}$

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Now, lets do the TD loss function

Deep Q Learning

$$\arg\min_{\theta_Q} \left[\sum_{s_0 \in S} \sum_{a_0 \in A} \right.$$

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Rewrite over the episode $\boldsymbol{x}$

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Rewrite over the episode $\boldsymbol{x}$

$$\arg\min_{\boldsymbol{\theta}_Q} \mathcal{L}(\boldsymbol{x}, \boldsymbol{\theta}_Q) = \arg\min_{\theta_Q} \Bigg[\sum_{s_i, a_i, d_i, s_{i+1} \in \boldsymbol{x}}$$

$$\left(Q(s_i, a_i, \theta_\pi, \boldsymbol{\theta}_Q) - \left(\hat{\mathbb{E}}[\mathcal{R}(s_{i+1}) \mid s_i, a_i] + \neg d_0 \gamma \max_{a \in A} Q(s_{i+1}, a, \theta_\pi, \boldsymbol{\theta}_Q) \right) \right)^2 \Bigg]$$

Deep Q Learning

$$\arg\min_{\boldsymbol{\theta}_Q} \mathcal{L}(\boldsymbol{x}, \boldsymbol{\theta}_Q) = \arg\min_{\theta_Q} \left[\sum_{s_i, a_i, d_i, s_{i+1} \in \boldsymbol{x}} \left(Q(s_i, a_i, \theta_\pi, \boldsymbol{\theta}_Q) - \left(\hat{\mathbb{E}}[\mathcal{R}(s_{i+1}) \mid s_i, a_i] + \neg d_0 \gamma \max_{a \in A} Q(s_{i+1}, a, \theta_\pi, \boldsymbol{\theta}_Q) \right) \right)^2 \right]$$

Deep Q Learning

$$\arg\min_{\boldsymbol{\theta}_Q} \mathcal{L}(\boldsymbol{x}, \boldsymbol{\theta}_Q) = \arg\min_{\theta_Q} \left[\sum_{s_i, a_i, d_i, s_{i+1} \in \boldsymbol{x}} \right.$$

$$\left. \left(Q(s_i, a_i, \theta_\pi, \boldsymbol{\theta}_Q) - \left(\hat{\mathbb{E}}[\mathcal{R}(s_{i+1}) \mid s_i, a_i] + \neg d_0 \gamma \max_{a \in A} Q(s_{i+1}, a, \theta_\pi, \boldsymbol{\theta}_Q) \right) \right)^2 \right]$$

Empirically compute expected reward

Deep Q Learning

$$\arg \min_{\boldsymbol{\theta}_Q} \mathcal{L}(\boldsymbol{x}, \boldsymbol{\theta}_Q) = \arg \min_{\theta_Q} \left[\sum_{s_i, a_i, d_i, s_{i+1} \in \boldsymbol{x}} \left(Q(s_i, a_i, \theta_\pi, \boldsymbol{\theta}_Q) - \left(\hat{\mathbb{E}}[\mathcal{R}(s_{i+1}) \mid s_i, a_i] + \neg d_0 \gamma \max_{a \in A} Q(s_{i+1}, a, \theta_\pi, \boldsymbol{\theta}_Q) \right) \right)^2 \right]$$

Empirically compute expected reward

$$\arg \min_{\boldsymbol{\theta}_Q} \mathcal{L}(\boldsymbol{x}, \boldsymbol{\theta}_Q) = \arg \min_{\theta_Q} \sum_{\substack{s_i, a_i, r_i, d_i, s_{i+1} \\ \in \boldsymbol{x}}} \left(Q(s_i, a_i, \theta_\pi, \boldsymbol{\theta}_Q) - \left(r_i + \neg d_0 \gamma \arg \max_{a \in A} Q(s_i, a, \theta_\pi, \boldsymbol{\theta}_Q) \right) \right)^2$$

Deep Q Learning

$$\arg\min_{\boldsymbol{\theta}_Q} \mathcal{L}(\boldsymbol{x}, \boldsymbol{\theta}_Q) = \arg\min_{\theta_Q}$$

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Do it over a batch

Deep Q Learning

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Do it over a batch

$$\arg\min_{\boldsymbol{\theta}_Q} \mathcal{L}(\boldsymbol{x}, \boldsymbol{\theta}_Q) = \arg\min_{\theta_Q} \sum_{\boldsymbol{x}_{[j]} \in \boldsymbol{X}}$$

$$\sum_{\substack{s_i, a_i, r_i, d_i, s_{i+1} \\ \in \boldsymbol{x}}} \left(Q(s_i, a_i, \theta_\pi, \boldsymbol{\theta}_Q) - \left(r_i + \neg d_0 \gamma \arg\max_{a \in A} Q(s_i, a, \theta_\pi, \boldsymbol{\theta}_Q) \right) \right)^2$$

Deep Q Learning

To summarize, our two loss functions:

Deep Q Learning

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$$\arg\min_{\boldsymbol{\theta}_Q} \mathcal{L}(\boldsymbol{X}, \boldsymbol{\theta}_Q) = \arg\min_{\theta_Q} \left[\sum_{\boldsymbol{x}_{[j]} \in \boldsymbol{X}} \sum_{\substack{s_i, a_i, r_i \\ \in \boldsymbol{x}_{[j]}}} \left(Q(s_i, a_i, \theta_\pi, \boldsymbol{\theta}_Q) - \sum_{t=i}^{\infty} \gamma^{t-i} r_t \right)^2 \right]$$

Deep Q Learning

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$$\arg\min_{\boldsymbol{\theta}_Q} \mathcal{L}(\boldsymbol{x}, \boldsymbol{\theta}_Q) = \arg\min_{\theta_Q} \sum_{\boldsymbol{x}_{[j]} \in \boldsymbol{X}}$$

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Deep Q Learning

Can optimize both loss functions using gradient descent

Deep Q Learning

Can optimize both loss functions using gradient descent

RL optimization is more difficult than supervised learning

Deep Q Learning

Can optimize both loss functions using gradient descent

RL optimization is more difficult than supervised learning

Supervised Learning:

Deep Q Learning

Can optimize both loss functions using gradient descent

RL optimization is more difficult than supervised learning

Supervised Learning:

- Static inputs

Deep Q Learning

Can optimize both loss functions using gradient descent

RL optimization is more difficult than supervised learning

Supervised Learning:

- Static inputs
- Static labels

Deep Q Learning

Can optimize both loss functions using gradient descent

RL optimization is more difficult than supervised learning

Supervised Learning:

- Static inputs
- Static labels
- Limited dataset

Deep Q Learning

Can optimize both loss functions using gradient descent

RL optimization is more difficult than supervised learning

Supervised Learning:

- Static inputs
- Static labels
- Limited dataset
  - Human can clean
  - Bad to overfit

Deep Q Learning

Can optimize both loss functions using gradient descent

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Supervised Learning:

- Static inputs
- Static labels
- Limited dataset
  - Human can clean
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Reinforcement Learning:

Deep Q Learning

Can optimize both loss functions using gradient descent

RL optimization is more difficult than supervised learning

Supervised Learning:

- Static inputs
- Static labels
- Limited dataset
  - Human can clean
  - Bad to overfit

Reinforcement Learning:

- Inputs change as θ_π changes

Deep Q Learning

Can optimize both loss functions using gradient descent

RL optimization is more difficult than supervised learning

Supervised Learning:

- Static inputs
- Static labels
- Limited dataset
  - Human can clean
  - Bad to overfit

Reinforcement Learning:

- Inputs change as θ_π changes
  - Visit new/different states

Deep Q Learning

Can optimize both loss functions using gradient descent

RL optimization is more difficult than supervised learning

Supervised Learning:

- Static inputs
- Static labels
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  - Human can clean
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Reinforcement Learning:

- Inputs change as θ_π changes
  - Visit new/different states
- Labels change as θ_π changes

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Supervised Learning:

- Static inputs
- Static labels
- Limited dataset
  - Human can clean
  - Bad to overfit

Reinforcement Learning:

- Inputs change as θ_π changes
  - Visit new/different states
- Labels change as θ_π changes
  - $\mathbb{E}[\mathcal{G}(\boldsymbol{\tau}) \mid \theta_\pi]$
- Infinite dataset

Deep Q Learning

Can optimize both loss functions using gradient descent

RL optimization is more difficult than supervised learning

Supervised Learning:

- Static inputs
- Static labels
- Limited dataset
  - Human can clean
  - Bad to overfit

Reinforcement Learning:

- Inputs change as θ_π changes
  - Visit new/different states
- Labels change as θ_π changes
  - $\mathbb{E}[\mathcal{G}(\boldsymbol{\tau}) \mid \theta_\pi]$
- Infinite dataset
  - Can always collect from env
  - Bad θ_π means bad dataset
  - Overfitting no problem

Experience Replay

Experience Replay

Optimization is difficult in RL

Experience Replay

Optimization is difficult in RL

Most RL papers train for 10M-10B environment steps

Experience Replay

Optimization is difficult in RL

Most RL papers train for 10M-10B environment steps

It takes a long time to train a deep Q function

Experience Replay

Optimization is difficult in RL

Most RL papers train for 10M-10B environment steps

It takes a long time to train a deep Q function

Let us see if we can improve training speed

Experience Replay

```
for epoch in range(num_epochs):
  terminated = False
  s = env.reset()
  episode = []
  # Step between 1 and infinity times to get one episode
  while not terminated:
    a = policy(s, theta_Q)
    next_s, r, d = env.step(action)
    episode.append([s, a, r, d, next_s])
  # Compute gradient over episode
  J = grad(L)(theta_Q, episode)
  theta_Q = update(theta_Q, grad)
```

Experience Replay

```
for epoch in range(num_epochs):
  terminated = False
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```

Question: Which part is slowest?

Experience Replay

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for epoch in range(num_epochs):
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  # Compute gradient over episode
  J = grad(L)(theta_Q, episode)
  theta_Q = update(theta_Q, grad)
```

Question: Which part is slowest? **Answer:** Collecting episodes

Experience Replay

```python
for epoch in range(num_epochs):
  terminated = False
  s = env.reset()
  episode = []
  # Step between 1 and infinity times to get one episode
  while not terminated:
    a = policy(s, theta_Q)
    next_s, r, d = env.step(action)
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  theta_Q = update(theta_Q, grad)
```

Experience Replay

```
for epoch in range(num_epochs):
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  # Step between 1 and infinity times to get one episode
  while not terminated:
    a = policy(s, theta_Q)
    next_s, r, d = env.step(action)
    episode.append([s, a, r, d, next_s])
  # Compute gradient over episode
  J = grad(L)(theta_Q, episode)
  theta_Q = update(theta_Q, grad)
```

Collect episode, train, throw away episode, start again

Experience Replay

What if we reuse episodes?

Experience Replay

What if we reuse episodes?

```
episodes = []
for epoch in range(num_epochs):
  terminated = False
  s = env.reset()
  episode = []
  while not terminated:
    a = policy(s, theta_Q)
    next_s, r, d = env.step(action)
    episode.append([s, a, r, d, next_s])
  episodes.append(episode)
  J = grad(L)(theta_Q, episodes) # Train over ALL episodes
  theta_Q = update(theta_Q, grad)
```

Experience Replay

When we reuse episodes, we call it **experience replay**

Experience Replay

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Store episodes in a **replay buffer**
(list)

Experience Replay

When we reuse episodes, we call it **experience replay**

Store episodes in a **replay buffer** (list)

$$B_t = \begin{bmatrix} \boldsymbol{s}_1 & \boldsymbol{a}_1 & \boldsymbol{r}_1 & \boldsymbol{d}_1 \\ \vdots & \vdots & \vdots & \vdots \\ \boldsymbol{s}_t & \boldsymbol{a}_t & \boldsymbol{r}_t & \boldsymbol{d}_t \end{bmatrix}$$

Experience Replay

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Create a dataset from the buffer

Experience Replay

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Create a dataset from the buffer

$$\boldsymbol{X}_t = \begin{bmatrix} \boldsymbol{s}_{31} & \boldsymbol{a}_{31} & \boldsymbol{r}_{31} & \boldsymbol{d}_{31} \\ \vdots & \vdots & \vdots & \vdots \\ \boldsymbol{s}_4 & \boldsymbol{a}_4 & \boldsymbol{r}_4 & \boldsymbol{d}_4 \end{bmatrix}$$

Experience Replay

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$$\boldsymbol{B}_t = \begin{bmatrix} \boldsymbol{s}_1 & \boldsymbol{a}_1 & \boldsymbol{r}_1 & \boldsymbol{d}_1 \\ \vdots & \vdots & \vdots & \vdots \\ \boldsymbol{s}_t & \boldsymbol{a}_t & \boldsymbol{r}_t & \boldsymbol{d}_t \end{bmatrix}$$

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$$\boldsymbol{X}_t = \begin{bmatrix} \boldsymbol{s}_{31} & \boldsymbol{a}_{31} & \boldsymbol{r}_{31} & \boldsymbol{d}_{31} \\ \vdots & \vdots & \vdots & \vdots \\ \boldsymbol{s}_4 & \boldsymbol{a}_4 & \boldsymbol{r}_4 & \boldsymbol{d}_4 \end{bmatrix}$$

Train on the dataset

Experience Replay

When we reuse episodes, we call it **experience replay**

Store episodes in a **replay buffer** (list)

$$\boldsymbol{B}_t = \begin{bmatrix} \boldsymbol{s}_1 & \boldsymbol{a}_1 & \boldsymbol{r}_1 & \boldsymbol{d}_1 \\ \vdots & \vdots & \vdots & \vdots \\ \boldsymbol{s}_t & \boldsymbol{a}_t & \boldsymbol{r}_t & \boldsymbol{d}_t \end{bmatrix}$$

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Train on the dataset

$$\arg\min_{\boldsymbol{\theta}_Q} \mathcal{L}(\boldsymbol{X}_t, \boldsymbol{\theta}_Q)$$

Experience Replay

When we reuse episodes, we call it **experience replay**

Store episodes in a **replay buffer** (list)

$$\boldsymbol{B}_t = \begin{bmatrix} \boldsymbol{s}_1 & \boldsymbol{a}_1 & \boldsymbol{r}_1 & \boldsymbol{d}_1 \\ \vdots & \vdots & \vdots & \vdots \\ \boldsymbol{s}_t & \boldsymbol{a}_t & \boldsymbol{r}_t & \boldsymbol{d}_t \end{bmatrix}$$

Create a dataset from the buffer

$$\boldsymbol{X}_t = \begin{bmatrix} \boldsymbol{s}_{31} & \boldsymbol{a}_{31} & \boldsymbol{r}_{31} & \boldsymbol{d}_{31} \\ \vdots & \vdots & \vdots & \vdots \\ \boldsymbol{s}_4 & \boldsymbol{a}_4 & \boldsymbol{r}_4 & \boldsymbol{d}_4 \end{bmatrix}$$

Train on the dataset

$$\arg\min_{\boldsymbol{\theta}_Q} \mathcal{L}(\boldsymbol{X}_t, \boldsymbol{\theta}_Q)$$

Humans do experience replay when they dream!

Experience Replay

On-policy algorithms must throw away episodes after training

Experience Replay

On-policy algorithms must throw away episodes after training

Must collect data using the current policy, cannot use experience replay

Experience Replay

On-policy algorithms must throw away episodes after training

Must collect data using the current policy, cannot use experience replay

Off-policy algorithms can reuse old episodes and use experience replay

Experience Replay

On-policy algorithms must throw away episodes after training

Must collect data using the current policy, cannot use experience replay

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In fact, for off policy algorithms, data can come from anywhere

Experience Replay

On-policy algorithms must throw away episodes after training

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Off-policy algorithms can reuse old episodes and use experience replay

In fact, for off policy algorithms, data can come from anywhere
- Previous policy

Experience Replay

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- Previous policy
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Question: Which is Q learning?

Experience Replay

On-policy algorithms must throw away episodes after training

Must collect data using the current policy, cannot use experience replay

Off-policy algorithms can reuse old episodes and use experience replay

In fact, for off policy algorithms, data can come from anywhere
- Previous policy
- Previous training run
- Human policy

Question: Which is Q learning?

Let us find out!

Experience Replay

Start with the Monte Carlo return

Experience Replay

Start with the Monte Carlo return

$$\arg\min_{\theta_Q} \left[\sum_{s_0 \in S} \sum_{a_0 \in A} \left(Q(s_0, a_0, \theta_\pi, \boldsymbol{\theta}_Q) - \sum_{t=0}^{\infty} \gamma^t \hat{\mathbb{E}}[\mathcal{R}(s_{t+1}) \mid s_0, a_0; \theta_\pi] \right)^2 \right]$$

Experience Replay

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$$\arg\min_{\theta_Q} \left[\sum_{s_0 \in S} \sum_{a_0 \in A} \left(Q(s_0, a_0, \theta_\pi, \boldsymbol{\theta}_Q) - \sum_{t=0}^{\infty} \gamma^t \hat{\mathbb{E}}[\mathcal{R}(s_{t+1}) \mid s_0, a_0; \theta_\pi] \right)^2 \right]$$

Question: On-policy or off-policy?

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Question: On-policy or off-policy? **Answer:** On-policy. Why?

Experience Replay

Start with the Monte Carlo return

$$\arg\min_{\theta_Q} \left[\sum_{s_0 \in S} \sum_{a_0 \in A} \left(Q(s_0, a_0, \theta_\pi, \boldsymbol{\theta}_Q) - \sum_{t=0}^{\infty} \gamma^t \hat{\mathbb{E}}[\mathcal{R}(s_{t+1}) \mid s_0, a_0; \theta_\pi] \right)^2 \right]$$

Question: On-policy or off-policy? **Answer:** On-policy. Why?

Our return is conditioned on the policy

Experience Replay

Start with the Monte Carlo return

$$\arg\min_{\theta_Q} \left[\sum_{s_0 \in S} \sum_{a_0 \in A} \left(Q(s_0, a_0, \theta_\pi, \boldsymbol{\theta}_Q) - \sum_{t=0}^{\infty} \gamma^t \hat{\mathbb{E}}[\mathcal{R}(s_{t+1}) \mid s_0, a_0; \theta_\pi] \right)^2 \right]$$

Question: On-policy or off-policy? **Answer:** On-policy. Why?

Our return is conditioned on the policy

If the policy changes, the return $r_0 + \gamma r_1 + \gamma^2 r_2 + \ldots$ is not valid!

Experience Replay

Start with the Monte Carlo return

$$\arg\min_{\theta_Q} \left[\sum_{s_0 \in S} \sum_{a_0 \in A} \left(Q(s_0, a_0, \theta_\pi, \boldsymbol{\theta}_Q) - \sum_{t=0}^{\infty} \gamma^t \hat{\mathbb{E}}[\mathcal{R}(s_{t+1}) \mid s_0, a_0; \theta_\pi] \right)^2 \right]$$

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If the policy changes, the return $r_0 + \gamma r_1 + \gamma^2 r_2 + \ldots$ is not valid!

Old episode gives us $\hat{\mathbb{E}}[\mathcal{R}(s_{t+1}) \mid s_0, a_0; \theta_{\text{old}}]$

Experience Replay

Start with the Monte Carlo return

$$\arg\min_{\theta_Q} \left[\sum_{s_0 \in S} \sum_{a_0 \in A} \left(Q(s_0, a_0, \theta_\pi, \boldsymbol{\theta}_Q) - \sum_{t=0}^{\infty} \gamma^t \hat{\mathbb{E}}[\mathcal{R}(s_{t+1}) \mid s_0, a_0; \theta_\pi] \right)^2 \right]$$

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Our return is conditioned on the policy

If the policy changes, the return $r_0 + \gamma r_1 + \gamma^2 r_2 + \ldots$ is not valid!

Old episode gives us $\hat{\mathbb{E}}[\mathcal{R}(s_{t+1}) \mid s_0, a_0; \theta_{\text{old}}]$

We need $\hat{\mathbb{E}}[\mathcal{R}(s_{t+1}) \mid s_0, a_0; \theta_\pi]$

Experience Replay

What about TD return?

Experience Replay

What about TD return?

$$\arg\min_{\boldsymbol{\theta}_Q} \left[\sum_{s_0 \in S} \sum_{a_0 \in A} \left(Q(s_0, a_0, \theta_\pi, \boldsymbol{\theta}_Q) - \left(\hat{\mathbb{E}}[\mathcal{R}(s_1) \mid s_0, a_0] + \neg d_0 \gamma \max_{a \in A} Q(s_1, a, \theta_\pi, \boldsymbol{\theta}_Q) \right) \right)^2 \right]$$

Experience Replay

What about TD return?

$$\arg\min_{\boldsymbol{\theta}_Q} \left[\sum_{s_0 \in S} \sum_{a_0 \in A} \left(Q(s_0, a_0, \theta_\pi, \boldsymbol{\theta}_Q) - \left(\hat{\mathbb{E}}[\mathcal{R}(s_1) \mid s_0, a_0] + \neg d_0 \gamma \max_{a \in A} Q(s_1, a, \theta_\pi, \boldsymbol{\theta}_Q) \right) \right)^2 \right]$$

Question: On-policy or off-policy?

Experience Replay

What about TD return?

$$\arg\min_{\boldsymbol{\theta}_Q} \left[\sum_{s_0 \in S} \sum_{a_0 \in A} \left(Q(s_0, a_0, \theta_\pi, \boldsymbol{\theta}_Q) - \left(\hat{\mathbb{E}}[\mathcal{R}(s_1) \mid s_0, a_0] + \neg d_0 \gamma \max_{a \in A} Q(s_1, a, \theta_\pi, \boldsymbol{\theta}_Q) \right) \right)^2 \right]$$

Question: On-policy or off-policy? **Answer:** Off-policy. Why?

Experience Replay

What about TD return?

$$\arg\min_{\boldsymbol{\theta}_Q} \left[\sum_{s_0 \in S} \sum_{a_0 \in A} \right.$$

$$\left. \left(Q(s_0, a_0, \theta_\pi, \boldsymbol{\theta}_Q) - \left(\hat{\mathbb{E}}[\mathcal{R}(s_1) \mid s_0, a_0] + \neg d_0 \gamma \max_{a \in A} Q(s_1, a, \theta_\pi, \boldsymbol{\theta}_Q) \right) \right)^2 \right]$$

Question: On-policy or off-policy? **Answer:** Off-policy. Why?

Experience Replay

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Question: On-policy or off-policy? **Answer:** Off-policy. Why?

Q function depends on θ_π, but reward does not!

Do we know $\arg\max_{a \in A} Q(s_1, a, \theta_\pi, \boldsymbol{\theta}_Q)$?

Experience Replay

What about TD return?

$$\arg\min_{\boldsymbol{\theta}_Q} \left[\sum_{s_0 \in S} \sum_{a_0 \in A} \left(Q(s_0, a_0, \theta_\pi, \boldsymbol{\theta}_Q) - \left(\hat{\mathbb{E}}[\mathcal{R}(s_1) \mid s_0, a_0] + \neg d_0 \gamma \max_{a \in A} Q(s_1, a, \theta_\pi, \boldsymbol{\theta}_Q) \right) \right)^2 \right]$$

Question: On-policy or off-policy? **Answer:** Off-policy. Why?

Q function depends on θ_π, but reward does not!

Do we know $\arg\max_{a \in A} Q(s_1, a, \theta_\pi, \boldsymbol{\theta}_Q)$? Yes! Just plug in s_1

Experience Replay

To summarize:

Experience Replay

To summarize:

Monte Carlo Q learning is on-policy

Experience Replay

To summarize:

Monte Carlo Q learning is on-policy

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MC needs more training data, but TD has harder optimization

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Target Networks

Usually, the target parameters are older parameters

```
theta_Q = ... # Initialize parameters
theta_T = theta_Q.copy()

for epoch in range(num_epochs):
  grad = grad(L)(theta_Q, theta_T, X)
  theta_Q = optimizer.update(theta_Q, grad)
  if epoch % 200 == 0:
    # Update target parameters
    theta_T = theta_Q.copy()
```

Deep Q Networks

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You have all the tools you need to implement DQN, except for one

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$$a = \arg\max_i \begin{bmatrix} Q(s, a = 1, \theta_\pi, \boldsymbol{\theta}_Q) \\ Q(s, a = 2, \theta_\pi, \boldsymbol{\theta}_Q) \\ \vdots \\ Q(s, a = i, \theta_\pi, \boldsymbol{\theta}_Q) \end{bmatrix}$$

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For each action, we must execute Q network $|A|$ times. Not efficient!

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This is $|A|$ times faster!

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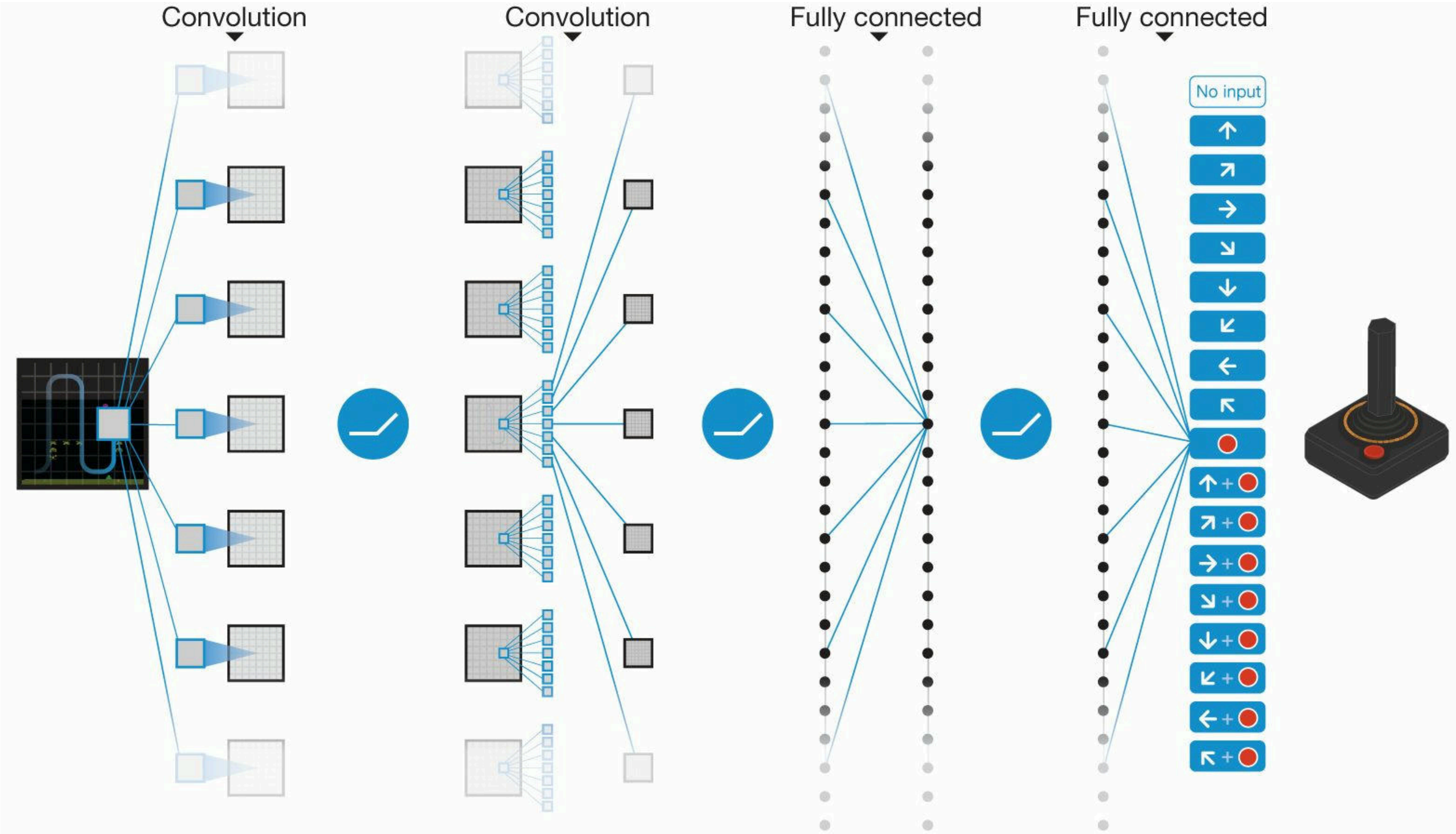

Deep Q Networks

```
Q = nn.Sequential([...])
theta_T = partition(Q, is_array)[0]
replay_buffer = deque(maxsize=50_000)
for epoch in range(num_epochs):
  while not terminated:
    a = random_action if epoch < k else epsilon_greedy(Q)
    s, r, d, next_s = env.step(a)
    replay_buffer.insert((s, a, r, d, next_s))
    X = random.sample(replay_buffer, batch_size)
    theta_Q, model = eqx.partition(Q, is_array)
    theta_Q = td_update(theta_Q, theta_T, Q, X)
    theta_T = copy(theta_Q) if epoch % j == 0 else theta_T
    Q = eqx.combine(theta_Q, model)
```

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Pokemon https://youtu.be/DcYLT37ImBY?si=AeR2WkQg4X-tWa5v