Value

CISC 7404 - Decision Making

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Review

Policy-Conditioned Returns

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Trajectory optimization is model-based algorithm

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Guaranteed optimal policy, given infinite compute

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Expensive to train, but very cheap to use

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Recall the return from trajectory optimization

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$$[\mathcal{G}(\boldsymbol{\tau}) \mid s_0, a_0, a_1, ...] = \sum_{t=0}^{\infty} \gamma^t \mathbb{E}[\mathcal{R}(s_{t+1}) \mid s_0, a_0, a_1, ...]$$

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- Random
- Picked by humans
- Maximize $\mathcal{G}$

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$$\pi : S \times \Theta \mapsto \Delta A$$

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Example policy, greedy policy

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$$\pi(a_t \mid s_t; \theta_\pi) = \begin{cases} 1 \text{ if } a_t = \arg\max_{a_t \in A} \mathbb{E}[\mathcal{G}(\boldsymbol{\tau}) \mid s_0, a_0, a_1, ...] \\ 0 \text{ otherwise} \end{cases}$$

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Must construct and evaluate decision tree at each timestep!

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Conditioning the return on actions is annoying

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Must compute infinitely many actions and outcomes for the return

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What if we condition on a policy, instead of specific actions?

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$$a_0 \sim \pi(\cdot \mid s_0; \theta_\pi), \quad a_1 \sim \pi(\cdot \mid s_1; \theta_\pi), \quad a_2 \sim \pi(\cdot \mid s_2; \theta_\pi), \quad ...$$

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Condition on distribution parameterized by θ_π instead of many actions

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Remember, $\pi(a \mid s; \theta_\pi)$ provides a distribution over the action space

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Now, return conditioned on the policy with θ_π

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How does $\mathbb{E}[\mathcal{R}(s_{t+1})]$ change when we condition on θ_π?

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Question: What changes when we condition on θ_π?

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Answer: State transition function

$$\mathrm{Tr}(s_{t+1} \mid s_t, a_t)$$

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Combine the policy distribution with next state distribution

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Write out the first few timesteps

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Write out the first few timesteps

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$$\Pr(s_2 \mid s_0; \theta_\pi) = \sum_{s_1 \in S} \sum_{a_1 \in A} \mathrm{Tr}(s_2 \mid s_1, a_1) \cdot \pi(a_1 \mid s_1; \theta_\pi)$$

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Derive a general form for $\Pr(s_{n+1} \mid s_0; \theta_\pi)$

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Derive a general form for $\Pr(s_{n+1} \mid s_0; \theta_\pi)$

$$\Pr(s_{n+1} \mid s_0; \theta_\pi) = \sum_{s_1, \ldots, s_n \in S} \prod_{t=0}^{n} \left(\sum_{a_t \in A} \mathrm{Tr}(s_{t+1} \mid s_t, a_t) \cdot \pi(a_t \mid s_t; \theta) \right)$$

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Plug back into our expected reward

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$$\mathbb{E}[\mathcal{R}(s_{t+1}) \mid s_0; \theta_\pi] = \sum_{s_{t+1} \in S} \mathcal{R}(s_{t+1}) \cdot \Pr(s_{t+1} \mid s_0; \theta_\pi)$$

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Need to plug expected reward back into expected discounted return

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- DQN
- DDPG/SAC
- A3C/PPO/GRPO

Goal: find the θ_π (policy parameters) to maximize the expected return

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We can compute

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To find the value of any state $S_a, S_b, S_c, \ldots$

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$$V(s_0, \theta_\pi) = \sum_{s_g \in \{S_x, S_y, S_z\}} \gamma^{t_g} \mathcal{R}(s_g) \cdot \Pr(s_g \mid s_0; \theta_\pi)$$

Where should you live?

TD Value Functions

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We call the following equation the **Monte Carlo** value function

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Replace infinite sum with value function

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First term is expected reward

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Compute the return with a single transition $s_0 \to s_1$

Evaluate infinite-depth decision tree with one function

TD Value Functions

To summarize, we can represent the value function in two ways:

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They produce the same result, but with different computation

Q Functions

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We saw two forms of the value function

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Special connection between an optimal policy and the value function

We can use the value function to find an optimal policy

Q Functions

Consider the Temporal Difference value function

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What if we wanted a mix of both?

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This expectation means:

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We can derive the Q function from the value function

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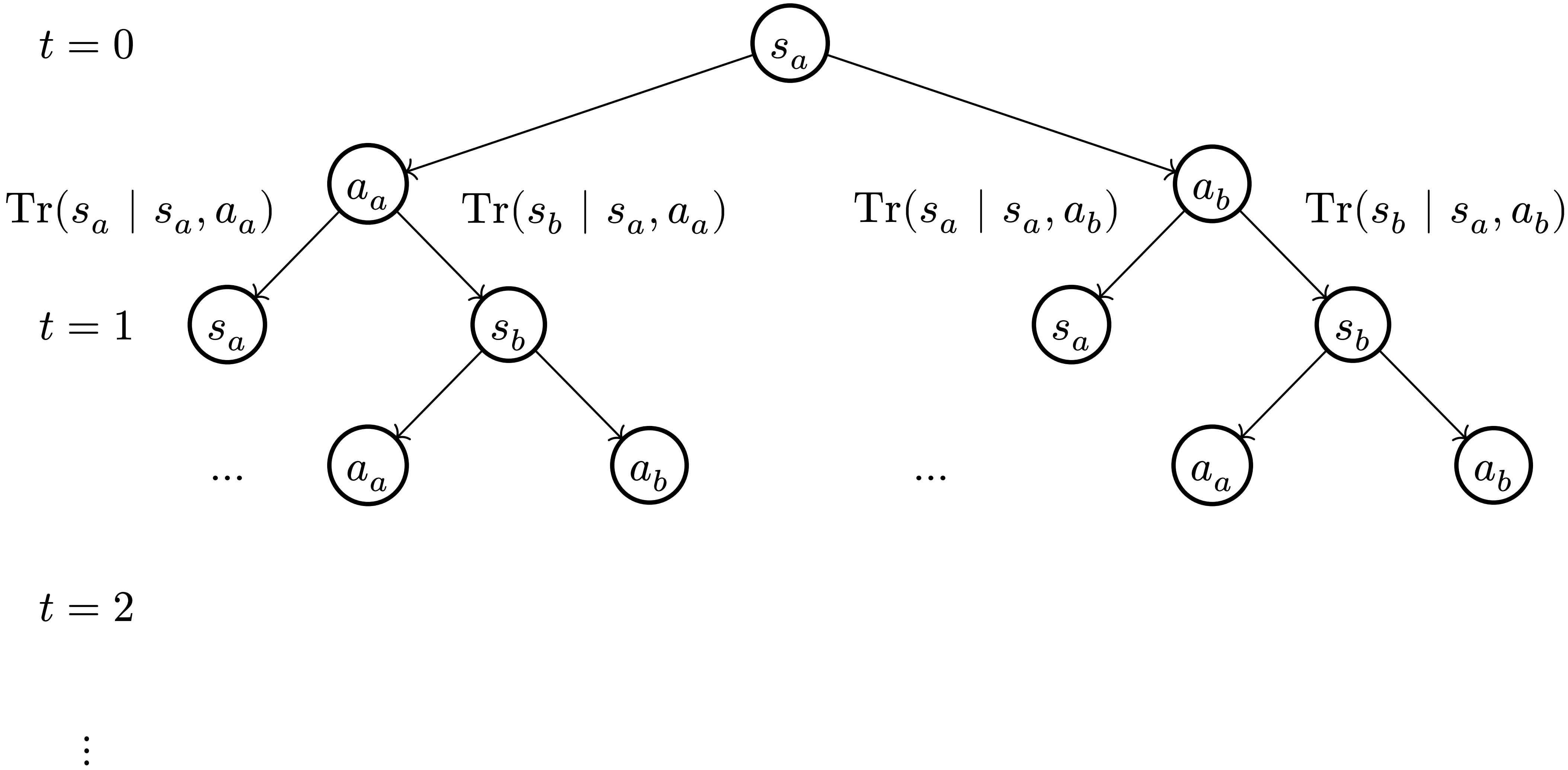
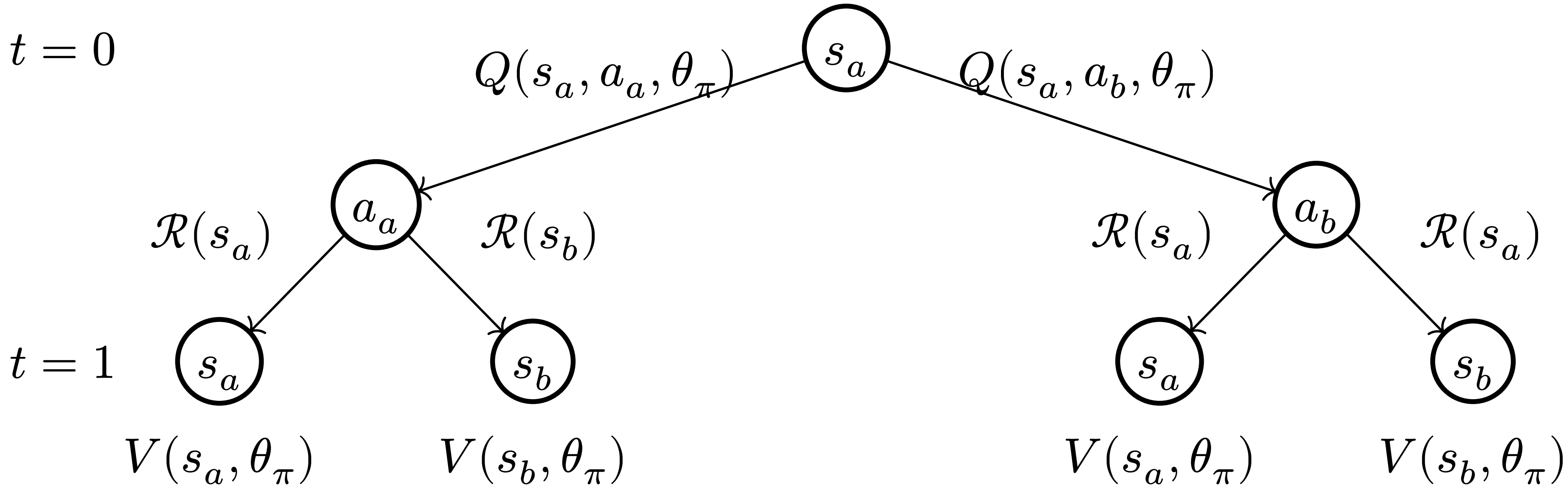
We collapsed the infinite decision tree into a single level

Q Functions


$t = 0$
s_a
a_a
a_b
$\mathrm{Tr}(s_a \mid s_a, a_a)$
$\mathrm{Tr}(s_b \mid s_a, a_a)$
$\mathrm{Tr}(s_a \mid s_a, a_b)$
$\mathrm{Tr}(s_b \mid s_a, a_b)$
$t = 1$
s_a
s_b
s_a
s_b
a_a
a_b
a_a
a_b
$t = 2$

Q Functions


$t = 0$
s_a
$Q(s_a, a_a, \theta_\pi)$
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a_b
$\mathcal{R}(s_a)$
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We now have all the information we need to implement Q learning

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Return following π

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Question: How do we find these terms?

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Answer: Empirical expectation from episode data $(s_t, a_t, \mathcal{R}(s_{t+1}))$

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$$\boldsymbol{E} = \begin{bmatrix} s_0 & s_1 & s_2 & \dots \\ a_0 & a_1 & a_2 & \dots \\ r_0 & r_1 & r_2 & \dots \end{bmatrix}^\top$$

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MC: $\gamma r_{t+1} + \gamma^2 r_{t+2} + \dots$

We know the right hand side, use it to learn the left hand side

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Last thing, we must collect episodes to train Q!

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Can run policy in environment to create episodes

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Can run policy in environment to create episodes

```
states, next_states, rewards, terminateds = [], [], [], []
state = environment.reset()
while not terminated:
    action = policy.sample(state)
    next_state, reward, terminated = environment.step(action)

    states.append(state), next_states.append(next_state), ...
    state = next_state

episode = (states, next_states, rewards, terminateds)
```

Q Learning

What policy do we sample actions from?

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$$\pi(a_0 \mid s_0; \theta_\pi) = \begin{cases} 1 \text{ if } a_0 = \arg\max_{a \in A} Q(s_0, a, \theta_\pi) \\ 0 \text{ otherwise} \end{cases}$$

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Epsilon greedy policy!

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Sample random action with probability ε

In the limit, we sample all possible actions in all states

Q Learning

Can we visualize Q learning?

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Navigation example, reward of 1 for reaching center tile

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https://user-images.githubusercontent.com/1883779/113412338-97430100-93d5-11eb-856c-ef0f420d1acb.gif

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https://mohitmayank.com/interactive_q_learning/q_learning.html

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- Defined training objective (TD and MC updates)

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Today and for homework, use a simple matrix

Q Learning

Model the Q function as a matrix

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Each state is a row, each action is a column in a matrix

Q Learning

Model the Q function as a matrix

Each state is a row, each action is a column in a matrix

$$\begin{bmatrix} Q(S_1, A_1) & Q(S_1, A_2) & \dots \\ Q(S_2, A_1) & Q(S_2, A_2) & \dots \\ \vdots & \vdots & \ddots \end{bmatrix}$$

Q Learning

Model the Q function as a matrix

Each state is a row, each action is a column in a matrix

$$\begin{bmatrix} Q(S_1, A_1) & Q(S_1, A_2) & \ldots \\ Q(S_2, A_1) & Q(S_2, A_2) & \ldots \\ \vdots & \vdots & \ddots \end{bmatrix}$$

$Q_{i,j}$ gives Q value for state $s = S_i$ and action $a = A_j$

Homework

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https://colab.research.google.com/drive/1xtBxAaVc3ax6_j59RC3NLQQPFcIEoau-?usp=sharing