Imitation Learning

CISC 7404 - Decision Making

Steven Morad

University of Macau

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TAs currently grading homework 1:

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- Grades will be released over the next week on Moodle

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  - He Enhao
  - Leonard Hangqin Zhuang
  - Qiao Yulin
  - Fu Zexin

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- 17 April
- 24 April (last lecture)

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Leaning towards 17 April so we can spend all of 24 on LLMs

Final Projects

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Project plans turned in

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- Legend of Zelda
- Super Mario (7x)
- LLM finetuning
- Honor of Kings (3x)
- Federated learning
- Sudoku
- Snake (2x)
- Health AI system
- Tetris
- PacMan (2x)
- Navigation
- StarCraft II
- Getting Over It
- Gymnax
- 2048
- Pokemon Double Battle

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Most plans look good

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https://youtu.be/8qGCleYV4cw?si=ynB0Idg5-TdAiAh9&t=74

Final Projects

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> This chapter compares … data on different addictions … from drug addiction to binge-eating disorders, gambling, and videogame addiction. … Based on these data, it is argued that **there is a hazard inherent in any rewarding operant behavior, no matter how apparently benign: that we may become genuinely "addicted" to any behavior that provides operant reward**. With this in mind, addiction is rightly seen as a possibility for any human being, not a product of the particular pharmacological or technological properties of any one particular substance or behavior.

Final Projects

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- Addict behavior policy maximizes the return!

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Humans are born with reward functions and cannot choose them

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We can select good reward functions for artificial agents

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LLMs are patient, intelligent, helpful because of the reward function

Finish DPG

Imitation Learning

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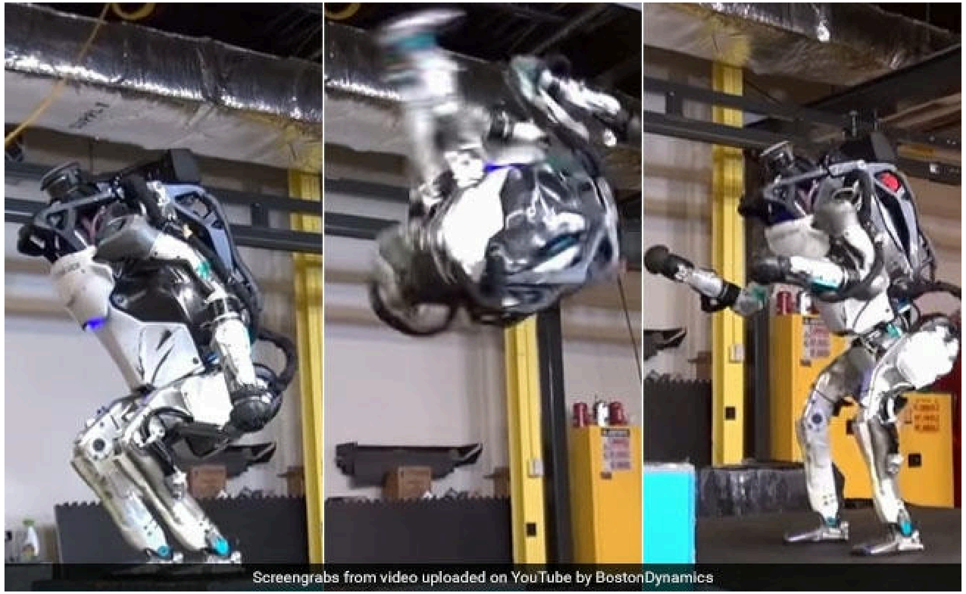

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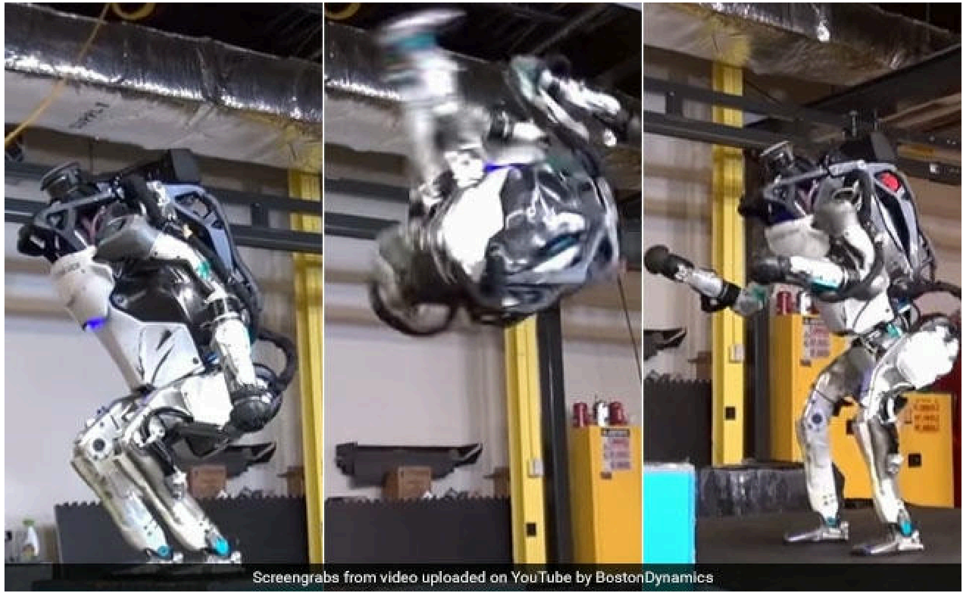

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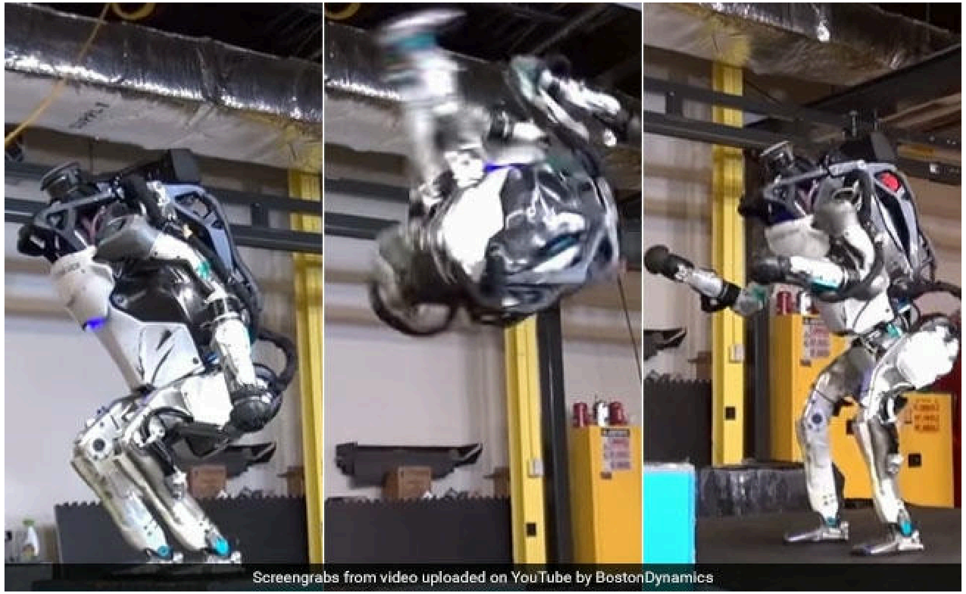

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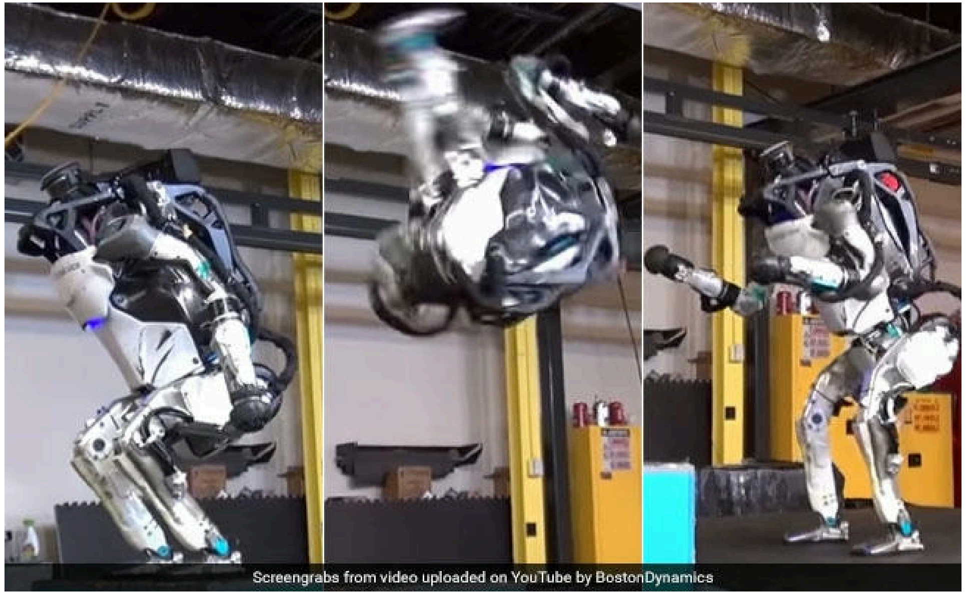

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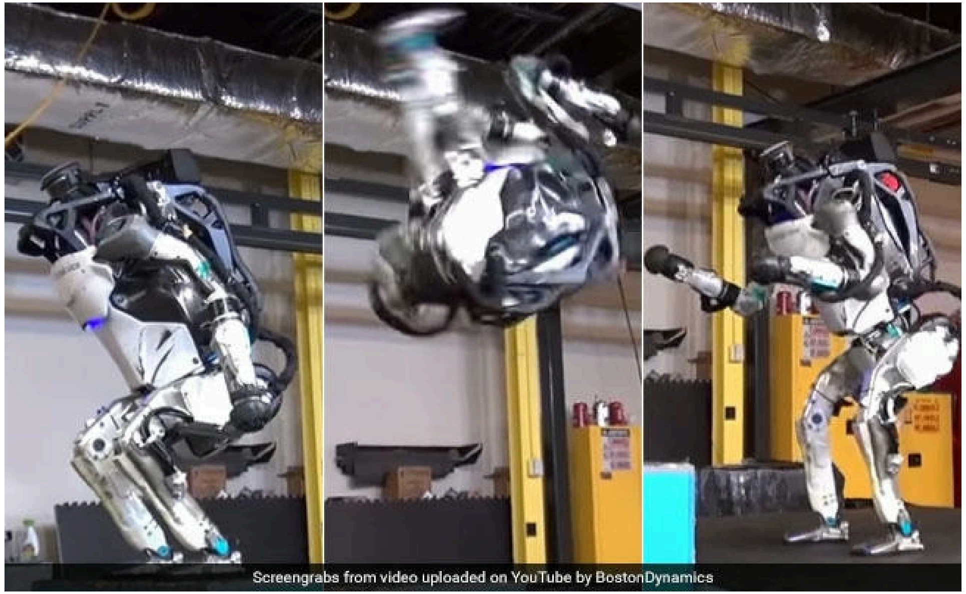

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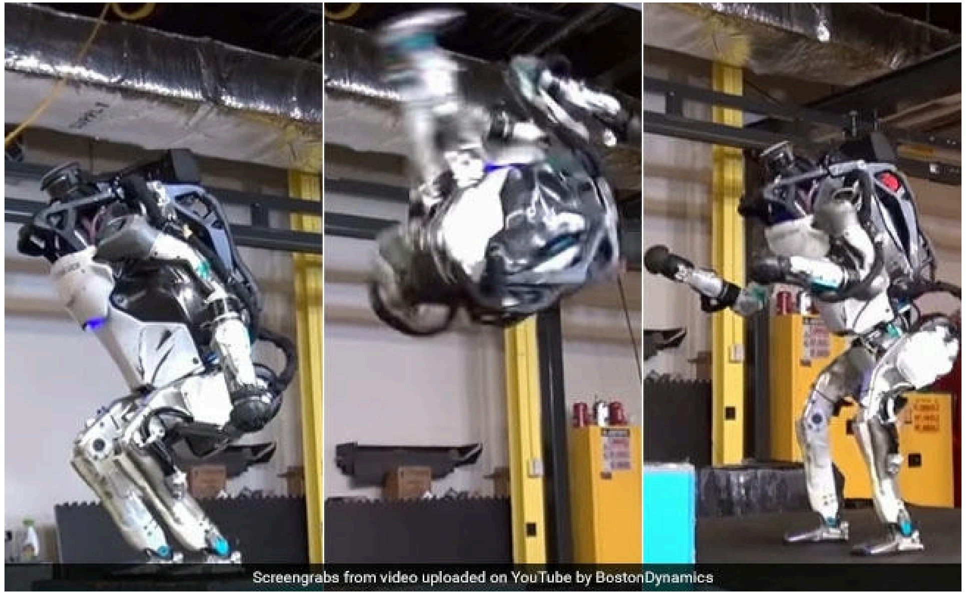

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$$\mathcal{R}(s) =$$

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It is very hard to write a reward function for these behaviors

Imitation Learning

It is often easier to demonstrate good behavior than create rewards

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How would a human do a backflip?

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How would a human do a backflip?

$$\pi(a_0 \mid s_0, \theta_{\text{human}}), \pi(a_1 \mid s_1, \theta_{\text{human}}), \pi(a_2 \mid s_2, \theta_{\text{human}})...$$

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Jump

Backward rotation

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Tuck legs

In **imitation learning** we learn to imitate an expert policy

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I want to introduce a formalism to model the problem

Imitation Learning

Definition: A Hidden-Reward MDP (HR-MDP) consists of:

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$$\boldsymbol{X} = [\boldsymbol{\tau}_1 \quad \boldsymbol{\tau}_2 \quad \dots] = \begin{bmatrix} \begin{bmatrix} (s_0, a_0) \\ (s_1, a_1) \\ \vdots \end{bmatrix} & \begin{bmatrix} (s_0, a_0) \\ (s_1, a_1) \\ \vdots \end{bmatrix} & \dots \end{bmatrix}$$

Imitation Learning

https://www.youtube.com/watch?v=4N4czAm61Fc

Behavioral Cloning

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Answer: KL divergence measures difference in distributions

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Minimize difference between $\underbrace{\pi\left(a \mid s; \theta_\beta\right)}_{\Pr(X)}$ and $\underbrace{\pi\left(a \mid s; \theta_\pi\right)}_{\Pr(Y)}$

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$$\arg\min_{\theta_\pi} \sum_{a \in A} \pi(a \mid s; \theta_\beta) \left[\log(\pi(a \mid s; \theta_\beta)) - \log(\pi(a \mid s; \theta_\pi))\right]$$

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First term is constant with respect to θ_π (only depends on θ_β)

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Behavioral cloning is a simple supervised learning algorithm!

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What if A is continuous (infinite)?

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Need to be careful how we model π, to make sure we can solve this

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Rewrite $\sqrt{...}$ as power

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Note: We derive the loss function for a Gaussian policy gradient loss the exact same way

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Behavioral Cloning

Definition: Behavioral cloning uses **supervised learning** for decision making, minimizing the cross-entropy between expert θ_β and our θ_π

$$\arg\min_{\theta_\pi} \sum_{s, a \in \boldsymbol{X}} -\pi(a \mid s; \theta_\beta) \log \pi(a \mid s; \theta_\pi)$$

For continuous actions, we pick special distributions so the cross entropy loss is tractable (Dirac delta θ_β, Gaussian θ_π)

Expert action

$$\arg\min_{\theta_\pi} \sum_{s \in \boldsymbol{X}} \frac{1}{2} \left(\log \sigma^2 + \frac{(a_* - \mu)^2}{\sigma^2} \right)$$

Policy outputs

Coding

Coding

Similar to policy gradient, just with different loss function

Coding

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The policies and methods change depending on action space

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The policies and methods change depending on action space

Discrete actions use a categorical distribution

Continuous actions usually use normal distribution

Start with discrete actions, then do continous

Coding

Implement a model for a categorical policy

Coding

Implement a model for a categorical policy

```
model = Sequential([
    Linear(state_size, hidden_size),
    Lambda(leaky_relu),
    Linear(hidden_size, hidden_size),
    Lambda(leaky_relu),
    # Output logits (real numbers)
    Linear(hidden_size, action_size),
])
```

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Identical to policy gradient

Coding

Next, implement discrete loss function

Coding

Next, implement discrete loss function

```
def bc_loss(model, states, actions):
    # Often, we can't know the expert action distribution
    # We only have the taken expert action
    # Taken action has p=1, all other actions p=0
    # Represent as a one-hot vector
    expert_probs = actions
    log_policy_probs = log_softmax(vmap(model)(states))
    # Log loss, can reduce over batch using mean or sum
    bce_loss = -sum(
        expert_probs * log_policy_probs, axis=1).mean()
    return bce_loss
```

Coding

Finally, to run our policy we sample actions from our policy

Coding

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```
def sample_action(model, state, key):
    z = model(state)
    # BE VERY CAREFUL, always read documentation
    # Sometimes takes UNNORMALIZED logits, sometimes probs
    action_probs = softmax(model, state)
    a = categorical(key, action_probs)
    a = categorical(key, z) # Does not even use pi
    return a
```

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Coding

Now, consider Gaussian policy (continuous actions)

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model = Sequential([
    Linear(state_size, hidden_size),
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    Linear(hidden_size, hidden_size),
    Lambda(leaky_relu),
    # Output mu and log_sigma
    Linear(hidden_size, 2 * action_size),
    Lambda(x: split(x, 2))
])
```

Coding

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model = Sequential([
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Same as policy gradient

Coding

Implement continuous loss function

Coding

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Use simplified cross entropy (Dirac-Gaussian)

Coding

Implement continuous loss function

Use simplified cross entropy (Dirac-Gaussian)

```
def bc_loss(model, states, actions):
    expert_probs = actions # Dirac delta
    mu, log_std = vmap(model)(states)
    # Gaussian CE, also called Gaus. Neg. Log Likelihood
    gnll_loss = log_std + 0.5 * (
        (mu - action)**2 / exp(log_std)**2
    )
    return gnll_loss
```

Coding

Next, we need to sample actions from our policy network

```
def sample_action(model, state, key):
    mu, log_sigma = model(state)
    # Reparameterization trick
    noise = random.normal(key, (action_size,))
    a = mu + exp(log_sigma) * noise
    return a
```

Coding

The training loop is much simpler than RL

Coding

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```
model = Sequential(...)
opt_state = ...
# Just supervised learning
for batch in dataset:
    states, actions = batch
    J = grad(bc_loss)(model, states, actions)
    update = optim.update(J, opt_state)
    model = apply_updates(update, model)
```

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```

Then deploy the policy

```
a_t = sample_action(model, s_t, key)
```

Applications

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Question: What are some disadvantages of BC?

Applications

Limitation: Imperfect expert

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Dataset is following an “expert” θ_β

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Humans are not reliable experts in many cases

Applications

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Humans are not reliable experts in many cases

You will learn a policy that drives like it is texting

Applications

Even where all the data is from a reliable “expert”, we have problems

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Consider a human surgeon

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Question: Any other issues?

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Limitation: Humans limited to small regions of state space

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In practice, BC policies generalizes much worse than RL policies

Small errors in the learned policy eventually drive the policy to out of distribution states

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One solution to out of distribution error is to collect more data

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DAgger: The agent asks an expert for help when it visit an out of distribution state

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Inverse RL: We learn the reward function that the expert is maximizing

$$\arg\max_{\theta_R} \mathbb{E}\left[\mathcal{G}(\boldsymbol{\tau}) \mid s_0; \theta_\beta\right]$$

$$= \arg\max_{\theta_R} \sum_{n=0}^{\infty} \gamma^n \sum_{s_{n+1} \in S} \underbrace{\mathcal{R}(s_{n+1}, \theta_R)}_{\text{Learnable}} \cdot \Pr(s_{n+1} \mid s_0; \theta_\beta)$$

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Then, we learn a policy θ_π using RL. This generalizes better than BC