Actor Critic II

CISC 7404 - Decision Making

Steven Morad

University of Macau

Quiz

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- After you are done, give me your exam and go relax outside, we resume class at 8:30

Quiz

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- Instructions are in both english and chinese, english instructions take precedence
- Good luck!

Quiz

- 在所有学生收起电脑/笔记/手机后,我会分发试卷。
- 如果在此之后仍有电脑/笔记/手机未收,将视为作弊。
- 试卷会背面朝下发下,在我宣布开始前请勿翻面。
- 试卷翻面后,我会简要说明每道题的注意事项。
- 说明结束后,你们有 75 分钟完成考试。
- 交卷后请到教室外休息,8:30 恢复上课。
- 试卷可能存在不同版本,细节略有差异。
- 若你的试卷上出现其他版本的答案,将被判定为作弊。
- 试卷说明为中英双语,若内容冲突以英文为准。
- 祝各位考试顺利!

Admin

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- MDPs, bandits, RL algorithms, even some control problems!

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- Imitation Learning
- Offline RL
- Memory and POMDPs
- Large Language Models

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Question: Should we replace a topic with something else?

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- Imitation learning
  - Sometimes, designing a reward function is hard
  - It is easier to demonstrate desired behavior to agents
    - Instead of reward for surgery, do what human does
    - Instead of reward for self driving, do what human does
  - With imitation learning, can learn behaviors without rewards
  - Closer to supervised learning, easier to train
  - Policies are not better than dataset/humans

Admin

- Offline RL
  - RL without exploration
  - How can we learn policies from a fixed dataset?
    - Learn surgery from surgical videos (no need to kill patients)
    - Learn driving from Xiaomi driving dataset (no need to crash cars)
  - Unlike imitation learning, can do **better** than dataset
  - Very new topic (2-3 years old)
    - Does not work very well (yet)

Admin

- Memory and POMDPs (my research focus)
  - So far, we focused on video games
    - MDP
  - Many interesting problems are not Markov
    - Think of robot with camera, not Markov
    - Almost every task has sensor noise, not Markov
  - Can we extend RL to work for virtually any problem?
    - Yes, requires long-term memory
    - LSTM, transformer, etc
  - May also have time to introduce world models
    - Dreamer, TD-MPC, etc

Admin

- Large Language Models
  - Can train LLMs using unsupervised learning
    - They only learn to predict next word
  - We use RL to teach them to interact with humans
    - Apply policy gradient to textual MDP
    - DeepSeek math/GRPO
    - RL-adjacent methods (DPO)

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Also a possibility to split lecture:

- E.g., 1 hour imitation learning, 1 hour something new

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Alternative topics:

- Multi-agent RL
- Model-based RL and world-models
- Evolutionary algorithms

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Homework 2 progress

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If you did not already start, you might be in trouble

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Experiments take a long time, start as soon as possible

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Harder and requires more debugging than `FrozenLake` assignment

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Those using Tencent AI Arena (Honor of Kings):

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- Cannot install new python libraries (Tencent security issue)
  - No `jax`, must use `torch`
  - You must learn Tencent's strange callback system
    - Prevents copy/pasting, so `torch` is ok

Review

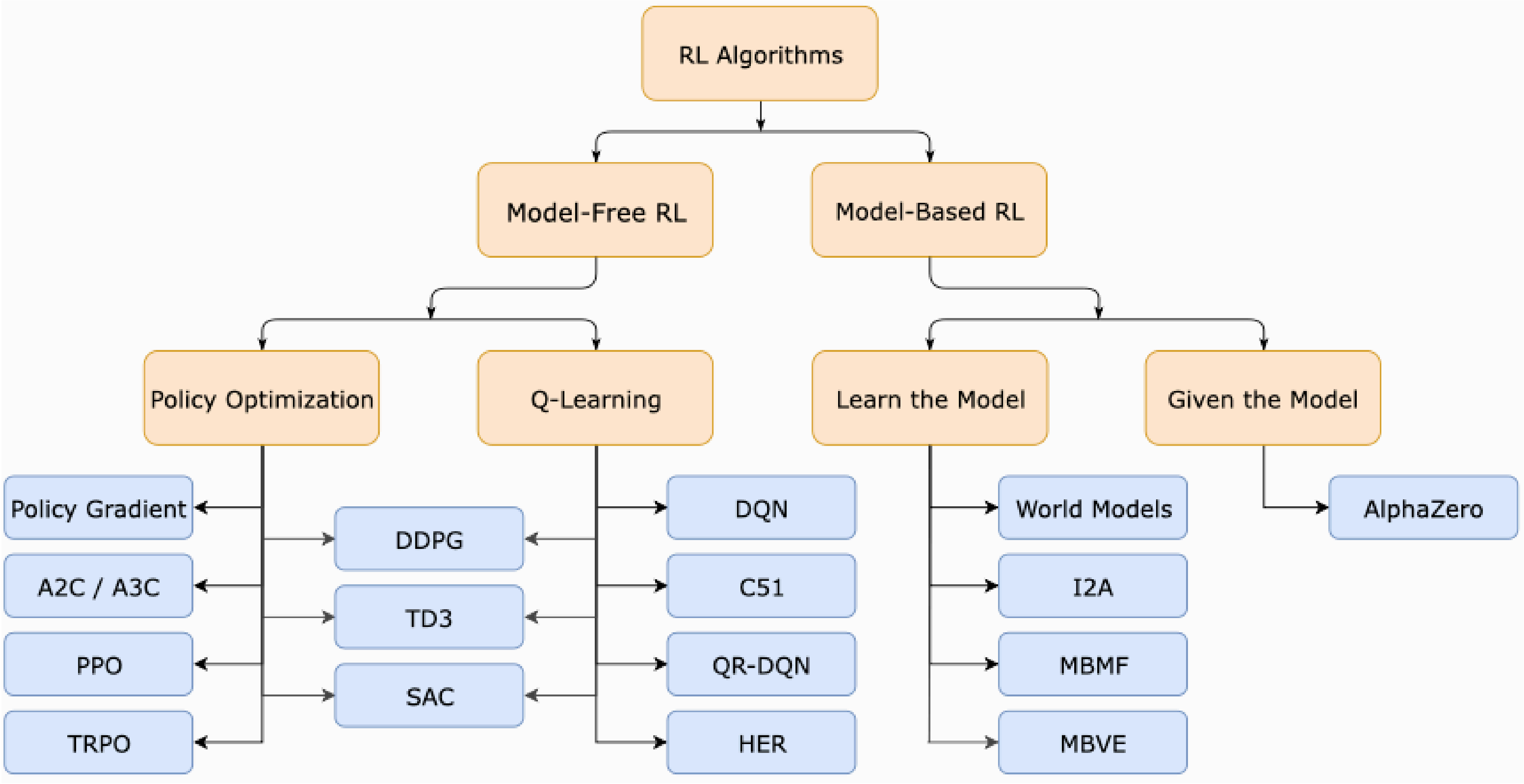
Actor Critic

Actor Critic

Alternative descriptions of actor critic algorithms

https://lilianweng.github.io/posts/2018-04-08-policy-gradient/

Actor Critic



Actor Critic

There are two approaches to actor critic

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Learn Q for a specific policy

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Deterministic Policy Gradient

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Question: Why did we introduce policy gradient methods?

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$$A = [0, 2\pi]^{12}$$

Deterministic Policy Gradient

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$$\pi(a_t \mid s_t; \theta_\pi) = \begin{cases} 1 \text{ if } a_t = \arg\max_{a_t \in A} Q(s_t, a_t, \theta_\pi) \\ 0 \text{ otherwise} \end{cases}$$

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Infinitely many a_t – compute Q for each and take $\arg\max$ over all

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Let us quickly review the Q function and value function

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Start with general form of Temporal Difference Q function

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$$Q(s_0, a_0, \theta_\pi) = \underbrace{\mathbb{E}[\mathcal{R}(s_{t+1}) \mid s_0, a_0]}_{\text{Reward for taking } a_0} + \gamma \underbrace{V(s_1, \theta_\pi)}_{\mathcal{G} \text{ following } \theta_\pi}$$

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Can we learn a different deterministic policy for continuous actions?

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Question: What method can we use to learn θ_π?

Deterministic Policy Gradient

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$$= \sum_{n=0}^{\infty} \gamma^n \sum_{s_{n+1} \in S} \mathcal{R}(s_{n+1}) \cdot \sum_{s_1, \ldots, s_n \in S} \prod_{t=0}^{n} \mathrm{Tr}(s_{t+1} \mid s_t, \mu(s_t, \theta_\mu))$$

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We must know ∇ Tr to find the deterministic policy gradient

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Stochastic policy gradient is very special – we do not need ∇ Tr

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We must know $\nabla \mathrm{Tr}$ to find the deterministic policy gradient

In model-free RL, we do not know $\nabla \mathrm{Tr}$ – it might not be differentiable!

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Let me explain what I mean

Deterministic Policy Gradient

With deterministic policy, μ inside Tr means chain rule

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With stochastic policy, π **not inside** Tr (product rule)

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$$\sum_{t=0}^{n} \sum_{a_t \in A} \nabla_{\theta_\pi} \left[\mathrm{Tr}(s_{t+1} \mid s_t, a_t) \cdot \log \pi(a_t \mid s_t; \theta_\pi)\right]$$

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We use stochastic policies in RL because of this

Deep Deterministic Policy Gradient

Deep Deterministic Policy Gradient

$$\nabla_{\theta_\mu} \mathbb{E}[\mathcal{G}(\boldsymbol{\tau}) \mid s_0; \theta_\mu]$$

Deep Deterministic Policy Gradient

$$\nabla_{\theta_\mu} \mathbb{E}[\mathcal{G}(\boldsymbol{\tau}) \mid s_0; \theta_\mu] \qquad \theta_{\pi, i+1} = \theta_{\pi, i} + \alpha \cdot \nabla_{\theta_\mu} \mathbb{E}[\mathcal{G}(\boldsymbol{\tau}) \mid s_0; \theta_\mu]$$

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Let us try to derive deterministic policy gradient again

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Let us try to derive deterministic policy gradient again

This time, take gradient of V instead of gradient of $\mathbb{E}[\mathcal{G}(\boldsymbol{\tau}) \mid s_0; \theta_\mu]$

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$$V(s_0, \theta_\mu) = \mathbb{E}[\mathcal{R}(s_1) \mid s_0; \theta_\mu] + \gamma V(s_0, \theta_\mu)$$

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Take the gradient of both sides

$$\nabla_{\theta_\mu} V(s_0, a_0, \theta_\mu) = \nabla_{\theta_\mu} Q(s_0, \mu(s_0, \theta_\mu), \theta_\mu)$$

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$$\nabla_{\theta_\mu} V(s_0, \theta_\mu) = \nabla_{\theta_\mu} Q(s_0, \mu(s_0, \theta_\mu), \theta_\mu)$$

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Let us inspect these terms more closely

Deep Deterministic Policy Gradient

How θ_μ changes a

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The second term is easy to compute, gradient of deterministic policy

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$$\nabla_{\theta_\mu} V(s_0, \theta_\mu) = \underbrace{\nabla_\mu Q(s_1, \mu(s_1, \theta_\mu), \theta_\mu)}_{\text{How } a \text{ changes } Q} \cdot \overbrace{\nabla_{\theta_\mu} \mu(s_1, \theta_\mu)}^{\text{How } \theta_\mu \text{ changes } a}$$

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Analytical solution for the first term is difficult (recursive definition)

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We can backpropagate through Q without worrying about recursion

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Writing the code makes it look easy

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How a changes Q

Writing the code makes it look easy

```
def V(s, Q_nn, mu_nn):
    a = mu_nn(s)
    return Q_nn(s, a)

# Learn the policy that maximizes V
# Make sure to differentiate w.r.t policy parameters!
J = grad(V, argnums=2)(states, Q_nn, mu_nn)
mu_nn = optimizer.update(mu_nn, J)
```

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$$Q(s_0, a_0, \theta_\mu, \theta_Q) = \mathbb{E}[\mathcal{R}(s_1) \mid s_0, a_0] + \gamma Q(s_1, \mu(s_1, \theta_\mu), \theta_\mu, \theta_Q)$$

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```
def V(s, Q_nn, mu_nn):
    a = mu_nn(s)
    return Q_nn(s, a)
# Before, we learned policy params to maximize Q
# Now, we learn params of Q following policy (argnums=2)
J = grad(V, argnums=1)(states, Q_nn, mu_nn)
Q_nn = optimizer.update(Q_nn, J)
```

Deep Deterministic Policy Gradient

Definition: Deep Deterministic Policy Gradient (DDPG) decomposes V into a deterministic policy μ and Q, learning them jointly

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$$\theta_{Q, i+1} = \arg \min_{\theta_{Q, i}}$$

$$\left(Q\left(s_0, a_0, \theta_{\mu, i}, \theta_{Q, i}\right) - \left(\hat{\mathbb{E}}\left[\mathcal{R}\left(s_1\right) \mid s_0, a_0\right] + \gamma Q\left(s_1, \mu\left(s_1, \theta_{\mu, i}\right), \theta_{\mu, i}, \theta_{Q, i}\right)\right)\right)$$

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Step 2: Learn θ_μ that maximizes Q

$$\theta_{\mu,i+1} = \theta_{\mu,i} + \alpha \cdot Q(s_0, \mu(s_0, \theta_\mu), \theta_{\mu,i}, \theta_{Q,i+1})$$

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Step 2: Learn θ_μ that maximizes Q

$$\theta_{\mu,i+1} = \theta_{\mu,i} + \alpha \cdot Q(s_0, \mu(s_0, \theta_\mu), \theta_{\mu,i}, \theta_{Q,i+1})$$

Repeat until convergence, $\theta_{\mu,i+1} = \theta_{\mu,i}, \quad \theta_{Q,i+1} = \theta_{Q,i}$

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Question: Is DDPG on-policy or off-policy?

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Question: Is DDPG on-policy or off-policy?

Answer: Depends on how we train Q:

Deep Deterministic Policy Gradient

$$\theta_{Q,i+1} = \arg\min_{\theta_{Q,i}}$$

$$\left(Q\left(s_0, a_0, \theta_{\mu,i}, \theta_{Q,i}\right) - \left(\hat{\mathbb{E}}[\mathcal{R}(s_1) \mid s_0, a_0] + \gamma Q\left(s_1, \mu\left(s_1, \theta_{\mu,i}\right), \theta_{\mu,i}, \theta_{Q,i}\right)\right)\right)$$

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Question: Is DDPG on-policy or off-policy?

Answer: Depends on how we train Q:

- TD Q is off-policy

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Almost **all** good off-policy actor-critic algorithms are based on DDPG

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$$\max_{\theta_\mu} Q(s, \mu(s, \theta_\mu), \theta_\mu) \approx \max_{a \in A} Q(s, a, \theta_\mu)$$

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Because μ is a neural network, it can generalize to continuous s, a

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Normal noise (infinite support) guarantees full action space coverage

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mu = Sequential([
    Linear(state_size, hidden_size),
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    Linear(hidden_size, hidden_size),
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BenBen: $A = [0, 2\pi]^{12}$, so `action_dims=12`

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```
def bound_action(action, lower, upper):
    return 0.5 * (upper + lower) + 0.5 * (upper - lower)
        * tanh(action)
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    return 0.5 * (upper + lower) + 0.5 * (upper - lower)
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def sample_action(mu, state, A_bounds, std):
    action = mu(state)
    noisy_action = action + normal(0, std) # Explore
    return bound_action(noisy_action, *A_bounds)
```

Coding

Now construct Q

Coding

Now construct Q

```
Q = Sequential([
    # Different from DQN network
    # Input action and state together
    Lambda(lambda s, a: concatenate(s, a)),
    Linear(state_size + action_dims, hidden_size),
    Lambda(leaky_relu),
    Linear(hidden_size, hidden_size),
    Lambda(leaky_relu),
    Linear(hidden_size, 1), # Single value for Q(s, a)
])
```

Coding

```
while not terminated:
    # Exploration: make sure actions within action space!
    action = sample_action(mu, state, bounds, std)
    transition = env.step(action)
    replay_buffer.append(transition)
    data = replay_buffer.sample()
    # Theta_pi params are in mu neural network
    # Argnums tells us differentiation variable
    J_Q = grad(Q_loss, argnums=0)(theta_Q, theta_T, mu, data)
    theta_Q = apply_updates(J_Q, ...)
    J_mu = grad(mu_loss, argnums=0)(mu, theta_Q, data)
    mu = apply_updates(J_mu, ...)
    if step % 200 == 0: # Target network necessary
        theta_T = theta_Q
```

Coding

```
def Q_loss(theta_Q, theta_T, theta_pi, data):
    Qnet = combine(Q, theta_Q)
    Tnet = combine(Q, theta_T) # Target network
    # Predict Q values for action we took
    prediction = vmap(Qnet)(data.state, data.action)
    # Now compute labels using TD error
    next_action = vmap(mu)(data.next_state)
    # NOTE: No max! Mu approximates argmax
    next_Q = vmap(Tnet)(data.next_state, next_action)
    label = data.reward + gamma * data.done * next_Q
    return (prediction - label) ** 2
```

Coding

```python
def mu_loss(mu, theta_Q, data):
    # Find the action that maximizes the Q function
    Qnet = combine(Q, theta_Q)
    # Instead of multiply, chain rule -- plug action into Q
    action = vmap(mu)(data.state)
    q_value = vmap(Qnet)(data.state, action)
    # Update the policy parameters to maximize the Q value
    # Gradient ascent but we min loss, use negative
    return -q_value
```

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We will very briefly cover max-entropy RL

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We will very briefly cover max-entropy RL

First, let us introduce entropy

Max Entropy RL

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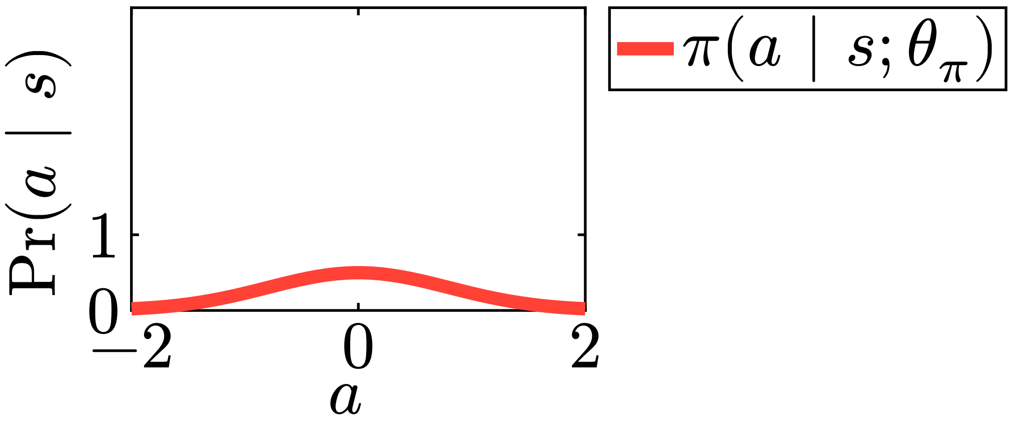

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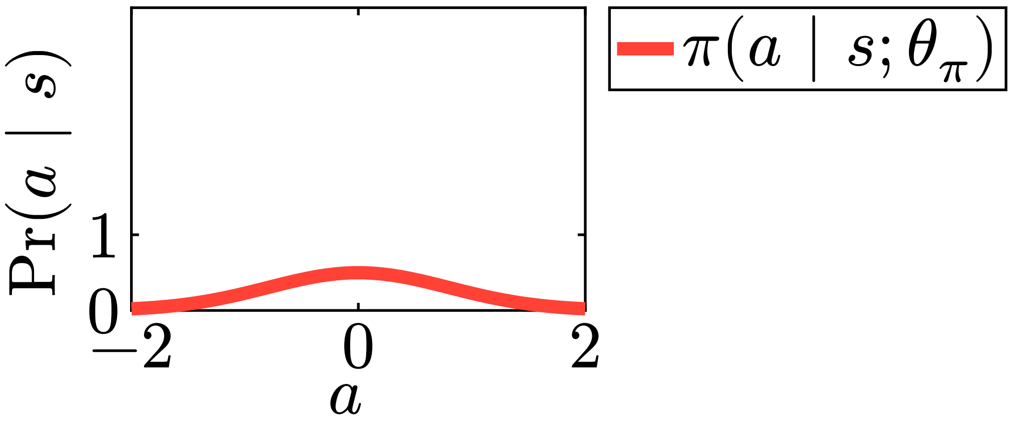

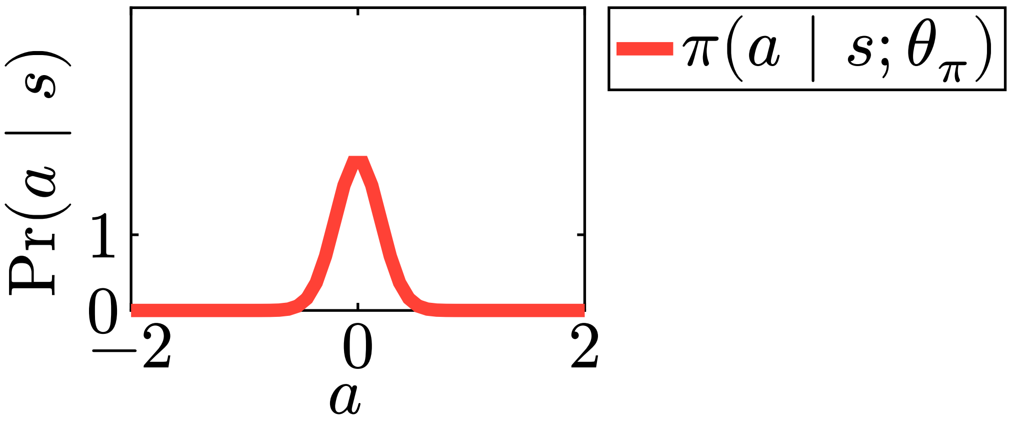

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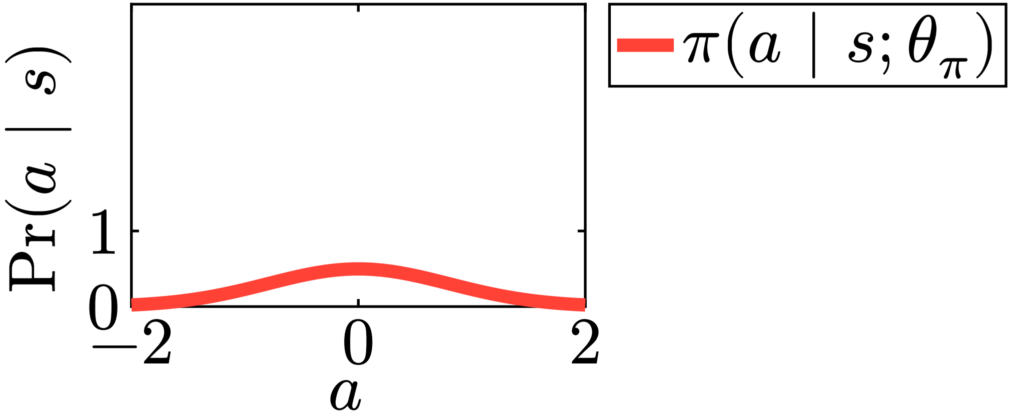

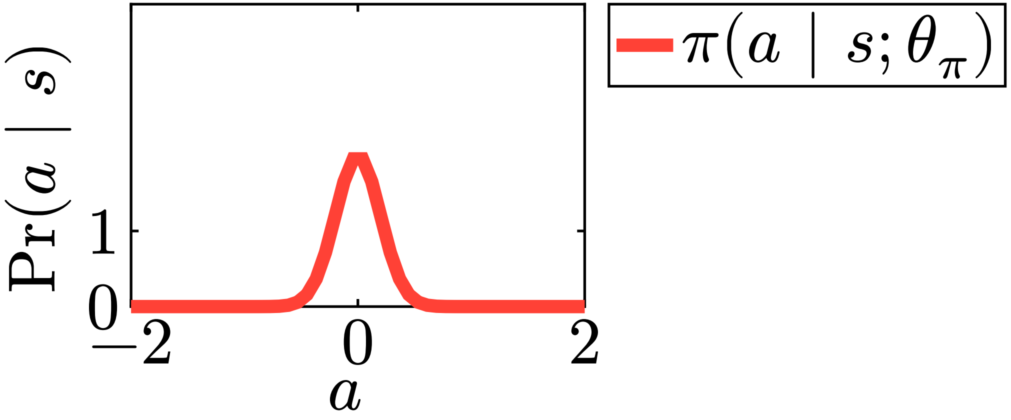

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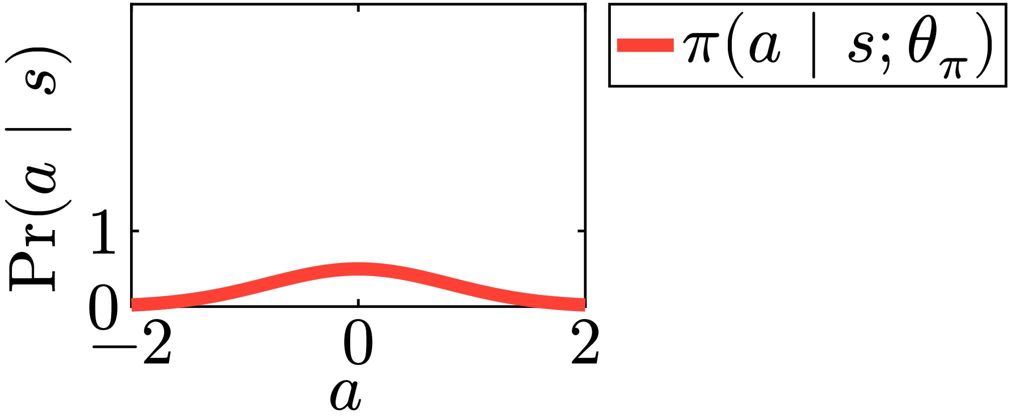

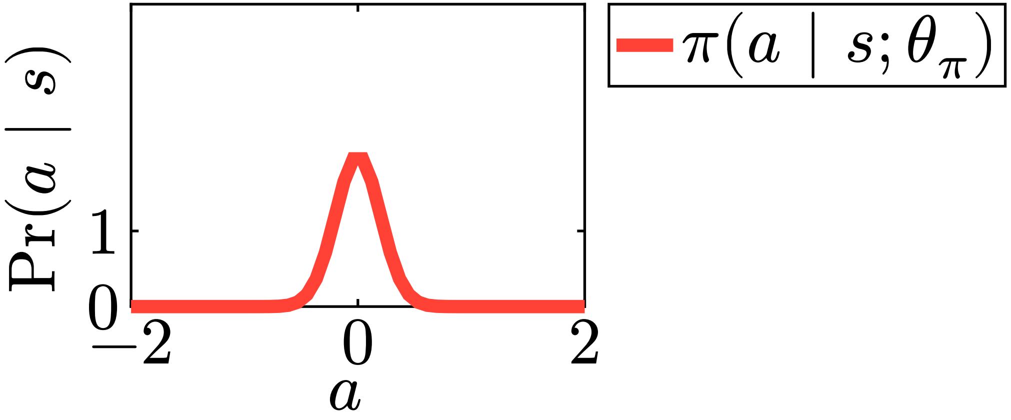

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Left policy, more uncertain/random

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We want a policy that is both random and maximizes the return

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- More stable training (explain further in offline RL)

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Mathematically, this is a "deterministic" policy

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If $\eta \sim \text{Normal}(0, 1)$ our function f is still deterministic

We call this the **reparameterization trick**, used in variational autoencoders (VAE)

Mathematically, this is a "deterministic" policy

$$\mathbb{E}[\mathcal{G}(\boldsymbol{\tau}) \mid s_0, \eta_0, \eta_1, ...; \theta_\mu]$$

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We get SAC!

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Repeat until convergence, $\theta_{\mu, i+1} = \theta_{\mu, i}, \quad \theta_{Q, i+1} = \theta_{Q, i}$

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Like PPO, SAC is complicated – uses many "implementation tricks"

- Often not documented
- CleanRL describes modern SAC, using tricks from 5+ papers
- https://docs.cleanrl.dev/rl-algorithms/sac/#implementation-details_1

Coding SAC could take an entire lecture, read CleanRL

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Introduces the concept, simple

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- Tuned DDPG can likely outperform untuned SAC

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- Much easier to debug

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  - Think about why it learned to do this (exploiting bugs in MDP)

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You must use your brain to be successful!