# Offline RL

## CISC 7404 - Decision Making

Steven Morad

University of Macau

# Admin

# Admin

Two more lectures after today

# Admin

Two more lectures after today

- Make sure you start on final projects!

# Admin

Two more lectures after today

- Make sure you start on final projects!

Homework 1 grading is done

# Admin

Two more lectures after today

- Make sure you start on final projects!

Homework 1 grading is done

- Results on moodle

# Admin

Two more lectures after today

- Make sure you start on final projects!

Homework 1 grading is done

- Results on moodle

Homework 2 grading deadline next Wednesday

# Admin

Quiz 1 scores uploaded

# Admin

Quiz 1 scores uploaded

- Mean grade 57/100

# Admin

Quiz 1 scores uploaded

- Mean grade 57/100
  - Modes at 25 and 85

# Admin

Quiz 1 scores uploaded

- Mean grade 57/100
  - Modes at 25 and 85
- Someone forget their name

# Admin

Quiz 1 scores uploaded

- Mean grade 57/100
  - Modes at 25 and 85
- Someone forget their name
  - If you have no score, come see me

# Admin

Quiz 1 scores uploaded

- Mean grade 57/100
  - Modes at 25 and 85
- Someone forget their name
  - If you have no score, come see me

Final quiz on 17 April (next week), format subject to change

# Admin

Quiz 1 scores uploaded

- Mean grade 57/100
  - Modes at 25 and 85
- Someone forget their name
  - If you have no score, come see me

Final quiz on 17 April (next week), format subject to change

- 1 question fundamental RL (V/Q/PG)

# Admin

Quiz 1 scores uploaded

- Mean grade 57/100
  - Modes at 25 and 85
- Someone forget their name
  - If you have no score, come see me

Final quiz on 17 April (next week), format subject to change

- 1 question fundamental RL (V/Q/PG)
- 1 question actor-critic

# Admin

Quiz 1 scores uploaded

- Mean grade 57/100
  - Modes at 25 and 85
- Someone forget their name
  - If you have no score, come see me

Final quiz on 17 April (next week), format subject to change

- 1 question fundamental RL (V/Q/PG)
- 1 question actor-critic
- 1 or 2 questions on new material (imitation/offline RL/POMDPs/etc)

# Review

# Offline RL

# Offline RL

**Review:** In on-policy RL, each iteration we collect a **new** dataset using our policy

# Offline RL

**Review:** In on-policy RL, each iteration we collect a **new** dataset using our policy

In off-policy RL, each iteration we **update** our dataset using our policy

# Offline RL

**Review:** In on-policy RL, each iteration we collect a **new** dataset using our policy

In off-policy RL, each iteration we **update** our dataset using our policy

In imitation learning, we are given a fixed **expert** dataset

**Question:** Did you find imitation learning interesting? Why?

# Offline RL

Imitation learning from a fixed dataset is attractive for many reasons

# Offline RL

Imitation learning from a fixed dataset is attractive for many reasons

- Fixed dataset results in much simpler code

# Offline RL

Imitation learning from a fixed dataset is attractive for many reasons

- Fixed dataset results in much simpler code
  - No need to collect new data, step environment, etc

# Offline RL

Imitation learning from a fixed dataset is attractive for many reasons

- Fixed dataset results in much simpler code
  - No need to collect new data, step environment, etc
- Easier and more stable to train (supervised learning)

# Offline RL

Imitation learning from a fixed dataset is attractive for many reasons

- Fixed dataset results in much simpler code
  - No need to collect new data, step environment, etc
- Easier and more stable to train (supervised learning)

But imitation learning (IL) also has disadvantages

# Offline RL

Imitation learning from a fixed dataset is attractive for many reasons

- Fixed dataset results in much simpler code
  - No need to collect new data, step environment, etc
- Easier and more stable to train (supervised learning)

But imitation learning (IL) also has disadvantages

- Only imitates, does not think or plan

# Offline RL

Imitation learning from a fixed dataset is attractive for many reasons

- Fixed dataset results in much simpler code
  - No need to collect new data, step environment, etc
- Easier and more stable to train (supervised learning)

But imitation learning (IL) also has disadvantages

- Only imitates, does not think or plan
  - Can only do as good as expert

# Offline RL

Imitation learning from a fixed dataset is attractive for many reasons

- Fixed dataset results in much simpler code
  - No need to collect new data, step environment, etc
- Easier and more stable to train (supervised learning)

But imitation learning (IL) also has disadvantages

- Only imitates, does not think or plan
  - Can only do as good as expert
  - Humans are usually bad "experts"

# Offline RL

Imitation learning from a fixed dataset is attractive for many reasons

- Fixed dataset results in much simpler code
  - No need to collect new data, step environment, etc
- Easier and more stable to train (supervised learning)

But imitation learning (IL) also has disadvantages

- Only imitates, does not think or plan
  - Can only do as good as expert
  - Humans are usually bad "experts"
  - We want to do **better** than humans

# Offline RL

Imitation learning from a fixed dataset is attractive for many reasons

- Fixed dataset results in much simpler code
  - No need to collect new data, step environment, etc
- Easier and more stable to train (supervised learning)

But imitation learning (IL) also has disadvantages

- Only imitates, does not think or plan
  - Can only do as good as expert
  - Humans are usually bad "experts"
  - We want to do **better** than humans

**Question:** Can we learn policies from fixed datasets that do better than the experts?

# Offline RL

In **offline RL** or **batch RL**, we learn without exploration

# Offline RL

In **offline RL** or **batch RL**, we learn without exploration

Like imitation learning, learn from a fixed dataset

# Offline RL

In **offline RL** or **batch RL**, we learn without exploration

Like imitation learning, learn from a fixed dataset

- Simpler implementation

# Offline RL

In **offline RL** or **batch RL**, we learn without exploration

Like imitation learning, learn from a fixed dataset

- Simpler implementation
- Can learn from large existing datasets without collecting it youurself

# Offline RL

In **offline RL** or **batch RL**, we learn without exploration

Like imitation learning, learn from a fixed dataset
- Simpler implementation
- Can learn from large existing datasets without collecting it youurself

Unlike imitation learning, offline RL can do **better** than the expert

# Offline RL

In **offline RL** or **batch RL**, we learn without exploration

Like imitation learning, learn from a fixed dataset

- Simpler implementation
- Can learn from large existing datasets without collecting it youurself

Unlike imitation learning, offline RL can do **better** than the expert

- Can learn an optimal policy from a random "expert"!

# Offline RL

In **offline RL** or **batch RL**, we learn without exploration

Like imitation learning, learn from a fixed dataset

- Simpler implementation
- Can learn from large existing datasets without collecting it youurself

Unlike imitation learning, offline RL can do **better** than the expert

- Can learn an optimal policy from a random “expert”!
- How is this possible without exploration?

# Offline RL

In **offline RL** or **batch RL**, we learn without exploration

Like imitation learning, learn from a fixed dataset
- Simpler implementation
- Can learn from large existing datasets without collecting it youurself

Unlike imitation learning, offline RL can do **better** than the expert
- Can learn an optimal policy from a random “expert”!
- How is this possible without exploration?

In imitation learning, we learn to imitate dataset trajectories

# Offline RL

In **offline RL** or **batch RL**, we learn without exploration

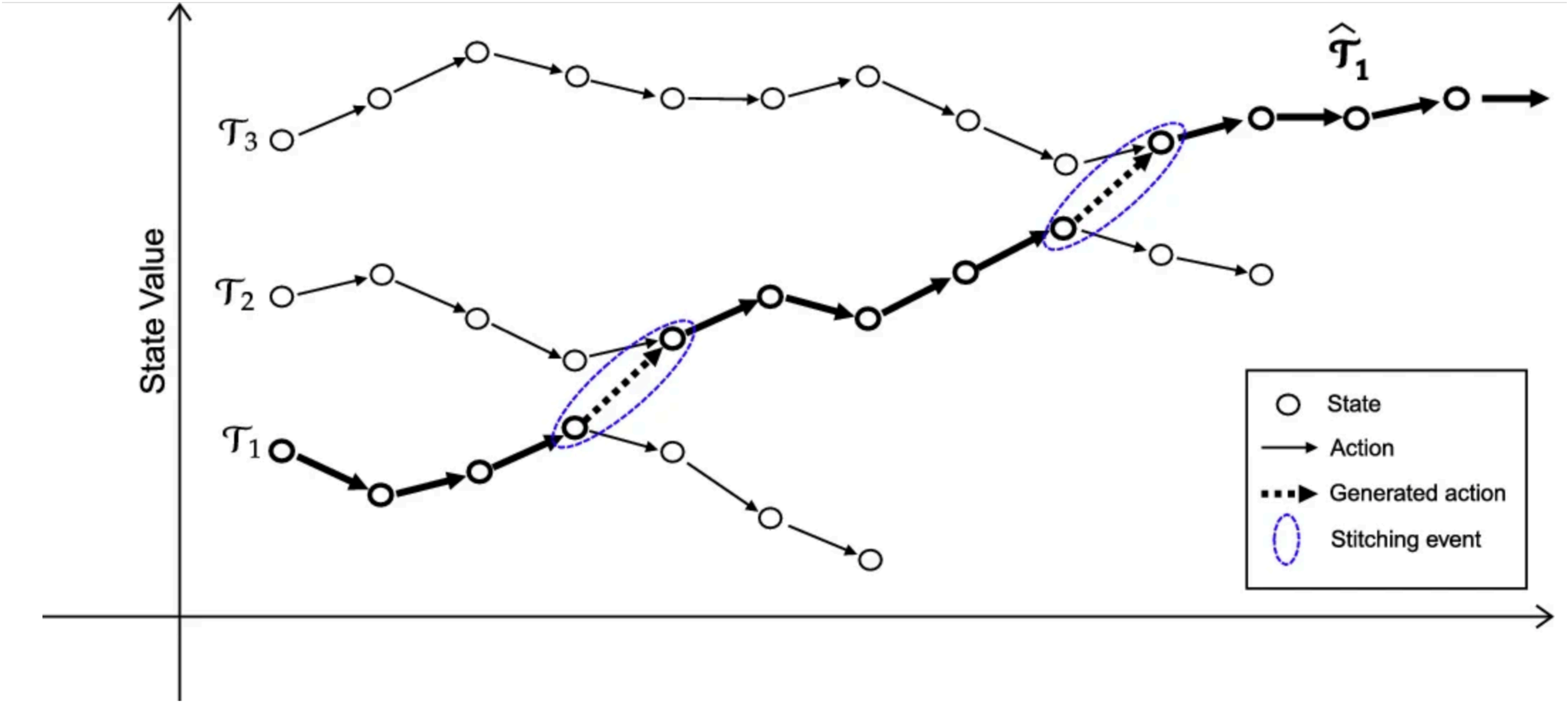
Like imitation learning, learn from a fixed dataset
- Simpler implementation
- Can learn from large existing datasets without collecting it youurself

Unlike imitation learning, offline RL can do **better** than the expert
- Can learn an optimal policy from a random “expert”!
- How is this possible without exploration?

In imitation learning, we learn to imitate dataset trajectories

In offline RL, we learn to **stitch** together subtrajectories into optimal trajectories

# Offline RL



State Value
$\mathcal{T}_3$
$\mathcal{T}_2$
$\mathcal{T}_1$
$\hat{\mathcal{T}}_1$
State
Action
Generated action
Stitching event

# Offline RL

Offline RL is a very new field

# Offline RL

Offline RL is a very new field

If we can make offline RL work reliably, it can be very powerful

# Offline RL

Offline RL is a very new field

If we can make offline RL work reliably, it can be very powerful

- Learn superhuman driving from existing Xiaomi dataset

# Offline RL

Offline RL is a very new field

If we can make offline RL work reliably, it can be very powerful

- Learn superhuman driving from existing Xiaomi dataset
- Learn superhuman surgery from existing surgery videos

# Offline RL

Offline RL is a very new field

If we can make offline RL work reliably, it can be very powerful

- Learn superhuman driving from existing Xiaomi dataset
- Learn superhuman surgery from existing surgery videos
- Learn language model finetuning from existing internet text

# Offline RL

Offline RL is a very new field

If we can make offline RL work reliably, it can be very powerful

- Learn superhuman driving from existing Xiaomi dataset
- Learn superhuman surgery from existing surgery videos
- Learn language model finetuning from existing internet text

I will also present my work on offline RL at ICRA next month

# Offline RL

Offline RL is a very new field

If we can make offline RL work reliably, it can be very powerful
- Learn superhuman driving from existing Xiaomi dataset
- Learn superhuman surgery from existing surgery videos
- Learn language model finetuning from existing internet text

I will also present my work on offline RL at ICRA next month

https://sites.google.com/view/llm-marl/home

# Offline RL

**Definition:** An **offline MDP** consists of:

# Offline RL

**Definition:** An **offline MDP** consists of:

- MDP with unknown $\mathcal{R}$, $\mathrm{Tr}$

# Offline RL

**Definition:** An **offline MDP** consists of:

- MDP with unknown $\mathcal{R}, \mathrm{Tr}$
- Dataset $\boldsymbol{X}$ of episodes

# Offline RL

**Definition:** An **offline MDP** consists of:

- MDP with unknown $\mathcal{R}$, $\mathrm{Tr}$
- Dataset $\boldsymbol{X}$ of episodes
  - Following a policy $\pi\left(a \mid s; \theta_\beta\right)$

# Offline RL

**Definition:** An **offline MDP** consists of:

- MDP with unknown $\mathcal{R}$, $\mathrm{Tr}$
- Dataset $\boldsymbol{X}$ of episodes
  - Following a policy $\pi\left(a \mid s; \theta_\beta\right)$

$$(S, A, \mathcal{R}, \mathrm{Tr}, \gamma, \boldsymbol{X})$$

# Offline RL

**Definition:** An **offline MDP** consists of:

- MDP with unknown $\mathcal{R}$, $\mathrm{Tr}$
- Dataset $\boldsymbol{X}$ of episodes
  - Following a policy $\pi(a \mid s; \theta_\beta)$

$$(S, A, \mathcal{R}, \mathrm{Tr}, \gamma, \boldsymbol{X})$$

$$\mathcal{R}(s_{t+1}) = ?$$

# Offline RL

**Definition:** An **offline MDP** consists of:

- MDP with unknown $\mathcal{R}$, $\mathrm{Tr}$
- Dataset $\boldsymbol{X}$ of episodes
  - Following a policy $\pi(a \mid s; \theta_\beta)$

$$(S, A, \mathcal{R}, \mathrm{Tr}, \gamma, \boldsymbol{X})$$

$$\mathcal{R}(s_{t+1}) = ? \qquad \mathrm{Tr}(s_{t+1} \mid s_t, a_t) = ?$$

# Offline RL

**Definition:** An **offline MDP** consists of:

- MDP with unknown $\mathcal{R}$, $\mathrm{Tr}$
- Dataset $\boldsymbol{X}$ of episodes
  - Following a policy $\pi\left(a \mid s; \theta_\beta\right)$

$$\left(S, A, \mathcal{R}, \mathrm{Tr}, \gamma, \boldsymbol{X}\right)$$

$$\mathcal{R}\left(s_{t+1}\right) = ? \qquad \mathrm{Tr}\left(s_{t+1} \mid s_t, a_t\right) = ?$$

$$\boldsymbol{X} = \begin{bmatrix} \boldsymbol{E}_{[1]} & \boldsymbol{E}_{[2]} & \dots \end{bmatrix}$$

# Offline RL

**Definition:** An **offline MDP** consists of:

- MDP with unknown $\mathcal{R}, \mathrm{Tr}$
- Dataset $\boldsymbol{X}$ of episodes
  - Following a policy $\pi(a \mid s; \theta_\beta)$

$$(S, A, \mathcal{R}, \mathrm{Tr}, \gamma, \boldsymbol{X})$$

$$\mathcal{R}(s_{t+1}) = ? \qquad \mathrm{Tr}(s_{t+1} \mid s_t, a_t) = ?$$

$$\boldsymbol{X} = \begin{bmatrix} \boldsymbol{E}_{[1]} & \boldsymbol{E}_{[2]} & \ldots \end{bmatrix} = \begin{bmatrix} \begin{bmatrix} s_0 & a_0 & d_0 & r_1 \\ s_1 & a_1 & d_1 & r_2 \\ \vdots & \vdots & \vdots & \end{bmatrix} & \begin{bmatrix} s_0 & a_0 & d_0 & r_1 \\ s_1 & a_1 & d_1 & r_2 \\ \vdots & \vdots & \vdots & \end{bmatrix} & \ldots \end{bmatrix}$$

# Offline RL

**Goal:** learn a policy that maximizes the return

# Offline RL

**Goal:** learn a policy that maximizes the return

$$\arg\max_{\theta_\pi} \mathbb{E}[\mathcal{G}(\boldsymbol{\tau}) \mid s_0; \theta_\pi]$$

# Offline RL

**Goal:** learn a policy that maximizes the return

$$\arg\max_{\theta_\pi} \mathbb{E}[\mathcal{G}(\boldsymbol{\tau}) \mid s_0; \theta_\pi]$$

There are two ways to approach offline RL

# Offline RL

**Goal:** learn a policy that maximizes the return

$$\arg\max_{\theta_\pi} \mathbb{E}[\mathcal{G}(\boldsymbol{\tau}) \mid s_0; \theta_\pi]$$

There are two ways to approach offline RL

- Improve behavior cloning with rewards

# Offline RL

**Goal:** learn a policy that maximizes the return

$$\arg\max_{\theta_\pi} \mathbb{E}[\mathcal{G}(\boldsymbol{\tau}) \mid s_0; \theta_\pi]$$

There are two ways to approach offline RL

- Improve behavior cloning with rewards
- Off-policy RL without exploration

# Offline RL

**Goal:** learn a policy that maximizes the return

$$\arg\max_{\theta_\pi} \mathbb{E}[\mathcal{G}(\boldsymbol{\tau}) \mid s_0; \theta_\pi]$$

There are two ways to approach offline RL

- Improve behavior cloning with rewards
- Off-policy RL without exploration

Let us begin with behavior cloning first

# Behavioral Cloning with Rewards

# Behavioral Cloning with Rewards

Recall the behavior cloning objective

# Behavioral Cloning with Rewards

Recall the behavior cloning objective

Want to minimize difference between learned and expert policy

# Behavioral Cloning with Rewards

Recall the behavior cloning objective

Want to minimize difference between learned and expert policy

$$\arg\min_{\theta_\pi} \sum_{s \in \boldsymbol{X}} \mathrm{KL}\left(\pi\left(a \mid s; \theta_\beta\right), \pi\left(a \mid s; \theta_\pi\right)\right)$$

# Behavioral Cloning with Rewards

Recall the behavior cloning objective

Want to minimize difference between learned and expert policy

$$\arg\min_{\theta_\pi} \sum_{s \in \boldsymbol{X}} \mathrm{KL}\left(\pi\left(a \mid s; \theta_\beta\right), \pi\left(a \mid s; \theta_\pi\right)\right)$$

From this, we derive the cross entropy loss

# Behavioral Cloning with Rewards

Recall the behavior cloning objective

Want to minimize difference between learned and expert policy

$$\arg\min_{\theta_\pi} \sum_{s \in \boldsymbol{X}} \mathrm{KL}\big(\pi\big(a \mid s; \theta_\beta\big), \pi\big(a \mid s; \theta_\pi\big)\big)$$

From this, we derive the cross entropy loss

$$\arg\min_{\theta_\pi} \sum_{s \in \boldsymbol{X}} \sum_{a \in A} -\pi\big(a \mid s; \theta_\beta\big) \log \pi\big(a \mid s; \theta_\pi\big)$$

# Behavioral Cloning with Rewards

$$\arg\min_{\theta_\pi} \sum_{s \in \boldsymbol{X}} \sum_{a \in A} -\pi(a \mid s; \theta_\beta) \log \pi(a \mid s; \theta_\pi)$$

# Behavioral Cloning with Rewards

$$\arg\min_{\theta_\pi} \sum_{s \in \boldsymbol{X}} \sum_{a \in A} -\pi(a \mid s; \theta_\beta) \log \pi(a \mid s; \theta_\pi)$$

Consider the following situation:

# Behavioral Cloning with Rewards

$$\arg\min_{\theta_\pi} \sum_{s \in \boldsymbol{X}} \sum_{a \in A} -\pi(a \mid s; \theta_\beta) \log \pi(a \mid s; \theta_\pi)$$

Consider the following situation:

- Two possible actions $A = \{a_+, a_-\}$

# Behavioral Cloning with Rewards

$$\arg\min_{\theta_\pi} \sum_{s \in \boldsymbol{X}} \sum_{a \in A} -\pi(a \mid s; \theta_\beta) \log \pi(a \mid s; \theta_\pi)$$

Consider the following situation:

- Two possible actions $A = \{a_+, a_-\}$
- Single state $s_0$ in the dataset, visited **twice**

# Behavioral Cloning with Rewards

$$\arg\min_{\theta_\pi} \sum_{s \in \boldsymbol{X}} \sum_{a \in A} -\pi(a \mid s; \theta_\beta) \log \pi(a \mid s; \theta_\pi)$$

Consider the following situation:

- Two possible actions $A = \{a_+, a_-\}$
- Single state $s_0$ in the dataset, visited **twice**
- First time, expert takes action $a_+$ in $s$

# Behavioral Cloning with Rewards

$$\arg\min_{\theta_\pi} \sum_{s \in \boldsymbol{X}} \sum_{a \in A} -\pi(a \mid s; \theta_\beta) \log \pi(a \mid s; \theta_\pi)$$

Consider the following situation:

- Two possible actions $A = \{a_+, a_-\}$
- Single state $s_0$ in the dataset, visited **twice**
- First time, expert takes action $a_+$ in $s$
- Second time, expert takes action $a_-$ in $s$

# Behavioral Cloning with Rewards

$$\arg\min_{\theta_\pi} \sum_{s \in \boldsymbol{X}} \sum_{a \in A} -\pi(a \mid s; \theta_\beta) \log \pi(a \mid s; \theta_\pi)$$

Consider the following situation:

- Two possible actions $A = \{a_+, a_-\}$
- Single state $s_0$ in the dataset, visited **twice**
- First time, expert takes action $a_+$ in $s$
- Second time, expert takes action $a_-$ in $s$

Expert must behave better in one state than the other!

# Behavioral Cloning with Rewards

$$\arg\min_{\theta_\pi} \sum_{s \in \boldsymbol{X}} \sum_{a \in A} -\pi(a \mid s; \theta_\beta) \log \pi(a \mid s; \theta_\pi)$$

Consider the following situation:

- Two possible actions $A = \{a_+, a_-\}$
- Single state $s_0$ in the dataset, visited **twice**
- First time, expert takes action $a_+$ in $s$
- Second time, expert takes action $a_-$ in $s$

Expert must behave better in one state than the other!

$$\arg\min_{\theta_\pi} \sum_{a \in \{a_+, a_-\}} -\pi(a \mid s_0; \theta_\beta) \log \pi(a \mid s_0; \theta_\pi)$$

# Behavioral Cloning with Rewards

$$\pi(a_+ \mid s_0; \theta_\beta) = 0.5 \qquad \pi(a_- \mid s_0; \theta_\beta) = 0.5$$

# Behavioral Cloning with Rewards

$$\pi\left(a_+ \mid s_0; \theta_\beta\right) = 0.5 \qquad \pi\left(a_- \mid s_0; \theta_\beta\right) = 0.5$$

**Question:** Which action is better behavior?

# Behavioral Cloning with Rewards

$$\pi\left(a_{+} \mid s_0; \theta_\beta\right) = 0.5 \qquad \pi\left(a_{-} \mid s_0; \theta_\beta\right) = 0.5$$

# Behavioral Cloning with Rewards

$$\pi(a_+ \mid s_0; \theta_\beta) = 0.5 \qquad \pi(a_- \mid s_0; \theta_\beta) = 0.5$$

**Question:** Two actions in same state, what policy $\theta_\pi$ does BC learn?

# Behavioral Cloning with Rewards

$\pi(a_+ \mid s_0; \theta_\beta) = 0.5$ $\quad$ $\pi(a_- \mid s_0; \theta_\beta) = 0.5$

**Question:** Two actions in same state, what policy $\theta_\pi$ does BC learn?

**Answer:** Randomly choose $a \in \{a_+, a_-\}$

# Behavioral Cloning with Rewards

$$\pi(a_+ \mid s_0; \theta_\beta) = 0.5 \qquad \pi(a_- \mid s_0; \theta_\beta) = 0.5$$

**Question:** Two actions in same state, what policy $\theta_\pi$ does BC learn?

**Answer:** Randomly choose $a \in \{a_+, a_-\}$

**Question:** Is this a good idea?

# Behavioral Cloning with Rewards

$$\pi\left(a_{+} \mid s_0; \theta_\beta\right) = 0.5 \qquad \pi\left(a_{-} \mid s_0; \theta_\beta\right) = 0.5$$

# Behavioral Cloning with Rewards

$$\pi(a_+ \mid s_0; \theta_\beta) = 0.5 \qquad \pi(a_- \mid s_0; \theta_\beta) = 0.5$$

**Question:** We know which action is better, how can we measure this?

# Behavioral Cloning with Rewards

$$\pi(a_+ \mid s_0; \theta_\beta) = 0.5 \qquad \pi(a_- \mid s_0; \theta_\beta) = 0.5$$

**Question:** We know which action is better, how can we measure this?

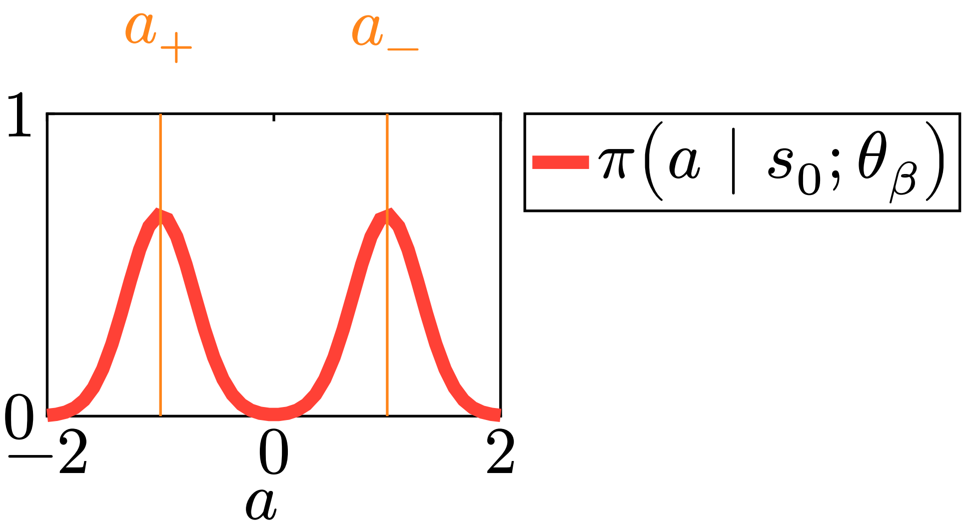
**Answer:** Reward! In BC, no reward. In offline RL, we have reward!

# Behavioral Cloning with Rewards

Expert has equal probability for both good and bad actions

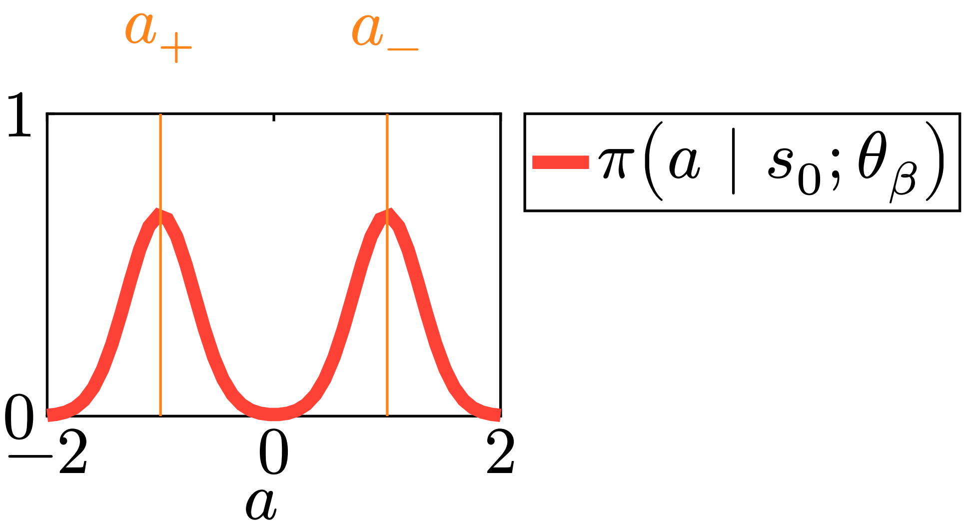
# Behavioral Cloning with Rewards

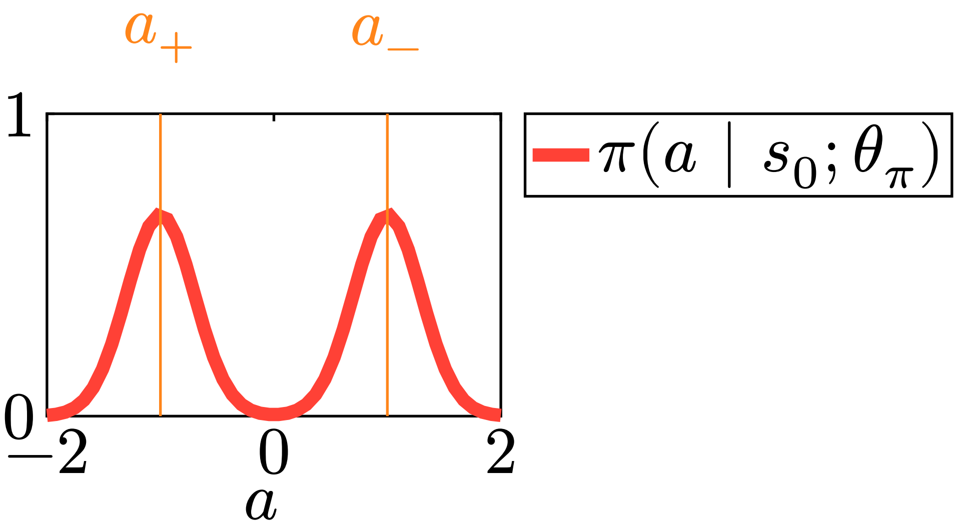
Expert has equal probability for both good and bad actions



# Behavioral Cloning with Rewards

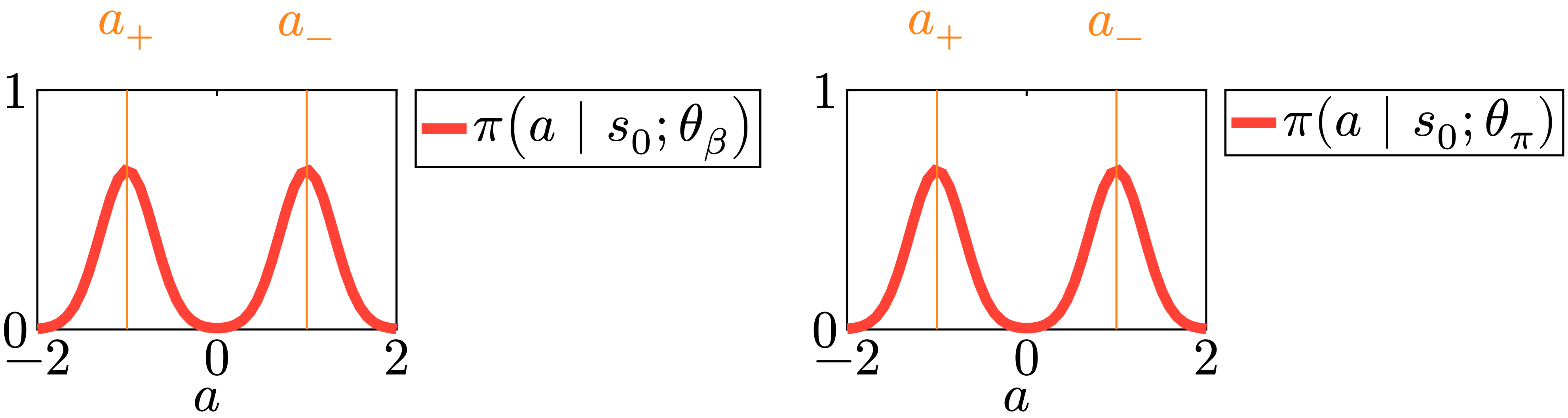
Expert has equal probability for both good and bad actions



# Behavioral Cloning with Rewards

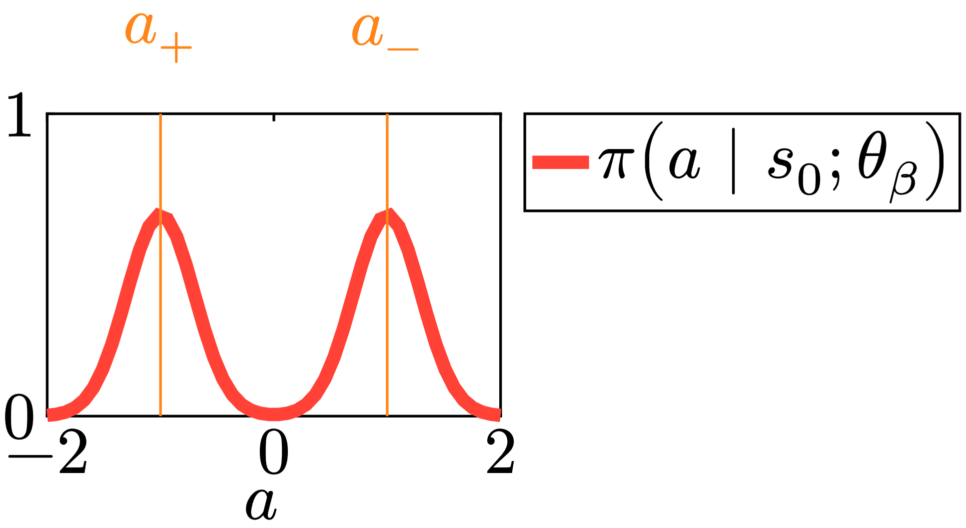
Expert has equal probability for both good and bad actions

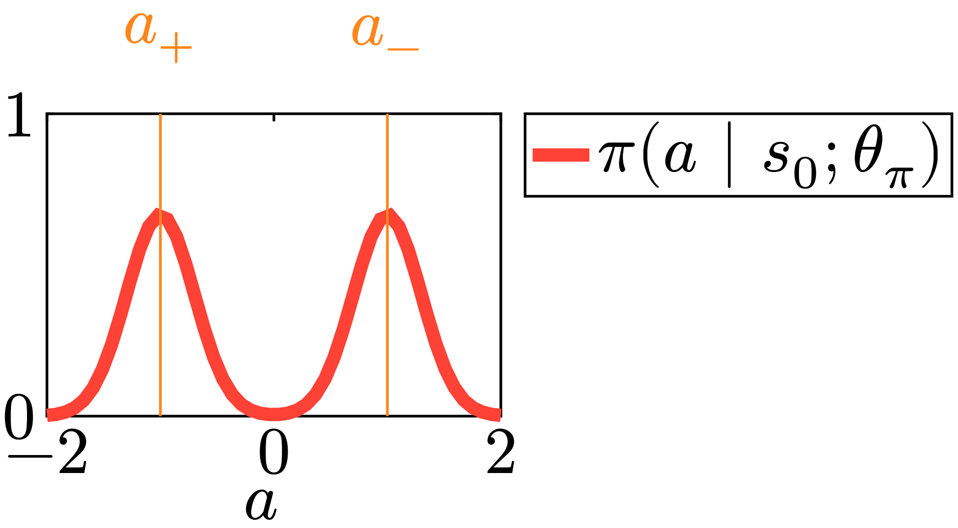



With IL, we can only imitate the expert

# Behavioral Cloning with Rewards

Expert has equal probability for both good and bad actions



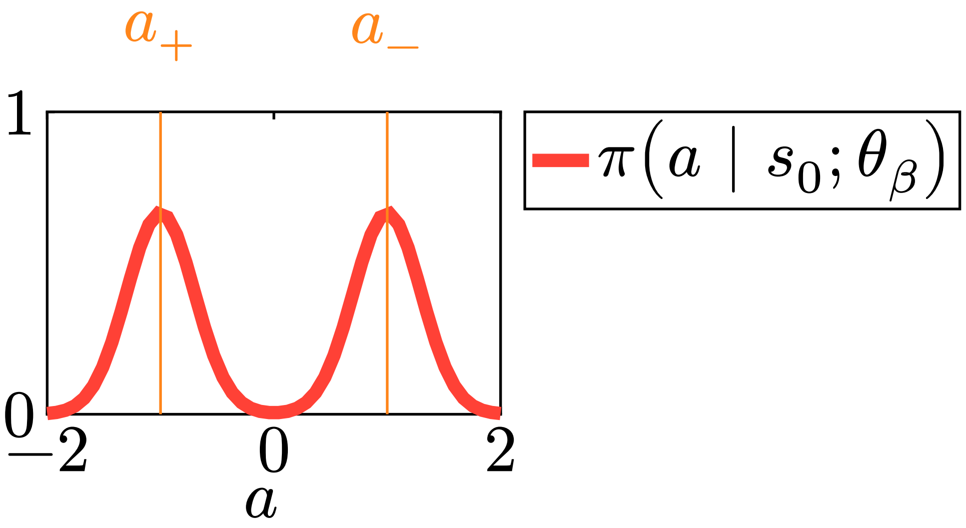



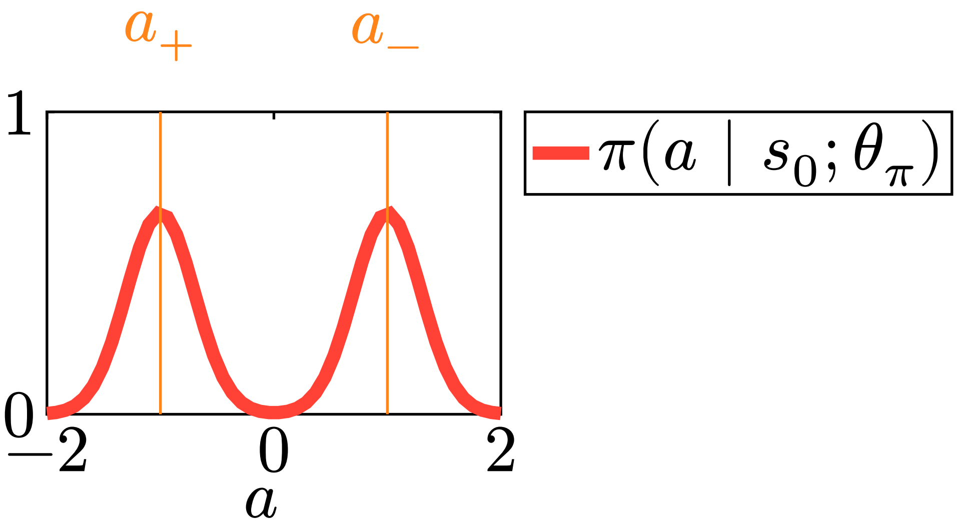

With IL, we can only imitate the expert

With offline RL, we have empirical rewards/returns, we can do better!

$$\hat{\mathbb{E}}[\mathcal{R}(s_1) \mid s_0, a_0] \qquad \hat{\mathbb{E}}[\mathcal{G}(\boldsymbol{\tau}) \mid s_0; \theta_\beta]$$

# Behavioral Cloning with Rewards

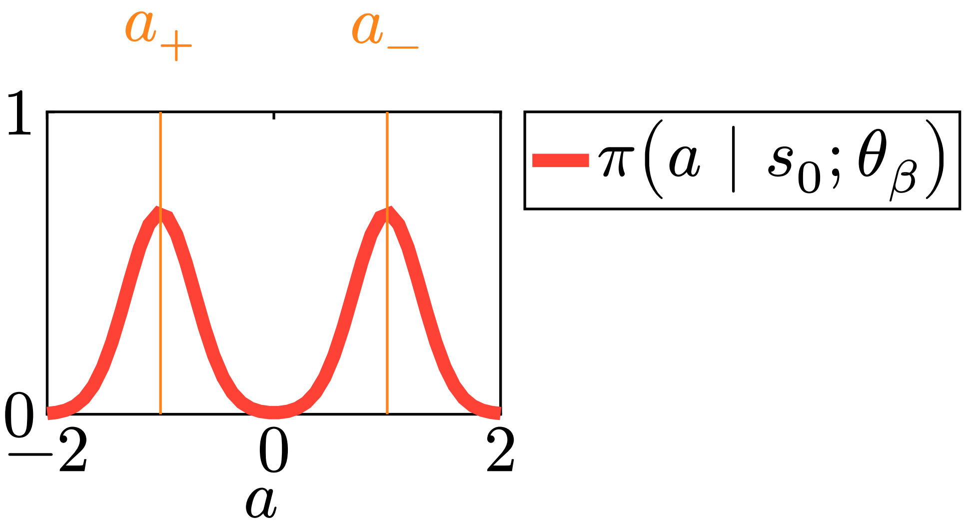


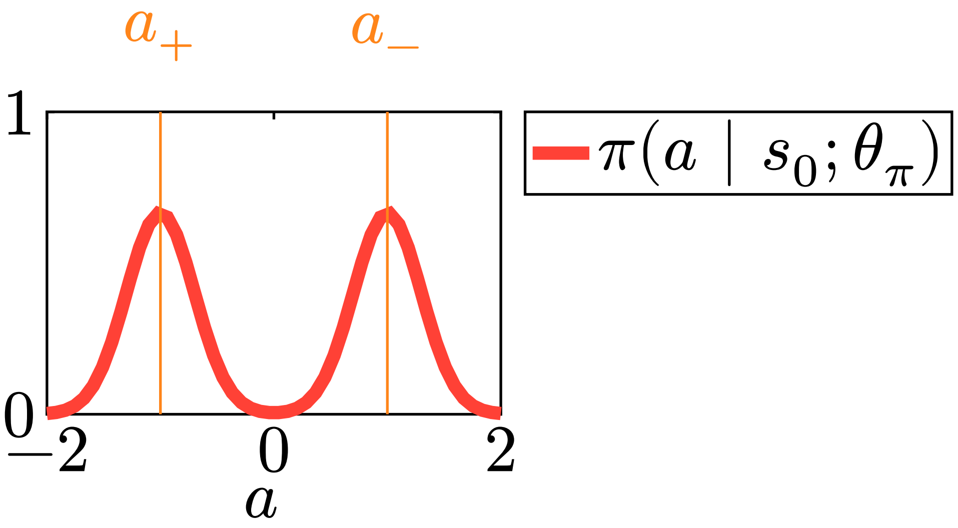




$$\hat{\mathbb{E}}\left[\mathcal{R}(s_1) \mid s_0; \theta_\beta\right]$$

$$\hat{\mathbb{E}}\left[\mathcal{G}(\boldsymbol{\tau}) \mid s_0; \theta_\beta\right]$$

# Behavioral Cloning with Rewards



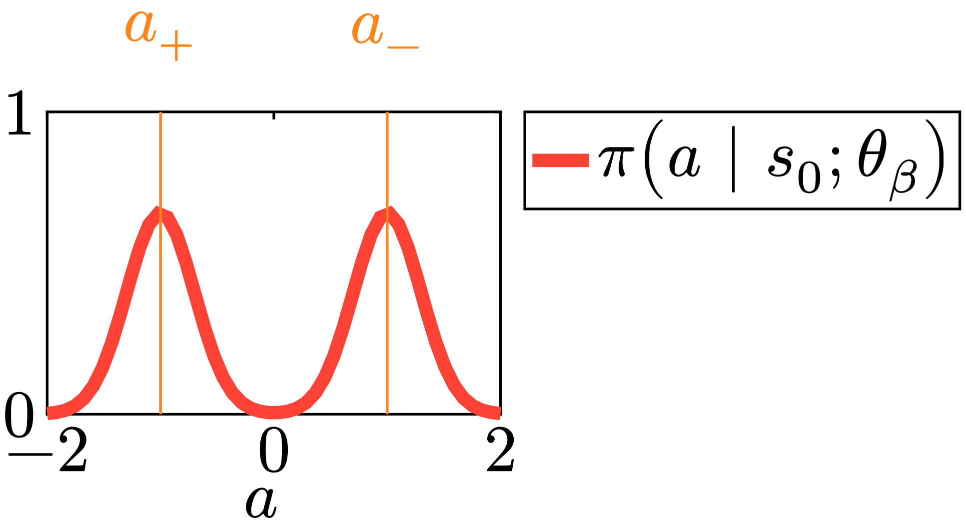



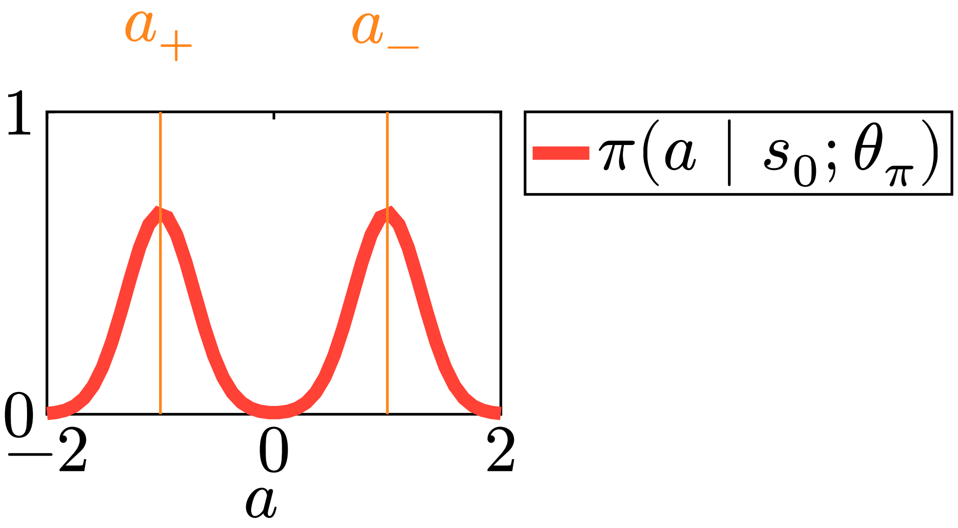

$$\hat{\mathbb{E}}[\mathcal{R}(s_1) \mid s_0; \theta_\beta]$$

$$\hat{\mathbb{E}}[\mathcal{G}(\boldsymbol{\tau}) \mid s_0; \theta_\beta]$$

**Question:** How should we change $\pi(a_+ \mid s_0; \theta_\pi)$ and $\pi(a_- \mid s_0; \theta_\pi)$?

# Behavioral Cloning with Rewards



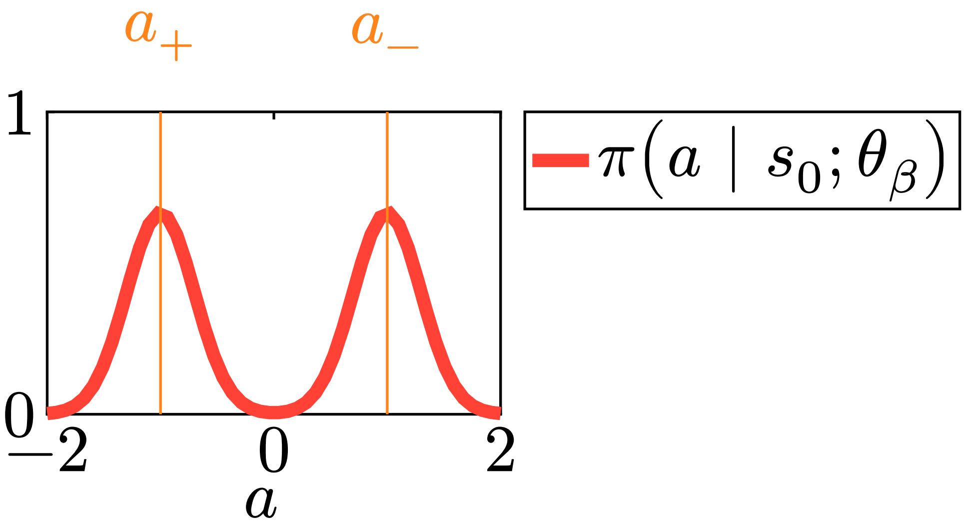




$$\hat{\mathbb{E}}[\mathcal{R}(s_1) \mid s_0; \theta_\beta] \qquad \hat{\mathbb{E}}[\mathcal{G}(\boldsymbol{\tau}) \mid s_0; \theta_\beta]$$

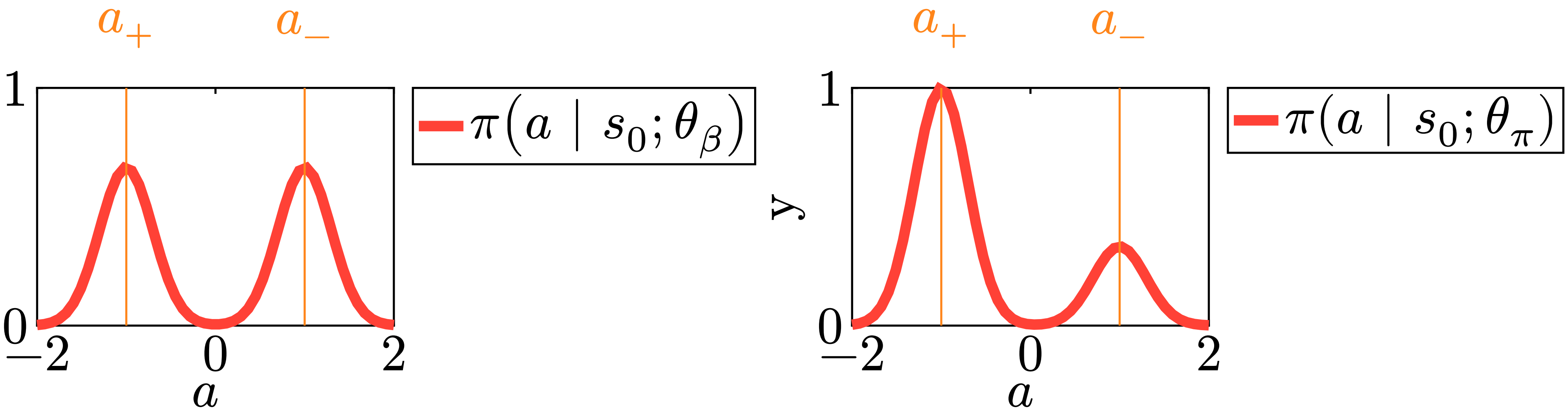
**Question:** How should we change $\pi(a_+ \mid s_0; \theta_\pi)$ and $\pi(a_- \mid s_0; \theta_\pi)$?

**Answer:** Increase probability of $a_+$, decrease probability of $a_-$

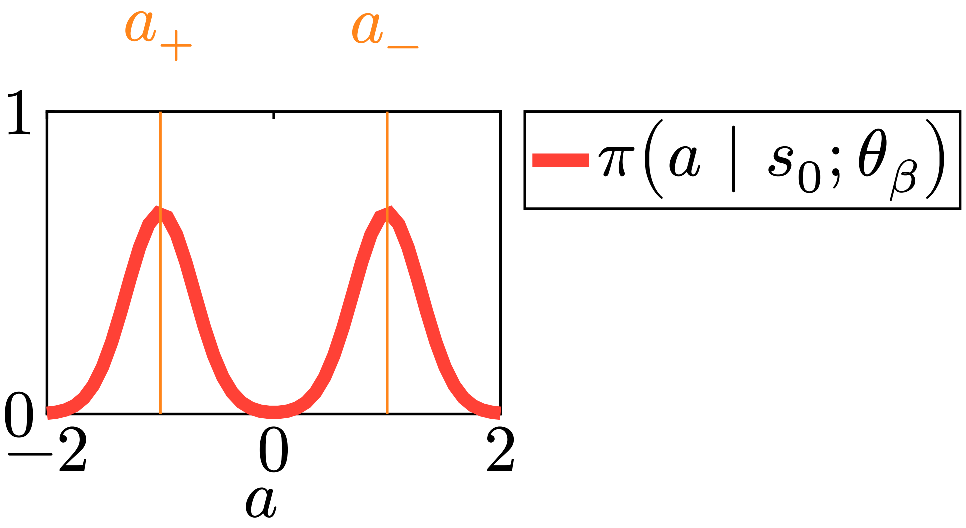
# Behavioral Cloning with Rewards

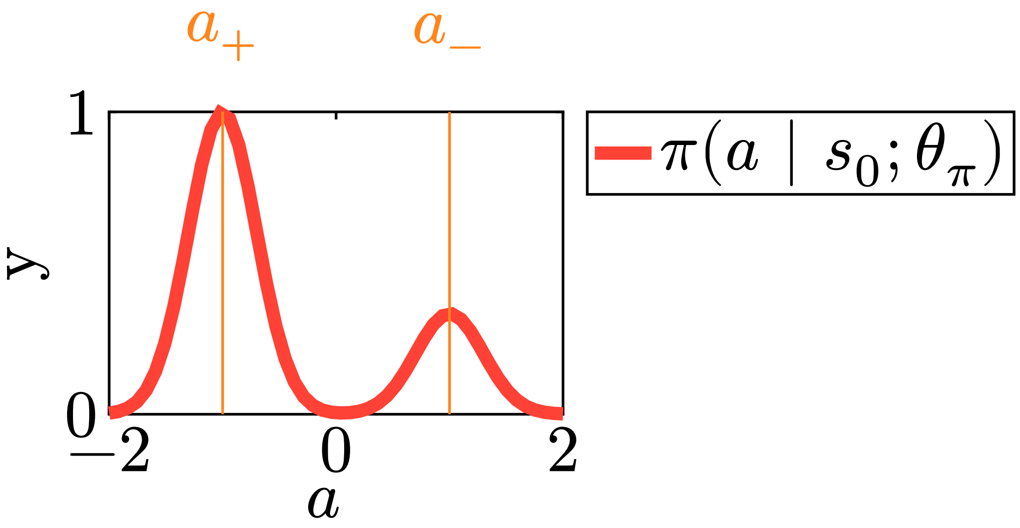


# Behavioral Cloning with Rewards



# Behavioral Cloning with Rewards




We want to reweight action probabilities based on reward or return

# Behavioral Cloning with Rewards

$$\arg\min_{\theta_\pi} \sum_{a \in \{a_+, a_-\}} -\pi(a \mid s_0; \theta_\beta) \log \pi(a \mid s_0; \theta_\pi)$$

Increase probability of $a_+$ and decrease probability of $a_-$ using reward

# Behavioral Cloning with Rewards

$$\arg\min_{\theta_\pi} \sum_{a \in \{a_+, a_-\}} -\pi(a \mid s_0; \theta_\beta) \log \pi(a \mid s_0; \theta_\pi)$$

Increase probability of $a_+$ and decrease probability of $a_-$ using reward

**Question:** How?

# Behavioral Cloning with Rewards

$$\arg\min_{\theta_\pi} \sum_{a \in \{a_+, a_-\}} -\pi(a \mid s_0; \theta_\beta) \log \pi(a \mid s_0; \theta_\pi)$$

Increase probability of $a_+$ and decrease probability of $a_-$ using reward

**Question:** How? Hint:

$$\arg\min_{\theta_\pi} -\pi(a_+ \mid s_0; \theta_\beta) \log \pi(a_+ \mid s_0; \theta_\pi) - \pi(a_- \mid s_0; \theta_\beta) \log \pi(a_- \mid s_0; \theta_\pi)$$

# Behavioral Cloning with Rewards

$$\arg\min_{\theta_\pi} \sum_{a \in \{a_+, a_-\}} -\pi(a \mid s_0; \theta_\beta) \log \pi(a \mid s_0; \theta_\pi)$$

Increase probability of $a_+$ and decrease probability of $a_-$ using reward

**Question:** How? Hint:

$$\arg\min_{\theta_\pi} -\pi(a_+ \mid s_0; \theta_\beta) \log \pi(a_+ \mid s_0; \theta_\pi) - \pi(a_- \mid s_0; \theta_\beta) \log \pi(a_- \mid s_0; \theta_\pi)$$

**Answer:** Reweight each action in the objective using the reward

# Behavioral Cloning with Rewards

$$\underset{\theta_\pi}{\arg\min} \sum_{a \in \{a_+, a_-\}} -\pi(a \mid s_0; \theta_\beta) \log \pi(a \mid s_0; \theta_\pi)$$

Increase probability of $a_+$ and decrease probability of $a_-$ using reward

**Question:** How? Hint:

$$\underset{\theta_\pi}{\arg\min} -\pi(a_+ \mid s_0; \theta_\beta) \log \pi(a_+ \mid s_0; \theta_\pi) - \pi(a_- \mid s_0; \theta_\beta) \log \pi(a_- \mid s_0; \theta_\pi)$$

**Answer:** Reweight each action in the objective using the reward

$$\underset{\theta_\pi}{\arg\min} -0.9\pi(a_+ \mid s_0; \theta_\beta) \log \pi(a_+ \mid s_0; \theta_\pi) - 0.1\pi(a_- \mid s_0; \theta_\beta) \log \pi(a_- \mid s_0; \theta_\pi)$$

# Behavioral Cloning with Rewards

$$\arg\min_{\theta_\pi} -0.9\pi\left(a_+ \mid s_0; \theta_\beta\right) \log \pi\left(a_+ \mid s_0; \theta_\pi\right) - 0.1\pi\left(a_- \mid s_0; \theta_\beta\right) \log \pi\left(a_- \mid s_0; \theta_\pi\right)$$

# Behavioral Cloning with Rewards

$$\arg\min_{\theta_\pi} -0.9\pi(a_+ \mid s_0; \theta_\beta) \log \pi(a_+ \mid s_0; \theta_\pi) - 0.1\pi(a_- \mid s_0; \theta_\beta) \log \pi(a_- \mid s_0; \theta_\pi)$$

More generally, use weights $w$

# Behavioral Cloning with Rewards

$$\arg\min_{\theta_\pi} -0.9\pi(a_+ \mid s_0; \theta_\beta) \log \pi(a_+ \mid s_0; \theta_\pi) - 0.1\pi(a_- \mid s_0; \theta_\beta) \log \pi(a_- \mid s_0; \theta_\pi)$$

More generally, use weights $w$

$$\arg\min_{\theta_\pi} -w_+\pi(a_+ \mid s_0; \theta_\beta) \log \pi(a_+ \mid s_0; \theta_\pi) - w_-\pi(a_- \mid s_0; \theta_\beta) \log \pi(a_- \mid s_0; \theta_\pi)$$

**Question:** What can we use for $w_+, w_-$?

# Behavioral Cloning with Rewards

$$\arg\min_{\theta_\pi} -0.9\pi(a_+ \mid s_0; \theta_\beta) \log \pi(a_+ \mid s_0; \theta_\pi) - 0.1\pi(a_- \mid s_0; \theta_\beta) \log \pi(a_- \mid s_0; \theta_\pi)$$

More generally, use weights $w$

$$\arg\min_{\theta_\pi} -w_+\pi(a_+ \mid s_0; \theta_\beta) \log \pi(a_+ \mid s_0; \theta_\pi) - w_-\pi(a_- \mid s_0; \theta_\beta) \log \pi(a_- \mid s_0; \theta_\pi)$$

**Question:** What can we use for $w_+, w_-$?

$$\hat{\mathbb{E}}[\mathcal{R}(s_1) \mid s_0; \theta_\beta]$$

# Behavioral Cloning with Rewards

**Definition:** In **Expectation Maximization Reinforcement Learning** (EMRL, Hinton), we reweight the behavior cloning objective using the reward

# Behavioral Cloning with Rewards

**Definition:** In **Expectation Maximization Reinforcement Learning** (EMRL, Hinton), we reweight the behavior cloning objective using the reward

$$\theta_\pi = \arg\min_{\theta_\pi} \left[ \underbrace{\sum_{s_0 \in \boldsymbol{X}} \sum_{a \in A} -\pi(a \mid s_0; \theta_\beta) \log \pi(a \mid s_0; \theta_\pi)}_{\text{BC objective}} \cdot \underbrace{\hat{\mathbb{E}}[\mathcal{R}(s_1) \mid s_0, a]}_{\text{Weighting}} \right]$$

# Behavioral Cloning with Rewards

$$\theta_\pi = \arg\min_{\theta_\pi} \left[ \sum_{s_0 \in \boldsymbol{X}} \sum_{a \in A} -\pi\left(a \mid s_0; \theta_\beta\right) \log \pi\left(a \mid s_0; \theta_\pi\right) \cdot \hat{\mathbb{E}}[\mathcal{R}(s_1) \mid s_0, a] \right]$$

# Behavioral Cloning with Rewards

$$\theta_\pi = \arg\min_{\theta_\pi} \left[ \sum_{s_0 \in \boldsymbol{X}} \sum_{a \in A} -\pi\left(a \mid s_0; \theta_\beta\right) \log \pi(a \mid s_0; \theta_\pi) \cdot \hat{\mathbb{E}}[\mathcal{R}(s_1) \mid s_0, a] \right]$$

Consider the simplified example, now with rewards $r$

# Behavioral Cloning with Rewards

$$\theta_\pi = \arg\min_{\theta_\pi} \left[ \sum_{s_0 \in \boldsymbol{X}} \sum_{a \in A} -\pi\left(a \mid s_0; \theta_\beta\right) \log \pi\left(a \mid s_0; \theta_\pi\right) \cdot \hat{\mathbb{E}}[\mathcal{R}(s_1) \mid s_0, a] \right]$$

Consider the simplified example, now with rewards $r$

$$\arg\min_{\theta_\pi} -r_+ \pi\left(a_+ \mid s_0; \theta_\beta\right) \log \pi\left(a_+ \mid s_0; \theta_\pi\right) - r_- \pi\left(a_- \mid s_0; \theta_\beta\right) \log \pi\left(a_- \mid s_0; \theta_\pi\right)$$

# Behavioral Cloning with Rewards

$$\theta_\pi = \arg\min_{\theta_\pi} \left[ \sum_{s_0 \in \boldsymbol{X}} \sum_{a \in A} -\pi(a \mid s_0; \theta_\beta) \log \pi(a \mid s_0; \theta_\pi) \cdot \hat{\mathbb{E}}[\mathcal{R}(s_1) \mid s_0, a] \right]$$

Consider the simplified example, now with rewards $r$

$$\arg\min_{\theta_\pi} -r_+ \pi(a_+ \mid s_0; \theta_\beta) \log \pi(a_+ \mid s_0; \theta_\pi) - r_- \pi(a_- \mid s_0; \theta_\beta) \log \pi(a_- \mid s_0; \theta_\pi)$$

**Question:** Are there any problems with EMRL?

# Behavioral Cloning with Rewards

$$\theta_\pi = \arg\min_{\theta_\pi} \left[ \sum_{s_0 \in \boldsymbol{X}} \sum_{a \in A} -\pi(a \mid s_0; \theta_\beta) \log \pi(a \mid s_0; \theta_\pi) \cdot \hat{\mathbb{E}}[\mathcal{R}(s_1) \mid s_0, a] \right]$$

Consider the simplified example, now with rewards $r$

$$\arg\min_{\theta_\pi} -{\color{red}r_+}\pi(a_+ \mid s_0; \theta_\beta) \log \pi(a_+ \mid s_0; \theta_\pi) - {\color{red}r_-}\pi(a_- \mid s_0; \theta_\beta) \log \pi(a_- \mid s_0; \theta_\pi)$$

**Question:** Are there any problems with EMRL?

Hint: What if the reward is negative?

# Behavioral Cloning with Rewards

$$\theta_\pi = \arg\min_{\theta_\pi} \left[ \sum_{s_0 \in \boldsymbol{X}} \sum_{a \in A} -\pi(a \mid s_0; \theta_\beta) \log \pi(a \mid s_0; \theta_\pi) \cdot \hat{\mathbb{E}}[\mathcal{R}(s_1) \mid s_0, a] \right]$$

Consider the simplified example, now with rewards $r$

$$\arg\min_{\theta_\pi} -{\color{red}r_+}\pi(a_+ \mid s_0; \theta_\beta) \log \pi(a_+ \mid s_0; \theta_\pi) - {\color{red}r_-}\pi(a_- \mid s_0; \theta_\beta) \log \pi(a_- \mid s_0; \theta_\pi)$$

**Question:** Are there any problems with EMRL?

Hint: What if the reward is negative?

Reweighting only makes sense with positive weights

# Behavioral Cloning with Rewards

$$\theta_\pi = \arg\min_{\theta_\pi} \left[ \sum_{s_0 \in \boldsymbol{X}} \sum_{a \in A} -\pi\left(a \mid s_0; \theta_\beta\right) \log \pi\left(a \mid s_0; \theta_\pi\right) \cdot \hat{\mathbb{E}}[\mathcal{R}(s_1) \mid s_0, a] \right]$$

Consider the simplified example, now with rewards $r$

$$\arg\min_{\theta_\pi} -{\color{red} r_+} \pi\left(a_+ \mid s_0; \theta_\beta\right) \log \pi\left(a_+ \mid s_0; \theta_\pi\right) - {\color{red} r_-} \pi\left(a_- \mid s_0; \theta_\beta\right) \log \pi\left(a_- \mid s_0; \theta_\pi\right)$$

**Question:** Are there any problems with EMRL?

Hint: What if the reward is negative?

Reweighting only makes sense with positive weights

EMRL only works with positive rewards!

# Behavioral Cloning with Rewards

$$\arg\min_{\theta_\pi} -r_+ \pi(a_+ \mid s_0; \theta_\beta) \log \pi(a_+ \mid s_0; \theta_\pi) - r_- \pi(a_- \mid s_0; \theta_\beta) \log \pi(a_- \mid s_0; \theta_\pi)$$

# Behavioral Cloning with Rewards

$$\arg\min_{\theta_\pi} -r_+ \pi(a_+ \mid s_0; \theta_\beta) \log \pi(a_+ \mid s_0; \theta_\pi) - r_- \pi(a_- \mid s_0; \theta_\beta) \log \pi(a_- \mid s_0; \theta_\pi)$$

$$\mathbb{E}[\mathcal{R}(s_1) \mid s_0, a_0] \in [-\infty, \infty]$$

# Behavioral Cloning with Rewards

$$\arg\min_{\theta_\pi} -r_+ \pi(a_+ \mid s_0; \theta_\beta) \log \pi(a_+ \mid s_0; \theta_\pi) - r_- \pi(a_- \mid s_0; \theta_\beta) \log \pi(a_- \mid s_0; \theta_\pi)$$

$$\mathbb{E}[\mathcal{R}(s_1) \mid s_0, a_0] \in [-\infty, \infty]$$

We want our algorithm to work with positive and negative rewards

# Behavioral Cloning with Rewards

$$\arg\min_{\theta_\pi} -r_+ \pi(a_+ \mid s_0; \theta_\beta) \log \pi(a_+ \mid s_0; \theta_\pi) - r_- \pi(a_- \mid s_0; \theta_\beta) \log \pi(a_- \mid s_0; \theta_\pi)$$

$$\mathbb{E}[\mathcal{R}(s_1) \mid s_0, a_0] \in [-\infty, \infty]$$

We want our algorithm to work with positive and negative rewards

Need a mapping between rewards and weights $f : [-\infty, \infty] \mapsto [0, \infty]$

# Behavioral Cloning with Rewards

$$\arg\min_{\theta_\pi} -r_+ \pi(a_+ \mid s_0; \theta_\beta) \log \pi(a_+ \mid s_0; \theta_\pi) - r_- \pi(a_- \mid s_0; \theta_\beta) \log \pi(a_- \mid s_0; \theta_\pi)$$

$$\mathbb{E}[\mathcal{R}(s_1) \mid s_0, a_0] \in [-\infty, \infty]$$

We want our algorithm to work with positive and negative rewards

Need a mapping between rewards and weights $f : [-\infty, \infty] \mapsto [0, \infty]$

**Question:** Any other properties for $f$?

# Behavioral Cloning with Rewards

$$\arg\min_{\theta_\pi} -r_+ \pi(a_+ \mid s_0; \theta_\beta) \log \pi(a_+ \mid s_0; \theta_\pi) - r_- \pi(a_- \mid s_0; \theta_\beta) \log \pi(a_- \mid s_0; \theta_\pi)$$

$$\mathbb{E}[\mathcal{R}(s_1) \mid s_0, a_0] \in [-\infty, \infty]$$

We want our algorithm to work with positive and negative rewards

Need a mapping between rewards and weights $f : [-\infty, \infty] \mapsto [0, \infty]$

**Question:** Any other properties for $f$? Hint: Does $f(r) = |r|$ work?

# Behavioral Cloning with Rewards

$$\arg\min_{\theta_\pi} -r_+ \pi(a_+ \mid s_0; \theta_\beta) \log \pi(a_+ \mid s_0; \theta_\pi) - r_- \pi(a_- \mid s_0; \theta_\beta) \log \pi(a_- \mid s_0; \theta_\pi)$$

$$\mathbb{E}[\mathcal{R}(s_1) \mid s_0, a_0] \in [-\infty, \infty]$$

We want our algorithm to work with positive and negative rewards

Need a mapping between rewards and weights $f : [-\infty, \infty] \mapsto [0, \infty]$

**Question:** Any other properties for $f$? Hint: Does $f(r) = |r|$ work?

**Answer:** $f$ must be **monotonic** to ensure we still maximize rewards!

# Behavioral Cloning with Rewards

$$\arg\min_{\theta_\pi} -r_+ \pi(a_+ \mid s_0; \theta_\beta) \log \pi(a_+ \mid s_0; \theta_\pi) - r_- \pi(a_- \mid s_0; \theta_\beta) \log \pi(a_- \mid s_0; \theta_\pi)$$

$$\mathbb{E}[\mathcal{R}(s_1) \mid s_0, a_0] \in [-\infty, \infty]$$

We want our algorithm to work with positive and negative rewards

Need a mapping between rewards and weights $f : [-\infty, \infty] \mapsto [0, \infty]$

**Question:** Any other properties for $f$? Hint: Does $f(r) = |r|$ work?

**Answer:** $f$ must be **monotonic** to ensure we still maximize rewards!

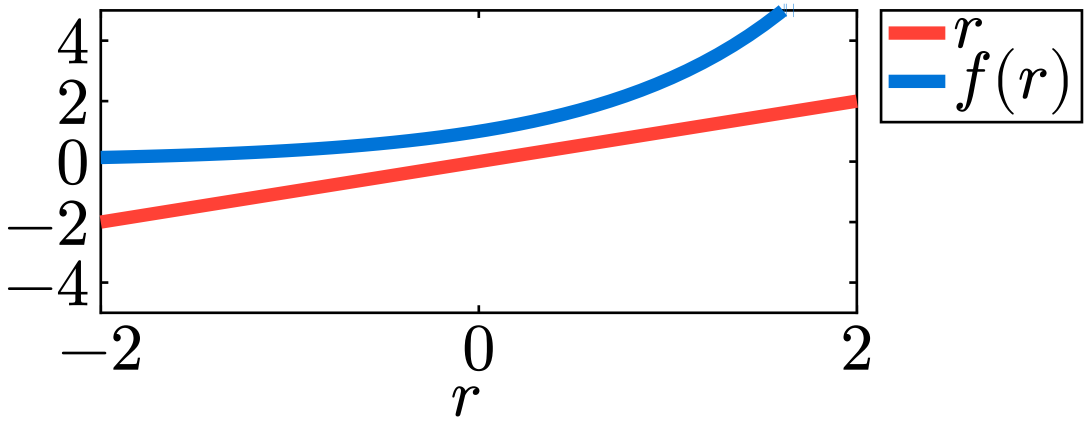
$$r_+ > r_- \Rightarrow f(r_+) > f(r_-)$$

# Behavioral Cloning with Rewards

**Question:** What functions $f : [-\infty, \infty] \mapsto [0, \infty]$ are monotonic?

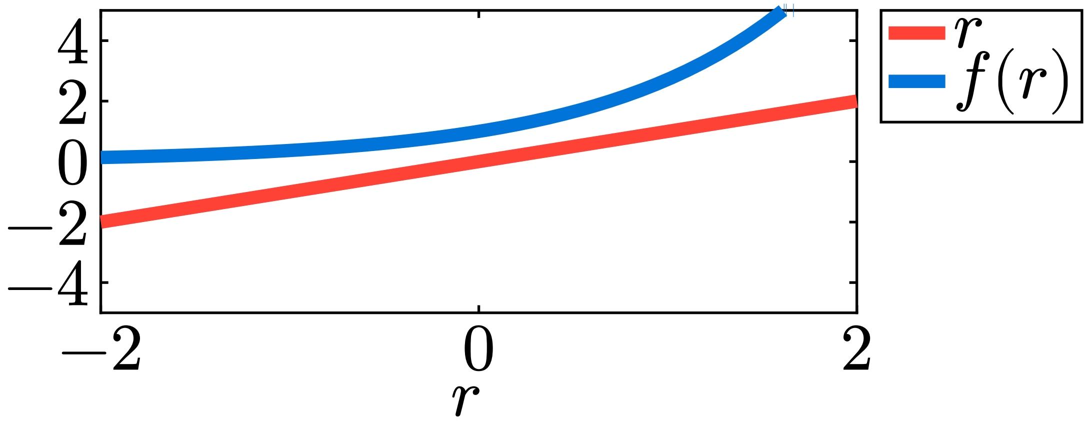
# Behavioral Cloning with Rewards

**Question:** What functions $f : [-\infty, \infty] \mapsto [0, \infty]$ are monotonic?



# Behavioral Cloning with Rewards

**Question:** What functions $f : [-\infty, \infty] \mapsto [0, \infty]$ are monotonic?




$$f(r) = e^r$$

# Behavioral Cloning with Rewards

**Definition:** Reward Weighted Regression (RWR) reweights the behavior cloning objective using the exponentiated reward

# Behavioral Cloning with Rewards

**Definition:** Reward Weighted Regression (RWR) reweights the behavior cloning objective using the exponentiated reward

$$\theta_\pi = \arg\min_{\theta_\pi} \sum_{s_0 \in \boldsymbol{X}} \sum_{a \in A} -\pi(a \mid s_0; \theta_\beta) \log \pi(a \mid s_0; \theta_\pi) \cdot \exp\left(\hat{\mathbb{E}}[\mathcal{R}(s_1) \mid s_0, a]\right)$$

# Behavioral Cloning with Rewards

**Definition:** Reward Weighted Regression (RWR) reweights the behavior cloning objective using the exponentiated reward

$$\theta_\pi = \arg\min_{\theta_\pi} \sum_{s_0 \in \boldsymbol{X}} \sum_{a \in A} -\pi(a \mid s_0; \theta_\beta) \log \pi(a \mid s_0; \theta_\pi) \cdot \exp\left(\hat{\mathbb{E}}[\mathcal{R}(s_1) \mid s_0, a]\right)$$

Consider an infinitely large and diverse dataset containing all $s, a$

# Behavioral Cloning with Rewards

**Definition:** Reward Weighted Regression (RWR) reweights the behavior cloning objective using the exponentiated reward

$$\theta_\pi = \arg\min_{\theta_\pi} \sum_{s_0 \in \boldsymbol{X}} \sum_{a \in A} -\pi(a \mid s_0; \theta_\beta) \log \pi(a \mid s_0; \theta_\pi) \cdot \exp\left(\hat{\mathbb{E}}[\mathcal{R}(s_1) \mid s_0, a]\right)$$

Consider an infinitely large and diverse dataset containing all $s, a$

Then, weights are proportional to Boltzmann distribution (softmax)

# Behavioral Cloning with Rewards

**Definition:** Reward Weighted Regression (RWR) reweights the behavior cloning objective using the exponentiated reward

$$\theta_\pi = \arg\min_{\theta_\pi} \sum_{s_0 \in \boldsymbol{X}} \sum_{a \in A} -\pi(a \mid s_0; \theta_\beta) \log \pi(a \mid s_0; \theta_\pi) \cdot \exp\left(\hat{\mathbb{E}}[\mathcal{R}(s_1) \mid s_0, a]\right)$$

Consider an infinitely large and diverse dataset containing all $s, a$

Then, weights are proportional to Boltzmann distribution (softmax)

$$\sum_{a_0 \in A} \exp\left(\hat{\mathbb{E}}[\mathcal{R}(s_1) \mid s_0, a_0]\right) \propto \sum_{a_i \in A} \frac{\exp\left(\hat{\mathbb{E}}[\mathcal{R}(s_1) \mid s_0, a_i]\right)}{\sum_{a_0 \in A} \exp\left(\hat{\mathbb{E}}[\mathcal{R}(s_1) \mid s_0, a_0]\right)}$$

# Behavioral Cloning with Rewards

$$\theta_\pi = \arg\min_{\theta_\pi} \sum_{s_0 \in \boldsymbol{X}} \sum_{a \in A} -\pi(a \mid s_0; \theta_\beta) \log \pi(a \mid s_0; \theta_\pi) \cdot \exp\left(\hat{\mathbb{E}}[\mathcal{R}(s_1) \mid s_0, a]\right)$$

# Behavioral Cloning with Rewards

$$\theta_\pi = \arg\min_{\theta_\pi} \sum_{s_0 \in \boldsymbol{X}} \sum_{a \in A} -\pi(a \mid s_0; \theta_\beta) \log \pi(a \mid s_0; \theta_\pi) \cdot \exp\left(\hat{\mathbb{E}}[\mathcal{R}(s_1) \mid s_0, a]\right)$$

RWR find $\theta_\pi$ that maximizes the reward

# Behavioral Cloning with Rewards

$$\theta_\pi = \arg\min_{\theta_\pi} \sum_{s_0 \in \boldsymbol{X}} \sum_{a \in A} -\pi(a \mid s_0; \theta_\beta) \log \pi(a \mid s_0; \theta_\pi) \cdot \exp\left(\hat{\mathbb{E}}[\mathcal{R}(s_1) \mid s_0, a]\right)$$

RWR find $\theta_\pi$ that maximizes the reward

**Question:** Do we maximize the reward in RL?

# Behavioral Cloning with Rewards

$$\theta_\pi = \arg\min_{\theta_\pi} \sum_{s_0 \in \boldsymbol{X}} \sum_{a \in A} -\pi(a \mid s_0; \theta_\beta) \log \pi(a \mid s_0; \theta_\pi) \cdot \exp\left(\hat{\mathbb{E}}[\mathcal{R}(s_1) \mid s_0, a]\right)$$

RWR find $\theta_\pi$ that maximizes the reward

**Question:** Do we maximize the reward in RL?

**Answer:** No, we maximize the return!

# Behavioral Cloning with Rewards

$$\theta_\pi = \arg\min_{\theta_\pi} \sum_{s_0 \in \boldsymbol{X}} \sum_{a \in A} -\pi(a \mid s_0; \theta_\beta) \log \pi(a \mid s_0; \theta_\pi) \cdot \exp\left(\hat{\mathbb{E}}[\mathcal{R}(s_1) \mid s_0, a]\right)$$

RWR find $\theta_\pi$ that maximizes the reward

**Question:** Do we maximize the reward in RL?

**Answer:** No, we maximize the return!

$$\theta_\pi = \arg\min_{\theta_\pi} \sum_{s_0 \in \boldsymbol{X}} \sum_{a \in A} -\pi(a \mid s_0; \theta_\beta) \log \pi(a \mid s_0; \theta_\pi) \cdot \underbrace{\exp\left(\hat{\mathbb{E}}\left[\mathcal{G}(\boldsymbol{\tau}) \mid s_0; \theta_\beta\right]\right)}_{\text{Replace reward with return}}$$

# Behavioral Cloning with Rewards

$$\theta_\pi = \arg\min_{\theta_\pi} \sum_{s_0 \in \boldsymbol{X}} \sum_{a \in A} -\pi(a \mid s_0; \theta_\beta) \log \pi(a \mid s_0; \theta_\pi) \cdot \exp\left(\hat{\mathbb{E}}\left[\mathcal{G}(\boldsymbol{\tau}) \mid s_0; \theta_\beta\right]\right)$$

This works in theory, but does not work well in practice

# Behavioral Cloning with Rewards

$$\theta_\pi = \arg\min_{\theta_\pi} \sum_{s_0 \in \boldsymbol{X}} \sum_{a \in A} -\pi(a \mid s_0; \theta_\beta) \log \pi(a \mid s_0; \theta_\pi) \cdot \exp\left(\hat{\mathbb{E}}\left[\mathcal{G}(\boldsymbol{\tau}) \mid s_0; \theta_\beta\right]\right)$$

This works in theory, but does not work well in practice

**Question:** Why does this fail in practice?

# Behavioral Cloning with Rewards

$$\theta_\pi = \arg\min_{\theta_\pi} \sum_{s_0 \in \boldsymbol{X}} \sum_{a \in A} -\pi(a \mid s_0; \theta_\beta) \log \pi(a \mid s_0; \theta_\pi) \cdot \exp\left(\hat{\mathbb{E}}\left[\mathcal{G}(\boldsymbol{\tau}) \mid s_0; \theta_\beta\right]\right)$$

This works in theory, but does not work well in practice

**Question:** Why does this fail in practice?

**Answer:**

# Behavioral Cloning with Rewards

$$\theta_\pi = \arg\min_{\theta_\pi} \sum_{s_0 \in \boldsymbol{X}} \sum_{a \in A} -\pi(a \mid s_0; \theta_\beta) \log \pi(a \mid s_0; \theta_\pi) \cdot \exp\left(\hat{\mathbb{E}}\left[\mathcal{G}(\boldsymbol{\tau}) \mid s_0; \theta_\beta\right]\right)$$

This works in theory, but does not work well in practice

**Question:** Why does this fail in practice?

**Answer:**

- Need infinite rewards to approximate Monte Carlo return

# Behavioral Cloning with Rewards

$$\theta_\pi = \arg\min_{\theta_\pi} \sum_{s_0 \in \boldsymbol{X}} \sum_{a \in A} -\pi(a \mid s_0; \theta_\beta) \log \pi(a \mid s_0; \theta_\pi) \cdot \exp\left(\hat{\mathbb{E}}\left[\mathcal{G}(\boldsymbol{\tau}) \mid s_0; \theta_\beta\right]\right)$$

This works in theory, but does not work well in practice

**Question:** Why does this fail in practice?

**Answer:**

- Need infinite rewards to approximate Monte Carlo return
- Returns can be big or small, causing overflows $\exp(\mathcal{G}(\boldsymbol{\tau})) \to 0, \infty$

# Behavioral Cloning with Rewards

$$\theta_\pi = \arg\min_{\theta_\pi} \sum_{s_0 \in \boldsymbol{X}} \sum_{a \in A} -\pi(a \mid s_0; \theta_\beta) \log \pi(a \mid s_0; \theta_\pi) \cdot \exp\left(\hat{\mathbb{E}}\left[\mathcal{G}(\boldsymbol{\tau}) \mid s_0; \theta_\beta\right]\right)$$

This works in theory, but does not work well in practice

**Question:** Why does this fail in practice?

**Answer:**

- Need infinite rewards to approximate Monte Carlo return
- Returns can be big or small, causing overflows $\exp(\mathcal{G}(\boldsymbol{\tau})) \to 0, \infty$

**Question:** Similar problems with actor critic, what was the solution?

# Behavioral Cloning with Rewards

$$\theta_\pi = \arg\min_{\theta_\pi} \sum_{s_0 \in \boldsymbol{X}} \sum_{a \in A} -\pi(a \mid s_0; \theta_\beta) \log \pi(a \mid s_0; \theta_\pi) \cdot \exp\left(\hat{\mathbb{E}}\left[\mathcal{G}(\boldsymbol{\tau}) \mid s_0; \theta_\beta\right]\right)$$

This works in theory, but does not work well in practice

**Question:** Why does this fail in practice?

**Answer:**

- Need infinite rewards to approximate Monte Carlo return
- Returns can be big or small, causing overflows $\exp(\mathcal{G}(\boldsymbol{\tau})) \to 0, \infty$

**Question:** Similar problems with actor critic, what was the solution?

- Introduce TD value function (remove infinite sum)

# Behavioral Cloning with Rewards

$$\theta_\pi = \arg\min_{\theta_\pi} \sum_{s_0 \in \boldsymbol{X}} \sum_{a \in A} -\pi(a \mid s_0; \theta_\beta) \log \pi(a \mid s_0; \theta_\pi) \cdot \exp\left(\hat{\mathbb{E}}\left[\mathcal{G}(\boldsymbol{\tau}) \mid s_0; \theta_\beta\right]\right)$$

This works in theory, but does not work well in practice

**Question:** Why does this fail in practice?

**Answer:**

- Need infinite rewards to approximate Monte Carlo return
- Returns can be big or small, causing overflows $\exp(\mathcal{G}(\boldsymbol{\tau})) \to 0, \infty$

**Question:** Similar problems with actor critic, what was the solution?

- Introduce TD value function (remove infinite sum)
- Introduce advantage (normalize return)

# Behavioral Cloning with Rewards

$$\theta_\pi = \arg\min_{\theta_\pi} \sum_{s_0 \in \boldsymbol{X}} \sum_{a \in A} -\pi(a \mid s_0; \theta_\beta) \log \pi(a \mid s_0; \theta_\pi) \cdot \exp\left(\hat{\mathbb{E}}\left[\mathcal{G}(\boldsymbol{\tau}) \mid s_0; \theta_\beta\right]\right)$$

# Behavioral Cloning with Rewards

$$\theta_\pi = \arg\min_{\theta_\pi} \sum_{s_0 \in \boldsymbol{X}} \sum_{a \in A} -\pi(a \mid s_0; \theta_\beta) \log \pi(a \mid s_0; \theta_\pi) \cdot \exp\left(\hat{\mathbb{E}}\left[\mathcal{G}(\boldsymbol{\tau}) \mid s_0; \theta_\beta\right]\right)$$

$$\theta_\pi = \arg\min_{\theta_\pi} \sum_{s_0 \in \boldsymbol{X}} \sum_{a \in A} -\pi(a \mid s_0; \theta_\beta) \log \pi(a \mid s_0; \theta_\pi) \cdot \underbrace{\exp\left(A(s, a, \theta_\beta)\right)}_{\text{Use advantage instead}}$$

# Behavioral Cloning with Rewards

$$\theta_\pi = \arg\min_{\theta_\pi} \sum_{s_0 \in \boldsymbol{X}} \sum_{a \in A} -\pi(a \mid s_0; \theta_\beta) \log \pi(a \mid s_0; \theta_\pi) \cdot \exp\left(\hat{\mathbb{E}}\left[\mathcal{G}(\boldsymbol{\tau}) \mid s_0; \theta_\beta\right]\right)$$

$$\theta_\pi = \arg\min_{\theta_\pi} \sum_{s_0 \in \boldsymbol{X}} \sum_{a \in A} -\pi(a \mid s_0; \theta_\beta) \log \pi(a \mid s_0; \theta_\pi) \cdot \underbrace{\exp(A(s, a, \theta_\beta))}_{\text{Use advantage instead}}$$

$$A(s, a, \theta_\beta) = Q(s, a, \theta_\beta) - V(s, \theta_\beta)$$

# Behavioral Cloning with Rewards

$$\theta_\pi = \arg\min_{\theta_\pi} \sum_{s_0 \in \boldsymbol{X}} \sum_{a \in A} -\pi\left(a \mid s_0; \theta_\beta\right) \log \pi\left(a \mid s_0; \theta_\pi\right) \cdot \exp\left(\hat{\mathbb{E}}\left[\mathcal{G}(\boldsymbol{\tau}) \mid s_0; \theta_\beta\right]\right)$$

$$\theta_\pi = \arg\min_{\theta_\pi} \sum_{s_0 \in \boldsymbol{X}} \sum_{a \in A} -\pi\left(a \mid s_0; \theta_\beta\right) \log \pi\left(a \mid s_0; \theta_\pi\right) \cdot \underbrace{\exp\left(A\left(s, a, \theta_\beta\right)\right)}_{\text{Use advantage instead}}$$

$$A\left(s, a, \theta_\beta\right) = Q\left(s, a, \theta_\beta\right) - V\left(s, \theta_\beta\right)$$

$$A\left(s_t, s_{t+1}, \theta_\beta\right) = -V\left(s_t, \theta_\beta\right) + r_t + \gamma V\left(s_{t+1}, \theta_\beta\right)$$

# Behavioral Cloning with Rewards

**Definition:** Monotonic Advantage Re-Weighted Imitation Learning (MARWIL) reweights the behavior cloning objective based on the advantage

# Behavioral Cloning with Rewards

**Definition:** Monotonic Advantage Re-Weighted Imitation Learning (MARWIL) reweights the behavior cloning objective based on the advantage

**Step 1:** Learn a value function for $\theta_\beta$

# Behavioral Cloning with Rewards

**Definition:** Monotonic Advantage Re-Weighted Imitation Learning (MARWIL) reweights the behavior cloning objective based on the advantage

**Step 1:** Learn a value function for $\theta_\beta$

$$\theta_V = \arg\min_{\theta_V} \left( V\left(s_0, \theta_\beta, \theta_V\right) - y \right)^2$$

# Behavioral Cloning with Rewards

**Definition:** Monotonic Advantage Re-Weighted Imitation Learning (MARWIL) reweights the behavior cloning objective based on the advantage

**Step 1:** Learn a value function for $\theta_\beta$

$$\theta_V = \arg\min_{\theta_V} \left(V\left(s_0, \theta_\beta, \theta_V\right) - y\right)^2$$

$$y = \hat{\mathbb{E}}[\mathcal{R}(s_1) \mid s_0, a_0] + \gamma V\left(s_1, \theta_\beta, \theta_V\right)$$

# Behavioral Cloning with Rewards

**Step 2:** Learn policy using weighted behavioral cloning

# Behavioral Cloning with Rewards

**Step 2:** Learn policy using weighted behavioral cloning

$$A(s_t, s_{t+1}, \theta_\beta, \theta_V) = -V(s_t, \theta_\beta, \theta_V) + \hat{\mathbb{E}}[\mathcal{R}(s_1) \mid s_0, a_0] + \gamma V(s_{t+1}, \theta_\beta, \theta_V)$$

# Behavioral Cloning with Rewards

**Step 2:** Learn policy using weighted behavioral cloning

$$A(s_t, s_{t+1}, \theta_\beta, \theta_V) = -V(s_t, \theta_\beta, \theta_V) + \hat{\mathbb{E}}[\mathcal{R}(s_1) \mid s_0, a_0] + \gamma V(s_{t+1}, \theta_\beta, \theta_V)$$

$$\theta_\pi = \arg\min_{\theta_\pi} \sum_{s \in \boldsymbol{X}} \sum_{a \in A} \underbrace{-\pi(a \mid s; \theta_\beta) \log \pi(a \mid s; \theta_\pi)}_{\text{BC objective}} \cdot \underbrace{\exp(A(s_t, s_{t+1}, \theta_\beta, \theta_V))}_{\text{Advantage reweighting}}$$

# Behavioral Cloning with Rewards

**Question:** Is MARWIL RL or supervised learning?

# Behavioral Cloning with Rewards

**Question:** Is MARWIL RL or supervised learning?

**Answer:** Both

# Behavioral Cloning with Rewards

**Question:** Is MARWIL RL or supervised learning?

**Answer:** Both

- Learn policy using supervised learning (SL)

# Behavioral Cloning with Rewards

**Question:** Is MARWIL RL or supervised learning?

**Answer:** Both

- Learn policy using supervised learning (SL)
- But weights require learning value function (RL)

# Behavioral Cloning with Rewards

**Question:** Is MARWIL RL or supervised learning?

**Answer:** Both

- Learn policy using supervised learning (SL)
- But weights require learning value function (RL)

**Question:** We used TD objective, can we also use MC objective?

# Behavioral Cloning with Rewards

**Question:** Is MARWIL RL or supervised learning?

**Answer:** Both

- Learn policy using supervised learning (SL)
- But weights require learning value function (RL)

**Question:** We used TD objective, can we also use MC objective?

$$V(s_0, \theta_\beta, \theta_V) = \hat{\mathbb{E}}[\mathcal{R}(s_1) \mid s_0; \theta_\beta] + \gamma V(s_1, \theta_\beta, \theta_V)$$

# Behavioral Cloning with Rewards

**Question:** Is MARWIL RL or supervised learning?

**Answer:** Both

- Learn policy using supervised learning (SL)
- But weights require learning value function (RL)

**Question:** We used TD objective, can we also use MC objective?

$$V(s_0, \theta_\beta, \theta_V) = \hat{\mathbb{E}}[\mathcal{R}(s_1) \mid s_0; \theta_\beta] + \gamma V(s_1, \theta_\beta, \theta_V)$$

$$V(s_0, \theta_\beta, \theta_V) = \sum_{t=0}^{\infty} \gamma^t \hat{\mathbb{E}}[\mathcal{R}(s_{t+1}) \mid s_0; \theta_\beta]$$

# Behavioral Cloning with Rewards

**Question:** Is MARWIL RL or supervised learning?

**Answer:** Both

- Learn policy using supervised learning (SL)
- But weights require learning value function (RL)

**Question:** We used TD objective, can we also use MC objective?

$$V(s_0, \theta_\beta, \theta_V) = \hat{\mathbb{E}}[\mathcal{R}(s_1) \mid s_0; \theta_\beta] + \gamma V(s_1, \theta_\beta, \theta_V)$$

$$V(s_0, \theta_\beta, \theta_V) = \sum_{t=0}^{\infty} \gamma^t \hat{\mathbb{E}}[\mathcal{R}(s_{t+1}) \mid s_0; \theta_\beta]$$

**Answer:** Yes, because we learn $V$ for $\theta_\beta$ not $\theta_\pi$

# Behavioral Cloning with Rewards

Add improvements to MARWIL to derive other offline RL algorithms

# Behavioral Cloning with Rewards

Add improvements to MARWIL to derive other offline RL algorithms

- Advantage Weighted Regression (AWR)

# Behavioral Cloning with Rewards

Add improvements to MARWIL to derive other offline RL algorithms

- Advantage Weighted Regression (AWR)
- Advantage Weighted Actor Critic (AWAC)

# Behavioral Cloning with Rewards

Add improvements to MARWIL to derive other offline RL algorithms

- Advantage Weighted Regression (AWR)
- Advantage Weighted Actor Critic (AWAC)
- Critic Regularized Regression (CRR)

# Behavioral Cloning with Rewards

Add improvements to MARWIL to derive other offline RL algorithms

- Advantage Weighted Regression (AWR)
- Advantage Weighted Actor Critic (AWAC)
- Critic Regularized Regression (CRR)
- Implicit Q learning (IQL)

# Behavioral Cloning with Rewards

Add improvements to MARWIL to derive other offline RL algorithms

- Advantage Weighted Regression (AWR)
- Advantage Weighted Actor Critic (AWAC)
- Critic Regularized Regression (CRR)
- Implicit Q learning (IQL)
- Maximum a Posteriori Optimization (MPO)

# The Deadly Triad

# The Deadly Triad

There are two standard approaches to offline RL

# The Deadly Triad

There are two standard approaches to offline RL

1. Reweight the BC loss using rewards/returns
2.

# The Deadly Triad

There are two standard approaches to offline RL

1. Reweight the BC loss using rewards/returns
2. Improve off-policy algorithms to work without exploration

# The Deadly Triad

There are two standard approaches to offline RL

1. Reweight the BC loss using rewards/returns
2. Improve off-policy algorithms to work without exploration

Finished first, now let us visit the second

# The Deadly Triad

**Goal:** Derive offline RL algorithm from an off-policy algorithm

# The Deadly Triad

**Goal:** Derive offline RL algorithm from an off-policy algorithm

**Question:** Why off-policy instead of on-policy algorithm?

# The Deadly Triad

**Goal:** Derive offline RL algorithm from an off-policy algorithm

**Question:** Why off-policy instead of on-policy algorithm?

On-policy requires collecting data with $\theta_\pi$

# The Deadly Triad

**Goal:** Derive offline RL algorithm from an off-policy algorithm

**Question:** Why off-policy instead of on-policy algorithm?

On-policy requires collecting data with $\theta_\pi$

- But we cannot do this! Dataset is collected with $\theta_\beta$
- $\theta_\beta \neq \theta_\pi$, cannot use on-policy method

# The Deadly Triad

**Goal:** Derive offline RL algorithm from an off-policy algorithm

**Question:** Why off-policy instead of on-policy algorithm?

On-policy requires collecting data with $\theta_\pi$

- But we cannot do this! Dataset is collected with $\theta_\beta$
- $\theta_\beta \neq \theta_\pi$, cannot use on-policy method

Off-policy can learn from any trajectories

# The Deadly Triad

**Goal:** Derive offline RL algorithm from an off-policy algorithm

**Question:** Why off-policy instead of on-policy algorithm?

On-policy requires collecting data with $\theta_\pi$

- But we cannot do this! Dataset is collected with $\theta_\beta$
- $\theta_\beta \neq \theta_\pi$, cannot use on-policy method

Off-policy can learn from any trajectories

- Trajectory collected following $\theta_\beta$
- Can use $\theta_\beta$ trajectory to update $\theta_\pi$

# The Deadly Triad

Ok, let us choose an off-policy algorithm to use

# The Deadly Triad

Ok, let us choose an off-policy algorithm to use

**Question:** Which off-policy RL algorithms do we know?

# The Deadly Triad

Ok, let us choose an off-policy algorithm to use

**Question:** Which off-policy RL algorithms do we know?

- Q learning

# The Deadly Triad

Ok, let us choose an off-policy algorithm to use

**Question:** Which off-policy RL algorithms do we know?

- Q learning
- DDPG (continuous Q learning)

# The Deadly Triad

Ok, let us choose an off-policy algorithm to use

**Question:** Which off-policy RL algorithms do we know?

- Q learning
- DDPG (continuous Q learning)
- Off-policy gradient (does not work well, ignore for now)

# The Deadly Triad

Ok, let us choose an off-policy algorithm to use

**Question:** Which off-policy RL algorithms do we know?

- Q learning
- DDPG (continuous Q learning)
- Off-policy gradient (does not work well, ignore for now)

**Question:** Temporal Difference or Monte Carlo Q learning?

# The Deadly Triad

Ok, let us choose an off-policy algorithm to use

**Question:** Which off-policy RL algorithms do we know?

- Q learning
- DDPG (continuous Q learning)
- Off-policy gradient (does not work well, ignore for now)

**Question:** Temporal Difference or Monte Carlo Q learning?

- MC is on-policy
- Only TD Q learning is off-policy

# The Deadly Triad

Recall the standard Q learning algorithm

# The Deadly Triad

Recall the standard Q learning algorithm

```
while not terminated:
    transition = env.step(action)
    buffer.append(transition)
    train_data = buffer.sample()
    J = grad(td_loss)(theta_Q, theta_T, Q, train_data)
    theta_Q = opt.update(theta_Q, J)
```

# The Deadly Triad

Recall the standard Q learning algorithm

```
while not terminated:
    transition = env.step(action)
    buffer.append(transition)
    train_data = buffer.sample()
    J = grad(td_loss)(theta_Q, theta_T, Q, train_data)
    theta_Q = opt.update(theta_Q, J)
```

**Question:** What can we do to make this offline? Without exploration?

# The Deadly Triad

Recall the standard Q learning algorithm

```
while not terminated:
    transition = env.step(action)
    buffer.append(transition)
    train_data = buffer.sample()
    J = grad(td_loss)(theta_Q, theta_T, Q, train_data)
    theta_Q = opt.update(theta_Q, J)
```

**Question:** What can we do to make this offline? Without exploration?

**Answer:**

- Put dataset into replay buffer
- Get rid of env code

# The Deadly Triad

```
for x in X:
    buffer.append(x) # Add dataset to replay buffer
# Comment out exploration code
# while not terminated:
    # transition = env.step(action)
    # buffer.append(transition)
for epoch in num_epochs:
    train_data = buffer.sample()
    J = grad(td_loss)(theta_Q, theta_T, Q, train_data)
    theta_Q = opt.update(theta_Q, J)
```

# The Deadly Triad

```python
for x in X:
    buffer.append(x) # Add dataset to replay buffer

for epoch in num_epochs:
    train_data = buffer.sample()
    J = grad(td_loss)(theta_Q, theta_T, Q, train_data)
    theta_Q = opt.update(theta_Q, J)
```

# The Deadly Triad

```
for x in X:
    buffer.append(x) # Add dataset to replay buffer

for epoch in num_epochs:
    train_data = buffer.sample()
    J = grad(td_loss)(theta_Q, theta_T, Q, train_data)
    theta_Q = opt.update(theta_Q, J)
```

**Question:** Will this work?

# The Deadly Triad

```
for x in X:
    buffer.append(x) # Add dataset to replay buffer

for epoch in num_epochs:
    train_data = buffer.sample()
    J = grad(td_loss)(theta_Q, theta_T, Q, train_data)
    theta_Q = opt.update(theta_Q, J)
```

**Question:** Will this work?

**Answer:** It can! But only for very simple problems

# The Deadly Triad

```
for x in X:
    buffer.append(x) # Add dataset to replay buffer

for epoch in num_epochs:
    train_data = buffer.sample()
    J = grad(td_loss)(theta_Q, theta_T, Q, train_data)
    theta_Q = opt.update(theta_Q, J)
```

**Question:** Will this work?

**Answer:** It can! But only for very simple problems

For harder/interesting problems, loss quickly becomes `NaN`

# The Deadly Triad

```
for x in X:
    buffer.append(x) # Add dataset to replay buffer

for epoch in num_epochs:
    train_data = buffer.sample()
    J = grad(td_loss)(theta_Q, theta_T, Q, train_data)
    theta_Q = opt.update(theta_Q, J)
```

**Question:** Will this work?

**Answer:** It can! But only for very simple problems

For harder/interesting problems, loss quickly becomes `NaN`

Let us investigate why

# The Deadly Triad

On assignment 2, many of you found issues with deep Q learning

# The Deadly Triad

On assignment 2, many of you found issues with deep Q learning

1. Q value would often become very large
2. 
3. 

# The Deadly Triad

On assignment 2, many of you found issues with deep Q learning

1. Q value would often become very large
2. Loss would become very large
3.

# The Deadly Triad

On assignment 2, many of you found issues with deep Q learning

1. Q value would often become very large
2. Loss would become very large
3. Loss/parameters become `NaN`

# The Deadly Triad

On assignment 2, many of you found issues with deep Q learning

1. Q value would often become very large
2. Loss would become very large
3. Loss/parameters become `NaN`

This was not your fault, it is a known problem in RL!

# The Deadly Triad

On assignment 2, many of you found issues with deep Q learning

1. Q value would often become very large
2. Loss would become very large
3. Loss/parameters become `NaN`

This was not your fault, it is a known problem in RL!

Call it the **deadly triad**, because it is caused by combining three factors:

# The Deadly Triad

On assignment 2, many of you found issues with deep Q learning

1. Q value would often become very large
2. Loss would become very large
3. Loss/parameters become `NaN`

This was not your fault, it is a known problem in RL!

Call it the **deadly triad**, because it is caused by combining three factors:

1. Function approximation (deep neural network)
2. 
3. 

# The Deadly Triad

On assignment 2, many of you found issues with deep Q learning

1. Q value would often become very large
2. Loss would become very large
3. Loss/parameters become `NaN`

This was not your fault, it is a known problem in RL!

Call it the **deadly triad**, because it is caused by combining three factors:

1. Function approximation (deep neural network)
2. Recursive bootstrapping (TD, $Q(s, a) = r + \gamma \max Q(s, a)$)
3.

# The Deadly Triad

On assignment 2, many of you found issues with deep Q learning

1. Q value would often become very large
2. Loss would become very large
3. Loss/parameters become `NaN`

This was not your fault, it is a known problem in RL!

Call it the **deadly triad**, because it is caused by combining three factors:

1. Function approximation (deep neural network)
2. Recursive bootstrapping (TD, $Q(s, a) = r + \gamma \max Q(s, a)$)
3. Off-policy learning/limited exploration

# The Deadly Triad

On assignment 2, many of you found issues with deep Q learning

1. Q value would often become very large
2. Loss would become very large
3. Loss/parameters become `NaN`

This was not your fault, it is a known problem in RL!

Call it the **deadly triad**, because it is caused by combining three factors:

1. Function approximation (deep neural network)
2. Recursive bootstrapping (TD, $Q(s, a) = r + \gamma \max Q(s, a)$)
3. Off-policy learning/limited exploration

Let us further investigate the deadly triad

# The Deadly Triad

Imagine a toy MDP

# The Deadly Triad

Imagine a toy MDP

$$S = \{s\}$$

# The Deadly Triad

Imagine a toy MDP

$$S = \{s\} \quad A = \{1, 2\}$$

# The Deadly Triad

Imagine a toy MDP

$$S = \{s\} \quad A = \{1, 2\} \quad Q(s, a, \theta_Q) = \theta_Q \cdot a$$

# The Deadly Triad

Imagine a toy MDP

$$S = \{s\} \quad A = \{1, 2\} \quad Q(s, a, \theta_Q) = \theta_Q \cdot a \quad \mathcal{R}(s) = 0$$

# The Deadly Triad

Imagine a toy MDP

$$S = \{s\} \quad A = \{1, 2\} \quad Q(s, a, \theta_Q) = \theta_Q \cdot a \quad \mathcal{R}(s) = 0 \quad \gamma = 1$$

Can update $\theta_Q$ using simple TD update

# The Deadly Triad

Imagine a toy MDP

$$S = \{s\} \quad A = \{1, 2\} \quad Q(s, a, \theta_Q) = \theta_Q \cdot a \quad \mathcal{R}(s) = 0 \quad \gamma = 1$$

Can update $\theta_Q$ using simple TD update

$$\theta_Q = \theta_Q - \underbrace{\left[Q(s, a, \theta_Q) - \left(r + \gamma \max_{a' \in A} Q(s', a', \theta_Q)\right)\right]}_{\text{Q error, can consider } \nabla \mathcal{L}}$$

# The Deadly Triad

Imagine a toy MDP

$$S = \{s\} \quad A = \{1, 2\} \quad Q(s, a, \theta_Q) = \theta_Q \cdot a \quad \mathcal{R}(s) = 0 \quad \gamma = 1$$

Can update $\theta_Q$ using simple TD update

$$\theta_Q = \theta_Q - \underbrace{\left[Q(s, a, \theta_Q) - \left(r + \gamma \max_{a' \in A} Q(s', a', \theta_Q)\right)\right]}_{\text{Q error, can consider } \nabla \mathcal{L}}$$

$$\theta_Q = \theta_Q - \left[Q(s, a, \theta_Q) - \max_{a' \in A} Q(s', a', \theta_Q)\right]$$

# The Deadly Triad

Imagine a toy MDP

$$S = \{s\} \quad A = \{1, 2\} \quad Q(s, a, \theta_Q) = \theta_Q \cdot a \quad \mathcal{R}(s) = 0 \quad \gamma = 1$$

Can update $\theta_Q$ using simple TD update

$$\theta_Q = \theta_Q - \underbrace{\left[Q(s, a, \theta_Q) - \left(r + \gamma \max_{a' \in A} Q(s', a', \theta_Q)\right)\right]}_{\text{Q error, can consider } \nabla \mathcal{L}}$$

$$\theta_Q = \theta_Q - \left[Q(s, a, \theta_Q) - \max_{a' \in A} Q(s', a', \theta_Q)\right]$$

Importantly, to simulate off-policy/offline RL, we have limited dataset

# The Deadly Triad

Imagine a toy MDP

$$S = \{s\} \quad A = \{1, 2\} \quad Q(s, a, \theta_Q) = \theta_Q \cdot a \quad \mathcal{R}(s) = 0 \quad \gamma = 1$$

Can update $\theta_Q$ using simple TD update

$$\theta_Q = \theta_Q - \underbrace{\left[Q(s, a, \theta_Q) - \left(r + \gamma \max_{a' \in A} Q(s', a', \theta_Q)\right)\right]}_{\text{Q error, can consider } \nabla \mathcal{L}}$$

$$\theta_Q = \theta_Q - \left[Q(s, a, \theta_Q) - \max_{a' \in A} Q(s', a', \theta_Q)\right]$$

Importantly, to simulate off-policy/offline RL, we have limited dataset

- We only have $s, a = 1$ in our dataset, not $s, a = 2$

# The Deadly Triad

Imagine a toy MDP

$$S = \{s\} \quad A = \{1, 2\} \quad Q(s, a, \theta_Q) = \theta_Q \cdot a \quad \mathcal{R}(s) = 0 \quad \gamma = 1$$

Can update $\theta_Q$ using simple TD update

$$\theta_Q = \theta_Q - \underbrace{\left[Q(s, a, \theta_Q) - \left(r + \gamma \max_{a' \in A} Q(s', a', \theta_Q)\right)\right]}_{\text{Q error, can consider } \nabla \mathcal{L}}$$

$$\theta_Q = \theta_Q - \left[Q(s, a, \theta_Q) - \max_{a' \in A} Q(s', a', \theta_Q)\right]$$

Importantly, to simulate off-policy/offline RL, we have limited dataset

- We only have $s, a = 1$ in our dataset, not $s, a = 2$

Let us perform some updates to $\theta_Q$ and see what happens

# The Deadly Triad

Initialize $\theta_Q = 1$, update for $a = 1$

# The Deadly Triad

Initialize $\theta_Q = 1$, update for $a = 1$

$$\theta_Q = \theta_Q - \left[Q\left(s, a, \theta_Q\right) - \max_{a' \in A} Q\left(s, a', \theta_Q\right)\right]$$

# The Deadly Triad

Initialize $\theta_Q = 1$, update for $a = 1$

$$\theta_Q = \theta_Q - \left[ Q(s, a, \theta_Q) - \max_{a' \in A} Q(s, a', \theta_Q) \right]$$

$$\theta_Q = \theta_Q - [Q(s, 1, \theta_Q) - (\max\{Q(s, 1, \theta_Q), Q(s, 2, \theta_Q)\})]$$

# The Deadly Triad

Initialize $\theta_Q = 1$, update for $a = 1$

$$\theta_Q = \theta_Q - \left[ Q(s, a, \theta_Q) - \max_{a' \in A} Q(s, a', \theta_Q) \right]$$

$$\theta_Q = \theta_Q - [Q(s, 1, \theta_Q) - (\max\{Q(s, 1, \theta_Q), Q(s, 2, \theta_Q)\})]$$

$$\theta_Q = \theta_Q - [Q(s, 1, \theta_Q) - (\max\{\theta_Q \cdot 1, \theta_Q \cdot 2\})]$$

# The Deadly Triad

Initialize $\theta_Q = 1$, update for $a = 1$

$$\theta_Q = \theta_Q - \left[Q(s, a, \theta_Q) - \max_{a' \in A} Q(s, a', \theta_Q)\right]$$

$$\theta_Q = \theta_Q - [Q(s, 1, \theta_Q) - (\max\{Q(s, 1, \theta_Q), Q(s, 2, \theta_Q)\})]$$

$$\theta_Q = \theta_Q - [Q(s, 1, \theta_Q) - (\max\{\theta_Q \cdot 1, \theta_Q \cdot 2\})]$$

$$\theta_Q = \theta_Q - [Q(s, 1, \theta_Q) - (\max\{1 \cdot 1, 1 \cdot 2\})]$$

# The Deadly Triad

Initialize $\theta_Q = 1$, update for $a = 1$

$$\theta_Q = \theta_Q - \left[ Q(s, a, \theta_Q) - \max_{a' \in A} Q(s, a', \theta_Q) \right]$$

$$\theta_Q = \theta_Q - [Q(s, 1, \theta_Q) - (\max\{Q(s, 1, \theta_Q), Q(s, 2, \theta_Q)\})]$$

$$\theta_Q = \theta_Q - [Q(s, 1, \theta_Q) - (\max\{\theta_Q \cdot 1, \theta_Q \cdot 2\})]$$

$$\theta_Q = \theta_Q - [Q(s, 1, \theta_Q) - (\max\{1 \cdot 1, 1 \cdot 2\})]$$

$$\theta_Q = 1 - [1 - 2]$$

# The Deadly Triad

Initialize $\theta_Q = 1$, update for $a = 1$

$$\theta_Q = \theta_Q - \left[Q(s, a, \theta_Q) - \max_{a' \in A} Q(s, a', \theta_Q)\right]$$

$$\theta_Q = \theta_Q - [Q(s, 1, \theta_Q) - (\max\{Q(s, 1, \theta_Q), Q(s, 2, \theta_Q)\})]$$

$$\theta_Q = \theta_Q - [Q(s, 1, \theta_Q) - (\max\{\theta_Q \cdot 1, \theta_Q \cdot 2\})]$$

$$\theta_Q = \theta_Q - [Q(s, 1, \theta_Q) - (\max\{1 \cdot 1, 1 \cdot 2\})]$$

$$\theta_Q = 1 - [1 - 2]$$

$$\theta_Q = 2$$

# The Deadly Triad

Repeat with $\theta_Q = 2$, update for $a = 1$

$$\theta_Q = \theta_Q - \left[Q\left(s, a, \theta_Q\right) - \max_{a' \in A} Q\left(s, a', \theta_Q\right)\right]$$

# The Deadly Triad

Repeat with $\theta_Q = 2$, update for $a = 1$

$$\theta_Q = \theta_Q - \left[ Q(s, a, \theta_Q) - \max_{a' \in A} Q(s, a', \theta_Q) \right]$$

$$\theta_Q = \theta_Q - [Q(s, 1, \theta_Q) - (\max\{Q(s, 1, \theta_Q), Q(s, 2, \theta_Q)\})]$$

# The Deadly Triad

Repeat with $\theta_Q = 2$, update for $a = 1$

$$\theta_Q = \theta_Q - \left[Q(s, a, \theta_Q) - \max_{a' \in A} Q(s, a', \theta_Q)\right]$$

$$\theta_Q = \theta_Q - [Q(s, 1, \theta_Q) - (\max\{Q(s, 1, \theta_Q), Q(s, 2, \theta_Q)\})]$$

$$\theta_Q = \theta_Q - [Q(s, 1, \theta_Q) - (\max\{\theta_Q \cdot 1, \theta_Q \cdot 2\})]$$

# The Deadly Triad

Repeat with $\theta_Q = 2$, update for $a = 1$

$$\theta_Q = \theta_Q - \left[Q(s, a, \theta_Q) - \max_{a' \in A} Q(s, a', \theta_Q)\right]$$

$$\theta_Q = \theta_Q - [Q(s, 1, \theta_Q) - (\max\{Q(s, 1, \theta_Q), Q(s, 2, \theta_Q)\})]$$

$$\theta_Q = \theta_Q - [Q(s, 1, \theta_Q) - (\max\{\theta_Q \cdot 1, \theta_Q \cdot 2\})]$$

$$\theta_Q = \theta_Q - [Q(s, 1, \theta_Q) - (\max\{2 \cdot 1, 2 \cdot 2\})]$$

# The Deadly Triad

Repeat with $\theta_Q = 2$, update for $a = 1$

$$\theta_Q = \theta_Q - \left[Q(s, a, \theta_Q) - \max_{a' \in A} Q(s, a', \theta_Q)\right]$$

$$\theta_Q = \theta_Q - [Q(s, 1, \theta_Q) - (\max\{Q(s, 1, \theta_Q), Q(s, 2, \theta_Q)\})]$$

$$\theta_Q = \theta_Q - [Q(s, 1, \theta_Q) - (\max\{\theta_Q \cdot 1, \theta_Q \cdot 2\})]$$

$$\theta_Q = \theta_Q - [Q(s, 1, \theta_Q) - (\max\{2 \cdot 1, 2 \cdot 2\})]$$

$$\theta_Q = 2 - [2 - 4]$$

# The Deadly Triad

Repeat with $\theta_Q = 2$, update for $a = 1$

$$\theta_Q = \theta_Q - \left[ Q(s, a, \theta_Q) - \max_{a' \in A} Q(s, a', \theta_Q) \right]$$

$$\theta_Q = \theta_Q - [Q(s, 1, \theta_Q) - (\max\{Q(s, 1, \theta_Q), Q(s, 2, \theta_Q)\})]$$

$$\theta_Q = \theta_Q - [Q(s, 1, \theta_Q) - (\max\{\theta_Q \cdot 1, \theta_Q \cdot 2\})]$$

$$\theta_Q = \theta_Q - [Q(s, 1, \theta_Q) - (\max\{2 \cdot 1, 2 \cdot 2\})]$$

$$\theta_Q = 2 - [2 - 4]$$

$$\theta_Q = 4$$

# The Deadly Triad

Repeat with $\theta_Q = 4$, update for $a = 1$

$$\theta_Q = \theta_Q - \left[ Q(s, a, \theta_Q) - \max_{a' \in A} Q(s, a', \theta_Q) \right]$$

# The Deadly Triad

Repeat with $\theta_Q = 4$, update for $a = 1$

$$\theta_Q = \theta_Q - \left[Q(s, a, \theta_Q) - \max_{a' \in A} Q(s, a', \theta_Q)\right]$$

$$\theta_Q = \theta_Q - \left[Q(s, 1, \theta_Q) - (\max\{Q(s, 1, \theta_Q), Q(s, 2, \theta_Q)\})\right]$$

# The Deadly Triad

Repeat with $\theta_Q = 4$, update for $a = 1$

$$\theta_Q = \theta_Q - \left[Q(s, a, \theta_Q) - \max_{a' \in A} Q(s, a', \theta_Q)\right]$$

$$\theta_Q = \theta_Q - [Q(s, 1, \theta_Q) - (\max\{Q(s, 1, \theta_Q), Q(s, 2, \theta_Q)\})]$$

$$\theta_Q = \theta_Q - [Q(s, 1, \theta_Q) - (\max\{\theta_Q \cdot 1, \theta_Q \cdot 2\})]$$

# The Deadly Triad

Repeat with $\theta_Q = 4$, update for $a = 1$

$$\theta_Q = \theta_Q - \left[Q(s, a, \theta_Q) - \max_{a' \in A} Q(s, a', \theta_Q)\right]$$

$$\theta_Q = \theta_Q - [Q(s, 1, \theta_Q) - (\max\{Q(s, 1, \theta_Q), Q(s, 2, \theta_Q)\})]$$

$$\theta_Q = \theta_Q - [Q(s, 1, \theta_Q) - (\max\{\theta_Q \cdot 1, \theta_Q \cdot 2\})]$$

$$\theta_Q = \theta_Q - [Q(s, 1, \theta_Q) - (\max\{4 \cdot 1, 4 \cdot 2\})]$$

# The Deadly Triad

Repeat with $\theta_Q = 4$, update for $a = 1$

$$\theta_Q = \theta_Q - \left[Q(s, a, \theta_Q) - \max_{a' \in A} Q(s, a', \theta_Q)\right]$$

$$\theta_Q = \theta_Q - [Q(s, 1, \theta_Q) - (\max\{Q(s, 1, \theta_Q), Q(s, 2, \theta_Q)\})]$$

$$\theta_Q = \theta_Q - [Q(s, 1, \theta_Q) - (\max\{\theta_Q \cdot 1, \theta_Q \cdot 2\})]$$

$$\theta_Q = \theta_Q - [Q(s, 1, \theta_Q) - (\max\{4 \cdot 1, 4 \cdot 2\})]$$

$$\theta_Q = 4 - [4 - 8]$$

# The Deadly Triad

Repeat with $\theta_Q = 4$, update for $a = 1$

$$\theta_Q = \theta_Q - \left[Q(s, a, \theta_Q) - \max_{a' \in A} Q(s, a', \theta_Q)\right]$$

$$\theta_Q = \theta_Q - [Q(s, 1, \theta_Q) - (\max\{Q(s, 1, \theta_Q), Q(s, 2, \theta_Q)\})]$$

$$\theta_Q = \theta_Q - [Q(s, 1, \theta_Q) - (\max\{\theta_Q \cdot 1, \theta_Q \cdot 2\})]$$

$$\theta_Q = \theta_Q - [Q(s, 1, \theta_Q) - (\max\{4 \cdot 1, 4 \cdot 2\})]$$

$$\theta_Q = 4 - [4 - 8]$$

$$\theta_Q = 8$$

# The Deadly Triad

Each time we update, $\theta_Q$ increases, even if $\mathcal{R}(s) = 0$

# The Deadly Triad

Each time we update, $\theta_Q$ increases, even if $\mathcal{R}(s) = 0$

Larger $\theta_Q$ means larger $Q(s, a, \theta_Q)$ for both $a = 1, a = 2$

# The Deadly Triad

Each time we update, $\theta_Q$ increases, even if $\mathcal{R}(s) = 0$

Larger $\theta_Q$ means larger $Q(s, a, \theta_Q)$ for both $a = 1, a = 2$

Eventually $\theta_Q \to \infty$

# The Deadly Triad

Each time we update, $\theta_Q$ increases, even if $\mathcal{R}(s) = 0$

Larger $\theta_Q$ means larger $Q(s, a, \theta_Q)$ for both $a = 1, a = 2$

Eventually $\theta_Q \to \infty$ $\quad$ $Q(s, a, \theta_Q) \to \infty$

# The Deadly Triad

Each time we update, $\theta_Q$ increases, even if $\mathcal{R}(s) = 0$

Larger $\theta_Q$ means larger $Q(s, a, \theta_Q)$ for both $a = 1, a = 2$

Eventually $\theta_Q \to \infty$ $\quad$ $Q(s, a, \theta_Q) \to \infty$

**Question:** Why does $\theta_Q$ keep increasing?

# The Deadly Triad

Each time we update, $\theta_Q$ increases, even if $\mathcal{R}(s) = 0$

Larger $\theta_Q$ means larger $Q(s, a, \theta_Q)$ for both $a = 1, a = 2$

Eventually $\theta_Q \to \infty$ $\qquad Q(s, a, \theta_Q) \to \infty$

**Question:** Why does $\theta_Q$ keep increasing?

1. (Function approximation) We share parameters for $a = 1$ and $a = 2$

2.

3.

# The Deadly Triad

Each time we update, $\theta_Q$ increases, even if $\mathcal{R}(s) = 0$

Larger $\theta_Q$ means larger $Q(s, a, \theta_Q)$ for both $a = 1, a = 2$

Eventually $\theta_Q \to \infty$ $Q(s, a, \theta_Q) \to \infty$

**Question:** Why does $\theta_Q$ keep increasing?

1. (Function approximation) We share parameters for $a = 1$ and $a = 2$
   - Updating $\theta_Q$ for $a = 1$ also updates for $a = 2$
2. 
3. 

# The Deadly Triad

Each time we update, $\theta_Q$ increases, even if $\mathcal{R}(s) = 0$

Larger $\theta_Q$ means larger $Q(s, a, \theta_Q)$ for both $a = 1, a = 2$

Eventually $\theta_Q \to \infty$ $\qquad$ $Q(s, a, \theta_Q) \to \infty$

**Question:** Why does $\theta_Q$ keep increasing?

1. (Function approximation) We share parameters for $a = 1$ and $a = 2$
   - Updating $\theta_Q$ for $a = 1$ also updates for $a = 2$
2. (Recursive bootstrap) updating $\theta_Q$ for $a = 1$ uses $\max_{a \in \{1,2\}}$ in label
3.

# The Deadly Triad

Each time we update, $\theta_Q$ increases, even if $\mathcal{R}(s) = 0$

Larger $\theta_Q$ means larger $Q(s, a, \theta_Q)$ for both $a = 1, a = 2$

Eventually $\theta_Q \to \infty$ $\qquad Q(s, a, \theta_Q) \to \infty$

**Question:** Why does $\theta_Q$ keep increasing?

1. (Function approximation) We share parameters for $a = 1$ and $a = 2$
   - Updating $\theta_Q$ for $a = 1$ also updates for $a = 2$
2. (Recursive bootstrap) updating $\theta_Q$ for $a = 1$ uses $\max_{a \in \{1,2\}}$ in label
3. (Off-policy) Trains on old data, does not contain $a = 2$

# The Deadly Triad

Each time we update, $\theta_Q$ increases, even if $\mathcal{R}(s) = 0$

Larger $\theta_Q$ means larger $Q(s, a, \theta_Q)$ for both $a = 1, a = 2$

Eventually $\theta_Q \to \infty$ $\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad$ $Q(s, a, \theta_Q) \to \infty$

**Question:** Why does $\theta_Q$ keep increasing?

1. (Function approximation) We share parameters for $a = 1$ and $a = 2$
   - Updating $\theta_Q$ for $a = 1$ also updates for $a = 2$
2. (Recursive bootstrap) updating $\theta_Q$ for $a = 1$ uses $\max_{a \in \{1,2\}}$ in label
3. (Off-policy) Trains on old data, does not contain $a = 2$
   - Eventually, greedy policy should visit $a = 2$

# The Deadly Triad

Each time we update, $\theta_Q$ increases, even if $\mathcal{R}(s) = 0$

Larger $\theta_Q$ means larger $Q(s, a, \theta_Q)$ for both $a = 1, a = 2$

Eventually $\theta_Q \to \infty$ $\quad$ $Q(s, a, \theta_Q) \to \infty$

**Question:** Why does $\theta_Q$ keep increasing?

1. (Function approximation) We share parameters for $a = 1$ and $a = 2$
   - Updating $\theta_Q$ for $a = 1$ also updates for $a = 2$
2. (Recursive bootstrap) updating $\theta_Q$ for $a = 1$ uses $\max_{a \in \{1,2\}}$ in label
3. (Off-policy) Trains on old data, does not contain $a = 2$
   - Eventually, greedy policy should visit $a = 2$

Offline RL guarantees case 3, because we will never explore

# The Deadly Triad

1. (Function approximation) We share parameters for $a = 1$ and $a = 2$
   - Updating $\theta_Q$ for $a = 1$ also updates for $a = 2$
2. (Recursive bootstrap) updating $\theta_Q$ for $a = 1$ uses $\max_{a \in \{1,2\}}$ in label
3. (Off-policy) Trains on old data, does not contain $a = 2$
   - Eventually, greedy policy should visit $a = 2$

# The Deadly Triad

1. (Function approximation) We share parameters for $a = 1$ and $a = 2$
    - Updating $\theta_Q$ for $a = 1$ also updates for $a = 2$
2. (Recursive bootstrap) updating $\theta_Q$ for $a = 1$ uses $\max_{a \in \{1,2\}}$ in label
3. (Off-policy) Trains on old data, does not contain $a = 2$
    - Eventually, greedy policy should visit $a = 2$

If we can prevent any of these, we can learn a Q function offline

# The Deadly Triad

1. (Function approximation) We share parameters for $a = 1$ and $a = 2$
   - Updating $\theta_Q$ for $a = 1$ also updates for $a = 2$
2. (Recursive bootstrap) updating $\theta_Q$ for $a = 1$ uses $\max_{a \in \{1,2\}}$ in label
3. (Off-policy) Trains on old data, does not contain $a = 2$
   - Eventually, greedy policy should visit $a = 2$

If we can prevent any of these, we can learn a Q function offline

1. Do not use neural network

# The Deadly Triad

1. (Function approximation) We share parameters for $a = 1$ and $a = 2$
   - Updating $\theta_Q$ for $a = 1$ also updates for $a = 2$
2. (Recursive bootstrap) updating $\theta_Q$ for $a = 1$ uses $\max_{a \in \{1,2\}}$ in label
3. (Off-policy) Trains on old data, does not contain $a = 2$
   - Eventually, greedy policy should visit $a = 2$

If we can prevent any of these, we can learn a Q function offline

1. Do not use neural network          Need nn for large state space

# The Deadly Triad

1. (Function approximation) We share parameters for $a = 1$ and $a = 2$
   - Updating $\theta_Q$ for $a = 1$ also updates for $a = 2$
2. (Recursive bootstrap) updating $\theta_Q$ for $a = 1$ uses $\max_{a \in \{1,2\}}$ in label
3. (Off-policy) Trains on old data, does not contain $a = 2$
   - Eventually, greedy policy should visit $a = 2$

If we can prevent any of these, we can learn a Q function offline

1. Do not use neural network

Need nn for large state space

2. Use MC (non-recursive) instead of TD (recursive)

# The Deadly Triad

1. (Function approximation) We share parameters for $a = 1$ and $a = 2$
   - Updating $\theta_Q$ for $a = 1$ also updates for $a = 2$
2. (Recursive bootstrap) updating $\theta_Q$ for $a = 1$ uses $\max_{a \in \{1,2\}}$ in label
3. (Off-policy) Trains on old data, does not contain $a = 2$
   - Eventually, greedy policy should visit $a = 2$

If we can prevent any of these, we can learn a Q function offline

| | |
|---|---|
| 1. Do not use neural network | Need nn for large state space |
| 2. Use MC (non-recursive) instead of TD (recursive) | MC is on-policy only, must use TD |

# The Deadly Triad

1. (Function approximation) We share parameters for $a = 1$ and $a = 2$
   - Updating $\theta_Q$ for $a = 1$ also updates for $a = 2$
2. (Recursive bootstrap) updating $\theta_Q$ for $a = 1$ uses $\max_{a \in \{1,2\}}$ in label
3. (Off-policy) Trains on old data, does not contain $a = 2$
   - Eventually, greedy policy should visit $a = 2$

If we can prevent any of these, we can learn a Q function offline

| | |
|---|---|
| 1. Do not use neural network | Need nn for large state space |
| 2. Use MC (non-recursive) instead of TD (recursive) | MC is on-policy only, must use TD |
| 3. Visit all possible states/actions | |

# The Deadly Triad

1. (Function approximation) We share parameters for $a = 1$ and $a = 2$
   - Updating $\theta_Q$ for $a = 1$ also updates for $a = 2$
2. (Recursive bootstrap) updating $\theta_Q$ for $a = 1$ uses $\max_{a \in \{1,2\}}$ in label
3. (Off-policy) Trains on old data, does not contain $a = 2$
   - Eventually, greedy policy should visit $a = 2$

If we can prevent any of these, we can learn a Q function offline

| | |
|---|---|
| 1. Do not use neural network | Need nn for large state space |
| 2. Use MC (non-recursive) instead of TD (recursive) | MC is on-policy only, must use TD |
| 3. Visit all possible states/actions | Not possible with fixed dataset |

# Constraining Q

# Constraining Q

So far, offline Q learning seems impossible

# Constraining Q

So far, offline Q learning seems impossible

Root problem is the out-of-distribution (OOD) TD update

# Constraining Q

So far, offline Q learning seems impossible

Root problem is the out-of-distribution (OOD) TD update

$$\max_{a \in A} Q(s, a); \quad (s, a) \notin \boldsymbol{X}$$

# Constraining Q

So far, offline Q learning seems impossible

Root problem is the out-of-distribution (OOD) TD update

$$\max_{a \in A} Q(s, a); \quad (s, a) \notin \boldsymbol{X}$$

We **overextrapolate** for state-action pairs missing from dataset

# Constraining Q

So far, offline Q learning seems impossible

Root problem is the out-of-distribution (OOD) TD update

$$\max_{a \in A} Q(s, a); \quad (s, a) \notin \boldsymbol{X}$$

We **overextrapolate** for state-action pairs missing from dataset

**Question:** What can we do about this?

# Constraining Q

So far, offline Q learning seems impossible

Root problem is the out-of-distribution (OOD) TD update

$$\max_{a \in A} Q(s, a); \quad (s, a) \notin \boldsymbol{X}$$

We **overextrapolate** for state-action pairs missing from dataset

**Question:** What can we do about this?

**Answer:** Ignore $Q$ for missing state-action pairs

# Constraining Q

So far, offline Q learning seems impossible

Root problem is the out-of-distribution (OOD) TD update

$$\max_{a \in A} Q(s, a); \quad (s, a) \notin \boldsymbol{X}$$

We **overextrapolate** for state-action pairs missing from dataset

**Question:** What can we do about this?

**Answer:** Ignore $Q$ for missing state-action pairs

# Constraining Q

We want to ignore $Q(s, a)$ where $s, a \notin \boldsymbol{X}$

# Constraining Q

We want to ignore $Q(s, a)$ where $s, a \notin \boldsymbol{X}$

For a finite discrete state space, this is easy!

# Constraining Q

We want to ignore $Q(s, a)$ where $s, a \notin \boldsymbol{X}$

For a finite discrete state space, this is easy!

Only consider actions we have in our dataset $\overline{A}(s)$

# Constraining Q

We want to ignore $Q(s, a)$ where $s, a \notin \boldsymbol{X}$

For a finite discrete state space, this is easy!

Only consider actions we have in our dataset $\overline{A}(s)$

$$\overline{A}(s) = \{a \mid (s, a) \in \boldsymbol{X}\}$$

# Constraining Q

We want to ignore $Q(s, a)$ where $s, a \notin \boldsymbol{X}$

For a finite discrete state space, this is easy!

Only consider actions we have in our dataset $\overline{A}(s)$

$$\overline{A}(s) = \{a \mid (s, a) \in \boldsymbol{X}\}$$

$$Q(s_0, a_0, \theta_Q) = \mathbb{E}[\mathcal{R}(s_1) \mid s_0, a_0] + \gamma \max_{a \in \overline{A}} Q(s_1, a, \theta_Q)$$

# Constraining Q

We want to ignore $Q(s, a)$ where $s, a \notin \boldsymbol{X}$

For a finite discrete state space, this is easy!

Only consider actions we have in our dataset $\overline{A}(s)$

$$\overline{A}(s) = \{a \mid (s, a) \in \boldsymbol{X}\}$$

$$Q(s_0, a_0, \theta_Q) = \mathbb{E}[\mathcal{R}(s_1) \mid s_0, a_0] + \gamma \max_{a \in \overline{A}} Q(s_1, a, \theta_Q)$$

We ignore actions missing from the dataset!

# Constraining Q

We want to ignore $Q(s, a)$ where $s, a \notin \boldsymbol{X}$

For a finite discrete state space, this is easy!

Only consider actions we have in our dataset $\overline{A}(s)$

$$\overline{A}(s) = \{a \mid (s, a) \in \boldsymbol{X}\}$$

$$Q(s_0, a_0, \theta_Q) = \mathbb{E}[\mathcal{R}(s_1) \mid s_0, a_0] + \gamma \max_{a \in \overline{A}} Q(s_1, a, \theta_Q)$$

We ignore actions missing from the dataset!

- This will fix deadly triad via (2. Recursive bootstrap)

# Constraining Q

We want to ignore $Q(s, a)$ where $s, a \notin \boldsymbol{X}$

For a finite discrete state space, this is easy!

Only consider actions we have in our dataset $\overline{A}(s)$

$$\overline{A}(s) = \{a \mid (s, a) \in \boldsymbol{X}\}$$

$$Q(s_0, a_0, \theta_Q) = \mathbb{E}[\mathcal{R}(s_1) \mid s_0, a_0] + \gamma \max_{a \in \overline{A}} Q(s_1, a, \theta_Q)$$

We ignore actions missing from the dataset!

- This will fix deadly triad via (2. Recursive bootstrap)

**Question:** What if the state space is continuous? Will this work?

# Constraining Q

Continuous state space means we cannot check $(s, a) \in \boldsymbol{X}$

# Constraining Q

Continuous state space means we cannot check $(s, a) \in \boldsymbol{X}$

- Every state $s$ in $\boldsymbol{X}$ will be different

# Constraining Q

Continuous state space means we cannot check $(s, a) \in \boldsymbol{X}$

- Every state $s$ in $\boldsymbol{X}$ will be different
  - Each state will only have one action

# Constraining Q

Continuous state space means we cannot check $(s, a) \in \boldsymbol{X}$

- Every state $s$ in $\boldsymbol{X}$ will be different
  - Each state will only have one action
  - $\max_{a \in \overline{A}}$ returns the action in the dataset

# Constraining Q

Continuous state space means we cannot check $(s, a) \in \boldsymbol{X}$

- Every state $s$ in $\boldsymbol{X}$ will be different
  - Each state will only have one action
  - $\max_{a \in \overline{A}}$ returns the action in the dataset
  - We will learn the behavior policy, not optimal policy

# Constraining Q

Continuous state space means we cannot check $(s, a) \in \boldsymbol{X}$

- Every state $s$ in $\boldsymbol{X}$ will be different
  - Each state will only have one action
  - $\max_{a \in \overline{A}}$ returns the action in the dataset
  - We will learn the behavior policy, not optimal policy

**Solution:** Continuous relaxation of $\overline{A}$

# Constraining Q

Continuous state space means we cannot check $(s, a) \in \boldsymbol{X}$

- Every state $s$ in $\boldsymbol{X}$ will be different
  - Each state will only have one action
  - $\max_{a \in \overline{A}}$ returns the action in the dataset
  - We will learn the behavior policy, not optimal policy

**Solution:** Continuous relaxation of $\overline{A}$

$$\overline{A}(s) = \{a \mid \pi(a \mid s; \theta_\beta) > \rho\}$$

# Constraining Q

Continuous state space means we cannot check $(s, a) \in \boldsymbol{X}$

- Every state $s$ in $\boldsymbol{X}$ will be different
  - Each state will only have one action
  - $\max_{a \in \overline{A}}$ returns the action in the dataset
  - We will learn the behavior policy, not optimal policy

**Solution:** Continuous relaxation of $\overline{A}$

$$\overline{A}(s) = \{a \mid \pi(a \mid s; \theta_\beta) > \rho\}$$

Only consider actions that our behavior policy might take

# Constraining Q

Continuous state space means we cannot check $(s, a) \in \boldsymbol{X}$

- Every state $s$ in $\boldsymbol{X}$ will be different
  - Each state will only have one action
  - $\max_{a \in \overline{A}}$ returns the action in the dataset
  - We will learn the behavior policy, not optimal policy

**Solution:** Continuous relaxation of $\overline{A}$

$$\overline{A}(s) = \{a \mid \pi(a \mid s; \theta_\beta) > \rho\}$$

Only consider actions that our behavior policy might take

We can learn $\theta_\beta$ using behavioral cloning

# Constraining Q

**Definition:** Batch Constrained Q Learning (BCQ) learns the behavior policy, only considers behavior policy actions in the TD update

# Constraining Q

**Definition:** Batch Constrained Q Learning (BCQ) learns the behavior policy, only considers behavior policy actions in the TD update

**Step 1:** Learn the behavior policy through BC

# Constraining Q

**Definition:** Batch Constrained Q Learning (BCQ) learns the behavior policy, only considers behavior policy actions in the TD update

**Step 1:** Learn the behavior policy through BC

$$\theta_\pi = \arg\min_{\theta_\pi} \sum_{s \in \boldsymbol{X}} \sum_{a \in A} -\pi(a \mid s; \theta_\beta) \log \pi(a \mid s; \theta_\pi)$$

# Constraining Q

**Definition:** Batch Constrained Q Learning (BCQ) learns the behavior policy, only considers behavior policy actions in the TD update

**Step 1:** Learn the behavior policy through BC

$$\theta_\pi = \arg \min_{\theta_\pi} \sum_{s \in \boldsymbol{X}} \sum_{a \in A} -\pi(a \mid s; \theta_\beta) \log \pi(a \mid s; \theta_\pi)$$

**Step 2:** Learn the Q function

# Constraining Q

**Definition:** Batch Constrained Q Learning (BCQ) learns the behavior policy, only considers behavior policy actions in the TD update

**Step 1:** Learn the behavior policy through BC

$$\theta_\pi = \arg \min_{\theta_\pi} \sum_{s \in \boldsymbol{X}} \sum_{a \in A} -\pi(a \mid s; \theta_\beta) \log \pi(a \mid s; \theta_\pi)$$

**Step 2:** Learn the Q function

$$\theta_{Q,i+1} = \arg \min_{\theta_{Q,i}} \left(Q(s_0, a_0, \theta_\pi, \theta_{Q,i}) - y\right)^2$$

# Constraining Q

**Definition:** Batch Constrained Q Learning (BCQ) learns the behavior policy, only considers behavior policy actions in the TD update

**Step 1:** Learn the behavior policy through BC

$$\theta_\pi = \arg\min_{\theta_\pi} \sum_{s \in \boldsymbol{X}} \sum_{a \in A} -\pi(a \mid s; \theta_\beta) \log \pi(a \mid s; \theta_\pi)$$

**Step 2:** Learn the Q function

$$\theta_{Q,i+1} = \arg\min_{\theta_{Q,i}} \left(Q(s_0, a_0, \theta_\pi, \theta_{Q,i}) - y\right)^2$$

$$y = \hat{\mathbb{E}}[\mathcal{R}(s_1) \mid s_0, a_0] + \gamma \max_{a \in \overline{A}} Q(s_1, a, \theta_\pi, \theta_{Q,i})$$

# Constraining Q

$$\theta_{Q,i+1} = \underset{\theta_{Q,i}}{\arg\min} \left(Q\left(s_0, a_0, \theta_\pi, \theta_{Q,i}\right) - y\right)^2$$

$$y = \hat{\mathbb{E}}[\mathcal{R}(s_1) \mid s_0, a_0] + \gamma \max_{a \in \overline{A}} Q\left(s_1, a, \theta_\pi, \theta_{Q,i}\right)$$

# Constraining Q

$$\theta_{Q,i+1} = \arg\min_{\theta_{Q,i}} \left(Q\left(s_0, a_0, \theta_\pi, \theta_{Q,i}\right) - y\right)^2$$

$$y = \hat{\mathbb{E}}[\mathcal{R}(s_1) \mid s_0, a_0] + \gamma \max_{a \in \overline{A}} Q\left(s_1, a, \theta_\pi, \theta_{Q,i}\right)$$

Constrain $\overline{A}$ to contain actions the behavior policy would take

# Constraining Q

$$\theta_{Q,i+1} = \underset{\theta_{Q,i}}{\arg\min} \left(Q\left(s_0, a_0, \theta_\pi, \theta_{Q,i}\right) - y\right)^2$$

$$y = \hat{\mathbb{E}}[\mathcal{R}(s_1) \mid s_0, a_0] + \gamma \max_{a \in \overline{A}} Q\left(s_1, a, \theta_\pi, \theta_{Q,i}\right)$$

Constrain $\overline{A}$ to contain actions the behavior policy would take

$$\overline{A} = \{a \mid \pi(a \mid s_1; \theta_\pi) > \rho\}$$

# Constraining Q

$$\theta_{Q,i+1} = \arg\min_{\theta_{Q,i}} \left(Q\left(s_0, a_0, \theta_\pi, \theta_{Q,i}\right) - y\right)^2$$

$$y = \hat{\mathbb{E}}[\mathcal{R}(s_1) \mid s_0, a_0] + \gamma \max_{a \in \overline{A}} Q\left(s_1, a, \theta_\pi, \theta_{Q,i}\right)$$

Constrain $\overline{A}$ to contain actions the behavior policy would take

$$\overline{A} = \{a \mid \pi(a \mid s_1; \theta_\pi) > \rho\}$$

where hyperparameter $\rho$ determines the level of extrapolation

# Constraining Q

$$\theta_{Q,i+1} = \underset{\theta_{Q,i}}{\arg\min} \left(Q\left(s_0, a_0, \theta_\pi, \theta_{Q,i}\right) - y\right)^2$$

$$y = \hat{\mathbb{E}}[\mathcal{R}(s_1) \mid s_0, a_0] + \gamma \max_{a \in \overline{A}} Q\left(s_1, a, \theta_\pi, \theta_{Q,i}\right)$$

Constrain $\overline{A}$ to contain actions the behavior policy would take

$$\overline{A} = \{a \mid \pi(a \mid s_1; \theta_\pi) > \rho\}$$

where hyperparameter $\rho$ determines the level of extrapolation

- Bigger $\rho$ means less extrapolation, less optimal

# Constraining Q

$$\theta_{Q,i+1} = \underset{\theta_{Q,i}}{\arg\min} \left(Q(s_0, a_0, \theta_\pi, \theta_{Q,i}) - y\right)^2$$

$$y = \hat{\mathbb{E}}[\mathcal{R}(s_1) \mid s_0, a_0] + \gamma \max_{a \in \overline{A}} Q(s_1, a, \theta_\pi, \theta_{Q,i})$$

Constrain $\overline{A}$ to contain actions the behavior policy would take

$$\overline{A} = \{a \mid \pi(a \mid s_1; \theta_\pi) > \rho\}$$

where hyperparameter $\rho$ determines the level of extrapolation

- Bigger $\rho$ means less extrapolation, less optimal
- Smaller $\rho$ means more extrapolation, more optimal

# Constraining Q

$$\theta_{Q,i+1} = \arg\min_{\theta_{Q,i}} \left(Q\left(s_0, a_0, \theta_\pi, \theta_{Q,i}\right) - y\right)^2$$

$$y = \hat{\mathbb{E}}[\mathcal{R}(s_1) \mid s_0, a_0] + \gamma \max_{a \in \overline{A}} Q\left(s_1, a, \theta_\pi, \theta_{Q,i}\right)$$

Constrain $\overline{A}$ to contain actions the behavior policy would take

$$\overline{A} = \{a \mid \pi(a \mid s_1; \theta_\pi) > \rho\}$$

where hyperparameter $\rho$ determines the level of extrapolation

- Bigger $\rho$ means less extrapolation, less optimal
- Smaller $\rho$ means more extrapolation, more optimal
- Too much extrapolation leads to deadly triad!

# Constraining Q

BCQ is a good algorithm, but it requires learning a policy using BC

# Constraining Q

BCQ is a good algorithm, but it requires learning a policy using BC

Can we do offline Q learning without learning the behavior policy?

# Constraining Q

BCQ is a good algorithm, but it requires learning a policy using BC

Can we do offline Q learning without learning the behavior policy?

What if we make Q small for OOD actions?

# Constraining Q

BCQ is a good algorithm, but it requires learning a policy using BC

Can we do offline Q learning without learning the behavior policy?

What if we make Q small for OOD actions?

$$\min_{a \notin \overline{A}} Q(s, a)$$

# Constraining Q

BCQ is a good algorithm, but it requires learning a policy using BC

Can we do offline Q learning without learning the behavior policy?

What if we make Q small for OOD actions?

$$\min_{a \notin \overline{A}} Q(s, a)$$

This still requires knowing $\overline{A}$ and use BC

# Constraining Q

BCQ is a good algorithm, but it requires learning a policy using BC

Can we do offline Q learning without learning the behavior policy?

What if we make Q small for OOD actions?

$$\min_{a \notin \overline{A}} Q(s, a)$$

This still requires knowing $\overline{A}$ and use BC

Can we do this without knowing $\overline{A}$?

# Constraining Q

**Solution:** Create two conflicting objectives:

# Constraining Q

**Solution:** Create two conflicting objectives:

- Learn $Q$ as usual with TD error

# Constraining Q

**Solution:** Create two conflicting objectives:

- Learn $Q$ as usual with TD error
- Minimize $Q$ values

# Constraining Q

**Solution:** Create two conflicting objectives:

- Learn $Q$ as usual with TD error
- Minimize $Q$ values

**Key idea:** This should force all $Q(s, a)$ to be small, unless $(s, a) \in \boldsymbol{X}$

# Constraining Q

**Solution:** Create two conflicting objectives:

- Learn $Q$ as usual with TD error
- Minimize $Q$ values

**Key idea:** This should force all $Q(s, a)$ to be small, unless $(s, a) \in \boldsymbol{X}$

$$\arg\min_{\theta_Q} \underbrace{\left(Q\left(s_0, a_0, \theta_\pi, \theta_Q\right) - y\right)^2}_{\text{TD error}} + \underbrace{\sum_{a \in A} Q\left(s_1, a, \theta_\pi, \theta_Q\right)}_{\text{Minimize Q}}$$

# Constraining Q

**Solution:** Create two conflicting objectives:

- Learn $Q$ as usual with TD error
- Minimize $Q$ values

**Key idea:** This should force all $Q(s, a)$ to be small, unless $(s, a) \in \boldsymbol{X}$

$$\arg\min_{\theta_Q} \underbrace{\left(Q\left(s_0, a_0, \theta_\pi, \theta_Q\right) - y\right)^2}_{\text{TD error}} + \underbrace{\sum_{a \in A} Q\left(s_1, a, \theta_\pi, \theta_Q\right)}_{\text{Minimize Q}}$$

$$y = \hat{\mathbb{E}}[\mathcal{R}(s_1) \mid s_0, a_0] + \gamma \max_{a \in \overline{A}} Q\left(s_1, a, \theta_\pi, \theta_{Q,i}\right)$$

# Constraining Q

$$\underset{\theta_Q}{\arg\min} \; \underbrace{\left(Q(s_0, a_0, \theta_\pi, \theta_Q) - y\right)^2}_{\text{TD error}} + \underbrace{\sum_{a \in A} Q(s_1, a, \theta_\pi, \theta_Q)}_{\text{Minimize Q}}$$

$$y = \hat{\mathbb{E}}[\mathcal{R}(s_1) \mid s_0, a_0] + \gamma \max_{a \in \overline{A}} Q(s_1, a, \theta_\pi, \theta_{Q,i})$$

However, the scale of TD error and Q values can be different

# Constraining Q

$$\arg\min_{\theta_Q} \underbrace{\left(Q(s_0, a_0, \theta_\pi, \theta_Q) - y\right)^2}_{\text{TD error}} + \underbrace{\sum_{a \in A} Q(s_1, a, \theta_\pi, \theta_Q)}_{\text{Minimize Q}}$$

$$y = \hat{\mathbb{E}}[\mathcal{R}(s_1) \mid s_0, a_0] + \gamma \max_{a \in \overline{A}} Q(s_1, a, \theta_\pi, \theta_{Q,i})$$

However, the scale of TD error and Q values can be different

Very sensitive to scale, we might just set all $Q(s, a) = -\infty$

# Constraining Q

$$\arg\min_{\theta_Q} \underbrace{\left(Q(s_0, a_0, \theta_\pi, \theta_Q) - y\right)^2}_{\text{TD error}} + \underbrace{\sum_{a \in A} Q(s_1, a, \theta_\pi, \theta_Q)}_{\text{Minimize Q}}$$

$$y = \hat{\mathbb{E}}[\mathcal{R}(s_1) \mid s_0, a_0] + \gamma \max_{a \in \overline{A}} Q(s_1, a, \theta_\pi, \theta_{Q,i})$$

However, the scale of TD error and Q values can be different

Very sensitive to scale, we might just set all $Q(s, a) = -\infty$

We need to balance the second term a little better

# Constraining Q

$$\arg\min_{\theta_Q} \underbrace{\left(Q(s_0, a_0, \theta_\pi, \theta_Q) - y\right)^2}_{\text{TD error}} + \underbrace{\sum_{a \in A} Q(s_1, a, \theta_\pi, \theta_Q)}_{\text{Minimize Q}}$$

$$y = \hat{\mathbb{E}}[\mathcal{R}(s_1) \mid s_0, a_0] + \gamma \max_{a \in \overline{A}} Q(s_1, a, \theta_\pi, \theta_{Q,i})$$

# Constraining Q

$$\arg\min_{\theta_Q} \underbrace{\left(Q(s_0, a_0, \theta_\pi, \theta_Q) - y\right)^2}_{\text{TD error}} + \underbrace{\sum_{a \in A} Q(s_1, a, \theta_\pi, \theta_Q)}_{\text{Minimize Q}}$$

$$y = \hat{\mathbb{E}}[\mathcal{R}(s_1) \mid s_0, a_0] + \gamma \max_{a \in A} Q(s_1, a, \theta_\pi, \theta_{Q,i})$$

We can subtract $Q$ for the action we take in the dataset!

# Constraining Q

$$\underset{\theta_Q}{\arg\min} \ \underbrace{\left(Q(s_0, a_0, \theta_\pi, \theta_Q) - y\right)^2}_{\text{TD error}} + \underbrace{\sum_{a \in A} Q(s_1, a, \theta_\pi, \theta_Q)}_{\text{Minimize Q}}$$

$$y = \hat{\mathbb{E}}[\mathcal{R}(s_1) \mid s_0, a_0] + \gamma \max_{a \in \overline{A}} Q(s_1, a, \theta_\pi, \theta_{Q,i})$$

We can subtract $Q$ for the action we take in the dataset!

$$\underset{\theta_Q}{\arg\min} \ \underbrace{\left(Q(s_0, a_0, \theta_\pi, \theta_Q) - y\right)^2}_{\text{TD error}} + \underbrace{\sum_{a \in A} Q(s_1, a, \theta_\pi, \theta_Q) - Q(s_1, a_1, \theta_\pi, \theta_Q)}_{\text{Minimize Q}}$$

This balances the second term to be closer to zero

# Constraining Q

$$\arg\min_{\theta_Q} \underbrace{\left(Q\left(s_0, a_0, \theta_\pi, \theta_Q\right) - y\right)^2}_{\text{TD error}} + \underbrace{\sum_{a \in A} Q\left(s_1, a, \theta_\pi, \theta_Q\right) - Q\left(s_1, a_1, \theta_\pi, \theta_Q\right)}_{\text{Minimize Q}}$$

# Constraining Q

$$\arg\min_{\theta_Q} \underbrace{\left(Q\left(s_0, a_0, \theta_\pi, \theta_Q\right) - y\right)^2}_{\text{TD error}} + \underbrace{\sum_{a \in A} Q\left(s_1, a, \theta_\pi, \theta_Q\right) - Q\left(s_1, a_1, \theta_\pi, \theta_Q\right)}_{\text{Minimize Q}}$$

While minimizing all $Q$ is useful, we care most about the biggest $Q$

# Constraining Q

$$\arg\min_{\theta_Q} \underbrace{\left(Q(s_0, a_0, \theta_\pi, \theta_Q) - y\right)^2}_{\text{TD error}} + \underbrace{\sum_{a \in A} Q(s_1, a, \theta_\pi, \theta_Q) - Q(s_1, a_1, \theta_\pi, \theta_Q)}_{\text{Minimize Q}}$$

While minimizing all $Q$ is useful, we care most about the biggest $Q$

We take $\max Q$, so we should emphasize minimizing $\max Q$

# Constraining Q

$$\arg\min_{\theta_Q} \underbrace{\left(Q(s_0, a_0, \theta_\pi, \theta_Q) - y\right)^2}_{\text{TD error}} + \underbrace{\sum_{a \in A} Q(s_1, a, \theta_\pi, \theta_Q) - Q(s_1, a_1, \theta_\pi, \theta_Q)}_{\text{Minimize Q}}$$

While minimizing all $Q$ is useful, we care most about the biggest $Q$

We take $\max Q$, so we should emphasize minimizing $\max Q$

Replace sum with logsumexp, a combination of max and sum

# Constraining Q

$$\arg\min_{\theta_Q} \underbrace{\left(Q\left(s_0, a_0, \theta_\pi, \theta_Q\right) - y\right)^2}_{\text{TD error}} + \underbrace{\sum_{a \in A} Q\left(s_1, a, \theta_\pi, \theta_Q\right) - Q\left(s_1, a_1, \theta_\pi, \theta_Q\right)}_{\text{Minimize Q}}$$

While minimizing all $Q$ is useful, we care most about the biggest $Q$

We take $\max Q$, so we should emphasize minimizing $\max Q$

Replace sum with logsumexp, a combination of max and sum

$$\underbrace{\left(Q\left(s_0, a_0, \theta_\pi, \theta_Q\right) - y\right)^2}_{\text{TD error}} + \underbrace{\log\left(\sum_{a \in A} \exp Q\left(s_1, a, \theta_\pi, \theta_Q\right)\right) - Q\left(s_1, a_1, \theta_\pi, \theta_Q\right)}_{\text{Minimize Q}}$$

# Constraining Q

**Definition:** Conservative Q Learning (CQL) learns a Q function that minimizes $Q$ for out of distribution actions

$$\theta_{Q,i+1} = \arg\min_{\theta_{Q,i}} \underbrace{\left(Q\left(s_0, a_0, \theta_\pi, \theta_{Q,i}\right) - y\right)^2}_{\text{TD error}} + z^2$$

# Constraining Q

**Definition:** Conservative Q Learning (CQL) learns a Q function that minimizes $Q$ for out of distribution actions

$$\theta_{Q,i+1} = \arg\min_{\theta_{Q,i}} \underbrace{\left(Q\left(s_0, a_0, \theta_\pi, \theta_{Q,i}\right) - y\right)^2}_{\text{TD error}} + z^2$$

$$y = \hat{\mathbb{E}}[\mathcal{R}(s_1) \mid s_0, a_0] + \gamma \max_{a \in A} Q\left(s_1, a, \theta_\pi, \theta_{Q,i}\right)$$

# Constraining Q

**Definition:** Conservative Q Learning (CQL) learns a Q function that minimizes $Q$ for out of distribution actions

$$\theta_{Q,i+1} = \arg\min_{\theta_{Q,i}} \underbrace{\left(Q\left(s_0, a_0, \theta_\pi, \theta_{Q,i}\right) - y\right)^2}_{\text{TD error}} + z^2$$

$$y = \hat{\mathbb{E}}[\mathcal{R}(s_1) \mid s_0, a_0] + \gamma \max_{a \in A} Q\left(s_1, a, \theta_\pi, \theta_{Q,i}\right)$$

$$z = \underbrace{\left( \log \sum_{a \in A} \exp Q\left(s_1, a, \theta_\pi, \theta_{Q,i}\right) \right)}_{\text{Minimize Q for all } a} - \underbrace{Q\left(s_1, a_1, \theta_\pi, \theta_{Q,i}\right)}_{\substack{\text{Push up Q for} \\ \text{in-distribution } a_1}}$$

# Conclusion

# Conclusion

Today, we looked at offline RL

- Like IL, but learns optimal policy instead of expert policy

Two standard approaches to offline RL

# Conclusion

Today, we looked at offline RL

- Like IL, but learns optimal policy instead of expert policy

Two standard approaches to offline RL

- BC with weighted objectives

# Conclusion

Today, we looked at offline RL

- Like IL, but learns optimal policy instead of expert policy

Two standard approaches to offline RL

- BC with weighted objectives
- Breaking the deadly triad with Q learning