Deep Value

CISC 7404 - Decision Making

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Admin

Admin

I noticed the last 1 hour of class everyone looks tired and sad

Three hours is a long time to pay attention, especially at night

Would you prefer:

- To have a long break (30 mins) in the middle?
- No breaks, end lecture after 2 or 2.5 hours
- Keep as-is (approximately 3 hours + 10 min break)

Admin

HW1 bug:

There was a bug in the `update_Q_TD0` starter code, thanks He Zhe!

Before:

```
terminateds = jnp.concatenate([jnp.zeros(states.shape[0],
dtype=bool), jnp.array([1], dtype=bool)])
```

After:

```
terminateds = jnp.concatenate([jnp.zeros(states.shape[0] - 1,
dtype=bool), jnp.array([1], dtype=bool)])
```

Admin

$$Q_{i+1}(s_0, a_0, \theta_\pi) = Q_i(s_0, a_0, \theta_\pi) - \alpha \cdot \eta$$

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$$\eta = Q_i(s_0, a_0, \theta_\pi) - \left(\hat{\mathbb{E}}[\mathcal{R}(s_1) \mid s_0, a_0] + \neg d \gamma \max_{a \in A} Q_i(s_1, a, \theta_\pi) \right)$$

Admin

$$Q_{i+1}(s_0, a_0, \theta_\pi) = Q_i(s_0, a_0, \theta_\pi) - \alpha \cdot \eta$$

Predicted value

$$\eta = \boxed{Q_i(s_0, a_0, \theta_\pi)} - \left(\hat{\mathbb{E}}[\mathcal{R}(s_1) \mid s_0, a_0] + \neg d \gamma \max_{a \in A} Q_i(s_1, a, \theta_\pi) \right)$$

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Empirical value

Admin

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Empirical value

小心! If s_1 is a terminal state, future value is 0 ($\neg d = \text{not terminated}$)

Admin

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Empirical value

小心! If s_1 is a terminal state, future value is 0 ($\neg d = \text{not terminated}$)

Without the $\neg d$ term, takes longer to train!

Admin

I thought about coding deep Q networks in class today

But I realize if I do this, then you will not learn as much

Learning to debug is the #1 skill to succeed in reinforcement learning

Instead, **you** will implement deep Q learning for your second homework

Homework 2:
- Deep Q learning
- Deep policy gradient

Will release after homework 1

Admin

Next quiz in 2-3 weeks

Focus on

- Returns
- Value functions
- Q learning
- Deep Q learning
- Policy gradient

Review

Deep Learning Review

Deep Learning Review

We model neural networks as parameterized functions

$$f : X \times \Theta \mapsto Y$$

Map an input $\boldsymbol{x} \in X$ and parameters $\boldsymbol{\theta} \in \Theta$ to output space Y

$$f(\boldsymbol{x}, \boldsymbol{\theta})$$

Deep Learning Review

Neural networks consist of **artificial neurons**

Deep Learning Review

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$$\boldsymbol{x} \in \mathbb{R}^{d_x}, \boldsymbol{\theta} \in \mathbb{R}^{d_x + 1}$$

Deep Learning Review

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σ is a nonlinearity like sigmoid

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$$\sigma(x) = \frac{1}{1 + e^{-x}}$$

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Or ReLU

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Or ReLU

$$\sigma(x) = \max(0, x)$$

Deep Learning Review

We combine individual neurons into a **layer**

Deep Learning Review

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$$f : \mathbb{R}^{d_x} \times \Theta \mapsto \mathbb{R}$$

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$$f : \mathbb{R}^{d_x} \times \Theta \mapsto \mathbb{R}$$

$$\Theta = \mathbb{R}^{d_x + 1}$$

d_y neurons (wide):

$$f : \mathbb{R}^{d_x} \times \Theta \mapsto \mathbb{R}^{d_y}$$

$$\Theta = \mathbb{R}^{(d_x + 1) \times d_y}$$

Deep Learning Review

For a single neuron

Deep Learning Review

For a single neuron

$$f\left(\begin{bmatrix} x_1 \\ \vdots \\ x_{d_x} \end{bmatrix}, \begin{bmatrix} \theta_0 \\ \theta_1 \\ \vdots \\ \theta_{d_x} \end{bmatrix}\right) = \sigma\left(\sum_{i=0}^{d_x} \theta_i \overline{x}_i\right)$$

Deep Learning Review

For a single neuron

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For a wide network

Deep Learning Review

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For a wide network

$$f\left(\begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_{d_x} \end{bmatrix}, \begin{bmatrix} \theta_{0,1} & \theta_{0,2} & \cdots & \theta_{0,d_y} \\ \theta_{1,1} & \theta_{1,2} & \cdots & \theta_{1,d_y} \\ \vdots & \vdots & \ddots & \vdots \\ \theta_{d_x,1} & \theta_{d_x,2} & \cdots & \theta_{d_x,d_y} \end{bmatrix}\right) = \begin{bmatrix} \sigma\left(\sum_{i=0}^{d_x} \theta_{i,1} \overline{x}_i\right) \\ \sigma\left(\sum_{i=0}^{d_x} \theta_{i,2} \overline{x}_i\right) \\ \vdots \\ \sigma\left(\sum_{i=0}^{d_x} \theta_{i,d_y} \overline{x}_i\right) \end{bmatrix}$$

Deep Learning Review

We can combine layers to create a **deep** neural network

Deep Learning Review

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A wide network:

$$f(\boldsymbol{x}, \boldsymbol{\theta}) = \sigma(\boldsymbol{\theta}^\top \overline{\boldsymbol{x}})$$

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$$f(\boldsymbol{x}, \boldsymbol{\theta}) = f_\ell(\dots f_2(f_1(\boldsymbol{x}, \boldsymbol{\varphi}), \boldsymbol{\psi})\dots \boldsymbol{\xi})$$

Deep Learning Review

Written another way

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$$z_1 = f_1(\boldsymbol{x}, \boldsymbol{\varphi}) = \sigma(\boldsymbol{\varphi}^\top \overline{\boldsymbol{x}})$$

Deep Learning Review

Written another way

$$\boldsymbol{z}_1 = f_1(\boldsymbol{x}, \boldsymbol{\varphi}) = \sigma(\boldsymbol{\varphi}^\top \overline{\boldsymbol{x}})$$

$$\boldsymbol{z}_2 = f_2(\boldsymbol{z}_{\mathbf{1}}, \boldsymbol{\psi}) = \sigma(\boldsymbol{\psi}^\top \overline{\boldsymbol{z}}_1)$$

Deep Learning Review

Written another way

$$\boldsymbol{z}_1 = f_1(\boldsymbol{x}, \boldsymbol{\varphi}) = \sigma(\boldsymbol{\varphi}^\top \overline{\boldsymbol{x}})$$

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Deep Learning Review

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We call each function a **layer**

Deep Learning Review

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$$\vdots$$

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We call each function a **layer**

A deep neural network is made of many layers

Deep Learning Review

We can create different models for different tasks

Deep Learning Review

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Standard tasks:

Deep Learning Review

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Standard tasks: Multi-layer perceptron (MLP)

Deep Learning Review

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Image tasks:

Deep Learning Review

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Standard tasks: Multi-layer perceptron (MLP)

Image tasks: Convolutional neural network (CNN)

Deep Learning Review

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Temporal tasks:

Deep Learning Review

We can create different models for different tasks

Standard tasks: Multi-layer perceptron (MLP)

Image tasks: Convolutional neural network (CNN)

Temporal tasks: Recurrent neural network (RNN)

Deep Learning Review

We can create different models for different tasks

Standard tasks: Multi-layer perceptron (MLP)

Image tasks: Convolutional neural network (CNN)

Temporal tasks: Recurrent neural network (RNN)

Image, temporal tasks: Transformer

Deep Learning Review

What functions can we represent using deep neural networks?

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Deep Learning Review

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It can approximate **any** continuous function $g(x)$ to precision η

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$$| \, g(\boldsymbol{x}) - f(\boldsymbol{x}, \boldsymbol{\theta}) \, | < \eta$$

Deep Learning Review

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Making the network deeper or wider decreases η

Deep Learning Review

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<u>Very powerful finding! The basis of deep learning.</u>

Deep Learning Review

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<u>Very powerful finding! The basis of deep learning.</u>

Although such $\boldsymbol{\theta}$ exists, it can be hard to find

Deep Learning Review

Finding $\boldsymbol{\theta}$ is an optimization problem

Deep Learning Review

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In particular, we optimize a **loss function**

Deep Learning Review

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$$\mathcal{L} : X \times Y \times \Theta \mapsto \mathbb{R}$$

Deep Learning Review

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$$\arg\min_{\boldsymbol{\theta}} \mathcal{L}(\boldsymbol{x}, \boldsymbol{y}, \boldsymbol{\theta})$$

Deep Learning Review

Finding $\boldsymbol{\theta}$ is an optimization problem

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Loss function measures the error between $f(\boldsymbol{x}, \boldsymbol{\theta})$ and desired $g(\boldsymbol{x}) = \boldsymbol{y}$

Deep Learning Review

Finding $\boldsymbol{\theta}$ is an optimization problem

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Loss function measures the error between $f(\boldsymbol{x}, \boldsymbol{\theta})$ and desired $g(\boldsymbol{x}) = \boldsymbol{y}$

In this class, we will build loss functions from two error functions

Deep Learning Review

Square error: The squared distance over a dataset of size n

Deep Learning Review

Square error: The squared distance over a dataset of size n

$$\sum_{i=1}^{n} \sum_{j=1}^{d_y} \left(f\left(\boldsymbol{x}_{[i]}, \boldsymbol{\theta}\right)_j - g(\boldsymbol{x})_j \right)^2 = \sum_{i=1}^{n} \sum_{j=1}^{d_y} \left(f\left(\boldsymbol{x}_{[i]}, \boldsymbol{\theta}\right)_j - y_{[i],j} \right)^2$$

Deep Learning Review

Square error: The squared distance over a dataset of size n

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Cross entropy error: The probabilistic error over a dataset of size n

Deep Learning Review

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Cross entropy error: The probabilistic error over a dataset of size n

$$-\sum_{i=1}^{n} \sum_{j=1}^{d_y} P\left(g\left(\boldsymbol{x}_{[i]}\right)_j \mid \boldsymbol{x}_{[i]} \right) \log f\left(\boldsymbol{x}_{[i]}, \boldsymbol{\theta}\right)_j = -\sum_{i=1}^{n} \sum_{j=1}^{d_y} P\left(y_{[i],j} \mid \boldsymbol{x}_{[i]} \right) \log f\left(\boldsymbol{x}_{[i]}, \boldsymbol{\theta}\right)_j$$

Deep Learning Review

Square error:

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Deep Learning Review

Square error:

$$\sum_{i=1}^{n} \sum_{j=1}^{d_y} \left(f\left(\boldsymbol{x}_{[i]}, \boldsymbol{\theta}\right)_j - y_{[i],j} \right)^2$$

Cross entropy error:

$$- \sum_{i=1}^{n} \sum_{j=1}^{d_y} P\left(y_{[i],j} \mid \boldsymbol{x}_{[i]}\right) \log f\left(\boldsymbol{x}_{[i]}, \boldsymbol{\theta}\right)_j$$

Question: Which one will we use for Q learning?

Answer: Predict a scalar (expected return), so square error (regression)

Deep Learning Review

We can use these errors in a loss function

$$\mathcal{L}(\boldsymbol{X}, \boldsymbol{Y}, \boldsymbol{\theta}) = \sum_{i=1}^{n} \sum_{j=1}^{d_y} \left(f\left(\boldsymbol{x}_{[i]}, \boldsymbol{\theta}\right)_j - y_{[i],j} \right)^2$$

Deep Learning Review

We can use these errors in a loss function

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Deep Learning Review

When we train, we search for neural network parameters θ

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$$\arg\min_{\boldsymbol{\theta}} \mathcal{L}(\boldsymbol{X}, \boldsymbol{Y}, \boldsymbol{\theta}) = \arg\min_{\boldsymbol{\theta}} \sum_{i=1}^{n} \sum_{j=1}^{d_y} \left(f\left(\boldsymbol{x}_{[i]}, \boldsymbol{\theta}\right)_j - y_{[i],j} \right)^2$$

Deep Learning Review

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Deep Learning Review

Question: Which search method do we use?

Deep Learning Review

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Answer: Gradient-based methods (gradient descent, Adam, etc)

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Deep Learning Review

function GRADIENT DESCENT($\boldsymbol{X}$, $\boldsymbol{Y}$, $\mathcal{L}$, t, α)

  ▷ Randomly initialize parameters

  $\boldsymbol{\theta} \leftarrow \text{Glorot}()$

  for $i \in 1 ... t$ **do**

    ▷ Compute the gradient of the loss

    $\boldsymbol{J} \leftarrow \nabla_{\boldsymbol{\theta}} \mathcal{L}(\boldsymbol{X}, \boldsymbol{Y}, \boldsymbol{\theta})$

    ▷ Update the parameters using the negative gradient

    $\boldsymbol{\theta} \leftarrow \boldsymbol{\theta} - \alpha \cdot \boldsymbol{J}$

  return $\boldsymbol{\theta}$

Deep Learning Review

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  return $\boldsymbol{\theta}$

Gradient descent computes $\nabla \mathcal{L}$ over all $\boldsymbol{X}$

Deep Learning Review

We can put it all together in jax and equinox

```
from jax import random
from equinox import nn

seed = random.key(0)
key, *net_keys= random.split(seed, 4)
net = nn.Sequential([
  nn.Linear(d_x, d_h, key=net_keys[0]),
  nn.Lambda(jax.nn.leaky_relu),
  nn.Linear(d_h, d_h, key=net_keys[1]),
  nn.Lambda(jax.nn.leaky_relu),
  nn.Linear(d_h, d_y, key=net_keys[2]),
])
```

Deep Learning Review

We can extract parameters using `eqx.partition`

```python
import equinox as eqx
# Get all arrays (parameters) in the network
theta, model = eqx.partition(net, eqx.is_array)
# Add one to every parameter
theta = jax.tree.map(theta, lambda x: x + 1)
# Put the new parameters back into network
net = eqx.combine(theta, model)
```

Deep Learning Review

We can define loss functions

```python
import equinox as eqx

def L_square(net, x, y):
  # vmap applies network to batch of data
  prediction = eqx.filter_vmap(net)(x)
  return ((prediction - y) ** 2).mean()

def L_cross_entropy(net, x, y):
  # Net outputs probabilities
  # And y is one-hot, e.g. [0, 0, 1, 0]
  prediction = eqx.filter_vmap(model)(x)
  return -(y * jnp.log(prediction)).sum(-1).mean()
```

Deep Learning Review

```
import optax
import equinox as eqx
opt = optimizer.adam(learning_rate=3e-4)
# Adam needs to track momentum and variance
opt_state = opt.init(eqx.filter(net, eqx.is_array))
# Gradient of loss function is a function
grad_L = eqx.filter_grad(L_square)
# Evaluate grad_L at x, y, theta to find J
J = grad_L(net, x, y)
# Compute parameter update using J (adam)
updates, opt_state = opt.update(
    grads, opt_state, params=eqx.filter(net, eqx.is_array)
)
net = eqx.apply_updates(net, updates) # Update params
```

Deep Learning Review

```python
def train_one_batch(net, dataset):
  x, y = batch
  grads = eqx.filter_grad(L_square)(net, x, y)
  updates, opt_state = opt.update(
    grads, opt_state, params=eqx.filter(net, eqx.is_array)
  )
  net = eqx.apply_updates(net, updates) # Update params
  return net, opt_state

for epoch in range(num_epochs):
  for batch in dataset:
    # Can use eqx.filter_jit(f) for speedup
    net, opt_state = train_one_batch(net, batch, opt_state)
```

Deep Learning Review

Dirty secret of deep learning:

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Dirty secret of deep learning: We do not understand deep learning

Biological inspiration, theoretical bounds and mathematical guarantees

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Today we experiment, and maybe tomorrow we discover the theory

Deep Q Learning

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After the homework and last lecture, you are experts in Q learning

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You quickly trained a Q function to solve a task

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Why introduce deep learning to Q learning?

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Why introduce deep learning to Q learning?

Consider an example problem

Deep Q Learning

Example: Learn a policy to pick up trash and put it in the bin

Deep Q Learning

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Deep Q Learning

Question: What is S?

Deep Q Learning

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- Camera image $\mathbb{R}^{256 \times 256 \times 3}$

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In the assignment, we store Q value for each state in a matrix

Question: What is the size of the matrix?

$$S \times A$$

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Let us consider a simplification, only have the map position

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Answer: There are infinitely many numbers between 0 and 1

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Answer: Discretize the space

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Discretize to 128×128 grid squares

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$$16384 \times A$$

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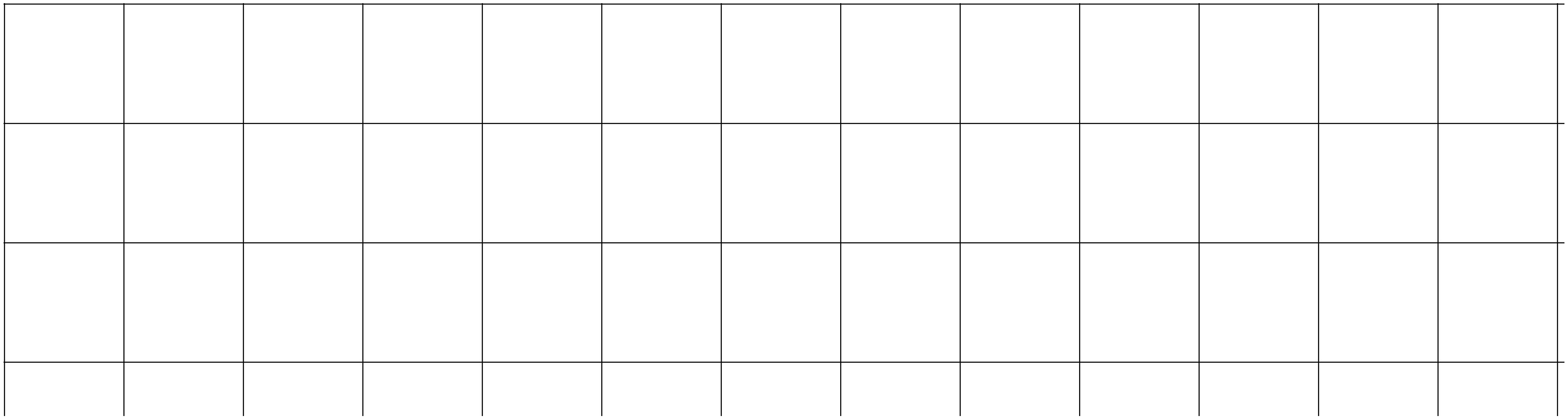
Very large but not infinite

Deep Q Learning



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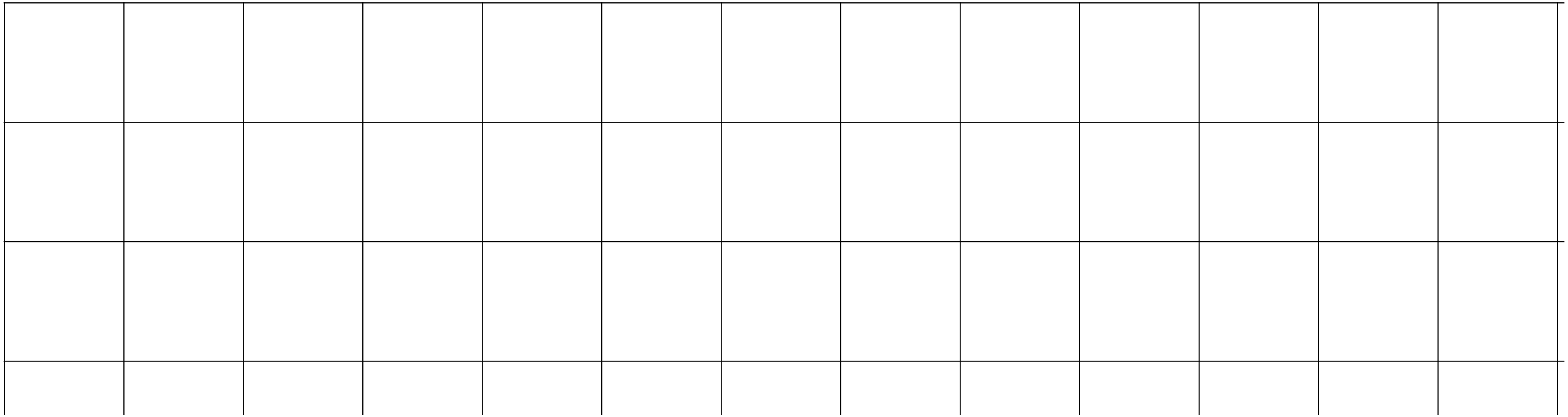

We must update Q for each s, a separately

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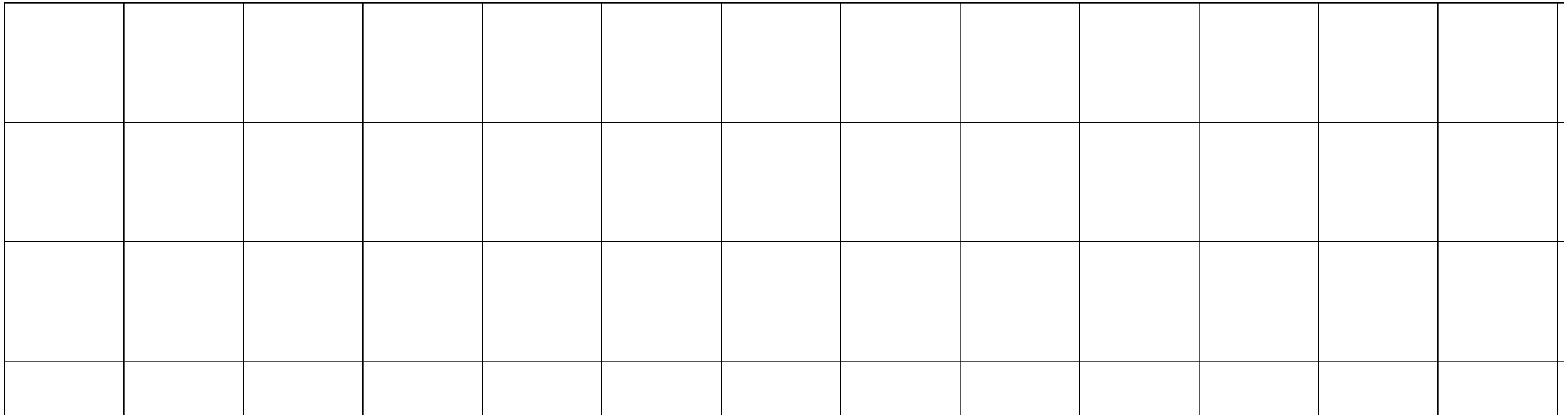

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With TD updates, updating one cell means we must update all cells

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Assume $\gamma = 0.99, \varepsilon = 0.1$

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This is the **minimum** number of samples to learn Q

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Simple Q learning works very well when the state space is small

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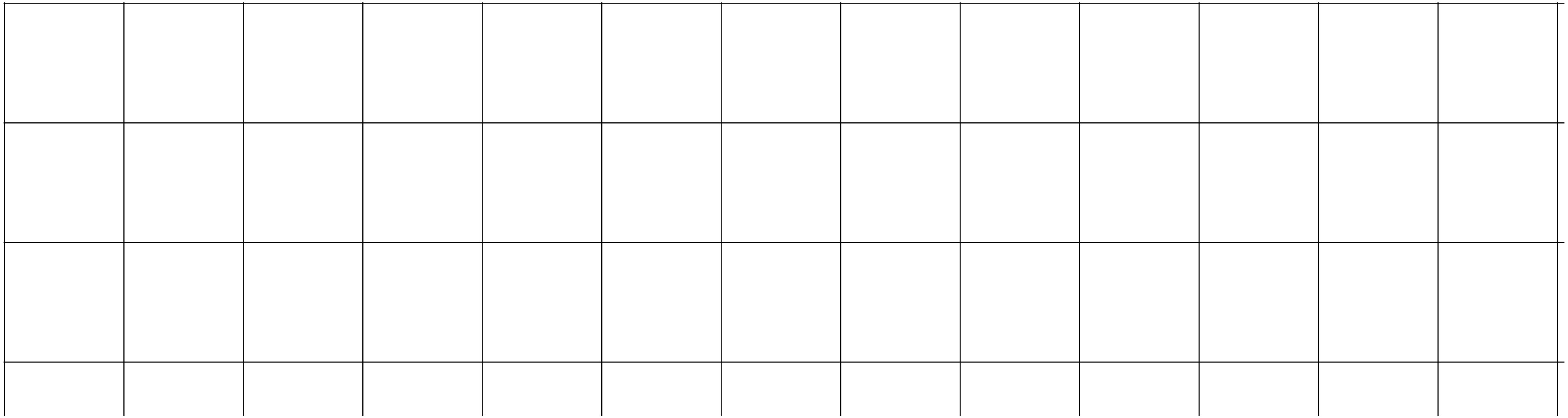

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But we will try!

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First, define the Q function as a deep neural network

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First, define the Q function as a deep neural network

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$$Q(s, a, \theta_\pi, \theta_Q)$$

The Q function estimates the policy-conditioned discounted return

Deep Q Learning

$$Q\left(\ s_0, a_0, \theta_\pi, \theta_Q\right) = \mathbb{E}[\mathcal{G}(\boldsymbol{\tau}) \mid s_0, a_0; \theta_\pi]$$

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Make this an optimization objective, so we can train a network

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Question: Missing anything? Hint: Other loss functions have sums?

Minimize over all states and actions

$$\arg\min_{\theta_Q} \left[\sum_{s_0 \in S} \sum_{a_0 \in A} \left(Q\left(s_0, a_0, \theta_\pi, \theta_Q\right) - \mathbb{E}[\mathcal{G}(\boldsymbol{\tau}) \mid s_0, a_0; \theta_\pi]\right)^2\right]$$

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Question: What are the two methods to compute $\mathbb{E}[\mathcal{G}(\boldsymbol{\tau}) \mid s_0, a_0; \theta_\pi]$?

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$$\underset{\theta_Q}{\arg\min} \left[\sum_{s_0 \in S} \sum_{a_0 \in A} \left(Q(s_0, a_0, \theta_\pi, \theta_Q) - \mathbb{E}[\mathcal{G}(\boldsymbol{\tau}) \mid s_0, a_0; \theta_\pi] \right)^2 \right]$$

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Answer: Monte Carlo and Temporal Difference

Deep Q Learning

Monte Carlo Objective:

$$\arg\min_{\theta_Q} \left[\sum_{s_0 \in S} \sum_{a_0 \in A} \left(Q(s_0, a_0, \theta_\pi, \theta_Q) - \sum_{t=0}^{\infty} \gamma^t \mathbb{E}[\mathcal{R}(s_{t+1}) \mid s_0, a_0; \theta_\pi] \right)^2 \right]$$

Temporal Difference Objective:

$$\arg\min_{\theta_Q} \left[\sum_{s_0 \in S} \sum_{a_0 \in A} \right.$$

$$\left. \left(Q(s_0, a_0, \theta_\pi, \theta_Q) - \left(\mathbb{E}[\mathcal{R}(s_1) \mid s_0, a_0] + \neg d \gamma \max_{a \in A} Q(s_1, a, \theta_\pi, \theta_Q) \right) \right)^2 \right]$$

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Still have expectations, which we do not know

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$$\left. \left(Q(s_0, a_0, \theta_\pi, \theta_Q) - \left(\hat{\mathbb{E}}[\mathcal{R}(s_1) \mid s_0, a_0] + \neg d \gamma \max_{a \in A} Q(s_1, a, \theta_\pi, \theta_Q) \right) \right)^2 \right]$$

Deep Q Learning

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Question: Which is harder to optimize?

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Question: Which is harder to optimize?

Answer: Temporal difference

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Deep Q Learning

Rewrite as loss functions to help with implementation

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The Monte Carlo loss uses an episode of states, actions, and rewards

Deep Q Learning

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The Monte Carlo loss uses an episode of states, actions, and rewards

$$\arg\min_{\boldsymbol{\theta}_Q} \mathcal{L}(\boldsymbol{x}, \boldsymbol{\theta}_Q) = \arg\min_{\theta_Q} \left[\sum_{s_i, a_i, r_i \in \boldsymbol{x}} \left(Q(s_i, a_i, \theta_\pi, \theta_Q) - \sum_{t=i}^{\infty} \gamma^t r_t \right)^2 \right]$$

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Can train over batch or dataset containing many episodes

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$$\sum_{\substack{s_i, a_i, r_i, s_{i+1} \\ \in \boldsymbol{x}}} \left(Q(s_i, a_i, \theta_\pi, \theta_Q) - \left(r_t + \neg d \gamma \arg \max_{a \in A} Q(s_i, a, \theta_\pi, \theta_Q) \right) \right)^2$$

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To summarize, our two loss functions:

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Deep Q Learning

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Deep Q Learning

Can optimize both loss functions using gradient descent

Deep Q Learning

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RL optimization is more difficult than supervised learning

Deep Q Learning

Can optimize both loss functions using gradient descent

RL optimization is more difficult than supervised learning

Supervised Learning:

- Static inputs
- Static labels
- Limited dataset
  - Human can clean
  - Bad to overfit

Reinforcement Learning:

- Inputs change as θ_π changes
  - Visit new/different states
- Labels change as θ_π changes
  - $\mathbb{E}[\mathcal{G}(\boldsymbol{\tau}) \mid \theta_\pi]$
- Infinite dataset
  - Can always collect from env
  - Bad θ_π means bad dataset
  - Overfitting no problem

Experience Replay

Experience Replay

Optimization is difficult in RL

Experience Replay

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Most RL papers train for 10M-10B environment steps

Experience Replay

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It takes a long time to train a deep Q function

Experience Replay

Optimization is difficult in RL

Most RL papers train for 10M-10B environment steps

It takes a long time to train a deep Q function

Let us see if we can improve training speed

Experience Replay

```
for epoch in range(num_epochs):
  terminated = False
  s = env.reset()
  episode = []
  # Step between 1 and infinity times to get one episode
  while not terminated:
    a = policy(s, theta_Q)
    next_s, r, d = env.step(action)
    episode.append([s, a, r, d, next_s])
  # Compute gradient over episode
  J = grad(L)(theta_Q, episode)
  theta_Q = update(theta_Q, grad)
```

Experience Replay

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Question: Which part is slowest?

Experience Replay

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```

Question: Which part is slowest? **Answer:** Collecting episodes

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for epoch in range(num_epochs):
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```

Collect episode, train, throw away episode, start again

Experience Replay

What if we reuse episodes?

Experience Replay

What if we reuse episodes?

```
episodes = []
for epoch in range(num_epochs):
  terminated = False
  s = env.reset()
  episode = []
  while not terminated:
    a = policy(s, theta_Q)
    next_s, r, d = env.step(action)
    episode.append([s, a, r, d, next_s])
  episodes.append(episode)
  J = grad(L)(theta_Q, episodes) # Train over ALL episodes
  theta_Q = update(theta_Q, grad)
```

Experience Replay

When we reuse episodes, we call it **experience replay**

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Store episodes in a **replay buffer**
(list)

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Store episodes in a **replay buffer** (list)

$$B_t = \begin{bmatrix} \boldsymbol{s}_1 & \boldsymbol{a}_1 & \boldsymbol{r}_1 & \boldsymbol{d}_1 \\ \vdots & \vdots & \vdots & \vdots \\ \boldsymbol{s}_t & \boldsymbol{a}_t & \boldsymbol{r}_t & \boldsymbol{d}_t \end{bmatrix}$$

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Create a dataset from the buffer

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Create a dataset from the buffer

$$\boldsymbol{X}_t = \begin{bmatrix} \boldsymbol{s}_{31} & \boldsymbol{a}_{31} & \boldsymbol{r}_{31} & \boldsymbol{d}_{31} \\ \vdots & \vdots & \vdots & \vdots \\ \boldsymbol{s}_4 & \boldsymbol{a}_4 & \boldsymbol{r}_4 & \boldsymbol{d}_4 \end{bmatrix}$$

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Train on the dataset

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Train on the dataset

$$\arg \min_{\boldsymbol{\theta}_Q} \mathcal{L}(\boldsymbol{X}_t, \boldsymbol{\theta}_Q)$$

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Train on the dataset

$$\underset{\boldsymbol{\theta}_Q}{\arg\min}\, \mathcal{L}(\boldsymbol{X}_t, \boldsymbol{\theta}_Q)$$

Humans do experience replay when they dream!

Experience Replay

On-policy algorithms must throw away episodes after training

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Must collect data using the current policy, cannot use experience replay

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Off-policy algorithms can reuse old episodes and use experience replay

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In fact, for off policy algorithms, data can come from anywhere

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Question: Which is Q learning?

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- Previous training run
- Human policy

Question: Which is Q learning?

Let us find out!

Experience Replay

Start with the Monte Carlo return

Experience Replay

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$$\arg\min_{\theta_Q} \left[\sum_{s_0 \in S} \sum_{a_0 \in A} \left(Q(s_0, a_0, \theta_\pi, \theta_Q) - \sum_{t=0}^{\infty} \gamma^t \mathbb{E}[\mathcal{R}(s_{t+1}) \mid s_0, a_0; \theta_\pi] \right)^2 \right]$$

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Question: On-policy or off-policy? **Answer:** On-policy. Why?

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Question: On-policy or off-policy? **Answer:** On-policy. Why?

Our return is conditioned on the policy

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Question: On-policy or off-policy? **Answer:** On-policy. Why?

Our return is conditioned on the policy

If the policy changes, the return $r_0 + \gamma r_1 + \gamma^2 r_2 + \ldots$ is not valid!

Experience Replay

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Old episode gives us $\mathbb{E}[\mathcal{R}(s_{t+1}) \mid s_0, a_0; \theta_{\text{old}}]$

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Old episode gives us $\mathbb{E}[\mathcal{R}(s_{t+1}) \mid s_0, a_0; \theta_{\text{old}}]$

We need $\mathbb{E}[\mathcal{R}(s_{t+1}) \mid s_0, a_0; \theta_\pi]$

Experience Replay

What about TD return?

Experience Replay

What about TD return?

$$\underset{\theta_Q}{\arg\min} \left[\sum_{s_0 \in S} \sum_{a_0 \in A} \left(Q(s_0, a_0, \theta_\pi, \theta_Q) - \left(\mathbb{E}[\mathcal{R}(s_1) \mid s_0, a_0] + \neg d \gamma \max_{a \in A} Q(s_1, a, \theta_\pi, \theta_Q) \right) \right)^2 \right]$$

Experience Replay

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$$\underset{\theta_Q}{\arg\min} \left[\sum_{s_0 \in S} \sum_{a_0 \in A} \right.$$

$$\left. \left(Q(s_0, a_0, \theta_\pi, \theta_Q) - \left(\mathbb{E}[\mathcal{R}(s_1) \mid s_0, a_0] + \neg d \gamma \max_{a \in A} Q(s_1, a, \theta_\pi, \theta_Q) \right) \right)^2 \right]$$

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Q function depends on θ_π, but reward does not!

Do we know $\arg\max_{a \in A} Q(s_1, a, \theta_\pi, \theta_Q)$?

Experience Replay

What about TD return?

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Question: On-policy or off-policy? **Answer:** Off-policy. Why?

Q function depends on θ_π, but reward does not!

Do we know $\arg\max_{a \in A} Q(s_1, a, \theta_\pi, \theta_Q)$? Yes! Just plug in s_1

Experience Replay

To summarize:

Experience Replay

To summarize:

Monte Carlo Q learning is on-policy

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Cannot reuse data, takes a long time to train

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Temporal Difference Q learning is special!

Experience Replay

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It is off-policy, can reuse data and train faster

Experience Replay

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MC needs more training data, but TD has harder optimization

Target Networks

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Target Networks

Usually, the target parameters are older parameters

```
theta_Q = ... # Initialize parameters
theta_T = theta_Q.copy()

for epoch in range(num_epochs):
  grad = grad(L)(theta_Q, theta_T, X)
  theta_Q = optimizer.update(theta_Q, grad)
  if epoch % 200 == 0:
    # Update target parameters
    theta_T = theta_Q.copy()
```

Deep Q Networks

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[1] Human-level control through deep reinforcement learning. *Nature.* 2014.

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You have all the tools you need to implement DQN, except for one

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$$a = \arg \max_i \begin{bmatrix} Q(s, a = 1, \theta_\pi, \theta_Q) \\ Q(s, a = 2, \theta_\pi, \theta_Q) \\ \vdots \\ Q(s, a = i, \theta_\pi, \theta_Q) \end{bmatrix}$$

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For each action, we must execute Q network $|A|$ times. Not efficient!

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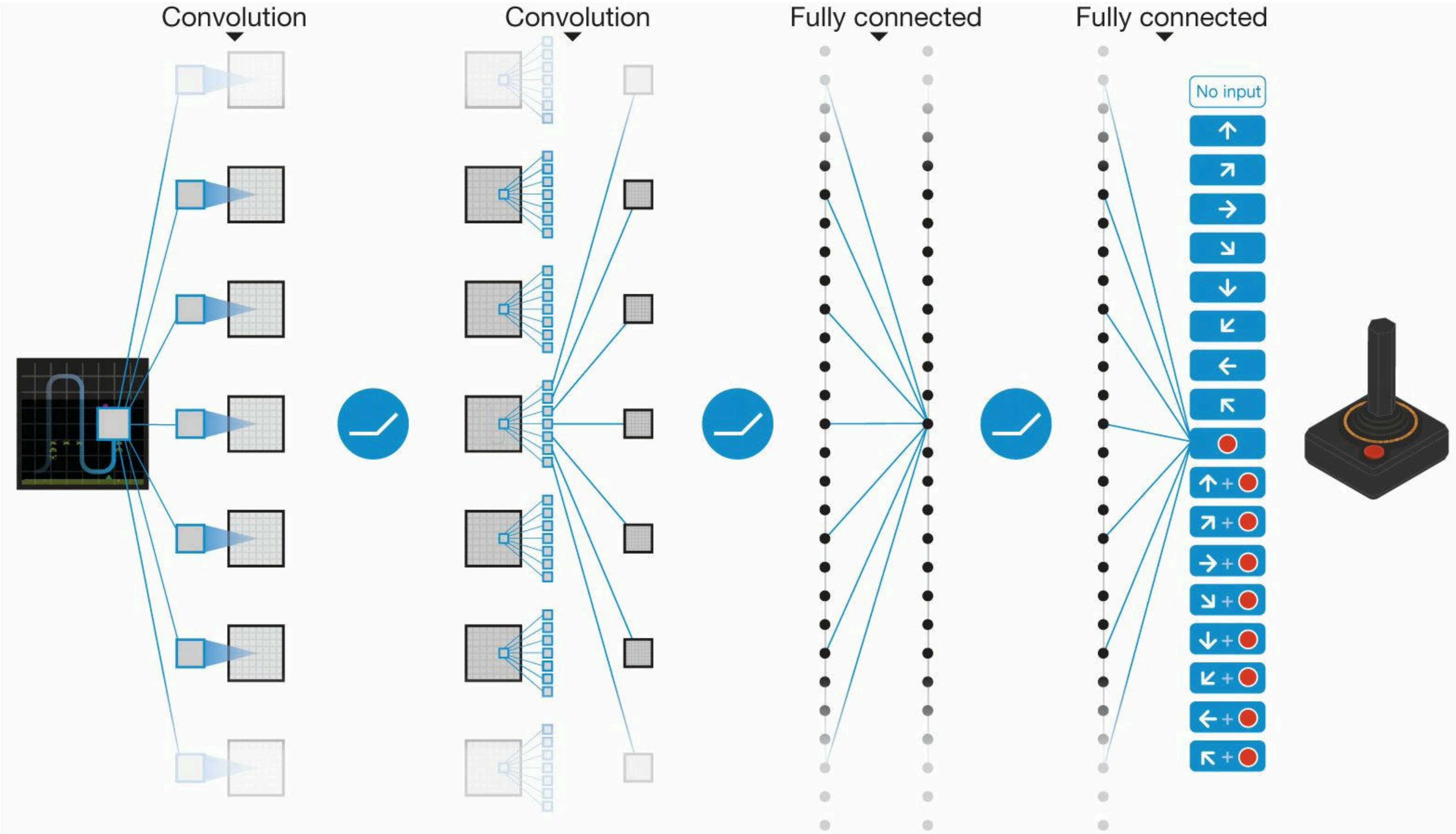
This is $|A|$ times faster!

Deep Q Networks



Deep Q Networks

```python
Q = nn.Sequential([...])
theta_T = partition(Q, is_array)[0]
replay_buffer = deque(maxsize=50_000)
for epoch in range(num_epochs):
  while not terminated:
    a = random_action if epoch < k else epsilon_greedy(Q)
    s, r, d, next_s = env.step(a)
    replay_buffer.insert((s, a, r, d, next_s))
    X = random.sample(replay_buffer, batch_size)
    theta_Q, model = eqx.partition(Q, is_array)
    theta_Q = td_update(theta_Q, theta_T, Q, X)
    theta_T = copy(theta_Q) if epoch % j == 0 else theta_T
    Q = eqx.combine(theta_Q, model)
```

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Finally, let us look at some successes of deep Q learning

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Pokemon https://youtu.be/DcYLT37ImBY?si=AeR2WkQg4X-tWa5v