Bandits

CISC 7404 - Decision Making

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Review

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In this course, we will focus primarily on reinforcement learning

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But reinforcement learning is a method, not a problem

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But reinforcement learning is a method, not a problem

The problem is **decision making**

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We will focus on **optimal** decision making

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Make the best possible decision, given the information we have

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- Best possible lawyer (what to argue?)

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With an optimal decision making machine, you can create:
- Best possible doctor (which medicine to give?)
- Best possible lawyer (what to argue?)
- Best possible scientist (what to research?)

Set Notation

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Let us review some notation I will use in the course

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If you ever get confused, come back to these slides

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Vectors

$$\boldsymbol{x} = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}$$

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$$\boldsymbol{x} = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}$$

Matrices

$$\boldsymbol{X} = \begin{bmatrix} x_{1,1} & x_{1,2} & \cdots & x_{1,n} \\ x_{2,1} & x_{2,2} & \cdots & x_{2,n} \\ \vdots & \vdots & \ddots & \vdots \\ x_{m,1} & x_{m,2} & \cdots & x_{m,n} \end{bmatrix}$$

Set Notation

We will represent **tensors** as nested vectors or matrices

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Each $\boldsymbol{x}_i$ is a vector

Set Notation

Same for matrices

Tensor of matrices

$$\boldsymbol{X} = \begin{bmatrix} \boldsymbol{x}_{1,1} & \boldsymbol{x}_{1,2} & \dots & \boldsymbol{x}_{1,n} \\ \boldsymbol{x}_{2,1} & \boldsymbol{x}_{2,2} & \dots & \boldsymbol{x}_{2,n} \\ \vdots & \vdots & \ddots & \vdots \\ \boldsymbol{x}_{m,1} & \boldsymbol{x}_{m,2} & \dots & \boldsymbol{x}_{m,n} \end{bmatrix}$$

Set Notation

Question: What is the difference between the following?

$$\boldsymbol{X} = \begin{bmatrix} x_{1,1} & x_{1,2} & \cdots & x_{1,n} \\ x_{2,1} & x_{2,2} & \cdots & x_{2,n} \\ \vdots & \vdots & \ddots & \vdots \\ x_{m,1} & x_{m,2} & \cdots & x_{m,n} \end{bmatrix}$$

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$\mathbb{R}$	Set of all real numbers $\{1, 2.03, \pi, ...\}$

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$\mathbb{Z}$	Set of all integers $\{-2, -1, 0, 1, 2, ...\}$

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$\mathbb{Z}$	Set of all integers $\{-2, -1, 0, 1, 2, \ldots\}$
$\mathbb{Z}_+$	Set of all **positive** integers $\{1, 2, \ldots\}$

Set Notation

$$[0, 1]$$

Closed interval

$0.0, 0.01, 0.00...1, 0.99, 1.0$

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$[0, 1]$ Closed interval

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$\{0, 1\}^{k \times k}$	A matrix of boolean values of shape k by k

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$$A \subseteq B \qquad A \text{ is a subset of } B$$

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Function Domain

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We will often use **set builder** notation

$$\{\underbrace{x + 1}_{\text{Function}} \mid \underbrace{x \in Z}_{\text{Domain}}\}$$

You can think of this as a for loop

```
output = {} # Set
for x in Z:
  output.insert(x + 1)
```

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```
output = {x + 1 for x in Z}
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Question: What functions could f be?

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I will define variables when possible

$$X \in \mathbb{R}^n; \Theta \in \mathbb{R}^{m \times n}; Y \in [0, 1]^{n \times m}$$

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Question: Any examples?

$$\frac{\mathrm{d}}{\mathrm{d}x} : \underbrace{(f : \mathbb{R} \mapsto \mathbb{R})}_{\text{Input function}} \mapsto \underbrace{(f' : \mathbb{R} \mapsto \mathbb{R})}_{\text{Output function}}$$

$$\frac{\mathrm{d}}{\mathrm{d}x} x^2 = 2x$$

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The max function returns the maximum of a function over a domain

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We want to make optimal decisions, so we will often take the minimum or maximum of functions

Exercises

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$$f(x) = -(x+1)^2$$

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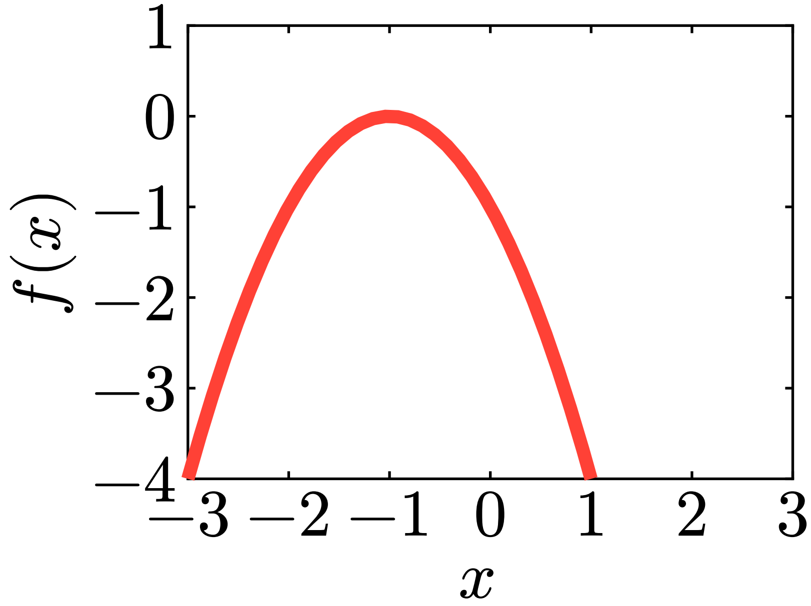

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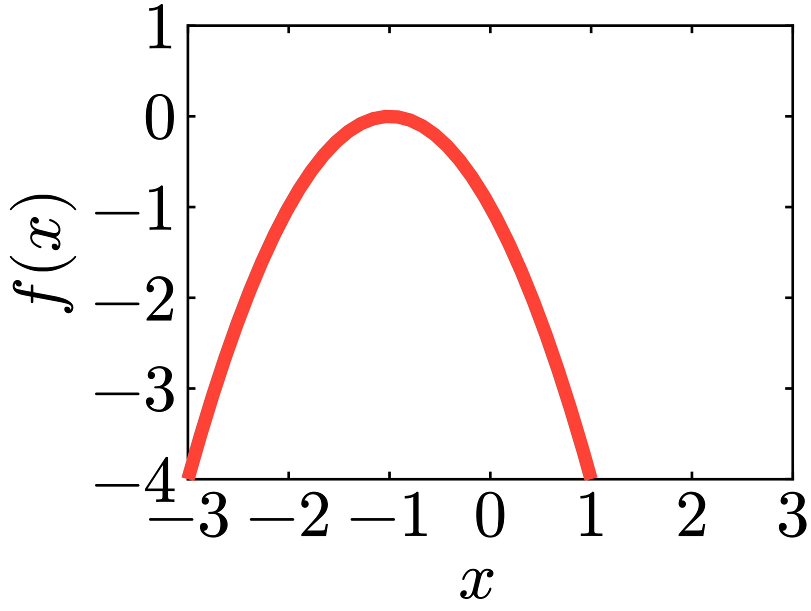

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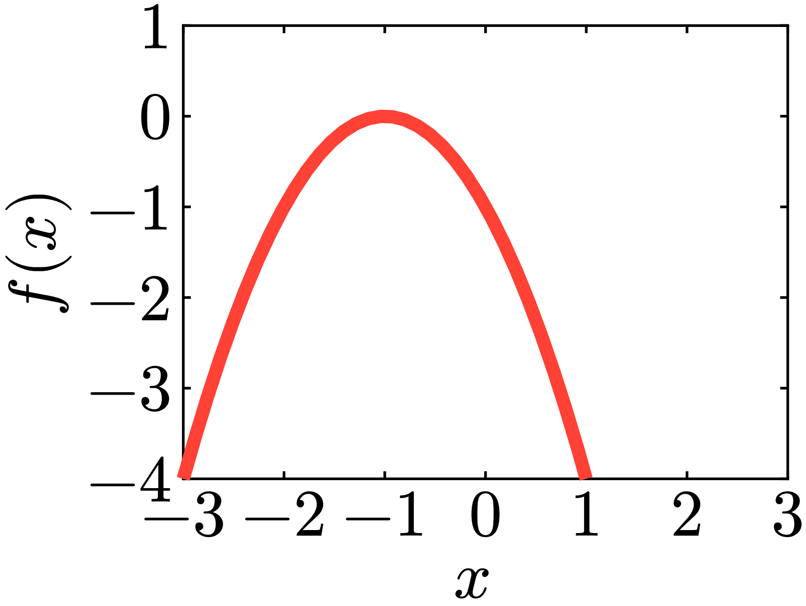

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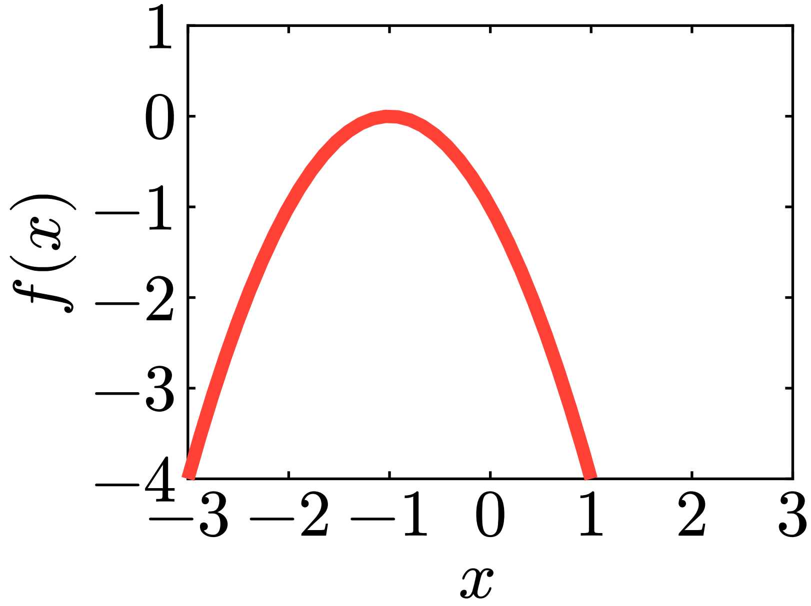

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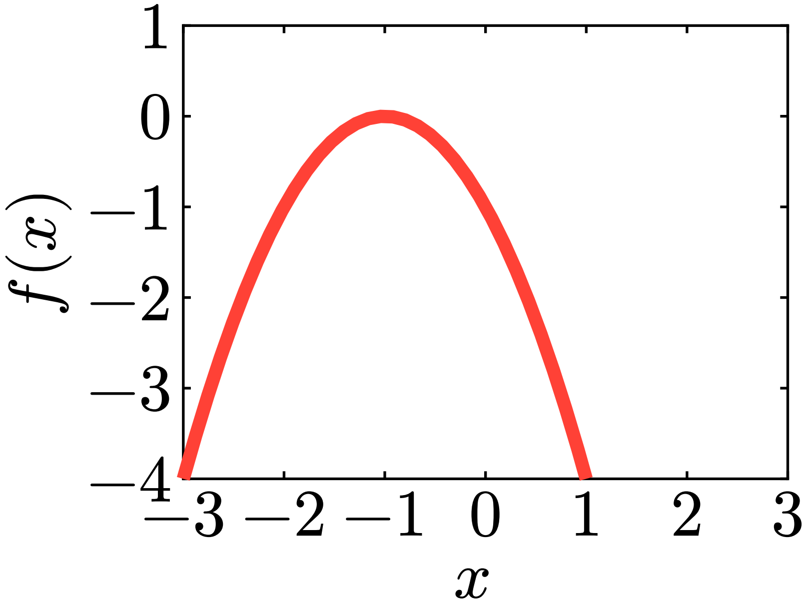

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$$0$$

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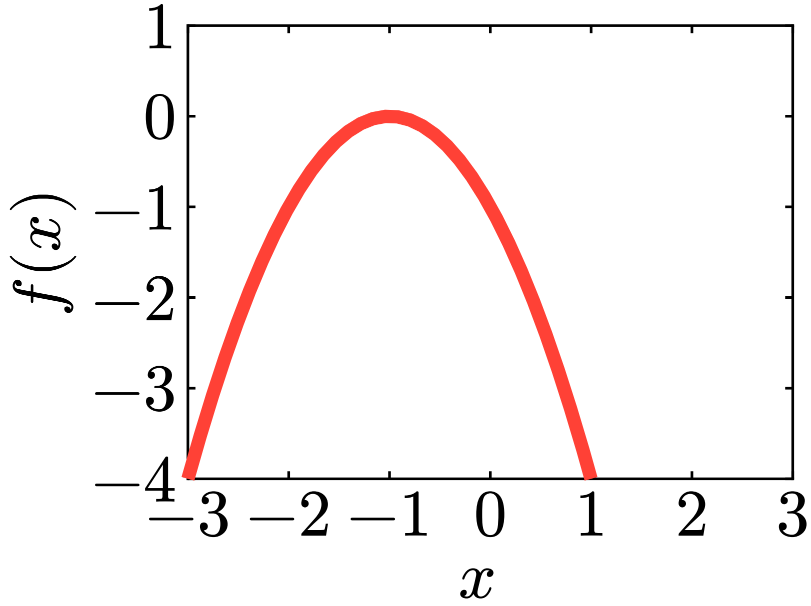

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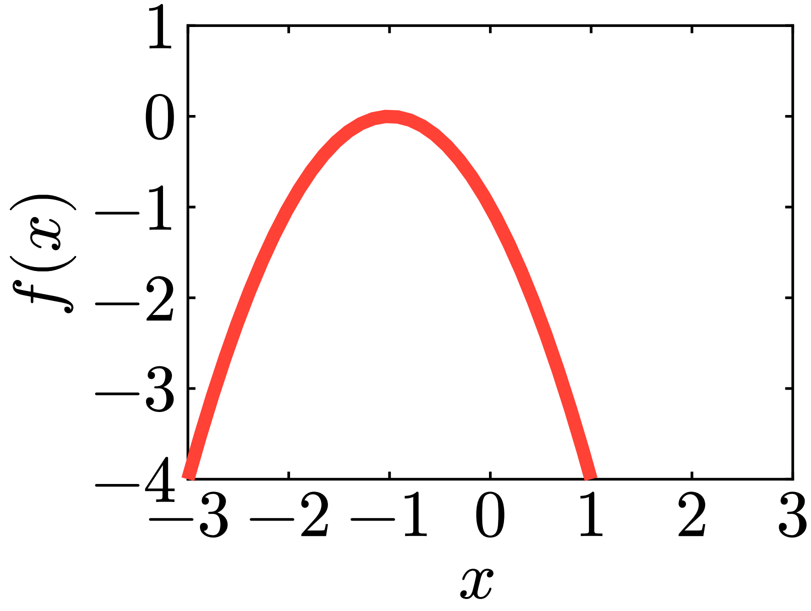

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$$\left\{x^{\frac{1}{2}} \mid x \in \mathbb{R}_+\right\}$$

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Question: What is this?

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Exercises

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Answer:

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Exercises

$$\left\{x^{\frac{1}{2}} \mid x \in \mathbb{R}_{+}\right\}$$

Question: What is this?

Answer:

- An infinitely large set
- The results of evaluating $f(x) = \sqrt{x}$ for all positive real numbers

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Today's lecture will be difficult

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If you can understand it, then reinforcement learning will be easy

Bandits

Bandits are the simplest decision making problem

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Question: What is a bandit?

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A bandit steals your money

Bandits

Here is the bandit we will focus on in this course

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This is a **one-armed** bandit

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Question: How does a one-armed bandit steal your money?

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Answer: You win less money than you put in

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Your chance of winning is $\frac{1}{200}$

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Let us see if we can make money playing this game

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First, we should briefly review probability theory

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For our bandit, we have two possible outcomes

$$\Omega = \{\text{win}, \text{lose}\}$$

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$$E \subseteq \Omega$$

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An **event** is a set of outcomes

$$E \subseteq \Omega$$

$$E_{\text{win}} = \{\text{win}\}; \quad E_{\text{lose}} = \{\text{lose}\}; \quad E_{\text{any}} = \{\text{win}, \text{lose}\}$$

Bandits

We define the probabilites over the outcome and event spaces

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Event probabilities do not always sum to one

$$E_{\text{win}} = \{\text{win}\} \qquad \sum_{\varepsilon \in E} \Pr(\varepsilon) \leq 1$$

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A **random variable** $\mathcal{X}$ maps an outcome to a real number

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Our bandit has two outcomes, lose (-10) or win (1000)

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We can combine probabilities and random variables to find out

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Question: What does $\mathbb{E}[\mathcal{X}] = -4.95$ mean?

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Negative reward means we lose money

Bandits

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Bandits

$$r_1 = -10$$
$$r_2 = -10$$
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If play the game more, the mean reward converges to the expectation

$$\lim_{n \to \infty} \sum_{t=1}^{n} r_t = n \cdot \mathbb{E}[\mathcal{X}] = -4.95n$$

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The more you play, the closer you get to $n \cdot \mathbb{E}[\mathcal{X}]$

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Question: Could a gambler find out $\mathbb{E}[\mathcal{X}]$?

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Gambler only has access to the rewards

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After playing enough, the gambler can approximate the expectation!

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Exercise: You start a new casino in Macau.

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- Near zero: The gambler wins sometimes and will continue to play

Multiarmed Bandits

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The bandit problem is useful for casino owners and gamblers

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We will make the problem more interesting

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You arrive at the Londoner with 1000 MOP and want to win money

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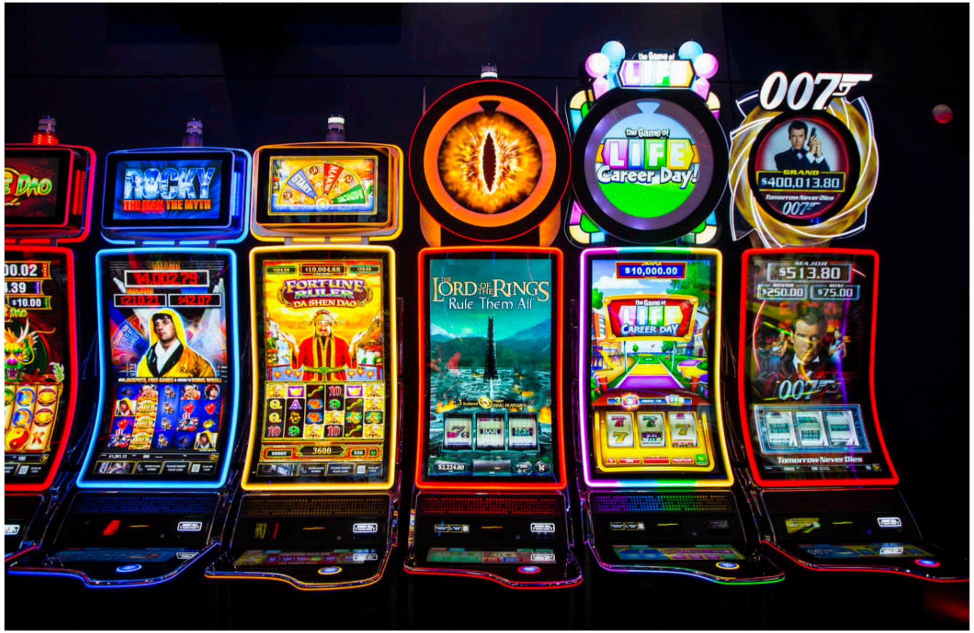

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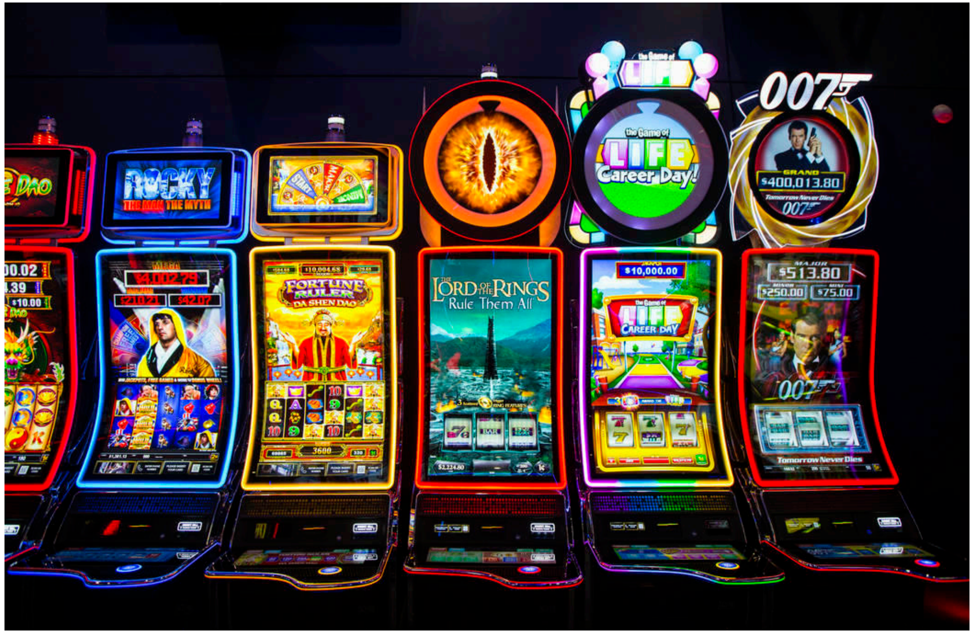

Question: Which machine do you play?

Multiarmed Bandits

We call this the **multi-armed bandit** problem

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You don't know the expected value of each arm. Which should you pull?

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We can find the best medicine while healing the most people

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TikTok select videos to maximize your $\mathbb{E}[$❤️$]$

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Which actions should you take to make the most money?

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- Pick a to estimate bandits $\quad \mathbb{E}[\mathcal{X}_a \mid a \in 1...k]$
- Pick a to make the most money $\quad \underset{a \in \{1...k\}}{\arg\max}\, \mathbb{E}[\mathcal{X}_a]$

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Exploration:

$$\mathbb{E}[\mathcal{X}_a \mid a \in \{1...k\}]$$

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Explore our options to improve our estimate of each random variable

Multiarmed Bandits

We have names for each goal

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It is important to understand the difference between exploration and exploitation! Any questions?

Multiarmed Bandits

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Question: How can we achieve both goals at once?

Answer: Sometimes choose a to explore, sometimes choose a to exploit

Multiarmed Bandits

$$u \sim \mathrm{uniform}([0, 1])$$

$$\text{if } u < 0.5 \text{ then } a \sim \mathrm{uniform}(\{1...k\})$$

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$\varepsilon \approx 0$ when we trust our estimates of $\mathbb{E}[\mathcal{X}]$

$\varepsilon \approx 1$ when we do not trust our estimates of $\mathbb{E}[\mathcal{X}]$

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- Sometimes it suggests study videos, to understand if you like study videos more

Questions?

Coding

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Let us code some multiarmed bandits!

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https://colab.research.google.com/drive/1cyNLRa-J8oe7pgy_gs2mcypZPqqaquoa