Homework 1, MAT 258A

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Problem 9 — Disk Packing into a Square Box

(a) Non-Intersection Condition

We show that two circular disks of radii r_1, r_2 centered at x_1, $x_2 \in \mathbb{R}^2$ do not intersect if and only if

$$\|x_1 - x_2\|_2 \geq r_1 + r_2.$$

Proof. The Euclidean distance between centers must be at least the sum of the radii to avoid overlap. If the distance is less, their perimeters intersect.

(b) Optimization Model Formulation

Let m sphere, each of fixed radius r_i, be placed in a box centered at the origin, with variable half-length $R > 0$. Let (x_i, y_i) be the center of disk i.

Variables:

- $(x_i, y_i, ...)$ for $i = 1, \ldots, m$
- $R > 0$: half-size of the enclosing box (to minimize)

Constraints:

$$\begin{aligned} &\text{Containment:} && -R + r_i \leq x_i, y_i, ... \leq R - r_i \quad \forall i \\ &\text{Non-overlap:} && \|x_i - x_j\|_2^2 \geq (r_i + r_j)^2 \quad \forall i < j \end{aligned}$$

Objective:

$$\min R$$

This is a nonconvex quadratic optimization since some convex combinations of optimal solutions will result in overlapping disks.

(c) Code Implementation and Results

We used `gurobipy` with the `NonConvex` flag to solve this model for $\mathbb{R}^2$, $m = 5$ and $m = 6$, with $r = 1$. The optimal R values obtained are:

- $m = 5$: $R^* = 2.414$
- $m = 6$: $R^* = 2.664$

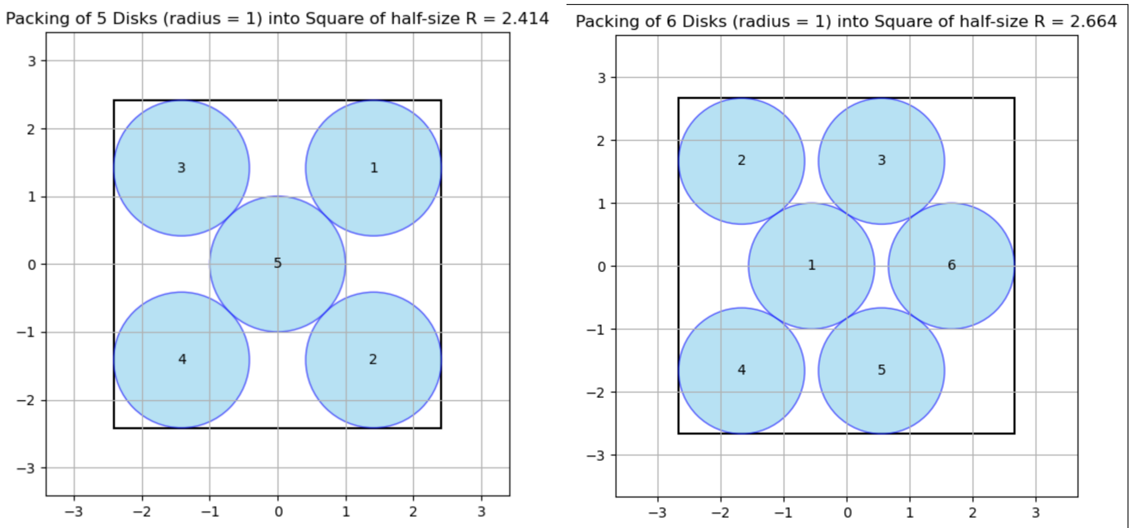

Figure 1: Optimal disk packings for $m = 5$ (left) and $m = 6$ (right) with $r = 1$

(d) Uniqueness Discussion

Due to the non-convex nature of the problem, multiple local minima may exist. The solution is not necessarily unique. Different initial guesses or solver tolerances may lead to different (but equally optimal) configurations.

Code, plots, and full writeup available at: `https://github.com/smoralesduarte/mat258a`