# Homework 1, MAT 258A

Santiago Morales
UC Davis
moralesduarte@ucdavis.edu

April 14, 2025

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## Problem 9 — Disk Packing into a Square Box

### (a) Non-Intersection Condition

We show that two circular disks of radii $r_1$, $r_2$ centered at $x_1$, $x_2 \in \mathbb{R}^2$ do not intersect if and only if

$$\|x_1 - x_2\|_2 \geq r_1 + r_2.$$

*Proof.* The Euclidean distance between centers must be at least the sum of the radii to avoid overlap. If the distance is less, their perimeters intersect.

### (b) Optimization Model Formulation

Let $m$ disks of fixed radius $r$ be placed in a square centered at the origin, with variable half-length $R > 0$. Let $(x_i, y_i)$ be the center of disk $i$.

**Variables:**

- $(x_i, y_i)$ for $i = 1, \dots, m$
- $R > 0$: half-size of the enclosing square (to minimize)

**Constraints:**

$$\begin{aligned} &\text{Containment:} && -R + r \leq x_i, y_i \leq R - r \quad \forall i \\ &\text{Non-overlap:} && \|x_i - x_j\|_2^2 \geq (2r)^2 \quad \forall i < j \end{aligned}$$

**Objective:**

$$\min R$$

This is a nonconvex quadratic optimization since some convex combinations of optimal solutions will result in overlapping disks.

### (c) Code Implementation and Results

We used `gurobipy` with the `NonConvex` flag to solve this model for $m = 5$ and $m = 6$, with $r = 1$. The optimal $R$ values obtained are:

- $m = 5$: $R^* = 2.414$
- $m = 6$: $R^* = 2.664$

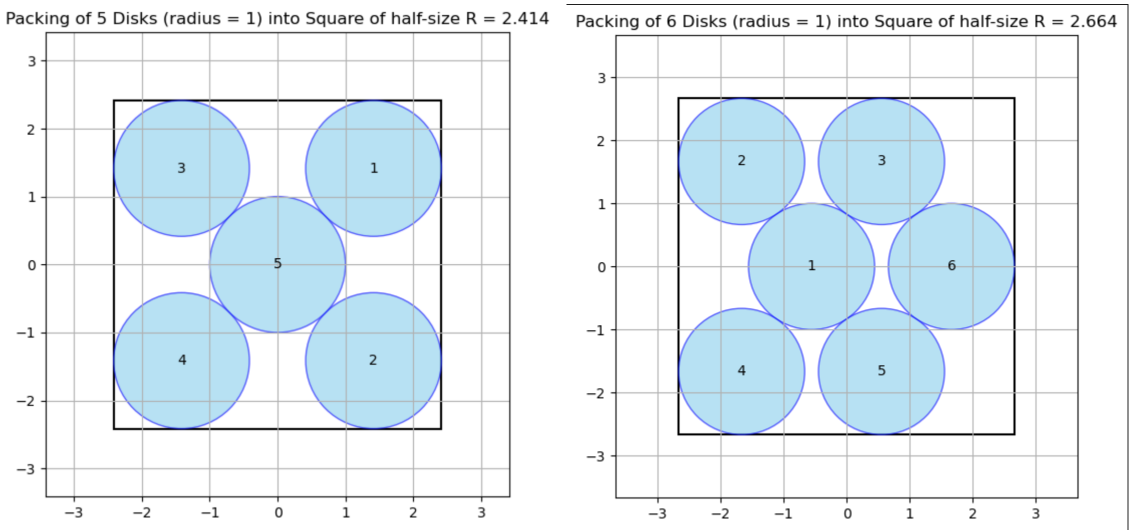


Figure 1: Optimal disk packings for $m = 5$ (left) and $m = 6$ (right) with $r = 1$

## (d) Uniqueness Discussion

Due to the non-convex nature of the problem, multiple local minima may exist. The solution is not necessarily unique. Different initial guesses or solver tolerances may lead to different (but equally optimal) configurations.

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Code, plots, and full writeup available at: `github.com/smorales/math258a-hw1`