Nori construction

Sophie Morel

January 2, 2026

The goal of this project is to prove Nori's factorization theorem.

Let Q be a quiver, $\mathcal{A}$ be an abelian category and $T : Q \to \mathcal{A}$ be a morphism of quivers.

Definition 1. A *factorization* of T is the data of an abelian category $\mathcal{B}$, a morphism of quivers $U :: Q \to \mathcal{B}$, an exact functor $R : \mathcal{B} \to \mathcal{A}$ and an isomorphism $\alpha : R \circ U \xrightarrow{\sim} T$.

If $(\mathcal{B}_1, U_1, R_1, \alpha_1)$ and $(\mathcal{B}_2, U_2, R_2, \alpha_2)$ are factorizations of T, a *morphism of factorizations* is the data of an exact functor $F : \mathcal{B}_1 \to \mathcal{B}_2$ and isomorphisms $\beta : F \circ U_1 \xrightarrow{\sim} U_2$ and $\gamma : R_2 \circ F \xrightarrow{\sim} R_1$ such that the two isomorphisms $R_2 \circ F \circ U_1 \xrightarrow{\gamma} R_1 \circ U_1 \xrightarrow{\alpha_1} T$ and $R_2 \circ F \circ U_1 \xrightarrow{\beta} R_2 \circ U_2 \xrightarrow{\alpha_2} T$ are equal.

Lemma 2. *Factorizations of T form a category.*

Definition 3. A *universal factorization* of T is an initial object of the category of factorizations of T.

Theorem 4 (Nori)**.** *There exists a universal factorization of T.*

There are several proofs of this theorem, and the one we give is due to Barbieri-Viale and Prest. It uses the following result, which follows from the existence of Freyd's free abelian category on a (pre)additive category.

Theorem 5. *Let Q be a quiver. Then there exists an abelian category $\mathcal{A}$ and a morphism of quivers $T : Q \to \mathcal{A}$ such that, for every abelian category $\mathcal{B}$, the functor $F \mapsto F \circ T$ is an equivalence from the category of exact functors $\mathcal{A} \to \mathcal{B}$ to the category of morphisms of quivers $Q \to \mathcal{B}$.*