Online Appendixes

A Estimation in the presence of false positive reports

In the main text, we follow all previous scale-up studies to date in assuming that there are never any false positive reports. In this appendix, we generalize our analysis to the situation where false positive reports are possible.

In Section 2, Equation 5, we discussed false positive reports in terms of in-reports: we explained that if there are no false positive reports, then $v_{i,F} = 0$ for all $i \notin H$. In this appendix, we will re-orient the analysis and focus on how false positives affect out-reports. Each individual i's out-reports can be divided into two groups: true positives, which actually connect to the hidden population ($y^{+}_{i,H}$); and false positives, which do not connect to the hidden population ($y^{-}_{i,H}$). Therefore,

$$y_{i,H} = y^{+}_{i,H} + y^{-}_{i,H}. \tag{A.1}$$

We can also define the aggregate quantities $y^{+}_{F,H} = \sum_{i \in F} y^{+}_{i,H}$ and $y^{-}_{F,H} = \sum_{i \in F} y^{-}_{i,H}$, so that

$$y_{F,H} = y^{+}_{F,H} + y^{-}_{F,H}. \tag{A.2}$$

Because the total number of true-positive out-reports must equal the total number of true-positive in-reports, it is the case that

$$y^{+}_{F,H} = v_{H,F} \tag{A.3}$$

where $y^{+}_{F,H}$ is the total number of true-positive out-reports and $v_{H,F}$ is the total number of true positive in-reports. Dividing both sides by $v_{H,F}$, and then multiplying both sides by N_H produces

$$N_H = \frac{y^+_{F,H}}{\bar{v}_{H,F}}. \tag{A.4}$$

In the main text, we introduce a strategy for estimating $\bar{v}_{H,F}$. If there was also a strategy for estimating $y^+_{F,H}$, then we could use Equation A.4 to estimate N_H, even if some reports are false positives. Unfortunately, we cannot typically estimate $y^+_{F,H}$ directly from F, since any attempt to do so would learn about $y_{F,H}$ instead. Therefore, we propose that researchers collect information about $y_{F,H}$ and then estimate an adjustment factor that relates $y_{F,H}$ to $y^+_{F,H}$. This approach leads us to introduce a new quantity called the *precision of out-reports*, η_F:

$$\eta_F = \frac{y^+_{F,H}}{y_{F,H}}. \tag{A.5}$$

The precision is useful because it relates the observed out-reports, $y_{F,H}$ to the true positive out-reports, $y^+_{F,H}$. It varies from 0, when none of the out-reports are true positives, to 1, when the out-reports are perfect. The precision allows us to derive an identity that relates out-reports to N_H:

$$N_H = \frac{\eta_F \; y_{F,H}}{\bar{v}_{H,F}}. \tag{A.6}$$

Equation A.6 then suggests the estimator:

$$\widehat{N}_H = \frac{\widehat{\eta}_F \; \widehat{y}_{F,H}}{\widehat{\bar{v}}_{H,F}}. \tag{A.7}$$

If we could find a consistent and essentially unbiased estimator for η_F, then we could use Equation A.7 to form a consistent and essentially unbiased estimator for N_H, even in the presence of false positive reports.

Unfortunately, we are not aware of a practical strategy for estimating the precision of out-reports. The most direct approach would be to interview each alter that a respondent reports as being in the hidden population. In other words, if a respondent reports knowing 3 drug injectors, researchers could try to interview these three people and see if they are actually drug injectors. Killworth et al. (2006) attempted a version of this procedure, which they called an "alter-chasing" study, but they later abandoned it because of the numerous logistical challenges that arose; see also Laumann (1969) for a related attempt. A second possible approach would be to conduct a census of a networked population where respondents are asked about themselves and specific people to whom they are connected. For example, Goel et al. (2010) collected responses about the political attitudes of thousands of interconnected people on Facebook, including respondents' attitudes as well as their beliefs about specific alters' attitudes. For a subset of respondents, they could compare i's belief about j's attitude with j's report of her own attitude in order to measure the precision. Unfortunately, we think it would be difficult to include a sufficiently large number of members of a stigmatized hidden population in this type of study.

We expect that the measurement of the precision of out-reports will pose a major challenge for future scale-up research, and we hope that practical solutions to this problem can be found. For the time being, we recommend that researchers show the impact that different values of the precision of out-reports would have on size estimates (Equation A.7).