University of Waterloo
Algebraic Geometry - Summer 2015
Assignment 3

Sina Motevalli 20455091

Problem

Part a

Part b

Part c

Problem 2

We have $\Gamma(\mathbb{A}^1) = k[t]$ and $\Gamma(X) = k[x,y]/ < y^2 - x^3 >= k[\bar{x},\bar{y}]$ with $\bar{x}^2 = \bar{y}^3$.
Suppose $\varphi : k[\bar{x},\bar{y}] \to k[t]$ is an isomorphism.

Problem 3

Part a

Note that ϕ^* sends $g + I(Y)$ to $g \circ \phi + I(X)$.
So ϕ^* is injective means $g \in I(Y) \iff g \circ \phi \in I(X)$.
Since $g \circ \phi \in I(X) \iff g \in I(\phi(X))$, we have that ϕ^* is injective if and only if

$$I(Y) = I(\phi(X))$$

which is equivalent to image of ϕ under X being dense in Y.

Part b

Assume ϕ has a polynomial left-inverse ψ.
Let $p + I(X)$ be an arbitrary element of the coordinate ring of X.
Note that $p \circ \psi \in k[y_1, ..., y_m]$.
We have $\phi^*(p \circ \psi + I(Y)) = p \circ \psi \circ \phi + I(x) = p(\psi \circ \phi) + I(X) = p + I(X)$.
Thus ϕ^* is surjective.
Conversely assume ϕ^* is surjective.
Since ϕ^* is surjective, there exist $g_i \in k[y_1, ..., y_m]$ such that $\phi(g_i + I(Y)) = x_i + I(X)$ for every $i \in \{1, 2, ..., n\}$.
So $(g_i \circ \phi) + I(X) = x_i + I(X) \to (g_i \circ \phi)(x) = x_i(x)\ \forall x \in X$.
Let $\psi = (g_1, g_2, ..., g_n)$, then we clearly have $\psi \circ \phi = id_X$.

Problem 4