University of Waterloo
Algebraic Geometry - Summer 2015
Assignment 2

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Problem 1

Part a

Let X be an irreducible algebraic set. Let $U \subset X$ be Zariski open. Let V be the closure of U. Assume for a contradiction that $X \neq V$ (ie U is not dense). Then we have: $X = V \cup U^c$, but as U is open U^c is closed and V is also closed, Thus X is reducible. Contradiction.

Part b

Consider the reducible algebraic set $X =< x^2 - 1 >= \{1, -1\}$ in $\mathbb{R}^2$. Then in the induced Zariski topology the set $\{1\}$ is both open and closed and therefore is not dense in X.

Problem 2

Problem 3

Problem 1 Part a

(i)

(ii)

(iii)

Part b

Problem 4

Part a

Let $V = V_1 \cup V_2 \cup .... \cup V_n$ and $W = W_1 \cup W_2 \cup ... \cup W_m$ be (unique) decompositions of V and W. We have $V_i \in W_1 \cup W_2 \cup ... \cup W_m$. Since V_i is irreducible and all W_j's are irreducible we have $V_i \subset w_{J_0}$ for some J_0.

Part b

Let $X = X_1 \cup X_2 \cup ... \cup X_n$ be the decomposition of the algebraic set X into irreducible components. Assume $X_i \subset \bigcup_{j \neq i} X_j$. So $X_i \subset X_{j_0}$ for some $j_0 \neq i$, this means that X_i is redundant and should not have existed in the decomposition which is a contradiction.