Object tracking assignment

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1 Introduction

This report describes an assignment to derive and implement an extended Kalman filter (EKF) for a given problem, to generate data from a true trajectory, and to evaluate the implemented algorithm.

2 Problem formulation

Suppose that you are on a boat equipped with a sonar and that you want to track an object that is passing by under the surface. The sonar that you have at your disposal measures fairly accurately in range (standard deviation ≈ 0.1 m), but poorly in azimuth (standard deviation ≈ 3 degrees).

Assume that the motion of the submarine vessel can be described by a constant-velocity (CV) model. The state vector for that model, at time k, is

$$\mathbf{x}_k = \begin{bmatrix} x_k \\ y_k \\ v_x \\ v_y \end{bmatrix}, \tag{1}$$

where x and y are Cartesian position states, and v_x and v_y corresponding velocities. The CV motion model is given by

$$\mathbf{x}_k = \begin{bmatrix} 1 & 0 & T_s & 0 \\ 0 & 1 & 0 & T_s \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \mathbf{x}_{k-1} + v_{k-1}, \tag{2}$$

where T_s is the sample interval and v_{k-1} is white Gaussian velocity noise with covariance matrix

$$\mathbf{Q} = q \cdot \begin{bmatrix} \frac{T_s^3}{3} & 0 & \frac{T_s^2}{2} & 0 \\ 0 & \frac{T_s^3}{3} & 0 & \frac{T_s^2}{2} \\ \frac{T_s^2}{2} & 0 & T_s & 0 \\ 0 & \frac{T_s^2}{2} & 0 & T_s \end{bmatrix}, \tag{3}$$

with scaling parameter $q \geq 0$.

Also the sonar measurements of range and angle can be assumed Gaussian with covariance matrix

$$\mathbf{R} = \begin{bmatrix} \sigma_r^2 & 0 \\ 0 & \sigma_\theta^2 \end{bmatrix}. \tag{4}$$

Here, r is range and θ azimuth angle.

At your disposal you have the true sequence of the object, given by the file *object_measurement.m*. The trajectory includes the x and y positions of the object, for each time step from 1 to 500, in increments of $T_s = 0.1$ s.

Assignments:

1. Write the prediction and measurement update steps (equations) of the problem in an extended Kalman filter (EKF) setting, including all necessary matrices.

2. Generate 100 random measurement sequences from the entire trajectory. A sonar measurement includes range and angle to the object plus additive Gaussian noise with the above-stated standard deviations.

3. Implement an EKF for the problem, with tuneable process noise parameter q. *Hint:* Let the initial state vector be the true x and y positions and set the initial velocities to 0.

4. For three choices of q, run a Monte-Carlo evaluation with at least 100 samples of the EKF and plot the root mean squared error in x and y position as well as x and y velocity.

5. What can be observed from the previous exercise? How does the process noise affect the performance?

The solutions to the assignments shall be handed in as a report, including figures as well as source code. The implementation should either be done in Matlab or Python.