The ROC Curve for a Binary Classifier

Confusion Matrix

A binary classifier can make two types of errors: positive cases that were incorrectly identified as negative and negative cases that were incorrectly identified as positive. Performance of such classifier is evaluated using the *confusion matrix*

$$
\begin{array}{ccc}
\multicolumn{3}{c}{Actual/Predicted} \\
 & No & Yes \\
No & a & b \\
Yes & c & d
\end{array}
$$

Accuracy

- a, d are the numbers of *correct* predictions of negative/positive examples
- b, c are the numbers of *incorrect* predictions of negative/positive examples
- The *accuracy* is the proportion of the predictions that are correct

$$\frac{a+d}{a+b+c+d}$$

True Positive Rate, False Positive Rate

- True positive rate is a proportion of positive cases that were correctly identify
$$TP = \frac{d}{d + c}$$
- False positive rate is a proportion of negative cases that were incorrectly identify
$$FP = \frac{b}{a + b}$$
- True negative rate is a proportion of negative cases that were correctly identify
$$TN = \frac{a}{a + b}$$
- False negative rate is a proportion of positive cases that were incorrectly identify
$$FN = \frac{c}{d + c}$$
- $Sensitivity = TP, Specificity = TN = 1 - FP$

The ROC

- The ROC curve is a graphic for displaying the two types of errors for all possible *thresholds*

$$Pr(Y = \text{"}Yes\text{"} | X = x) > threshold$$

- *Threshold* is the cutoff imposed on the predicted probabilities for assigning observation to each class

The ROC, AUC

The overall performance of a classifier is given by the area under the ROC curve (AUC). An ideal ROC curve will hug the top left corner, so the larger the AUC the better classifier.