

# ACCURATE MODELING OF MICROCAVITY LASERS WITH SYMMETRY LINES BASED ON MULLER'S INTEGRAL EQUATIONS

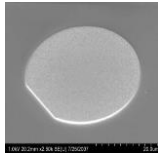
Microcavities with a symmetry line (depart from a circle but not too far):

- ✓To keep low thresholds
- ✓To improve directionality

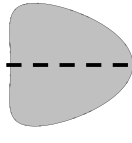
limaçon cut disk kite



H.Cao



R. Dubertrand et al.



## Lasing Eigenvalue problem

in 2-D:  $U = E_z$  or  $H_z$ ,  $U \neq 0$

- Helmholtz equation off the boundaries

$$(\Delta + k^2 n^2)U(r, \varphi) = 0$$

$$\sqrt{\epsilon} = v = \alpha_{eff} - i\gamma$$

$\alpha_{eff}$  is effective refractive index

$\gamma$  is threshold material gain

- Boundary conditions

- Sommerfeld radiation condition at infinity

The real-valued  $(\kappa_j, \gamma_j)$  are the eigenparameters

## The Muller Integral Equations

$$\text{MIEs: } \oint_{\Gamma} A(\vec{r}, \vec{r}') \varphi(\vec{r}') d\Gamma' - \oint_{\Gamma} B(\vec{r}, \vec{r}') \psi(\vec{r}') d\Gamma' = 0,$$

$$(\eta_1 + \eta_2) \psi(\vec{r}) / 2\eta_0 + \oint_{\Gamma} C(\vec{r}, \vec{r}') \varphi(\vec{r}') d\Gamma' - \oint_{\Gamma} D(\vec{r}, \vec{r}') \psi(\vec{r}') d\Gamma' = 0, \quad \vec{r} \in \Gamma,$$

Kernels:

$$A(\vec{r}, \vec{r}') = \partial G(\vec{r}, \vec{r}') / \partial n' - \partial G(\vec{r}', \vec{r}) / \partial n,$$

$$B(\vec{r}, \vec{r}') = G(\vec{r}, \vec{r}') - \eta_1 / \eta_0 G(\vec{r}', \vec{r})$$

$$C(\vec{r}, \vec{r}') = \partial^2 G(\vec{r}, \vec{r}') / \partial n \partial n' - \partial^2 G(\vec{r}', \vec{r}) / \partial n \partial n,$$

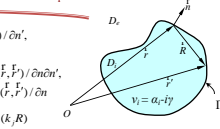
$$D(\vec{r}, \vec{r}') = \partial G(\vec{r}, \vec{r}') / \partial n - \eta_1 / \eta_0 \partial G(\vec{r}', \vec{r}) / \partial n$$

$$\text{Green's function: } G(\vec{r}, \vec{r}') = \frac{i}{4} H_0^{(1)}(kR)$$

- Integration over a closed contour

- Fredholm 2-nd kind equations

- Equivalency to original problem: no spurious eigenvalues



## Discretization of IEs

Extraction of logarithmic part from kernels:

$$F(t, \tau) = F_1(t, \tau) \ln[4 \sin^2((t - \tau)/2)] + F_2(t, \tau), \quad F = A, B, C, D,$$

For the logarithmic parts use the quadrature with trigonometric polynomials:

$$\int_0^{2\pi} \ln[4 \sin^2((t - \tau)/2)] F_1(t, \tau) f(\tau) L(\tau) d\tau = \sum_{p=0}^{2N-1} P_p^{(N)}(t) F_1(t, t_p) f(t_p) L(t_p)$$

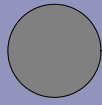
$$P_p^{(N)}(t) = -(2\pi/N) \sum_{m=1}^{N-1} \cos[m(t - t_p)] / [m - \pi \cos[N(t - t_p)]] / N^2$$

For regular parts use trapezoid formula:

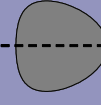
$$\int_0^{2\pi} F_2(\vec{r}, \vec{r}') f(\vec{r}') d\Gamma' = (\pi/N) \sum_{p=0}^{2N-1} F_2(t, t_p) f(t_p) L(t_p)$$

$$x(t) = a(\cos t + \delta \cos 2t - \delta),$$

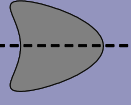
$$y(t) = a \sin t$$



$\delta = 0.0$

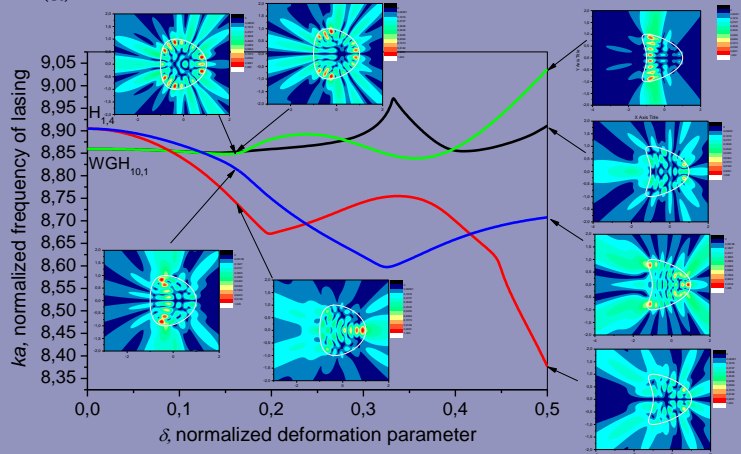


$\delta = 0.165$



$\delta = 0.5$

(a)



(b)

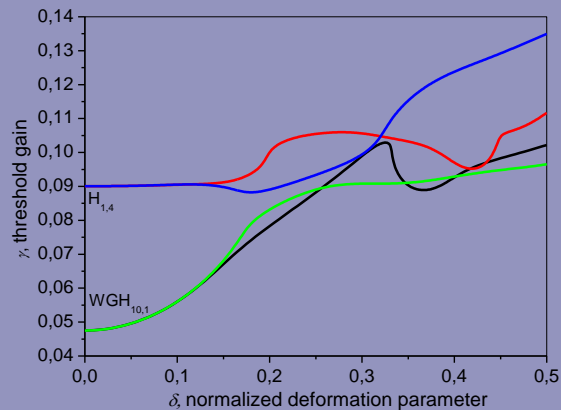


Fig. 1. Dependences of lasing frequencies (a) and threshold gains (b) on normalized deformation parameter for doublets of modes WGH<sub>10,1</sub> (black and green lines) and H<sub>1,4</sub> (blue and red lines). Inserts are modal field patterns at corresponding values of marked by arrows. Other parameters: N = 50, = 1.5.

## Conclusions

- By deforming the disk into a kite resonator one splits the modes into doublets of even and odd modes
- The directivity of emission of the modes increases however at the expense of higher thresholds
- WG modes keep their favorable features in a convex kite

**Elena I. Smotrova\*, Iryna Gozhyk\*\*, Melanie Lebental\*\*, Alexander I. Nosich\***

\*LMNO, Institute of Radiophysics and Electronics NASU, Kharkiv, Ukraine

\*\*LPQM, Ecole Normale Supérieure de Cachan CNRS, UMR 8537, Cachan, France