Nyström-method Analysis of Active Spiral Subwavelength 2-D Microresonators

E.I. Smotrova[1], A.I. Nosich[1]
P. Sewell[2], T. Benson[2], J. Ctyroky[3]

Lasing eigenvalue problem

Time dependence is assumed as $\sim e^{-ikct}, k = \omega / c$, $U = E_z$ or H_z

$$\left(\Delta + k_{i,j}^2\right) U_{i,e}(\vec{r}) = 0, \quad \vec{r} \in D_i \cup D_e$$

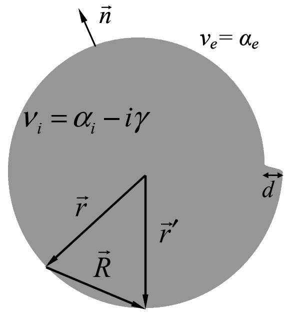
$$U_i\big|_\Gamma = U_e\big|_\Gamma; \quad \eta_i \left.\frac{\partial U_i}{\partial n}\right|_\Gamma = \eta_e \left.\frac{\partial U_e}{\partial n}\right|_\Gamma$$

U_e is satisfied Sommerfeld rad.cond. at ∞

$$k_{i,e} = k\nu_{i,e}; \quad \nu_i = \alpha_i - i\gamma; \quad \nu_e = \alpha_e$$

$$\eta_{i,e} = 1/\nu_{i,e}^2 \text{ (Hpol) or 1 (Epol)}$$

(k_s, γ_s) are eigenparameters



Muller's integral equations

$$\varphi(\vec{r}) - \int_\Gamma \varphi(\vec{r}') A(\vec{r}, \vec{r}') dl' + \int_\Gamma \psi(\vec{r}') B(\vec{r}, \vec{r}') dl' = 0$$

$$\frac{\eta_i + \eta_e}{2\eta_e} \psi(\vec{r}) - \int_\Gamma \varphi(\vec{r}') C(\vec{r}, \vec{r}') dl' + \int_\Gamma \psi(\vec{r}') D(\vec{r}, \vec{r}') dl' = 0$$

Here the unknown functions $\varphi(r)$ and $\psi(r)$ are the values of the field function and its normal derivative, respectively on the contour and kernel are given as

$$A(\vec{r}, \vec{r}') = \frac{\partial G_i(\vec{r}, \vec{r}')}{\partial n'} - \frac{\partial G_e(\vec{r}, \vec{r}')}{\partial n'}; \quad B(\vec{r}, \vec{r}') = G_i(\vec{r}, \vec{r}') - \frac{\eta_i}{\eta_e} G_e(\vec{r}, \vec{r}')$$

$$C(\vec{r}, \vec{r}') = \frac{\partial^2 G_i(\vec{r}, \vec{r}')}{\partial n \partial n'} - \frac{\partial^2 G_e(\vec{r}, \vec{r}')}{\partial n \partial n'}; \quad D(\vec{r}, \vec{r}') = \frac{\partial G_i(\vec{r}, \vec{r}')}{\partial n} - \frac{\eta_i}{\eta_e} \frac{\partial G_e(\vec{r}, \vec{r}')}{\partial n}$$

$G_j(\vec{r}, \vec{r}')$ is the 2-D Green's functions

Nyström method

Separating the logarithmic parts from kernels as follows:

$$F(\vec{r}, \vec{r}') = F_1(\vec{r}, \vec{r}') \ln\left[4\sin^2 \frac{t - \tau}{2}\right] + F_2(\vec{r}, \vec{r}'), \quad F = A, B, C, D$$

and functions are given by

$$A_1(\vec{r}, \vec{r}') = (-1/4\pi)\left[k_i J_1(k_i R) - k_e J_1(k_e R)\right](\vec{R} \cdot \vec{n}')/R$$

$$B_1(\vec{r}, \vec{r}') = (-1/4\pi)\left[J_0(k_i R) - (\eta_i/\eta_e) J_0(k_e R)\right]$$

$$C_1(\vec{r}, \vec{r}') = (1/4\pi)\left[k_i^2 J_2(k_i R) - k_e^2 J_2(k_e R)\right](\vec{R} \cdot \vec{n}')(\vec{R} \cdot \vec{n})/R^2 - (1/4\pi)\left[k_i J_1(k_i R) - k_e J_1(k_e R)\right](\vec{n}' \cdot \vec{n})/R$$

$$D_1(\vec{r}, \vec{r}') = (1/4\pi)\left[k_i J_1(k_i R) - (\eta_i/\eta_e) k_e J_1(k_e R)\right](\vec{R} \cdot \vec{n})/R$$

$$\vec{r}(t) = \{x(t), y(t)\} \quad t \in [0, 2\pi] \quad t_p = \pi p / N, \; p = 0, 1, \ldots, 2N-1$$

$$\int_0^{2\pi} \ln\left[4\sin^2 \frac{t - \tau}{2}\right] F_1(\vec{r}, \vec{r}') f(\vec{r}') dl' = \sum_{p=0}^{2N-1} P_p^{(N)}(t) F_1(t, t_p) f(t_p) L(t_p)$$

$$P_p^{(N)}(t) = -(2\pi/N) \sum_{m=1}^{2N-1} \cos\left[m(t - t_p)\right]/m - \pi \cos\left[N(t - t_p)\right]/N^2$$

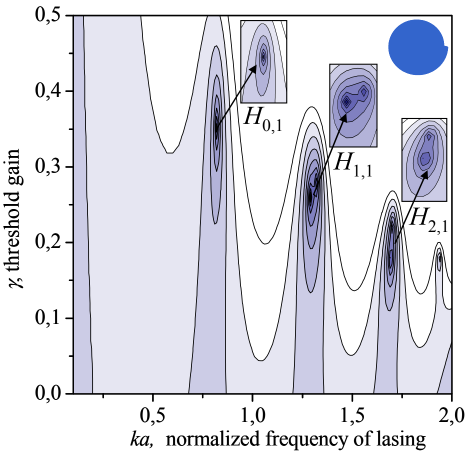
$$\int_0^{2\pi} F_2(\vec{r}, \vec{r}') f(\vec{r}') dl' = (\pi/N) \sum_{p=1}^{2N-2} F_2(t, t_p) f(t_p) L(t_p) + (\pi/2N)\left[F_2(t, t_0) f(t_0) L(t_0) + F_2(t, t_N) f(t_N) L(t_N)\right]$$

$$L(t) = \sqrt{(dx/dt)^2 + (dy/dt)^2}; \quad f = \varphi, \psi$$

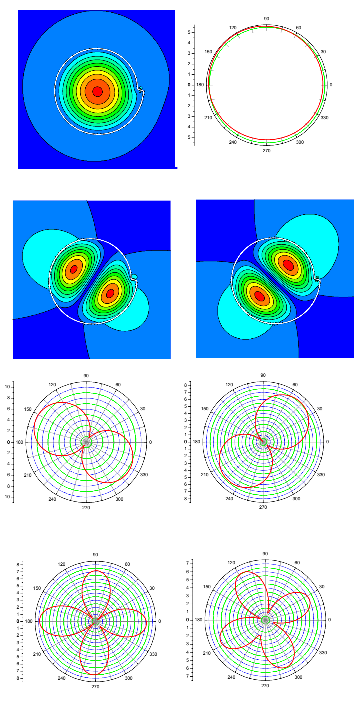
Numerical results

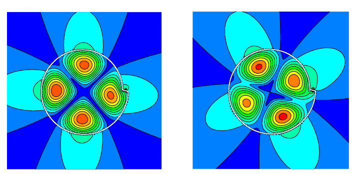
Spiral-shaped contour parameterization with a smooth function (β is the step tilt angle):

$$r(t) = \begin{cases} 1 - \delta/4\pi\left[(2\pi - \beta)/\beta t - \pi/\beta^2 t^2 - \pi\right], t \in [0, \beta), & \delta = d/a \\ 1 + \delta/4\pi t, \quad t \in [\beta, 2\pi - \beta] \\ 1 + \delta/4\pi\left[(2\pi - \beta)/\beta(2\pi - t) - \pi/\beta^2(2\pi - t)^2 + \pi\right], t \in (2\pi - \beta, 2\pi] \end{cases}$$



Determinant absolute value relief on the plane (ka, γ): $d = 0.3a$, $\beta = 0.03$, α_i=2.63, α_e=1, N=50.



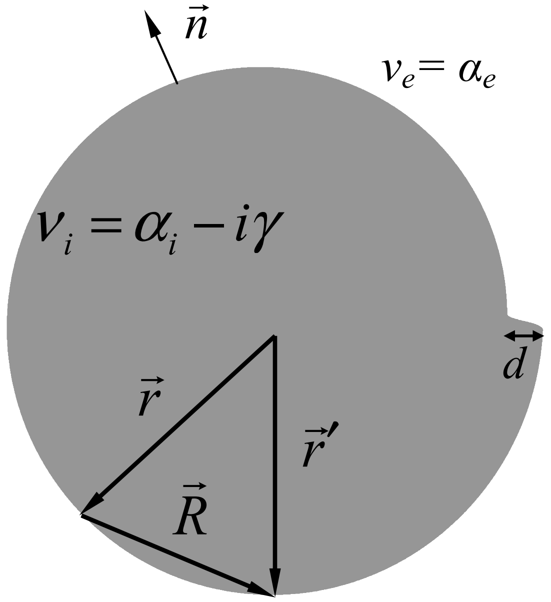


Near- and far-field patterns for the lowest modes in a spiral-shape cavity

Conclusions

Equivalence ➢ no spurious eigenvalues
Smooth or integrable kernels ➢ reliable discretization
Controllable accuracy
Smooth parameterization for a spiral-shaped contour
Modes in a spiral cavity split into doublets

[1]Institute of Radio-Physics and Electronics NASU, Kharkov, 61085 Ukraine
[2]George Green Institute for Electromagnetics Research, University of Nottingham, Nottingham NG7 2RD, UK
[3]Institute of Photonics and Electronics ASCR v.v.i., 182 51 Prague 8, Czech Republic



$\vec{n}$
$\nu_e = \alpha_e$
$\nu_i = \alpha_i - i\gamma$
d
$\vec{r}$
$\vec{r}'$
$\vec{R}$