CONCLUSION

In the dissertation, a novel approach has been elaborated to the solution of an important problem of electromagnetics that is the development of a linear electromagnetic model able to quantify not only the frequencies but also the thresholds of emission of natural electromagnetic field states in dielectric resonators with active regions. In the framework of this model, modifies eigenvalue problems for several important types of 2-D open dielectric resonators have been studied. Among them there are stand-alone fully and partially active circular resonators, coupled circular resonators, and also fully active non-circular resonators.

Summarizing the results of the work one can make the following general conclusions:

1. Until recently, linear electromagnetic analysis of microlaser dielectric resonators has been conducted using the passive-cavity model, i.e. neglecting the presence of the active region. This has prevented from determining the thresholds of lasing of the natural modes and, therefore, made difficult an adequate interpretation of experiments and elaboration of the recommendations towards the reduction of the thresholds.
2. In this dissertation, the problem about finding the natural model of the open dielectric resonators has been formulated, for the first time, in a modified form that enables one to take into account the presence of the active region in resonator and, thanks to this modification, to determine the spectra and thresholds of lasing of the natural nodes as the elements of eigenvalues.
3. We have established, on the basis of the Maxwell equations, a simple analytical link between the threshold of lasing in an open resonator and the mode quality factor and the overlap coefficient of the active region and the mode field.
4. Efficient numerical algorithms have been developed for the computation of the lasing spectra and thresholds, and also the modal fields in the near and far zone,

for the active circular resonators, cyclic photonic molecules of such resonators, and also 2-D resonators with arbitrary smooth contours.

5. It has been found that in a stand-alone thin microdisk there are lower—order modes with high thresholds of lasing and the whispering-gallery modes with exponentially low thresholds.
6. We have shown that by collecting the microdisks into cyclic photonic molecules one can achieve the reduction of the thresholds of lasing for the supermodes (optically coupled modes) built on both the lower-type modes and on the whispering-gallery modes.
7. It has been discovered that in the configuration active disk + passive ring reflector the threshold of lasing for a supermode can be both lower and higher than in a stand-alone disk. This depends on the overlap between the active region and the mode field. The threshold rapidly grows up if the field is pulled into the passive regions of reflector.
8. We have quantified the splitting of the modes into the doublets due to the deformation of a thin microdisk to a spiral resonator. Here the directionality of the emission of the perturbed whispering-gallery modes improves however their thresholds grow up. The main factor is the height of the step on the contour in terms of wavelengths.
9. The obtained results have enabled us to consider the shape and location of active region as engineering parameters, which can be used for manipulation of the thresholds of lasing in the open dielectric resonators. They also show the ways of lowering the thresholds and improve the directionality using the shape of the contour (for the stand-alone resonators) and the symmetry properties (for the coupled resonators).

The development of the mathematical and numerical models and their verification has been based on the following aspects:

- Introduction of the active region filled in with an active material, the gain of which is characterized by the “negative absorption;” on the active-region boundary the continuity of the field tangential components is requested,

- Formulation of the mathematically correct eigenvalue problem with the eigenvalues consisting of the pairs of real numbers: frequencies and thresholds of lasing of the natural modes; thresholds are understood as imaginary parts of the refractive index in the active region needed to make the frequency real,
- Use of widely known approximate technique of effective refractive index for the reduction of dimensionality of the problems for with thin dielectric disks,
- Reduction of the eigenvalue problems to the equivalent transcendental equations or determinantal equations of the Fredholm second kind that guarantees the discreteness of the eigenvalues and convergence of numerical algorithms of their computation,
- Use, in the problem of the modes in a resonator with arbitrary smooth contour, of the Muller boundary integral equations and exponentially convergent quadrature technique for the discretization,
- Application of the secant method (a.k.a. two-parametric Newton method) for the iterative search of the eigenvalues as the roots of obtained transcendental of determinantal equations,
- Numerical control of the power conservation law (Optical Theorem) for the natural modes of active dielectric resonators,
- Systematic check of the fulfillment of the boundary conditions for the mode fields, and also of the limiting behavior of the eigenvalues when the resonator geometry transforms to canonic configurations.

Проведенные исследования открывают новое направление в электродинамике активных резонаторов. Развитый в работе подход можно применять к разнообразным моделям любых лазеров. Разработанные алгоритмы и выявленные закономерности могут использоваться для интерпретации экспериментальных данных и для поиска перспективных конфигураций путем предварительного численного моделирования.

For the further research, we suggest the development of the full 3-D model of active disk resonator using the Muller equations, and also the modeling of the 2-D periodic open resonators and resonators with arbitrarily shaped active regions.