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ELECTROMAGNETIC FIELDS AND EMISSION THRESHOLDS OF STAND-ALONE AND COUPLED TWO-DIMENSIONAL DIELECTRIC RESONATORS WITH ACTIVE REGIONS

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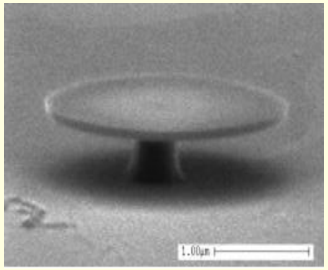
The object of research in the thesis is the effect of radiation of monochromatic electromagnetic waves from stand-alone and coupled dielectric resonators with active regions.

Specifically, we study the natural electromagnetic fields in two-dimensional (2-D) models of stand-alone and coupled dielectric resonators with active regions and their spectra of natural frequencies and associated material thresholds.

Methods of research used in the thesis are the following: theory of boundary-value problems of electromagnetics, which imply that the natural modes are the solutions of the homogeneous time-harmonic Maxwell equations with rigorous boundary conditions and radiation condition at infinity. Dimensionality of these problems was reduced to 2-Dusing widely known approximate method of effective refractive index. For each of considered configurations, the obtained 2-D problems were equivalently reduced to homogeneous matrix equations of the Fredholm second kind. The eigenvalues as the roots of corresponding determinantal equations were found numerically with controlled accuracy using two-parametric iterative Newton algorithm.

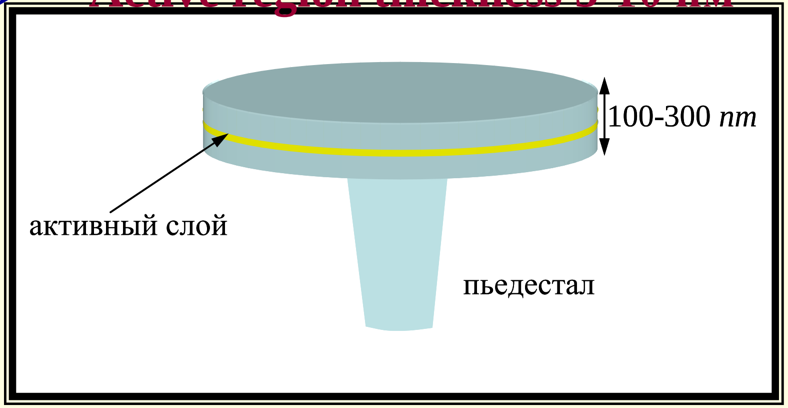
Structure and basic properties of microdisk lasers

G. Gayral, *Appl. Phys. Lett.*, 1999; H. Cao, *Appl. Phys. Lett.*, 2000



Dimensions:

- **Diameter 2-50 мкм**
- **Thickness 50-200 нм**
- **Active region thickness 5-10 нм**



Semiconductor systems

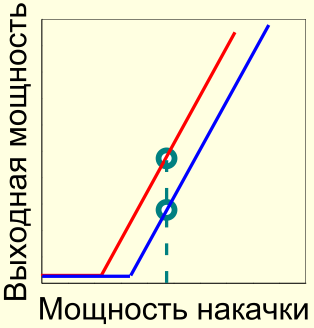
- GaAs/InP - λ = 1550 nm
- ZnSe/CdS - λ = 510 nm
- GaAs-AlAs/InAs – λ = 970 nm

Pumping

- Optical (photopump)
- Injection of carriers

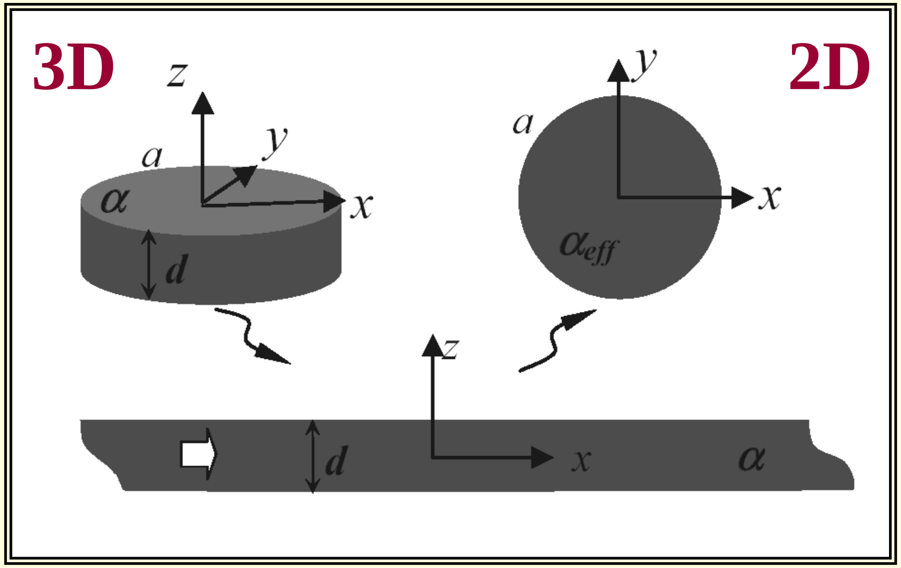
Basic properties

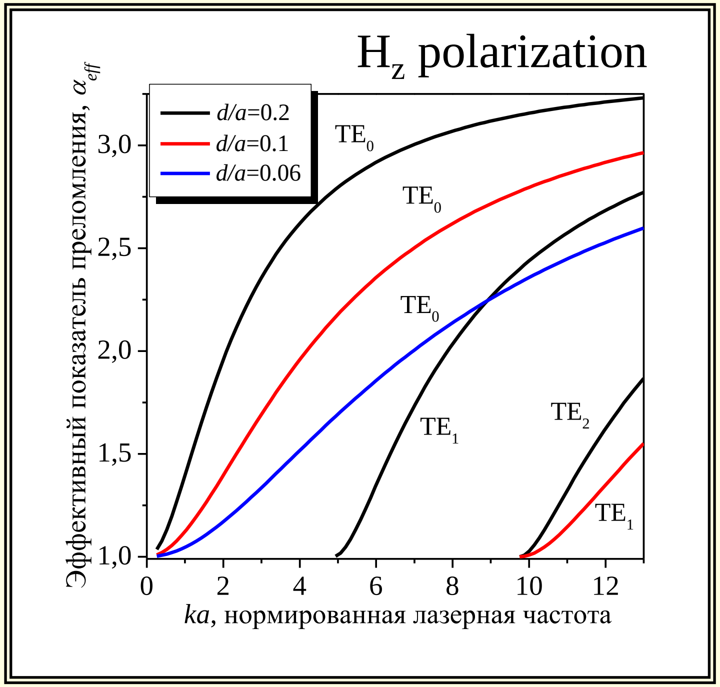
- **Ultra-low thresholds of lasing**
- **Equidistant frequency spectrum**
- **Emission in the disk plane**



Dimensionality reduction: effective index

$$\alpha \to \alpha_{eff}(\lambda, d, \text{mode \#})$$





For a disk of GaAs with thickness of 0.1 of the radius: $\alpha = 3.374, \lambda = 1.55\mu m$

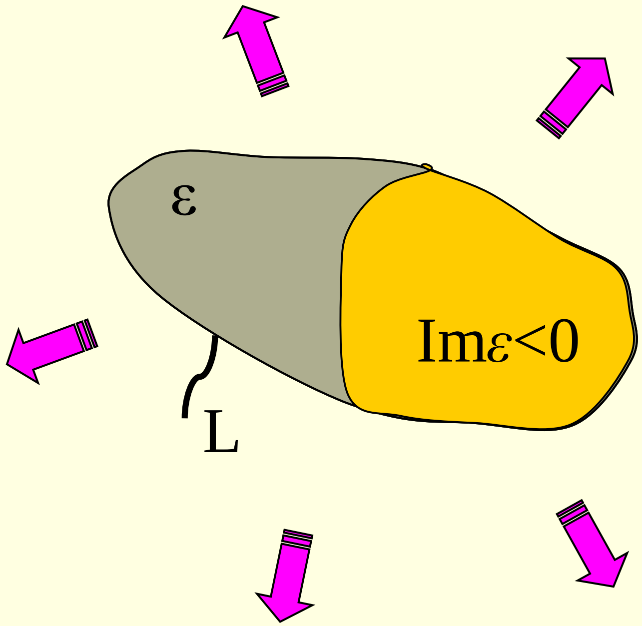
E_z polarization $\alpha_{eff}^{E} = 1.31$

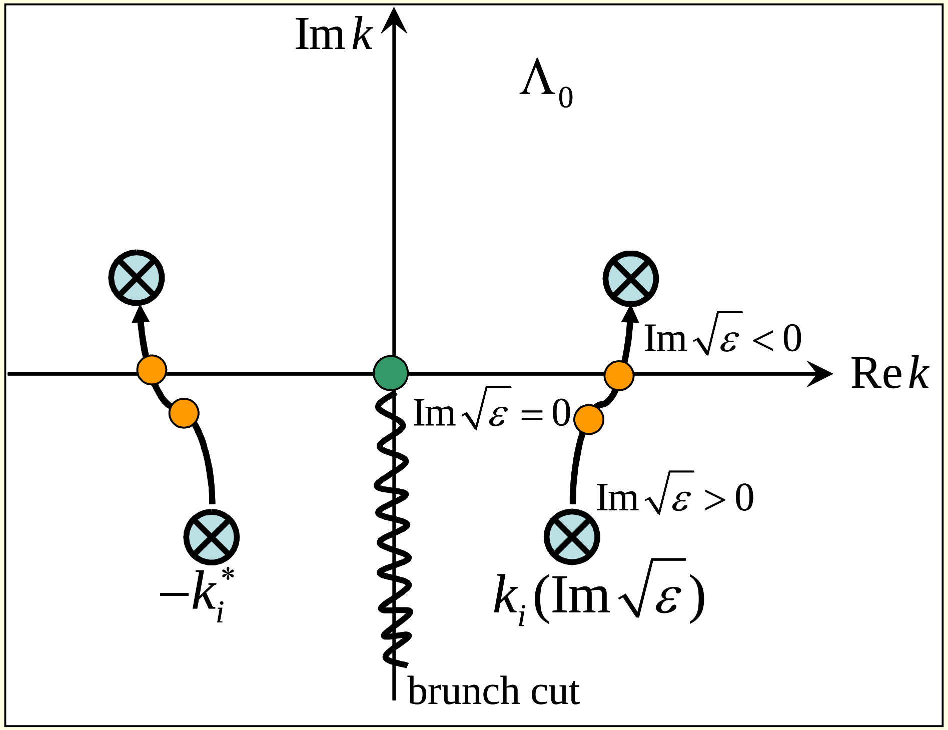
H_z polarization $\alpha_{eff}^{H} = 2.63$

in a thin disk H-polarized modes can be studied

Lasing from the viewpoint of Maxwell equations

The lasing **= existence of purely real natural frequency, $k_j = \mathrm{Re}\,k_j$, for the eigenstate of electromagnetic field (no decay in time). This is possible only if the open resonator contains an *active region*.**





2-D lasing eigenvalue problem

- Helmholtz equation off theboundaries

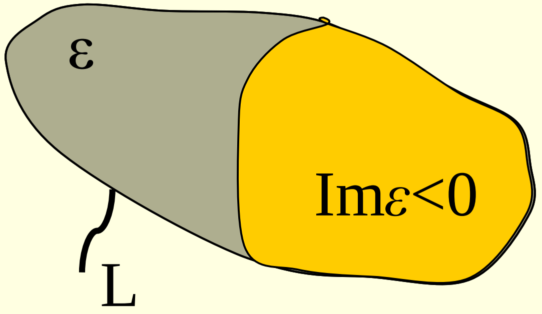
$\sim e^{-i\omega t}$

$$\left(\Delta + k^2 \nu^2\right) U(r,\varphi) = 0$$

$$\sqrt{\varepsilon} = \nu = \alpha_{eff} - i\gamma$$

α_{eff} = effective refractive index

γ = threshold of lasing



- Continuity conditions at the boundary

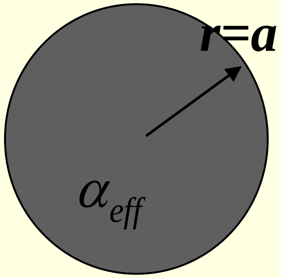
- Sommerfeld condition at ∞

Eigenvalues are the pairs of real numbers: $(\kappa_j\,,\ \gamma_j)$

Uniformly active circular resonator

Separation of variables for the filed in circular dielectric cavity

$$U_{mn}(r,\varphi)=\begin{cases}\dfrac{H_m^{(1)}(\kappa_{mn})}{J_m(\kappa_{mn}\nu_{mn})}J_m(\kappa_{mn}\nu_{mn}\rho)\begin{Bmatrix}\cos m\varphi\\ \sin m\varphi\end{Bmatrix},\rho<1\\ H_m^{(1)}(\kappa_{mn}\rho)\begin{Bmatrix}\cos m\varphi\\ \sin m\varphi\end{Bmatrix},\rho=r/a>1\end{cases}$$



$\kappa_{mn}=k_{mn}a,\quad \nu_{mn}=\alpha_{eff}-i\gamma_{mn}$ All modes are twice degenerate (m > 0)

Characteristic equation for the *m*-th mode family: (m=0,1,2,…):

$$J_m'(\kappa\nu)H_m^{(1)}(\kappa)-\beta^{H,E}\nu H_m'^{(1)}(\kappa)J_m(\kappa\nu)=0$$

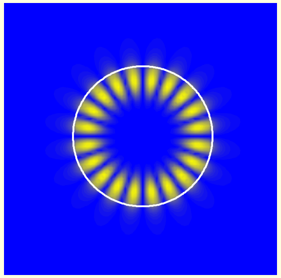
$$\beta^{H,E}=\nu^{-2}\ or\ 1\quad (U=H_z\ or\ U=E_z)$$

Fundamental properties of the modes of active circular resonator

Asymptotic analysis of the characteristic equations:

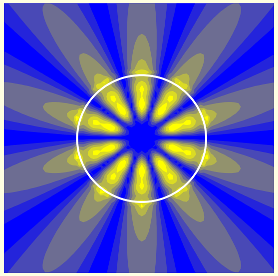
Equidistant frequency spectrum: $\kappa_{mn}^{E,H} \sim \dfrac{\pi}{2\alpha}\left(m + 2n \pm \dfrac{1}{2}\right)$

Whispering gallery modes: exponentially low thresholds of lasing



$$\gamma_{mn}^{E,H} \sim C_{E,H} e^{-2m\ln(2m/\kappa_{mn}^{E,H})}$$

Lower modes: high thresholds of lasing, $\sim 10^{-1}$

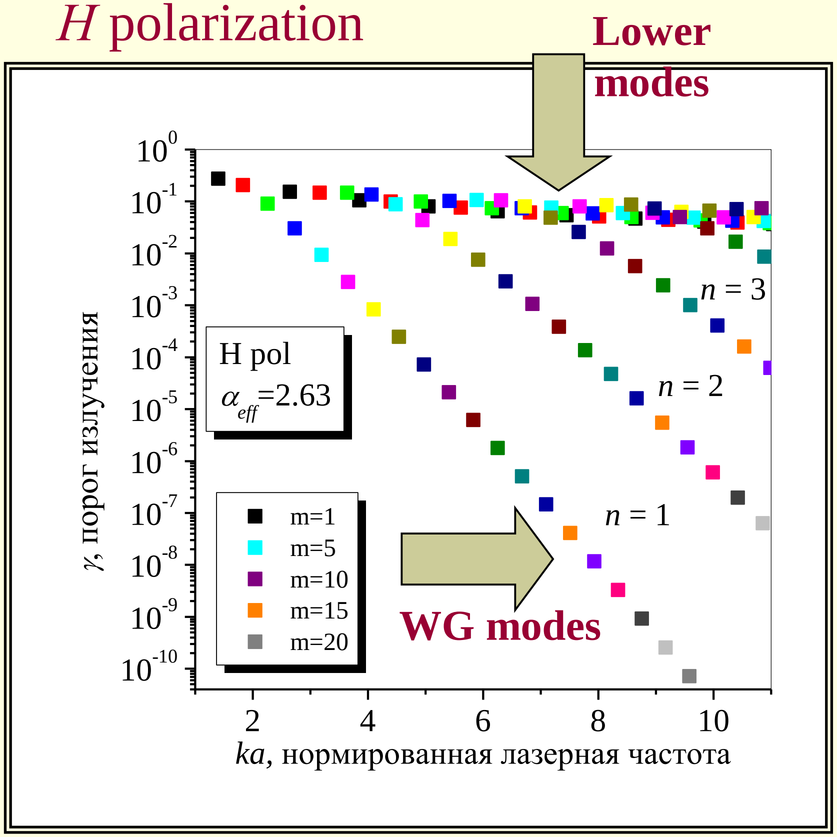


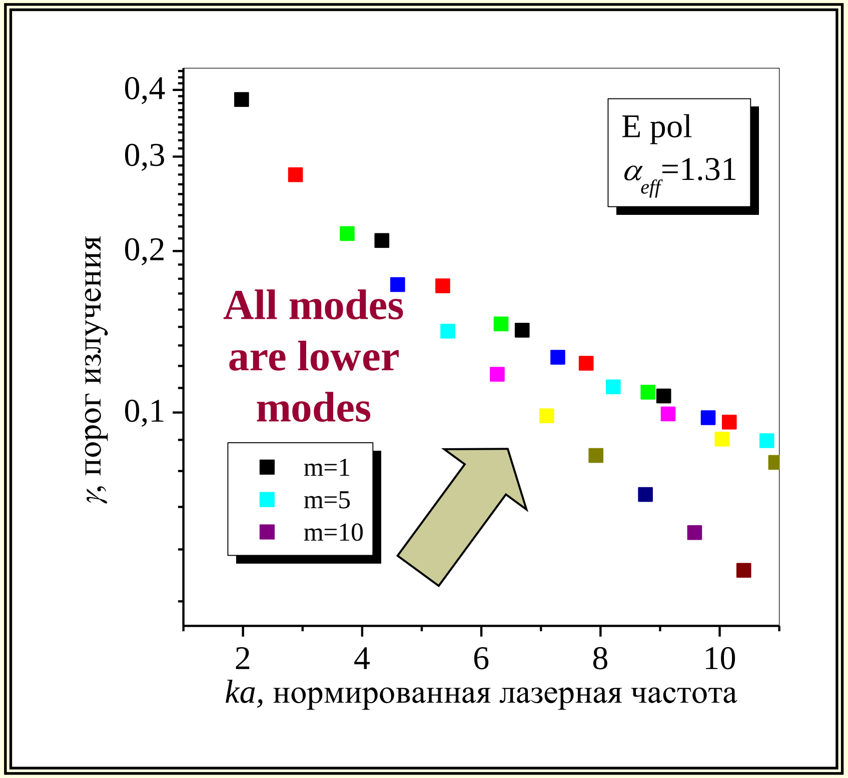
$$\gamma_{mn}^{E,H} \sim \pi \ln[(\alpha+1)/(\alpha-1)]/2\kappa_{mn}^{E,H}$$

Frequencies and thresholds of lasing of a uniformly active circular cavity

H polarization

E polarization



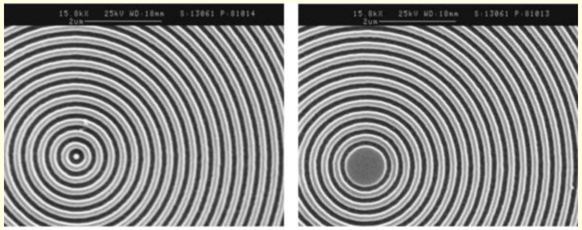


Direct quantification of the lasing thresholds for a uniformly active circular resonator as a 2-D model of microdisk

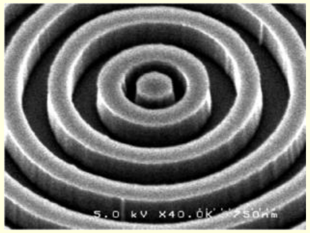
E. Smotrova, et al., *OQE*, vol. 36, 2004; *JSTQE*, vol. 11, 2005

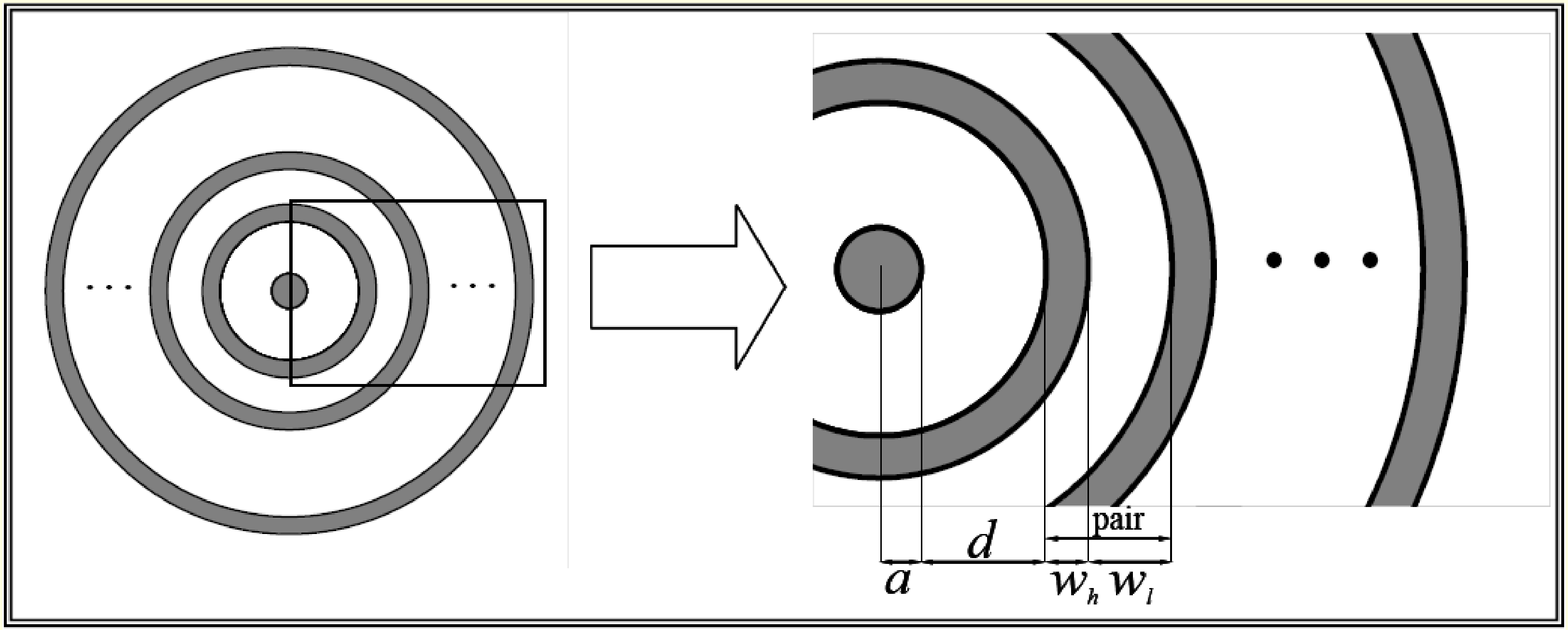
Active microdisk inside a Bragg reflector

J. Appl. Phys., Vol. 96, No. 6, 2004



Appl. Phys. Letts, vol 86, 2005

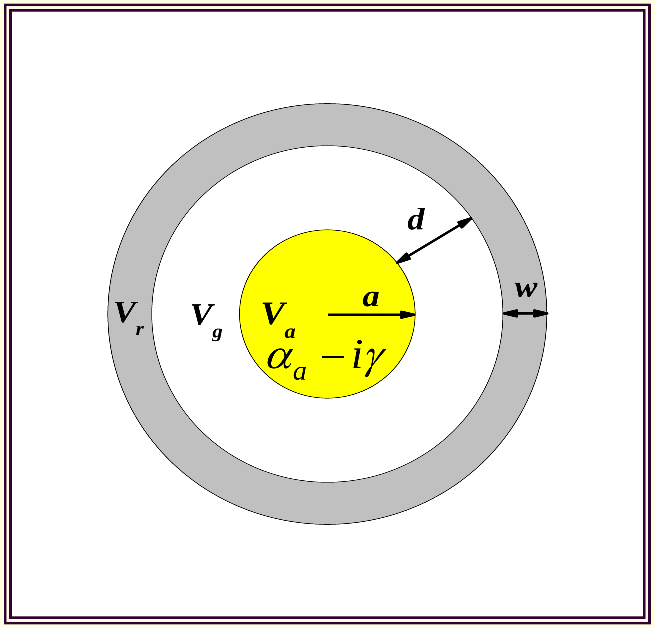




Circular active cavity inside a passive ring

$\kappa = ka$

H polarization



Separation of variables in partial regions:

$$H_z(r,\varphi) = \sum_{m=0}^{\infty} \left[A_m^s J_m(k\nu_s r) + B_m^s H_m(k\nu_s r) \right] \cos m\varphi$$

Interface continuity conditions yield ($s = 1,2,\ldots M$)

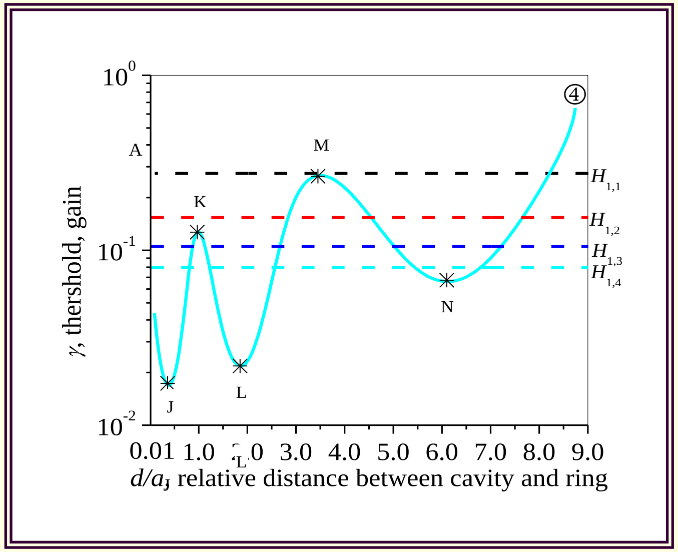
$$A_m^{s+1} J_m(\kappa\nu_{s+1}\rho_s) + B_m^{s+1} H_m(\kappa\nu_{s+1}\rho_s) = A_m^s J_m(\kappa\nu_s\rho_s) + B_m^s H_m(\kappa\nu_s\rho_s)$$

$$\left[A_m^{s+1} J'_m(\kappa\nu_{s+1}\rho_s) + B_m^{s+1} H'_m(\kappa\nu_{s+1}\rho_s) \right] / \nu_{s+1} = \left[A_m^s J'_m(\kappa\nu_s\rho_s) + B_m^s H'_m(\kappa\nu_s\rho_s) \right] / \nu_s$$

Characteristic equentions:

$$Det[C^{(m)}(\kappa,\gamma)] = 0, \quad m = 0,1,2,\ldots$$

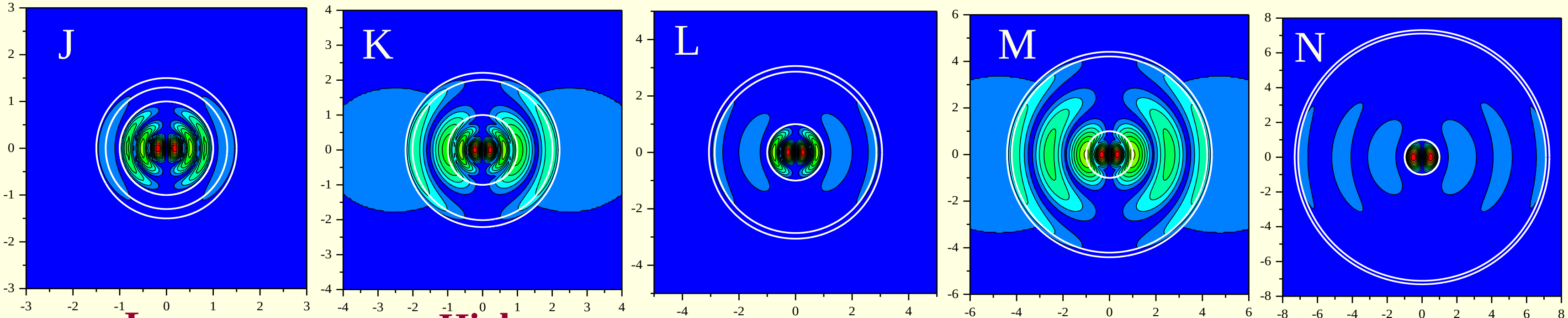
Near fields of dipole-type supermodes for active disk in passive ring



Near fields $|H_z|$ at the points marked with letters from J to N

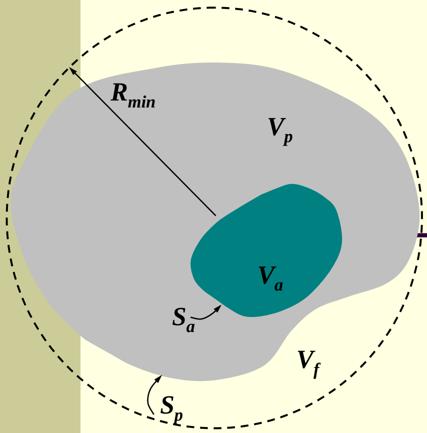
Low threshold | **High threshold** | **Low threshold**

$H_{1,3,1,0}$ $H_{1,2,2,0}$ $H_{1,2,2,0}$ $H_{1,1,3,0}$ $H_{1,1,3,0}$



Low threshold | **High threshold**

$w=0.2a$, $\alpha_a=\alpha_r=2.63$, $\alpha_g=1$.



Optical Theorem for lasers: basic formulas

Poynting theorem for complex frequencies:

$$(1/2)\oint_S \vec{E}\times\vec{H}^* ds = (i/2)\int_V \left(k^*\varepsilon^* Z_0^{-1}\,|\vec{E}|^2 - k\mu Z_0\,|\vec{H}|^2\right)dv,$$

Lasing Problem:

k=Re k, μ=1, ε=Re ε in $V^{(p)}$ & $\varepsilon=\nu^2=(\alpha-i\gamma)^2$ in $V^{(a)}$

$$(Z_0/2)\,\mathrm{Re}\oint_S \vec{E}_j\times\vec{H}_j^* ds = \gamma_j k_j \alpha_a \int_{V_a} |\vec{E}_j(\vec{R},k_j,\gamma_j)|^2 dv,$$

j is modal index. **Radiation losses and the power generated in the active region are balanced for each mode**

Overlap coefficients: basic formulas

Overlap coefficients:

$$\Gamma_j^{(f)} = W_j^{(f)} / W_j \le 1, \qquad W_j^{(f)}(k_j,\gamma_j) = \int_{V_f} \alpha_f^2 \,|\vec{E}_j(\vec{R},k_j,\gamma_j)|^2 dv, \qquad f = a, g, r$$

Эффективный модовый объем = сумма частичных модовых объемов:

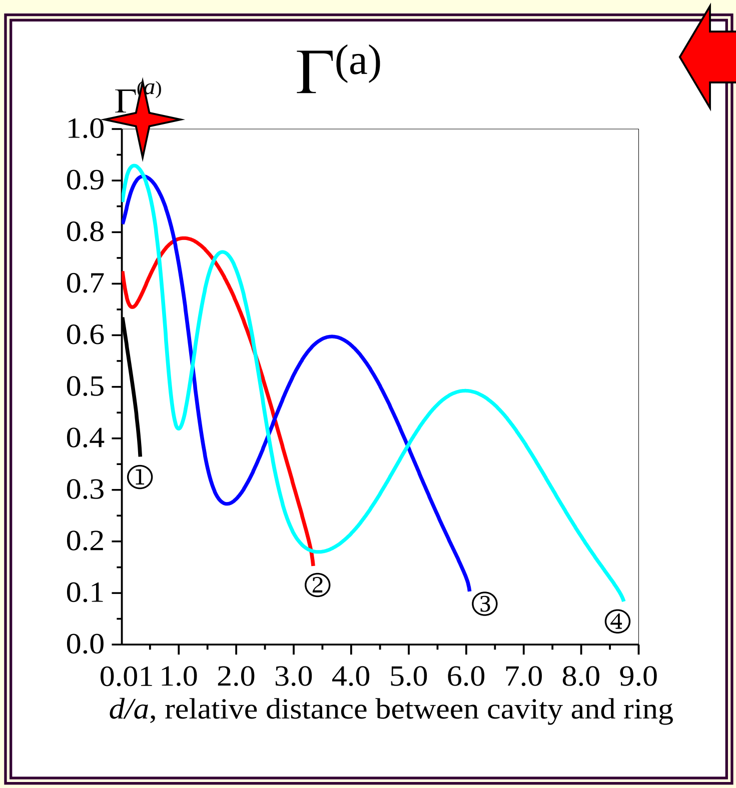
$$W_j(k_j,\gamma_j) = \int_V \alpha^2 \,|\vec{E}_j|^2 dv = \int_{V_a} \alpha_a^2 \,|\vec{E}_j|^2 dv + \int_{V_g} \alpha_g^2 \,|\vec{E}_j|^2 dv + \int_{V_r} \alpha_r^2 \,|\vec{E}_j|^2 dv$$

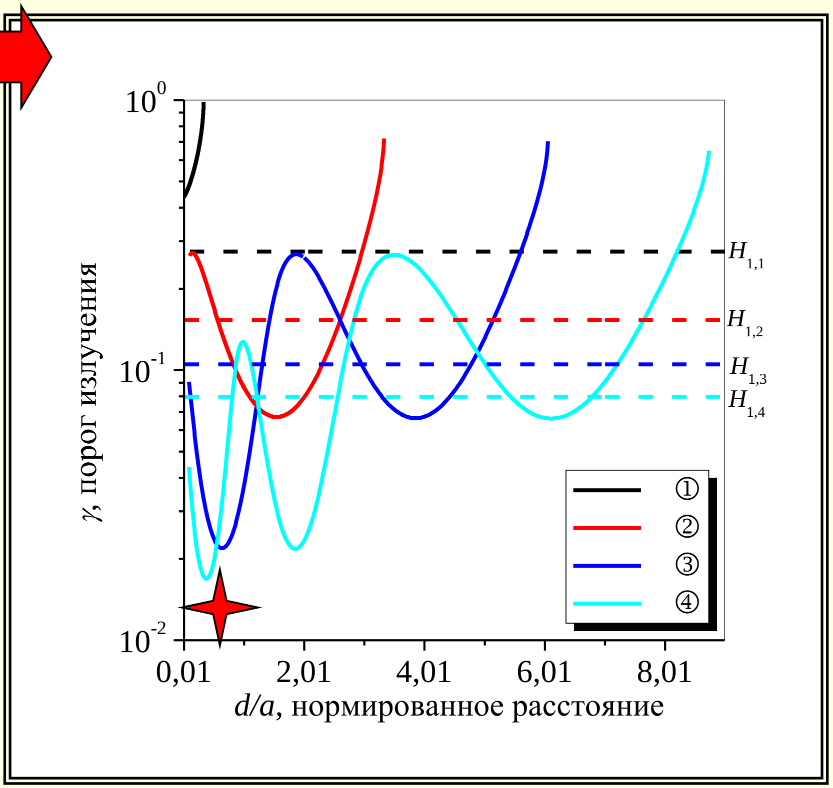
Optical Theorem for lasers:

$$\gamma_j = \frac{(Z_0/2)\,\mathrm{Re}\oint_S \vec{E}_j \times \vec{H}_j^* ds}{k_j \alpha_a \int_{V_a} |\vec{E}_j(\vec{R},k_j,\gamma_j)|^2 dv} = \frac{\alpha_a P_j}{\Gamma_j^{(a)} k_j W_j} = \frac{\alpha_a}{\Gamma_j^{(a)} \tilde{Q}_j}$$

Optical Theorem is satisfied with machine precision if κ and γ have been found from the lasing eigenvalue problem

Active disk in a passive ring: overlap coefficients



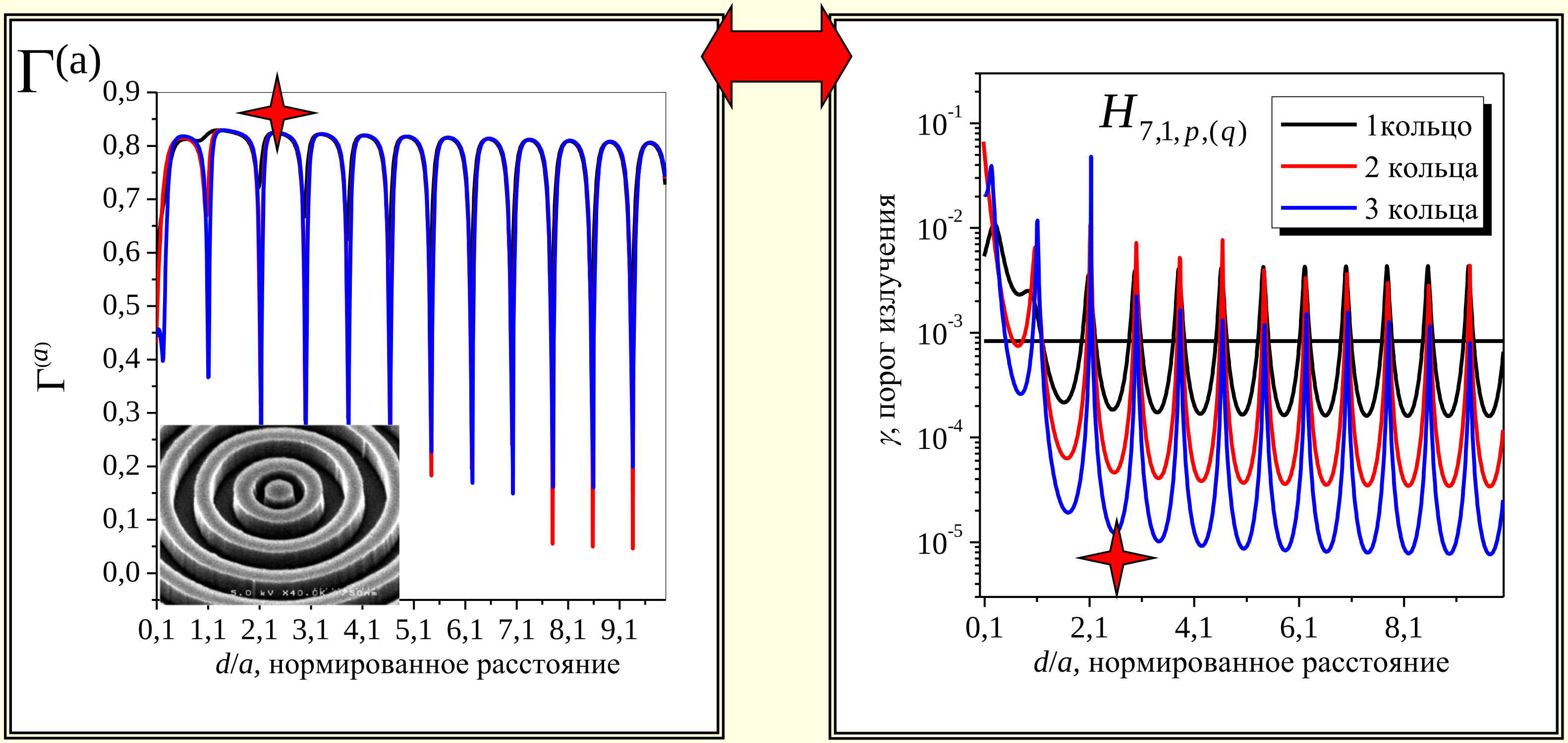


Low thresholds are caused by good overlap of supermode E-field with active region

$$w=0.2a,\ \alpha_a=\alpha_r=2.63,\ \alpha_g=1$$

Smotrova, et al., *JOSA A*, vol. 25, 2008

Active disk in a Bragg reflector: overlap coefficients and supermode thresholds

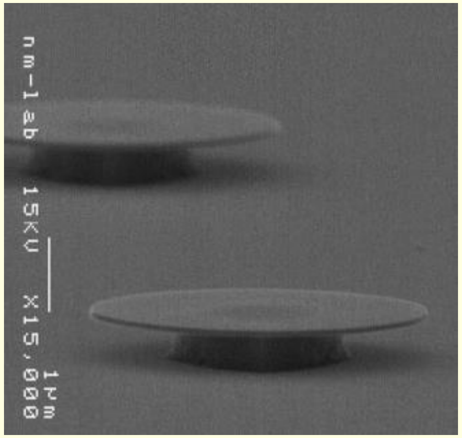


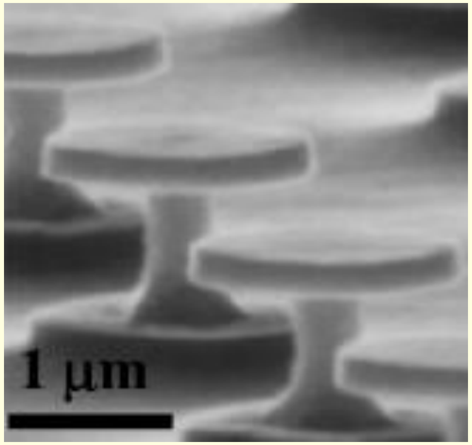
Overlap coefficient of active region behaves in inverse manner with respect to the supermode threshold.

$w_h/w_l=1,\ w_h=0.2a,\ \alpha_a=\alpha_h=2.63,\ \alpha_l=1$

Coupled microcavity lasers as photonic molecules

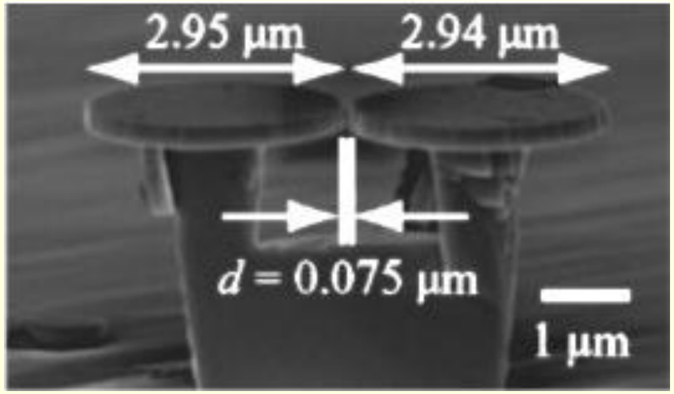
First experiment: 1996



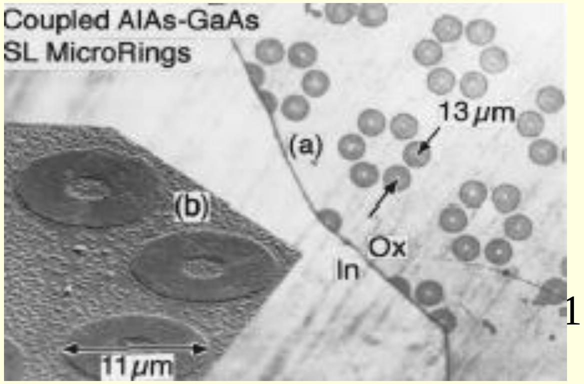


Zwiller et al.,
New J. of Physics,2004

Petter et al.,
Appl. Phys. Letts., 2002

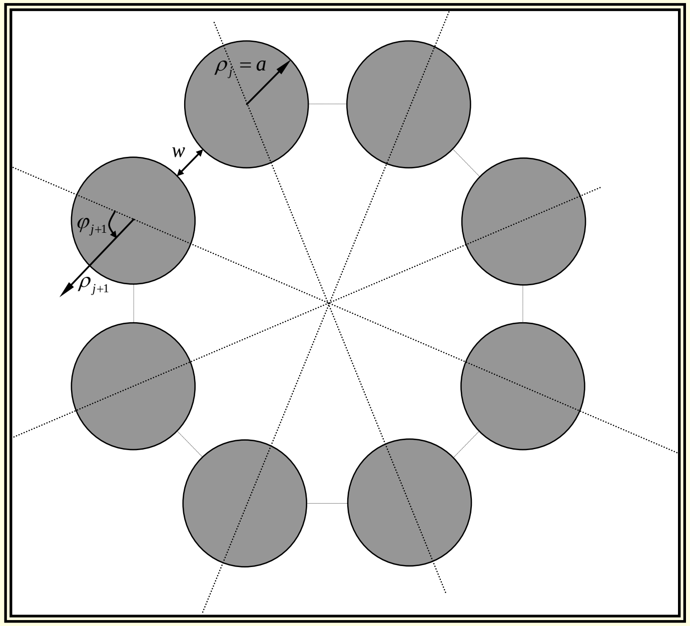


Nakagawa et al., *Appl. Phys. Letts*., 2005



Evans, et al., Appl. Phys. Letts, 1996

Cyclic photonic molecule of *M* active disks



Partial separation of variables in the local coordinates:

$$\rho_j < a, \quad j = 1,\dots M$$

$$U(\rho_j, \varphi_j) = \sum_{p=(0)1}^{\infty} A_p^j J_p(k\nu\rho_j) S_p(\varphi_j)$$

$$\rho_j > a, \quad j = 1,\dots M$$

$$U(\rho, \varphi) = \sum_{j=1}^{M} \sum_{p=(0)1}^{\infty} B_p^j H_p^{(1)}(k\rho_j) S_p(\varphi_j)$$

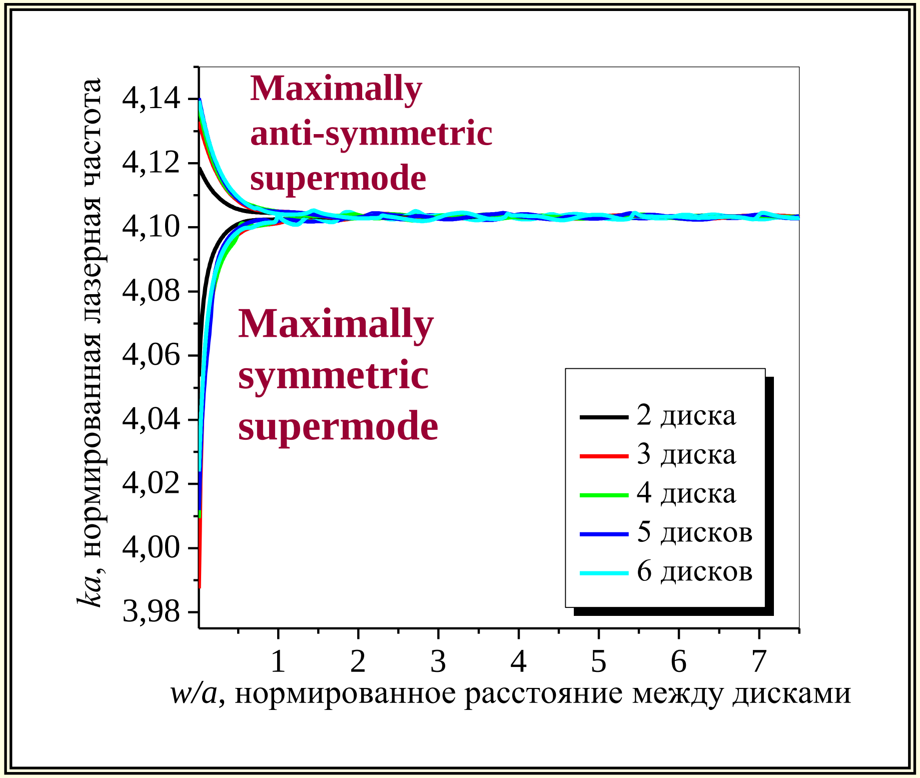
Continuity conditions and addition theorems lead to the **Fredholm 2-nd kind matrix equations** for each modal class in terms of symmetry

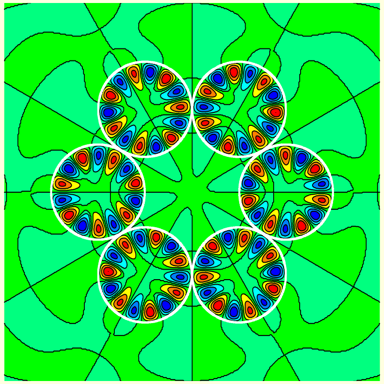
$$(I + A(\kappa, \gamma))X = 0$$

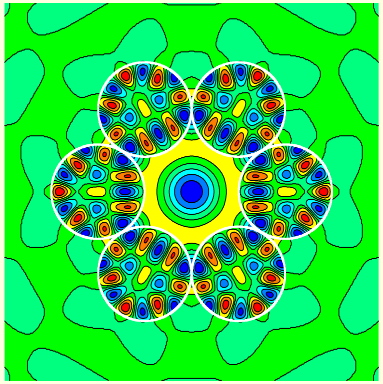
Characteristic equations: $Det(I + A(\kappa, \gamma)) = 0$

Frequencies of lasing for supermodes of photonic molecule of *M* active disks

Coupled modes (supermodes) in CPM form close multiplets

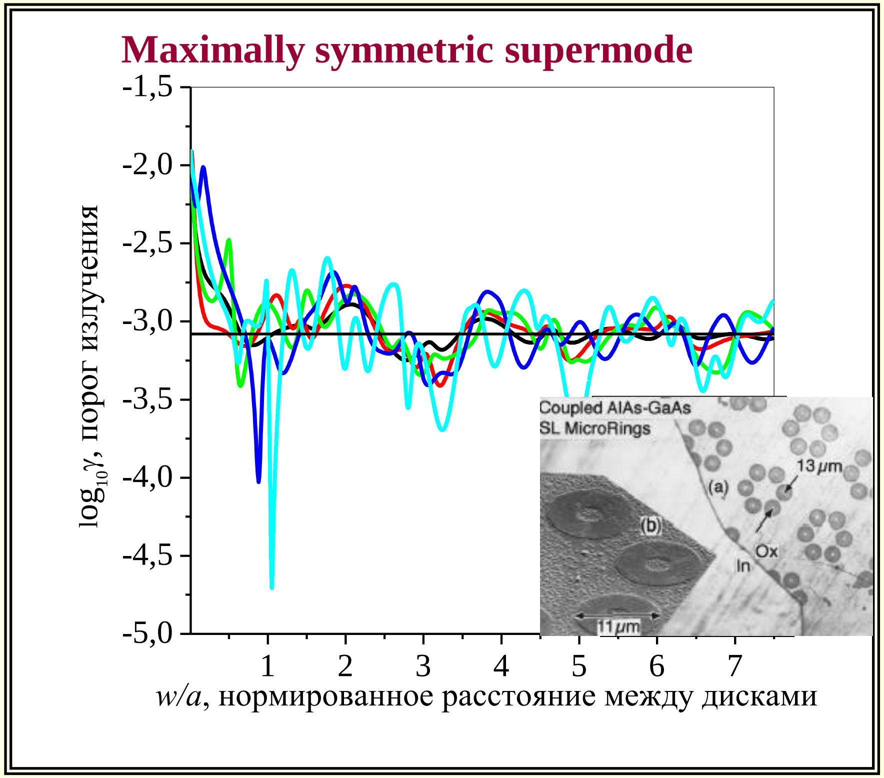


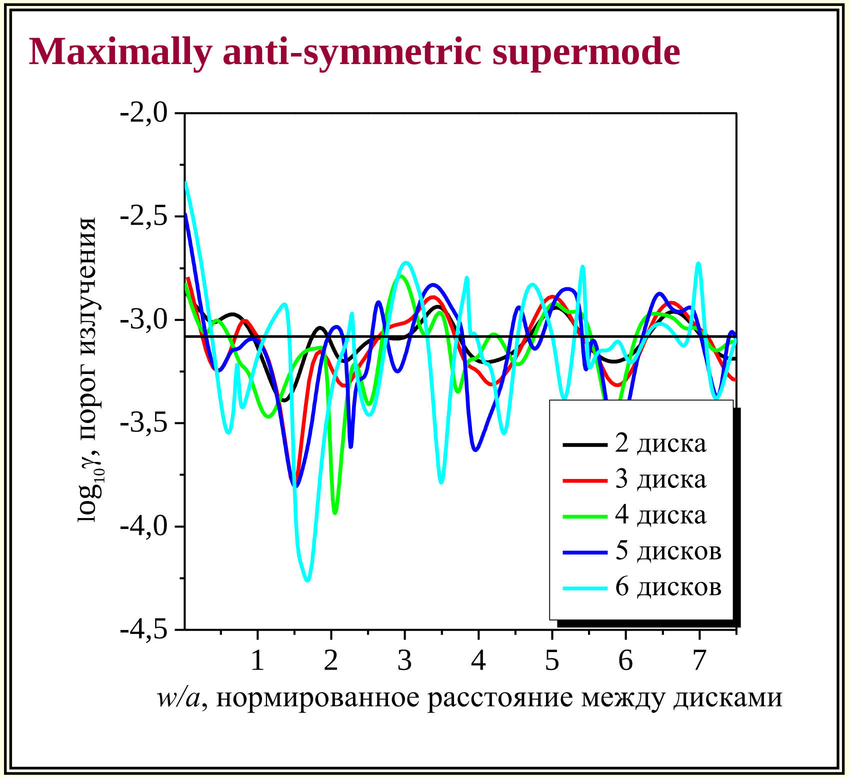




All modes of a cyclic photonic molecule made of *M* disks split to the classes according to the symmetry: *M+1* (if *M* is odd) or *M+2* (if *M* is even)

Thresholds of lasing for the WG supermodes of photonic molecule of *M* active disks



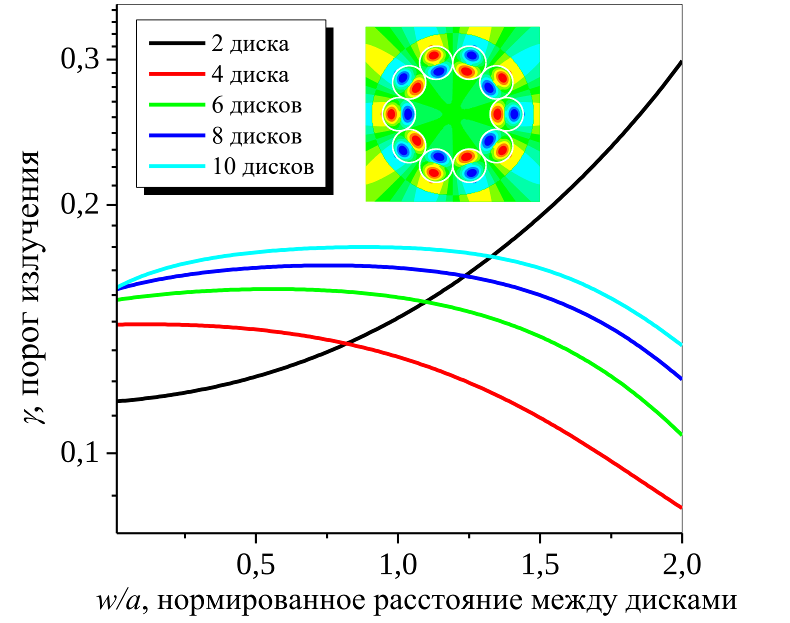


The threshold of lasing for any supermode in a cyclic photonic molecule made of *M* active disks can be lowered with respect to a stand-alone disk; the larger *M* the better.

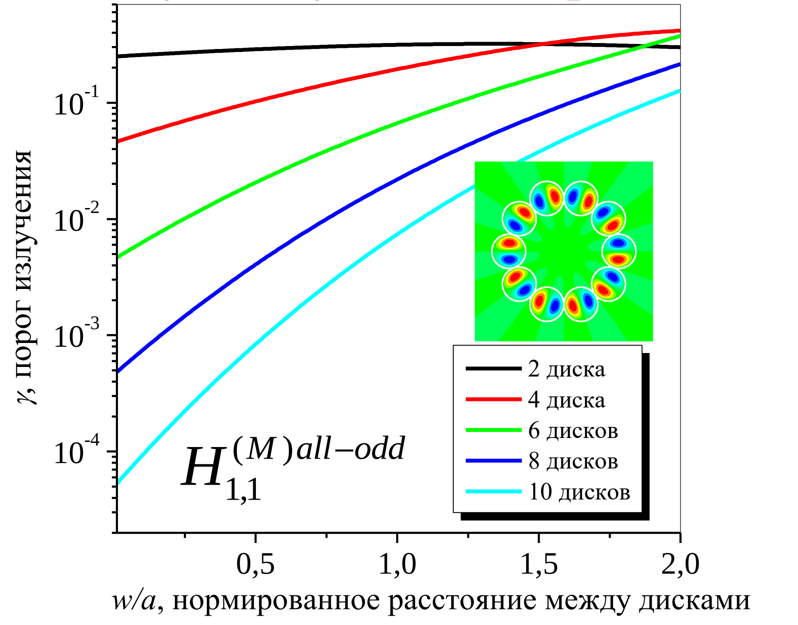
E. Smotrova, et al., *Optics Lett*, 2006

Thresholds of lasing for the dipole-type supermodes of photonic molecule of *M* active disks

Maximally anti-symmetric supermode



Maximally anti-symmetric supermode



The thresholds of lasing for anti-symmetric supermodes built on the lowest modes (non WG modes) in a cyclic photonic molecule can be considerably lowered with respect to a stand-alone disk; the larger *M* and the closer are the disks the better.

E. Smotrova, et al., *PTL*, 2006

Muller boundary integral equations

$$\varphi(\vec{r}) + \int_\Gamma A(\vec{r},\vec{r}\,')\varphi(\vec{r}\,')dl' - \int_\Gamma B(\vec{r},\vec{r}\,')\psi(\vec{r}\,')dl' = 0,$$

$$(\eta_i + \eta_e)\psi(\vec{r})/2\eta_e + \int_\Gamma C(\vec{r},\vec{r}\,')\varphi(\vec{r}\,')dl' - \int_\Gamma D(\vec{r},\vec{r}\,')\psi(\vec{r}\,')dl' = 0, \quad \vec{r} \in \Gamma,$$

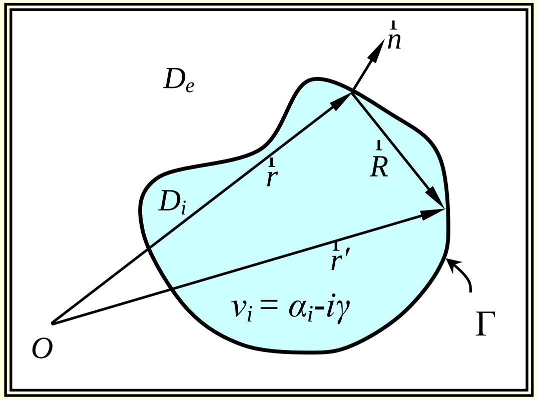
$$A(\vec{r},\vec{r}\,') = \partial G_i(\vec{r},\vec{r}\,')/\partial n' - \partial G_e(\vec{r},\vec{r}\,')/\partial n',$$

$$B(\vec{r},\vec{r}\,') = G_i(\vec{r},\vec{r}\,') - \eta_i/\eta_e G_e(\vec{r},\vec{r}\,')$$

$$C(\vec{r},\vec{r}\,') = \partial^2 G_i(\vec{r},\vec{r}\,')/\partial n \partial n' - \partial^2 G_e(\vec{r},\vec{r}\,')/\partial n \partial n',$$

$$D(\vec{r},\vec{r}\,') = \partial G_i(\vec{r},\vec{r}\,')/\partial n - \eta_i/\eta_e \partial G_e(\vec{r},\vec{r}\,')/\partial n$$

$$G_j(\vec{r},\vec{r}\,') = \frac{i}{4} H_0^{(1)}(k_j R)$$



- **Integration over the closed contour**
- **Smooth or integrable kernels: Fredholm nature**
- **2-nd kind equations**
- **Equivalency to original problem: no spurious eigenvalues**

Discretization of IEs using quadratures

Extraction of logarithmic part from kernels:

$$F(t,\tau)=F_1(t,\tau)\ln[4\sin^2((t-\tau)/2)]+F_2(t,\tau),\quad F=A,B,C,D,$$

For the **logarithmic** parts use the quadrature with **trigonometric polynomials:**

$$\int_0^{2\pi}\ln[4\sin^2((t-\tau)/2)]F_1(t,\tau)f(\tau)L(\tau)d\tau=\sum_{p=0}^{2N-1}P_p^{(N)}(t)F_1(t,t_p)f(t_p)L(t_p)$$

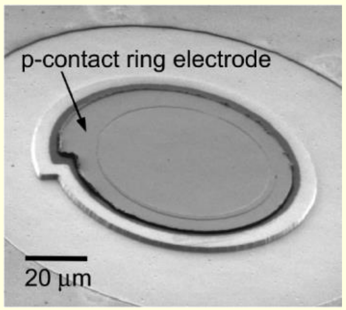
$$P_p^{(N)}(t)=-(2\pi/N)\sum_{m=1}^{N-1}\cos\left[m(t-t_p)\right]/m-\pi\cos\left[N(t-t_p)\right]/N^2$$

For **regular** parts use **trapezoid formula:**

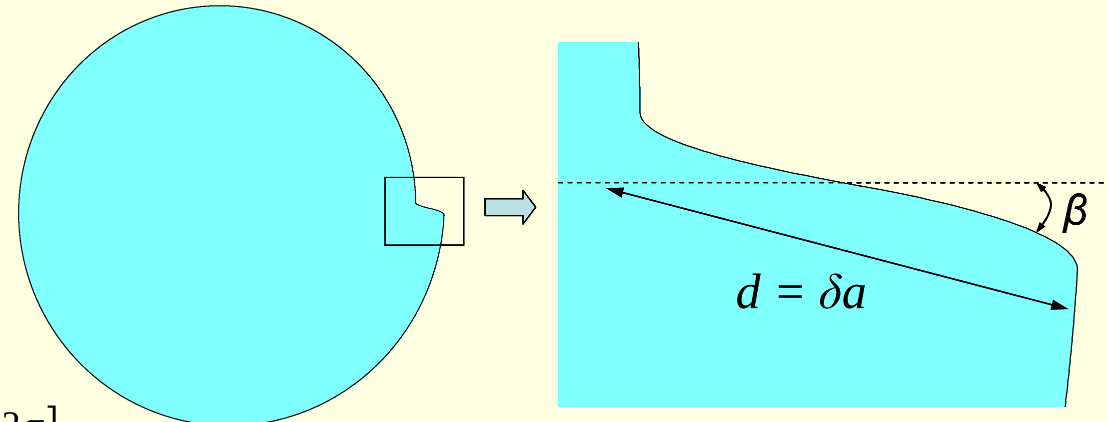
$$\int_0^{2\pi}F_2(\vec{r},\vec{r}\,')f(\vec{r}\,')dl'=(\pi/N)\sum_{p=0}^{2N-1}F_2(t,t_p)f(t_p)L(t_p)$$

Active spiral-shaped microcavity

M. Kneissl, et al., *Appl. Phys. Lett.*, vol. 84, 2004



Parameterization of the contour



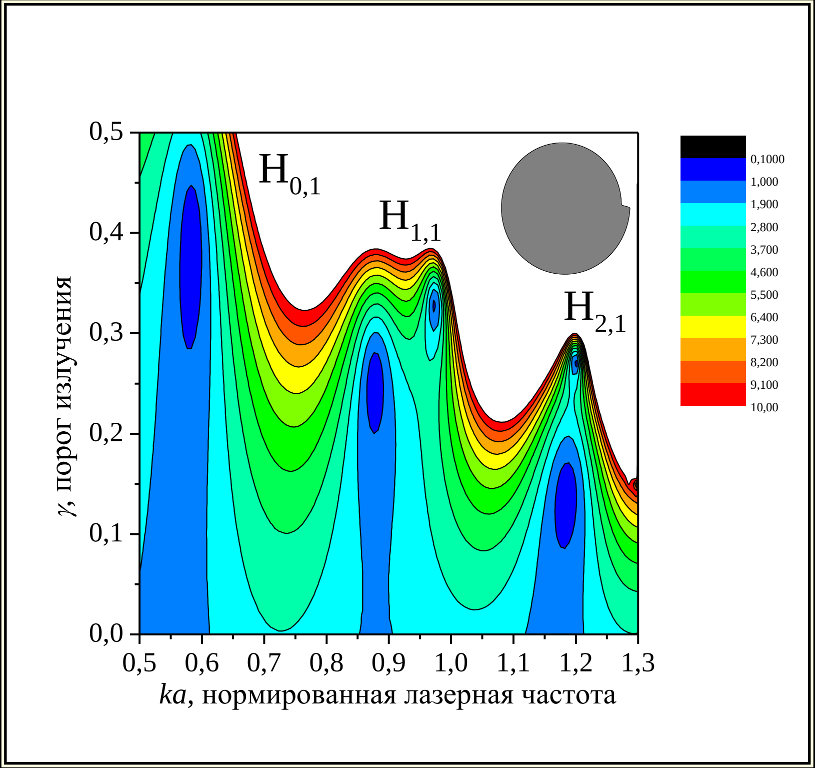
$$\vec{r}(t) = \{r(t)\cos(t), r(t)\sin(t)\}, \quad t \in [0, 2\pi]$$

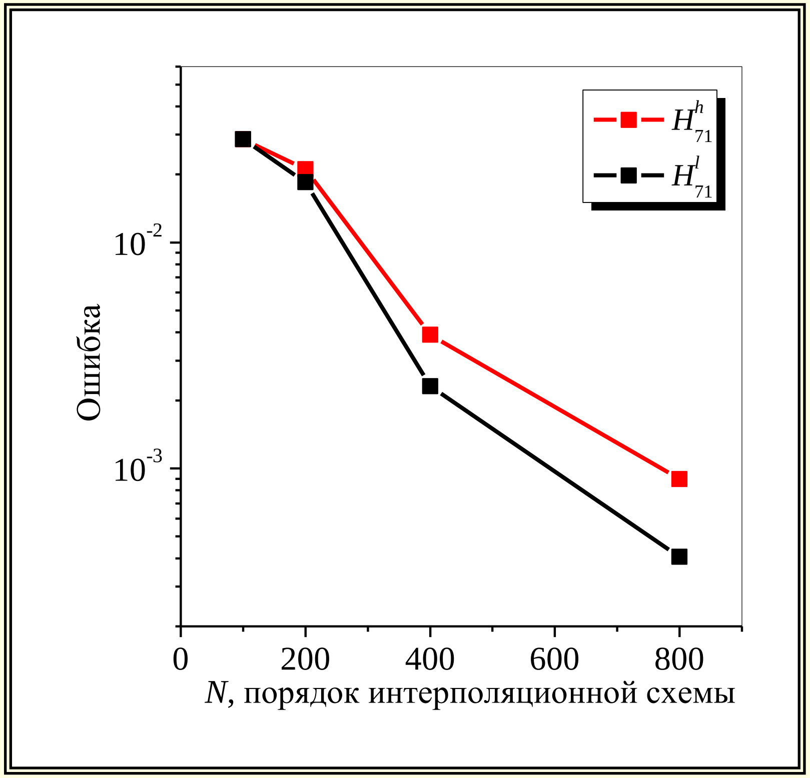
$$r(t) = \begin{cases} 1 + \delta/4\pi t, & t \in [\beta, 2\pi - \beta] \\ 1 - \delta/4\pi\left[t(2\pi - \beta)/\beta - t^2\pi/\beta^2 - \pi\right], & t \in [0, \beta) \\ 1 + \delta/4\pi\left[(2\pi - t)(2\pi - \beta)/\beta - (2\pi - t)^2\pi/\beta^2 + \pi\right], & t \in (2\pi - \beta, 2\pi], \end{cases}$$

a is the minimum radius of resonator,
δ is the normalized to a step height,
β is the angle of the step inclination (counted from the "horizontal" direction)

Quadrature formula error and initial guess: minima of determinant

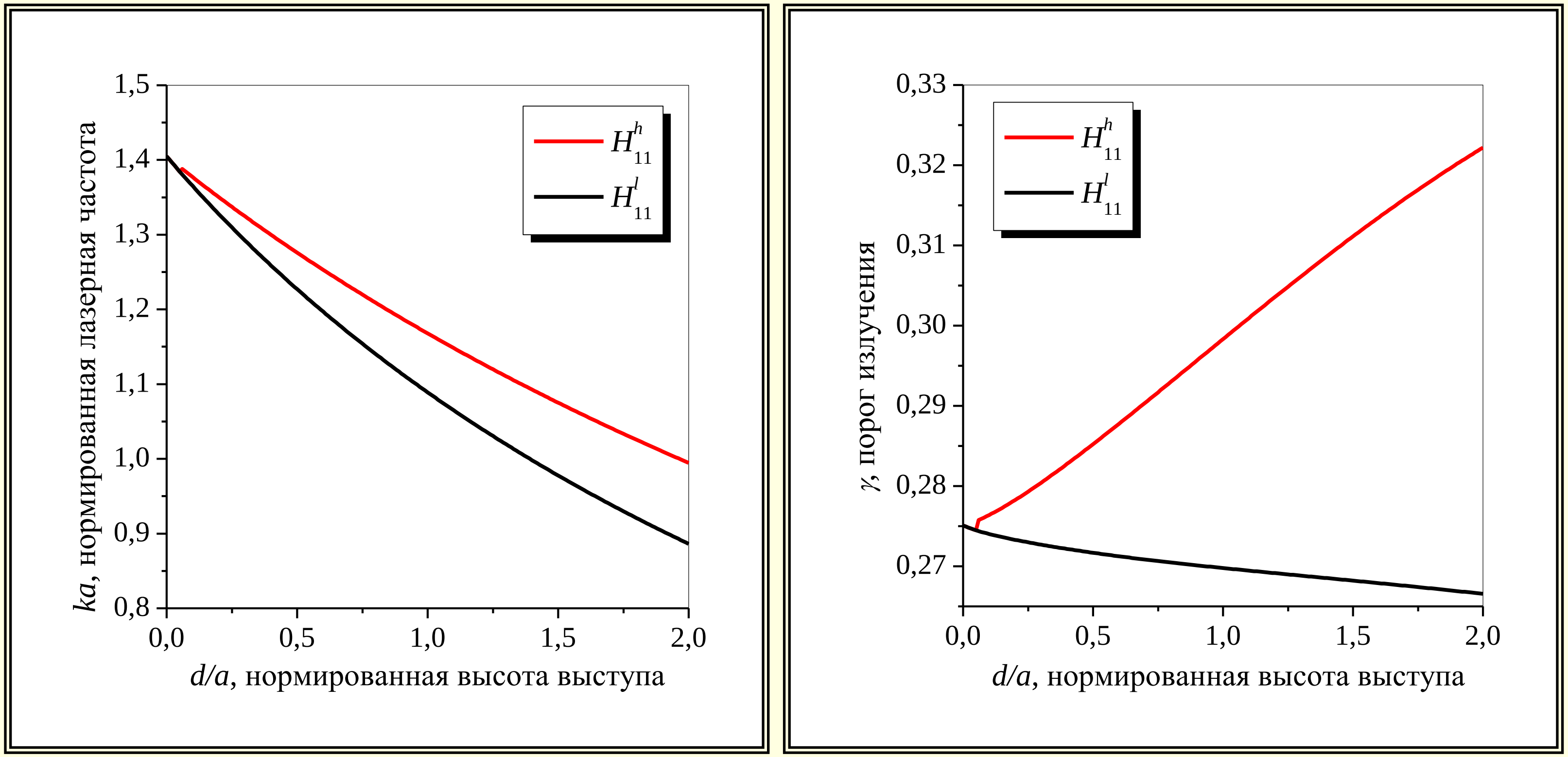
Resonator parameters: $d = 1.0a$, $\beta = \pi/100$, $\alpha_i = 2.63$, $\alpha_e = 1$





Equal-value lines for the matrix determinant of the discretized IE.
The roots are found by the 2-parametric Newton method. The error in the computation of the eigenvalue as a function of interpolation scheme.

Splitting of modes to doublets: lowest modes

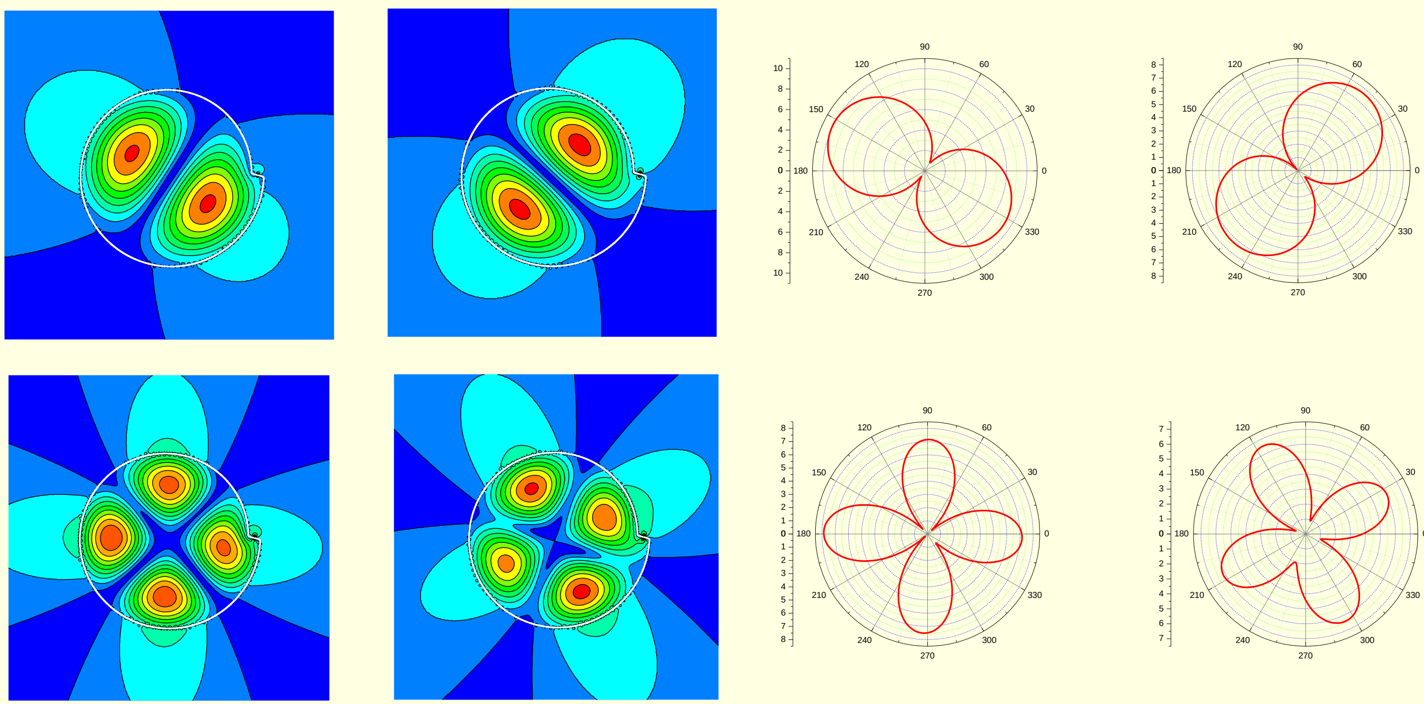


Mode of the H_{11} doublet frequencies and thresholds as a function of the step height in the spiral microlaser; $\beta=\pi/100$, $n=200$, $\alpha=2.63$

Smotrova et al., *Optics Letters*, vol. 34, no 24, 2009

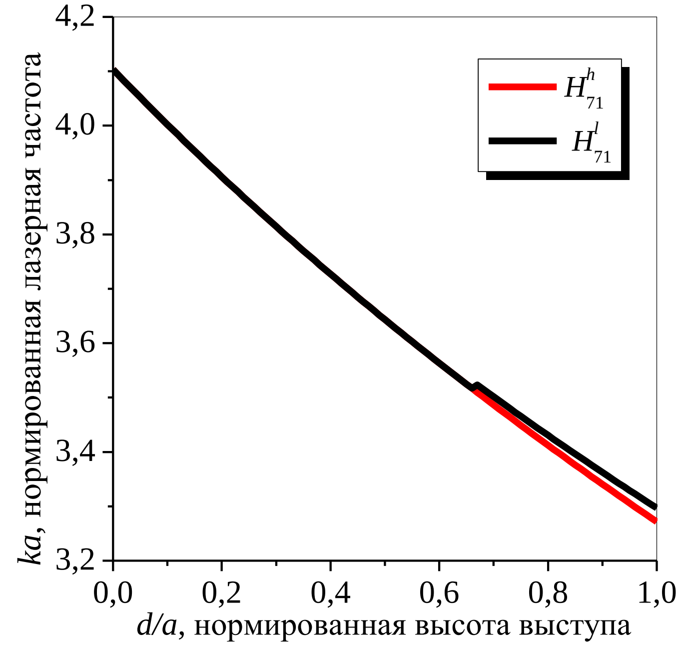
Near and far fields of the lowest modes in a spiral microlaser

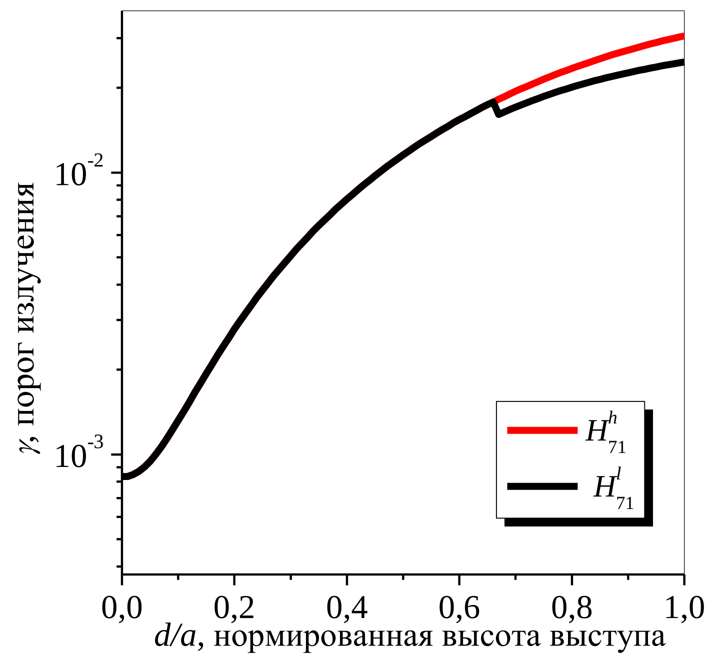
$d = 0.3a,\ \beta = \pi/100,\ \alpha_i = 2.63,\ \alpha_e = 1,\ N = 100$ **Small step**



The fields of the lowest modes are not much perturbed by the step on the contour. The most interesting is the observation that the modes of doublet are rotated differently.

Splitting of modes to doublets: whispering gallery modes



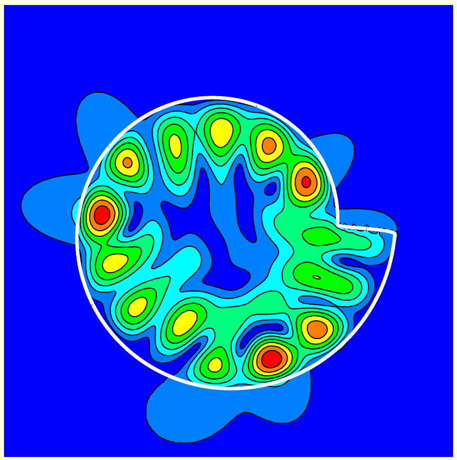


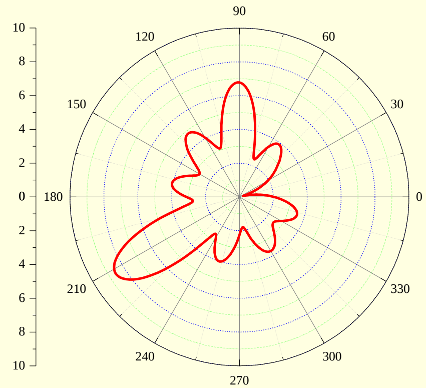
Mode of the H_{71} doublet frequencies and thresholds as a function of the step height in the spiral microlaser; $\beta=\pi/100$, n=400, α=2.63

Near and far fields of the WG modes in a spiral microlaser

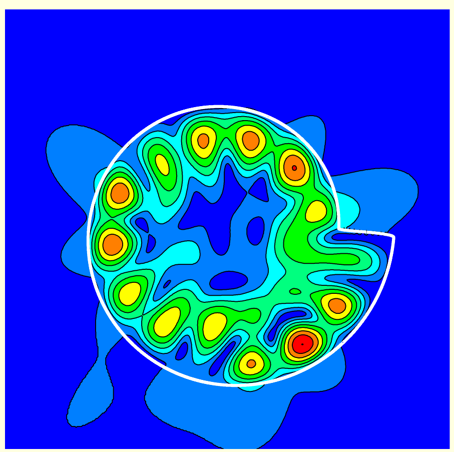
$d = 1.0a,\ \beta = \pi/100,\ \alpha_i = 2.63,\ \alpha_e = 1$

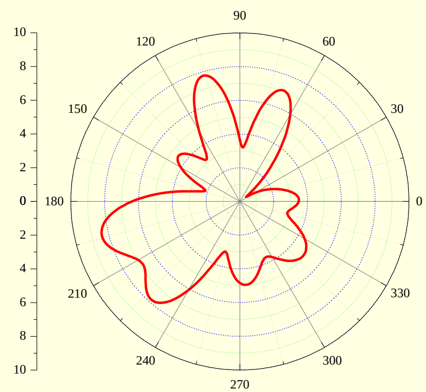
$H^{l}_{7,1}$





$H^{h}_{7,1}$





Large step

The fields of the WG mode doublet are strongly perturbed by the step on the contour. Main beam appears. Directivity of emission ~ 5

Conclusions

- **The eigenvalue/eigenmode problem for an open resonator has been reformulated to account for the presence of the active region and, based on this, to determine both their frequencies and thresholds of lasing.**
- **A link has been established between the dielectric resonator mode threshold of lasing and its Q-factor and overlap coefficient of its E-field with the active region.**
- **Efficient numerical algorithms have been developed for computing the modes of stand-alone circular resonators, cyclic photonic molecules of them, and 2-D resonators with arbitrary smooth contours.**
- **It has been found that in a stand-alone circular resonator (2-D disk model) there are lower modes with high thresholds and WG modes with exponentially low thresholds.**
- **It has been shown that by gathering the microdisks into cyclic photonic molecules one can lower the thresholds of lasing of supermodes (coupled modes) built on the lower modes and on the WG modes.**
- **It has been revealed that in the structure active disk + passive ringlike reflector the thresholds of any supermode can be both lower and higher that in a stand-alone disk. This depends on the overlap between the E-field and the active region. The threshold jumps up if the field is pulled into the passive regions.**
- **It has been demonstrated that deforming the disk into a spiral resonator one splits the modes into doublets of standing waves. Here, the directivity of emission of the WG modes increases however at the expense of higher thresholds. The main factor is the step height in terms of wavelengths.**