

## 1 Shane's Part

In two dimensions, one can model the macroscopic behavior of the system by

$$\frac{\partial \rho}{\partial t} = D \left( \frac{\partial^2 \rho}{\partial^2 x} + \frac{\partial^2 \rho}{\partial^2 y} \right) + \rho(1 - \rho), \quad (1)$$

where  $\rho(t, x, y)$  is the density of cancer cell at time  $t$  at location  $(x, y)$ , and  $D$  is the diffusion coefficient. One can model the microscopic behavior of the system by

$$dX_t = \sigma dB_t, \quad (2)$$

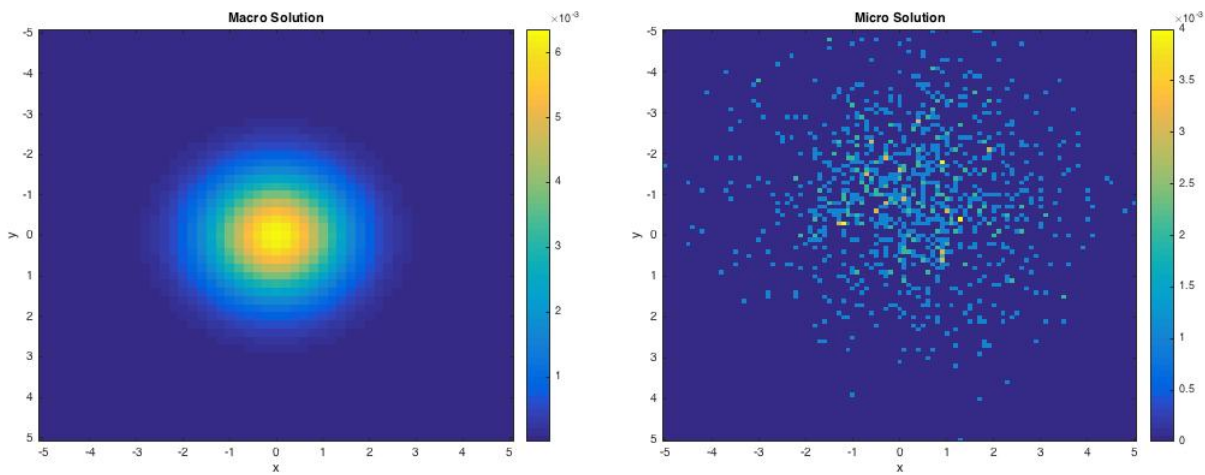
where  $\sigma \in \mathbb{R}$ , and  $B_t$  denotes a standard two-dimensional Brownian motion, with  $B(0) = (0, 0)$ . Thus,

$$\int_0^{\Delta t} dX_t = \sigma \int_0^{\Delta t} dB_t \implies x_{\Delta t} - x_0 = \sigma \mathbf{Z} \quad (3)$$

where  $\mathbf{Z} \sim \mathcal{N}_2(0, \Delta t \mathbb{I}_2)$ , where  $\mathbb{I}_p$  is the  $p \times p$  identity matrix. We conclude that

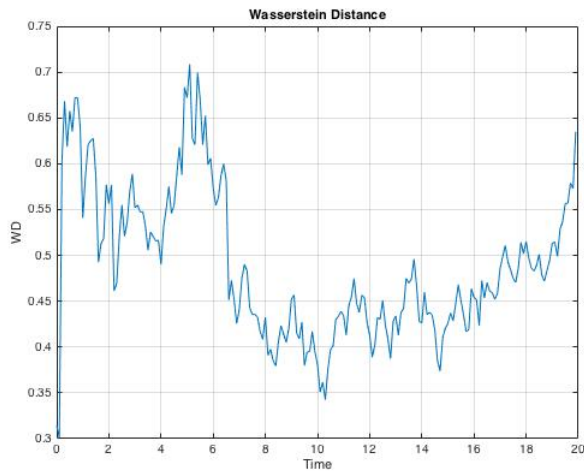
$$x_{\Delta t} = x_0 + \sigma \sqrt{\Delta t} \mathbf{W}, \quad (4)$$

where  $\mathbf{W} \sim \mathcal{N}_2(0, \mathbb{I}_2)$ . In Figures 1a and 1b we plot the level curves of the solution to (1) and (4) respectively. We use  $N = 10^3, L = M = 5, \Delta x = \Delta y = .2$  at  $t = 0$ , where  $x \in [-L, L], y \in [-M, M]$ . In Figure 1c, we plot the WD between Micro and Macro as a function of  $t$ , where we use  $N = 10^3, \Delta t = .01, t = 1, L = M = 2, \Delta x = \Delta y = .5$



(a) Macro Solution

(b) Micro Solution



(c) Wasserstein Distance

Figure 1: 2D Figures