1 Shane's Part

In two dimensions, one can model the macroscopic behavior of the system by

$$\frac{\partial \rho}{\partial t} = D\Big(\frac{\partial^2 \rho}{\partial^2 x} + \frac{\partial^2 \rho}{\partial^2 y}\Big) + \rho(1-\rho), \tag{1}$$

where $\rho(t, x, y)$ is the density of cancer cell at time t at location (x, y), and D is the diffusion coefficient. One can model the microscopic behavior of the system by

$$dX_t = \sigma dB_t, \tag{2}$$

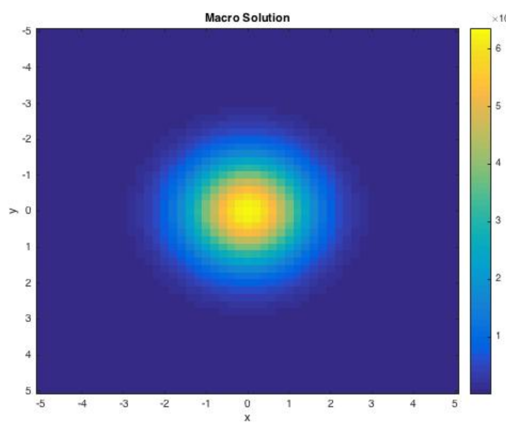
where $\sigma = \sqrt{2D}$, and B_t denotes a standard two-dimensional Brownian motion, with $B(0) = (0, 0)$. Thus,

$$\int_0^{\Delta t} dX_t = \sigma \int_0^{\Delta t} dB_t \implies x_{\Delta t} - x_0 = \sigma \mathbf{Z} \tag{3}$$

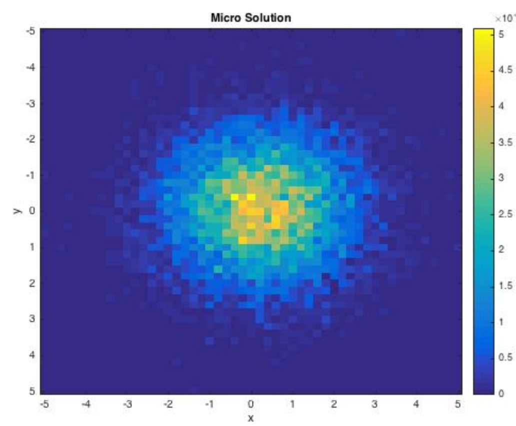
where $\mathbf{Z} \sim \mathcal{N}_2((0, 0), \Delta t \mathbb{I}_2)$, where $\mathbb{I}_p$is the $p \times p$ identity matrix. We conclude that

$$x_{\Delta t} = x_0 + \sigma\sqrt{\Delta t}\mathbf{W}, \tag{4}$$

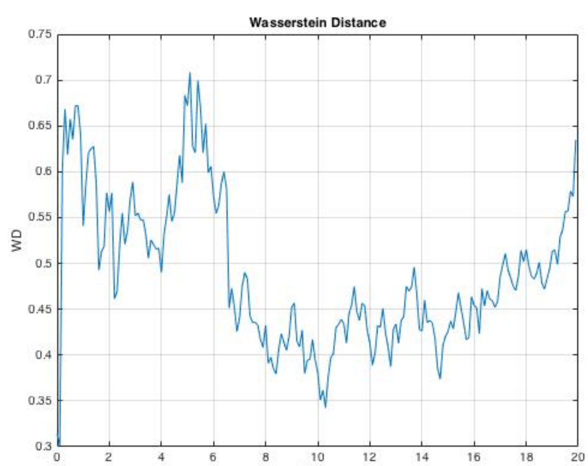
where $\mathbf{W} \sim \mathcal{N}_2((0, 0), \mathbb{I}_2)$. In Figures 1a and 1b we plot the level curves of the solution to (1) and (4) respectively. We use $N = 10^4, L = M = 5, \Delta x = \Delta y = .2$ at $t = 0$, where $x \in [-L, L],\ y \in [-M, M]$. In Figure 1c, we plot the Wasserstein Distance between the solutions of (1) and (2) as a function of t, where we use $N = 10^3,\ \Delta t = .05,\ t = 20, L = M = 2,\ \Delta x = \Delta y = .5$.



(a) Macro Solution



(b) Micro Solution



(c) Wasserstein Distance

Figure 1: 2D WD Data