1. (a) With A fixed and B random, the ANOVA model is

$$Y_{ijk} = \mu_{\cdot} + \alpha_i + \beta_j + (\alpha\beta)_{ij} + \epsilon_{ijk},$$

where $\mu_{\cdot}$ and α_i are fixed parameters, and β_j and $(\alpha\beta)_{ij}$ are random variables with mean 0 and variances σ^2_β and $\frac{a-1}{a}\sigma^2_{\alpha\beta}$, respectively.

(b) With $a = 3$, $b = 4$, and $n = 4$, the mean square table is

Mean Sq	df	Expected MS (A fixed, B random)
MSA	$a-1$	$\sigma^2 + n\sigma^2_{\alpha\beta} + \frac{bn}{a-1}\sum_i \alpha_i^2$
MSB	$b-1$	$\sigma^2 + an\sigma^2_\beta$
$MSAB$	$(a-1)(b-1)$	$\sigma^2 + n\sigma^2_{\alpha\beta}$
MSE	$ab(n-1)$	σ^2

(c) The hypotheses are $H_0 : \sigma^2_{\alpha\beta} = 0$ versus $H_a : \sigma^2_{\alpha\beta} > 0$. The test statistic is 0.064 with p-value 0.9988. This is not significant evidence of interaction between number of coats and batches.

(d) The hypotheses are $H_0 : \alpha_1 = \alpha_2 = \alpha_3$ versus Ha : at least one $\neq$. From the table of expected mean squares, the F ratio for this test is $MSA/MSAB = 75.194/0.309 = 243.35 > f_{2,6}(.95) = 5.14$. So, we have significant evidence of an effect due to number of coats.

(e) The hypotheses are $H_0 : \sigma^2_\beta = 0$ versus $Ha : \sigma^2_\beta > 0$. From the table of expected mean squares, the F ratio for this test is $MSB/MSE = 50.95/4.82 = 10.56 > f_{3,36}(.95) = 2.87$. So, we have significant evidence of an effect due to batches.

(f) The estimates for these group means are found by

```
tapply(y,A,mean)
       1        2        3
73.10625 76.79375 76.92500
```

and the standard error for each pair is $\sqrt{2MSAB/bn} = .1965$. With Bonferroni $\alpha' = .10/2 = .05$ (for two intervals), the critical value is $t_6(1 - .05/2) = 2.45$. Comparing 6 vs 8, the mean difference in market value is within

$$73.10625 - 76.79375 \pm 2.45(.1965) = (-4.17, -3.21)$$

and, simultaneously, the mean difference comparing 6 vs 10 is within

$$73.10625 - 76.925 \pm 2.45(.1965) = (-4.30, -3.34)$$

with 90% confidence.

2. (a) The nested model with random effects is

$$Y_{ijk} = \mu_{\cdot\cdot} + \alpha_i + \beta_{j(i)} + \epsilon_{ijk}$$

where $\mu_{\cdot\cdot}$ is the overall mean, $\alpha_i \sim N(0, \sigma^2_\alpha)$, $\beta_{j(i)} \sim N(0, \sigma^2_\beta)$, and $\epsilon_{ijk} \sim N(0, \sigma^2)$.

(b) Since batches are nested within sites, the last two SS combine to give the SS for batches:

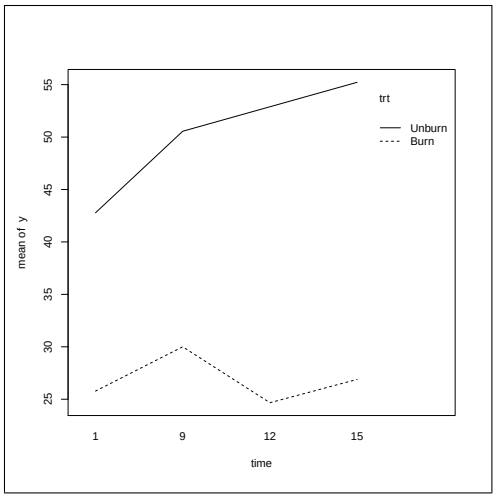
$$SSB(A) = 0.01153 + 0.44249 = .45402$$

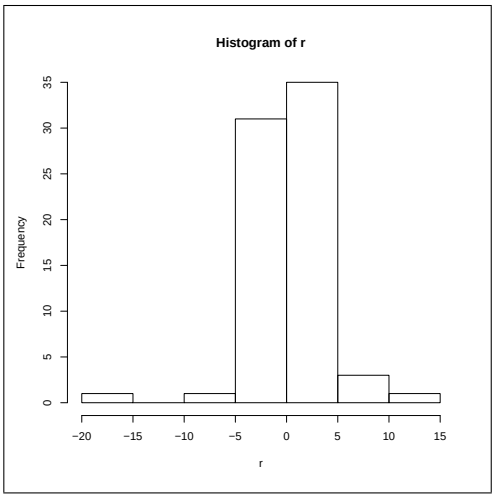
Now, with $a = 2$, $b = 3$, and $n = 5$,

Source	df	Mean Sq	Expected MS
A	1	0.01825	$\sigma^2 + 5\sigma^2_\beta + 15\sigma^2_\alpha$
B(A)	4	0.11351	$\sigma^2 + 5\sigma^2_\beta$
Error	24	0.01209	σ^2

(c) The test statistic for site is $F = MSA/MSB(A) = .01825/.11351 = .161$, which is less than its critical value $f_{1,4}(.95) = 7.71$. So, there is not a significant effect due to sites. The test statistic and critical value for batches would be $F = MSB(A)/MSE = .11351/0.01209 = 9.39$, which exceeds $f_{4,24}(.95) = 2.78$, so we have significant evidence for an effect (variation) due to batches.

3. (a) The interaction plot (left below) shows two lines with rather different slopes; the unburned response seems to increase over time, while the burned response seems to fluctuate in both directions.





(b) The repeated measures model for this situation is

$$Y_{ijk} = \rho_{i(j)} + \alpha_j + \beta_k + (\alpha\beta)_{jk} + \epsilon_{ijk}$$

where $\rho_{i(j)} \sim N(0, \sigma^2_\rho)$ is the random effect due to plot, nested within the burn groups, α_j and β_k are each fixed effects due to burn group and time, respectively, with $(\alpha\beta)_{jk}$ their interaction term. Each combination of these occurs with each other, so they are crossed.

(c) This models assumes that the covariance between such observations is σ^2_ρ, regardless of how close together in time the observations are. While it is appropriate to model some nonzero covariance for these (since they are together in the same plot, it may not be appropriate to assume time points far apart are equally related as those closer together.

(d) The histogram (above right) shows at least one outlier in the lower end that is not consistent with normality. Most of the observations seem to be clustered around the mean, but this would appear different if more bins were chosen for the histogram.

(e) With the plot term serving as error (not the same error as those for time), the test statistic is

$$\frac{9964}{222.8} = 44.72$$

with critical value `qf(.95,1,3)=10.13`, which is significant evidence of an effect between burn and unburned groups.