chapter

9 Testing the Difference Between Two Means, Two Variances, and Two Proportions

Outline

Objectives

After completing this chapter, you should be able to

1. Test the difference between two large sample means, using the *z* test.
2. Test the difference between two variances or standard deviations.
3. Test the difference between two means for small independent samples.
4. Test the difference between two means for small dependent samples.
5. Test the difference between two proportions.

Statistics Today

To Vaccinate or Not to Vaccinate? Small or Large?

Influenza is a serious disease among the elderly, especially those living in nursing homes. Those residents are more susceptible to influenza than elderly persons living in the community because the former are usually older and more debilitated, and they live in a closed environment where they are exposed more so than community residents to the virus if it is introduced into the home. Three researchers decided to investigate the use of vaccine and its value in determining outbreaks of influenza in small nursing homes.

These researchers surveyed 83 licensed homes in seven counties in Michigan. Part of the study consisted of comparing the number of people being vaccinated in small nursing homes (100 or fewer beds) with the number in larger nursing homes (more than 100 beds). Unlike the statistical methods presented in Chapter 8, these researchers used the techniques explained in this chapter to compare two sample proportions to see if there was a significant difference in the vaccination rates of patients in small nursing homes compared to those in larger nursing homes.

Source: Nancy Arden, Arnold S. Monto, and Suzanne E. Ohmit, "Vaccine Use and the Risk of Outbreaks in a Sample of Nursing Homes during an Influenza Epidemic," *American Journal of Public Health* 85, no. 3 (March 1995), pp. 399–401. Copyright 1995 by the American Public Health Association.

9–1 Introduction

The basic concepts of hypothesis testing were explained in Chapter 8. With the z, t, and χ^2 tests, a sample mean, variance, or proportion can be compared to a specific population mean, variance, or proportion to determine whether the null hypothesis should be rejected.

There are, however, many instances when researchers wish to compare two sample means, using experimental and control groups. For example, the average lifetimes of two different brands of bus tires might be compared to see whether there is any difference in tread wear. Two different brands of fertilizer might be tested to see whether one is better than the other for growing plants. Or two brands of cough syrup might be tested to see whether one brand is more effective than the other.

In the comparison of two means, the same basic steps for hypothesis testing shown in Chapter 8 are used, and the z and t tests are also used. When comparing two means by using the t test, the researcher must decide if the two samples are *independent* or *dependent.* The concepts of independent and dependent samples will be explained in Sections 9–4 and 9–5.

Furthermore, when the samples are independent, there are two different formulas that can be used depending on whether or not the variances are equal. To determine if the variances are equal, use the F test shown in Section 9–3. Finally, the z test can be used to compare two proportions, as shown in Section 9–6.

9–2 Testing the Difference Between Two Means: Large Samples

Objective 1. Test the difference between two large sample means, using the z test.

Suppose a researcher wishes to determine whether there is a difference in the average age of nursing students who enroll in a nursing program at a community college and those who enroll in a nursing program at a university. In this case, the researcher is not interested in the average age of all beginning nursing students; instead, he is interested in *comparing* the means of the two groups. His research question is: Does the mean age of nursing students who enroll at a community college differ from the mean age of nursing students who enroll at a university? Here, the hypotheses are

$$H_0\colon \mu_1 = \mu_2$$
$$H_1\colon \mu_1 \neq \mu_2$$

where

μ_1 = mean age of all beginning nursing students at the community college
μ_2 = mean age of all beginning nursing students at the university

Another way of stating the hypotheses for this situation is

$$H_0\colon \mu_1 - \mu_2 = 0$$
$$H_1\colon \mu_1 - \mu_2 \neq 0$$

If there is no difference in population means, subtracting them will give a difference of zero. If they are different, subtracting will give a number other than zero. Both methods of stating hypotheses are correct; however, the first method will be used in this book.

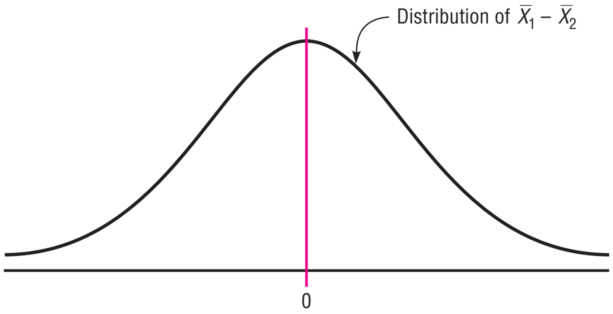
Assumptions for the Test to Determine the Difference Between Two Means

1. The samples must be independent of each other. That is, there can be no relationship between the subjects in each sample.
2. The populations from which the samples were obtained must be normally distributed, and the standard deviations of the variable must be known, or the sample sizes must be greater than or equal to 30.

The theory behind testing the difference between two means is based on selecting pairs of samples and comparing the means of the pairs. The population means need not be known.

All possible pairs of samples are taken from populations. The means for each pair of samples are computed and then subtracted, and the differences are plotted. If both populations have the same mean, then most of the differences will be zero or close to zero. Occasionally, there will be a few large differences due to chance alone, some positive and others negative. If the differences are plotted, the curve will be shaped like the normal distribution and have a mean of zero, as shown in Figure 9–1.

Figure 9–1
Differences of Means of Pairs of Samples



Unusual Stats

Adult children who live with their parents spend more than 2 hours a day doing household chores. According to a study, daughters contribute about 17 hours a week and sons about 14.4 hours. Source: Reprinted with permission from *Psychology Today* magazine. Copyright © 1995 (Sussex Publishers, Inc.).

The variance of the difference $\bar{X}_1 - \bar{X}_2$ is equal to the sum of the individual variances of $\bar{X}_1$ and $\bar{X}_2$. That is,

$$\sigma^2_{\bar{X}_1 - \bar{X}_2} = \sigma^2_{\bar{X}_1} + \sigma^2_{\bar{X}_2}$$

where $\sigma^2_{\bar{X}_1} = \dfrac{\sigma_1^2}{n_1}$ and $\sigma^2_{\bar{X}_2} = \dfrac{\sigma_2^2}{n_2}$

So the standard deviation of $\bar{X}_1 - \bar{X}_2$ is

$$\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}$$

Formula for the *z* Test for Comparing Two Means from Independent Populations

$$z = \frac{(\bar{X}_1 - \bar{X}_2) - (\mu_1 - \mu_2)}{\sqrt{\dfrac{\sigma_1^2}{n_1} + \dfrac{\sigma_2^2}{n_2}}}$$

This formula is based on the general format of

$$\text{Test value} = \frac{(\text{observed value}) - (\text{expected value})}{\text{standard error}}$$

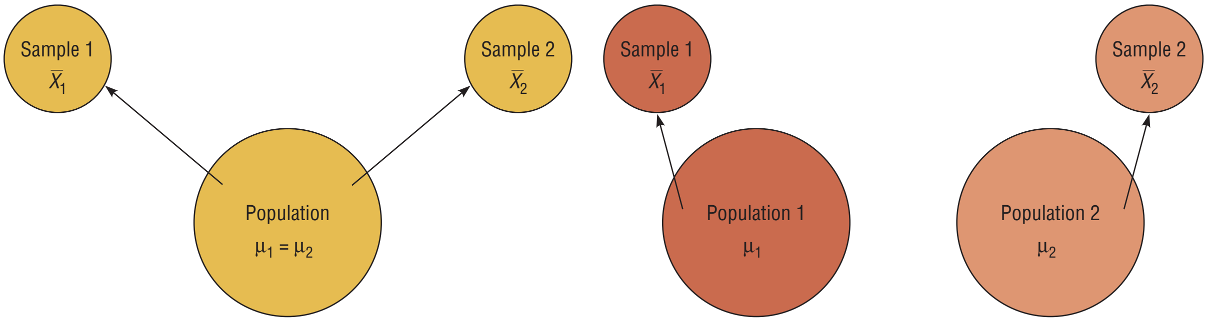
where $\bar{X}_1 - \bar{X}_2$ is the observed difference, and the expected difference $\mu_1 - \mu_2$ is zero when the null hypothesis is $\mu_1 = \mu_2$, since that is equivalent to $\mu_1 - \mu_2 = 0$. Finally, the standard error of the difference is

$$\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}$$

In the comparison of two sample means, the difference may be due to chance, in which case the null hypothesis will not be rejected, and the researcher can assume that the means of the populations are basically the same. The difference in this case is not significant. See Figure 9–2(a). On the other hand, if the difference is significant, the null hypothesis is rejected and the researcher can conclude that the population means are different. See Figure 9–2(b).

Figure 9–2

Hypothesis-Testing Situations in the Comparison of Means



(a) Difference is not significant

Do not reject H_0: $\mu_1 = \mu_2$ since $\overline{X}_1 - \overline{X}_2$ is not significant.

(b) Difference is significant

Reject H_0: $\mu_1 = \mu_2$ since $\overline{X}_1 - \overline{X}_2$ is significant.

These tests can also be one-tailed, using the following hypotheses:

Right-tailed			Left-tailed		
H_0: $\mu_1 \le \mu_2$ H_1: $\mu_1 > \mu_2$	or	H_0: $\mu_1 - \mu_2 \le 0$ H_1: $\mu_1 - \mu_2 > 0$	H_0: $\mu_1 \ge \mu_2$ H_1: $\mu_1 < \mu_2$	or	H_0: $\mu_1 - \mu_2 \ge 0$ H_1: $\mu_1 - \mu_2 < 0$

The same critical values used in Section 8–3 are used here. They can be obtained from Table E in Appendix C.

If σ_1^2 and σ_2^2 are not known, the researcher can use the variances obtained from each sample s_1^2 and s_2^2, but both sample sizes must be 30 or more. The formula then is

$$z = \frac{(\overline{X}_1 - \overline{X}_2) - (\mu_1 - \mu_2)}{\sqrt{\dfrac{s_1^2}{n_1} + \dfrac{s_2^2}{n_2}}}$$

provided that $n_1 \ge 30$ and $n_2 \ge 30$.

When one or both sample sizes are less than 30 and σ_1 and σ_2 are unknown, the t test must be used, as shown in Section 9–4.

The basic format for hypothesis testing using the traditional method is reviewed here.

STEP 1 State the hypotheses and identify the claim.

STEP 2 Find the critical value(s).

STEP 3 Compute the test value.

STEP 4 Make the decision.

STEP 5 Summarize the results.

Example 9–1

A survey found that the average hotel room rate in New Orleans is \$88.42 and the average room rate in Phoenix is \$80.61. Assume that the data were obtained from two samples of 50 hotels each and that the standard deviations were \$5.62 and \$4.83, respectively. At $\alpha = 0.05$, can it be concluded that there is a significant difference in the rates?

Source: *USA TODAY.*

Solution

STEP 1 State the hypotheses and identify the claim.

$$H_0\colon \mu_1 = \mu_2 \qquad \text{and} \qquad H_1\colon \mu_1 \neq \mu_2 \text{ (claim)}$$

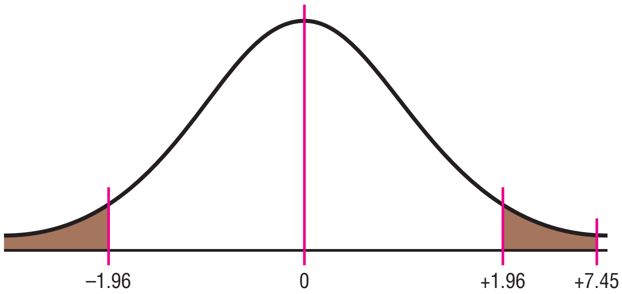
STEP 2 Find the critical values. Since $\alpha = 0.05$, the critical values are $+1.96$ and -1.96.

STEP 3 Compute the test value.

$$z = \frac{(\overline{X}_1 - \overline{X}_2) - (\mu_1 - \mu_2)}{\sqrt{\dfrac{s_1^2}{n_1} + \dfrac{s_2^2}{n_2}}} = \frac{(88.42 - 80.61) - 0}{\sqrt{\dfrac{5.62^2}{50} + \dfrac{4.83^3}{50}}} = 7.45$$

STEP 4 Make the decision. Reject the null hypothesis at $\alpha = 0.05$, since $7.45 > 1.96$. See Figure 9–3.

Figure 9–3
Critical and Test Values for Example 9–1



STEP 5 Summarize the results. There is enough evidence to support the claim that the means are not equal. Hence, there is a significant difference in the rates.

The *P*-values for this test can be determined by using the same procedure shown in Section 8–3. For example, if the test value for a two-tailed test is 1.40, then the *P*-value obtained from Table E is 0.1616. This value is obtained by looking up the area for $z = 1.40$, which is 0.4192. Then 0.4192 is subtracted from 0.5000 to get 0.0808. Finally, this value is doubled to get 0.1616 since the test is two-tailed. If $\alpha = 0.05$, the decision would be to not reject the null hypothesis, since $P\text{-value} > \alpha$.

The *P*-value method for hypothesis testing for this chapter also follows the same format as stated in Chapter 8. The steps are reviewed here.

STEP 1 State the hypotheses and identify the claim.

STEP 2 Compute the test value.

STEP 3 Find the *P*-value.

STEP 4 Make the decision.

STEP 5 Summarize the results.

Example 9–2 illustrates these steps.

Example 9–2

A researcher hypothesizes that the average number of sports that colleges offer for males is greater than the average number of sports that colleges offer for females. A sample of the number of sports offered by colleges is shown. At $\alpha = 0.10$, is there enough evidence to support the claim?

Males					Females				
6	11	11	8	15	6	8	11	13	8
6	14	8	12	18	7	5	13	14	6
6	9	5	6	9	6	5	5	7	6
6	9	18	7	6	10	7	6	5	5
15	6	11	5	5	16	10	7	8	5
9	9	5	5	8	7	5	5	6	5
8	9	6	11	6	9	18	13	7	10
9	5	11	5	8	7	8	5	7	6
7	7	5	10	7	11	4	6	8	7
10	7	10	8	11	14	12	5	8	5

Source: *USA TODAY*.

Solution

STEP 1 State the hypotheses and identify the claim:

$$H_0: \mu_1 \leq \mu_2$$
$$H_1: \mu_1 > \mu_2 \text{ (claim)}$$

STEP 2 Compute the test value. Using a calculator or the formulas in Chapter 3, find the mean and standard deviation for each data set.

For the males $\overline{X}_1 = 8.6$ and $s_1 = 3.3$

For the females $\overline{X}_2 = 7.9$ and $s_2 = 3.3$

Substitute in the formula

$$z = \frac{(\overline{X}_1 - \overline{X}_2) - (\mu_1 - \mu_2)}{\sqrt{\dfrac{s_1^2}{n_1} + \dfrac{s_2^2}{n_2}}} = \frac{(8.6 - 7.9) - 0}{\sqrt{\dfrac{3.3^2}{50} + \dfrac{3.3^2}{50}}} = 1.06^*$$

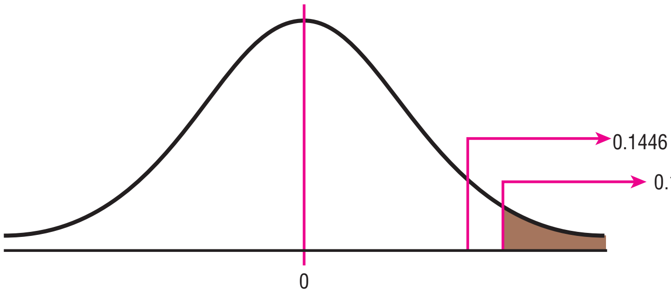
STEP 3 Find the *P*-value. For $z = 1.06$, the area is 0.3554, and $0.5000 - 0.3554 = 0.1446$ or a *P*-value of 0.1446.

STEP 4 Make the decision. Since the *P*-value is larger than α (that is, $0.1446 > 0.10$), the decision is to not reject the null hypothesis. See Figure 9–4.

STEP 5 Summarize the results. There is not enough evidence to support the claim that colleges offer more sports for males than they do for females.

Figure 9–4

P-Value and α Value for Example 9–2



*Note: Calculator results may differ due to rounding.

Sometimes, the researcher is interested in testing a specific difference in means other than zero. For example, he or she might hypothesize that the nursing students at a community college are, on the average, 3.2 years older than those at a university. In this case, the hypotheses are

$$H_0: \mu_1 - \mu_2 \leq 3.2 \qquad \text{and} \qquad H_1: \mu_1 - \mu_2 > 3.2$$

The formula for the z test is still

$$z = \frac{(\bar{X}_1 - \bar{X}_2) - (\mu_1 - \mu_2)}{\sqrt{\dfrac{\sigma_1^2}{n_1} + \dfrac{\sigma_2^2}{n_2}}}$$

where $\mu_1 - \mu_2$ is the hypothesized difference or expected value. In this case, $\mu_1 - \mu_2 = 3.2$.

Confidence intervals for the difference between two means can also be found. When one is hypothesizing a difference of 0, if the confidence interval contains 0, the null hypothesis is not rejected. If the confidence interval does not contain 0, the null hypothesis is rejected.

Confidence intervals for the difference between two means can be found by using this formula:

Formula for Confidence Interval for Difference Between Two Means: Large Samples

$$(\bar{X}_1 - \bar{X}_2) - z_{\alpha/2}\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}} < \mu_1 - \mu_2 < (\bar{X}_1 - \bar{X}_2) + z_{\alpha/2}\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}$$

When $n_1 \geq 30$ and $n_2 \geq 30$, s_1^2 and s_2^2 can be used in place of σ_1^2 and σ_2^2.

Example 9–3

Find the 95% confidence interval for the difference between the means for the data in Example 9–1.

Solution

Substitute in the formula, using $z_{\alpha/2} = 1.96$.

$$(\bar{X}_1 - \bar{X}_2) - z_{\alpha/2}\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}} < \mu_1 - \mu_2 < (\bar{X}_1 - \bar{X}_2) + z_{\alpha/2}\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}$$

$$(88.42 - 80.61) - 1.96\sqrt{\frac{5.62^2}{50} + \frac{4.83^2}{50}} < \mu_1 - \mu_2 < (88.42 - 80.61) + 1.96\sqrt{\frac{5.62^2}{50} + \frac{4.83^2}{50}}$$

$$7.81 - 2.05 < \mu_1 - \mu_2 < 7.81 + 2.05$$

$$5.76 < \mu_1 - \mu_2 < 9.86$$

Since the confidence interval does not contain zero, the decision is to reject the null hypothesis, which agrees with the previous result.

Exercises 9–2

1. Explain the difference between testing a single mean and testing the difference between two means.

2. When a researcher selects all possible pairs of samples from a population in order to find the difference between the means of each pair, what will be the shape of the distribution of the differences when the original distributions are normally distributed? What will be the mean of the distribution? What will be the standard deviation of the distribution?

3. What two assumptions must be met when one is using the z test to test differences between two means? When can the sample standard deviations s_1 and s_2 be used in place of the population standard deviations σ_1 and σ_2?

4. Show two different ways to state that the means of two populations are equal.

For Exercises 5 through 17, perform each of the following steps.

a. State the hypotheses and identify the claim.
b. Find the critical value(s).
c. Compute the test value.
d. Make the decision.
e. Summarize the results.

Use the traditional method of hypothesis testing unless otherwise specified.

5. A researcher wishes to see if the average length of the major rivers in the United States is the same as the average length of the major rivers in Europe. The data (in miles) of a sample of rivers are shown. At $\alpha = 0.01$, is there enough evidence to reject the claim?

United States			Europe		
729	560	434	481	724	820
329	332	360	532	357	505
450	2315	865	1776	1122	496
330	410	1036	1224	634	230
329	800	447	1420	326	626
600	1310	652	877	580	210
1243	605	360	447	567	252
525	926	722	824	932	600
850	310	430	634	1124	1575
532	375	1979	565	405	2290
710	545	259	675	454	
300	470	425			

Source: *The World Almanac and Book of Facts.*

6. A study was conducted to see if there was a difference between spouses and significant others in coping skills when living with or caring for a person with multiple sclerosis. These skills were measured by questionnaire responses. The results of the two groups are given on one factor, ambivalence. At $\alpha = 0.10$, is there a difference in the means of the two groups?

Spouses	Significant others
$\bar{X}_1 = 2.0$	$\bar{X}_2 = 1.7$
$s_1 = 0.6$	$s_2 = 0.7$
$n_1 = 120$	$n_2 = 34$

Source: Elsie E. Gulick, "Coping Among Spouses or Significant Others of Persons with Multiple Sclerosis," *Nursing Research.*

7. A medical researcher wishes to see whether the pulse rates of smokers are higher than the pulse rates of nonsmokers. Samples of 100 smokers and 100 nonsmokers are selected. The results are shown here. Can the researcher conclude, at $\alpha = 0.05$, that smokers have higher pulse rates than nonsmokers?

Smokers	Nonsmokers
$\bar{X}_1 = 90$	$\bar{X}_2 = 88$
$s_1 = 5$	$s_2 = 6$
$n_1 = 100$	$n_2 = 100$

8. At age 9 the average weight (21.3 kg) and the average height (124.5 cm) for both boys and girls are exactly the same. A random sample of 9-year-olds yielded these results. Estimate the mean difference in height between boys and girls with 95% confidence. Does your interval support the given claim?

	Boys	Girls
Sample size	60	50
Mean height, cm	123.5	126.2
Sample variance	98	120

Source: www.healthepic.com.

9. Using data from the "Noise Levels in an Urban Hospital" study cited in Exercise 19 in Exercise set 7–2, test the claim that the noise level in the corridors is higher than that in the clinics. Use $\alpha = 0.02$. The data are shown here.

Corridors	Clinics
$\bar{X}_1 = 61.2$ dBA	$\bar{X}_2 = 59.4$ dBA
$s_1 = 7.9$	$s_2 = 7.9$
$n_1 = 84$	$n_2 = 34$

Source: M. Bayo, A. Garcia, and A. Garcia, "Noise Levels in an Urban Hospital and Workers' Subjective Responses," *Archives of Environmental Health* 50, no. 3.

10. A real estate agent compares the selling prices of homes in two suburbs of Seattle to see whether there is a difference in price. The results of the study are shown here. Is there evidence to reject the claim that the average cost of a home in both locations is the same? Use $\alpha = 0.01$.

Suburb 1	Suburb 2
$\bar{X}_1 = \$63,255$	$\bar{X}_2 = \$59,102$
$s_1 = \$5,602$	$s_2 = \$4,731$
$n_1 = 35$	$n_2 = 40$

11. In a study of women science majors, the following data were obtained on two groups, those who left their profession within a few months after graduation (leavers) and those who remained in their profession after they graduated (stayers). Test the claim that those who stayed had a higher science grade-point average than those who left. Use $\alpha = 0.05$.

Leavers	Stayers
$\bar{X}_1 = 3.16$	$\bar{X}_2 = 3.28$
$s_1 = 0.52$	$s_2 = 0.46$
$n_1 = 103$	$n_2 = 225$

Source: Paula Rayman and Belle Brett, "Women Science Majors: What Makes a Difference in Persistence after Graduation?" *The Journal of Higher Education.*

12. A survey of 1000 students nationwide showed a mean ACT score of 21.4. A survey of 500 Ohio scores showed a mean of 20.8. If the standard deviation in each case is 3, can we conclude that Ohio is below the national average? Use $\alpha = 0.05$.

Source: Report of WFIN radio.

13. A school administrator hypothesizes that colleges spend more for male sports than they do for female sports. A sample of two different colleges is selected, and the annual expenses (in dollars) per student at each school are shown. At $\alpha = 0.01$, is there enough evidence to support the claim?

Males

7,040	6,576	1,664	12,919	8,605
22,220	3,377	10,128	7,723	2,063
8,033	9,463	7,656	11,456	12,244
6,670	12,371	9,626	5,472	16,175
8,383	623	6,797	10,160	8,725
14,029	13,763	8,811	11,480	9,544
15,048	5,544	10,652	11,267	10,126
8,796	13,351	7,120	9,505	9,571
7,551	5,811	9,119	9,732	5,286
5,254	7,550	11,015	12,403	12,703

Females

10,333	6,407	10,082	5,933	3,991
7,435	8,324	6,989	16,249	5,922
7,654	8,411	11,324	10,248	6,030
9,331	6,869	6,502	11,041	11,597
5,468	7,874	9,277	10,127	13,371
7,055	6,909	8,903	6,925	7,058
12,745	12,016	9,883	14,698	9,907
8,917	9,110	5,232	6,959	5,832
7,054	7,235	11,248	8,478	6,502
7,300	993	6,815	9,959	10,353

Source: *USA TODAY.*

14. Is there a difference in average miles traveled for each of two taxi companies during a randomly selected week? The data are shown. Use $\alpha = 0.05$. Assume that the populations are normally distributed. Use the *P*-value method.

Moonview Cab Company	Starlight Taxi Company
$\bar{X}_1 = 837$	$\bar{X}_2 = 753$
$\sigma_1 = 30$	$\sigma_2 = 40$
$n_1 = 35$	$n_2 = 40$

15. In the study cited in Exercise 11, the researchers collected the data shown here on a self-esteem questionnaire. At $\alpha = 0.05$, can it be concluded that there is a difference in the self-esteem scores of the two groups? Use the *P*-value method.

Leavers	Stayers
$\bar{X}_1 = 3.05$	$\bar{X}_2 = 2.96$
$s_1 = 0.75$	$s_2 = 0.75$
$n_1 = 103$	$n_2 = 225$

Source: Paula Rayman and Belle Brett, "Women Science Majors: What Makes a Difference in Persistence after Graduation?" *The Journal of Higher Education.*

16. The dean of students wants to see whether there is a significant difference in ages of resident students and commuting students. She selects a sample of 50 students from each group. The ages are shown here. At $\alpha = 0.05$, decide if there is enough evidence to reject the claim of no difference in the ages of the two groups. Use the standard deviations from the samples and the *P*-value method.

Resident students

22	25	27	23	26	28	26	24
25	20	26	24	27	26	18	19
18	30	26	18	18	19	32	23
19	19	18	29	19	22	18	22
26	19	19	21	23	18	20	18
22	21	19	21	21	22	18	20
19	23						

Commuter students

18	20	19	18	22	25	24	35
23	18	23	22	28	25	20	24
26	30	22	22	22	21	18	20
19	26	35	19	19	18	19	32
29	23	21	19	36	27	27	20
20	21	18	19	23	20	19	19
20	25						

17. Two groups of students are given a problem-solving test, and the results are compared. Find the 90% confidence interval of the true difference in means.

Mathematics majors	Computer science majors
$\bar{X}_1 = 83.6$	$\bar{X}_2 = 79.2$
$s_1 = 4.3$	$s_2 = 3.8$
$n_1 = 36$	$n_2 = 36$

18. It is commonly felt by people in northwest Ohio that on Interstate 75 drivers from Michigan drive faster than drivers from Ohio. To examine the claim, a class checks the speed of 50 Michigan drivers and finds the mean to be 67 miles per hour with a standard deviation of 8 miles per hour. They check the speed of 50 Ohio drivers and find the mean to be 64 miles per hour with a standard deviation of 7 miles per hour. If we let $\alpha = 0.05$, is the perception that Michigan drivers are faster correct?

19. Two brands of cigarettes are selected and their nicotine content is compared. The data are shown here. Find the 99% confidence interval of the true difference in the means.

Brand A	Brand B
$\bar{X}_1 = 28.6$ milligrams	$\bar{X}_2 = 32.9$ milligrams
$\sigma_1 = 5.1$ milligrams	$\sigma_2 = 4.4$ milligrams
$n_1 = 30$	$n_2 = 40$

20. Two brands of batteries are tested and their voltage is compared. The data follow. Find the 95% confidence interval of the true difference in the means. Assume that both variables are normally distributed.

Brand X	Brand Y
$\bar{X}_1 = 9.2$ volts	$\bar{X}_2 = 8.8$ volts
$\sigma_1 = 0.3$ volt	$\sigma_2 = 0.1$ volt
$n_1 = 27$	$n_2 = 30$

Extending the Concepts

21. A researcher claims that students in a private school have exam scores that are at most 8 points higher than those of students in public schools. Random samples of 60 students from each type of school are selected and given an exam. The results are shown. At $\alpha = 0.05$, test the claim.

Private school	Public school
$\bar{X}_1 = 110$	$\bar{X}_2 = 104$
$s_1 = 15$	$s_2 = 15$
$n_1 = 60$	$n_2 = 60$

Technology Step by Step

MINITAB Step by Step

Test the Difference Between Two Means: Large Independent Samples

MINITAB will calculate the test statistic and *P*-value for differences between the means for two populations when the population standard deviations are unknown.

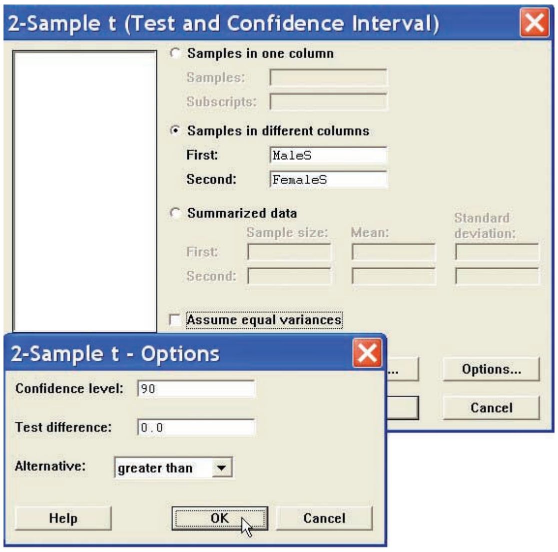
For Example 9–2, is the average number of sports for men higher than the average number for women?

1. Enter the data for Example 9–2 into C1 and C2. Name the columns **MaleS** and **FemaleS.**
2. Select **Stat>Basic Statistics>2-Sample t.**
3. Click the button for Samples in different columns.

There is one sample in each column.

4. Click in the box for First:. Double-click C1 MaleS in the list.

5. Click in the box for Second:, then double-click C2 FemaleS in the list. Do not check the box for Assume equal variances. MINITAB will use the large sample formula. The completed dialog box is shown.
6. Click [Options].
   a) Type in **90** for the Confidence level and **0** for the Test mean.
   b) Select greater than for the Alternative. This option affects the *P*-value. It must be correct.
7. Click [OK] twice. Since the *P*-value is greater than the significance level, 0.172 > 0.1, do not reject the null hypothesis.



Two-Sample T-Test and CI: MaleS, FemaleS

```
Two-sample  T  for  MaleS  vs FemaleS
          N   Mean  StDev  SE Mean
MaleS    50   8.56   3.26     0.46
FemaleS  50   7.94   3.27     0.46
Difference = mu (MaleS) - mu (FemaleS)
Estimate for difference: 0.620000
90% lower bound for difference: -0.221962
T-Test of difference = 0 (vs >): T-Value = 0.95 P-Value = 0.172  DF = 97
```

TI-83 Plus
Step by Step

Hypothesis Test for the Difference Between Two Means and z Distribution (Data)

1. Enter the data values into L_1 and L_2.
2. Press **STAT** and move the cursor to TESTS.
3. Press **3** for 2-SampZTest.
4. Move the cursor to Data and press **ENTER.**
5. Type in the appropriate values.
6. Move the cursor to the appropriate alternative hypothesis and press **ENTER.**
7. Move the cursor to Calculate and press **ENTER.**

Hypothesis Test for the Difference Between Two Means and z Distribution (Statistics)

1. Press **STAT** and move the cursor to TESTS.
2. Press **3** for 2-SampZTest.
3. Move the cursor to Stats and press **ENTER.**
4. Type in the appropriate values.
5. Move the cursor to the appropriate alternative hypothesis and press **ENTER.**
6. Move the cursor to Calculate and press **ENTER.**

Confidence Interval for the Difference Between Two Means and z Distribution (Data)

1. Enter the data values into L_1 and L_2.
2. Press **STAT** and move the cursor to TESTS.
3. Press **9** for 2-SampZInt.
4. Move the cursor to Data and press **ENTER.**
5. Type in the appropriate values.
6. Move the cursor to Calculate and press **ENTER.**

Confidence Interval for the Difference Between Two Means and z Distribution (Statistics)

1. Press **STAT** and move the cursor to TESTS.
2. Press **9** for 2-SampZInt.
3. Move the cursor to Stats and press **ENTER.**
4. Type in the appropriate values.
5. Move the cursor to Calculate and press **ENTER.**

Excel Step by Step

z Test for the Difference Between Two Means

Excel has a two-sample z test in its Data Analysis tools. To perform a z test for the difference between the means of two populations, given two independent samples, do this:

1. Enter the first sample data set in column A.
2. Enter the second sample data set in column B.
3. If the population variances are not known but $n \geq 30$ for both samples, use the formulas =VAR(A1:A*n*) and =VAR(B1:B*n*), where A*n* and B*n* are the last cells with data in each column, to find the variances of the sample data sets.
4. Select **Tools>Data Analysis** and choose z-Test: Two Sample for Means.
5. Enter the ranges for the data in columns A and B and enter **0** for Hypothesized Mean Difference.
6. If the population variances are known, enter them for Variable 1 and Variable 2. Otherwise, use the sample variances obtained in step 3.
7. Specify the confidence level Alpha.
8. Specify a location for output, and click [OK].

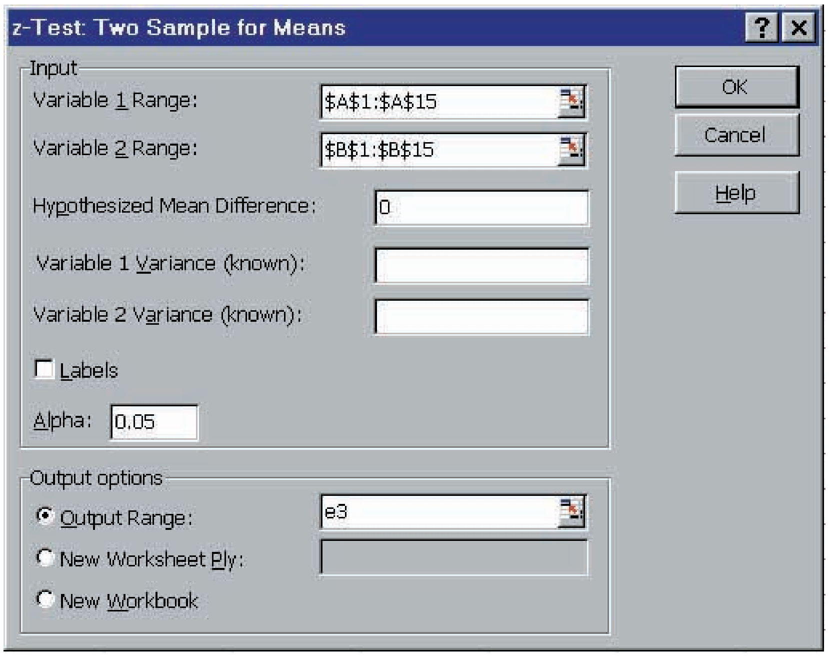
Example XL9–1

Test the claim that the two population means are equal, using the sample data provided here, at $\alpha = 0.05$. Assume the population variances are $\sigma_A^2 = 10.067$ and $\sigma_B^2 = 7.067$.

Set A	10	2	15	18	13	15	16	14	18	12	15	15	14	18	16
Set B	5	8	10	9	9	11	12	16	8	8	9	10	11	7	6

The two-sample z test dialog box is shown on next page (before the variances are entered); the results appear in the table that Excel generates. Note that the P-value and critical z value are

Two-Sample z Test Dialog Box



provided for both the one-tailed test and the two-tailed test. The P-values here are expressed in scientific notation: 7.09045E-06 = 7.09045×10^{-6} = 0.00000709045. Because this value is less than 0.05, we reject the null hypothesis and conclude that the population means are not equal.

z-Test: Two Sample for Means		
	Variable 1	*Variable 2*
Mean	14.06666667	9.266666667
Known Variance	10.067	7.067
Observations	15	15
Hypothesized Mean Difference	0	
z	4.491149228	
P(Z<=z) one-tail	3.54522E-06	
z Critical one-tail	1.644853	
P(Z<=z) two-tail	7.09045E-06	
z Critical two-tail	1.959961082	

9–3 Testing the Difference Between Two Variances

In addition to comparing two means, statisticians are interested in comparing two variances or standard deviations. For example, is the variation in the temperatures for a certain month for two cities different?

In another situation, a researcher may be interested in comparing the variance of the cholesterol of men with the variance of the cholesterol of women. For the comparison of two variances or standard deviations, an ***F* test** is used. The F test should not be confused with the chi-square test, which compares a single sample variance to a specific population variance, as shown in Chapter 8.

Objective 2. Test the difference between two variances or standard deviations.

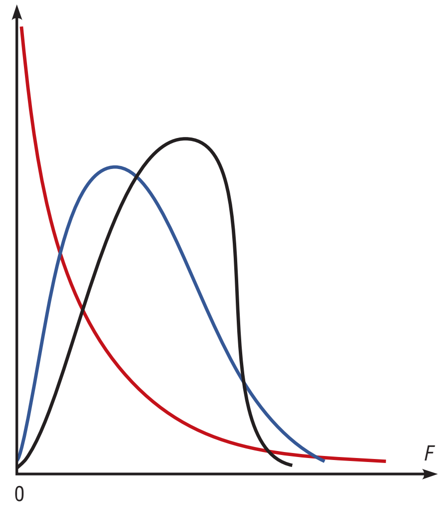
If two independent samples are selected from two normally distributed populations in which the variances are equal ($\sigma_1^2 = \sigma_2^2$) and if the variances s_1^2 and s_2^2 are compared as $\frac{s_1^2}{s_2^2}$, the sampling distribution of the variances is called the ***F* distribution.**

Characteristics of the *F* Distribution

1. The values of F cannot be negative, because variances are always positive or zero.
2. The distribution is positively skewed.
3. The mean value of F is approximately equal to 1.
4. The F distribution is a family of curves based on the degrees of freedom of the variance of the numerator and the degrees of freedom of the variance of the denominator.

Figure 9–5 shows the shapes of several curves for the F distribution.

Figure 9–5
The *F* Family of Curves



Formula for the *F* Test

$$F = \frac{s_1^2}{s_2^2}$$

where s_1^2 is the larger of the two variances.

The F test has two terms for the degrees of freedom: that of the numerator, $n_1 - 1$, and that of the denominator, $n_2 - 1$, where n_1 is the sample size from which the larger variance was obtained.

When one is finding the F test value, *the larger of the variances is placed in the numerator of the F formula;* this is not necessarily the variance of the larger of the two sample sizes.

Table H in Appendix C gives the F critical values for $\alpha = 0.005, 0.01, 0.025, 0.05,$ and 0.10 (each α value involves a separate table in Table H). These are one-tailed values; if a two-tailed test is being conducted, then the $\alpha/2$ value must be used. For example, if a two-tailed test with $\alpha = 0.05$ is being conducted, then the $0.05/2 = 0.025$ table of Table H should be used.

Example 9–4

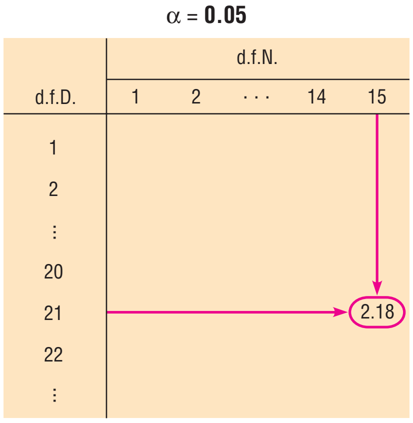
Find the critical value for a right-tailed F test when $\alpha = 0.05$, the degrees of freedom for the numerator (abbreviated d.f.N.) are 15, and the degrees of freedom for the denominator (d.f.D.) are 21.

Solution

Since this test is right-tailed with $\alpha = 0.05$, use the 0.05 table. The d.f.N. is listed across the top, and the d.f.D. is listed in the left column. The critical value is found where the row and column intersect in the table. In this case, it is 2.18. See Figure 9–6.

Figure 9–6

Finding the Critical Value in Table H for Example 9–4



As noted previously, when the F test is used, the larger variance is always placed in the numerator of the formula. When one is conducting a two-tailed test, α is split; and even though there are two values, only the right tail is used. The reason is that the F test value is always greater than or equal to 1.

Example 9–5

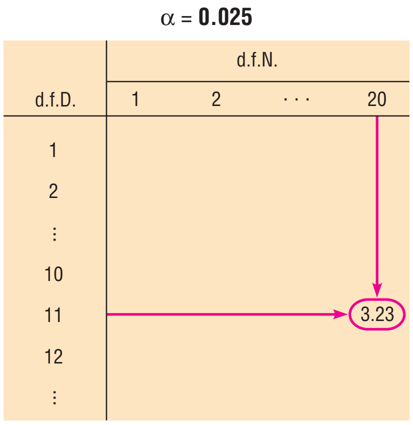
Find the critical value for a two-tailed F test with $\alpha = 0.05$ when the sample size from which the variance for the numerator was obtained was 21 and the sample size from which the variance for the denominator was obtained was 12.

Solution

Since this is a two-tailed test with $\alpha = 0.05$, the $0.05/2 = 0.025$ table must be used. Here, d.f.N. $= 21 - 1 = 20$, and d.f.D. $= 12 - 1 = 11$; hence, the critical value is 3.23. See Figure 9–7.

Figure 9–7

Finding the Critical Value in Table H for Example 9–5



When the degrees of freedom values cannot be found in the table, the closest value on the smaller side should be used. For example, if d.f.N. = 14, this value is between the given table values of 12 and 15; therefore, 12 should be used, to be on the safe side.

When one is testing the equality of two variances, these hypotheses are used:

Right-tailed	Left-tailed	Two-tailed
H_0: $\sigma_1^2 \leq \sigma_2^2$	H_0: $\sigma_1^2 \geq \sigma_2^2$	H_0: $\sigma_1^2 = \sigma_2^2$
H_1: $\sigma_1^2 > \sigma_2^2$	H_1: $\sigma_1^2 < \sigma_2^2$	H_1: $\sigma_1^2 \neq \sigma_2^2$

There are four key points to keep in mind when one is using the F test.

Notes for the Use of the *F* Test

1. The larger variance should always be designated as s_1^2 and placed in the numerator of the formula.

$$F = \frac{s_1^2}{s_2^2}$$

2. For a two-tailed test, the α value must be divided by 2 and the critical value placed on the right side of the F curve.
3. If the standard deviations instead of the variances are given in the problem, they must be squared for the formula for the F test.
4. When the degrees of freedom cannot be found in Table H, the closest value on the smaller side should be used.

Assumptions for Testing the Difference Between Two Variances

1. The populations from which the samples were obtained must be normally distributed. (*Note:* The test should not be used when the distributions depart from normality.)
2. The samples must be independent of each other.

Remember also that in tests of hypotheses using the traditional method, these five steps should be taken:

STEP 1 State the hypotheses and identify the claim.

STEP 2 Find the critical value.

STEP 3 Compute the test value.

STEP 4 Make the decision.

STEP 5 Summarize the results.

Example 9–6

A medical researcher wishes to see whether the variance of the heart rates (in beats per minute) of smokers is different from the variance of heart rates of people who do not smoke. Two samples are selected, and the data are as shown. Using $\alpha = 0.05$, is there enough evidence to support the claim?

Smokers	Nonsmokers
$n_1 = 26$	$n_2 = 18$
$s_1^2 = 36$	$s_2^2 = 10$

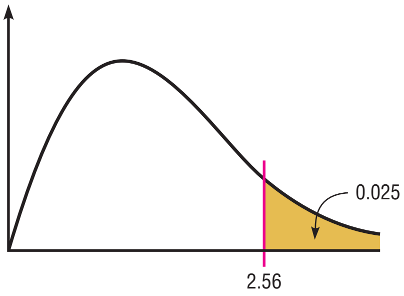
Solution

STEP 1 State the hypotheses and identify the claim.

$$H_0: \sigma_1^2 = \sigma_2^2 \quad \text{and} \quad H_1: \sigma_1^2 \neq \sigma_2^2 \text{ (claim)}$$

STEP 2 Find the critical value. Use the 0.025 table in Table H since $\alpha = 0.05$ and this is a two-tailed test. Here, d.f.N. = 26 − 1 = 25, and d.f.D. = 18 − 1 = 17. The critical value is 2.56 (d.f.N. = 24 was used). See Figure 9–8.

Figure 9–8
Critical Value for Example 9–6



STEP 3 Compute the test value.

$$F = \frac{s_1^2}{s_2^2} = \frac{36}{10} = 3.6$$

STEP 4 Make the decision. Reject the null hypothesis, since $3.6 > 2.56$.

STEP 5 Summarize the results. There is enough evidence to support the claim that the variance of the heart rates of smokers and nonsmokers is different.

Example 9–7

An instructor hypothesizes that the standard deviation of the final exam grades in her statistics class is larger for the male students than it is for the female students. The data from the final exam for the last semester are shown. Is there enough evidence to support her claim, using $\alpha = 0.01$?

Males	Females
$n_1 = 16$	$n_2 = 18$
$s_1 = 4.2$	$s_2 = 2.3$

Solution

STEP 1 State the hypotheses and identify the claim.

$$H_0: \sigma_1^2 \leq \sigma_2^2 \quad \text{and} \quad H_1: \sigma_1^2 > \sigma_2^2 \text{ (claim)}$$

STEP 2 Find the critical value. Here, d.f.N. = 16 − 1 = 15, and d.f.D. = 18 − 1 = 17. From the 0.01 table, the critical value is 3.31.

STEP 3 Compute the test value.

$$F = \frac{s_1^2}{s_2^2} = \frac{4.2^2}{2.3^2} = 3.33$$

STEP 4 Make the decision. Reject the null hypothesis, since $3.33 > 3.31$.

STEP 5 Summarize the results. There is enough evidence to support the claim that the standard deviation of the final exam grades for the male students is larger than that for the female students.

Finding P-values for the F test statistic is somewhat more complicated since it requires looking through all the F tables (Table H in Appendix C) using the specific d.f.N. and d.f.D. values. For example, suppose that a certain test has $F = 3.58$, d.f.N. $= 5$, and d.f.D. $= 10$. To find the P-value interval for $F = 3.58$, one must first find the corresponding F values for d.f.N. $= 5$ and d.f.D. $= 10$ for α equal to 0.005 on page 733, 0.01 on page 734, 0.025 on page 735, 0.05 on page 736, and 0.10 on page 737 in Table H. Then make a table as shown.

α	0.10	0.05	0.025	0.01	0.005
F	2.52	3.33	4.24	5.64	6.87
Reference page	737	736	735	734	733

Now locate the two F values that the test value 3.58 falls between. In this case, 3.58 falls between 3.33 and 4.24, corresponding to 0.05 and 0.025. Hence, the P-value for a right-tailed test for $F = 3.58$ falls between 0.025 and 0.05 (that is, $0.025 < P\text{-value} < 0.05$). For a right-tailed test, then, one would reject the null hypothesis at $\alpha = 0.05$ but not at $\alpha = 0.01$. The P-value obtained from a calculator is 0.0481. Remember that for a two-tailed test the values found in Table H for α must be doubled. In this case, $0.05 < P\text{-value} < 0.10$ for $F = 3.58$.

Once you understand the concept, you can dispense with making a table as shown and find the P-value directly from Table H.

Example 9–8

The CEO of an airport hypothesizes that the variance for the number of passengers for U.S. airports is greater than the variance for the number of passengers for foreign airports. At $\alpha = 0.10$, is there enough evidence to support the hypothesis? The data in millions of passengers per year are shown for selected airports. Use the P-value method. Assume the variable is normally distributed.

U.S. airports		**Foreign airports**	
36.8	73.5	60.7	51.2
72.4	61.2	42.7	38.6
60.5	40.1		

Source: Airports Council International.

Solution

STEP 1 State the hypotheses and identify the claim.

$$H_0\colon \sigma_1^2 \le \sigma_2^2 \quad \text{and} \quad H_1\colon \sigma_1^2 > \sigma_2^2 \text{ (claim)}$$

STEP 2 Compute the test value. Using the formula in Chapter 3 or a calculator, find the variance for each group.

$$s_1^2 = 246.38 \quad \text{and} \quad s_2^2 = 95.87$$

Substitute in the formula and solve.

$$F = \frac{s_1^2}{s_2^2} = \frac{246.38}{95.87} = 2.57$$

STEP 3 Find the *P*-value in Table H, using d.f.N. = 5 and d.f.D. = 3.

α	0.10	0.05	0.025	0.01	0.005
F	5.31	9.01	14.88	28.24	45.39

Since 2.57 is less than 5.31, the *P*-value is greater than 0.10. (The *P*-value obtained from a calculator is 0.234.)

STEP 4 Make the decision. The decision is do not reject the null hypothesis since *P*-value > 0.10.

STEP 5 Summarize the results. There is not enough evidence to support the claim that the variance in the number of passengers for U.S. airports is greater than the variance for the number of passengers for foreign airports.

If the exact degrees of freedom are not specified in Table H, the closest smaller value should be used. For example, if $\alpha = 0.05$ (right-tailed test), d.f.N. = 18, and d.f.D. = 20, use the column d.f.N. = 15 and the row d.f.D. = 20 to get $F = 2.20$.

Note: It is not necessary to place the larger variance in the numerator when one is performing the F test. Critical values for left-tailed hypotheses tests can be found by interchanging the degrees of freedom and taking the reciprocal of the value found in Table H.

Also, one should use caution when performing the F test since the data can run contrary to the hypotheses on rare occasions. For example, if the hypotheses are H_0: $\sigma_1^2 \le \sigma_2^2$ and H_1: $\sigma_1^2 > \sigma_2^2$, but if $s_1^2 < s_2^2$, then the F test should not be performed and one would not reject the null hypothesis.

Exercises 9–3

1. When one is computing the F test value, what condition is placed on the variance that is in the numerator?

2. Why is the critical region always on the right side in the use of the F test?

3. What are the two different degrees of freedom associated with the F distribution?

4. What are the characteristics of the F distribution?

5. Using Table H, find the critical value for each.

a. Sample 1: $s_1^2 = 128$, $n_1 = 23$
Sample 2: $s_2^2 = 162$, $n_2 = 16$
Two-tailed, $\alpha = 0.01$

b. Sample 1: $s_1^2 = 37$, $n_1 = 14$
Sample 2: $s_2^2 = 89$, $n_2 = 25$
Right-tailed, $\alpha = 0.01$

c. Sample 1: $s_1^2 = 232$, $n_1 = 30$
Sample 2: $s_2^2 = 387$, $n_2 = 46$
Two-tailed, $\alpha = 0.05$

d. Sample 1: $s_1^2 = 164$, $n_1 = 21$
Sample 2: $s_2^2 = 53$, $n_2 = 17$
Two-tailed, $\alpha = 0.10$

e. Sample 1: $s_1^2 = 92.8$, $n_1 = 11$
Sample 2: $s_2^2 = 43.6$, $n_2 = 11$
Right-tailed, $\alpha = 0.05$

6. (ans) Using Table H, find the *P*-value interval for each F test value.

a. $F = 2.97$, d.f.N. = 9, d.f.D. = 14, right-tailed
b. $F = 3.32$, d.f.N. = 6, d.f.D. = 12, two-tailed
c. $F = 2.28$, d.f.N. = 12, d.f.D. = 20, right-tailed
d. $F = 3.51$, d.f.N. = 12, d.f.D. = 21, right-tailed
e. $F = 4.07$, d.f.N. = 6, d.f.D. = 10, two-tailed
f. $F = 1.65$, d.f.N. = 19, d.f.D. = 28, right-tailed
g. $F = 1.77$, d.f.N. = 28, d.f.D. = 28, right-tailed
h. $F = 7.29$, d.f.N. = 5, d.f.D. = 8, two-tailed

For Exercises 7 through 20, perform the following steps. Assume that all variables are normally distributed.

a. State the hypotheses and identify the claim.
b. Find the critical value.
c. Compute the test value.
d. Make the decision.
e. Summarize the results.

Use the traditional method of hypothesis testing unless otherwise specified.

7. The standard deviation for the number of weeks 15 *New York Times* hardcover fiction books spent on their bestseller list is 6.17 weeks. The standard deviation for the 15 *New York Times* hardcover nonfiction list is 13.12 weeks. At $\alpha = 0.10$, can we conclude that there is a difference in the variances?

Source: *The New York Times.*

8. A mathematics professor claims that the variance on placement tests when the students use graphing calculators will be smaller than the variance on placement tests when the students use scientific calculators. A randomly selected group of 50 students who used graphing calculators had a variance of 32, and a randomly selected group of 40 students who used scientific calculators had a variance of 37. Is the professor correct, using $\alpha = 0.05$?

9. A tax collector wishes to see if the variances of the values of the tax-exempt properties are different for two large cities. The values of the tax-exempt properties for two samples are shown. The data are given in millions of dollars. At $\alpha = 0.05$, is there enough evidence to support the tax collector's claim that the variances are different?

City A				City B			
113	22	14	8	82	11	5	15
25	23	23	30	295	50	12	9
44	11	19	7	12	68	81	2
31	19	5	2	20	16	4	5

10. In the hospital study cited in Exercise 19 in Exercise set 7–2, it was found that the standard deviation of the sound levels from 20 areas designated as "casualty doors" was 4.1 dBA and the standard deviation of 24 areas designated as operating theaters was 7.5 dBA. At $\alpha = 0.05$, can one substantiate the claim that there is a difference in the standard deviations?

Source: M. Bayo, A. Garcia, and A. Garcia, "Noise Levels in an Urban Hospital and Workers' Subjective Responses," *Archives of Environmental Health.*

11. The number of calories contained in 1/2 cup servings of randomly selected flavors of ice cream from two national brands are listed here. At the 0.05 level of significance, is there sufficient evidence to conclude that the variance in the number of calories differs between the two brands?

Brand A		Brand B	
330	300	280	310
310	350	300	370
270	380	250	300
310	300	290	310

Source: *The Doctor's Pocket Calorie, Fat and Carbohydrate Counter.*

12. A researcher wishes to see if the variance in the number of vehicles passing through the toll booths during a fiscal year on the Pennsylvania Turnpike is different from the variance in the number of vehicles passing through the toll booths on the expressways in Pennsylvania during the same year. The data are shown. At $\alpha = 0.05$, can it be concluded that the variances are different?

PA Turnpike	PA expressways
3,694,560	2,774,251
719,934	204,369
3,768,285	456,123
1,838,271	1,068,107
3,358,175	3,534,092
6,718,905	235,752
3,469,431	499,043
4,420,553	2,016,046
920,264	253,956
1,005,469	826,710
1,112,722	133,619
2,855,109	3,453,745

Source: *Pittsburgh Post-Gazette.*

13. The standard deviation of the average waiting time to see a doctor for non-life-threatening problems in the emergency room at an urban hospital is 32 minutes. At a second hospital the standard deviation is 28 minutes. If 100 patients were used in each sample, can we conclude that the second hospital has a smaller standard deviation? Use 0.05 for α.

14. The number of grams of carbohydrates contained in 1-ounce servings of randomly selected chocolate and nonchocolate candy is listed here. Is there sufficient evidence to conclude that the variance in carbohydrate content varies between chocolate and nonchocolate candy? Use $\alpha = 0.10$.

Chocolate:	29	25	17	36	41	25	32	29
	38	34	24	27	29			
Nonchocolate:	41	41	37	29	30	38	39	10
	29	55	29					

Source: *The Doctor's Pocket Calorie, Fat and Carbohydrate Counter.*

15. The yearly tuition costs in dollars for random samples of medical schools that specialize in research and in primary care are listed below. At $\alpha = 0.05$, can it be concluded that a difference between the variances of the two groups exists?

Research			Primary care		
30,897	34,280	31,943	26,068	21,044	30,897
34,294	31,275	29,590	34,208	20,877	29,691
20,618	20,500	29,310	33,783	33,065	35,000
21,274			27,297		

Source: *U.S. News & World Report Best Graduate Schools.*

16. A researcher wishes to see if the variance of the areas in square miles for counties in Indiana is less than the variance of the areas for counties in Iowa. A random sample of counties is selected, and the data are shown. At $\alpha = 0.01$, can it be concluded that the variance of the areas for counties in Indiana is less than the variance of the areas for counties in Iowa?

Indiana				Iowa			
406	393	396	485	640	580	431	416
431	430	369	408	443	569	779	381
305	215	489	293	717	568	714	731
373	148	306	509	571	577	503	501
560	384	320	407	568	434	615	402

Source: *The World Almanac and Book of Facts.*

17. Test the claim that the variance of heights of tall buildings in Denver is equal to the variance in heights of tall buildings in Detroit at $\alpha = 0.10$. The data are given in feet.

Denver			Detroit		
714	698	544	620	472	430
504	438	408	562	448	420
404			534	436	

Source: *The World Almanac and Book of Facts.*

18. A researcher claims that the variation in the salaries of elementary school teachers is greater than the variation in the salaries of secondary school teachers. A sample of the salaries of 30 elementary school teachers has a variance of \$8324, and a sample of the salaries of 30 secondary school teachers has a variance of \$2862. At $\alpha = 0.05$, can the researcher conclude that the variation in the elementary school teachers' salaries is greater than the variation in the secondary teachers' salaries? Use the *P*-value method.

19. The weights in ounces of a sample of running shoes for men and women are shown below. Calculate the variances for each sample, and test the claim that the variances are equal at $\alpha = 0.05$. Use the *P*-value method.

Men			Women		
11.9	10.4	12.6	10.6	10.2	8.8
12.3	11.1	14.7	9.6	9.5	9.5
9.2	10.8	12.9	10.1	11.2	9.3
11.2	11.7	13.3	9.4	10.3	9.5
13.8	12.8	14.5	9.8	10.3	11.0

20. Upright vacuum cleaners have either a hard body type or a soft body type. Shown are the weights in pounds of a sample of each type. At $\alpha = 0.05$, can the claim that there is a difference in the variances of the weights of the two types be substantiated?

Hard body types				Soft body types			
21	17	17	20	24	13	11	13
16	17	15	20	12	15		
23	16	17	17				
13	15	16	18				
18							

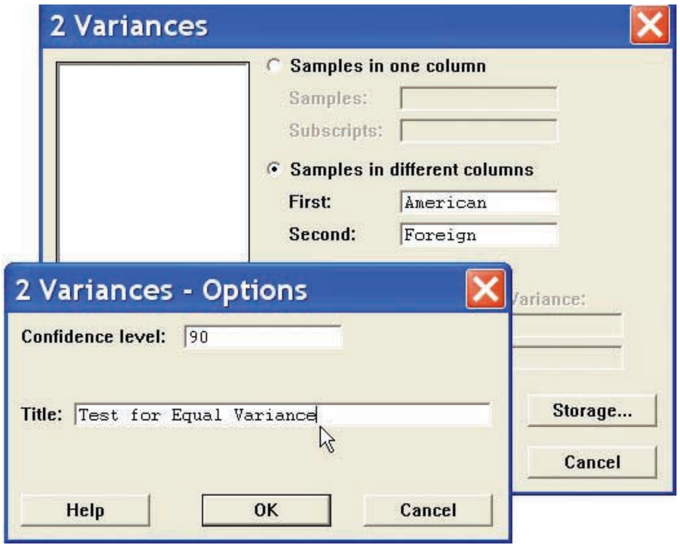
Technology Step by Step

MINITAB Step by Step

Test for the Difference Between Two Variances

For Example 9–8, test the hypothesis that the variance in the number of passengers for American and foreign airports is different. Use the *P*-value approach.

American airports	Foreign airports
36.8	60.7
72.4	42.7
60.5	51.2
73.5	38.6
61.2	
40.1	



1. Enter the data into two columns of MINITAB.
2. Name the columns American and Foreign.
   a) Select **Stat>Basic Statistics>2-Variances.**
   b) Click the button for Samples in different columns.
   c) Click in the text box for First, then double-click C1 American.
   d) Double-click C2 Foreign, then click on [Options]. The dialog box is shown. Change the Confidence level to **90** and type an appropriate title. In this dialog, we cannot specify a left- or right-tailed test.
3. Click [OK] twice. A graph window will open that includes a small window that says $F = 2.57$ and the P-value is 0.437. Divide this two-tailed P-value by two for a one-tailed test.

There is not enough evidence in the sample to conclude there is greater variance in the number of passengers in American airports compared to foreign airports.

TI-83 Plus Step by Step

Hypothesis Test for the Difference Between Two Variances (Data)

1. Enter the data values into L_1 and L_2.
2. Press **STAT** and move the cursor to TESTS.
3. Press **D** (**ALPHA** $\mathbf{X^{-1}}$) for 2-SampFTest.
4. Move the cursor to Data and press **ENTER.**
5. Type in the appropriate values.
6. Move the cursor to the appropriate alternative hypothesis and press **ENTER.**
7. Move the cursor to Calculate and press **ENTER.**

Hypothesis Test for the Difference Between Two Variances (Statistics)

1. Press **STAT** and move the cursor to TESTS.

2. Press **D (ALPHA X^{-1})** for 2-SampFTest.
3. Move the cursor to Stats and press **ENTER.**
4. Type in the appropriate values.
5. Move the cursor to the appropriate alternative hypothesis and press **ENTER.**
6. Move the cursor to Calculate and press **ENTER.**

Excel
Step by Step

F Test for the Difference Between Two Variances

Excel has a two-sample *F* test in its Data Analysis tools. To perform an *F* test for the difference between the variances of two populations given two independent samples:

1. Enter the first sample data set in column A.
2. Enter the second sample data set in column B.
3. Select **Tools>Data Analysis** and choose F-Test Two-Sample for Variances.
4. Enter the ranges for the data in columns A and B.
5. Specify the confidence level, Alpha.
6. Specify a location for output, and click [OK].

Example XL9–2

At $\alpha = 0.05$, test the hypothesis that the two population variances are equal using the sample data provided here.

Set A	63	73	80	60	86	83	70	72	82
Set B	86	93	64	82	81	75	88	63	63

The results appear in the table that Excel generates, shown here. For this example, the output shows that the null hypothesis cannot be rejected at an alpha level of 0.05.

F-Test Two-Sample for Variances		
	Variable 1	*Variable 2*
Mean	74.33333333	77.22222222
Variance	82.75	132.9444444
Observations	9	9
df	8	8
F	0.622440451	
P(F<=f) one-tail	0.258814151	
F Critical one-tail	0.290858004	

9–4

Testing the Difference Between Two Means: Small Independent Samples

In Section 9–2, the *z* test was used to test the difference between two means when the population standard deviations were known and the variables were normally or approximately normally distributed, or when both sample sizes were greater than or equal to 30. In many situations, however, these conditions cannot be met—that is, the population standard deviations are not known, and one or both sample sizes are less than 30. In these cases, a *t* test is used to test the difference between means when the two samples are independent and when the samples are taken from two normally or approximately normally distributed populations. Samples are **independent samples** when they are not related.

Objective 3. Test the difference between two means for small independent samples.

There are actually two different options for the use of t tests. *One option is used when the variances of the populations are not equal, and the other option is used when the variances are equal.* To determine whether two sample variances are equal, the researcher can use an F test, as shown in Section 9–3.

Note, however, that not all statisticians are in agreement about using the F test before using the t test. Some believe that conducting the F and t tests at the same level of significance will change the overall level of significance of the t test. Their reasons are beyond the scope of this textbook.

Formulas for the *t* Tests—For Testing the Difference Between Two Means—Small Independent Samples

Variances are assumed to be unequal:

$$t = \frac{(\bar{X}_1 - \bar{X}_2) - (\mu_1 - \mu_2)}{\sqrt{\dfrac{s_1^2}{n_1} + \dfrac{s_2^2}{n_2}}}$$

where the degrees of freedom are equal to the smaller of $n_1 - 1$ or $n_2 - 1$.

Variances are assumed to be equal:

$$t = \frac{(\bar{X}_1 - \bar{X}_2) - (\mu_1 - \mu_2)}{\sqrt{\dfrac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2}}\sqrt{\dfrac{1}{n_1} + \dfrac{1}{n_2}}}$$

where the degrees of freedom are equal to $n_1 + n_2 - 2$.

When the variances are unequal, the first formula

$$t = \frac{(\bar{X}_1 - \bar{X}_2) - (\mu_1 - \mu_2)}{\sqrt{\dfrac{s_1^2}{n_1} + \dfrac{s_2^2}{n_2}}}$$

follows the format of

$$\text{Test value} = \frac{(\text{observed value}) - (\text{expected value})}{\text{standard error}}$$

where $\bar{X}_1 - \bar{X}_2$ is the observed difference between sample means and where the expected value $\mu_1 - \mu_2$ is equal to zero when no difference between population means is hypothesized. The denominator $\sqrt{s_1^2/n_1 + s_2^2/n_2}$ is the standard error of the difference between two means. Since mathematical derivation of the standard error is somewhat complicated, it will be omitted here.

When the variances are assumed to be equal, the second formula

$$t = \frac{(\bar{X}_1 - \bar{X}_2) - (\mu_1 - \mu_2)}{\sqrt{\dfrac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2}}\sqrt{\dfrac{1}{n_1} + \dfrac{1}{n_2}}}$$

also follows the format of

$$\text{Test value} = \frac{(\text{observed value}) - (\text{expected value})}{\text{standard error}}$$

For the numerator, the terms are the same as in the first formula. However, a note of explanation is needed for the denominator of the second test statistic. Since both populations are assumed to have the same variance, the standard error is computed with what is called a pooled estimate of the variance. A **pooled estimate of the variance** is a weighted average of the variance using the two sample variances and the *degrees of freedom* of each variance as the weights. Again, since the algebraic derivation of the standard error is somewhat complicated, it is omitted.

In summary, then, to use the t test, first use the F test to determine whether the variances are equal. Then use the appropriate t test formula. This procedure involves two five-step processes.

Example 9–9

The average size of a farm in Indiana County, Pennsylvania, is 191 acres. The average size of a farm in Greene County, Pennsylvania, is 199 acres. Assume the data were obtained from two samples with standard deviations of 38 acres and 12 acres, respectively, and sample sizes of 8 and 10, respectively. Can it be concluded at $\alpha = 0.05$ that the average size of the farms in the two counties is different? Assume the populations are normally distributed.

Source: *Pittsburgh Tribune-Review.*

Solution

Here we will use the F test to determine whether the variances are equal. The null hypothesis is that the variances are equal.

STEP 1 State the hypotheses and identify the claim.

$$H_0\colon \sigma_1^2 = \sigma_2^2 \text{ (claim)} \qquad \text{and} \qquad H_1\colon \sigma_1^2 \neq \sigma_2^2$$

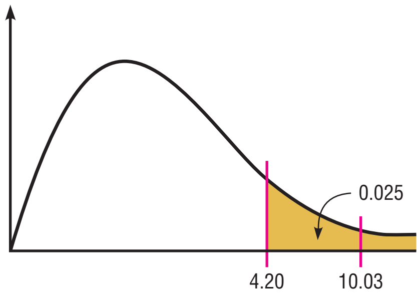
STEP 2 Find the critical value. The critical value for the F test found in Table H (Appendix C) for $\alpha = 0.05$ is 4.20, since there are 7 and 9 degrees of freedom. (*Note:* Use the 0.025 table.)

STEP 3 Compute the test value.

$$F = \frac{s_1^2}{s_2^2} = \frac{38^2}{12^2} = 10.03$$

STEP 4 Make the decision. Reject the null hypothesis since 10.03 falls in the critical region. See Figure 9–9.

Figure 9–9
Critical and F Test Values for Example 9–9



STEP 5 Summarize the results. It can be concluded that the variances are not equal.

Since the variances are not equal, the first formula will be used to test the equality of the means.

STEP 1 State the hypotheses and identify the claim for the means.

$$H_0\colon \mu_1 = \mu_2 \qquad \text{and} \qquad H_1\colon \mu_1 \neq \mu_2 \text{ (claim)}$$

STEP 2 Find the critical values. Since the test is two-tailed, since $\alpha = 0.05$, and since the variances are unequal, the degrees of freedom are the smaller of $n_1 - 1$ or $n_2 - 1$. In this case, the degrees of freedom are $8 - 1 = 7$. Hence, from Table F, the critical values are $+2.365$ and -2.365.

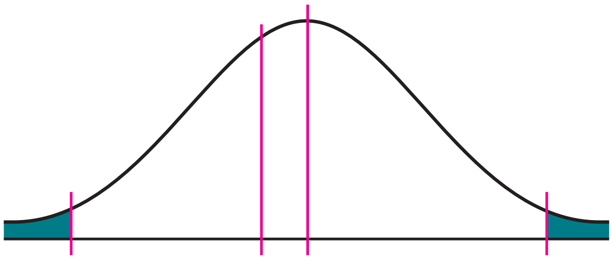
STEP 3 Compute the test value. Since the variances are unequal, use the first formula.

$$t = \frac{(\bar{X}_1 - \bar{X}_2) - (\mu_1 - \mu_2)}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}} = \frac{(191 - 199) - 0}{\sqrt{\frac{38^2}{8} + \frac{12^2}{10}}} = -0.57$$

STEP 4 Make the decision. Do not reject the null hypothesis, since $-0.57 > -2.365$. See Figure 9–10.

Figure 9–10

Critical and Test Values for Example 9–9



STEP 5 Summarize the results. There is not enough evidence to support the claim that the average size of the farms is different.

Example 9–10

A researcher wishes to determine whether the salaries of professional nurses employed by private hospitals are higher than those of nurses employed by government-owned hospitals. She selects a sample of nurses from each type of hospital and calculates the means and standard deviations of their salaries. At $\alpha = 0.01$, can she conclude that the private hospitals pay more than the government hospitals? Assume that the populations are approximately normally distributed. Use the *P*-value method.

Private	Government
$\bar{X}_1 = \$26{,}800$	$\bar{X}_2 = \$25{,}400$
$s_1 = \$600$	$s_2 = \$450$
$n_1 = 10$	$n_2 = 8$

Solution

The *F* test will be used to determine whether the variances are equal. The null hypothesis is that the variances are equal.

STEP 1 State the hypotheses and identify the claim.

$$H_0\colon \sigma_1^2 = \sigma_2^2 \text{ (claim)} \qquad \text{and} \qquad H_1\colon \sigma_1^2 \neq \sigma_2^2$$

STEP 2 Compute the test value.

$$F = \frac{s_1^2}{s_2^2} = \frac{600^2}{450^2} = 1.78$$

STEP 3 Find the *P*-value in Table H, using d.f.N. = 9 and d.f.D. = 7. Since $1.78 < 2.72$, *P*-value > 0.20. (The *P*-value obtained from a calculator is 0.460.)

STEP 4 Make the decision. Do not reject the null hypothesis since P-value > 0.01 (the α value).

STEP 5 Summarize the results. There is not enough evidence to reject the claim that the variances are equal; therefore, the second formula is used to test the difference between the two means, as shown next.

STEP 1 State the hypotheses and identify the claim.

$$H_0\colon \mu_1 \le \mu_2 \qquad \text{and} \qquad H_1\colon \mu_1 > \mu_2 \text{ (claim)}$$

STEP 2 Compute the test value. Use the second formula, since the variances are assumed to be equal.

$$t = \frac{(\bar{X}_1 - \bar{X}_2) - (\mu_1 - \mu_2)}{\sqrt{\dfrac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2}}\sqrt{\dfrac{1}{n_1} + \dfrac{1}{n_2}}}$$

$$= \frac{(26{,}800 - 25{,}400) - 0}{\sqrt{\dfrac{(10 - 1)(600)^2 + (8 - 1)(450)^2}{10 + 8 - 2}}\sqrt{\dfrac{1}{10} + \dfrac{1}{8}}}$$

$$= 5.47$$

STEP 3 Find the P-value, using Table F. The P-value for $t = 5.47$ with d.f. $= 16$ (that is, $10 + 8 - 2$) is P-value < 0.005. (The P-value obtained from a calculator is 0.00002.)

STEP 4 Make the decision. Since P-value < 0.01 (the α value), the decision is to reject the null hypothesis.

STEP 5 Summarize the results. There is enough evidence to support the claim that the salaries paid to nurses employed by private hospitals are higher than those paid to nurses employed by government-owned hospitals.

When raw data are given in the exercises, use your calculator or the formulas in Chapter 3 to find the means and variances for the data sets. Then follow the procedures shown in this section to test the hypotheses.

Confidence intervals can also be found for the difference between two means with these formulas:

Confidence Intervals for the Difference of Two Means: Small Independent Samples

Variances unequal

$$(\bar{X}_1 - \bar{X}_2) - t_{\alpha/2}\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}} < \mu_1 - \mu_2 < (\bar{X}_1 - \bar{X}_2) + t_{\alpha/2}\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}$$

d.f. = smaller value of $n_1 - 1$ or $n_2 - 1$

Variances equal

$$(\bar{X}_1 - \bar{X}_2) - t_{\alpha/2}\sqrt{\frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2}} \cdot \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}$$

$$< \mu_1 - \mu_2 < (\bar{X}_1 - \bar{X}_2) + t_{\alpha/2}\sqrt{\frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2}} \cdot \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}$$

d.f. $= n_1 + n_2 - 2$

Remember that when one is testing the difference between two means from independent samples, two different statistical test formulas can be used. One formula is used when the variances are equal, the other when the variances are not equal. As shown in Section 9–3, some statisticians use an F test to determine whether the two variances are equal.

Example 9–11

Find the 95% confidence interval for the data in Example 9–9.

Solution

Substitute in the formula

$$(\bar{X}_1 - \bar{X}_2) - t_{\alpha/2}\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}} < \mu_1 - \mu_2$$

$$< (\bar{X}_1 - \bar{X}_2) + t_{\alpha/2}\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}$$

$$(191 - 199) - 2.365\sqrt{\frac{38^2}{8} + \frac{12^2}{10}} < \mu_1 - \mu_2$$

$$< (191 - 199) + 2.365\sqrt{\frac{38^2}{8} + \frac{12^2}{10}}$$

$$-41.02 < \mu_1 - \mu_2 < 25.02$$

Since 0 is contained in the interval, the decision is not to reject the null hypothesis H_0: $\mu_1 = \mu_2$.

Exercises 9–4

For Exercises 1 through 11, perform each of these steps. Assume that all variables are normally or approximately normally distributed. Be sure to test for equality of variance first.

a. State the hypotheses and identify the claim.
b. Find the critical value(s).
c. Compute the test value.
d. Make the decision.
e. Summarize the results.

Use the traditional method of hypothesis testing unless otherwise specified.

1. A real estate agent wishes to determine whether tax assessors and real estate appraisers agree on the values of homes. A random sample of the two groups appraised 10 homes. The data are shown here. Is there a significant difference in the values of the homes for each group? Let $\alpha = 0.05$. Find the 95% confidence interval for the difference of the means.

Real estate appraisers	Tax assessors
$\bar{X}_1 = \$83,256$	$\bar{X}_2 = \$88,354$
$s_1 = \$3,256$	$s_2 = \$2,341$
$n_1 = 10$	$n_2 = 10$

2. A researcher suggests that male nurses earn more than female nurses. A survey of 16 male nurses and 20 female nurses reports these data. Is there enough evidence to support the claim that male nurses earn more than female nurses? Use $\alpha = 0.05$.

Female	Male
$\bar{X}_2 = \$23,750$	$\bar{X}_1 = \$23,800$
$s_2 = \$250$	$s_1 = \$300$
$n_2 = 20$	$n_1 = 16$

3. An agent claims that there is no difference between the pay of safeties and linebackers in the NFL. A survey of 15 safeties found an average salary of $501,580, and a survey of 15 linebackers found an average salary of $513,360. If the standard deviation in each case was $20,000, is the agent correct? Use $\alpha = 0.05$.

Source: NFL Players Assn./*USA TODAY.*

4. A researcher estimates that high school girls miss more days of school than high school boys. A sample of 16 girls showed that they missed an average of 3.9 days of school per school year; a sample of 22 boys showed that they missed an average of 3.6 days of school per year. The standard deviations are 0.6 and 0.8, respectively. At $\alpha = 0.01$, is there enough evidence to support the researcher's claim?

5. A health care worker wishes to see if the average number of family day care homes per county is greater than the average number of day care centers per county. The number of centers for a selected sample of counties is shown. At $\alpha = 0.01$, can it be concluded that the average number of family day care homes is greater than the average number of day care centers?

Number of family day care homes			**Number of day care centers**		
25	57	34	5	28	37
42	21	44	16	16	48

Source: *Pittsburgh Tribune Review.*

6. A researcher wishes to test the claim that, on average, more juveniles than adults are classified as missing persons. Records for the last 5 years are shown. At $\alpha = 0.10$, is there enough evidence to support the claim?

Juveniles	65,513	65,934	64,213	61,954	59,167
Adults	31,364	34,478	36,937	35,946	38,209

Source: *USA TODAY.*

7. The local branch of the Internal Revenue Service spent an average of 21 minutes helping each of 10 people prepare their tax returns. The standard deviation was 5.6 minutes. A volunteer tax preparer spent an average of 27 minutes helping 14 people prepare their taxes. The standard deviation was 4.3 minutes. At $\alpha = 0.02$, is there a difference in the average time spent by the two services? Find the 98% confidence interval for the two means.

8. Females and males alike from the general adult population volunteer an average of 4.2 hours per week. A random sample of 20 female college students and 18 male college students indicated these results concerning the amount of time spent in volunteer service per week. At the 0.01 level of significance, is there sufficient evidence to conclude that a difference exists between the mean number of volunteer hours per week for male and female college students?

	Male	**Female**
Sample mean	2.5	3.8
Sample variance	2.2	3.5
Sample size	18	20

Source: *N.Y. Times Almanac.*

9. The average monthly premium paid by 12 administrators for hospitalization insurance is \$56. The standard deviation is \$3. The average monthly premium paid by 27 nurses is \$63. The standard deviation is \$5.75. At $\alpha = 0.05$, do the nurses pay more for hospitalization insurance? Use the *P*-value method.

10. Health Care Knowledge Systems reported that an insured woman spends on average 2.3 days in the hospital for a routine childbirth, while an uninsured woman spends on average 1.9 days. Assume two samples of 16 women each were used and the standard deviations are both equal to 0.6 day. At $\alpha = 0.01$, test the claim that the means are equal. Find the 99% confidence interval for the differences of the means. Use the *P*-value method.

Source: Michael D. Shook and Robert L. Shook, *The Book of Odds.*

11. The times (in minutes) it took six white mice to learn to run a simple maze and the times it took six brown mice to learn to run the same maze are given here. At $\alpha = 0.05$, does the color of the mice make a difference in their learning rate? Find the 95% confidence interval for the difference of the means. Use the *P*-value method.

White mice	18	24	20	13	15	12
Brown mice	25	16	19	14	16	10

12. A random sample of enrollments from medical schools that specialize in research and from those that are noted for primary care is listed. Find the 90% confidence interval for the difference in the means.

Research				**Primary care**			
474	577	605	663	783	605	427	728
783	467	670	414	546	474	371	107
813	443	565	696	442	587	293	277
692	694	277	419	662	555	527	320
884							

Source: *U.S. News & World Report Best Graduate Schools.*

13. The out-of-state tuitions (in dollars) for random samples of both public and private 4-year colleges in a New England state are listed. Find the 95% confidence interval for the difference in the means.

Private		**Public**	
13,600	13,495	7,050	9,000
16,590	17,300	6,450	9,758
23,400	12,500	7,050	7,871
		16,100	

Source: *N.Y. Times Almanac.*

Technology Step by Step

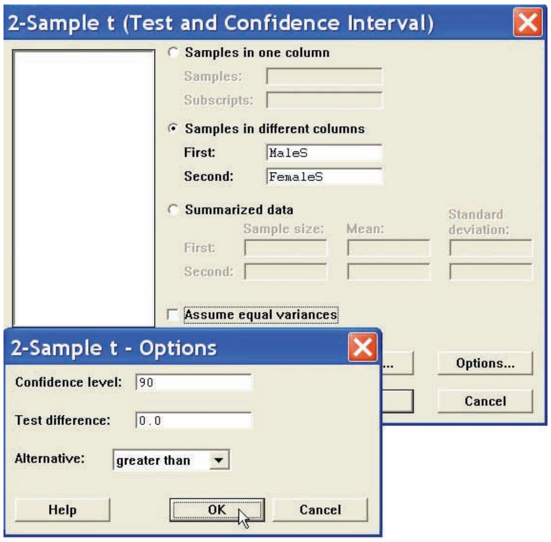
MINITAB
Step by Step

Test the Difference Between Two Means: Small Independent Samples with Equal Variance

In Section 9–3 we determined that the variance in the number of passengers at American and foreign airports was not different. To test the hypothesis that the mean number of passengers is the same, we continue using the data from Example 9–8.

American airports	Foreign airports
36.8	60.7
72.4	42.7
60.5	51.2
73.5	38.6
61.2	
40.1	

1. Enter the data into two columns of a MINITAB worksheet.
2. Name the columns **American** and **Foreign.**
3. Select **Stat>Basic Statistics>2-Sample t.**
4. Click on Samples in different columns.
5. Click in the box for First, then double-click C1 American.
6. Double-click C2 Foreign for the Second.
7. Check the box for Assume equal variances. The pooled standard deviation formula from Section 9–4 will be used to calculate the test statistic and *P*-value.
8. Click [OK]. The session window is shown. The *P*-value for the difference is 0.335. Do not reject the null hypothesis. There is no significant difference in the mean number of passengers at American airports compared to foreign airports.



Two-Sample T-Test and CI: American, Foreign

```
Two-sample T for American vs Foreign

          N  Mean   StDev    SE Mean
American  6  57.4     15.7       6.4
Foreign   4  48.30     9.79      4.9

Difference = mu (American) - mu (Foreign)
Estimate for difference:  9.11667
90% CI for difference:  (-7.42622,  25.65955)

T-Test of difference = 0 (vs not =) : T-Value = 1.02  P-Value = 0.335  DF = 8
      Both use Pooled StDev = 13.7819
```

TI-83 Plus Step by Step

Hypothesis Test for the Difference Between Two Means and *t* Distribution (Data)

1. Enter the data values into L_1 and L_2.
2. Press **STAT** and move the cursor to TESTS.
3. Press **4** for 2-SampTTest.
4. Move the cursor to Data and press **ENTER.**
5. Type in the appropriate values.
6. Move the cursor to the appropriate alternative hypothesis and press **ENTER.**
7. On the line for Pooled, move the cursor to No (standard deviations are assumed not equal) or Yes (standard deviations are assumed equal) and press **ENTER**.
8. Move the cursor to Calculate and press **ENTER.**

Hypothesis Test for the Difference Between Two Means and *t* Distribution (Statistics)

1. Press **STAT** and move the cursor to TESTS.
2. Press **4** for 2-SampTTest.
3. Move the cursor to Stats and press **ENTER.**
4. Type in the appropriate values.
5. Move the cursor to the appropriate alternative hypothesis and press **ENTER.**
6. On the line for Pooled, move the cursor to No (standard deviations are assumed not equal) or Yes (standard deviations are assumed equal) and press **ENTER.**
7. Move the cursor to Calculate and press **ENTER.**

Confidence Interval for the Difference Between Two Means and *t* Distribution (Data)

1. Enter the data values into L_1 and L_2.
2. Press **STAT** and move the cursor to TESTS.
3. Press **0** for 2-SampTInt.
4. Move the cursor to Data and press **ENTER.**
5. Type in the appropriate values.

6. On the line for Pooled, move the cursor to No (standard deviations are assumed not equal) or Yes (standard deviations are assumed equal) and press **ENTER.**
7. Move the cursor to Calculate and press **ENTER.**

Confidence Interval for the Difference Between Two Means and *t* Distribution (Statistics)

1. Press **STAT** and move the cursor to TESTS.
2. Press **0** for 2-SampTInt.
3. Move the cursor to Stats and press **ENTER.**
4. Type in the appropriate values.
5. On the line for Pooled, move the cursor to No (standard deviations are assumed not equal) or Yes (standard deviations are assumed equal) and press **ENTER**.
6. Move the cursor to Calculate and press **ENTER.**

Excel
Step by Step

Testing the Difference Between Two Means: Small Independent Samples

Excel has a two-sample *t* test in its Data Analysis tools. To perform the *t* test for the difference between means, see Example XL9–3.

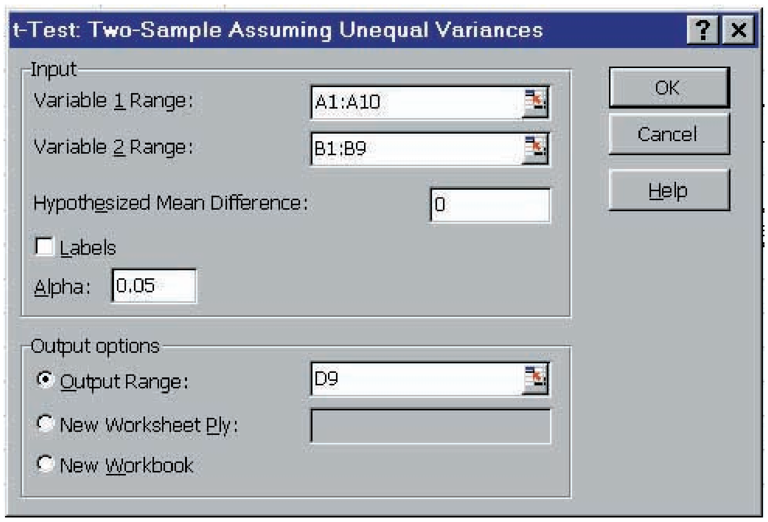
Example XL9–3

Test the hypothesis that there is no difference between population means based on these sample data. Assume the population variances are not equal. Use $\alpha = 0.05$.

Set A	32	38	37	36	36	34	39	36	37	42
Set B	30	36	35	36	31	34	37	33	32	

1. Enter the 10-number data set A in column A.
2. Enter the 9-number data set B in column B.
3. Select **Tools>Data Analysis** and choose t-Test: Two-Sample Assuming Unequal Variances.
4. Enter the data ranges, hypothesized mean difference (here, 0), and α.
5. Select a location for output and click [OK].

Two-Sample *t* Test in Excel



t-Test: Two-Sample Assuming Unequal Variances	*Variable 1*	*Variable 2*
Mean	36.7	33.77777778
Variance	7.344444444	5.944444444
Observations	10	9
Hypothesized Mean Difference	0	
df	17	
t Stat	2.474205364	
P(T<=t) one-tail	0.012095	
t Critical one-tail	1.739606432	
P(T<=t) two-tail	0.024189999	
t Critical two-tail	2.109818524	

The output reports both one- and two-tailed *P*-values.

If the variances are equal, use the two-sample *t* test assuming equal variances procedure.

9–5 Testing the Difference Between Two Means: Small Dependent Samples

Objective 4. Test the difference between two means for small dependent samples.

In Section 9–4, the *t* test was used to compare two sample means when the samples were independent. In this section, a different version of the *t* test is explained. This version is used when the samples are dependent. Samples are considered to be **dependent samples** when the subjects are paired or matched in some way.

For example, suppose a medical researcher wants to see whether a drug will affect the reaction time of its users. To test this hypothesis, the researcher must pretest the subjects in the sample first. That is, they are given a test to ascertain their normal reaction times. Then after taking the drug, the subjects are tested again, using a posttest. Finally, the means of the two tests are compared to see whether there is a difference. Since the same subjects are used in both cases, the samples are *related;* subjects scoring high on the pretest will generally score high on the posttest, even after consuming the drug. Likewise, those scoring lower on the pretest will tend to score lower on the posttest. To take this effect into account, the researcher employs a *t* test using the differences between the pretest values and the posttest values. Thus only the gain or loss in values is compared.

Here are some other examples of dependent samples. A researcher may want to design an SAT preparation course to help students raise their test scores the second time they take the SAT exam. Hence, the differences between the two exams are compared. A medical specialist may want to see whether a new counseling program will help subjects lose weight. Therefore, the preweights of the subjects will be compared with the postweights.

Besides samples in which the same subjects are used in a pre–post situation, there are other cases where the samples are considered dependent. For example, students might be matched or paired according to some variable that is pertinent to the study; then one student is assigned to one group, and the other student is assigned to a second group. For instance, in a study involving learning, students can be selected and paired according to their IQs. That is, two students with the same IQ will be paired. Then one will be assigned to one sample group (which might receive instruction by computers), and the other student will be assigned to another sample group (which might receive instruction by the lecture-discussion method). These assignments will be done randomly. Since a student's IQ is important to learning, it is a variable that should be controlled. By matching subjects on IQ, the researcher can eliminate the variable's influence, for the most part. Matching, then, helps to reduce type II error by eliminating extraneous variables.

Speaking of STATISTICS

In this study, two groups of children were compared. One group dined with their family daily, and the other did not. Suggest a hypothesis for the study. Comment on what variables were used for comparison.

Do Your Kids Have Dinner With You?

If so, they probably eat better than those who don't dine with their folks. In a recent Harvard Medical School study of 16,000 children ages 9 to 14, 24% of those who dined daily with their family got the recommended five servings of fruits and vegetables, compared with 13% of those who rarely or never shared meals at home. They also ate less fried food, drank less soda, and consumed more calcium, fiber, iron and vitamins C and E. Says Matthew W. Gillman, M.D., lead investigator of the study at Harvard, "There are two possible explanations. When kids eat with their parents, there may be more nutritious food on the table. Or maybe there's a discussion of healthful eating."

— JEANNIE RALSTON *in* *Ladies' Home Journal*

Two notes of caution should be mentioned. First, when subjects are matched according to one variable, the matching process does not eliminate the influence of other variables. Matching students according to IQ does not account for their mathematical ability or their familiarity with computers. Since not all variables influencing a study can be controlled, it is up to the researcher to determine which variables should be used in matching. Second, when the same subjects are used for a pre–post study, sometimes the knowledge that they are participating in a study can influence the results. For example, if people are placed in a special program, they may be more highly motivated to succeed simply because they have been selected to participate; the program itself may have little effect on their success.

When the samples are dependent, a special *t* test for dependent means is used. This test employs the difference in values of the matched pairs. The hypotheses are:

Two-tailed	Left-tailed	Right-tailed
H_0: $\mu_D = 0$	H_0: $\mu_D \geq 0$	H_0: $\mu_D \leq 0$
H_1: $\mu_D \neq 0$	H_1: $\mu_D < 0$	H_1: $\mu_D > 0$

where μ_D is the symbol for the expected mean of the difference of the matched pairs.

The general procedure for finding the test value involves several steps. First, find the differences of the values of the pairs of data.

$$D = X_1 - X_2$$

Second, find the mean $\bar{D}$ of the differences, using the formula

$$\bar{D} = \frac{\Sigma D}{n}$$

where n is the number of data pairs. Third, find the standard deviation s_D of the differences, using the formula

$$s_D = \sqrt{\frac{\Sigma D^2 - \frac{(\Sigma D)^2}{n}}{n - 1}}$$

Fourth, find the estimated standard error $s_{\bar{D}}$ of the differences, which is

$$s_{\bar{D}} = \frac{s_D}{\sqrt{n}}$$

Finally, find the test value, using the formula

$$t = \frac{\bar{D} - \mu_D}{s_D/\sqrt{n}} \qquad \text{with d.f.} = n - 1$$

The formula in the final step follows the basic format of

$$\text{Test value} = \frac{(\text{observed value}) - (\text{expected value})}{\text{standard error}}$$

where the observed value is the mean of the differences. The expected value μ_D is zero if the hypothesis is $\mu_D = 0$. The standard error of the difference is the standard deviation of the difference, divided by the square root of the sample size. Both populations must be normally or approximately normally distributed. Example 9–12 illustrates the hypothesis-testing procedure in detail.

Example 9–12

A physical education director claims by taking a special vitamin, a weight lifter can increase his strength. Eight athletes are selected and given a test of strength, using the standard bench press. After two weeks of regular training, supplemented with the vitamin, they are tested again. Test the effectiveness of the vitamin regimen at $\alpha = 0.05$. Each value in these data represents the maximum number of pounds the athlete can bench-press. Assume that the variable is approximately normally distributed.

Athlete	1	2	3	4	5	6	7	8
Before (X_1)	210	230	182	205	262	253	219	216
After (X_2)	219	236	179	204	270	250	222	216

Solution

STEP 1 State the hypotheses and identify the claim. In order for the vitamin to be effective, the before weights must be significantly less than the after weights; hence, the mean of the differences must be less than zero.

$$H_0: \mu_D \geq 0 \quad \text{and} \quad H_1: \mu_D < 0 \text{ (claim)}$$

STEP 2 Find the critical value. The degrees of freedom are $n - 1$. In this case, d.f. $= 8 - 1 = 7$. The critical value for a left-tailed test with $\alpha = 0.05$ is -1.895.

STEP 3 Compute the test value.

a. Make a table.

		A	B
Before (X_1)	**After (X_2)**	$D = X_1 - X_2$	$D^2 = (X_1 - X_2)^2$
210	219		
230	236		
182	179		
205	204		
262	270		
253	250		
219	222		
216	216		

b. Find the differences and place the results in column A.

$$\begin{aligned}
210 - 219 &= -9 \\
230 - 236 &= -6 \\
182 - 179 &= +3 \\
205 - 204 &= +1 \\
262 - 270 &= -8 \\
253 - 250 &= +3 \\
219 - 222 &= -3 \\
216 - 216 &= \underline{0} \\
\Sigma D &= -19
\end{aligned}$$

c. Find the mean of the differences.

$$\bar{D} = \frac{\Sigma D}{n} = \frac{-19}{8} = -2.375$$

d. Square the differences and place the results in column B.

$$\begin{aligned}
(-9)^2 &= 81 \\
(-6)^2 &= 36 \\
(+3)^2 &= 9 \\
(+1)^2 &= 1 \\
(-8)^2 &= 64 \\
(+3)^2 &= 9 \\
(-3)^2 &= 9 \\
0^2 &= \underline{0} \\
\Sigma D^2 &= 209
\end{aligned}$$

The completed table is shown next.

Before (X_1)	After (X_2)	A $D = X_1 - X_2$	B $D^2 = (X_1 - X_2)^2$
210	219	−9	81
230	236	−6	36
182	179	+3	9
205	204	+1	1
262	270	−8	64
253	250	+3	9
219	222	−3	9
216	216	0	0
		$\Sigma D = -19$	$\Sigma D^2 = 209$

e. Find the standard deviation of the differences.

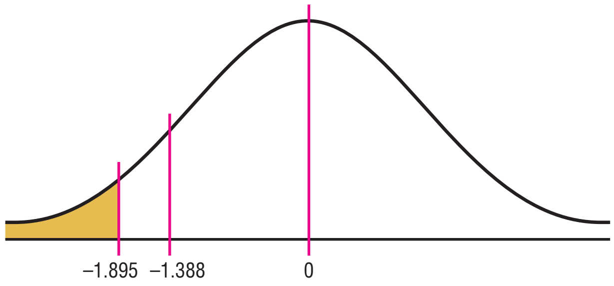
$$s_D = \sqrt{\frac{\Sigma D^2 - \frac{(\Sigma D)^2}{n}}{n-1}} = \sqrt{\frac{209 - \frac{(-19)^2}{8}}{8-1}} = 4.84$$

f. Find the test value.

$$t = \frac{\overline{D} - \mu_D}{s_D/\sqrt{n}} = \frac{-2.375 - 0}{4.84/\sqrt{8}} = -1.388$$

STEP 4 Make the decision. The decision is not to reject the null hypothesis at $\alpha = 0.05$, since $-1.388 > -1.895$, as shown in Figure 9–11.

Figure 9–11
Critical and Test Values for Example 9–12



STEP 5 Summarize the results. There is not enough evidence to support the claim that the vitamin increases the strength of weight lifters.

The formulas for this t test are summarized next.

Formulas for the *t* Test for Dependent Samples

$$t = \frac{\overline{D} - \mu_D}{s_D/\sqrt{n}}$$

with d.f. $= n - 1$ and where

$$\overline{D} = \frac{\Sigma D}{n} \quad \text{and} \quad s_D = \sqrt{\frac{\Sigma D^2 - \frac{(\Sigma D)^2}{n}}{n-1}}$$

Example 9–13

A dietitian wishes to see if a person's cholesterol level will change if the diet is supplemented by a certain mineral. Six subjects were pretested, and then they took the mineral supplement for a 6-week period. The results are shown in the table. (Cholesterol level is measured in milligrams per deciliter.) Can it be concluded that the cholesterol level has been changed at $\alpha = 0.10$? Assume the variable is approximately normally distributed.

Subject	1	2	3	4	5	6
Before (X_1)	210	235	208	190	172	244
After (X_2)	190	170	210	188	173	228

Solution

STEP 1 State the hypotheses and identify the claim. If the diet is effective, the before cholesterol levels should be different from the after levels.

$$H_0\colon \mu_D = 0 \qquad \text{and} \qquad H_1\colon \mu_D \neq 0 \text{ (claim)}$$

STEP 2 Find the critical value. The degrees of freedom are 5. At $\alpha = 0.10$, the critical values are ± 2.015.

STEP 3 Compute the test value.

a. Make a table.

Before (X_1)	**After (X_2)**	**A** $D = X_1 - X_2$	**B** $D^2 = (X_1 - X_2)^2$
210	190		
235	170		
208	210		
190	188		
172	173		
244	228		

b. Find the differences and place the results in column A.

$$\begin{aligned}
210 - 190 &= 20 \\
235 - 170 &= 65 \\
208 - 210 &= -2 \\
190 - 188 &= 2 \\
172 - 173 &= -1 \\
244 - 228 &= \underline{16} \\
\Sigma D &= 100
\end{aligned}$$

c. Find the mean of the differences.

$$\overline{D} = \frac{\Sigma D}{n} = \frac{100}{6} = 16.7$$

d. Square the differences and place the results in column B.

$$\begin{aligned}
(20)^2 &= 400 \\
(65)^2 &= 4225 \\
(-2)^2 &= 4 \\
(2)^2 &= 4 \\
(-1)^2 &= 1 \\
(16)^2 &= \underline{256} \\
\Sigma D^2 &= 4890
\end{aligned}$$

Then complete the table as shown.

Before (X_1)	After (X_2)	A $D = X_1 - X_2$	B $D^2 = (X_1 - X_2)^2$
210	190	20	400
235	170	65	4225
208	210	−2	4
190	188	2	4
172	173	−1	1
244	228	16	256
		$\Sigma D = 100$	$\Sigma D^2 = 4890$

e. Find the standard deviation of the differences.

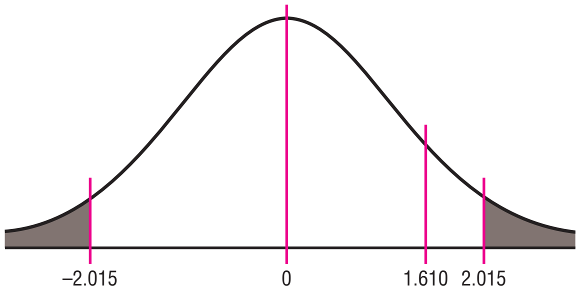
$$s_D = \sqrt{\frac{\Sigma D^2 - \frac{(\Sigma D)^2}{n}}{n-1}} = \sqrt{\frac{4890 - \frac{(100)^2}{6}}{5}} = 25.4$$

f. Find the test value.

$$t = \frac{\bar{D} - \mu_D}{s_D/\sqrt{n}} = \frac{16.7 - 0}{25.4/\sqrt{6}} = 1.610$$

STEP 4 Make the decision. The decision is not to reject the null hypothesis, since the test value 1.610 is in the noncritical region, as shown in Figure 9–12.

Figure 9–12
Critical and Test Values for Example 9–13



STEP 5 Summarize the results. There is not enough evidence to support the claim that the mineral changes a person's cholesterol level.

The steps for this t test are summarized in the Procedure Table:

Procedure Table

Testing the Difference Between Means for Dependent Samples

STEP 1 State the hypotheses and identify the claim.

STEP 2 Find the critical value(s).

STEP 3 Compute the test value.

a. Make a table, as shown.

X_1	X_2	A $D = X_1 - X_2$	B $D^2 = (X_1 - X_2)^2$
⋮	⋮		
		$\Sigma D =$ ______	$\Sigma D^2 =$ ______

b. Find the differences and place the results in column A.

$$D = X_1 - X_2$$

c. Find the mean of the differences.

$$\bar{D} = \frac{\Sigma D}{n}$$

d. Square the differences and place the results in column B. Complete the table.

$$D^2 = (X_1 - X_2)^2$$

e. Find the standard deviation of the differences.

$$s_D = \sqrt{\frac{\Sigma D^2 - \frac{(\Sigma D)^2}{n}}{n - 1}}$$

f. Find the test value.

$$t = \frac{\bar{D} - \mu_D}{s_D/\sqrt{n}} \qquad \text{with d.f.} = n - 1$$

STEP 4 Make the decision.

STEP 5 Summarize the results.

The P-values for the t test are found in Table F. For a two-tailed test with d.f. $= 5$ and $t = 1.610$, the P-value is found between 1.476 and 2.015; hence, $0.10 < P\text{-value} < 0.20$. Thus, the null hypothesis cannot be rejected at $\alpha = 0.10$.

If a specific difference is hypothesized, this formula should be used

$$t = \frac{\bar{D} - \mu_D}{s_D/\sqrt{n}}$$

where μ_D is the hypothesized difference.

For example, if a dietitian claims that people on a specific diet will lose an average of 3 pounds in a week, the hypotheses are

$$H_0: \mu_D = 3 \qquad \text{and} \qquad H_1: \mu_D \neq 3$$

The value 3 will be substituted in the test statistic formula for μ_D.

Confidence intervals can be found for the mean differences with this formula.

Confidence Interval for the Mean Difference

$$\bar{D} - t_{\alpha/2}\frac{s_D}{\sqrt{n}} < \mu_D < \bar{D} + t_{\alpha/2}\frac{s_D}{\sqrt{n}}$$

d.f. $= n - 1$

Example 9–14

Find the 90% confidence interval for the data in Example 9–13.

Solution

Substitute in the formula

$$\bar{D} - t_{\alpha/2}\frac{s_D}{\sqrt{n}} < \mu_D < \bar{D} + t_{\alpha/2}\frac{s_D}{\sqrt{n}}$$

$$16.7 - 2.015 \cdot \frac{25.4}{\sqrt{6}} < \mu_D < 16.7 + 2.015 \cdot \frac{25.4}{\sqrt{6}}$$

$$16.7 - 20.89 < \mu_D < 16.7 + 20.89$$

$$-4.19 < \mu_D < 37.59$$

Since 0 is contained in the interval, the decision is not to reject the null hypothesis H_0: $\mu_D = 0$.

Exercises 9–5

1. Classify each as independent or dependent samples.

a. Heights of identical twins
b. Test scores of the same students in English and psychology
c. The effectiveness of two different brands of aspirin
d. Effects of a drug on reaction time, measured by a "before" and an "after" test
e. The effectiveness of two different diets on two different groups of individuals

For Exercises 2 through 10, perform each of these steps. Assume that all variables are normally or approximately normally distributed.

a. State the hypotheses and identify the claim.
b. Find the critical value(s).
c. Compute the test value.
d. Make the decision.
e. Summarize the results.

Use the traditional method of hypothesis testing unless otherwise specified.

2. A program for reducing the number of days missed by food handlers in a certain restaurant chain was conducted. The owners hypothesized that after the program the workers would miss fewer days of work due to illness. The table shows the number of days 10 workers missed per month before and after completing the program. Is there enough evidence to support the claim, at $\alpha = 0.05$, that the food handlers missed fewer days after the program?

Before	2	3	6	7	4	5	3	1	0	0
After	1	4	3	8	3	3	1	0	1	0

3. As an aid for improving students' study habits, nine students were randomly selected to attend a seminar on the importance of education in life. The table shows the number of hours each student studied per week before and after the seminar. At $\alpha = 0.10$, did attending the seminar increase the number of hours the students studied per week?

Before	9	12	6	15	3	18	10	13	7
After	9	17	9	20	2	21	15	22	6

4. A doctor is interested in determining whether a film about exercise will change 10 persons' attitudes about exercise. The results of his questionnaire are shown. A higher numerical value shows a more favorable attitude toward exercise. Is there enough evidence to support the claim, at $\alpha = 0.05$, that there was a change in attitude?

Speaking of STATISTICS

Here is a study that involves two groups of children. What was the sample size for each group? State possible null and alternative hypotheses for this study. What statistical test was probably used? What was the decision? Explain your answer.

Pedaling a solution for couch-potato kids

By Nanci Hellmich
USA TODAY

Getting kids to exercise instead of vegging out in front of the TV has long been a problem for parents. But researchers have figured out how to get kids moving while watching TV.

Obesity researchers at St. Luke's-Roosevelt Hospital Center in New York City wanted to know what would happen if kids had to ride a stationary bike to keep the TV working. So they designed and built TV-cycles.

They randomly assigned 10 overweight, sedentary children ages 8 to 12, to a bicycle that required the child to pedal for TV time or a bicycle that was in front of the TV but not necessary for its operation.

They found that:

➢ The children who had to pedal to watch TV biked an average of an hour a week compared with eight minutes for the others.

➢ The treatment group watched one hour of TV a week, while the other children watched for an average of more than 20 hours.

➢ The treatment group significantly decreased overall body fat.

"This was a non-nagging approach" to get kids to exercise, says lead investigator David Allison, associate professor of medical psychology at Columbia University College of Physicians and Surgeons. "We told parents to just let the bicycle do the work."

One problem: It was tough on parents not to be able to watch TV, and they had to find ways to occupy their kids.

"This is an example of changing the environment to promote activity. We need to think of how we can take this concept and make it work for more people," Allison says.

The study was discussed Sunday at the Experimental Biology meeting in Washington, D.C.

Source: *USA TODAY,* April 19, 1999. Used with permission.

Find the 95% confidence interval for the difference of the two means.

Before	12	11	14	9	8	6	8	5	4	7
After	13	12	10	9	8	8	7	6	5	5

5. Students in a statistics class were asked to report the number of hours they slept on weeknights and on weekends. At $\alpha = 0.05$, is there sufficient evidence that there is a difference in the mean number of hours slept?

Student	1	2	3	4	5	6	7	8
Hours, Sun.–Thurs.	8	5.5	7.5	8	7	6	6	8
Hours, Fri.–Sat.	4	7	10.5	12	11	9	6	9

6. College students at a professional golf camp played one round of golf on the opening day of camp. After a week of intensive instruction, they played a second round of golf. Based on the scores for a random sample of golfers, can it be concluded that the scores improved? (Remember, lower golf scores are better.) Use $\alpha = 0.05$.

Golfer	1	2	3	4	5	6	7
First day	80	72	78	68	75	69	72
Last day	78	75	72	70	72	68	73

7. A composition teacher wishes to see whether a new grammar program will reduce the number of grammatical errors her students make when writing a two-page essay. The data are shown here. At $\alpha = 0.025$, can it be concluded that the number of errors has been reduced?

Student	1	2	3	4	5	6
Errors before	12	9	0	5	4	3
Errors after	9	6	1	3	2	3

8. Incoming first-year students at a particular college are required to take a mathematics placement exam. As an experiment, six randomly selected students are given the exam, and then they participate in a 3-hour refresher class. These six students are then given a retest. Is there sufficient evidence at the 0.05 level of significance that the refresher class helped?

Student	1	2	3	4	5	6
Before	10	16	12	12	18	20
After	12	15	15	12	17	20

9. A researcher wanted to compare the pulse rates of identical twins to see whether there was any difference. Eight sets of twins were selected. The rates are given in the table as number of beats per minute. At $\alpha = 0.01$, is there a significant difference in the average pulse rates of twins? Find the 99% confidence interval for the difference of the two. Use the *P*-value method.

Twin A	87	92	78	83	88	90	84	93
Twin B	83	95	79	83	86	93	80	86

10. A reporter hypothesizes that the average assessed values of land in a large city have changed during a 5-year period. A random sample of wards is selected, and the data (in millions of dollars) are shown. At $\alpha = 0.05$, can it be concluded that the average taxable assessed values have changed? Use the *P*-value method.

Ward	1994	1999
A	184	161
B	414	382
C	22	22
D	99	109
E	116	120
F	49	52
G	24	28
H	50	50
I	282	297
J	25	40
K	141	148
L	45	56
M	12	20
N	37	38
O	9	9
P	17	19

Source: *Pittsburgh Tribune Review.*

Extending the Concepts

11. Instead of finding the mean of the differences between X_1 and X_2 by subtracting $X_1 - X_2$, one can find it by finding the means of X_1 and X_2 and then subtracting the means. Show that these two procedures will yield the same results.

Technology Step by Step

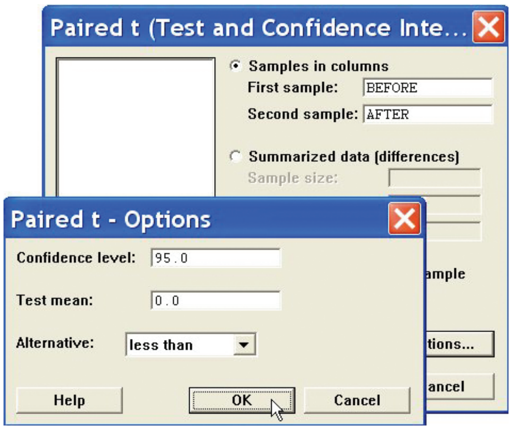
MINITAB Step by Step

Test the Difference Between Two Means: Small Dependent Samples

For Example 9–12, test the effectiveness of the vitamin regimen. Is there a difference in the strength of the athletes after the treatment?

1. Enter the data into C1 and C2. Name the columns **Before** and **After.**
2. Select **Stat>Basic Statistics>Paired t.**
3. Double-click C1 Before for First sample.
4. Double-click C2 After for Second sample. The second sample will be subtracted from the first. The differences are not stored or displayed.
5. Click [Options].

6. Change the Alternative to less than.
7. Click [OK] twice.



Paired T-Test and CI: BEFORE, AFTER

```
Paired T for BEFORE - AFTER
              N      Mean     StDev    SE Mean
BEFORE        8   222.125    25.920      9.164
AFTER         8   224.500    27.908      9.867
Difference    8  -2.37500   4.83846    1.71065

95% upper bound for mean difference: 0.86597
T-Test of mean difference = 0 (vs < 0) : T-Value = -1.39  P-Value = 0.104.
```

Since the *P*-value is 0.104, do not reject the null hypothesis. The sample difference of −2.38 in the strength measurement is not statistically significant.

TI-83 Plus Step by Step

Hypothesis Test for the Difference Between Two Means: Dependent Samples

1. Enter the data values into L_1 and L_2.
2. Move the cursor to the top of the L_3 column so that L_3 is highlighted.
3. Type $\mathbf{L_1 - L_2}$, then press **ENTER.**
4. Press **STAT** and move the cursor to TESTS.
5. Press **2** for TTest.
6. Move the cursor to Data and press **ENTER.**
7. Type in the appropriate values, using **0** for μ_0 and $\mathbf{L_3}$ for the list.
8. Move the cursor to the appropriate alternative hypothesis and press **ENTER.**
9. Move the cursor to Calculate and press **ENTER.**

Confidence Interval for the Difference Between Two Means: Dependent Samples

1. Enter the data values into L_1 and L_2.
2. Move the cursor to the top of the L_3 column so that L_3 is highlighted.
3. Type $\mathbf{L_1 - L_2}$, then press **ENTER.**
4. Press **STAT** and move the cursor to TESTS.
5. Press **8** for TInterval.
6. Move the cursor to Stats and press **ENTER.**
7. Type in the appropriate values, using $\mathbf{L_3}$ for the list.
8. Move the cursor to Calculate and press **ENTER.**

Excel
Step by Step

t Test for the Difference Between Two Means: Small Dependent Samples

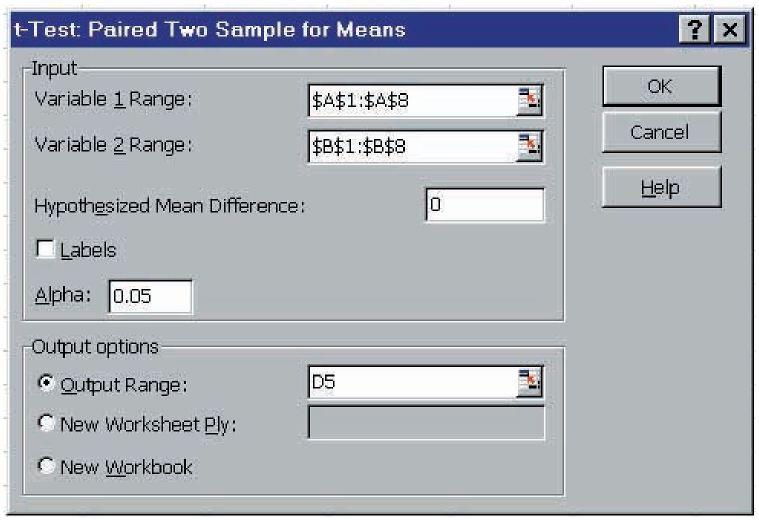
Example XL9–4

Test the hypothesis that there is no difference in population means, based on these sample paired data. Use $\alpha = 0.05$.

Set A	33	35	28	29	32	34	30	34
Set B	27	29	36	34	30	29	28	24

1. Enter the eight-number data set A in column A.
2. Enter the eight-number data set B in column B.
3. Select **Tools>Data Analysis** and choose t-Test: Paired Two Sample for Means.
4. Enter the data ranges and hypothesized mean difference (here, zero), and α.
5. Select a location for output and click [OK].

Dialog Box for Paired-Data *t* Test



The screen shows a *P*-value of 0.3253988 for the two-tailed case. This is greater than the confidence level 0.05, so we fail to reject the null hypothesis.

t-Test: Paired Two Sample for Means		
	Variable 1	*Variable 2*
Mean	31.875	29.625
Variance	6.696428571	14.55357143
Observations	8	8
Pearson Correlation	-0.757913399	
Hypothesized Mean Difference	0	
df	7	
t Stat	1.057517468	
P(T<=t) one-tail	0.1626994	
t Critical one-tail	1.894577508	
P(T<=t) two-tail	0.3253988	
t Critical two-tail	2.36462256	

9–6

Testing the Difference Between Proportions

Objective 5. Test the difference between two proportions.

The z test with some modifications can be used to test the equality of two proportions. For example, a researcher might ask: Is the proportion of men who exercise regularly less than the proportion of women who exercise regularly? Is there a difference in the percentage of students who own a personal computer and the percentage of non-students who own one? Is there a difference in the proportion of college graduates who pay cash for purchases and the proportion of non-college graduates who pay cash?

Recall from Chapter 7 that the symbol $\hat{p}$ ("p hat") is the sample proportion used to estimate the population proportion, denoted by p. For example, if in a sample of 30 college students, 9 are on probation, then the sample proportion is $\hat{p} = \frac{9}{30}$, or 0.3. The population proportion p is the number of all students who are on probation, divided by the number of students who attend the college. The formula for $\hat{p}$ is

$$\hat{p} = \frac{X}{n}$$

where

X = number of units that possess the characteristic of interest
n = sample size

When one is testing the difference between two population proportions p_1 and p_2, the hypotheses can be stated thus, if no difference between the proportions is hypothesized.

$$H_0\text{: } p_1 = p_2 \qquad \text{or} \qquad H_0\text{: } p_1 - p_2 = 0$$
$$H_1\text{: } p_1 \neq p_2 \qquad \qquad H_1\text{: } p_1 - p_2 \neq 0$$

Similar statements using $\geq$ and $<$ or $\leq$ and $>$ can be formed for one-tailed tests.

For two proportions, $\hat{p}_1 = X_1/n_1$ is used to estimate p_1 and $\hat{p}_2 = X_2/n_2$ is used to estimate p_2. The standard error of the difference is

$$\sigma_{(\hat{p}_1-\hat{p}_2)} = \sqrt{\sigma^2_{p_1} + \sigma^2_{p_2}} = \sqrt{\frac{p_1q_1}{n_1} + \frac{p_2q_2}{n_2}}$$

where $\sigma^2_{p_1}$ and $\sigma^2_{p_2}$ are the variances of the proportions, $q_1 = 1 - p_1$, $q_2 = 1 - p_2$, and n_1 and n_2 are the respective sample sizes.

Since p_1 and p_2 are unknown, a weighted estimate of p can be computed by using the formula

$$\bar{p} = \frac{n_1\hat{p}_1 + n_2\hat{p}_2}{n_1 + n_2}$$

and $\bar{q} = 1 - \bar{p}$. This weighted estimate is based on the hypothesis that $p_1 = p_2$. Hence, $\bar{p}$ is a better estimate than either $\hat{p}_1$ or $\hat{p}_2$, since it is a combined average using both $\hat{p}_1$ and $\hat{p}_2$.

Since $\hat{p}_1 = X_1/n_1$ and $\hat{p}_2 = X_2/n_2$, $\bar{p}$ can be simplified to

$$\bar{p} = \frac{X_1 + X_2}{n_1 + n_2}$$

Finally, the standard error of the difference in terms of the weighted estimate is

$$\sigma_{(\hat{p}_1-\hat{p}_2)} = \sqrt{\bar{p}\,\bar{q}\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}$$

The formula for the test value is shown next.

Formula for the z Test for Comparing Two Proportions

$$z = \frac{(\hat{p}_1 - \hat{p}_2) - (p_1 - p_2)}{\sqrt{\bar{p}\,\bar{q}\left(\dfrac{1}{n_1} + \dfrac{1}{n_2}\right)}}$$

where

$$\bar{p} = \frac{X_1 + X_2}{n_1 + n_2} \qquad \hat{p}_1 = \frac{X_1}{n_1}$$

$$\bar{q} = 1 - \bar{p} \qquad \hat{p}_2 = \frac{X_2}{n_2}$$

This formula follows the format

$$\text{Test value} = \frac{(\text{observed value}) - (\text{expected value})}{\text{standard error}}$$

There are two requirements for use of the z test: (1) The samples must be independent of each other, and (2) n_1p_1 and n_1q_1 must be 5 or more, and n_2p_2 and n_2q_2 must be 5 or more.

Example 9–15

In the nursing home study mentioned in the chapter-opening "Statistics Today," the researchers found that 12 out of 34 small nursing homes had a resident vaccination rate of less than 80%, while 17 out of 24 large nursing homes had a vaccination rate of less than 80%. At $\alpha = 0.05$, test the claim that there is no difference in the proportions of the small and large nursing homes with a resident vaccination rate of less than 80%.

Source: Nancy Arden, Arnold S. Monto, and Suzanne E. Ohmit, "Vaccine Use and the Risk of Outbreaks in a Sample of Nursing Homes during an Influenza Epidemic," *American Journal of Public Health.*

Solution

Let $\hat{p}_1$ be the proportion of the small nursing homes with a vaccination rate of less than 80% and $\hat{p}_2$ be the proportion of the large nursing homes with a vaccination rate of less than 80%. Then

$$\hat{p}_1 = \frac{X_1}{n_1} = \frac{12}{34} = 0.35 \qquad \text{and} \qquad \hat{p}_2 = \frac{X_2}{n_2} = \frac{17}{24} = 0.71$$

$$\bar{p} = \frac{X_1 + X_2}{n_1 + n_2} = \frac{12 + 17}{34 + 24} = \frac{29}{58} = 0.5$$

$$\bar{q} = 1 - \bar{p} = 1 - 0.5 = 0.5$$

Now, follow the steps in hypothesis testing.

STEP 1 State the hypotheses and identify the claim.

$$H_0\colon p_1 = p_2 \text{ (claim)} \qquad \text{and} \qquad H_1\colon p_1 \neq p_2$$

STEP 2 Find the critical values. Since $\alpha = 0.05$, the critical values are $+1.96$ and -1.96.

STEP 3 Compute the test value.

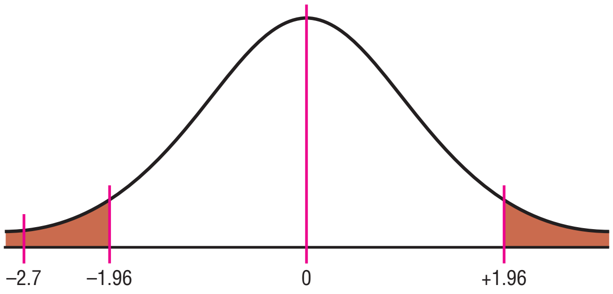
$$z = \frac{(\hat{p}_1 - \hat{p}_2) - (p_1 - p_2)}{\sqrt{\bar{p}\,\bar{q}\left(\dfrac{1}{n_1} + \dfrac{1}{n_2}\right)}}$$

$$= \frac{(0.35 - 0.71) - 0}{\sqrt{(0.5)(0.5)\left(\dfrac{1}{34} + \dfrac{1}{24}\right)}} = \frac{-0.36}{0.1333} = -2.7$$

STEP 4 Make the decision. Reject the null hypothesis, since $-2.7 < -1.96$. See Figure 9–13.

Figure 9–13

Critical and Test Values for Example 9–15



STEP 5 Summarize the results. There is enough evidence to reject the claim that there is no difference in the proportions of small and large nursing homes with a resident vaccination rate of less than 80%.

Example 9–16

A sample of 50 randomly selected men with high triglyceride levels consumed 2 tablespoons of oat bran daily for 6 weeks. After 6 weeks, 60% of the men had lowered their triglyceride level. A sample of 80 men consumed 2 tablespoons of wheat bran for 6 weeks. After 6 weeks, 25% had lower triglyceride levels. Is there a significant difference in the two proportions, at the 0.01 significance level?

Solution

Since the statistics are given in percentages, $\hat{p}_1 = 60\%$, or 0.60, and $\hat{p}_2 = 25\%$, or 0.25. To compute $\bar{p}$, one must find X_1 and X_2.

Since $\hat{p}_1 = X_1/n_1$, $X_1 = \hat{p}_1 \cdot n_1$, and since $\hat{p}_2 = X_2/n_2$, $X_2 = \hat{p}_2 \cdot n_2$.

$$X_1 = (0.60)(50) = 30 \qquad X_2 = (0.25)(80) = 20$$

$$\bar{p} = \frac{X_1 + X_2}{n_1 + n_2} = \frac{30 + 20}{50 + 80} = \frac{50}{130} = 0.385$$

$$\bar{q} = 1 - \bar{p} = 1 - 0.385 = 0.615$$

STEP 1 State the hypotheses and identify the claim.

$$H_0\colon p_1 = p_2 \qquad \text{and} \qquad H_1\colon p_1 \neq p_2 \text{ (claim)}$$

STEP 2 Find the critical values. Since $\alpha = 0.01$, the critical values are $+2.58$ and -2.58.

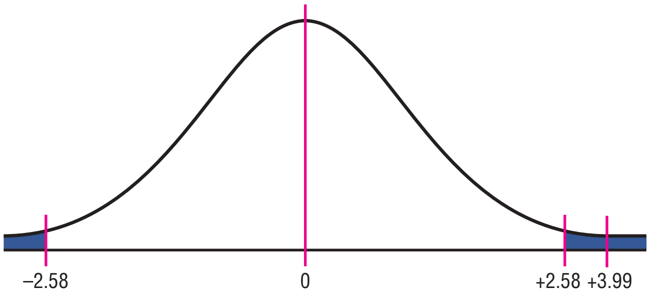
STEP 3 Compute the test value.

$$z = \frac{(\hat{p}_1 - \hat{p}_2) - (p_1 - p_2)}{\sqrt{\bar{p}\,\bar{q}\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}$$

$$= \frac{(0.60 - 0.25) - 0}{\sqrt{(0.385)(0.615)\left(\frac{1}{50} + \frac{1}{80}\right)}} = 3.99$$

STEP 4 Make the decision. Reject the null hypothesis, since $3.99 > 2.58$. See Figure 9–14.

Figure 9–14
Critical and Test Values for Example 9–16



STEP 5 Summarize the results. There is enough evidence to support the claim that there is a difference in proportions.

The *P*-value for the difference of proportions can be found from Table E, as shown in Section 8–3. For Example 9–16, 3.99 is beyond 3.09; hence, the null hypothesis can be rejected since the *P*-value is less than 0.001.

The formula for the confidence interval for the difference between two proportions is shown next.

Confidence Interval for the Difference Between Two Proportions

$$(\hat{p}_1 - \hat{p}_2) - z_{\alpha/2}\sqrt{\frac{\hat{p}_1\hat{q}_1}{n_1} + \frac{\hat{p}_2\hat{q}_2}{n_2}} < p_1 - p_2 < (\hat{p}_1 - \hat{p}_2) + z_{\alpha/2}\sqrt{\frac{\hat{p}_1\hat{q}_1}{n_1} + \frac{\hat{p}_2\hat{q}_2}{n_2}}$$

Example 9–17

Find the 95% confidence interval for the difference of proportions for the data in Example 9–15.

Solution

$$\hat{p}_1 = \frac{12}{34} = 0.35 \qquad \hat{q}_1 = 0.65$$

$$\hat{p}_2 = \frac{17}{24} = 0.71 \qquad \hat{q}_2 = 0.29$$

Substitute in the formula.

$$(\hat{p}_1 - \hat{p}_2) - z_{\alpha/2}\sqrt{\frac{\hat{p}_1\hat{q}_1}{n_1} + \frac{\hat{p}_2\hat{q}_2}{n_2}} < p_1 - p_2$$
$$< (\hat{p}_1 - \hat{p}_2) + z_{\alpha/2}\sqrt{\frac{\hat{p}_1\hat{q}_1}{n_1} + \frac{\hat{p}_2\hat{q}_2}{n_2}}$$
$$(0.35 - 0.71) - 1.96\sqrt{\frac{(0.35)(0.65)}{34} + \frac{(0.71)(0.29)}{24}}$$
$$< p_1 - p_2 < (0.35 - 0.71) + 1.96\sqrt{\frac{(0.35)(0.65)}{34} + \frac{(0.71)(0.29)}{24}}$$
$$-0.36 - 0.242 < p_1 - p_2 < -0.36 + 0.242$$
$$-0.602 < p_1 - p_2 < -0.118$$

Since 0 is not contained in the interval, the decision is to reject the null hypothesis H_0: $p_1 = p_2$.

Exercises 9–6

1a. Find the proportions $\hat{p}$ and $\hat{q}$ for each.
a. $n = 48, X = 34$
b. $n = 75, X = 28$
c. $n = 100, X = 50$
d. $n = 24, X = 6$
e. $n = 144, X = 12$

1b. Find each X, given $\hat{p}$.
a. $\hat{p} = 0.16, n = 100$
b. $\hat{p} = 0.08, n = 50$
c. $\hat{p} = 6\%, n = 80$
d. $\hat{p} = 52\%, n = 200$
e. $\hat{p} = 20\%, n = 150$

2. Find $\bar{p}$ and $\bar{q}$ for each.
a. $X_1 = 60, n_1 = 100, X_2 = 40, n_2 = 100$
b. $X_1 = 22, n_1 = 50, X_2 = 18, n_2 = 30$
c. $X_1 = 18, n_1 = 60, X_2 = 20, n_2 = 80$
d. $X_1 = 5, n_1 = 32, X_2 = 12, n_2 = 48$
e. $X_1 = 12, n_1 = 75, X_2 = 15, n_2 = 50$

For Exercises 3 through 14, perform these steps.
a. State the hypotheses and identify the claim.
b. Find the critical value(s).
c. Compute the test value.
d. Make the decision.
e. Summarize the results.

Use the traditional method of hypothesis testing unless otherwise specified.

3. A sample of 150 people from a certain industrial community showed that 80 people suffered from a lung disease. A sample of 100 people from a rural community showed that 30 suffered from the same lung disease. At $\alpha = 0.05$, is there a difference between the proportions of people who suffer from the disease in the two communities?

4. According to the U.S. Department of Education, 51.9% of women and 46.7% of men receive financial aid in undergraduate school. A random sample of undergraduate students revealed these results. At $\alpha = 0.01$, is there sufficient evidence to conclude that the difference in proportions has changed?

	Women	**Men**
Sample size	250	300
Number receiving aid	200	180

Source: U.S. Department of Education, National Center for Education Statistics.

5. Labor statistics indicate that 77% of cashiers and servers are women. A random sample of cashiers and servers in a large metropolitan area found that 112 of 150 cashiers and 150 of 200 servers were women. At the 0.05 level of significance, is there sufficient evidence to conclude that a difference exists between the proportion of servers and the proportion of cashiers who are women?

Source: *N.Y. Times Almanac.*

6. In Cleveland, a sample of 73 mail carriers showed that 10 had been bitten by an animal during one week. In Philadelphia, in a sample of 80 mail carriers, 16 had received animal bites. Is there a significant difference in the proportions? Use $\alpha = 0.05$. Find the 95% confidence interval for the difference of the two proportions.

7. A survey found that 83% of the men questioned preferred computer-assisted instruction to lecture and 75% of the women preferred computer-assisted instruction to lecture. There were 100 individuals in each sample. At $\alpha = 0.05$, test the claim that there is no difference in the proportion of men and the proportion of women who favor computer-assisted instruction over lecture. Find the 95% confidence interval for the difference of the two proportions.

8. In a sample of 200 surgeons, 15% thought the government should control health care. In a sample of 200 general practitioners, 21% felt this way. At $\alpha = 0.10$, is there a difference in the proportions? Find the 90% confidence interval for the difference of the two proportions.

9. In a sample of 80 Americans, 55% wished that they were rich. In a sample of 90 Europeans, 45% wished that they were rich. At $\alpha = 0.01$, is there a difference in the proportions? Find the 99% confidence interval for the difference of the two proportions.

10. In a sample of 200 men, 130 said they used seat belts. In a sample of 300 women, 63 said they used seat belts. Test the claim that men are more safety-conscious than women, at $\alpha = 0.01$. Use the *P*-value method.

11. A survey of 80 homes in a Washington, D.C., suburb showed that 45 were air-conditioned. A sample of 120 homes in a Pittsburgh suburb showed that 63 had air conditioning. At $\alpha = 0.05$, is there a difference in the two proportions? Find the 95% confidence interval for the difference of the two proportions.

12. A recent study showed that in a sample of 100 people, 30% had visited Disneyland. In another sample of 100 people, 24% had visited Disney World. Are the proportions of people who visited each park different? Use $\alpha = 0.02$ and the *P*-value method.

13. A sample of 200 teenagers shows that 50 believe that war is inevitable, and a sample of 300 people over age 60 shows that 93 believe war is inevitable. Is the proportion of teenagers who believe war is inevitable different from the proportion of people over age 60 who do? Use $\alpha = 0.01$. Find the 99% confidence interval for the difference of the two proportions.

14. In a sample of 50 high school seniors, 8 had their own cars. In a sample of 75 college freshmen, 20 had their own cars. At $\alpha = 0.05$, can it be concluded that a higher proportion of college freshmen have their own cars? Use the *P*-value method.

15. Find the 99% confidence interval for the difference in the population proportions for the data of a study in which 80% of the 150 Republicans surveyed favored the bill for a salary increase and 60% of the 200 Democrats surveyed favored the bill for a salary increase.

16. The Miami county commissioners feel that a higher percentage of women work there than in neighboring Greene County. To test this, they randomly select 1000 women in each county and find that in Miami, 622 women work and in Greene, 594 work. Using $\alpha = 0.05$, do you think the Miami County commissioners are correct?

Source: 2000 U.S. Census/*Dayton Daily News.*

17. In a sample of 100 store customers, 43 used a MasterCard. In another sample of 100, 58 used a Visa card. At $\alpha = 0.05$, is there a difference in the proportion of people who use each type of credit card?

18. Find the 95% confidence interval for the true difference in proportions for the data of a study in which 40% of the 200 males surveyed opposed the death penalty and 56% of the 100 females surveyed opposed the death penalty.

19. The percentages of adults 25 years of age and older who have completed 4 or more years of college are 23.6% for females and 27.8% for males. A random sample of women and men who were 25 years old or older was surveyed with these results. Estimate the true difference in proportions with 95% confidence, and compare your interval with the *Almanac*'s statistics.

	Women	Men
Sample size	350	400
No. who completed 4 or more years	100	115

Source: *N.Y. Times Almanac.*

Extending the Concepts

20. If there is a significant difference between p_1 and p_2 and between p_2 and p_3, can one conclude that there is a significant difference between p_1 and p_3?

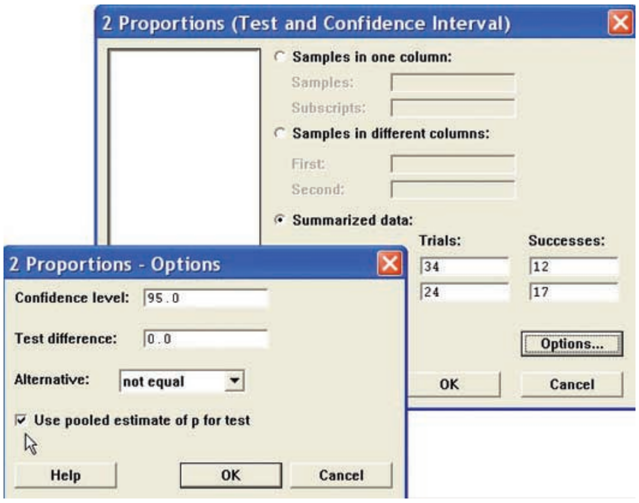
Technology Step by Step

MINITAB
Step by Step

Test the Difference Between Two Proportions

For Example 9–15, test for a difference in the resident vaccination rates between small and large nursing homes.

1. This test does not require data. It doesn't matter what is in the worksheet.
2. Select **Stat>Basic Statistics>2 Proportions.**
3. Click the button for Summarized data.
4. Press **TAB** to move cursor to the First sample box for Trials.
   a) Enter **34, TAB,** then enter **12.**
   b) Press **TAB** or click in the second sample text box for trials.
   c) Enter **24, TAB,** then enter **17.**
5. Click on [Options]. Check the box for Use pooled estimate of p for test. The Confidence level should be 95%, and the Test difference should be 0.
6. Click [OK] twice. The results are shown in the session window.



Test and CI for Two Proportions

```
Sample    X    N   Sample p
1        12   34   0.352941
2        17   24   0.708333

Difference = p (1) - p (2)
Estimate for difference:  -0.355392
95% CI for difference:  (-0.598025, -0.112759)
Test for difference = 0 (vs not = 0):  Z = -2.67 P-Value = 0.008
```

The *P*-value of the test is 0.008. Reject the null hypothesis. The difference is statistically significant. Thirty-five percent of all small nursing homes, compared to 71% of all large nursing homes, have an immunization rate of 80%. We can't tell why, only that there is a difference!

TI-83 Plus Step by Step

Hypothesis Test for the Difference Between Two Proportions

1. Press **STAT** and move the cursor to TESTS.
2. Press **6** for 2-PropZTEST.
3. Type in the appropriate values.
4. Move the cursor to the appropriate alternative hypothesis and press **ENTER.**
5. Move the cursor to Calculate and press **ENTER.**

Confidence Interval for the Difference Between Two Proportions

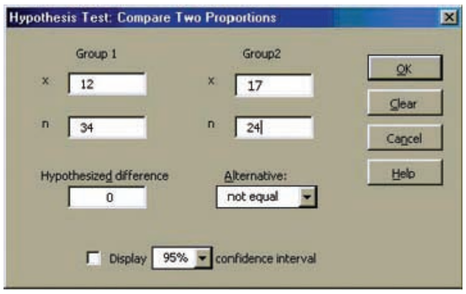
1. Press **STAT** and move the cursor to TESTS.
2. Press **B (ALPHA APPS)** for 2-PropZInt.
3. Type in the appropriate values.
4. Move the cursor to Calculate and press **ENTER.**

Excel
Step by Step

Testing the Difference Between Two Proportions

Excel does not have a procedure to test the difference between two proportions. However, you may test the difference between two proportions by using the Mega-Stat Add-in available on your CD and Online Learning Center. If you have not installed this add-in, do so by following the instructions on page 24.

We will use the summary information from Example 9–15.



1. Select **Mega-Stat>Hypothesis tests>Compare Two Independent Proportions.**
2. Enter the information as indicated in the figure and click [OK].

Note: This hypothesis test is conducted using the *P*-value method; thus no significance level is required.

9–7 Summary

Many times researchers are interested in comparing two population parameters, such as means or proportions. This comparison can be accomplished by using special z and t tests. If the samples are independent and the variances are known, the z test is used. The z test is also used when the variances are unknown but both sample sizes are 30 or more. If the variances are not known and one or both sample sizes are less than 30, the t test must be used. For independent samples, a further requirement is that one must determine whether the variances of the populations are equal. The F test is used to determine whether the variances are equal. Different formulas are used in each case. If the samples are dependent, the t test for dependent samples is used. Finally, a z test is used to compare two proportions.

Important Terms

dependent samples 464	*F* test 444	independent samples 454	pooled estimate of the variance 456
F distribution 445			

Important Formulas

Formula for the z test for comparing two means from independent populations:

$$z = \frac{(\bar{X}_1 - \bar{X}_2) - (\mu_1 - \mu_2)}{\sqrt{\dfrac{\sigma_1^2}{n_1} + \dfrac{\sigma_2^2}{n_2}}}$$

Formula for the confidence interval for difference of two means (large samples):

$$(\bar{X}_1 - \bar{X}_2) - z_{\alpha/2}\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}} < \mu_1 - \mu_2 < (\bar{X}_1 - \bar{X}_2) + z_{\alpha/2}\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}$$

Formula for the F test for comparing two variances:

$$F = \frac{s_1^2}{s_2^2} \quad \begin{array}{l} \text{d.f.N.} = n_1 - 1 \\ \text{d.f.D.} = n_2 - 1 \end{array}$$

Formula for the t test for comparing two means (small independent samples, variances not equal):

$$t = \frac{(\bar{X}_1 - \bar{X}_2) - (\mu_1 - \mu_2)}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}$$

and d.f. $=$ the smaller of $n_1 - 1$ or $n_2 - 1$.

Formula for the t test for comparing two means (independent samples, variances equal):

$$t = \frac{(\bar{X}_1 - \bar{X}_2) - (\mu_1 - \mu_2)}{\sqrt{\frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{(n_1 + n_2 - 2)}}\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}$$

and d.f. $= n_1 + n_2 - 2$.

Formula for the confidence interval for the difference of two means (small independent samples, variances unequal):

$$(\bar{X}_1 - \bar{X}_2) - t_{\alpha/2}\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}} < \mu_1 - \mu_2 < (\bar{X}_1 - \bar{X}_2) + t_{\alpha/2}\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}$$

and d.f. $=$ smaller of $n_1 - 1$ and $n_2 - 2$.

Formula for the confidence interval for the difference of two means (small independent samples, variances equal):

$$(\bar{X}_1 - \bar{X}_2) - t_{\alpha/2}\sqrt{\frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2}} \cdot \sqrt{\frac{1}{n_1} + \frac{1}{n_2}} < \mu_1 - \mu_2 < (\bar{X}_1 - \bar{X}_2) + t_{\alpha/2}\sqrt{\frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2}} \cdot \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}$$

and d.f. $= n_1 + n_2 - 2$.

Formula for the t test for comparing two means from dependent samples:

$$t = \frac{\bar{D} - \mu_D}{s_D/\sqrt{n}}$$

where $\bar{D}$ is the mean of the differences

$$\bar{D} = \frac{\Sigma D}{n}$$

and s_D is the standard deviation of the differences,

$$s_D = \sqrt{\frac{\Sigma D^2 - \frac{(\Sigma D)}{n}}{n - 1}}$$

Formula for confidence interval for the mean of the difference for dependent samples:

$$\bar{D} - t_{\alpha/2}\frac{s_D}{\sqrt{n}} < \mu_D < \bar{D} + t_{\alpha/2}\frac{s_D}{\sqrt{n}}$$

and d.f. $= n - 1$.

Formula for the z test for comparing two proportions:

$$z = \frac{(\hat{p}_1 - \hat{p}_2) - (p_1 - p_2)}{\sqrt{\bar{p}\,\bar{q}\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}$$

where

$$\bar{p} = \frac{X_1 + X_2}{n_1 + n_2} \qquad \hat{p}_1 = \frac{X_1}{n_1}$$

$$\bar{q} = 1 - \bar{p} \qquad \hat{p}_2 = \frac{X_2}{n_2}$$

Formula for confidence interval for the difference of two proportions:

$$(\hat{p}_1 - \hat{p}_2) - z_{\alpha/2}\sqrt{\frac{\hat{p}_1\hat{q}_1}{n_1} + \frac{\hat{p}_2\hat{q}_2}{n_2}} < p_1 - p_2 < (\hat{p}_1 - \hat{p}_2) + z_{\alpha/2}\sqrt{\frac{\hat{p}_1\hat{q}_1}{n_1} + \frac{\hat{p}_2\hat{q}_2}{n_2}}$$

Review Exercises

For each exercise, perform these steps. Assume that all variables are normally or approximately normally distributed.

a. State the hypotheses and identify the claim.
b. Find the critical value(s).
c. Compute the test value.
d. Make the decision.
e. Summarize the results.

Use the traditional method of hypothesis testing unless otherwise specified.

1. Two groups of drivers are surveyed to see how many miles per week they drive for pleasure trips.

The data are shown. At $\alpha = 0.01$, can it be concluded that single drivers do more driving for pleasure trips on average than married drivers?

Single drivers					Married drivers				
106	110	115	121	132	97	104	138	102	115
119	97	118	122	135	133	120	119	136	96
110	117	116	138	142	139	108	117	145	114
115	114	103	98	99	140	136	113	113	150
108	117	152	147	117	101	114	116	113	135
154	86	115	116	104	115	109	147	106	88
107	133	138	142	140	113	119	99	108	105

2. An educator wishes to compare the variances of the amount of money spent per pupil in two states. The data are given below. At $\alpha = 0.05$, is there a significant difference in the variances of the amounts the states spend per pupil?

State 1	State 2
$s_1^2 = \$585$	$s_2^2 = \$261$
$n_1 = 18$	$n_2 = 16$

3. In the hospital study cited in Exercise 19 in Exercise Section 7–2, the standard deviation of the noise levels of the 11 intensive care units was 4.1 dBA, and the standard deviation of the noise levels of 24 nonmedical care areas, such as kitchens and machine rooms, was 13.2 dBA. At $\alpha = 0.10$, is there a significant difference between the standard deviations of these two areas?

Source: M. Bayo, A. Garcia, and A. Garcia, "Noise Levels in an Urban Hospital and Workers' Subjective Responses," *Archives of Environmental Health.*

4. A researcher wants to compare the variances of the heights (in inches) of major league baseball players with those of players in the minor leagues. A sample of 25 players from each league is selected, and the variances of the heights for each league are 2.25 and 4.85, respectively. At $\alpha = 0.10$, is there a significant difference between the variances of the heights for the two leagues?

5. A traffic safety commissioner believes the variation in the number of speeding tickets given on Route 19 is greater than the variation in the number of speeding tickets given on Route 22. Ten weeks are randomly selected; the standard deviation of the number of tickets issued for Route 19 is 6.3, and the standard deviation of the number of tickets issued for Route 22 is 2.8. At $\alpha = 0.05$, can the commissioner conclude that the variance of speeding tickets issued on Route 19 is greater than the variance of speeding tickets issued on Route 22? Use the *P*-value method.

6. The variations in the number of absentees per day in two schools are being compared. A sample of 30 days is selected; the standard deviation of the number of absentees in school A is 4.9, and for school B it is 2.5. At $\alpha = 0.01$, can one conclude that there is a difference in the two standard deviations?

7. A researcher claims the variation in the number of days that factory workers miss per year due to illness is greater than the variation in the number of days that hospital workers miss per year. A sample of 42 workers from a large hospital has a standard deviation of 2.1 days, and a sample of 65 workers from a large factory has a standard deviation of 3.2 days. Test the claim at $\alpha = 0.10$.

8. The average price of 15 cans of tomato soup from different stores is \$0.73, and the standard deviation is \$0.05. The average price of 24 cans of chicken noodle soup is \$0.91, and the standard deviation is \$0.03. At $\alpha = 0.01$, is there a significant difference in price?

9. The average temperatures for a 25-day period for Birmingham, Alabama, and Chicago, Illinois, are shown. Based on the samples, at $\alpha = 0.10$, can it be concluded that it is warmer in Birmingham?

Birmingham					Chicago				
78	82	68	67	68	70	74	73	60	77
75	73	75	64	68	71	72	71	74	76
62	73	77	78	79	71	80	65	70	83
74	72	73	78	68	67	76	75	62	65
73	79	82	71	66	66	65	77	66	64

10. A sample of 15 teachers from Rhode Island has an average salary of \$35,270, with a standard deviation of \$3256. A sample of 30 teachers from New York has an average salary of \$29,512, with a standard deviation of \$1432. Is there a significant difference in teachers' salaries between the two states? Use $\alpha = 0.02$. Find the 99% confidence interval for the difference of the two means.

11. The average income of 16 families who reside in a large metropolitan city is \$54,356, and the standard deviation is \$8256. The average income of 12 families who reside in a suburb of the same city is \$46,512, with a standard deviation of \$1311. At $\alpha = 0.05$, can one conclude that the income of the families who reside within the city is greater than that of those who reside in the suburb? Use the *P*-value method.

12. In an effort to improve the vocabulary of 10 students, a teacher provides a weekly 1-hour tutoring session for them. A pretest is given before the sessions and a posttest is given afterward. The results are shown in the table. At $\alpha = 0.01$, can the teacher conclude that the tutoring sessions helped to improve the students' vocabularies?

Before	1	2	3	4	5	6	7	8	9	10
Pretest	83	76	92	64	82	68	70	71	72	63
Posttest	88	82	100	72	81	75	79	68	81	70

13. In an effort to increase production of an automobile part, the factory manager decides to play music in the manufacturing area. Eight workers are selected, and the number of items each produced for a specific day is recorded. After one week of music, the same workers are monitored again. The data are given in the table. At $\alpha = 0.05$, can the manager conclude that the music has increased production?

Worker	1	2	3	4	5	6	7	8
Before	6	8	10	9	5	12	9	7
After	10	12	9	12	8	13	8	10

14. St. Petersburg, Russia, has 207 foggy days out of 365 days while Stockholm, Sweden, has 166 foggy days out of 365. At $\alpha = 0.02$, can it be concluded that the proportions of foggy days for the two cities are different? Find the 98% confidence interval for the difference of the two proportions.

Source: Jack Williams, *USA TODAY.*

15. In a recent survey of 50 apartment residents, 32 had microwave ovens. In a survey of 60 homeowners, 24 had microwave ovens. At $\alpha = 0.05$, test the claim that the proportions are equal. Find the 95% confidence interval for the difference of the two proportions.

Statistics Today

To Vaccinate or Not to Vaccinate? Small or Large?—Revisited

Using a z test to compare two proportions, the researchers found that the proportion of residents in smaller nursing homes who were vaccinated (80.8%) was statistically greater than that of residents in large nursing homes who were vaccinated (68.7%). Using statistical methods presented in later chapters, they also found that the larger size of the nursing home and the lower frequency of vaccination were significant predictions of influenza outbreaks in nursing homes.

Data Analysis

The Data Bank is found in Appendix D, or on the World Wide Web by following links from www.mhhe.com/math/stat/bluman/.

1. From the Data Bank, select a variable and compare the mean of the variable for a random sample of at least 30 men with the mean of the variable for the random sample of at least 30 women. Use a z test.

2. Repeat the experiment in Exercise 1, using a different variable and two samples of size 15. Compare the means by using a t test. Assume that the variances are equal.

3. Compare the proportion of men who are smokers with the proportion of women who are smokers. Use the data in the Data Bank. Choose random samples of size 30 or more. Use the z test for proportions.

4. Using the data from Data Set XIV, test the hypothesis that the means of the weights of the players for two professional football teams are equal. Use an α value of your choice. Be sure to include the five steps of hypothesis testing. Use a z test.

5. For the same data used in Exercise 4, test the equality of the variances of the weights.

6. Using the data from Data Set XV, test the hypothesis that the means of the sizes of earthquakes of the two hemispheres are equal. Select an α value and use a t test.

Chapter Quiz

Determine whether each statement is true or false. If the statement is false, explain why.

1. When one is testing the difference between two means for small samples, it is not important to distinguish whether the samples are independent of each other.

2. If the same diet is given to two groups of randomly selected individuals, the samples are considered to be dependent.

3. When computing the F test value, one always places the larger variance in the numerator of the fraction.

4. Tests for variances are always two-tailed.

Select the best answer.

5. To test the equality of two variances, one would use a(n) ______ test.

a. z *c.* Chi-square
b. t *d.* F

6. To test the equality of two proportions, one would use a(n) ______ test.

a. z *c.* Chi-square
b. t *d.* F

7. The mean value of the F is approximately equal to

a. 0 *c.* 1
b. 0.5 *d.* It cannot be determined.

8. What test can be used to test the difference between two small sample means?

a. z *c.* Chi-square
b. t *d.* F

Complete these statements with the best answer.

9. If one hypothesizes that there is no difference between means, this is represented as H_0: ______ .

10. When one is testing the difference between two means, a(n) ______ estimate of the variances is used when the variances are equal.

11. When the t test is used for testing the equality of two means, the populations must be ______ .

12. The values of F cannot be ______ .

13. The formula for the F test for variances is ______ .

For each of these problems, perform the following steps.

a. State the hypotheses and identify the claim.
b. Find the critical value(s).
c. Compute the test value.
d. Make the decision.
e. Summarize the results.

Use the traditional method of hypothesis testing unless otherwise specified.

14. A researcher wishes to see if there is a difference in the cholesterol levels of two groups of men. A random sample of 30 men between the ages of 25 and 40 is selected and tested. The average level is 223. A second sample of 25 men between the ages of 41 and 56 is selected and tested. The average of this group is 229. The population standard deviation for both groups is 6. At $\alpha = 0.01$, is there a difference in the cholesterol levels between the two groups? Find the 99% confidence interval for the difference of the two means.

15. The data shown are the rental fees (in dollars) for two random samples of apartments in a large city. At $\alpha = 0.10$, can it be concluded that the average rental fee for apartments in the East is greater than the average rental fee in the West?

East					West				
495	390	540	445	420	525	400	310	375	750
410	550	499	500	550	390	795	554	450	370
389	350	450	530	350	385	395	425	500	550
375	690	325	350	799	380	400	450	365	425
475	295	350	485	625	375	360	425	400	475
275	450	440	425	675	400	475	430	410	450
625	390	485	550	650	425	450	620	500	400
685	385	450	550	425	295	350	300	360	400

Source: *Pittsburgh Post-Gazette.*

16. A politician wishes to compare the variances of the amount of money spent for road repair in two different counties. The data are given here. At $\alpha = 0.05$, is there a significant difference in the variances of the amounts spent in the two counties? Use the P-value method.

County A	County B
$s_1 = \$11,596$	$s_2 = \$14,837$
$n_1 = 15$	$n_2 = 18$

17. A researcher wants to compare the variances of the heights (in inches) of 4-year college basketball players with those of players in junior colleges. A sample of 30 players from each type of school is selected, and the variances of the heights for each type are 2.43 and 3.15, respectively. At $\alpha = 0.10$, is there a significant difference between the variances of the heights in the two types of schools?

18. The data shown are based on a survey taken in February and July and indicate the number of hours per day of household television usage. At $\alpha = 0.05$, test the claim that there is no difference in the standard deviations of the number of hours that televisions are used.

February			July		
7.6	9.3	8.2	7.4	10.3	9.4
7.4	7.9	6.8	4.6	7.3	7.1
7.5	7.1	6.4	6.8	7.7	8.2
4.3	10.6	9.8	5.4	6.2	7.1

19. The variances of the amount of fat in two different types of ground beef are compared. Eight samples of the first type, Super Lean, have a variance of 18.2 grams; 12 of the second type, Ultimate Lean, have a variance of 9.4 grams. At $\alpha = 0.10$, can it be concluded that there is a difference in the variances of the two types of ground beef?

20. It is hypothesized that the variations of the number of days that high school teachers miss per year due to illness are greater than the variations of the number of days that nurses miss per year. A sample of 56 high school teachers has a standard deviation of 3.4 days, while a sample of 70 nurses has a standard deviation of 2.8. Test the hypothesis at $\alpha = 0.10$.

21. The variations in the number of retail thefts per day in two shopping malls are being compared. A sample of 21 days is selected. The standard deviation of the number of retail thefts in mall A is 6.8, and for mall B it is 5.3. At $\alpha = 0.05$, can it be concluded that there is a difference in the two standard deviations?

22. The average price of a sample of 12 bottles of diet salad dressing taken from different stores is \$1.43. The standard deviation is \$0.09. The average price of a sample of 16 low-calorie frozen desserts is \$1.03. The standard deviation is \$0.10. At $\alpha = 0.01$, is there a significant difference in price? Find the 99% confidence interval of the difference in the means.

23. The data shown represent the number of accidents people had when using jet skis and other types of wet bikes. At $\alpha = 0.05$, can it be concluded that the average number of accidents per year has increased during the last 5 years?

1987–1991			1992–1996		
376	650	844	1650	2236	3002
1162	1513		4028	4010	

Source: *USA TODAY.*

24. A sample of 12 chemists from Washington state shows an average salary of \$39,420 with a standard deviation of \$1659, while a sample of 26 chemists from New Mexico has an average salary of \$30,215 with a standard deviation of \$4116. Is there a significant difference between the two states in chemists' salaries at $\alpha = 0.02$? Find the 98% confidence interval of the difference in the means.

25. The average income of 15 families who reside in a large metropolitan East Coast city is \$62,456. The standard deviation is \$9652. The average income of 11 families who reside in a rural area of the Midwest is \$60,213, with a standard deviation of \$2009. At $\alpha = 0.05$, can it be concluded that the families who live in the cities have a higher income than those who live in the rural areas? Use the *P*-value method.

26. In an effort to improve the mathematical skills of 10 students, a teacher provides a weekly 1-hour tutoring session for the students. A pretest is given before the sessions, and a posttest is given after. The results are shown here. At $\alpha = 0.01$, can it be concluded that the sessions help to improve the students' mathematical skills?

Student	1	2	3	4	5	6	7	8	9	10
Pretest	82	76	91	62	81	67	71	69	80	85
Posttest	88	80	98	80	80	73	74	78	85	93

27. To increase egg production, a farmer decided to increase the amount of time the lights in his hen house were on. Ten hens were selected, and the number of eggs each produced was recorded. After one week of lengthened light time, the same hens were monitored again. The data are given here. At $\alpha = 0.05$, can it be concluded that the increased light time increased egg production?

Hen	1	2	3	4	5	6	7	8	9	10
Before	4	3	8	7	6	4	9	7	6	5
After	6	5	9	7	4	5	10	6	9	6

28. In a sample of 80 workers from a factory in city A, it was found that 5% were unable to read, while in a sample of 50 workers in city B, 8% were unable to read. Can it be concluded that there is a difference in the proportions of nonreaders in the two cities? Use $\alpha = 0.10$. Find the 90% confidence interval for the difference of the two proportions.

29. In a recent survey of 45 apartment residents, 28 had phone answering machines. In a survey of 55 homeowners, 20 had phone answering machines. At $\alpha = 0.05$, test the claim that the proportions are equal. Find the 95% confidence interval for the difference of the two proportions.

Critical Thinking Challenges

1. The study cited in the article shown on the next page used twins and found that, in men, a strong craving for sweets is linked to a tendency toward alcoholism. Based on the results of the study, answer the following questions.

a. Why do you think twins were used in the study?
b. What was the sample size?
c. What are the variables in the study?
d. How do you think the variables were measured?
e. How could a population be defined?

Study links cravings for sweets, alcohol

Studies on twins suggest that, in men, a strong craving for sweets is linked to a tendency to alcoholism, and the cause may be genetic, researchers say. Researchers at the University of North Carolina at Chapel Hill, led by David Overstreet, studied 19 pairs of male twins, none of whom had been diagnosed as alcoholic. "Those individuals who reported drinking more alcohol on occasion and having more alcohol-related problems also had problems with controlling how many sweets they ate," Overstreet says. The finding could lead to a screening test for youngsters and might allow early alcohol education and intervention. The researchers note that not everyone with a sweet tooth became an alcoholic, but the men who liked the most intense sweets also tended to like alcohol more.

Source: *USA TODAY,* Nov. 8, 2000. Reprinted with permission.

2. In the study shown, the researcher concluded that falsehoods are remembered longer than truths. After you read the study, answer these questions.

a. What was the sample size?

b. How did the researcher differentiate the truths from the falsehoods?

c. State possible null and alternative hypotheses for the study.

d. What type of instrument could have been used to measure the information remembered?

e. What statistical test could have been used to arrive at the conclusion?

Falsehoods linger longer than truth

Misinformation—the wrong stuff—is more likely to be remembered than the real, factual thing, a small study has found. While the memory of actual information tends to blur as time goes by, "not only do people recall more misinformation over time, but they tend to judge it as more vivid," says Peter Frost, a researcher with Rivier College in Nashua, N.H. The false information takes time to register: It comes to mind more readily a while after the event than immediately after it, he says. Frost did his research with 100 college students, who were asked to remember facts from a series of slides showing a crime. They also heard a narrative with some inaccurate information about what the slides portrayed. A week later, they were as apt to remember as fact what they heard in the false narrative as what they saw in the slides. Such research helps explain how "false memories" of events become established, Frost says. His study appears in *Psychonomic Bulletin & Review*.

Source: *USA TODAY,* December 11, 2000. Reprinted with permission.

Data Projects

Where appropriate, use MINITAB, the TI-83 Plus, or a computer program of your choice to complete the following exercises.

1. Choose a variable for which you would like to determine if there is a difference in the averages for two groups. Make sure that the samples are independent. For example, you may wish to see if men see more movies or spend more money on lunch than women. Select a sample of data values (10 to 50) and complete the following:

a. Write a brief statement of the purpose of the study.
b. Define the population.
c. State the hypotheses for the study.
d. Select an α value.
e. State how the sample was selected.
f. Show the raw data.
g. Decide which statistical test is appropriate and compute the test statistic (z or t). Why is the test appropriate?
h. Find the critical value(s).
i. State the decision.
j. Summarize the results.

2. Choose a variable that will permit the use of dependent samples. For example, you might wish to see if a person's weight has changed after a diet. Select a sample of data (10 to 50) value pairs (e.g., before and after), and then complete the following:

a. Write a brief statement of the purpose of the study.
b. Define the population.
c. State the hypotheses for the study.
d. Select an α value.
e. State how the sample was selected.
f. Show the raw data.
g. Decide which statistical test is appropriate and compute the test statistic (z or t). Why is the test appropriate?
h. Find the critical value(s).
i. State the decision.
j. Summarize the results.

3. Choose a variable that will enable you to compare proportions of two groups. For example, you might want to see if the proportion of first-year students who buy used books is lower than (or higher than or the same as) the proportion of sophomores who buy used books. After collecting 30 or more responses from the two groups, complete the following:

a. Write a brief statement of the purpose of the study.
b. Define the population.
c. State the hypotheses for the study.
d. Select an α value.
e. State how the sample was selected.
f. Show the raw data.
g. Decide which statistical test is appropriate and compute the test statistic (z or t). Why is the test appropriate?
h. Find the critical value(s).
i. State the decision.
j. Summarize the results.

You may use the following websites to obtain raw data:

http://www.mhhe.com/math/stat/bluman/
http://lib.stat.cmu.edu/DASL
http://www.statcan.ca/english/

Hypothesis-Testing Summary 1

1. Comparison of a sample mean with a specific population mean.

Example: H_0: $\mu = 100$

a. Use the z test when σ is known:

$$z = \frac{\overline{X} - \mu}{\sigma/\sqrt{n}}$$

b. Use the t test when σ is unknown:

$$t = \frac{\overline{X} - \mu}{s/\sqrt{n}} \qquad \text{with d.f.} = n - 1$$

2. Comparison of a sample variance or standard deviation with a specific population variance or standard deviation.

Example: H_0: $\sigma^2 = 225$

Use the chi-square test:

$$\chi^2 = \frac{(n-1)s^2}{\sigma^2} \qquad \text{with d.f.} = n - 1$$

3. Comparison of two sample means.

Example: H_0: $\mu_1 = \mu_2$

a. Use the z test when the population variances are known:

$$z = \frac{(\overline{X}_1 - \overline{X}_2) - (\mu_1 - \mu_2)}{\sqrt{\dfrac{\sigma_1^2}{n_1} + \dfrac{\sigma_2^2}{n_2}}}$$

b. Use the t test for independent samples when the population variances are unknown and the sample variances are unequal:

$$t = \frac{(\bar{X}_1 - \bar{X}_2) - (\mu_1 - \mu_2)}{\sqrt{\dfrac{s_1^2}{n_1} + \dfrac{s_2^2}{n_2}}}$$

with d.f. = the smaller of $n_1 - 1$ or $n_2 - 1$.

c. Use the t test for independent samples when the population variances are unknown and assumed to be equal:

$$t = \frac{(\bar{X}_1 - \bar{X}_2) - (\mu_1 - \mu_2)}{\sqrt{\dfrac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2}}\sqrt{\dfrac{1}{n_1} + \dfrac{1}{n_2}}}$$

with d.f. $= n_1 + n_2 - 2$.

d. Use the t test for means for dependent samples:

Example: H_0: $\mu_D = 0$

$$t = \frac{\bar{D} - \mu_D}{s_D/\sqrt{n}} \qquad \text{with d.f.} = n - 1$$

where n = number of pairs.

4. Comparison of a sample proportion with a specific population proportion.

Example: H_0: $p = 0.32$

Use the z test:

$$z = \frac{X - \mu}{\sigma} \qquad \text{or} \qquad z = \frac{\hat{p} - p}{\sqrt{pq/n}}$$

5. Comparison of two sample proportions.

Example: H_0: $p_1 = p_2$

Use the z test:

$$z = \frac{(\hat{p}_1 - \hat{p}_2) - (p_1 - p_2)}{\sqrt{\bar{p}\,\bar{q}\left(\dfrac{1}{n_1} + \dfrac{1}{n_2}\right)}}$$

where

$$\bar{p} = \frac{X_1 + X_2}{n_1 + n_2} \qquad \hat{p}_1 = \frac{X_1}{n_1}$$

$$\bar{q} = 1 - \bar{p} \qquad \hat{p}_2 = \frac{X_2}{n_2}$$

6. Comparison of two sample variances or standard deviations.

Example: H_0: $\sigma_1^2 = \sigma_2^2$

Use the F test:

$$F = \frac{s_1^2}{s_2^2}$$

where

s_1^2 = larger variance d.f.N. $= n_1 - 1$

s_2^2 = smaller variance d.f.D. $= n_2 - 1$