HW6

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Problem 1)

```
data <- read.table('pearls.txt', header=T, colClasses = c('numeric', 'factor','factor','factor'))

mu <- mean(data$value)
mu_i <- tapply(data$value, data$no_coats, mean)
mu_j <- tapply(data$value, data$batch, mean)

tau_b <- mu - mu_j
tau_a <- mu - mu_i
```

(a)

$Y_{ijk} = \mu_{..} + \alpha_i + \beta_j + (\alpha\beta)_{ij} + \epsilon_{ijk}$

the terms $\alpha_i, (\alpha\beta)_{ij}, \epsilon_{ijk}$ are random and $\mu_{..}, \beta_j$ are fixed.

(b)

Mean sq	df	Expected MS
MSA	$a-1=2$	$\sigma^2 + n\sigma^2_{\alpha\beta} + \frac{bn}{a-1}\sum_j \alpha_i^2$
MSB	$b-1=3$	$\sigma^2 + an\sigma^2_{\beta}$
MSAB	$(a-1)(b-1)=6$	$\sigma^2 + n\sigma^2_{\alpha\beta}$
MSE	$ab(n-1)=36$	σ^2

$\sigma^2 = 4.832$

$\sigma^2_{\alpha\beta} = -1.1285$

$\sigma_\beta = 3.844$

$E\{MSAB\} = 4.823$

$E\{MSB\} = 50.951$

$E\{MSA\} = 81.2488333$

(c)

```
fit2 <- lm(value ~ no_coats + batch + no_coats:batch, data)
anova(fit2)
```

```
## Analysis of Variance Table
```

```
##
## Response: value
##                 Df  Sum Sq Mean Sq F value    Pr(>F)
## no_coats         2 150.388  75.194  15.591 1.327e-05 ***
## batch            3 152.852  50.951  10.564 3.984e-05 ***
## no_coats:batch   6   1.852   0.309   0.064    0.9988
## Residuals       36 173.625   4.823
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

$H_0 : \sigma^2_{\alpha\beta} = 0$

$H_a : \sigma^2_{\alpha\beta} > 0$

$F* = \frac{MSAB}{MSE} = 0.0627666 < F = 2.363751$

concluding null, interaction term is not significant.

d)

$H_0 : \sigma^2_{\alpha} = 0$

$H_a : \sigma^2_{\alpha} > 0$

$F^* = \frac{75.194}{0.309} = 243.3462783$

$F = 5.1432528$

$F^* > F$ concluding H_a the effect of coat factor is significant.

e)

Mean sq	df	Expected MS
MSA	$a-1=2$	$\sigma^2 + bn\sigma^2_\alpha$
MSB	$b-1=3$	$\sigma^2 + n\sigma^2_{\alpha\beta} + \frac{an}{b-1}\sum_j \beta_j^2$
MSAB	$(a-1)(b-1)=6$	$\sigma^2 + n\sigma^2_{\alpha\beta}$
MSE	$ab(n-1)=36$	σ^2

$\sigma^2 = 4.832$

$\sigma^2_{\alpha\beta} = -1.1285$

$\sigma_\alpha = 4.3981875$

$E\{MSAB\} = 4.823$

$E\{MSA\} = 75.194$

$E\{MSB\} = 72.7570741$

f)

```
mu = tapply(data$value, data$no_coat, mean)
l1 = mu[2] - mu[1]
l2 = mu[3] - mu[1]
```

```
s = sqrt((0.309/(4*4))*(2))
t = qt(1- 0.1/(2*2),  3*4*3)
```

$B = 2.028094$

$s\{\hat{D}\} = 0.1965324$

$\mu_6 - \mu_8 = 3.6875$

$\mu_6 - \mu_{10} = 3.81875$

$3.2889137 \leq \mu_8 - \mu_6 \leq 4.0860863$

$3.4201637 \leq \mu_8 - \mu_{10} \leq 4.2173363$

Problem 2)

a)

$Y_{ijk} = \mu_{..} + \alpha_i + \beta_{j(i)} + \epsilon_{ijk}$

$\mu_{..} constant variable$

α_i : constant related to factor A and subject to $\sum \alpha_i = 0$

$\beta_{j(i)} : random normal variable with mean 0$

$epsilon_{ijk} independet N(0, \sigma^2)$

$i = 1, \ldots, 2; j = 1, \ldots, 3; k = 1, \ldots, 5$

b)

$SSA = 0.01825$

$SSB(A) = 0.01153 + 0.44249 = 0.45402$

$SSE = 0.29020$

Source	SS	df	MS	$E\{MS\}$
Factor A	0.01825	a-1=1	0.01825	$\sigma^2 + bn\frac{\sum \alpha_i^2}{a-1} + n\sigma_\beta^2$
Factor B(in A)	0.45402	a(b-1)=4	0.113505	$\sigma^2 + n\sigma_\beta^2$
Error	0.29020	ab(n-1)=24	0.0120917	σ^2
Total	0.47227	abn-1=29	0.0162852	

assuming $\alpha = 0.05$

for factor A:

$F^* = MSA/MSB(A) = 0.1607859$

$F = 7.7086474$

$F^* < F$

concluding H_0, there's no considerable variability in different sites.

for factor B:

$F^* = MSB(A)/MSE = 9.3870434$

$F = 2.7762893$

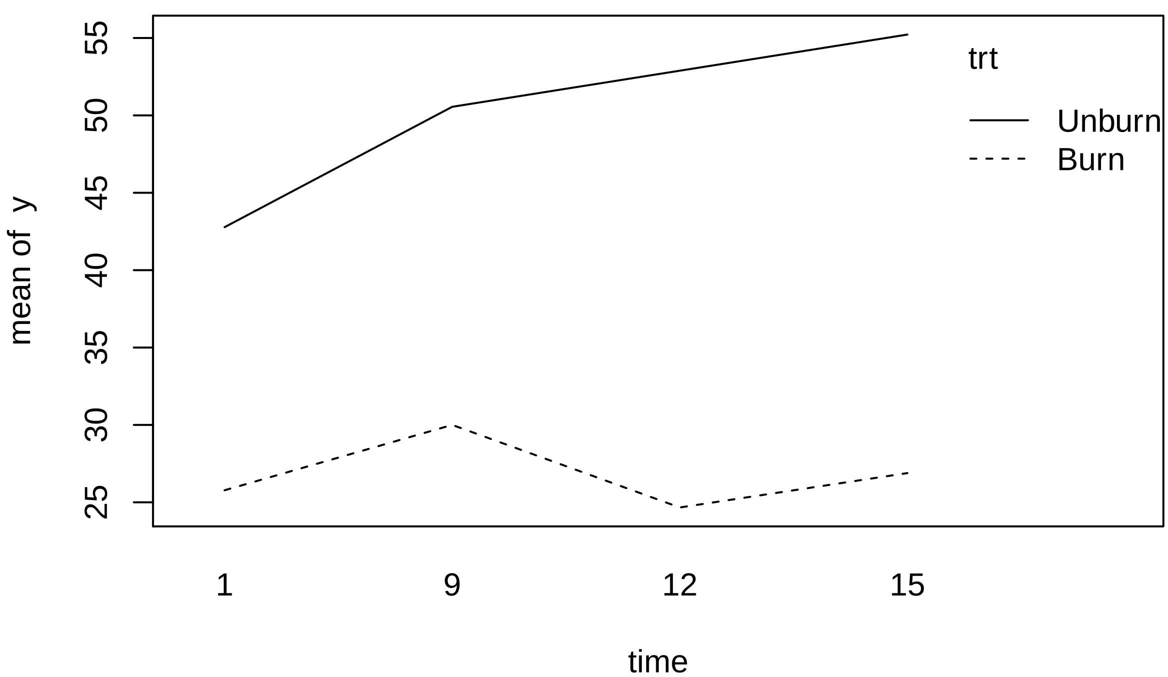
$F^* > F$ concluding H_a, there is considerable batch to batch difference.

Problem 3)

```r
data = read.table('floral.dat',header=T,sep='\t')
trt = as.factor(data$trt)
plot = as.factor(data$plot)
time = as.factor(data$time)
y = data$resp
```

a)

```r
interaction.plot(time, trt, y)
```



The mean of y for unburned region is higher compared to the burned region. The mean of Y increases with time for unburned region. For the burned region the the mean of y increases with time between 1-9 and deacreases between 9-12 and again increases between 12-15.

b)

$Y_{ijk} = \mu_{...} + \rho_{i(j)} + \alpha_j = \beta_k + (\alpha\beta)_{jk} + \epsilon_{ijk}$

$\mu_{...}$ is constant

$\rho_{i(j)}$ random $\sim N(0, \sigma_p^2)$, nested with in factor A.

α_j burn/no burn, fixed

β_k : Time factor, fixed

c)

That the covariance for the same plot has compound symmetry and all covariances should be similar in magnitude. Yes.

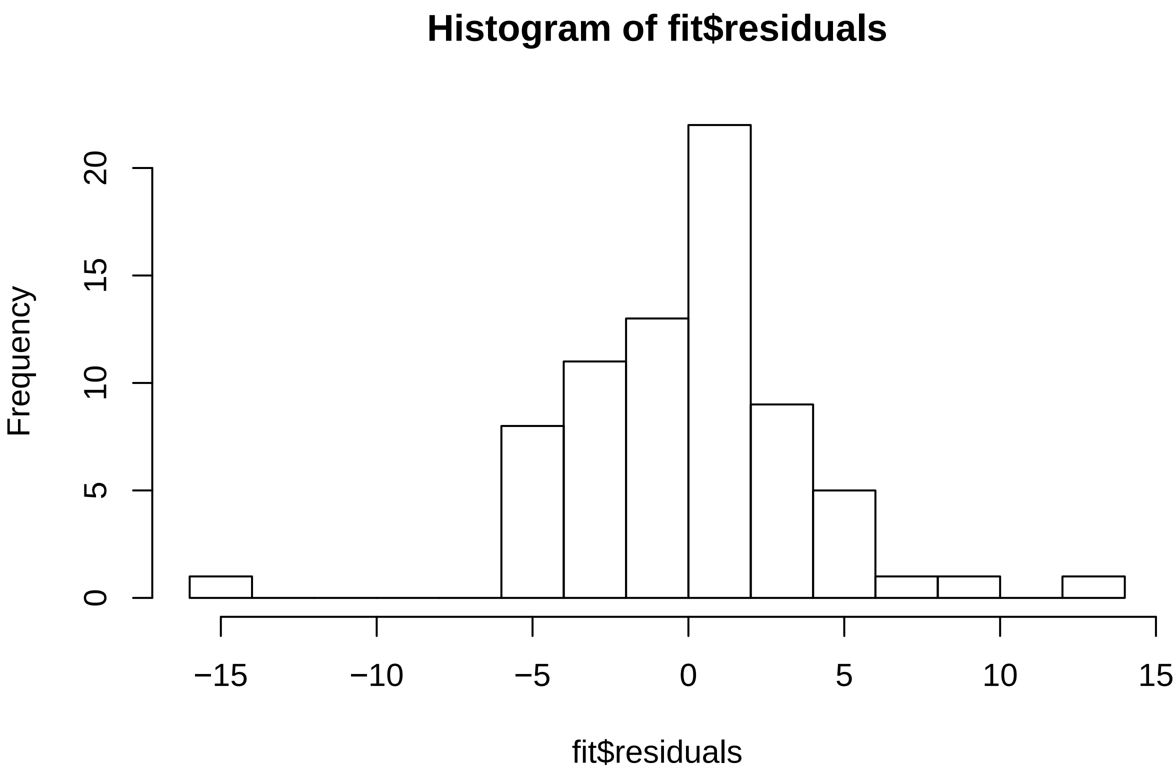
d)

Yes, except for the two extreme points in left and right the rest of the points seems to be normal.

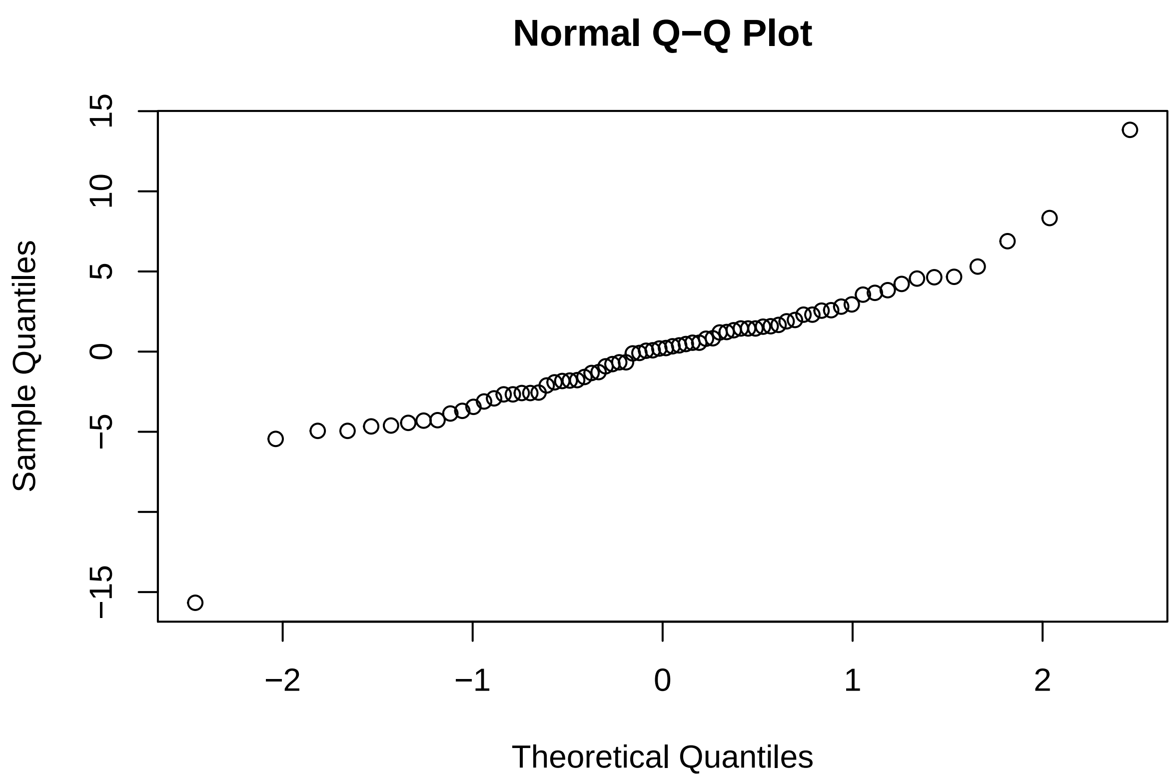
```r
fit <- lm(y ~ trt/plot + trt + time + trt:time)
anova(fit)
```

```
## Analysis of Variance Table
##
## Response: y
##           Df Sum Sq Mean Sq  F value    Pr(>F)
## trt        1 9964.0  9964.0 446.7287 < 2.2e-16 ***
## time       3  496.0   165.3   7.4132 0.0003540 ***
## trt:plot  16 3564.1   222.8   9.9870  2.43e-10 ***
## trt:time   3  434.6   144.9   6.4949 0.0008879 ***
## Residuals 48 1070.6    22.3
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```r
hist(fit$residuals, 20)
```




```r
qqnorm(fit$residuals)
```



e)

$H_0 : all \alpha_j = 0$ H_a : not all$\alpha_j$equal zero

$F^* = \frac{MSA}{MSS(A)} = 44.7217235$

$F = 4.4939985$

$F^* > F$ we conclude H_a. That treatment effect exist and burned areas are significantly different.