HW6

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Problem 1)

```r
data <- read.table('pearls.txt', header=T, colClasses = c('numeric', 'factor','factor','factor'))
```

(a)

$Y_{ijk} = \mu_{..} + \alpha_i + \beta_j + (\alpha\beta)_{ij} + \epsilon_{ijk}$

the terms $\alpha_i, (\alpha\beta)_{ij}, \epsilon_{ijk}$ are random and $\mu_{..}, \beta_j$ are fixed.

(b)

Mean sq	df	Expected MS
MSA	$a-1=2$	$\sigma^2 + n\sigma^2_{\alpha\beta} + \frac{bn}{a-1}\sum_j \alpha_i^2$
MSB	$b-1=3$	$\sigma^2 + an\sigma^2_{\beta}$
MSAB	$(a-1)(b-1)=6$	$\sigma^2 + n\sigma^2_{\alpha\beta}$
MSE	$ab(n-1)=36$	σ^2

(c)

```r
library(lme4)
```

```
## Loading required package: Matrix
```

```r
fit <- lmer(value ~ no_coats + (1|batch), data=data)
summary(fit)
```

```
## Linear mixed model fit by REML ['lmerMod']
## Formula: value ~ no_coats + (1 | batch)
##    Data: data
##
## REML criterion at convergence: 207.9
##
## Scaled residuals:
##     Min      1Q  Median      3Q     Max
## -2.1991 -0.6281  0.1033  0.6555  1.3638
##
## Random effects:
##  Groups   Name        Variance Std.Dev.
##  batch    (Intercept) 3.898    1.974
##  Residual             4.178    2.044
## Number of obs: 48, groups:  batch, 4
```

```
##
## Fixed effects:
##             Estimate Std. Error t value
## (Intercept)  73.1062     1.1116   65.77
## no_coats2     3.6875     0.7227    5.10
## no_coats3     3.8188     0.7227    5.28
##
## Correlation of Fixed Effects:
##           (Intr) n_cts2
## no_coats2 -0.325
## no_coats3 -0.325  0.500
```

```
fit2 <- lm(value ~ no_coats + batch + no_coats:batch, data)
anova(fit2)
```

```
## Analysis of Variance Table
##
## Response: value
##                Df  Sum Sq Mean Sq F value    Pr(>F)
## no_coats        2 150.388  75.194  15.591 1.327e-05 ***
## batch           3 152.852  50.951  10.564 3.984e-05 ***
## no_coats:batch  6   1.852   0.309   0.064    0.9988
## Residuals      36 173.625   4.823
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

$H_0 : \sigma^2_{\alpha\beta} = 0$

$H_a : \sigma^2_{\alpha\beta} > 0$

$F* = \frac{MSAB}{MSE} = 0.0627666 < F = 2.363751$

concluding null, interaction term is not significant.

d)

$H_0 : \sigma^2_{\alpha} = 0$

$H_a : \sigma^2_{\alpha} > 0$

$F^* = \frac{75.194}{0.309} = 243.3462783$

$F = 5.1432528$

$F^* > F$ concluding H_a the effect of coat factor is significant.

e)

Mean sq	df	Expected MS
MSA	$a-1=2$	$\sigma^2 + bn\sigma^2_\alpha$
MSB	$b-1=3$	$\sigma^2 + n\sigma^2_{\alpha\beta} + \frac{an}{b-1}\sum_j \beta_j^2$
MSAB	$(a-1)(b-1)=6$	$\sigma^2 + n\sigma^2_{\alpha\beta}$
MSE	$ab(n-1)=36$	σ^2

f)

```
mu = tapply(data$value, data$no_coat, mean)
l1 = mu[2] - mu[1]
l2 = mu[3] - mu[1]
s = sqrt((0.309/(4*4))*(2))
t = qt(1- 0.1/(2*2),  3*4*3)
```

$B = 2.028094$

$s\{\hat{D}\} = 0.1965324$

$\mu_6 - \mu_8 = 3.6875$

$\mu_6 - \mu_{10} = 3.81875$

$3.2889137 \leq \mu_8 - \mu_6 \leq 4.0860863$

$3.4201637 \leq \mu_8 - \mu_{10} \leq 4.2173363$

Problem 2)

a)

$Y_{ijk} = \mu_{..} + \alpha_i + \beta_{j(i)} + \epsilon_{ijk}$

$\mu_{..} constant variable$

α_i : constant related to factor A and subject to $\sum \alpha_i = 0$

$\beta_{j(i)} : random normal variable with mean 0$

$epsilon_{ijk} independet N(0, \sigma^2)$

$i = 1, \ldots, 2; j = 1, \ldots, 3; k = 1, \ldots, 5$

b)

$SSA = 0.01825$

$SSB(A) = 0.01153 + 0.44249 = 0.45402$

$SSE = 0.29020$

Source	SS	df	MS	$E\{MS\}$
Factor A	0.01825	a-1=1	0.01825	$\sigma^2 + bn\frac{\sum \alpha_i^2}{a-1} + n\sigma_\beta^2$
Factor B(in A)	0.45402	a(b-1)=4	0.113505	$\sigma^2 + n\sigma_\beta^2$
Error	0.29020	ab(n-1)=24	0.0120917	σ^2
Total	0.47227	abn-1=29	0.0162852	

assuming $\alpha = 0.05$

for factor A:

$F^* = MSA/MSB(A) = 0.1607859$

$F = 7.7086474$

$F^* < F$

concluding H_0, there's no considerable variability in different sites.

for factor B:

$F^* = MSB(A)/MSE = 9.3870434$

$F = 2.7762893$

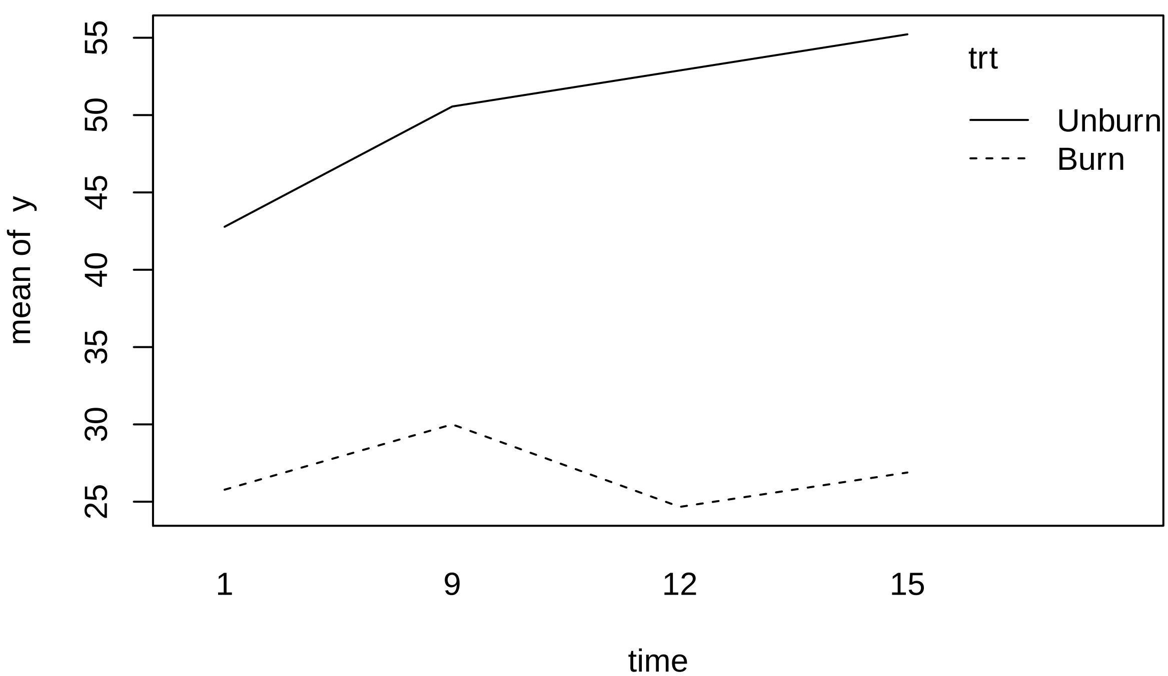
$F^* > F$ concluding H_a, there is considerable batch to batch difference.

Problem 3)

```
data = read.table('floral.dat',header=T,sep='\t')
trt = as.factor(data$trt)
plot = as.factor(data$plot)
time = as.factor(data$time)
y = data$resp
```

a)

```
interaction.plot(time, trt, y)
```



The mean of y for unburned region is higher compared to the burned region. The mean of Y increases with time for unburned region. For the the burned region the the mean of y increases with time between 1-9 and deacreases between 9-12 and again increases between 12-15.

b)

$Y_{ijk} = \mu_{...} + \rho_{i(j)} + \alpha_j = \beta_k + (\alpha\beta)_{jk} + \epsilon_{ijk}$

$\mu_{...}$ is constant

$\rho_{i(j)}$ random $\sim N(0, \sigma_p^2)$, nested with in factor A.

α_j burn/no burn, fixed

β_k : Time factor, fixed

c)

That they have constant covariance. ## d)

e)