Major League Baseball No-Hitters

The following analysis was excerpted from the Data Camp course "Statistical Thinking in Python"

How often do we get no-hitters?

The number of games played between each no-hitter in the modern era (1901-2015) of Major League Baseball is stored in the array `nohitter_times`.

If you assume that no-hitters are described as a Poisson process, then the time between no-hitters is Exponentially distributed. As you have seen, the Exponential distribution has a single parameter, which we will call τ, the typical interval time. The value of the parameter τ that makes the exponential distribution best match the data is the mean interval time (where time is in units of number of games) between no-hitters.

Compute the value of this parameter from the data. Then, use `np.random.exponential()` to "repeat" the history of Major League Baseball by drawing inter-no-hitter times from an exponential distribution with the τ you found and plot the histogram as an approximation to the PDF.

In [129]:

```
import numpy as np
import matplotlib.pyplot as plt
import seaborn as sns
```

In [130]:

```
nohitter_times = np.array([843, 1613, 1101,  215,  684,  814,  278,  324,  161,  219,  545,
        715,  966,  624,   29,  450,  107,   20,   91, 1325,  124, 1468,
        104, 1309,  429,   62, 1878, 1104,  123,  251,   93,  188,  983,
        166,   96,  702,   23,  524,   26,  299,   59,   39,   12,    2,
        308, 1114,  813,  887,  645, 2088,   42, 2090,   11,  886, 1665,
       1084, 2900, 2432,  750, 4021, 1070, 1765, 1322,   26,  548, 1525,
         77, 2181, 2752,  127, 2147,  211,   41, 1575,  151,  479,  697,
        557, 2267,  542,  392,   73,  603,  233,  255,  528,  397, 1529,
       1023, 1194,  462,  583,   37,  943,  996,  480, 1497,  717,  224,
        219, 1531,  498,   44,  288,  267,  600,   52,  269, 1086,  386,
        176, 2199,  216,   54,  675, 1243,  463,  650,  171,  327,  110,
        774,  509,    8,  197,  136,   12, 1124,   64,  380,  811,  232,
        192,  731,  715,  226,  605,  539, 1491,  323,  240,  179,  702,
        156,   82, 1397,  354,  778,  603, 1001,  385,  986,  203,  149,
        576,  445,  180, 1403,  252,  675, 1351, 2983, 1568,   45,  899,
       3260, 1025,   31,  100, 2055, 4043,   79,  238, 3931, 2351,  595,
        110,  215,    0,  563,  206,  660,  242,  577,  179,  157,  192,
        192, 1848,  792, 1693,   55,  388,  225, 1134, 1172, 1555,   31,
       1582, 1044,  378, 1687, 2915,  280,  765, 2819,  511, 1521,  745,
       2491,  580, 2072, 6450,  578,  745, 1075, 1103, 1549, 1520,  138,
       1202,  296,  277,  351,  391,  950,  459,   62, 1056, 1128,  139,
        420,   87,   71,  814,  603, 1349,  162, 1027,  783,  326,  101,
        876,  381,  905,  156,  419,  239,  119,  129,  467])
```

In [131]:

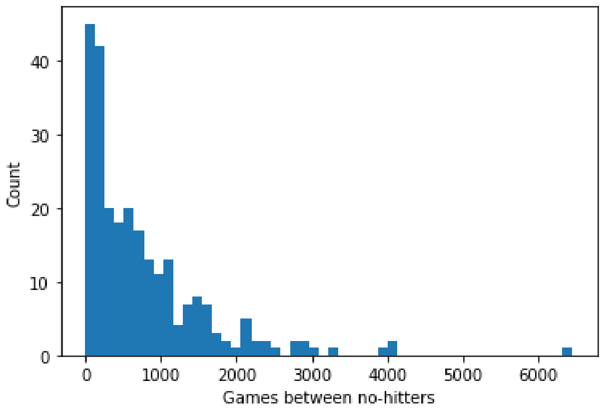
```
#Print summary statistcs
print('The array count is',len(nohitter_times))
print('The mean is',np.mean(nohitter_times))
print('The median is',np.median(nohitter_times))
print('The variance is',np.var(nohitter_times))
print('The standard deviation is',np.std(nohitter_times))
print('The percentiles are',np.percentile(nohitter_times,[2.5, 25, 50, 75, 97.5]))
#np.sort(nohitter_times)
```

```
The array count is 251
The mean is 763.0358565737051
The median is 524.0
The variance is 707938.0664433899
The standard deviation is 841.3905552378098
The percentiles are [  20.75  190.    524.   1063.   2911.25]
```

Visualize the data first

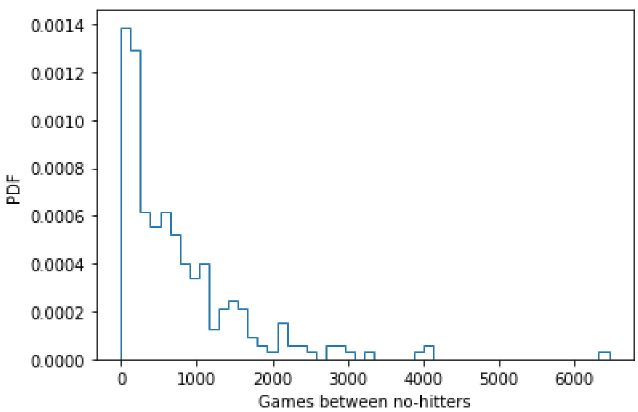
In [132]:

```
# Plot a histogram and label axes
_ = plt.hist(nohitter_times, bins=50)
_ = plt.xlabel('Games between no-hitters')
_ = plt.ylabel('Count')

# Show the plot
plt.show()
```



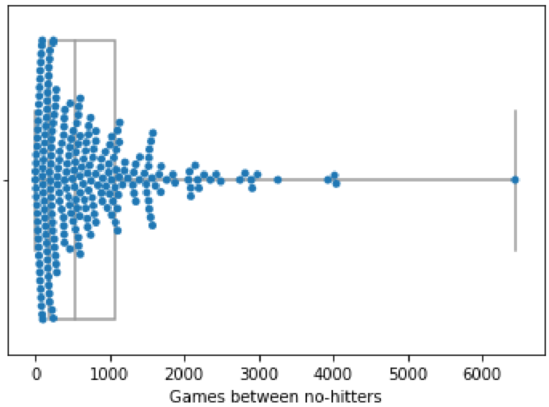
In [133]:

```
# Plot the PDF and label axes
_ = plt.hist(nohitter_times,
             bins=50, density=True, histtype='step')
_ = plt.xlabel('Games between no-hitters')
_ = plt.ylabel('PDF')

# Show the plot
plt.show()
```



In [134]:

```
# Plot a combined boxplot and swarmplot
_ = sns.boxplot(x = nohitter_times,whis=np.inf,color='white')
_ = sns.swarmplot(x = nohitter_times)

# Label the axes
_ = plt.xlabel('Games between no-hitters')

# Show the plot
plt.show()
```



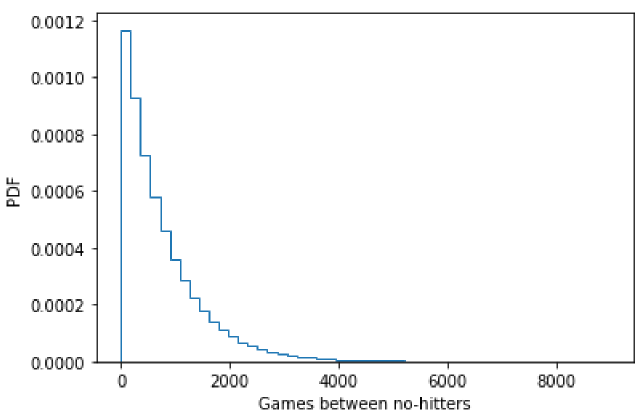
Determine if the time between no-hitters is exponentialy distributed

In [135]:

```
# Seed random number generator
np.random.seed(42)

# Compute mean no-hitter time: tau
tau = np.mean(nohitter_times)

# Draw out of an exponential distribution with parameter tau: inter_nohitter_time
inter_nohitter_times = np.random.exponential(tau, 100000)
```

In [136]:

```
# Plot the PDF and label axes
_ = plt.hist(inter_nohitter_times,
             bins=50, density=True, histtype='step')
_ = plt.xlabel('Games between no-hitters')
_ = plt.ylabel('PDF')

# Show the plot
plt.show()
```



Nice work! We see the typical shape of the Exponential distribution, going from a maximum at 0 and decaying to the right.

Do the data follow our story?

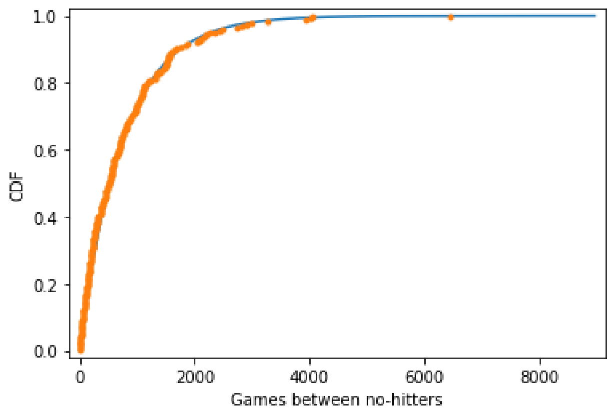
You have modeled no-hitters using an Exponential distribution. Create an ECDF of the real data. Overlay the theoretical CDF with the ECDF from the data. This helps you to verify that the Exponential distribution describes the observed data.

In [137]:

```
def ecdf(data):
    """Compute ECDF for a one-dimensional array of measurements."""
    # Number of data points: n
    n = len(data)

    # x-data for the ECDF: x
    x = np.sort(data)

    # y-data for the ECDF: y
    y = np.arange(1, n+1) / n

    return x, y
```

In [138]:

```
# Create an ECDF from real data: x, y
x, y = ecdf(nohitter_times)

# Create a CDF from theoretical samples: x_theor, y_theor
x_theor, y_theor = ecdf(inter_nohitter_times)

# Overlay the plots
plt.plot(x_theor, y_theor)
plt.plot(x, y, marker='.', linestyle='none')

# Margins and axis labels
plt.margins(.02)
plt.xlabel('Games between no-hitters')
plt.ylabel('CDF')

# Show the plot
plt.show()
```



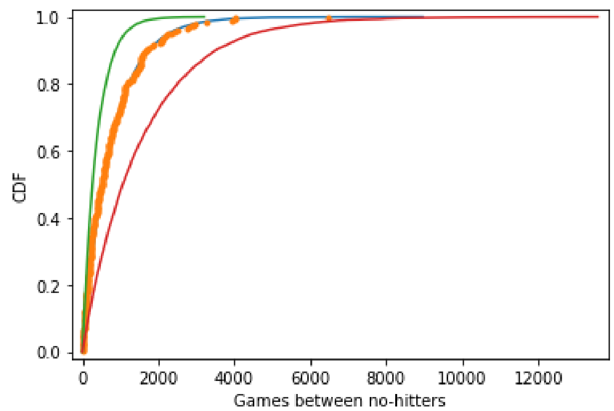
It looks like no-hitters in the modern era of Major League Baseball are Exponentially distributed. Based on the story of the Exponential distribution, this suggests that they are a random process; when a no-hitter will happen is independent of when the last no-hitter was.

How is this parameter optimal?

Now sample out of an exponential distribution with τ being twice as large as the optimal τ. Do it again for τ half as large. Make CDFs of these samples and overlay them with your data. You can see that they do not reproduce the data as well. Thus, the τ you computed from the mean inter-no-hitter times is optimal in that it best reproduces the data.

In [139]:

```
# Plot the theoretical CDFs
plt.plot(x_theor, y_theor)
plt.plot(x, y, marker='.', linestyle='none')
plt.margins(0.02)
plt.xlabel('Games between no-hitters')
plt.ylabel('CDF')

# Take samples with half tau: samples_half
samples_half = np.random.exponential(tau/2,10000)

# Take samples with double tau: samples_double
samples_double = np.random.exponential(2*tau,10000)

# Generate CDFs from these samples
x_half, y_half = ecdf(samples_half)
x_double, y_double = ecdf(samples_double)

# Plot these CDFs as lines
_ = plt.plot(x_half, y_half)
_ = plt.plot(x_double, y_double)

# Show the plot
plt.show()
```



Great work! Notice how the value of tau given by the mean matches the data best. In this way, tau is an optimal parameter.

Confidence interval on the rate of no-hitters

Consider again the inter-no-hitter intervals for the modern era of baseball. Generate 10,000 bootstrap replicates of the optimal parameter □τ. Plot a histogram of your replicates and report a 95% confidence interval.

In [140]:

```
def bootstrap_replicate_1d(data, func):
    """Generate bootstrap replicate of 1D data."""
    bs_sample = np.random.choice(data, len(data))
    return func(bs_sample)
```

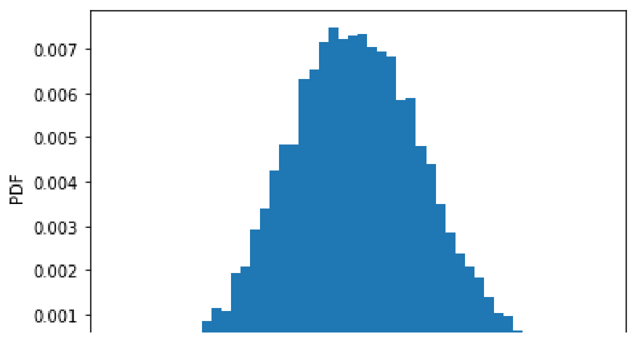
In [141]:

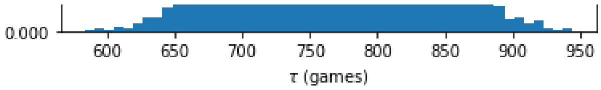
```
def draw_bs_reps(data, func, size=1):
    """Draw bootstrap replicates."""

    # Initialize array of replicates: bs_replicates
    bs_replicates = np.empty(shape=size)

    # Generate replicates
    for i in range(size):
        bs_replicates[i] = bootstrap_replicate_1d(data,func)

    return bs_replicates
```

In [142]:

```
# Draw bootstrap replicates of the mean no-hitter time (equal to tau): bs_replicates
bs_replicates = draw_bs_reps(nohitter_times, np.mean,10000)

# Compute the 95% confidence interval: conf_int
conf_int = np.percentile(bs_replicates,[2.5,97.5])

# Print the confidence interval
print('95% confidence interval of the bootstrap replicates=',conf_int, 'games')

# Plot the histogram of the replicates
_ = plt.hist(bs_replicates, bins=50, density=True)
_ = plt.xlabel(r'$\tau$ (games)')
_ = plt.ylabel('PDF')

# Show the plot
plt.show()
```

```
95% confidence interval of the bootstrap replicates= [661.97370518 870.59501992] games
```





This gives you an estimate of what the typical time between no-hitters is. It could be anywhere between 660 and 870 games.

A time-on-website analog

It turns out that you already did a hypothesis test analogous to an A/B test where you are interested in how much time is spent on the website before and after an ad campaign. The frog tongue force (a continuous quantity like time on the website) is an analog. "Before" = Frog A and "after" = Frog B. Let's practice this again with something that actually is a before/after scenario.

We return to the no-hitter data set. In 1920, Major League Baseball implemented important rule changes that ended the so-called dead ball era. Importantly, the pitcher was no longer allowed to spit on or scuff the ball, an activity that greatly favors pitchers. In this problem you will perform an A/B test to determine if these rule changes resulted in a slower rate of no-hitters (i.e., longer average time between no-hitters) using the difference in mean inter-no-hitter time as your test statistic. The inter-no-hitter times for the respective eras are stored in the arrays `nht_dead` and `nht_live`, where "nht" is meant to stand for "no-hitter time."

In [143]:

```
nht_dead = np.array([  -1,  894,   10,  130,    1,  934,   29,    6,  485,  254,  372,
         81,  191,  355,  180,  286,   47,  269,  361,  173,  246,  492,
        462, 1319,   58,  297,   31, 2970,  640,  237,  434,  570,   77,
        271,  563, 3365,   89,    0,  379,  221,  479,  367,  628,  843,
       1613, 1101,  215,  684,  814,  278,  324,  161,  219,  545,  715,
        966,  624,   29,  450,  107,   20,   91, 1325,  124, 1468,  104,
       1309,  429,   62, 1878, 1104,  123,  251,   93,  188,  983,  166,
         96,  702,   23,  524,   26,  299,   59,   39,   12,    2,  308,
       1114,  813,  887])

nht_live = np.array([ 645, 2088,   42, 2090,   11,  886, 1665, 1084, 2900, 2432,  750,
       4021, 1070, 1765, 1322,   26,  548, 1525,   77, 2181, 2752,  127,
       2147,  211,   41, 1575,  151,  479,  697,  557, 2267,  542,  392,
         73,  603,  233,  255,  528,  397, 1529, 1023, 1194,  462,  583,
         37,  943,  996,  480, 1497,  717,  224,  219, 1531,  498,   44,
        288,  267,  600,   52,  269, 1086,  386,  176, 2199,  216,   54,
        675, 1243,  463,  650,  171,  327,  110,  774,  509,    8,  197,
        136,   12, 1124,   64,  380,  811,  232,  192,  731,  715,  226,
        605,  539, 1491,  323,  240,  179,  702,  156,   82, 1397,  354,
        778,  603, 1001,  385,  986,  203,  149,  576,  445,  180, 1403,
        252,  675, 1351, 2983, 1568,   45,  899, 3260, 1025,   31,  100,
       2055, 4043,   79,  238, 3931, 2351,  595,  110,  215,    0,  563,
        206,  660,  242,  577,  179,  157,  192,  192, 1848,  792, 1693,
         55,  388,  225, 1134, 1172, 1555,   31, 1582, 1044,  378, 1687,
       2915,  280,  765, 2819,  511, 1521,  745, 2491,  580, 2072, 6450,
        578,  745, 1075, 1103, 1549, 1520,  138, 1202,  296,  277,  351,
```

```
        391,  950,  459,   62, 1056, 1128,  139,  420,   87,   71,  814,
        603, 1349,  162, 1027,  783,  326,  101,  876,  381,  905,  156,
        419,  239,  119,  129,  467])
```

In [144]:

```
def diff_of_means(data_1, data_2):
    """Difference in means of two arrays."""

    # The difference of means of data_1, data_2: diff
    diff = np.mean(data_1) - np.mean(data_2)

    return diff
```

In [145]:

```
def permutation_sample(data1, data2):
    """Generate a permutation sample from two data sets."""

    # Concatenate the data sets: data
    data = np.concatenate((data1,data2))

    # Permute the concatenated array: permuted_data
    permuted_data = np.random.permutation(data)

    # Split the permuted array into two: perm_sample_1, perm_sample_2
    perm_sample_1 = permuted_data[:len(data1)]
    perm_sample_2 = permuted_data[len(data1):]

    return perm_sample_1, perm_sample_2
```

In [146]:

```
def draw_perm_reps(data_1, data_2, func, size=1):
    """Generate multiple permutation replicates."""

    # Initialize array of replicates: perm_replicates
    perm_replicates = np.empty(size)

    for i in range(size):
        # Generate permutation sample
        perm_sample_1, perm_sample_2 = permutation_sample(data_1,data_2)

        # Compute the test statistic
        perm_replicates[i] = func(perm_sample_1, perm_sample_2)

        return perm_replicates
```

In [147]:

```
# Compute the observed difference in mean inter-no-hitter times: nht_diff_obs
nht_diff_obs = diff_of_means(nht_dead,nht_live)
print(nht_diff_obs)
```

```
-345.0011367942402
```

In [148]:

```
# Acquire 10,000 permutation replicates of difference in mean no-hitter time: perm_replicates
perm_replicates = draw_perm_reps(nht_dead,nht_live,diff_of_means,size=10000)
perm_replicates
```

Out[148]:

```
array([1.34282683e+002, 8.95283735e-312, 1.06161801e-051, ...,
       1.03753786e-322, 4.94065646e-324, 9.88131292e-323])
```

In [149]:

```
# Compute and print the p-value: p
```

```python
p = np.sum(perm_replicates <= nht_diff_obs) / len(perm_replicates)

print('p-val =', p)
```

```
p-val = 0.0341
```

```
C:\Users\smpet\Anaconda3\lib\site-packages\ipykernel_launcher.py:2: RuntimeWarning: invalid value
encountered in less_equal
```

Your p-value is 0.0001, which means that only one out of your 10,000 replicates had a result as extreme as the actual difference between the dead ball and live ball eras. This suggests strong statistical significance. Watch out, though, you could very well have gotten zero replicates that were as extreme as the observed value. This just means that the p-value is quite small, almost certainly smaller than 0.001.

Was 2015 anomalous?

1990 and 2015 featured the most no-hitters of any season of baseball (there were seven). Given that there are on average 251/115 no-hitters per season, what is the probability of having seven or more in a season?

In [150]:

```python
# Draw 10,000 samples out of Poisson distribution: n_nohitters
n_nohitters = np.random.poisson(251/115, size=10000)


# Compute number of samples that are seven or greater: n_large
n_large = np.sum(n_nohitters >= 7)
print(n_large)

# Compute probability of getting seven or more: p_large
p_large = n_large / 10000

# Print the result
print('Probability of seven or more no-hitters:', p_large)
```

```
70
Probability of seven or more no-hitters: 0.007
```

The result is about 0.007. This means that it is not that improbable to see a 7-or-more no-hitter season in a century. We have seen two in a century and a half, so it is not unreasonable.

If you have a story, you can simulate it!

Sometimes, the story describing our probability distribution does not have a named distribution to go along with it. In these cases, fear not! You can always simulate it. We'll do that in this and the next exercise.

In earlier exercises, we looked at the rare event of no-hitters in Major League Baseball. *Hitting the cycle* is another rare baseball event. When a batter hits the cycle, he gets all four kinds of hits, a single, double, triple, and home run, in a single game. Like no-hitters, this can be modeled as a Poisson process, so the time between hits of the cycle are also Exponentially distributed.

How long must we wait to see both a no-hitter *and then* a batter hit the cycle? The idea is that we have to wait some time for the no-hitter, and then after the no-hitter, we have to wait for hitting the cycle. Stated another way, what is the total waiting time for the arrival of two different Poisson processes? The total waiting time is the time waited for the no-hitter, plus the time waited for the hitting the cycle.

Now, you will write a function to sample out of the distribution described by this story.

In [151]:

```
def successive_poisson(tau1, tau2, size=1):
    """Compute time for arrival of 2 successive Poisson processes."""
    # Draw samples out of first exponential distribution: t1
    t1 = np.random.exponential(tau1,size=size)

    # Draw samples out of second exponential distribution: t2
    t2 = np.random.exponential(tau2,size=size)

    return t1 + t2
```

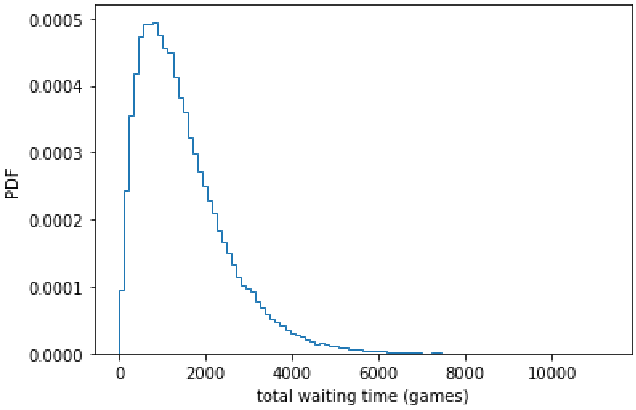
Great work! We'll put the function to use in the next exercise.

Distribution of no-hitters and cycles

Now, you'll use your sampling function to compute the waiting time to observe a no-hitter and hitting of the cycle. The mean waiting time for a no-hitter is 764 games, and the mean waiting time for hitting the cycle is 715 games.

In [152]:

```python
# Draw samples of waiting times
waiting_times = successive_poisson(764, 715, size=100000)

# Make the histogram
_ = plt.hist(waiting_times, bins=100, histtype='step',
             density=True)

# Label axes
_ = plt.xlabel('total waiting time (games)')
_ = plt.ylabel('PDF')

# Show the plot
plt.show()
```



Great work! Notice that the PDF is peaked, unlike the waiting time for a single Poisson process. For fun (and enlightenment), I encourage you to also plot the CDF.