Bernoulli Trials - Simulating Bank Loans

The following analysis was excerpted from the Data Camp course "Statistical Thinking in Python"

How many defaults might we expect?

Let's say a bank made 100 mortgage loans. It is possible that anywhere between 0 and 100 of the loans will be defaulted upon. You would like to know the probability of getting a given number of defaults, given that the probability of a default is `p = 0.05` . To investigate this, you will do a simulation. You will perform 100 Bernoulli trials using the `perform_bernoulli_trials()` function you wrote in the previous exercise and record how many defaults we get. Here, a success is a default. (Remember that the word "success" just means that the Bernoulli trial evaluates to `True` , i.e., did the loan recipient default?) You will do this for another 100 Bernoulli trials. And again and again until we have tried it 1000 times. Then, you will plot a histogram describing the probability of the number of defaults.

In [9]:

```
import numpy as np
import matplotlib.pyplot as plt
```

In [5]:

```
def perform_bernoulli_trials(n, p):
    """Perform n Bernoulli trials with success probability p
    and return number of successes."""
    # Initialize number of successes: n_success
    n_success = 0

    # Perform trials
    for i in range(n):
        # Choose random number between zero and one: random_number
        random_number = np.random.random()

        # If less than p, it's a success so add one to n_success
        if random_number < p:
            n_success += 1

    return n_success
```

In [11]:

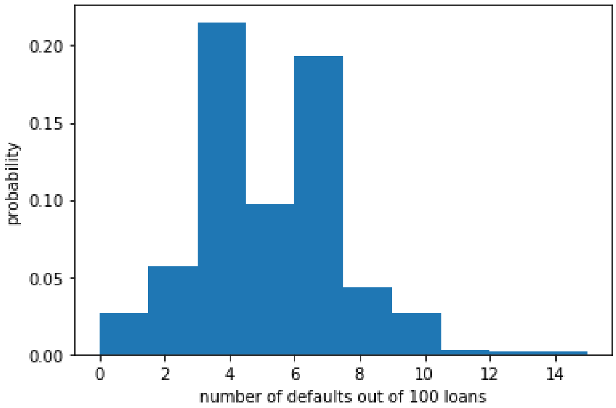
```
# Seed random number generator
np.random.seed(42)

# Initialize the number of defaults: n_defaults
n_defaults = np.empty(1000)

# Compute the number of defaults
for i in range(1000):
    n_defaults[i] = perform_bernoulli_trials(100,.05)
```

```python
# Plot the histogram with default number of bins; label your axes
_ = plt.hist(n_defaults, density=True)
_ = plt.xlabel('number of defaults out of 100 loans')
_ = plt.ylabel('probability')

# Show the plot
plt.show()
```



Nice work! This is actually not an optimal way to plot a histogram when the results are known to be integers. We will revisit this in forthcoming exercises.

Will the bank fail?

Plot the number of defaults you got from the previous exercise, in your namespace as `n_defaults`, as a CDF. The `ecdf()` function you wrote in the first chapter is available.

If interest rates are such that the bank will lose money if 10 or more of its loans are defaulted upon, what is the probability that the bank will lose money?

In [14]:

```python
def ecdf(data):
    """Compute ECDF for a one-dimensional array of measurements."""
    # Number of data points: n
    n = len(data)

    # x-data for the ECDF: x
    x = np.sort(data)

    # y-data for the ECDF: y
    y = np.arange(1, n+1) / n

    return x, y
```

In [15]:

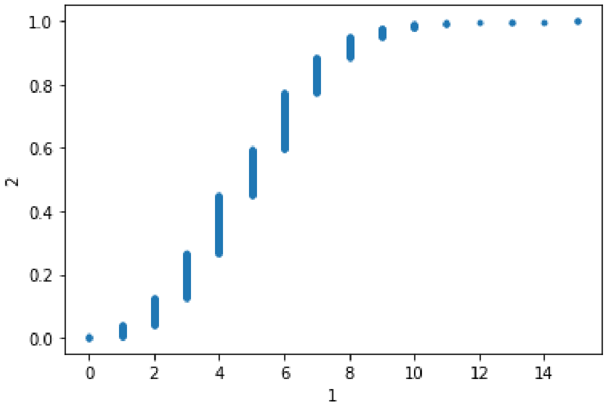
```python
# Compute ECDF: x, y
x, y = ecdf(n_defaults)
```

```python
# Plot the ECDF with labeled axes
_ = plt.plot(x, y, marker='.', linestyle='none')
_ = plt.xlabel('1')
_ = plt.ylabel('2')

# Show the plot
plt.show()

# Compute the number of 100-loan simulations with 10 or more defaults: n_lose_money
n_lose_money = np.sum(n_defaults >= 10)

# Compute and print probability of losing money
print('Probability of losing money =', n_lose_money / len(n_defaults))
```




```
Probability of losing money = 0.022
```

As we might expect, we most likely get 5/100 defaults. But we still have about a 2% chance of getting 10 or more defaults out of 100 loans.

Plotting the Binomial PMF

As mentioned in the video, plotting a nice looking PMF requires a bit of matplotlib trickery that we will not go into here. Instead, we will plot the PMF of the Binomial distribution as a histogram with skills you have already learned. The trick is setting up the edges of the bins to pass to `plt.hist()` via the `bins` keyword argument. We want the bins centered on the integers. So, the edges of the bins should be `-0.5, 0.5, 1.5, 2.5, ...` up to `max(n_defaults) + 1.5`. You can generate an array like this using `np.arange()` and then subtracting `0.5` from the array.

You have already sampled out of the Binomial distribution during your exercises on loan defaults, and the resulting samples are in the NumPy array `n_defaults`.

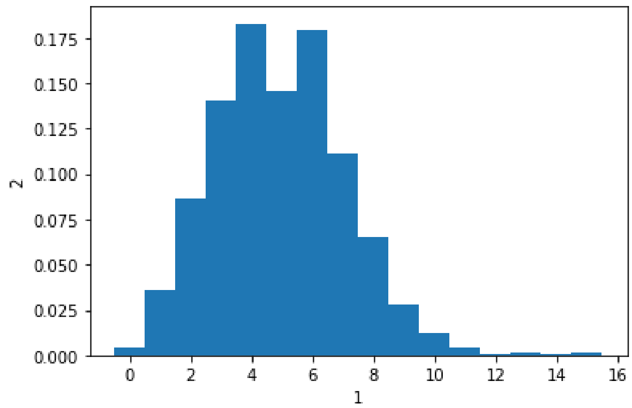
In [18]:

```
# Compute bin edges: bins
bins = np.arange(0, max(n_defaults) + 1.5) - 0.5

# Generate histogram
_ = plt.hist(n_defaults, density=True, bins=bins)

# Label axes
_ = plt.xlabel('1')
_ = plt.ylabel('2')


# Show the plot
plt.show()
```



Great work!