Literacy vs Fertility

The following analysis was excerpted from the Data Camp course "Statistical Thinking in Python"

EDA of literacy/fertility data

In the next few exercises, we will look at the correlation between female literacy and fertility (defined as the average number of children born per woman) throughout the world. For ease of analysis and interpretation, we will work with the *il*literacy rate.

It is always a good idea to do some EDA ahead of our analysis. To this end, plot the fertility versus illiteracy and compute the Pearson correlation coefficient. The Numpy array `illiteracy` has the illiteracy rate among females for most of the world's nations. The array `fertility` has the corresponding fertility data.

In [24]:

```
import numpy as np
import matplotlib.pyplot as plt
```

In [25]:

```
def pearson_r(x, y):
    """Compute Pearson correlation coefficient between two arrays."""
    # Compute correlation matrix: corr_mat
    corr_mat = np.corrcoef(x,y)

    # Return entry [0,1]
    return corr_mat[0,1]
```

In [26]:

```
illiteracy = np.array([ 9.5, 49.2,  1. , 11.2,  9.8, 60. , 50.2, 51.2,  0.6,  1. ,  8.5,
        6.1,  9.8,  1. , 42.2, 77.2, 18.7, 22.8,  8.5, 43.9,  1. ,  1. ,
        1.5, 10.8, 11.9,  3.4,  0.4,  3.1,  6.6, 33.7, 40.4,  2.3, 17.2,
        0.7, 36.1,  1. , 33.2, 55.9, 30.8, 87.4, 15.4, 54.6,  5.1,  1.1,
       10.2, 19.8,  0. , 40.7, 57.2, 59.9,  3.1, 55.7, 22.8, 10.9, 34.7,
       32.2, 43. ,  1.3,  1. ,  0.5, 78.4, 34.2, 84.9, 29.1, 31.3, 18.3,
       81.8, 39. , 11.2, 67. ,  4.1,  0.2, 78.1,  1. ,  7.1,  1. , 29. ,
        1.1, 11.7, 73.6, 33.9, 14. ,  0.3,  1. ,  0.8, 71.9, 40.1,  1. ,
        2.1,  3.8, 16.5,  4.1,  0.5, 44.4, 46.3, 18.7,  6.5, 36.8, 18.6,
       11.1, 22.1, 71.1,  1. ,  0. ,  0.9,  0.7, 45.5,  8.4,  0. ,  3.8,
        8.5,  2. ,  1. , 58.9,  0.3,  1. , 14. , 47. ,  4.1,  2.2,  7.2,
        0.3,  1.5, 50.5,  1.3,  0.6, 19.1,  6.9,  9.2,  2.2,  0.2, 12.3,
        4.9,  4.6,  0.3, 16.5, 65.7, 63.5, 16.8,  0.2,  1.8,  9.6, 15.2,
       14.4,  3.3, 10.6, 61.3, 10.9, 32.2,  9.3, 11.6, 20.7,  6.5,  6.7,
        3.5,  1. ,  1.6, 20.5,  1.5, 16.7,  2. ,  0.9])
```

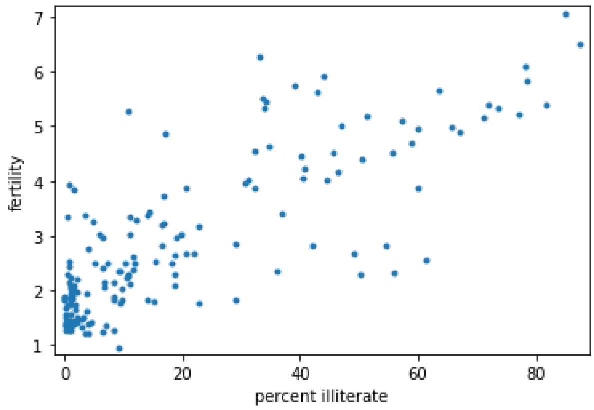
In [27]:

```
fertility = np.array([1.769, 2.682, 2.077, 2.132, 1.827, 3.872, 2.288, 5.173, 1.393,
       1.262, 2.156, 3.026, 2.033, 1.324, 2.816, 5.211, 2.1  , 1.781,
       1.822, 5.908, 1.881, 1.852, 1.39 , 2.281, 2.505, 1.224, 1.361,
       1.468, 2.404, 5.52 , 4.058, 2.223, 4.859, 1.267, 2.342, 1.579,
```

```
       6.254, 2.334, 3.961, 6.505, 2.53 , 2.823, 2.498, 2.248, 2.508,
       3.04 , 1.854, 4.22 , 5.1  , 4.967, 1.325, 4.514, 3.173, 2.308,
       4.62 , 4.541, 5.637, 1.926, 1.747, 2.294, 5.841, 5.455, 7.069,
       2.859, 4.018, 2.513, 5.405, 5.737, 3.363, 4.89 , 1.385, 1.505,
       6.081, 1.784, 1.378, 1.45 , 1.841, 1.37 , 2.612, 5.329, 5.33 ,
       3.371, 1.281, 1.871, 2.153, 5.378, 4.45 , 1.46 , 1.436, 1.612,
       3.19 , 2.752, 3.35 , 4.01 , 4.166, 2.642, 2.977, 3.415, 2.295,
       3.019, 2.683, 5.165, 1.849, 1.836, 2.518, 2.43 , 4.528, 1.263,
       1.885, 1.943, 1.899, 1.442, 1.953, 4.697, 1.582, 2.025, 1.841,
       5.011, 1.212, 1.502, 2.516, 1.367, 2.089, 4.388, 1.854, 1.748,
       2.978, 2.152, 2.362, 1.988, 1.426, 3.29 , 3.264, 1.436, 1.393,
       2.822, 4.969, 5.659, 3.24 , 1.693, 1.647, 2.36 , 1.792, 3.45 ,
       1.516, 2.233, 2.563, 5.283, 3.885, 0.966, 2.373, 2.663, 1.251,
       2.052, 3.371, 2.093, 2.   , 3.883, 3.852, 3.718, 1.732, 3.928])
```

In [28]:

```python
# Plot the illiteracy rate versus fertility
_ = plt.plot(illiteracy, fertility, marker='.', linestyle='none')

# Set the margins and label axes
plt.margins(.02)
_ = plt.xlabel('percent illiterate')
_ = plt.ylabel('fertility')

# Show the plot
plt.show()

# Show the Pearson correlation coefficient
print(pearson_r(illiteracy, fertility))
```




```
0.8041324026815341
```

You can see the correlation between illiteracy and fertility by eye, and by the substantial Pearson correlation coefficient of 0.8. It is difficult to resolve in the scatter plot, but there are many points around near-zero illiteracy and about 1.8 children/woman.

Linear regression

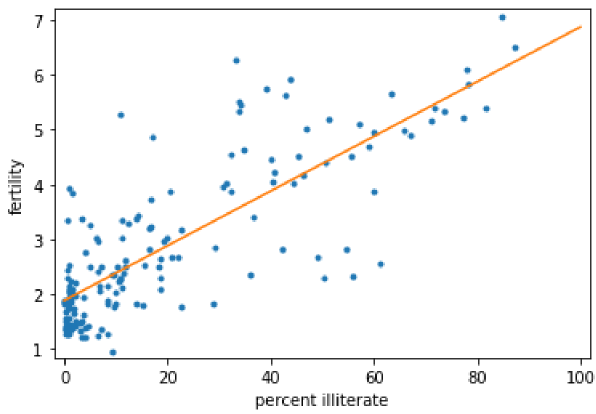
We will assume that fertility is a linear function of the female illiteracy rate. That is, $f = ai + b$, where a is the slope and b is the intercept. We can think of the intercept as the minimal fertility rate, probably somewhere between one and two. The slope tells us how the fertility rate varies with illiteracy. We can find the best fit line using `np.polyfit()` .

Plot the data and the best fit line. Print out the slope and intercept. (Think: what are their units?)

In [29]:

```
# Plot the illiteracy rate versus fertility
_ = plt.plot(illiteracy, fertility, marker='.', linestyle='none')
plt.margins(0.02)

# Perform a linear regression using np.polyfit(): a, b
a, b = np.polyfit(illiteracy, fertility, 1)

# Print the results to the screen
print('slope =', a, 'children per woman / percent illiterate')
print('intercept =', b, 'children per woman')

# Make theoretical line to plot
x = np.array([0,100])
y = a * x + b

# Add regression line to your plot
_ = plt.plot(x, y)
_ = plt.xlabel('percent illiterate')
_ = plt.ylabel('fertility')


# Draw the plot
plt.show()
```

```
slope = 0.04979854809063423 children per woman / percent illiterate
intercept = 1.888050610636557 children per woman
```



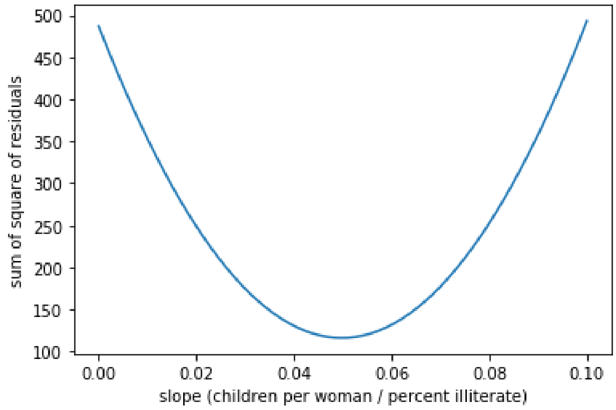
Great work!

How is it optimal?

The function `np.polyfit()` that you used to get your regression parameters finds the *optimal* slope and intercept. It is optimizing the sum of the squares of the residuals, also known as RSS (for residual sum of squares). In this exercise, you will plot the function that is being optimized, the RSS, versus the slope parameter `a` . To do this, fix the intercept to be what you found in the optimization. Then, plot the RSS vs. the slope. Where is it minimal?

In [30]:

```
# Specify slopes to consider: a_vals
a_vals = np.linspace(0, 0.1, 200)

# Initialize sum of square of residuals: rss
rss = np.empty_like(a_vals)

# Compute sum of square of residuals for each value of a_vals
for i, a in enumerate(a_vals):
    rss[i] = np.sum((fertility - a*illiteracy - b)**2)

# Plot the RSS
plt.plot(a_vals, rss, '-')
plt.xlabel('slope (children per woman / percent illiterate)')
plt.ylabel('sum of square of residuals')

plt.show()
```



Great work! Notice that the minimum on the plot, that is the value of the slope that gives the minimum sum of the square of the residuals, is the same value you got when performing the regression.

Pairs bootstrap of literacy/fertility data

Using the function you just wrote, perform pairs bootstrap to plot a histogram describing the estimate of the slope from the illiteracy/fertility data. Also report the 95% confidence interval of the slope. The data is available to you in the NumPy arrays `illiteracy` and `fertility` .

In [31]:

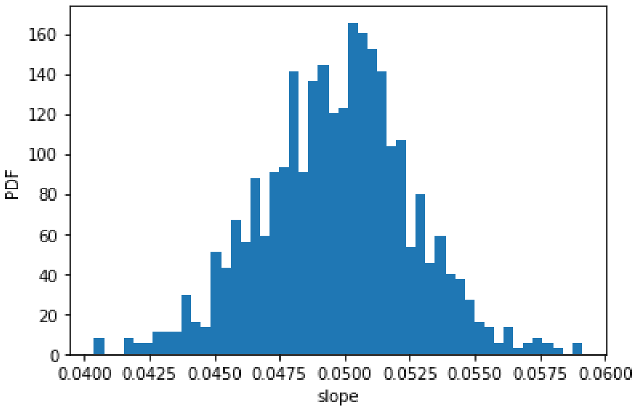
```
def draw_bs_pairs_linreg(x, y, size=1):
    """Perform pairs bootstrap for linear regression."""

    # Set up array of indices to sample from: inds
```

```
    inds = np.arange(len(x))

    # Initialize replicates: bs_slope_reps, bs_intercept_reps
    bs_slope_reps = np.empty(size)
    bs_intercept_reps = np.empty(size)

    # Generate replicates
    for i in range(size):
        bs_inds = np.random.choice(inds, size=len(inds))
        bs_x, bs_y = x[bs_inds], y[bs_inds]
        bs_slope_reps[i], bs_intercept_reps[i] = np.polyfit(bs_x,bs_y,1)

    return bs_slope_reps, bs_intercept_reps
```

In [32]:

```
# Generate replicates of slope and intercept using pairs bootstrap
bs_slope_reps, bs_intercept_reps = draw_bs_pairs_linreg(illiteracy, fertility, size=1000)

# Compute and print 95% CI for slope
print(np.percentile(bs_slope_reps, [2.5,97.5]))

# Plot the histogram
_ = plt.hist(bs_slope_reps, bins=50, density=True)
_ = plt.xlabel('slope')
_ = plt.ylabel('PDF')
plt.show()
```

```
[0.04384997 0.05522836]
```



Great work!

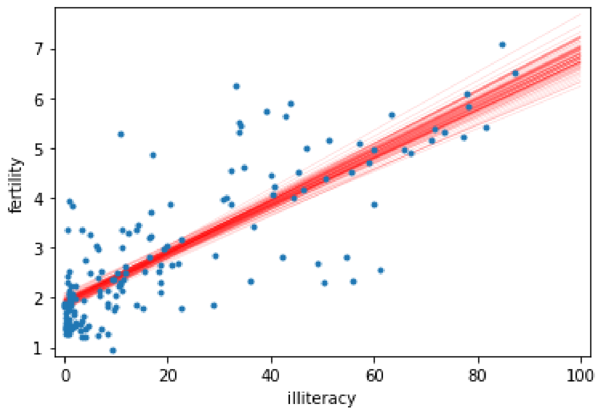
Plotting bootstrap regressions

A nice way to visualize the variability we might expect in a linear regression is to plot the line you would get from each bootstrap replicate of the slope and intercept. Do this for the first 100 of your bootstrap replicates of the slope and intercept (stored as `bs_slope_reps` and `bs_intercept_reps`).

In [33]:

```
# Generate array of x-values for bootstrap lines: x
x = np.array([0,100])
```

```
# Plot the bootstrap lines
for i in range(100):
    _ = plt.plot(x,
                 bs_slope_reps[i] * x + bs_intercept_reps[i],
                 linewidth=0.5, alpha=0.2, color='red')

# Plot the data
_ = plt.plot(illiteracy, fertility, marker='.',linestyle='none')

# Label axes, set the margins, and show the plot
_ = plt.xlabel('illiteracy')
_ = plt.ylabel('fertility')
plt.margins(0.02)
plt.show()
```



Great work! You now have some serious chops for parameter estimation. Let's move on to hypothesis testing!

Hypothesis test on Pearson correlation

The observed correlation between female illiteracy and fertility may just be by chance; the fertility of a given country may actually be totally independent of its illiteracy. You will test this hypothesis. To do so, permute the illiteracy values but leave the fertility values fixed. This simulates the hypothesis that they are totally independent of each other. For each permutation, compute the Pearson correlation coefficient and assess how many of your permutation replicates have a Pearson correlation coefficient greater than the observed one.

In [34]:

```
# Compute observed correlation: r_obs
r_obs = pearson_r(illiteracy,fertility)

# Initialize permutation replicates: perm_replicates
perm_replicates = np.empty(10000)

# Draw replicates
for i in range(10000):
    # Permute illiteracy measurments: illiteracy_permuted
    illiteracy_permuted = np.random.permutation(illiteracy)

    # Compute Pearson correlation
    perm_replicates[i] = pearson_r(illiteracy_permuted,fertility)
```

```python
# Compute p-value: p
p = np.sum(perm_replicates >= r_obs) / len(perm_replicates)
print('p-val =', p)
```

```
p-val = 0.0
```

You got a p-value of zero. In hacker statistics, this means that your p-value is very low, since you never got a single replicate in the 10,000 you took that had a Pearson correlation greater than the observed one. You could try increasing the number of replicates you take to continue to move the upper bound on your p-value lower and lower.