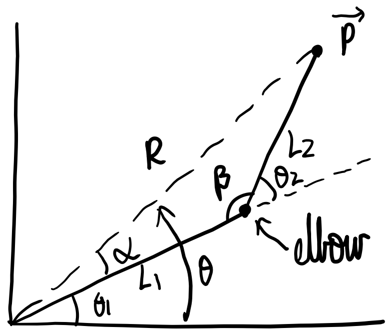
Forward Kinematics



$$\vec{elbow} = (L_1 \cos\theta_1 , L_1 \sin\theta_1)$$

$$\vec{P} = \vec{elbow} + (L_2 \cos(\theta_1+\theta_2), L_2 \sin(\theta_1+\theta_2))$$

$$end = (R, \theta)$$

$$\theta = \theta_1 + \alpha$$

$$\beta = \pi - \theta_2$$

$$R^2 = L_1^2 + L_2^2 - 2 L_1 L_2 \cos\beta$$

$$\frac{R}{\sin\beta} = \frac{L_2}{\sin\alpha}$$

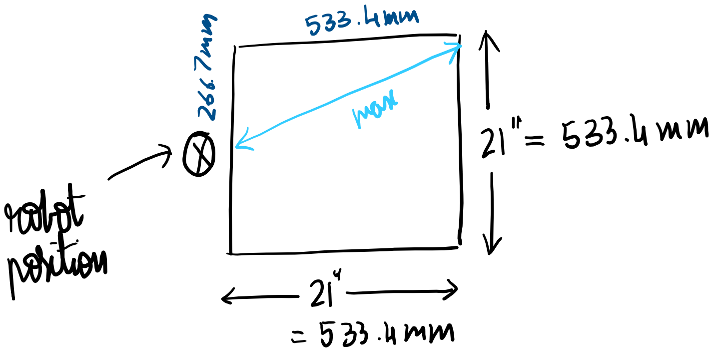
$$\Rightarrow \cos\beta = \frac{R^2 - L_1^2 - L_2^2}{-2L_1L_2}$$

$$\beta = \cos^{-1}\left(\frac{R^2 - L_1^2 - L_2^2}{-2L_1L_2}\right)$$

$$\sin\alpha = \frac{L_2 \sin\beta}{R}$$

$$\alpha = \sin^{-1}\left(\frac{L_2 \sin\beta}{R}\right)$$

for a standard chess board



$R_{max} = 596.359$ mm (required)

max = 596.359

if $L_1 = 298.179\,mm, \quad L_2 = 298.179\,mm$

$\rightarrow R = L_1 + L_2 = R_{max}$

$\rightarrow \alpha = 0^\circ, \quad \beta = 180^\circ$

$\rightarrow \theta = \theta_1$

$\rightarrow \theta_2 = 0^\circ$

For my fusion model I had started with dimensions —

$L_1 = 153$ mm $\quad L_2 = 148$ mm

from the model, when $\theta_1 = 78.7^\circ$ and $\theta_2 = 104.6^\circ$

$\Rightarrow R = 184$ mm

on calculating α and β,

$$\beta = \cos^{-1}\left(\frac{184^2 - 153^2 - 148^2}{-2(153)(148)}\right) = \cos^{-1}(0.2530) = 75.35^\circ$$

$$\alpha = \sin^{-1}\left(\frac{(148)(0.9675)}{184}\right) = \sin^{-1}(0.7782) = 51.10^\circ$$

from the model,

$\alpha = 50.97^\circ \qquad \beta = 75.384^\circ$