Forward Guidance in an Uncertain World

Smriti Verma

November 2025

Click here for the latest version of the paper

Abstract

I investigate the impact of state-contingent forward guidance in monetary policy under incomplete markets and aggregate uncertainty. I develop a novel recursive solution framework that enables a quantitative study of forward guidance contingent on macroeconomic states—a question previously unexplored in the literature. Results from a global solution approach, without assuming perfect foresight, reveal that state-contingent forward guidance generates substantially smaller effects on current output, consumption, and inflation than non-state-contingent guidance which is implemented irrespective of the future state of the economy. Households, facing aggregate and idiosyncratic risks with incomplete markets, increase precautionary savings and respond less to state-contingent policy announcements. State-contingent forward guidance leaves room for uncertainty about expected future rate shifts, only partially achieving its purpose of increasing predictability of future policy, thus limiting its effectiveness. A global solution method in my work allows for a robust analysis of state-contingent forward guidance with aggregate uncertainty, infeasible in the existing frameworks in the literature, and unexplored in prior studies, making my work a first-order contribution to the literature on the study of forward guidance and more generally, a methodological innovation allowing the study of broader monetary policy issues in stochastic environments.

1 Introduction

Forward guidance in monetary policy refers to communication by central banks about the future path of short-term interest rates, aiming to influence expectations and economic decisions today. Over the last two decades, forward guidance has evolved from terse statements about the future economic outlook and anticipated policy response to more explicit commitments, where the central bank announces target ranges contingent on macroeconomic targets and future states of the economy. In this paper, I examine the effects of this evolved form of state-contingent forward guidance, specifically under conditions of incomplete markets. I develop a stochastic dynamic general equilibrium model with aggregate uncertainty to quantify how announcements about future interest rate cuts, contingent on the realization of particular economic states, affect current macroeconomic variables. My results show these effects to be half the size of the effects of non-state-contingent forward guidance.

Prior literature has emphasized on the “forward guidance puzzle”, where standard DSGE models predicted implausibly large effects of future policy announcements on present macroeconomic variables. Extending this line of inquiry, my analysis reveals that state-contingent forward guidance—where future policy paths depend explicitly on the state of the economy—has substantially smaller effects on output, consumption, and inflation compared to non-state-contingent guidance.

This diminished impact arises because households, facing uninsurable idiosyncratic income risks and aggregate uncertainty, exhibit strong precautionary savings and borrowing constraints. These factors attenuate their responsiveness to policy announcements that only partially resolve uncertainty about future interest rates. Through this framework, this paper contributes a new methodological approach to study state-contingent forward guidance and highlights the importance of nonlinearities and uncertainty in shaping the efficacy of forward guidance.

McKay, Nakamura, and Steinsson (2016) show that the power of forward guidance is highly sensitive to the assumption of complete markets—when agents are subject to uninsurable income risks and borrowing constraints, the precautionary savings motive tempers their current response to changes in future interest rates. I augment this framework with aggregate uncertainty to study how state-contingent announcements about future policy rate cuts bears on current macroeconomic outcomes. My results demonstrate that state-contingent forward guidance is nearly half as effective in influencing current output, consumption, and inflation, as non-state-contingent forward guidance. Importantly, even non-state-contingent forward guidance, in the presence of aggregate uncertainty and incomplete markets, tends to have contemporaneous effects that are smaller than those predicted by McKay, Nakamura, and Steinsson (2016) under foresight.

Time-based forward guidance, widely studied in the literature, is prone to misinterpretation by the public as the central bank’s dismal outlook for the economy instead of a continuation of its expansionary stance. This poses the threat of destabilizing long-term inflation expectations producing outcomes that run counter to the intended objective of forward guidance. As such, the US Federal Reserve has moved away from time-based announcements and instead gives target ranges within which it intends to maintain interest rates, contingent on specific macroeconomic targets, such as the unemployment rate (employment targets), long-term inflation expectations, labor market conditions, the evolving outlook of the economy, and the balance of risks. The objective of this paper is to study forward guidance in its present form and analyze its effects on contemporaneous macroeconomic outcomes in light of the "forward guidance puzzle".

I augment a one-asset heterogeneous-agent New Keynesian model without capital, with aggregate uncertainty to study the effects of state-contingent forward guidance. To allow for nonlinearities in household and firm behavior that may exist outside of steady state due to the presence of aggregate uncertainty as well as to accurately compute elasticities of aggregate functions outside of steady-state, I solve the model recursively using Backward Induction—a global solution method due to Reiter (2010)—to arrive at a stochastic general equilibrium for the economy. Households' policy functions and aggregate economic responses, thus, incorporate all relevant individual and aggregate state variables, aptly capturing both individual and aggregate risks and their importance in macroeconomic dynamics. I use this global stochastic general equilibrium to study the effects of forward guidance about future real interest rates in the economy on current consumption, output, and inflation, without perfect foresight.

The study of aggregate uncertainty in heterogeneous-agent New Keynesian models has been limited to linear approximations around the steady state [1] and the study of dynamics under the assumption of perfect foresight [2]. While computationally fast, locally linearized methods rely on the assumption that the steady state behavior of agents is relevant even in the presence of aggregate uncertainty to some order of approximation. My work shows local approximations around the steady state may not accurately represent the elasticities of aggregate responses to price changes in the presence of aggregate uncertainty.

A household's individual endogenous state is the bond holdings it has, and its individual exogenous state is the idiosyncratic income shock it faces. The actual aggregate endogenous state is the distribution of households, and the aggregate exogenous state is the shock to aggregate productivity. Due to aggregate uncertainty, there is no stationary distribution of households in this economy; instead, there is a distribution of such distributions. I solve the model using a novel algorithm that

[1] Adrien Auclert, Bence Bardóczy, Matthew Rognlie, and Ludwig Straub (2021); Cosmin L. Ilut, Ralph Luetticke, and Martin Schneider (2025), Christian Bayer, Ralph Luetticke, Lien Pham-Dao, and Volker Tjaden (2018)

[2] McKay, Nakamura, Steinsson (2016)

utilizes the Backward Induction method [3] to solve for households' decision rules for consumption and labor, using the aggregate bond holdings by households, a moment of the actual distribution of households, as the approximate aggregate endogenous state for the economy. This is a stark departure from existing work in the literature of forward guidance, and HANK more generally, and a first-order contribution, providing a new light in which to study a variety of questions related to monetary policy in the presence of aggregate uncertainty.

The literature on forward guidance has studied the effect of forward guidance "shocks" on contemporaneous aggregate consumption and inflation, typically under the assumption of perfect foresight. Odyssean forward guidance by the Fed comprises indications on the duration for which the Fed intends to keep monetary policy accommodative, even if circumstances in the future would warrant and justify otherwise. However, the assumption of perfect foresight greatly simplifies the problem that the central bank faces before it makes such announcements because the likelihood of an aggregate shock or even the risk of such shock is assumed away with the assumption of perfect foresight.

Forward guidance can be an effective tool of monetary policy when the monetary authority finds its hands tied in the face of significant economic risks, such as a lurking recession or a build up of inflationary pressures and the effective lower bound on interest rates binds. Forward guidance, by anchoring long term inflation expectations, becomes an effective tool to influence real economic activity in times when more conventional policies are constrained. My work lends to a better understanding of how the monetary authority's messaging affects macroeconomic outcomes in uncertain times and informs such guidance on conditioning based on thresholds that may improve its effectiveness.

The risk of economic uncertainty, such as the risk of a recession or a recession itself, underscores the importance of forward guidance as a tool of conventional monetary policy. But the effect of such state-contingent commitment on consumption and output when the economy faces the risk of aggregate shocks, can be studied only with the explicit modeling of aggregate shocks and solving the model globally to allow aggregate risks to interact with individual risks, in the absence of perfect foresight, which is akin to the environment in which agents in the real world operate.

To the best of my knowledge, there is no work in the literature that has studied the effect of state-contingent forward guidance on current macroeconomic variables of interest, in the presence of aggregate uncertainty using a global solution method without perfect foresight, so that the stochastic nature of the economy is preserved. In a baseline model, I benchmark the effect of non-state-contingent forward guidance and

[3] Reiter (2010)

compare the outcomes of state-contingent forward guidance with this baseline case. I conduct several experiments of state-contingent forward guidance—a future rate cut, an increase in future interest rates, a rate cut over a longer horizon—and compare the results with their respective non-state-contingent counterpart to check for robustness of my results. Finally, I conduct some sensitivity analyses with alternate parameter values to check the sensitivity of these results to some critical parameters.

Forward guidance about a 50 basis points rate cut 20 quarters ahead contingent on a certain state of the economy being realized causes an increase in current output by 2.5 basis points whereas current consumption increases by 3 basis points, and inflation practically remains unchanged, increasing only by 0.25 basis points. The effects of a state-contingent forward guidance are smaller by a factor of 0.5 compared to the effects of the same forward guidance given unconditionally, that is, an announcement of a 50 basis points rate cut 20 quarters into the future irrespective of the future state of the economy. This result demonstrates the salience of aggregate uncertainty in determining the efficacy of forward guidance contingent on the state of the economy.

My results indicate that the effects of forward guidance in the presence of aggregate uncertainty and incomplete markets, even when the guidance is non-state-contingent, are relatively smaller than the effects of forward guidance with incomplete markets under perfect foresight, as in McKay, Nakamura, and Steinsson (2016). For the same magnitude and horizon of forward guidance, current output and consumption increase by 5 and 7 basis points, respectively, in my environment, compared to an increase of 10 basis points under their perfect foresight environment. The difference in the impact on inflation is, however, much larger. While they report a 30 basis points increase in current inflation on such announcement, non-state-contingent forward guidance in my model only causes a 0.5 basis point increase in inflation at the time of announcement.

Forward guidance tied to specific economic conditions only partially reduces uncertainty about future policy rates. Households, even when informed about a future rate cut, cannot significantly boost current spending because they remain exposed to both aggregate economic shocks and unpredictable changes in their own incomes. Borrowing constraints further restrict their ability to increase consumption in anticipation of future policy changes, since the risk of reaching their borrowing limits shortens their planning horizon and mutes their response further to guidance about distant future rates.

Effect of state-contingent forward guidance on inflation is smaller by half compared to non-state-contingent guidance, but the magnitude of these effects are very small in both state-contingent and non-state-contingent guidance. This is likely the result of quadratic adjustment costs faced by firms in setting prices, which prevents large movements in prices. In contrast, in McKay, Nakamura, and Steinsson, the firms face nominal rigidities as in Calvo (1983) so that firms have precautionary pricing incentives to cover for periods when they would not be able to reset prices, and hence make large adjustment in prices when they are able to reset prices.

My results highlight how aggregate uncertainty crucially shapes the effectiveness of state-contingent forward guidance. I develop a novel framework and solution method, absent in the existing literature, to study this question. This method utilizes a stochastic equilibrium rather than a deterministic steady state or perfect foresight, to model forward guidance in the presence of aggregate uncertainty. This approach better captures elasticities of macroeconomic aggregates in presence of aggregate risks beyond the steady states. My framework can be used to analyze the impact of various policy actions in an environment with aggregate uncertainty. My findings guide how forward guidance communication can be tailored to effectively influence current output, consumption, and inflation.

The remainder of this paper is organized as follows. In section 2, I discuss the related literature and where my work fits in, section 3 lays out the model environment, with section 4 describing the decision problems faced by the agents in the economy, section 5 detailing a recursive formulation of the economy, and section 6 defining the equilibrium for this economy, followed by a brief discussion of the choice of parameters for the model in section 7. Further, I discuss the methods used for computation in section 8 and present the results and its discussion in section 9, and finally, section 10 concludes.

2 Related Literature

The literature on forward guidance has been driven by the "forward guidance puzzle" (Del Negro, Giannoni, and Patterson 2012) – estimated DSGE models, while successfully explained the effects of contemporaneous monetary policy actions on key macroeconomic variables, grossly exaggerated the effects of future policy announcements. The central bank's announcement of lower interest rates, eight quarters in the future, implied a massive stimulus, with the impact on both output and inflation several times larger than that implied by a same-sized contemporaneous drop in the policy rate (Del Negro et al 2013). Gurkaynak, Sack, and Swanson (2005) measure the effects of a "future path of policy", closely associated with FOMC statements, on bond yields and stock prices, and find that both, monetary policy actions and statements have important but differing effects on asset prices, with FOMC statements having a much larger impact on longer-term Treasury yields. In a similar flavor, Campbell, Fisher, and Justiniano (2012), and Milani and Treadwell (2012) studied the impact of forward guidance shocks on the economy using small- or medium-scale DSGE models estimated on a pre-Great Recession sample, and both papers find that forward guidance shocks play a large role in explaining movements in the policy rate, and these contribute significantly to business cycle fluctuations.

Campbell et al. (2012) assessed the impact of exogenous changes in monetary policy on private expectations and found the sign to be opposite to the predictions in theory so that accommodative monetary policy was associated with lower projections for inflation and activity. In view of these considerations, the FOMC replaced the calendar-based language in the December 2012 statement and switched to announcing policy intentions that were tied to economic performance on inflation and real activity, and the state of the economy more generally.

Campbell et al. (2017) use a full information, rational expectations framework and find that FOMC forward guidance has mixed effects. The main goal of their study is to assess whether FOMC improved economic performance since the financial crisis using Odyssean forward guidance, and to quantify the impact of Odyssean FOMC forward guidance on macroeconomic outcomes since the financial crisis that unwound from 2007 to 2009. They find that puzzling responses of private-sector forecasts to FOMC announcements can be attributed entirely to Delphic forward guidance, even though a large fraction of the variability in Federal Funds Futures rates on days with FOMC announcement remained unexplained by their measure of FOMC private information. Interestingly, while their findings suggest that purely rule-based policy would have ameliorated the recession and kept inflation closer to target in the years immediately following the crisis, than FOMC forward guidance did in practice, the Fed's introduction of calendar-based Odyssean forward guidance starting toward the end of 2011 appeared to have boosted real activity and moved inflation closer to target.

A large chunk of the forward guidance literature has focused on resolving the forward guidance puzzle. The forward guidance puzzle - large contemporaneous effects as a result of a future policy announcement - arises from intertemporal substitution being the main channel of transmission of forward guidance into the real economy. As a result, departure from the complete markets, full-information rational expectations framework alleviates the forward guidance puzzle. The macroeconomic resolution of the puzzle comes from adding heterogeneity such as in Mckay, Nakamura, Steinsson (2016), Hagedorn et al. (2019), Acharya and Dogra (2020). McKay-Nakamura-Steinsson (2016) (MNS) show that forward guidance has substantially less power to stimulate the economy in the presence of uninsurable risk and borrowing constraints. It is not realistic to assume agents increase their consumption by the same amount in response to a future interest rate cut as they do to a current interest rate cut because of two reasons, borrowing constraints, and precautionary savings. People face some risk of hitting a borrowing limit which shortens their planning horizon, as interest rate changes that occur after they hit a borrowing constraint, are irrelevant to them. Presence of uninsurable income risks lead to precautionary savings, which temper households' responses to future interest rate shocks. As a result, power of forward guidance is substantially muted in the incomplete markets model compared to the standard complete-markets New Keynesian model.

Other departures from full information set ups and rational expectations include Chung, Herbst, and Kiley (2014), Carlstrom, Fuerst, and Paustian (2015), and Kiley (2016) which use sticky information models à la Mankiw and Reis (2002); Angeletos and Lian (2018) who relax the assumption of common knowledge; and other works that depart from rational expectations in the form of bounded rationality (Gabaix 2015), finite planning horizons (Woodford 2019), or by modeling the cognitive process of expectation formation and use the concept of reflective equilibrium (Garcia-Schmidt and Woodford 2019).

Given how critically the efficacy of forward guidance depends on the credibility of the central bank's commitment to announcements about future policy, several papers discuss imperfect credibility of the central bank to resolve the forward guidance puzzle (Bodenstein, Hebden, and Nunes 2012; Haberis, Harrison, and Waldron 2014; Campbell et al. 2019; Bernanke 2020).

My work contributes to the literature on forward guidance by introducing aggregate uncertainty enabling the study of state-contingent forward guidance which is its most evolved form. The use of a global solution method, unprecedented in the literature on forward guidance, and in HANK models more generally, allows the stochastic dynamic general equilibrium to capture the nonlinearities in households' responses in presence of aggregate risks and uncertainty. To the best of my knowledge, mine is the first work that solves for a fully stochastic general equilibrium solution to a HANK model with aggregate uncertainty, and leverages the model to study the effect of state-contingent forward guidance, which is a first-order contribution to the literature on forward guidance.

My results question the existence of the forward guidance puzzle, when aggregate economic risks are accounted for and the stochastic nature of the economy is preserved in examining the effects of forward guidance. Guidance that is contingent on future economic conditions has half the impact of non-state-contingent guidance; however, even non-state-contingent guidance has effects much smaller than those predicted under perfect foresight in the literature [4]. My model and methodology offer a framework to study a variety of monetary policy issues in the presence of aggregate

[4]McKay, Nakamura, and Steinsson (2016)

uncertainty when nonlinearities can play an important role in driving outcomes. My results provide insights for how communication of forward guidance can be modified and conditioned to enhance its efficacy in an uncertain world.

3 Model

Households There is a continuum of a unit mass of ex-ante identical households with preferences given by

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[\frac{c_{h,t}^{1-\sigma}}{1-\sigma} - \chi \frac{n_{h,t}^{1+\psi}}{1+\psi} \right] \tag{1}$$

where $c_{h,t}$ is consumption of household h at time t and $n_{h,t}$ is labor supply of household h at time t. Each household faces an uninsurable idiosyncratic productivity shock $e_{h,t}$ which follows a Markov chain with transition probabilities $\Pr(e_{h,t+1}|e_{h,t})$, and earns real labor income before tax $w_t e_{h,t} n_{h,t}$. By assumption, the initial cross-sectional distribution of idiosyncratic productivities is equal to the ergodic distribution of this Markov chain.

In addition to holding stakes in the firm, households also have access to a risk-free real bond b that yields real interest rate r_t between periods t and $t+1$. Borrowing constraints prevent households from taking negative bond positions. A household's period t budget constraint is given by

$$c_t + q_t \cdot b_{t+1} \leq b_t + (1-\tau_t) \cdot w_t \cdot e_t \cdot n_t + d_t \tag{2}$$

where c_t is real consumption, q_t is the price of the bond between period t and $t+1$, b_{t+1} and b_t are the real risk-free bond holdings of the household in period $t+1$ and period t, respectively, w_t is the real wage rate, and d_t is the real dividend earnings of the household.

Final-good producer A competitive representative final good producer aggregates a continuum of intermediate inputs indexed by $j \in [0,1]$ through a Dixit-Stiglitz constant elasticity aggregator as follows

$$Y_t = \left(\int_0^1 y_{j,t}^{\frac{\varepsilon-1}{\varepsilon}} \, dj \right)^{\frac{\varepsilon}{\varepsilon-1}} \tag{3}$$

where Y_t is the quantity of the final good produced by the final good firm at time t, $y_{j,t}$ is the quantity of the intermediate good produced by the firm j in period t, and ε is the price elasticity of substitution across the varieties of the intermediate good.

Profit maximization by the final good producer implies the following demand for a typical intermediate input variety j

$$y_{j,t} = \left(\frac{p_{j,t}}{P_t}\right)^{-\varepsilon} Y_t \tag{4}$$

where $p_{j,t}$ is the price set by the producer of intermediate input variety j, and P_t is the aggregate price index given by

$$P_t = \left(\int_0^1 p_{j,t}^{1-\varepsilon} dj\right)^{\frac{1}{1-\varepsilon}} \tag{5}$$

Intermediate goods producers Each intermediate input variety j is produced by a monopolistically competitive producer according to the production function

$$y_{j,t} = z_t \; n_{j,t} \tag{6}$$

where z_t is the aggregate total factor productivity (TFP) common across all intermediate firms, and $n_{j,t}$ denotes the amount of labor hired by firm j in period t.

Intermediate goods producers hire labor at real wage w_t in a competitive labor market such that wage equals the marginal revenue product of labor.

Each intermediate producer chooses its price to maximize lifetime profits subject to price adjustment costs as in Rotemberg (1982). These adjustment costs are quadratic in the rate of price change $\frac{p_{j,t}}{p_{j,t-1}}$ and are expressed as a fraction of real aggregate output Y_t as

$$\Theta_t = \frac{\theta}{2}\left(\frac{p_{j,t}}{p_{j,t-1}} - \bar{\pi} - 1\right)^2 Y_t \tag{7}$$

where $\theta > 0$ is the cost of the price adjustment.

Profits produced by the intermediate firms are distributed immediately to the households such that each household receives an equal share d_t. Households cannot trade their stakes in the firms.

Aggregate uncertainty The source of aggregate uncertainty is a shock to the exogenous aggregate state of the economy, i.e., the total factor productivity (z_t) in the intermediate firms' production functions.The exogenous shock to TFP, φ, can be decomposed into two components, the exogenous idiosyncratic shock to TFP (η), and, the risk of a disaster shock (Δ).

Idiosyncratic shock to total factor productivity The idiosyncratic shock to TFP is given by the following AR(1) process:

$$log\ z_{t+1} = \rho_z\ log\ z_t + \eta_t \tag{8}$$

where $\eta \sim N(0, \sigma_z^2)$ which can be discretized using the Rouwenhorst method so that $z \in \{z_1, z_2, z_3\}$ and follows a Markov chain with transition probabilities $\Pr(z_{t+1} = z_j | z_t = z_i) = \pi_{ij}^z$ giving the probability of transition from the state z_i in period t to the state z_j in period $t+1$. The transition probability matrix for the idiosyncratic shock to TFP is given by

$$\Pi^z = \begin{array}{c} \\ z = z_1 \\ z = z_2 \\ z = z_3 \end{array} \begin{array}{c} \begin{array}{ccc} z = z_1 & z = z_2 & z = z_3 \end{array} \\ \begin{pmatrix} \pi_{11}^z & \pi_{12}^z & \pi_{13}^z \\ \pi_{21}^z & \pi_{22}^z & \pi_{23}^z \\ \pi_{31}^z & \pi_{32}^z & \pi_{33}^z \end{pmatrix} \end{array}$$

Fiscal authority The government levies a proportional tax on household labor income at the rate τ_t and and holds a stock of outstanding government debt with real face value B_t such that positive values of B denote government debt. The government budget constraint is given as

$$q_t \cdot B_{t+1} = B_t + G_t - \tau_t \cdot w_t \int e \cdot n_t \cdot \mu_t\ d\mu_t \tag{9}$$

which can be rewritten as

$$q_t \cdot B_{t+1} = B_t + G_t - T_t \tag{10}$$

where where G_t is government spending in period t, and $T_t = \tau_t \cdot w_t \int e \cdot n_t \cdot \mu_t\ d\mu_t$. The government's budget constraint can be written in terms of the primary surplus $S_t \equiv T_t - G_t$ as

$$q_t \cdot B_{t+1} = B_t - S_t \tag{11}$$

The government adjusts primary surplus as per the following fiscal rule to stabilize debt

$$S_t = \rho_b(B_t - \bar{B}) + \bar{S} \tag{12}$$

where $\rho_b > 0$ is the fiscal responsiveness to debt deviations, and $\bar{S}$ is a target primary surplus given as $\bar{T} - \bar{G}$.

Monetary authority The monetary authority sets nominal interest rates using a Taylor rule given by

$$i_t = r^* + \pi_t + \alpha\,(\pi_t - \bar{\pi}) \tag{13}$$

where r^* is the long-run equilibrium real interest rate, and $\bar{\pi}$ is the monetary authority's target inflation rate. where the coefficient α must be greater than 1 for determinacy.

The relationship between the current real interest rate r_t, the expected future inflation rate π_{t+1}, and the current nominal interest rate i_t, is given by the Fisher relation as

$$1 + r_t = \frac{1 + i_t}{\mathbb{E}_t \left[1 + \pi_{t+1}\right]} \tag{14}$$

where π_{t+1} is the net inflation rate given as $\pi_{t+1} \equiv \frac{P_{t+1}}{P_t} - 1$ and P_t is the aggregate price level. The real interest rate and the expected future inflation rate imply a nominal interest rate determined by the Fisher equation, such that the the real return on assets r_t must be consistent with equilibrium in the bond market.

4 Decision Problems

4.1 Decision problem of the household

Households face the following decision problem in this economy

$$\max_{\{c_{h,t}, n_{h,t}, b_{h,t+1}\}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[\frac{c_{h,t}^{1-\sigma}}{1-\sigma} - \chi \frac{n_{h,t}^{1+\psi}}{1+\psi}\right] \tag{15}$$

subject to their period budget constraint,

$$c_t + q_t \cdot b_{t+1} \leq b_t + (1 - \tau_t) \cdot w_t \cdot e_t \cdot n_t + d_t \tag{16}$$

the borrowing constraint,

$$b_{t+1} \geq \underline{\mathrm{b}} \tag{17}$$

the non-negativity constraint,

$$c_t \geq 0 \tag{18}$$

Each household chooses consumption and labor optimally by satisfying its Euler equation given by

$$u_c'\left(c_{h,t}, n_{h,t}\right) = \beta \mathbb{E}_t \left[\left(1 + r_{t+1}\right) u_c'\left(c_{h,t+1}, n_{h,t+1}\right)\right] \tag{19}$$

which can be written in terms of the discount bond price as

$$q_t = \beta \mathbb{E}_t \left[\frac{u_c'\left(c_{h,t+1}, n_{h,t+1}\right)}{u_c'\left(c_{h,t}, n_{h,t}\right)}\right] \tag{20}$$

and its labor-leisure condition, given by

$$\chi \cdot n_{h,t}^{1/\eta} = c_{h,t}^{-\sigma} \cdot w \tag{21}$$

which, together with the household's budget constraint, yield the optimal choices of consumption and leisure for each household.

4.2 Decision problem of the final good producer

The final good producer faces the following profit maximization problem

$$\max_{y_{j,t}} \left\{ P_t \left(\int_0^1 y_{j,t}^{\frac{\varepsilon-1}{\varepsilon}} \, dj \right)^{\frac{\varepsilon}{\varepsilon-1}} - \int_0^1 p_{j,t} y_{j,t} \, dj \right\} \tag{22}$$

which yields the following demand for each intermediate good variety j

$$y_{j,t} = \left(\frac{p_{j,t}}{P_t} \right)^{-\varepsilon} Y_t \tag{23}$$

where $p_{j,t}$ is the price charged by firm producing intermediate input variety j in period t and P_t is the aggregate price index in period t given by

$$P_t = \left(\int_0^1 p_{j,t}^{1-\varepsilon} dj \right)^{\frac{1}{1-\varepsilon}} \tag{24}$$

4.3 Decision problem of intermediate goods producers

Intermediate goods producers face a common real wage w_t. Due to the presence of nominal rigidities, intermediate firms are unable to adjust prices each period to maximize their profit. Therefore, they act to minimize cost. The cost minimization problem faced by intermediate firms, subject to producing enough to meet demand, is given as

$$\min_{n_{j,t}} \{ w_t n_{j,t} \} \tag{25}$$

subject to

$$z_t n_{j,t} \geq y_{j,t} \tag{26}$$

and

$$y_{j,t} = \left(\frac{p_{j,t}}{P_t} \right)^{-\varepsilon} Y_t \tag{27}$$

Cost minimization by intermediate goods producers implies that marginal cost is identical across intermediate goods firms and is given by

$$mc_t = \frac{w_t}{z_t} \tag{28}$$

Each intermediate firm seeks to maximize its expected present discounted value of future profits for its shareholders. In doing so, each intermediate producer solves the following maximization problem

$$\max_{p_{j,t}} \mathbb{E}_t \left[\sum_{s=0}^{\infty} \mathcal{Q}_{t\to t+s} \frac{P_t}{P_{t+s}} \tilde{V}_{j,t+s} \right] \tag{29}$$

subject to

$$y_{j,t} = \left(\frac{p_{j,t}}{P_t} \right)^{-\varepsilon} Y_t \tag{30}$$

where $\mathcal{Q}_{t\to t+s}$ is the s-period ahead real stochastic discount factor and $\mathcal{Q}_{t\to t+s}\frac{P_t}{P_{t+s}}$ is the s-period ahead nominal stochastic discount factor. The firm discounts future profits by the inverse of the real-interest rate in this economy. Hence, we can substitute for $\mathcal{Q}_{t\to t+s} = \frac{1}{1+r_t} = q_t$, which is the price of the discount bond in the economy. Further, $\tilde{V}_{j,t+s}$ is the s-period ahead nominal profit for an intermediate producer of a typical input variety j, that is, nominal profit at time $t+s$, with price $p_{j,t}$ chosen at time t, and is given by its per-period nominal flow profit in period $t+s$ net of nominal price adjustment cost in period $t+s$ as

$$\tilde{V}_{j,t+s} = p_{j,t+s} y_{j,t+s} - mc_{t+s} y_{j,t+s} P_t + s - \frac{\theta}{2} \left(\frac{p_{j,t+s}}{p_{j,t+s-1}} - \bar{\pi} - 1 \right)^2 Y_{t+s} P_{t+s} \tag{31}$$

Substituting for $\tilde{V}_{j,t+s}$ from equation (31) and $y_{j,t}$ from the constraint in equation (30) in equation (29), the problem of an intermediate producer of a typical input variety j can be rewritten as

$$\begin{aligned} \max_{p_{j,t}} \mathbb{E}_t \Bigg[\sum_{s=0}^{\infty} q_t \frac{P_t}{P_{t+s}} \Bigg(& p_{j,t+s} \left(\frac{p_{j,t+s}}{P_{t+s}} \right)^{-\varepsilon} Y_{t+s} - mc_{t+s} \left(\frac{p_{j,t+s}}{P_{t+s}} \right)^{-\varepsilon} Y_{t+s} P_{t+s} \\ & - \frac{\theta}{2} \left(\frac{p_{j,t+s}}{p_{j,t+s-1}} - \bar{\pi} - 1 \right)^2 Y_{t+s} P_{t+s} \Bigg) \Bigg] \end{aligned} \tag{32}$$

The solution to this problem satisfies

$$\varepsilon \frac{p_{j,t}}{P_t}\left(\frac{p_{j,t}}{P_t}\right)^{-\varepsilon-1} Y_t = \left(\frac{p_{j,t}}{P_t}\right)^{-\varepsilon} Y_t + mc_t \varepsilon Y_t \left(\frac{p_{j,t}}{P_t}\right)^{-\varepsilon-1} - \theta\left(\frac{p_{j,t}}{p_{j,t-1}} - \bar{\pi} - 1\right)\frac{P_t}{p_{j,t-1}} Y_t$$
$$+ \theta \mathbb{E}_t\left[\frac{q_t}{P_{t+1}/P_t}\left(\frac{p_{j,t+1}}{p_{j,t}} - \bar{\pi} - 1\right)\left(\frac{p_{j,t+1}}{p_{j,t}}\right)^2 Y_{t+1}\right] \tag{33}$$

In equilibrium, all intermediate input producers behave identically: each of them faces the same marginal cost, therefore, each intermediate goods producer sets the same price and produces the same level of intermediate input variety j. Imposing symmetry in equilibrium, i.e. $p_{j,t} = P_t$, the intermediate goods producers' optimality condition for price setting in equation (33) can be written in terms of inflation rates by defining $\frac{P_{t+1}}{P_t} = \pi_{t+1} + 1$ as

$$\frac{\theta}{\varepsilon - 1}\left(\pi_t - \bar{\pi}\right)\pi_t = \frac{\varepsilon}{\varepsilon - 1} mc_t - 1 + \theta \mathbb{E}_t\left[q_t\left(\pi_{t+1} - \bar{\pi}\right)\pi_{t+1}\frac{Y_{t+1}}{Y_t}\right] \tag{34}$$

or,

$$\left(\pi_t - \bar{\pi}\right)\pi_t = \frac{1-\varepsilon}{\theta} + \frac{w_t}{z_t}\frac{\varepsilon}{\theta} + \mathbb{E}_t\left[q_t\left(\pi_{t+1} - \bar{\pi}\right)\pi_{t+1}\frac{Y_{t+1}}{Y_t}\right] \tag{35}$$

This is the New Keynesian Phillips Curve.

Aggregate production Integrating demand for intermediate good variety j across firms in equation 4, and using production function of intermediate goods producers in equation 6, we have

$$\begin{aligned} \int_0^1 y_{j,t}\ dj &= \int_0^1 \left(\frac{p_{j,t}}{P_t}\right)^{-\varepsilon} Y_t\ dj \\ \int_0^1 z_t n_{j,t}\ dj &= Y_t \int_0^1 \left(\frac{p_{j,t}}{P_t}\right)^{-\varepsilon}\ dj \end{aligned} \tag{36}$$

Since all firms are identical in the demand they face for their variety of the intermediate good, and the adjustment cost they must pay to adjust prices each period, all intermediate firms adjust to the same reset price every period, which then becomes the aggregate price level for the economy. Hence, setting $p_{j,t} = p_t = P_t$, we have

$$\begin{aligned} z_t \ N_t &= Y_t \int_0^1 \left(\frac{P_t}{P_t}\right)^{-\varepsilon} \ dj \\ z_t \ N_t &= Y_t \\ \text{or, } Y_t &= z_t \ N_t \end{aligned} \tag{37}$$

5 Recursive formulation of the economy

In this section, Iwill define the economy described above in a recursive formulation, which is essential for a state-contingent exposition of monetary policy in this environment. Let φ denote the aggregate exogenous state of the economy, representing the effective TFP. The endogenous aggregate state of the economy is the distribution of households across the state space, denoted by μ. The households have an endogenous individual state given by their individual choice of bond holding, denoted by b, and an exogenous individual state which is the idiosyncratic shock they face, denoted by e.

Household's decision problem The decision problem facing households is as follows.

$$\begin{aligned} V\left(e_i, b; \varphi_i, \mu\right) = \max_{\{c, b_i, n\}} \Bigg\{ & \frac{c^{1-\sigma}}{1-\sigma} - \chi \frac{n^{1+\psi}}{1+\psi} \\ & + \beta \sum_{\varphi_j'} Pr(\varphi_j' | \varphi_i) \sum_{e_j'} Pr(e_j' | e_i) \ V'\left(e_j', b'; \varphi_j', \mu'\right) \Bigg\} \end{aligned} \tag{38}$$

subject to their budget constraint,

$$c + q\left(\varphi_i, \mu\right) \cdot b' \leq b + (1 - \tau) \, w\left(\varphi_i, \mu\right) \cdot e_i \cdot n_i + d\left(\varphi_i, \mu\right) \tag{39}$$

the borrowing constraint,

$$b' \geq \underline{\mathrm{b}} \tag{40}$$

the non-negativity constraint,

$$c \geq 0 \tag{41}$$

and, the law of motion for the distribution of households,

$$\mu' = \Lambda\left(\mu, \varphi_i\right). \tag{42}$$

The solution to this problem yields the decision rule for households' consumption, $c(e_i, b; \varphi_i, \mu)$, the decision rule for households' bond holdings, b, given by $g(e_i, b; \varphi_i, \mu)$, and the decision rule for the households' labor supply choice, $n(e_i, b; \varphi_i, \mu)$.

Each household's optimal choices of consumption and labor satisfy their Euler equation given by

$$u_c'(c, n) = \beta \mathbb{E}\left[\left(1 + r'\right) u_c'\left(c', n'\right)\right] \tag{43}$$

which can be written in terms of the discount bond price as

$$q = \beta \mathbb{E}\left[\frac{u_c'(c', n')}{u_c'(c, n)}\right] \tag{44}$$

and its labor-leisure condition, given by

$$\chi \cdot n^{1/\eta} = c^{-\sigma} \cdot w(\varphi_i, \mu). \tag{45}$$

Together with the budget constraint of the household, these conditions pin down each household's optimal choices of consumption and leisure.

Decision problem of intermediate goods producers The individual state of an intermediate input producer is given by the relative price of the last period's price set by the producer to the aggregate price level last period, $\frac{p_{j,-1}}{P_{-1}}$. An intermediate firm's problem in real terms can then be defined in recursive form as

$$V_f\left(\frac{p_{j,-1}}{P_{-1}}; \varphi_i, \mu\right) = \max_{p_j}\left\{\frac{p_j y_j}{P} - w(\varphi_i, \mu) n_j - \frac{\theta}{2}\left(\frac{p_j/P}{p_{j,-1}/P_{-1}} \cdot \frac{P}{P_{-1}} - 1\right)^2 Y(\varphi_i, \mu)\right. \tag{46}$$

$$\left. + \beta \sum_{\varphi_j'} q(\varphi_i, \mu)\ Pr\left(\varphi_j' | \varphi_i\right)\ V_f\left(\frac{p_j}{P}; \varphi_j', \mu'\right)\right\}$$

or,

$$V_f\left(\frac{p_{j,-1}}{P_{-1}}; \varphi_i, \mu\right) = \max_{p_j}\left\{\frac{p_j y_j}{P} - w(\varphi_i, \mu) n_j - \frac{\theta}{2}\left(\frac{p_j/P}{p_{j,-1}/P_{-1}} \cdot (1 + \pi(\varphi_i, \mu)) - (\bar{\pi} + 1)\right)^2 Y(\varphi_i, \mu)\right. \tag{47}$$

$$\left. + \beta \sum_{\varphi_j'} q(\varphi_i, \mu)\ Pr\left(\varphi_j' | \varphi_i\right)\ V_f\left(\frac{p_j}{P}; \varphi_j', \mu'\right)\right\}$$

subject to

$$y_j = \left(\frac{p_j}{P}\right)^{-\varepsilon} Y\left(\varphi_i, \mu\right) \tag{48}$$

$$y_j = z \cdot n_j \tag{49}$$

where p_j is the price chosen by the firm producing input variety j when resetting its price in the current period, θ is the cost of price adjustment, and ε is the price elasticity of substitution across varieties of the intermediate goods.

To simplify the intermediate firms' problem in recursive form, Idefine the price of intermediate goods in terms of a relative price. Let $p \equiv p_j/P$ be the relative price for an intermediate firm producing variety j, where p_j is the price of variety j. Let $p_0 \equiv p^*/P$ be the ratio of the current optimal reset price, p^*, set by intermediate firms to the current aggregate price level P; in equilibrium, p_0 will be 1 - as all intermediate firms are identical, all firms face the same demand for their respective varieties and reset to the same optimal price, which becomes the aggregate price level. The net inflation rate in the current period is defined as $\pi \equiv \frac{P-P_{-1}}{P_{-1}}$ where P_{-1} is the aggregate price level in the previous period.

Substituting the constraints into the problem yields the following value of an intermediate firm

$$V_f\left(p_{-1}; \varphi_i, \mu\right) = \max_p \left\{ p^{1-\varepsilon} Y - \frac{w\left(\varphi_i, \mu\right)}{z} p^{-\varepsilon} Y - \frac{\theta}{2} \left(\frac{p}{p_{-1}} \cdot \left(1 + \pi\left(\varphi_i, \mu\right)\right) - \left(\bar{\pi} + 1\right) \right)^2 Y\left(\varphi_i, \mu\right) P \right. \tag{50}$$

$$\left. + \beta \sum_{\varphi_j'} q\left(\varphi_i, \mu\right) \; Pr\left(\varphi_j' | \varphi_i\right) \; V_f\left(p; \varphi_j', \mu'\right) \right\}$$

The solution to the intermediate firm's problem yields the New Keynesian Phillips Curve (NKPC) given in equation 35 which can be rewritten in recursive form as

$$\left(\pi\left(\varphi_i, \mu\right) - \bar{\pi}\right) \cdot \pi\left(\varphi_i, \mu\right) = \frac{1-\varepsilon}{\theta} + \frac{w\left(\varphi_i, \mu\right)}{z} \frac{\varepsilon}{\theta} \tag{51}$$

$$+ \mathbb{E}\left[q\left(\varphi_i, \mu\right) \left(\pi'\left(\varphi_j', \mu'\right) - \bar{\pi}\right) \cdot \pi'\left(\varphi_j', \mu'\right) \cdot \frac{Y'\left(\varphi_j', \mu'\right)}{Y\left(\varphi_i, \mu\right)} \right]$$

Evolution of inflation The New Keynesian Phillips curve in equation 51 above summarizes the firms' optimal responses to the decision problems faced by them in this economy, and is the equation for the evolution of inflation in this economy.

Aggregate production Aggregate production in this economy given by equation 36 can be rewritten as

$$Y(\varphi_i, \mu) = z \cdot N(\varphi_i, \mu) \tag{52}$$

Fiscal authority The government's budget constraint in equation 9 is given as

$$\begin{aligned} q(\varphi_i, \mu) \cdot B' = \; & B \; + \; G(\varphi_i, \mu) \\ & - \; \tau w(\varphi_i, \mu) \int e_i \cdot n(e_i, b; \varphi_i, \mu) \cdot d\mu(e_i, b; \varphi_i, \mu) \end{aligned} \tag{53}$$

which can be rewritten as

$$q(\varphi_i, \mu) \cdot B'(\varphi_i, \mu) = B(\varphi_i, \mu) + G(\varphi_i, \mu) - T(\varphi_i, \mu) \tag{54}$$

where $T = \tau w(\varphi_i, \mu) N(\varphi_i, \mu)$ is tax revenues, $N = \int e_i \cdot n(e_i, b; \varphi_i, \mu) \cdot d\mu(e_i, b; \varphi_i, \mu)$ is effective aggregate labor supply, $G(\varphi_i, \mu)$ is an exogenous government spending function. Further, aggregate bonds are the sum of bonds held across individuals.

$$B(\varphi_i, \mu) = \int b(e_i, b; \varphi_i, \mu) \, d\mu(e_i, b; \varphi_i, \mu) \tag{55}$$

where $\mu(e_i, b; \varphi_i, \mu)$ represents the distribution of households across the state space.

Defining the primary surplus as $S(\varphi_i, \mu) \equiv T(\varphi_i, \mu) - G(\varphi_i, \mu)$, we can write the government's budget constraint as:

$$q(\varphi_i, \mu) \cdot B'(\varphi_i, \mu) = B(\varphi_i, \mu) - S(\varphi_i, \mu) \tag{56}$$

To stabilize debt, the fiscal authority adjusts the primary balance according to a simple feedback rule (centered around the target):

$$S(\varphi_i, \mu) = \rho_b \left(B(\varphi_i, \mu) - \bar{B} \right) + \bar{S} \tag{57}$$

where $\rho_b > 0$ is fiscal responsiveness to debt deviations, $\bar{B}$ is target debt, $\bar{S}$ is target primary surplus given as $\bar{T} - \bar{G}$. At the target, we have,

$$q\bar{B} = \bar{B} - \bar{S} \tag{58}$$

which, together with the period budget constraint of the government expressed in primary surplus terms, yields the law of motion for debt deviations

$$B'(\varphi_i, \mu) - \bar{B} = \frac{1 - \rho_b}{q} \left(B(\varphi_i, \mu) - \bar{B} \right) \tag{59}$$

and the evolution of government spending as

$$G(\varphi_i,\mu) = T(\varphi_i,\mu) - \left[\bar{S} + \rho_b(B(\varphi_i,\mu) - \bar{B})\right]. \tag{60}$$

which can be rewritten as

$$G(\varphi_i,\mu) = T(\varphi_i,\mu) - \left[\bar{T} - \bar{G} + \rho_b(B(\varphi_i,\mu) - \bar{B})\right]. \tag{61}$$

Monetary authority The monetary authority sets nominal interest rates following the Taylor rule

$$i(\varphi_i,\mu) = r^* + \pi(\varphi_i,\mu) + \alpha\Bigg(\pi(\varphi_i,\mu) - \bar{\pi}\Bigg). \tag{62}$$

where $i(\varphi_i,\mu)$ is nominal interest rate function, r^* is the target real interest rate, $\pi(\varphi_i,\mu)$ is the inflation rate function, $\bar{\pi}$ is the inflation target, and α: policy response coefficient on inflation gap.

The relationship between nominal interest rates, real interest rates, and expected future inflation given by the Fisher equation in equation 14 can be written in recursive form as

$$1 + r(\varphi_i,\mu) = \frac{1 + i(\varphi_i,\mu)}{\mathbb{E}\left[1 + \pi\left(\varphi_j',\mu'\right)\right]}. \tag{63}$$

6 Equilibrium

The distribution of households over individual and aggregate endogenous and exogenous states at date t is given by $\mu_t(e_i,b)$, and it evolves according to

$$\mu_{t+1}\left(e_j',\mathcal{B}\right) = \int_{\{(e,b):g_t(e,b;\varphi_i,\mu)\in\mathcal{B}\}} Pr\left(e_j'|e_i\right) d\mu_t(e,b_i) \text{ given } \varphi_i,\varphi_j' \tag{64}$$

for all sets $\mathcal{B} \subset \mathbb{R}$.

In equilibrium, individual labor supply decisions are consistent with the aggregate such that aggregate labor supply is given by

$$L \equiv \int e \cdot n_t(e_i,b;\varphi_i,\mu) \; d\mu(e_i,b;\varphi_i,\mu) \tag{65}$$

and labor market clearing requires labor supply and labor demand to equal,

$$L = N \tag{66}$$

Aggregate bond holding is given by,

$$\bar{B} = \int g_t \left(e_i, b; \varphi_i, \mu\right) \ d\mu \left(e_i, b; \varphi_i, \mu\right) \tag{67}$$

These are held by the government as debt, such that bond market clearing requires

$$\bar{B} = B \tag{68}$$

where B is aggregate public debt.

The aggregate dividends paid by the intermediate goods firms in real terms is given by

$$d\left(\varphi_i, \mu\right) = Y\left(\varphi_i, \mu\right) - w\left(\varphi_i, \mu\right) \cdot N\left(\varphi_i, \mu\right) - \frac{\theta}{2}\bigg(\pi\left(\varphi_i, \mu\right) - \bar{\pi}\bigg)^2 \cdot Y\left(\varphi_i, \mu\right) \tag{69}$$

Finally, the goods market clears when the aggregate resource constraint holds as

$$Y\left(\varphi_i, \mu\right) = C\left(\varphi_i, \mu\right) + G\left(\varphi_i, \mu\right) - \frac{\theta}{2}\bigg(\pi\left(\varphi_i, \mu\right) - \bar{\pi}\bigg)^2 \cdot Y\left(\varphi_i, \mu\right) \tag{70}$$

where $C \equiv \int c\left(e_i, b; \varphi_i, \mu\right) \ d\mu\left(e_i, b; \varphi_i, \mu\right)$ denotes aggregate consumption from consistency between individual consumption decisions and aggregate consumption in equilibrium.

Definition of a recursive competitive equilibrium A recursive competitive equilibrium in this economy consists of decision rules and value functions:
$\left\{c_t\left(e_i, b; \varphi_i, \mu\right), g_t\left(e_i, b; \varphi_i, \mu\right), n_t\left(e_i, b; \varphi_i, \mu\right), V_t\left(e_i, b; \varphi_i, \mu\right), V_{f_t}\left(\frac{p_{j,-1}}{P_{-1}}; \varphi_i, \mu\right)\right\}_{t=0}^{\infty}$ that solve the problems of households and firms, respectively, and distributions of households $\{\mu_t\left(e_i, b\right)\}_{t=0}^{\infty}$ that evolve according to the law of motion in equation 64 for given φ_i, φ_j', and individual decisions are consistent with the aggregates as in equations 67 and 68 for the bond market, equations 65 and 66 for the labor market, and equation 70 for the goods market, which must also hold as the aggregate resource constraint for the economy. In addition, the government's budget constraint in equation 9, the Fisher relation in equation 63, the optimal price setting equation for the intermediate firms, which yields the New Keynesian Phillips Curve in equation 51, which shows the evolution of inflation, the equation for aggregate dividends in equation 69, and the aggregate resource constraint in equation 70, along with the monetary policy rule in equation 62 must be satisfied.

7 Parameters

A period in this model economy is a quarter of a calendar year. The subjective discount factor *beta* is set at 0.985. The coefficient of relative risk aversion is set to 2. The inverse of the Frisch elasticity of labor supply is 2 based on the findings in Chetty (2012) and as in Mckay, Nakamura, Steinsson (2016), and the preference parameter for households for labor supply is set to 100 to achieve a labor supply of a third.

The desired mark-up for intermediate firms, defined by $\mu \equiv \frac{\varepsilon}{\varepsilon-1}$ is set to 1.2 following Christiano, Eichenbaum, and Rebello (2011) and Mckay, Nakamura, and Steinsson (2016). The target inflation rate is set to 2 percent per annum. The cost of price adjustment, θ, is chosen to be 250. In Rotemberg pricing, firms face quadratic costs when adjusting prices, and the parameter θ controls the magnitude of these costs. When aggregate risks are present, shocks can be large and volatile, leading to potentially explosive price dynamics if the adjustment cost is too low. A high θ acts as a damping mechanism, preventing prices from changing too rapidly and ensuring that the model can be solved numerically without encountering instability or explosive paths. In HANK models, where household heterogeneity and incomplete markets amplify the effects of aggregate shocks, the need for a high adjustment cost parameter is even more pronounced to maintain equilibrium and avoid computational breakdowns. With the value of θ in the baseline calibration, the model can better handle large shocks and aggregate uncertainty, as the adjustment cost penalizes rapid price changes and stabilizes the system. This is particularly important for models with forward-looking agents and heterogeneous households, where the propagation of shocks can be highly nonlinear and sensitive to parameter values. In an alternate parameterization, I compute the model with a value of θ equal to 150.

The idiosyncratic income shocks for a quarterly AR(1) process have a persistence parameter of 0.966 and a standard deviation of innovation of 0.1, following the quarterly estimates used by McKay, Nakamura, and Steinsson (2016). The AR(1) shock process is discretized to a three-point Markov chain using the Rouwenhorst (1995) method.

The shock to Total Factor Productivity is also a quarterly AR(1) process, with a persistence parameter of 0.89, and the standard deviation of innovation being 0.015. The AR(1) shock process for total factor productivity is discretized also to a three-point Markov chain, again using the Rouwenhorst (1995) method.

The coefficient on debt deviations ρ_b in the fiscal rule is set at 0.08 for the baseline model; in an alternate calibration it is set to 0.12. For stability, this value must be greater than the real interest rate so that primary surplus rises faster than interest costs on outstanding debt.

Parameter	Description	Value
β	Subjective discount factor	0.985
η	Inverse of Frisch elasticity	2
χ	Coefficient on labor	100
σ	Coefficient of relative risk aversion	2
τ	Proportional tax rate	0.30
μ	Markup	1.2
θ	Price adjustment cost parameter	250
ρ_b	Coefficient on debt deviation (fiscal rule)	0.08
$\bar{\pi}$	Inflation target	1.005
α	Coefficient on inflation gap	1.5
r^*	Intercept in Taylor rule	1.005
ρ_z	Persistence of aggregate shock	0.89
σ_ϵ	Std of innov in aggregate shock	0.015
ρ_e	Persistence of idiosyncratic shock	0.966
σ_e	Std of innov in idiosyncratic shock	0.1

Table 1: Parameter values in the baseline model

The coefficient on the inflation gap in the Taylor rule is chosen to be 1.5. For an active monetary policy in place, nominal rates must rise more than one-for-one with inflation, thus raising real rates and stabilizing inflation. This condition, $\alpha > 1$, known as the Taylor principle is essential for a determinate equilibrium. In it is equal to 1, real rates will not change with inflation causing inflation to drift, leading to indeterminacy.

8 Computation

We compute a steady state for this economy where households face idiosyncratic income shocks but there are no shocks to TFP. As the aggregate of household savings is held by the government as public debt, Ibisect the government's budget constraint to achieve an equilibrium bond price for this economy. I solve the households' problem using value function iteration and solve for the households' saving and consumption decision rules using the endogenous grid method.

In computing the stochastic equilibrium, I use the Backward Induction method due to Reiter (2010). The stochastic economy has an exogenous aggregate state, which is the TFP, and an endogenous aggregate state, which is the distribution of households in the economy. As the distribution of households is an infinite dimensional object, I approximate the aggregate endogenous state of the economy with the aggregate bond holdings, m. For approximating this, I use the steady-state distribution of households as the initial proxy distribution, and discretize the approximate endogenous state space over levels of bond holdings, such that the distribution yields a mean bond holding level equal to the value at that grid point. In other words, the dicretized grid for the levels of aggregate bond holdings is obtained around the mean of the steady-state level of bond holdings given the stationary distribution of households in the steady-state. The proxy distributions are computed at each point in the discretized aggregate state space, by adjusting the steady-state distribution in a way such that the resulting distribution yields a mean level of bond holdings equal to the grid value of the approximate aggregate state, which here is the level of aggregate bond holdings in economy m.

Utilizing these proxy distributions, I solve the real economy in the model in one run of backward iterations, on a discrete grid of points in the aggregate state space, while enforcing consistency between individual and aggregate solutions in each step of the backward iteration, which is done separately at each point on the grid of aggregate states (Reiter 2010).

Households' decision rules are solved for using the endogenous grid method on the grid for individual endogenous and exogenous states, that is, the bond holdings of households and the idiosyncratic income shocks, respectively. Consistency is achieved between individual and aggregate decisions, when the expected level of aggregate bond holdings in the economy becomes equal to that obtained from its law of motion. Once such consistency is achieved, for an initial guess for the price of bonds q function, and an initial guess of the inflation function over the aggregate state space, I iterate on wages to clear the labor market. The partial equilibrium wage function obtained from iterating over the real economy until labor demand equals labor supply, therefore, clears both the labor and the goods market, for the initially guessed rate of inflation. The asset market is cleared by Walras' law. Using these partial equilibrium wages, and the output obtained from the aggregate resource constraint, for the initial inflation rate, I use the New Keynesian Phillips Curve to update current inflation, over expectations of future inflation and output. The general equilibrium is obtained once inflation, wages, the law of motion for aggregate bond holdings, and households' value functions have all converged up to a certain tolerance set for each of these. This method yields the stochastic equilibrium in this economy.

Variable	Steady-state values
Real interest rate (r^*)	1.93 percent
Bond price (q^*)	0.9952
Debt (B^*)	0.52
Output (Y^*)	0.3324
Consumption (C^*)	0.2518
Labor (efficiency units) (L^*, N^*)	0.3324
Raw labor hours (H^*)	0.3121
Wage (w^*)	0.833
Government spending (G^*)	0.081
Dividend (D^*)	0.0554
Inflation (π^*)	1.005
Primary deficit $(G - \tau wL)/Y$	-0.75 percent
Debt-to-GDP (B/Y)	157.47 percent

Table 2: Steady-state values

Percentiles of wealth	1	5	10	50	90	< 0
Share of households	0.057	0.249	0.443	1.000	1.001	0.467

Table 3: Wealth distribution of households in steady-state

9 Results

9.1 Steady-State

The deterministic steady state, obtained by bisecting the government's budget constraint, yields an equilibrium bond price q of 0.9952, which implies a quarterly risk-free real interest rate of 0.50 percent, which yields an annualized risk-free real interest rate of 1.93 percent per annum. In steady-state, the primary deficit-to-GDP ratio, given by $(G - \tau \cdot w \cdot e \cdot n)/Y$, is around -0.75 percent, whereas the deficit-to-GDP ratio, given by $(G - (1-q) \cdot B + G - \tau \cdot w \cdot e \cdot n)/Y$, is 0 percent in the model. This was around 6.4 percent in 2024 for the US economy, and the 50-year long-run average has been close to 3.6 percent. The steady-state, yields a debt-to-GDP ratio of about 157.47 percent, which is around 123 percent for the US economy in 2024. Government spending in the model economy in the steady-state is about 24.3 percent of GDP, where federal government spending was equal to 23 percent of the total gross domestic product (GDP) in the US economy in FY 2024.

The distributions of wealth in the steady state are given in Table 3. The share of negative assets, that is, households with zero or negative net worth, in the model economy is around 46 percent, which for the US economy is close to 11 percent. This is to match the share of borrowing constrained households in McKay, Nakamura, and Steinsson (2016). For the purposes of this paper, my objective is to achieve a plausible distribution of wealth to study the effect of forward guidance in the presence of aggregate uncertainty. Hence, the model is not calibrated to match the wealth distributions from the data in the US economy. The need to insure against income shocks is small among the wealthiest households, and hence their consumption responses do not affect, in any significant way, the efficacy of forward guidance on aggregate consumption. The steady-state wealth distribution in the model yields a Gini coefficient of 0.717. The steady state results are summarized in Table 2.

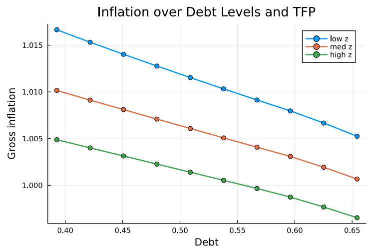
9.2 Stochastic Equilibrium and Forward Guidance

The two key findings of my work are: one, the presence of aggregate uncertainty with incomplete markets makes non-state-contingent forward guidance far less than in the absence of aggregate risks; and two, the effects of state-contingent forward guidance are nearly half the size of the effects of non-state-contingent forward guidance, highlighting the role of uncertainty in determining the efficacy of forward guidance.

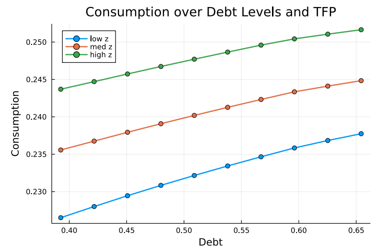
The stochastic general equilibrium of my model forms the basis for the conduct of all forward guidance experiments using this model. Figure 1 shows the aggregate functions plotted in the aggregate state space. In doing the forward guidance experiment, the stochastic dynamic general equilibrium forms the basis of the response of aggregate functions in the forward guidance experiments that follow.

In the baseline experiment, the central bank announces a 50 basis points (bps) rate cut 20 periods ahead in the future, irrespective of the future state of the economy. This causes current output to increase by 5 bps and current consumption to increase by 7 basis points. These effects are much smaller than those seen in McKay, Nakamura, and Steinsson (2016) where markets are incomplete, but agents have perfect foresight. While there are difference in modeling choices between their model and my framework, the presence of aggregate economic uncertainty and the internalization of the stochastic nature of the economy by the households is the key difference in the environment in which the economy operates. They show a 10 bps increase in consumption (and output, since output is consumption in their economy) and a 30 basis point increase in inflation on impact. My results show that even when forward guidance is non-state-contingent, the effects on contemporaneous variables are muted in the presence of aggregate shocks.

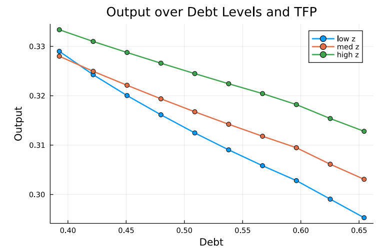
In order to isolate the effect of the forward guidance experiment, while agents expect there to be aggregate economic shocks, these shocks are not actually realized in the duration of the forward guidance experiment. Despite the fact that no aggregate



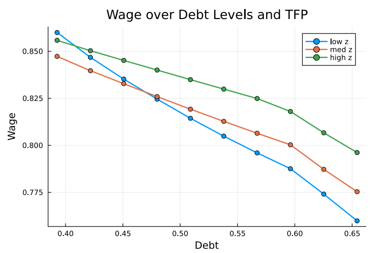
(a) Inflation



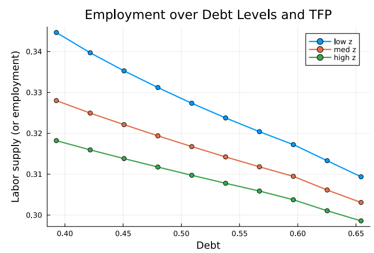
(b) Consumption



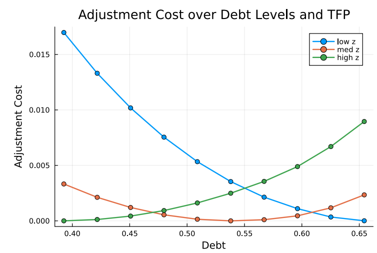
(c) Output



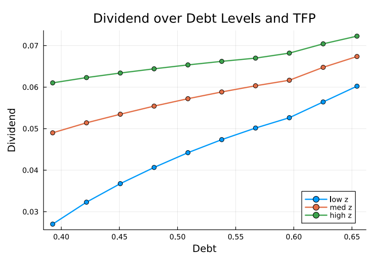
(d) Wage



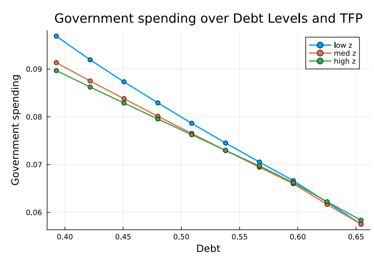
(e) Labor



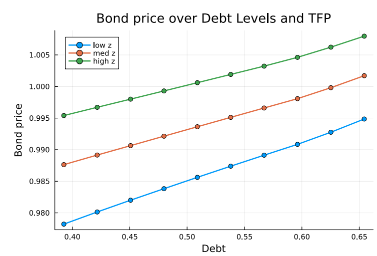
(f) Adj cost



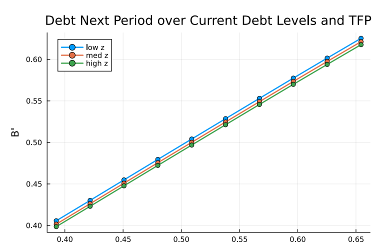
(g) Dividend



(h) Gov't spending



(i) Bond price



(j) Savings

Figure 1: Stochastic equilibrium

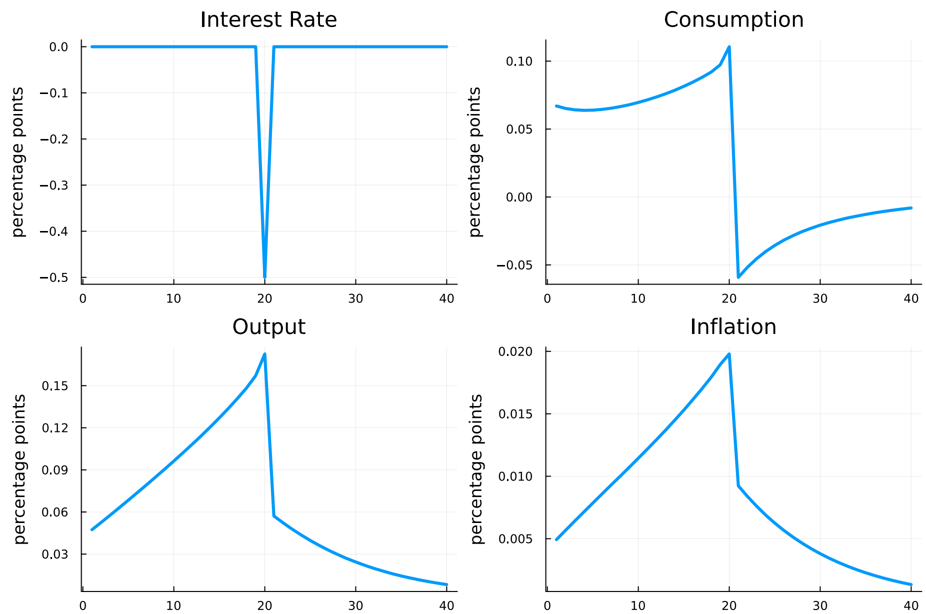


Figure 2: Enter Caption

shocks are realized in the economy, we see that the power of forward guidance is significantly diminished when there are aggregate risks and uncertainty present in the economy. This further underscores the salience of nonlinearities in household behavior when households internalize the stochastic nature of the economy, even though they do not actually experience such shocks.

Over time, the effects on output, consumption, and inflation gradually increase until the date of implementation of the policy. Table 4 shows the differences in the effect of non-state-contingent forward guidance on output, consumption, and inflation, at the date of impact and on the date of the policy implementation, in my model compared to the results in McKay et al (2016). Figure 2 shows the results for this baseline experiment.

(In basis points)	**Variable**	**Date of policy announcement**	**Date of policy implementation**
My model with aggregate uncertainty	Output	5	17
	Consumption	7	12
	Inflation	0.5	2
McKay et al (2016)	Output	10	17
	Consumption	10	17
	Inflation	30	-5

Table 4: Effects of a non-state-contingent forward guidance about a 50 bps rate cut 20 periods ahead

When real interest rates are reduced, the cost of borrowing falls, making borrowing for consumption more attractive. As a result, demand for consumption increases, causing production by firms to increase, which in turn causes firms to demand more labor to meet increased production demands. This is the direct intertemporal substitution effect of an interest rate cut. In general equilibrium, the increased demand for labor by firms drives up wages, causing labor supply to increase. Hence, households' labor incomes increase, therefore, causing an additional increase in demand for consumption and adding to the direct effects of an interest rate cut. These have been referred to as the "indirect" effects" or the "general equilibrium" effects of a real interest rate cut (Kaplan, Moll, and Violante (2016)).

McKay, Nakamura, and Steinsson (2016) suggested a resolution to the puzzle by having incomplete markets and household heterogeneity. As households face idiosyncratic shocks to income, there is greater uncertainty about their future incomes and wealth positions, causing precautionary savings motives to escalate. Additionally, when households are subject to borrowing constraints in the presence of such idiosyncratic income risks, communications about changes in interest rates far in the future dampen the current consumption response of households—those who experience negative income shocks persistently may run the risk of hitting their borrowing limits sooner. To smooth consumption over their lifetimes, they respond very little to announcements about future interest rate changes.

My results show an even smaller response in output, consumption, and inflation when households face exacerbated risks from the presence of aggregate economic uncertainty, in addition to the individual risks from idiosyncratic shocks to their income. This further mutes their response to an announcement about future interest rate cuts.

Now we turn to a state-contingent forward guidance experiment wherein the central bank announces a 50 basis points rate cut 20 periods ahead, only when a certain state of the economy is realized. Since no existing work in the literature has the environment to study state-contingency in forward guidance, these results shed light on questions that previous works in the literature could not have answered. Table 5 compares the results from a state-contingent forward guidance to a non-state-contingent forward guidance over the same horizon and the same magnitude of the rate cut in both cases. Making forward guidance state-contingent reduces its power to affect current output, consumption, and inflation by almost half, compared to when the guidance is non-state-contingent and would take effect irrespective of the state of the economy.

Aggregate uncertainty amplifies the heterogeneous exposure of households, already experiencing idiosyncratic income shocks, to aggregate risks. This amplifies their precautionary savings motives, causing households to save more and the trans-

(In basis points)	Variable	Date of policy announcement	Date of policy implementation
State-contingent forward guidance	Output	5	17
	Consumption	7	12
	Inflation	0.5	2
Non-contingent forward guidance	Output	2.5	14
	Consumption	3	11
	Inflation	0.5	1.5

Table 5: Effects of state-contingent versus non-state-contingent forward guidance about a 50 bps rate cut 20 periods ahead

mission of a policy shock to aggregate consumption and output to be weaker. This is an additional source of consumption volatility in this economy. This force becomes particularly stronger when forward guidance does not resolve the uncertainty around expected future policy rates, when it is contingent on a certain state of the economy in the future. This in part defeats the purpose of forward guidance, as the policy announcement is tied to a certain economic state in the future and agents are uncertain about future economic conditions. This further tempers households' responses to such state-contingent announcements, making them much less powerful than announcements that are expressly contingent on time. While time-contingent forward guidance is fraught with the dangers of destabilizing long run inflation expectations, my work provides evidence that there is need to balance the objectives of eliminating uncertainty about future policy rates and keeping inflation expectations anchored so that forward guidance can be more effective in influencing current macroeconomic variables and improving the predictability of monetary policy actions, which is the core objective of forward guidance.

(in basis points)	Variable	Date of policy announcement	Date of policy implementation
Different fiscal rule (high $\theta_b = 0.125$)	Output	5	20
	Consumption	7	11-12
	Inflation	0.5	2.5
Lower price adj cost parameter ($\theta = 150$)	Output	4-5	16
	Consumption	6-7	10
	Inflation	0.5	2.7

Table 6: Non-state-contingent forward guidance about a 50 bps rate cut 20 periods ahead with alternate parameters

The effects of guidance about a future rate increase has a symmetric impact on current output, consumption, and inflation. A non-state-contingent forward guidance about a rate increase by 50 bps 20 periods in the future, causes output, consumption, and inflation to fall by 5 bps, 7 bps, and 0.5 bps, respectively. Additionally, guidance over a longer horizon causes much smaller effects than that over a shorter horizon. My results are robust to alternate calibrations for the fiscal policy parameter and the price adjustment cost parameter, as shown in Table 6.

10 Conclusion

In this paper, I study the effect of forward guidance in an incomplete market setting with aggregate uncertainty. Households face risks from uninsurable idiosyncratic income shocks which are compounded by the presence of aggregate uncertainty.

Given the state-contingent nature of forward guidance about future policy rates based on the expected state of the economy in the future, adopted by the US Federal Reserve in recent years, the study of such state-contingent forward guidance is only feasible in the presence aggregate uncertainty, which has been unexplored in the literature so far.

I solve the model with aggregate uncertainty to find a global stochastic general equilibrium for the economy and utilize this to study the effect of state-contingent and non-state-contingent forward guidance where the monetary authority announces a 50 bps real interest rate cut 20 periods ahead contingent on a particular state of the economy or unconditionally, respectively.

I find that non-state-contingent forward guidance in the presence of aggregate uncertainty and without perfect foresight has relatively smaller effects than a similar forward guidance expressly contingent only on time, irrespective of the state of the economy under perfect foresight. Further, the power of forward guidance to impact current output, consumption, and inflation, are significantly muted when the guidance is state-contingent. Specifically, state-contingent forward guidance is only half as effective as non-state-contingent forward guidance in affecting current real economic activity and current inflation.

My results underscore the importance of incorporating aggregate uncertainty for a meaningful study of forward guidance in its present state-contingent form. The presence of aggregate uncertainty causes the power of forward guidance to fall substantially, even when such guidance is not contingent on future states of the economy. This highlights the presence of nonlinearities in household behavior when the economy faces risks of aggregate uncertainty, necessitating a global solution and elimination of perfect foresight to accurately capture such nonlinearities in presence of aggregate risks. Heightened risks lead to large precautionary savings, and borrowing constraints further lessen the response of households to such announcements, particularly so when these announcements are contingent on future economic circumstances as this leaves room for uncertainty about expected future rates. Therefore, forward guidance has significantly diminished power in the presence of aggregate uncertainty and when markets are incomplete.

My work augments the existing literature on forward guidance by allowing for the study of state-contingent forward guidance, which is not possible in existing frameworks in the literature. I solve for a stochastic dynamic general equilibrium which is a global solution to the model with aggregate uncertainty, without relying on approximations around the steady-state outcomes or on perfect foresight. My model and results point to a new direction for further research and exploration to better inform forward guidance announcements to have more power and influence on aggregate macroeconomic variables of interest. This, in my view, is a first-order contribution to academic and policy-relevant research.

References

[1] S. Acharya, E. Challe, and K. Dogra, "Optimal monetary policy according to hank," *American Economic Review*, vol. 113, no. 7, pp. 1741–1782, 2023.

[2] G.-M. Angeletos and C. Lian, "Forward guidance and heterogeneous beliefs," *American Economic Review*, vol. 108, no. 9, pp. 2477–2512, 2018.

[3] A. Auclert, B. Bardóczy, M. Rognlie, and L. Straub, "Using the sequence-space jacobian to solve and estimate heterogeneous-agent models," *Econometrica*, vol. 89, no. 3, pp. 1329–1356, 2021.

[4] C. Bayer, R. Luetticke, L. Pham-Dao, and V. Tjaden, "Precautionary savings, illiquid assets, and the aggregate consequences of wealth taxes," tech. rep., CEPR Discussion Paper No. DP13528, 2019.

[5] B. S. Bernanke, "The new tools of monetary policy," *American Economic Review*, vol. 110, no. 4, pp. 943–983, 2020.

[6] M. Bodenstein, J. Hebden, and R. Nunes, "Imperfect credibility and the forward guidance puzzle," tech. rep., Board of Governors of the Federal Reserve System, International Finance Discussion Papers No. 1061, 2012.

[7] J. R. Campbell, J. D. M. Fisher, A. Justiniano, and L. Melosi, "Forward guidance and macroeconomic outcomes since the financial crisis," in *NBER Macroeconomics Annual 2016* (M. Eichenbaum and J. A. Parker, eds.), vol. 31, pp. 283–357, University of Chicago Press, 2017.

[8] C. T. Carlstrom, T. S. Fuerst, and M. Paustian, "Optimal monetary policy in a model with agency costs," *Journal of Money, Credit and Banking*, vol. 47, no. S2, pp. 265–308, 2015.

[9] R. Chetty, "Bounds on elasticities with optimization frictions: A synthesis across microeconomic contexts," *Econometrica*, vol. 80, no. 6, pp. 2127–2181, 2012.

[10] L. J. Christiano, M. Eichenbaum, and S. Rebelo, "When is the government spending multiplier large?," *Journal of Political Economy*, vol. 119, no. 1, pp. 78–121, 2011.

[11] H. Chung, E. Herbst, and M. T. Kiley, "Effective monetary accommodation and the reversal rate," tech. rep., Board of Governors of the Federal Reserve System, Finance and Economics Discussion Series 2014-44, 2014.

[12] M. Del Negro, M. P. Giannoni, and C. Patterson, "The forward guidance puzzle," tech. rep., Federal Reserve Bank of New York, Staff Report No. 574, 2012.

[13] M. Del Negro, M. P. Giannoni, and C. Patterson, "The forward guidance puzzle," tech. rep., Federal Reserve Bank of New York, Staff Report No. 574, Revised, 2013.

[14] X. Gabaix, "A sparsity-based model of bounded rationality," *Quarterly Journal of Economics*, vol. 129, no. 4, pp. 1661–1710, 2015.

[15] M. Garcia-Schmidt and M. Woodford, "Are low interest rates deflationary? a paradox of perfect-foresight analysis," *American Economic Review*, vol. 109, no. 1, pp. 86–126, 2019.

[16] R. S. Gürkaynak, B. Sack, and E. T. Swanson, "Do actions speak louder than words? the response of asset prices to monetary policy actions and statements," *International Journal of Central Banking*, vol. 1, no. 1, pp. 55–93, 2005.

[17] A. Haberis, R. Harrison, and M. Waldron, "Monetary policy, forward guidance and the yield curve," tech. rep., Bank of England Working Paper No. 485, 2014.

[18] M. Hagedorn, I. Manovskii, K. Mitman, and S. Sehran, "Unemployment benefits and unemployment in the great recession: The role of macro effects," *American Economic Review*, vol. 109, no. 8, pp. 2957–2993, 2019.

[19] C. L. Ilut, R. Luetticke, and M. Schneider, "Title forthcoming." Forthcoming (update publication details as available), 2025.

[20] G. Kaplan, B. Moll, and G. L. Violante, "Monetary policy according to hank," *American Economic Review*, vol. 108, no. 3, pp. 697–743, 2018.

[21] M. T. Kiley, "Macroeconomic effects of federal reserve forward guidance," tech. rep., Board of Governors of the Federal Reserve System, Finance and Economics Discussion Series 2016-061, 2016.

[22] N. G. Mankiw and R. Reis, “Sticky information versus sticky prices: A proposal to replace the new keynesian phillips curve,” *Quarterly Journal of Economics*, vol. 117, no. 4, pp. 1295–1328, 2002.

[23] A. McKay, E. Nakamura, and J. Steinsson, “The power of forward guidance revisited,” *American Economic Review*, vol. 106, no. 10, pp. 3133–3158, 2016.

[24] F. Milani and J. Treadwell, “The effects of forward guidance in dsge models: A comparison of strategies,” *Economics Letters*, vol. 116, no. 2, pp. 172–175, 2012.

[25] M. Reiter, “Global solution methods for heterogeneous agent models,” *Journal of Economic Dynamics and Control*, vol. 34, no. 1, pp. 80–90, 2010.

[26] J. J. Rotemberg, “Monopolistic price adjustment and aggregate output,” *Review of Economic Studies*, vol. 49, no. 4, pp. 517–531, 1982.

[27] G. L. Violante, “Title forthcoming.” Please insert full citation for the specific publication used, 2021.

[28] M. Woodford, “Monetary policy analysis in models with heterogeneous agents,” in *Handbook of Macroeconomics* (J. B. Taylor and H. Uhlig, eds.), vol. 2B, pp. 867–928, Elsevier, 2019.