Forward Guidance in an Uncertain World

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Abstract

I investigate the impact of state-contingent forward guidance in monetary policy under incomplete markets and aggregate uncertainty. I develop a novel recursive solution framework using a global solution method that enables a thorough quantitative study of forward guidance contingent on macroeconomic states—a question previously unexplored in the literature. My results reveal that state-contingent forward guidance generates substantially smaller effects on current output, consumption, and inflation than non-state-contingent guidance, which is implemented irrespective of the future state of the economy. Households facing aggregate and idiosyncratic risks have stronger precautionary savings than in the absence of aggregate uncertainty and respond even less to state-contingent policy announcements: state-contingent forward guidance leaves room for uncertainty about future rate shifts and only partially increases the predictability of future policy, thereby limiting its effectiveness. A global solution method in my work allows for a robust analysis of state-contingent forward guidance with aggregate uncertainty, unexplored in prior studies, and infeasible with methods in the literature that rely on local approximations around the steady state or on perfect foresight, making my work a significant contribution to the literature on forward guidance and, more generally, a methodological innovation allowing for the study of broader monetary policy issues in dynamic stochastic environments.

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1 Introduction

Forward guidance in monetary policy refers to central banks' communication about the future path of short-term interest rates, intended to influence long-run inflation expectations and economic decisions today. Over the last two decades, forward guidance has evolved from terse statements about the central bank's future economic outlook and anticipated policy response to more explicit commitments, where the central bank announces target ranges contingent on macroeconomic targets and future states of the economy. I examine the effects of this evolved form of state-contingent forward guidance in an incomplete markets environment. I develop a stochastic dynamic general equilibrium model with aggregate uncertainty to quantify how announcements about future interest rate cuts, contingent on the realization of particular economic states, affect current macroeconomic variables.

Prior literature has emphasized the "forward guidance puzzle", where standard DSGE models predicted implausibly large effects of future policy announcements on present macroeconomic variables [1]. Extending this line of inquiry, my analysis reveals that state-contingent forward guidance—where future policy paths depend explicitly on the state of the economy—has substantially smaller effects on output, consumption, and inflation compared to non-state-contingent guidance, which is implemented irrespective of the future state of the economy.

This diminished impact arises because households facing uninsurable idiosyncratic income risks, borrowing constraints, and aggregate uncertainty exhibit strong precautionary savings. These factors attenuate their responsiveness to policy announcements that only partially resolve uncertainty about future interest rates. To better understand this behavior, I introduce a new methodological approach for the study of state-contingent forward guidance, highlighting the importance of nonlinearities and uncertainty in shaping its efficacy.

McKay, Nakamura, and Steinsson (2016) show that the power of forward guidance is highly sensitive to the assumption of complete markets—when agents face uninsurable income risks and borrowing constraints, the precautionary saving motive tempers their contemporaneous response to changes in future interest rates. I augment this framework with aggregate uncertainty to examine how state-contingent announcements of future policy rate cuts affect current macroeconomic outcomes. My results demonstrate that state-contingent forward guidance is nearly half as effective in influencing current output and consumption as non-state-contingent forward guidance, which is implemented irrespective of the future state of the economy. Importantly,

[1] Del Negro, Giannoni, and Patterson (2012); Gurkaynak, Sack, and Swanson (2005); Campbell, Fisher, and Justiniano (2012); Milani and Treadwell (2012); Campbell et al. (2017)

even non-state-contingent forward guidance, in the presence of aggregate uncertainty and incomplete markets, tends to have contemporaneous effects that are smaller than those predicted by McKay, Nakamura, and Steinsson (2016) under perfect foresight. This is because households tend to retain savings to hedge against the risk of future economic shocks, characteristic of an uncertain world.

Forward guidance contingent on a future calendar date, independent of the state of the economy, is referred to in the literature as calendar-based forward guidance. The US Federal Reserve started using such calendar-based forward guidance in August 2011. However, calendar-based forward guidance posed the threat of destabilizing long-term inflation expectations because the public was likely to misinterpret the central bank's announcement to hold interest rates low until a future date as the bank's dismal outlook for future economic conditions. This risked producing outcomes that run counter to the very objective of forward guidance: anchoring long-run inflation expectations that would shift real rates without changing the nominal rate. To overcome this drawback, the US Federal Reserve moved away from "date-based" guidance [2] to state-contingent forward guidance in December 2012, when it started announcing target ranges for interest rates contingent on specific macroeconomic targets, such as the unemployment rate, long-term inflation expectations, labor market conditions, the evolving outlook of the economy, and the balance of risks [3].

The objective of this paper is to study forward guidance in its present state-contingent form and analyze its effects on contemporaneous macroeconomic outcomes in light of the "forward guidance puzzle". To the best of my knowledge, no study in the existing literature has examined this question or developed a framework that accounts for aggregate uncertainty using a global solution method to investigate the effects of state-contingent forward guidance, thereby allowing the stochastic steady state to differ from the deterministic one.

The study of aggregate uncertainty in heterogeneous-agent New Keynesian models has mostly been limited to local approximations around the steady state [4] and the study of dynamics under the assumption of perfect foresight [5]. While computationally fast, local perturbation methods rely on the assumption that agents' steady-state

[2] FOMC Press Release on December 12, 2012 used this nomenclature to describe previous guidance contingent on a future calendar date

[3] The FOMC Press Release on December 12, 2012 stated that it "currently anticipates that this exceptionally low range for the federal funds rate will be appropriate at least as long as the unemployment rate remains above 6-1/2 percent, inflation between one and two years ahead is projected to be no more than a half percentage point above the Committee's 2 percent longer-run goal, and longer-term inflation expectations continue to be well anchored."

[4] Adrien Auclert, Bence Bardóczy, Matthew Rognlie, and Ludwig Straub (2021); Cosmin L. Ilut, Ralph Luetticke, and Martin Schneider (2025), Christian Bayer, Ralph Luetticke, Lien Pham-Dao, and Volker Tjaden (2018)

[5] McKay, Nakamura, Steinsson (2016)

behavior is relevant even in the presence of aggregate uncertainty, to some order of approximation. My work shows that local approximations around the steady state may not accurately capture the elasticities of aggregate responses to price changes in the presence of aggregate uncertainty.

I augment a one-asset, heterogeneous-agent New Keynesian model without capital, in which nominal rigidities arise due to price adjustment costs faced by intermediate firms, as in Rotemberg (1982). To allow for nonlinearities in household and firm behavior that may exist outside the steady state, I solve the model recursively using Backward Induction—a global solution method due to Reiter (2010)—to compute the stochastic general equilibrium. Households' policy functions and aggregate economic responses, thus, incorporate all relevant individual and actual or approximated aggregate state variables, aptly capturing both individual and aggregate risks and their importance in macroeconomic dynamics. I use this global stochastic general equilibrium to study the effects of state-contingent and non-state-contingent forward guidance on current macroeconomic variables.

A household's individual endogenous state is its bond holdings, and its individual exogenous state is the idiosyncratic income shock it faces. The aggregate exogenous state is the shock to aggregate productivity, and the aggregate endogenous state is the distribution of households. For computation, I use aggregate bond holdings by households—a moment of the actual distribution of households—to approximate the economy's aggregate endogenous state.

In a baseline exercise, I benchmark the effects of non-state-contingent forward guidance and compare its outcomes with those of state-contingent guidance. I conduct several experiments—future rate cuts, increases in future interest rates, rate cuts over a longer horizon—and compare the results for state-contingent guidance with their non-state-contingent counterparts to assess the robustness of the reduced effectiveness of state-contingent forward guidance. Finally, I conduct the baseline experiment for state-contingent and non-state-contingent forward guidance using alternative parameter values to assess the sensitivity of my results to critical parameters.

Forward guidance about a 50-basis-point rate cut 20 quarters ahead, contingent on a certain state of the economy, increases current output by 2.5 basis points and current consumption by 3 basis points. The effects of a state-contingent forward guidance are half the magnitude of the effects of the same forward guidance but without being contingent on the future state of the economy. This result demonstrates the importance of aggregate uncertainty in determining the efficacy of forward guidance when it is contingent on the state of the economy.

Additionally, the presence of aggregate economic uncertainty tempers the power of forward guidance even when such guidance is not contingent on the future state of the economy, as my results demonstrate, in comparison to the effects of forward guidance with incomplete markets under perfect foresight, as in McKay, Nakamura, and Steinsson (2016). For the same magnitude and horizon of forward guidance, current output and consumption increase by 5 and 7 basis points, respectively, in my environment, compared to 10 basis points in their perfect-foresight environment.

Forward guidance tied to specific economic conditions only partially reduces uncertainty about future policy rates. Households are unable to increase consumption significantly to take advantage of this announced rate cut because they remain exposed to unpredictable changes in their incomes and aggregate economic shocks, while also being uncertain about future policy rates. Borrowing constraints further restrict their ability to increase consumption in anticipation of partly uncertain future policy changes, since the risk of reaching their borrowing limits shortens their planning horizon and mutes their response further to guidance about distant future rates. Aggregate consumption and output, accordingly, increase very little at the date of impact when the forward guidance is state-contingent.

The remainder of this paper is organized as follows. In section 2, I discuss the related literature and where my work fits in, section 3 lays out the model environment, with section 4 describing the decision problems faced by the agents in the economy, section 5 detailing a recursive formulation of the economy, and section 6 defining the equilibrium for this economy, followed by a brief discussion of the choice of parameters for the model in section 7. Further, I discuss the methods used for computation in section 8 and present the results and their discussion in section 9, and finally, section 10 concludes.

2 Related Literature

The literature on forward guidance has been driven by the "forward guidance puzzle" (Del Negro, Giannoni, and Patterson 2012) – estimated DSGE models, while successfully explaining the effects of contemporaneous monetary policy actions on key macroeconomic variables, grossly exaggerated the effects of future policy announcements. The central bank's announcement of lower interest rates eight quarters in the future implied massive stimulus, with the impact on both output and inflation several times larger than that of a same-sized contemporaneous drop in the policy rate (Del Negro et al, 2013). Gurkaynak, Sack, and Swanson (2005) measure the effects of a "future path of policy", closely associated with FOMC statements, on bond yields and stock prices, and find that both monetary policy actions and statements have important but differing effects on asset prices, with FOMC statements having a much larger impact on longer-term Treasury yields. In a similar flavor, Campbell, Fisher, and Justiniano (2012), and Milani and Treadwell (2012) studied the impact of forward guidance shocks on the economy using small- or medium-scale DSGE models estimated on a pre-Great Recession sample, and both papers find that forward guidance shocks play a large role in explaining movements in the policy rate, and these contribute significantly to business cycle fluctuations.

Campbell et al. (2012) assessed the impact of exogenous changes in monetary policy on private expectations and found the sign was the opposite of the theoretical predictions, so that accommodative monetary policy was associated with lower projections of inflation and activity. In view of these considerations, the FOMC replaced the calendar-based language in the December 2012 statement and switched to announcing policy intentions tied to economic performance on inflation and real activity, and to the state of the economy more generally.

Campbell et al. (2017) use a full-information, rational-expectations framework and find that FOMC forward guidance has mixed effects. The main goal of their study is to assess whether the FOMC improved economic performance since the financial crisis through Odyssean forward guidance, and to quantify the impact of Odyssean FOMC forward guidance on macroeconomic outcomes since the financial crisis, which unwound from 2007 to 2009. They find that puzzling responses of private-sector forecasts to FOMC announcements can be attributed entirely to Delphic forward guidance, even though a large fraction of the variability in Federal Funds Futures rates on days with FOMC announcements remained unexplained by their measure of FOMC private information. Interestingly, while their findings suggest that purely rule-based policy would have ameliorated the recession and kept inflation closer to target in the years immediately following the crisis, than FOMC forward guidance did in practice, the Fed's introduction of calendar-based Odyssean forward guidance starting toward the end of 2011 appeared to have boosted real activity and moved inflation closer to target.

A large chunk of the forward guidance literature has focused on resolving the forward guidance puzzle. The forward guidance puzzle—large contemporaneous effects from a future policy announcement—arises from intertemporal substitution as the main channel through which forward guidance is transmitted into the real economy. As a result, departure from the complete markets, full-information rational expectations framework alleviates the forward guidance puzzle. The macroeconomic resolution of the puzzle comes from adding heterogeneity, such as in McKay, Nakamura, and Steinsson (2016), Hagedorn et al. (2019), and Acharya and Dogra (2020). McKay-Nakamura-Steinsson (2016) (MNS) show that forward guidance has substantially less power to stimulate the economy in the presence of uninsurable risk and borrowing constraints. It is not realistic to assume that agents increase their consumption by the same amount in response to a future interest rate cut as they do in response to a current rate cut, for two reasons: borrowing constraints and precautionary savings. People face a risk of hitting a borrowing limit, which shortens their planning horizon, because interest rate changes that occur after they hit a borrowing limit are irrelevant to them. The presence of uninsurable income risks leads to precautionary savings, which temper households' responses to future interest rate shocks. As a result, the power of forward guidance is substantially muted in the incomplete markets model compared to the standard complete-markets New Keynesian model.

Other departures from full information set ups and rational expectations include Chung, Herbst, and Kiley (2014), Carlstrom, Fuerst, and Paustian (2015), and Kiley (2016) which use sticky information models à la Mankiw and Reis (2002); Angeletos and Lian (2018) who relax the assumption of common knowledge; and other works that depart from rational expectations in the form of bounded rationality (Gabaix 2015), finite planning horizons (Woodford 2019), or by modeling the cognitive process of expectation formation and use the concept of reflective equilibrium (Garcia-Schmidt and Woodford 2019).

Given how critically the efficacy of forward guidance depends on the credibility of the central bank's commitment to announcements about future policy, several papers discuss the imperfect credibility of the central bank to resolve the forward guidance puzzle (Bodenstein, Hebden, and Nunes 2012; Haberis, Harrison, and Waldron 2014; Campbell et al. 2019; Bernanke 2020).

My work contributes to the literature on forward guidance by introducing aggregate uncertainty using a global solution method, thereby enabling the study of state-contingent forward guidance in its most evolved form. The use of a global solution method, unprecedented in the literature on forward guidance and in HANK models more generally, allows the stochastic dynamic general equilibrium to capture nonlinearities in households' responses in the presence of aggregate risks and uncertainty. To the best of my knowledge, mine is the first work to solve for a fully stochastic general equilibrium in a HANK model with aggregate uncertainty and to leverage the model to study the effects of state-contingent forward guidance, thereby making a significant contribution to the literature on forward guidance.

My results question the existence of the forward guidance puzzle when aggregate economic risks are accounted for to preserve the economy's stochastic nature in examining the effects of forward guidance, especially when it is state-contingent. Guidance that is contingent on future economic conditions is half as effective as the non-state-contingent guidance; however, even non-state-contingent guidance has effects much smaller than those predicted under perfect foresight in the literature [6]. My model

[6] McKay, Nakamura, and Steinsson (2016)

and methodology offer a framework for studying a variety of monetary policy issues in the presence of aggregate uncertainty, when nonlinearities can play an important role in shaping aggregate macroeconomic dynamics. My results provide insights into how forward guidance communication can be tailored to enhance its efficacy in an uncertain world.

3 Model

Households There is a continuum of a unit mass of ex-ante identical households with preferences given by

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[\frac{c_{h,t}^{1-\sigma}}{1-\sigma} - \chi \frac{n_{h,t}^{1+\psi}}{1+\psi} \right] \tag{1}$$

where $c_{h,t}$ is consumption of household h at time t and $n_{h,t}$ is labor supply of household h at time t. Each household faces an uninsurable idiosyncratic productivity shock $e_{h,t}$ which follows a Markov chain with transition probabilities $\Pr(e_{h,t+1}|e_{h,t})$, and earns real labor income before tax $w_t e_{h,t} n_{h,t}$. By assumption, the initial cross-sectional distribution of idiosyncratic productivities is equal to the ergodic distribution of this Markov chain.

In addition to holding stakes in the firm, households also have access to a risk-free real bond b that yields real interest rate r_t between periods t and $t+1$. Borrowing constraints prevent households from taking negative bond positions. A household's period t budget constraint is given by

$$c_t + q_t \cdot b_{t+1} \leq b_t + (1-\tau_t) \cdot w_t \cdot e_t \cdot n_t + d_t \tag{2}$$

where c_t is real consumption, q_t is the price of the bond between period t and $t+1$, b_{t+1} and b_t are the real risk-free bond holdings of the household in period $t+1$ and period t, respectively, w_t is the real wage rate, and d_t is the real dividend earnings of the household.

Final-good producer A competitive representative final good producer aggregates a continuum of intermediate inputs indexed by $j \in [0,1]$ through a Dixit-Stiglitz constant elasticity aggregator as follows

$$Y_t = \left(\int_0^1 y_{j,t}^{\frac{\varepsilon-1}{\varepsilon}} \, dj \right)^{\frac{\varepsilon}{\varepsilon-1}} \tag{3}$$

where Y_t is the quantity of the final good produced by the final good firm at time t, $y_{j,t}$ is the quantity of the intermediate good produced by the firm j in period t, and ε is the price elasticity of substitution across the varieties of the intermediate good. Profit maximization by the final good producer implies the following demand for a typical intermediate input variety j

$$y_{j,t} = \left(\frac{p_{j,t}}{P_t}\right)^{-\varepsilon} Y_t \tag{4}$$

where $p_{j,t}$ is the price set by the producer of intermediate input variety j, and P_t is the aggregate price index given by

$$P_t = \left(\int_0^1 p_{j,t}^{1-\varepsilon} dj\right)^{\frac{1}{1-\varepsilon}} \tag{5}$$

Intermediate goods producers Each intermediate input variety j is produced by a monopolistically competitive producer according to the production function

$$y_{j,t} = z_t \, n_{j,t} \tag{6}$$

where z_t is the aggregate total factor productivity (TFP) common across all intermediate firms, and $n_{j,t}$ denotes the amount of labor hired by firm j in period t.

Intermediate goods producers hire labor at real wage w_t in a competitive labor market such that wage equals the marginal revenue product of labor.

Each intermediate producer chooses its price to maximize lifetime profits subject to price adjustment costs as in Rotemberg (1982). These adjustment costs are quadratic in the rate of price change $\frac{p_{j,t}}{p_{j,t-1}}$ and are expressed as a fraction of real aggregate output Y_t as

$$\Theta_t = \frac{\theta}{2}\left(\frac{p_{j,t}}{p_{j,t-1}} - \bar{\pi} - 1\right)^2 Y_t \tag{7}$$

where $\theta > 0$ is the cost of the price adjustment.

Profits produced by the intermediate firms are distributed immediately to the households such that each household receives an equal share d_t. Households cannot trade their stakes in the firms.

Aggregate uncertainty The source of aggregate uncertainty is a shock to the exogenous aggregate state of the economy, i.e., the total factor productivity (z_t) in the intermediate firms' production functions.The exogenous shock to TFP, φ, can be decomposed into two components, the exogenous idiosyncratic shock to TFP (η), and, the risk of a disaster shock (Δ).

Idiosyncratic shock to total factor productivity The idiosyncratic shock to TFP is given by the following AR(1) process:

$$log\ z_{t+1} = \rho_z\ log\ z_t + \eta_t \tag{8}$$

where $\eta \sim N(0, \sigma_z^2)$ which can be discretized using the Rouwenhorst method so that $z \in \{z_1, z_2, z_3\}$ and follows a Markov chain with transition probabilities $\Pr(z_{t+1} = z_j | z_t = z_i) = \pi_{ij}^z$ giving the probability of transition from the state z_i in period t to the state z_j in period $t + 1$. The transition probability matrix for the idiosyncratic shock to TFP is given by

$$\Pi^z = \begin{array}{c c} & \begin{array}{ccc} z = z_1 & z = z_2 & z = z_3 \end{array} \\ \begin{array}{c} z = z_1 \\ z = z_2 \\ z = z_3 \end{array} & \begin{pmatrix} \pi_{11}^z & \pi_{12}^z & \pi_{13}^z \\ \pi_{21}^z & \pi_{22}^z & \pi_{23}^z \\ \pi_{31}^z & \pi_{32}^z & \pi_{33}^z \end{pmatrix} \end{array}$$

Fiscal authority The government levies a proportional tax on household labor income at the rate τ_t and holds a stock of outstanding government debt with real face value B_t such that positive values of B denote government debt. The government budget constraint is given as

$$q_t \cdot B_{t+1} = B_t + G_t - \tau_t \cdot w_t \int e \cdot n_t \cdot \mu_t\ d\mu_t \tag{9}$$

which can be rewritten as

$$q_t \cdot B_{t+1} = B_t + G_t - T_t \tag{10}$$

where where G_t is government spending in period t, and $T_t = \tau_t \cdot w_t \int e \cdot n_t \cdot \mu_t\ d\mu_t$. The government's budget constraint can be written in terms of the primary surplus $S_t \equiv T_t - G_t$ as

$$q_t \cdot B_{t+1} = B_t - S_t \tag{11}$$

The government adjusts primary surplus as per the following fiscal rule to stabilize debt

$$S_t = \rho_b(B_t - \bar{B}) + \bar{S} \tag{12}$$

where $\rho_b > 0$ is the fiscal responsiveness to debt deviations, and $\bar{S}$ is a target primary surplus given as $\bar{T} - \bar{G}$.

Monetary authority The monetary authority sets nominal interest rates using a Taylor rule given by

$$i_t = r^* + \pi_t + \alpha\left(\pi_t - \bar{\pi}\right) \tag{13}$$

where r^* is the long-run equilibrium real interest rate, and $\bar{\pi}$ is the monetary authority's target inflation rate. where the coefficient α must be greater than 1 for determinacy.

The relationship between the current real interest rate r_t, the expected future inflation rate π_{t+1}, and the current nominal interest rate i_t, is given by the Fisher relation as

$$1 + r_t = \frac{1 + i_t}{\mathbb{E}_t\left[1 + \pi_{t+1}\right]} \tag{14}$$

where π_{t+1} is the net inflation rate given as $\pi_{t+1} \equiv \frac{P_{t+1}}{P_t} - 1$ and P_t is the aggregate price level. The real interest rate and the expected future inflation rate imply a nominal interest rate determined by the Fisher equation, such that the real return on assets r_t must be consistent with equilibrium in the bond market.

4 Decision Problems

4.1 Decision problem of the household

Households face the following decision problem in this economy

$$\max_{\{c_{h,t}, n_{h,t}, b_{h,t+1}\}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[\frac{c_{h,t}^{1-\sigma}}{1-\sigma} - \chi \frac{n_{h,t}^{1+\psi}}{1+\psi}\right] \tag{15}$$

subject to their period budget constraint,

$$c_t + q_t \cdot b_{t+1} \leq b_t + (1 - \tau_t) \cdot w_t \cdot e_t \cdot n_t + d_t \tag{16}$$

the borrowing constraint,

$$b_{t+1} \geq \underline{\mathrm{b}} \tag{17}$$

the non-negativity constraint,

$$c_t \geq 0 \tag{18}$$

Each household chooses consumption and labor optimally by satisfying its Euler equation given by

$$u_c'\left(c_{h,t}, n_{h,t}\right) = \beta \mathbb{E}_t\left[\left(1 + r_{t+1}\right) u_c'\left(c_{h,t+1}, n_{h,t+1}\right)\right] \tag{19}$$

which can be written in terms of the discount bond price as

$$q_t = \beta \mathbb{E}_t \left[\frac{u_c'\left(c_{h,t+1}, n_{h,t+1}\right)}{u_c'\left(c_{h,t}, n_{h,t}\right)} \right] \tag{20}$$

and its labor-leisure condition, given by

$$\chi \cdot n_{h,t}^{1/\eta} = c_{h,t}^{-\sigma} \cdot w \tag{21}$$

which, together with the household's budget constraint, yield the optimal choices of consumption and leisure for each household.

4.2 Decision problem of the final good producer

The final good producer faces the following profit maximization problem

$$\max_{y_{j,t}} \left\{ P_t \left(\int_0^1 y_{j,t}^{\frac{\varepsilon-1}{\varepsilon}} \; dj \right)^{\frac{\varepsilon}{\varepsilon-1}} - \int_0^1 p_{j,t} y_{j,t} \; dj \right\} \tag{22}$$

which yields the following demand for each intermediate good variety j

$$y_{j,t} = \left(\frac{p_{j,t}}{P_t} \right)^{-\varepsilon} Y_t \tag{23}$$

where $p_{j,t}$ is the price charged by firm producing intermediate input variety j in period t and P_t is the aggregate price index in period t given by

$$P_t = \left(\int_0^1 p_{j,t}^{1-\varepsilon} dj \right)^{\frac{1}{1-\varepsilon}} \tag{24}$$

4.3 Decision problem of intermediate goods producers

Intermediate goods producers face a common real wage w_t. Due to nominal rigidities, intermediate firms are unable to adjust prices each period to maximize their profits. Therefore, they act to minimize cost. The cost minimization problem faced by intermediate firms, subject to producing enough to meet demand, is given as

$$\min_{n_{j,t}} \{ w_t n_{j,t} \} \tag{25}$$

subject to

$$z_t n_{j,t} \geq y_{j,t} \tag{26}$$

and

$$y_{j,t} = \left(\frac{p_{j,t}}{P_t}\right)^{-\varepsilon} Y_t \tag{27}$$

Cost minimization by intermediate goods producers implies that marginal cost is identical across intermediate goods firms and is given by

$$mc_t = \frac{w_t}{z_t} \tag{28}$$

Each intermediate firm seeks to maximize its expected present discounted value of future profits for its shareholders. In doing so, each intermediate producer solves the following maximization problem

$$\max_{p_{j,t}} \mathbb{E}_t \left[\sum_{s=0}^{\infty} \mathcal{Q}_{t\to t+s} \frac{P_t}{P_{t+s}} \tilde{V}_{j,t+s}\right] \tag{29}$$

subject to

$$y_{j,t} = \left(\frac{p_{j,t}}{P_t}\right)^{-\varepsilon} Y_t \tag{30}$$

where $\mathcal{Q}_{t\to t+s}$ is the s-period ahead real stochastic discount factor and $\mathcal{Q}_{t\to t+s}\frac{P_t}{P_{t+s}}$ is the s-period ahead nominal stochastic discount factor. The firm discounts future profits by the inverse of the real interest rate in this economy. Hence, we can substitute for $\mathcal{Q}_{t\to t+s} = \frac{1}{1+r_t} = q_t$, which is the price of the discount bond in the economy. Further, $\tilde{V}_{j,t+s}$ is the s-period ahead nominal profit for an intermediate producer of a typical input variety j, that is, nominal profit at time $t+s$, with price $p_{j,t}$ chosen at time t, and is given by its per-period nominal flow profit in period $t+s$ net of nominal price adjustment cost in period $t+s$ as

$$\tilde{V}_{j,t+s} = p_{j,t+s} y_{j,t+s} - mc_{t+s} y_{j,t+s} P_t + s - \frac{\theta}{2}\left(\frac{p_{j,t+s}}{p_{j,t+s-1}} - \bar{\pi} - 1\right)^2 Y_{t+s} P_{t+s} \tag{31}$$

Substituting for $\tilde{V}_{j,t+s}$ from equation (31) and $y_{j,t}$ from the constraint in equation (30) in equation (29), the problem of an intermediate producer of a typical input variety j can be rewritten as

$$\begin{aligned}\max_{p_{j,t}} \mathbb{E}_t \Bigg[\sum_{s=0}^{\infty} q_t \frac{P_t}{P_{t+s}} \Bigg(& p_{j,t+s}\left(\frac{p_{j,t+s}}{P_{t+s}}\right)^{-\varepsilon} Y_{t+s} - mc_{t+s}\left(\frac{p_{j,t+s}}{P_{t+s}}\right)^{-\varepsilon} Y_{t+s} P_{t+s} \\ & - \frac{\theta}{2}\left(\frac{p_{j,t+s}}{p_{j,t+s-1}} - \bar{\pi} - 1\right)^2 Y_{t+s} P_{t+s}\Bigg)\Bigg]\end{aligned} \tag{32}$$

The solution to this problem satisfies

$$
\begin{aligned}
\varepsilon \frac{p_{j,t}}{P_t}\left(\frac{p_{j,t}}{P_t}\right)^{-\varepsilon-1} Y_t = \left(\frac{p_{j,t}}{P_t}\right)^{-\varepsilon} Y_t + mc_t \varepsilon Y_t \left(\frac{p_{j,t}}{P_t}\right)^{-\varepsilon-1} - \theta\left(\frac{p_{j,t}}{p_{j,t-1}} - \bar{\pi} - 1\right)\frac{P_t}{p_{j,t-1}} Y_t \\
+ \theta \mathbb{E}_t\left[\frac{q_t}{P_{t+1}/P_t}\left(\frac{p_{j,t+1}}{p_{j,t}} - \bar{\pi} - 1\right)\left(\frac{p_{j,t+1}}{p_{j,t}}\right)^2 Y_{t+1}\right]
\end{aligned}
\tag{33}
$$

In equilibrium, all intermediate input producers behave identically: each of them faces the same marginal cost; therefore, each intermediate goods producer sets the same price and produces the same level of intermediate input variety j. Imposing symmetry in equilibrium, i.e. $p_{j,t} = P_t$, the intermediate goods producers' optimality condition for price setting in equation (33) can be written in terms of inflation rates by defining $\frac{P_{t+1}}{P_t} = \pi_{t+1} + 1$ as

$$
\frac{\theta}{\varepsilon - 1}\left(\pi_t - \bar{\pi}\right)\pi_t = \frac{\varepsilon}{\varepsilon - 1} mc_t - 1 + \theta \mathbb{E}_t\left[q_t\left(\pi_{t+1} - \bar{\pi}\right)\pi_{t+1}\frac{Y_{t+1}}{Y_t}\right]
\tag{34}
$$

or,

$$
\left(\pi_t - \bar{\pi}\right)\pi_t = \frac{1-\varepsilon}{\theta} + \frac{w_t}{z_t}\frac{\varepsilon}{\theta} + \mathbb{E}_t\left[q_t\left(\pi_{t+1} - \bar{\pi}\right)\pi_{t+1}\frac{Y_{t+1}}{Y_t}\right]
\tag{35}
$$

This is the New Keynesian Phillips Curve.

Aggregate production Integrating demand for intermediate good variety j across firms in equation 4, and using production function of intermediate goods producers in equation 6, we have

$$
\begin{aligned}
\int_0^1 y_{j,t}\ dj = \int_0^1 \left(\frac{p_{j,t}}{P_t}\right)^{-\varepsilon} Y_t\ dj \\
\int_0^1 z_t n_{j,t}\ dj = Y_t \int_0^1 \left(\frac{p_{j,t}}{P_t}\right)^{-\varepsilon}\ dj
\end{aligned}
\tag{36}
$$

Since all firms face identical demand for their variety of the intermediate good and identical adjustment costs, they all adjust to the same reset price each period, which then becomes the aggregate price level for the economy. Hence, setting $p_{j,t} = p_t = P_t$, we have

$$
\begin{aligned}
z_t \ N_t &= Y_t \int_0^1 \left(\frac{P_t}{P_t}\right)^{-\varepsilon} \ dj \\
z_t \ N_t &= Y_t \\
\text{or, } Y_t &= z_t \ N_t
\end{aligned}
\tag{37}
$$

5 Recursive formulation of the economy

In this section, I will redefine the economy described above in a recursive formulation, which is essential for a state-contingent exposition of monetary policy in this environment. Let φ denote the aggregate exogenous state of the economy, representing the effective TFP. The endogenous aggregate state of the economy is the distribution of households across the state space, denoted by μ. The households have an endogenous individual state, given by their bond-holding choice, b, and an exogenous individual state, the idiosyncratic shock e.

Household's decision problem The decision problem facing households is as follows.

$$
\begin{aligned}
V\left(e_i, b; \varphi_i, \mu\right) = \max_{\{c, b_i, n\}} \Bigg\{ & \frac{c^{1-\sigma}}{1-\sigma} - \chi \frac{n^{1+\psi}}{1+\psi} \\
& + \beta \sum_{\varphi_j'} Pr(\varphi_j' | \varphi_i) \sum_{e_j'} Pr(e_j' | e_i) \ V'\left(e_j', b'; \varphi_j', \mu'\right) \Bigg\}
\end{aligned}
\tag{38}
$$

subject to their budget constraint,

$$
c + q\left(\varphi_i, \mu\right) \cdot b' \leq b + (1-\tau) \, w\left(\varphi_i, \mu\right) \cdot e_i \cdot n_i + d\left(\varphi_i, \mu\right) \tag{39}
$$

the borrowing constraint,

$$
b' \geq \underline{\mathrm{b}} \tag{40}
$$

the non-negativity constraint,

$$
c \geq 0 \tag{41}
$$

and the law of motion for the distribution of households,

$$
\mu' = \Lambda\left(\mu, \varphi_i\right). \tag{42}
$$

The solution to this problem yields the decision rule for households' consumption,

$c(e_i, b; \varphi_i, \mu)$, the decision rule for households' bond holdings, b, given by $g(e_i, b; \varphi_i, \mu)$, and the decision rule for the households' labor supply choice, $n(e_i, b; \varphi_i, \mu)$.

Each household's optimal choices of consumption and labor satisfy their Euler equation given by

$$u'_c(c, n) = \beta \mathbb{E}\left[\left(1 + r'\right) u'_c\left(c', n'\right)\right] \tag{43}$$

which can be written in terms of the discount bond price as

$$q = \beta \mathbb{E}\left[\frac{u'_c(c', n')}{u'_c(c, n)}\right] \tag{44}$$

and its labor-leisure condition, given by

$$\chi \cdot n^{1/\eta} = c^{-\sigma} \cdot w(\varphi_i, \mu). \tag{45}$$

Together with the household's budget constraint, these conditions pin down each household's optimal choices of consumption and leisure.

Decision problem of intermediate goods producers The individual state of an intermediate input producer is given by the relative price of the last period's price set by the producer to the aggregate price level last period, $\frac{p_{j,-1}(\varphi_i,\mu)}{P_{-1}(\varphi_i,\mu)}$ which is written with the state-dependence suppressed for clarity in the equations below as $\frac{p_{j,-1}}{P_{-1}}$. An intermediate firm's problem in real terms can then be defined in recursive form as

$$V_f\left(\frac{p_{j,-1}}{P_{-1}}; \varphi_i, \mu\right) = \max_{p_j}\left\{\frac{p_j y_j}{P} - w(\varphi_i, \mu) n_j - \frac{\theta}{2}\left(\frac{p_j/P}{p_{j,-1}/P_{-1}} \cdot \frac{P}{P_{-1}} - 1\right)^2 Y(\varphi_i, \mu) \right. \tag{46}$$
$$\left. + \beta \sum_{\varphi'_j} q(\varphi_i, \mu) \; Pr\left(\varphi'_j | \varphi_i\right) \; V_f\left(\frac{p_j}{P}; \varphi'_j, \mu'\right)\right\}$$

or,

$$V_f\left(\frac{p_{j,-1}}{P_{-1}}; \varphi_i, \mu\right) = \max_{p_j}\left\{\frac{p_j y_j}{P} - w(\varphi_i, \mu) n_j - \frac{\theta}{2}\left(\frac{p_j/P}{p_{j,-1}/P_{-1}} \cdot (1 + \pi(\varphi_i, \mu)) - (\bar{\pi} + 1)\right)^2 Y(\varphi_i, \mu) \right. \tag{47}$$
$$\left. + \beta \sum_{\varphi'_j} q(\varphi_i, \mu) \; Pr\left(\varphi'_j | \varphi_i\right) \; V_f\left(\frac{p_j}{P}; \varphi'_j, \mu'\right)\right\}$$

subject to

$$y_j = \left(\frac{p_j}{P}\right)^{-\varepsilon} Y(\varphi_i, \mu) \tag{48}$$

$$y_j = z \cdot n_j \tag{49}$$

where p_j is the price chosen by the firm producing input variety j when resetting its price in the current period, θ is the cost of price adjustment, and ε is the price elasticity of substitution across varieties of the intermediate goods.

To simplify the intermediate firms' problem in recursive form, I define the price of intermediate goods in terms of a relative price. Let $p \equiv p(\varphi_i, \mu) \equiv p_j(\varphi_i, \mu)/P(\varphi_i, \mu)$ be the relative price for an intermediate firm producing variety j, where $p_j(\varphi_i, \mu)$ is the price of variety j. Let $p_0 \equiv p_0(\varphi_i, \mu) \equiv p^*(\varphi_i, \mu)/P(\varphi_i, \mu)$ be the ratio of the current optimal reset price, $p^*(\varphi_i, \mu)$, set by intermediate firms to the current aggregate price level $P(\varphi_i, \mu)$; in equilibrium, $p_0(\varphi_i, \mu)$ will be 1—as all intermediate firms are identical, all firms face the same demand for their respective varieties and reset to the same optimal price, which becomes the aggregate price level. The net inflation rate in the current period is defined as $\pi \equiv \frac{P - P_{-1}}{P_{-1}}$ where P_{-1} is the aggregate price level in the previous period.

Substituting the constraints into the problem yields the following value of an intermediate firm

$$V_f(p_{-1}; \varphi_i, \mu) = \max_p \Bigg\{ p^{1-\varepsilon} Y - \frac{w(\varphi_i, \mu)}{z} p^{-\varepsilon} Y - \frac{\theta}{2}\left(\frac{p}{p_{-1}} \cdot (1 + \pi(\varphi_i, \mu)) - (\bar{\pi} + 1)\right)^2 Y(\varphi_i, \mu) P \tag{50}$$

$$+ \beta \sum_{\varphi'_j} q(\varphi_i, \mu) \;\; Pr\left(\varphi'_j | \varphi_i\right) \;\; V_f\left(p; \varphi'_j, \mu'\right) \Bigg\}$$

The solution to the intermediate firm's problem yields the New Keynesian Phillips Curve (NKPC) given in equation 35, which can be rewritten in recursive form as

$$\Big(\pi(\varphi_i, \mu) - \bar{\pi}\Big) \cdot \pi(\varphi_i, \mu) = \frac{1-\varepsilon}{\theta} + \frac{w(\varphi_i, \mu)}{z} \frac{\varepsilon}{\theta} \tag{51}$$

$$+ \mathbb{E}\left[q(\varphi_i, \mu) \Big(\pi'\left(\varphi'_j, \mu'\right) - \bar{\pi}\Big) \cdot \pi'\left(\varphi'_j, \mu'\right) \cdot \frac{Y'\Big(\varphi'_j, \mu'\Big)}{Y(\varphi_i, \mu)} \right]$$

Evolution of inflation The New Keynesian Phillips curve in equation 51 above summarizes the firms' optimal responses to the decision problems they face in this economy and is the equation for the evolution of inflation in this economy.

Aggregate production Aggregate production in this economy, given by equation 36, can be rewritten as

$$Y(\varphi_i, \mu) = z \cdot N(\varphi_i, \mu) \tag{52}$$

Fiscal authority The government's budget constraint in equation 9 is given as

$$\begin{aligned} q(\varphi_i, \mu) \cdot B' = \; & B \; + \; G(\varphi_i, \mu) \\ & - \; \tau w(\varphi_i, \mu) \int e_i \cdot n(e_i, b; \varphi_i, \mu) \cdot d\mu(e_i, b; \varphi_i, \mu) \end{aligned} \tag{53}$$

which can be rewritten as

$$q(\varphi_i, \mu) \cdot B'(\varphi_i, \mu) = B(\varphi_i, \mu) + G(\varphi_i, \mu) - T(\varphi_i, \mu) \tag{54}$$

where $T = \tau w(\varphi_i, \mu) N(\varphi_i, \mu)$ is tax revenues, $N = \int e_i \cdot n(e_i, b; \varphi_i, \mu) \cdot d\mu(e_i, b; \varphi_i, \mu)$ is effective aggregate labor supply, $G(\varphi_i, \mu)$ is an exogenous government spending function. Further, aggregate bonds are the sum of bonds held across individuals.

$$B(\varphi_i, \mu) = \int b(e_i, b; \varphi_i, \mu) \, d\mu(e_i, b; \varphi_i, \mu) \tag{55}$$

where $\mu(e_i, b; \varphi_i, \mu)$ represents the distribution of households across the state space.

Defining the primary surplus as $S(\varphi_i, \mu) \equiv T(\varphi_i, \mu) - G(\varphi_i, \mu)$, we can write the government's budget constraint as:

$$q(\varphi_i, \mu) \cdot B'(\varphi_i, \mu) = B(\varphi_i, \mu) - S(\varphi_i, \mu) \tag{56}$$

To stabilize debt, the fiscal authority adjusts the primary balance according to a simple feedback rule (centered around the target):

$$S(\varphi_i, \mu) = \rho_b \left(B(\varphi_i, \mu) - \bar{B} \right) + \bar{S} \tag{57}$$

where $\rho_b > 0$ is fiscal responsiveness to debt deviations, $\bar{B}$ is target debt, $\bar{S}$ is target primary surplus given as $\bar{T} - \bar{G}$. At the target, we have,

$$q\bar{B} = \bar{B} - \bar{S} \tag{58}$$

which, together with the period budget constraint of the government expressed in primary surplus terms, yields the law of motion for debt deviations

$$B'(\varphi_i, \mu) - \bar{B} = \frac{1 - \rho_b}{q} \left(B(\varphi_i, \mu) - \bar{B} \right) \tag{59}$$

and the evolution of government spending as

$$G\left(\varphi_i,\mu\right) = T\left(\varphi_i,\mu\right) - \left[\bar{S} + \rho_b(B\left(\varphi_i,\mu\right) - \bar{B})\right]. \tag{60}$$

which can be rewritten as

$$G\left(\varphi_i,\mu\right) = T\left(\varphi_i,\mu\right) - \left[\bar{T} - \bar{G} + \rho_b(B\left(\varphi_i,\mu\right) - \bar{B})\right]. \tag{61}$$

Monetary authority The monetary authority sets nominal interest rates following the Taylor rule

$$i\left(\varphi_i,\mu\right) = r^* + \pi\left(\varphi_i,\mu\right) + \alpha\Bigg(\pi\left(\varphi_i,\mu\right) - \bar{\pi}\Bigg). \tag{62}$$

where $i\left(\varphi_i,\mu\right)$ is nominal interest rate function, r^* is the target real interest rate, $\pi\left(\varphi_i,\mu\right)$ is the inflation rate function, $\bar{\pi}$ is the inflation target, and α: policy response coefficient on inflation gap.

The relationship between nominal interest rates, real interest rates, and expected future inflation given by the Fisher equation in equation 14 can be written in recursive form as

$$1 + r\left(\varphi_i,\mu\right) = \frac{1 + i\left(\varphi_i,\mu\right)}{\mathbb{E}\left[1 + \pi\left(\varphi_j',\mu'\right)\right]}. \tag{63}$$

6 Equilibrium

The distribution of households over individual and aggregate endogenous and exogenous states at date t is given by $\mu_t\left(e_i, b\right)$, and it evolves according to

$$\mu_{t+1}\left(e_j', \mathcal{B}\right) = \int_{\{(e,b):g_t(e,b;\varphi_i,\mu)\in\mathcal{B}\}} Pr\left(e_j'|e_i\right) d\mu_t\left(e, b_i\right) \text{ given } \varphi_i, \varphi_j' \tag{64}$$

for all sets $\mathcal{B} \subset \mathbb{R}$.

In equilibrium, individual labor supply decisions are consistent with the aggregate, such that aggregate labor supply is given by

$$L \equiv \int e \cdot n_t\left(e_i, b; \varphi_i, \mu\right)\ d\mu\left(e_i, b; \varphi_i, \mu\right) \tag{65}$$

and labor market clearing requires labor supply and labor demand to be equal,

$$L = N \tag{66}$$

Aggregate bond holding is given by,

$$\bar{B} = \int g_t\left(e_i, b; \varphi_i, \mu\right)\ d\mu\left(e_i, b; \varphi_i, \mu\right) \tag{67}$$

These are held by the government as debt, such that bond market clearing requires

$$\bar{B} = B \tag{68}$$

where B is aggregate public debt.

The aggregate dividend paid by the intermediate goods firms in real terms is given by

$$d\left(\varphi_i, \mu\right) = Y\left(\varphi_i, \mu\right) - w\left(\varphi_i, \mu\right) \cdot N\left(\varphi_i, \mu\right) - \frac{\theta}{2}\left(\pi\left(\varphi_i, \mu\right) - \bar{\pi}\right)^2 \cdot Y\left(\varphi_i, \mu\right) \tag{69}$$

Finally, the goods market clears when the aggregate resource constraint holds as

$$Y\left(\varphi_i, \mu\right) = C\left(\varphi_i, \mu\right) + G\left(\varphi_i, \mu\right) - \frac{\theta}{2}\left(\pi\left(\varphi_i, \mu\right) - \bar{\pi}\right)^2 \cdot Y\left(\varphi_i, \mu\right) \tag{70}$$

where $C \equiv \int c\left(e_i, b; \varphi_i, \mu\right)\ d\mu\left(e_i, b; \varphi_i, \mu\right)$ denotes aggregate consumption from consistency between individual consumption decisions and aggregate consumption in equilibrium.

Definition of a recursive competitive equilibrium A recursive competitive equilibrium in this economy consists of decision rules and value functions: $\left\{c_t\left(e_i, b; \varphi_i, \mu\right), g_t\left(e_i, b; \varphi_i, \mu\right), n_t\left(e_i, b; \varphi_i, \mu\right), V_t\left(e_i, b; \varphi_i, \mu\right), V_{f_t}\left(\frac{p_{j,-1}}{P_{-1}}; \varphi_i, \mu\right)\right\}_{t=0}^{\infty}$ that solve the problems of households and firms, respectively, and distributions of households $\{\mu_t\left(e_i, b\right)\}_{t=0}^{\infty}$ that evolve according to the law of motion in equation 64 for given φ_i, φ_j', and individual decisions are consistent with the aggregates as in equations 67 and 68 for the bond market, equations 65 and 66 for the labor market, and equation 70 for the goods market, which must also hold as the aggregate resource constraint for the economy. In addition, the government's budget constraint in equation 9, the Fisher relation in equation 63, the optimal price setting equation for the intermediate firms, which yields the New Keynesian Phillips Curve in equation 51, which shows the evolution of inflation, the equation for aggregate dividends in equation 69, and the aggregate resource constraint in equation 70, along with the monetary policy rule in equation 62 must be satisfied.

7 Parameters

A period in this model economy is a quarter of a calendar year. The subjective discount factor *beta* is set at 0.985. The coefficient of relative risk aversion is set to 2. The inverse of the Frisch elasticity of labor supply is 2, based on Chetty (2012) and McKay, Nakamura, and Steinsson (2016). The preference parameter for households' labor supply is set to 100 to achieve a labor supply of a third.

The desired mark-up for intermediate firms, defined by $\mu \equiv \frac{\varepsilon}{\varepsilon - 1}$, is set to 1.2 following Christiano, Eichenbaum, and Rebello (2011) and McKay, Nakamura, and Steinsson (2016). The target inflation rate is set to 2 percent per annum. The cost of the price adjustment, θ, is set to 250. In Rotemberg pricing, firms face quadratic costs when adjusting prices, and the parameter θ controls the magnitude of these costs. When aggregate risks are present, shocks can be large and volatile, leading to potentially explosive price dynamics if the adjustment cost is too low. A high θ acts as a damping mechanism, preventing prices from changing too rapidly and ensuring that the model can be solved numerically without encountering instability or explosive paths. In HANK models, where household heterogeneity and incomplete markets amplify the effects of aggregate shocks, the need for a high adjustment cost parameter is even more pronounced to maintain equilibrium and avoid computational breakdowns. With the value of θ in the baseline calibration, the model can better handle large shocks and aggregate uncertainty, as the adjustment cost penalizes rapid price changes and stabilizes the system. This is particularly important for models with forward-looking agents and heterogeneous households, where the propagation of shocks can be highly nonlinear and sensitive to parameter values. In an alternate parameterization, I compute the model with θ set to 150.

The idiosyncratic income shocks for a quarterly AR(1) process have a persistence parameter of 0.966 and a standard deviation of innovation of 0.1, following the quarterly estimates used by McKay, Nakamura, and Steinsson (2016). The AR(1) shock process is discretized to a three-point Markov chain using the Rouwenhorst (1995) method.

The shock to Total Factor Productivity is also a quarterly AR(1) process, with a persistence parameter of 0.89, and the standard deviation of innovation being 0.015. The AR(1) shock process for total factor productivity is also discretized to a three-point Markov chain, again using the Rouwenhorst (1995) method.

The coefficient on debt deviations, ρ_b, in the fiscal rule is set to 0.08 in the baseline model; in an alternate calibration, it is set to 0.12. For stability, this value must be greater than the real interest rate so that primary surplus rises faster than interest costs on outstanding debt.

Parameter	Description	Value
β	Subjective discount factor	0.985
η	Inverse of Frisch elasticity	2
χ	Coefficient on labor	100
σ	Coefficient of relative risk aversion	2
τ	Proportional tax rate	0.30
μ	Markup	1.2
θ	Price adjustment cost parameter	250
ρ_b	Coefficient on debt deviation (fiscal rule)	0.08
$\bar{\pi}$	Inflation target	1.005
α	Coefficient on inflation gap	1.5
r^*	Intercept in Taylor rule	1.005
ρ_z	Persistence of aggregate shock	0.89
σ_ϵ	Std of innov in aggregate shock	0.015
ρ_e	Persistence of idiosyncratic shock	0.966
σ_e	Std of innov in idiosyncratic shock	0.1

Table 1: Parameter values in the baseline model

The coefficient on the inflation gap in the Taylor rule is chosen to be 1.5. For an active monetary policy to be in place, nominal rates must rise more than one-for-one with inflation, thereby raising real rates and stabilizing inflation. This condition, $\alpha > 1$, known as the Taylor principle, is essential for a determinate equilibrium. If it equals 1, real rates will not change with inflation, causing inflation to drift and leading to indeterminacy.

8 Computation

I compute a steady state for this economy in which households face idiosyncratic income shocks but none to TFP. Since household savings are held by the government as public debt, I bisect the government's budget constraint to determine an equilibrium bond price for this economy. I solve the households' problem using value function iteration and determine their saving and consumption decision rules using the endogenous grid method.

In computing the stochastic equilibrium, I use the Backward Induction method as described by Reiter (2010). The stochastic economy has an exogenous aggregate state—TFP—and an endogenous aggregate state—the distribution of households in the economy. Since household distributions are infinite-dimensional, I approximate the economy's aggregate endogenous state by the aggregate bond holdings, m. To approximate this, I use an initial proxy distribution and discretize the approximate endogenous state space into levels of bond holdings, so that the distribution yields a mean bond holding level equal to the value at that grid point. In other words, the discretized grid for the levels of aggregate bond holdings is centered on the target mean, given the proxy distribution of households. The proxy distributions are computed at each point in the discretized aggregate state space, by adjusting the steady-state distribution in a way such that the resulting distribution yields a mean level of bond holdings equal to the grid value of the approximate aggregate state, which here is the level of aggregate bond holdings in economy m.

Utilizing these proxy distributions, I solve the real economy in the model in one run of backward iterations, on a discrete grid of points in the aggregate state space, while enforcing consistency between individual and aggregate solutions in each step of the backward iteration, which is done separately at each point on the grid of aggregate states (Reiter 2010).

Households' decision rules are solved for using the endogenous grid method on grids of individual endogenous and exogenous states, namely households' bond holdings and idiosyncratic income shocks, respectively. Consistency is achieved between individual and aggregate decisions when the expected level of aggregate bond holdings in the economy becomes equal to that obtained from its law of motion. Once such consistency is achieved, I iterate on wages to clear the labor market, using an initial guess for the bond price q function and an initial guess for the inflation function over the aggregate state space. The partial equilibrium wage function obtained by iterating over the real economy until labor demand equals labor supply clears both the labor and the goods markets for the initially guessed rate of inflation. The asset market is cleared by Walras' law. Using these partial equilibrium wages, and the output obtained from the aggregate resource constraint, for the initial inflation rate, I use the New Keynesian Phillips Curve to update current inflation, over expectations of future inflation and output. The general equilibrium is obtained once inflation, wages, the law of motion for aggregate bond holdings, and households' value functions have all converged up to a certain tolerance set for each of these. This method yields the stochastic equilibrium in this economy.

9 Results

9.1 Steady-State

The deterministic steady state, obtained by bisecting the government's budget constraint, yields an equilibrium bond price q of 0.9952, implying a quarterly risk-free real interest rate of 0.50 percent and an annualized risk-free real interest rate of

Variable	Steady-state values
Real interest rate (r^*)	1.93 percent
Bond price (q^*)	0.9952
Debt (B^*)	0.52
Output (Y^*)	0.3324
Consumption (C^*)	0.2518
Labor (efficiency units) (L^*, N^*)	0.3324
Raw labor hours (H^*)	0.3121
Wage (w^*)	0.833
Government spending (G^*)	0.081
Dividend (D^*)	0.0554
Inflation (π^*)	1.005
Primary deficit $(G - \tau w L)/Y$	-0.75 percent
Debt-to-GDP (B/Y)	157.47 percent

Table 2: Steady-state values

Percentiles of wealth	1	5	10	50	90	< 0
Share of households	0.057	0.249	0.443	1.000	1.001	0.467

Table 3: Wealth distribution of households in steady-state

1.93 percent per annum. In steady-state, the primary deficit-to-GDP ratio, given by $(G - \tau \cdot w \cdot e \cdot n)/Y$, is around -0.75 percent, whereas the deficit-to-GDP ratio, given by $(G - (1-q) \cdot B + G - \tau \cdot w \cdot e \cdot n)/Y$, is 0 percent in the model. This was around 6.4 percent in 2024 for the US economy, and the 50-year long-run average has been close to 3.6 percent. The steady-state yields a debt-to-GDP ratio of about 157.47 percent, which is around 123 percent for the US economy in 2024. Government spending in the model economy in the steady state is about 24.3 percent of GDP, whereas federal government spending was 23 percent of total gross domestic product (GDP) in the US economy in FY 2024.

The distributions of wealth in the steady state are given in Table 3. The share of negative assets, that is, households with zero or negative net worth, in the model economy is around 46 percent, which for the US economy is close to 11 percent. This is to match the share of borrowing-constrained households in McKay, Nakamura, and Steinsson (2016). For the purposes of this paper, my objective is to achieve a plausible distribution of wealth to study the effects of forward guidance in the presence of aggregate uncertainty. Hence, the model is not calibrated to match the wealth distributions observed in the US economy. The need to insure against income shocks is small among the wealthiest households, and hence their consumption responses do not affect, in any significant way, the efficacy of forward guidance on aggregate consumption. The steady-state wealth distribution in the model yields a Gini coefficient of 0.717. The steady state results are summarized in Table 2.

9.2 Stochastic Equilibrium and Forward Guidance

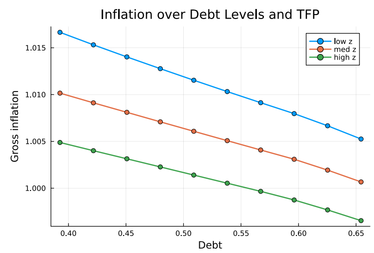
The two key findings of my work are: one, the presence of aggregate uncertainty with incomplete markets makes non-state-contingent forward guidance far less than in the absence of aggregate risks; and two, the effects of state-contingent forward guidance are nearly half the size of the effects of non-state-contingent forward guidance, highlighting the role of uncertainty in determining the efficacy of forward guidance.

The stochastic general equilibrium of my model serves as the basis for all forward guidance experiments conducted with this model. Figure 1 shows the aggregate functions plotted in the aggregate state space. In the forward guidance experiment, the stochastic dynamic general equilibrium serves as the basis for the responses of aggregate functions to the policy announcements.

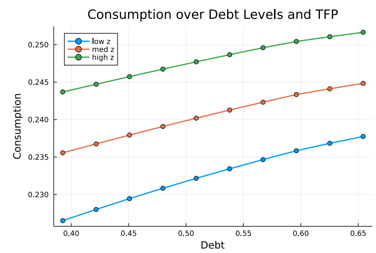
9.2.1 Non-state-contingent forward guidance with aggregate uncertainty

In the baseline experiment, the central bank announces a 50-basis-point (bps) rate cut 20 periods ahead, irrespective of the future state of the economy. This causes the current output to increase by 5 bps and the current consumption to increase by 7 bps. These effects are much smaller than those seen in McKay, Nakamura, and Steinsson (2016), where markets are incomplete, but agents have perfect foresight. While there are differences in modeling choices between their model and my framework, the presence of aggregate economic uncertainty and the internalization of the stochastic nature of the economy by households are the key differences in the environments in which the two economies operate. They show a 10 bps increase in consumption (and output, since output is consumption in their economy) and a 30 basis point increase in inflation on impact. My results show that even when forward guidance is non-state-contingent, the effects on contemporaneous variables are muted in the presence of aggregate shocks.

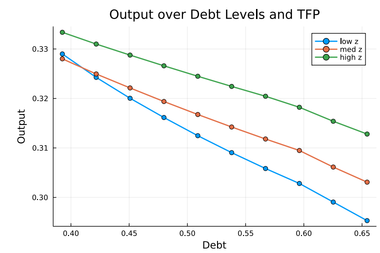
In order to isolate the effects of the forward guidance experiment, while agents expect there to be aggregate economic shocks, these shocks are not actually realized in the duration of the forward guidance experiment. I find that the power of forward guidance is significantly diminished when aggregate risks and uncertainty are present in the economy, even when shocks are not realized. This further underscores the salience of nonlinearities in household behavior when households internalize the



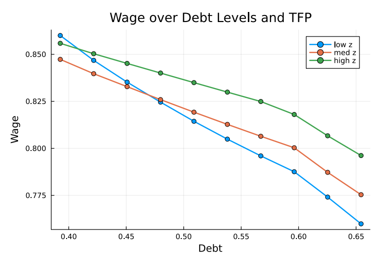
(a) Inflation



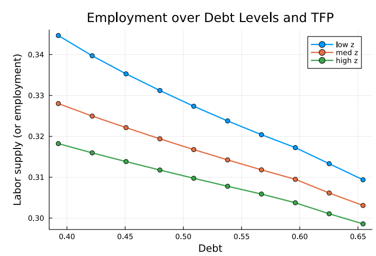
(b) Consumption



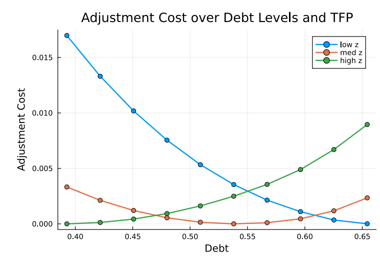
(c) Output



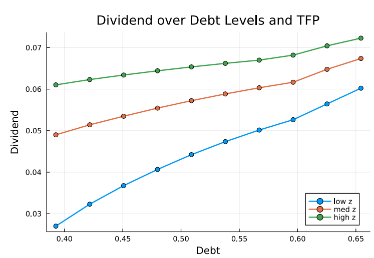
(d) Wage



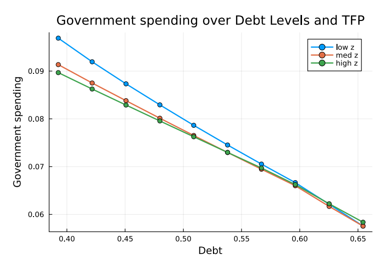
(e) Labor



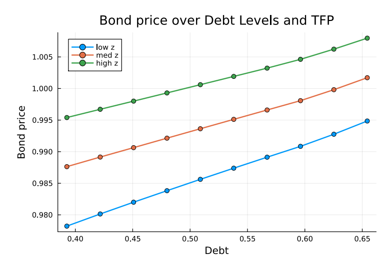
(f) Adj cost



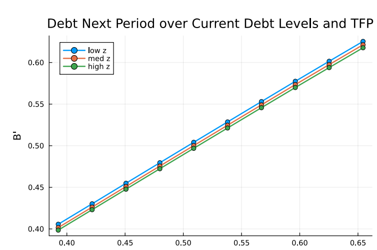
(g) Dividend



(h) Gov't spending



(i) Bond price



(j) Savings

Figure 1: Stochastic equilibrium

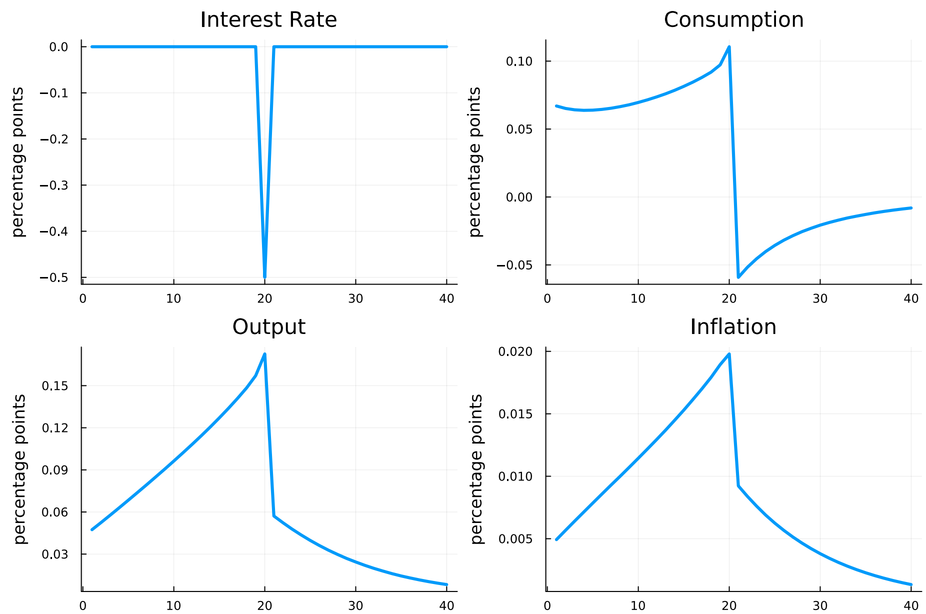


Figure 2: Non-state-contingent forward guidance about a 50 basis points rate cut 20 periods ahead

stochastic nature of the economy, even though they do not actually experience such shocks.

Over time, the effects on output, consumption, and inflation gradually increase until the policy is implemented. Table 4 shows the differences in the effects of non-state-contingent forward guidance on output, consumption, and inflation, at the date of impact and on the date of the policy implementation, in my model compared to the results in McKay et al (2016). Figure 2 shows the results for this baseline experiment.

(In basis points)	**Variable**	**Date of policy announcement**	**Date of policy implementation**
My model with aggregate uncertainty	Output	5	17
	Consumption	7	12
	Inflation	0.5	2
McKay et al (2016)	Output	10	17
	Consumption	10	17
	Inflation	30	-5

Table 4: Effects of a non-state-contingent forward guidance about a 50 basis points rate cut 20 periods ahead

When real interest rates are reduced, the cost of borrowing falls, making borrowing for consumption more attractive. As a result, demand for consumption increases, prompting firms to increase production, which in turn prompts them to demand more labor to meet the increased production. This is the direct intertemporal substitution effect of an interest rate cut. In a general equilibrium, increased labor demand by firms drives up wages, leading to higher labor supply. Hence, households' labor incomes increase, therefore, causing an additional increase in demand for consumption and adding to the direct effects of an interest rate cut. These have been referred to as the "indirect" or the "general equilibrium" effects of a real interest rate cut (Kaplan, Moll, and Violante, 2016).

McKay, Nakamura, and Steinsson (2016) suggested a resolution to the puzzle by having incomplete markets and household heterogeneity. As households face idiosyncratic income shocks, there is greater uncertainty about their future incomes and wealth positions, leading to greater precautionary savings. Additionally, when households are subject to borrowing constraints in the presence of such idiosyncratic income risks, communications about changes in interest rates far in the future dampen households' current consumption responses—those who experience negative income shocks persistently may run the risk of hitting their borrowing limits sooner. To smooth consumption over their lifetimes, they respond very little to announcements about future interest rate changes.

My results show an even smaller response in output, consumption, and inflation when households face exacerbated risks stemming from aggregate economic uncertainty, in addition to the individual risks from idiosyncratic income shocks. This further mutes their response to an announcement about future interest rate cuts.

A part of the forward guidance puzzle was that forward guidance over longer time horizons had larger effects on current output and consumption than guidance over shorter horizons. This is also eliminated in my results—non-state-contingent forward guidance about a 50-bps rate cut 40 periods ahead has smaller effects than the same guidance 20 periods ahead. These results are summarized in Table 5.

(In basis points)	**Variable**	**Date of policy announcement**	**Date of policy implementation**
Short horizon (20-period ahead)	Output	5	17
	Consumption	7	12
	Inflation	0.5	2
Long horizon (40-period ahead)	Output	2.5	17.5
	Consumption	3.5	11
	Inflation	0.25	2.1

Table 5: Effects of a non-state-contingent forward guidance about a 50 basis points rate cut at different horizons

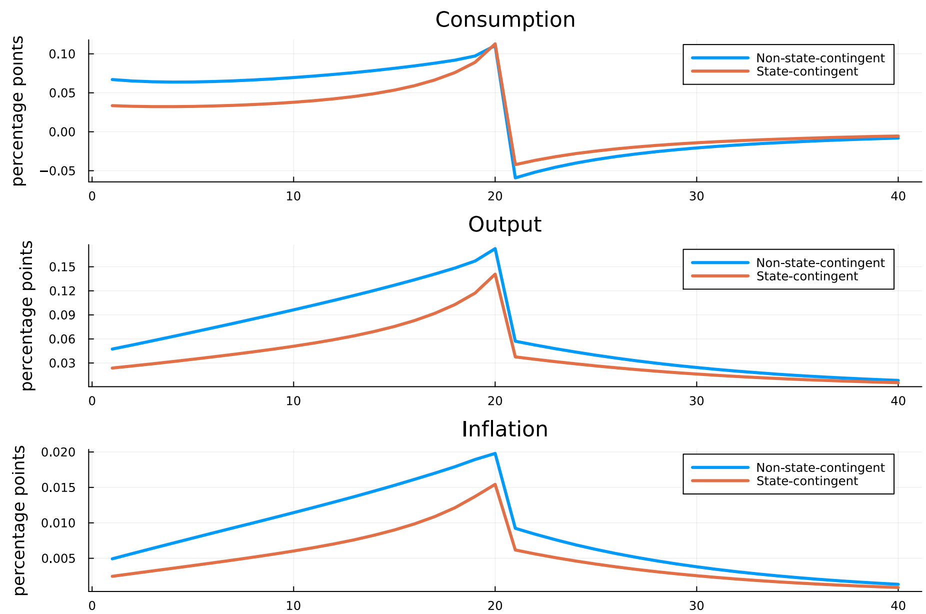


Figure 3: State-contingent versus non-state-contingent forward guidance about a 50 basis points rate cut 20 periods ahead

9.2.2 State-contingent forward guidance

Now I turn to a state-contingent forward guidance experiment in which the central bank announces a 50 basis point rate cut 20 periods ahead, only when a certain state of the economy is realized. Since no prior work in the literature provides a framework for studying state-contingency in forward guidance, these results shed light on questions that prior work could not address. Table 6 compares the results from a state-contingent forward guidance to a non-state-contingent forward guidance over the same horizon and the same magnitude of the rate cut in both cases. Making forward guidance state-contingent reduces its power to affect current output, consumption, and inflation by almost half compared to non-state-contingent guidance, which would take effect irrespective of the state of the economy. These results are shown in Figure 3 and summarized in Table 6.

Aggregate uncertainty amplifies the heterogeneous exposure of households that are already experiencing idiosyncratic income shocks to aggregate risks. This amplifies their precautionary savings motives, causing households to save more, and the transmission of a policy shock to aggregate consumption and output to be weaker. This is an additional source of consumption volatility in this economy. This force becomes particularly strong when forward guidance does not resolve the uncertainty about expected future policy rates, especially when it is contingent on a specific future state of the economy. This, in part, defeats the purpose of forward guidance,

(In basis points)	Variable	Date of policy announcement	Date of policy implementation
Non-state-contingent forward guidance	Output	5	17
	Consumption	7	12
	Inflation	0.5	2
State-contingent forward guidance	Output	2.5	14
	Consumption	3	11
	Inflation	0.5	1.5

Table 6: Effects of state-contingent versus non-state-contingent forward guidance about a 50 bps rate cut 20 periods ahead

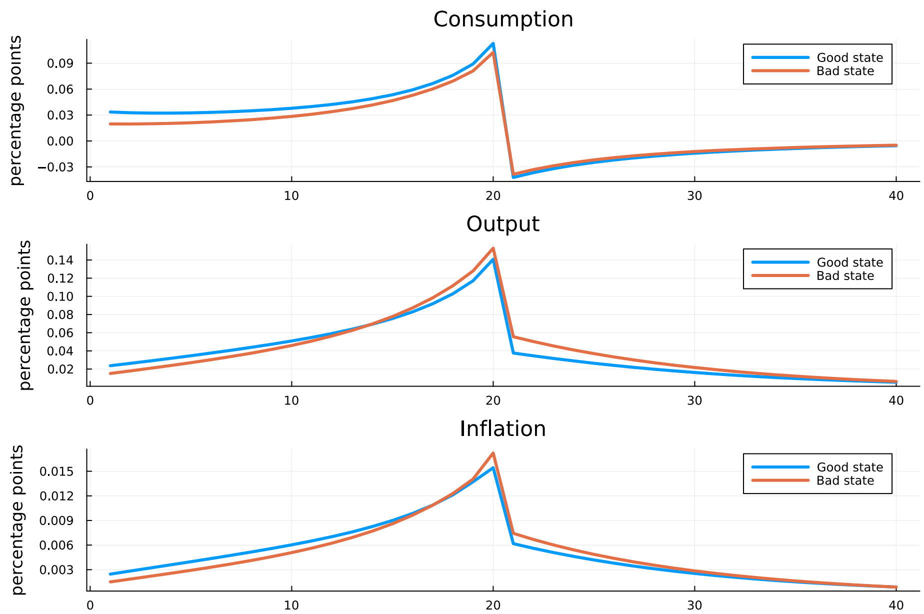


Figure 4: State-contingent forward guidance about a 50 basis points rate cut 20 periods ahead in bad times versus in good times

as the policy announcement is tied to a particular future economic state, and agents are uncertain about future economic conditions. This further tempers households' responses to such state-contingent announcements, making them much less powerful than those that are expressly time-contingent. While time-contingent forward guidance is fraught with the dangers of destabilizing long run inflation expectations, my work provides evidence that there is need to balance the objectives of eliminating uncertainty about future policy rates and keeping inflation expectations anchored so that forward guidance can be more effective in influencing current macroeconomic variables and improving the predictability of monetary policy actions, which is the core objective of forward guidance.

It is worth noting that the power of forward guidance is diminished further when

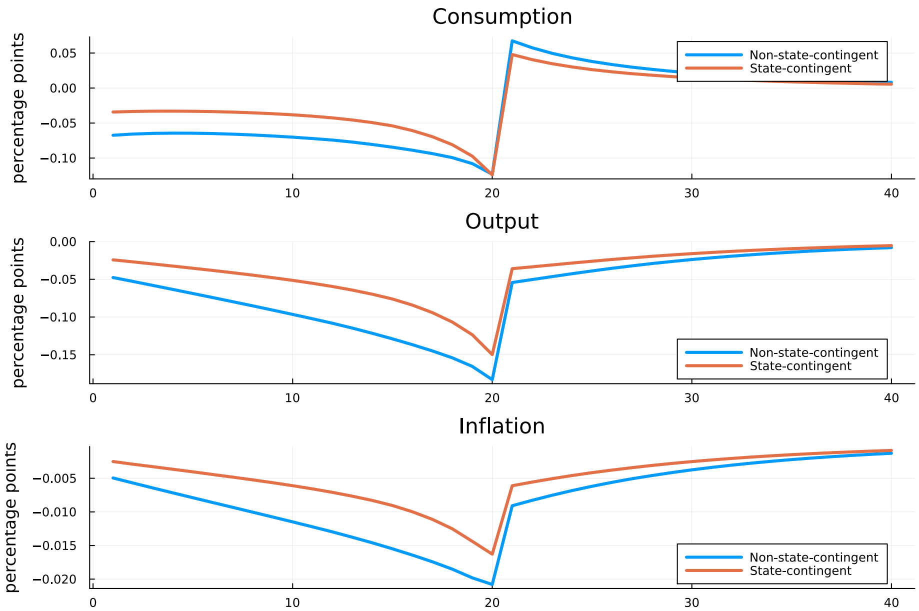


Figure 5: State-contingent versus non-state-contingent forward guidance about a 50 basis points rate increase 20 periods ahead

the state-contingent guidance is communicated in poorer aggregate economic conditions, compared to when relatively stronger aggregate economic conditions prevail. This is depicted in Figure 4.

9.2.3 Robustness and sensitivity

As a robustness check, I find that the effects of state-contingent forward guidance on a future rate increase on current output, consumption, and inflation are symmetric with those of a rate cut. These results are shown in Figure 5. Non-state-contingent forward guidance about a future rate increase by 50 basis points in period 20 causes output, consumption, and inflation to fall by 5 bps, 7 bps, and 0.5 bps, respectively. In contrast, when such guidance is state-contingent, output falls by about 2 bps, consumption by 3 bps, and inflation by only 0.3 bps, on impact at the date of the policy announcement. This experiment serves as a robustness check for one of the main results of my paper.

My results are robust to the forward guidance's time horizon. State-contingent forward guidance over a longer horizon has half the efficacy of non-state-contingent guidance over the same horizon. Figure 6 shows the results of a state-contingent versus a non-state-contingent forward guidance over a longer horizon, where the policy guidance is for 40 periods in the future, depicting the effects of state-contingent guidance to be half the effects of a non-state-contingent guidance at the date of

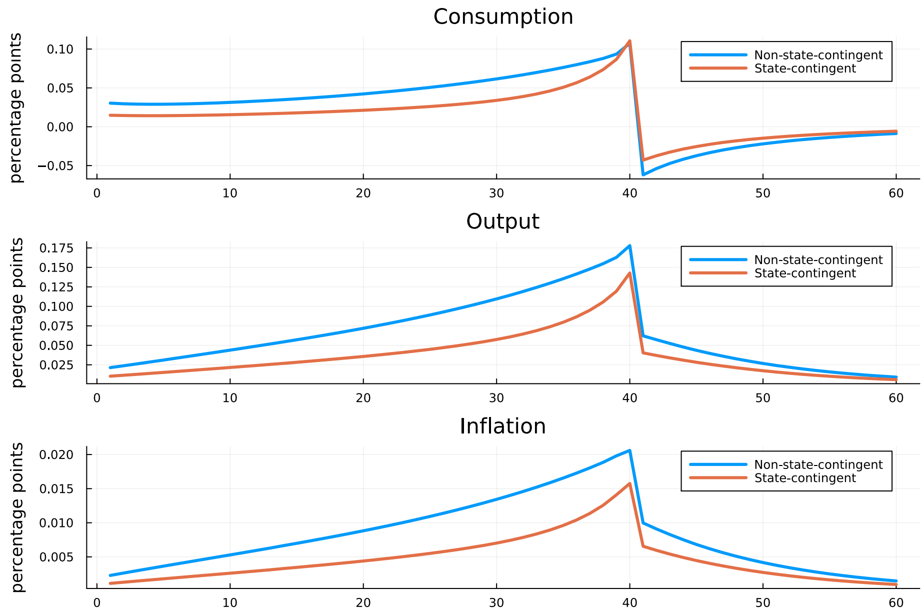


Figure 6: State-contingent versus non-state-contingent forward guidance about a 50 basis points rate cut 40 periods ahead

impact.

(in basis points)	**Variable**	**Date of policy announcement**	**Date of policy implementation**
Non-state-contingent	Output	4.5	16
	Consumption	6.5	10
	Inflation	0.75	2.7
State-contingent	Output	2.5	13
	Consumption	3.5	10
	Inflation	0.45	2.3

Table 7: Non-state-contingent versus state-contingent forward guidance about a 50 bps rate cut 20 periods ahead when price adjustment cost parameter $\theta = 150$

The two main findings of my paper are also robust to alternative calibrations for the fiscal rule and the price adjustment cost. A tighter fiscal policy rule and a lower price adjustment cost parameter both yield comparable results for non-state-contingent forward guidance on current macroeconomic variables. Additionally, state-contingent forward guidance is also similarly less effective than the non-state-contingent forward guidance by the same magnitude in these alternate calibrations as in the baseline experiment. The results from these alternate calibrations are shown in Tables 7 and Table 8, and Figure 7 and Figure 8 for comparison.

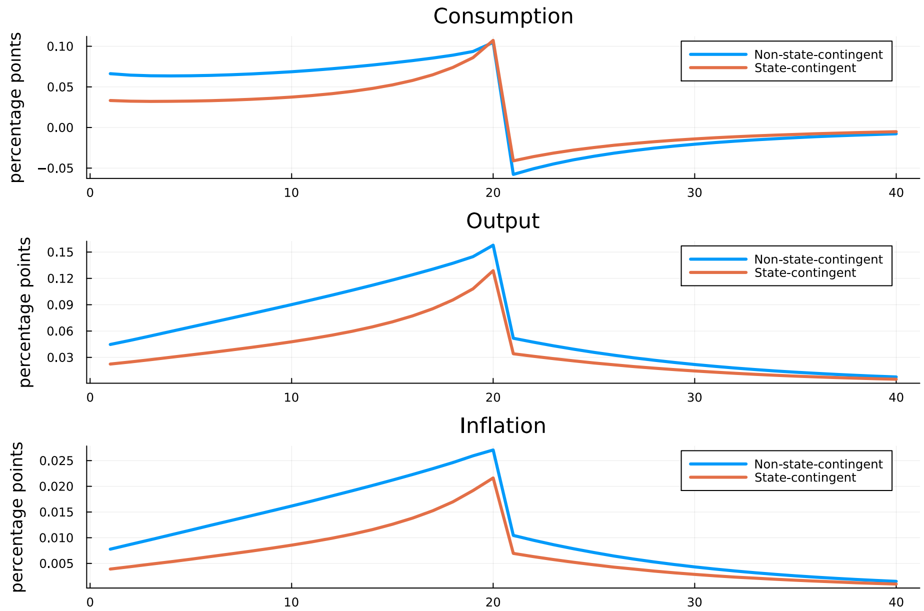


Figure 7: Non-state-contingent versus state-contingent forward guidance about a 50 bps rate cut 20 periods ahead when price adjustment cost parameter $\theta = 150$

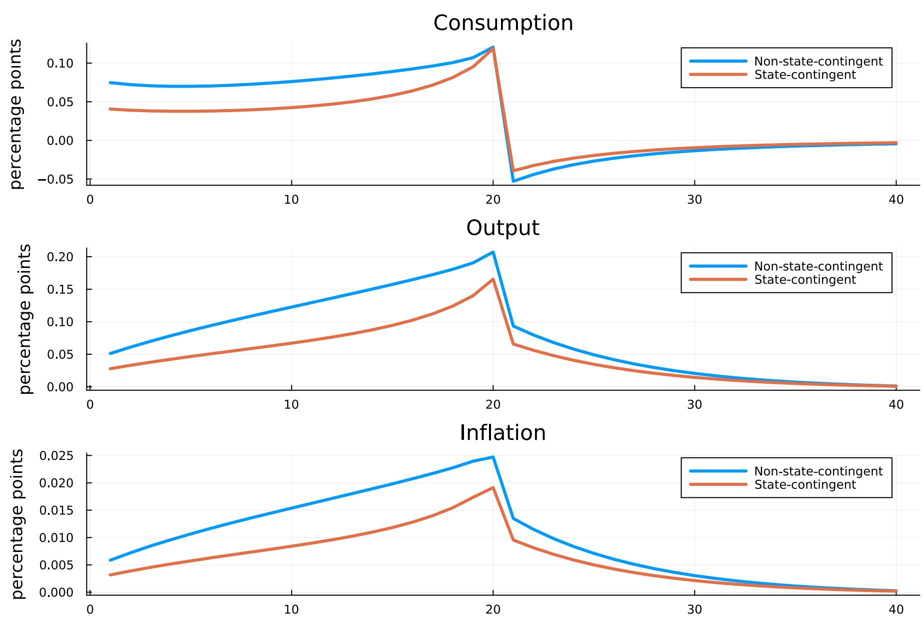


Figure 8: Non-state-contingent versus state-contingent forward guidance about a 50 bps rate cut 20 periods ahead when fiscal rule coefficient on debt deviation $\rho_B = 0.125$

(in basis points)	Variable	Date of policy announcement	Date of policy implementation
Non-state-contingent	Output	5	20
	Consumption	7	11.5
	Inflation	0.5	2.5
State-contingent	Output	2.5	16.5
	Consumption	3.5	12.5
	Inflation	0.45	1.9

Table 8: Non-state-contingent versus state-contingent forward guidance about a 50 bps rate cut 20 periods ahead when fiscal rule coefficient on debt deviation $\rho_B = 0.125$

10 Conclusion

I study the effects of forward guidance in an incomplete market setting with aggregate uncertainty. Households face risks from uninsurable idiosyncratic income shocks, which are compounded by aggregate uncertainty.

Given the state-contingent nature of forward guidance about future policy rates based on the expected state of the economy in the future, adopted by the US Federal Reserve in recent years, the study of such state-contingent forward guidance is only feasible in the presence of aggregate uncertainty, which has been unexplored in the literature so far.

I solve the model with aggregate uncertainty to find a global stochastic general equilibrium for the economy without relying on perfect foresight or approximations around the deterministic steady-state, to study the effects of state-contingent and non-state-contingent forward guidance where the monetary authority announces a 50 bps real interest rate cut 20 periods ahead, contingent on a particular state of the economy or irrespective of the state of the economy, respectively.

I find that non-state-contingent forward guidance in the presence of aggregate uncertainty and without perfect foresight has relatively smaller effects than a similar forward guidance expressly contingent only on time, irrespective of the state of the economy under perfect foresight. Further, the power of forward guidance to impact current output, consumption, and inflation is significantly muted when the guidance is state-contingent. Specifically, state-contingent forward guidance is only half as effective as non-state-contingent forward guidance in affecting current real economic activity and current inflation.

My results underscore the importance of incorporating aggregate uncertainty for a meaningful study of forward guidance in its contemporary state-contingent form. The presence of aggregate uncertainty substantially reduces the power of forward guidance, even when it is not contingent on future states of the economy. This highlights the presence of nonlinearities in household behavior when the economy faces aggregate uncertainty, necessitating a global solution and the elimination of perfect foresight to accurately capture them. Heightened risks lead to large precautionary savings, and borrowing constraints further reduce households' response to such announcements, particularly when they are contingent on future economic circumstances, as this leaves room for uncertainty about expected future rates. Therefore, forward guidance with incomplete markets has significantly diminished power in the presence of aggregate uncertainty, more so when the guidance is contingent on the future state of the economy.

My work augments the existing literature on forward guidance by enabling the study of state-contingent forward guidance, previously infeasible with the existing frameworks. This allows for a meaningful study of a variety of issues in monetary policy in the presence of aggregate uncertainty, while fully preserving the economy's stochastic nature.

References

[1] S. Acharya, E. Challe, and K. Dogra, "Optimal monetary policy according to hank," *American Economic Review*, vol. 113, no. 7, pp. 1741–1782, 2023.

[2] G.-M. Angeletos and C. Lian, "Forward guidance and heterogeneous beliefs," *American Economic Review*, vol. 108, no. 9, pp. 2477–2512, 2018.

[3] A. Auclert, B. Bardóczy, M. Rognlie, and L. Straub, "Using the sequence-space jacobian to solve and estimate heterogeneous-agent models," *Econometrica*, vol. 89, no. 3, pp. 1329–1356, 2021.

[4] C. Bayer, R. Luetticke, L. Pham-Dao, and V. Tjaden, "Precautionary savings, illiquid assets, and the aggregate consequences of wealth taxes," tech. rep., CEPR Discussion Paper No. DP13528, 2019.

[5] B. S. Bernanke, "The new tools of monetary policy," *American Economic Review*, vol. 110, no. 4, pp. 943–983, 2020.

[6] M. Bodenstein, J. Hebden, and R. Nunes, "Imperfect credibility and the forward guidance puzzle," tech. rep., Board of Governors of the Federal Reserve System, International Finance Discussion Papers No. 1061, 2012.

[7] J. R. Campbell, J. D. M. Fisher, A. Justiniano, and L. Melosi, "Forward guidance and macroeconomic outcomes since the financial crisis," in *NBER Macroeconomics Annual 2016* (M. Eichenbaum and J. A. Parker, eds.), vol. 31, pp. 283–357, University of Chicago Press, 2017.

[8] C. T. Carlstrom, T. S. Fuerst, and M. Paustian, "Optimal monetary policy in a model with agency costs," *Journal of Money, Credit and Banking*, vol. 47, no. S2, pp. 265–308, 2015.

[9] R. Chetty, "Bounds on elasticities with optimization frictions: A synthesis across microeconomic contexts," *Econometrica*, vol. 80, no. 6, pp. 2127–2181, 2012.

[10] L. J. Christiano, M. Eichenbaum, and S. Rebelo, "When is the government spending multiplier large?," *Journal of Political Economy*, vol. 119, no. 1, pp. 78–121, 2011.

[11] H. Chung, E. Herbst, and M. T. Kiley, "Effective monetary accommodation and the reversal rate," tech. rep., Board of Governors of the Federal Reserve System, Finance and Economics Discussion Series 2014-44, 2014.

[12] M. Del Negro, M. P. Giannoni, and C. Patterson, "The forward guidance puzzle," tech. rep., Federal Reserve Bank of New York, Staff Report No. 574, 2012.

[13] M. Del Negro, M. P. Giannoni, and C. Patterson, "The forward guidance puzzle," tech. rep., Federal Reserve Bank of New York, Staff Report No. 574, Revised, 2013.

[14] X. Gabaix, "A sparsity-based model of bounded rationality," *Quarterly Journal of Economics*, vol. 129, no. 4, pp. 1661–1710, 2015.

[15] M. Garcia-Schmidt and M. Woodford, "Are low interest rates deflationary? a paradox of perfect-foresight analysis," *American Economic Review*, vol. 109, no. 1, pp. 86–126, 2019.

[16] R. S. Gürkaynak, B. Sack, and E. T. Swanson, "Do actions speak louder than words? the response of asset prices to monetary policy actions and statements," *International Journal of Central Banking*, vol. 1, no. 1, pp. 55–93, 2005.

[17] A. Haberis, R. Harrison, and M. Waldron, "Monetary policy, forward guidance and the yield curve," tech. rep., Bank of England Working Paper No. 485, 2014.

[18] M. Hagedorn, I. Manovskii, K. Mitman, and S. Sehran, "Unemployment benefits and unemployment in the great recession: The role of macro effects," *American Economic Review*, vol. 109, no. 8, pp. 2957–2993, 2019.

[19] C. L. Ilut, R. Luetticke, and M. Schneider, "Title forthcoming." Forthcoming (update publication details as available), 2025.

[20] G. Kaplan, B. Moll, and G. L. Violante, "Monetary policy according to hank," *American Economic Review*, vol. 108, no. 3, pp. 697–743, 2018.

[21] M. T. Kiley, "Macroeconomic effects of federal reserve forward guidance," tech. rep., Board of Governors of the Federal Reserve System, Finance and Economics Discussion Series 2016-061, 2016.

[22] N. G. Mankiw and R. Reis, "Sticky information versus sticky prices: A proposal to replace the new keynesian phillips curve," *Quarterly Journal of Economics*, vol. 117, no. 4, pp. 1295–1328, 2002.

[23] A. McKay, E. Nakamura, and J. Steinsson, "The power of forward guidance revisited," *American Economic Review*, vol. 106, no. 10, pp. 3133–3158, 2016.

[24] F. Milani and J. Treadwell, "The effects of forward guidance in dsge models: A comparison of strategies," *Economics Letters*, vol. 116, no. 2, pp. 172–175, 2012.

[25] M. Reiter, "Global solution methods for heterogeneous agent models," *Journal of Economic Dynamics and Control*, vol. 34, no. 1, pp. 80–90, 2010.

[26] J. J. Rotemberg, "Monopolistic price adjustment and aggregate output," *Review of Economic Studies*, vol. 49, no. 4, pp. 517–531, 1982.

[27] G. L. Violante, "Title forthcoming." Please insert full citation for the specific publication used, 2021.

[28] M. Woodford, "Monetary policy analysis in models with heterogeneous agents," in *Handbook of Macroeconomics* (J. B. Taylor and H. Uhlig, eds.), vol. 2B, pp. 867–928, Elsevier, 2019.