Forward Guidance in an Uncertain World: A Recursive Approach

Smriti Verma

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Abstract

I examine the effect of forward guidance in monetary policy, with incomplete markets in the presence of aggregate uncertainty. Using a global solution method to solve for a recursive equilibrium in this economy, my results suggest that the forward guidance puzzle is far from resolved. When households face risks from idiosyncratic income shocks, aggregate uncertainty makes the precautionary saving motive of households even stronger than it is in steady state where households are only trying to insure against idiosyncratic income shocks. With aggregate uncertainty, announcements about future interest rate cuts unconditional on the state of the economy cause stronger consumption and output responses than purported by perfect foresight analyses. The effects are slightly tempered when such announcement is conditioned on the economic outlook for the economy being a certain kind at such future date, but the effects are still large. My results highlight the importance of studying monetary policy in a stochastic dynamic general equilibrium, especially when markets are incomplete. Further, in the presence of aggregate uncertainty, aggregate savings in the stochastic equilibrium are higher than in steady state as households respond to business cycle risks. This result requires a global solution to the model. A separate contribution of this paper is to present such a solution method, which has not existed in the literature on monetary policy and forward guidance so far.

1 Introduction

Has the Forward Guidance puzzle really been resolved? "Forward guidance" in monetary policy is communication about future interest rates by the monetary authority.

Forward guidance has become a conventional and prime tool of monetary policy in recent years. The "forward guidance puzzle" [1] has been the focus of the literature on forward guidance - estimated DSGE models, while successfully explaining the effects of contemporaneous monetary policy actions on key macroeconomic variables, seemed to grossly exaggerate the effects of future policy announcements. This paper attempts to revisit and reexamine the “forward guidance puzzle” in a new light, previously unexplored in the literature, by adding aggregate uncertainty to a model of heterogeneous households with incomplete markets and finding a global solution to the model without using linearized methods that use approximations around the steady state. My results indicate that the forward guidance puzzle may be far from resolved, as I find relatively large effects of forward guidance on contemporaneous output, consumption, and inflation.

McKay, Nakamura, and Steinsson (2016) show that the power of forward guidance is highly sensitive to the assumption of complete markets - when agents are subject to uninsurable income risks and borrowing constraints, the precautionary savings motive tempers their current response to changes in future interest rates. In this paper, I study how, in the absence of perfect foresight, aggregate uncertainty affects households' responses to state-contingent interest rate policy announcements by the monetary authority.

My results indicate that while aggregate uncertainty exacerbates income risks faced by households in making consumption-saving choices, if forward guidance alleviates uncertainty for households by committing to a certain interest rate adjustment in the future irrespective of the state of the economy, the power of forward guidance can be significantly large, even with incomplete markets, as their precautionary saving motives get tempered due to knowledge of future interest rate changes with certainty. If the forward guidance is contingent on a certain state of the economy at the time of the rate cut, it has relatively less power, but still larger than effects shown shown in perfect foresight analyses [2]. The effect of forward guidance on aggregate output and consumption also critically hinges on the government's fiscal rule that it uses to accommodate the effects of forward guidance in the period up to the date of the promised rate cut.

Time-based forward guidance, studied in the literature, poses the threat of destabilizing long-term inflation expectations. Announcements to keep policy rates low until until some time in the future is subject to misinterpretation by the public, as the monetary authority's dismal outlook for the economy until such date, rather than

[1] Del Negro, Giannoni and Patterson 2012

[2] McKay, Nakamura, Steinsson (2016) use perfect foresight to study the effect of a forward guidance experiment where households respond to an announcement of a 50 basis point cut in the real interest rate 20 periods in the future.

continuation of its expansionary stance even in good times to boost the economy. As such, the US Federal Reserve has moved away from time-based announcements and instead gives target ranges within which it intends to maintain interest rates, conditional on specific macroeconomic targets, such as the unemployment rate (employment targets), long-term inflation expectations, labor market conditions, the evolving outlook of the economy, and the balance of risks. The model I develop in this paper is with the objective of studying such state-contingent forward guidance given by the monetary authority, in the presence of aggregate risks and uncertainty, as the economy is subject to in the real world.

Price adjustment costs faced by monopolistically competitive intermediate firms setting prices each period, is the source of nominal rigidities in the economy in my model. Shocks to the total factor productivity in the production function of intermediate firms is the source of aggregate uncertainty in the model economy. The fiscal authority taxes labor income, holds public debt, and has expenditures that do not provide any utility to households. The monetary authority sets real interest rates using a monetary policy function.

To preserve nonlinearities in household and firm behavior due to the presence of aggregate uncertainty as well as to accurately compute elasticities of aggregate functions outside of steady-state, I solve the model recursively using Backward Induction - a global solution method due to Reiter (2010) - to arrive at a stochastic equilibrium for the economy. A recursive general equilibrium provides for households' decision rules to account for both idiosyncratic and aggregate risks, by using all available individual and aggregate state variables in computing such policy functions. This allows modeling the transmission of such shocks in the economy in a way that aptly captures their importance in macroeconomic dynamics. I use such a global stochastic general equilibrium to study the effects of forward guidance about future real interest rates in the economy on current consumption, output, and inflation, without the assumption that households and firms have perfect foresight.

The study of aggregate uncertainty in heterogeneous-agent New Keynesian models has been limited to linear approximations around the steady state [3] and the study of dynamics under the assumption of perfect foresight [4], so that there is no aggregate risk or uncertainty per se. While computationally fast, these methods cannot take account of non-linearities in household and firm behavior outside of the steady state in the presence of aggregate uncertainty. Local approximations around the steady state may not accurately estimate elasticities of households' consumption and labor supply

[3] Adrien Auclert, Bence Bardóczy, Matthew Rognlie, and Ludwig Straub (2021); Cosmin L. Ilut, Ralph Luetticke, and Martin Schneider (2025), Christian Bayer, Ralph Luetticke, Lien Pham-Dao, and Volker Tjaden (2018)

[4] McKay, Nakamura, Steinsson (2016)

responses, outside of steady state, especially when aggregate uncertainty causes there to be non-linearities when the economy is not in steady state.

I address this significant technical challenge in the literature by meaningfully interacting household heterogeneity from income shocks and aggregate uncertainty by computing a global solution which is a stochastic general equilibrium for a heterogeneous-agent New Keynesian (HANK) economy with aggregate uncertainty.

A household's individual endogenous state is the bond holdings it has, and its individual exogenous state is the idiosyncratic income shock it faces. The actual aggregate endogenous state is the distribution of households, and the aggregate exogenous state is the TFP shock. Due to aggregate uncertainty, there is no stationary distribution of households in this economy; instead, there is a distribution of such distributions.

I solve the model using the Backward Induction method due to Reiter (2010) to solve for households' decision rules regarding consumption and labor, using the aggregate bond holdings by households, an approximation to the actual distribution of households, as the approximate aggregate endogenous state for the economy. This is a stark departure from existing work in the literature of forward guidance, and HANK more generally, and a first-order contribution to the existing literature, providing a new light in which to study questions related to monetary policy in a HANK framework.

The literature on forward guidance has studied the effect of forward guidance "shocks" on contemporaneous aggregate consumption and inflation, typically under the assumption of perfect foresight. Odyssean forward guidance by the Fed comprises indications on the duration for which the Fed intends to keep monetary policy accommodative, even if circumstances in the future would warrant and justify otherwise. However, the assumption of perfect foresight greatly simplifies the problem that the central bank faces before it makes such announcements because the likelihood of an aggregate shock or even the risk of such shock is assumed away with the assumption of perfect foresight.

Forward guidance comes handy as an instrument of monetary policy when the monetary authority finds its hands tied in the face of aggregate risks, such as a lurking recession or upcoming periods of high inflation. The risk of aggregate uncertainty, such as the risk of a recession or a recession itself, underscores the importance of forward guidance as a tool of conventional monetary policy. In such times, steering inflation expectations in a calculative manner can yield larger and more immediate outcomes than actual changes in contemporaneous real interest rates. The point of forward guidance is to commit to a future path of policy and then stand by the policy avowed. But what is the effect of such state-contingent commitment on consumption and output when the economy faces the risk of aggregate shocks, can be studied only with the explicit modeling of aggregate shocks and solving the model globally to keep risks from aggregate uncertainty intact, in the absence of perfect foresight, which is akin to the environment in which agents in the real world operate.

To the best of my knowledge, there is no work in the literature that looks at the effect of state-contingent forward guidance on macroeconomic variables of interest, in a recursive economy in the presence of aggregate uncertainty. The use of a global solution method makes the dynamic stochastic general equilibrium solution to the model accurate and relevant irrespective of the steady state without relying on linear approximations around a stationary equilibrium for deciphering such effects when the economy is subject to aggregate uncertainty. I devise a method utilizing the global stochastic equilibrium to study the paths of transition of macroeconomic aggregates in an exercise that is analogous to perfect foresight exercises in the literature, but without the assumption of perfect foresight, so that the stochastic nature of the economy is preserved.

For comparison with forward guidance policy experiments done in perfect foresight in the literature, I study the effects of an announcement of a 50 basis points cut in the real interest rate 20 periods in the future. I find that output rises by 0.36 percent and consumption by 0.19 percent on the date of the announcement of forward guidance which is unconditional on the state of the economy in period 20. However, when forward guidance is conditional on the state of the economy in period 20 being the same as the state in which such announcement is being made, then the effects are comparatively less pronounced; output rises by 0.0925 percent on impact, that is, on the date of the announcement of such forward guidance, while consumption rises by 0.096 percent immediately.

Forward guidance about a future interest rate cut ameliorates the heightened risks that households face in the presence of aggregate uncertainty, from idiosyncratic shocks to income, due to market incompleteness. Households' savings fall and consumption increases, as the need to save for the future is tempered by the knowledge of the future rate cut, which would make borrowing cheaper in the future, even if households realized negative income shocks then. This eases the precautionary savings among households and the horizon for which households worry about is shortened temporarily until the rate cut comes into effect. To smooth consumption, they consume more from the date of such communication to the date when the rate cut is implemented. The large effects of forward guidance on consumption and output are an outcome of the heightened response of households when forward guidance provides them relief from the augmented risks they face from idiosyncratic income shocks in the presence of aggregate uncertainty in ordinary times in the absence of such guidance about future rates. Firms respond to the increase in present consumption demand and to the announcement of the future rate cut by initially increasing output. Since large jumps in prices are costly to firms due to quadratic costs to price adjustments, firms gradually increase prices in expectation of higher demand in the future, which causes current inflation to rise as well.

My results point toward the resurrection of the forward guidance puzzle that the literature seems to have largely believed to have resolved. If forward guidance has such power to affect contemporaneous macroeconomic outcomes, my model can be used to study the effects of important policy actions and can also provide direction on how such communication about future interest rates may be modified to have desired effects on output, consumption, and inflation in the economy.

The remainder of this paper is organized as follows. In section 2, I discuss the related literature and where my work fits in, section 3 lays out the model environment, with section 4 describing the decision problems faced by the agents in the economy, section 5 detailing a recursive formulation of the economy, and section 6 defining the equilibrium for this economy, followed by a brief discussion of the choice of parameters for the model in section 7. Further, I discuss the methods used for computation in section 8 and present the results and its discussion in section 9, and finally, section 10 concludes.

2 Related Literature

The literature on forward guidance has been driven by the "forward guidance puzzle" (Del Negro, Giannoni, and Patterson 2012) – estimated DSGE models, while successfully explained the effects of contemporaneous monetary policy actions on key macroeconomic variables, grossly exaggerated the effects of future policy announcements. The central bank's announcement of lower interest rates, eight quarters in the future, implied a massive stimulus, with the impact on both output and inflation several times larger than that implied by a same-sized contemporaneous drop in the policy rate (Del Negro et al 2013). Gurkaynak, Sack, Swanson (2005) measure the effects of a "future path of policy", closely associated with FOMC statements, on bond yields and stock prices, and find that both, monetary policy actions and statements have important but differing effects on asset prices, with FOMC statements having a much larger impact on longer-term Treasury yields. In a similar flavor, Campbell, Fisher, and Justiniano (2012), and Milani and Treadwell (2012) studied the impact of forward guidance shocks on the economy using small- or medium-scale DSGE models estimated on a pre-Great Recession sample, and both papers find that forward guidance shocks play a large role in explaining movements in the policy rate, and these contribute significantly to business cycle fluctuations.

Campbell et al. (2012) assessed the impact of exogenous changes in monetary policy on private expectations and found the sign to be opposite to the predictions in theory so that accommodative monetary policy was associated with lower projections for inflation and activity. In view of these considerations, the FOMC replaced the calendar-based language in the December 2012 statement and switched to announcing policy intentions that were tied to economic performance on inflation and real activity, and the state of the economy more generally.

Campbell et al. (2017) use a full information, rational expectations framework and find that FOMC forward guidance has mixed effects. The main goal of their study is to assess whether FOMC improved economic performance since the financial crisis using Odyssean forward guidance, and to quantify the impact of Odyssean FOMC forward guidance on macroeconomic outcomes since the financial crisis that unwound from 2007 to 2009. They find that puzzling responses of private-sector forecasts to FOMC announcements can be attributed entirely to Delphic forward guidance, even though a large fraction of the variability in Federal Funds Futures rates on days with FOMC announcement remained unexplained by their measure of FOMC private information. Interestingly, while their findings suggest that purely rule-based policy would have ameliorated the recession and kept inflation closer to target in the years immediately following the crisis, than FOMC forward guidance did in practice, the Fed's introduction of calendar-based Odyssean forward guidance starting toward the end of 2011 appeared to have boosted real activity and moved inflation closer to target.

A large chunk of the forward guidance literature has focused on resolving the forward guidance puzzle. The forward guidance puzzle - large contemporaneous effects as a result of a future policy announcement - arises from intertemporal substitution being the main channel of transmission of forward guidance into the real economy. As a result, departure from the complete markets, full-information rational expectations framework alleviates the forward guidance puzzle. The macroeconomic resolution of the puzzle comes from adding heterogeneity such as in Mckay, Nakamura, Steinsson (2016), Hagedorn et al. (2019), Acharya and Dogra (2020). McKay-Nakamura-Steinsson (2016) (MNS) show that forward guidance has substantially less power to stimulate the economy in the presence of uninsurable risk and borrowing constraints. It is not realistic to assume agents increase their consumption by the same amount in response to a future interest rate cut as they do to a current interest rate cut because of two reasons, borrowing constraints, and precautionary savings. People face some risk of hitting a borrowing limit which shortens their planning horizon, as interest rate changes that occur after they hit a borrowing constraint, are irrelevant to them.

Presence of uninsurable income risks lead to precautionary savings, which temper households' responses to future interest rate shocks. As a result, power of forward guidance is substantially muted in the incomplete markets model compared to the standard complete-markets New Keynesian model.

Other departures from full information set ups and rational expectations include Chung, Herbst, and Kiley (2014), Carlstrom, Fuerst, and Paustian (2015), and Kiley (2016) which use sticky information models à la Mankiw and Reis (2002); Angeletos and Lian (2018) who relax the assumption of common knowledge; and other works that depart from rational expectations in the form of bounded rationality (Gabaix 2015), finite planning horizons (Woodford 2019), or by modeling the cognitive process of expectation formation and use the concept of reflective equilibrium (Garcia-Schmidt and Woodford 2019).

Given how critically the efficacy of forward guidance depends on the credibility of the central bank's commitment to announcements about future policy, several papers discuss imperfect credibility of the central bank to resolve the forward guidance puzzle (Bodenstein, Hebden, and Nunes 2012; Haberis, Harrison, and Waldron 2014; Campbell et al. 2019; Bernanke 2020).

My work contributes to this literature by introducing aggregate uncertainty in the study of state-contingent forward guidance, using a global solution method to account for non-linearities in households' consumption and labor supply responses due to the presence of aggregate uncertainty, and to study transitions in the aggregates in the absence of perfect foresight. To the best of my knowledge, mine is the first work that solves for a fully stochastic general equilibrium solution to a HANK model with aggregate uncertainty, and leverages the model to the study of state-contingent forward guidance, which is also a first-order contribution to the literature.

My work augments the literature and sheds new light on the "forward guidance puzzle" in a setting that has not been studied previously, using methods that are novel. My results for a forward guidance policy experiment question if the forward guidance puzzle is really resolved as the literature claims, because in my model environment and using global solution methods, the contemporaneous macroeconomic aggregate outcomes show substantive response to forward guidance, as was the case in the original puzzle which was thought to only exist in representative agent economies. My model and methodology provide a platform for answering many interesting questions that involve aggregate uncertainty and monetary policy with the use of HANK models, where non-linearities can play an important role in driving outcomes. My results provide an enhanced understanding for policymakers on what kind of communication about future interest rates could yield desirable contemporaneous outcomes for the economy.

3 Model

Households There is a continuum of a unit mass of ex-ante identical households with preferences given by

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[\frac{c_{h,t}^{1-\sigma}}{1-\sigma} - \chi \frac{n_{h,t}^{1+\psi}}{1+\psi} \right] \tag{1}$$

where $c_{h,t}$ is consumption of household h at time t and $n_{h,t}$ is labor supply of household h at time t. Each household faces an uninsurable idiosyncratic productivity shock $e_{h,t}$ which follows a Markov chain with transition probabilities $\Pr(e_{h,t+1}|e_{h,t})$, and earns real labor income before tax $w_t e_{h,t} n_{h,t}$. By assumption, the initial cross-sectional distribution of idiosyncratic productivities is equal to the ergodic distribution of this Markov chain.

In addition to holding stakes in the firm, households also have access to a risk-free real bond b that yields real interest rate r_t between periods t and $t+1$. Borrowing constraints prevent households from taking negative bond positions. A household's period t budget constraint is given by

$$c_t + q_t \cdot b_{t+1} \leq b_t + (1-\tau_t) \cdot w_t \cdot e_t \cdot n_t + d_t \tag{2}$$

where c_t is real consumption, q_t is the price of the bond between period t and $t+1$, b_{t+1} and b_t are the real risk-free bond holdings of the household in period $t+1$ and period t, respectively, w_t is the real wage rate, and d_t is the real dividend earnings of the household.

Final-good producer A competitive representative final good producer aggregates a continuum of intermediate inputs indexed by $j \in [0,1]$ through a Dixit-Stiglitz constant elasticity aggregator as follows

$$Y_t = \left(\int_0^1 y_{j,t}^{\frac{\varepsilon-1}{\varepsilon}} \, dj \right)^{\frac{\varepsilon}{\varepsilon-1}} \tag{3}$$

where Y_t is the quantity of the final good produced by the final good firm at time t, $y_{j,t}$ is the quantity of the intermediate good produced by the firm j in period t, and ε is the price elasticity of substitution across the varieties of the intermediate good. Profit maximization by the final good producer implies the following demand for a typical intermediate input variety j

$$y_{j,t} = \left(\frac{p_{j,t}}{P_t} \right)^{-\varepsilon} Y_t \tag{4}$$

where $p_{j,t}$ is the price set by the producer of intermediate input variety j, and P_t is the aggregate price index given by

$$P_t = \left(\int_0^1 p_{j,t}^{1-\varepsilon} dj \right)^{\frac{1}{1-\varepsilon}} \tag{5}$$

Intermediate goods producers Each intermediate input variety j is produced by a monopolistically competitive producer according to the production function

$$y_{j,t} = z_t \; n_{j,t} \tag{6}$$

where z_t is the aggregate total factor productivity (TFP) common across all intermediate firms, and $n_{j,t}$ denotes the amount of labor hired by firm j in period t.

Intermediate goods producers hire labor at real wage w_t in a competitive labor market such that wage equals the marginal revenue product of labor.

Each intermediate producer chooses its price to maximize lifetime profits subject to price adjustment costs as in Rotemberg (1982). These adjustment costs are quadratic in the rate of price change $\frac{p_{j,t}}{p_{j,t-1}}$ and are expressed as a fraction of real aggregate output Y_t as

$$\Theta_t = \frac{\theta}{2} \left(\frac{p_{j,t}}{p_{j,t-1}} - \bar{\pi} - 1 \right)^2 Y_t \tag{7}$$

where $\theta > 0$ is the cost of the price adjustment.

Profits produced by the intermediate firms are distributed immediately to the households such that each household receives an equal share d_t. Households cannot trade their stakes in the firms.

Aggregate uncertainty The source of aggregate uncertainty is a shock to the exogenous aggregate state of the economy, i.e., the total factor productivity (z_t) in the intermediate firms' production functions.The exogenous shock to TFP, φ, can be decomposed into two components, the exogenous idiosyncratic shock to TFP (η), and, the risk of a disaster shock (Δ).

Idiosyncratic shock to total factor productivity The idiosyncratic shock to TFP is given by the following AR(1) process:

$$log \; z_{t+1} = \rho_z \; log \; z_t + \eta_t \tag{8}$$

where $\eta \sim N(0, \sigma_z^2)$ which can be discretized using the Rouwenhorst method so that $z \in \{z_1, z_2, z_3\}$ and follows a Markov chain with transition probabilities $\Pr(z_{t+1} =$

$z_j | z_t = z_i) = \pi^z_{ij}$ giving the probability of transition from the state z_i in period t to the state z_j in period $t+1$. The transition probability matrix for the idiosyncratic shock to TFP is given by

$$\Pi^z = \begin{array}{c} \\ z = z_1 \\ z = z_2 \\ z = z_3 \end{array} \begin{array}{c} \begin{array}{ccc} z = z_1 & z = z_2 & z = z_3 \end{array} \\ \begin{pmatrix} \pi^z_{11} & \pi^z_{12} & \pi^z_{13} \\ \pi^z_{21} & \pi^z_{22} & \pi^z_{23} \\ \pi^z_{31} & \pi^z_{32} & \pi^z_{33} \end{pmatrix} \end{array}$$

Fiscal authority The government levies a proportional tax on household labor income at the rate τ_t and and holds a stock of outstanding government debt with real face value B_t such that positive values of B denote government debt. The government budget constraint is given as

$$q_t \cdot B_{t+1} = B_t + G_t - \tau_t \cdot w_t \int e \cdot n_t \cdot \mu_t \; d\mu_t \tag{9}$$

where G_t is exogenous government spending in period t.

Monetary authority The monetary authority sets nominal interest rates using a Taylor rule given by

$$i_t = r^* + \alpha \left(\pi_t - \bar{\pi} \right) \tag{10}$$

where r^* is the long-run equilibrium real interest rate, and $\bar{\pi}$ is the monetary authority's target inflation rate. This Taylor rule for nominal interest rates translates into a rule for real interest rates or equivalently the bond price in the economy, and can be re-written as

$$r = r^* + \alpha \left(\pi - \bar{\pi} \right) - \pi \tag{11}$$

where the coefficient α must be greater than 1 for determinacy, and $r_t = \frac{1}{q_t} - 1$.

The relationship between the real interest rate r_t, the expected future inflation rate π_{t+1}, and the nominal interest rate i_t, is given by the Fisher relation as

$$1 + r_t = \frac{1 + i_t}{\mathbb{E}_t \left[1 + \pi_{t+1} \right]} \tag{12}$$

where π_{t+1} is the net inflation rate given as $\pi_{t+1} \equiv \frac{P_{t+1}}{P_t} - 1$ and P_t is the aggregate price level. The real interest rate and the expected future inflation rate imply a nominal interest rate determined by the Fisher equation, such that the the real return on assets r_t must be consistent with equilibrium in the bond market.

4 Decision Problems

4.1 Decision problem of the household

Households face the following decision problem in this economy

$$\max_{\{c_{h,t},n_{h,t},b_{h,t+1}\}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[\frac{c_{h,t}^{1-\sigma}}{1-\sigma} - \chi \frac{n_{h,t}^{1+\psi}}{1+\psi} \right] \tag{13}$$

subject to their period budget constraint,

$$c_t + q_t \cdot b_{t+1} \leq b_t + (1-\tau_t) \cdot w_t \cdot e_t \cdot n_t + d_t \tag{14}$$

the borrowing constraint,

$$b_{t+1} \geq \underline{\mathrm{b}} \tag{15}$$

the non-negativity constraint,

$$c_t \geq 0 \tag{16}$$

Each household chooses consumption and labor optimally by satisfying its Euler equation given by

$$u_c'(c_{h,t}, n_{h,t}) = \beta \mathbb{E}_t \left[(1+r_{t+1}) \, u_c'(c_{h,t+1}, n_{h,t+1}) \right] \tag{17}$$

which can be written in terms of the discount bond price as

$$q_t = \beta \mathbb{E}_t \left[\frac{u_c'(c_{h,t+1}, n_{h,t+1})}{u_c'(c_{h,t}, n_{h,t})} \right] \tag{18}$$

and its labor-leisure condition, given by

$$\chi \cdot n_{h,t}^{1/\eta} = c_{h,t}^{-\sigma} \cdot w \tag{19}$$

which, together with the household's budget constraint, yield the optimal choices of consumption and leisure for each household.

4.2 Decision problem of the final good producer

The final good producer faces the following profit maximization problem

$$\max_{y_{j,t}} \left\{ P_t \left(\int_0^1 y_{j,t}^{\frac{\varepsilon-1}{\varepsilon}} \, dj \right)^{\frac{\varepsilon}{\varepsilon-1}} - \int_0^1 p_{j,t} y_{j,t} \, dj \right\} \tag{20}$$

which yields the following demand for each intermediate good variety j

$$y_{j,t} = \left(\frac{p_{j,t}}{P_t}\right)^{-\varepsilon} Y_t \tag{21}$$

where $p_{j,t}$ is the price charged by firm producing intermediate input variety j in period t and P_t is the aggregate price index in period t given by

$$P_t = \left(\int_0^1 p_{j,t}^{1-\varepsilon} dj\right)^{\frac{1}{1-\varepsilon}} \tag{22}$$

4.3 Decision problem of intermediate goods producers

Intermediate goods producers face a common real wage w_t. Due to the presence of nominal rigidities, intermediate firms are unable to adjust prices each period to maximize their profit. Therefore, they act to minimize cost. The cost minimization problem faced by intermediate firms, subject to producing enough to meet demand, is given as

$$\min_{n_{j,t}} \{w_t n_{j,t}\} \tag{23}$$

subject to

$$z_t n_{j,t} \geq y_{j,t} \tag{24}$$

and

$$y_{j,t} = \left(\frac{p_{j,t}}{P_t}\right)^{-\varepsilon} Y_t \tag{25}$$

Cost minimization by intermediate goods producers implies that marginal cost is identical across intermediate goods firms and is given by

$$mc_t = \frac{w_t}{z_t} \tag{26}$$

Each intermediate firm seeks to maximize its expected present discounted value of future profits for its shareholders. In doing so, each intermediate producer solves the following maximization problem

$$\max_{p_{j,t}} \mathbb{E}_t \left[\sum_{s=0}^{\infty} \mathcal{Q}_{t\to t+s} \frac{P_t}{P_{t+s}} \tilde{V}_{j,t+s}\right] \tag{27}$$

subject to

$$y_{j,t} = \left(\frac{p_{j,t}}{P_t}\right)^{-\varepsilon} Y_t \tag{28}$$

where $\mathcal{Q}_{t\to t+s}$ is the s-period ahead real stochastic discount factor and $\mathcal{Q}_{t\to t+s}\frac{P_t}{P_{t+s}}$

is the s-period ahead nominal stochastic discount factor. The firm discounts future profits by the inverse of the real-interest rate in this economy. Hence, we can substitute for $\mathcal{Q}_{t\to t+s} = \frac{1}{1+r_t} = q_t$, which is the price of the discount bond in the economy. Further, $\tilde{V}_{j,t+s}$ is the s-period ahead nominal profit for an intermediate producer of a typical input variety j, that is, nominal profit at time $t+s$, with price $p_{j,t}$ chosen at time t, and is given by its per-period nominal flow profit in period $t+s$ net of nominal price adjustment cost in period $t+s$ as

$$\tilde{V}_{j,t+s} = p_{j,t+s}y_{j,t+s} - mc_{t+s}y_{j,t+s}P_t + s - \frac{\theta}{2}\left(\frac{p_{j,t+s}}{p_{j,t+s-1}} - \bar{\pi} - 1\right)^2 Y_{t+s}P_{t+s} \tag{29}$$

Substituting for $\tilde{V}_{j,t+s}$ from equation (29) and $y_{j,t}$ from the constraint in equation (28) in equation (27), the problem of an intermediate producer of a typical input variety j can be rewritten as

$$\begin{aligned}\max_{p_{j,t}} \mathbb{E}_t \Bigg[\sum_{s=0}^{\infty} q_t \frac{P_t}{P_{t+s}} \Bigg(& p_{j,t+s}\left(\frac{p_{j,t+s}}{P_{t+s}}\right)^{-\varepsilon} Y_{t+s} - mc_{t+s}\left(\frac{p_{j,t+s}}{P_{t+s}}\right)^{-\varepsilon} Y_{t+s}P_{t+s} \\ & - \frac{\theta}{2}\left(\frac{p_{j,t+s}}{p_{j,t+s-1}} - \bar{\pi} - 1\right)^2 Y_{t+s}P_{t+s}\Bigg)\Bigg]\end{aligned} \tag{30}$$

The solution to this problem satisfies

$$\begin{aligned}\varepsilon\frac{p_{j,t}}{P_t}\left(\frac{p_{j,t}}{P_t}\right)^{-\varepsilon-1} Y_t = \left(\frac{p_{j,t}}{P_t}\right)^{-\varepsilon} Y_t + mc_t\varepsilon Y_t\left(\frac{p_{j,t}}{P_t}\right)^{-\varepsilon-1} - \theta\left(\frac{p_{j,t}}{p_{j,t-1}} - \bar{\pi} - 1\right)\frac{P_t}{p_{j,t-1}}Y_t \\ + \theta\mathbb{E}_t\left[\frac{q_t}{P_{t+1}/P_t}\left(\frac{p_{j,t+1}}{p_{j,t}} - \bar{\pi} - 1\right)\left(\frac{p_{j,t+1}}{p_{j,t}}\right)^2 Y_{t+1}\right]\end{aligned} \tag{31}$$

In equilibrium, all intermediate input producers behave identically: each of them faces the same marginal cost, therefore, each intermediate goods producer sets the same price and produces the same level of intermediate input variety j. Imposing symmetry in equilibrium, i.e. $p_{j,t} = P_t$, the intermediate goods producers' optimality condition for price setting in equation (31) can be written in terms of inflation rates by defining $\frac{P_{t+1}}{P_t} = \pi_{t+1} + 1$ as

$$\frac{\theta}{\varepsilon - 1}(\pi_t - \bar{\pi})\pi_t = \frac{\varepsilon}{\varepsilon - 1}mc_t - 1 + \theta\mathbb{E}_t\left[q_t(\pi_{t+1} - \bar{\pi})\pi_{t+1}\frac{Y_{t+1}}{Y_t}\right] \tag{32}$$

or,

$$(\pi_t - \bar{\pi})\,\pi_t = \frac{1-\varepsilon}{\theta} + \frac{w_t}{z_t}\frac{\varepsilon}{\theta} + \mathbb{E}_t\left[q_t\,(\pi_{t+1} - \bar{\pi})\,\pi_{t+1}\frac{Y_{t+1}}{Y_t}\right] \tag{33}$$

This is the New Keynesian Phillips Curve.

Aggregate production Integrating demand for intermediate good variety j across firms in equation 4, and using production function of intermediate goods producers in equation 6, we have

$$\begin{aligned}\int_0^1 y_{j,t}\ dj &= \int_0^1 \left(\frac{p_{j,t}}{P_t}\right)^{-\varepsilon} Y_t\ dj \\ \int_0^1 z_t n_{j,t}\ dj &= Y_t \int_0^1 \left(\frac{p_{j,t}}{P_t}\right)^{-\varepsilon}\ dj\end{aligned} \tag{34}$$

Since all firms are identical in the demand they face for their variety of the intermediate good, and the adjustment cost they must pay to adjust prices each period, all intermediate firms adjust to the same reset price every period, which then becomes the aggregate price level for the economy. Hence, setting $p_{j,t} = p_t = P_t$, we have

$$\begin{aligned}&z_t\ N_t = Y_t \int_0^1 \left(\frac{P_t}{P_t}\right)^{-\varepsilon}\ dj \\ &z_t\ N_t = Y_t \\ &\text{or, } Y_t = z_t\ N_t\end{aligned} \tag{35}$$

5 Recursive formulation of the economy

In this section, we will define the economy described above in a recursive formulation, which is essential for a state-contingent exposition of monetary policy in this environment. Let φ denote the aggregate exogenous state of the economy, representing the effective TFP. The endogenous aggregate state of the economy is the distribution of households across the state space, denoted by μ. The households have an endogenous individual state given by their individual choice of bond holding, denoted by b, and an exogenous individual state which is the idiosyncratic shock they face, denoted by e.

Household's decision problem The decision problem facing households is as follows.

$$V\left(e_i, b; \varphi_i, \mu\right) = \max_{\{c, b_i, n\}} \left\{ \frac{c^{1-\sigma}}{1-\sigma} - \chi \frac{n^{1+\psi}}{1+\psi} \right. \tag{36}$$

$$\left. + \beta \sum_{\varphi'_j} Pr(\varphi'_j|\varphi_i) \sum_{e'_j} Pr(e'_j|e_i)\ V'\left(e'_j, b'; \varphi'_j, \mu'\right) \right\}$$

subject to their budget constraint,

$$c + q\left(\varphi_i, \mu\right) \cdot b' \leq b + (1-\tau)\, w\left(\varphi_i, \mu\right) \cdot e_i \cdot n_i + d\left(\varphi_i, \mu\right) \tag{37}$$

the borrowing constraint,

$$b' \geq \underline{\mathrm{b}} \tag{38}$$

the non-negativity constraint,

$$c \geq 0 \tag{39}$$

and, the law of motion for the distribution of households,

$$\mu' = \Lambda\left(\mu, \varphi_i\right). \tag{40}$$

The solution to this problem yields the decision rule for households' consumption, $c\left(e_i, b; \varphi_i, \mu\right)$, the decision rule for households' bond holdings, b, given by $g\left(e_i, b; \varphi_i, \mu\right)$, and the decision rule for the households' labor supply choice, $n\left(e_i, b; \varphi_i, \mu\right)$.

Decision problem of intermediate goods producers The individual state of an intermediate input producer is given by the relative price of the last period's price set by the producer to the aggregate price level last period, $\frac{p_{j,-1}}{P_{-1}}$. An intermediate firm's problem in real terms can then be defined in recursive form as

$$V_f\left(\frac{p_{j,-1}}{P_{-1}}; \varphi_i, \mu\right) = \max_{p_j} \left\{ \frac{p_j y_j}{P} - w\left(\varphi_i, \mu\right) n_j - \frac{\theta}{2}\left(\frac{p_j/P}{p_{j,-1}/P_{-1}} \cdot \frac{P}{P_{-1}} - 1\right)^2 Y\left(\varphi_i, \mu\right) \right. \tag{41}$$

$$\left. + \beta \sum_{\varphi'_j} q\left(\varphi_i, \mu\right)\ Pr\left(\varphi'_j|\varphi_i\right)\ V_f\left(\frac{p_j}{P}; \varphi'_j, \mu'\right) \right\}$$

or,

$$V_f\left(\frac{p_{j,-1}}{P_{-1}};\varphi_i,\mu\right)=\max_{p_j}\left\{\frac{p_j y_j}{P}-w\left(\varphi_i,\mu\right)n_j-\frac{\theta}{2}\left(\frac{p_j/P}{p_{j,-1}/P_{-1}}\cdot\left(1+\pi\left(\varphi_i,\mu\right)\right)-\left(\bar{\pi}+1\right)\right)^2 Y\left(\varphi_i,\mu\right)\right.$$

$$\left.+\,\beta\sum_{\varphi_j'}q\left(\varphi_i,\mu\right)\ Pr\left(\varphi_j'|\varphi_i\right)\ V_f\left(\frac{p_j}{P};\varphi_j',\mu'\right)\right\} \tag{42}$$

subject to

$$y_j=\left(\frac{p_j}{P}\right)^{-\varepsilon}Y\left(\varphi_i,\mu\right) \tag{43}$$

$$y_j=z\cdot n_j \tag{44}$$

where p_j is the price chosen by the firm producing input variety j when resetting its price in the current period, θ is the cost of price adjustment, and ε is the price elasticity of substitution across varieties of the intermediate goods.

To simplify the intermediate firms' problem in recursive form, we define the price of intermediate goods in terms of a relative price. Let $p\equiv p_j/P$ be the relative price for an intermediate firm producing variety j, where p_j is the price of variety j. Let $p_0\equiv p^*/P$ be the ratio of the current optimal reset price, p^*, set by intermediate firms to the current aggregate price level P; in equilibrium, p_0 will be 1 - as all intermediate firms are identical, all firms face the same demand for their respective varieties and reset to the same optimal price, which becomes the aggregate price level. The net inflation rate in the current period is defined as $\pi\equiv\frac{P-P_{-1}}{P_{-1}}$ where P_{-1} is the aggregate price level in the previous period.

Substituting the constraints into the problem yields the following value of an intermediate firm

$$V_f\left(p_{-1};\varphi_i,\mu\right)=\max_{p}\left\{p^{1-\varepsilon}Y-\frac{w\left(\varphi_i,\mu\right)}{z}p^{-\varepsilon}Y-\frac{\theta}{2}\left(\frac{p}{p_{-1}}\cdot\left(1+\pi\left(\varphi_i,\mu\right)\right)-\left(\bar{\pi}+1\right)\right)^2 Y\left(\varphi_i,\mu\right)P\right.$$

$$\left.+\,\beta\sum_{\varphi_j'}q\left(\varphi_i,\mu\right)\ Pr\left(\varphi_j'|\varphi_i\right)\ V_f\left(p;\varphi_j',\mu'\right)\right\} \tag{45}$$

The solution to the intermediate firm's problem yields the New Keynesian Phillips

Curve (NKPC) given in equation 33 which can be rewritten in recursive form as

$$\Big(\pi\left(\varphi_i,\mu\right)-\bar{\pi}\Big)\cdot\pi\left(\varphi_i,\mu\right)=\frac{1-\varepsilon}{\theta}+\frac{w\left(\varphi_i,\mu\right)}{z}\frac{\varepsilon}{\theta}$$
$$+\mathbb{E}\left[q\left(\varphi_i,\mu\right)\Big(\pi'\left(\varphi_j',\mu'\right)-\bar{\pi}\Big)\cdot\pi'\left(\varphi_j',\mu'\right)\cdot\frac{Y'\Big(\varphi_j',\mu'\Big)}{Y\left(\varphi_i,\mu\right)}\right] \tag{46}$$

Evolution of inflation The New Keynesian Phillips curve in equation 46 above summarizes the firms' optimal responses to the decision problems faced by them in this economy, and is the equation for the evolution of inflation in this economy.

Aggregate production Aggregate production in this economy given by equation 34 can be rewritten as

$$Y\left(\varphi_i,\mu\right)=z\cdot N\left(\varphi_i,\mu\right) \tag{47}$$

Fiscal authority The government's budget constraint in equation 9 is given as

$$q\left(\varphi_i,\mu\right)\cdot B' = \ B \ + \ G\left(\varphi_i,\mu\right)$$
$$- \ \tau w\left(\varphi_i,\mu\right)\int e_i\cdot n\left(e_i,b;\varphi_i,\mu\right)\cdot\mu\left(e_i,b;\varphi_i,\mu\right) \tag{48}$$

where $G\left(\varphi_i,\mu\right)$ is an exogenous government spending function. Further, aggregate bonds are the sum of bonds held across individuals.

$$B\left(\varphi_i,\mu\right)=\int b\left(e_i,b;\varphi_i,\mu\right)d\mu\left(e_i,b;\varphi_i,\mu\right) \tag{49}$$

where $\mu\left(e_i,b;\varphi_i,\mu\right)$ represents the distribution of households across the state space.

Monetary authority The Taylor rule followed by the monetary authority for setting nominal interest rates can be defined as a function as

$$i\left(\varphi_i,\mu\right)=r^*+\alpha\Big(\pi\left(\varphi_i,\mu\right)-\bar{\pi}\Big). \tag{50}$$

Transforming the Taylor rule into a rule setting real interest rates in the economy, we can write

$$r\left(\varphi_i,\mu\right)=r^*+\alpha\Big(\pi\left(\varphi_i,\mu\right)-\bar{\pi}\Big)-\pi\left(\varphi_i,\mu\right) \tag{51}$$

where $r\left(\varphi_i,\mu\right)=\frac{1}{q(\varphi_i,\mu)}-1$.

The relationship between nominal interest rates, real interest rates, and expected future inflation given by the Fisher equation in equation 12 can be written in recursive form as

$$1 + r(\varphi_i, \mu) = \frac{1 + i(\varphi_i, \mu)}{\mathbb{E}\left[1 + \pi\left(\varphi'_j, \mu'\right)\right]}. \tag{52}$$

6 Equilibrium

The distribution of households over individual and aggregate endogenous and exogenous states at date t is given by $\mu_t(e_i, b)$, and it evolves according to

$$\mu_{t+1}\left(e'_j, \mathcal{B}\right) = \int_{\{(e,b):g_t(e,b;\varphi_i,\mu)\in\mathcal{B}\}} Pr\left(e'_j|e_i\right) d\mu_t(e, b_i) \text{ given } \varphi_i, \varphi'_j \tag{53}$$

for all sets $\mathcal{B} \subset \mathbb{R}$.

In equilibrium, individual labor supply decisions are consistent with the aggregate such that aggregate labor supply is given by

$$L \equiv \int e \cdot n_t(e_i, b; \varphi_i, \mu) \;\; d\mu(e_i, b; \varphi_i, \mu) \tag{54}$$

and labor market clearing requires labor supply and labor demand to equal,

$$L = N \tag{55}$$

Aggregate bond holding is given by,

$$\bar{B} = \int g_t(e_i, b; \varphi_i, \mu) \;\; d\mu(e_i, b; \varphi_i, \mu) \tag{56}$$

These are held by the government as debt, such that bond market clearing requires

$$\bar{B} = B \tag{57}$$

where B is aggregate public debt.

The aggregate dividends paid by the intermediate goods firms in real terms is given by

$$d(\varphi_i, \mu) = Y(\varphi_i, \mu) - w(\varphi_i, \mu) \cdot N(\varphi_i, \mu) - \frac{\theta}{2}\left(\pi(\varphi_i, \mu) - \bar{\pi}\right)^2 \cdot Y(\varphi_i, \mu) \tag{58}$$

Finally, the goods market clears when the aggregate resource constraint holds as

$$Y(\varphi_i, \mu) = C(\varphi_i, \mu) + G(\varphi_i, \mu) - \frac{\theta}{2}\left(\pi(\varphi_i, \mu) - \bar{\pi}\right)^2 \cdot Y(\varphi_i, \mu) \tag{59}$$

where $C \equiv \int c(e_i, b; \varphi_i, \mu) \;\; d\mu(e_i, b; \varphi_i, \mu)$ denotes aggregate consumption from consistency between individual consumption decisions and aggregate consumption in equilibrium.

Definition of a recursive competitive equilibrium A recursive competitive equilibrium in this economy consists of decision rules and value functions:

$$\left\{c_t\left(e_i, b; \varphi_i, \mu\right), g_t\left(e_i, b; \varphi_i, \mu\right), n_t\left(e_i, b; \varphi_i, \mu\right), V_t\left(e_i, b; \varphi_i, \mu\right), V_{f_t}\left(\frac{p_{j,-1}}{P_{-1}}; \varphi_i, \mu\right)\right\}_{t=0}^{\infty}$$

that solve the problems of households and firms, respectively, and distributions of households $\{\mu_t\left(e_i, b\right)\}_{t=0}^{\infty}$ that evolve according to the law of motion in equation 53 for given φ_i, φ_j', and individual decisions are consistent with the aggregates as in equations 56 and 57 for the bond market, equations 54 and 55 for the labor market, and equation 59 for the goods market, which must also hold as the aggregate resource constraint for the economy. In addition, the government's budget constraint in equation 9, the Fisher relation in equation 52, the optimal price setting equation for the intermediate firms, which yields the New Keynesian Phillips Curve in equation 46, which shows the evolution of inflation, the equation for aggregate dividends in equation 58, and the aggregate resource constraint in equation 59, along with the monetary policy rule in equation 51 must be satisfied.

7 Calibration

A period in this model economy is a quarter of a calendar year. The subjective discount factor *beta* is set at 0.99. The coefficient of relative risk aversion is set to 2. The inverse of the Frisch elasticity of labor supply is 2 based on the findings in Chetty (2012) and as in Mckay, Nakamura, Steinsson (2016), and the preference parameter for households for labor supply is set to 150.

The desired mark-up for intermediate firms, defined by $\mu \equiv \frac{\varepsilon}{\varepsilon-1}$ is set to 1.2 following Christiano, Eichenbaum, and Rebello (2011) and Mckay, Nakamura, and Steinsson (2016). The steady-state inflation rate is set to 2 percent per annum. The cost of price adjustment, θ, is set to 150.

The idiosyncratic income shocks for a quarterly AR(1) process have a persistence parameter of 0.9 and a standard deviation of innovation of 0.025. The AR(1) shock process is discretized to a five-point Markov chain using the Rouwenhorst (1995) method.

The shock to Total Factor Productivity is also a quarterly AR(1) process, with a persistence parameter of 0.89, and the standard deviation of innovation being 0.015. The AR(1) shock process for total factor productivity is discretized to a three-point Markov chain, also using the Rouwenhorst (1995) method.

Parameter	Description	Value
β	Subjective discount factor	0.99
σ	Coefficient of relative risk aversion	2
χ	Coefficient on preference for labor	150
$1/\eta$	Frisch elasticity	1/2
μ	Mark-up	1.2
θ	Price adjustment cost	150
τ	Proportional tax rate	0.33
$\bar{\pi}$	Steady-state gross inflation rate	1.005
α	Coefficient on the Taylor rule	2.0

Table 1: Baseline Calibration

8 Computation

We compute a steady state for this economy where households face idiosyncratic income shocks but there are no shocks to TFP. As the aggregate of household savings is held by the government as public debt, we bisect the government's budget constraint to achieve an equilibrium bond price for this economy. I solve the households' problem using value function iteration and solve for the households' saving and consumption decision rules using the endogenous grid method.

In computing the stochastic equilibrium, I use the Backward Induction method due to Reiter (2010). The stochastic economy has an exogenous aggregate state, which is the TFP, and an endogenous aggregate state, which is the distribution of households in the economy. As the distribution of households is an infinite dimensional object, I approximate the aggregate endogenous state of the economy with the aggregate bond holdings, m. For approximating this, I use the steady-state distribution of households as the initial proxy distribution, and discretize the approximate endogenous state space over levels of bond holdings, such that the distribution yields a mean bond holding level equal to the value at that grid point. In other words, the dicretized grid for the levels of aggregate bond holdings is obtained around the mean of the steady-state level of bond holdings given the stationary distribution of households in the steady-state. The proxy distributions are computed at each point in the discretized aggregate state space, by adjusting the steady-state distribution in a way such that the resulting distribution yields a mean level of bond holdings equal to the grid value of the approximate aggregate state, which here is the level of aggregate bond holdings in economy m.

Utilizing these proxy distributions, I solve the real economy in the model in one run of backward iterations, on a discrete grid of points in the aggregate state space, while enforcing consistency between individual and aggregate solutions in each step of the backward iteration, which is done separately at each point on the grid of aggregate states (Reiter 2010).

Households' decision rules are solved for using the endogenous grid method on the grid for individual endogenous and exogenous states, that is, the bond holdings of households and the idiosyncratic income shocks, respectively. Consistency is achieved between individual and aggregate decisions, when the expected level of aggregate bond holdings in the economy becomes equal to that obtained from its law of motion. Once such consistency is achieved, for any given price of bonds q and an initial guess of inflation π in the economy, I iterate on wages to clear the labor market. As the production function of intermediate producers is linear in labor, the marginal product of labor is fixed and intermediate producers are willing to supply all that is demanded at the going wages (???). The partial equilibrium wage function obtained from iterating over the real economy until labor demand equals labor supply, therefore, clears both the labor and the goods market, for the initially guessed rate of inflation. The asset market is cleared by Walras' law. Using these partial equilibrium wages, and the output obtained from the aggregate resource constraint, for the initial inflation rate, I use the New Keynesian Phillips Curve to update current inflation, over expectations of future inflation and output. The general equilibrium is obtained once inflation, wages, the law of motion for aggregate bond holdings, and households' value functions have all converged up to a certain tolerance set for each of these. This method yields the stochastic equilibrium in this economy.

9 Results

9.1 Steady-State

In steady-state, bisecting the government's budget constraint results in an equilibrium bond price q of 0.9926, which is a quarterly risk-free real interest rate of 0.75 percent, which yields an annualized risk-free real interest rate of 3.02 percent per annum. In steady-state, the primary deficit-to-GDP ratio, given by $\left(G - \tau \cdot w \cdot e \cdot n\right)/Y$, is around 1.13 percent, whereas the deficit-to-GDP ratio, given by $\left(G - (1-q) \cdot B + G - \tau \cdot w \cdot e \cdot n\right)/Y$, is 2.28 percent in this model economy, which was around 6.4 percent in 2024 for the US economy, and the 50-year long-run average for which has been close to 3.6 percent for the US economy. The model economy, in steady-state, yields a debt-to-GDP ratio of about 153 percent, which is around 123 percent for the US economy in 2024. Government spending in the model economy in the steady-state is about 28.5 percent of GDP, where federal government spending was equal to 23 percent of the total gross domestic product (GDP) in the US economy in FY 2024. The steady-state inflation

Percentiles of wealth	1	5	10	50	90	< 0
Share of households	0.059	0.220	0.370	0.974	1.047	22.9

Table 2: Wealth distribution of households in steady-state

is set to 2 percent per annum, which is equivalent to 0.5 percent quarterly inflation rate. The distributions of wealth in the steady state are given in Table 2. The share of negative assets, that is households with zero or negative net worth, in the model economy is around 23 percent, which for the US economy is close to 11 percent. The steady state distribution for this economy also does not match the right tail of the wealth distribution in the US economy. However, for the purposes of this paper, we do not attempt to explain the distribution of wealth in the US economy; instead, the objective is to achieve a plausible distribution of wealth to study the effect of forward guidance in the presence of aggregate uncertainty. Hence, the model is not calibrated to match the wealth distributions from the data in the US economy. As for the effects of forward guidance and the precautionary saving motives of households in driving the effects of forward guidance, the need to insure against income shocks is small among the wealthiest households, and hence their consumption responses do not affect, in any significant way, the efficacy of forward guidance on aggregate consumption. The steady-state wealth distribution in the model yields a Gini coefficient of 0.627.

9.2 Stochastic Equilibrium and Forward Guidance

The key finding of my work is that forward guidance about an interest rate cut by 50 basis points, 20 periods ahead in the future, causes a rise in output by 36 basis points and a rise in consumption by 19 basis points relative to the risky stationary equilibrium at the date of the announcement of forward guidance. Over time, at the date of the rate cut, the increase in output is by 17 basis points, and in consumption is 24 basis points above its long run mean values in a risky world. These effects are much larger than a similar exercise done in perfect foresight in the literature [5].

The transmission of guidance about future interest rates to real consumption and labor supply is as for any conventional instrument of monetary policy. When real interest rates are reduced, the cost of borrowing falls, making borrowing for consumption more attractive. As a result, demand for consumption increases, causing production by firms to increase, which in turn causes firms to demand more labor to meet increased production demands. This is the direct intertemporal substitution effect of an interest rate cut. In general equilibrium, the increased demand for labor

[5] McKay, Nakamura, and Steinsson (2016) find the effect of such forward guidance to be 0.1 percent when markets are incomplete, compared to 0.25 percent in a representative-agent economy.

by firms drives up wages, causing labor supply to increase. Hence, households' labor incomes increase, therefore, causing an additional increase in demand for consumption and adding to the direct effects of an interest rate cut. These have been referred to as the "indirect" effects or the general equilibrium effects of a real interest rate cut (Kaplan, Moll, and Violante (2016)).

A proposed resolution of the "forward guidance puzzle" came from McKay, Nakamura, and Steinsson (2016) who study the effects of forward guidance about monetary policy with incomplete markets and household heterogeneity. As households face idiosyncratic shocks to income, there is greater uncertainty about their future incomes and wealth positions, causing precautionary savings motives to escalate. Additionally, when households are subject to borrowing constraints in the presence of such idiosyncratic income risks, communications about changes in interest rates far in the future dampen the current consumption response of households - those who experience negative income shocks persistently may run the risk of hitting their borrowing limits. To smooth consumption over their lifetimes, they respond very little to announcements about future interest rate changes.

For comparability with forward guidance experiments done in a perfect foresight setting in the literature, I do a simple policy experiment where the monetary authority announces a 50 basis points real interest rate cut, 20 periods in the future from the date of such announcement. I study the effects of such forward guidance about the real interest rate, on current output, consumption, and inflation, and their movement over time in the run-up to the date of the actual implementation of the policy.

To dissociate the effects of such forward guidance from the effects of the aggregate shocks that households face in ordinary times, and for comparison with exercises in perfect foresight, I shut down the TFP shock for the period between the date of announcement of the policy and the date of its implementation. While I compute household decision rules which still account for the risk of aggregate uncertainty, the households do not actually experience the realization of any such shock during this period.

Aggregate uncertainty amplifies the heterogeneous exposure of households, already experiencing idiosyncratic income shocks, to aggregate risks. Outside of the steady state, the risks due to idiosyncratic income shocks faced by households are exacerbated by the presence of aggregate uncertainty in the economy. This amplifies their precautionary savings motives, causing households to save more and the transmission of a policy shock to aggregate consumption and output to be weaker. This is an additional source of consumption volatility in this economy.

However, unconditional forward guidance - communication about a rate cut in the future irrespective of the then state of the economy - alleviates uncertainty for households when aggregate shocks occur, allowing them to rein in their precautionary savings, causing aggregate consumption to respond strongly to such forward guidance.

When the economy knows about a rate cut in the future with certainty, given credibility of the monetary authority in its commitment to implement the communicated rate cut, agents are aware that borrowing is going to be cheaper in the future. This stimulates borrowing and spending and overall demand increases. Households saving aggressively to hedge themselves against risks of idiosyncratic income shocks amplified by the presence of aggregate uncertainty, now know that they will be able to borrow easily when rates are cut at a certain date in the future, and hence reduce their current savings substantively. This causes demand for consumption to increase in the period leading up to the interest rate cut. Aggregate price levels increase due to the increased demand. Firms respond to this increased demand initially by increasing output as prices are sticky, and the quadratic costs on price changes make it costly for them to make large jumps. As firms foresee an increase in prices in the future due to the guidance about a future rate cut, they gradually increase prices balancing the desire to meet optimal prices and the cost of adjusting prices. Higher adjustment costs from change in prices as the date of the rate cut approaches, lead to more production, higher wages, and thus more inflation.

The government's reaction to forward guidance plays an important role in the effect of forward guidance on aggregate consumption, output, and inflation. Communications about future rate cuts benefit borrowers in the economy. The government is a net borrower in the economy. Indications about a future rate cut make borrowing more lucrative, and governments see an inflow of resources, which they can spend on consumption, or use to reduce net taxes on consumers, or make transfers to households (Violante (2021)). An increase in government spending directly increases consumption demand, hence labor demand, and amplifies the household-driven effects of a rate cut in the economy. If instead the government were to reduce taxes imposed on households or make transfers to households, the amplification would occur through the households who would have more resources to spend, causing enlarged effects on output, employment, and inflation in the economy.

The results of the forward guidance experiment hinge on the fiscal policy wielded by the fiscal authority. In the baseline model, the government maintains an exogenous and constant level of government spending at the steady-state level, across all points in the aggregate state space.

When the government follows a fiscal policy rule such that government spending is fixed at an exogenous constant value, the announcement of a rate cut in period 20 from the date of such guidance causes output to rise by 0.36 percent and consumption to rise by 0.19 percent on the date of the announcement. Over time, leading up

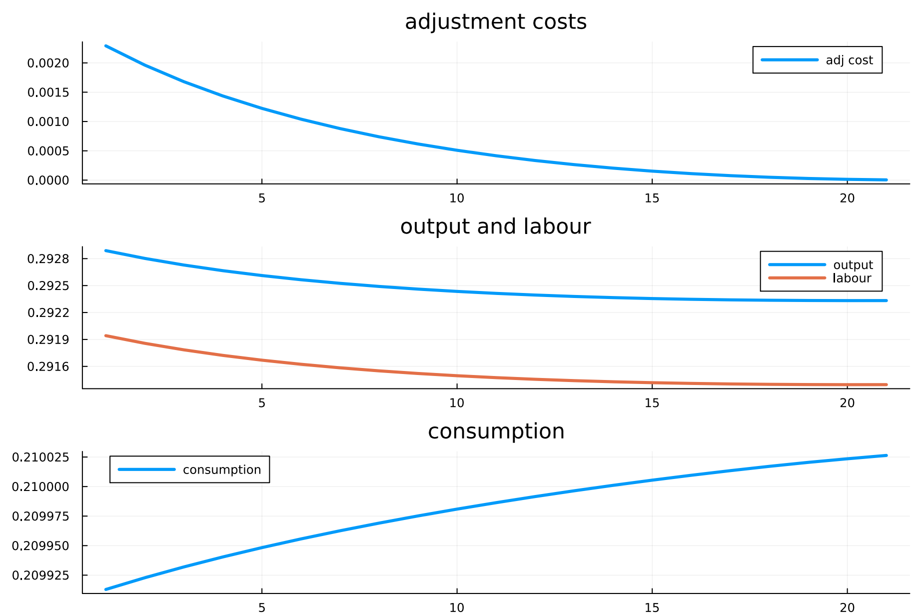

Figure 1: Effects of unconditional forward guidance for 20 periods in the future

to the date of the implementation of the rate cut, output gradually falls, whereas consumption rises. On the date of the actual rate cut, output ends up 0.17 basis points above its mean value, whereas consumption is 0.24 basis points above its mean. The fiscal rule of the government to maintain a constant level of government spending makes it look as if the government maintains a stable level of debt. Although not entirely comparable due to modeling differences, this is larger than the effect seen under perfect foresight, for the same experiment in McKay, Nakamura, and Steinsson (2016), where the corresponding changes in output on the date of the announcement is 0.1 percent when markets are incomplete, solved under the assumption of perfect foresight.

When forward guidance about future interest rate adjustments are unconditional, that is, when the rate cut is announced independent of the future outlook of the economy, such that the real interest rate across the aggregate state space would see a cut by 50 bps, then such announcement works to lessen the uncertainty that households face about the future, as they expect the rate cut irrespective of what the state of the economy might be in 20 periods from now. This is a substantive reassurance, especially when households that are trying to hedge against their idiosyncratic income shocks in the presence of aggregate uncertainty, know with certainty how interest rates will change in 20 periods from now. For this reason, saving motives are tempered and households actually consume more now and during this period before the rate cut is implemented, in anticipation of such rate cut.

Now let us look at another experiment where the monetary authority gives forward guidance conditional on the future state of the economy. Suppose the monetary authority announces that real interest rates in 20 periods from now, will see a 50 bps cut, only if the economy is in a state as it is at the time of the announcement; in all other states, status quo will be maintained. This leaves households uncertain about whether such rate cut will be implemented or not depending on what course the economy takes during this time. In this case, because of the presence of aggregate uncertainty and, hence, the inability of forward guidance conditional on the state of the economy to eliminate uncertainty about the future interest rates, brings back some of the precautionary savings motive of households at the date of such announcement, as well as during the period leading up to the date when the proposed policy could be implemented, depending on what state the economy ends up in.

In this case, output rises by 0.0925 percent on impact, that is, on the date of the announcement of such forward guidance, while consumption rises by 0.096 percent on impact. This is much less than the effect we see in the case where forward guidance mitigates uncertainty about the future by being unconditional in nature. On the date when such policy is implemented, conditional on the state of the economy on this date being the same as the state on the date of the announcement, the rise in output is almost by 0.17 percent, while rise in consumption is by about 0.24 percent. These effects are lower than in the experiment when forward guidance eliminates uncertainty about future interest rates, by being pervasive regardless of the state of the economy in period 20. In this case, inflation rises to 0.68 percent on the date of the announcement, and eventually falls to 0.52 percent in period 20, its long-run average being 0.5 percent for the state on which the forward guidance is conditioned. This is an increase of 0.18 percentage points.

The fiscal authority's reaction to such forward guidance can make a significant difference to the effect of forward guidance. I experiment with an alternate fiscal rule where the government adjusts its spending based on the following rule:

$$G = G_{ss} - 0.1\,(B - B_{ss}) \tag{60}$$

By using this fiscal rule, the government accommodates changes in current debt caused by forward guidance by changing government spending, such that when current debt exceeds its steady-state level, government spending falls below the steady-state level of spending.

Re-examining unconditional forward guidance, where the monetary authority indicates that real rates will be cut by 50 bps at date 20, irrespective of the state of the economy in period 20, we see different outcomes for output and consumption.

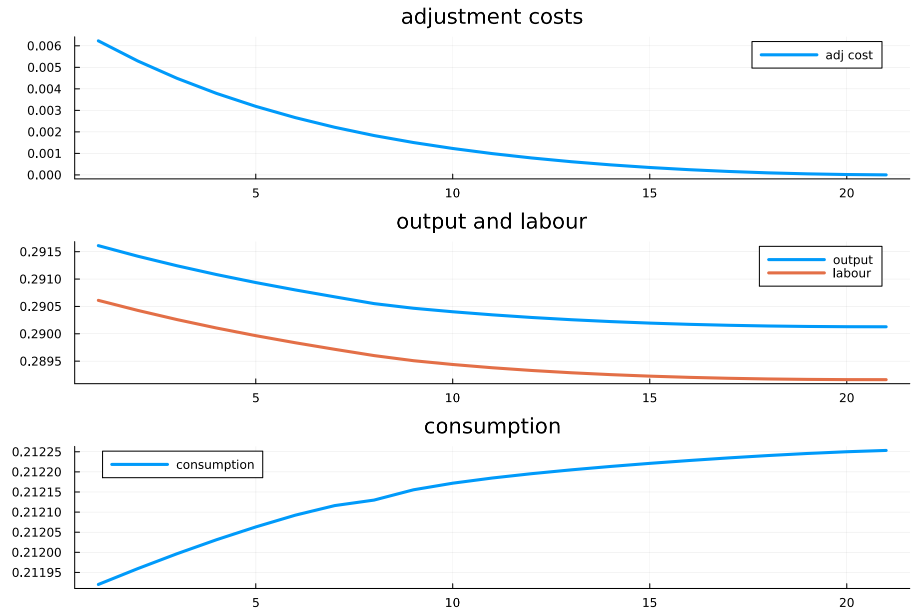

Figure 2: Effect of forward guidance of a 50 bps cut in real interest rate at date 20

When such forward guidance is announced, output increases immediately by 0.68 percent, whereas consumption increases only by 0.071 percent on impact. The increase in output gradually falls to about 0.17 percent, and that in consumption gradually increases to 0.23 percent. This is shown in figure 2.

Output rises by a lot more than in the incomplete markets case in perfect foresight in McKay, Nakamura, Steinsson (2016) [6]. This is because of a large effect on inflation, which causes firms to increase prices because there is now a higher probability of a rate cut at date 20. Inflation rises from 0.55 percent to 1.41 percent. Higher adjustment costs lead to more production, higher wages and thus more inflation. This, in turn, raises adjustment costs.

With this fiscal rule in place, there is not much difference between the impacts of conditional and unconditional forward guidance on output and consumption. However, inflation rises by much less, to only 1.02 percent, which too is a relatively large impact compared to the incomplete markets case in McKay, Nakamura, and Steinsson (2016). They, however, have no government spending in their model, and hold debt constant at the steady state level, such that the government maintains a balanced budget in every period.

These results point to a resurrection of the forward guidance puzzle. The more pertinent question to ask is if there is a puzzle at all. The effects of forward guidance

[6] In McKay, Nakamura, Steinsson (2016) output rises by 0.1 percent in incomplete markets, and by 0.25 percent in complete markets.

seem to be very sensitive to the kind of fiscal policy or rule used by the government to accommodate the monetary policy action. This is well-known in the literature - effects of monetary policy are contingent on how the government responds to it and how the effects of such policy are absorbed. Alves et al. (2020) makes this point about the criticality of the government's response in determining the magnitude of the effect of monetary policy on aggregate consumption and output.

10 Conclusion

In this paper, I study the effect of forward guidance in an incomplete market setting with aggregate uncertainty. Households face risks from uninsurable idiosyncratic income shocks which are compounded by the presence of aggregate uncertainty. Monopolistically competitive firms face price adjustment costs as in Rotemberg (1982) in resetting prices every period which is a source of nominal rigidities in the economy.

Given the state-contingent nature of forward guidance on monetary policy based on the expected state of the economy in the future, adopted by the US Federal Reserve in recent years, studying the effect of forward guidance in the presence aggregate uncertainty is a natural extension of the existing literature on forward guidance.

I solve the model to find a global solution for the economy which is a stochastic equilibrium, and utilize that to study the effect of a forward guidance experiment where the monetary authority announces a 50 bps real interest rate in 20 periods from now, and examine how aggregate consumption, output, and inflation respond to such forward guidance about future interest rates. The results show that unconditional forward guidance has large effects on aggregate output and consumption even in the presence of incomplete markets, when there is aggregate uncertainty. Forward guidance that conditions such a rate cut in the future on a certain state of the economy being realized in period 20 has relatively smaller effects, but these are still quite large in comparison to a similar experiment conducted in perfect foresight in the literature [7].

My results suggest a re-emergence of the forward guidance "puzzle" when aggregate uncertainty is introduced in the environment and the model is solved globally to preserve nonlinearities in agents' behavior and elasticities of aggregate functions outside of steady state to study the effects of forward guidance in such a setting.

My findings contradict previous findings [8] in the literature that the forward guidance puzzle is largely resolved, in a setting with incomplete markets. Instead, I find that forward guidance can be significantly powerful in affecting contemporaneous

[7] McKay, Nakamura, and Steinsson (2016)
[8] McKay, Nakamura, and Steinsson (2016)

output, consumption, and inflation, even with incomplete markets, the degree of its power depending on the state-based nature of such announcements.

My work augments the existing literature on HANK by taking it in a previously unexplored direction, that is, having aggregate uncertainty modeled in a heterogeneous agent New Keynesian framework, and finding a global solution that captures the nonlinear consumption and labor supply responses of households outside of steady state, in the absence of perfect foresight, in the presence of such aggregate risks. My model and results point to a new direction for further research and exploration to inform such policy announcements to have more power and influence on aggregate macroeconomic variables of interest. This, in my view, is a first-order contribution to academic and policy-relevant research.

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