

# Forward Guidance in an Uncertain World

Smriti Verma\*

31 December 2025

[Click here for the latest version of the paper](#)

## Abstract

I investigate the impact of state-contingent forward guidance in monetary policy under incomplete markets and aggregate uncertainty. I develop a novel recursive approach using a global solution method that enables a thorough quantitative study of forward guidance contingent on macroeconomic states—a question that has not been addressed previously in the literature. My results reveal that state-contingent forward guidance generates substantially smaller effects on current output, consumption, and inflation than non-state-contingent or date-based guidance, which is implemented based on a future calendar date, irrespective of future macroeconomic conditions. Households facing aggregate and idiosyncratic risks have stronger precautionary motives for saving than in the absence of aggregate uncertainty and respond even less to state-contingent policy announcements: state-contingent forward guidance leaves room for uncertainty about future rate shifts and only partially increases the predictability of future policy, thereby limiting its effectiveness. A global solution method in my work allows for a more accurate analysis of state-contingent forward guidance under aggregate uncertainty, unexplored in prior studies, and infeasible with methods in the literature that rely on local approximations around the steady state or on perfect foresight, making my work a significant contribution to the literature on forward guidance and, more generally, a methodological innovation allowing for the study of broader monetary policy issues in dynamic stochastic environments.

---

\*Ohio State University (email: verma.211@osu.edu). I sincerely thank my advisor, Aubhik Khan, for his invaluable advice and guidance. I am immensely thankful to Julia Thomas, Kyle Dempsey, and Gabriel Mihalache for their valuable comments and discussions. All errors are, however, my own.

# 1 Introduction

Forward guidance in monetary policy refers to central banks’ communication about the future path of short-term interest rates, intended to influence long-run inflation expectations and economic decisions today. Over the last two decades, forward guidance has evolved from terse statements about the central bank’s future economic outlook and anticipated policy response to more explicit commitments, in which the central bank announces target ranges contingent on future macroeconomic conditions or on targets for unemployment or inflation. I examine the effects of this evolved form of state-contingent forward guidance in an incomplete markets environment under aggregate uncertainty. I develop a stochastic dynamic general equilibrium model with aggregate uncertainty to quantify how announcements about future interest rate shifts, contingent on the realization of specific macroeconomic states, affect current macroeconomic variables.

Prior literature has emphasized the “forward guidance puzzle”, where standard DSGE models predicted implausibly large effects of announcements of future policy on present macroeconomic variables <sup>1</sup>. Extending this line of inquiry, my analysis reveals that state-contingent forward guidance—where future policy rate paths depend explicitly on the state of the economy—has substantially smaller effects on output, consumption, and inflation compared to non-state-contingent or date-based guidance, which is conditional only on a future calendar date and is implemented irrespective of the future state of the economy.

This diminished impact arises because households facing uninsurable idiosyncratic income risks, borrowing constraints, and aggregate uncertainty exhibit stronger precautionary savings than in the absence of aggregate uncertainty. These factors attenuate their responsiveness to policy announcements that only partially resolve uncertainty about future interest rates. To better understand this behavior, I introduce a new methodological approach that uses a global solution method to study state-contingent forward guidance, highlighting the importance of nonlinearities and uncertainty in shaping its efficacy.

McKay, Nakamura, and Steinsson (2016) show that the power of forward guidance is highly sensitive to the assumption of complete markets—when agents face uninsurable income risks and borrowing constraints, the precautionary saving motive tempers their contemporaneous response to changes in future interest rates. I augment this framework with aggregate uncertainty to examine how state-contingent announcements of future policy rate changes affect current macroeconomic outcomes.

---

<sup>1</sup>Del Negro, Giannoni, and Patterson (2012); Gurkaynak, Sack, and Swanson (2005); Campbell, Fisher, and Justiniano (2012); Milani and Treadwell (2012); Campbell et al. (2017)

My results demonstrate that state-contingent forward guidance is nearly half as effective in influencing current output, consumption, and inflation as date-based forward guidance, which is implemented irrespective of the future state of the economy. Importantly, even non-state-contingent or date-based forward guidance, in the presence of aggregate uncertainty and incomplete markets, tends to have contemporaneous effects that are smaller than those predicted by McKay, Nakamura, and Steinsson (2016) under perfect foresight. This is because households tend to hold savings to hedge against possible future adverse economic shocks, reflecting an uncertain world.

The US Federal Reserve started using date-based<sup>2</sup> forward guidance in August 2011. However, date-based forward guidance posed the risk of destabilizing long-term inflation expectations because the public was likely to misinterpret the central bank’s announcement of holding interest rates low until a future date as the bank’s dismal outlook for future economic conditions. This risked producing outcomes that run counter to the very objective of forward guidance: anchoring long-run inflation expectations, thereby shifting real rates without changing the nominal rate. To overcome this drawback, the US Federal Reserve moved away from date-based guidance to state-contingent forward guidance in December 2012, when it started announcing target ranges for interest rates contingent on specific macroeconomic conditions or targets, such as the unemployment rate, long-term inflation expectations, labor market conditions, the evolving outlook of the economy, and the balance of risks<sup>3</sup>.

A flavor of state contingency in Federal Open Market Committee (FOMC) statements can be seen in this excerpt from the FOMC Press Release on 29 January 2025: “In support of its goals, the Committee decided to maintain the target range for the federal funds rate at 4-1/4 to 4-1/2 percent. In considering the extent and timing of additional adjustments to the target range for the federal funds rate, the Committee will carefully assess incoming data, the evolving outlook, and the balance of risks.”. The release further announces: “The Committee would be prepared to adjust the stance of monetary policy as appropriate if risks emerge that could impede the attainment of the Committee’s goals. The Committee’s assessments will take into account a wide range of information, including readings on labor market conditions, inflation pressures and inflation expectations, and financial and international developments.”. These elements began to appear in the FOMC press releases over a

---

<sup>2</sup>Federal Open Market Committee (FOMC) Press Release on December 12, 2012 used this nomenclature to describe previous guidance contingent on a future calendar date

<sup>3</sup>The FOMC Press Release on December 12, 2012 stated that it “currently anticipates that this exceptionally low range for the federal funds rate will be appropriate at least as long as the unemployment rate remains above 6-1/2 percent, inflation between one and two years ahead is projected to be no more than a half percentage point above the Committee’s 2 percent longer-run goal, and longer-term inflation expectations continue to be well anchored.”

decade ago and have since been used in very similar language in its announcements regarding monetary policy.

The objective of this paper is to study forward guidance in its contemporary state-contingent form and analyze its effects on contemporaneous macroeconomic outcomes in light of the “forward guidance puzzle”. To the best of my knowledge, no study in the existing literature has examined this question or developed a framework that accounts for aggregate uncertainty using a global solution method to investigate the effects of state-contingent forward guidance, thereby allowing the stochastic steady state to differ from the deterministic one.

The study of aggregate uncertainty in heterogeneous-agent New Keynesian models has mostly been limited to local approximations around the steady state <sup>4</sup> and the study of dynamics under the assumption of perfect foresight <sup>5</sup>. While computationally fast, local perturbation methods rely on the assumption that agents’ steady-state behavior is relevant even in the presence of aggregate uncertainty, to some order of approximation. My work shows that local approximations around the steady state may not accurately capture the elasticities of aggregate responses to price changes in the presence of aggregate uncertainty.

I augment a one-asset, heterogeneous-agent New Keynesian model without capital, in which nominal rigidities arise due to price adjustment costs faced by intermediate firms, as in Rotemberg (1982). To allow for nonlinearities in household and firm behavior that may exist outside the steady state, I solve the model recursively using Backward Induction—a global solution method due to Reiter (2010)—to compute the stochastic general equilibrium. Households’ policy functions and aggregate economic responses, thus, incorporate all relevant individual and actual or approximated aggregate state variables, thereby aptly capturing both individual and aggregate risks and their importance in macroeconomic dynamics. I use this globally computed stochastic general equilibrium to study the effects of state-contingent and date-based forward guidance on current consumption, output, and inflation.

A household’s individual endogenous state is its bond holdings, and its individual exogenous state is the idiosyncratic income shock it faces. The aggregate exogenous state is the shock to aggregate productivity, and the aggregate endogenous state is the distribution of households. For computation, I use aggregate bond holdings by households—a moment of the actual distribution of households—to approximate the economy’s aggregate endogenous state.

In the baseline exercise, I compare the effects of forward guidance of a 50-basis-

---

<sup>4</sup>Adrien Auclert, Bence Bardóczy, Matthew Rognlie, and Ludwig Straub (2021); Cosmin L. Ilut, Ralph Luetticke, and Martin Schneider (2025), Christian Bayer, Ralph Luetticke, Lien Pham-Dao, and Volker Tjaden (2018)

<sup>5</sup>McKay, Nakamura, Steinsson (2016)

point rate cut 20 periods in the future, when the guidance is state-contingent with those when it is date-based. A comparison of the effects of state-contingent and date-based forward guidance on current macroeconomic variables highlights the role of uncertainty in shaping the effectiveness of such guidance. I conduct the baseline experiment for state-contingent and date-based forward guidance, using alternative values of two crucial parameters to assess the sensitivity of my results.

State-contingent forward guidance is an announcement of a 50-basis-point rate cut 20 periods in the future, conditional on the aggregate economic circumstances remaining unchanged from the time of the policy announcement to its implementation. Such an announcement increases current output by 2.5 basis points and current consumption by 3 basis points. The magnitude of the effects of state-contingent forward guidance is nearly half that of a similar guidance when it is purely date-based. This result demonstrates the importance of aggregate uncertainty in assessing the efficacy of forward guidance that is contingent on future macroeconomic conditions and the state of the economy.

In an alternative form of state-contingent forward guidance, the central bank announces that it will cut rates if and when the economy enters a recession and will keep rates low until the economy recovers. This is referred to as “lower-for-longer” forward guidance, where the central bank promises to cut rates in the future, conditional on being in a recession, and promises extended durations of low rates until the economy is out of the recessionary state. This form of state-contingent forward guidance is also shown to have even smaller effects compared to a corresponding date-based announcement.

Additionally, the presence of aggregate economic uncertainty tempers the power of forward guidance, even when such guidance is date-based, as my results demonstrate, relative to the effects of forward guidance with incomplete markets under perfect foresight, as in McKay, Nakamura, and Steinsson (2016). For the same magnitude and horizon of forward guidance, current output and consumption increase by 5 and 7 basis points, respectively, in my environment, compared to 10 basis points in their perfect-foresight environment. The difference in the impact on inflation, however, is much larger. While they report a 30-basis-point increase in current inflation following such an announcement, date-based forward guidance in my model only causes a 0.5-basis-point increase in inflation at the time of the announcement.

Forward guidance tied to specific economic conditions only partially reduces uncertainty about future policy rates. Households cannot increase consumption significantly to take advantage of this announced rate cut because they remain exposed to unpredictable income shocks and aggregate economic shocks, and are uncertain about future policy rates. Borrowing constraints further restrict their ability to in-

crease consumption in anticipation of partly uncertain future policy changes, since the risk of reaching their borrowing limits shortens their planning horizon and mutes their response further to guidance about distant future rates. Aggregate consumption and output, accordingly, increase less at the announcement date when the forward guidance is state-contingent.

The presence of aggregate economic uncertainty necessitates a passive fiscal policy that accommodates the growing stock of household savings driven by enhanced precautionary motives. It does so by adjusting government spending to keep public debt from spiraling. In the forward guidance experiments, to disentangle the effects of forward guidance, I suppress the realization of aggregate shocks in the economy. However, since households internalize the stochastic nature of the economy through their decision rules computed under aggregate uncertainty, they save not in response to materializing shocks, but due to heightened perceived future risks. Thus, in an uncertain world, a passive fiscal policy plays a crucial role in accommodating a large and growing supply of savings, helping stabilize debt at sustainable levels.

The remainder of this paper is organized as follows. In section 2, I discuss the related literature and where my work fits in, section 3 lays out the model environment, with section 4 describing the decision problems faced by the agents in the economy, section 5 detailing a recursive formulation of the economy, and section 6 defining the equilibrium for this economy, followed by a brief discussion of the choice of parameters for the model in section 7. Further, I discuss the methods used for computation in section 8 and present the results and their discussion in section 9, and finally, section 10 concludes.

## 2 Related Literature

The literature on forward guidance has been driven by the “forward guidance puzzle” (Del Negro, Giannoni, and Patterson 2012) – estimated DSGE models, while successfully explaining the effects of contemporaneous monetary policy actions on key macroeconomic variables, grossly exaggerated the effects of announcements of future policy. The central bank’s announcement of lower interest rates eight quarters in the future implied massive stimulus, with the impact on both output and inflation several times larger than that of a same-sized contemporaneous drop in the policy rate (Del Negro et al, 2013). Gurkaynak, Sack, and Swanson (2005) measure the effects of a “future path of policy”, closely associated with FOMC statements, on bond yields and stock prices, and find that both monetary policy actions and statements have important but differing effects on asset prices, with FOMC statements having a much larger impact on longer-term Treasury yields. In a similar flavor, Campbell,

Fisher, and Justiniano (2012), and Milani and Treadwell (2012) studied the impact of forward guidance shocks on the economy using small- or medium-scale DSGE models estimated on a pre-Great Recession sample, and both papers find that forward guidance shocks play a large role in explaining movements in the policy rate, and these contribute significantly to business cycle fluctuations.

Campbell et al. (2012) assessed the impact of exogenous changes in monetary policy on private expectations and found the sign was the opposite of the theoretical predictions, so that accommodative monetary policy was associated with lower projections of inflation and activity. In view of these considerations, the FOMC replaced the calendar-based language in the December 2012 statement and switched to announcing policy intentions tied to economic performance on inflation and real activity, and to the state of the economy more generally.

Campbell et al. (2017) use a full-information, rational-expectations framework and find that FOMC forward guidance has mixed effects. The main goal of their study is to assess whether the FOMC improved economic performance since the financial crisis through Odyssean <sup>6</sup> forward guidance, and to quantify the impact of Odyssean FOMC forward guidance on macroeconomic outcomes since the financial crisis, which unwound from 2007 to 2009. They find that puzzling responses of private-sector forecasts to FOMC announcements can be attributed entirely to Delphic <sup>7</sup> forward guidance, even though a large fraction of the variability in Federal Funds Futures rates on days with FOMC announcements remained unexplained by their measure of FOMC private information. Interestingly, while their findings suggest that purely rule-based policy would have ameliorated the recession and kept inflation closer to target in the years immediately following the crisis, than FOMC forward guidance did in practice, the Fed’s introduction of calendar-based Odyssean forward guidance starting toward the end of 2011 appeared to have boosted real activity and moved inflation closer to target.

A large portion of the forward guidance literature has focused on resolving the forward guidance puzzle. The forward guidance puzzle—large contemporaneous effects from a future policy announcement—arises from intertemporal substitution as the main channel through which forward guidance is transmitted into the real economy. As a result, departure from the complete markets, full-information rational expectations framework alleviates the forward guidance puzzle. The macroeconomic resolution of the puzzle comes from adding heterogeneity, such as in McKay, Nakamura, and Steinsson (2016), Hagedorn et al. (2019), and Acharya and Dogra (2020).

---

<sup>6</sup>Odyssean forward guidance is a statement about the path of future policy rates

<sup>7</sup>Delphic guidance is a statement about the future outlook of the economy and anticipated policy response

McKay-Nakamura-Steinsson (2016) (MNS) show that forward guidance has substantially less power to stimulate the economy in the presence of uninsurable risk and borrowing constraints. It is not realistic to assume that agents increase their consumption by the same amount in response to a future interest rate cut as they do in response to a current rate cut, for two reasons: borrowing constraints and precautionary savings. People face a risk of hitting a borrowing limit, which shortens their planning horizon, because interest rate changes that occur after they hit a borrowing limit are irrelevant to them. The presence of uninsurable income risks leads to precautionary savings, which temper households' responses to future interest rate shocks. As a result, the power of forward guidance is substantially muted in the incomplete markets model compared to the standard complete-markets New Keynesian model.

Other departures from full information set ups and rational expectations include Chung, Herbst, and Kiley (2014), Carlstrom, Fuerst, and Paustian (2015), and Kiley (2016) which use sticky information models à la Mankiw and Reis (2002); Angeletos and Lian (2018) who relax the assumption of common knowledge; and other works that depart from rational expectations in the form of bounded rationality (Gabaix 2015), finite planning horizons (Woodford 2019), or by modeling the cognitive process of expectation formation and use the concept of reflective equilibrium (Garcia-Schmidt and Woodford 2019).

Given how critically the efficacy of forward guidance depends on the credibility of the central bank's commitment to announcements about future policy, several papers discuss the imperfect credibility of the central bank to resolve the forward guidance puzzle (Bodenstein, Hebden, and Nunes 2012; Haberis, Harrison, and Waldron 2014; Campbell et al. 2019; Bernanke 2020).

My work contributes to the literature on forward guidance by introducing aggregate uncertainty and using a global solution method, thereby enabling the study of state-contingent forward guidance in its most evolved form. The use of a global solution method, unprecedented in the literature on forward guidance and in HANK models more generally, enables the stochastic dynamic general equilibrium to capture nonlinearities in households' responses to shocks under aggregate economic uncertainty. To the best of my knowledge, mine is the first work to solve for a fully stochastic general equilibrium in a HANK model with aggregate uncertainty and to leverage the model to study the effects of state-contingent forward guidance, making a notable contribution to the literature on forward guidance and HANK.

My results question the existence of the forward guidance puzzle when aggregate economic risks are adequately accounted for, thereby preserving the economy's stochastic nature, in examining the effects of forward guidance, especially when it is state-contingent. My model and methodology provide a rich, general framework



for studying a variety of monetary policy issues in more realistic, uncertain economic environments, where nonlinearities can play a significant role in shaping aggregate macroeconomic dynamics. My results provide insights into how forward guidance communication can be tailored to enhance its efficacy in an uncertain world.

### 3 Model

**Households** There is a continuum of a unit mass of ex-ante identical households with preferences given by

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[ \frac{c_{h,t}^{1-\sigma}}{1-\sigma} - \chi \frac{n_{h,t}^{1+\psi}}{1+\psi} \right] \quad (1)$$

where  $c_{h,t}$  is consumption of household  $h$  at time  $t$  and  $n_{h,t}$  is labor supply of household  $h$  at time  $t$ . Each household faces an uninsurable idiosyncratic productivity shock  $e_{h,t}$  which follows a Markov chain with transition probabilities  $\Pr(e_{h,t+1}|e_{h,t})$ , and earns real labor income before tax  $w_t e_{h,t} n_{h,t}$ . By assumption, the initial cross-sectional distribution of idiosyncratic productivities is equal to the ergodic distribution of this Markov chain.

In addition to holding stakes in the firm, households also have access to a risk-free real bond  $b$  with price  $q_t$ . A household's period  $t$  budget constraint is given by

$$c_t + q_t \cdot b_{t+1} \leq b_t + (1 - \tau_t) \cdot w_t \cdot e_t \cdot n_t + d_t \quad (2)$$

where  $c_t$  is real consumption,  $q_t$  is the price of the bond between period  $t$  and  $t + 1$ ,  $b_{t+1}$  and  $b_t$  are the real risk-free bond holdings of the household in period  $t + 1$  and period  $t$ , respectively,  $w_t$  is the real wage rate, and  $d_t$  is the real dividend earnings of the household.

**Final-good producer** A competitive representative final good producer aggregates a continuum of intermediate inputs indexed by  $j \in [0, 1]$  through a Dixit-Stiglitz constant elasticity aggregator as follows

$$Y_t = \left( \int_0^1 y_{j,t}^{\frac{\varepsilon-1}{\varepsilon}} dj \right)^{\frac{\varepsilon}{\varepsilon-1}} \quad (3)$$

where  $Y_t$  is the quantity of the final good produced by the final good firm at time  $t$ ,  $y_{j,t}$  is the quantity of the intermediate good produced by the firm  $j$  in period  $t$ , and  $\varepsilon$  is the price elasticity of substitution across the varieties of the intermediate good. Profit maximization by the final good producer implies the following demand for a

typical intermediate input variety  $j$

$$y_{j,t} = \left( \frac{p_{j,t}}{P_t} \right)^{-\varepsilon} Y_t \quad (4)$$

where  $p_{j,t}$  is the price set by the producer of intermediate input variety  $j$ , and  $P_t$  is the aggregate price index given by

$$P_t = \left( \int_0^1 p_{j,t}^{1-\varepsilon} dj \right)^{\frac{1}{1-\varepsilon}} \quad (5)$$

**Intermediate goods producers** Each intermediate input variety  $j$  is produced by a monopolistically competitive producer according to the production function

$$y_{j,t} = z_t n_{j,t} \quad (6)$$

where  $z_t$  is the aggregate total factor productivity (TFP) common across all intermediate firms, and  $n_{j,t}$  denotes the amount of labor hired by firm  $j$  in period  $t$ .

Intermediate goods producers hire labor at real wage  $w_t$  in a competitive labor market such that wage equals the marginal revenue product of labor.

Each intermediate producer chooses its price to maximize lifetime profits subject to price adjustment costs as in Rotemberg (1982). These adjustment costs are quadratic in the rate of price change  $\frac{p_{j,t}}{p_{j,t-1}}$  and are expressed as a fraction of real aggregate output  $Y_t$  as

$$\Theta_t = \frac{\theta}{2} \left( \frac{p_{j,t}}{p_{j,t-1}} - \bar{\pi} - 1 \right)^2 Y_t \quad (7)$$

where  $\theta > 0$  is the cost of the price adjustment.

Profits produced by the intermediate firms are distributed immediately to the households such that each household receives an equal share  $d_t$ . Households cannot trade their stakes in the firms.

**Aggregate uncertainty** The source of aggregate uncertainty is a shock to the exogenous aggregate state of the economy, i.e., the total factor productivity ( $z_t$ ) in the intermediate firms' production functions, which follows an AR(1) process .

$$\log z_{t+1} = \rho_z \log z_t + \eta_t \quad (8)$$

where  $\eta \sim N(0, \sigma_z^2)$  which can be discretized using the Rouwenhorst method so that  $z \in \{z_1, z_2, z_3\}$  and follows a Markov chain with transition probabilities  $\Pr(z_{t+1} = z_j | z_t = z_i) = \pi_{ij}^z$  giving the probability of transition from the state  $z_i$  in period  $t$  to

the state  $z_j$  in period  $t + 1$ .

**Fiscal authority** The government levies a proportional tax on household labor income at the rate  $\tau_t$  and holds a stock of outstanding government debt with real face value  $B_t$  such that positive values of  $B$  denote government debt. The government budget constraint is given as

$$q_t \cdot B_{t+1} = B_t + G_t - \tau_t \cdot w_t \int e \cdot n_t \cdot \mu_t d\mu_t \quad (9)$$

which can be rewritten as

$$q_t \cdot B_{t+1} = B_t + G_t - T_t \quad (10)$$

where  $G_t$  is government spending in period  $t$ , and  $T_t = \tau_t \cdot w_t \int e \cdot n_t \cdot \mu_t d\mu_t$ . The government's budget constraint can be written in terms of the primary surplus  $S_t \equiv T_t - G_t$  as

$$q_t \cdot B_{t+1} = B_t - S_t \quad (11)$$

The government adjusts primary surplus by changing government spending as per the following fiscal rule to stabilize debt

$$S_t = \rho_b(B_t - \bar{B}) + \bar{S} \quad (12)$$

where  $\rho_b > 0$  is the fiscal responsiveness to debt deviations, and  $\bar{S}$  is a target primary surplus given as  $\bar{T} - \bar{G}$ .

**Monetary authority** The monetary authority sets nominal interest rates using a Taylor rule given by <sup>8</sup>

$$i_t = r^* + \pi_t + \alpha(\pi_t - \bar{\pi}) \quad (13)$$

where  $r^*$  is the long-run equilibrium real interest rate, and  $\bar{\pi}$  is the monetary authority's target inflation rate. where the coefficient  $\alpha$  must be greater than 1 for determinacy.

The relationship between the current real interest rate  $r_t$ , the expected future inflation rate  $\pi_{t+1}$ , and the current nominal interest rate  $i_t$ , is given by the Fisher relation as

$$1 + r_t = \frac{1 + i_t}{\mathbb{E}_t[1 + \pi_{t+1}]} \quad (14)$$

where  $\pi_{t+1}$  is the net inflation rate given as  $\pi_{t+1} \equiv \frac{P_{t+1}}{P_t} - 1$  and  $P_t$  is the aggregate

---

<sup>8</sup>See Kaplan et al. (2018)

price level. The real interest rate and the expected future inflation rate imply a nominal interest rate determined by the Fisher equation, such that the real return on assets  $r_t$  must be consistent with equilibrium in the bond market.

## 4 Decision Problems

### 4.1 Decision problem of the household

Households face the following decision problem in this economy

$$\max_{\{c_{h,t}, n_{h,t}, b_{h,t+1}\}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[ \frac{c_{h,t}^{1-\sigma}}{1-\sigma} - \chi \frac{n_{h,t}^{1+\psi}}{1+\psi} \right] \quad (15)$$

subject to their period budget constraint,

$$c_t + q_t \cdot b_{t+1} \leq b_t + (1 - \tau_t) \cdot w_t \cdot e_t \cdot n_t + d_t \quad (16)$$

the borrowing constraint,

$$b_{t+1} \geq \underline{b} ; \underline{b} < 0 \quad (17)$$

the non-negativity constraint,

$$c_t \geq 0 \quad (18)$$

Each household chooses consumption and labor optimally by satisfying its Euler equation given by

$$u'_c(c_{h,t}, n_{h,t}) = \beta \mathbb{E}_t \left[ (1 + r_{t+1}) u'_c(c_{h,t+1}, n_{h,t+1}) \right] \quad (19)$$

which can be written in terms of the discount bond price as

$$q_t = \beta \mathbb{E}_t \left[ \frac{u'_c(c_{h,t+1}, n_{h,t+1})}{u'_c(c_{h,t}, n_{h,t})} \right] \quad (20)$$

and its labor-leisure condition, given by

$$\chi \cdot n_{h,t}^\psi = c_{h,t}^{-\sigma} \cdot w \quad (21)$$

which, together with the household's budget constraint, yield the optimal choices of consumption and leisure for each household.

## 4.2 Decision problem of the final good producer

The final good producer faces the following profit maximization problem

$$\max_{y_{j,t}} \left\{ P_t \left( \int_0^1 y_{j,t}^{\frac{\varepsilon-1}{\varepsilon}} dj \right)^{\frac{\varepsilon}{\varepsilon-1}} - \int_0^1 p_{j,t} y_{j,t} dj \right\} \quad (22)$$

which yields the following demand for each intermediate good variety  $j$

$$y_{j,t} = \left( \frac{p_{j,t}}{P_t} \right)^{-\varepsilon} Y_t \quad (23)$$

where  $p_{j,t}$  is the price charged by firm producing intermediate input variety  $j$  in period  $t$  and  $P_t$  is the aggregate price index in period  $t$  given by

$$P_t = \left( \int_0^1 p_{j,t}^{1-\varepsilon} dj \right)^{\frac{1}{1-\varepsilon}} \quad (24)$$

## 4.3 Decision problem of intermediate goods producers

Intermediate goods producers face a common real wage  $w_t$ . Due to nominal rigidities, intermediate firms are unable to adjust prices each period to maximize their profits. Therefore, they act to minimize cost. The cost minimization problem faced by intermediate firms, subject to producing enough to meet demand, is given as

$$\min_{n_{j,t}} \{ w_t n_{j,t} \} \quad (25)$$

subject to

$$z_t n_{j,t} \geq y_{j,t} \quad (26)$$

and

$$y_{j,t} = \left( \frac{p_{j,t}}{P_t} \right)^{-\varepsilon} Y_t \quad (27)$$

Cost minimization by intermediate goods producers implies that marginal cost is identical across intermediate goods firms and is given by

$$mc_t = \frac{w_t}{z_t} \quad (28)$$

Each intermediate firm seeks to maximize its expected present discounted value of future profits for its shareholders. In doing so, each intermediate producer solves the following maximization problem

$$\max_{p_{j,t}} \mathbb{E}_t \left[ \sum_{s=0}^{\infty} \mathcal{Q}_{t \rightarrow t+s} \frac{P_t}{P_{t+s}} \tilde{V}_{j,t+s} \right] \quad (29)$$

subject to

$$y_{j,t} = \left( \frac{p_{j,t}}{P_t} \right)^{-\varepsilon} Y_t \quad (30)$$

where  $\mathcal{Q}_{t \rightarrow t+s}$  is the  $s$ -period ahead real stochastic discount factor and  $\mathcal{Q}_{t \rightarrow t+s} \frac{P_t}{P_{t+s}}$  is the  $s$ -period ahead nominal stochastic discount factor. The firm discounts future profits by the inverse of the real interest rate in this economy. Hence, we can substitute for  $\mathcal{Q}_{t \rightarrow t+s} = \prod_{k=0}^{s-t} \frac{1}{1+r_{t+k}}$ , which can be expressed in terms of the discount bond price as  $\mathcal{Q}_{t \rightarrow t+s} = \prod_{k=0}^{s-1} q_{t+k}$ . Further,  $\tilde{V}_{j,t+s}$  is the  $s$ -period ahead nominal profit for an intermediate producer of a typical input variety  $j$ , that is, nominal profit at time  $t+s$ , with price  $p_{j,t}$  chosen at time  $t$ , and is given by its per-period nominal flow profit in period  $t+s$  net of nominal price adjustment cost in period  $t+s$  as

$$\tilde{V}_{j,t+s} = p_{j,t+s} y_{j,t+s} - mc_{t+s} y_{j,t+s} P_{t+s} - \frac{\theta}{2} \left( \frac{p_{j,t+s}}{p_{j,t+s-1}} - \bar{\pi} - 1 \right)^2 Y_{t+s} P_{t+s} \quad (31)$$

Substituting for  $\tilde{V}_{j,t+s}$  from equation (31) and  $y_{j,t}$  from the constraint in equation (30) in equation (29), the problem of an intermediate producer of a typical input variety  $j$  can be rewritten as

$$\max_{p_{j,t}} \mathbb{E}_t \left[ \sum_{s=0}^{\infty} \mathcal{Q}_{t \rightarrow t+s} \frac{P_t}{P_{t+s}} \left( p_{j,t+s} \left( \frac{p_{j,t+s}}{P_{t+s}} \right)^{-\varepsilon} Y_{t+s} - mc_{t+s} \left( \frac{p_{j,t+s}}{P_{t+s}} \right)^{-\varepsilon} Y_{t+s} P_{t+s} - \frac{\theta}{2} \left( \frac{p_{j,t+s}}{p_{j,t+s-1}} - \bar{\pi} - 1 \right)^2 Y_{t+s} P_{t+s} \right) \right] \quad (32)$$

The solution to this problem satisfies

$$\begin{aligned} \varepsilon \frac{p_{j,t}}{P_t} \left( \frac{p_{j,t}}{P_t} \right)^{-\varepsilon-1} Y_t &= \left( \frac{p_{j,t}}{P_t} \right)^{-\varepsilon} Y_t + mc_t \varepsilon Y_t \left( \frac{p_{j,t}}{P_t} \right)^{-\varepsilon-1} - \theta \left( \frac{p_{j,t}}{p_{j,t-1}} - \bar{\pi} - 1 \right) \frac{P_t}{p_{j,t-1}} Y_t \\ &\quad + \theta \mathbb{E}_t \left[ \frac{q_t}{P_{t+1}/P_t} \left( \frac{p_{j,t+1}}{p_{j,t}} - \bar{\pi} - 1 \right) \left( \frac{p_{j,t+1}}{p_{j,t}} \right)^2 Y_{t+1} \right] \end{aligned} \quad (33)$$

In equilibrium, all intermediate input producers behave identically: each of them faces the same marginal cost; therefore, each intermediate goods producer sets the same price and produces the same level of intermediate input variety  $j$ . Imposing

symmetry in equilibrium, i.e.  $p_{j,t} = P_t$ , the intermediate goods producers' optimality condition for price setting in equation (33) can be written in terms of inflation rates by defining  $\frac{P_{t+1}}{P_t} = \pi_{t+1} + 1$  as

$$\frac{\theta}{\varepsilon - 1} (\pi_t - \bar{\pi}) \pi_t = \frac{\varepsilon}{\varepsilon - 1} mc_t - 1 + \theta \mathbb{E}_t \left[ q_t (\pi_{t+1} - \bar{\pi}) \pi_{t+1} \frac{Y_{t+1}}{Y_t} \right] \quad (34)$$

or,

$$(\pi_t - \bar{\pi}) \pi_t = \frac{1 - \varepsilon}{\theta} + \frac{w_t \varepsilon}{z_t \theta} + \mathbb{E}_t \left[ q_t (\pi_{t+1} - \bar{\pi}) \pi_{t+1} \frac{Y_{t+1}}{Y_t} \right] \quad (35)$$

This is the New Keynesian Phillips Curve.

**Aggregate production** Integrating demand for intermediate good variety  $j$  across firms in equation 4, and using production function of intermediate goods producers in equation 6, we have

$$\begin{aligned} \int_0^1 y_{j,t} dj &= \int_0^1 \left( \frac{p_{j,t}}{P_t} \right)^{-\varepsilon} Y_t dj \\ \int_0^1 z_t n_{j,t} dj &= Y_t \int_0^1 \left( \frac{p_{j,t}}{P_t} \right)^{-\varepsilon} dj \end{aligned} \quad (36)$$

Since all firms face identical demand for their variety of the intermediate good and identical adjustment costs, they all adjust to the same reset price each period, which then becomes the aggregate price level for the economy. Hence, setting  $p_{j,t} = p_t = P_t$ , we have

$$\begin{aligned} z_t N_t &= Y_t \int_0^1 \left( \frac{P_t}{P_t} \right)^{-\varepsilon} dj \\ \text{or, } Y_t &= z_t N_t \end{aligned} \quad (37)$$

## 5 Recursive formulation of the economy

In this section, I will redefine the economy described above in a recursive formulation, which is essential for a state-contingent exposition of monetary policy in this environment. Let  $\mathbf{s}_t$  denote the aggregate exogenous state of the economy, representing the effective TFP. The endogenous aggregate state of the economy is the distribution of households across the state space, denoted by  $\mu$ . The households have an endogenous individual state, given by their bond-holding choice,  $b$ , and an exogenous individual

state, the idiosyncratic shock  $e$ .

**Household's decision problem** The decision problem facing households is as follows.

$$V(e_i, b; z_i, \mu) = \max_{\{c, b_i, n\}} \left\{ \frac{c^{1-\sigma}}{1-\sigma} - \chi \frac{n^{1+\psi}}{1+\psi} \right. \\ \left. + \beta \sum_{z'_j} Pr(z'_j | z_i) \sum_{e'_j} Pr(e'_j | e_i) V'(e'_j, b'; z'_j, \mu') \right\} \quad (38)$$

subject to their budget constraint,

$$c + q(z_i, \mu) \cdot b' \leq b + (1 - \tau) w(z_i, \mu) \cdot e_i \cdot n_i + d(z_i, \mu) \quad (39)$$

the borrowing constraint,

$$b' \geq \underline{b} \quad (40)$$

the non-negativity constraint,

$$c \geq 0 \quad (41)$$

and the law of motion for the distribution of households,

$$\mu' = \Lambda(\mu, z_i). \quad (42)$$

The solution to this problem yields the decision rule for households' consumption,  $c(e_i, b; z_i, \mu)$ , the decision rule for households' bond holdings,  $b$ , given by  $g(e_i, b; z_i, \mu)$ , and the decision rule for the households' labor supply choice,  $n(e_i, b; z_i, \mu)$ .

Each household's optimal choices of consumption and labor satisfy their Euler equation given by

$$u'_c(c, n) = \beta \mathbb{E}[(1 + r') u'_c(c', n')] \quad (43)$$

which can be written in terms of the discount bond price as

$$q = \beta \mathbb{E} \left[ \frac{u'_c(c', n')}{u'_c(c, n)} \right] \quad (44)$$

and its labor-leisure condition, given by

$$\chi \cdot n^\psi = c^{-\sigma} \cdot w(z_i, \mu). \quad (45)$$

Together with the household's budget constraint, these conditions pin down each household's optimal choices of consumption and leisure.



**Decision problem of intermediate goods producers** The individual state of an intermediate input producer is given by the relative price of the last period's price set by the producer to the aggregate price level last period,  $\frac{p_{j,-1}(z_i, \mu)}{P_{-1}(z_i, \mu)}$  which is written with the state-dependence suppressed for clarity in the equations below as  $\frac{p_{j,-1}}{P_{-1}}$ . An intermediate firm's problem in real terms can then be defined in recursive form as

$$V_f \left( \frac{p_{j,-1}}{P_{-1}}; z_i, \mu \right) = \max_{p_j} \left\{ \frac{p_j y_j}{P} - w(z_i, \mu) n_j - \frac{\theta}{2} \left( \frac{p_j/P}{p_{j,-1}/P_{-1}} \cdot \frac{P}{P_{-1}} - (1 + \bar{\pi}) \right)^2 Y(z_i, \mu) \right. \\ \left. + \sum_{z'_j} q(z_i, \mu) \Pr(z'_j | z_i) V_f \left( \frac{p_j}{P}; z'_j, \mu' \right) \right\} \quad (46)$$

or,

$$V_f \left( \frac{p_{j,-1}}{P_{-1}}; z_i, \mu \right) = \max_{p_j} \left\{ \frac{p_j y_j}{P} - w(z_i, \mu) n_j - \frac{\theta}{2} \left( \frac{p_j/P}{p_{j,-1}/P_{-1}} \cdot (1 + \pi(z_i, \mu)) - (1 + \bar{\pi}) \right)^2 Y(z_i, \mu) \right. \\ \left. + \sum_{z'_j} q(z_i, \mu) \Pr(z'_j | z_i) V_f \left( \frac{p_j}{P}; z'_j, \mu' \right) \right\} \quad (47)$$

subject to

$$y_j = \left( \frac{p_j}{P} \right)^{-\varepsilon} Y(z_i, \mu) \quad (48)$$

$$y_j = z \cdot n_j \quad (49)$$

where  $p_j$  is the price chosen by the firm producing input variety  $j$ ,  $\theta$  is the cost of price adjustment, and  $\varepsilon$  is the price elasticity of substitution across varieties of the intermediate goods.

To simplify the intermediate firms' problem in recursive form, I define the price of intermediate goods in terms of a relative price. Let  $p \equiv p(z_i, \mu) \equiv p_j(z_i, \mu) / P(z_i, \mu)$  be the relative price for an intermediate firm producing variety  $j$ , where  $p_j(z_i, \mu)$  is the price of variety  $j$ . Let  $p_0 \equiv p_0(z_i, \mu) \equiv p^*(z_i, \mu) / P(z_i, \mu)$  be the ratio of the current optimal reset price,  $p^*(z_i, \mu)$ , set by intermediate firms to the current aggregate price level  $P(z_i, \mu)$ ; in equilibrium,  $p_0(z_i, \mu)$  will be 1—as all intermediate firms are identical, all firms face the same demand for their respective varieties and

reset to the same optimal price, which becomes the aggregate price level. The net inflation rate in the current period is defined as  $\pi \equiv \frac{P-P_{-1}}{P_{-1}}$  where  $P_{-1}$  is the aggregate price level in the previous period.

Substituting the constraints into the problem yields the following value of an intermediate firm

$$V_f(p_{-1}; z_i, \mu) = \max_p \left\{ p^{1-\varepsilon} Y - \frac{w(z_i, \mu)}{z} p^{-\varepsilon} Y - \frac{\theta}{2} \left( \frac{p}{p_{-1}} \cdot (1 + \pi(z_i, \mu)) - (\bar{\pi} + 1) \right)^2 Y(z_i, \mu) \right. \\ \left. + \sum_{z'_j} q(z_i, \mu) \Pr(z'_j | z_i) V_f(p; z'_j, \mu') \right\} \quad (50)$$

The solution to the intermediate firm's problem yields the New Keynesian Phillips Curve (NKPC) given in equation 35, which can be rewritten in recursive form as

$$\left( \pi(z_i, \mu) - \bar{\pi} \right) \cdot \pi(z_i, \mu) = \frac{1-\varepsilon}{\theta} + \frac{w(z_i, \mu)}{z} \frac{\varepsilon}{\theta} \\ + \mathbb{E} \left[ q(z_i, \mu) \left( \pi'(z'_j, \mu') - \bar{\pi} \right) \cdot \pi'(z'_j, \mu') \cdot \frac{Y'(z'_j, \mu')}{Y(z_i, \mu)} \mid (z_i, \mu_i) \right] \quad (51)$$

**Evolution of inflation** The New Keynesian Phillips curve in equation 51 above summarizes the firms' optimal responses to the decision problems they face in this economy and is the equation for the evolution of inflation in this economy.

**Aggregate production** Aggregate production in this economy, given by equation 36, can be rewritten as

$$Y(z_i, \mu) = z \cdot N(z_i, \mu) \quad (52)$$

**Fiscal authority** The government's budget constraint in equation 9 is given as

$$q(z_i, \mu) \cdot B' = B + G(z_i, \mu) \\ - \tau w(z_i, \mu) \int e_i \cdot n(e_i, b; z_i, \mu) \cdot d\mu(e_i, b; z_i, \mu) \quad (53)$$

which can be rewritten as

$$q(z_i, \mu) \cdot B'(z_i, \mu) = B(z_i, \mu) + G(z_i, \mu) - T(z_i, \mu) \quad (54)$$

where  $T = \tau w(z_i, \mu) N(z_i, \mu)$  is tax revenues,  $N = \int e_i \cdot n(e_i, b; z_i, \mu) \cdot d\mu(e_i, b; z_i, \mu)$  is effective aggregate labor supply,  $G(z_i, \mu)$  is an exogenous government spending function. Further, aggregate bonds are the sum of bonds held across individuals.

$$B(z_i, \mu) = \int b(e_i, b; z_i, \mu) d\mu(e_i, b; z_i, \mu) \quad (55)$$

where  $\mu(e_i, b; z_i, \mu)$  represents the distribution of households across the state space.

Defining the primary surplus as  $S(z_i, \mu) \equiv T(z_i, \mu) - G(z_i, \mu)$ , we can write the government's budget constraint as:

$$q(z_i, \mu) \cdot B'(z_i, \mu) = B(z_i, \mu) - S(z_i, \mu) \quad (56)$$

To stabilize debt, the fiscal authority adjusts the primary balance according to a simple feedback rule (centered around the target):

$$S(z_i, \mu) = \rho_b (B(z_i, \mu) - \bar{B}) + \bar{S} \quad (57)$$

where  $\rho_b > 0$  is fiscal responsiveness to debt deviations,  $\bar{B}$  is target debt,  $\bar{S}$  is target primary surplus given as  $\bar{T} - \bar{G}$ . At the target, we have,

$$q\bar{B} = \bar{B} - \bar{S} \quad (58)$$

which, together with the period budget constraint of the government expressed in primary surplus terms, yields the law of motion for debt deviations

$$B'(z_i, \mu) - \bar{B} = \frac{1 - \rho_b}{q} (B(z_i, \mu) - \bar{B}) \quad (59)$$

and the evolution of government spending as

$$G(z_i, \mu) = T(z_i, \mu) - [\bar{S} + \rho_b (B(z_i, \mu) - \bar{B})] \quad (60)$$

which can be rewritten as

$$G(z_i, \mu) = T(z_i, \mu) - [\bar{T} - \bar{G} + \rho_b (B(z_i, \mu) - \bar{B})] \quad (61)$$

**Monetary authority** The monetary authority sets nominal interest rates following the Taylor rule

$$i(z_i, \mu) = r^* + \pi(z_i, \mu) + \alpha \left( \pi(z_i, \mu) - \bar{\pi} \right). \quad (62)$$

where  $i(z_i, \mu)$  is nominal interest rate function,  $r^*$  is the target real interest rate,  $\pi(z_i, \mu)$  is the inflation rate function,  $\bar{\pi}$  is the inflation target, and  $\alpha$ : policy response coefficient on inflation gap.

The relationship between nominal interest rates, real interest rates, and expected future inflation given by the Fisher equation in equation 14 can be written in recursive form as

$$1 + r(z_i, \mu) = \frac{1 + i(z_i, \mu)}{\mathbb{E} \left[ 1 + \pi(z'_j, \mu') \right]}. \quad (63)$$

## 6 Equilibrium

The distribution of households over individual and aggregate endogenous and exogenous states at date  $t$  is given by  $\mu_t(e_i, b)$ , and it evolves according to

$$\mu_{t+1}(e'_j, \mathcal{B}) = \int_{\{(e,b): g_t(e,b; z_i, \mu) \in \mathcal{B}\}} \Pr(e'_j | e_i) d\mu_t(e, b_i) \text{ given } z_i, z'_j \quad (64)$$

for all sets  $\mathcal{B} \subset \mathbb{R}$ .

In equilibrium, individual labor supply decisions are consistent with the aggregate, so that aggregate labor supplied by households equals the labor demanded by firms, given by

$$N = \int e \cdot n_t(e_i, b; z_i, \mu) d\mu(e_i, b; z_i, \mu) \quad (65)$$

Bond market clearing requires aggregate savings of the households in discount bonds to be held by the government as public debt, given as

$$B = \int g_t(e_i, b; z_i, \mu) d\mu(e_i, b; z_i, \mu) \quad (66)$$

The aggregate dividend paid by the intermediate goods firms in real terms is given by

$$d(z_i, \mu) = Y(z_i, \mu) - w(z_i, \mu) \cdot N(z_i, \mu) - \frac{\theta}{2} \left( \pi(z_i, \mu) - \bar{\pi} \right)^2 \cdot Y(z_i, \mu) \quad (67)$$

Finally, the goods market clears when the aggregate resource constraint holds as

$$Y(z_i, \mu) = C(z_i, \mu) + G(z_i, \mu) - \frac{\theta}{2} \left( \pi(z_i, \mu) - \bar{\pi} \right)^2 \cdot Y(z_i, \mu) \quad (68)$$

where  $C \equiv \int c(e_i, b; z_i, \mu) d\mu(e_i, b; z_i, \mu)$  denotes aggregate consumption from consistency between individual consumption decisions and aggregate consumption in equilibrium.

**Definition of a recursive competitive equilibrium** A recursive competitive equilibrium in this economy consists of decision rules and value functions:

$\left\{ c_t(e_i, b; z_i, \mu), g_t(e_i, b; z_i, \mu), n_t(e_i, b; z_i, \mu), V_t(e_i, b; z_i, \mu), V_{f_t}\left(\frac{p_{i,t}-1}{P-1}; z_i, \mu\right) \right\}_{t=0}^{\infty}$  that solve the problems of households and firms, respectively, and distributions of households  $\{\mu_t(e_i, b)\}_{t=0}^{\infty}$  that evolve according to the law of motion in equation 64 for given  $z_i, z'_j$ , and individual decisions are consistent with the aggregates as in equations 66 for the bond market, equations 65 for the labor market, and equation 68 for the goods market, which must also hold as the aggregate resource constraint for the economy. In addition, the government's budget constraint in equation 9, the Fisher relation in equation 63, the optimal price setting equation for the intermediate firms, which yields the New Keynesian Phillips Curve in equation 51, which shows the evolution of inflation, the equation for aggregate dividends in equation 67, and the aggregate resource constraint in equation 68, along with the monetary policy rule in equation 62 must be satisfied.

## 7 Parameters

A period in this model economy is a quarter of a calendar year. The subjective discount factor *beta* is set at 0.985 as in McKay, Nakamura, and Steinsson (2016). The coefficient of relative risk aversion is set to 2. The inverse of the Frisch elasticity of labor supply is 2, based on Chetty (2012) and McKay et al. (2016). The preference parameter for households' labor supply is set to 100 to achieve a labor supply of a third.

The desired mark-up for intermediate firms, defined by  $\mu \equiv \frac{\varepsilon}{\varepsilon-1}$ , is set to 1.2 following Christiano, Eichenbaum, and Rebello (2011) and McKay, Nakamura, and Steinsson (2016). The target inflation rate is set to 2 percent per annum. The cost of the price adjustment,  $\theta$ , is set to 250. In Rotemberg pricing, firms face quadratic costs when adjusting prices, and the parameter  $\theta$  controls the magnitude of these costs. When aggregate risks are present, shocks can be large and volatile, leading to potentially explosive price dynamics if the adjustment cost is too low. A high  $\theta$  acts as a damping mechanism, preventing prices from changing too rapidly and ensuring that the model can be solved numerically without encountering instability or explosive paths. In HANK models, where household heterogeneity and incomplete markets amplify the effects of aggregate shocks, the need for a high adjustment cost parameter is even more pronounced to maintain equilibrium and avoid computational breakdowns. With the value of  $\theta$  in the baseline calibration, the model can better handle large shocks and aggregate uncertainty, as the adjustment cost penalizes rapid price changes and stabilizes the system. This is particularly important for models

| Parameter         | Description                                 | Value |
|-------------------|---|-------|
| $\beta$           | Subjective discount factor                  | 0.985 |
| $\eta$            | Inverse of Frisch elasticity                | 2     |
| $\chi$            | Coefficient on labor                        | 100   |
| $\sigma$          | Coefficient of relative risk aversion       | 2     |
| $\tau$            | Proportional tax rate                       | 0.30  |
| $\mu$             | Markup                                      | 1.2   |
| $\theta$          | Price adjustment cost parameter             | 250   |
| $\rho_b$          | Coefficient on debt deviation (fiscal rule) | 0.08  |
| $\bar{\pi}$       | Inflation target                            | 1.005 |
| $\alpha$          | Coefficient on inflation gap                | 1.5   |
| $r^*$             | Intercept in Taylor rule                    | 1.005 |
| $\rho_z$          | Persistence of aggregate shock              | 0.89  |
| $\sigma_\epsilon$ | Std of innov in aggregate shock             | 0.015 |
| $\rho_e$          | Persistence of idiosyncratic shock          | 0.966 |
| $\sigma_e$        | Std of innov in idiosyncratic shock         | 0.1   |

Table 1: Parameter values in the baseline model

with forward-looking agents and heterogeneous households, where the propagation of shocks can be highly nonlinear and sensitive to parameter values. In an alternate parameterization, I compute the model with  $\theta$  set to 150. It should be noted that a higher value of the price adjustment cost parameter  $\theta$  increases the degree of price rigidity in the model, which would warrant a larger impact on the real economy, but the effects of state-contingent forward guidance on consumption and output remain small relative to non-state-contingent guidance, despite the high degree of price stickiness, highlighting the role of aggregate uncertainty in dampening these effects when forward guidance is state-contingent.

The idiosyncratic income shocks for a quarterly AR(1) process have a persistence parameter of 0.966 and a standard deviation of innovation of 0.1, following the quarterly estimates used by McKay, Nakamura, and Steinsson (2016). The AR(1) shock process is discretized to a three-point Markov chain using the Rouwenhorst (1995) method.

The shock to Total Factor Productivity is also a quarterly AR(1) process, with a persistence parameter of 0.89, and the standard deviation of innovation being 0.015. The AR(1) shock process for total factor productivity is also discretized to a three-point Markov chain, again using the Rouwenhorst (1995) method.

The coefficient on debt deviations,  $\rho_b$ , in the fiscal rule is set to 0.08 in the baseline model; in an alternate calibration, it is set to 0.12. For stability, this value must be greater than the real interest rate so that primary surplus rises faster than interest

costs on outstanding debt.

The coefficient on the inflation gap in the Taylor rule is chosen to be 1.5. For an active monetary policy to be in place, nominal rates must rise more than one-for-one with inflation, thereby raising real rates and stabilizing inflation. This condition,  $\alpha > 1$ , known as the Taylor principle, is essential for a determinate equilibrium. If it equals 1, real rates will not change with inflation, leading to inflationary drift and hence indeterminacy.

As permitted by the environment, parameter values are chosen to match those of McKay et al. (2016) for comparability of results under a perfect-foresight environment.

## 8 Computation

In computing the stochastic equilibrium, I use the Backward Induction method as described by Reiter (2010) to solve the households' problem yielding their decision rules. The stochastic economy has an exogenous aggregate state—TFP—and an endogenous aggregate state—the distribution of households in the economy. Since household distributions are infinite-dimensional, I approximate the economy's aggregate endogenous state by the aggregate bond holdings,  $m$ . To approximate this, I use an initial proxy distribution and discretize the approximate endogenous state space into levels of bond holdings, so that the distribution yields a mean bond holding level equal to the value at that grid point. In other words, the discretized grid for the levels of aggregate bond holdings is centered on the target mean, given the proxy distribution of households. The proxy distributions are computed at each point in the discretized aggregate state space, by adjusting the steady-state distribution in a way such that the resulting distribution yields a mean level of bond holdings equal to the grid value of the approximate aggregate state, which here is the level of aggregate bond holdings in economy  $m$ .

Utilizing these proxy distributions, I solve the real economy in the model in one run of backward iterations, on a discrete grid of points in the aggregate state space, while enforcing consistency between individual and aggregate solutions in each step of the backward iteration, which is done separately at each point on the grid of aggregate states (Reiter 2010).

Households' decision rules are solved for using the endogenous grid method on grids of individual endogenous and exogenous states, namely households' bond holdings and idiosyncratic income shocks, respectively. Consistency is achieved between individual and aggregate decisions when the expected level of aggregate bond holdings in the economy becomes equal to that obtained from its law of motion. Once

such consistency is achieved, I iterate on wages to clear the labor market, using an initial guess for the bond price  $q$  function and an initial guess for the inflation function over the aggregate state space. The partial equilibrium wage function obtained by iterating over the real economy until labor demand equals labor supply clears both the labor and the goods markets for the initially guessed rate of inflation. The asset market is cleared by Walras' law. Using these partial equilibrium wages, and the output obtained from the aggregate resource constraint, for the initial inflation rate, I use the New Keynesian Phillips Curve to update current inflation, over expectations of future inflation and output. The Taylor rule gives the nominal interest rate and the Fisher equation determines the bond price every time the inflation function is updated. The general equilibrium is obtained once inflation, wages, the law of motion for aggregate bond holdings, and households' value functions have all converged up to a certain tolerance set for each of these. This method yields the stochastic equilibrium in this economy.

The Backward Induction method is global because it approximates policy and value functions over a wide range of state variables, including large movements in aggregates and distributions, rather than only in a neighborhood of the deterministic steady state. It uses projection methods in high-dimensional state spaces and enforces equilibrium conditions along entire simulated or specified paths that can move substantially away from the deterministic steady state. When the recursive competitive equilibrium is stable and unique, iterating the backward (value/policy) and forward (distribution/price) steps converges to a fixed-point path consistent with agents' expectations and market clearing, delivering a global nonlinear approximation to the stochastic equilibrium of the model (Reiter 2010).

## 9 Results

### 9.1 Stochastic equilibrium and forward guidance

The key finding of my work is that state-contingent forward guidance is nearly half as effective as date-based forward guidance in affecting current macroeconomic variables. Depending on the form of state-contingency, the effectiveness of state-contingent forward guidance can be even lower, highlighting the importance of aggregate uncertainty in determining the efficacy of forward guidance.

A passive fiscal policy plays an essential role in managing debt sustainability in an economy characterized by aggregate uncertainty, as in this paper. Households save for precautionary reasons in anticipation of aggregate economic shocks, even if such shocks do not materialize. In such circumstances, a fiscal rule that stabilizes debt



and prevents it from spiraling, by adjusting government spending to accommodate the large stock of savings, is essential for an equilibrium under volatile economic conditions.

To delineate the effects of forward guidance, the aggregate shocks are suppressed during the period between the policy announcement and its implementation. However, as households' decision rules, computed in the presence of aggregate shocks, internalize the risks of aggregate economic uncertainty, they perceive the possibility of adverse future events and raise savings in anticipation. Therefore, a stabilizing fiscal rule is necessary to prevent debt from spiraling out of control.

### 9.1.1 State-Contingent Forward Guidance

The baseline experiment in this paper is a state-contingent forward guidance, namely, an announcement by the central bank of a 50-basis-point rate cut 20 periods ahead, contingent on the aggregate economic state remaining unchanged from the time of the policy announcement to the date of its implementation. Since no prior work in the literature provides a framework for studying state-contingency in forward guidance, these results shed light on questions that have not been previously addressed. Table 2 compares the results from a state-contingent forward guidance to a non-state-contingent or a date-based forward guidance where the same announcement is made of a 50-basis-point rate cut 20 periods ahead, but irrespective of the future state of the economy. State-contingent forward guidance is only half as powerful in affecting current output, consumption, and inflation as date-based guidance. These results are shown in Figure 1 and summarized in Table 2. The results of an announcement of a 50-basis-point rate hike 20 periods ahead, are roughly symmetric in terms of the effectiveness of state-contingent forward guidance vis-a-vis a similar date-based guidance independent of the state of the economy<sup>9</sup>.

| (In basis points)   | Variable    | At policy announcement | At policy implementation |
|---------------------|-------------|------------------------|--------------------------|
| Date-based FG       | Output      | 5                      | 17                       |
|                     | Consumption | 7                      | 12                       |
|                     | Inflation   | 0.5                    | 2                        |
| State-contingent FG | Output      | 2.5                    | 14                       |
|                     | Consumption | 3                      | 11                       |
|                     | Inflation   | 0.25                   | 1.5                      |

Table 2: State-contingent versus date-based FG of a 50-basis-point rate cut 20 periods ahead

<sup>9</sup>Results available on request

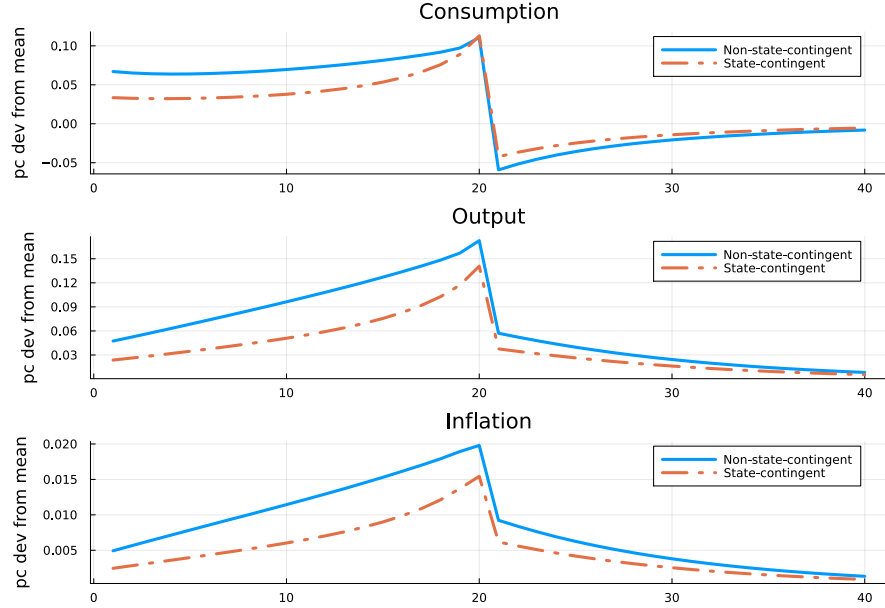


Figure 1: State-contingent versus date-based FG of a 50-basis-point rate cut 20 periods ahead

To isolate the effects of the forward guidance experiment, agents expect aggregate economic shocks, as reflected in households' value functions and decision rules, but these shocks are not realized during the experiment.

The monetary authority's announcement of a reduction in nominal interest rates, given expectations of future inflation, implies a decrease in the real interest rate. When real interest rates fall, the cost of borrowing falls, lowering the relative price of current consumption. As a result, consumption demand increases. This is a positive demand shock. Because of nominal price rigidities, prices cannot adjust instantaneously or fully to equilibrate demand and supply, prompting firms to increase production immediately. This requires firms to demand more labor to produce more output to meet the increased demand. This is the direct intertemporal substitution effect of an interest rate cut. In a general equilibrium, increased labor demand by firms drives up wages, thus increasing labor supply. Hence, households' labor incomes increase, thereby adding to the direct effects of an interest rate cut and further increasing consumption demand. These have been referred to as the "indirect" or the "general equilibrium" effects of a real interest rate cut (Kaplan, Moll, and Violante, 2016). These are the effects of an actual rate cut. However, an announcement of a future rate cut causes these cogwheels to start turning now, in anticipation of the future rate cut.

In a classical economy, without nominal rigidities, equilibrium prices would adjust instantaneously and completely to offset the increase in consumption demand,

leaving output unchanged and inflation rising fully on the shock date (the policy announcement date), bringing the economy back to equilibrium instantaneously. An announcement about a future interest rate cut stimulates current aggregate demand, creating a positive demand shock. However, with price rigidities, nominal prices adjust slowly. Firms increase prices slightly because they face quadratic adjustment costs that cause price stickiness, while increasing production in response to the demand shock. As output increases, households' incomes increase. In the absence of capital, households have fewer mechanisms to smooth consumption over time by adjusting investment, so consumption tends to move with current income. Over time, as firms gradually raise prices, inflation and output continue to increase moderately until the rate cut is implemented.

**Why state-contingent forward guidance is less effective.** The announcement of a cut in interest rates 20 quarters ahead makes future borrowing cheaper and current savings more expensive. In response, households would like to save less and consume more now, to smooth consumption over their lifetimes. If markets were complete, such a response would be larger to fully exploit the future rate cut, starting now and continuing until its implementation. However, as households face heightened risks from uninsurable idiosyncratic income shocks and aggregate economic uncertainty, there is greater uncertainty about their future incomes and wealth positions due to market incompleteness and aggregate uncertainty, thereby substantially increasing their precautionary saving motive.

Aggregate uncertainty amplifies the consumption risk of households already experiencing idiosyncratic income shocks. This amplifies households' precautionary savings motives, leading them to save more and thereby weakening the transmission of a forward guidance policy shock to current aggregate consumption and output. To smooth lifetime consumption, households save relatively more than they would in the absence of aggregate uncertainty, in turn causing public debt to be larger.

Additionally, households approaching their borrowing limit may be constrained from borrowing in the future by the time the rate cut is implemented; the current consumption response of these households is relatively less affected by the announcement of the future rate cut as their planning horizon is shortened and any rate cut that is likely to happen after they hit their borrowing limit becomes irrelevant for them in changing their current consumption and savings choices. Together, these effects significantly dampen the current impact of an announcement of a future rate cut under incomplete markets and aggregate economic uncertainty.

Importantly, households respond not only to the risks they face from income shocks and economic uncertainty, but also to expected movements in future pol-

icy rates, and, to a second order, to uncertainty about these future rate movements. State-contingent forward guidance partially resolves the uncertainty and unpredictability of future policy, making current consumption spending move by much less than under a similar date-based guidance known to be implemented with certainty.

The difference between consumption and output in my model arises from government spending by the fiscal authority and from the Rotemberg price adjustment costs incurred by intermediate firms that draw on aggregate resources. As households borrow increasingly in response to the announcement about a future interest rate cut, to take advantage of the lower relative price of consumption today, aggregate savings become negative, and government debt falls below its target, spurring government spending to increase to keep the government's budget constraint in balance, while satisfying the fiscal rule. The dynamics of debt and government spending, together with adjustment costs, drive the difference in the responses of consumption and output to the forward guidance announcement.

To keep debt stable, the government uses a fiscal rule centered on a target debt level, which adjusts spending to accommodate deviations from the target, thereby ensuring debt stability and preventing increases in saving from causing debt to increase unboundedly. This is an additional source of consumption volatility in this economy. This force becomes particularly strong when forward guidance is state-contingent and does not resolve the uncertainty about expected future policy rates. This, in part, defeats the purpose of forward guidance, as the policy announcement is tied to a particular future economic state, and agents are uncertain about future economic conditions due to aggregate risks and uncertainty. This further tempers households' responses to state-contingent announcements, making them much less potent than comparable date-based announcements.

Figure 2 shows the higher precautionary motive to save at play when forward guidance is state-contingent: when the monetary authority gives forward guidance of a rate cut in the future, the fall in savings by households in response to forward guidance is relatively less when the guidance is state-contingent because precautionary motives to save are higher in this case compared to when the guidance is purely date-based and is not contingent on the future state of the economy.

While date-based forward guidance is fraught with the dangers of destabilizing long run inflation expectations, my work provides evidence that there is need to balance the objectives of eliminating uncertainty about future policy rates and keeping inflation expectations anchored so that forward guidance can be more effective in influencing current macroeconomic variables and improving the predictability of monetary policy actions, which is the core objective of forward guidance.

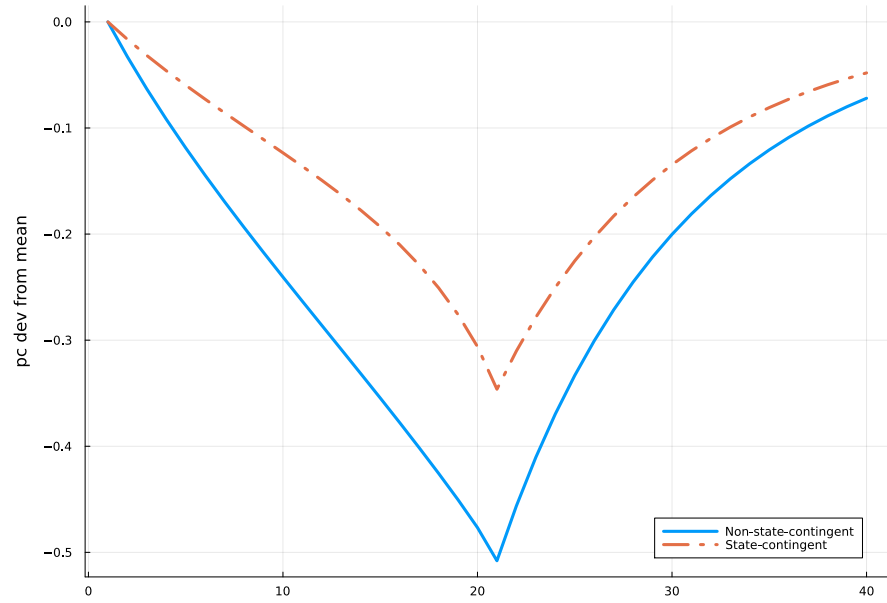


Figure 2: Aggregate savings response to FG of a 50-basis-point rate cut 20 periods ahead

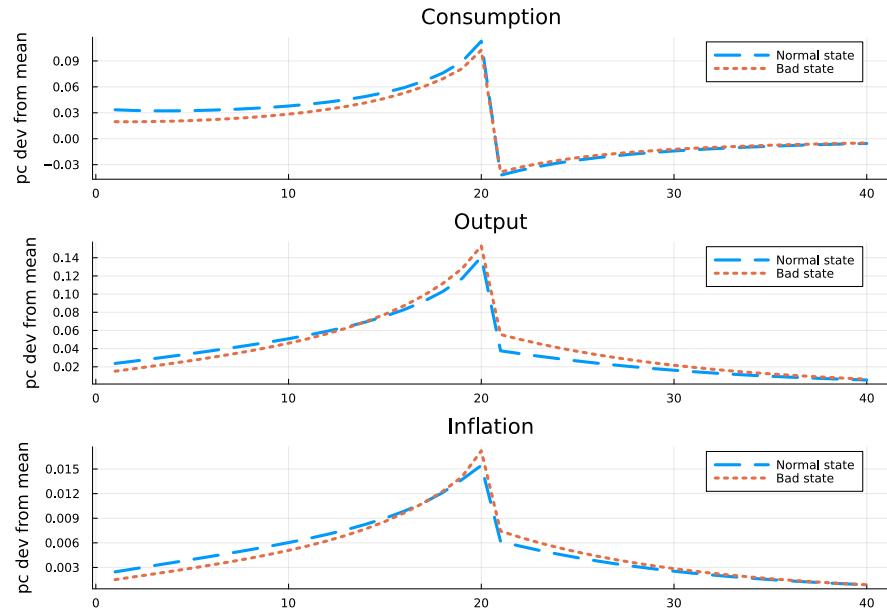


Figure 3: State-contingent forward guidance in bad times versus in normal times

(State-contingent FG given in bad times is conditional on the aggregate economic state remaining bad at the date of policy implementation, whereas state-contingent FG given in ordinary times is contingent on the economic state remaining normal at the time of policy implementation)

It is worth noting that the power of forward guidance is further diminished when state-contingent guidance is communicated in poorer aggregate economic conditions, compared to when normal aggregate economic conditions prevail. This is depicted in Figure 3.

**Does the forward guidance puzzle exist?** My results show that the “forward guidance puzzle” may not exist in the case of state-contingent forward guidance, once aggregate economic uncertainty is adequately accounted for. Even for purely date-based forward guidance, I find that its power is significantly diminished in the presence of aggregate risks and economic uncertainty, even when such shocks are not realized.

A date-based forward guidance, where the central bank announces a 50-basis-point rate cut 20 periods ahead, irrespective of the future state of the economy, increases the current output by 5 basis points and the current consumption by 7 basis points. These effects are much smaller than those observed in McKay et al. (2016), who resolve the puzzle by introducing incomplete markets, but with agents having perfect foresight. While there are differences in modeling choices between their model and my environment, the presence of aggregate economic uncertainty and the internalization of the economy’s stochastic nature by households are the key differences in the environments in which the two economies operate. They show a 10-basis-point increase in consumption (and output, since output is consumption in their economy) and a 30-basis-point increase in inflation on the policy announcement date. My results show that even when forward guidance is non-state-contingent, its effects on contemporaneous variables are attenuated by aggregate economic uncertainty. This demonstrates nonlinearities in household behavior when the stochastic nature of the economy is adequately accounted for, as households internalize economic uncertainty and anticipate such shocks in responding to the forward guidance policy.

Table 3 shows the effects of date-based forward guidance on current output, consumption, and inflation, in my model compared to the results in McKay et al (2016).

| (In basis points)                   | Variable    | At policy announcement | At policy implementation |
|-------------------------------------|-------------|------------------------|--------------------------|
| My model with aggregate uncertainty | Output      | 5                      | 17                       |
|                                     | Consumption | 7                      | 12                       |
|                                     | Inflation   | 0.5                    | 2                        |
| McKay et al (2016)                  | Output      | 10                     | 17                       |
|                                     | Consumption | 10                     | 17                       |
|                                     | Inflation   | 30                     | -5                       |

Table 3: Date-based FG of a 50-basis-point rate cut 20 periods ahead

**Longer horizons make forward guidance less powerful.** A part of the forward guidance puzzle was that forward guidance over longer time horizons predicted larger effects on current output and consumption than guidance over shorter horizons. This is also eliminated in my results. My results show that guidance over a longer horizon is less effective in stimulating current output, consumption, and inflation, under both state-contingent forward guidance and date-based guidance. A comparison of the results of state-contingent versus date-based guidance over the short and long horizons is summarized in Table 4. State-contingent forward guidance over a longer horizon has half the efficacy of date-based guidance over the same horizon.

| (In basis points):<br>50-basis-point rate<br>cut... | Type of FG             | Variable    | At policy<br>announcement | At policy<br>implementation |
|---|------------------------|-------------|---------------------------|-----------------------------|
| Short Horizon: ...20<br>periods ahead               | Date-based FG          | Output      | 5                         | 17                          |
|   |                        | Consumption | 7                         | 12                          |
|   |                        | Inflation   | 0.5                       | 2                           |
|   | State-contingent<br>FG | Output      | 2.5                       | 14                          |
|   |                        | Consumption | 3                         | 11                          |
|   |                        | Inflation   | 0.25                      | 1.5                         |
| Long Horizon: ...40<br>periods ahead                | Date-based FG          | Output      | 2.5                       | 17.5                        |
|   |                        | Consumption | 3.5                       | 11                          |
|   |                        | Inflation   | 0.25                      | 2.1                         |
|   | State-contingent<br>FG | Output      | 1.5                       | 14                          |
|   |                        | Consumption | 1.5                       | 11                          |
|   |                        | Inflation   | 0.1                       | 1.6                         |

Table 4: State-contingent versus date-based forward guidance over different horizons

### 9.1.2 “Lower-for-longer” state-contingent forward guidance

Here, I examine an alternative form of state-contingent forward guidance and compare the results with those of a corresponding data-based guidance.

The state-contingent forward guidance here is an announcement by the monetary authority to lower interest rates by 50 basis points if and when the economy enters a recession, marked by a drop in aggregate productivity, and to keep rates low as long as the economy remains in the low-productivity state. These are often referred to as “lower-for-longer” policies, where the central bank commits to lower rates when the economy experiences a recession, and promises to keep rates low until the economy recovers from the recessionary state or until aggregate economic conditions improve.

In this experiment, the economy slips into the lower-productivity state in period 20. It remains in the recessionary state for 6 periods before recovering, marked by an increase in aggregate productivity from the low level to the median level of produc-

tivity in the model. The real current macroeconomic effects of such state-contingent forward guidance are shown in Figure ??, in comparison with a corresponding date-based guidance wherein the central bank announces a 50-basis-point rate cut starting in period 20 and lasting 6 periods, irrespective of the macroeconomic conditions in the economy.

State-contingent forward guidance here is only about 20 to 33 percent as effective as a date-based announcement, in its impact on current consumption, output, and inflation. These results confirm my finding that effectiveness of forward guidance is significantly muted when such guidance is state-contingent, i.e., it is conditional on future macroeconomic conditions, highlighting the importance of aggregate uncertainty in the study on forward guidance.

### 9.1.3 Sensitivity

I test the sensitivity of my results to the choice of two crucial parameter values, one, the fiscal responsiveness of debt deviations measured by the coefficient  $\rho_b$ , and two, the size of the price adjustment cost parameter  $\theta$ , which determines the scale of the quadratic adjustment cost faced by firms when resetting their prices each period, and hence the degree of price stickiness in the economy.

The role of the fiscal rule has been emphasized above in keeping debt from growing unboundedly due to households' desire to self-insure against perceived future risks, whether or not these risks materialize. The fiscal rule directs government spending to fall when debt overshoots its target. For stability, government spending should fall faster than the cost of servicing existing debt. The parameter on debt deviation in the fiscal rule determines the fiscal responsiveness to debt deviations, namely, how aggressively government spending will be reduced to bring debt back to its targeted level. The baseline parameterization of the model uses  $\rho_b = 0.08$  in the fiscal rule. Tightening the fiscal rule further, while ensuring government spending remains positive, allows a value of  $\rho_b = 0.12$ . While debt stabilizes more quickly due to a stronger response to government spending, it has no significant bearing on the attenuated effects of state-contingent versus date-based forward guidance on current macroeconomic variables, nor on the magnitude of such effects individually. This is shown in Table 5.

The price adjustment cost parameter is a measure of price stickiness in the economy. The presence of aggregate uncertainty warrants a relatively large price-adjustment cost parameter, which serves as a damping mechanism, preventing large price jumps and thereby preventing explosive price paths and instability in the model.

In the baseline model, the price adjustment cost parameter is set to  $\theta = 250$ . Despite a high degree of price stickiness in the model, which biases it toward larger



| (in basis points)<br>50-basis-point rate cut 20<br>periods ahead | Variable    | At policy<br>announcement | At policy im-<br>plementation |
|--|-------------|---------------------------|-------------------------------|
| Date-based FG  | Output      | 5                         | 20                            |
|  | Consumption | 7                         | 11.5                          |
|  | Inflation   | 0.5                       | 2.5                           |
| State-contingent FG  | Output      | 2.5                       | 16.5                          |
|  | Consumption | 3.5                       | 12.5                          |
|  | Inflation   | 0.45                      | 1.9                           |

Table 5: Date-based versus state-contingent forward guidance with  $\rho_B = 0.12$

effects on the real economy, both state-contingent and date-based forward guidance yield relatively smaller effects on aggregate macroeconomic variables, with the former yielding relatively even smaller effects than the latter.

A lower price adjustment cost parameter reduces price stickiness and weakens the real effects of forward guidance: when prices are more flexible, the economy adjusts more via prices and less via quantities; inflation responds more to a given output gap, so smaller quantity adjustments are needed to restore equilibrium. Hence, for a given forward-guidance shock, a lower price adjustment cost implies larger movements in inflation, smaller movements in real activity, and a smaller output gain from a given policy announcement of a future rate cut. This is shown in the results from the alternate value of  $\theta = 0.12$  in Table 6. The inflation response to an announcement of a 50-basis-point rate cut 20 periods in the future is slightly more pronounced when the price adjustment cost is lower, and output and consumption responses are somewhat muted compared to the baseline case. State-contingent forward guidance, however, remains only about half as effective as date-based guidance.

| (in basis points)<br>50-basis-point rate cut 20<br>periods ahead | Variable    | At policy<br>announcement | At policy im-<br>plementation |
|--|-------------|---------------------------|-------------------------------|
| Date-based FG  | Output      | 4.5                       | 16                            |
|  | Consumption | 6.5                       | 10                            |
|  | Inflation   | 0.75                      | 2.7                           |
| State-contingent FG  | Output      | 2.5                       | 13                            |
|  | Consumption | 3.5                       | 10                            |
|  | Inflation   | 0.45                      | 2.3                           |

Table 6: Date-based versus state-contingent forward guidance with  $\theta = 150$

Thus, the main finding of my paper is robust to alternative parameter values of the fiscal responsiveness to debt deviations and the price adjustment cost. A tighter fiscal policy rule and a lower price-adjustment cost parameter both yield

comparable results regarding the relative effectiveness of state-contingent versus date-based forward guidance for current macroeconomic variables.

## 10 Conclusion

I study the effects of forward guidance in an incomplete-markets setting under aggregate uncertainty, where households face uninsurable idiosyncratic income shocks as well as aggregate economic risks.

Given the state-contingent nature of forward guidance adopted by the US Federal Reserve in recent years, studying forward guidance contingent on future macroeconomic conditions is feasible only with the explicit modeling of aggregate uncertainty, which has not been explored in the literature to date.

I solve the model with aggregate uncertainty, globally, to obtain a stochastic dynamic general equilibrium. I use this stochastic general equilibrium to study state-contingent forward guidance, in which the monetary authority announces a 50-basis-point interest rate cut 20 periods ahead, conditional on the aggregate macroeconomic state remaining unchanged between the policy announcement and its implementation. I compare the results of this experiment with a date-based non-state-contingent forward guidance announcement that announces a similar rate cut for the same time horizon, to be implemented independent of the future macroeconomic state. I also examine the effects of an alternative form of state-contingency, in which the central bank announces a 50-basis-point rate cut if the economy enters a recession and promises to keep rates low until the economy returns to normal conditions. This is referred to as a “lower-for-longer” forward guidance, in which the central bank announces to lower rates in the future and keep rates low for an extended period, conditional on the future macroeconomic circumstances.

My findings show that state-contingent forward guidance is nearly half as effective as date-based guidance, and, depending on the form of state-contingency, the impact of such guidance on current output, consumption, and inflation is even lower than that of a corresponding date-based guidance. Even date-based guidance has less power in the presence of aggregate uncertainty than the effects reported in the literature under perfect foresight.

Because markets are incomplete, households face uninsurable income shocks, making their future incomes and wealth uncertain. These risks are exacerbated by aggregate economic uncertainty. These heightened risks facing households aggravate their precautionary motives to save. Households are also subject to a borrowing constraint, and those closer to their borrowing limits when forward guidance is issued are even less responsive to such announcements, as the future rate cut would be irrelevant

to them. Furthermore, households respond not only to the risks they face but also to expected movements in future interest rates and to uncertainty about future policy. State-contingent forward guidance partly improves the predictability of future policy because the economy’s path unfolds only with the resolution of uncertainty. For this reason, households are much less confident about future policy rate changes. As a result, the motive to build up buffer savings to hedge against possible future adverse events becomes much stronger, attenuating their consumption response to state-contingent forward guidance announcements.

The government’s fiscal policy, driven by the fiscal rule, accommodates the increased supply of savings in an economy with aggregate uncertainty, reflecting stronger precautionary motives to save among households as a mechanism for self-insuring against future adverse economic events. In the absence of a fiscal rule that stabilized debt, increased savings would cause public debt to grow unboundedly.

My results underscore the importance of aggregate uncertainty for a meaningful study of forward guidance, and, specifically, of state-contingent forward guidance as it currently exists. The presence of aggregate uncertainty substantially reduces the power of forward guidance, even when it is not contingent on future macroeconomic conditions. This highlights nonlinearities in household behavior when the economy faces aggregate uncertainty, necessitating a global solution to accurately capture these in aggregate responses to announcements regarding future policy.

My work augments the existing literature on forward guidance by enabling the study of state-contingent forward guidance, which is previously unexplored and, in part, infeasible with existing methods that rely on local approximations to the steady state or on perfect-foresight analyses. My work, therefore, expands the understanding of forward guidance in its current form. It also makes a methodological contribution, enabling a meaningful study of a range of monetary policy issues in more realistic and uncertain economic environments.

The relationship between forward guidance, asset prices, and term premiums is one such issue that I aim to explore using my model and solution approach. In related future work, I also plan to examine the effectiveness of forward guidance under aggregate uncertainty, with heterogeneity in inflation expectation formation. I am extending my existing work to investigate the efficacy of state-contingent forward guidance at the zero lower bound (ZLB), where more conventional policies are constrained, and forward guidance becomes a potent tool to boost the real economy. This opens avenues to explore the effects of state-contingent forward guidance in both high- and low-uncertainty environments, which my work and solution approach enable me to uncover accurately.

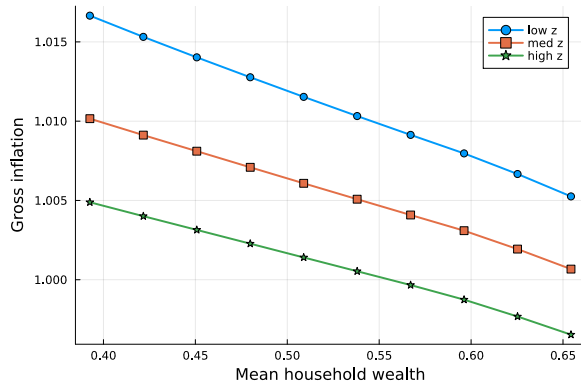
## References

- [1] S. Acharya, E. Challe, and K. Dogra, “Optimal monetary policy according to hank,” *American Economic Review*, vol. 113, no. 7, pp. 1741–1782, 2023.
- [2] G.-M. Angeletos and C. Lian, “Forward guidance and heterogeneous beliefs,” *American Economic Review*, vol. 108, no. 9, pp. 2477–2512, 2018.
- [3] A. Auclert, B. Bardóczy, M. Rognlie, and L. Straub, “Using the sequence-space jacobian to solve and estimate heterogeneous-agent models,” *Econometrica*, vol. 89, no. 3, pp. 1329–1356, 2021.
- [4] C. Bayer, R. Luetticke, L. Pham-Dao, and V. Tjaden, “Precautionary savings, illiquid assets, and the aggregate consequences of wealth taxes,” tech. rep., CEPR Discussion Paper No. DP13528, 2019.
- [5] B. S. Bernanke, “The new tools of monetary policy,” *American Economic Review*, vol. 110, no. 4, pp. 943–983, 2020.
- [6] M. Bodenstein, J. Hebden, and R. Nunes, “Imperfect credibility and the forward guidance puzzle,” tech. rep., Board of Governors of the Federal Reserve System, International Finance Discussion Papers No. 1061, 2012.
- [7] J. R. Campbell, J. D. M. Fisher, A. Justiniano, and L. Melosi, “Forward guidance and macroeconomic outcomes since the financial crisis,” in *NBER Macroeconomics Annual 2016* (M. Eichenbaum and J. A. Parker, eds.), vol. 31, pp. 283–357, University of Chicago Press, 2017.
- [8] C. T. Carlstrom, T. S. Fuerst, and M. Paustian, “Optimal monetary policy in a model with agency costs,” *Journal of Money, Credit and Banking*, vol. 47, no. S2, pp. 265–308, 2015.
- [9] R. Chetty, “Bounds on elasticities with optimization frictions: A synthesis across microeconomic contexts,” *Econometrica*, vol. 80, no. 6, pp. 2127–2181, 2012.
- [10] L. J. Christiano, M. Eichenbaum, and S. Rebelo, “When is the government spending multiplier large?,” *Journal of Political Economy*, vol. 119, no. 1, pp. 78–121, 2011.
- [11] H. Chung, E. Herbst, and M. T. Kiley, “Effective monetary accommodation and the reversal rate,” tech. rep., Board of Governors of the Federal Reserve System, Finance and Economics Discussion Series 2014-44, 2014.
- [12] M. Del Negro, M. P. Giannoni, and C. Patterson, “The forward guidance puzzle,” tech. rep., Federal Reserve Bank of New York, Staff Report No. 574, 2012.

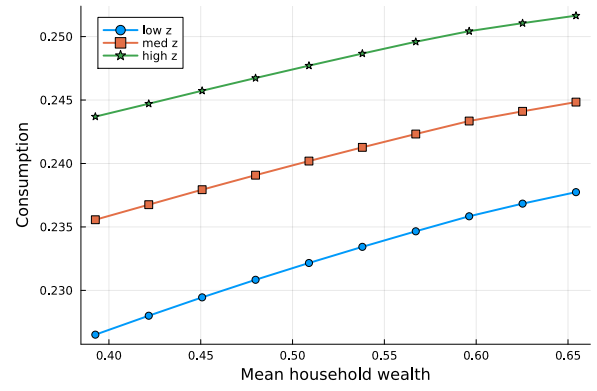
- [13] M. Del Negro, M. P. Giannoni, and C. Patterson, “The forward guidance puzzle,” tech. rep., Federal Reserve Bank of New York, Staff Report No. 574, Revised, 2013.
- [14] X. Gabaix, “A sparsity-based model of bounded rationality,” *Quarterly Journal of Economics*, vol. 129, no. 4, pp. 1661–1710, 2015.
- [15] M. Garcia-Schmidt and M. Woodford, “Are low interest rates deflationary? a paradox of perfect-foresight analysis,” *American Economic Review*, vol. 109, no. 1, pp. 86–126, 2019.
- [16] R. S. Gürkaynak, B. Sack, and E. T. Swanson, “Do actions speak louder than words? the response of asset prices to monetary policy actions and statements,” *International Journal of Central Banking*, vol. 1, no. 1, pp. 55–93, 2005.
- [17] A. Haberis, R. Harrison, and M. Waldron, “Monetary policy, forward guidance and the yield curve,” tech. rep., Bank of England Working Paper No. 485, 2014.
- [18] M. Hagedorn, I. Manovskii, K. Mitman, and S. Sehran, “Unemployment benefits and unemployment in the great recession: The role of macro effects,” *American Economic Review*, vol. 109, no. 8, pp. 2957–2993, 2019.
- [19] C. L. Ilut, R. Luetticke, and M. Schneider, “Title forthcoming.” Forthcoming (update publication details as available), 2025.
- [20] G. Kaplan, B. Moll, and G. L. Violante, “Monetary policy according to hank,” *American Economic Review*, vol. 108, no. 3, pp. 697–743, 2018.
- [21] M. T. Kiley, “Macroeconomic effects of federal reserve forward guidance,” tech. rep., Board of Governors of the Federal Reserve System, Finance and Economics Discussion Series 2016-061, 2016.
- [22] N. G. Mankiw and R. Reis, “Sticky information versus sticky prices: A proposal to replace the new keynesian phillips curve,” *Quarterly Journal of Economics*, vol. 117, no. 4, pp. 1295–1328, 2002.
- [23] A. McKay, E. Nakamura, and J. Steinsson, “The power of forward guidance revisited,” *American Economic Review*, vol. 106, no. 10, pp. 3133–3158, 2016.
- [24] F. Milani and J. Treadwell, “The effects of forward guidance in dsge models: A comparison of strategies,” *Economics Letters*, vol. 116, no. 2, pp. 172–175, 2012.
- [25] M. Reiter, “Global solution methods for heterogeneous agent models,” *Journal of Economic Dynamics and Control*, vol. 34, no. 1, pp. 80–90, 2010.
- [26] J. J. Rotemberg, “Monopolistic price adjustment and aggregate output,” *Review of Economic Studies*, vol. 49, no. 4, pp. 517–531, 1982.

- [27] G. L. Violante, “Title forthcoming.” Please insert full citation for the specific publication used, 2021.
- [28] M. Woodford, “Monetary policy analysis in models with heterogeneous agents,” in *Handbook of Macroeconomics* (J. B. Taylor and H. Uhlig, eds.), vol. 2B, pp. 867–928, Elsevier, 2019.

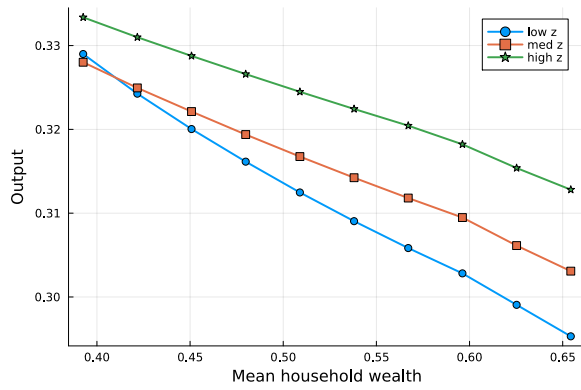
## A Stochastic equilibrium



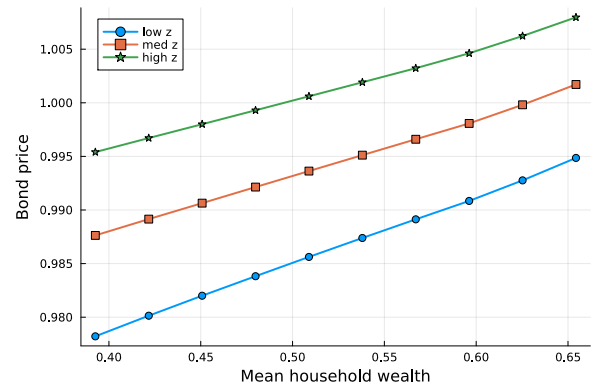
(a) Inflation



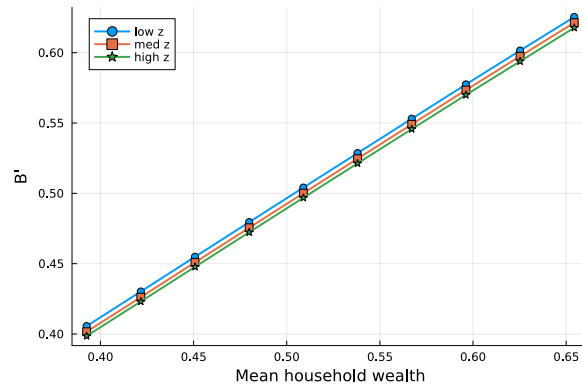
(b) Consumption



(c) Output



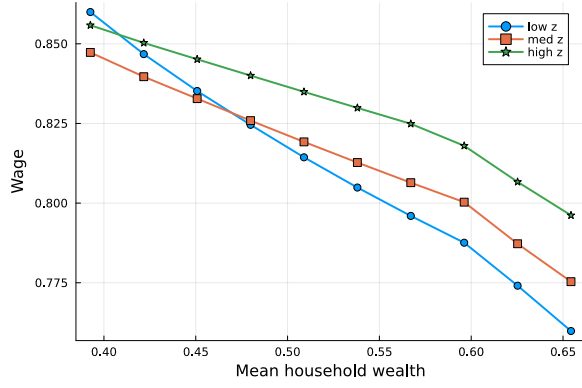
(d) Bond price



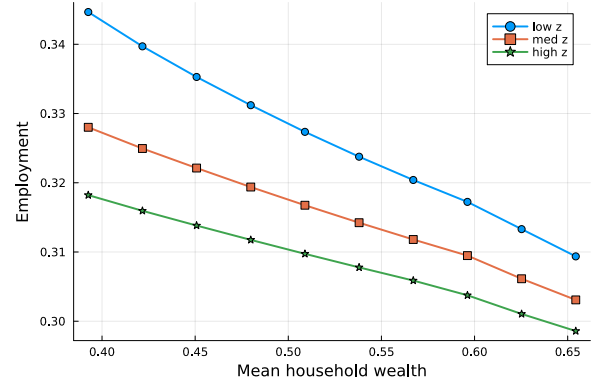
(e) Savings

Figure 4: Stochastic equilibrium: aggregate functions in the aggregate state space characterizing the stochastic general equilibrium

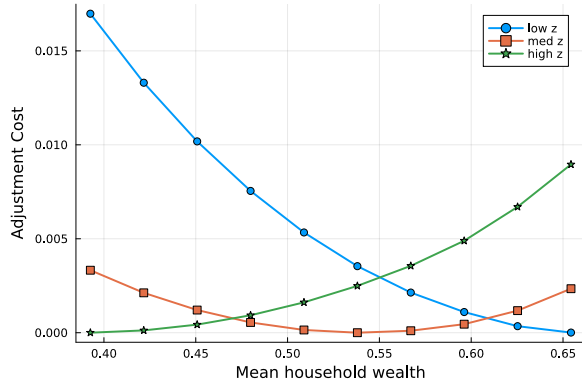




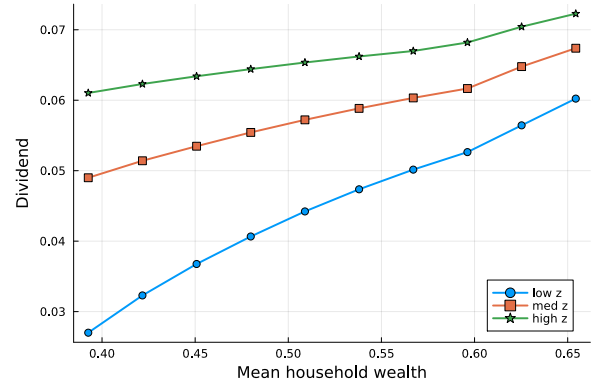
(a) Wage



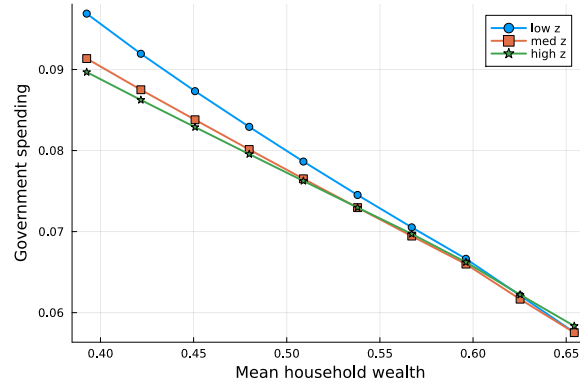
(b) Employment



(c) Adjustment cost



(d) Dividend



(e) Gov't spending

Figure 5: Stochastic equilibrium: aggregate functions in the aggregate state space characterizing the stochastic general equilibrium

## B A note on the fiscal rule

The fiscal authority in this economy uses a passive fiscal rule to stabilize debt. Households save for precautionary motives when markets are incomplete. When aggregate uncertainty is high, households have greater motivation to save for precautionary reasons, due to idiosyncratic income shocks and aggregate economic risks. As consumption risks rise, in the absence of capital, households tend to smooth lifetime consumption by saving more aggressively. This leads to a large and growing supply of savings in the economy, which translates into an expanding public debt burden. In this environment, it is essential for the government to wield a fiscal rule to stabilize the economy's debt. In this economy, the government follows a fiscal rule centered on a target debt level, lowering government spending whenever debt overshoots the target. This prevents debt from growing unboundedly when households expect to realize aggregate economic shocks.

In the forward guidance experiment shown in the paper, to disentangle the effects of forward guidance announcements, the realization of aggregate shocks in the economy is suppressed during the period between the policy announcement and its implementation. When the macroeconomic environment becomes more uncertain, households internalize risks about the future economic outlook, as reflected in their decision rules computed under aggregate uncertainty. They respond by cutting current consumption and increasing savings to self-insure, not in response to shocks that materialize, but due to heightened perceived future risks.

As a result, savings continue to rise over time in the presence of aggregate uncertainty, even when shocks are not realized, because households build precautionary wealth as a buffer against possible future adverse events, thereby depressing consumption. This accumulation is driven by the anticipation of risk, rather than the actual realization of shocks, leading to persistently elevated savings levels. As more households save more and consume less, aggregate demand decreases, potentially impeding recovery from downturns and reinforcing expectations of future risks.

Therefore, a fiscal rule is required that stabilizes debt amid aggregate economic uncertainty. In my model, the government adjusts its spending in response to deviations in debt from its target level, namely, decreasing spending when debt exceeds the target, holding the proportional labor income tax rate constant.

## C Steady-State

The deterministic steady state yields an equilibrium bond price  $q$  of 0.9952, implying a quarterly risk-free real interest rate of 0.50 percent and an annualized risk-free real interest rate of 1.93 percent per annum. The steady-state yields a debt-to-GDP ratio of about 157.47 percent, which is around 123 percent for the US economy in 2024 <sup>10</sup>. Government spending in the model economy in the steady state is about 24.3 percent of GDP, whereas federal government spending was 23 percent of total gross domestic product (GDP) in the US economy in FY 2024.

The distributions of wealth in the steady state are given in Table 8. I use the income process for idiosyncratic shocks from McKay et al. (2016). For the purposes of this paper, my objective is to achieve a plausible distribution of wealth to study the effects of forward guidance in the presence of aggregate uncertainty. Hence, the model is not calibrated to match the wealth distributions observed in the US economy. The steady-state wealth distribution in the model yields a Gini coefficient of 0.717. The steady state results are summarized in Table 7.

| Variable                                | Steady-state values |
|---|---------------------|
| Real interest rate ( $r^*$ )            | 1.93 percent        |
| Bond price ( $q^*$ )                    | 0.9952              |
| Debt ( $B^*$ )                          | 0.52                |
| Output ( $Y^*$ )                        | 0.3324              |
| Consumption ( $C^*$ )                   | 0.2518              |
| Labor (efficiency units) ( $L^*, N^*$ ) | 0.3324              |
| Raw labor hours ( $H^*$ )               | 0.3121              |
| Wage ( $w^*$ )                          | 0.833               |
| Government spending ( $G^*$ )           | 0.081               |
| Dividend ( $D^*$ )                      | 0.0554              |
| Inflation ( $\pi^*$ )                   | 1.005               |
| Primary deficit $(G - \tau w L)/Y$      | -0.75 percent       |
| Debt-to-GDP ( $B/Y$ )                   | 157.47 percent      |

Table 7: Steady-state values

<sup>10</sup>In steady-state, the primary deficit-to-GDP ratio, given by  $(G - \tau \cdot w \cdot e \cdot n)/Y$ , is around -0.75 percent, whereas the deficit-to-GDP ratio, given by  $(G - (1 - q) \cdot B + G - \tau \cdot w \cdot e \cdot n)/Y$ , is 0 percent in the model. This was around 6.4 percent in 2024 for the US economy, and the 50-year long-run average has been close to 3.6 percent. However, such a primary deficit is necessary to deliver a steady state debt-to-asset ratio of 157 percent close to the 2024 value for the US economy.

|                       |       |       |       |       |       |       |
|-----------------------|-------|-------|-------|-------|-------|-------|
| Percentiles of wealth | 1     | 5     | 10    | 50    | 90    | $< 0$ |
| Share of households   | 0.057 | 0.249 | 0.443 | 1.000 | 1.001 | 0.467 |

Table 8: Wealth distribution of households in steady-state