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KL-divergence

A measure of similarity between two probability distributions

$$D_{\mathrm{KL}}(Q\|P) = \int_{-\infty}^{\infty} \mathrm{d}x\, q(x)\, \log\left(\frac{q(x)}{p(x)}\right)$$

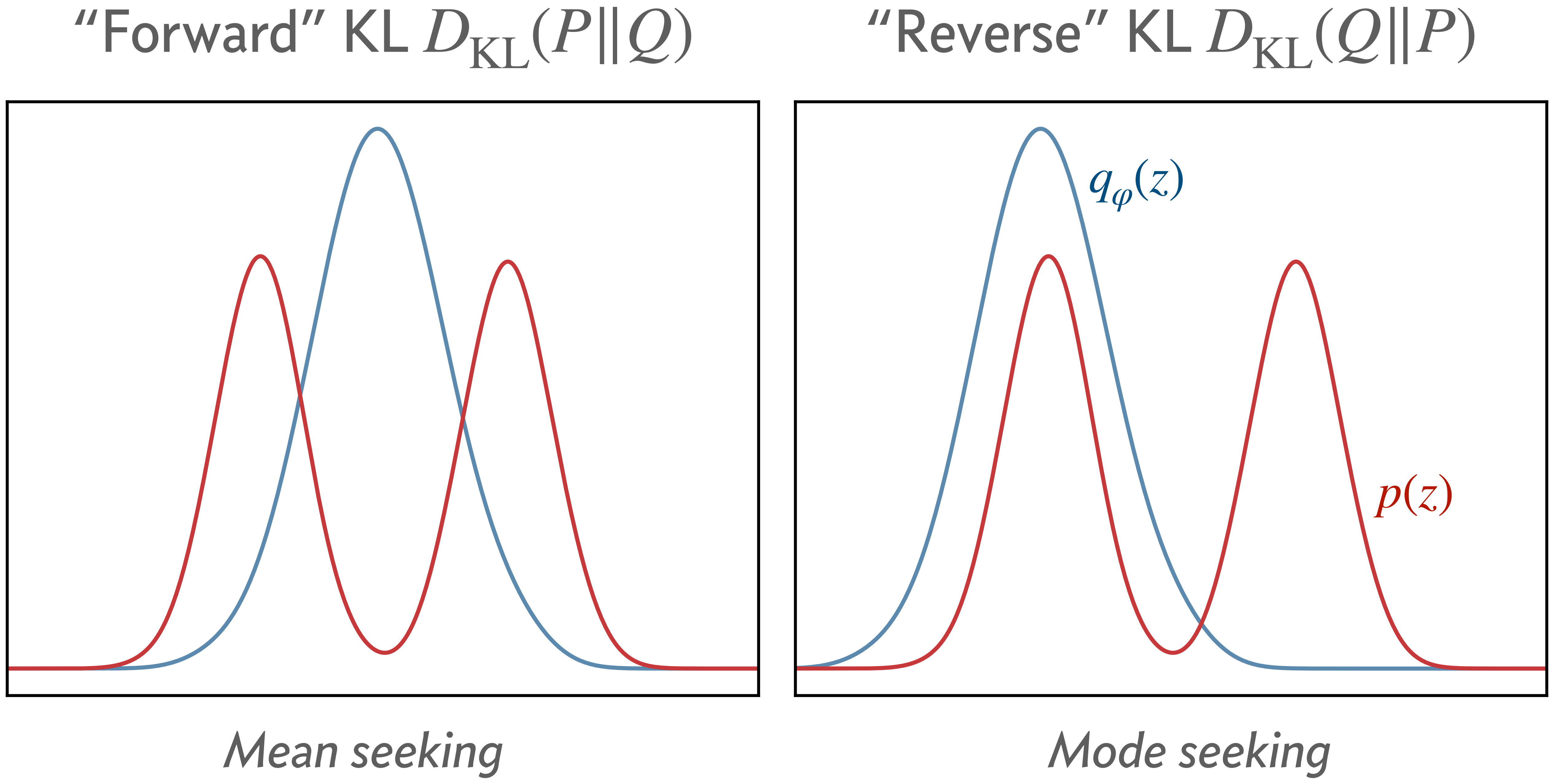
Not symmetric!

$$D_{\mathrm{KL}}(Q \| P) \neq D_{\mathrm{KL}}(P \| Q)$$

Maximum-likelihood inference is equivalent to minimizing the *forward* KL

Forward KL

$$D_{\mathrm{KL}}(P_{\mathscr{D}} \| Q_\varphi) = -\left\langle \log q_\varphi(z) \right\rangle_{z \sim p_{\mathscr{D}}(z)} + \mathrm{const}\,.$$


"Forward" KL $D_{\mathrm{KL}}(P\|Q)$
"Reverse" KL $D_{\mathrm{KL}}(Q\|P)$
$q_{\varphi}(z)$
$p(z)$
Mean seeking
Mode seeking

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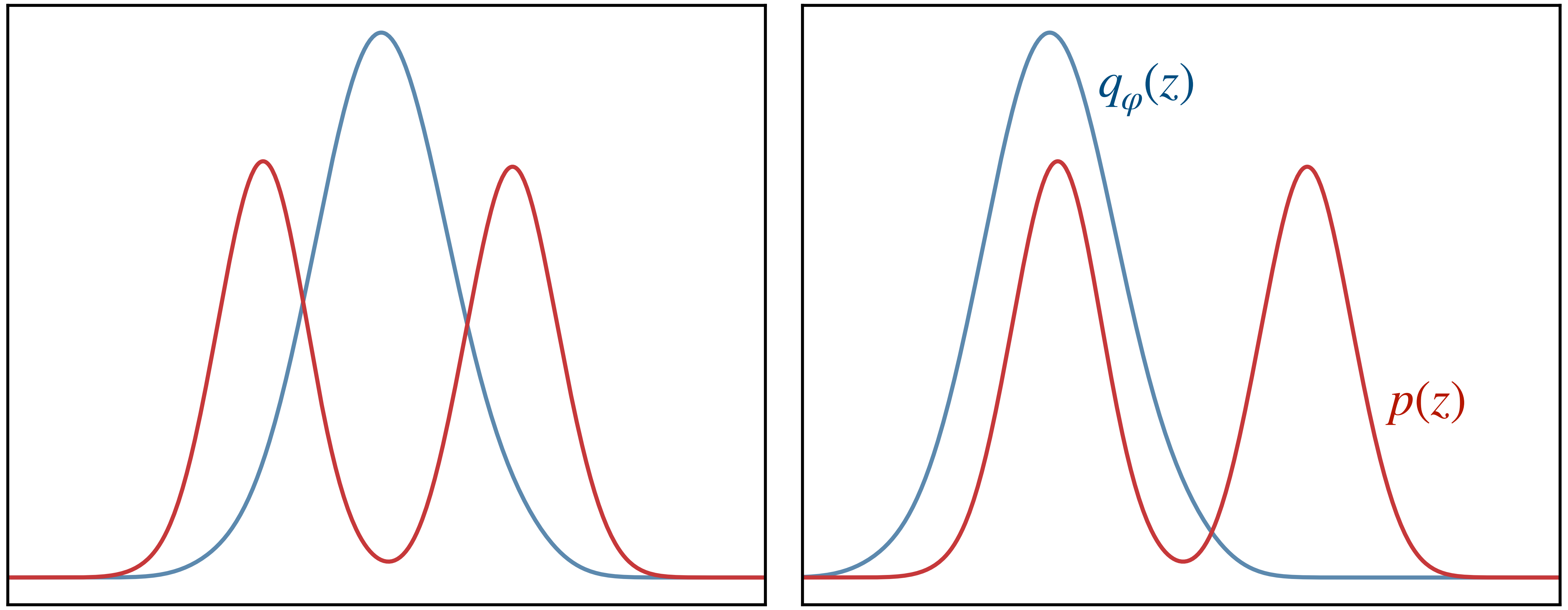

Mean seeking   *Mode seeking*

Forward KL

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Variational inference

The intractability of $p(x)$ is closely related to the intractability of the *posterior* $p(z \mid x)$

$$p(z \mid x) = \frac{p(x, z)}{p(x)} \quad \text{(Bayes' theorem)}$$