/62

40

Continuous-time/SDE formulation

$$
\begin{aligned}
x_t &= \sqrt{1 - \beta(t)\Delta_t}\, x_{t-1} + \sqrt{\beta(t)\Delta_t}\,\mathcal{N}(0, \Box) \\
&\approx x_{t-1} - \frac{\beta(t)\Delta_t}{2} x_{t-1} + \sqrt{\beta(t)\Delta_t}\,\mathcal{N}(0, \Box)
\end{aligned}
$$

In the limit of infinite time steps, $\Delta_t \to 0$ and the forward diffusion process can be written as

Which is an update rule corresponding to the Euler-Murayama discretization of the stochastic differential equation (SDE)

$$\mathrm{d}x_t = -\frac{1}{2}\beta(t)x_t \, \mathrm{d}t + \sqrt{\beta(t)}\mathrm{d}w_t$$

Continuous-time/SDE formulation

In the limit of infinite time steps, $\Delta_t \to 0$ and the forward diffusion process can be written as

$$
\begin{aligned}
x_t &= \sqrt{1-\beta(t)\Delta_t}\,x_{t-1} + \sqrt{\beta(t)\Delta_t}\,\mathcal{N}(0,\mathbb{I}) \\
&\approx x_{t-1} - \frac{\beta(t)\Delta_t}{2}x_{t-1} + \sqrt{\beta(t)\Delta_t}\,\mathcal{N}(0,\mathbb{I})
\end{aligned}
$$

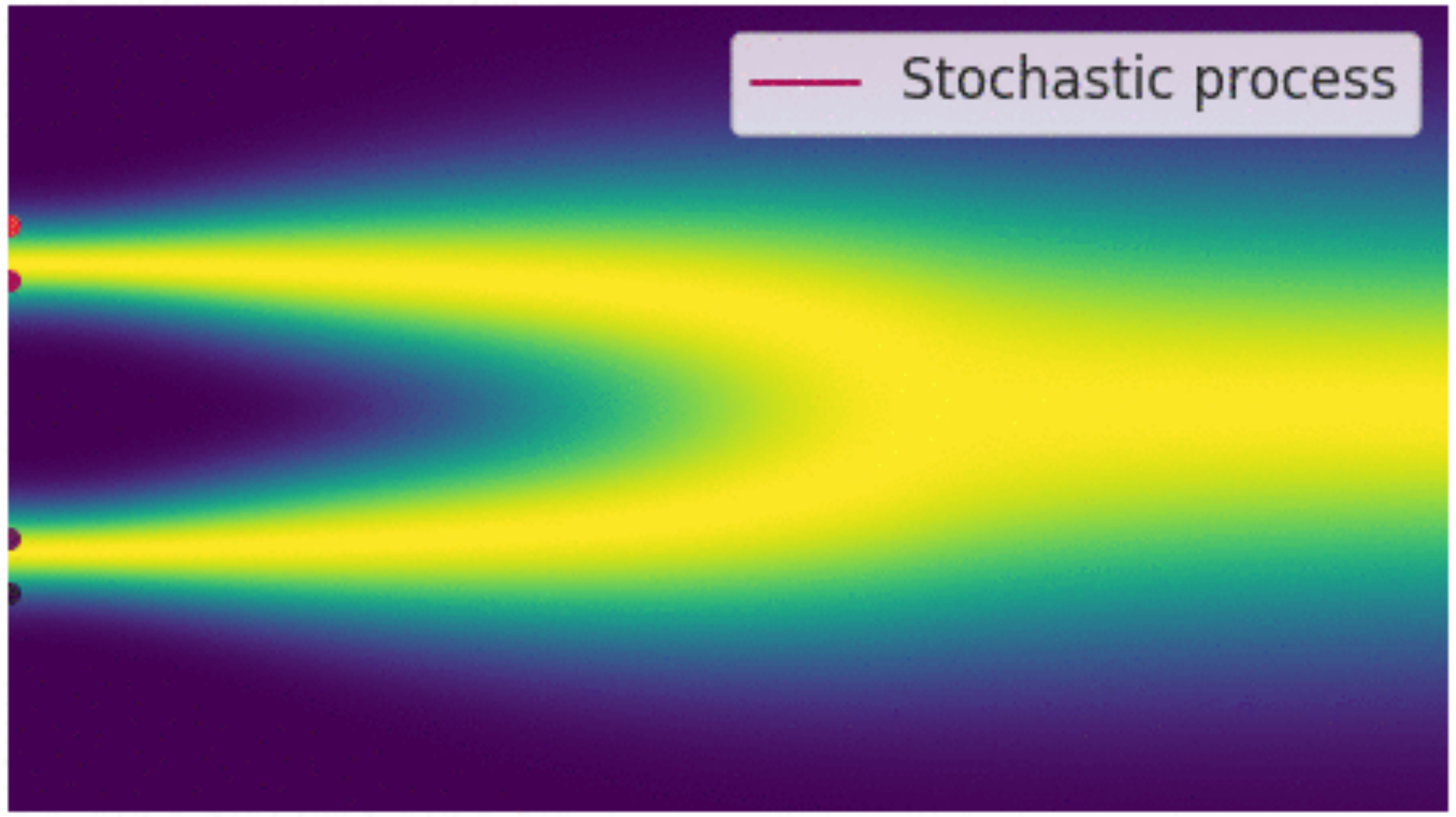
Which is an update rule corresponding to the Euler-Murayama discretization of the stochastic differential equation (SDE)

$$
\mathrm{d}x_t = -\frac{1}{2}\beta(t)x_t\,\mathrm{d}t + \sqrt{\beta(t)}\,\mathrm{d}w_t
$$

Continuous-time/SDE formulation

The forward diffusion process defined by an SDE

[Song et al 2021]



$$\mathrm{d}x_t = -\frac{1}{2}\beta(t)x_t\,\mathrm{d}t + \sqrt{\beta(t)}\mathrm{d}w_t$$