



Siddhant Mishra-Sharma (MIT/AI FI) Summer School



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Formally: *expected excess “surprise” from using  $Q$  as a model when the actual distribution is  $P$*

19

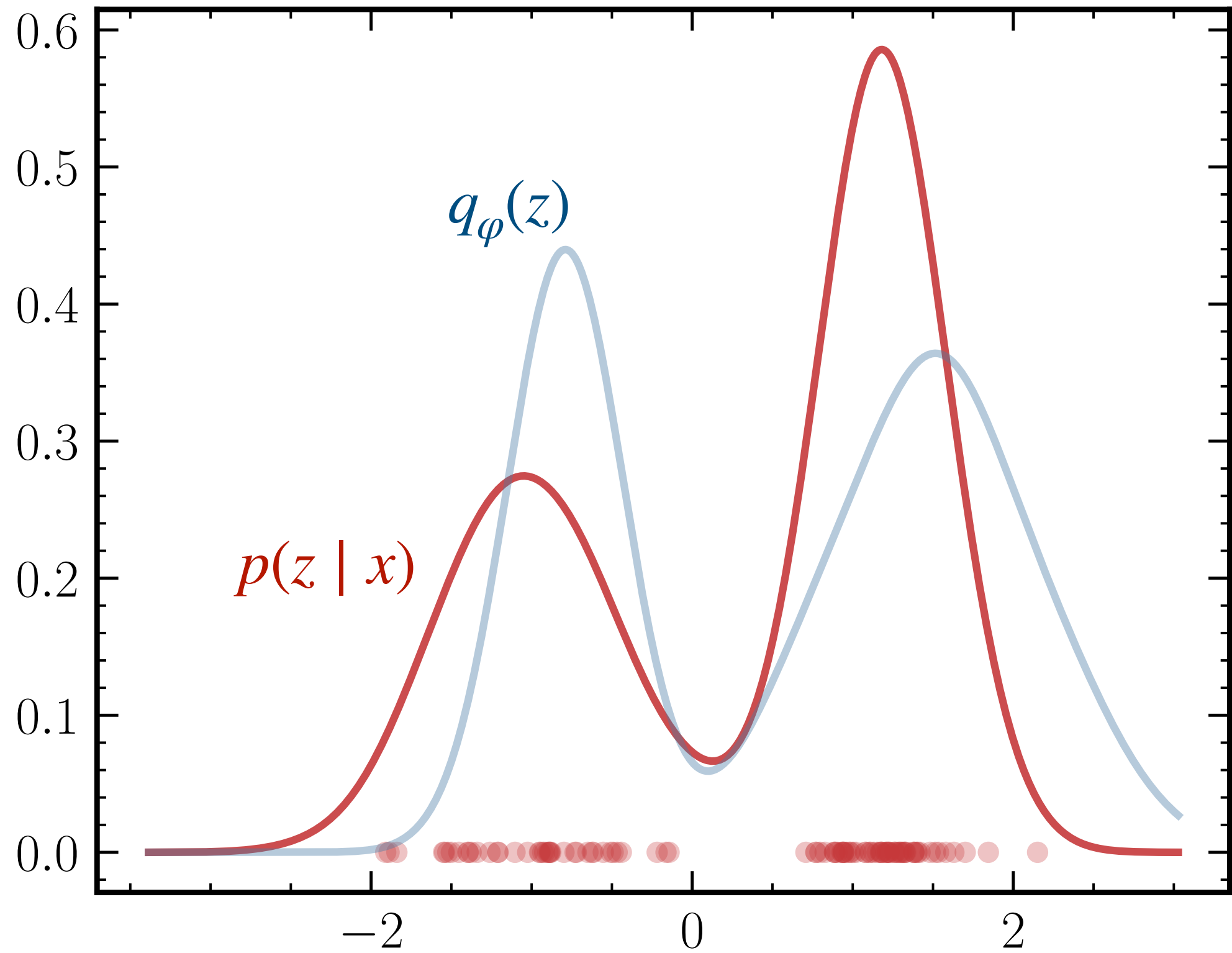


A measure of similarity between two probability distributions

$$D_{\text{KL}}(P\|Q) = \int_{-\infty}^{\infty} \mathrm{d}x \, p(x) \log \left( \frac{p(x)}{q(x)} \right)$$

$$= \left\langle \log \frac{p(x)}{q(x)} \right\rangle_{x \sim p(x)}$$





Kullback-Leibler (KL) divergence

$$= - \underbrace{\left\langle \log q(x) \right\rangle_{p(x)}}_{\text{Cross-entropy } \mathbb{H}(P, Q)} + \underbrace{\left\langle \log p(x) \right\rangle_{p(x)}}_{\text{(Self-)entropy } \mathbb{H}(P)}$$

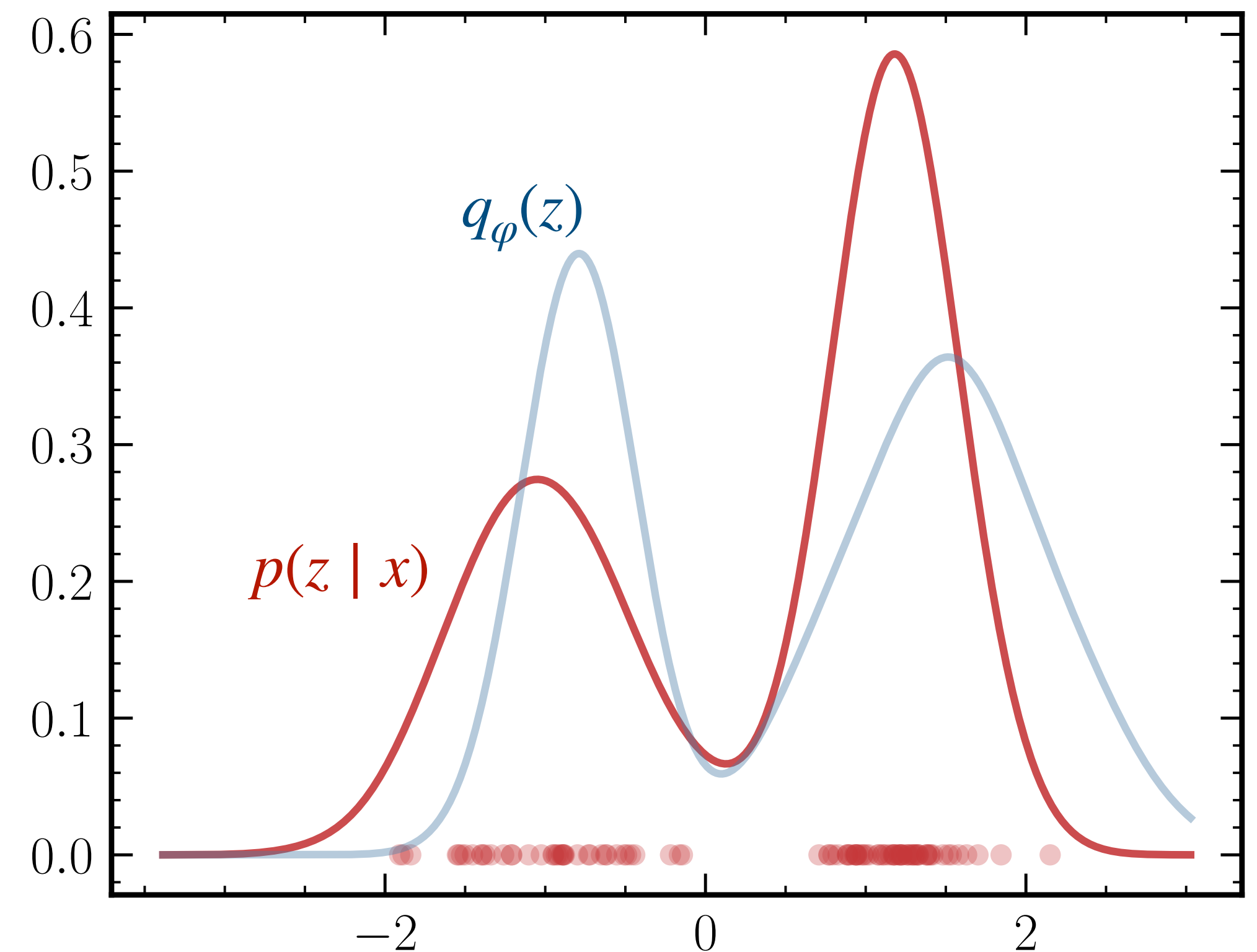
Cross-entropy  $\mathbb{H}(P, Q)$  — (Self-)entropy  $\mathbb{H}(P)$

# Kullback–Leibler (KL) divergence

A measure of similarity between two probability distributions

$$\begin{aligned} D_{\text{KL}}(P\|Q) &= \int_{-\infty}^{\infty} dx p(x) \log \left( \frac{p(x)}{q(x)} \right) \\ &= \left\langle \log \frac{p(x)}{q(x)} \right\rangle_{x \sim p(x)} \\ &= \underbrace{- \langle \log q(x) \rangle_{p(x)}}_{\text{Cross-entropy } \mathbb{H}(P, Q)} + \underbrace{\langle \log p(x) \rangle_{p(x)}}_{\text{(Self-)entropy } \mathbb{H}(P)} \end{aligned}$$

Formally: *expected excess “surprise” from using  $Q$  as a model when the actual distribution is  $P$*



# KL-divergence

A measure of similarity between two probability distributions

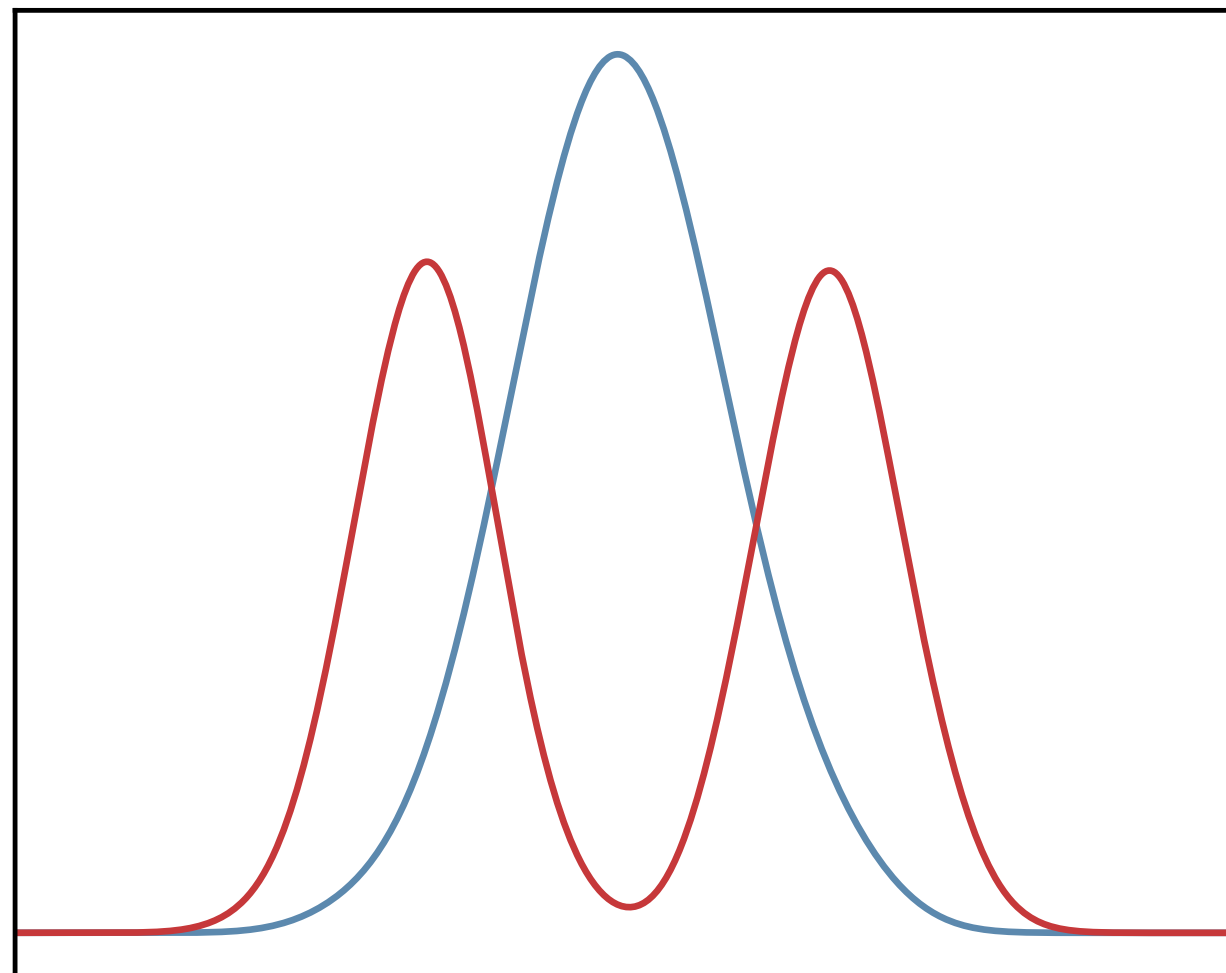
$$D_{\text{KL}}(Q\|P) = \int_{-\infty}^{\infty} dx \, q(x) \log \left( \frac{q(x)}{p(x)} \right)$$

Not symmetric!  $D_{\text{KL}}(Q\|P) \neq D_{\text{KL}}(P\|Q)$

“True” distribution



“Forward” KL  $D_{\text{KL}}(P\|Q)$



“Reverse” KL  $D_{\text{KL}}(Q\|P)$

