Normalizing flows

Multiple flow transformation can be easily composed for e.g. expressivity

$f(z_2)$

$f(z_1)$

$p(z_T) \longrightarrow \quad z_T \qquad z_{T-1} \quad \cdots \quad z_1 \qquad x \quad \longleftarrow \quad p(x)$

$f^{-1}(z_1)$

$f^{-1}(x)$
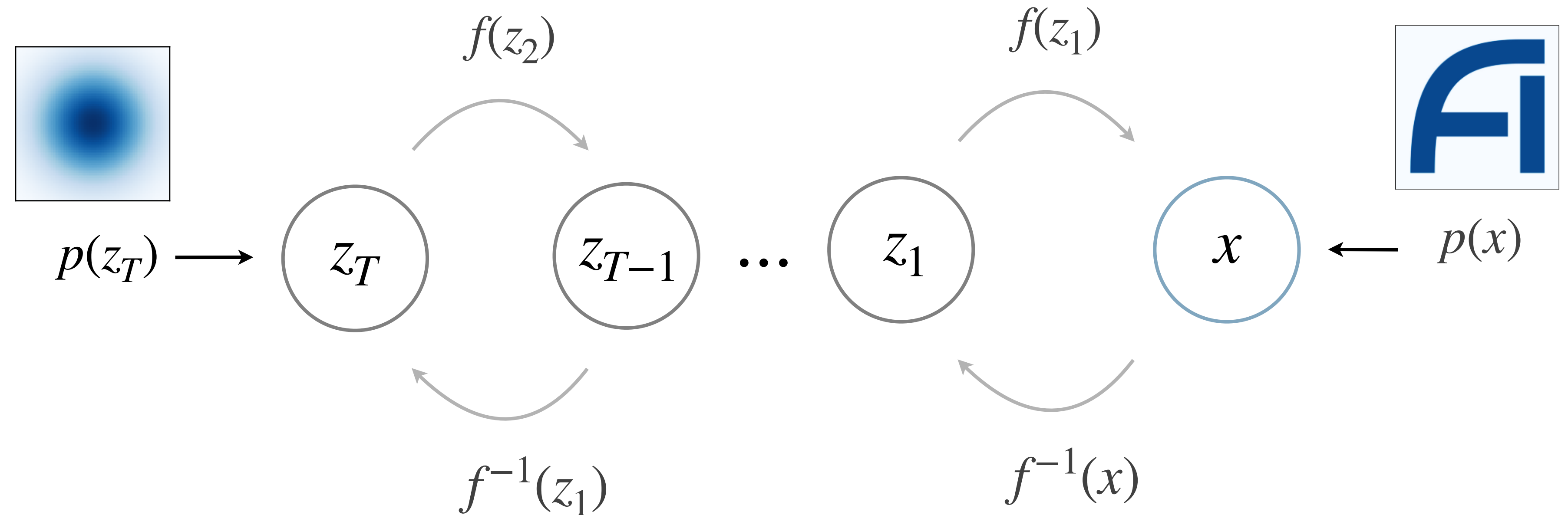
Computing $p(x)$: *change-of-variables formula*

$$\int p(x)dx = \int p(z)dz = 1$$

$$p(x) = p(z) \left| \frac{dz}{dx} \right| = p\left(f^{-1}(x)\right) \left| \frac{df^{-1}}{dx} \right| = p\left(f^{-1}(x)\right) |\det \nabla f|^{-1}$$

Train using maximum-likelihood objective

$$\varphi^* = \left\langle \arg\max_{\varphi} p\left(f_{\varphi}^{-1}(x)\right) |\det \nabla f_{\varphi}|^{-1} \right\rangle_{x \sim p(x)}$$

Simple flow transformations

Example: *Affine coupling flow* [RealNVP; Dinh et al 2016]

$f(z)$

z_1 x_1

z_2 x_2

$\vdots$ $\vdots$

z_D x_D