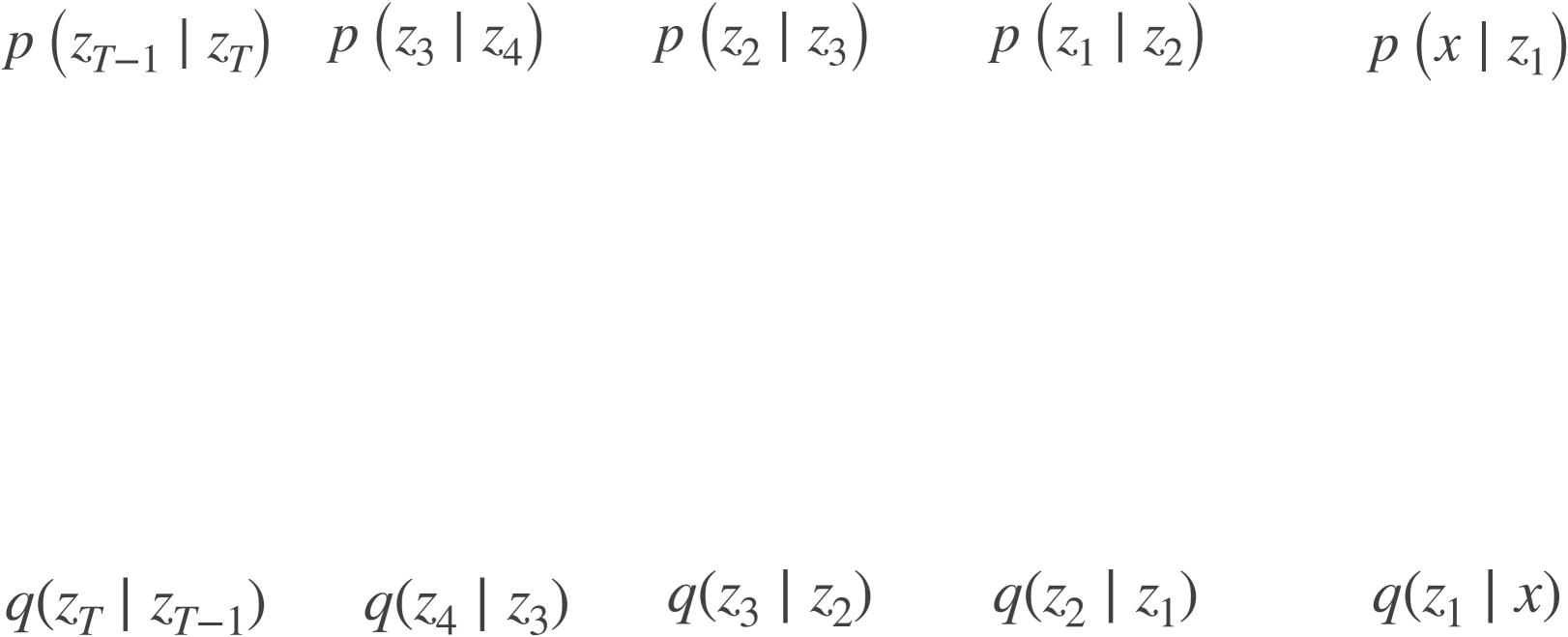
#### Siddharth Mishra-Sharma (MIT/IAIFI) | IAIFI Summer School

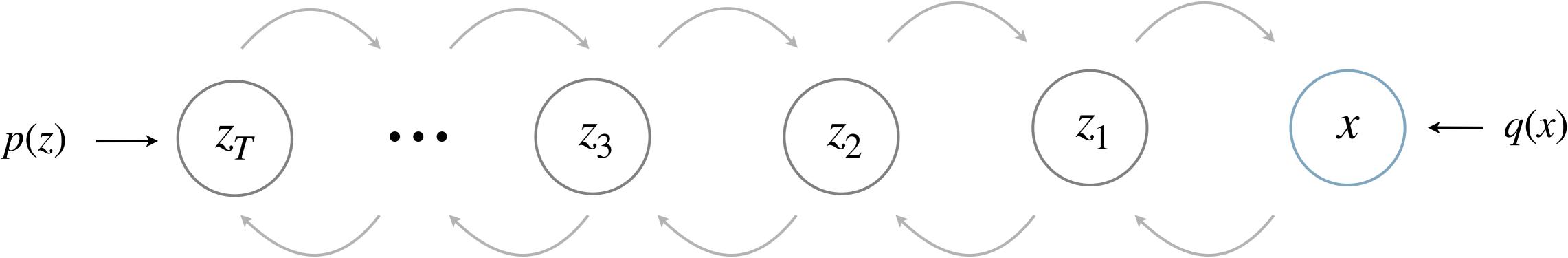


# Towards diffusion: (Markovian) hierarchical VAEs



Reverse process  $p\left(x, z_1, z_2, \dots, z_T\right) = p\left(z_T\right)p\left(z_{T-1} \mid z_T\right) \dots p\left(z_1 \mid z_2\right)p\left(x \mid z_1\right)$ 

Forward process 
$$q\left(x,z_{1},z_{2},\cdots,z_{T}\right)=q(x)\,q\left(z_{1}\mid x\right)\,q\left(z_{2}\mid z_{1}\right)\cdots q\left(z_{T}\mid z_{T-1}\right)$$



## Towards diffusion: (Markovian) hierarchical VAEs

Reverse process  $p(x, z_1, z_2, \dots, z_T) = p(z_T) p(z_{T-1} \mid z_T) \dots p(z_1 \mid z_2) p(x \mid z_1)$ 

$$p(z_{T-1} | z_T) \quad p(z_3 | z_4) \quad p(z_2 | z_3) \quad p(z_1 | z_2) \quad p(x | z_1)$$

$$p(z) \longrightarrow (z_T) \qquad (z_3) \qquad (z_2) \qquad (z_1) \qquad (z_1) \qquad (z_2) \qquad (z_2) \qquad (z_3) \qquad (z_2) \qquad (z_1) \qquad (z_2) \qquad (z_2) \qquad (z_3) \qquad (z_3) \qquad (z_2) \qquad (z_2) \qquad (z_3) \qquad (z_3) \qquad (z_2) \qquad (z_3) \qquad (z_3) \qquad (z_4) \qquad$$

Forward process q(x)

$$q(x, z_1, z_2, \dots, z_T) = q(x) q(z_1 | x) q(z_2 | z_1) \dots q(z_T | z_{T-1})$$

## Towards diffusion: (Markovian) hierarchical VAEs

#### Diffusion models can be seen as hierarchical VAEs with a few restrictions:

• The forward (encoding) distribution prescribed as a Markov chain of Gaussians; it is not learned

$$q\left(z_{t} \mid z_{t-1}\right) = \mathcal{N}\left(z_{t}; \alpha_{t} z_{t-1}, \beta_{t}\right)$$

Distributions of latents at the final timestep T is a standard (unit) Gaussian

$$q(z_T \mid z_{T-1}, ...x) = \mathcal{N}(z_T; 0, \mathbb{I})$$

The dimensionality of latents is the same as the data dimensionality

$$\dim(z_t) = \dim(x)$$

