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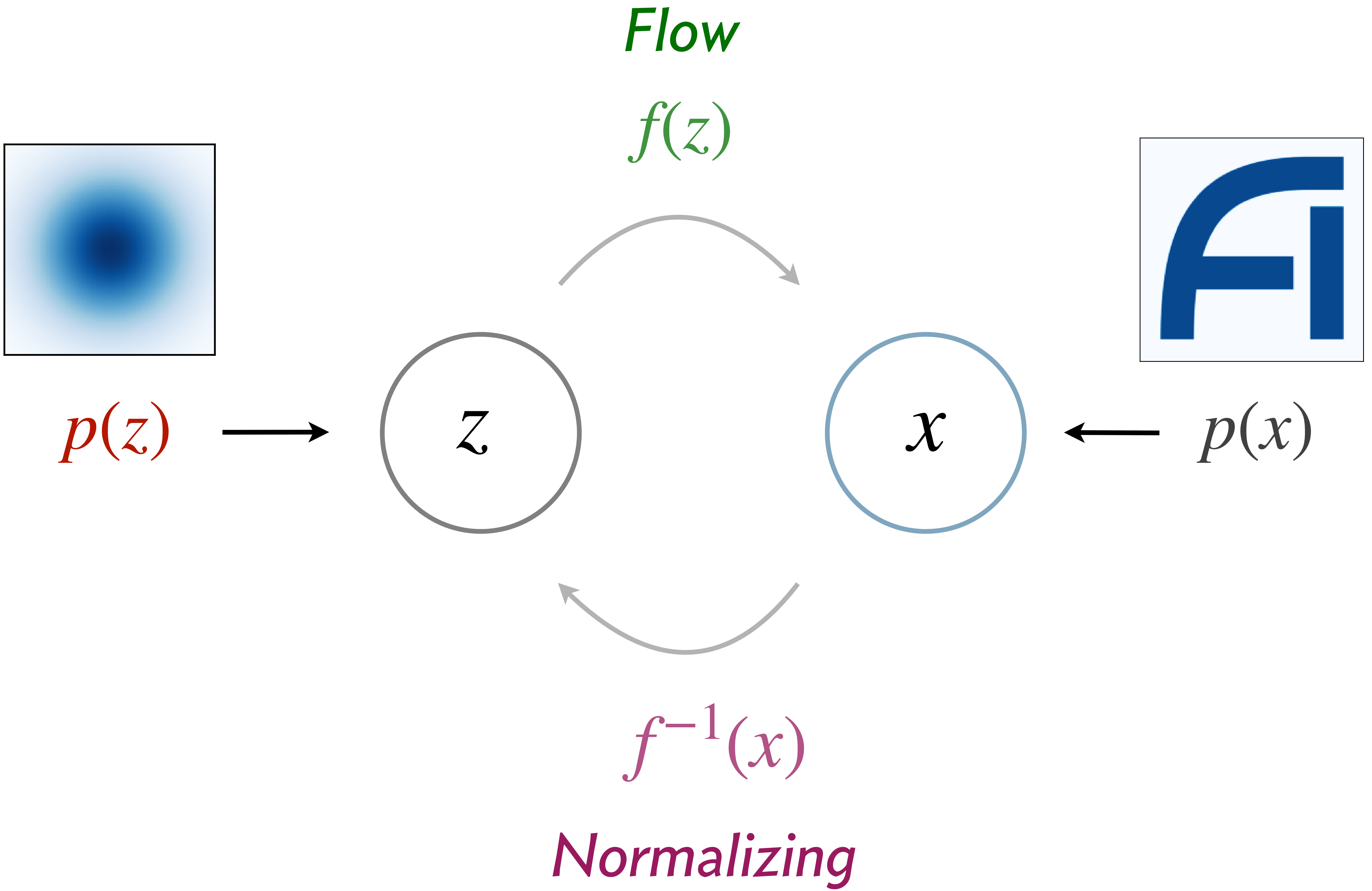
Normalizing flows



Flow
$f(z)$
$p(z)$
z
x
$p(x)$
$f^{-1}(x)$
Normalizing

The distribution $p(z)$ should

- *Have an easy-to-evaluate density*
- *Be easy to sample from* $z \sim p(z)$

Typically

$$p(z) = \mathcal{N}(0, \mathbb{I})$$

The function f should be

- *One-to-one*
- *Differentiable*
- *Invertible*

Diffeomorphism

- *Tractable* f^{-1} *and* $\det \nabla f$

- Constrained form of the transformation can limit the expressivity of flows compared to e.g. diffusion models.
- However, for certain physics applications the transformation can be restricted in a specific, desired way; *see Miranda Cheng's lectures on Wednesday!*

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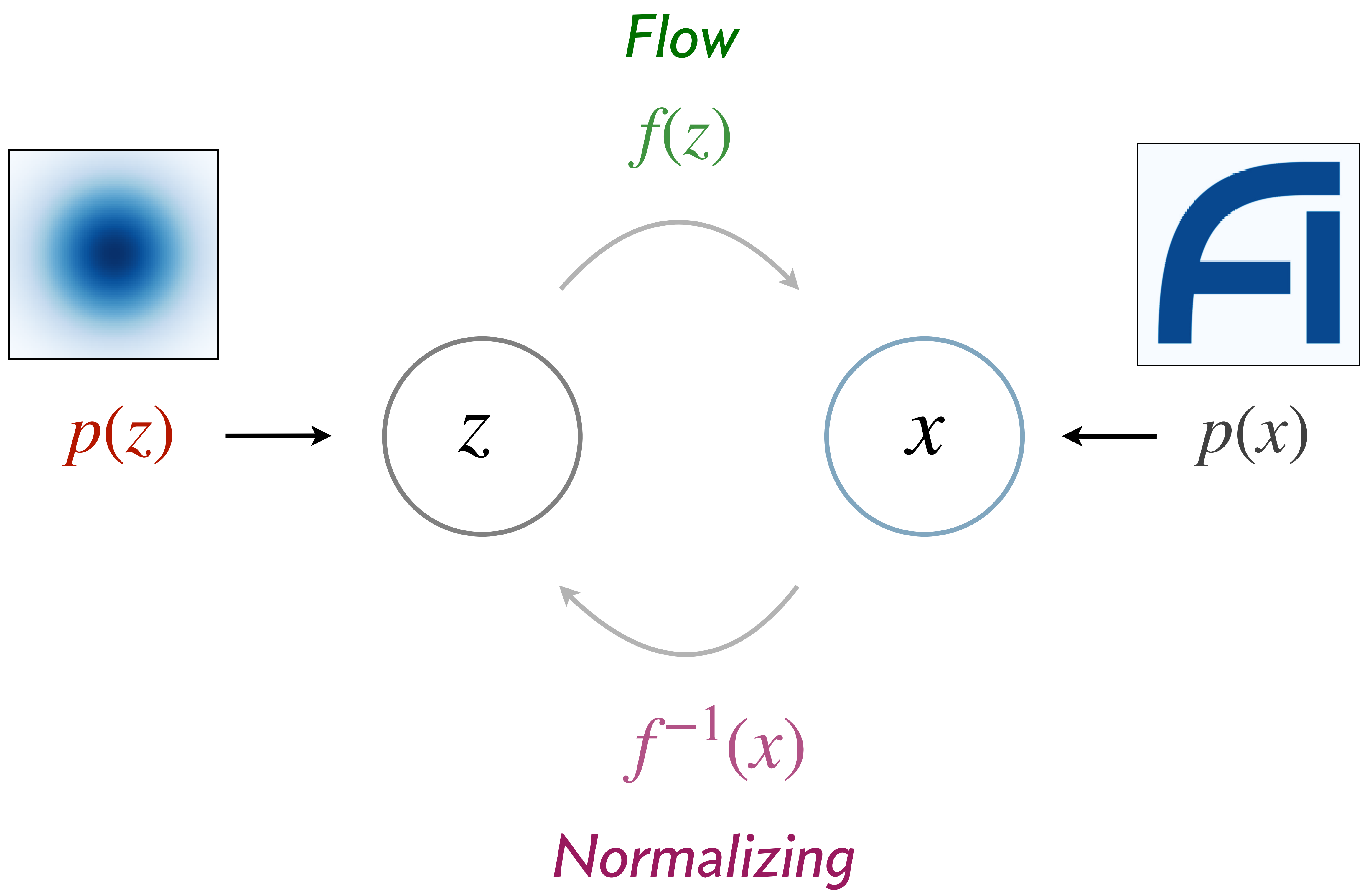
$\left.\begin{array}{l}\text{One-to-one}\\ \text{Differentiable}\\ \text{Invertible}\end{array}\right\}$ Diffeomorphism



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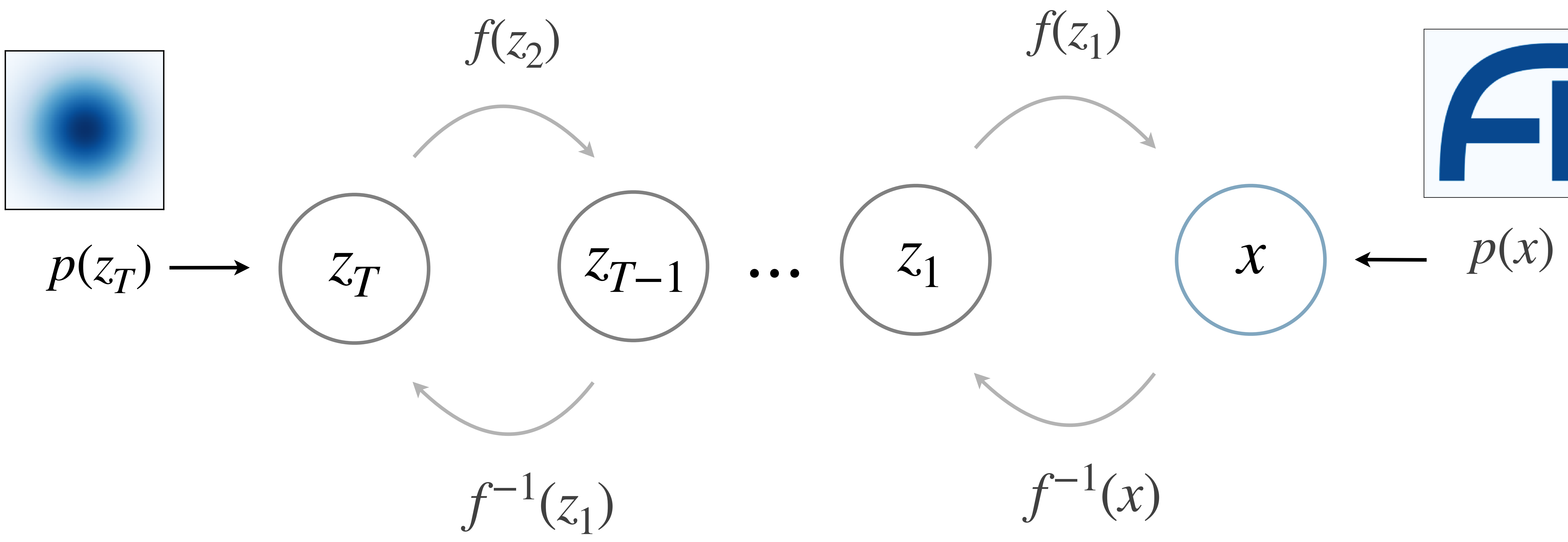
Normalizing flows

Multiple flow transformation can be easily composed for e.g. expressivity



Computing $p(x)$: *change-of-variables formula*

$$\int p(x)dx = \int p(z)dz = 1$$

$$p(x) = \pi(z)\left|\frac{dz}{dx}\right| = p\left(f^{-1}(x)\right)\left|\frac{df^{-1}}{dx}\right| = p\left(f^{-1}(x)\right)|\det \nabla f|^{-1}$$

Train using maximum-likelihood objective

$$\varphi^* = \left\langle \arg\max_{\varphi} p\left(f_\varphi^{-1}(x)\right)|\det \nabla f_\varphi|^{-1} \right\rangle_{x\sim p(x)}$$