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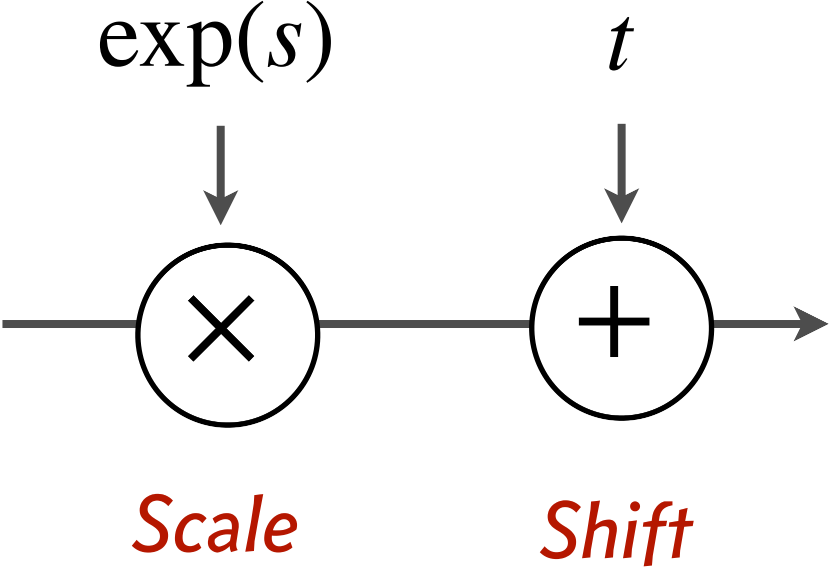
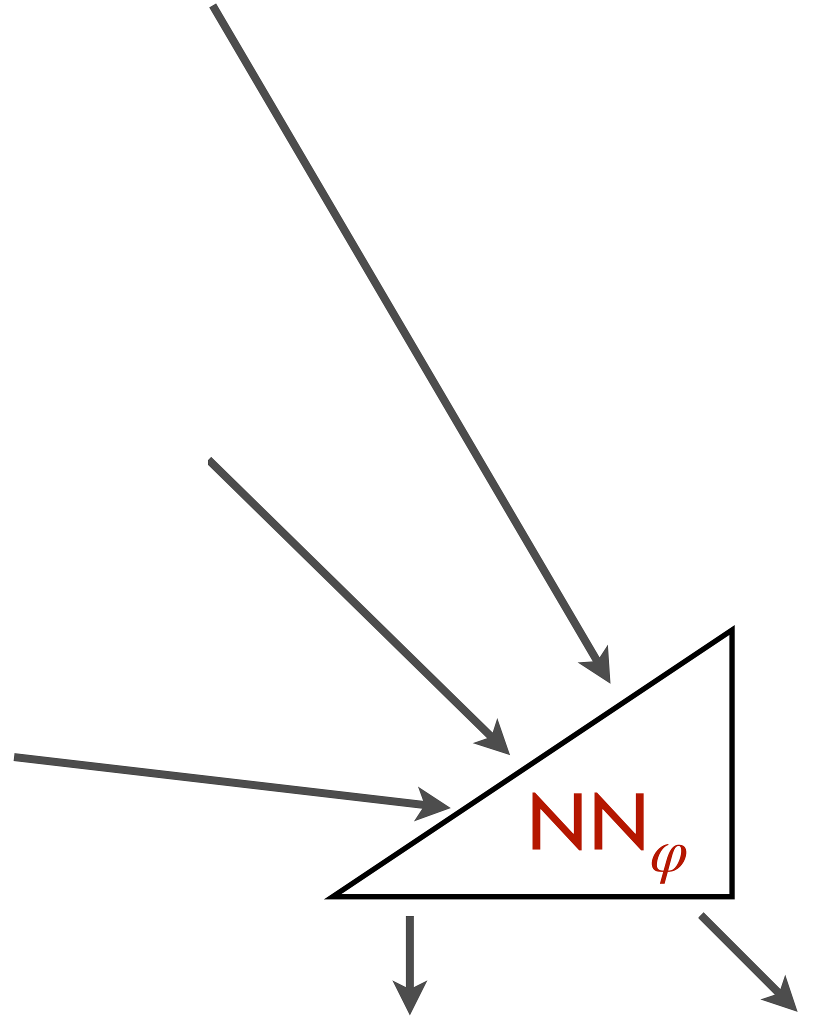
Simple flow transformations

Example: *Affine coupling flow*

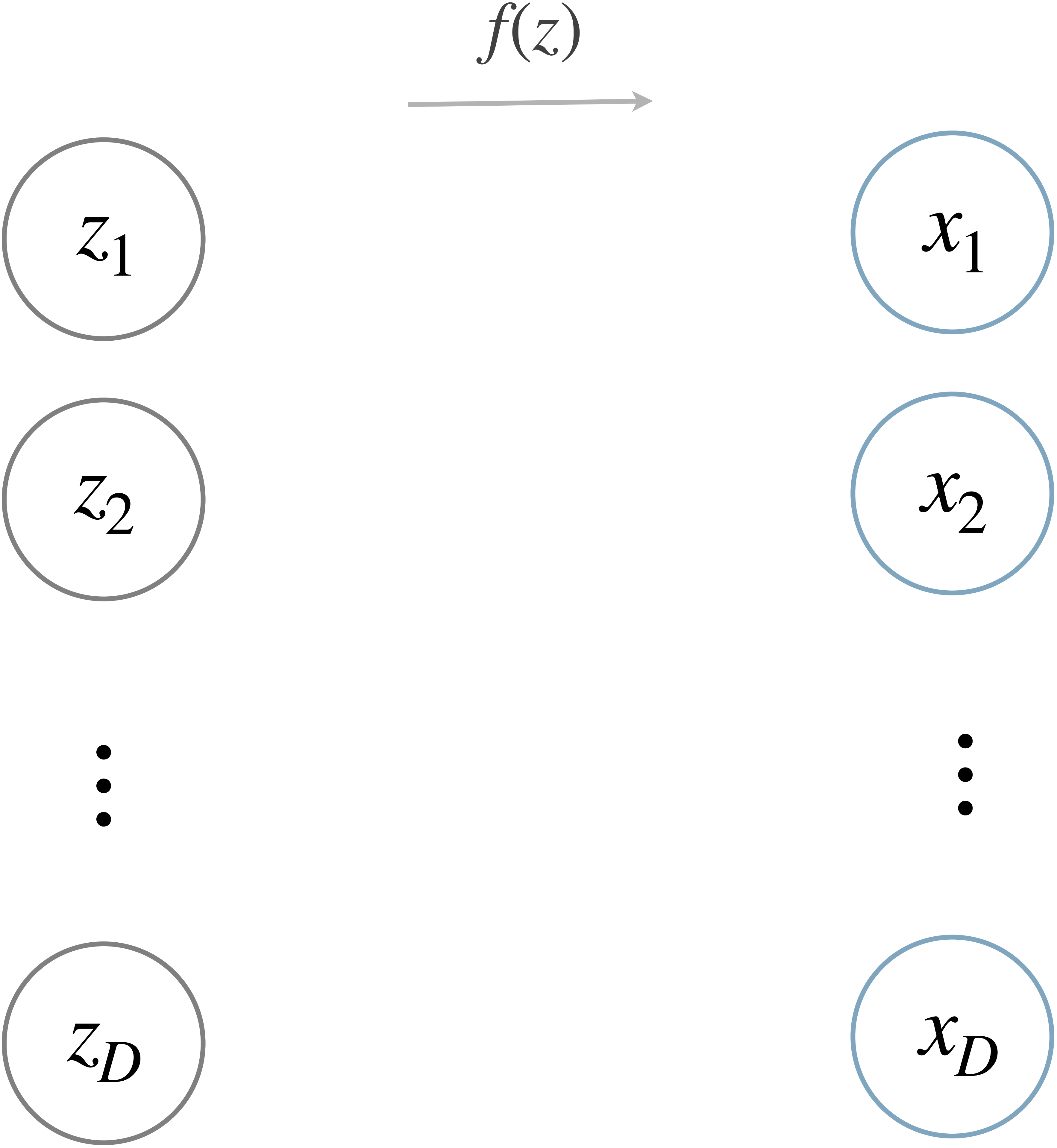


$\exp(s)$
t
Scale
Shift



$z_{d+1:D}$
transform
conditioned on
$z_{1:d}$

$z_{1:d}$ don't change



$f(z)$
z_1
z_2
z_D
x_1
x_2
x_D

Transformation ✅

$$x_{d+1:D} = z_{d+1:D} \odot \exp\left(s\left(x_{1:d}\right)\right) + t\left(x_{1:d}\right)$$

$$\Updownarrow$$

Inverse ✅

$$z_{d+1:D} = \left(x_{d+1:D} - t\left(x_{1:d}\right)\right) \odot \exp\left(-s\left(x_{1:d}\right)\right)$$

Jacobian determinant ✅

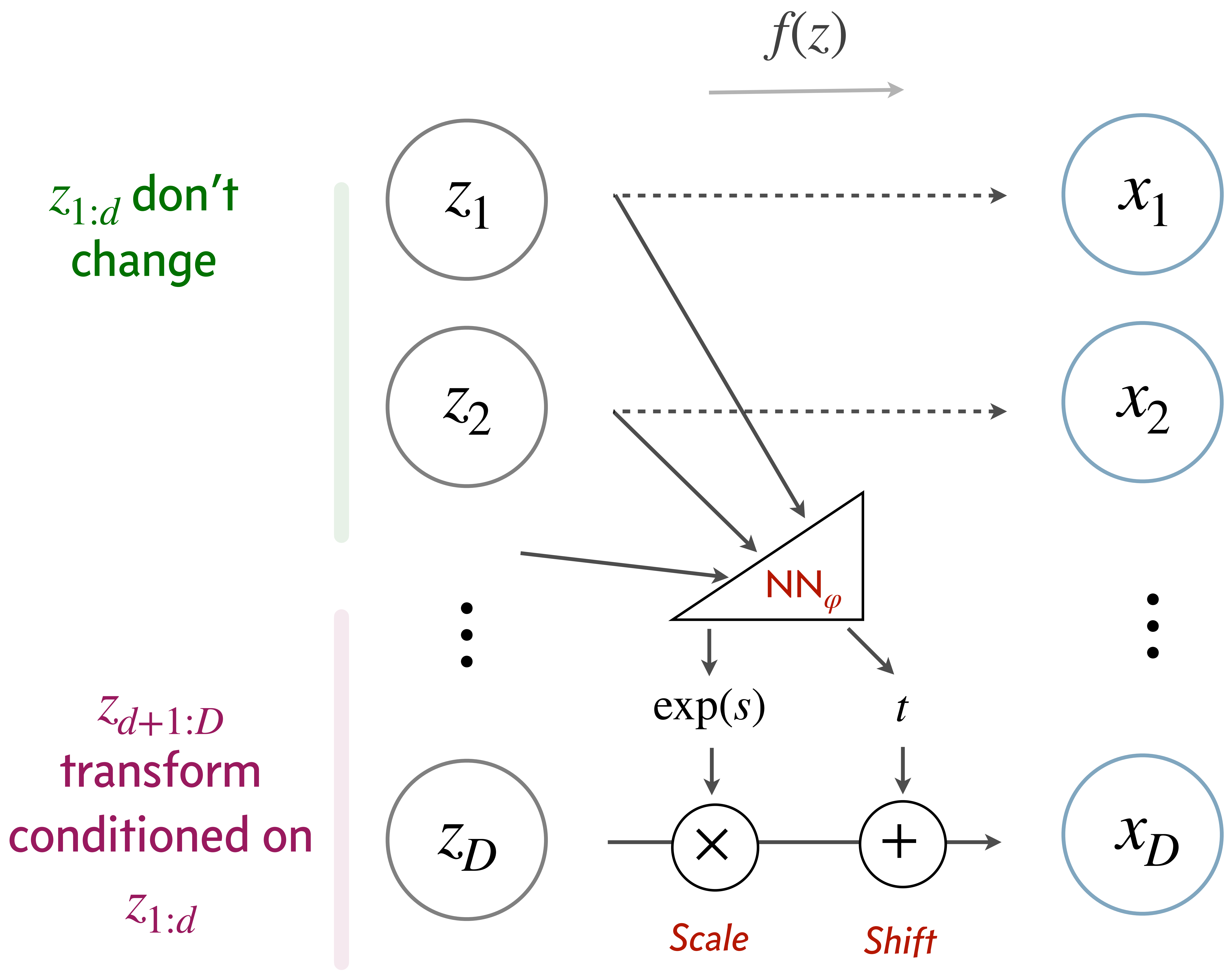
$$\det(\nabla f) = \prod_{j=1}^{D-d} \exp\left(s\left(z_{1:d}\right)\right)_j = \exp\left[\sum_{j=1}^{D-d} s\left(z_{1:d}\right)_j\right]$$

+ Switch up order of transformed variables at every transformation

[Dinh et al 2016]

Simple flow transformations

Example: *Affine coupling flow* [Dinh et al 2016]



Transformation ✅

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⇕

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+ Switch up order of transformed variables at every transformation

Continuous-time normalizing flows

Parameterize the transformation by a neural ODE

ODE with reversible dynamics

$$\frac{\mathrm{d}z}{\mathrm{d}t} = f\left(z(t)\right)$$