/70

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Continuous-time/SDE formulation

$$
\begin{aligned}
x_t &= \sqrt{1 - \beta(t)\Delta_t}\, x_{t-1} + \sqrt{\beta(t)\Delta_t}\,\mathcal{N}(0, \mathbf{I}) \\
&\approx x_{t-1} - \frac{\beta(t)\Delta_t}{2} x_{t-1} + \sqrt{\beta(t)\Delta_t}\,\mathcal{N}(0, \mathbf{I})
\end{aligned}
$$

In the limit of infinite time steps, $\Delta_t \to 0$ and the forward diffusion process can be written as

Which is an update rule corresponding to the Euler-Murayama discretization of the stochastic differential equation (SDE)

$$\mathrm{d}x_t = -\frac{1}{2}\beta(t)x_t\,\mathrm{d}t + \sqrt{\beta(t)}\mathrm{d}w_t$$

$$x_t = \sqrt{1 - \beta_t} \cdot x_{t-1} + \sqrt{\beta_t} \cdot \varepsilon$$

Continuous-time/SDE formulation

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$$\begin{aligned} x_t &= \sqrt{1-\beta(t)\Delta_t}\, x_{t-1} + \sqrt{\beta(t)\Delta_t}\,\mathcal{N}(0,\mathbb{I}) \\ &\approx x_{t-1} - \frac{\beta(t)\Delta_t}{2} x_{t-1} + \sqrt{\beta(t)\Delta_t}\,\mathcal{N}(0,\mathbb{I}) \end{aligned}$$

Which is an update rule corresponding to the Euler-Murayama discretization of the stochastic differential equation (SDE)

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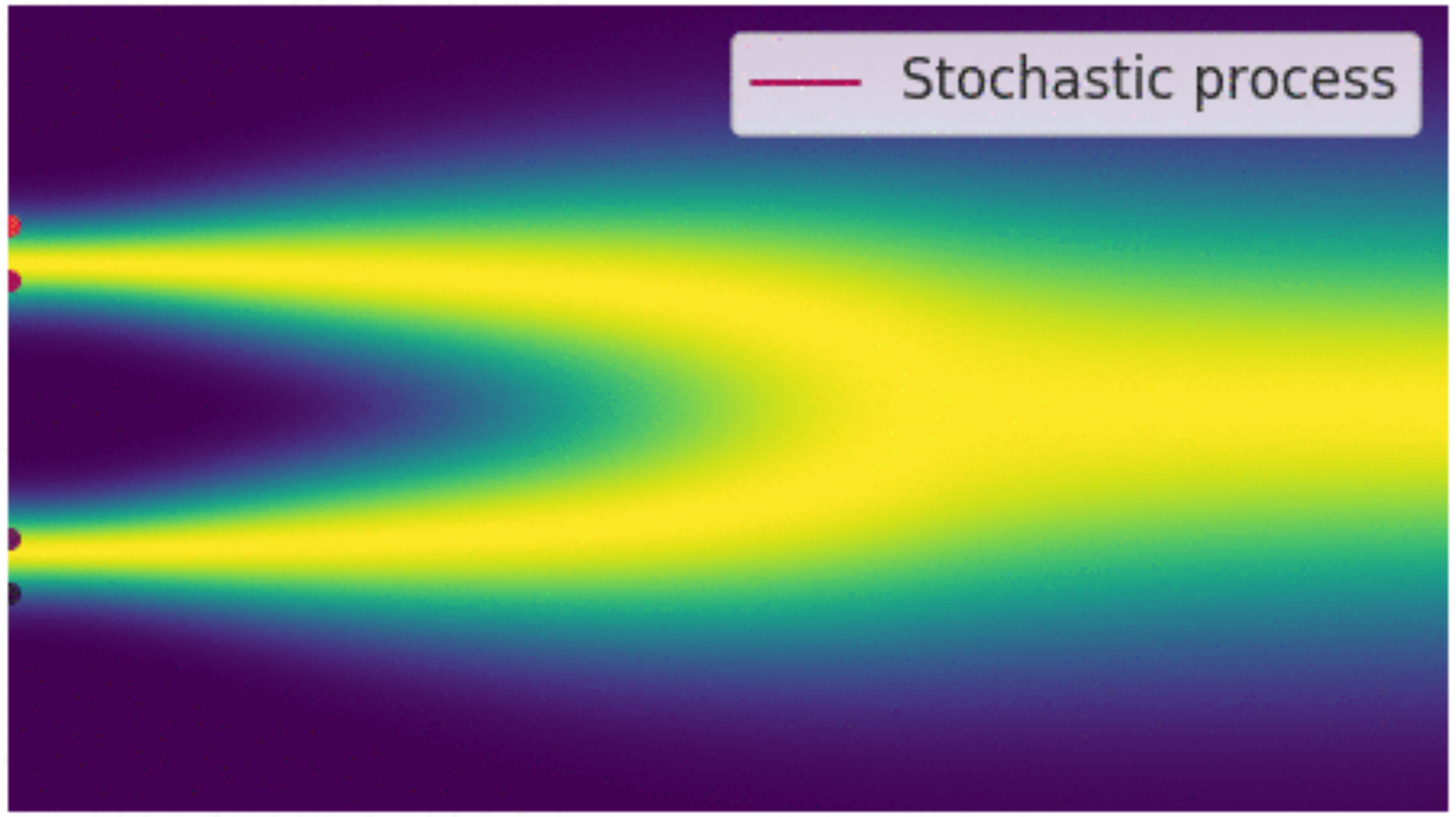
Continuous-time/SDE formulation

[Song et al 2021]

https://yang-song.net/blog/2021/score/

The forward diffusion process defined by an SDE



$$\mathrm{d}x_t = -\frac{1}{2}\beta(t)x_t\,\mathrm{d}t + \sqrt{\beta(t)}\mathrm{d}w_t$$