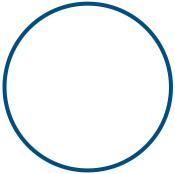
### Siddharth Mishra-Sharma (MIT/IAIFI) | IAIFI Summer School

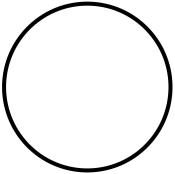


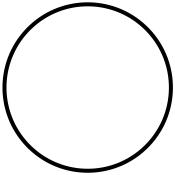
# The forward process and diffusion kernel

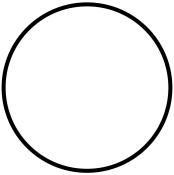
$$\sum_{t=2}^{T} \left\langle D_{\mathrm{KL}} \left( q \left( z_{t-1} \mid z_{t}, x \right) \parallel p_{\theta} \left( z_{t-1} \mid z_{t} \right) \right) \right\rangle_{q(z_{t}\mid x)}$$

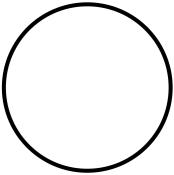
















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### Variance-preserving noise schedule

$$q\left(z_{t} \mid z_{t-1}\right) = \mathcal{N}\left(\sqrt{1-\beta_{t}} \cdot z_{t-1}, \beta_{t}\right)$$

$$z_t = \sqrt{1 - \beta_t} \cdot z_{t-1} + \sqrt{\beta_t} \cdot \varepsilon$$

### Predict arbitrary timestep without Markovian sampling

 $q\left(z_t \mid x\right)$ 

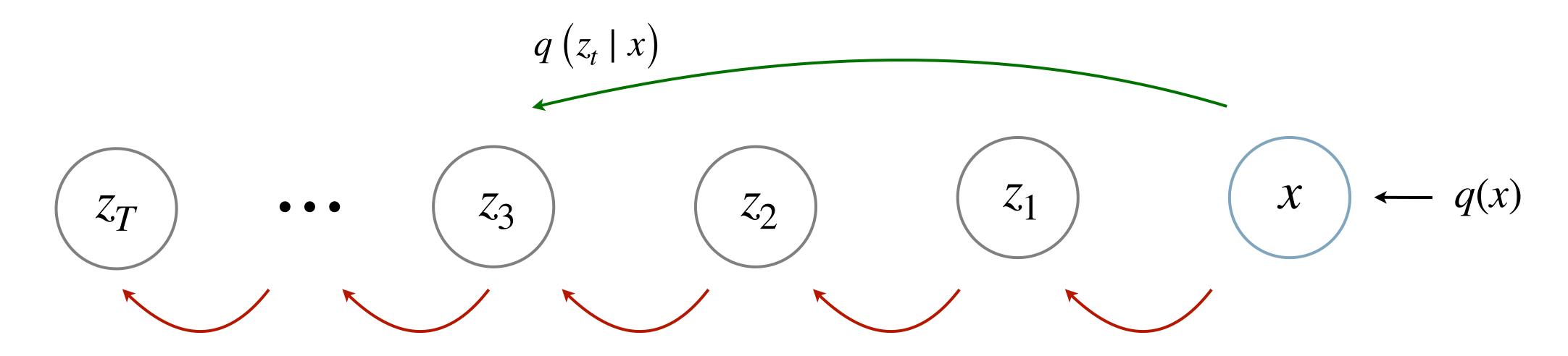
#### Diffusion kernel

$$q(z_t \mid x) = \mathcal{N}\left(\sqrt{\overline{\alpha_t}} \cdot x, \sqrt{1 - \overline{\alpha_t}}\right)$$
$$\bar{\alpha}_t = \prod_{i=1}^t \alpha_i$$

# The forward process and diffusion kernel

Predict arbitrary timestep without Markovian sampling

$$\sum_{t=2}^{T} \left\langle D_{KL} \left( q \left( z_{t-1} \mid z_{t}, x \right) \parallel p_{\theta} \left( z_{t-1} \mid z_{t} \right) \right) \right\rangle_{q(z_{t}\mid x)}$$



## Variance-preserving noise schedule

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#### Diffusion kernel

$$q(z_t \mid x) = \mathcal{N}\left(\sqrt{\overline{\alpha_t}} \cdot x, \sqrt{1 - \overline{\alpha_t}}\right)$$
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# The denoising objective

Given the nature of the forward (noising) process,  $q\left(z_{t-1} \mid z_t, x\right)$  can be computed analytically

$$q\left(z_{t-1} \mid z_t, x\right) = \mathcal{N}\left(z_{t-1}; \mu_q(x_t, x_0), \sigma_q(t)\mathbb{I}\right)$$

$$\sum_{t=2}^{T} \left\langle D_{\mathrm{KL}} \left( q \left( z_{t-1} \mid z_{t}, x \right) \parallel p_{\theta} \left( z_{t-1} \mid z_{t} \right) \right) \right\rangle_{q(z_{t}\mid x)}$$