Siddharth Mishra-Sharma (MIT/IAIFI) | IAIFI Summer School



Simple objectives as a weighted sum of ELBOs

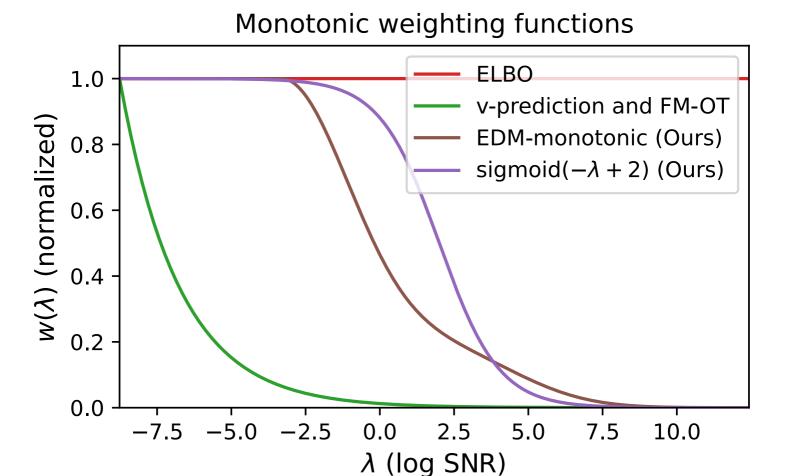
Kingma and Gao (2023) showed that common objectives can be written as a weighted sum (across different noise levels) of ELBOs

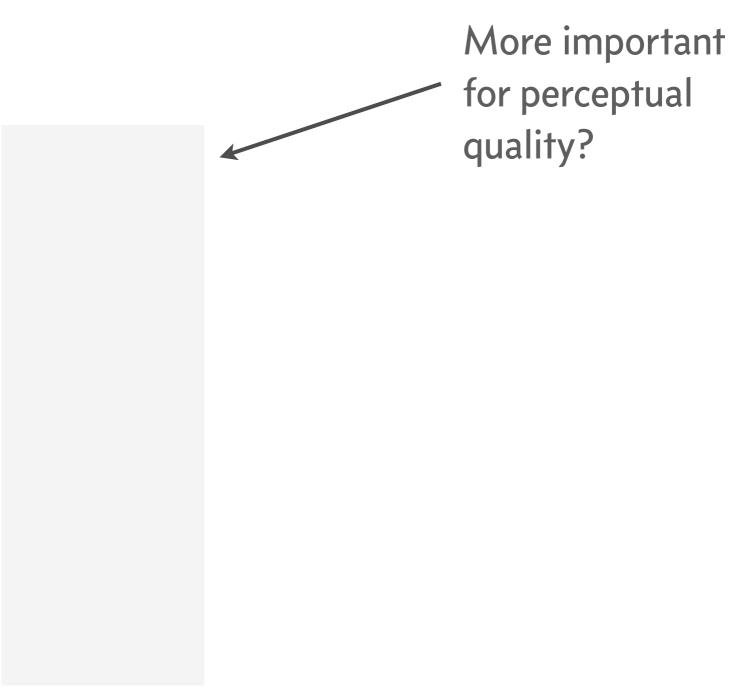
$$L_{w}(x) = \left\langle w(t) \cdot w_{\text{ML}}(t) \mid \mid \epsilon - \hat{\epsilon}_{\theta} \left(z_{t}, t \right) \mid \mid^{2} \right\rangle_{t,\epsilon}$$
Additional weighting Weighting for ELBO/

ML objective

Additional weighting

($w_{\rm ML}^{-1}$ for ϵ -prediction)

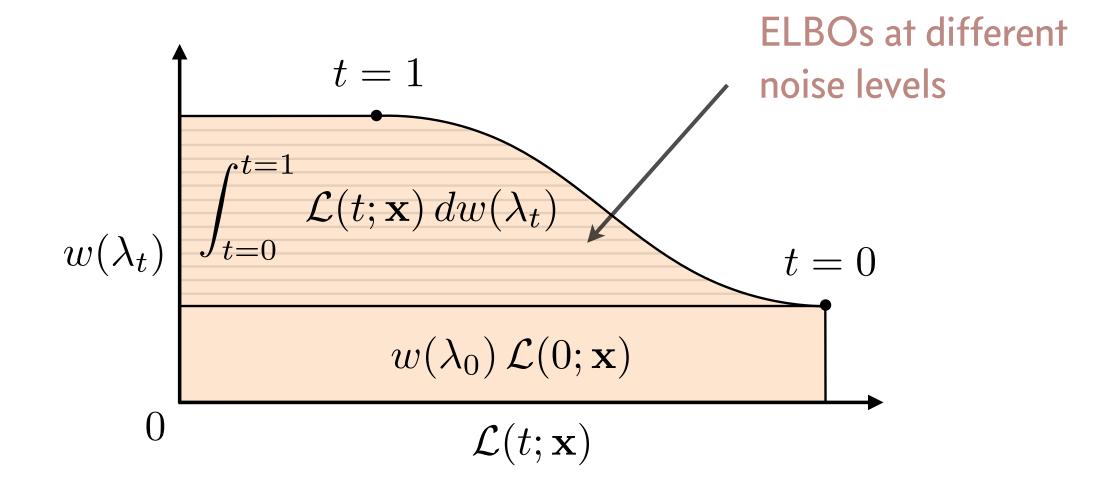




 $L(t;x) \equiv D_{\mathrm{KL}} \left(q \left(z_{t:1} \mid x \right) || p \left(z_{t:1} \right) \right)$

$$L_w(x) \propto \int_0^1 \frac{d}{dt} w(t) L(t; x) dt + w(t_0) L(0; x)$$

Interpretation: data augmentation with additive Gaussian noise / data-distribution smoothing



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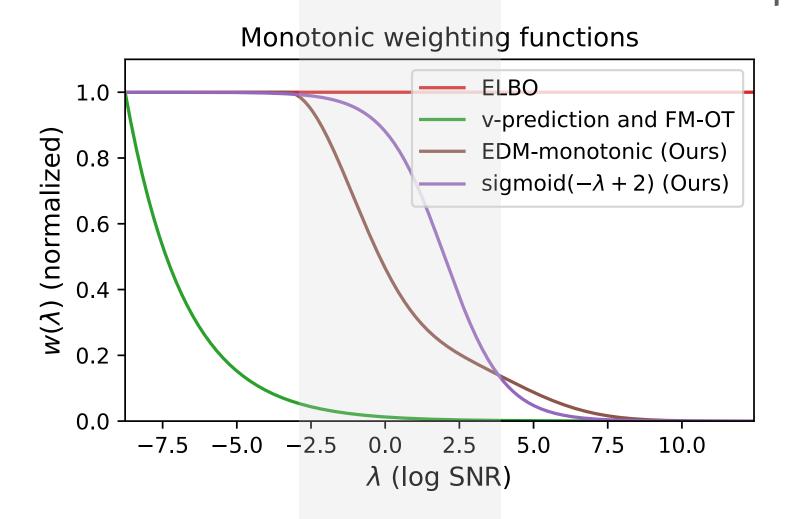
Weighting for ELBO/ ML objective

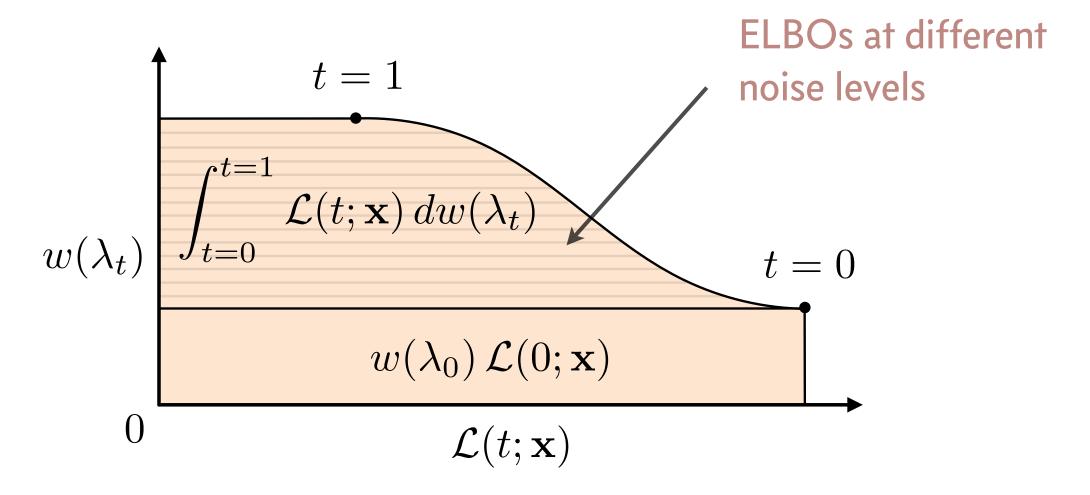
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More important for perceptual quality?





Continuous-time/SDE formulation

$$x_t = \sqrt{1 - \beta_t} \cdot x_{t-1} + \sqrt{\beta_t} \cdot \varepsilon$$

In the limit of infinite time steps, $\Delta_t \to 0$ and the forward diffusion process can be written as

$$x_{t} = \sqrt{1 - \beta(t)\Delta_{t}}x_{t-1} + \sqrt{\beta(t)\Delta_{t}}\mathcal{N}(0,\mathbb{I})$$

$$\approx x_{t-1} - \frac{\beta(t)\Delta_{t}}{2}x_{t-1} + \sqrt{\beta(t)\Delta_{t}}\mathcal{N}(0,\mathbb{I})$$