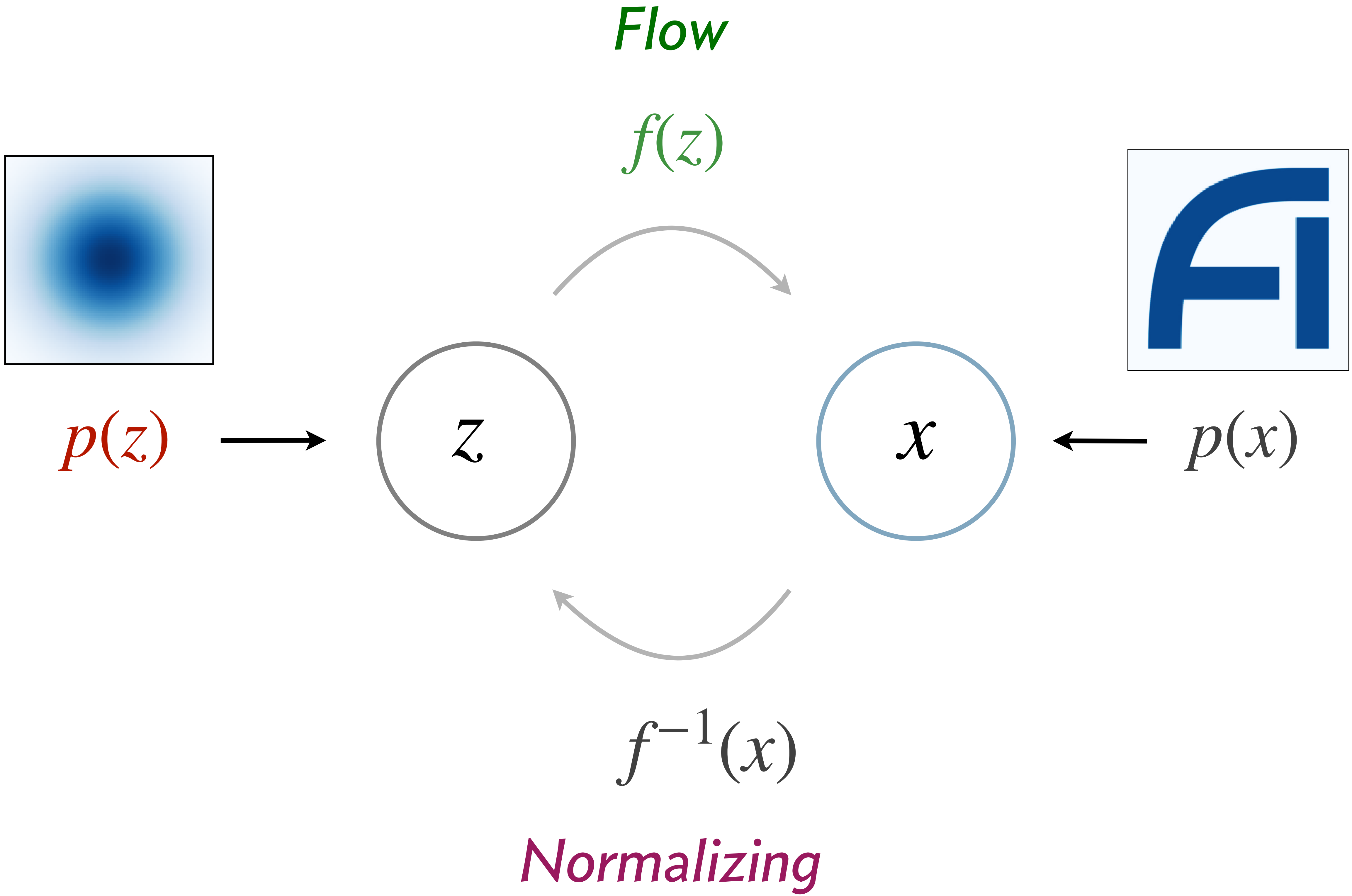
/62

56

# Normalizing flows


Flow
$f(z)$
$p(z)$
$z$
$x$
$p(x)$
$f^{-1}(x)$
Normalizing

## Data likelihood

$$p(x) = p\left(f^{-1}(x)\right) |\det \nabla f|^{-1}$$

The distribution $p(z)$ should

- *Have an easy-to-evaluate density*
- *Be easy to sample from* $z \sim p(z)$

Typically

$$p(z) = \mathcal{N}(0, \mathbb{I})$$

The function $f$ should be

- *One-to-one*
- *Differentiable*
- *Invertible*

Diffeomorphism

- *Tractable* $f^{-1}$ *and* $\det \nabla f$

# Normalizing flows

The distribution $p(z)$ should

- *Have an easy-to-evaluate density*
- *Be easy to sample from $z \sim p(z)$*

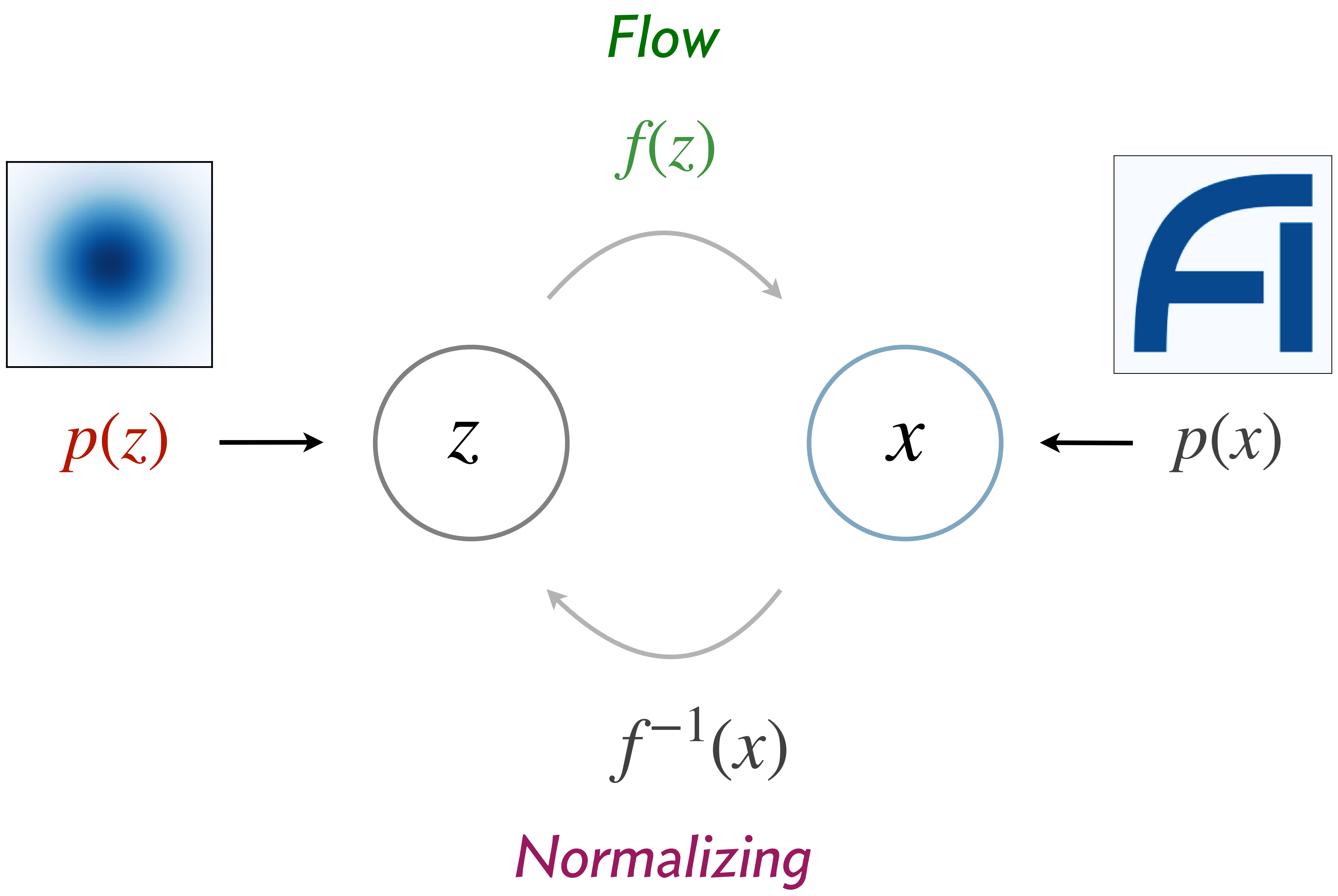
Typically

$$p(z) = \mathcal{N}(0, \mathbb{I})$$

The function $f$ should be

- *One-to-one*
- *Differentiable*
- *Invertible*
- *Tractable $f^{-1}$ and $\det \nabla f$*

$\left.\begin{matrix} \textit{One-to-one} \\ \textit{Differentiable} \\ \textit{Invertible} \end{matrix}\right\}$ Diffeomorphism

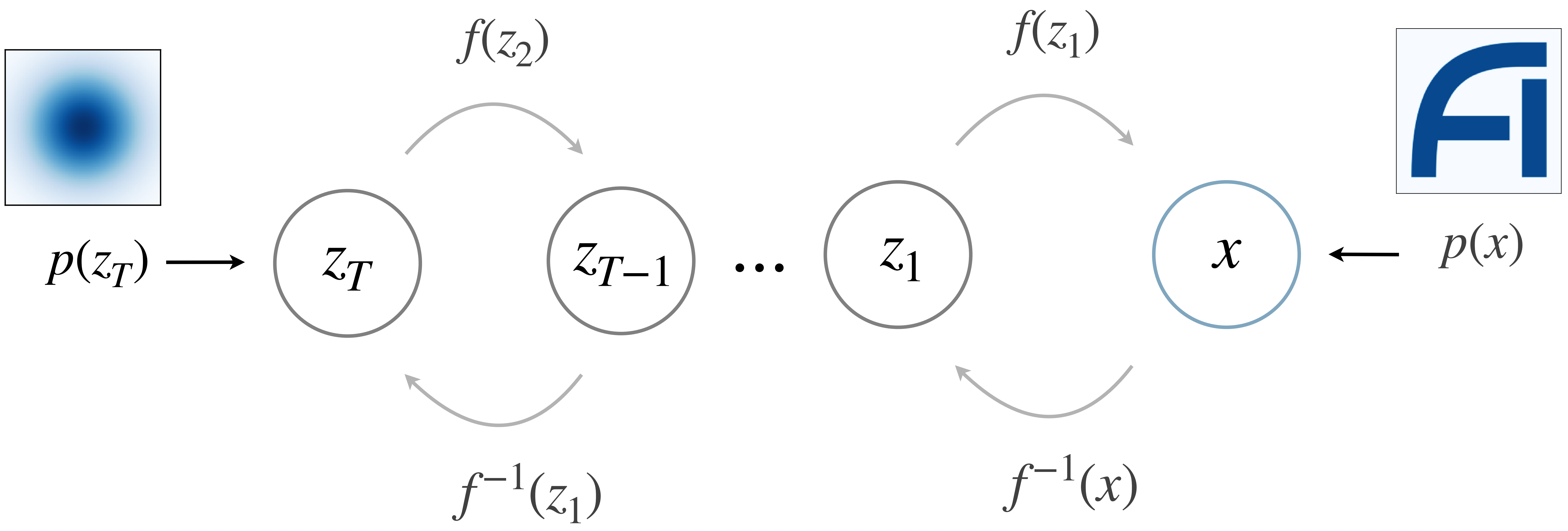



Data likelihood

$$p(x) = p\left(f^{-1}(x)\right) |\det \nabla f|^{-1}$$

# Normalizing flows

Multiple flow transformation can be easily composed for e.g. more expressivity




Computing $p(x)$: *change-of-variables formula*

$$\int p(x)dx = \int p(z)dz = 1$$

$$p(x) = \pi(z)\left|\frac{dz}{dx}\right| = p\left(f^{-1}(x)\right)\left|\frac{df^{-1}}{dx}\right| = p\left(f^{-1}(x)\right)|\det \nabla f|^{-1}$$

Train using maximum-likelihood objective

$$\phi^* = \left\langle \arg\max_{\phi} p\left(f_\varphi^{-1}(x)\right)|\det \nabla f_\varphi|^{-1} \right\rangle_{x\sim p(x)}$$