

Siddhant Mishra-Sharma (MIT/AI FI) Summer School

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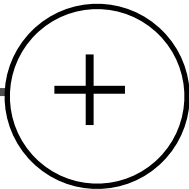
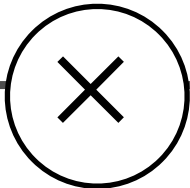
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Simple flow transformations

Example: *Affine coupling flow*

$\exp(s)$

t

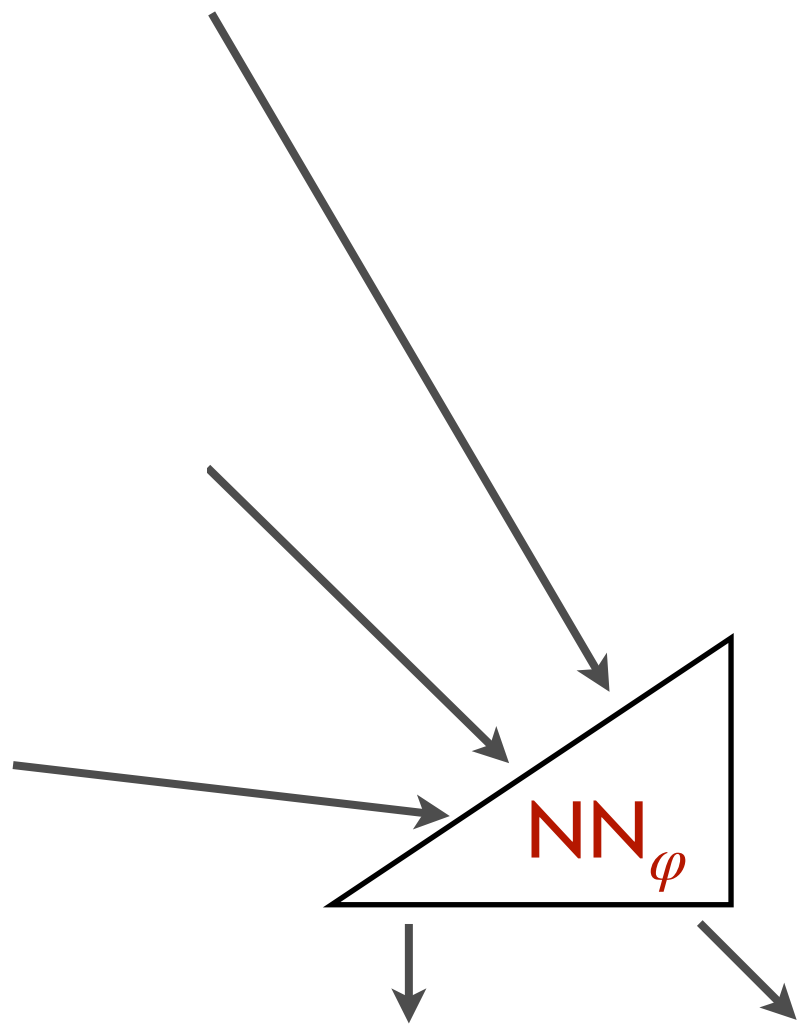


Scale

Shift

$z_{d+1:D}$
transform
conditioned on

$z_{1:d}$



$z_{1:d}$ don't
change



$f(z)$



z_1

x_1

z_2

x_2

\vdots

\vdots

z_D

x_D

Transformation 

$$x_{d+1:D} = z_{d+1:D} \odot \exp \left(s \left(x_{1:d} \right) \right) + t \left(x_{1:d} \right)$$



Inverse 

$$z_{d+1:D} = \left(x_{d+1:D} - t(x_{1:d}) \right) \odot \exp \left(-s(x_{1:d}) \right)$$

Jacobian determinant 

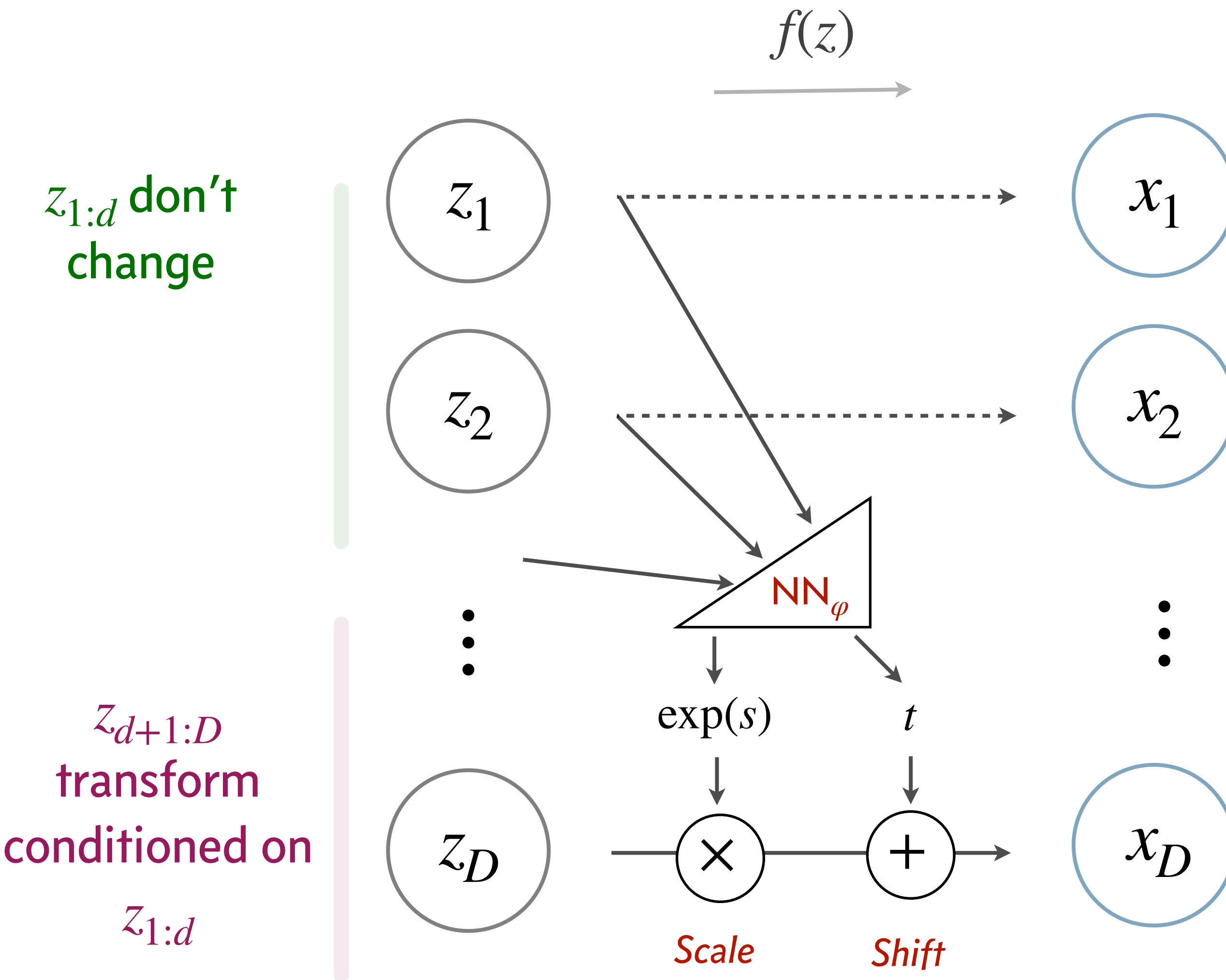
$$\det(\nabla f) = \prod_{j=1}^{D-d} \exp \left(s(z_{1:d})_j \right) = \exp \left(\sum_{j=1}^{D-d} s(z_{1:d})_j \right)$$

+ Switch up order of transformed
variables at every transformation

[Dinkel et al 2016]

Simple flow transformations

Example: *Affine coupling flow* [Dinh et al 2016]



Transformation ✓

$$x_{d+1:D} = z_{d+1:D} \odot \exp \left(s \left(x_{1:d} \right) \right) + t \left(x_{1:d} \right)$$



Inverse ✓

$$z_{d+1:D} = \left(x_{d+1:D} - t \left(x_{1:d} \right) \right) \odot \exp \left(-s \left(x_{1:d} \right) \right)$$

Jacobian determinant ✓

$$\det(\nabla f) = \prod_{j=1}^{D-d} \exp \left(s \left(z_{1:d} \right) \right)_j = \exp \left(\sum_{j=1}^{D-d} s \left(z_{1:d} \right)_j \right)$$

+ Switch up order of transformed variables at every transformation

Continuous-time normalizing flows

Parameterize the transformation by a neural ODE

ODE with reversible dynamics

$$\frac{dz}{dt} = f(z(t))$$

