Revision

Roots of polynomials

n^{th} order polynomial

$$f_n(x) = a_0 + a_1 x + a_2 x + \cdots + a_n x^n$$

If a's are real

(i) n roots (complex/real)

(ii) if n is odd - at least one real root

(iii) Complex roots occur in conjugate pairs

1 Evaluation of polynomials

a. $f_3(x) = a_0 + a_1 \cdot x + a_2 \cdot x \cdot x \cdot + a_3 x \cdot x \cdot x$

n^{th} order — n additions

$\frac{n(n+1)}{2}$ multiplications

b. $f_3(x) = a_0 + x \cdot (a_1 + x \cdot (a_2 + x \cdot a_3))$

n — addition

n - multiplication

2. Division of polynomials

$$f_3(x) = x^3 - 13x - 12$$

Divide $x^2 - x - 1$

$$
\begin{array}{r}
x + 1 \\
x^2 - x - 1 \;\big)\; \cancel{x^3} + 0x^2 - 13x - 12 \\
-\;\cancel{x^3} \mp x^2 \mp x \qquad\qquad \\
\hline
x^2 - 12x - 12 \\
-\; x^2 \mp x \mp 1 \\
\hline
-11x - 11
\end{array}
$$

$$f_n(x) = (x^2 - rx - s)\, f_{n-2}(x) + R$$

3. Deflation of polynomials

Let's assume that we have determined 's' to be a root of $f_n(x)$

$$f_n(x) = (x - s)\, f_{n-1}(x) = 0$$

✓

3. Effective degree of polynomial

$$f(x) = x^{12} - 6x^{8} + 4x^{4} + 1 = 0$$

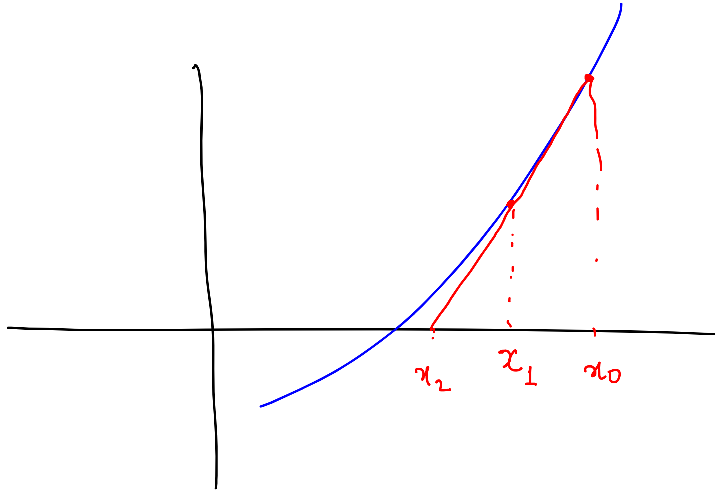
$$= (x^4)^3 - (6x^4)^2 + 4x^4 + 1 = 0$$

but x^4 it's a cubic polynomial

$$f(x) = x^6 + 3x^4 + 2x^2 = 0$$

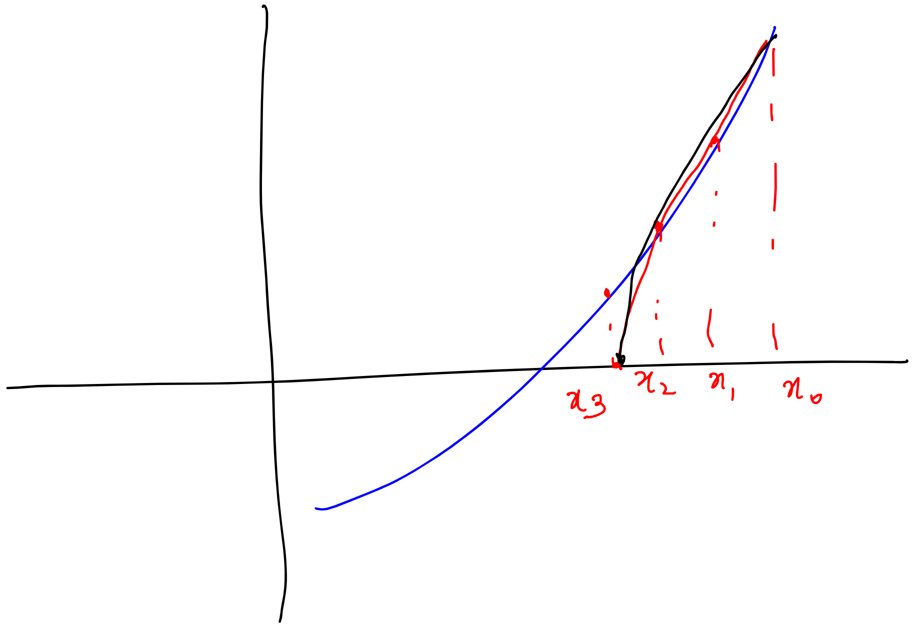
Roots of Polynomial

1. Müller Method



secant

Instead of straight line a parabola is fitted



To determine x_3

$$f(x) = ax^2 + bx + c$$

$$f(x) = a(x-x_2)^2 + b(x-x_2) + c$$

$$x_3 - x_2 = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

Equivalent expression

$$\boxed{x_3 - x_2 = \frac{-2c}{b \pm \sqrt{b^2-4ac}}}$$

2. Bairstow's Method

$$f_n(x) = (x^2 - rx - s) f_{n-2}(x) + R$$

Start with a guess value of r and s

$$= \underbrace{(x^2 - r^* x - s^*)} f_{n-2}(x) = 0$$

To find r^* and s^* — Newton-Raphson method

System of Linear Equations

n - equations

n - unknowns

E_1 : $a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n = b_1$

E_2 : $a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n = b_2$

$\vdots$

E_n : $a_{n1}x_1 + a_{n2}x_2 + \cdots + a_{nn}x_n = b_n$

$$Ax = b$$

$b = 0 \rightarrow$ Homogeneous

$b \neq 0 \rightarrow$ non-homogeneous

Coefficient matrix

$$A = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & & & \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{bmatrix}_{n \times n}$$

$$x = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}_{n \times 1} \qquad b = \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_n \end{bmatrix}_{n \times 1}$$

Augmented matrix

$$\tilde{A} = [A \;\; b]$$

All vectors will be column vector

Two methods

1. **Direct Method**

Gives exact answers (ignore round off errors) in finite number of steps

Gauss elimination

$n \lessapprox 1000$

2. **Indirect method**

Iterative approach gives approximate answers

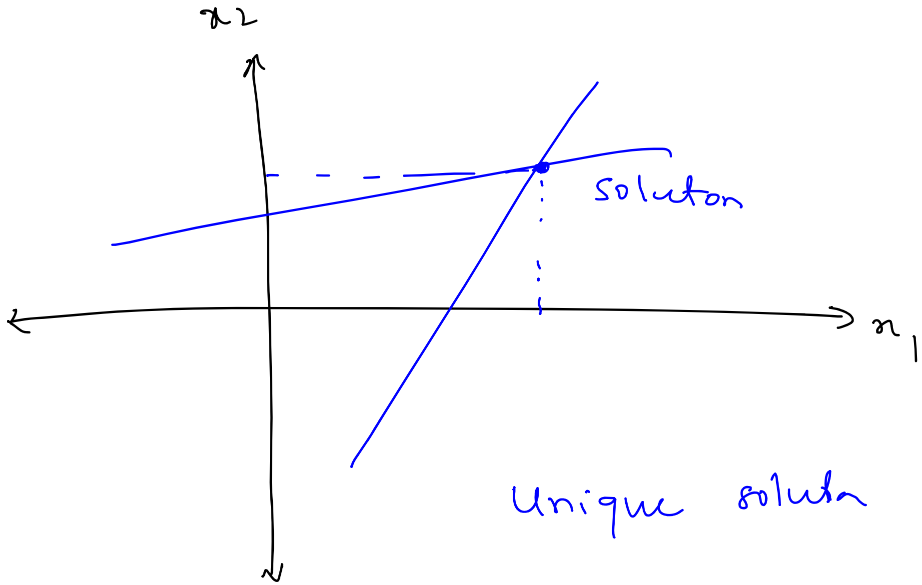
large system ($n > 10{,}000$)

Graphical interpretation

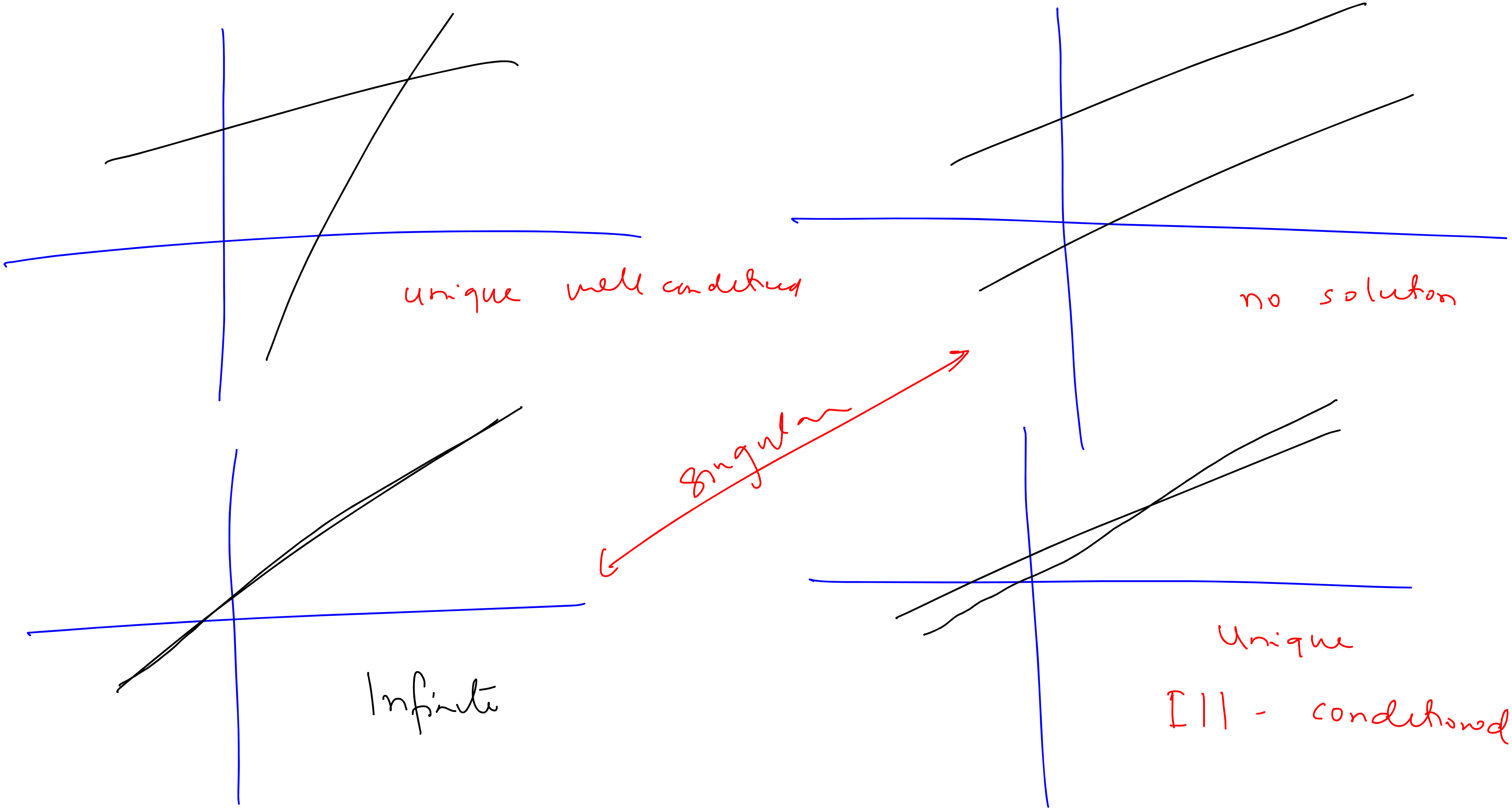
2-variables

$$a_{11}x_1 + a_{12}x_2 = b_1$$

$$a_{21}x_1 + a_{22}x_2 = b_2$$



Unique solution


unique well conditioned
no solution
singular
Infinite
Unique
Ill-conditioned