ESO 208A: Computational Methods in Engineering

Tutorial 1

Truncation error and estimation of error bound

1. Use the second order Taylor series approximation of $f(x) = e^x \cos(x)$ at $x = 0$ to :

   i. Approximate the function values at $x = 0.5$ and $x = 1$

   ii. Estimate the true error for both the approximations and compare them with the upper bound of truncation errors obtained from Taylor's theorem.

   iii. Approximate $\int_0^1 f(x)\mathrm{d}x$ using Taylor's series. Determine an upper bound for the error in and compare it with true error.

Propagation of data error

2. Consider the expression $z = x^2 y - xy^2$ where x and y are measured quantities used for estimating z. If the measured values of x and y are 3 and 2, respectively, and their measurement errors are $\delta x = \delta y = 0.1$,

   i. Estimate error in z by using first order error analysis

   ii. Recalculate error in z by second order analysis and comment on the usefulness of higher order error analysis.

Round-off error

3. Consider the following function, where x is very large

$$f(x) = \sqrt{(x+1)} - \sqrt{x}$$

   i. Calculate condition number for the problem

   ii. Estimate the value function for $x = 208208$ by performing operations with six significant digits and calculate the corresponding relative error (use double precision calculation to obtain the true solution that you will need for calculating relative error).

   iii. How can we reduce the relative error?