True value = Approx. value + error

error = True − Approx

$$\text{Relative error} = \frac{\text{True} - \text{Approx}}{\text{True value}}$$

1. LHC to OAT

$d = 800$ m

$\tilde{d} = 1000$ m

$e = -200$ m

$e_r = -\frac{200}{800} = -\frac{1}{4}$

2. Camples to railway station

$d = 14.7$ km

$\tilde{x} = 1.5$ km

$e = 0.3$ km

$e_r = \frac{0.3}{1.5}$

SIGNIFICANT DIGITS/FIGURES

228.15 — 5

108.01

00.034

3400 — 4 ?

34.50 — 4

3.40×10^3

3.400×10^3

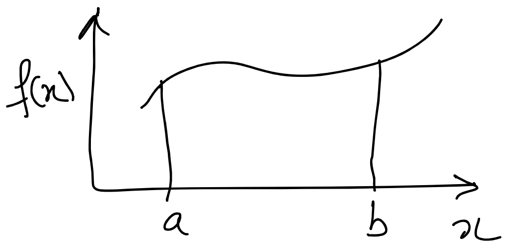
Errors

- Model error
- Data error
- Truncation error
- Round-off error

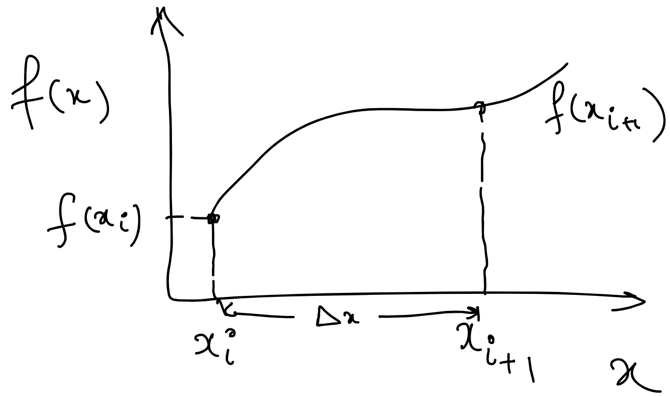
$\left.\begin{matrix}\text{Truncation error} \\ \text{Round-off error}\end{matrix}\right\}$ Finite nature of computers

Truncation error – Error committed when a limiting process is truncated before one has reached the limiting value

$$\int_a^b f(x)\,dx \approx \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i)\,\Delta x_i$$



Function approximation



Taylor series

$$f(x_{i+1}) = f(x_i) + \Delta x\, f'(x_i) + \frac{\Delta x^2}{2!} f''(x_i) + \dots + \frac{\Delta x^n}{n!} f^{n}(x_i) + R_n$$

$$R_n = \frac{\Delta x^{n+1}}{(n+1)!} f^{n+1}(\xi) \qquad x_i \le \xi \le x_{i+1}$$

Example

$$f(x) = -0.1x^4 - 0.15x^3 - 0.5x^2 - 0.25x + 1.2$$

$x_i = 0 \quad f(x_i = 0) = 1.2$

$\Delta x = 1$

$x_{i+1} = 1 \quad f(x_{i+1} = 1) = 0.2$

1. Zero $\quad f(x_{i+1}) = f(x_i) = 1.2$

$$e = 0.2 - 1.2 = -1.0$$

2. First order $\quad f(x_{i+1}) = f(x_i) + \Delta x f'(x_i)$

$$f'(x) = -0.4x^3 - 0.45x^2 - 0.1x - 0.25 \Big|_{x=0}$$

$$= -0.25$$

$$f(x_{i+1}) = 1.2 - 0.25$$

$$= 0.95$$

$$e = -0.75$$

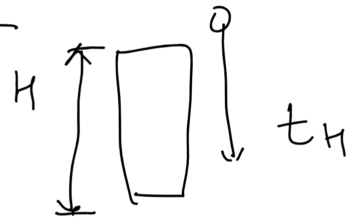
Data error

$$y = f(x)$$

$$\tilde{x} = x - e$$

$$\tilde{y} = f(\tilde{x})$$

Example



$$t_H = \sqrt{\frac{2H}{g}}$$

$$\Rightarrow \quad g = \frac{2H}{t_H^2} = f(H, t)$$

$$\tilde{t} = t \pm \Delta t$$

$$\tilde{H} = H \pm \Delta H$$

$$\tilde{g} = g \pm \Delta g$$

$H = 660 \pm 0.01\,m$

$t = 11.65 \pm 0.01\,s$

$$\Delta g = \frac{\partial f}{\partial t}\Delta t + \frac{\partial f}{\partial H}\Delta H$$

$$\frac{\partial f}{\partial t} = \frac{\partial}{\partial t}\left(\frac{2H}{t^2}\right) = \frac{-4H}{t^3}$$

$$\frac{\partial f}{\partial H} = \frac{\partial}{\partial H}\left(\frac{2H}{t^2}\right) = \frac{2}{t^2}$$

$$\boxed{\Delta g = \frac{-4H}{t^3}\Delta t + \frac{2}{t^2}\Delta H}$$

Taylor series

$$f(x_0 + \Delta x) = f(x_0) + \Delta x \left.\frac{df}{dx}\right|_{x_0} + \ldots$$

$$f(x_0 + \Delta x, z_0 + \Delta z) = f(x_0, z_0) + \Delta x \left.\frac{\partial f}{\partial x}\right|_{x_0} + \Delta z \left.\frac{\partial f}{\partial z}\right|_{z_0} + \ldots$$

$$f(x_0 + \Delta x, z_0 + \Delta z) - f(x_0, z_0)$$

$$\Delta f = \Delta x \left.\frac{\partial f}{\partial x}\right|_{x_0} + \Delta z \left.\frac{\partial f}{\partial z}\right|_{z_0}$$

$$f(x_1, x_2, x_3, \ldots x_n)$$

$$\Delta f = \sum_{i=1}^{n} \left.\frac{\partial f}{\partial x_i}\right|_{x_i}$$