MIDTERM EXAMINATION
MTH101- Calculus And Analytical Geometry

Question No: 1 (Marks: 1) - Please choose one

If f is a twice differentiable function at a stationary point x_0 and $f''(x_0) < 0$ then f has relative ……. At x_0

- Minima
- **Maxima**
- None of these

Note:Maxima(If Maxima refers to local maximum)

Question No: 2 (Marks: 1) - Please choose one

A line $x = x_0$ is called ---------- for the graph of a function f if $f(x) \to +\infty \; or \; f(x) \to -\infty$ as x approaches x_0 from the right or from the left

- Horizontal asymptotes
- None of these
- **Vertical asymptotes**

Question No: 3 (Marks: 1) - Please choose one

A line $y = y_0$ is called a ……. for the graph f if

$$\lim_{x \to +\infty} f(x) = y_0 \quad or \quad \lim_{x \to -\infty} f(x) = y_0$$

- Vertical asymptotes
- Horizontal asymptotes
- **None of these**

Question No: 4 (Marks: 1) - Please choose one

According to Power-Rule of differentiation, if $f(x) = x^n$ where n is a real number, then $\frac{d}{dx}[x^n] =$

- x^{n-1}
- $n x^{n-1}$

- $n\,x^{n+1}$
- $(n-1)x^{n+1}$

Question No: 5 (Marks: 1) - Please choose one

If $y=\frac{1}{1-x}$ then $\frac{dy}{dx}=$

- 1
- -1
- $\frac{1}{(1-x)^2}$
- $\frac{-1}{(1-x)^2}$

Question No: 6 (Marks: 1) - Please choose one

If $xy=4$ then $\frac{dy}{dx}=$

- 0
- $\frac{-1}{x^2}$
- $\frac{4}{x^2}$
- $\frac{-4}{x^2}$

Question No: 7 (Marks: 1) - Please choose one

If $2x-y=-3$ then $\frac{dy}{dx}=$

- 2
- -2
- 0
- -3

Question No: 8 (Marks: 1) - Please choose one

$\frac{d}{dx}[\sec x]=$ _____

- $\frac{1}{1+\sin^2 x}$
- $\frac{-\sin x}{1+\sin^2 x}$
- $\frac{1}{1-\sin^2 x}$
- $\frac{\sin x}{1-\sin^2 x}$

Question No: 9 (Marks: 1) - Please choose one

30^0 = _____

- $\frac{\pi}{3}$
- $\frac{\pi}{4}$
- $\frac{\pi}{6}$
- $\frac{\pi}{2}$

Question No: 10 (Marks: 1) - Please choose one

Consider a function $h(x)$ and a constant c then

$\frac{d}{dx}\left((c)\{h(x)\}\right)$ = _____

- 0
- $\frac{d}{dx}\left(h(x)\right)$

- $\frac{d}{dx}(h(cx))$
- $c\frac{d}{dx}(h(x))$

Question No: 11 (Marks: 1) - Please choose one

$\frac{d}{dx}[\text{cosec } x] =$ _____

- $\frac{1}{1+\cos^2 x}$
- $\frac{-\cos x}{1-\cos^2 x}$
- $\frac{-\cos x}{1-\cos^2 x}$
- $\frac{1}{1-\cos^2 x}$

Question No: 12 (Marks: 1) - Please choose one

Chain rule is a rule for differentiating _____ of functions.

- **Product**
- Sum
- Difference
- Composition

Question No: 13 (Marks: 1) - Please choose one

$\frac{d}{dx}[x^n] = nx^{n-1}$

The power rule, holds if n is _____

- **An integer**
- A rational number
- An irrational number
- All of the above

Question No: 14 (Marks: 1) - Please choose one

Let a function f be defined on an interval, and let x_1 and x_2 denote points in that

interval. If $f(x_1) < f(x_2)$ whenever $x_1 < x_2$ then which of the following statement is correct?

- f **is an increasing function.**
- f is a decreasing function.
- f is a constant function.

Question No: 15 (Marks: 1) - Please choose one

If $f''(x) > 0$ on an open interval (a,b), then which of the following statement is correct?

- f **is concave up on (a, b).**
- f is concave down on (a, b).
- f is linear on (a, b).

Question No: 16 (Marks: 1) - Please choose one

If $f''(x) < 0$ on an open interval (a,b) then which of the following statement is correct?

- f is concave up on (a, b).
- f **is concave down on (a, b)**
- f is linear on (a, b).

Question No: 17 (Marks: 1) - Please choose one

If $x > 0$ *then* $\frac{d}{dx}[\ln x] =$ _____

- 1
- x
- $\frac{1}{x}$
- $\ln \frac{1}{x}$

Question No: 18 (Marks: 1) - Please choose one

If $b > 0$ *then* $\frac{d}{dx}[b^x] =$ _____

- 0
- xb^{x-1}
- $\ln b$
- $b^x \ln b$

Question No: 19 (Marks: 1) - Please choose one

Let $y = (x^3 + 2x)^{37}$. Which of the following is correct?

- $\frac{dy}{dx} = (37)(x^3 + 2x)^{36}$
- $\frac{dy}{dx} = 111x^2(x^3 + 2x)^{36}$
- $\frac{dy}{dx} = (111x^2 + 74)(x^3 + 2x)^{36}$
- $\frac{dy}{dx} = (111x^2 + 74)(x^3 + 2x)^{38}$

Question No: 20 (Marks: 1) - Please choose one

What is the base of natural logarithm?

- **2.71**
- 10
- 5
- Any real number

Question No: 21 (Marks: 1) - Please choose one

Let x_0 be critical points of the function f. Those critical points for which $f'(x_0) = 0$ are called _____ of f

- Local points
- End points
- **Stationary points**

Question No: 22 (Marks: 1) - Please choose one

$\log_b a^r =$ _____

- $a \log_b r$

- $r \log_b a$
- $\frac{\log_b a}{\log_b r}$
-
- $\log_b a + \log_b r$

Question No: 23 (Marks: 1) - Please choose one

$\log_b \frac{1}{c} =$ _____

- $\log_b c$
- $1 - \log_b c$
- $-\log_b c$
- $1 + \log_b c$

Question No: 24 (Marks: 1) - Please choose one

$\log_b \frac{1}{t} =$ _____

- $\log_b t$
- $1 - \log_b t$
- $1 + \log_b t$
- $-\log_b t$

Question No: 25 (Marks: 3)

If $f(x) = x^4 - 8x^2$, determine all relative extrema for the function. Using First Derivative Test.

Solution:

$$f = x^4 - 8x^2$$
$$f` = 4x^3 - 16x^1$$
$$f` = 0$$
$$4x^3 - 16x = 0$$
$$x(4x^2 - 16) = 0$$
$$x = 0$$
$$4x^2 - 16 = 0$$
$$x^2 = \frac{16}{4}$$
$$x^2 = 4$$
$$x = \pm 2$$

Re*lactive* extreama $(0, \pm 2)$....*Ans*

Question No: 26 (Marks: 5)

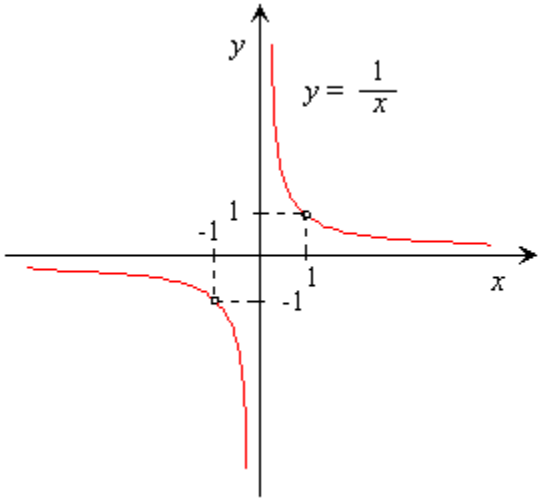
Differentiate $y = x^{-2}(4 + 3x^{-3})$

Solution:

$$y = 4x^{-2} + 3x^{-3}.x^{-2}$$
$$= 4x^{-2} + 3x^{-5}$$
$$\frac{dy}{dx} = 4\frac{d}{dx}(x^{-2}) + 3\frac{d}{dx}(x^{-5})$$
$$= 4(-2)x^{-2-1} + 3\frac{d}{dx} - 5x^{-5-1}$$
$$= -8x^{-3} + (-15x^{-6})$$
$$\frac{dy}{dx} = -8x^{-3} - 15x^{-6}.....\text{Ans}$$

Question No: 27 (Marks: 10)

Determine the intervals in which the graph of the function $f(x) = \frac{1}{x}$ is concave upward or downward.



Solution:

$$f(x) = \frac{1}{x}$$

$$f`(x) = -\frac{1}{x^2}$$

X	1	2	3	4
F(x)	-1	-0.25	-0.11	-0.625

Conclusion:

$f^{`(x)}$ is increasing when x is from $(0,\infty)$

so,

It is concave up.

$f^{`(x)}$ is decreasing when x is from $(-\infty,0)$

so,

It is concave down