

Fast Nonconvex Deconvolution of Calcium Imaging Data

Jewell & al.

Mini-Project: ML for Time Series

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Context: The Inverse Problem

Scientific Context

- **Goal:** Decipher "neural code" from calcium fluorescence.
- **Challenge:** Spike inference is an inverse deconvolution problem on noisy signals.

Limitations of L_1

- Traditional Lasso methods introduce "shrinkage bias".
- Systematically underestimates spike amplitudes.

Objective: Implement Jewell & Witten (2018) L_0 optimization.

Key Model

$$y_t = c_t + \epsilon_t$$

$$c_t = \gamma c_{t-1} + s_t$$

γ is assumed known

The Objective Function

$$\min_{\mathbf{c}_{1:T}} \left\{ \frac{1}{2} \sum_{t=1}^T (y_t - c_t)^2 + \lambda \sum_{t=2}^T \mathbb{I}(c_t > \gamma c_{t-1}) \right\}$$

Algorithm: Functional Dynamic Programming

- Solves the non-convex problem exactly using the $AR(1)$ structure.
- **Cost-to-Go** $Cost_s^*(\alpha)$:

$$Cost_s^*(\alpha) = \min \{ Cost_{s-1}^*(\alpha/\gamma), \min_{\alpha' \leq \alpha/\gamma} Cost_{s-1}^*(\alpha') + \lambda \} + \frac{1}{2} (y_s - \alpha)^2 \quad (1)$$

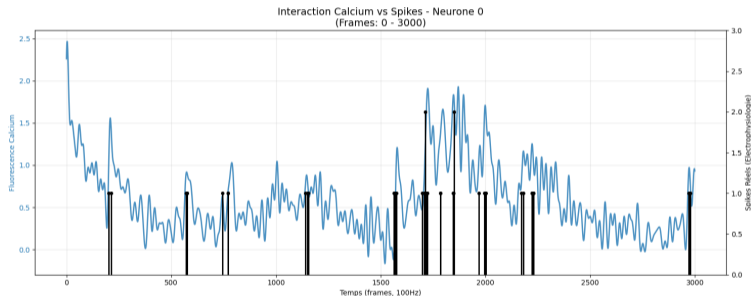
- Prunes the search space to find the global optimum efficiently.

Data: Benchmarking and Synthetic Stress-Tests

Figure: Trace example from Spikefinder
: Fluorescence vs. Ground Truth
Spikes.

Observations:

- **Ground Truth:** Spikes recorded via electrophysiology.
- **Dynamics:** Signal follows AR(1) exponential decay.
- **Preprocessing:** 3,000 timestep windows to manage non-NaN values.



Qualitative Comparison: L_0 vs L_1

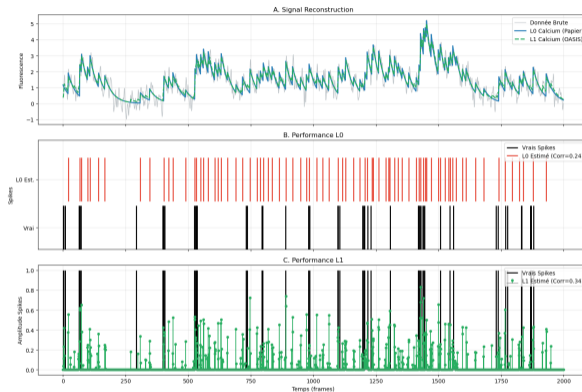


Figure: Comparison of L_0 and L_1 algorithms. Note the shrinkage bias in L_1 (Green) vs exact recovery in L_0 (Black).

Quantitative Analysis: Distance Metrics

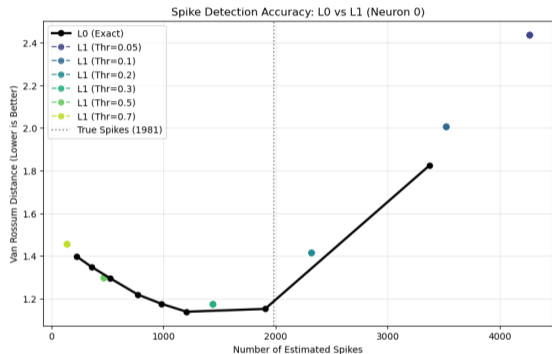


Figure: Van Rossum Distance. L_0 achieves a lower minimum.

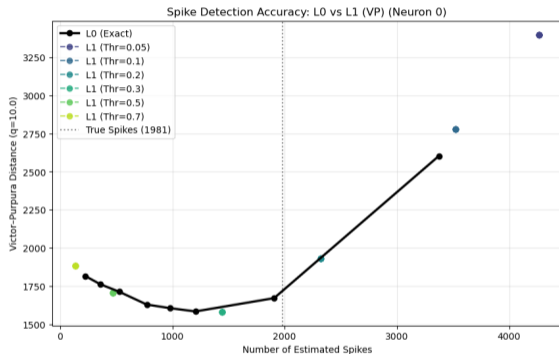


Figure: Victor-Purpura Distance. L_0 is closer to the ground truth (number of spikes).

Comparison with Change Point Detection (PELT)

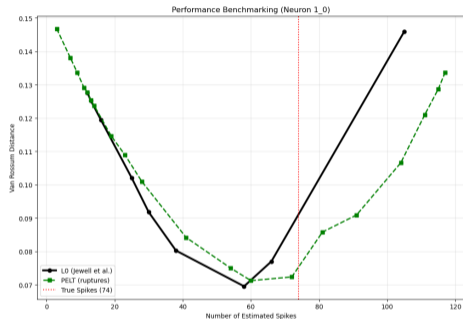


Figure: Benchmark: Van Rossum error comparison between L_0 and PELT.

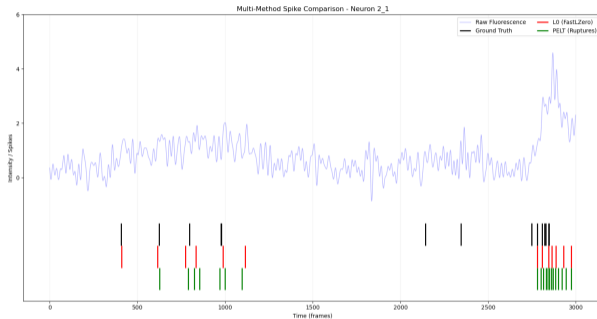


Figure: Activity Trace: Predicted spikes vs. ground truth on Neuron 2_1.

Robustness: Sensitivity to Decay (γ)

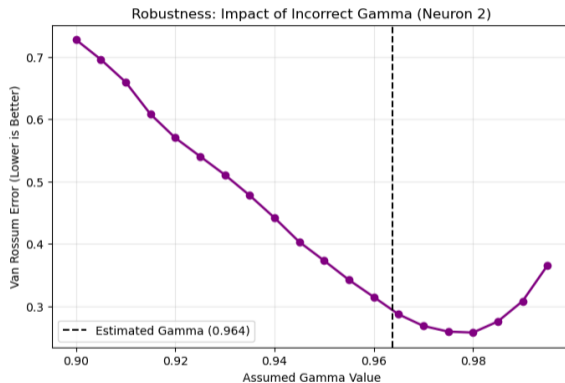


Figure: Impact of incorrect γ . The algorithm is brittle; deviations of ± 0.01 cause significant error increases.

Structural Failure Modes

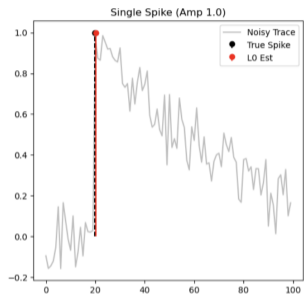


Figure: Single Spike (Baseline). Perfect recovery of amplitude 1.0.

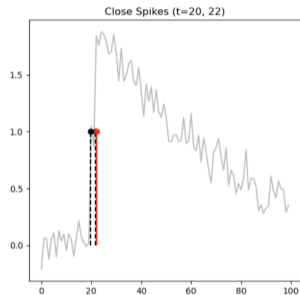


Figure: Close Spikes ($\Delta t = 2$). Merged into one "compromise" event.

Finding: The L_0 penalty creates an *artificial refractory period*.

Summary of Findings

- **Accuracy:** The L_0 framework provides superior reconstruction fidelity over L_1 relaxations (OASIS) by eliminating systematic shrinkage bias in spike amplitudes.
- **Efficiency:** The functional pruning algorithm developed for L_0 is highly efficient, processing fluorescence traces of 100,000 timesteps linearly.
- **Sensitivity:** The model is brittle regarding parameter precision; minor deviations in the decay constant γ can lead to "ringing" artifacts or spike under-counting.
- **Structural Limits:** The algorithm exhibits a "Temporal Resolution Limit" where high-frequency bursts ($ISI < 2$ frames) are merged into single events because a single penalty λ is mathematically cheaper than paying for multiple spikes 4.