Eddington Acceleration

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INTRODUCTION

One of the most challenging computational tasks is to simulate the interaction of radiation with matter. The steady–state, one–group, isotropically–scattering, fixed–source Linear Boltzmann Equation in planar geometry is:

$$\mu \frac{\partial \psi}{\partial x}(x,\mu) + \Sigma_t(x)\psi(x,\mu) = \frac{\Sigma_s(x)}{2}\int_{-1}^{1} \psi(x,\mu')d\mu' + \frac{Q(x)}{2} \quad (1)$$

where $\mu = \cos\theta$ is the cosine of the angle of flight θ relative to the x–axis, $\Sigma_t(x)$ and $\Sigma_s(x)$ the total and scattering cross sections, $Q(x)$ the isotropic fixed–source and $\psi(x,\mu)$ the angular flux [1]. In the Discrete–Ordinates (S_N) angular discretization, μ takes values from Gauss Legendre quadrature. The scalar flux, ϕ, is then

$$\phi(x) = \sum_{n=1}^{N} w_n \psi_n(x) \quad (2)$$

where $\psi_n(x) = \psi(x,\mu_n)$ and w_n the quadrature weights [2]. The S_N equations are then

$$\mu_n \frac{\mathrm{d}\psi_n}{\mathrm{d}x}(x) + \Sigma_t(x)\psi_n(x) = \frac{\Sigma_s(x)}{2}\sum_{n=1}^{N} w_n\psi_n(x) + \frac{Q(x)}{2}. \quad (3)$$

In the Source Iteration (SI) scheme, the right hand side of Eq. 3 is lagged. In other words,

$$\mu_n \frac{\mathrm{d}\psi_n^{\ell+1}}{\mathrm{d}x}(x) + \Sigma_t(x)\psi_n^{\ell+1}(x) = \frac{\Sigma_s(x)}{2}\sum_{n=1}^{N} w_n\psi_n^{\ell}(x) + \frac{Q(x)}{2}. \quad (4)$$

Equation 4 is iterated until the flux converges. If $\phi^0(x) = 0$ then, ϕ^ℓ is the scalar flux of particles that have undergone $\ell - 1$ collisions [1]. Thus, the number of iterations until convergence is directly linked to the number of collisions in a particle's lifetime. Typically, SI becomes increasingly slow to converge as the ratio of Σ_s to Σ_t approaches unity.

Fortunately, the regime where SI is slow to converge is also the regime where Diffusion Theory is more accurate. A popular method of accelerating SI is Diffusion Synthetic Acceleration (DSA) where a transport sweep is conducted and then a diffusion solve is used to generate a correction factor. Standard DSA requires correction schemes such as the Source Correction, Diffusion Coefficient, and Removal Correction schemes presented in [3] to prevent instability in highly scattering regimes with coarse spatial grids.

Lawrence Livermore National Laboratory (LLNL) is developing a high–order radiation–hydrodynamics code. The hydrodynamics portion is discretized using the Mixed–Hybrid Finite Element Method (MHFEM), where values are taken to be constant within a cell with discontinuous jumps at the cell edges. MHFEM is particularly suited for hydrodynamics but not for radiation transport. This work seeks to efficiently accelerate S_N calculations with a scheme that is both robust and compatible with MHFEM hydrodynamics.

EDDINGTON ACCELERATION

The zeroth and first angular moments of Eq. 1 are

$$\frac{\mathrm{d}}{\mathrm{d}x}J(x) + \Sigma_a(x)\phi(x) = Q(x) \quad (5a)$$

$$\frac{\mathrm{d}}{\mathrm{d}x}\langle\mu^2\rangle(x)\phi(x) + \Sigma_t J = 0 \quad (5b)$$

where $J = \int_{-1}^{1} \mu\, \psi(x,\mu)\,\mathrm{d}\mu$ is the current and

$$\langle\mu^2\rangle(x) = \frac{\int_{-1}^{1}\mu^2\psi(x,\mu)\,\mathrm{d}\mu}{\int_{-1}^{1}\psi(x,\mu)\,\mathrm{d}\mu} \quad (6)$$

the Eddington factor. When $\langle\mu^2\rangle = \frac{1}{3}$, Eqs. 5a and 5b are equivalent to Diffusion Theory.

The proposed acceleration scheme is:

1. Compute $\psi_n(x)$ with S_N with an arbitrary spatial discretization
2. Compute $\langle\mu^2\rangle(x)$
3. Interpolate $\langle\mu^2\rangle(x)$ onto the MHFEM grid
4. Solve the moment equations with the preconditioned $\langle\mu^2\rangle(x)$ using MHFEM.

This scheme allows the S_N equations and moment equations to be solved with different spatial discretizations. This means S_N can be discretized using normal methods such as Linear Discontinuous Galerkin or Diamond Differencing while the moment equations can be solved on the same grid as the hydrodynamics.

This method differs from DSA in that two solutions are generated: one from S_N and one from the moment equations. The solution of the moment equations will be used because the moment equations are conservative while S_N is not.

RESULTS

ACKNOWLEDGMENTS

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