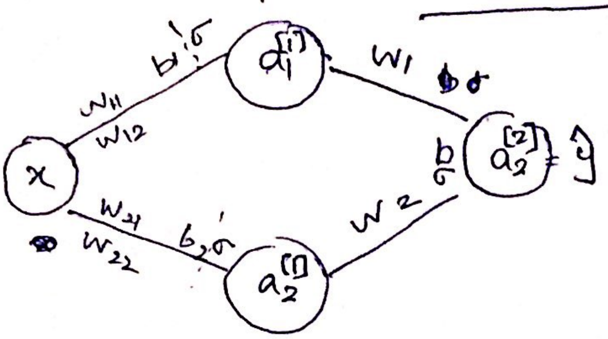
①

Homework-3



$$z_1^{[1]} = w_1^{[1]} x + b_1 = [20 \quad 20]\begin{bmatrix}-1\\-1\end{bmatrix} + -30$$

$$= -40 - 30 = -70$$

$$z_2^{[1]} = w_2^{[1]} x + b_2 = [-20 \quad -20]\begin{bmatrix}-1\\-1\end{bmatrix} + 10$$

$$= \underline{\underline{50}}$$

$$a_1^{[1]} = \sigma(z_1) = 0$$

$$a_2^{[1]} = \sigma(z_2) = 1$$

$$z^{[2]} = w_1^{[2]} a_1^{[1]} + w_2^{[2]} a_2^{[1]} + b^{[2]}$$

$$= 20 \times 0 + 20 \times 1 + -10$$

$$= 30$$

$$\hat{y} = a^{[2]} = \sigma(z^{[2]}) = \sigma(30) = \underline{\underline{1}}$$

② Model architecture

$$x \xrightarrow[b^{[1]}]{W^{[1]}} z^{[1]} \xrightarrow{RELU} a^{[1]} \xrightarrow[b^{[2]}]{W^{[2]}} z^{[2]} \xrightarrow{sigmoid} a^{[2]}$$

Equations

$$z^{[1]} = W^{[1]}x + b^{[1]}$$

$$a^{[1]} = RELU[z^{[1]}] \quad :- \quad a^{[1]} = \begin{cases} z^{[1]} & z^{[1]} > 0 \\ 0 & \text{otherwise} \end{cases}$$

$$z^{[2]} = W^{[2]}a^{[1]} + b^{[2]}$$

$$a^{[2]} = \sigma(z^{[2]}) \quad :- \quad a^{[2]} = \frac{1}{1+e^{-z^{[2]}}}$$

$$= \hat{y}$$

$$L(a^{[2]}, y) = -[y \log \hat{y} + (1-y)\log(1-\hat{y})]$$

$$= -[y \log a^{[2]} + (1-y)\log(1-a^{[2]})]$$

$$\frac{\partial L}{\partial a^{[2]}} = \frac{-y}{a^{[2]}} + \frac{(1-y)}{1-a^{[2]}} = \frac{-y + ya^{[2]} + a^{[2]} - ya^{[2]}}{a^{[2]}(1-a^{[2]})}$$

$$= \frac{a^{[2]} - y}{a^{[2]}(1-a^{[2]})} \quad \text{——— ①}$$

Similarly

$$\frac{\partial L}{\partial z^{[2]}} = \frac{\partial L}{\partial a^{[2]}} \times \frac{\partial a^{[2]}}{\partial z^{[2]}} \quad \text{——— ②}$$

$$\frac{\partial a^{[2]}}{\partial z^{[2]}} = \sigma(z^{[2]})(1-\sigma(z^{[2]})) \quad [\because \sigma'(z) = \sigma(z)(1-\sigma(z))]$$

$$= \frac{1}{(1+e^{-z[2]})}\left[1 - \frac{1}{1+e^{-z[2]}}\right] = \frac{e^{-z[2]}}{(1+e^{-z[2]})^2}$$

$$= a^{[2]}(1-a^{[2]}) \qquad \text{--- (3)}$$

From (1), (2) and (3)

$$\frac{\partial L}{\partial z^{[2]}} = \frac{a^{[2]}-y}{a^{[2]}(1-a^{[2]})} \times \frac{e^{-z[2]}}{(1+e^{-z[2]})^2}$$

$$\frac{\partial L}{\partial Z^{[2]}} = \frac{a^{[2]}-y}{a^{[2]}(1-a^{[2]})} \times a^{[2]}(1-a^{[2]})$$

$$= a^{[2]}-y$$

$$\frac{\partial L}{\partial w^{[2]}} = \frac{\partial L}{\partial Z^{[2]}} \times \frac{\partial Z^{[2]}}{\partial w^{[2]}}$$

$$= (a^{[2]}-y) \times a^{[1]}$$

$$[\because Z^{[2]} = w^{[2]}a^{[1]} + b^{[2]}]$$

$$\frac{\partial z^{[2]}}{\partial w^{[2]}} = a^{[1]}$$

$$\frac{\partial Z^{[2]}}{\partial b^{[2]}} = 1$$

$$\frac{\partial L}{\partial b^{[2]}} = \frac{\partial L}{\partial Z^{[2]}} \times \frac{\partial Z^{[2]}}{\partial b^{[2]}}$$

$$= a^{[2]}-y$$

$$\frac{\partial L}{\partial a^{[1]}} = \frac{\partial L}{\partial z^{[2]}} \times \frac{\partial z^{[2]}}{\partial a^{[1]}} = (a^{[2]} - y) \times w^{[2]}$$

$\left[\cancel{z^{[2]} = w^{[2]} a^{[1]} + b}\right.$

$[\because z^{[2]} = w^{[2]} a^{[1]} + b^{[2]}$

$$\frac{\partial z^{[2]}}{\partial a^{[1]}} = w^{[2]}$$

$$\frac{\partial L}{\partial z^{[1]}} = \frac{\partial L}{\partial a^{[1]}} \times \frac{\partial a^{[1]}}{\partial z^{[1]}}$$

$$= (a^{[2]} - y)\, w^{[2]} \times 1_{\{z^{[1]} > 0\}}$$

$$\left[\; a^{[1]} = \begin{cases} z^{[1]} & z^{[1]} > 0 \\ 0 & \text{otherwise} \end{cases} \qquad \frac{\partial a^{[1]}}{\partial z^{[1]}} = \begin{cases} 1 & z^{[1]} > 0 \\ 0 & \text{otherw} \end{cases} \right.$$

$$\frac{\partial L}{\partial w^{[1]}} = \frac{\partial L}{\partial z^{[1]}} \times \frac{\partial z^{[1]}}{\partial w^{[1]}}$$

$$= (a^{[2]} - y)\, w^{[2]} \times 1_{\{z^{[1]} > 0\}} \times \cancel{x}\, x$$

$$\left[\; z^{[1]} = w^{[1]} x + b^{[1]} \qquad \frac{\partial z^{[1]}}{\partial w^{[1]}} = x \qquad \frac{\partial z^{[1]}}{\partial b^{[1]}} = 1 \right.$$

$$\frac{\partial L}{\partial b^{[1]}} = \frac{\partial L}{\partial z^{[1]}} \times \frac{\partial z^{[1]}}{\partial b^{[1]}}$$

$$= (a^{[2]} - y)\, w^{[2]} \times 1_{z^{[1]} > 0}$$

for N observations Total loss is given by

$$J = \frac{1}{N} \left[\sum_i L(a^{[2]}, y_i) \right)$$

$$\frac{\partial J}{\partial w^{[2]}} = \frac{1}{N} \left[A^{[2]} - y \right] \times A^{[1]}$$

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$$\frac{\partial J}{\partial b^{[2]}} = \frac{1}{N}(A^{[2]} - y)$$

$$\frac{\partial J}{\partial W^{[1]}} = \frac{1}{N}(A^{[2]} - y)\, W^{[2]} \times 1_{\{Z^{[1]} > 0\}} \times X$$

$$\frac{\partial J}{\partial b^{[1]}} = \frac{1}{N}(A^{[2]} - y)$$