Progress thus far

- Delay & Sum is done, with decent results. The sound localization is good, albeit a bit fuzzy.
- After playing around with β_{PM} for a little bit, I got very good results using Pressure Matching. The dark bands are bigger and there are 4 of them now, and the bright bands are more directed. However, I needed to multiply the entire $\mathbf{q}$ vector by scalar g to get the same magnitudes I got with D&S.
- Neither Amplitude Contrast Maximization (ACM) nor Energy Difference Maximization (EDM) have been developed yet. I have been having trouble calculation the regularization parameters (β_{ACM} and α) for both methods. This is likely due to an issue with the eigenvalue routine, which should hopefully get fixed soon. Talking with Dr. Osborne on this would also really help.
- The overall plan for the next few days is to get all four methods working here, in a Python Jupyter notebook, a purely synthetic environment. After that, I will start building the actual beamformer and make my final decision regarding what should be doing the DSP (an FPGA, processor, or my laptop). That choice will likely come down to time, I think. Or how much lab service I end up doing.

In [426]:
```
import numpy as np
import matplotlib.pyplot as plt
from math import exp, pi, e
```

Formulas for Delay & Sum

$$Z(\mathbf{x}_m, \mathbf{y}_\ell, \omega) = \frac{e^{-j\frac{\omega}{c}||\mathbf{x}_m - \mathbf{y}_\ell||}}{4\pi \; || \; \mathbf{x}_m - \mathbf{y}_\ell \; ||}$$

$$\mathbf{p}(\omega) = \mathbf{Z}(\omega)\mathbf{q}(\omega)s(\omega)$$

$$\mathbf{Z}_{M\times L}(\omega) = \begin{bmatrix} Z(\mathbf{x}_1, \mathbf{y}_1, \omega) & \dots & Z(\mathbf{x}_1, \mathbf{y}_L, \omega) \\ \vdots & \ddots & \vdots \\ Z(\mathbf{x}_M, \mathbf{y}_1, \omega) & \dots & Z(\mathbf{x}_M, \mathbf{y}_L, \omega) \end{bmatrix}$$

$$\mathbf{q}_{DAS} = \mathbf{\Gamma}\mathbf{z}_B^*$$

$$\mathbf{\Gamma} = \begin{bmatrix} \gamma_1 & 0 & \dots & 0 \\ 0 & \gamma_2 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & \gamma_L \end{bmatrix}$$

$$\gamma_\ell = \frac{16\pi^2 \; || \; \mathbf{x}_B - \mathbf{y}_\ell \; ||^2}{L}$$

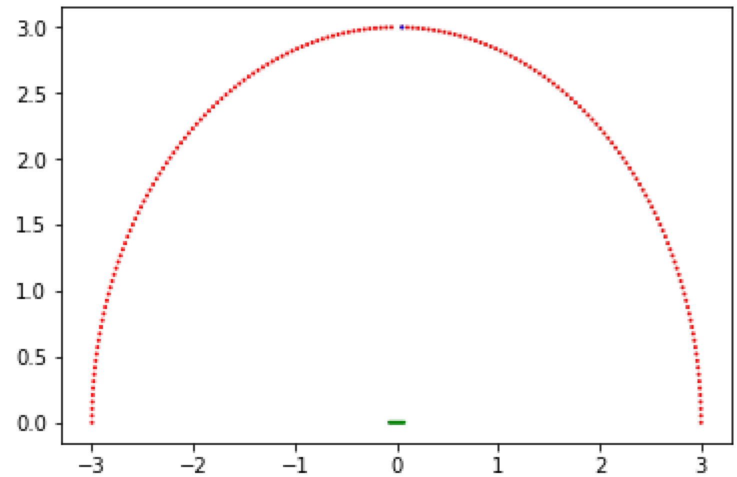
In [474]:
```
c = 343 #speed of sound
l2 = lambda arr: np.sqrt(np.sum(arr**2))


Zf = lambda x_m, y_l, omega: e**(-1j * omega / c * l2(x_m - y_l)) / (4 * pi * l2(
```

In [475]:
```
delta, delta_theta = 0.02, 1 * pi / 180 #sources 20 mm apart, validation points 1
M, L = 181, 8 #181 validation points, 8 sources
R = 3 # 3 meters
x = np.zeros((M, 3))
for i in range(M):
    x[i] = np.array([R*np.cos(delta_theta*(i)), R*np.sin(delta_theta*(i)), 0])
print('\n'.join(['{:d}> {}'.format(i, x[i]) for i in range(M)]))
x[1: M//2+1], x[0] = x[:M//2], x[M//2]
y = np.zeros((L, 3))
for l in range(L):
    y[l] = np.array([(l-3.5)*delta, 0, 0])
print(x, y)
```

```
 [ -2.78155156e+00   1.12381978e+00   0.00000000e+00]
 [ -2.80074128e+00   1.07510385e+00   0.00000000e+00]
 [ -2.81907786e+00   1.02606043e+00   0.00000000e+00]
 [ -2.83655573e+00   9.76704463e-01   0.00000000e+00]
 [ -2.85316955e+00   9.27050983e-01   0.00000000e+00]
 [ -2.86891427e+00   8.77115114e-01   0.00000000e+00]
 [ -2.88378509e+00   8.26912067e-01   0.00000000e+00]
 [ -2.89777748e+00   7.76457135e-01   0.00000000e+00]
 [ -2.91088718e+00   7.25765687e-01   0.00000000e+00]
 [ -2.92311019e+00   6.74853163e-01   0.00000000e+00]
 [ -2.93444280e+00   6.23735072e-01   0.00000000e+00]
 [ -2.94488155e+00   5.72426986e-01   0.00000000e+00]
 [ -2.95442326e+00   5.20944533e-01   0.00000000e+00]
 [ -2.96306502e+00   4.69303395e-01   0.00000000e+00]
 [ -2.97080421e+00   4.17519303e-01   0.00000000e+00]
 [ -2.97763845e+00   3.65608030e-01   0.00000000e+00]
 [ -2.98356569e+00   3.13585390e-01   0.00000000e+00]
 [ -2.98858409e+00   2.61467228e-01   0.00000000e+00]
 [ -2.99269215e+00   2.09269421e-01   0.00000000e+00]
 [ -2.99588860e+00   1.57007869e-01   0.00000000e+00]
```

In [476]:
```
print(np.asarray(np.concatenate((x, y))[:, 0]).shape)
plt.scatter(np.asarray(np.concatenate((x, y))[:, 0]), np.concatenate((x, y))[:, 1
plt.show()
```

```
(189,)
```



In [486]:
```
f = 3000 #3 kHz signal
omega = 2 * pi * f #angular frequency
b = M//2

print(omega, b)

z_b = np.asmatrix(np.vectorize(complex)(np.zeros((L, 1))))
Z_d = np.asmatrix(np.vectorize(complex)(np.zeros((M-1, L))))
print(z_b.shape, Z_d.shape)
for l in range(L):
    print(x[b] - y[l], l2(x[b] - y[l]), Zf(x[b], y[l], omega))
    z_b[l, 0] = Z(x[b], y[l], omega)
n = 0
for m in range(0, M):
    if m == M//2: continue
    for l in range(L):
        print(n, m, l)
        Z_d[n, l] = Z(x[n], y[l], omega)
    n += 1
gamma = lambda x_b, y, l: 16 * (pi ** 2) * l2(x_b - y[l])**2 / L
Gamma = np.asmatrix(np.zeros((L, L)))
for l in range(L):
    Gamma[l, l] = gamma(x[b], y, l)
print(z_b, Gamma)
```

```
50 50 2
50 50 3
50 50 4
50 50 5
50 50 6
50 50 7
51 51 0
51 51 1
51 51 2
51 51 3

51 51 4
51 51 5
51 51 6
51 51 7
52 52 0
52 52 1
52 52 2
52 52 3
52 52 4
52 52 5
```

In [499]:
```
q_DAS = Gamma * np.conjugate(z_b)
```

In [500]:
```
print(q_DAS)
```

```
[[-0.20405013+4.71117287j]
 [-0.01010674+4.71440292j]
 [ 0.14930517+4.71108159j]
 [ 0.27406334+4.70471356j]
 [ 0.36421000+4.6980447j ]
 [ 0.41985128+4.69305916j]
 [ 0.44108448+4.69098116j]
 [ 0.42795313+4.69227772j]]
```

In [501]:
```
Z = np.asmatrix(np.vectorize(complex)(np.zeros((M, L))))
print(Z)
for m in range(M):
    for l in range(L):
        Z[m, l] = Zf(x[m], y[l], omega)
print(Z)
```

```
[[ 0.+0.j  0.+0.j  0.+0.j ...,  0.+0.j  0.+0.j  0.+0.j]
 [ 0.+0.j  0.+0.j  0.+0.j ...,  0.+0.j  0.+0.j  0.+0.j]
 [ 0.+0.j  0.+0.j  0.+0.j ...,  0.+0.j  0.+0.j  0.+0.j]
 ...,
 [ 0.+0.j  0.+0.j  0.+0.j ...,  0.+0.j  0.+0.j  0.+0.j]
 [ 0.+0.j  0.+0.j  0.+0.j ...,  0.+0.j  0.+0.j  0.+0.j]
 [ 0.+0.j  0.+0.j  0.+0.j ...,  0.+0.j  0.+0.j  0.+0.j]]
[[ -1.14703023e-03-0.02648299j  -5.68415159e-05-0.02651437j
    8.40054803e-04-0.02650656j ...,   2.36391255e-03-0.0264236j
    2.48359946e-03-0.02641335j   2.40957934e-03-0.02641975j]
 [  1.54083275e-02+0.02084422j  -1.16423297e-02+0.02334941j
   -2.62620903e-02+0.00024055j ...,   2.65065659e-02+0.0039125j
    8.61732776e-03+0.02556198j  -1.89835720e-02+0.01942331j]
 [  1.53964431e-02+0.02085311j  -1.16519688e-02+0.02334468j
   -2.62621885e-02+0.00023402j ...,   2.65075162e-02+0.00390577j
    8.62818320e-03+0.02555824j  -1.89718461e-02+0.01943462j]
 ...,
 [ -1.89366317e-02+0.01946851j   8.66073632e-03+0.025547j
    2.65103565e-02+0.00388558j ...,  -2.62624734e-02+0.00021443j
   -1.16808717e-02+0.02333044j   1.53607628e-02+0.02087974j]
 [ -1.89718461e-02+0.01943462j   8.62818320e-03+0.02555824j
    2.65075162e-02+0.00390577j ...,  -2.62621885e-02+0.00023402j
   -1.16519688e-02+0.02334468j   1.53964431e-02+0.02085311j]
 [ -1.89835720e-02+0.01942331j   8.61732776e-03+0.02556198j
    2.65065659e-02+0.0039125j  ...,  -2.62620903e-02+0.00024055j
   -1.16423297e-02+0.02334941j   1.54083275e-02+0.02084422j]]
```

In [502]:
```
p = Z * q_DAS
print(p)
```

```
[[  1.00000000e+00 +4.77618352e-18j]
 [ -2.24530648e-01 -1.28799003e-02j]
 [ -2.24499938e-01 -1.28739379e-02j]
 [ -2.24407168e-01 -1.28560273e-02j]
 [ -2.24250422e-01 -1.28260989e-02j]
 [ -2.24026507e-01 -1.27840374e-02j]
 [ -2.23730950e-01 -1.27296838e-02j]
 [ -2.23358002e-01 -1.26628372e-02j]
 [ -2.22900633e-01 -1.25832575e-02j]
 [ -2.22350537e-01 -1.24906686e-02j]
 [ -2.21698130e-01 -1.23847629e-02j]
 [ -2.20932556e-01 -1.22652049e-02j]
 [ -2.20041686e-01 -1.21316369e-02j]
 [ -2.19012131e-01 -1.19836842e-02j]
 [ -2.17829245e-01 -1.18209615e-02j]
 [ -2.16477139e-01 -1.16430796e-02j]
 [ -2.14938698e-01 -1.14496521e-02j]
 [ -2.13195602e-01 -1.12403035e-02j]
 [ -2.11228347e-01 -1.10146772e-02j]
 [ -2.09016281e-01 -1.07724436e-02j]
```

In [503]:
```
M_test = 800 * 400
x_test = np.ones((M_test, 3))
n = 0
for i in range(-400, 400):
    for j in range(0, 400):
        x_test[n] = np.array([i / 100, j / 100, 0])
        n += 1
print(x_test)
Z_test = np.asmatrix(np.vectorize(complex)(np.zeros((M_test, L))))
for m in range(M_test):
    for l in range(L):
        try:
            Z_test[m, l] = Zf(x_test[m], y[l], omega)
        except:
            Z_test[m, l] = 0
print(Z_test)
p_test = Z_test * q_DAS
print('>>', p_test)
print('\n\n>>', x_test, y)
```

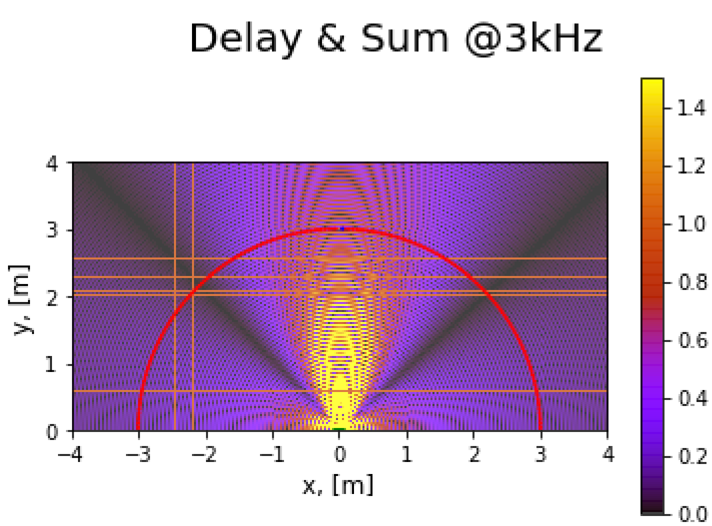
```
[[-4.    0.    0.  ]
 [-4.    0.01  0.  ]
 [-4.    0.02  0.  ]
 ...,
 [ 3.99  3.97  0.  ]
 [ 3.99  3.98  0.  ]
 [ 3.99  3.99  0.  ]]
[[-0.01415315-0.01448099j -0.01923294+0.00599694j -0.00338024+0.01975763j
  ...,  0.00027128-0.01974441j -0.01737877-0.00916799j
  -0.01598478+0.01125946j]
 [-0.01416322-0.01447105j -0.01922870+0.0060103j  -0.00336655+0.01975991j
  ...,  0.00025782-0.01974453j -0.01738494-0.00915617j
  -0.01597713+0.01127022j]
 [-0.01419341-0.01444117j -0.01921593+0.00605036j -0.00332548+0.01976667j
  ...,  0.00021743-0.01974483j -0.01740338-0.00912068j
  -0.01595412+0.01130245j]
 ...,
 [-0.00709451+0.01208554j -0.01359476+0.00354495j -0.01216731-0.00709489j
  ...,  0.01365331-0.00387155j  0.01249317+0.00680733j
   0.00419675+0.01363189j]
 [-0.00204135+0.0138471j  -0.01125075+0.00838551j -0.01392665-0.00198404j
  ...,  0.01114799-0.00875306j  0.01412564+0.00154013j
   0.00906690+0.010987j  ]
 [ 0.00330375+0.01358355j -0.00725792+0.01198883j -0.01362893+0.00341299j
  ...,  0.00697658-0.01231724j  0.01363005-0.00395169j
   0.01255904+0.00668404j]]
>> [[ 0.01385280-0.17130394j]
 [ 0.01373503-0.17131278j]
 [ 0.01338170-0.17133881j]
 ...,
 [ 0.01837335-0.00133758j]
 [ 0.01716767-0.0083957j ]
 [ 0.01324572-0.01471528j]]
```

```
>> [[-4.    0.    0.  ]
 [-4.    0.01  0.  ]
 [-4.    0.02  0.  ]
 ...,
 [ 3.99  3.97  0.  ]
 [ 3.99  3.98  0.  ]
 [ 3.99  3.99  0.  ]] [[-0.07  0.    0.  ]
 [-0.05  0.    0.  ]
 [-0.03  0.    0.  ]
 [-0.01  0.    0.  ]
 [ 0.01  0.    0.  ]
 [ 0.03  0.    0.  ]
 [ 0.05  0.    0.  ]
 [ 0.07  0.    0.  ]]
```

In [504]:

```
output = np.ones((400,800))
for n in range(x_test.shape[0]):
    output[399 - int(x_test[n, 1]*100), int(x_test[n, 0]*100+400)] = p_test[n, 0]
fig = plt.figure()
plt.scatter(np.concatenate((x, y))[:, 0], np.concatenate((x, y))[:, 1], c=['r']*(
implot = plt.imshow(np.abs(output), interpolation='nearest', cmap='gnuplot', exte
plt.colorbar()
fig.suptitle("Delay & Sum @3kHz", fontsize=20)
plt.xlabel("x, [m]", fontsize=12)
plt.ylabel("y, [m]", fontsize=12)
plt.show()
```

```
C:\Users\sarki\Anaconda3\lib\site-packages\ipykernel_launcher.py:3: ComplexWarn
ing: Casting complex values to real discards the imaginary part
  This is separate from the ipykernel package so we can avoid doing imports unt
il
```



Formulas for Pressure matching

$$\min_{\mathbf{q}} J_{PM} = \min_{\mathbf{q}} (\mathbf{e}_{PM}^H \mathbf{e}_{PM} + \beta_{PM} E_q)$$

$$\mathbf{e}_{PM} = \hat{\mathbf{p}} - \mathbf{Z}\mathbf{q}$$

$$E_{\mathbf{q}} = \mathbf{q}^H \mathbf{q}$$

$$\mathbf{q}_{PM} = (\mathbf{Z}^H \mathbf{Z} + \beta_{PM} \mathbf{I})^{-1} \mathbf{Z}^H \hat{\mathbf{p}}$$

In [527]:
```
E_ref = (4 * pi * R)**2 / L
sigma = 25
epsilon_beta, beta_min, E_max = 10e-6, 10e-20, sigma * E_ref
p_b_hat = 1
p_hat = np.asmatrix(np.array([p_b_hat] + [0]*(M-1))).T
g = 30 # weakness compensation

W = lambda q: (p_b_hat / np.dot(z_b.T, q))[0, 0]
E = lambda q: np.dot(np.conjugate(q).T, q)[0, 0]
```

In [528]:
```
### Calculate Tikhonov Regularization Parameter ###

beta = beta_min
print('a')
print(np.conjugate(Z).T.shape, Z.shape, np.asmatrix(np.eye(L)).shape, np.linalg.i
q_temp = np.linalg.inv(np.conjugate(Z).T * Z + beta * np.asmatrix(np.eye(L))) * n

# np.dot(np.dot(np.linalg.inv(np.dot(np.conjugate(ZZ).T, ZZ)
#                                 + beta * np.eye(L)), np.conjugate(ZZ).T), p_hat)

print('b')
q_hat = W(q_temp) * q_temp
n = 0
while E(q_hat) > E_max:
    if n % 1000 == 0: print(beta, E(q_hat), E_max, E(q_hat) > E_max)
    beta += epsilon_beta
    q_temp = np.linalg.inv(np.conjugate(Z).T * Z + beta * np.asmatrix(np.eye(L)))
    q_hat = W(q_temp) * q_temp
    n += 1

print(beta, E(q_hat), E_max, E(q_hat) > E_max)
beta_PM = beta
```

```
a
(8, 181) (181, 8) (8, 8) (8, 8) (181, 1)
b
1e-19 (4578904.81551+0j) 4441.321980490211 True
0.00011000000000000001 (4194.65314199+0j) 4441.321980490211 False
```

In [529]:
```
q_PM = g * np.linalg.inv(np.conjugate(Z).T * Z + beta_PM * np.asmatrix(np.eye(L))

            #g * np.dot(np.dot(np.linalg.inv(np.dot(np.conjugate(ZZ).T, ZZ) + bet

p = Z * q_PM
print(p)
```

```
[[ 0.96503735 -4.52415883e-15j]
 [ 0.15481745 +8.00011972e-03j]
 [ 0.15441202 +7.97633575e-03j]
 [ 0.15319672 +7.90512154e-03j]
 [ 0.15117459 +7.78688975e-03j]
 [ 0.14835065 +7.62232700e-03j]
 [ 0.14473205 +7.41239205e-03j]
 [ 0.14032805 +7.15831318e-03j]
 [ 0.13515014 +6.86158480e-03j]
 [ 0.12921211 +6.52396304e-03j]
 [ 0.12253017 +6.14746045e-03j]
 [ 0.11512304 +5.73433944e-03j]
 [ 0.10701209 +5.28710465e-03j]
 [ 0.09822149 +4.80849386e-03j]
 [ 0.08877831 +4.30146754e-03j]
 [ 0.07871270 +3.76919678e-03j]
 [ 0.06805801 +3.21504958e-03j]
 [ 0.05685098 +2.64257534e-03j]
 [ 0.04513179 +2.05548749e-03j]
```

In [530]:
```
M_test = 800 * 400
x_test = np.ones((M_test, 3))
n = 0
for i in range(-400, 400):
    for j in range(0, 400):
        x_test[n] = np.array([i / 100, j / 100, 0])
        n += 1
print(x_test)
Z_test = np.asmatrix(np.vectorize(complex)(np.zeros((M_test, L))))
for m in range(M_test):
    for l in range(L):
        try:
            Z_test[m, l] = Zf(x_test[m], y[l], omega)
        except:
            Z_test[m, l] = 0
print(Z_test)
p_test2 = Z_test * q_PM
print('>>', p_test2, p_test2.shape)
print('\n\n>>', x_test, y)
```

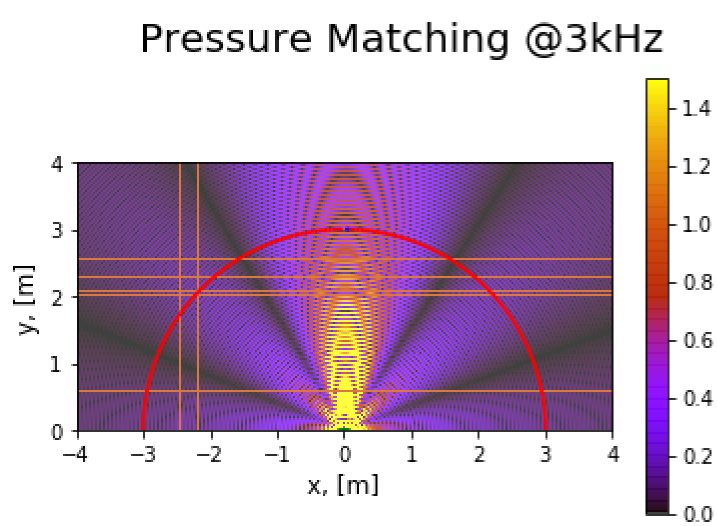
```
[[-4.    0.    0.  ]
 [-4.    0.01  0.  ]
 [-4.    0.02  0.  ]
 ...,
 [ 3.99  3.97  0.  ]
 [ 3.99  3.98  0.  ]
 [ 3.99  3.99  0.  ]]
[[-0.01415315-0.01448099j -0.01923294+0.00599694j -0.00338024+0.01975763j
  ...,  0.00027128-0.01974441j -0.01737877-0.00916799j
  -0.01598478+0.01125946j]
 [-0.01416322-0.01447105j -0.01922870+0.0060103j  -0.00336655+0.01975991j
  ...,  0.00025782-0.01974453j -0.01738494-0.00915617j
  -0.01597713+0.01127022j]
 [-0.01419341-0.01444117j -0.01921593+0.00605036j -0.00332548+0.01976667j
  ...,  0.00021743-0.01974483j -0.01740338-0.00912068j
  -0.01595412+0.01130245j]
 ...,
 [-0.00709451+0.01208554j -0.01359476+0.00354495j -0.01216731-0.00709489j
  ...,  0.01365331-0.00387155j  0.01249317+0.00680733j
   0.00419675+0.01363189j]
 [-0.00204135+0.0138471j  -0.01125075+0.00838551j -0.01392665-0.00198404j
  ...,  0.01114799-0.00875306j  0.01412564+0.00154013j
   0.00906690+0.010987j  ]
 [ 0.00330375+0.01358355j -0.00725792+0.01198883j -0.01362893+0.00341299j
  ...,  0.00697658-0.01231724j  0.01363005-0.00395169j
   0.01255904+0.00668404j]]
>> [[-0.01002116+0.10207439j]
 [-0.00995062+0.10207621j]
 [-0.00973900+0.10208139j]
 ...,
 [-0.11297123-0.01446874j]
 [-0.10973761+0.0292232j ]
 [-0.09024771+0.06837764j]] (320000, 1)
```

```
>> [[-4.    0.    0.  ]
 [-4.    0.01  0.  ]
 [-4.    0.02  0.  ]
 ...,
 [ 3.99  3.97  0.  ]
 [ 3.99  3.98  0.  ]
 [ 3.99  3.99  0.  ]] [[-0.07  0.    0.  ]
 [-0.05  0.    0.  ]
 [-0.03  0.    0.  ]
 [-0.01  0.    0.  ]
 [ 0.01  0.    0.  ]
 [ 0.03  0.    0.  ]
 [ 0.05  0.    0.  ]
 [ 0.07  0.    0.  ]]
```

In [531]:

```
output = np.ones((400,800))
for n in range(x_test.shape[0]):
    output[399 - int(x_test[n, 1]*100), int(x_test[n, 0]*100+400)] = p_test2[n, 0
fig = plt.figure()
plt.scatter(np.concatenate((x, y))[:, 0], np.concatenate((x, y))[:, 1], c=['r']*(l
implot = plt.imshow(np.abs(output), interpolation='nearest', cmap='gnuplot', exte
plt.colorbar()
fig.suptitle("Pressure Matching @3kHz", fontsize=20)
plt.xlabel("x, [m]", fontsize=12)
plt.ylabel("y, [m]", fontsize=12)
plt.show()
```

```
C:\Users\sarki\Anaconda3\lib\site-packages\ipykernel_launcher.py:3: ComplexWarn
ing: Casting complex values to real discards the imaginary part
  This is separate from the ipykernel package so we can avoid doing imports unt
il
```



In [532]:
```
print(q_DAS, q_PM, np.average(q_DAS / q_PM))
```

```
[[-0.20405013+4.71117287j]
 [-0.01010674+4.71440292j]
 [ 0.14930517+4.71108159j]
 [ 0.27406334+4.70471356j]
 [ 0.36421000+4.6980447j ]
 [ 0.41985128+4.69305916j]
 [ 0.44108448+4.69098116j]
 [ 0.42795313+4.69227772j]] [[-4.24528812+22.55133075j]
 [ 8.15669008-26.35215524j]
 [-5.11258791+25.04097817j]
 [-2.15854695 -3.25158063j]
 [ 4.61473470 -2.94830762j]
 [ 1.05705848+25.38077933j]
 [-4.01363225-26.99611305j]
 [ 2.96923682+22.83613293j]] (-0.126521345014+0.0148289148266j)
```

Acoustic Contract Maximization Formulas

$$E_B = \frac{1}{\rho_0 c^2} \mid p_B \mid^2 = \frac{1}{\rho_0 c^2} \mathbf{q}^H \mathbf{R}_B \mathbf{q}$$

$$E_D = \frac{1}{\rho_0 c^2} \frac{1}{M_D} \mid\mid \mathbf{p}_D \mid\mid^2 = \frac{1}{\rho_0 c^2} \mathbf{q}^H \mathbf{R}_D \mathbf{q}$$

$$\mathbf{R}_B = \mathbf{z}_B^H \mathbf{z}_B$$

$$\mathbf{R}_D = \mathbf{Z}_D^H \mathbf{Z}_D / M_D$$

$$AC = \frac{E_B}{E_D} = \frac{\mathbf{q}^H \mathbf{R}_B \mathbf{q}}{\mathbf{q}^H \mathbf{R}_D \mathbf{q}}$$

$$\max_{\mathbf{q}} J_{ACM} = \max_{\mathbf{q}} \frac{\mathbf{q}^H \mathbf{R}_B \mathbf{q}}{\mathbf{q}^H \mathbf{R}_D \mathbf{q}}$$

$$\mathbf{q}_{ACM} = \Phi(\mathbf{R}_D^{-1} \mathbf{R}_B)$$

However, $\mathbf{R}_D^{-1}$ is rather unsavory and often ill-conditioned to inversion. Rather, we would use

$$\mathbf{q}_{ACM} = \Phi\Big((\mathbf{R}_D + \beta_{ACM} \mathbf{I})^{-1} \mathbf{R}_B \Big)$$

where $\Phi(\mathbf{A})$ is the eigenvector corresponding to the maximum eigenvalue of matrix $\mathbf{A}$.

In [559]:
```
E_ref = (4 * pi * R)**2 / L
sigma = 25
epsilon_beta, beta_min, E_max = 10e-9, 10e-20, sigma * E_ref
p_b_hat = 1
p_hat = np.asmatrix(np.array([p_b_hat] + [0]*(M-1))).T

W = lambda q: (p_b_hat / np.dot(z_b.T, q))[0, 0]
#W = lambda q: 1 #temporary....
E = lambda q: np.dot(np.conjugate(q).T, q)[0, 0]
def PHI(A):
    B = A.real
    w, v = np.linalg.eig(B)
    print(w)
    i = np.argmax(w)
    print(i)
    print(v[:, i])
    print(B * v[:, i] / v[:, i])
    return v[:, i]

M_D = M - 1
R_D = np.dot(np.conjugate(Z_d).T, Z_d) / (M-1)
R_B = np.dot(np.conjugate(z_b).T, z_b)

print(np.conjugate(z_b).T, z_b)

print(R_B)

print(type(R_D), type(R_B))

print(R_D.shape, R_B.shape)
```

```
[[ -1.14703023e-03+0.02648299j  -5.68415159e-05+0.02651437j
    8.40054803e-04+0.02650656j   1.54249351e-03+0.02647924j
    2.05033734e-03+0.02644786j   2.36391255e-03+0.0264236j
    2.48359946e-03+0.02641335j   2.40957934e-03+0.02641975j]] [[ -1.14703023e-0
3-0.02648299j]
 [ -5.68415159e-05-0.02651437j]
 [  8.40054803e-04-0.02650656j]
 [  1.54249351e-03-0.02647924j]
 [  2.05033734e-03-0.02644786j]
 [  2.36391255e-03-0.0264236j ]
 [  2.48359946e-03-0.02641335j]
 [  2.40957934e-03-0.02641975j]]
[[ 0.00562764+0.j]]
<class 'numpy.matrixlib.defmatrix.matrix'> <class 'numpy.matrixlib.defmatrix.ma
trix'>
(8, 8) (1, 1)
```

In [562]:
```
### Calculate ACM Regularization Parameter ###


beta = beta_min
print('a')
q_temp = PHI(np.linalg.inv(R_D + beta * np.asmatrix(np.eye(L))) * R_B[0, 0])
# q_temp = PHI(np.dot(np.linalg.inv(R_D + beta * np.eye(L)), R_B))
print('b')
q_hat = W(q_temp) * q_temp
n = 0
print(z_b.T, q_temp, z_b.T * q_temp)

print(W(q_temp), q_hat)
print("E =", E(q_hat))
return
while E(q_hat) > E_max:
    if n % 1000 == 0:
        print(beta, E(q_hat), E_max, E(q_hat) > E_max)
    beta += epsilon_beta
    q_temp = PHI(np.linalg.inv(R_D + beta * np.asmatrix(np.eye(L))) * R_B[0, 0])
    q_hat = W(q_temp) * q_temp
    n += 1

print(beta, E(q_hat), E_max, E(q_hat) > E_max)
beta_ACM = beta
```

```
a
[  3.85007763e+07   2.33990395e+05   3.61908532e+03   1.15959324e+02
   9.88286292e+00   4.29325815e+00   2.99844711e+00   3.10027200e+00]
0
[[ 0.03028093]
 [-0.15464089]
 [ 0.38014602]
 [-0.57506416]
 [ 0.5750379 ]
 [-0.38009401]
 [ 0.15460617]
 [-0.03027141]]
[[ 38500776.30020554]
 [ 38500776.30020554]
 [ 38500776.30020548]
 [ 38500776.30020548]
 [ 38500776.30020548]
 [ 38500776.30020548]
 [ 38500776.30020547]
 [ 38500776.30020548]]
b
[[ -1.14703023e-03-0.02648299j  -5.68415159e-05-0.02651437j
    8.40054803e-04-0.02650656j   1.54249351e-03-0.02647924j
    2.05033734e-03-0.02644786j   2.36391255e-03-0.0264236j
    2.48359946e-03-0.02641335j   2.40957934e-03-0.02641975j]] [[ 0.03028093]
 [-0.15464089]
 [ 0.38014602]
 [-0.57506416]
 [ 0.5750379 ]
```

```
 [-0.38009401]
 [ 0.15460617]
 [-0.03027141]] [[ -2.08125453e-06 +2.00105028e-07j]]
1 [[ 0.03028093]
 [-0.15464089]
 [ 0.38014602]
 [-0.57506416]
 [ 0.5750379 ]
 [-0.38009401]
 [ 0.15460617]
 [-0.03027141]]
E = 1.0

  File "<ipython-input-562-fd5cb8b912e5>", line 15
    return
          ^
SyntaxError: 'return' outside function
```

In [410]:

```
q_ACM = PHI(np.dot(np.linalg.inv(R_D), R_B))
print(q_ACM, q_DAS, q_DAS.real / q_ACM.real)
```

```
[-0.03056138 +3.46882534e-05j  0.15525338 +6.72881479e-06j
 -0.38045526 -1.33860403e-04j  0.57467685 +0.00000000e+00j
 -0.57465195 +5.41708986e-04j  0.38040639 -9.41969248e-04j
 -0.15522097 +7.24904088e-04j  0.03055282 -2.36279763e-04j] [[-4.42774546+1.611
11273j -4.42310489+1.62357719j -4.43059043+1.60341753j
  -4.44975458+1.55046996j -4.47941986+1.46432173j -4.51766439+1.34435495j
  -4.56180116+1.18981816j -4.60835469+0.99992665j]] [[ 144.88042254  -28.489587
71   11.6454965    -7.74305515    7.79501369
   -11.87588991   29.3890781  -150.8323745 ]]
```

In [413]:
```
M_test = 800 * 400
x_test = np.ones((M_test, 3))
n = 0
for i in range(-400, 400):
    for j in range(0, 400):
        x_test[n] = np.array([i / 100, j / 100, 0])
        n += 1
print(x_test)
ZZ_test = np.vectorize(complex)(np.zeros((M_test, L)))
for m in range(M_test):
    for l in range(L):
        try:
            ZZ_test[m, l] = Z(x_test[m], y[l], omega)
        except:
            ZZ_test[m, l] = 0
print(ZZ_test)
p_test = np.dot(ZZ_test, q_ACM.T)
print('>>', p_test, p_test.shape)
print('\n\n>>', x_test, y)
```

```
[[-4.    0.    0.  ]
 [-4.    0.01  0.  ]
 [-4.    0.02  0.  ]
 ...,
 [ 3.99  3.97  0.  ]
 [ 3.99  3.98  0.  ]
 [ 3.99  3.99  0.  ]]
[[ -1.22167916e-02 +1.61480861e-02j   8.94701020e-03 +1.80504902e-02j
    2.00446263e-02 +5.55163640e-05j ...,  -1.94300831e-02 +3.51952905e-03j
   -5.50079608e-03 +1.88630578e-02j   1.43501319e-02 +1.32801509e-02j]
 [ -1.22053688e-02 +1.61566395e-02j   8.95963577e-03 +1.80441545e-02j
    2.00445965e-02 +4.15323070e-05j ...,  -1.94275999e-02 +3.53287071e-03j
   -5.48787841e-03 +1.88667576e-02j   1.43591223e-02 +1.32703426e-02j]
 [ -1.21710649e-02 +1.61822508e-02j   8.99748531e-03 +1.80250947e-02j
    2.00444488e-02 -4.19159442e-07j ...,  -1.94200953e-02 +3.57288492e-03j
   -5.44911071e-03 +1.88778040e-02j   1.43860533e-02 +1.32408811e-02j]
 ...,
 [  1.29958148e-02 -5.24415624e-03j   1.28983411e-02 +5.56927750e-03j
    5.14539576e-03 +1.31112826e-02j ...,  -1.30084970e-02 -5.67280682e-03j
   -5.25325409e-03 -1.32220353e-02j   5.57023837e-03 -1.31306361e-02j]
 [  1.00380666e-02 -9.75431055e-03j   1.40294079e-02 +2.68074806e-04j
    9.72632718e-03 +1.01629990e-02j ...,  -1.41711966e-02 -2.66411240e-04j
   -9.90793098e-03 -1.01852206e-02j   9.69116900e-05 -1.42447708e-02j]
 [  5.59438307e-03 -1.28113377e-02j   1.30667884e-02 -5.06640627e-03j
    1.28436598e-02 +5.69532832e-03j ...,  -1.31766618e-02 +5.17326929e-03j
   -1.30416426e-02 -5.59549850e-03j  -5.38376418e-03 -1.31689423e-02j]]
>> [  1.47576456e-05 -2.61536238e-06j   1.47538823e-05 -2.62518670e-06j
   1.47425539e-05 -2.65463586e-06j ...,   3.76992311e-06 +3.07837368e-06j
   4.72108447e-06 +1.43089564e-06j   4.97513263e-06 -4.82560764e-07j] (320000,)


>> [[-4.    0.    0.  ]
 [-4.    0.01  0.  ]
 [-4.    0.02  0.  ]
```

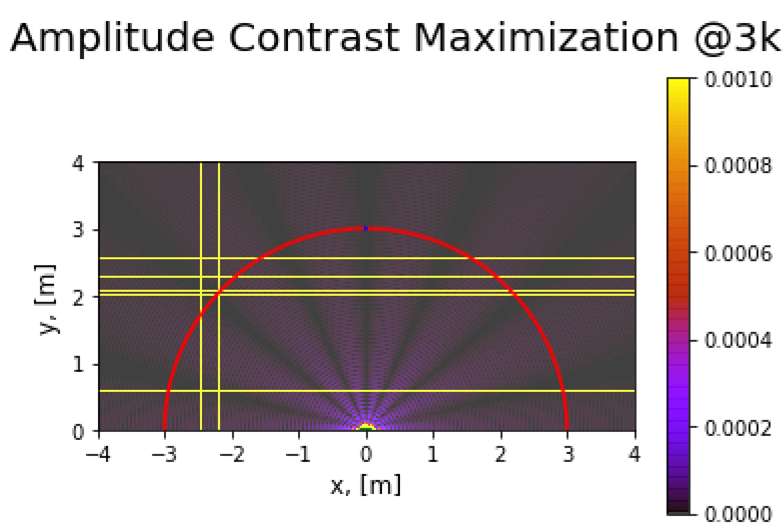
```
 ...,
 [ 3.99  3.97  0.  ]
 [ 3.99  3.98  0.  ]
 [ 3.99  3.99  0.  ]] [[-0.07  0.    0.  ]
 [-0.05  0.    0.  ]
 [-0.03  0.    0.  ]
 [-0.01  0.    0.  ]
 [ 0.01  0.    0.  ]
 [ 0.03  0.    0.  ]
 [ 0.05  0.    0.  ]
 [ 0.07  0.    0.  ]]
```

In [418]:

```python
output = np.ones((400,800))
for n in range(x_test.shape[0]):
    output[399 - int(x_test[n, 1]*100), int(x_test[n, 0]*100+400)] = p_test[n]
print(np.max(output))
fig = plt.figure()
plt.scatter(np.concatenate((x, y))[:, 0], np.concatenate((x, y))[:, 1], c=['r']*(
implot = plt.imshow(np.abs(output), interpolation='nearest', cmap='gnuplot', vmin
plt.colorbar()
fig.suptitle("Amplitude Contrast Maximization @3kHz", fontsize=20)
plt.xlabel("x, [m]", fontsize=12)
plt.ylabel("y, [m]", fontsize=12)
plt.show()
```

```
C:\Users\sarki\Anaconda3\lib\site-packages\ipykernel_launcher.py:3: ComplexWarn
ing: Casting complex values to real discards the imaginary part
  This is separate from the ipykernel package so we can avoid doing imports unt
il

2.33394760509
```



The above plot makes no sense, likely because it omits the regularization parameter β_{ACM}. This cements the importance of regularization

Equations for Energy Difference

Maximization

$$\max_{\mathbf{q}} \frac{E_B - \alpha E_D}{E_{\mathbf{q}}} = \max_{\mathbf{q}} \frac{\mathbf{q}^H(\mathbf{R}_B - \alpha \mathbf{R}_D)\mathbf{q}}{\mathbf{q}^H\mathbf{q}}$$

$$\mathbf{q}_{EDM} = \Phi(\mathbf{R}_B - \alpha \mathbf{R}_D)$$

In [384]:

```
E_ref = (4 * pi * R)**2 / L
sigma = 1.05
epsilon_alpha, alpha_max, E_max = 1e2, 1e6, sigma * E_ref
p_b_hat = 1
p_hat = np.array([0]*(M//2) + [1] + [0]*(M//2))

W = lambda q: p_b_hat / np.dot(z_b.T, q)
E = lambda q: np.dot(np.conjugate(q).T, q)
def PHI(A):
    w, v = np.linalg.eig(A)
    i = np.argmax(w)
    return v[:, i]

M_D = M - 1
R_D = np.dot(np.conjugate(Z_d).T, Z_d).real / (M_D)
R_B = np.dot(np.conjugate(z_b).T, z_b).real

print(M_D, R_D, R_B)
```

```
180 [[  7.03820003e-04   5.03863000e-04   7.16140042e-05  -2.45788624e-04
   -2.36252144e-04   4.55922445e-06   1.96757577e-04   1.57841092e-04]
 [  5.03863000e-04   7.03684463e-04   5.03774310e-04   7.15699207e-05
   -2.45807129e-04  -2.36245596e-04   4.59739389e-06   1.96806324e-04]
 [  7.16140042e-05   5.03774310e-04   7.03611504e-04   5.03730421e-04
    7.15322220e-05  -2.45847472e-04  -2.36260054e-04   4.63597547e-06]
 [ -2.45788624e-04   7.15699207e-05   5.03730421e-04   7.03601088e-04
    5.03731310e-04   7.15008879e-05  -2.45909677e-04  -2.36295526e-04]
 [ -2.36252144e-04  -2.45807129e-04   7.15322220e-05   5.03731310e-04
    7.03653209e-04   5.03776978e-04   7.14759017e-05  -2.45993776e-04]
 [  4.55922445e-06  -2.36245596e-04  -2.45847472e-04   7.15008879e-05
    5.03776978e-04   7.03767895e-04   5.03867449e-04   7.14572501e-05]
 [  1.96757577e-04   4.59739389e-06  -2.36260054e-04  -2.45909677e-04
    7.14759017e-05   5.03867449e-04   7.03945207e-04   5.04002770e-04]
 [  1.57841092e-04   1.96806324e-04   4.63597547e-06  -2.36295526e-04
   -2.45993776e-04   7.14572501e-05   5.04002770e-04   7.04185240e-04]] 0.00562
764336198
```

In [385]:

```
### Calculate alpha ###

alpha = alpha_max
q_temp = PHI(R_B - alpha * R_D)
q_hat = W(q_temp) * q_temp
n = 0
while E(q_hat) > E_max and alpha >= 0:
    if n % 1000:
        print(alpha, E(q_hat), E_max, E(q_hat) > E_max)
    alpha = alpha - epsilon_alpha
    q_temp = PHI(R_B - alpha * R_D)
    q_hat = W(q_temp) * q_temp
    n += 1

print(alpha, E(q_hat), E_max, E(q_hat) > E_max)
alpha_EDM = alpha
```

```
80300.0 (202233112841+0j) 186.53552318058888 True
80200.0 (202233112834+0j) 186.53552318058888 True
80100.0 (202233112813+0j) 186.53552318058888 True
80000.0 (202233112797+0j) 186.53552318058888 True
79900.0 (202233112779+0j) 186.53552318058888 True
79800.0 (202233112773+0j) 186.53552318058888 True
79700.0 (202233112747+0j) 186.53552318058888 True
79600.0 (202233112737+0j) 186.53552318058888 True
79500.0 (202233112719+0j) 186.53552318058888 True
79400.0 (202233112698+0j) 186.53552318058888 True
79300.0 (202233112692+0j) 186.53552318058888 True
79200.0 (202233112673+0j) 186.53552318058888 True
79100.0 (202233112651+0j) 186.53552318058888 True
79000.0 (202233112635+0j) 186.53552318058888 True
78900.0 (202233112631+0j) 186.53552318058888 True
78800.0 (202233112603+0j) 186.53552318058888 True
78700.0 (202233112596+0j) 186.53552318058888 True
78600.0 (202233112566+0j) 186.53552318058888 True
78500.0 (202233112561+0j) 186.53552318058888 True
78400.0 (202233112542+0j) 186.53552318058888 True
```

In [453]:
```
q_EDM = PHI(R_D - alpha * R_B)
print(q_EDM, q_DAS, np.average(q_DAS / q_EDM))
p = np.dot(ZZ, q_EDM.T)
print(p)
print(np.max(p))
```

```
[-0.41100165-0.01978789j -0.28569511+0.03771158j  0.11410313+0.08442244j
  0.46737839+0.07169087j  0.47335254+0.j          0.12701134-0.07695935j
 -0.27611268-0.09927769j -0.41072655-0.05482335j] [[ 0.42795313+4.69227772j  0.
44108448+4.69098116j  0.41985128+4.69305916j
   0.36421000+4.6980447j   0.27406334+4.70471356j  0.14930517+4.71108159j
  -0.01010674+4.71440292j -0.20405013+4.71117287j]] (0.239559987472+2.246323529
83j)
[  9.46335080e-03-0.05666284j   9.46404126e-03-0.05667323j
   9.46610277e-03-0.05670436j   9.46950572e-03-0.05675607j
   9.47420084e-03-0.05682807j   9.48011927e-03-0.05692001j
   9.48717269e-03-0.05703137j   9.49525349e-03-0.05716158j
   9.50423500e-03-0.05730991j   9.51397171e-03-0.05747554j
   9.52429964e-03-0.05765754j   9.53503664e-03-0.05785486j
   9.54598288e-03-0.05806633j   9.55692129e-03-0.05829066j
   9.56761816e-03-0.05852647j   9.57782377e-03-0.05877221j
   9.58727311e-03-0.05902627j   9.59568667e-03-0.05928687j
   9.60277138e-03-0.05955215j   9.60822158e-03-0.0598201j
   9.61172011e-03-0.06008861j   9.61293954e-03-0.06035546j
   9.61154345e-03-0.06061829j   9.60718786e-03-0.06087465j
   9.59952276e-03-0.06112197j   9.58819373e-03-0.06135761j
   9.57284371e-03-0.06157878j   9.55311479e-03-0.06178265j
   9.52865022e-03-0.06196627j   9.49909643e-03-0.06212664j
   9.46410512e-03-0.06226069j   9.42333552e-03-0.0623653j
   9.37645663e-03-0.0624373j    9.32314955e-03-0.0624735j
   9.26310984e-03-0.06247071j   9.19604994e-03-0.06242572j
   9.12170154e-03-0.06233538j   9.03981798e-03-0.06219655j
   8.95017661e-03-0.06200617j   8.85258114e-03-0.06176124j
   8.74686383e-03-0.0614589j    8.63288767e-03-0.06109638j
   8.51054842e-03-0.0606711j    8.37977654e-03-0.06018062j
   8.24053888e-03-0.05962273j   8.09284033e-03-0.05899541j
   7.93672514e-03-0.05829694j   7.77227812e-03-0.05752582j
   7.59962553e-03-0.05668088j   7.41893582e-03-0.05576128j
   7.23041999e-03-0.05476649j   7.03433177e-03-0.05369639j
   6.83096750e-03-0.05255121j   6.62066565e-03-0.0513316j
   6.40380618e-03-0.05003863j   6.18080948e-03-0.0486738j
   5.95213511e-03-0.04723906j   5.71828020e-03-0.04573681j
   5.47977762e-03-0.0441699j    5.23719386e-03-0.04254166j
   4.99112668e-03-0.04085586j   4.74220252e-03-0.03911672j
   4.49107370e-03-0.03732892j   4.23841542e-03-0.03549757j
   3.98492263e-03-0.03362816j   3.73130674e-03-0.03172662j
   3.47829216e-03-0.02979921j   3.22661288e-03-0.02785255j
   2.97700886e-03-0.02589355j   2.73022246e-03-0.02392939j
   2.48699482e-03-0.02196749j   2.24806231e-03-0.02001544j
   2.01415299e-03-0.018081j     1.78598310e-03-0.01617197j
   1.56425373e-03-0.01429626j   1.34964753e-03-0.0124617j
   1.14282554e-03-0.01067611j   9.44424216e-04-0.00894718j
   7.55052574e-04-0.00728241j   5.75289495e-04-0.00568909j
   4.05681238e-04-0.00417425j   2.46739113e-04-0.00274456j
   9.89373526e-05-0.00140634j  -3.72888181e-05-0.00016547j
  -1.61544923e-04+0.00097264j  -2.73478774e-04+0.00200309j
```

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  -3.72781809e-04+0.0029215j   -4.59190239e-04+0.00372411j
  -5.32485994e-04+0.00440773j  -5.92497473e-04+0.00496983j
  -6.39100105e-04+0.00540851j  -6.72216711e-04+0.00572254j
  -6.91817689e-04+0.00591135j  -6.97920991e-04+0.00597505j
  -6.90591928e-04+0.00591442j  -6.69942780e-04+0.00573089j
  -6.36132220e-04+0.00542653j  -5.89364548e-04+0.00500405j
  -5.29888745e-04+0.00446674j  -4.57997330e-04+0.00381849j
  -3.74025038e-04+0.00306368j  -2.78347308e-04+0.00220725j
  -1.71378586e-04+0.00125454j  -5.35704455e-05+0.00021135j
   7.45904714e-05-0.00091617j   2.12584688e-04-0.00212155j
   3.59862297e-04-0.00339805j   5.15845525e-04-0.0047387j
   6.79931451e-04-0.00613638j   8.51494864e-04-0.00758385j
   1.02989125e-03-0.00907379j   1.21445990e-03-0.01059888j
   1.40452708e-03-0.01215183j   1.59940935e-03-0.01372545j
   1.79841684e-03-0.01531264j   2.00085667e-03-0.0169065j
   2.20603629e-03-0.01850033j   2.41326688e-03-0.02008767j
   2.62186666e-03-0.02166235j   2.83116419e-03-0.02321849j
   3.04050153e-03-0.02475056j   3.24923737e-03-0.02625339j
   3.45674993e-03-0.02772216j   3.66243978e-03-0.02915246j
   3.86573247e-03-0.03054028j   4.06608092e-03-0.03188201j
   4.26296770e-03-0.03317446j   4.45590691e-03-0.03441485j
   4.64444605e-03-0.03560082j   4.82816739e-03-0.03673039j
   5.00668928e-03-0.037802j     5.17966712e-03-0.03881446j
   5.34679399e-03-0.03976699j   5.50780119e-03-0.04065911j
   5.66245832e-03-0.04149073j   5.81057326e-03-0.04226207j
   5.95199183e-03-0.04297365j   6.08659717e-03-0.04362627j
   6.21430902e-03-0.04422099j   6.33508267e-03-0.04475914j
   6.44890776e-03-0.04524223j   6.55580692e-03-0.04567198j
   6.65583422e-03-0.04605031j   6.74907346e-03-0.04637926j
   6.83563643e-03-0.04666101j   6.91566096e-03-0.04689786j
   6.98930892e-03-0.04709219j   7.05676421e-03-0.04724647j
   7.11823065e-03-0.04736321j   7.17392981e-03-0.04744496j
   7.22409897e-03-0.04749429j   7.26898890e-03-0.04751377j
   7.30886180e-03-0.04750598j   7.34398924e-03-0.04747346j
   7.37465011e-03-0.04741873j   7.40112865e-03-0.04734425j
   7.42371258e-03-0.04725245j   7.44269127e-03-0.04714567j
   7.45835399e-03-0.04702622j   7.47098825e-03-0.04689629j
   7.48087826e-03-0.04675801j   7.48830347e-03-0.04661343j
   7.49353715e-03-0.0464645j    7.49684517e-03-0.04631308j
   7.49848479e-03-0.04616093j   7.49870358e-03-0.04600971j
   7.49773844e-03-0.04586098j   7.49581465e-03-0.04571621j
   7.49314512e-03-0.04557674j   7.48992958e-03-0.04544385j
   7.48635397e-03-0.04531868j   7.48258984e-03-0.04520228j
   7.47879382e-03-0.0450956j    7.47510716e-03-0.04499949j
   7.47165538e-03-0.0449147j    7.46854790e-03-0.04484186j
   7.46587775e-03-0.04478153j   7.46372136e-03-0.04473414j
   7.46213836e-03-0.04470005j   7.46117146e-03-0.04467949j
   7.46084631e-03-0.04467262j]
(0.0096129395363-0.0603554565737j)
```

I gave up on EDM after seeing this maximum value. I need to get the regularization parameters figured out.

In [ ]: