Bi-Weekly Update

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1 Introduction

This is a section which outlines the fundamental formulas behind the "Delay Sum" (DAS) and "Pressure Matching" (PM) methods for sound localization.

1.1 Math Fundamentals

The relationship between L sources and M control points is

$$\mathbf{p}(\omega) = \mathbf{Z}(\omega)\mathbf{q}(\omega)s(\omega) \tag{1}$$

where $\mathbf{p}(\omega)$ are the sound pressure at the M control points, $\mathbf{q}(\omega)$ is the vector of weights for the L sources, $\mathbf{Z}(\omega)$ is the transfer matrix, and $s(\omega)$ is set to 1 at each angular velocity ω.

The transfer matrix $\mathbf{Z}$ is

$$\mathbf{Z}_{M\times L}(\omega) = \begin{bmatrix} Z(\mathbf{x}_1, \mathbf{y}_1, \omega) & \dots & Z(\mathbf{x}_1, \mathbf{y}_L, \omega) \\ \vdots & \ddots & \vdots \\ Z(\mathbf{x}_M, \mathbf{y}_1, \omega) & \dots & Z(\mathbf{x}_M, \mathbf{y}_L, \omega) \end{bmatrix} \tag{2}$$

Where the transfer function is defined as

$$Z(\mathbf{x}_m, \mathbf{y}_\ell, \omega) = \frac{e^{-j\frac{\omega}{c}||\mathbf{x}_m - \mathbf{y}_\ell||}}{4\pi \ || \ \mathbf{x}_m - \mathbf{y}_\ell \ ||} \tag{3}$$

For simplicity's sake, ω will be omitted in all further equations.

1.2 DAS

The equation that gives us the $\mathbf{q}$ using the DAS method is

$$\mathbf{q}_{DAS} = \mathbf{\Gamma}\mathbf{z}_B^* \tag{4}$$

Where $[\cdot]^*$ is the operation of complex conjugate. The vector $\mathbf{z}_B$ is the transfer vector for the singular bright point and the matrix $\mathbf{\Gamma}$ is defined as

$$\mathbf{\Gamma} = \begin{bmatrix} \gamma_1 & 0 & \dots & 0 \\ 0 & \gamma_2 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & \gamma_L \end{bmatrix} \tag{5}$$

Where $\gamma_\ell = \frac{16\pi^2 ||\mathbf{x}_B - \mathbf{y}_\ell||^2}{L}$.

1.3 PM

The equation that allows us to derive $\mathbf{q}$ using the PM method is

$$\min_{\mathbf{q}} J_{PM} = \min_{\mathbf{q}} (\mathbf{e}_{PM}^H \mathbf{e}_{PM} + \beta_{PM} E_q) \tag{6}$$

Where $\mathbf{e}_{PM} = \hat{\mathbf{p}} - \mathbf{Z}\mathbf{q}$. The Tikhonov regularization parameter β_{PM} is given by gradually increasing β_{PM} until this inequality is satisfied

$$E_{\mathbf{q}} = \mathbf{q}^H \mathbf{q} \leq E_{max} \tag{7}$$

The final relationship used to calculate the vector $\mathbf{q}_{PM}$ is

$$\mathbf{q}_{PM} = (\mathbf{Z}^H \mathbf{Z} + \beta_{PM} \mathbf{I})^{-1} \mathbf{Z}^H \hat{\mathbf{p}} \tag{8}$$

2 Conclusion

From tinkering with the algorithms in a Jupyter notebook, I've elucidated some of the intricacies of the two algorithms. Notably:

1. $\mathbf{q}_{DAS}$ is much, much faster to compute than $\mathbf{q}_{PM}$
2. PM has much better directive performance than DAS

3. both algorithms perform better at higher frequencies and worse at lower frequencies

The following images highlight points 2 and 3:

Figure 1: Delay Sum

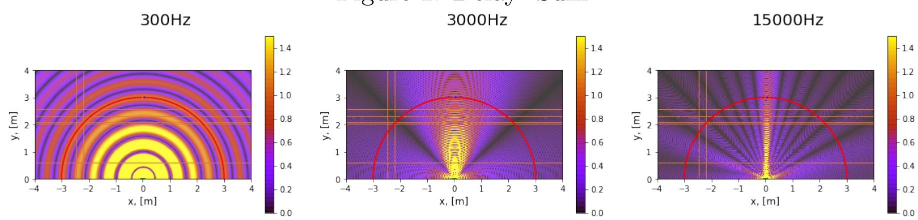

Figure 2: Pressure Matching