Finite Mixture Model

Introduction

Imagine we randomly choose a distribution to sample from. Let k denote which distribution we choose.

- Distribution 1
  - Parameter Ω_1
  - Probability of choosing distribution 1 is π_1
  - $x|k = 1, \Omega_1 \ \sim \Pr(x|k = 1, \Omega_1)$
- Distribution 2
  - Parameter Ω_2
  - Probability of choosing distribution 2 is π_2
  - $x|k = 2, \Omega_2 \ \sim \Pr(x|k = 2, \Omega_2)$

$\vdots$

- Distribution C
  - Parameter Ω_C
  - Probability of choosing distribution C is π_C
  - $x|k = C, \Omega_C \ \sim \Pr(x|k = C, \Omega_C)$

In general we have data $\mathbf{x} = \{x_1, x_2, ...x_n\}$. Suppose we know which distribution each sample was sampled from $\mathbf{k} = \{k_1, k_2, ...k_n\}$. We know the parameters of each distribution $\boldsymbol{\Omega} = \{\Omega_1, \Omega_2, ...\Omega_C\}$. Finally we know the probability of choosing each distribution $\boldsymbol{\pi} = \{\pi_1, \pi_2, ...\pi_C\}$. We can represent each sample as:

$$x_i|k_i, \boldsymbol{\Omega} \ \sim \Pr(x_i|k_i, \boldsymbol{\Omega})$$

Bayes Theorm

$$\Pr(\mathbf{k}, \boldsymbol{\Omega}|\mathbf{x}) \propto \Pr(\mathbf{x}|\mathbf{k}, \boldsymbol{\Omega})\Pr(\mathbf{k}, \boldsymbol{\Omega}, \boldsymbol{\pi})$$

Prior

We often times assume independence between the some of the parameters

$$\Pr(\mathbf{k}, \boldsymbol{\Omega}, \boldsymbol{\pi}) = \Pr(\mathbf{k}|\boldsymbol{\pi})\Pr(\boldsymbol{\pi})\Pr(\boldsymbol{\Omega})$$

$$\begin{aligned}\Pr(\mathbf{k}, \boldsymbol{\Omega}, \boldsymbol{\pi}) &= \Pr(\mathbf{k}, \boldsymbol{\pi})\Pr(\boldsymbol{\Omega}) \\ &\propto \Pr(\mathbf{k}|\boldsymbol{\pi})\Pr(\boldsymbol{\pi})\Pr(\boldsymbol{\Omega})\end{aligned}$$

Prior $\mathbf{Pr}(\mathbf{k}|\boldsymbol{\pi})$

todo

Prior $\mathbf{Pr}(\boldsymbol{\pi})$

todo

Prior $\mathbf{Pr}(\boldsymbol{\Omega})$

todo

Likelihood

todo : We want a general form for $\Pr(\mathbf{x}|\mathbf{k}, \boldsymbol{\Omega})$