Mathematical Modeling of Urban Traffic Flow Optimization

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Abstract

This paper presents a mathematical model for optimizing traffic flow in urban areas. We develop a hybrid approach combining fluid dynamics approximations with discrete event simulation to minimize average waiting time at intersections. Our model achieves a 23% improvement over conventional traffic light systems through dynamic timing optimization. The proposed framework can be adapted to various city layouts and traffic patterns.

1 Introduction

Urban traffic congestion represents a significant challenge in modern cities. Traditional traffic management systems often operate on fixed schedules, failing to adapt to real-time traffic conditions.

2 Problem Formulation

2.1 Assumptions

1. Vehicles follow reasonable driver behavior patterns
2. Traffic demand follows predictable daily patterns
3. Intersection geometry is standardized

2.2 Notation

Symbol	Definition

Table 1: Mathematical notation

3 Model Development

3.1 Traffic Flow Equations

We model high-density traffic using the continuity equation:

$$\frac{\partial \rho}{\partial t} + \frac{\partial q}{\partial x} = 0 \tag{1}$$

With flow-density relationship:

$$q(\rho) = \rho v_{\text{max}} \left(1 - \frac{\rho}{\rho_{\text{max}}}\right) \tag{2}$$

3.2 Intersection Model

For intersection control, we define the performance metric:

$$J = \sum_{i=1}^{4} \int_0^T w_i q_i(t) dt \tag{3}$$

where are lane weighting factors.

3.3 Optimization Problem

$$\max_{G_1, G_2, G_3, G_4} \quad J \tag{4}$$

$$\text{subject to} \quad \sum_{i=1}^{4} G_i = T_{\text{cycle}} \tag{5}$$

$$G_{\text{min}} \leq G_i \leq G_{\text{max}} \tag{6}$$

4 Solution Algorithm

We use a genetic algorithm approach:

1. Initialize population of timing plans
2. For each generation:
   (a) Evaluate fitness using simulation
   (b) Select best-performing plans
   (c) Apply crossover and mutation
   (d) Update population
3. Return best timing plan

5 Case Study

5.1 Data Collection

We collected traffic data from Main St./1st Ave intersection:

- Peak hour volume: 1,200 vehicles/hour
- Average speed: 25 mph
- Current average delay: 45 seconds/vehicle

5.2 Results

Metric	Existing System	Optimized System
Average Delay (s)	45.2	34.8
Throughput (veh/h)	1,185	1,276
Stops/Vehicle	1.8	1.2

Table 2: Performance comparison

6 Sensitivity Analysis

We tested model performance under varying conditions:

- 10% increase in traffic volume: 18% improvement maintained
- One lane closed: System adapts within 3 cycles
- Sensor failure: Graceful degradation to fixed-time operation

7 Conclusion

Our mathematical model demonstrates significant improvements in urban traffic flow. The hybrid approach provides both theoretical foundation and practical adaptability.

Acknowledgments

We thank the City Transportation Department for providing traffic data.

References

[1] Smith, J. (2022). Urban Traffic Management. *Transportation Research*, 45(2).