Optimizing Questions via Abductive reasoning in a non-monotonic logic setting

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1 Introduction

The scenario we will consider here is that we have a theory T (ie. a set of rules) written in a non-monotonic first order logic, and our aim is to solicit some set of facts F from a user to determine whether or not $T \cup F \vdash_{NM} G$, for some fixed goal G. Here the entailment relation $\vdash_{NM}$ is assumed to be some fixed appropriate non-monotonic entailment relation. Assume that we have access to a set of true/false questions Q in order to do this and we want to ask as few questions as possible, but (with respect to the questions Q available), the truth value of G must be established 'beyond doubt'.

2 Transformation to an abduction problem

Here is one possible approach to this problem of minimizing the number of questions asked. Given some $q \in Q$, let q_f denote the fact that would become true, were the user to answer 'yes' to the question q. Then one can start of by finding a minimal set $S \subseteq Q$ such that S_f, is a minimal solution to the problem:

$T \cup S_f \vdash_{NM} G$.

This now becomes an abuductive reasoning problem that can be solved in a non-monotonic reasoning engine like ASP using things like choice rules and weak constraints. Having obtained this set S, we pick a question q in S and ask it to the user. Then depending on the users response, we either augment the theory T, by setting $T_1 = T \cup \{q_f\}$ or setting $T_1 = T \cup \{\neg q_f\}$, depending on the user's response. Having done this, we again ask for a minimal $S_1 \subseteq Q$ such that the following problem is solved:

$T_1 \cup S_{1f} \vdash_{NM} G$.

3 Accounting for defeasibility and termination of the process

As we carry on with this process, augmenting the current theory as we go along, there will eventually be some integer m such that one of two things happens.

Either we have that no additional questions need to be asked for G to be entailed, ie:
$T_m \vdash_{NM} G$ or no additional response to a question can cause G to be entailed. Ie:
there is no set $S_m \subseteq Q$ such that $S_{mf} \cup T_m \vdash_{NM} G$.

If we are in the latter case, then we are done and we have established that the truth value of G is false and no additional information from the user can make it true.

In the former case, since we are in a non-monotonic setting we need to now check for additional information that the user can provide that may defeat the entailment of G. So we repeat the previous process, but now our goal is *not* G.

So our initial theory is now T_m, and we are seeking some minimal $S' \subseteq Q$ such that $T_m \cup S'_f \vdash_{NM}$ *not* G.

If eventually we find that there is some $n > m$, such that $T_n \vdash_{NM}$ *not* G, then we switch the goal back to G and our initial theory becomes T_n.

We keep doing this process of adding facts to our theory according to user responses and switching between G and *not* G as the goal until we have got a theory T_k such that either:

$T_k \vdash_{NM} G$ and there is no $S \subseteq Q$ such that $T_k \cup S_f \vdash_{NM}$ *not* G.

or we have $T_k \vdash_{NM}$ *not* G and there is no $S \subseteq Q$ such that $T_k \cup S_f \vdash_{NM} G$.

4 Treatment of negation

The treatment of negation needs some careful consideration within this set-up. For example if a 'no' answer to a question q is just intended to mean that q_f is not known to be true rather than the classical negation of q_f, then our set of questions Q, needs to contain a question q', which is such that were the user to answer yes to q', the fact that would be made true would indeed be the classical negation of q_f.