L4 to ASP translation for rules with inequalities and existential variables in preconditions

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1 Introduction

In the previous document we gave L4 to ASP translations for L4 rules that did not have inequalities or existential variables. We will now describe translations for rules that have existential quantifiers in preconditions as well as inequalities in the preconditions. The only real extension is to the translation from section 4 in the previous document, ie. the translation needed to automatically generate questions.

2 Running example

Consider the raw L4 rule: $\forall X \ (q(X) \text{ and } \exists Y \text{ st. } r(X,Y) \text{ and } X < Y \text{ then } p(X))$
In prolog like syntax this rule would read as:

```
p(X):-r(X,Y),q(X),X<Y.
```

2.1 Section 3 translation - Forward Reasoning

First assume that the rule above has rule id: a_1. The section 3 (forward reasoning) translation is as follows:

```
according_to(a_1,p(X)):-legally_holds(r(X,Y)),legally_holds(q(X)),legally_holds(ineq_a_1(X,Y)).
legally_holds(ineq_a_1(X,Y)):-legally_holds(r(X,Y)),legally_holds(q(X)),X<Y.
```

Here $ineq\_a\_1$ is a fresh predicate that does not occur elsewhere in the rules.

2.2 Section 4 translation - Question generation

Here we use skolemization to construct the appropriate 'explains' rules for the given raw rules as follows:

```
explains(r(X,ext_vbl_a_1_1(X)),p(X),N+1):-query(p(X),N).
explains(ineq_a_1(X,ext_vbl_a_1_1(X)),p(X),N+1):-query(p(X),N).
explains(q(X),p(X),N+1):-query(p(X),N).
```

Here $ext\_vbl\_a\_1\_1$ is a fresh skolem function.
Optionally, we can also add the constraint:

```
:-user_input(pos,ineq_a_1(X,Y)).
```

2.3 Section 5 translation - Justification graph generation

There are two versions possible of this translation, one is:

```
caused_by(pos,legally_holds(r(X,Y)),according_to(a_1,p(X)),N+1):-
according_to(a_1,p(X)),legally_holds(q(X)),legally_holds(r(X,Y)),X<Y,justify(p(X),N).

caused_by(pos,legally_holds(q(X)),according_to(a_1,p(X)),N+1):-
according_to(a_1,p(X)),legally_holds(q(X)),legally_holds(r(X,Y)),X<Y,justify(p(X),N).
```

The other possible version is:

```
caused_by(pos,legally_holds(r(X,Y)),according_to(a_1,p(X)),N+1):-
according_to(a_1,p(X)),legally_holds(q(X)),legally_holds(r(X,Y)),
legally_holds(ineq_a_1(X,Y)),justify(p(X),N).

caused_by(pos,legally_holds(q(X)),according_to(a_1,p(X)),N+1):-
according_to(a_1,p(X)),legally_holds(q(X)),legally_holds(r(X,Y)),
legally_holds(ineq_a_1(X,Y)),justify(p(X),N).

caused_by(pos,legally_holds(ineq_a_1(X,Y)),according_to(a_1,p(X)),N+1):-
according_to(a_1,p(X)),legally_holds(q(X)),legally_holds(r(X,Y)),
legally_holds(ineq_a_1(X,Y)),justify(p(X),N).
```

References