# Notes on the first part of the course

## Investigating the optical spectra of galaxies

We have 3 images of different galaxies and their correspondent spectra.

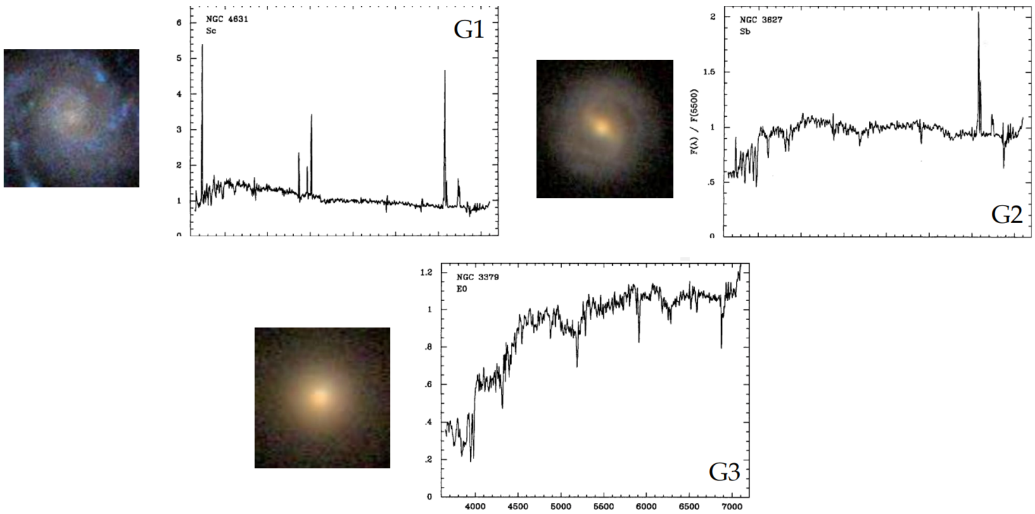


Figure 1: 3 galaxies and their spectra. The fluxes are normalized at $\lambda = 5500$Å, and the y-axis is different for each spectrum.

### Part 1

First off, we want to **find and list the relevant features of these spectra**. Only observe without interpretation at first! It is to notice that in figure 1 we only see a part of the total spectra of the galaxies; taking spectra lower than $\lambda = 3500$Å is a problem because of the atmosphere, also the atmosphere creates problems in the redder part.
We can divide the spectra in 3 different parts: the continuum, the emission lines and the absorption lines. The continuum is almost at a constant value for G1, meanwhile for G2 and G3 there is a drop in the bluer part of the spectrum. When present, emission and absorption lines are all at the same wavelength, but with different intensities.
We notice that the three galaxies have the peak of emission at different wavelengths:

$$\lambda_{G1} \sim 4000\text{Å} \qquad \lambda_{G2} \sim 4500\text{Å} \qquad \lambda_{G3} \sim 6000\text{Å}$$

### Part 2

Now that we observed the galaxies spectra, we can ask ourselves: **What are the physical properties of the galaxies which determine the galaxies' optical spectral shapes? And why do they differ from each other?**
To answer this, we first focus on a single component, the continuum, and we try to identify the physical process that determine the continuum spectra. A process that produces continuum spectra is the bremsstrahlung, a free-free process; but it emits in x-ray, so it cannot be responsible for these spectra.

Another possible process is the black-body spectrum. We made the hypothesis that `the spectra we see are a superposition of stars spectra` and the assumption that *the stars emit as a perfect black-body*. Other assumptions we can make are that *all the stars have the same metallicity and are born at the same time*, also *we are not accounting for dust*. Knowing the dependencies of the black-body emission spectrum, we have a relative physical variable: the temperature.
The Stefan-Boltzmann law gives us the radial emittance of a black-body, namely the total energy radiated per unit surface area and per unit time: $j^* = \sigma_B \cdot T_{eff}^4$, where $\sigma_B = 5.67 \times 10^{-8} ergs^{-1}cm^{-2}K^{-4}$ is the Stefan-Boltzmann constant and $T_{eff}$ is the effective temperature of the star. Therefore, the luminosity (total energy radiated per unit time) of a black-body is: $L = 4\pi R^2 \sigma_B T_{eff}^4$, where $R$ is the radius of the star.
We can link all the variables using another assumption: *the stars we are considering all lie in the main sequence*. Why are we making this assumption? Because for stars in the main sequence we know from stellar physics the scale relations between variables, simplifying our problem. The relation between mass and luminosity for stars on the ZAMS is

$$L \propto M^{3.5} \tag{1}$$

The scale relation between the radius of a star and its mass depends on the type of nuclear process inside the star.

$$\text{pp chain} \begin{cases} M < 1.3M_\odot \\ R \propto M^{0.5} \end{cases} \qquad \text{CNO cycle} \begin{cases} M > 1.3M_\odot \\ R \propto M^{0.8} \end{cases} \tag{2}$$

Using equations 1 and 2 we can relate the mass of a star to its effective temperature

$$\text{pp chain} \longrightarrow T \propto M^{0.6} \qquad \text{CNO cycle} \longrightarrow T \propto M^{0.5} \tag{3}$$

Now we have a correlation between the mass of a star and its effective temperature, that is directly linked with the black-body emission it produces.

$$B(\nu, T) = \frac{h\nu^3}{c^2} \frac{1}{e^{\frac{h\nu}{KT}} - 1} \tag{4}$$

The peak of a black-body distribution is given by the Wien's displacement law

$$\lambda_{max} = 29000\text{Å}\left(\frac{T}{1000K}\right)^{-1} \propto \begin{cases} M^{-0.6} & \text{pp chain} \\ M^{-0.5} & \text{CNO cycle} \end{cases} \tag{5}$$

So, we see that for a hotter star we have the peak at lower wavelengths.
Considering now a whole galaxy as a superposition of stars spectra, as in our hypothesis, we can see from equation 1 that the luminosity is directly proportional to the mass. Therefore, the more massive stars contribute more to the total luminosity. Examine now the evolution of the galaxy in time, assuming that *we don't have star formation*, so that we had a *single burst* at the beginning. At $t \sim 0Myr$ we have all kind of stars on the main sequence from low mass to high mass; after some time at $t \sim 1Myr$ we only have low mass stars. Why is that? Because stars have a main sequence lifetime that goes as

$$\tau \propto M^{-2.5} \qquad \tau = \left(\frac{M}{M_\odot}\right)^{-2.5} \tau_\odot \tag{6}$$

and as a result, higher mass stars live less than lighter stars; since we don't have star formation we are left with low mass stars after some time.
For galaxies that are young, the luminosity is dominated by the high mass stars leading to have higher temperature and peaks at lower wavelengths respectively to older galaxies that have stars with lower mass.
Observing the peaks of the spectra in figure 1 we can say that the G1 galaxy is younger than the G2 galaxy that is younger than the G3 galaxy.

## Test the model

So far, we made a testable hypothesis and developed a model based on a set of assumptions that now we want to test.
To do this, we write a code in Python and we later compare the results with our observations.
As we saw in equation 3 every mass is linked with a particular black-body spectrum, so to have a superposition of them we have to consider an array of masses. Obviously, we can't account for every mass that a star can have, so we have to bin them. We assume that *the minimum mass of a star is constant* and fixed at $M_{min} = 0.1M_\odot$. For the maximum mass, instead, we saw that it changes through time if we assume *a single burst*; but we can fix the maximum mass at $t = 0yr$ as $M_{max}(t = 0) = 100M_\odot$.
For the bin size of the array $dM$, we have to compromise between accuracy and computational power; not too tight because otherwise the script will take ages, but not too large due to possible inaccuracies.
To overlap the single spectra, we have to know how many stars have a certain temperature, aka: how many stars have a certain mass. For this, we assume that *the Initial Mass Function (IMF) of the stars is given by the Salpeter mass function:* $\frac{dN}{dM} \propto M^{-2.35}$. Taking the bin size $dM$ as constant, we can neglect it and the number of stars with a certain mass is given by

$$N = \left(\frac{M}{M_\odot}\right)^{-2.35} \tag{7}$$

For a given age of the galaxy, we compute its spectrum in the following way:
We iterate over the array of the masses and for each mass we compute the luminosity, the radius, the temperature and the lifetime using relations 1, 2, 3 and 6. If the main sequence lifetime of the star exceeds the age of the galaxy, we then we compute its black-body spectrum with `astropy.modeling.physical_models.BlackBody`, a function which returns the black-body spectrum in units of $erg\,cm^{-2}\,s^{-1}\,\text{Å}^{-1}\,sr^{-1}$ (so, a spectral radiance or specific intensity). Afterward, we multiply the flux by the number of stars (equation 7) that have the same temperature (aka same mass). We also multiply by the luminosity produced by a star of that specific mass and divide by the bolometric flux to normalize it. In the end, the flux will be in the strange units of $L_\odot sr^{-1}\text{Å}^{-1}$. Once the iteration is over, we sum all the fluxes and we normalize the total flux by the flux at $\lambda = 5500\text{Å}$ to compare our results with observations.

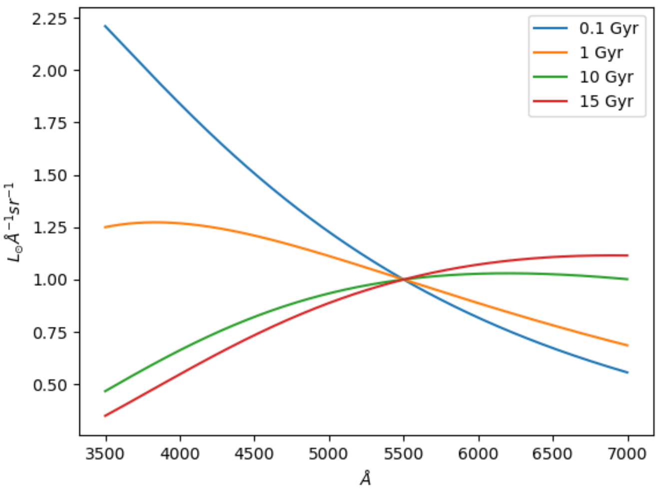


Figure 2: Fluxes of galaxies with different ages.

From the figure 2 we can see that older galaxies have the peak more toward the red and younger galaxies have the peak more toward the blue, as our model predicted.
In conclusion, we can say that by looking at the continuum, we can measure the age of the galaxy we are studying. The spectra in figure 2 don't resemble the observed in spectra in figure 1. To forward our investigation, we can relax some assumptions.

## Dust Attenuation

We made the assumption that the model spectrum is the emitted one, but we saw that the observed spectra are different! We can relax the assumption that the continuum is given only by the superposition of black-body emission of stars, and now consider also dust. Dust is composed by "grains" of things (often carbon and silicon compounds) that are opaque to optical light, so dust absorbs radiation[1]. We can describe dust attenuation as

$$f_{obs}(\lambda) = f_{intrinsic}(\lambda) 10^{-0.4 A_\lambda} \tag{8}$$

where

$$A_\lambda = k(\lambda) E(B - V) \tag{9}$$

The $E$ stands for "excess". Excess of what? Of color; represented by $(B - V)$ in this case. $B$ and $V$ are two bands, but they can be different; this is only an example. The color excess is the difference, due to absorption, between the observed color and its intrinsic color index. Note that the $A_\lambda$ is a magnitude, that is why in equation 8 it is written as the exponent of 10. The $k(\lambda)$ is in function of the wavelength.

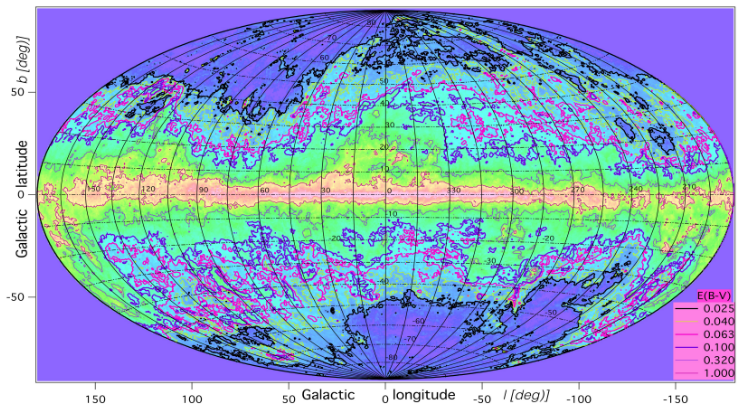


Figure 3: Color excess map of the Milky Way. The $E(B - V)$ values are in logarithm, the counturs are $log(E(B - V))$=0 (thin pink), -0.5 (thin violet), -1 (thick violet), -1.2 (thick pink), -1.4 (yellow) and -1.6 (black). Lallement et al (2013), based on Schlegel et al (1998).

In figure 3 we can see the color excess caused by the Milky Way.
Now, how much is $k(\lambda)$? We don't know the exact value; the form of the extinction curve depends on the composition of the ISM, which varies from galaxy to galaxy.

[1]Later, it can re-emits radiation in the infrared.

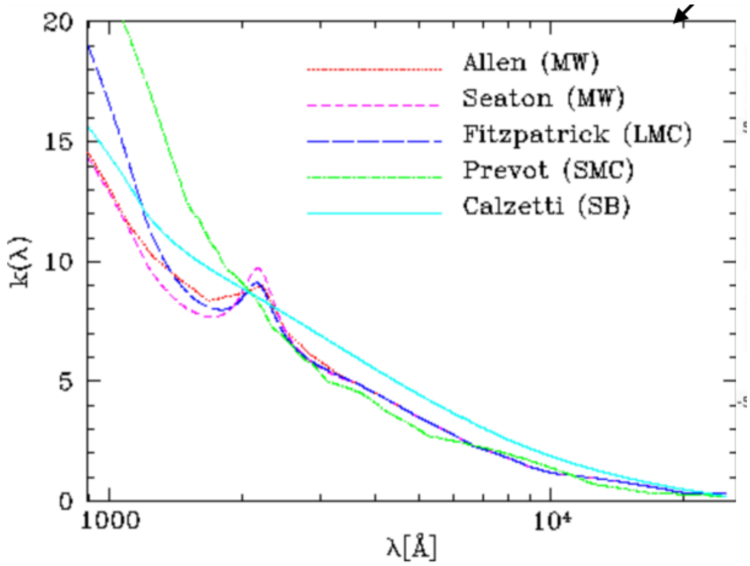


Figure 4: Extinction curves for MW= Milky Way, LMC=Large Magellanic Cloud and SB=star burst galaxies.

In figure 4 we have different curves; we will use the one from Calzetti et al(2000) for starburst galaxies:

$$k_\lambda = \begin{cases} 2.659(-2.156 + \frac{1.509}{\lambda} - \frac{0.198}{\lambda^2} + \frac{0.011}{\lambda^3}) + R_V & \text{for } 0.12\mu m \leq \lambda \leq 0.63\mu m \\ 2.659(-1.857 + \frac{1.040}{\lambda}) + R_V & \text{for } 0.63\mu m \leq \lambda \leq 2.20\mu m \end{cases} \tag{10}$$

with $R_V = 4.05$.
The dust extinction is more efficient for smaller wavelengths; it means that we absorb more blue components, resulting in a spectrum that is more red. This is the reddening in few words.
Now, we can correct our model using equation 8 to modify the fluxes computed previously. It is to note that we modify the second line of equation 10 with $R_V = 4.03$ to avoid the sudden jump at 6300Å.

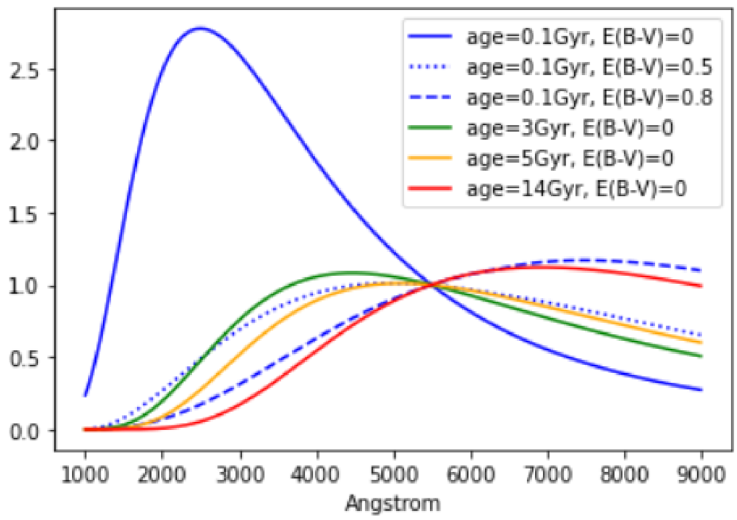


Figure 5: Fluxes from galaxies with different ages, accounting also for dust attenuation. Credits: this code

From figure 5 we can see that a galaxy with an age of $0.1Gyr$ and an excess color of $E(B-V) = 0.8$ can be easily mistaken for a galaxy without excess color, but with an age of $14Gyr$. It is not simple to determine if a galaxy is old without dust attenuation or young with dust attenuation; we have degeneracy between dust and age!
How can we break this degeneracy?

## Absorption Lines

We assumed that the spectrum of a star was a perfect black-body; but that is not true; we can relax that assumption. Beside the continuum, there are absorption lines present in a spectrum.

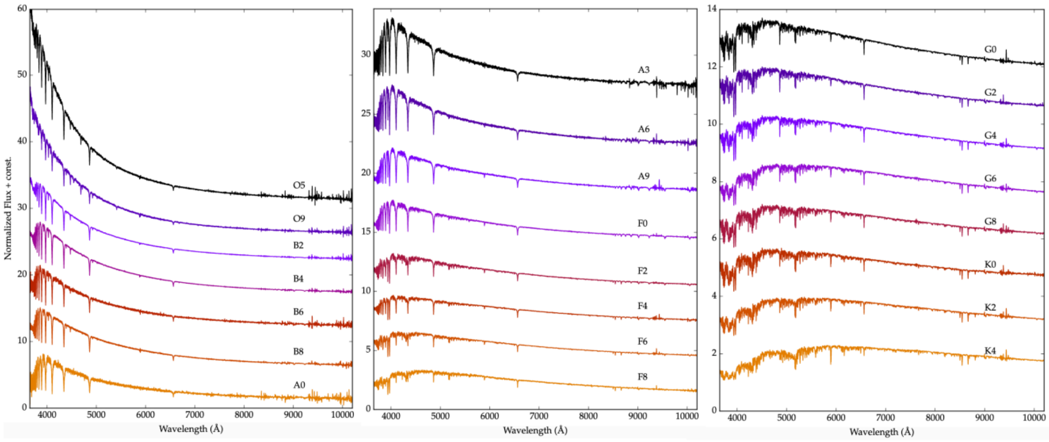


Figure 6: Spectra of stars of different spectral type.

Stars can be divided in different spectral types; the continuum can be described as a black-body and from that we divide stars from the hottest $O$ to the coldest $K$. We can see from figure 6 that different spectral types have different absorption lines. The most important absorption lines are the one from the Balmer series of the hydrogen ($n \geq 3 \longrightarrow n = 2$).

$$\lambda_{H-\alpha} \sim 6563\text{Å} \qquad \lambda_{H-\beta} \sim 4861\text{Å} \qquad \lambda_{H-\gamma} \sim 4341\text{Å} \qquad \lambda_{H-\infty} \sim 3646\text{Å}$$

We can make the assumption that *the absorption lines are due to hydrogen*. So, if we see Balmer lines we know that in the stellar photosphere there is hydrogen, and if we don't see them it means that there is no hydrogen present. From figure 6 we notice that hotter stars like the O/B/A type present Balmer lines, coldest stars instead don't. Knowing the link between temperature→mass→age, we can say that hotter stars that present the Balmer series are young because there is still hydrogen in the photosphere to absorb radiation, colder stars that don't present the Balmer series are older. Comparing the features of the spectra in figure 6 with the observed spectra in figure 1 we conclude that the galaxy $G3$ is not young and dusty, but old and dustless, having similar features to a k-star.

## Emission Lines

Another way to break the degeneracy is to consider emission lines. In particular, we see emission lines from "blue" galaxies ($G1$ and $G12$) of figure 1. The physical process that causes the emission lines can be the de-excitation of hydrogen atoms that live in the interstellar medium (ISM). A hydrogen atom goes into an excited state after recombination, then it de-excites back to the ground state. To have ionized hydrogen, namely electrons and protons that later recombine together, we need ionizing photons. We made the hypothesis that `the ionizing photons come from young and hot stars` . Hotter stars have a higher luminosity. To have a lot of young stars, we need to have star formation.
Summarizing, and focusing only on the $H-\alpha$ emission line because it's the easiest and

more evident: the $H-\alpha$ luminosity is proportional to the ionization rate that is proportional to the number of young stars that is proportional to the stellar mass produced in a short timescale (so, the SFR). All of this assuming *that we have fewer photons than atoms*; so the emission depends only on the number of photons and we have density bound $H_{II}$ regions outside the star where the ionized gas is confined. "Density bound" because the high density of gas surrounding the ionized region prevents the ionized hydrogen from diffusing or dispersing into the surrounding space[2].

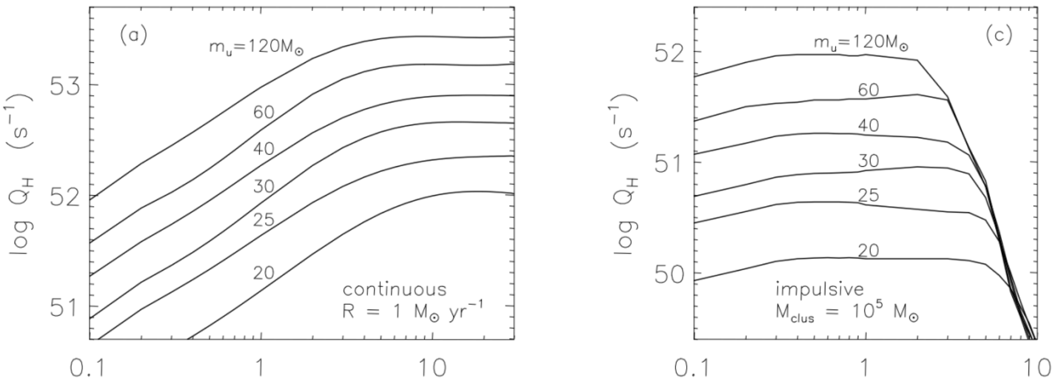


Figure 7: Relation between the surface temperature of stars and the ionizing photons production rate. $Q_H$ is the number of ionizing photons per second. Left: continuous SFR Right: burst of SFR

From figure 7 we see that stars with a higher temperature (young stars) produce more ionizing photons. The star formation rate depends on two variables: the mass and the time.

## Metallicity

"Metals" in astrophysics refers to all the elements that are neither hydrogen nor helium. Metals come from previous stars; the ISM is continuously enrich with metals from supernovae. To estimate the stellar properties $(L, T, R)$ as a function of mass we have rescaled everything to solar units, so we have assumed that all stars have solar composition or metallicity (Z). We have a degeneracy with different metallicities and ages; the rule of thumb is that if we have a percentage change of $\Delta A/\Delta Z = 3/2$ for two populations, then they will appear almost identical. To solve this degeneracy, it is possible to use spectroscopic indices like $H-\beta$ lines or some chemical abundances.

[2]Think like a Strömgren Sphere

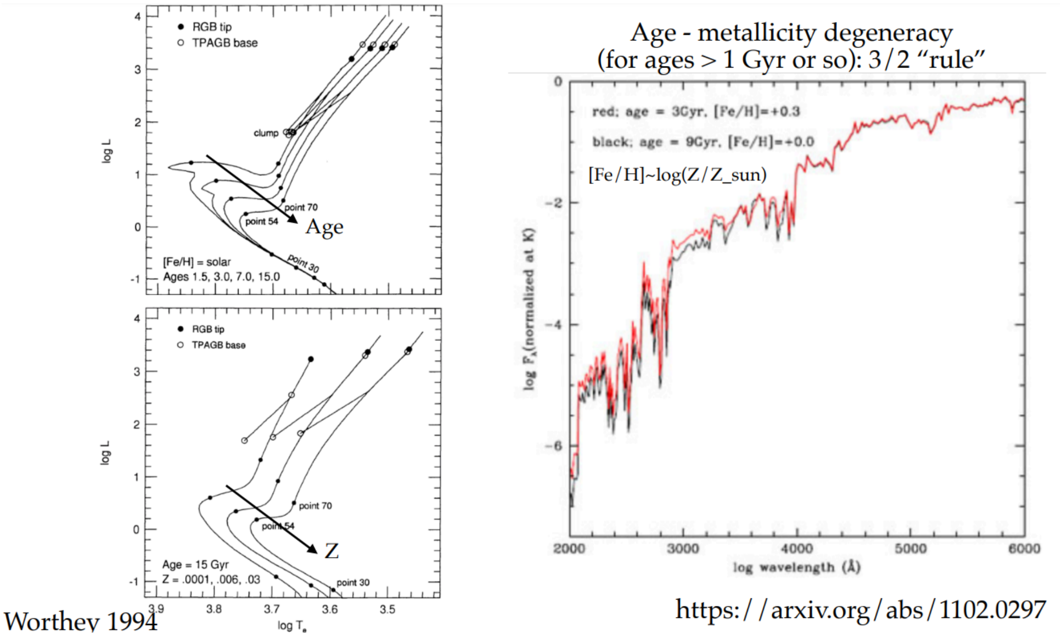


Figure 8: High-left: the tip of the AGB dims steadily with age. Bottom-left: steady shift to cooler temperatures with increasing metallicity. Right: example of age-metallicity degeneracy from Chavez et al. (2011).

## Quantitative Comparison

Finally, we want to have a quantitative comparison between our model and the data. To do this, we need to convert our model into physical units. The original units were $L_{\odot}\, sr^{-1}\text{Å}^{-1}$. First off, we multiply this by the solar luminosity $L_{\odot} = 3.828 \times 10^{33} \frac{erg}{s}$ and then we take care of the steradians multiplying by $4\pi$. In this way the units will be the one of a specific luminosity: $erg\, s^{-1}\, \text{Å}^{-1}$.

Always in the assumption of *a single burst of star formation*, we need to compute more carefully the number of stars in each star bins. Before we computed $N_{stars} = m^{-\alpha}$ with $\alpha = 2.35$; but we neglected a constant which sets the local stellar density. In fact, assuming always *a Salpeter IMF*, the initial mass function is

$$\xi(M) = \frac{dN}{dM} = \xi_0 M^{-2.35}$$

So, the total number of stars in a bin $dM$ would be

$$N_{stars} = \xi_0 M^{-\alpha} dM$$

To find what is $\xi_0$ we can compute the integral to find the total mass in stars born within the range we are considering. Integrating between the minimum and maximum stellar mass we set, we find the total mass of the galaxy[3].

$$M_{tot} \equiv \int_{M_{min}}^{M_{max}} M \cdot \xi(M) dM =$$
$$= \int_{0.1}^{100} M^{1-\alpha} \cdot \xi_0 dM =$$
$$= \xi_0 \left[ \frac{M^{1-\alpha+1}}{1-\alpha+1} \right]_{0.1}^{100} =$$
$$= \xi_0 \frac{100^{-0.35} - 0.1^{-0.35}}{-0.35} = \xi_0 A$$

Then $\xi_0$ is equal to $\xi_0 = \frac{M_{tot}}{A} = \frac{M_{tot}}{\frac{100^{-0.35}-0.1^{-0.35}}{-0.35}}$ and the number of stars in a bin is

$$N_{stars} = \frac{M_{tot}}{\frac{100^{-0.35}-0.1^{-0.35}}{-0.35}} M^{-\alpha} dM$$

where $dM$ is constant and is computed as $(M_{max} - M_{min})/n_{bins}$.
Now let's look at our data. We have specific fluxes and the observed wavelengths, but to make a comparison we want them in the same units of our model.
For each galaxy spectra we find their redshift using emission lines and computing it as: $z = \frac{\lambda_{obs}}{\lambda_{em}} - 1$.
Since the wavelengths are the observed one, we divide them by $1 + z$ to obtain the emitted wavelengths.
To convert the specific flux to a specific luminosity we use the definition of luminosity distance

$$L = 4\pi \cdot F \cdot D_L^2$$

taking care to multiply it by $1 + z$ considering it is a specific luminosity (i.e. per unit Å).
The luminosity distance value is obtained from the redshift using the function `cosmo.luminosity_distance(z)`.
Finally, we try to understand which mass and age our galaxies have according to our model. The results are in figure 9.

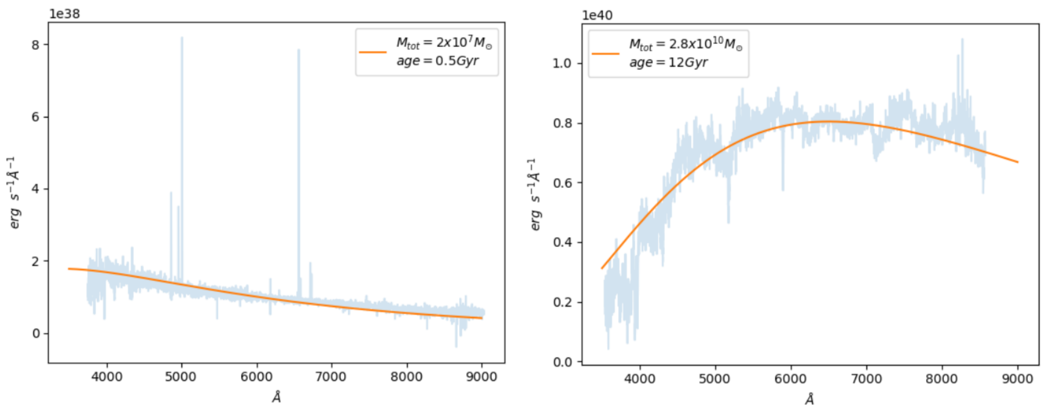


Figure 9: In light blue, the spectra of galaxy S1 (left) and S4 (right). In orange, the prediction of our model with age and mass written in the legend.

[3]If we had constant SFR, the equation would be $M_{tot} = \int_{M_{min}}^{M_{max}} \dot{M}(t) dt$