DERIVING THE OPEN AND CLOSED BOX EQUATIONS FOR OUR MODEL

WE KNOW FROM THE FOLLOWING ASSUMPTIONS:

- SINGLE BURST . $SFR = \varepsilon M_{gas}^{\alpha}$ w/ $\alpha = 1$, $\varepsilon =$ const. that

$$SFR = \varepsilon M_{gas} \qquad M_* = \int_{t_0}^{t} dt \; SFR(t) \qquad Z_{neb} = \frac{M(O)}{M(H)}$$

- ASSUMING ALL METALS ARE MOSTLY IN STARS . AND GAS IS MOSTLY HYDROGEN

$$Z = \frac{M_Z}{M_{gas}} \qquad M_Z = \text{metals in Galaxy}$$

CLOSED BOX

$$\dot{M}_Z = \underbrace{y\,SNR}_{\text{YIELD DUE TO SN EXPL.}} - \underbrace{Z\,SFR}_{\text{STAR FORMATION}}$$

- $\underline{\text{ASSUMING}}$ ONLY TYPE II

$SNR = SFR$

$$\dot{M}_Z = [y - Z]\,SFR \Rightarrow \dot{M}_Z + \frac{M_Z}{M_g} SFR = y\,SFR$$

$$\Rightarrow \dot{M}_Z + M_Z \varepsilon = y\,SFR$$

which IS OF THE FORM

$$\dot{x} + \tilde{A}x = B \Rightarrow x(t) = e^{-A(t)} \int_{t_0}^{t} B(t)\, e^{A(t)}\, dt$$

WITH $A(t) = \int_{t_0}^{t} \tilde{A}(t)\, dt = \varepsilon(t - t_0)$ IF $\varepsilon =$ const.

THE FOLLOWING SOLUTION WILL THEN BE

$$M_Z(t) = e^{-\varepsilon(t-t_0)} \int_{t_0}^{t} y\, SFR\, e^{\varepsilon(t-t_0)}\, dt$$

$$= e^{-\varepsilon(t-t_0)} \int_{t_0}^{t} y\, \varepsilon\, M_g(t)\, e^{\varepsilon(t-t_0)}\, dt$$

BUT FROM $\frac{d}{dt} M_g = -SFR$ WE FOUND $M_g(t) = \overbrace{M_g(t_0)}^{M_0} e^{-\varepsilon(t-t_0)}$

SO NOW

$$M_Z(t) = e^{-\varepsilon(t-t_0)} \int_{t_0}^{t} y\,\varepsilon\, M_g(t_0)\, e^{-\varepsilon(t-t_0)}\, e^{\varepsilon(t-t_0)}\, dt$$

$$= e^{-\varepsilon(t-t_0)}\, M_0\, y\,\varepsilon\,(t-t_0)$$

SO $M_Z(t) = M_Z^{closed} = y\, M_0\, \varepsilon(t-t_0)\, e^{-\varepsilon(t-t_0)}$

WHICH MEANS

$$\left\{ Z^{CLOSED} = \left(\frac{M_Z}{M_g}\right)^{CLOSED} = y\,\varepsilon\,(t-t_0) \right\} \qquad \varepsilon = .25\ Gyr^{-1}$$

OPEN BOX

- HOMOGENEOUS METALS DISTRIBUTION IN YOUR GALAXY
- GAS PRESTEEN $Z_{IGM} = 0$ (no $\dot{M}_Z^{in}$)
- $\dot{M}_g^{in} = \dot{M}_g^{out}$ $\Rightarrow$ this way $M_g = \varepsilon\, SFR$

WITH THIS WE HAVE ONLY $\dot{M}_Z^{out}$ due to SUPERNOVAE FEEDBACK = CONST ASSUMPT.

$$\dot{M}_Z = [y - Z]\, SFR - Z\eta\, SFR$$

WHERE AGAIN WE ASSUME SNR = SFR AND η IS THE FEEDBACK RATE

$$\dot{M}_Z + Z(1+\eta)\, SFR = y\, SFR$$

WE KNOW SOLUTION $\quad x(t) = e^{-\int a(t)} \int_{t_0}^{t} B(t)\, e^{\int a(t)\, dt}$

now $\quad A(t) = \int_{t_0}^{t} a(t)\, dt = ? = (1+\eta)\,\varepsilon\,(t-t_0)$

$$\dot{M}_Z + M_Z(1+\eta)\,\varepsilon = y\, SFR$$

WHICH HAS SOLUTION $(1+\eta)\varepsilon = \alpha$

$$M_z^{OPEN}(t) = M_z^{OPEN} = e^{-\alpha(t-t_0)} \int_{t_0}^{t} y \cdot SFR \; e^{\alpha(t-t_0)} \, dt$$

FROM OUR ASSUMPT. $\varepsilon M_g = SFR$

$$M_z^{OPEN} = e^{-\alpha(t-t_0)} \int_{t_0}^{t} y \; \varepsilon M_g(t_0) \, e^{-\varepsilon(t-t_0)} \; e^{\varepsilon(1+\eta)(t-t_0)} \, dt$$

IF $y, \varepsilon \not\propto t$

$$M_z^{OPEN} = e^{-\alpha(t-t_0)} \int_{t_0}^{t} e^{\varepsilon\eta(t-t_0)} \, dt \; \varepsilon y M_0$$

SMALLER TERMS

$$\simeq \varepsilon y \, M_0 \, e^{-\varepsilon(1+\eta)(t-t_0)} \frac{1}{\varepsilon\eta} e^{\varepsilon\eta(t-t_0)} + (\dots)$$

$$= y/\eta \; \varepsilon \; \underbrace{\frac{1}{\varepsilon} M_0 \, e^{-\varepsilon(t-t_0)}}_{M_z^{CLOSED}/y(t-t_0)} = \frac{\varepsilon}{\eta}\left(\frac{1}{(t-t_0)} M_z^{CLOSED} - 1\right)$$

$$\boxed{M_z^{OPEN} = \frac{\varepsilon}{\eta} \frac{1}{(t-t_0)} M_z^{CLOSED}}$$

BIG RESULT

HAVING SAID THIS WE NOW CAN USE METALLICITY, SINCE $M_{gas}^{CLOSED} = M_{gas}^{OPEN}$

$$\boxed{Z^{OPEN} = \frac{\varepsilon}{\eta} \frac{1}{(t-t_0)} Z^{CLOSED}}$$

but $Z^{CLOSED} = y(t-t_0)\frac{1}{\varepsilon}$

$$Z^{OPEN} = y/\eta$$

DOESN'T DEPEND ON $(t-t_0)$ AGE OF THE GALAXY

t_0 = time obs. = NOW SO $M_0 > M_g$

HAVING SAID THIS, WE NOW HAVE THAT WE NEED TO EXPRESS THIS INTO STELLAR MASS.

MA CON L'ULTIMA EQ. WE JUST NEED THE CLOSED BOX ONE

SO WE CAN SAY ALREADY

$$Z^{OPEN} = y\,\eta^{-1} \propto \eta^{-1}(M_*)$$

NOW Z^{CLOSED}

WE KNOW
$$M_* = \int_{t_0}^{t} SFR\, dt = \varepsilon M_0 \int_{t_0}^{t} e^{-\varepsilon(t-t_0)}\, dt$$
$$= \varepsilon M_0 \left(-\frac{1}{\varepsilon}\right) e^{-\varepsilon(t-t_0)} \Big|_{t_0}^{t}$$
$$= \cancel{\varepsilon} M_0 \left(-\frac{1}{\cancel{\varepsilon}}\right)\left(e^{-\varepsilon(t-t_0)} - 1\right)$$
$$= M_0 \left(1 - e^{-\varepsilon(t-t_0)}\right)$$

SO $M_* = M_0\left(1 - e^{-\varepsilon(t-t_0)}\right)$ WHICH MEANS, SINCE $Z^{CLOSED} = y\,\varepsilon(t-t_0)$

$$\Rightarrow \quad M_*/M_0 = 1 - e^{-\varepsilon(t-t_0)} \quad \Rightarrow \quad e^{-\varepsilon(t-t_0)} = 1 - \frac{M_*}{M_0}$$

$$\Rightarrow \quad \varepsilon(t-t_0) = \log\left(\frac{1}{1-\frac{M_*}{M_0}}\right)$$

$\frac{y}{\eta+1}$ $\frac{1}{1-\alpha_Z}$

AND SO

$$\boxed{Z^{CLOSED} = y \log\left(\frac{1}{1-\frac{M_*}{M_0}}\right)}$$

$$\boxed{Z^{OPEN} = y\,\eta^{-1}(M_*) \propto \eta^{-1}(M_*)}$$

$\eta+1$?

Q: HOW DO WE DEFINE M_0?

IF $M_0 \gg M_{GAS}$

- $Z^{CLOSED} \propto y \frac{M_*}{M_0}$

$\frac{M_*}{M_0} < 1$

Q: ASSUMPTIONS CORRECT?

$\eta \propto M_*^{-1/3}$ FROM DISPENSE / DAVÉ ANALYTIC

SMALL MASSES

$y \sim 0.01$ (TREMONTI ET AL.)