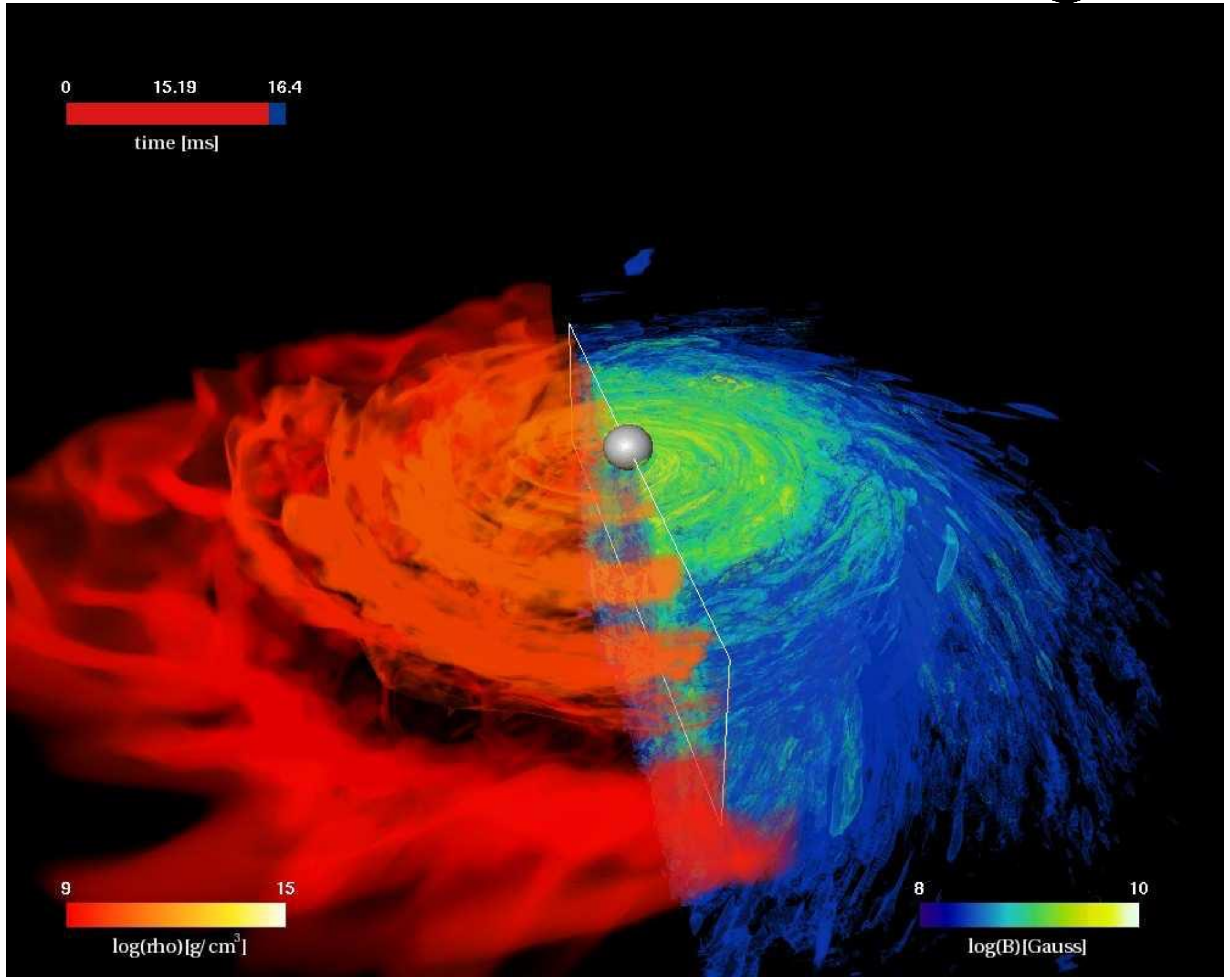
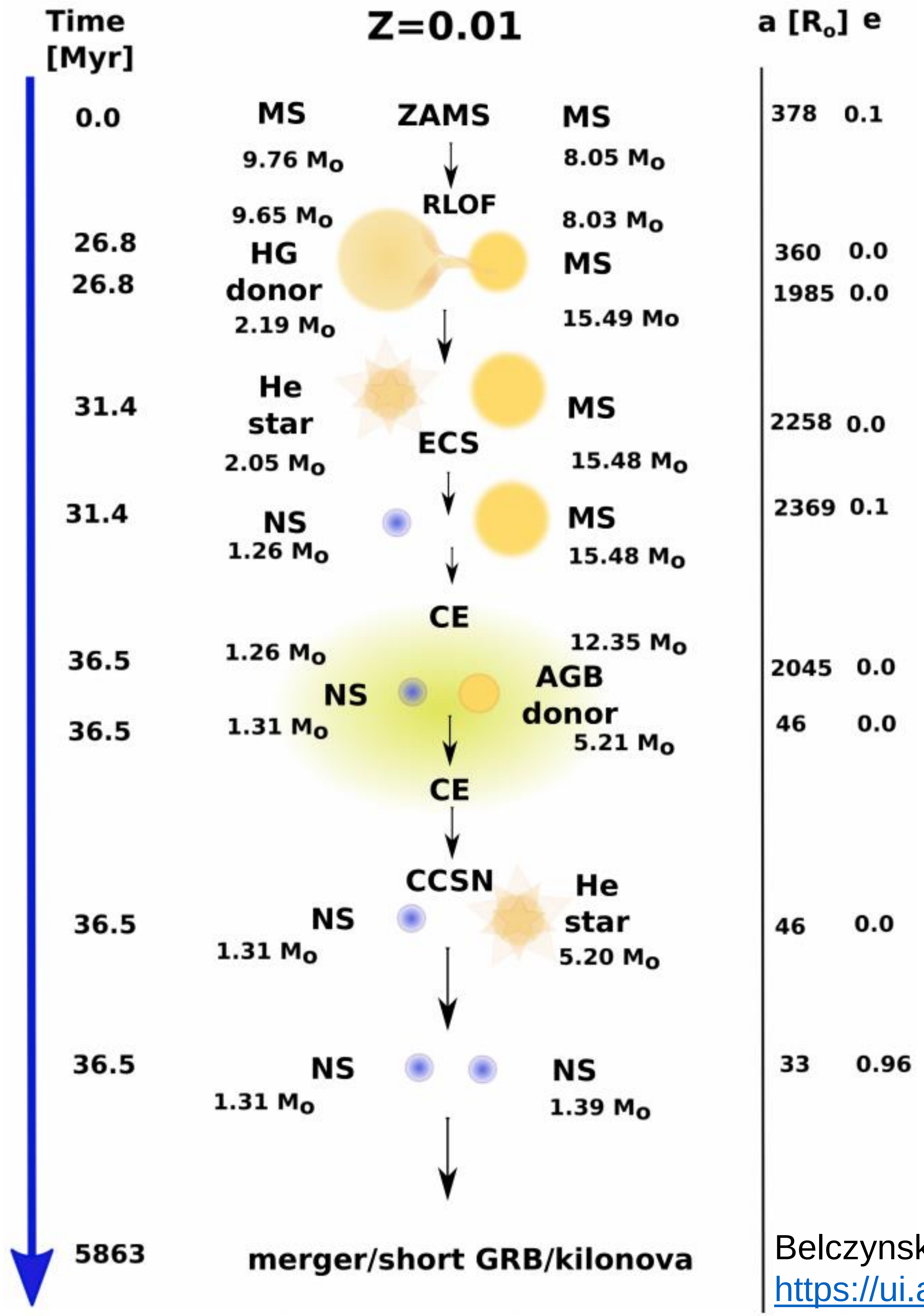
General Relativistic Simulations of NS-NS and NS-BH Mergers



Bruno Giacomazzo

www.brunogiacomazzo.org

NS Binary Formation



Several possible formation channels

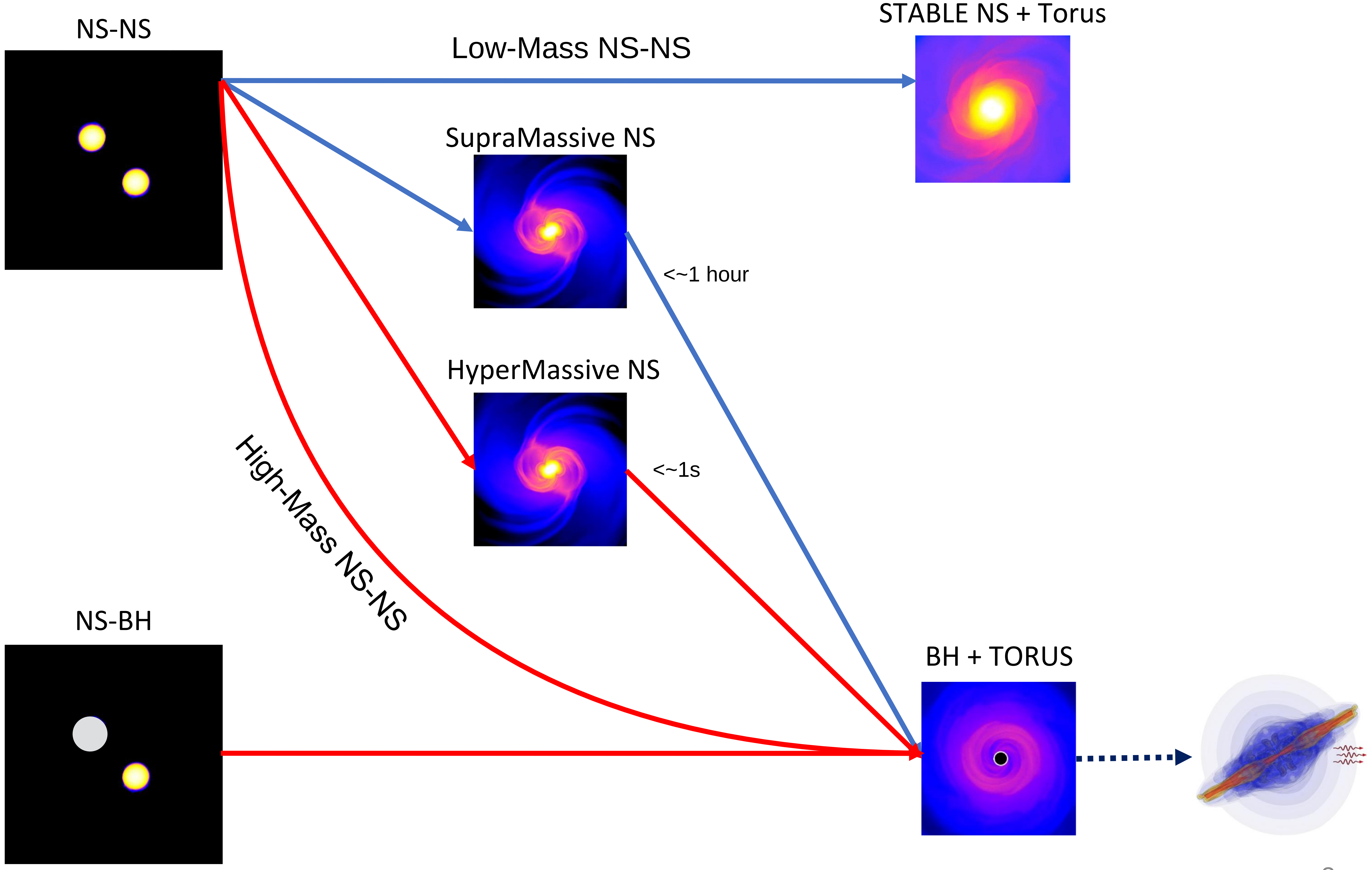
Requires two stars with masses between ~8 and ~20 M$_\odot$

System needs to survive both SN explosions and common envelope phase

Similar evolution (different progenitors) will lead to NS-BH and BH-BH binaries

Belczynski et al 2018
https://ui.adsabs.harvard.edu/abs/2018A%26A...615A..91B/abstract

Neutron Star Binary Mergers



Computing GWs in Simulations

Spin-Weighted Spherical Harmonics

- GWs are usually decomposed in their different "modes"

$$h(t,\boldsymbol{x}) \equiv h_+ - ih_\times = \sum_{l=2}^{\infty} \sum_{m=-l}^{l} h_{lm}(t,r)\ {}_{(-2)}Y_{lm}(\theta,\phi)$$

- Where ${}_sY_{lm}(\theta,\phi)$ are the spin-weighted spherical harmonics (s=0 corresponds to the "standard" spherical harmonics)
- h_{20} is for example the dominant mode for an axisymmetric collapse
- h_{22} is the dominant one for a typical inspiral signal

Moncrief Formalism

- Gauge invariant wavefunctions $Q^{\times}_{lm}$ and Q^{+}_{lm} are computed on spherical surfaces (see "thorn" Extract in the Einstein Toolkit, https://ui.adsabs.harvard.edu/abs/2012CQGra..29k5001L)
- It assumes the background metric to be Schwarzschild
- One can then compute the GW signal:

$$h = h_+ - i h_\times = \frac{1}{\sqrt{2}r} \sum_{l=2}^{\infty} \sum_{m=-l}^{l} \left(Q^{+}_{lm} - i \int_{-\infty}^{t} Q^{\times}_{lm}(t')\, dt' \right) {}_{(-2)}Y_{lm}(\theta, \phi)$$

Weyl Scalar

- A more accurate and general method uses the Weyl scalar Ψ_4 (see code WeylScal4 in the Einstein Toolkit):

$$\Psi_4 = R_{ijkl} n^i \bar{m}^j n^k \bar{m}^l + 2R_{0jkl}\left(n^0 \bar{m}^j n^k \bar{m}^l - \bar{m}^0 n^j n^k \bar{m}^l\right)$$

$$+R_{0j0l}(n^0 \bar{m}^j n^0 \bar{m}^l + \bar{m}^0 n^j \bar{m}^0 n^l - 2n^0 \bar{m}^j \bar{m}^0 n^l)$$

where $l^\mu \equiv \frac{1}{\sqrt{2}}(u^\mu + \tilde{r}^\mu), n^\mu \equiv \frac{1}{\sqrt{2}}(u^\mu - \tilde{r}^\mu), m^\mu \equiv \frac{1}{\sqrt{2}}(\tilde{\theta}^\mu + i\tilde{\phi}^\mu)$ and u^μ is a unit time-like vector.

Weyl Scalar

- One can then compute the GW signal:

$$h = h_+ - ih_\times = -\int_{-\infty}^{t} dt' \int_{-\infty}^{t'} \Psi_4 \, dt''$$

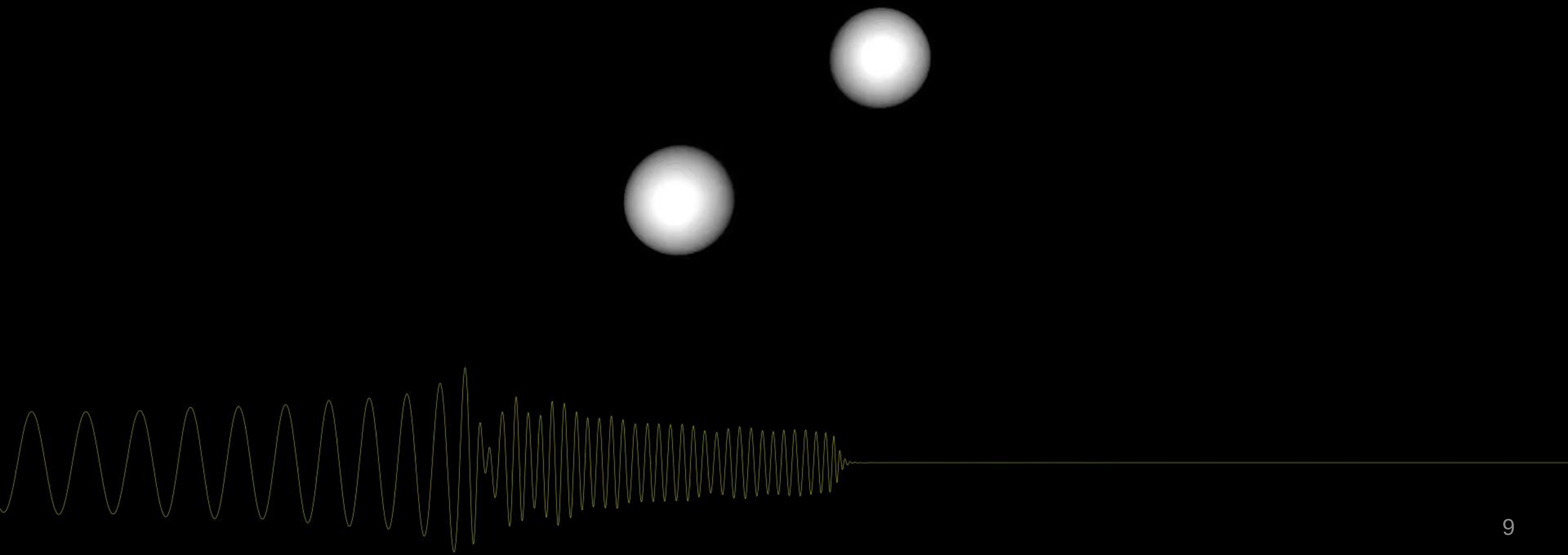
- This integration is usually done in Fourier space for more accurate results

  (see Reisswig & Pollney 2011, https://ui.adsabs.harvard.edu/abs/2011CQGra..28s5015R)

- Kuibit library already implements the necessary tools

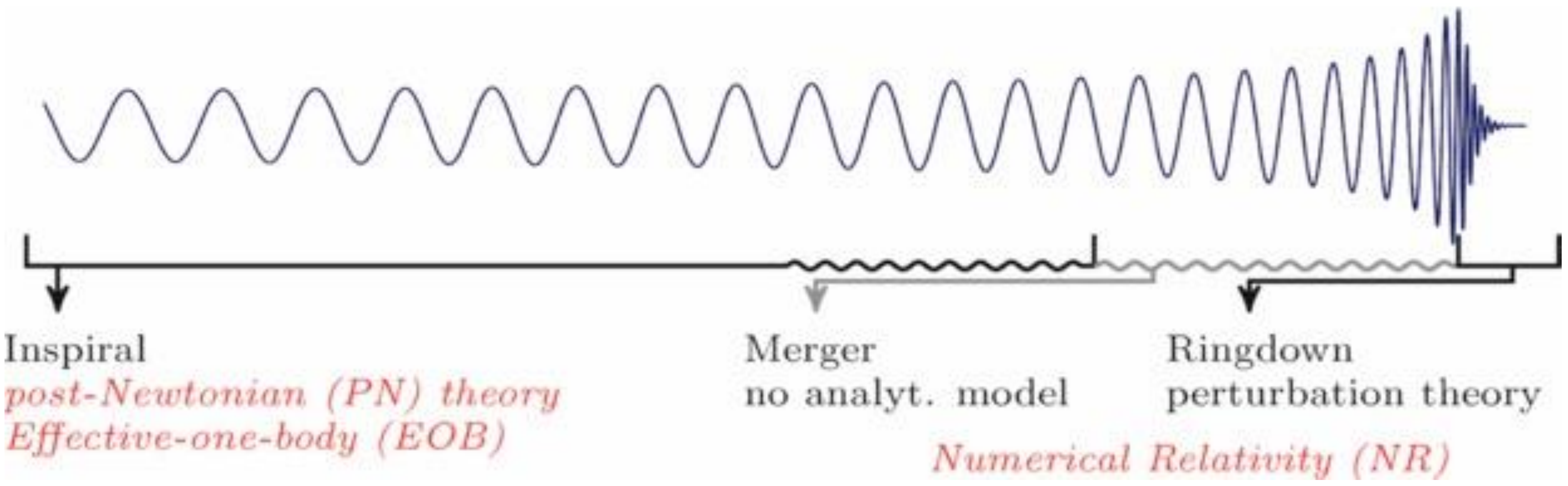
Kawamura et al 2016
Movie by W. Kastaun

$t = 0.0$ ms



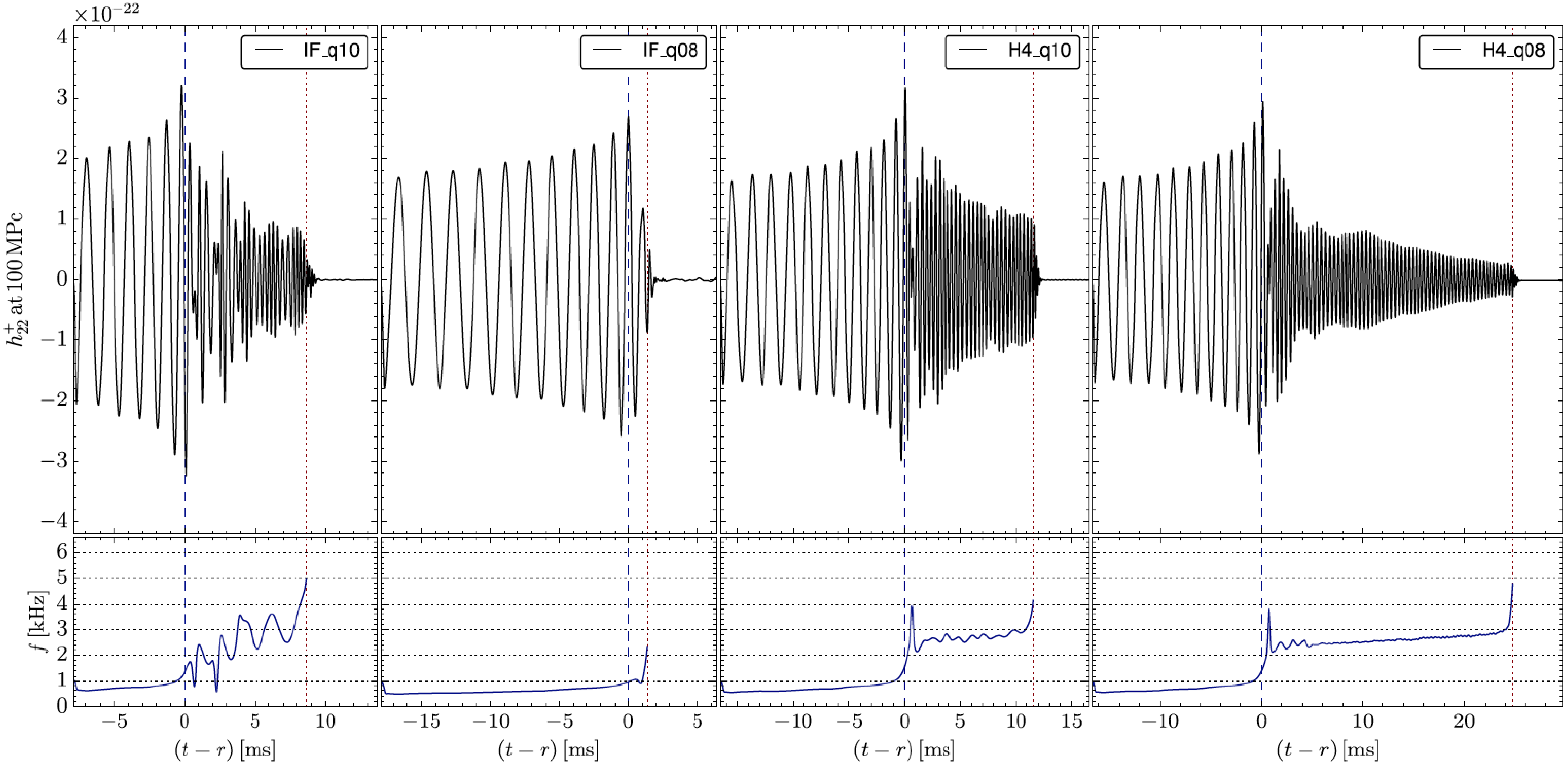
BH-BH merger signal

Figure 1 from Frank Ohme 2012 Class. Quantum Grav. 29 124002



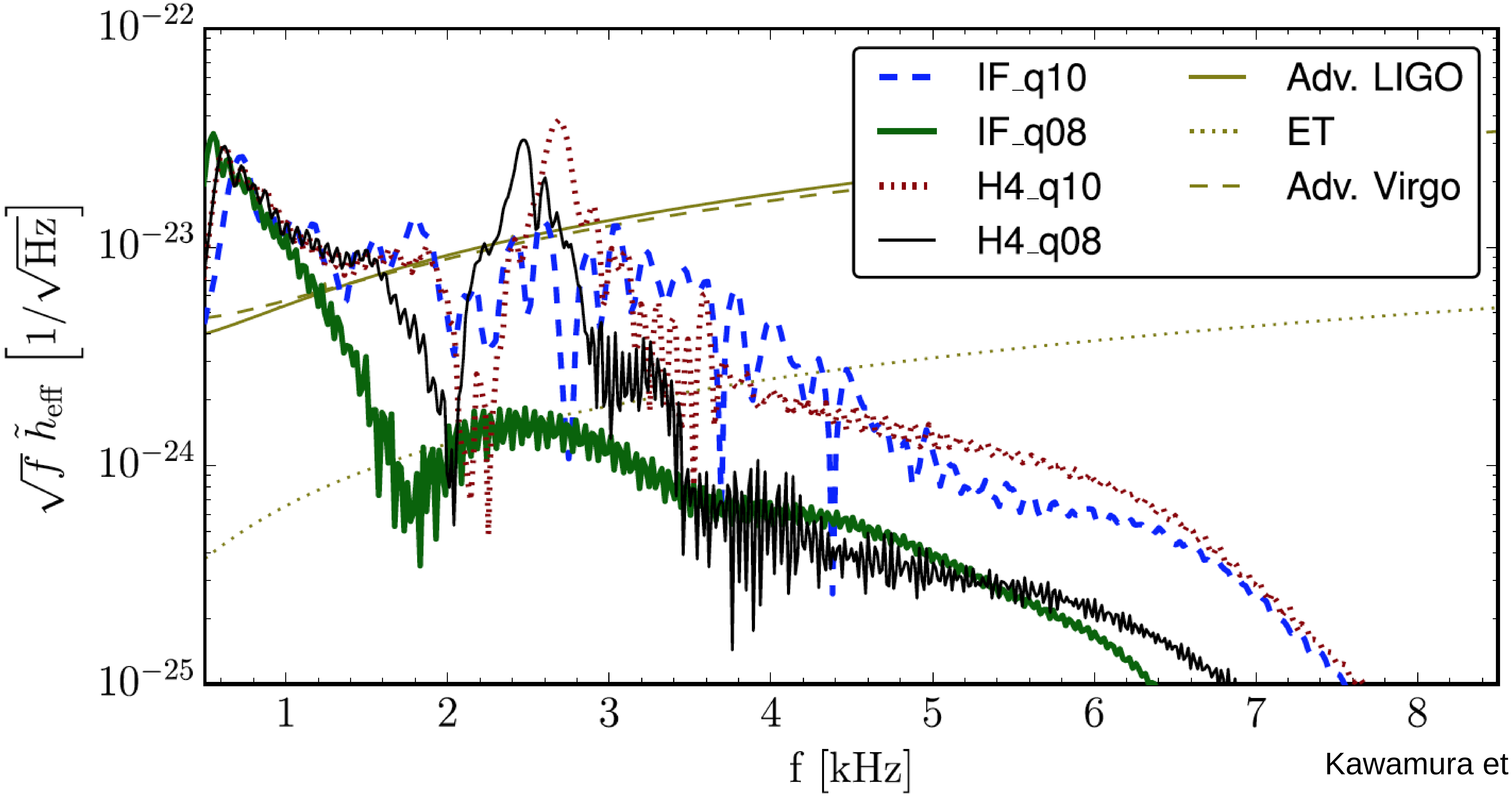
in NS-NS we have a new phase between merger and ringdown

GWs from Binary Neutron Stars



Kawamura et al 2016

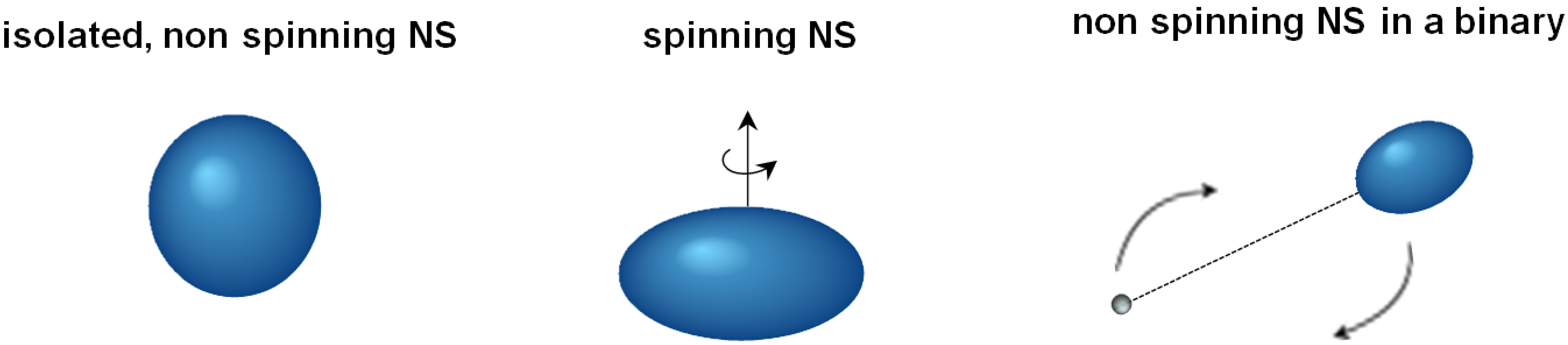
GWs: detectability



Kawamura et al 2016

Matter Effects on BNS GW signals

Tidal Deformability



companion

https://compstar.uni-frankfurt.de/outreach/short-articles/i-love-q-universality-in-properties-of-neutron-stars/

For a recent review, see Dietrich, Hinderer & Samajdar 2020
http://arxiv.org/abs/2004.02527

Newtonian Theory:

- external quadrupolar tidal field $\mathcal{E}_{ij} = \frac{\partial^2 \Phi_{ext}}{\partial x^i \partial x^j}$
- induced quadrupole moment $Q_{ij} = \int \delta\rho(\boldsymbol{x}) \left(x_i x_j - \frac{1}{3} r^2 \delta_{ij} \right) d^3x$
- the **dimensionless Love number** k_2 is then introduced by $Q_{ij} = -\frac{2}{3G} k_2 R^5 \mathcal{E}_{ij}$
- in general, it needs to be computed numerically
- important to note that for a rigid body $k_2 = 0$

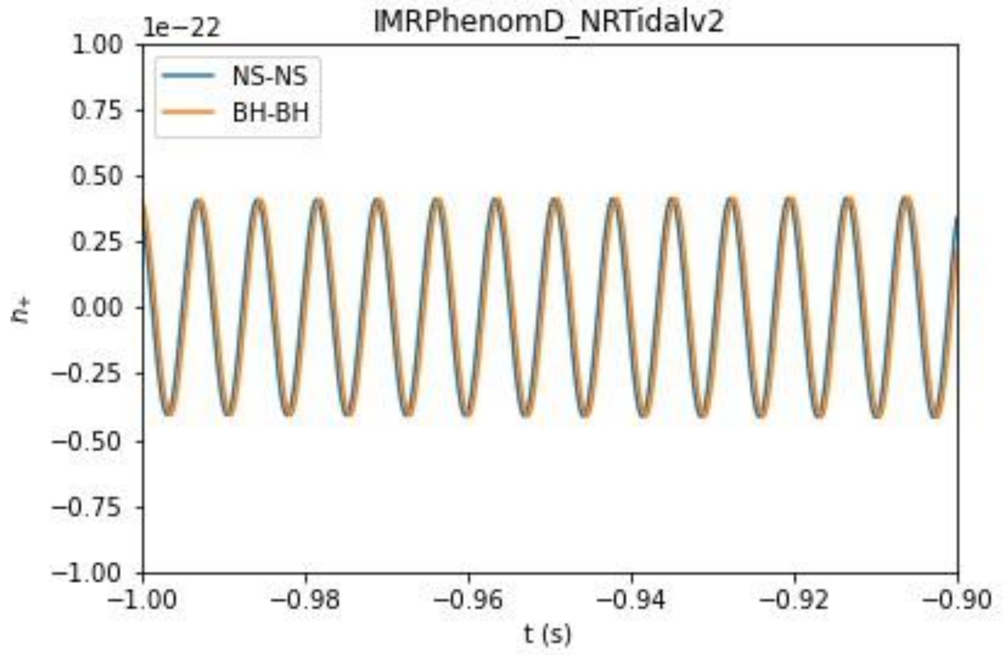
General Relativity:

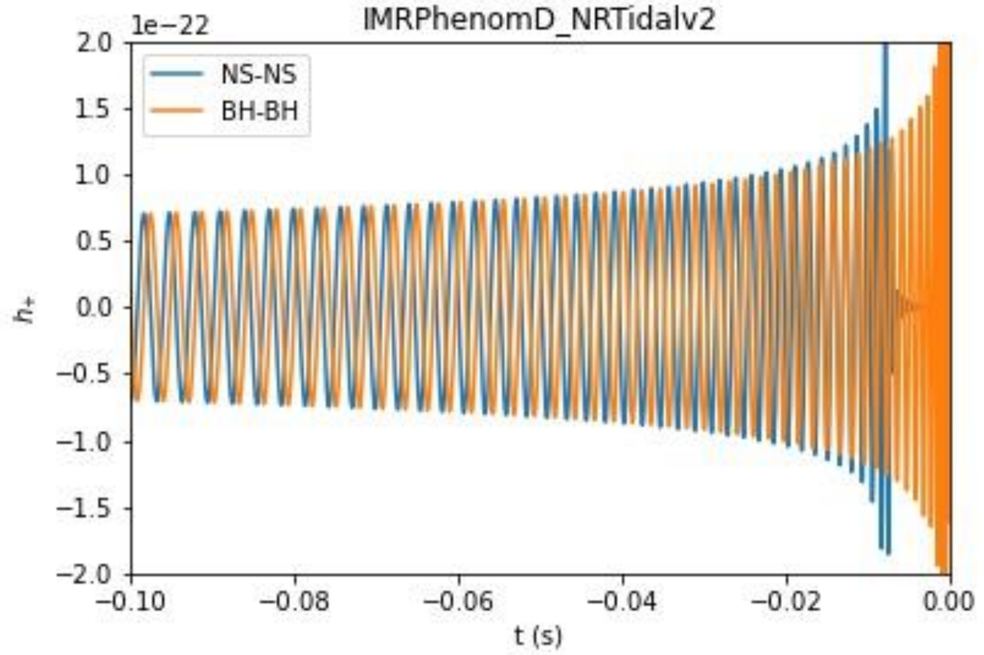
- An important quantity that can be measured is the **dimensionless tidal deformability**:

$$\Lambda = \frac{2}{3} k_2 \left[\left(\frac{c^2}{G} \right) \left(\frac{R}{m} \right) \right]^5$$

- In BNS systems one can more easily extract a combination of the tidal deformabilities of the two NSs:

$$\widetilde{\Lambda} = \frac{16}{13} \frac{(m_1 + 12m_2) m_1^4 \Lambda_1 + (m_2 + 12m_1) m_2^4 \Lambda_2}{(m_1 + m_2)^5}$$


IMRPhenomD_NRTidalv2
1e−22
NS-NS
BH-BH
h_+
t (s)


IMRPhenomD_NRTidalv2
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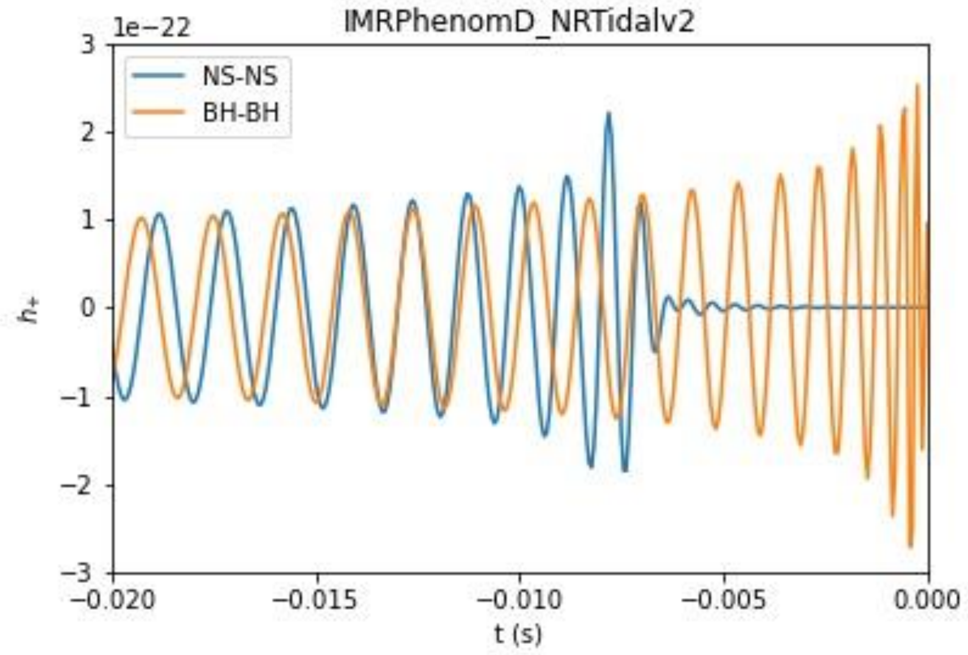
h_+
t (s)


IMRPhenomD_NRTidalv2
1e−22
NS-NS
BH-BH
h_+
t (s)

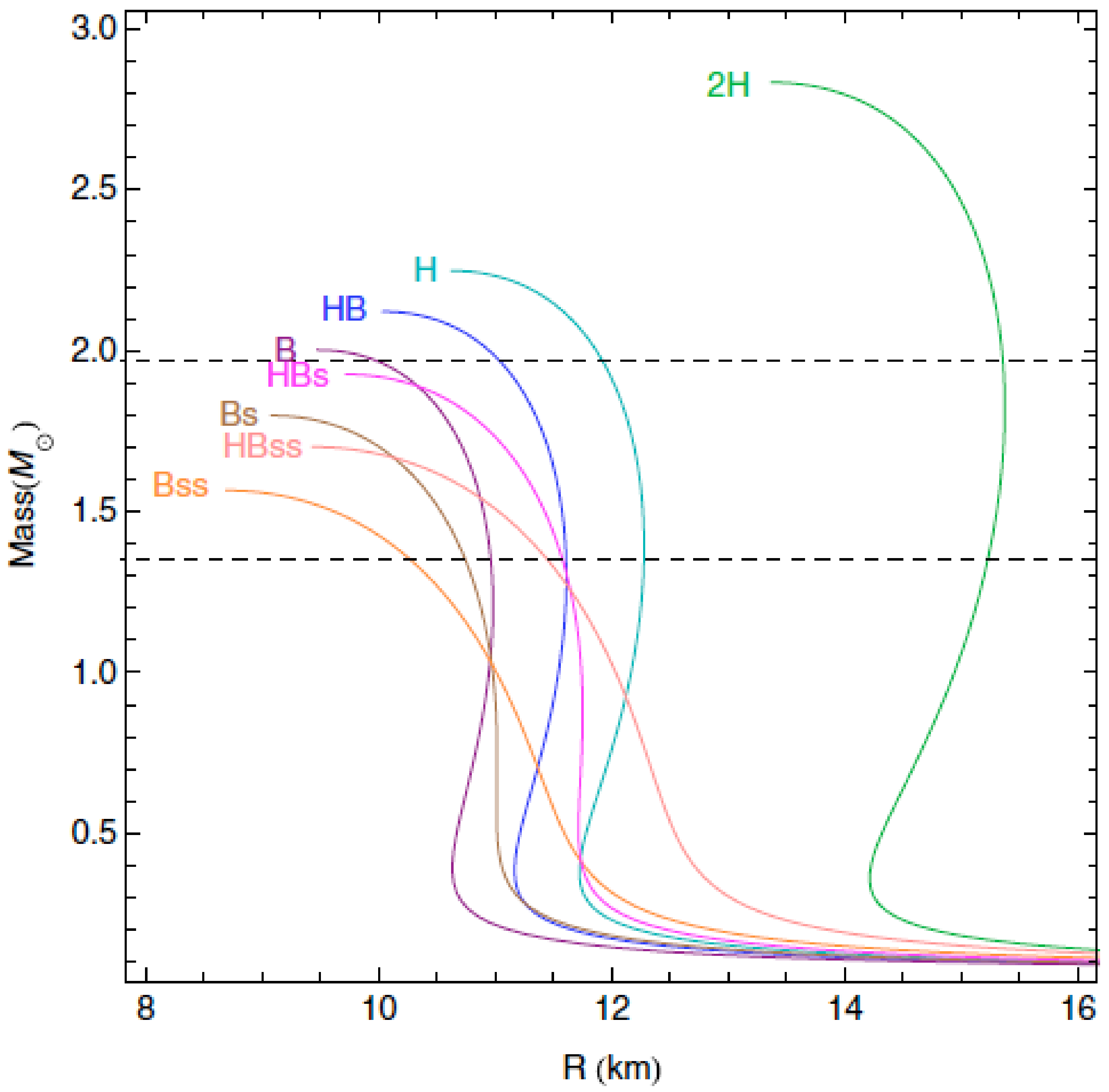
MATTER EFFECTS ON BNS GWS

(Read et al 2013, PRD 88, 044042)

We used the Whisky and SACRA codes to perform the first multi-code study of EOS effects on merger waveforms for equal-mass systems

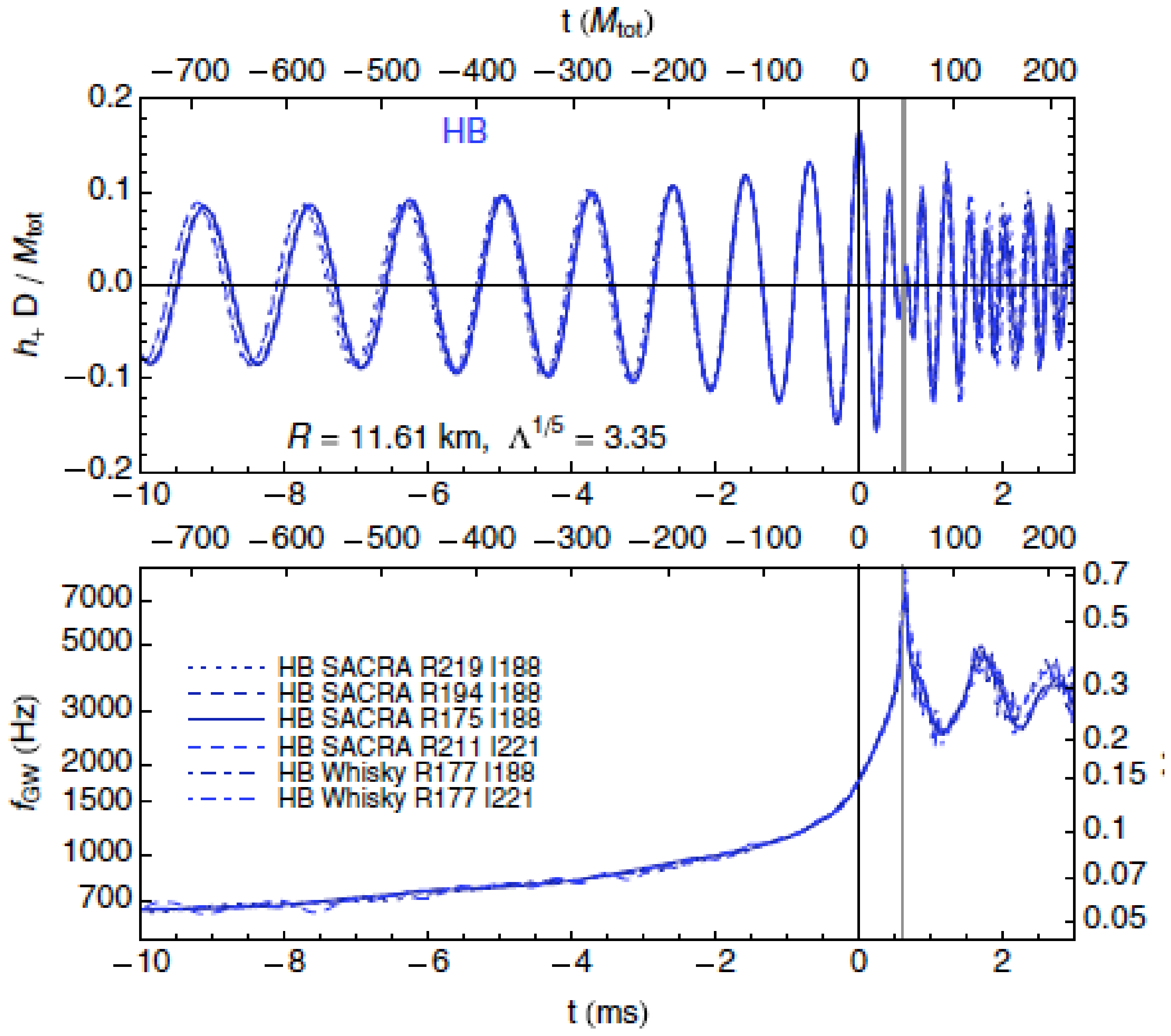
Used an extended set of piecewise polytropic EOSs

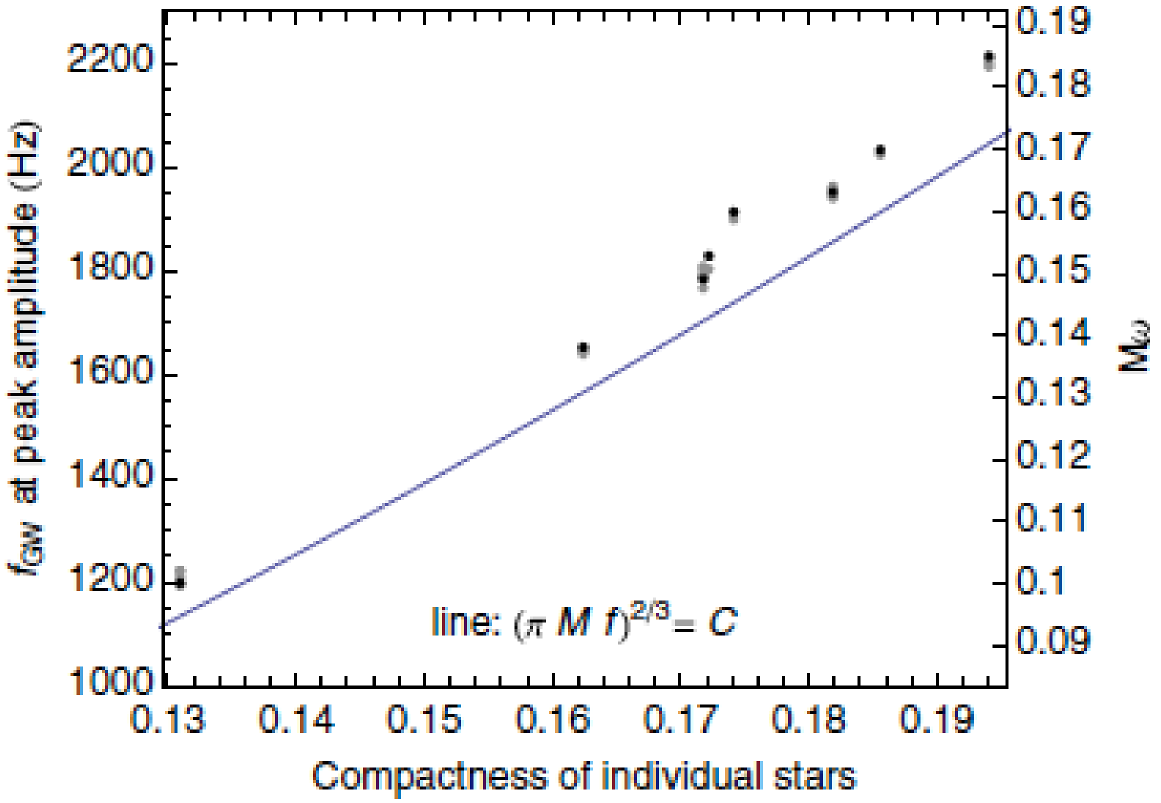
Estimated numerical errors by comparing between the codes and using different resolutions.



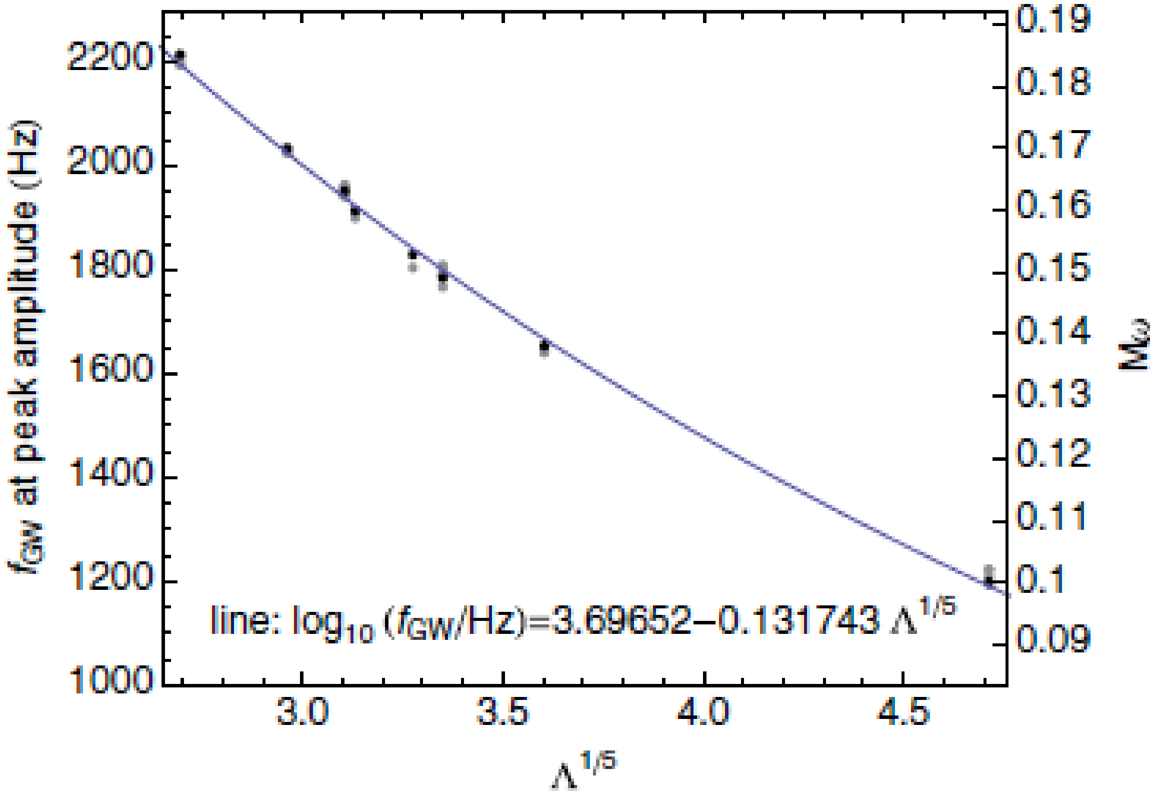
MATTER EFFECTS ON BNS GWS

(Read et al 2013, PRD 88, 044042)

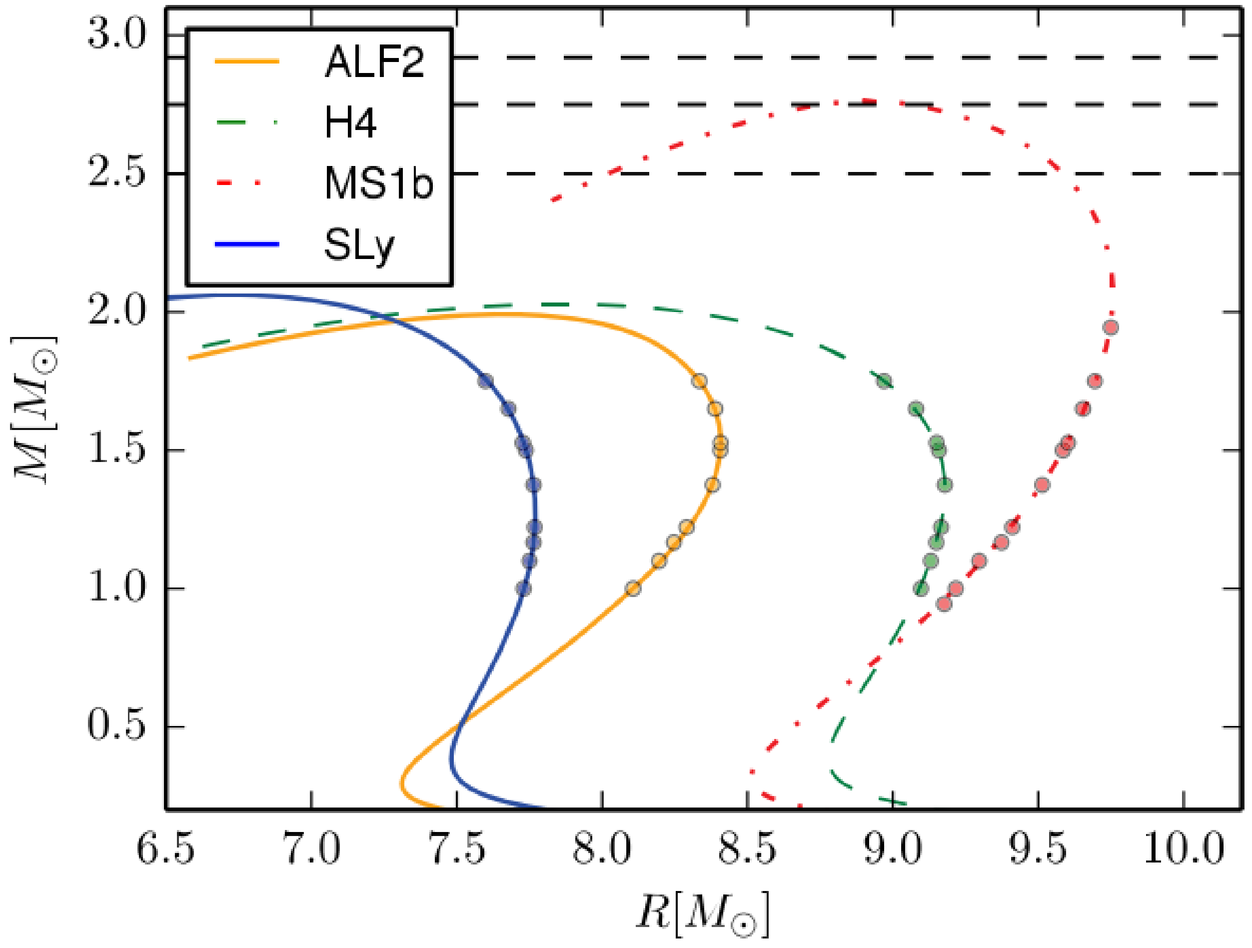




GW frequency at merger is well correlated with tidal deformability and NS compactness (see also Bernuzzi et al 2014).

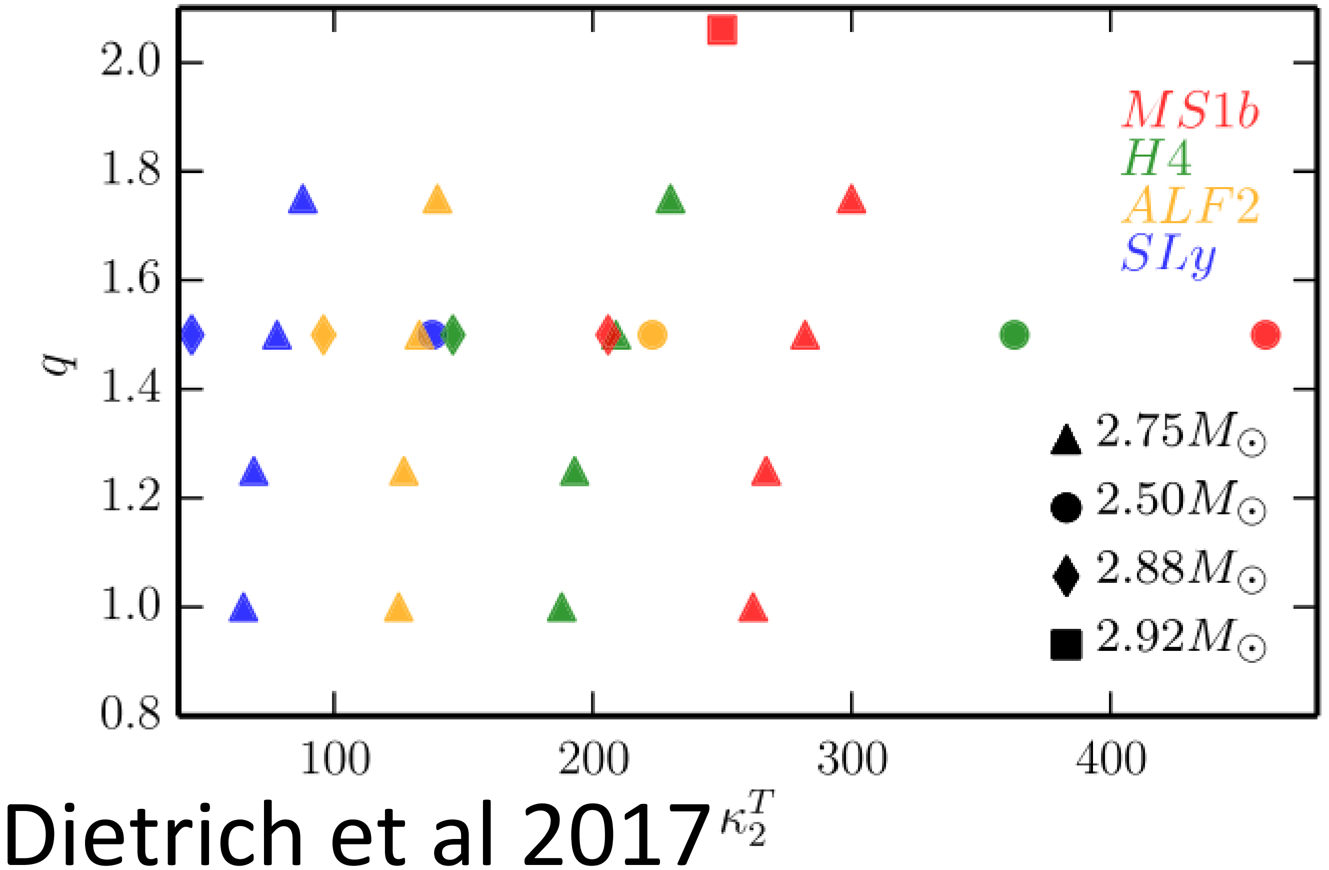


GW: EFFECT of EOS and MASS RATIO



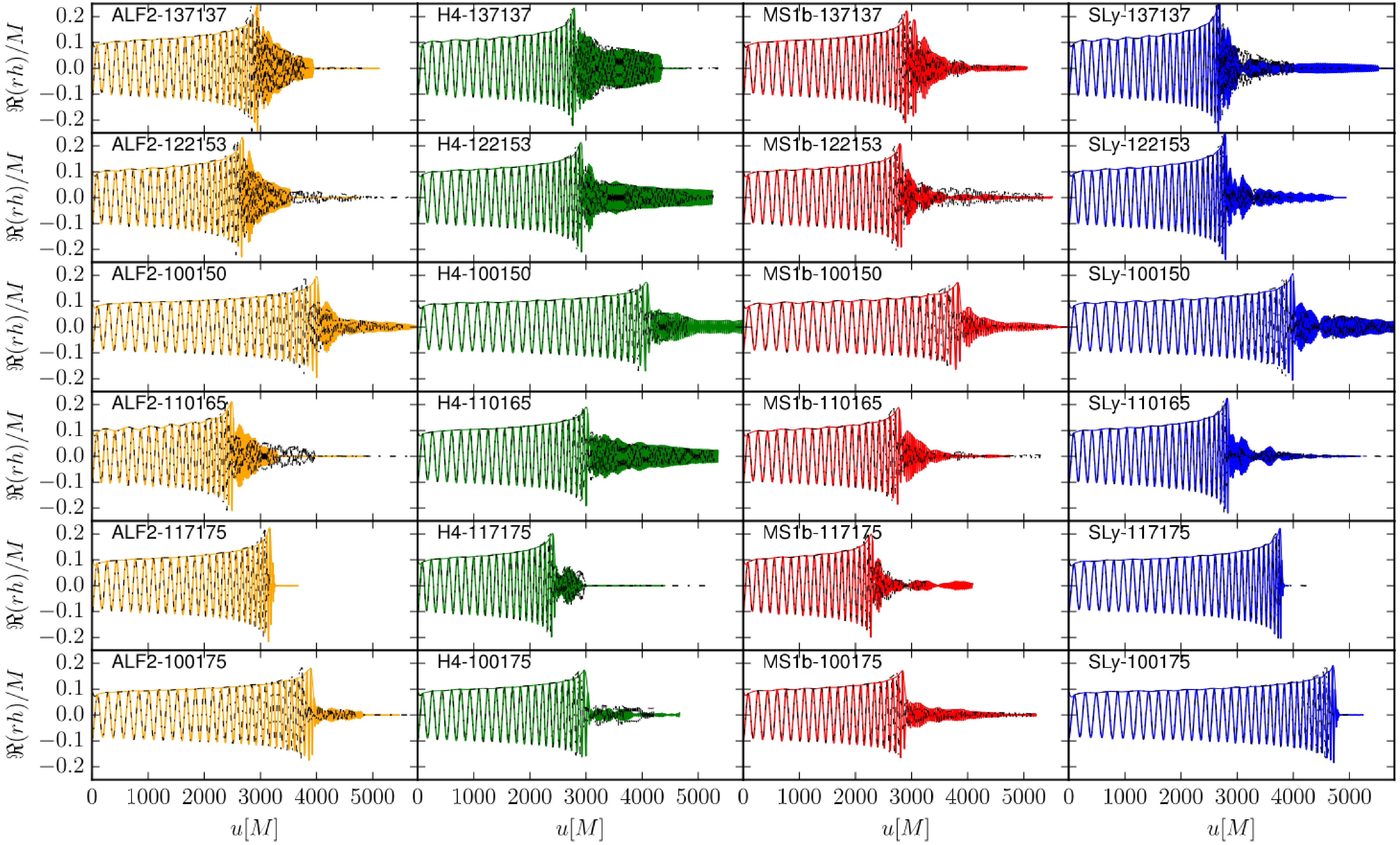
Considered 4 different piece-wise polytropic EOSs

Built models with mass ratios from 1 to ~0.5.



Dietrich et al 2017

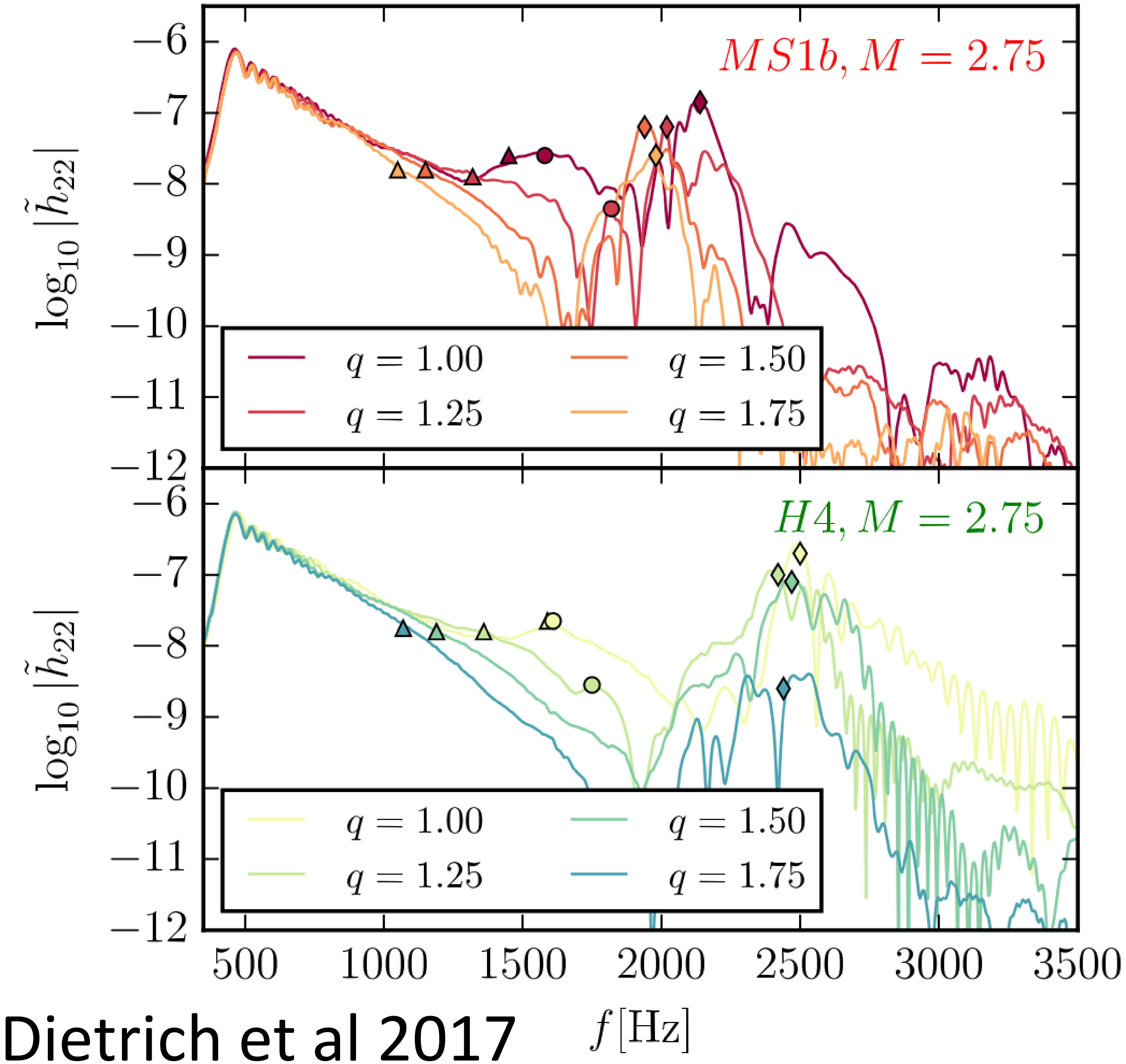
GW: EFFECT of EOS and MASS RATIO



Dietrich et al 2017

$$u \equiv t - r_* = t - \left[r_{extr} + 2M \ln\left(\frac{r_{extr}}{2M} - 1\right)\right]$$

GW: EFFECT of EOS and MASS RATIO



Dietrich et al 2017

Merger frequency is largest for equal masses and decreases for decreasing q.

For MS1b it decreases from 1.45 kHz to 0.9 kHz when q goes from 1 to 1/1.75 (0.57).

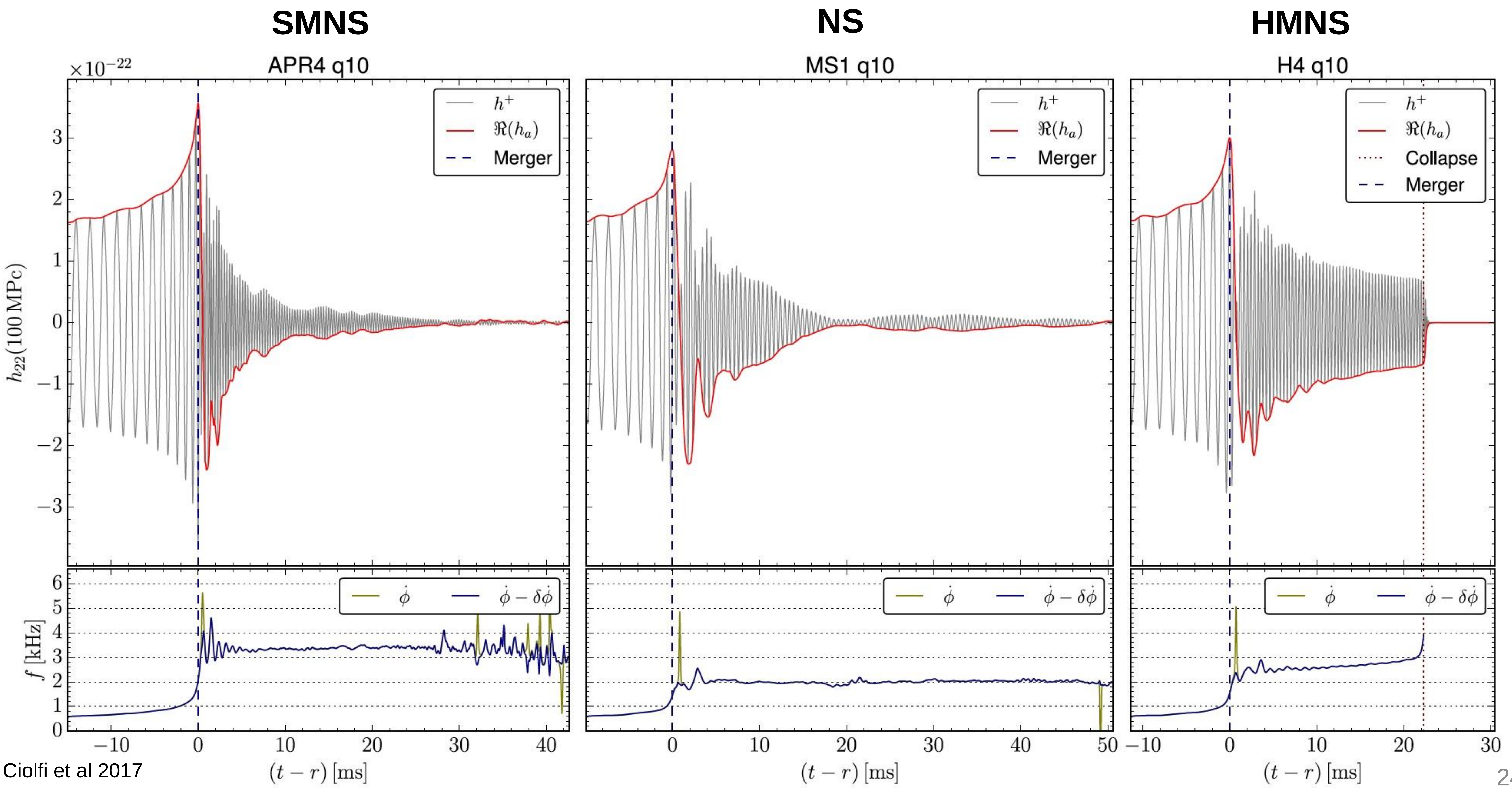
GWs in the INSPIRAL (Recap)

- Dominant Parameters:
  - Masses and Mass Ratios
  - Equation of State (Tidal Deformability)

- Minor corrections (maybe):
  - Spin (only relevant if $\chi > 0.05$). Fastest spinning NS observed in an NS-NS system (PSR J0737-3039A) has $\chi \sim 0.02$ $(\mathrm{P} \sim 22.7\ ms)$.
  - Eccentricity. This is relevant only for BNS systems formed via dynamical capture in star clusters and globular clusters.

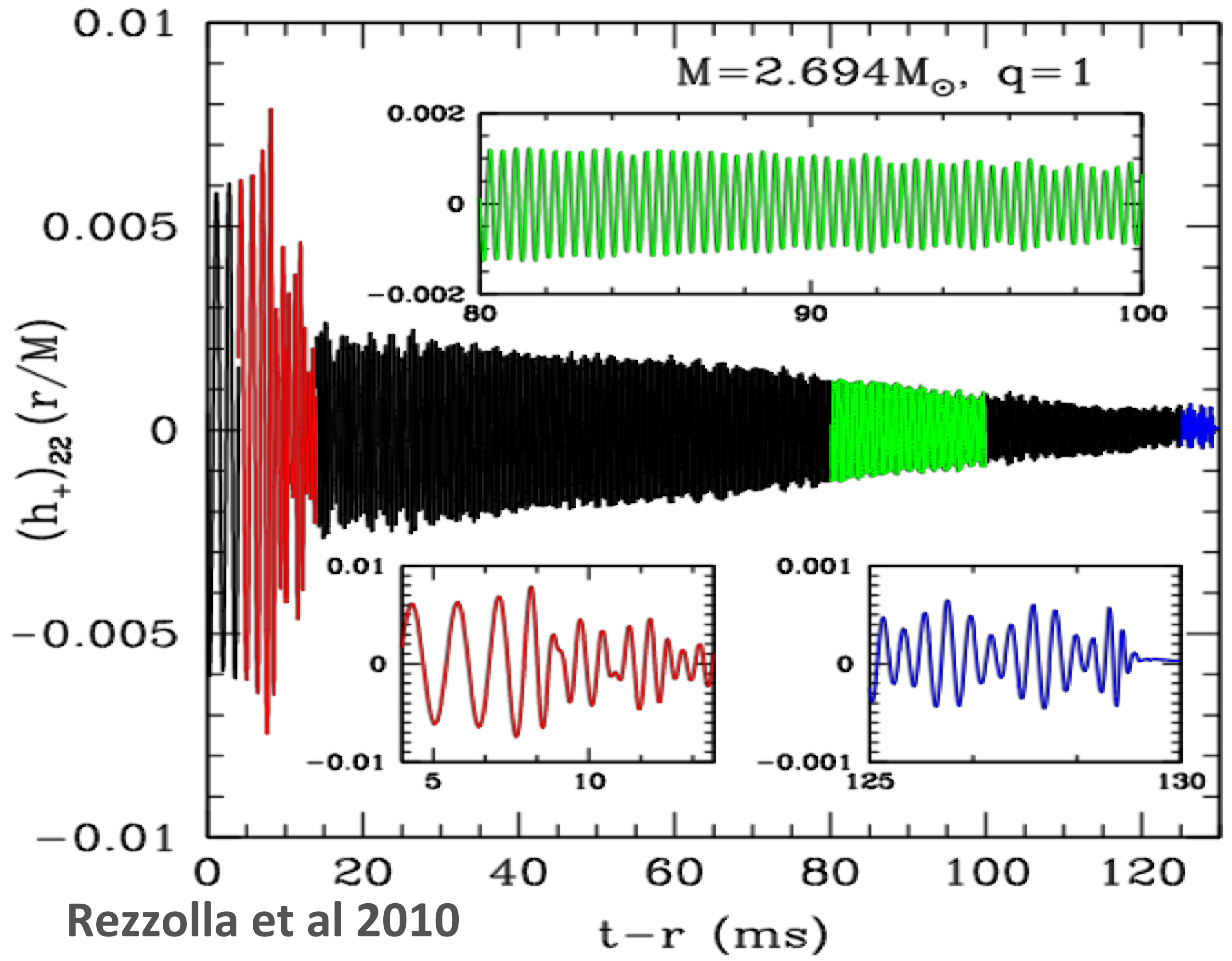
POST-MERGER GW SIGNAL

GW: EOS Effects on the Post-Merger Phase

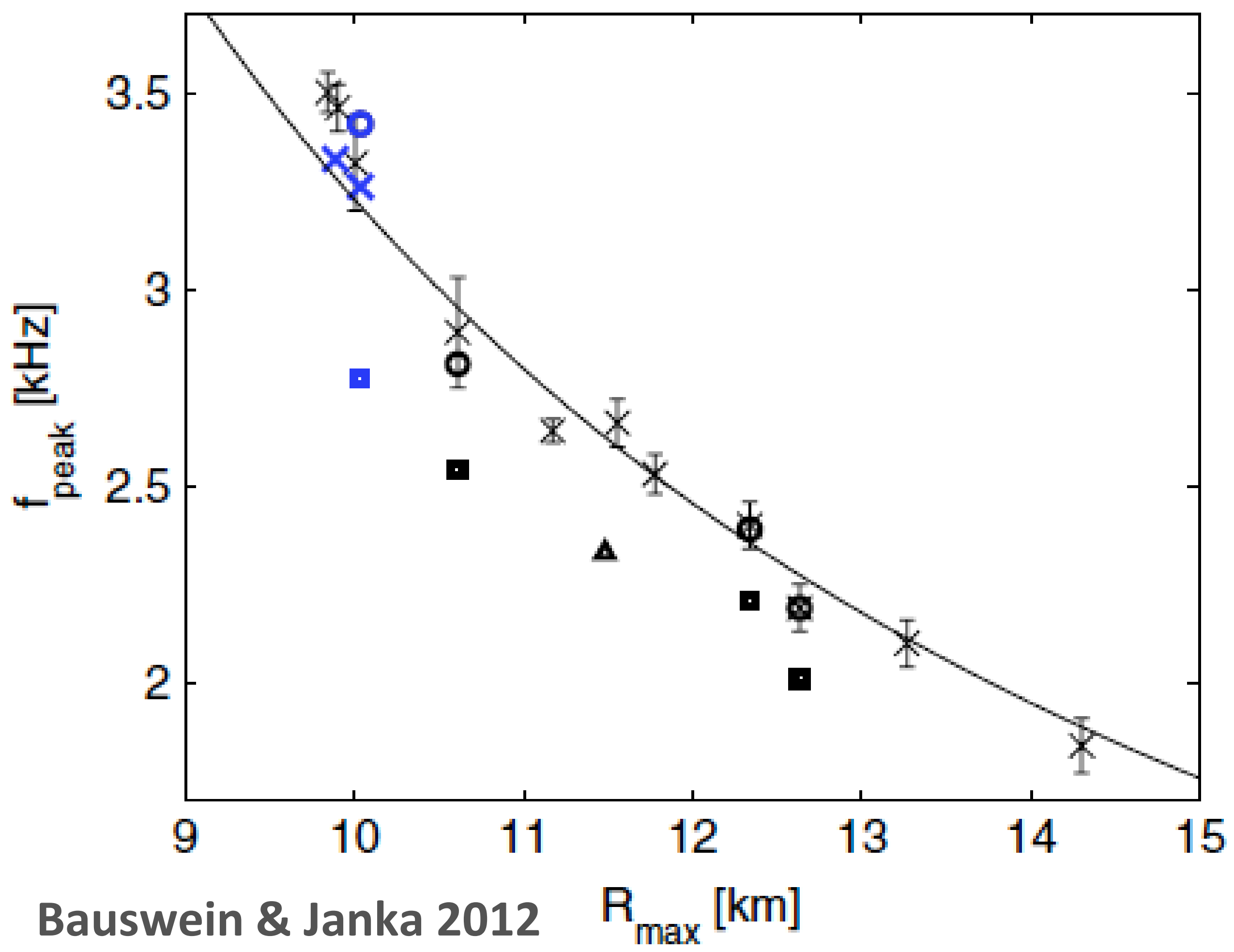


Ciolfi et al 2017

GW: EOS Effects on the Post-Merger Phase



Rezzolla et al 2010



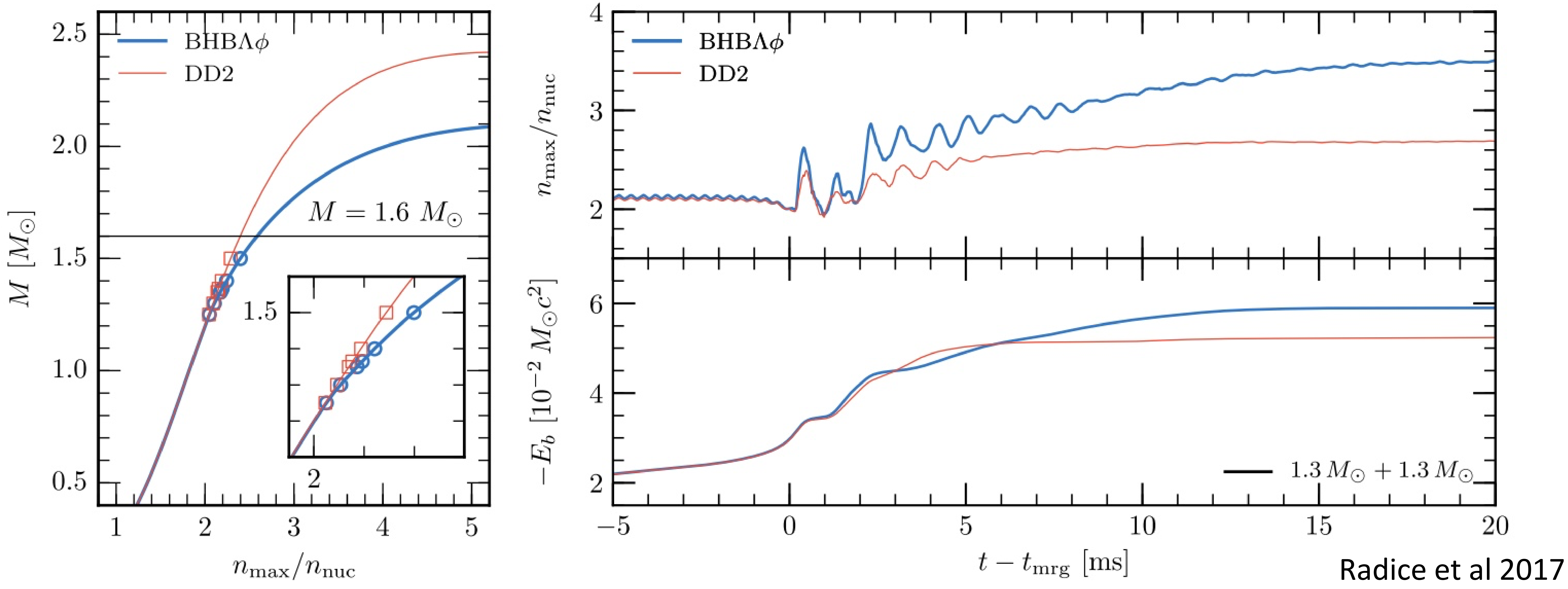
Bauswein & Janka 2012

For smaller NS masses, a long-lived NS may be formed

Bauswein & Janka 2012, Hotokezaka et al 2013: frequency peak in GWs emitted after merger can constrain EOS

High sensitivities at f>~1Khz required for post-merge signal!

GW: EOS Effects on the Post-Merger Phase

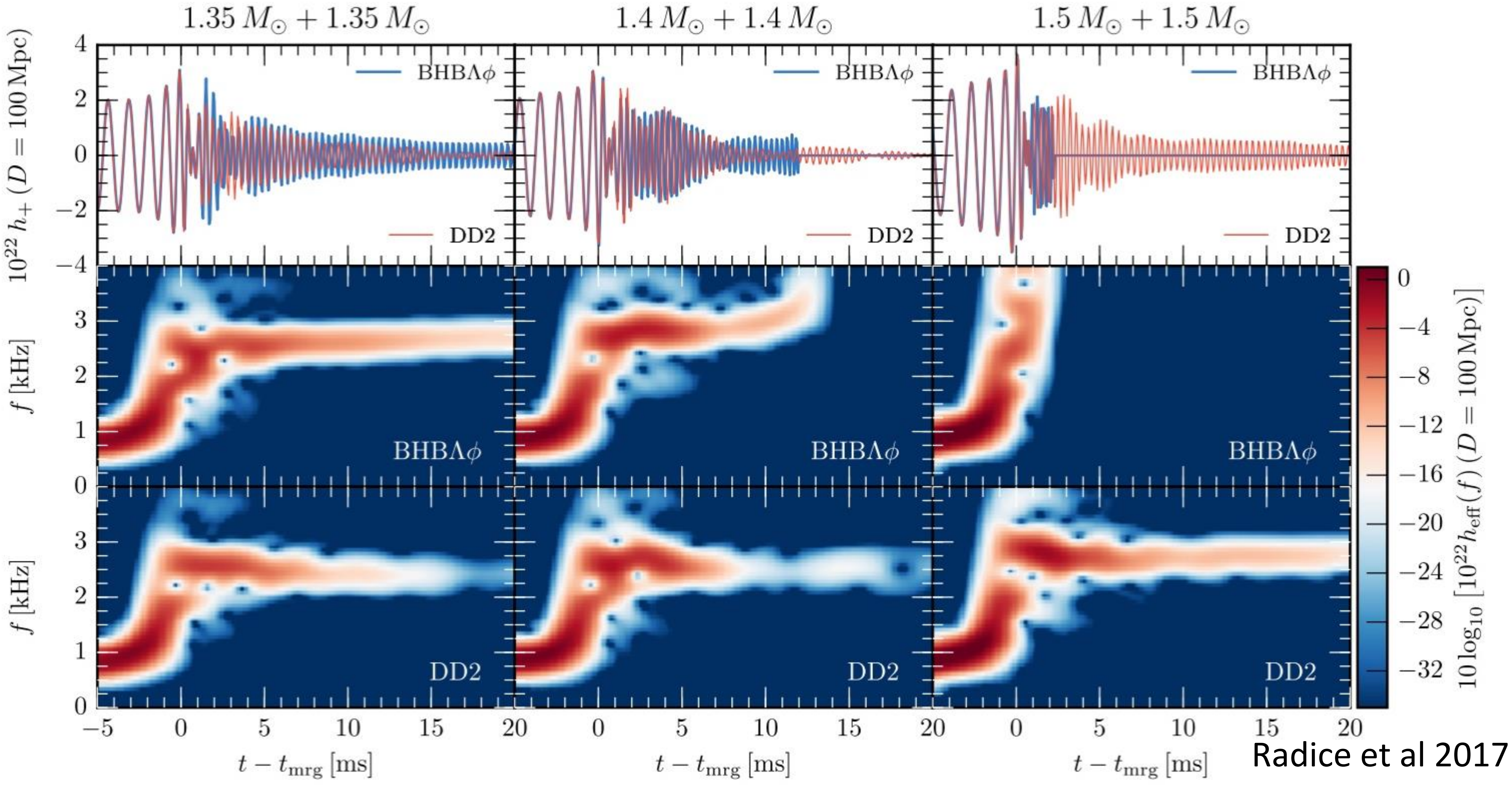


Radice et al 2017

EOS identical at “low” (inspiral) densities, but different at post-merger densities (phase transition effects).

GW: EOS Effects on the Post-Merger Phase

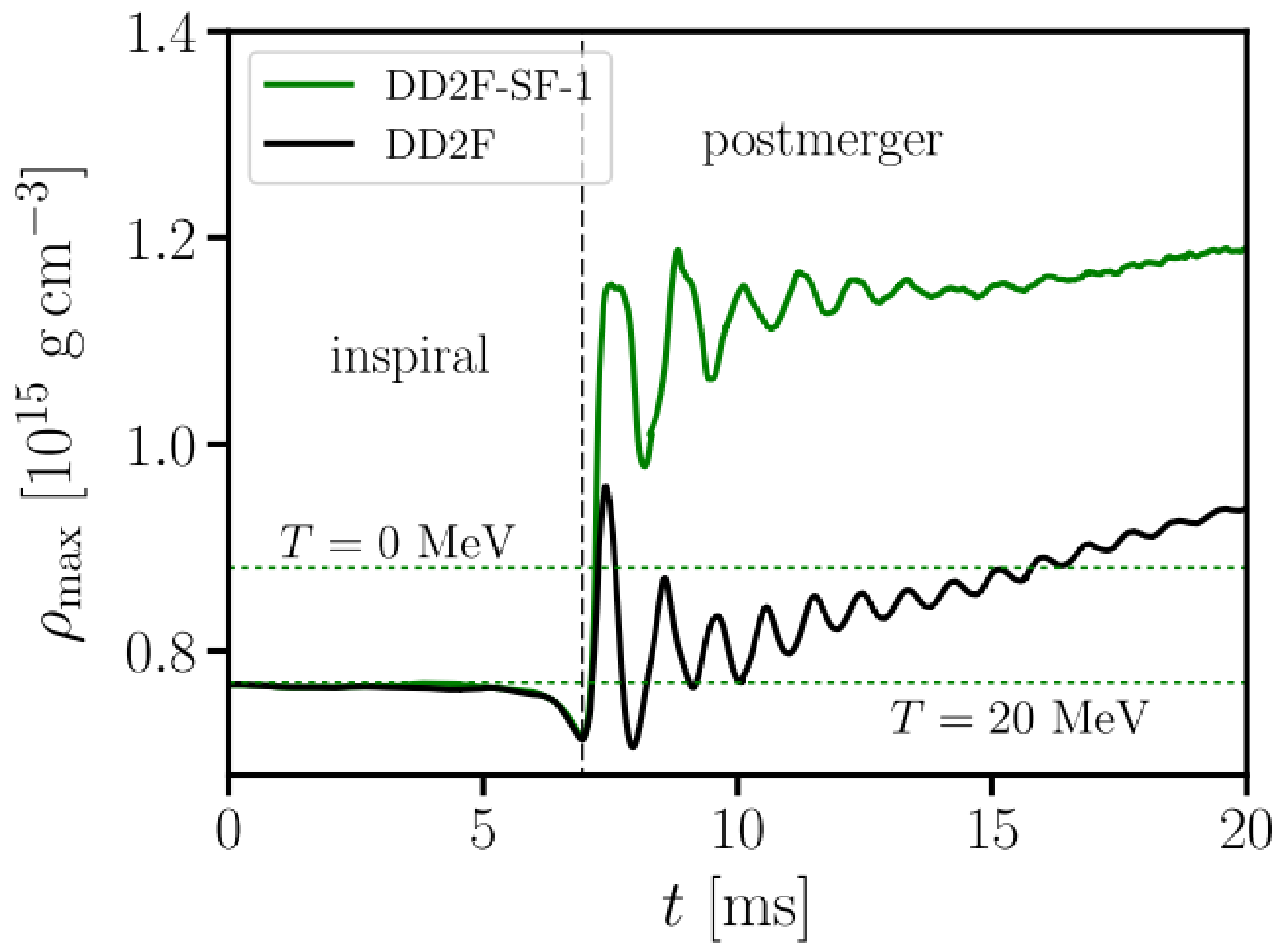
Same post-merger frequencies. Difficult to distinguish between the two, unless collapse to BH is detected.

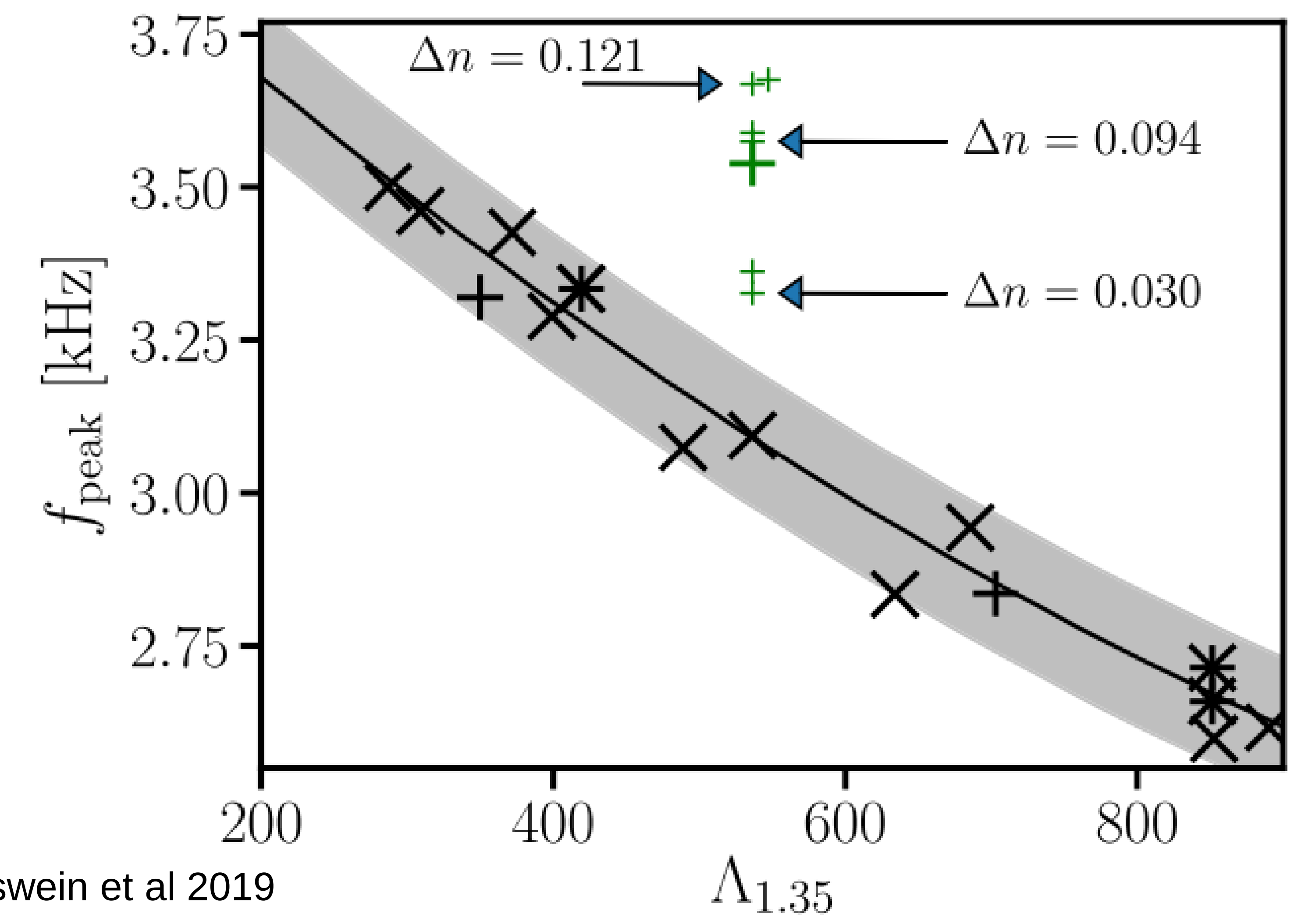


Radice et al 2017

Effects are more evident in post-merger luminosities and phase evolution (see also Bernuzzi et al 2016).

Phase transitions in the post-merger



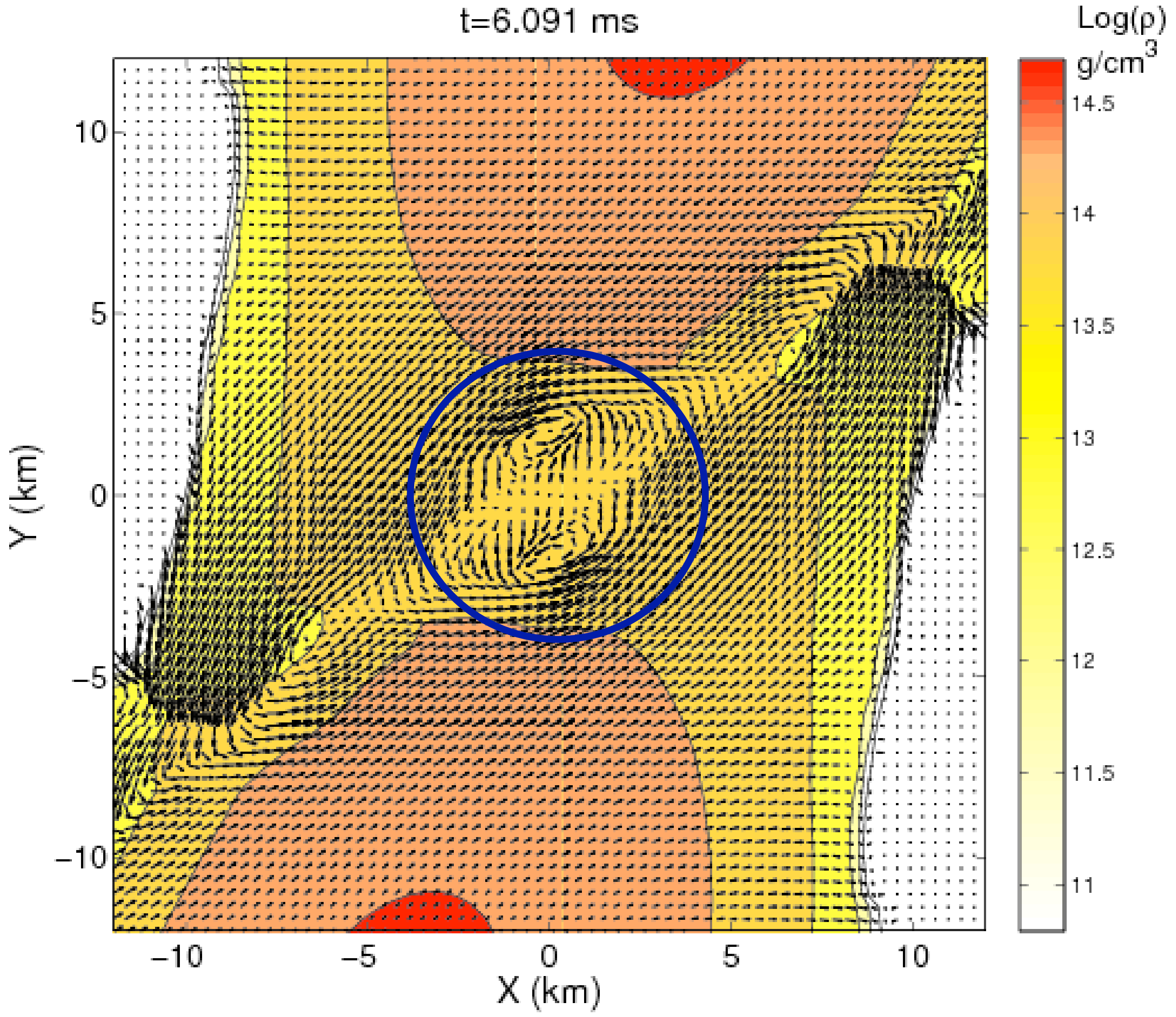


Bauswein et al 2019

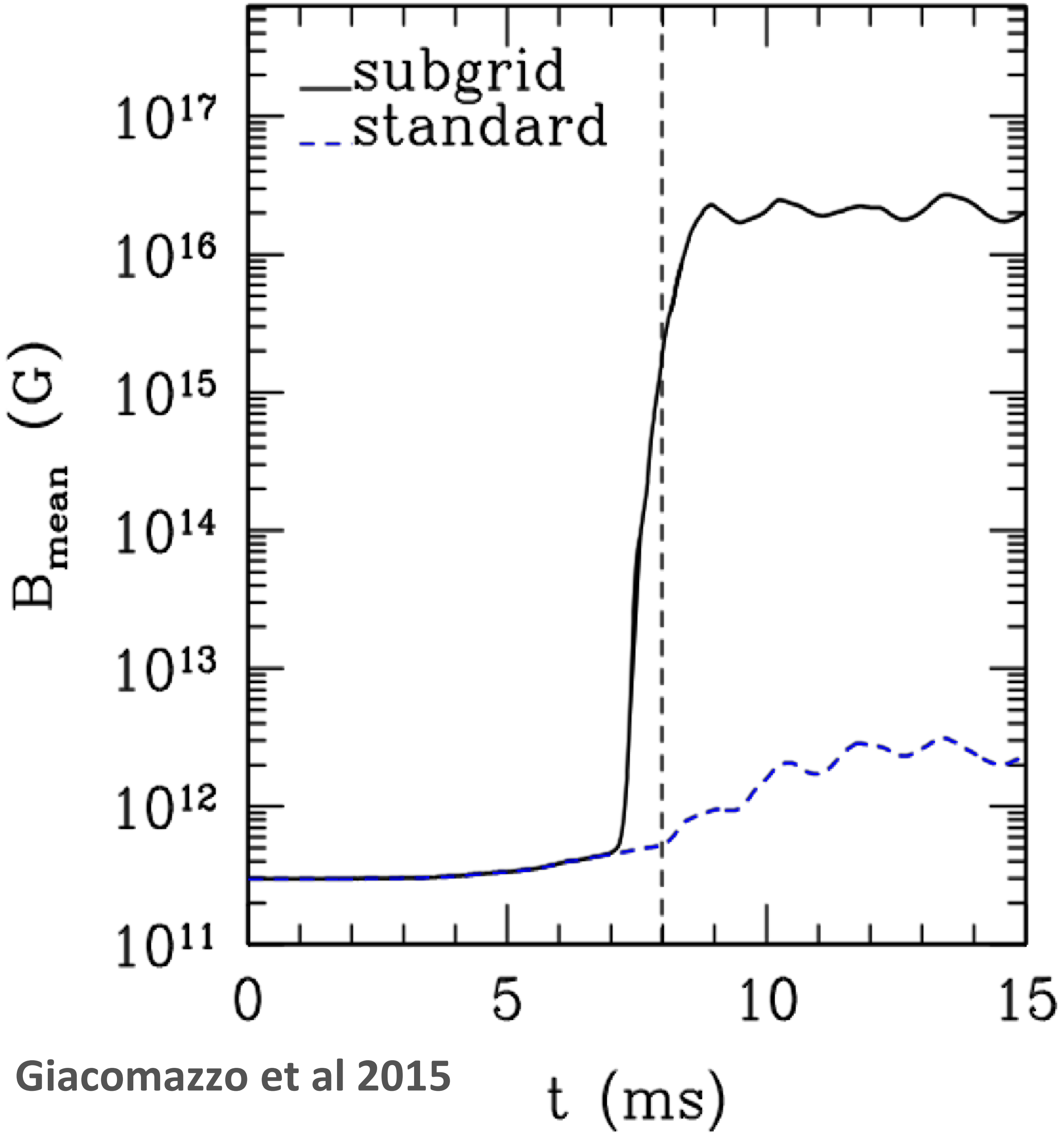
A phase transition to a deconfined-quark-matter core affects significantly the post-merger GW peak.

KH INSTABILITY AND MAGNETIC FIELDS

During the merger a shear interface forms and it develops a Kelvin-Helmholtz instability which produces a series of vortices.



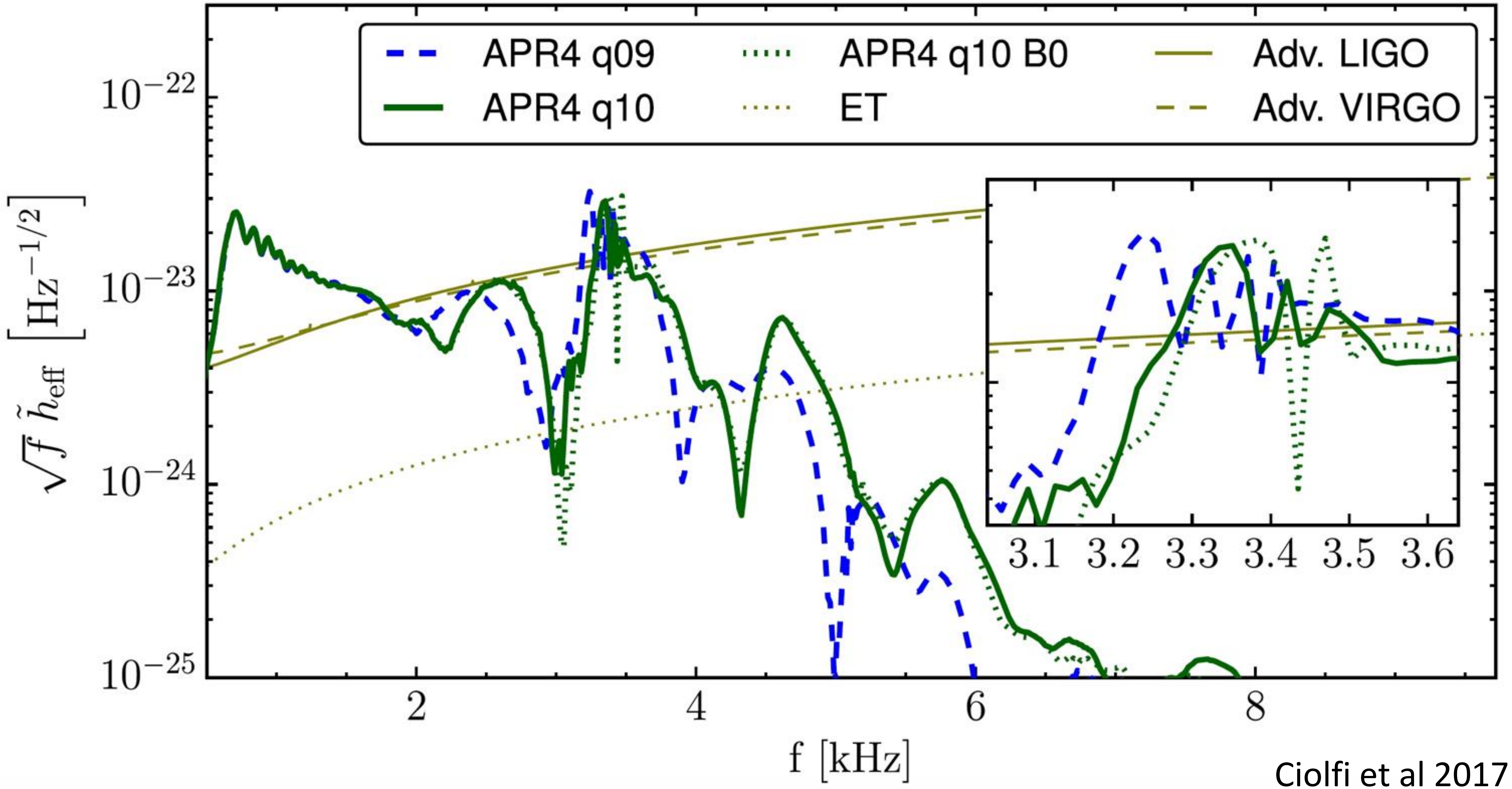
Baiotti et al 2008



Giacomazzo et al 2015

After merger the magnetic field may grow up to equipartition with the kinetic energy of the turbulent fluid ($B \sim 10^{16}$G).

Magnetic field effects on GWs



Ciolfi et al 2017

Evolved "low-mass" BNS with high magnetic fields. Difference in the post-merger peak of less than ~100 Hz.

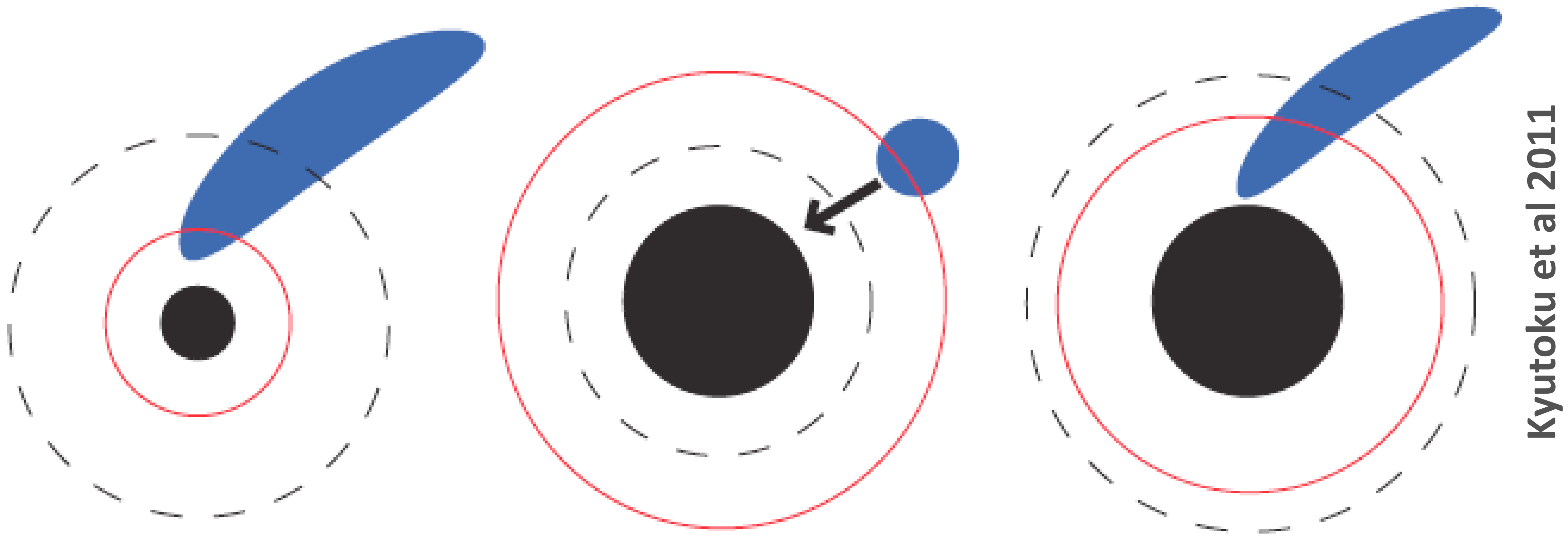
GWs in the POST-MERGER (Recap)

- Dominant Parameters:
  - Equation of State (high density, high temperature, possible phase transition)

- Minor corrections (maybe):
  - Magnetic field. Even if amplified up to $\sim 10^{16}$ G it does not seem to affect post-merger GW frequency. It may dump down the amplitude of the signal though making it more difficult to detect.

NS-BH MERGERS

BH-NS: Classification of GWs

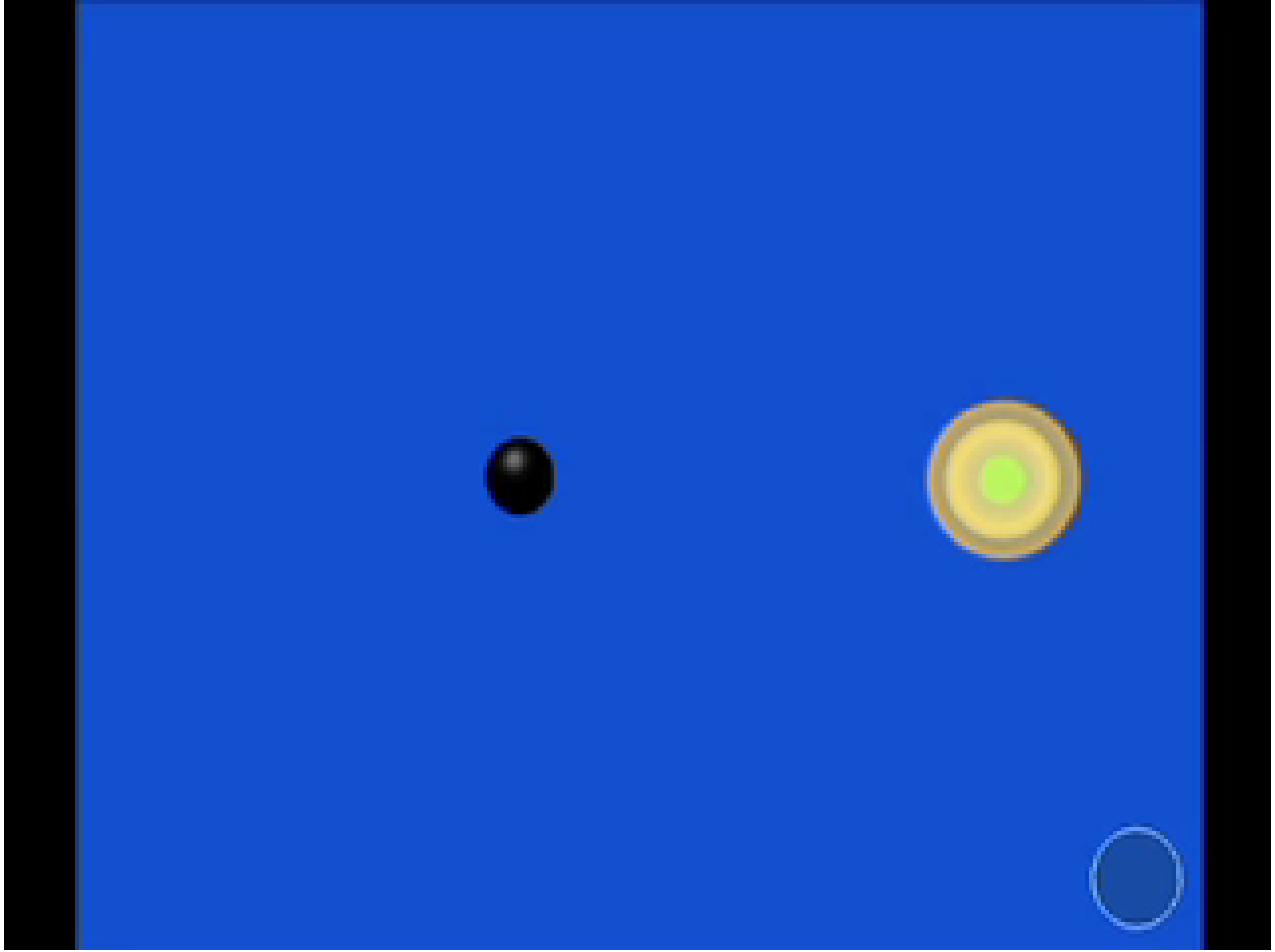


type I: NS disrupted outside ISCO. Only inspiral.

type II: no disruption. GWs very similar to BBH and composed by inspiral, merger and ringdown.

type III: mass transfer near ISCO. Both inspiral and merger are present in the GWs.

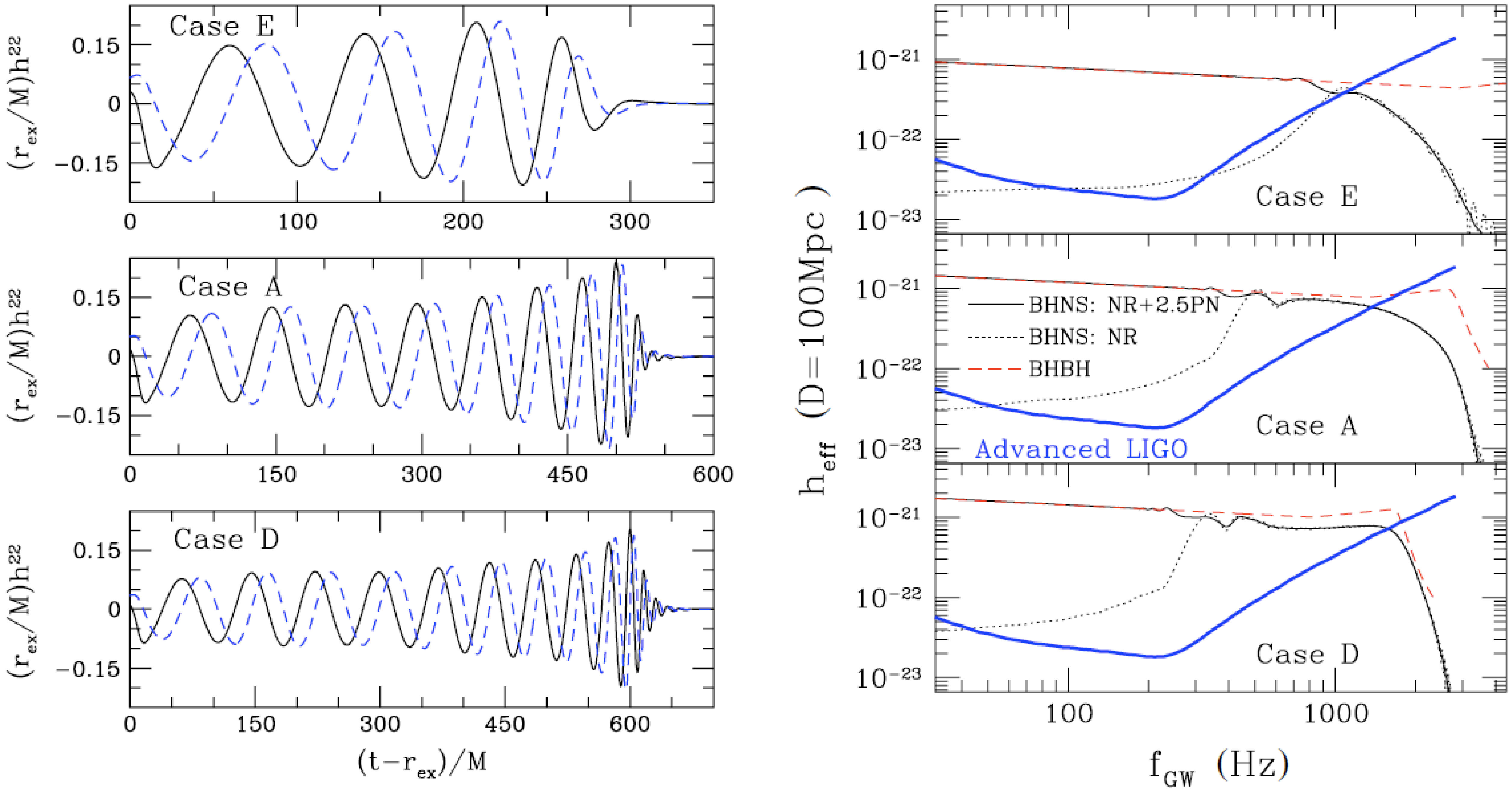
Classification depends on mass-ratio, BH spin, and NS compactness



http://research.physics.illinois.edu/cta/movies/cbm/bhns.html

GW FROM BH-NS (NO SPIN)

Etienne et al 2009



E: Q=1

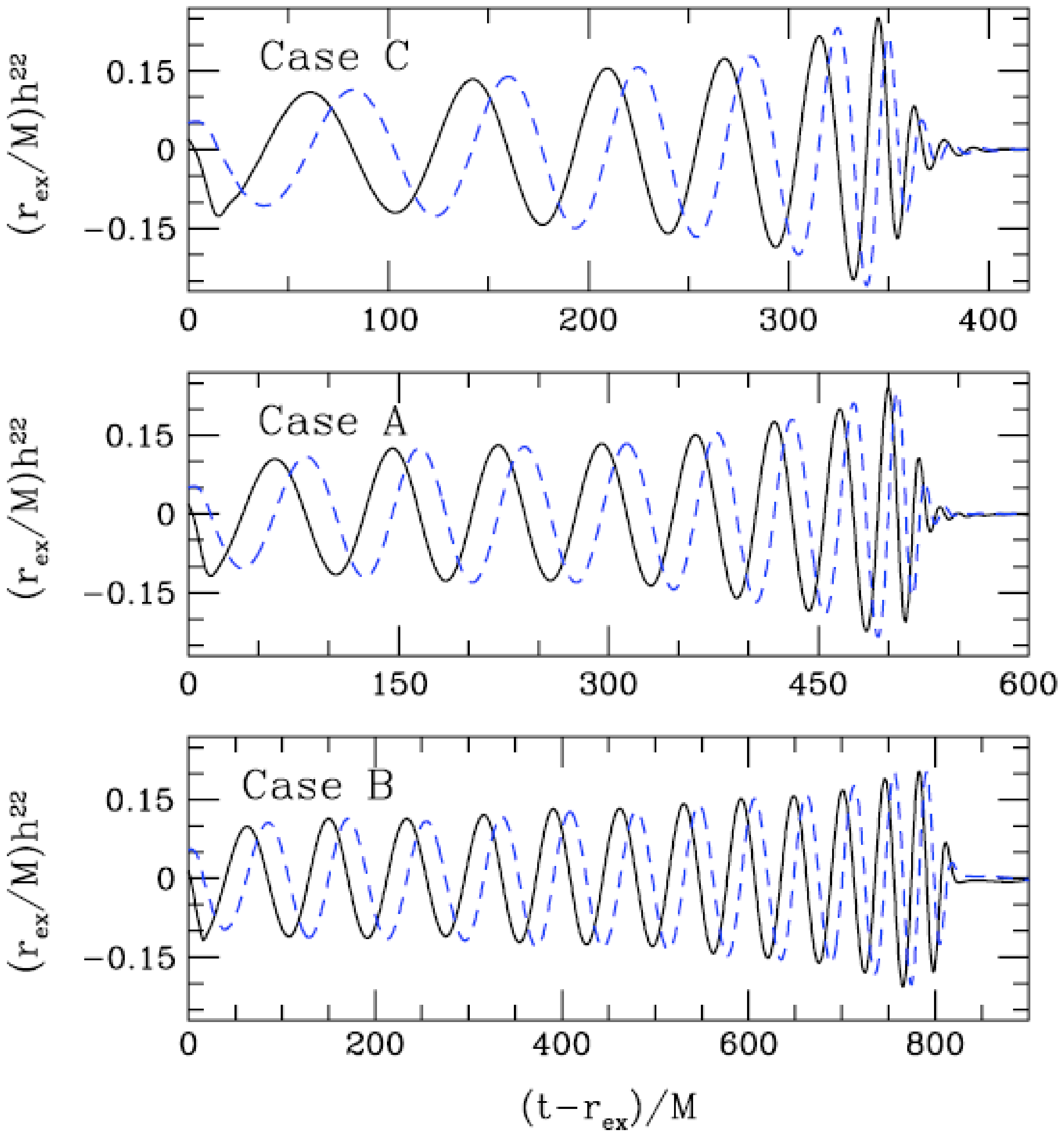
A: Q=3

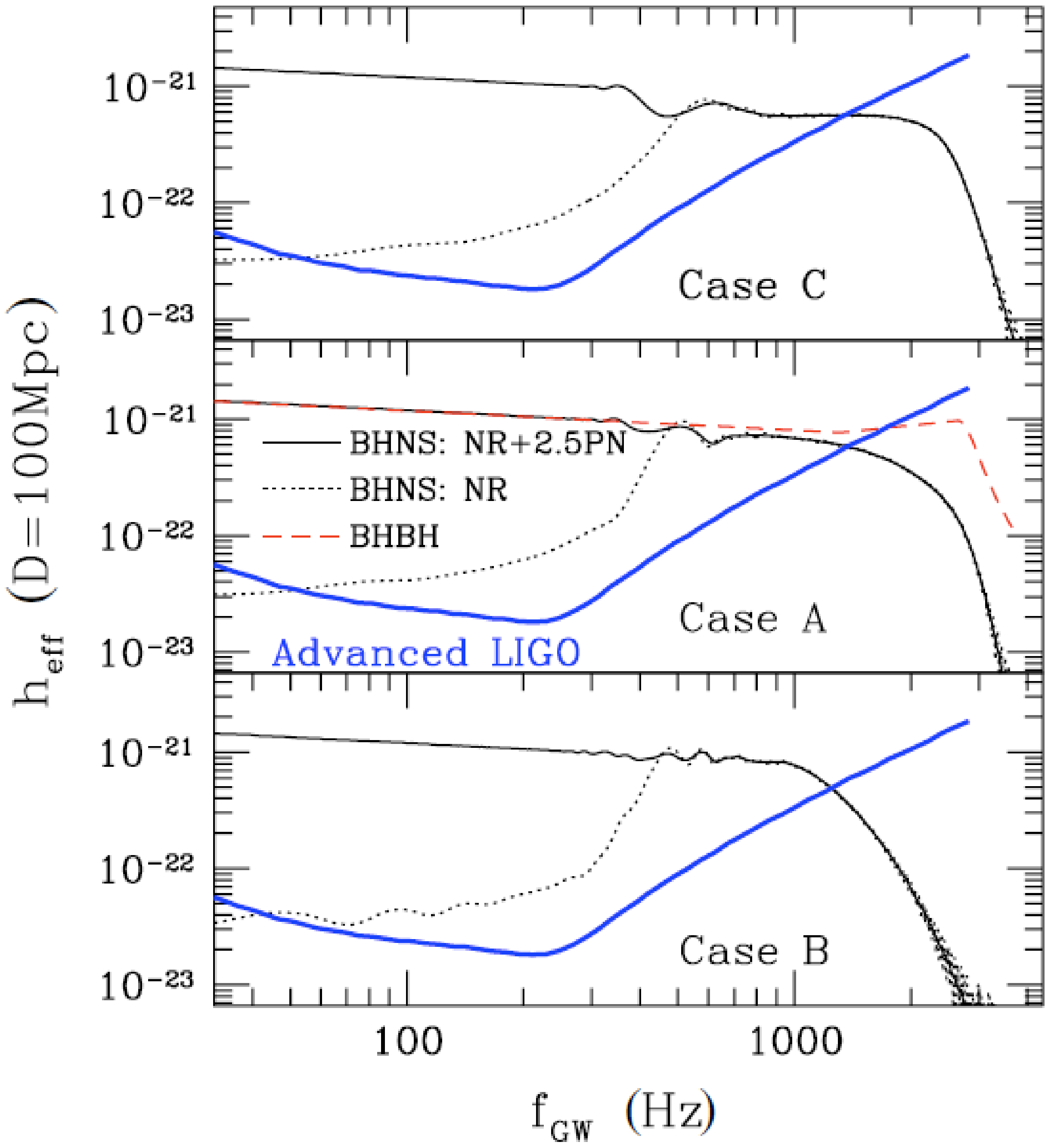
D: Q=5

Difficult to detect difference with BBH if low spin and high Q. Note how when increasing Q the frequency cutoff gets close to the one for BBH.

GW from BH-NS: role of BH spin

Etienne et al 2009





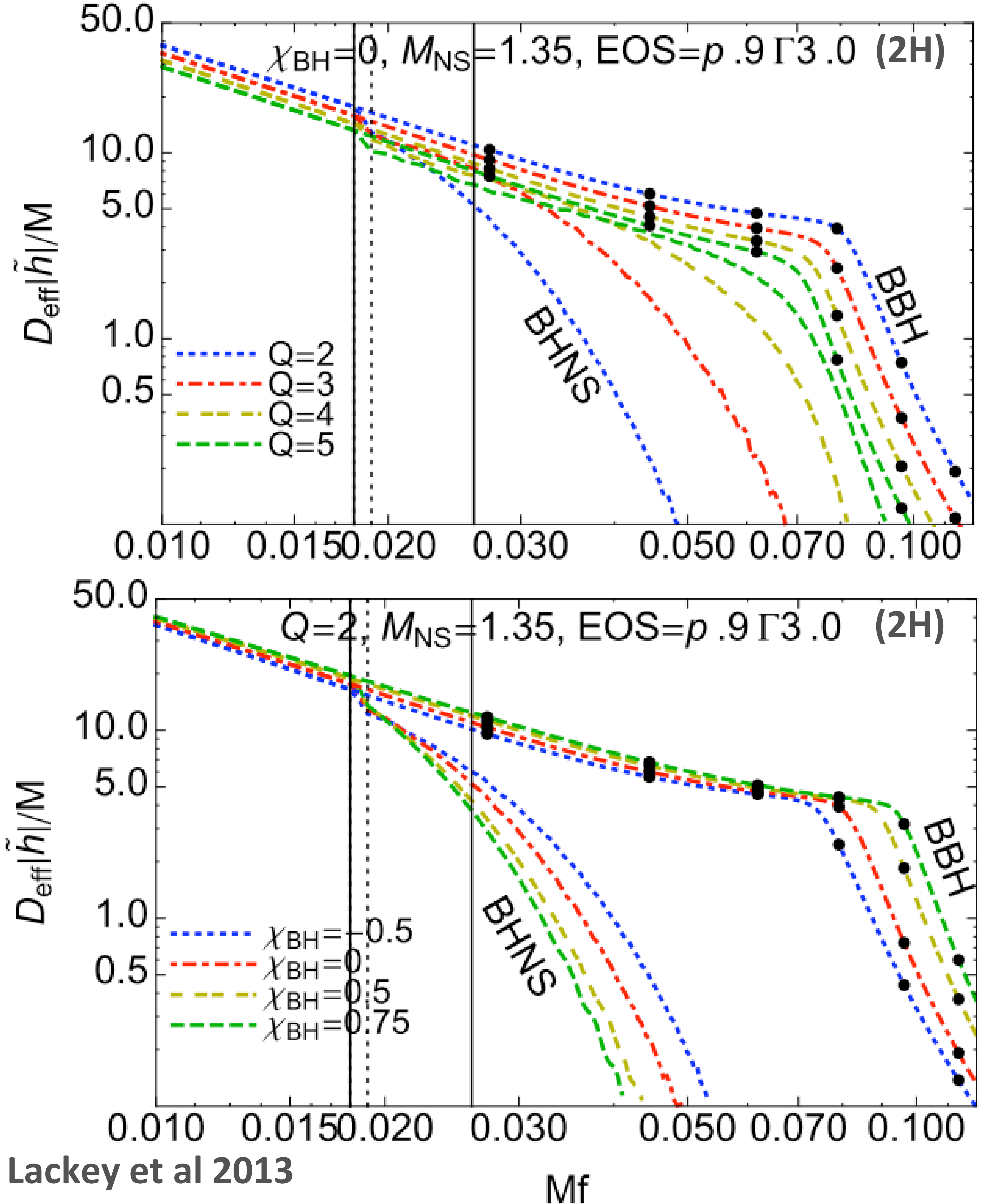
C: Q=3, a=-0.5

A: Q=3, a=0

B: Q=3, a=0.75

Ringdown signal gets smaller with higher BH spin because of larger disk formation.

NS-BH: matter effects

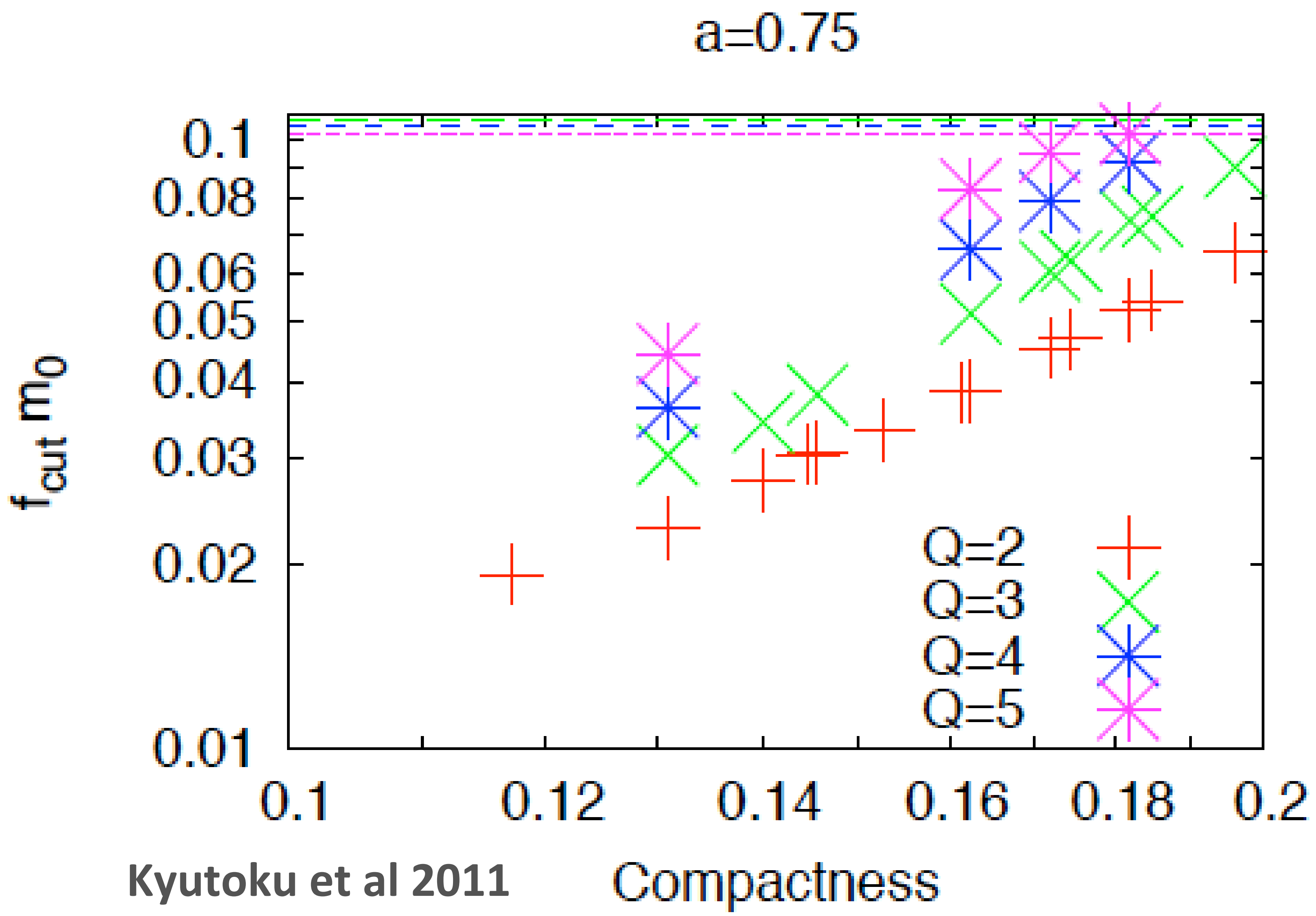


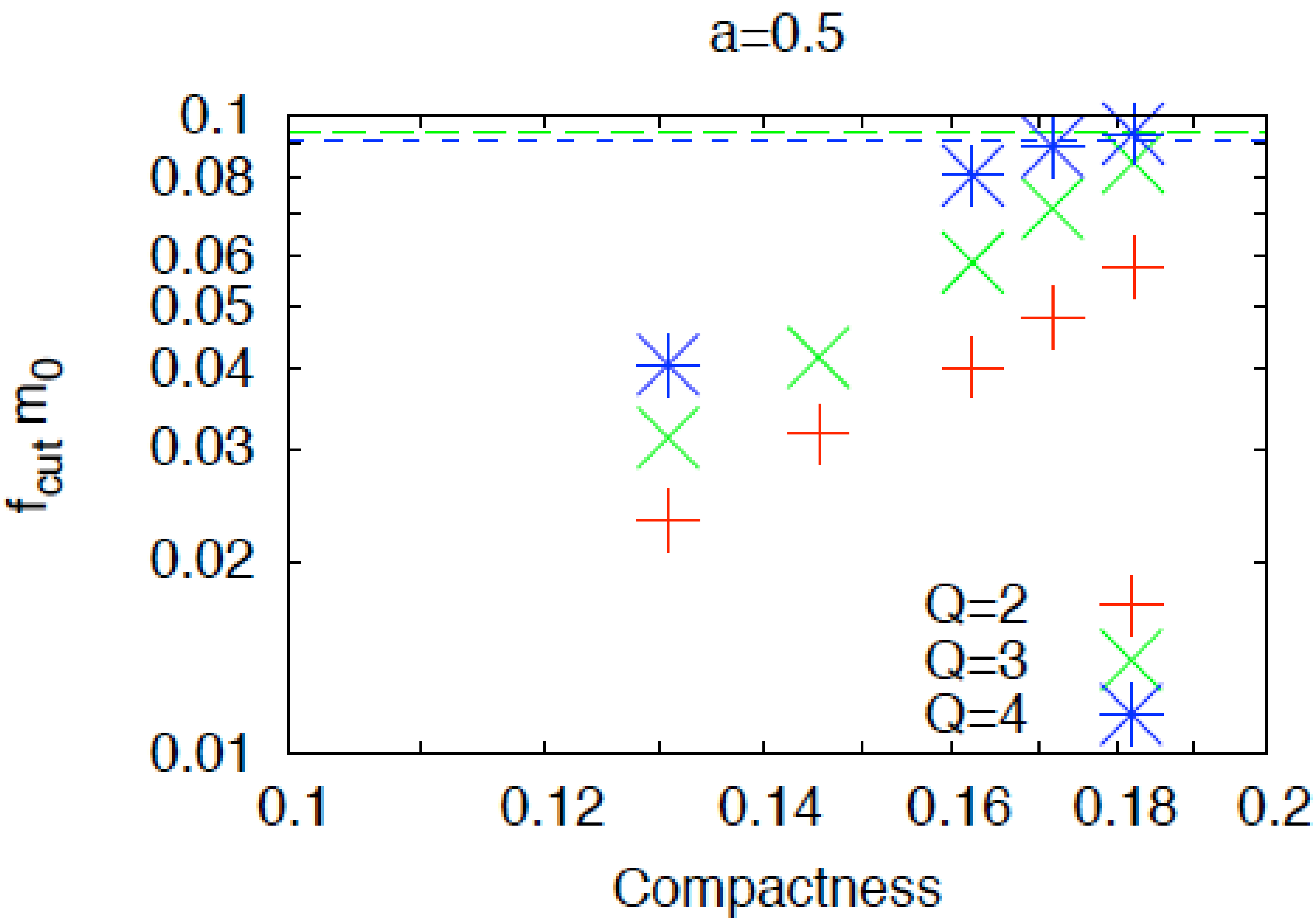
Lackey et al 2013

Lackey et al 2013 performed 134 simulations of NS-BH mergers with different EOS, BH masses and spins

higher Q and small spin reduce difference with BBH GWs

NS-BH: EOS effects





Kyutoku et al 2011

NS compactness influence the GW frequency cutoff.

Some Review Articles

- Shibata & Taniguchi 2011
  https://link.springer.com/article/10.12942/lrr-2011-6
- Faber & Rasio 2012
  https://link.springer.com/article/10.12942/lrr-2012-8
- Paschalidis 2017
  https://ui.adsabs.harvard.edu/abs/2017CQGra..34h4002P/abstract
- The Physics and Astrophysics of Neutron Stars (2018)
  https://link.springer.com/book/10.1007/978-3-319-97616-7
- Dietrich, Hinderer & Samajdar 2021
  https://ui.adsabs.harvard.edu/abs/2021GReGr..53...27D/abstract
- Foucart 2020
  https://www.frontiersin.org/articles/10.3389/fspas.2020.00046/full
- Ciolfi 2020
  https://www.frontiersin.org/articles/10.3389/fspas.2020.00027/full

Waveform Catalogues

- CoRe database:
  http://www.computational-relativity.org/gwdb/

- SACRA Gravitational Waveform Data Bank:
  https://www2.yukawa.kyoto-u.ac.jp/~nr_kyoto/SACRA_PUB/catalog.html

- Riccardo Ciolfi's BNS GW database:
  https://bitbucket.org/ciolfir/bns-waveforms/src/master/

- SXS Gravitational Waveform Database:
  https://data.black-holes.org/waveforms/index.html