Chapter 7 Exponential Logarithmic Functions

7.0 Pre-assessment

Match each of the vocabulary terms on the left with the appropriate letter and definition on the right.

1. exponent.
2. function.
3. relation.
4. variable.

A. a symbol used to represent one or more numbers.

B. the set of counting numbers and their opposites

C. a relation with at most one y-value for each x-value.

D. the number of times the base of a power is used as a factor.

E. a set of ordered pairs.

Simplify each expression.

5. $x^2(x^3)(x)$
6. $3y^{-1}(5x^2y^2)$
7. $\dfrac{a^{-2}b^3}{a^4b^{-1}}$.
8. $(3x)^2(4x^3)$

Use the simple interest formula, $I = Prt$, where I is the interest, P is the initial amount (principal), and r is the interest rate..

9. Find the simple interest on an investment of \$3000 at 3% for 2 years..

10. A savings account of \$2000 earned \$90 simple interest in 3 years. Find the interest rate.

Solve each equation for x.

11. $\dfrac{x}{2} = 3y - 4$
12. $y = \dfrac{3}{4}x - \dfrac{1}{2}$

1. D 2. C 3. E 4. A 5. x^6 6. $15x^2y$ 7. b^4/a^6 8. $36x^5$ 9. \$180

10. 1.5% 11. $6y - 8$ 12. $(4y + 2)/3$

7.1 Exponential Functions, growth and Decay

Objective: Write and evaluate exponential expressions to model growth and decay situations.

Moore's law, a rule used in the computer industry, states that the number of transistors per integrated circuit (the processing power) doubles every year. Beginning in the early days of integrated circuits, the growth in capacity may be approximated by this table.

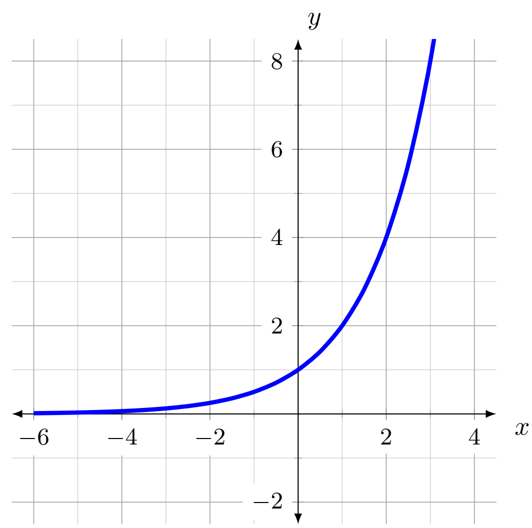
Transistors per Integrated Chip							
Year	1965	1966	1967	1968	1969	1970	1971
Transistors	60	120	240	480	960	1920	3840

$\times 2$ $\times 2$ $\times 2$ $\times 2$ $\times 2$ $\times 2$

Definition 7.1.1. Functions with a variable exponent are known as **exponential functions**. The parent exponential function is $f(x) = b^x$, where the **base** b is a constant and the exponent x is the independent variable.

$$f(x) = b^x, \text{ where } b > 0, b \neq 1.$$

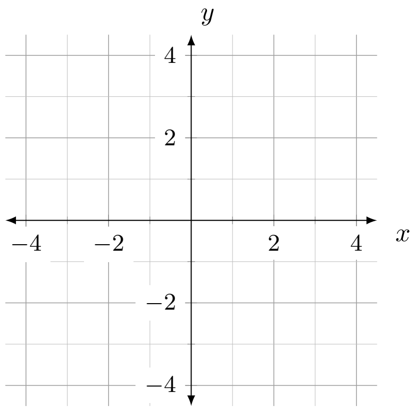
x	-2	-1	0	1	2	3
$\mathbf{f(x) = 2^x}$						



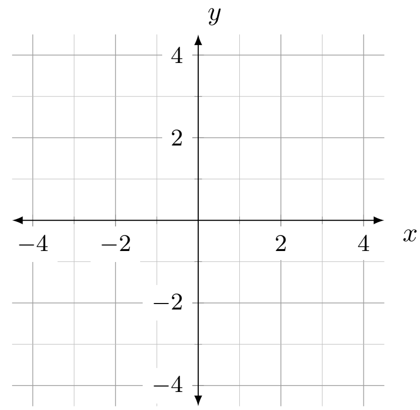
Definition 7.1.2. An **asymptote** is a line that a graphed function approaches as the value of x gets very large or very small. A function of the form $f(x) = ab^x$, with $a > 0$ and $b > 1$, is an **exponential growth** function, which increases as x increases. When $0 < b < 1$, the function is called an **exponential decay** function, which decreases as x increases.

Example 1. Tell whether the function shows growth or decay. Then graph.

(a) $f(x) = 3^x$

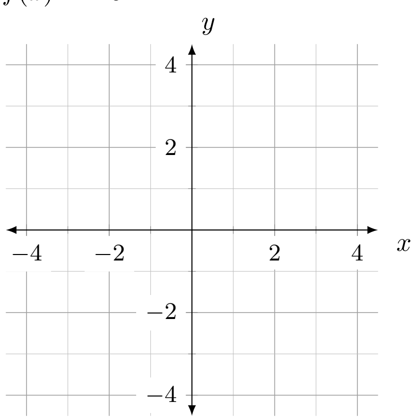


(b) $g(x) = 2\left(\frac{1}{2}\right)^2$

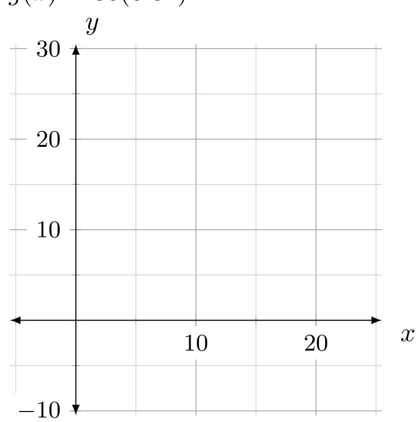


Example 2. Use a table and graphing calculator to sketch the exponential functions. Tell whether the function shows growth or decay.

(a) $f(x) = 1.5^x$



(b) $g(x) = 30(0.8^x)$



Exponential Growth Model

$$A(x) = P(1 \pm r)^t$$

Where A is the final amount, P is the principal (initial amount), r is the rate of increase or decrease, and t is time in compounding periods.

Example 3. (Calculator) Tyler purchased a rare 1959 Gibson Les Paul guitar in 2000 for \$12,000. Experts estimate that its value will increase by 14% per year. Use a calculator to graph to find when the value of the guitar will be \$60,000.

7.1 (day 1) Homework: page 493 2-5, 7-10

7.1 Exponential Functions (day 2)

Objective: Write and evaluate exponential expressions to model growth and decay situations.

Often we have a need to find when two equations are equal or when a function reaches a specific value. This is where we will use the `INTERSECT` function in the calculate menu.

Step 1: Press $\boxed{Y=}$ and use `Y1` for one side of the equation to solve and `Y2` for the other side of the equation.

Step 2: Press $\boxed{\text{GRAPH}}$ and use $\boxed{\text{WINDOW}}$ to set the `Xmin` and `Xmax` as well as the `Ymin` and `Ymax` to fit the interestion within the window.

Step 3: Use $\boxed{\text{2nd}}$ $\boxed{\text{Trace}}$ (the calculate menu) and select option 5. `insersect`. Press $\boxed{\text{ENTER}}$ on `Y1` and use the up arrow to select `Y2`. Use the arrow keys to guess the approximate intersection of the two curves.

Example 4. In 1081, the Australian humpback whale population was 350 and has increased at a rate of about 14% each year since then. Write a function to model population growth. Use a graph to predict when the population will reach $20,000$.

Example 5. The value of a truck bought new for $\$28,000$ decreases 9.5% each year. Write an exponential function, and graph the function. Use the graph to predict when the value will fall to $\$5,000$.

You Try It! 1. A motor scooter purchased for $\$1,000$ depreciates at an annual rate of 15%. Write an exponential function, and graph the function. Use the graph to predict when the value will fall below $100.

Example 6. The amount of freight transported by rail in the United States was about 580 billion *ton-miles* in 1960 and has been increasing at a rate of 2.32% per year since then.

a. Write a function representing the amount of freight, in billions of ton-miles, transported annually (let 1960 = year 0).

b. Graph the function.

c. In what year would you predict that the number of ton-miles woud have exceeded or would exceed 1 trillion (1000 billion)?

You Try It! 2. A quantity of insulin used to regulate sugar in the bloodstream breaks down by about 5% each minute. A body-weight adjusted does is generally 10 units.

a. Write a function representing the amount of the dose that remains after t minutes.

b. Graph the function.

c. About how much insulin remains after 10 minutes?

d. About how long does it take for half the dose to remain?

7.1 (day 2) Homework: page 494 15, 17, 21, 22, 27

7.2 Inverse of Relations and Functions

Objective: Graph and recognize inverses of relations and functions. Find Inverses of functions

Definition 7.2.1. An **inverse relation** is a relation that swaps x and y in every ordered pair of a given relation. This is the same as reflecting a graph over the line $y = x$.

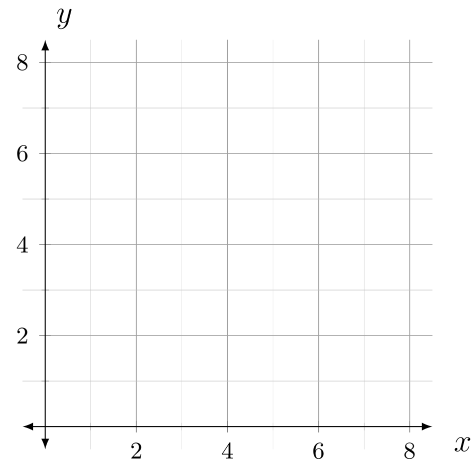
Example 1. Graph the relation and connect the points. Then graph the inverse relation, identify the domain and range of each relation.

Relation

Domain:

Range:

x	0	1	2	4	8
y	2	4	5	6	7



Inverse Relation

Domain:

Range:

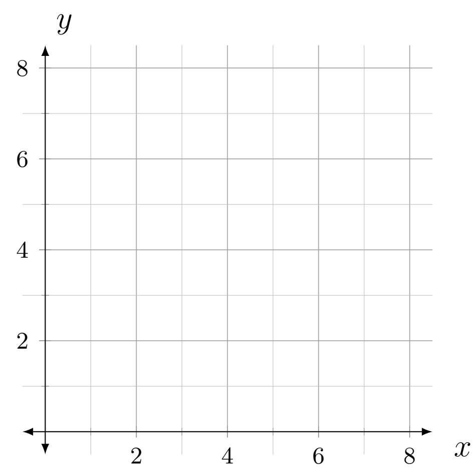
You Try It! 3. Graph the relation and connect the points. Then graph the inverse relation, identify the domain and range of each relation.

Relation

Domain:

Range:

x	1	3	4	5	6
y	0	1	2	3	5



Inverse Relation

Domain:

Range:

Definition 7.2.2. When a relation is also a function, you can write the inverse of the function $f(x)$ as $\mathbf{f^{-1}(x)}$. This notation does not indicate a reciprocal. Functions that *undo* each other are **inverse functions**.

Input	$\longrightarrow$	Function	$\longrightarrow$	**Output**
3		$\mathbf{f(x) = x + 6}$		**9**
Input	$\longrightarrow$	Inverse Function	$\longrightarrow$	**Output**
9		$\mathbf{f^{-1}(x) = x - 6}$		**3**

Example 2. Use inverse operations to write the inverse of the following functions.

(a) $f(x) = \dfrac{x}{3}$

(b) $g(x) = x - \dfrac{2}{3}$

You Try It! 4. Use inverse operations to write the inverse of the following functions.

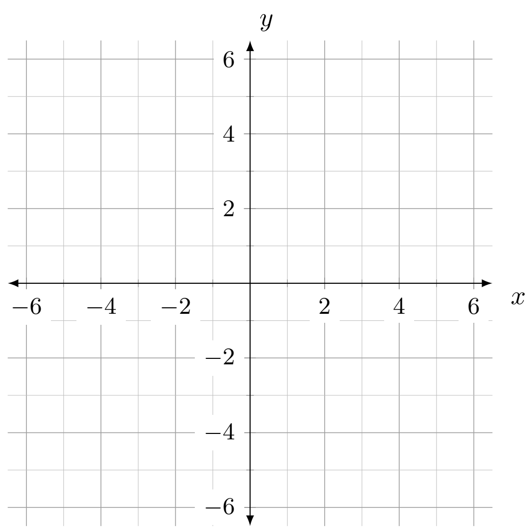
(a) $f(x) = -5x$

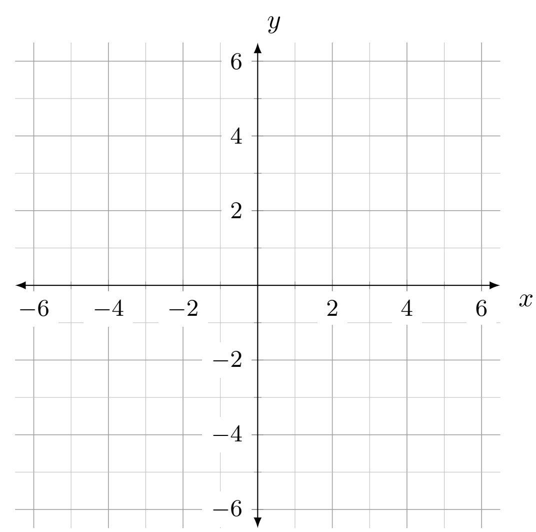
(b) $g(x) = x + 5$

Example 3. Write and graph the inverse of each function.

(a) $f(x) = 3x + 6$

(b) $g(x) = \dfrac{2}{3}x + 2$





7.2 Homework (day 1): page 501 2-16 all

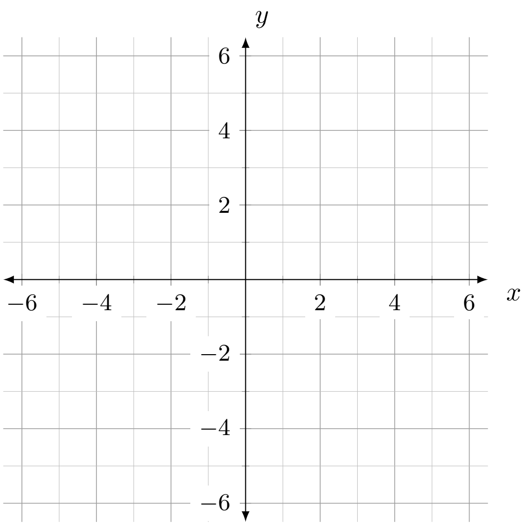
7.2 Inverses of Relations and Functions (day 2)

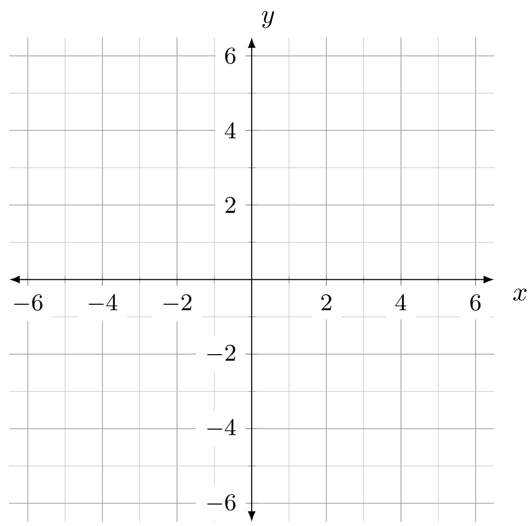
Objective: Find the inverses of functions.

Example 4. Write and graph the inverse of each function.

(a) $f(x) = \frac{3}{5}x - 4$

(b) $g(x) = \frac{3}{4}x + 3$





Example 5. A clerk needs to price a digital camera returned by a customer. THe customer paid a total of \$103.14, which included a gift-wrapping charge of \$3 and 8% sales tax. What price should the clerk mark on the tag?

Example 6. To make tea, use $\frac{1}{6}$ teaspoon of tea per ounce of water plus a teaspoon for the pot. Use the inverse to find the number of ounces of water needed if 7 teaspoons of tea are used.

Example 7. Tell whether each statement is sometimes, always, or never true.

The inverse of an ordered pair on a graph is its reflection over the line $y = x$

The inverse of a llinear function is a linear function.

The inverse of a line with positive slope is a line with negative slope.

The inverse of a line with slope greater than 1 is a line with slope less than one.

The inverse of the inverse of a point (x, y) is the original point.

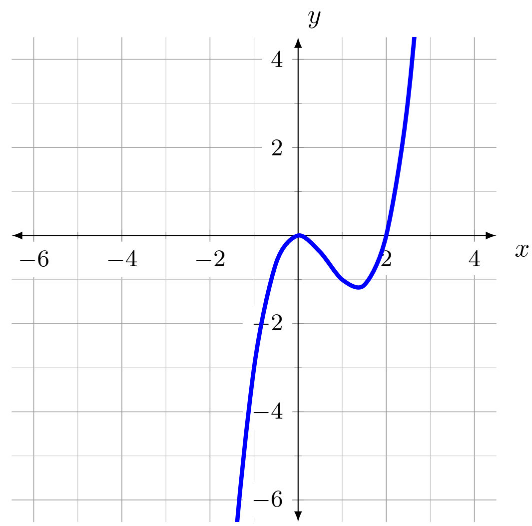
Example 8. What is the inverse of $f(x) = 3$ (Hint: Write this function as $y = 0x + 3$.) Is the inverse a function? Explain.

Example 9. State whether the inverse is a function.

(a)

x	1	3	4	5	6
y	5	1	5	10	17

(b)



(c) $(1, 2), (2, 5), (3, 8), (4, 11)$

(d) $f(x) = x^2 + 2x - 3$

$\boxed{\textbf{7.2 Homework (day 2)}\text{: page 501 \quad 17-28 all}}$

7.3 Logarithmic Functions

Objective: Write equivalent forms for exponential and logarithmic functions.

Definition 7.3.1. A **logarithim** is the exponent to which a specified base is raised to obtain a given value.

$$b^x = \qquad \log_b a = x$$

Example 1. Write each exponential equation in logarithmic form.

	Exponential Equation	Logarithmic Form
(a)	$2^6 = 64$	$\log_2 64 =$ ______
(b)	$4^1 = 4$	$\log_4 4 =$ ______
(c)	$5^0 = 1$	$\log_5 1 =$ ______
(d)	$5^{-2} = 0.04$	$\log_5 0.04 =$ ______
(e)	$3^x = 81$	$\log_3 81 =$ ______

(f) $9^2 = 81$

(g) $3^3 = 27$

(h) $x^0 = 1$ $(x \neq 0)$

Example 2. Write each logarithmic equation in exponential form.

	Logarithmic Form	Exponential Equation
(a)	$\log_1 0100 = 2$	$10^2 =$ ______
(b)	$\log_7 49 = 2$	$7^2 =$ ______
(c)	$\log_8 0.125 = -1$	$8^{-1} =$ ______
(d)	$\log_5 5 = 1$	$5^1 =$ ______
(e)	$\log_1 21 = 0$	$12^0 =$ ______

(f) $\log_{10} 10 = 1$

(g) $\log_{12} 144 = 2$

(h) $\log_{\frac{1}{2}} 8 = -3$

Special Properties of Logarithms for any base b such that $b > 0$ and $b \neq 1$		
Logarithmic Form	**Exponential Form**	**example**
$\log_b b = 1$	$b^1 = b$	$\log_{10} 10 = 1$ and $10^1 = 10$
$\log_b 1 = 0$	$b^0 = 1$	$\log_{10} 1 = 0$ and $10^0 = 1$

Definition 7.3.2. A logarithm with base 10 is called a **common logarithm**. If no base is written the base is assumed to be 10, $\log 5 = \log_{10} 5$.

Example 3. Use mental math to evaluate the following logarithms.

(a) $\log 1000$ (b) $\log_4 \frac{1}{4}$ (c) $\log 0.00001$ (d) $\log_{25} 0.04$ (e) $\log_2 8$

You Try It! 5. Rewrite each equation in exponential form.

(a) $\log_6 36 = 2$

(b) $\log_{289} 17 = -2$

(c) $\log_{14} \frac{1}{196} = -2$

(d) $\log_3 81 = 4$

You Try It! 6. Rewrite each equation in logarithmic form.

(a) $64^{\frac{1}{2}} = 8$

(b) $12^2 = 144$

(c) $9^{-2} = \frac{1}{81}$

(d) $\left(\frac{1}{12}\right)^2 = \frac{1}{144}$

$\boxed{\textbf{7.3 (day 1) Homework}\text{: page 509 1-13 all, 17-27 odds Adv. 40, 41, 45}}$

7.3 (day 2)

Objective: Write, evaluate, and graph logarithmic functions.

You Try It! 7.

(a)

(b)

Example 4.

(a)

(b)

Example 5.

(a)

(b)

$\boxed{\textbf{7.3 (day 2) Homework}\text{: page}}$

7.4 Properties of Logarithms

Objective: Use properties to simplify logarithmic expressions.

Example 1.

(a)

(b)

Example 2.

(a)

(b)

You Try It! 8.

(a)

(b)

7.4 (day 1) Homework: page

7.4 (day 2) Properties of Logarithms

Objective: Translate between logarithms in any base.

Example 3.

(a)

(b)

You Try It! 9.

(a)

(b)

7.4 (day 2) Homework: page

7.6 The Natural Base, e

Objective: Use the number e to write and graph exponential functions representing real-world.

Example 1.

(a)

(b)

You Try It! 10.

(a)

(b)

Definition 7.6.1.

Example 2.

(a)

(b)

$\boxed{\textbf{7.6 Homework (day 1)}\text{: page}}$

7.6 The Natural Base, e (day 2)

Objective: Solve equations and problems involving e or natural logarithms.

Example 3.

Example 4.

(a)

(b)

7.6 Homework (day 2): page

7.7 Transforming Exponential and Logarithmic Functions

Objective: Transform exponential and logarithmic functions by changing parameters..

Example 1.

(a)

(b)

Example 2.

(a)

(b)

7.7 Homework (day 1): page

7.7 Transforming Exponential and Logarithmic Functions (day 2)

Objective: Describe the effects of changes in the coefficients of exponential and logarithmic functions.

Example 3.

(a)

(b)

Example 4.

Example 5.

7.7 Homework (day 2): page

Chapter 7 Review (day 1)

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Ch 7 Review (day 2)