Chapter 6 Polynomial Functions

6.0 Pre-assessment

Match each of the vocabulary terms on the left with the appropriate letter and definition on the right.

1. coefficent
2. like terms
3. root of an equation
4. x-intercept
5. maximum of a function

A. the y-value of the highest point of the graph of the function

B. the horizontal number line that divides the coordinate plane

C. the numerical factor in a term

D. a value of the variable that makes the equation true

E. terms that contain the same variables raised to the same powers

F. the x-coordinate of a point where the graph intersects the x-axis.

Evaluate each expression.

6. 6^4 7. -5^4 8. $(-1)^5$ 9. $\left(-\frac{2}{3}\right)^2$

Evaluate each expression for the given value of the variable.

10. $x^4 - 5x^2 - 6x - 8$ for $x = 3$

11. $2x^3 - 3x^2 - 29x - 30$ for $x = -2$

12. $2x^3 - x^2 - 8x + 4$ for $x = \frac{1}{2}$

13. $3x^4 + 5x^3 + 6x^2 + 4x - 1$ for $x = -1$

Multiply or divide.

14. $2x^3y \cdot 4x^2$ 15. $-a^2b \cdot ab^4$ 16. $\frac{-7t^4}{3t^2}$ 17. $\frac{3p^3q^2r}{12pt^4}$

1. C 2. E 3. D 4. F 5. A 6. 1296 7. -625 8. -1 9. 4/9

10. 10 11. 0 12. 0 13. -1 14. $8x^5y$ 15. $-a^3b^5$ 16. $-\frac{7}{3}t^2$ 17. $\frac{p^2q^2r}{4t^4}$

6.1 Polynomials

Objective: Identify and classify polynomials

Definition 6.1.1. A **monomial** is a number or a product of numbers and variables with whole number exponents. A **polynomial** is a monomial or a sum or difference of monomials. The **degree of a monomial** is the sum of the exponents of the variables.

Polynomials: $3x^4$ $\quad 2z^{12}+9z^3$ $\quad \frac{1}{2}a^7$ $\quad 0.15x^{101}$ $\quad 3t^2-t^3$

Not Polynominals: 3^x $\quad |2b^3-6b|$ $\quad \frac{8}{5y^2}$ $\quad \frac{1}{2}\sqrt{x}$ $\quad m^{0.75}-m$

Example 1. Identify the degree of each monomial.

(a) x^4

(b) 12

(c) $4a^2b$

(d) x^3y^4z

Definition 6.1.2. The **degree of a polynomial** is given by the term with the greatest degree. A polynomial is in standard when its terms are written in decending order of degree. The **leading coefficent** the coefficent of the first term in standard form.

$$5x^{3}+8x^2+3x-17$$

Definition 6.1.3. A polynomial with two terms is called a **binomial**, and a polynomial with three terms is called a **trinomial**.

Classifying Polynomials by Degree		
Name	**Degree**	**Example**
Constant	0	-9
Linear	1	$x-4$
Quadratic	2	x^2+3x-1
Cubic	3	x^3+2x^2+x+1
Quartic	4	$2x^4+x^3+3x^2+4x-1$
Quintic	5	$7x^5+x^4-x^3+3x^2+2x-1$

Example 2. Rewrite each polynomial in standard form. The identify the leading coefficent, degree, and number of terms. Name the polynomial.

(a.) $2x+4x^3-1$

Standard Form:

Leading Coefficent:

Degree:

Terms:

Name:

(b.) $7x^3-11x+x^5-2$

Standard Form:

Leading Coefficent:

Degree:

Terms:

Name:

Example 3. Add or subtract. Write you answer in standard form.

(a.) $(3x^2+7+x)+(14x^3+2+x^2-x)$

(b.) $(1-x^2)-(3x^2+2x-5)$

6.1 (day 1) Homework: page 410 1-13 all Adv. 47-49

Polynomials (day 2)

Objective: Evaluate and Graph Polynomials

You Try It! 1. Add or subtract. Write your answer in standard form.

(a) $(-36x^2 + 6x - 11) + (6x^2 + 16x^3 - 5)$

(b) $(5x^3 + 12 + 6x^2) + (15x^2 + 3x - 2)$

Example 4. Cardiac output is the amount of blood pumped through the heard. The output is measured by a technique called dye dilution. A doctor injects dye into a vein near the heart and measured the amount of dye in the arteries over time.

The cardiac output of a particular patient can be approximated by the function

$$f(t) = 0.0056t^3 - 0.22t^2 + 2.33t,$$

where $f(t)$ represents the concentration of dye (in milligrams per liter).

(a) Evaluate $f(t)$ for $t = 0$ and $t = 3$.

(b) Describe what the values of the function in part (a) represent.

Example 5. Graph each polynomial on a graphing calculator. Describe the graph, and identify the number of real zeros.

(a) $f(x) = x^3 - x$

(b) $f(x) = -3x^3 + 2x + 1$

(c) $h(x) = x^4 - 8x^2 + 1$

(d) $k(x) = x^4 + x^3 - x^2 + 2x - 3$

6.1 (day 2) Homework: page 410 31-39 all Adv. 41, 42

6.2 Multiplying Polynomials

Objective: To Multiply Polynomials and Binomial Expansion

Example 1. Find each product.

(a) $3x^2(x^3+4)$

(b) $ab(a^3+3ab^2-b^3)$

Example 2. Find each product.

(a) $(x-2)(1+3x-x^2)$

(b) $(x^2+3x-5)(x^2-x+1)$

Binomial Expansion

Example 3. Find the product.

$(x+y)^3$

Binomial Expansion		Pascal's Triangle (Coefficients)
$(a+b)^0 =$	1	1
$(a+b)^1 =$	$1a + 1b$	1 1
$(a+b)^2 =$	$1a^2 + 2ab + 1b^2$	1 2 1
$(a+b)^3 =$	$1a^3 + 3a^2b + 3ab^2 + 1b^3$	1 3 3 1
$(a+b)^4 =$	$1a^4 + 4a^3b + 6a^2b^2 + 4ab^3 + 1b^4$	1 4 6 4 1
$(a+b)^5 =$	$1a^5 + 5a^4b + 10a^3b^2 + 10a^2b^3 + 5ab^4 + 1b^5$	1 5 10 10 5 1

Example 4. Expand each expression using Pascal's triangle.

(a) $(y-3)^4$

(b) $(4z+5)^3$

6.2 Homework: page 418 1-13 odd, Adv. 52, 56

6.3 Dividing Polynomials

Objective: Use long and synthetic division to divide polynomials.

Example 1. Divide using arithmetic long division.

(a) $12\overline{)277}$

You Try It! 2. Divide.

(b) $8\overline{)347}$

Example 2. Divide using long division.

(a) $(4x^2 + 3x^3 + 10) \div (x - 2)$

(b) $(15x^2 + 8x - 12) \div (3x + 1)$

Example 3. Divide using synthetic division.

(a) $(4x^2 - 12x + 9) \div \left(x + \dfrac{1}{2}\right)$

(b) $(6x^2 - 5x - 6) \div (x + 3)$

Example 4. Use synthetic substitution to evaluate the polynomial for the given value.

(a) $P(x) = x^3 - 4x^2 + 3x - 5$ for $x = 4$

(b) $P(x) = 4x^4 + 2x^3 + 3x + 5$ for $x = -\dfrac{1}{2}$

6.3 (day 1) Homework: page 426 1-12 all, Adv. 53, 57

6.3 (day 2)

Objective: Use long and synthetic division to divide polynomials.

You Try It! 3. Divide using long division.

(a) $(2x^2 + 7x + 7) \div (x + 2)$

(b) $(x^2 + 5x - 28) \div (x - 3)$

Example 5. Divide using synthetic division.

(a) $(x^2 - 3x - 18) \div (x - 6)$

(b) $(x^4 - 7x^3 + 9x^2 - 22x + 25) \div (x + 3)$

Remainder Theorem	
Theorem	**Example**
If the polynomial function $P(x)$ is divided by $x - \mathbf{a}$, then the remainder r is $P(\mathbf{a})$.	Divide $x^3 - 4x^2 + 5x + 1$ by $x - 3$ $$\begin{array}{r\|rrrr} 3 & 1 & -4 & 5 & 1 \\ & \downarrow & 3 & -3 & 6 \\ \hline & 1 & -1 & 2 & \boxed{7} \end{array}$$ $P(3) = 7$

Example 6. Use synthetic substitution to evaluate the polynomial for the given value.

(a) $P(x) = x^3 + 3x^2 + 4$ for $x = -3$

(b) $P(x) = 5x^2 + 9x + 3$ for $x = \frac{1}{5}$

6.3 (day 2) Homework: page 426 13, 17, 19, 23, 25, 27, 43, 44

6.2 & 6.3 Review

Objective: Multiply and Divide Polynomials

Find each product.

1. $3x^2(2x^2 + 9x - 6)$
2. $(2x + 5y)(3x^2 - 4xy + 2y^2)$

Expand each expression. (Use Pascal's triangle)

3. $(x - 3y)^3$
4. $(x - 2)^5$

Divide.

5. $7\overline{)647}$
6. $9\overline{)3452}$

Use long division to divide the polynomials. Write as Quotient + Remainder/Divisor.

7. $(2x^2 + 3x - 20) \div (x - 2)$
8. $(x^4 + 6x^3 + 6x^2) \div (x + 5)$

Use synthetic division to divide the polynomials. Write as Quotient + Remainder/Divisor.

9. $x^4 - 3x^3 - 7x - 14) \div (x - 4)$

10. $(x^2 + 9x + 6) \div (x + 8)$

Use synthetic substitution (The Remainder Theorem) to evaluate the polynomial for the given value.

11. $P(x) = 4x^3 - 5x^2 - x + 2$ for $x = -1$

12. $P(x) = 25x^2 - 16$ for $x = \dfrac{4}{5}$

13. $P(x) = 4x^3 - 5x^2 - x + 2$ for $x = -1$

14. $P(x) = 25x^2 - 16$ for $x = \dfrac{4}{5}$

6.4 Factoring Polynomials

Objective: Use the Factor Theorem to determine factors of a polynomial.

Factor Theorem	
Theorem	**Example**
For any polynomial $P(x)$, $(x-a)$ is a factor of $P(x)$ if and only if $P(a) = 0$.	Because $P(1) = 1^2 - 1 = 0$, $(x-1)$ is a factor of $P(x) = x^2 - 1$.

Example 1. Determine whether the given binomial is a factor of the polynomial $P(x)$.

(a) $(x-3)$; $P(x) = x^2 + 2x - 3$

(b) $(x+4)$; $P(x) = 2x^4 + 8x^3 + 2x + 8$

Example 2. Factor by grouping.

(a) $x^3 + 3x^2 - 4x - 12$

(b) $x^3 - 2x^2 - 9x + 18$

You Try It! 4. Factor by grouping

(a) $2x^3 + x^2 + 8x + 4$

(b) $8y^3 - 4y^2 - 50y + 25$

6.4 (day 1) Homework: page 433 1-9 all, 17-19, Adv. 41, 43

6.4 (day 2) Factoring

Objective: Factor the sum and difference of two cubes.

Factoring The Sum and Difference of Two Cubes	
Method	**Algebra**
Sum of two cubes	$a^3+b^3=(a+b)(a^2-ab+b^2)$
Difference of two cubes	$a^3-b^3=(a-b)(a^2+ab+b^2)$

S · **O** · **A** · **P**

same opposite always positive

$$(a \pm b)^3 = (a \pm b)(a^2 \mp ab + b^2)$$

Example 3. Factor each expression using sum or difference of cubes.

(a) $5x^4 + 40x$

(b) $8y^3 - 27$

You Try It! 5. Factor each expression using sum or difference of cubes.

(a) $8 + z^6$

(b) $2x^5 - 16x^2$

6.4 (day 2) Homework: page 433 10-15 all, 21-31 odds

6.4 Review of Factoring

Objective: Factor using the sum and difference of two cubes, difference of square, grouping, and GCF.

Factor using the greatest common factor (GCF).

1. $2x^5 - 6x^3$
2. $5x^3 - 10x$
3. $14x^3 - 49x^2 - 28x$
4. $27x^5 - 18x^4 + 9x^3$

Factor using difference of squares.

5. $q^2 - r^2$
6. $25a^2 - 64b^2$
7. $81x^2 - 100y^2$
8. $x^4 - y^4$
9. $a^6 - b^6$
10. $4x^4 - 9y^6$

Factor using sum and difference of cubes.

5. $x^3 - y^3$

6. $r^3 + s^3$

7. $8a^3 - 27b^3$

8. $64x^3 + 125y^3$

9. $a^6 - b^6$

10. $x^6 + y^6$

Factor using grouping.

5. $6x^3 + 2x^2 + 9x + 3$

6. $7x^3 - 35x^2 + 8x - 40$

7. $4x^3 + 8x^2 - 9x - 18$

8. $16x^3 - 64x^2 - 25x + 100$

6.4 Factoring Review Homework: page 433 20-30 evens, 33-38 all

6.5 Finding Real Roots of Polynomial Equations

Objective: Identify the multiplicity of roots, Use the Rational Root Theorem to solve polynomial equations.

Example 1. Solve each polynomial equation by factoring.

(a) $3x^5 + 18x^4 + 27x^3 = 0$

(b) $x^4 - 13x^2 = -36$

You Try It! 6. Solve each polynomial equation by factoring.

(a) $2x^6 - 10x^5 - 12x^4 = 0$

(b) $x^3 - 2x^2 - 25x = -50$

Definition 6.5.1. The **multiplicity** of root r is the number of times that $x - r$ is a factor of $P(x)$. Even multiplicity means the graph "touches" the x-axis at the root but does not cross. Odd multiplicity means the graph crosses the x-axis at the root.

Example 2. Identify the roots of each equation. State the multiplicity of each root.

(a) $x^3 - 9x^2 + 27x - 27 = 0$

(b) $-2x^3 - 12x^2 + 30x + 200 = 0$

6.5 Homework (day 1): page 442   2-9 all, 15-21 odds

6.6

Objective:

Definition 6.6.1. Definition

Example 1. Example

6.6 Homework: page

6.7

Objective:

Definition 6.7.1. Definition

Example 1. Example

6.7 Homework: page

6.8

Objective:

Definition 6.8.1. Definition

Example 1. Example

6.8 Homework: page

6.9

Objective:

Definition 6.9.1. Definition

Example 1. Example

6.9 Homework: page