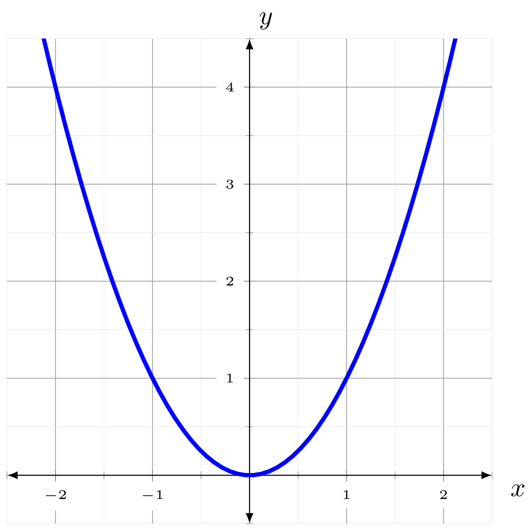
Chapter 5 Quadratic Functions

5.1 Using Transformations to Graph Quadratic Functions

Objective: Transform quadratic functions. Use the vertex form to graph quadratics.

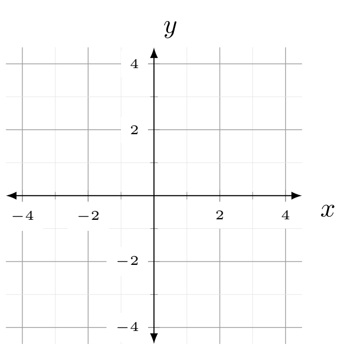
Definition 5.1.1. A **quadratic function** is a function that can be written in the form $f(x) = ax^2 + bx + c$ where $a \neq 0$. A graph of the quadratic parent function is shown below. Fill in the table below using a graphing calculator.



x	y_1
-2	
-1	
0	
1	
2	

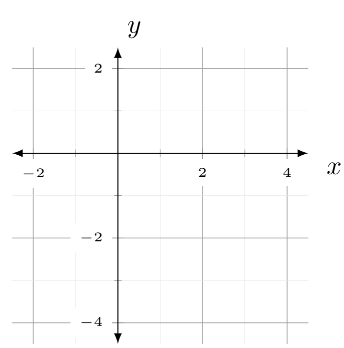
Example 1. Graph using a graphing calculator table.

(a) $f(x) = x^2 - 6x + 8$



x	y_1
1	
2	
3	
4	
5	

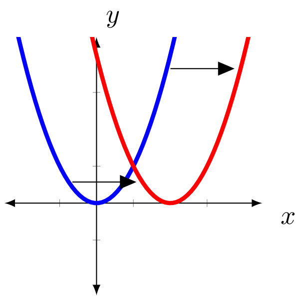
(b) $f(x) = -x^2 + 6x - 8$



x	y_1
1	
2	
3	
4	
5	

<table>
<tr><th colspan="2">Translations of Quadratic Functions</th></tr>
<tr><th>Horizontal Translation</th><th>Vertical Translations</th></tr>
<tr><td>

Horizontal Shift of $|h|$ Units



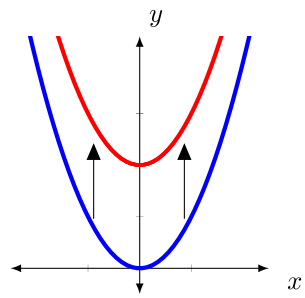
$f(x) = x^2$

$f(x - h) = (x - h)^2$

Moves left for $h < 0$
Moves right for $h > 0$

</td><td>

Vertical Shift of $|k|$ Units



$f(x) = x^2$

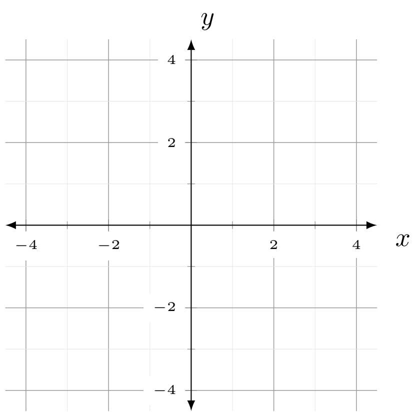
$f(x) + k = x^2 + k$

Moves up for $k > 0$
Moves down for $k < 0$

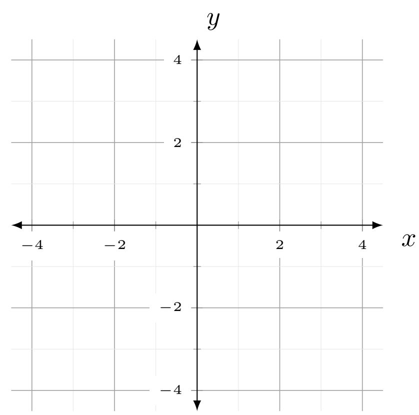
</td></tr>
</table>

Example 2. Using the graph of $f(x) = x^2$ as a guide, describe the transformations, and then graph each function.

(a) $f(x) = (x + 3)^2 + 1$

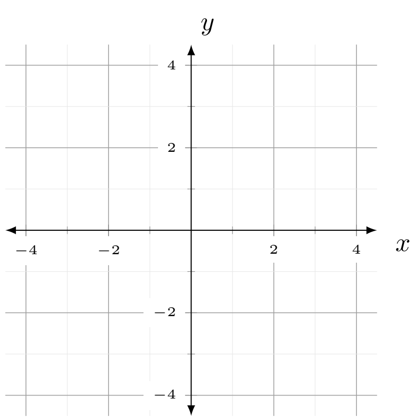


(b) $f(x) = (x - 2)^2 - 1$

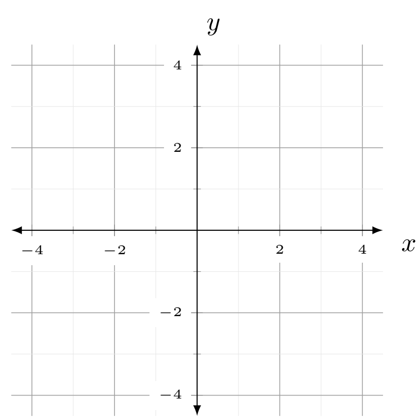


You Try It! 1. Using the graph of $f(x) = x^2$ as a guide, describe the transformations, and then graph each function.

(a) $f(x) = (x - 1)^2 + 2$



(b) $f(x) = (x + 2)^2 - 3$



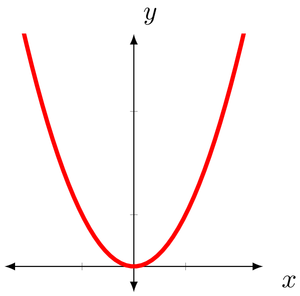
5.1 (day 1) Homework: page 320 2-7, 17-22 all

5.1 (Day 2) Using Transformations to Graph Quadratic Function

Objective: Transform quadratic functions.

Reflections of Quadratic Functions

Reflection Across y-axis



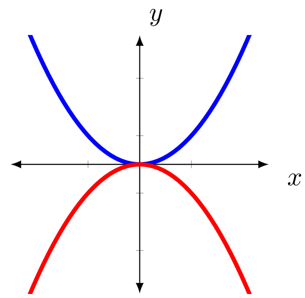
Input values change.

$f(x) = x^2$

$f(-x) = (-x)^2 = x^2$

The function $f(x) = x^2$ is its own reflection across the y-axis

Reflection Across x-axis



Input values change.

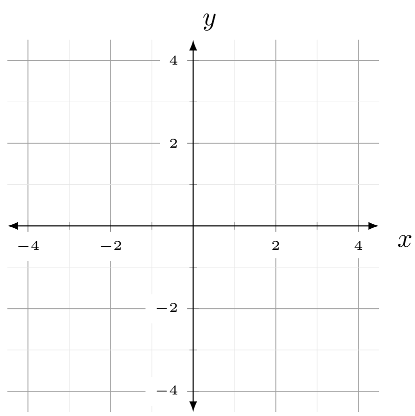
$f(x) = x^2$

$-f(x) = -(x^2) = -x^2$

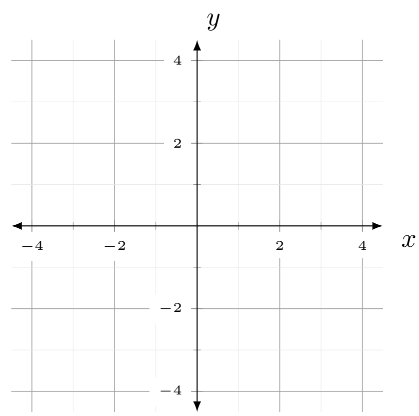
The function is flipped across the x-axis.

Example 3. Using the graph of $f(x) = x^2$ as a guide, describe the transformations, and then graph each function.

(a) $f(x) = (-(x+1))^2 - 3$

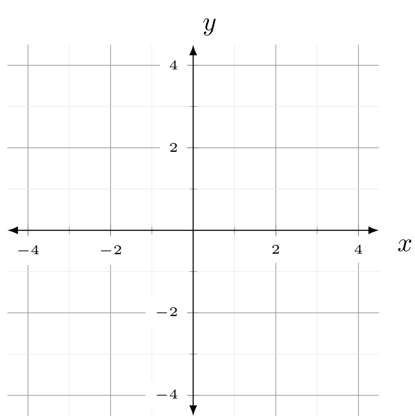


(b) $f(x) = -(x-1)^2 + 2$

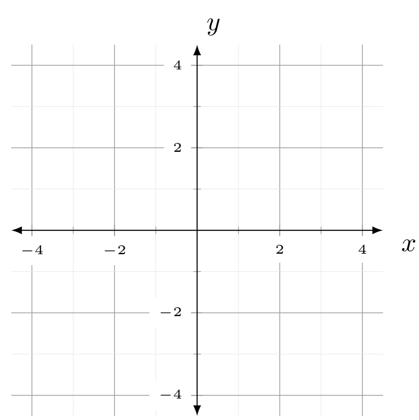


You Try It! 2. Using the graph of $f(x) = x^2$ as a guide, describe the transformations, and then graph each function.

(a) $f(x) = -(x+2)^2 + 1$

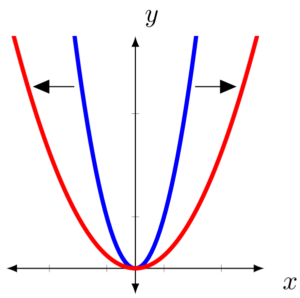


(b) $f(x) = (-(x-2))^2 - 4$



Stretches and Compressions of Quadratic Functions

Horizontal Stretch/Compression by a Factor of $|b|$



Input values change.

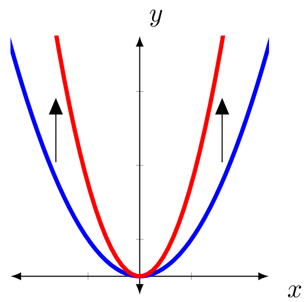
$f(x) = x^2$

$f\left(\frac{1}{b}x\right) = \left(\frac{1}{b}x\right)^2$

$|b| > 1$ stretches away from the y-axis

$0 < |b| < 1$ compresses toward the y-axis

Vertical Stretch/Compression by a Factor of $|a|$



Input values change.

$f(x) = x^2$

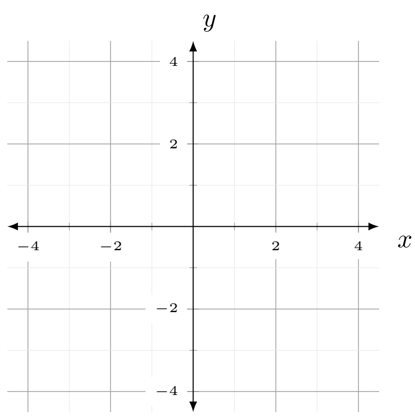
$af(x) = a(x^2)$

$|a| > 1$ stretches away from the x-axis

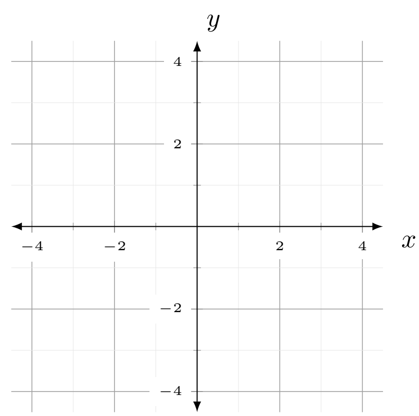
$0 < |a| < 1$ compresses toward the x-axis

Example 4. Using the graph of $f(x) = x^2$ as a guide, describe the transformations, and then graph each function.

(a) $g(x) = -4x^2$

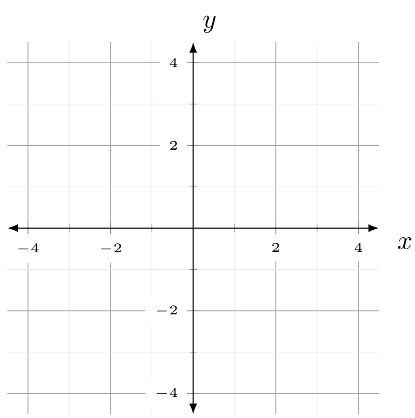


(b) $g(x) = -\left(\frac{1}{2}x\right)^2$

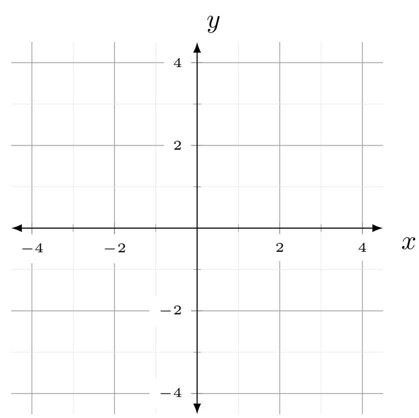


You Try It! 3. Using the graph of $f(x) = x^2$ as a guide, describe the transformations, and then graph each function.

(a) $g(x) = (2x)^2$



(b) $h(x) = -\frac{1}{2}(x-1)^2$



5.1 (day 2) Homework: page 320 23-28 all, 33, 35, 37, Adv. 29, 30, 39-41 all

5.2 Properties of Quadratics in Standard Form

Objective: Define, identify, and graph quadratic functions. Use the vertex and standard form to graph quadratics.

Definition 5.2.1. If a parabola opens upward, it has a lowest point. If a parabola opens downward, it has a highest point. This lowest or highest point is called the **vertex** of the parabola.

Vertex Form of a Quadratic Function

$$\boxed{f(x) = \mathbf{a}\big(x - \mathbf{h}\big)^2 + \mathbf{k}}$$

a indicates a reflection across the x-axis and/or a vertical stretch or compression.

h indicates a horizontal translation.

k indicates a vertical translation.

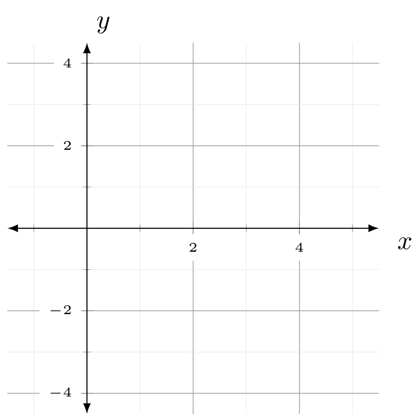
In **vertex form** of a parabola the vertex can be read as **h** horizontal units and **k** vertical units from the origin, the vertex of the parabola is at $(\mathbf{h}, \mathbf{k})$.

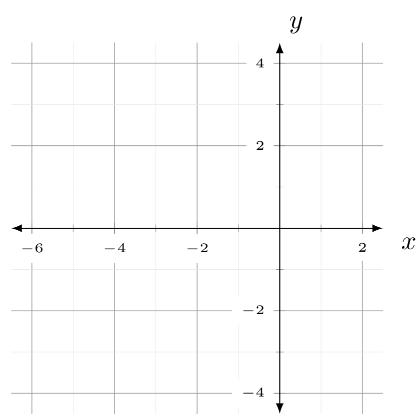
Example 1. Use the description to write the equation in vertex form.

(a) The parent function $f(x) = x^2$ is vertically compressed by a factor of $\frac{1}{3}$ and translated 2 units right and 4 units down to create g.

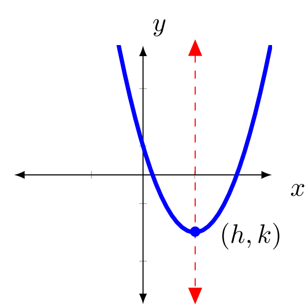
(b) The parent function $f(x) = x^2$ is reflected across the x-axis and translated 5 units left and 1 unit up to create g.

Example 2. Graph the equations found in the previous examples.





Definition 5.2.2. The **axis of symmetry** is the line through the vertex of a parabola that divides the parabola into two congruent halves.

Words	Algebra	Graph
The axis of symmetry is a vertical line through the vertex of the function's graph.	The quadratic function $f(x) = a(x-h)^2 + k$ has the axis of symmetry $x = h$.	

Properties of a Parabola

For $f(x) = ax^2 + bx + c$, where a, b, and c are real numbers and $a \neq 0$, the parabola has these properties:

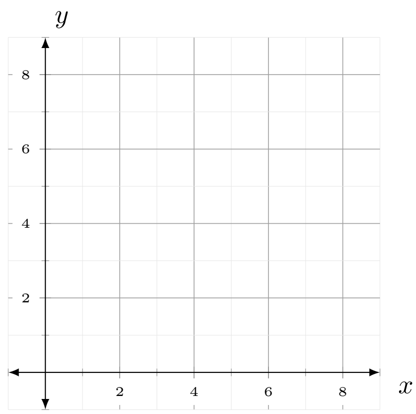
Parabola **opens** upward if $\mathbf{a} > \mathbf{0}$ and downward if $\mathbf{a} < \mathbf{0}$.
The **axis of symmetry** is the vertical line $\mathbf{x} = -\dfrac{\mathbf{b}}{\mathbf{2a}}$.
The **vertex** is the point $\left(-\dfrac{\mathbf{b}}{\mathbf{2a}}, \mathbf{f}\left(-\dfrac{\mathbf{b}}{\mathbf{2a}}\right)\right)$.
The **y-intercept** is $\mathbf{c}$.

Example 3. For each function **(a)** determine whether the graph opens upward or downward. **(b)** Find the axis of symmetry. **(c)** Find the vertex. **(d)** Find the y-intercept. **(e)** Graph the function.

$f(x) = x^2 - 4x + 6$

(a) Opens: (c) Vertex:

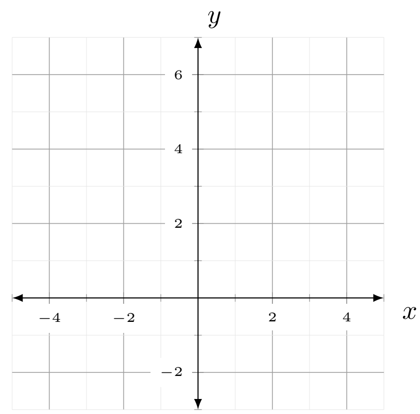
(b) Axis: (d) y-int:

(e) 

$g(x) = -4x^2 - 12x - 3$

(a) Opens: (c) Vertex:

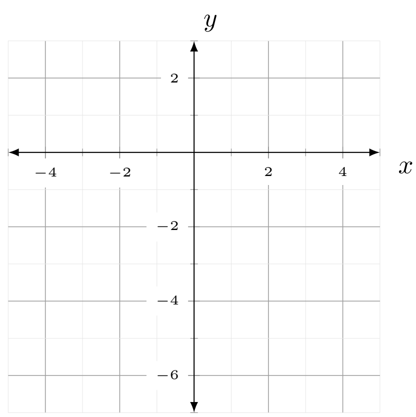
(b) Axis: (d) y-int:

(e) 

$h(x) = -2x^2 - 4x$

(a) Opens: (c) Vertex:

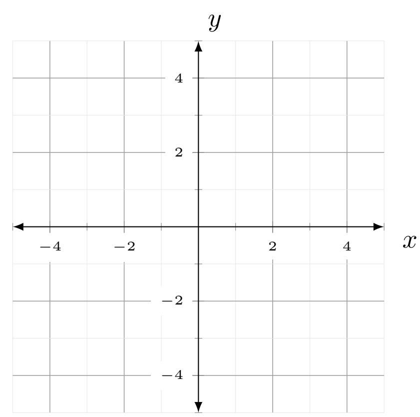
(b) Axis: (d) y-int:

(e) 

$p(x) = x^2 + 3x - 1$

(a) Opens: (c) Vertex:

(b) Axis: (d) y-int:

(e) 

5.2 (day 1) Homework: page 328 2-7, 13, 15, 17 Adv. 39, 42, 43

5.2 (Day 2) Properties of Quadratic Functions in Standard Form

Objective: Identify and use maximums and minimums of quadratic functions to solve problems.

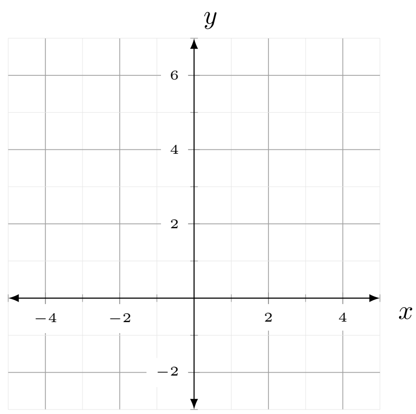
You Try It! 4. For each function **(a)** determine whether the graph opens upward or downward. **(b)** Find the axis of symmetry. **(c)** Find the vertex. **(d)** Find the y-intercept. **(e)** Graph the function.

$f(x) = x^2 - 4x + 5$

(a) Opens: (c) Vertex:

(b) Axis: (d) y-int:

(e)

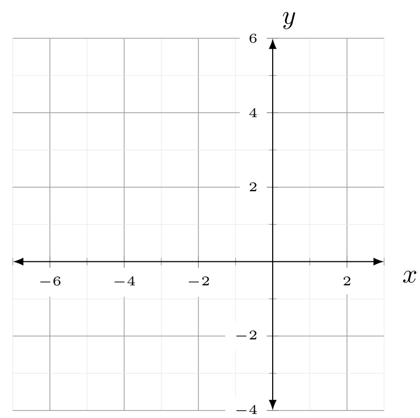


$g(x) = x^2 + 6x + 6$

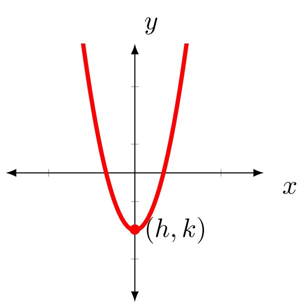
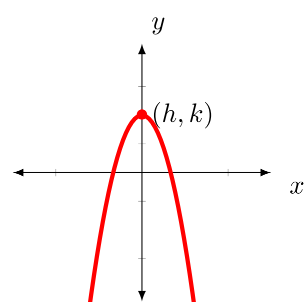
(a) Opens: (c) Vertex:

(b) Axis: (d) y-int:

(e)



Minimum and Maximum Values

Opens Upward	Opens Downward
When a parabola opens upward, the y-value of the vertex is the **minimum value**.	When a parabola opens downward, the y-value of the vertex is the **maximum value**.
$D: \left\{x \vert x \in \mathbb{R}\right\}$ $R: \left\{y \vert y \geq k\right\}$	$D: \left\{x \vert x \in \mathbb{R}\right\}$ $R: \left\{y \vert y \leq k\right\}$
	
The domain is all real number, $\mathbb{R}$. The range is all values greater than or equal to the minimum.	The domain is all real numbers, $\mathbb{R}$. The range is all values less than or equal to the maximum.

Example 4. Find the minimum or maximum value of each function. Then state the domain and range of the function.

(a) $f(x) = 2x^2 - 2x + 5$

Max/Min:

Domain:

Range:

(b) $f(x) = x^2 - 6x + 3$

Max/Min:

Domain:

Range:

Example 5. The power p in horsepower (hp) generated by a speedboat engine operating at r revolutions per minute (rpm) can be modeled by the function $p(r) = -0.0000147r^2 + 0.18r - 251$. What is the maximum power of this engine to the nearest horsepower? At how many revolutions per minute must the engine be operating to achieve this power?

Example 6. The highway mileage m in miles per gallon for a compact car is approximated by $m(s) = -0.025s^2 + 2.45s - 30$, where s is the speed in miles per hour. What is the maximum mileage for this compact car to the nearest tenth of a mile per gallon? What speed results in this mileage?

5.2 (day 2) Homework: page 328 8-10 all, 12-30 multiples of 3, Adv. 35, 40

5.3 Solving Quadratic Equations by Graphing and Factoring

Objective: Solve quadratic equations by graphing and factoring.
Determine a quadratic function from its roots.

Definition 5.3.1. A **zero of a function** is a value of the input x that makes the output $f(x)$ equal zero. The zeros of a function are the x-intercepts.

Example 1. Find the zeros of the following functions by using a graph and table.

(a) $f(x) = x^2 + 2x - 3$

(b) $g(x) = -x^2 - 2x + 3$

Definition 5.3.2. The solutions to a quadratic equation of the form $ax^2 + bx + c = 0$ are called **roots**. The **roots of an equation** are the values of the variable that make the equation true.

Note: Functions have *zeros* or x-intercepts. Equations have *solutions* or *roots*.

Zero Product Property		
WORDS	**NUMBERS**	**ALGEBRA**
If the product of two quantities equals zero, at least one of the quantities equals zero.	$3(0) = 0$ $0(4) = 0$	For all real numbers a and b, if $ab = 0$, then $a = 0$. or $b = 0$

Example 2. Find the zeros of each function by factoring.

(a) $f(x) = x^2 - 8x + 12$

(b) $g(x) = 3x^2 + 12x$

(c) $h(x) = x^2 - 5x - 6$

(d) $k(x) = x^2 - 8x$

You Try It! 1. Find the zeros of each function by factoring.

(a) $f(x) = x^2 - 5x + 6$

(b) $g(x) = 4x^2 - 8x$

(c) $h(x) = x^2 + x - 12$

(d) $k(x) = x^2 - 5x$

Special Products and Factors	
Difference of Two Squares	**Perfect Square Trinomial**
$a^2 - b^2 = (a+b)(a-b)$	$a^2 - 2ab + b^2 = (a-b)^2$ $a^2 + 2ab + b^2 = (a+b)^2$

Example 3. Find the roots of each equation by factoring.

(a) $9x^2 = 1$

(b) $40x = 8x^2 + 50$

You Try It! 2. Find the roots of each equation by factoring.

(a) $x^2 - 4x = -4$

(b) $25x^2 = 9$

5.3 Homework: page 338 1-14 all (not 6, 10, or 11) 54, 55 Adv. 63, 65

5.3 (Day 2) Solving Quadratic Equations by Graphing and Factoring

Objective: Solve quadratic equations by factoring.

Factor quadratics with leading coefficent

Example 4. Factor.

(a) $3p^2 - 2p - 5$

(b) $2n^2 + 3n - 9$

(c) $3n^2 - 8n + 4$

(d) $5n^2 + 19n + 12$

(e) $2v^2 + 11v + 5$

(f) $2n^2 + 5n + 2$

You Try It! 3. Factor.

(a) $7a^2 + 53a + 28$

(b) $9k^2 + 66k + 21$

(c) $15n^2 - 27n - 6$

(d) $5x^2 - 18x + 9$

5.3 (day 2) Homework

Factor each of the given quadratics completely.

1. $2x^2 - 5x + 2$
2. $3x^2 + 13x + 4$
3. $4n^2 - 15n - 25$
4. $4x^2 - 35x + 49$
5. $4n^2 - 17n + 4$
6. $6x^2 + 7x - 49$
7. $6x^2 + 37x + 6$
8. $-6a^2 - 25a - 25$
9. $6n^2 + 5n - 6$
10. $16b^2 + 60b - 100$
11. $15x^2 - x - 2$
12. $10x^2 - x - 21$

Solutions: 1. $(x-2)(2x-1)$ 2. $(3x+1)(x+4)$ 3. $(n-5)(4n+5)$ 4. $(x-7)(4x-7)$ 5. $(n-4)(4n-1)$ 6. $(3x-7)(2x+7)$ 7. $(x+6)(6x+1)$ 8. $-(2a+5)(3a+5)$ 9. $(2n+3)(3n-2)$ 10. $4(b+5)(4b-5)$ 11. $(5x-2)(3x+1)$ 12. $(2x-3)(5x+7)$

5.3 (Day 3) More Factoring Practice

Objective: Solve quadratic equations by factoring.

Example 5. Factor.

(a) $7x^2 - 45x - 28$

(b) $2b^2 + 17b + 21$

(c) $30n^2b - 87nb + 30b$

(d) $x^2 - 16x + 63$

(e) $5p^2 - p - 18$

(f) $28n^4 + 16n^3 - 80n^2$

You Try It! 4. Factor.

(a) $x^2 - 7x - 18$

(b) $p^2 - 5p - 14$

(c) $7x^2 - 31x - 20$

(d) $7k^2 + 9k$

5.3 (day 3) Homework

Factor each of the given quadratics completely. If not factorable state, not factorable.

1. $m^2 - 9m + 8$
2. $7x^2 - 32x - 60$
3. $3b^3 - 5b^2 + 2b$
4. $9r^2 - 5r - 10$
5. $9p^2r + 73pr + 70r$
6. $9x^2 + 7x - 56$
7. $4x^3 + 43x^2 + 30x$
8. $10m^2 + 89m - 9$
9. For what values of b is the expression factorable?
   $x^2 + bx + 12$
10. Name four values of b which make the expression factorable:
    $x^2 - 3x + b$

Solutions: 1. $(m-1)(m-8)$ 2. $(7x+10)(x-6)$ 3. $b(3b-2)(b-1)$ 4. *not factorable* 5. $r(p+7)(9p+10)$ 6. *not factoable* 7. $x(x+10)(4x+3)$ 8. $(m+9)(10m-1)$ 9. $13, 8, 7, -13, -8, -7$ 10. Answers vary $0, 2, -4, -10, -18$

5.4 Completing the Square

Objective: Solve quadratic equations by completing the square.

Square-Root Property		
WORDS	**NUMBERS**	**ALGEBRA**
To solve a quadratic equation, you can take the square root of both sides. Be sure to consider the positive and negative square roots.	$x^2 = 15$ $\lvert x\rvert = \sqrt{15}$	If $x^2 = a$ and a is a nonnegative real number, then $x = \pm\sqrt{a}$

Example 1. Solve the equation.

(a) $3x^2 - 4 = 68$

(b) $x^2 - 10x + 25 = 27$

You Try It! 1. Solve each equation.

(a) $4x^2 - 20 = 5$

(b) $x^2 + 8x + 16 = 49$

Completing the Square		
WORDS	**NUMBERS**	**ALGEBRA**
To complete the square of $x^2 + bx$, add $\left(\frac{b}{2}\right)^2$.	$x^2 + 6x +$ ______ $x^2 + 6x + \left(\frac{6}{2}\right)^2$ $x^2 + 6x + 9$ $(x+3)^2$	$x^2 + bx +$ ______ $x^2 + bx + \left(\frac{b}{2}\right)^2$ $\left(x - \frac{b}{2}\right)^2$

Example 2. Complete the square for each expression. Write the resulting expression as a binomial squared.

(a) $x^2 - 2x +$ ______

(b) $x^2 + 5x +$ ______

(c) $x^2 - 10x +$ ______

You Try It! 2. Complete the square for each expression. Write the resulting expression as a binomial squared.

(a) $x^2 + 4x +$ ______

(b) $x^2 + 3x +$ ______

(c) $x^2 - 8x +$ ______

Example 3. Solve each equation by completing the square.

(a) $x^2 = 27 - 6x$

(b) $2x^2 + 8x = 12$

Example 4. Write each function in vertex form, and identify the vertex.

(a) $f(x) = x^2 + 10x - 13$

(b) $g(x) = 2x^2 - 8x + 3$

You Try It! 3. Solve the equation by completing the square.

$3x^2 - 24x = 27$

You Try It! 4. Write each function in vertex form, and identify the vertex.

$f(x) = x^2 + 24x + 145$

5.4 Homework: page 345 1-19 odd, Adv. 63, 73, 74

5.5 Complex Numbers and Roots

Objective: Define and use imaginary and complex numbers.

Definition 5.5.1. The **imaginary unit** i is defined as $\sqrt{-1}$. You can use the imaginary unit to write the square root of any negative number.

Imaginary Numbers		
WORDS	**NUMBERS**	**ALGEBRA**
An **imaginary number** is the square root of a negative number. Imaginary numbers can be written in the form bi, where b is a real number and i is the imaginary unit	$\sqrt{-1} = i$ $\sqrt{-2} = \sqrt{-1}\sqrt{2} = i\sqrt{2}$ $\sqrt{-4} = \sqrt{-1}\sqrt{4} = 2i$ $(\sqrt{-1})^2 = i^2 = -1$	If b is a positive real number, the $\sqrt{-b} = i\sqrt{b}$ and $\sqrt{-b^2} = bi$. $(\sqrt{-b})^2 = -b$.

Example 1. Express each number in terms of i.

(a) $3\sqrt{-16}$

(b) $-\sqrt{-75}$

You Try It! 1. Express each number in terms of i.

(a) $\sqrt{-12}$

(b) $2\sqrt{-36}$

Example 2. Solve each equation.

(a) $x^2 = -81$

(b) $3x^2 + 75 = 0$

You Try It! 2. Solve each equation.

(a) $x^2 = -36$

(b) $x^2 + 48 = 0$

Definition 5.5.2. A **complex number** is a number that can be written in the form $a + bi$, where a and b are real numbers and $i = \sqrt{-1}$. The set of real numbers is a subset of the set of complex numbers.

Every complex number has a **real part** **a** and an **imaginary part** **b**.

$$\mathbf{a} + \mathbf{b}i$$

Example 3. Find the values of x and y that make the equation true.

(a) $3x - 5i = 6 - (10y)i$

(b) $2x - 6i = -8 + (20y)i$

Example 4. Find the zeros of each function.

(a) $f(x) = x^2 - 2x + 5$

(b) $g(x) = x^2 + 10x + 35$

Definition 5.5.3. The **complex conjugate** of any complex number $a + bi$ is the complex number $a - bi$.

Example 5. Find each complex conjugate.

(a) $2i - 15$

(b) $-4i$

You Try It! 3. Find each complex conjugate.

(a) $9 - i$

(b) $i + \sqrt{3}$

5.5 Homework: page 353 3-17 odds, 29, 30, 47, 49, 51

5.6 The Quadratic Formula

Objective: Solve quadratic equations using the Quadratic Formula.

Note: We have learned how find zeros of a quadratic function and roots of a quadratic equation by graphing, factoring, and completing the square. What we are going to learn here is how to find the roots numerically using the ***quadratic formula***

Numbers	**Algebra**
$3x^2 + 5x + 1 = 0$	$ax^2 + bx + c = 0 \ (a \neq 0)$

The Quadratic Formula
If $ax^2 + bx + c = 0$ $(a \neq 0)$, then the solutions, or roots, are $$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Example 1. Find the zeros of $f(x)$ by using the Quadratic Formula.

(a) $f(x) = x^2 + 10x + 2$

(b) $f(x) = x^2 + 3x - 7$

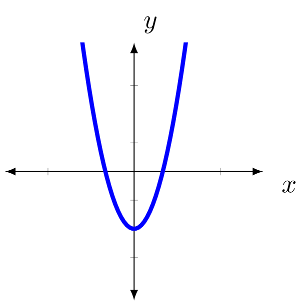
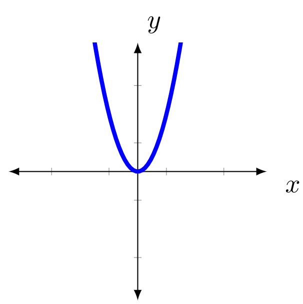
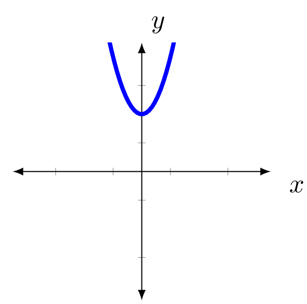
Example 2. Find the zeros of $f(x)$ by using the Quadratic Formula.

(a) $f(x) = 2x^2 - x + 2$

(b) $f(x) = x^2 - 4x + 13$

Definition 5.6.1. The **discriminant** is part of the Quadratic Formula that you can use to determine the number of real roots of a quadratic equation.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \longleftarrow \text{Discriminant}$$

Discriminant		
The **discriminant** of the quadratic equation $ax^2 + bx + c = 0$ $(a \neq 0)$ is $\mathbf{b^2 - 4ac}$.		
$\mathbf{b^2 - 4ac > 0}$	$\mathbf{b^2 - 4ac = 0}$	$\mathbf{b^2 - 4ac < 0}$
two distinct real solutions	one distinct real solution	two distinct nonreal complex solutions
		

Example 3. Find the type and number of solutions for each equation.

(a) $x^2 - 6x = -7$

(b) $x^2 - 6x = -9$

(c) $x^2 - 6x = -11$

5.6 Homework: page 361 3-13 odd, 14-16 all

TI-83/84 Quadratic Formula Program

Step 1. Press PRGM arrow over to **NEW** press ENTER. Name the program "QUADSOLV" using the letters above each of the keys.

Step 2. Once in the program press PRGM and arrow over to **I/O** (input/output) and select **ClrHome**. Press ENTER to start the next line.

Step 3. On the next line press PRGM arrow over to **I/O** and select **Disp** (display) and using the ALPHA key type **"$\mathbf{AX^2+BX+C=0}$"** in quotations. Use 2nd MATH (TEST key) to find the equal sign "=". Press ENTER to start a new line.

Step 4. On the next line press PRGM and arrow over to **I/O** and select **Prompt**. Back in the program use the ALPHA key to type **A, B, C** after the Prompt including commas. Press ENTER to start the next line.

Step 5. On the next line type $\mathbf{B^2-4AC}$ STO → **D**. Press ENTER to start a new line.

Step 6. On the next line press MODE and select **Float 4**. Press ENTER to start a new line.

Step 7. On the next line press MODE and select $\mathbf{a+bi}$. Press ENTER to start a new line.

Step 8. On the next line type $(-\mathbf{B}+\sqrt{(}\mathbf{D}))/(\mathbf{2A})$ STO→ **X**. Press ENTER to start a new line.

Step 9. On the next line type $(-\mathbf{B}-\sqrt{(}\mathbf{D}))/(\mathbf{2A})$ STO→ **Y**. Press ENTER to start a new line.

Step 10. On the next line press PRGM arrow over to **I/O** and select **Disp**. Use the ALPHA key to type **"ROOTS EQUAL:"** in quotations.

Step 11. On the next line press PRGM arrow over to **I/O** and select **Disp** and type **X**, **Y**.

Below is the typed program as viewed from your screen.

```
  PROGRAM:QUADSOLV
: ClrHome
: Disp ''AX²+BX+C=0''
: Prompt A,B,C
: B²−4AC→D
: Fix 4
: a+bi
: (−B+√(D))/(2A)→X
: (−B−√(D))/(2A)→Y
: Disp ''ROOTS EQUAL:''
: Disp X, Y
```

To run the program on your calculator exit the program window by pressing 2nd MODE (QUIT key). You should be at the Home Screen. Press the PRGM key and select **QUADSOLV** and press ENTER. To test that your program runs correctly use **A=1 B=-10 C=29**. Your calculator should read:

```
AX²+BX+C=0
A=?1
B=?−10
C=?29
ROOTS EQUAL:
                 5.0000+2.0000i
                 5.0000−2.0000i
                           Done
```

If you get `ERR: NONREAL ANS` you may need to manually change the mode to complex numbers by pressing MODE and selecting **a+bi**.

5.6 The Quadratic Formula (day 2) Homework

Use your `QUADSOLV` program on your calculator to solve the problems below.

Find the zeros of each function by using the Quadratic Formula.

1. $f(x) = 3x^2 - 10x + 3$
2. $g(x) = x^2 + 6x$
3. $h(x) = x(x-3) - 4$
4. $g(x) = -x^2 - 2x + 9$
5. $p(x) = 2x^2 - 7x - 8$
6. $f(x) = 7x^2 - 3$
7. $r(x) = x^2 + x + 1$
8. $h(x) = -x^2 - x - 1$
9. $f(x) = 2x^2 + 8$
10. $f(x) = 2x^2 + 7x - 13$

Find the type and number of solutions for each equation.

11. $2x^2 + 5 = 2x$
12. $2x^2 - 3x = 8$
13. $2x^2 - 16x = -32$
14. $4x^2 - 28x = -49$

Solve each equation by any method.

15. $x^2 = 7$
16. $x^2 - 4x - 21 = 0$
17. $6x^2 = 150$
18. $4x^2 - 4x - 1 = 0$

Solutions: 1. $x = 3, 0.3333$ 2. $x = 0, -6$ 3. $x = 4, -1$ 4. $x = -4.1623, 2.1623$ 5. $x = 4.4075, -0.9075$ 6. $x = \pm 0.6547$, 7. $x = -0.5 \pm 0.8660i$ 8. $x = -0.5 \pm 0.8660i$ 9. $x = \pm 2i$ 10. $x = 1.3423, -4.8423$ 11. Two non-real 12. Two non-real 13. One real 14. One real 15. $x = \pm i\sqrt{7} = \pm 2.6458i$ 16. $x = -3, 7$ 17. $x = \pm 5$ 18. 1.2071, -0.2071

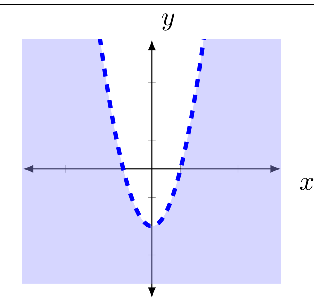
5.7 Solving Quadratic Inequalities

Objective: Solve quadratic inequalities by using tables and graphs.

Definition 5.7.1. A **quadratic inequality in two variables** can be written in one of the following forms, where a, b, and c are real numbers and $a \neq 0$. Its solution set is a set of ordered pairs (x, y) so that:

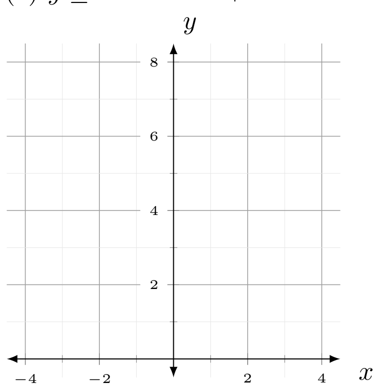
$$y < ax^2 + bx + c \qquad y > ax^2 + bx + c$$
$$y \leq ax^2 + bx + c \qquad y \geq ax^2 + bx + c$$

Graphing Quadratic Inequalities
To graph a quadratic inequality
1. Graph the parabola that defines the boundary.
2. Use a solid parabola for $y \leq$ and $y \geq$ and a dashed parabola for $y <$ and $y >$.
3. Shade above the parabola for $y >$ or $\geq$ and below the parabola for $y \leq$ or $<$.

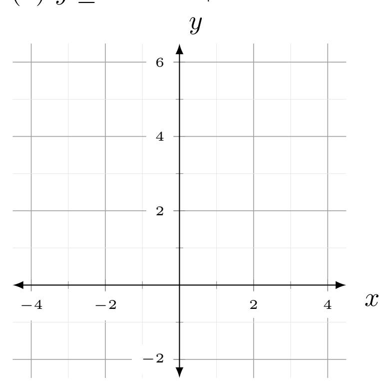


Example 1. Graph each of the following quadratic inequalities.

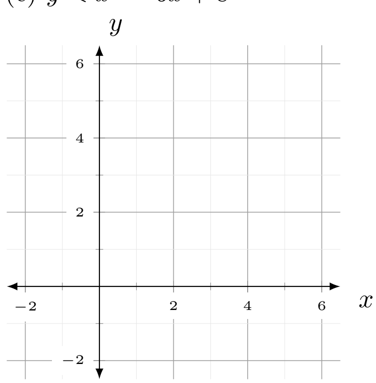
(a) $y \leq -2x^2 - 4x + 6$



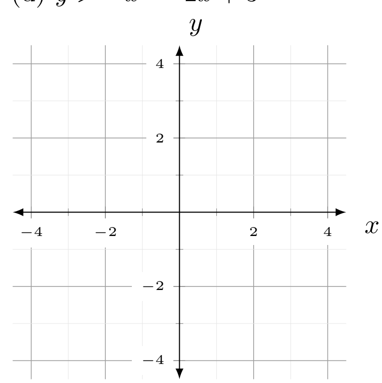
(b) $y \geq x^2 - 2x + 1$



(c) $y < x^2 - 6x + 8$

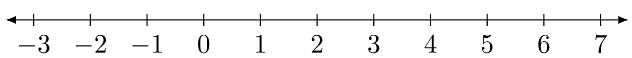


(d) $y > -x^2 - 2x + 3$

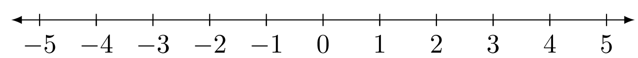


Example 2. Solve the inequalities using algebra.

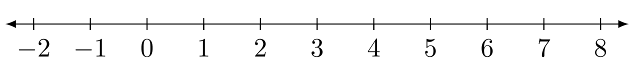
(a) $x^2 - 4x + 1 > 6$



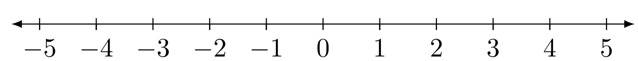
(b) $x^2 - x + 5 < 7$



(c) $x^2 - 6x + 8 \leq 3$

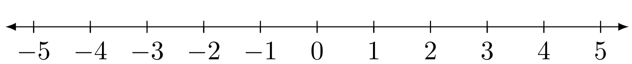


(d) $-2x^2 + 3x + 7 < 2$

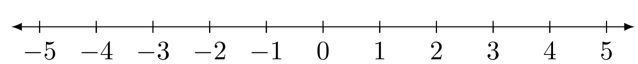


You Try It! 1. Solve the inequalities using algebra.

(a) $x^2 - 6x + 10 \geq 2$



(b) $x^2 - 1 < 3$

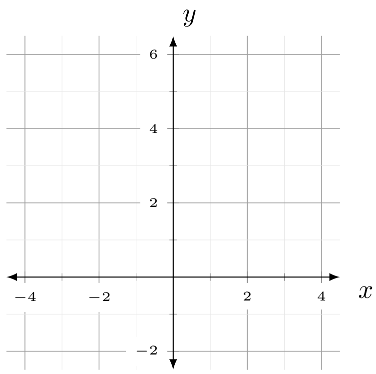


5.7 Homework: page 370 2-4, 8-10, 13-21 odds

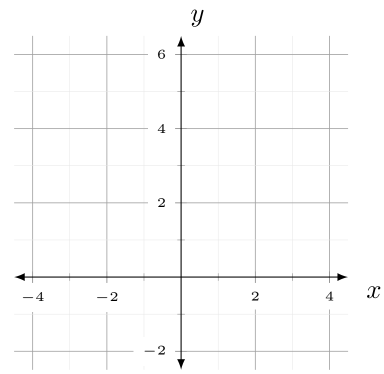
5.7 Solving Quadratic Inequalities Homework (day 2)

Graph each of the following quadratic inequalities. Objective: Solve quadratic inequalities by using tables and graphs.

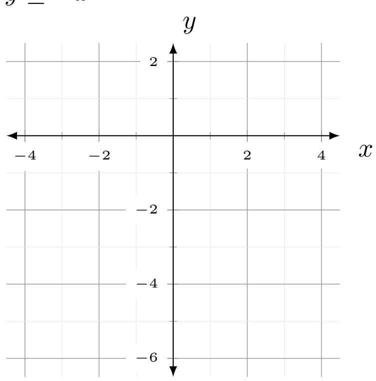
1. $y \geq 2x^2$



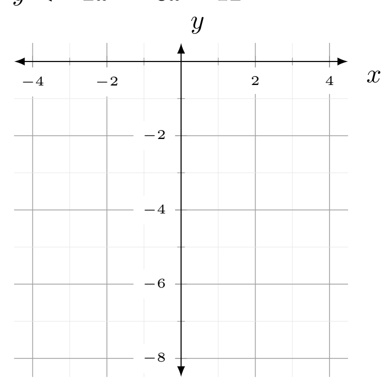
2. $y > 3x^2$



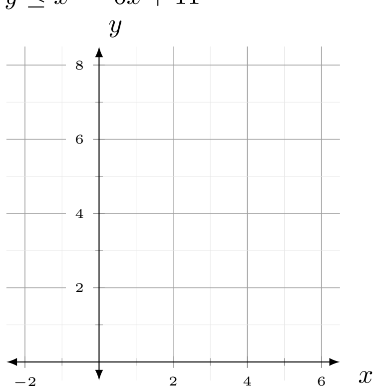
3. $y \leq -x^2$



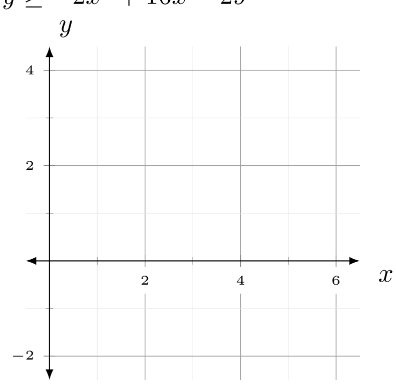
4. $y < -2x^2 - 8x - 12$



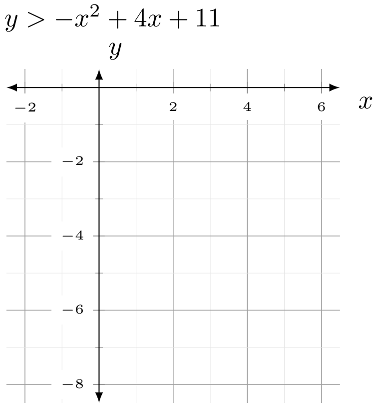
5. $y \leq x^2 - 6x + 11$



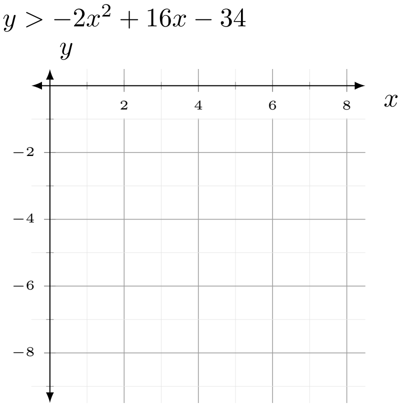
6. $y \geq -2x^2 + 16x - 29$



7. $y > -x^2 + 4x + 11$

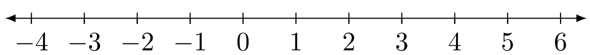


8. $y > -2x^2 + 16x - 34$

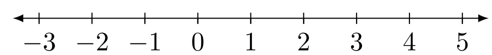


Solve the inequalities using using any method.
Write your answer on the **number line** and in **interval notation**.

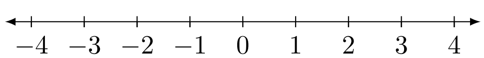
9. $x^2 - 3x - 10 < 0$



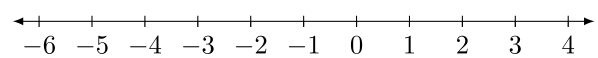
10. $x^2 - 3x - 4 < 0$



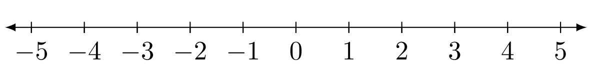
11. $-x^2 + x + 6 > 0$



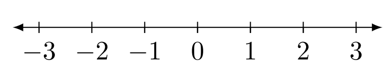
12. $x^2 + 3x \geq 10$



13. $x^2 - 12 \leq -x$



14. $x^2 + x - 2 > 0$



5.8 Curve Fitting with Quadratic

Objective: Use quadratic functions to model data.

x	-3	-2	-1	0	1	2	3
$f(x) = x^2$	9	4	1	0	1	4	9

1st differences -5 -3 -1 1 3 5
2nd differences 2 2 2 2 2

Constant 2nd Differences

Example 1. Determined whether each data set could represent a quadratic function. Explain.

(a)

x	0	2	4	6	8
y	12	10	9	9	10

(b)

x	-2	-1	0	1	2
y	1	2	4	8	16

You Try It! 2. Determined whether each data set could represent a quadratic function. Explain.

(a)

x	3	4	5	6	7
y	11	21	35	53	75

(b)

x	10	9	8	7	6
y	6	8	10	12	14

Definition 5.8.1. A **quadratic model** is a quadratic function that represents a real data set. Models are useful for making estimates.You can apply statistical methods to make a quadratic model for a given data set using **quadratic regression**.

Quadratic Regression on TI-Calculator

Step 1: Enter data into your TI-83/84 graphing calculator by selecting STAT and select `1:Edit...`. Enter the all the x-coordinates in the list `L1` and all the corresponding y-coordinates in the `L2`.

Step 2: Select STAT arrow over to `CALC` and arrow down to `5:QuadReg`. Next press 2nd STAT (`LIST` button) and select `L1` press , and again press 2nd STAT (`LIST`) select `L2` and finally press ENTER. Write down the the quadratic using coefficents a, b, and c.

Step 3: Press 2nd $y =$ (`STAT PLOT` button). Select `1:Plot1...Off`. Arrow over `On` and select it. Arrow down to `YList:` and press 2nd STAT (`LIST`) and select `L2`. On TI-83 you want to use 2nd 1 to get `L1` and 2nd 2 to get `L2`.

Step 4: Press $y =$ and type the quadratic equation found in Step 2 using the coefficents. Next press GRAPH.

Example 2. Write a quadratic function that fits each set of points.

(a) $(0,5)$, $(2,1)$, $(3,2)$

(b) $(0,-3)$, $(1,0)$, $(2,1)$

Example 3. Use a graphing calculator to find the quadratic of best fit of the following data.

(a) $(1,2)$, $(2,3)$, $(3,8)$, $(4,19)$, $(5,40)$

(b) $(1,1)$, $(2,-3)$, $(3,-12)$, $(4,-20)$, $(5,-24)$

$\boxed{\textbf{5.8 Homework}\text{: page 377 \quad 2-5, 7,9}}$

5.9 Operations with Complex Numbers

Objective: Perform operations with complex numbers

Definition 5.9.1. The **complex plane** is a set of coordinate axes in which the horizontal axis represents real numbers and the vertical axis represents imaginary numbers.

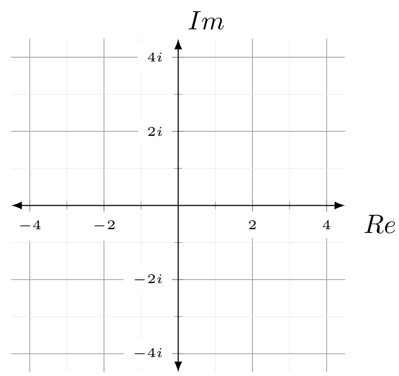
Example 1. Graph each complex number on the complex plane.

(a) $-3+0i$

(b) $-3i$

(c) $4+3i$

(d) $-2+4i$



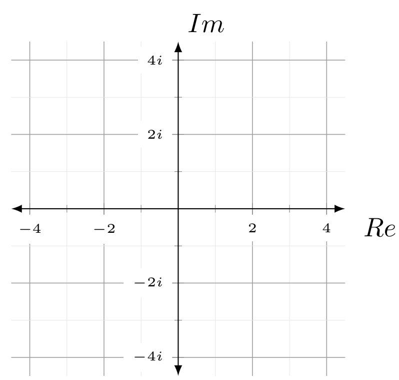
You Try It! 3. Graph each complex number on the complex plane.

(a) $3+0i$

(b) $2i$

(c) $-2-i$

(d) $3+2i$



Definition 5.9.2. The **absolute value of a complex number** $a+bi$ is the distance from the origin to the point (a,b) in the complex plane, and is denoted $|a+bi|$ and is calculated as shown below.

$$|a+bi| = \sqrt{a^2+b^2}$$

Example 2. Find each absolute value.

(a) $|-9+i|$

(b) $|6|$

(c) $|-4i|$

Example 3. Add or subtract. Write the result in the form $a + bi$

(a) $(-2 + 4i) + (3 - 11i)$

(b) $(4 - i) - (5 + 8i)$

Example 4. Multiply. Write the result in the form $a + bi$

(a) $(5 - 6i)(4 - 3i)$

(b) $(7 + 2i)(7 - 2i)$

Powers of i		
$i^1 = i$	$i^5 = i^4 \cdot i = 1 \cdot i = i$	$i^9 = i$
$i^2 = -1$	$i^6 = i^4 \cdot i^2 = 1 \cdot (-1) = -1$	$i^{10} = -1$
$i^3 = i^2 \cdot i = -i$	$i^7 = i^4 \cdot i^3 = 1 \cdot (-i) = -i$	$i^{11} = -i$
$i^4 = i^2 \cdot i^2 = -1 \cdot -1 = 1$	$i^8 = i^4 \cdot i^4 = 1 \cdot 1 = 1$	$i^{12} = 1$

Example 5. Simplify.

(a) $-3i^{12}$

(b) i^{25}

Example 6. Simplify.

(a) $\dfrac{3 + 7i}{8i}$

(b) $\dfrac{5 + i}{2 - 4i}$

$\boxed{\textbf{5.9 Homework}\text{: page 386 3-35 odds}}$

Chapter 5 Review (day 1)

Vertex Form of a Quadratic: $f(x) = a(x-h)^2+k$ with vertex (h,k)

Standard Form: $f(x) = ax^2 + bx + c$

Axis of Symmetry: $x = \dfrac{-b}{2a}$

Vertex: $\left(\dfrac{-b}{2a}, f\left(\dfrac{-b}{2a}\right)\right)$

y-intercept: $y = c$

Difference of Squares: $a^2 - b^2 = (a-b)(a+b)$

Sum/Diff of Cubes: $a^3 \pm b^3 = (a \pm b)(a^2 \mp ab + b^2)$

Quadratic Formula: $\dfrac{-b \pm \sqrt{b^2-4ac}}{2a}$

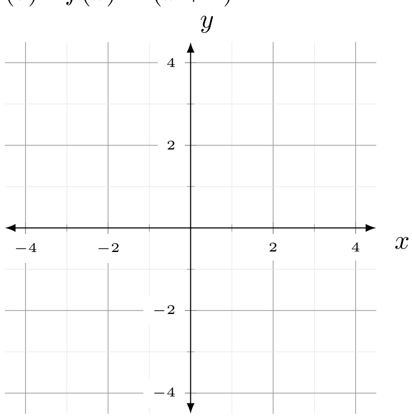
Discriminant: $b^2 - 4ac$

Complex Conjuagates: $a + bi$, $a - bi$

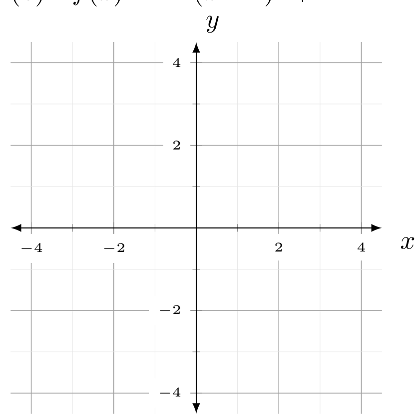
Absolute Value: $|a+bi| = \sqrt{a^2+b^2}$

1. Using the graph of $f(x) = x^2$ as a guide, describe the transformations, and then graph each function.

(a) $f(x) = (x+2)^2 - 1$



(b) $f(x) = -2(x-2)^2 + 4$



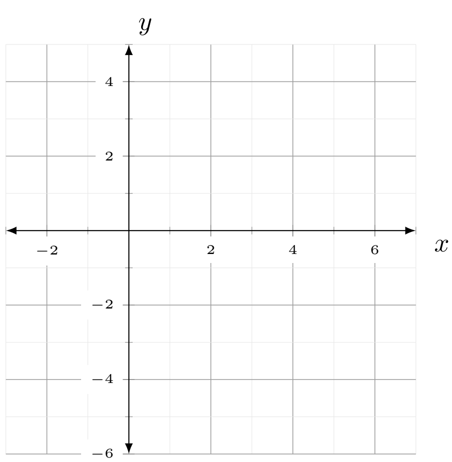
2. For each function **(a)** determine whether the graph opens upward or downward. **(b)** Find the axis of symmetry. **(c)** Find the vertex. **(d)** Find the y-intercept. **(e)** Graph the function.

$$f(x) = x^2 - 6x + 4$$

(a) Opens: (c) Vertex:

(b) Axis: (d) y-int:

(e)

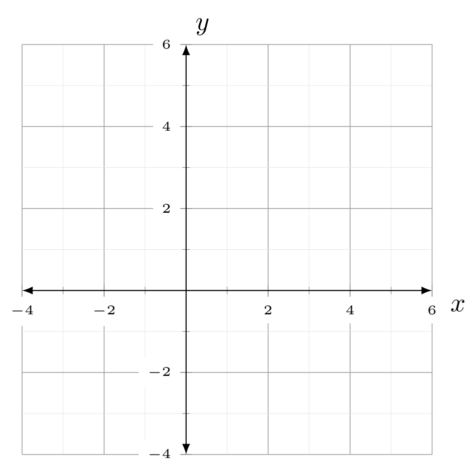


$$g(x) = -2x^2 + 4x + 3$$

(a) Opens: (c) Vertex:

(b) Axis: (d) y-int:

(e)



Find the zeros of each function by factoring.

3. $4x^2 - 28x + 49$

4. $6x^2 + 7x - 49$

5. $4n^2 - 48n - 25$

6. $2x^2 - 5x + 2$

7. $4n^2 - 6n - 4$

8. $3x^2 + x - 4$

Solve each equation by completing the square.

11. $x^2 + 8x = -5$

12. $x^2 - 10x = 21$

Simplify.

13. $4x^2 + 196 = 0$

14. $3x^2 + 30 = -45$

Find the complex conjugate.

15. $4 - 7i$

16. $3i - 1$

Simplify.

17. $(3 + 2i)(4 - 5i)$

18. $(1 + 3i) - (i - 4)$

19. $\dfrac{4 - 3i}{1 - 6i}$

20. i^{103}

Chapter 5 Review (day 2)

Find the minimum or maximum value of each function. Then state the domain and range of the function.

21. $f(x) = -3x^2 - 2x + 5$

    Max/Min:

    Domain:

    Range:

22. $f(x) = x^2 - 6x - 2$

    Max/Min:

    Domain:

    Range:

Find the roots of the following quadratic equations by factoring.

23. $0 = -2x^2 + 5x + 12$

24. $0 = 3x^2 + 11x - 4$

25. $-2x^2 - 3x + 2 = 0$

26. $-4x^2 - 2x + 12 = 0$

Find the zeros of $f(x)$ by using the Quadratic Formula.

27. $f(x) = 2x^2 + 10x + 25$

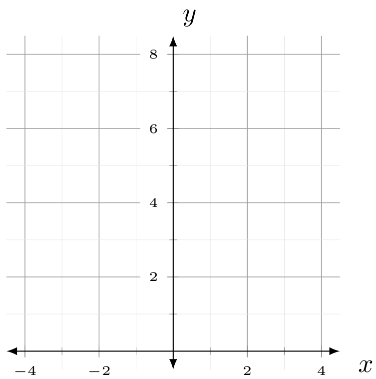
28. $g(x) = 3x^2 + 20x - 7$

29. $h(x) = -4x^2 + 8x - 5$

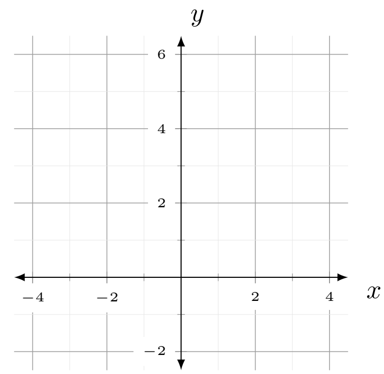
30. $f(x) = x^2 - 17x + 60$

Graph each of the following quadratic inequalities.

31. $y \geq -2x^2 + 8$

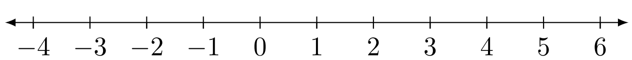


32. $y > x^2 + 2x + 3$

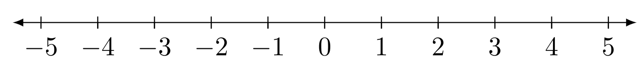


Solve the inequalities using algebra.

33. $x^2 - 2x + 1 > 16$



34. $-x^2 - x + 5 \leq 11$



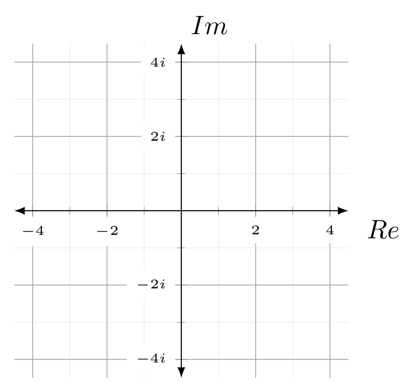
35. Graph each complex number on the complex plane.

(a) $3 + 2i$

(b) $2 - 2i$

(c) $4 + 3i$

(d) $-3 - 2i$



Find each absolute value.

36. $|8 + 2i|$

37. $|6 - 3i|$

38. $|4 - 5i|$