Chapter 6 Polynomial Functions

6.0 Pre-assessment

Match each of the vocabulary terms on the left with the appropriate letter and definition on the right.

1. coefficent
2. like terms
3. root of an equation
4. x-intercept
5. maximum of a function

A. the y-value of the highest point of the graph of the function

B. the horizontal number line that divides the coordinate plane

C. the numerical factor in a term

D. a value of the variable that makes the equation true

E. terms that contain the same variables raised to the same powers

F. the x-coordinate of a point where the graph intersects the x-axis.

Evaluate each expression.

6. 6^4
7. -5^4
8. $(-1)^5$
9. $\left(-\frac{2}{3}\right)^2$

Evaluate each expression for the given value of the variable.

10. $x^4 - 5x^2 - 6x - 8$ for $x = 3$
11. $2x^3 - 3x^2 - 29x - 30$ for $x = -2$
12. $2x^3 - x^2 - 8x + 4$ for $x = \frac{1}{2}$
13. $3x^4 + 5x^3 + 6x^2 + 4x - 1$ for $x = -1$

Multiply or divide.

14. $2x^3y \cdot 4x^2$
15. $-a^2b \cdot ab^4$
16. $\dfrac{-7t^4}{3t^2}$
17. $\dfrac{3p^3q^2r}{12pt^4}$

10. 10 11. 0 12. 0 13. -1 14. $8x^5y$ 15. $-a^3b^5$ 16. $-\frac{7}{3}t^2$ 17. $\frac{p^2q^2r}{4t^4}$

1. C 2. E 3. D 4. F 5. A 6. 1296 7. -625 8. -1 9. 4/9

6.1 Polynomials

Objective: Identify and classify polynomials

Definition 6.1.1. A **monomial** is a number or a product of numbers and variables with whole number exponents. A **polynomial** is a monomial or a sum or difference of monomials. The **degree of a monomial** is the sum of the exponents of the variables.

Polynomials:	$3x^4$	$2z^{12} + 9z^3$	$\frac{1}{2}a^7$	$0.15x^{101}$	$3t^2 - t^3$
Not Polynominals:	3^x	$\lvert 2b^3 - 6b \rvert$	$\frac{8}{5y^2}$	$\frac{1}{2}\sqrt{x}$	$m^{0.75} - m$

Example 1. Identify the degree of each monomial.

(a) x^4

(b) 12

(c) $4a^2b$

(d) x^3y^4z

Definition 6.1.2. The **degree of a polynomial** is given by the term with the greatest degree. A polynomial is in standard when its terms are written in decending order of degree. The **leading coefficent** the coefficent of the first term in standard form.

$$5x^{3} + 8x^2 + 3x - 17$$

Definition 6.1.3. A polynomial with two terms is called a **binomial**, and a polynomial with three terms is called a **trinomial**.

Classifying Polynomials by Degree		
Name	**Degree**	**Example**
Constant	0	-9
Linear	1	$x - 4$
Quadratic	2	$x^2 + 3x - 1$
Cubic	3	$x^3 + 2x^2 + x + 1$
Quartic	4	$2x^4 + x^3 + 3x^2 + 4x - 1$
Quintic	5	$7x^5 + x^4 - x^3 + 3x^2 + 2x - 1$

Example 2. Rewrite each polynomial in standard form. The identify the leading coefficent, degree, and number of terms. Name the polynomial.

(a.) $2x + 4x^3 - 1$

Standard Form:

Leading Coefficent:

Degree:

Terms:

Name:

(b.) $7x^3 - 11x + x^5 - 2$

Standard Form:

Leading Coefficent:

Degree:

Terms:

Name:

Example 3. Add or subtract. Write you answer in standard form.

(a.) $(3x^2 + 7 + x) + (14x^3 + 2 + x^2 - x)$

(b.) $(1 - x^2) - (3x^2 + 2x - 5)$

6.1 (day 1) Homework: page 410 1-13 all Adv. 47-49

Polynomials (day 2)

Objective: Evaluate and Graph Polynomials

You Try It! 1. Add or subtract. Write your answer in standard form.

(a) $(-36x^2 + 6x - 11) + (6x^2 + 16x^3 - 5)$

(b) $(5x^3 + 12 + 6x^2) + (15x^2 + 3x - 2)$

Example 4. Cardiac output is the amount of blood pumped through the heard. The output is measured by a technique called dye dilution. A doctor injects dye into a vein near the heart and measured the amount of dye in the arteries over time.

The cardiac output of a particular patient can be approximated by the function

$$f(t) = 0.0056t^3 - 0.22t^2 + 2.33t,$$

where $f(t)$ represents the concentration of dye (in milligrams per liter).

(a) Evaluate $f(t)$ for $t = 0$ and $t = 3$.

(b) Describe what the values of the function in part (a) represent.

Example 5. Graph each polynomial on a graphing calculator. Describe the graph, and identify the number of real zeros.

(a) $f(x) = x^3 - x$

(b) $f(x) = -3x^3 + 2x + 1$

(c) $h(x) = x^4 - 8x^2 + 1$

(d) $k(x) = x^4 + x^3 - x^2 + 2x - 3$

6.1 (day 2) Homework: page 410 31-39 all Adv. 41, 42

6.2 Multiplying Polynomials

Objective: To Multiply Polynomials and Binomial Expansion

Example 1. Find each product.

(a) $3x^2(x^3+4)$

(b) $ab(a^3+3ab^2-b^3)$

Example 2. Find each product.

(a) $(x-2)(1+3x-x^2)$

(b) $(x^2+3x-5)(x^2-x+1)$

Binomial Expansion

Example 3. Find the product.

$(x+y)^3$

Binomial Expansion		Pascal's Triangle (Coefficients)
$(a+b)^0 =$	1	1
$(a+b)^1 =$	$1a + 1b$	1 1
$(a+b)^2 =$	$1a^2 + 2ab + 1b^2$	1 2 1
$(a+b)^3 =$	$1a^3 + 3a^2b + 3ab^2 + 1b^3$	1 3 3 1
$(a+b)^4 =$	$1a^4 + 4a^3b + 6a^2b^2 + 4ab^3 + 1b^4$	1 4 6 4 1
$(a+b)^5 =$	$1a^5 + 5a^4b + 10a^3b^2 + 10a^2b^3 + 5ab^4 + 1b^5$	1 5 10 10 5 1

Example 4. Expand each expression using Pascal's triangle.

(a) $(y-3)^4$

(b) $(4z+5)^3$

6.2 Homework: page 418 1-13 odd, Adv. 52, 56

6.3 Dividing Polynomials

Objective: Use long and synthetic division to divide polynomials.

Example 1. Divide using arithmetic long division.

(a) $12\overline{)277}$

You Try It! 2. Divide.

(b) $8\overline{)347}$

Example 2. Divide using long division.

(a) $(4x^2 + 3x^3 + 10) \div (x - 2)$

(b) $(15x^2 + 8x - 12) \div (3x + 1)$

Example 3. Divide using synthetic division.

(a) $(4x^2 - 12x + 9) \div \left(x + \frac{1}{2}\right)$

(b) $(6x^2 - 5x - 6) \div (x + 3)$

Example 4. Use synthetic substitution to evaluate the polynomial for the given value.

(a) $P(x) = x^3 - 4x^2 + 3x - 5$ for $x = 4$

(b) $P(x) = 4x^4 + 2x^3 + 3x + 5$ for $x = -\frac{1}{2}$

6.3 (day 1) Homework: page 426 1-12 all, Adv. 53, 57

6.3 (day 2)

Objective: Use long and synthetic division to divide polynomials.

You Try It! 3. Divide using long division.

(a) $(2x^2 + 7x + 7) \div (x + 2)$

(b) $(x^2 + 5x - 28) \div (x - 3)$

Example 5. Divide using synthetic division.

(a) $(x^2 - 3x - 18) \div (x - 6)$

(b) $(x^4 - 7x^3 + 9x^2 - 22x + 25) \div (x + 3)$

Remainder Theorem	
Theorem	**Example**
If the polynomial function $P(x)$ is divided by $x - \mathbf{a}$, then the remainder r is $P(\mathbf{a})$.	Divide $x^3 - 4x^2 + 5x + 1$ by $x - 3$ $\begin{array}{c\|cccc} \mathbf{3} & 1 & -4 & 5 & 1 \\ & \downarrow & 3 & -3 & 6 \\ \hline & 1 & -1 & 2 & \boxed{\mathbf{7}} \end{array}$ $P(\mathbf{3}) = \mathbf{7}$

Example 6. Use synthetic substitution to evaluate the polynomial for the given value.

(a) $P(x) = x^3 + 3x^2 + 4$ for $x = -3$

(b) $P(x) = 5x^2 + 9x + 3$ for $x = \dfrac{1}{5}$

6.3 (day 2) Homework: page 426   13, 17, 19, 23, 25, 27, 43, 44

6.2 & 6.3 Review

Objective: Multiply and Divide Polynomials

Find each product.

1. $3x^2(2x^2 + 9x - 6)$
2. $(2x + 5y)(3x^2 - 4xy + 2y^2)$

Expand each expression. (Use Pascal's triangle)

3. $(x - 3y)^3$
4. $(x - 2)^5$

Divide.

5. $7\overline{)647}$
6. $9\overline{)3452}$

Use long division to divide the polynomials. Write as Quotient + Remainder/Divisor.

7. $(2x^2 + 3x - 20) \div (x - 2)$
8. $(x^4 + 6x^3 + 6x^2) \div (x + 5)$

Use synthetic division to divide the polynomials. Write as Quotient + Remainder/Divisor.

9. $x^4 - 3x^3 - 7x - 14) \div (x - 4)$

10. $(x^2 + 9x + 6) \div (x + 8)$

Use synthetic substitution (The Remainder Theorem) to evaluate the polynomial for the given value.

11. $P(x) = 4x^3 - 5x^2 - x + 2$ for $x = -1$

12. $P(x) = 25x^2 - 16$ for $x = \dfrac{4}{5}$

13. $P(x) = 4x^3 - 5x^2 - x + 2$ for $x = -1$

14. $P(x) = 25x^2 - 16$ for $x = \dfrac{4}{5}$

6.4 Factoring Polynomials

Objective: Use the Factor Theorem to determine factors of a polynomial.

Factor Theorem	
Theorem	**Example**
For any polynomial $P(x)$, $(x-a)$ is a factor of $P(x)$ if and only if $P(a)=0$.	Because $P(1)=1^2-1=0$, $(x-1)$ is a factor of $P(x)=x^2-1$.

Example 1. Determine whether the given binomial is a factor of the polynomial $P(x)$.

(a) $(x-3)$; $P(x)=x^2+2x-3$

(b) $(x+4)$; $P(x)=2x^4+8x^3+2x+8$

Example 2. Factor by grouping.

(a) $x^3+3x^2-4x-12$

(b) $x^3-2x^2-9x+18$

You Try It! 4. Factor by grouping

(a) $2x^3+x^2+8x+4$

(b) $8y^3-4y^2-50y+25$

6.4 (day 1) Homework: page 433 1-9 all, 17-19, Adv. 41, 43

6.4 (day 2) Factoring

Objective: Factor the sum and difference of two cubes.

Factoring The Sum and Difference of Two Cubes	
Method	**Algebra**
Sum of two cubes	$a^3+b^3=(a+b)(a^2-ab+b^2)$
Difference of two cubes	$a^3-b^3=(a-b)(a^2+ab+b^2)$

S · **O** · **A** · **P**

same opposite always positive

$$(a \pm b)^3 = (a \pm b)(a^2 \mp ab + b^2)$$

Example 3. Factor each expression using sum or difference of cubes.

(a) $5x^4+40x$

(b) $8y^3-27$

You Try It! 5. Factor each expression using sum or difference of cubes.

(a) $8+z^6$

(b) $2x^5-16x^2$

6.4 (day 2) Homework: page 433 10-15 all, 21-31 odds

6.4 Review of Factoring

Objective: Factor using the sum and difference of two cubes, difference of square, grouping, and GCF.

Factor using the greatest common factor (GCF).

1. $2x^5 - 6x^3$
2. $5x^3 - 10x$
3. $14x^3 - 49x^2 - 28x$
4. $27x^5 - 18x^4 + 9x^3$

Factor using difference of squares.

5. $q^2 - r^2$
6. $25a^2 - 64b^2$
7. $81x^2 - 100y^2$
8. $x^4 - y^4$
9. $a^6 - b^6$
10. $4x^4 - 9y^6$

Factor using sum and difference of cubes.

5. $x^3 - y^3$
6. $r^3 + s^3$
7. $8a^3 - 27b^3$
8. $64x^3 + 125y^3$
9. $a^6 - b^6$
10. $x^6 + y^6$

Factor using grouping.

5. $6x^3 + 2x^2 + 9x + 3$
6. $7x^3 - 35x^2 + 8x - 40$
7. $4x^3 + 8x^2 - 9x - 18$
8. $16x^3 - 64x^2 - 25x + 100$

6.4 Factoring Review Homework: page 433 20-30 evens, 33-38 all

6.5 Finding Real Roots of Polynomial Equations

Objective: Identify the multiplicity of roots, Use the Rational Root Theorem to solve polynomial equations.

Example 1. Solve each polynomial equation by factoring. Check your answer using **Desmos**.

(a) $3x^5 + 18x^4 + 27x^3 = 0$

(b) $x^4 - 13x^2 = -36$

You Try It! 6. Solve each polynomial equation by factoring.

(a) $2x^6 - 10x^5 - 12x^4 = 0$

(b) $x^3 - 2x^2 - 25x = -50$

Definition 6.5.1. The **multiplicity** of root r is the number of times that $x - r$ is a factor of $P(x)$. Even multiplicity means the graph "touches" the x-axis at the root but does not cross. Odd multiplicity means the graph crosses the x-axis at the root.

Example 2. Identify the roots of each equation. State the multiplicity of each root.

(a) $x^3 - 9x^2 + 27x - 27 = 0$

(b) $-2x^3 - 12x^2 + 30x + 200 = 0$

6.5 Homework (day 1): page 442 2-9 all, 15-21 odds

6.5 Finding Real Roots of Polynomial Equations (day 2)

Theorem 6.5.1. (Rational Root Theorem)If the polynomial $P(x)$ has integer coefficents, then every rational root of the polynomial equation $P(x) = 0$ can be written in the form $\frac{p}{q}$, where p is a factor of the constant term of $P(x)$ and q is a factor of the leading coefficient of $P(x)$.

Example 3. Owen, a popcorn producer, is designing a new box for popcorn distribution. The marketing department has required a box with the width 2 inches less than the length and with the height 5 inches greater than the length. The volume of each box must be 24 cubic inches. What is the length of the box?

Example 4. Identify all of the real roots of:

(a) $4x^4 - 21x^3 + 18x^2 + 19x - 6 = 0$

(b) $2x^3 - 3x^2 - 10x - 4 = 0$

6.5 Homework (day 2): page 442 8, 9 ,10, 22, 25

6.6 Fundamental Theorem of Algebra

Objective: Use the Fundamental Theorem of Algebra and corollary to write a polynomial equation given roots.

The following statements are equivalent:
A real number r is a root of the polynomial equation $P(x) = 0$.
$P(r) = 0$
r is an x-intercept of the graph of $P(x)$.
$x - r$ is a factor of $P(x)$.
When you divide the rule for $P(x)$ by $x - r$, the remainder is 0.
r is a zero of $P(x)$

Example 1. Write the simplest polynomial function with the given zeros.

(a) $-3, \frac{1}{2}$, and 1

(b) $-2, 2$, and 4

Theorem 6.6.1. (The Fundamental Theorem of Algebra) Every polynomial function of degree $n \geq 1$ has at least one zero, where a zero may be a complex number.

Theorem 6.6.2. (FTA Corollary) Every polynomial function of degree $n \geq 1$ has exactly n zeros, including multiplicities.

Example 2. Solve each polynomial by finding all roots.

(a) $x^4 + x^3 + 2x^2 + 4x - 8 = 0$

(b) $x^4 + 4x^3 - x^2 + 16x - 20 = 0$

6.6 Homework (day 1): page 449 1-6, 11-16

6.6 Fundamental Theorem of Algebra (day 2)

Objective: Use the Fundamental Theorem of Algebra and corollary to write a polynomial equation given roots.

Theorem 6.6.3. If $\mathbf{a} + \mathbf{bi}$ is a root of a polynomial equation with real-number coefficients, then $\mathbf{a} - \mathbf{bi}$ is also a root.

Example 3. Write the simplest polynomial function with the given zeros.

(a) $1 + i$, $\sqrt{2}$, and 3

(b) $2i$, $1 + \sqrt{2}$ and 3

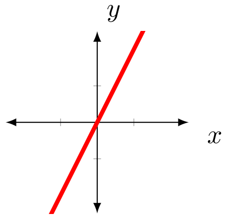
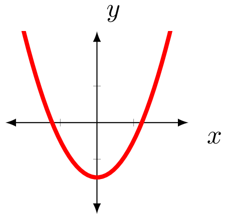
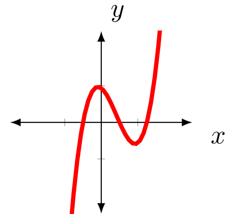
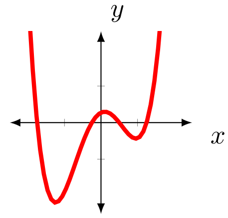
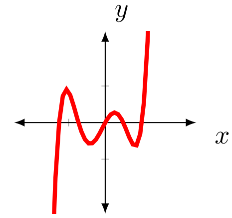
Example 4. An engineering class is designed model rockets for competition. The body of the rocket must be cylindrical with a cone-shaped top. The cylindrical portion must be 60 cm tall, and the height of the cone must be twice the radius. The volume of the paylode region must be $558\pi cm^3$ in order to hold the cargo. Find the radius of the rocket.

Example 5. A grain silo is in the shape of a cylinder with a hemisphere on top. The cylinder is 20 ft tall. The volume of the silo is 2106π cubic feet. Find the radius of the silo.

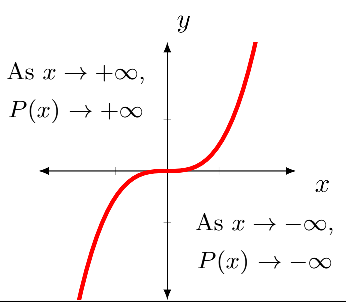
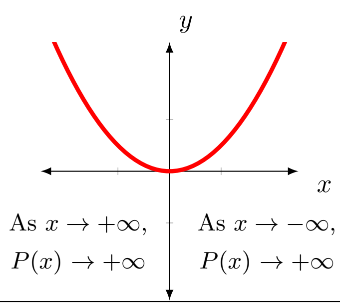
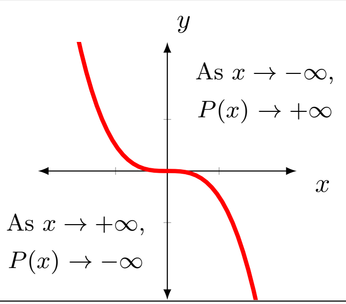
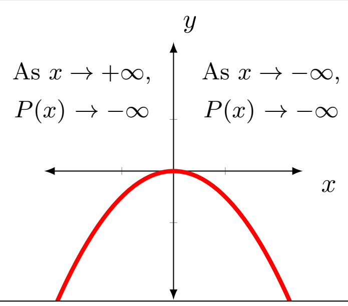
$\boxed{\textbf{6.6 Homework (day 2)}\text{: page 449 7-10, 23}}$

6.7 Investing Graphs of Polynomial Functions

Objective: Use properties of end behavior to analyze, describe, and graph polynomial functions.

Graphs of Polynomial Functions				
Linear function Degree 1	Quadratic function Degree 2	Cubic function Degree 3	Quartic function Degree 4	Quintic function Degree 5
				

Definition 6.7.1. **End behavior** is a description of the values of a function (y-values) as x approaches positive infinity ($x \longrightarrow \infty$) or negative infinity ($x \longrightarrow \infty$).

Polynomial End Behavior		
P(x) has..	**Odd Degree**	**Even Degree**
Leading Coefficient $a > 0$		
Leading Coefficient $a < 0$		

Example 1. Identify the leading coefficient, degree and end behavior.

(a) $P(x) = -4x^3 - 3x^2 + 5x + 6$

Degree:

Leading Coefficient

End Behavior: As $x \longrightarrow +\infty$

As $x \longrightarrow -\infty$

(b) $R(x) = x^6 - 7x^5 + x^3 - 2$

Degree:

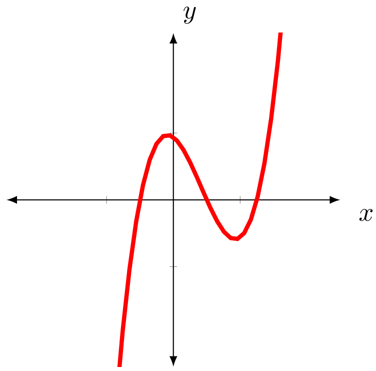
Leading Coefficient

End Behavior: As $x \longrightarrow +\infty$

As $x \longrightarrow -\infty$

Example 2. Identify whether the function graphed has an odd or even degree and a positive or negative leading coefficient.

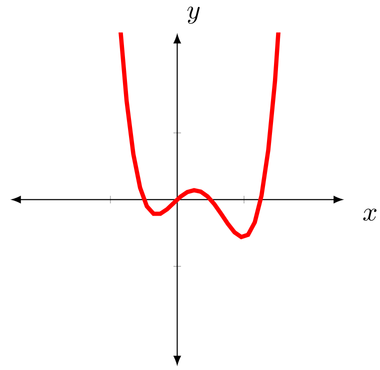
(a)



Degree:

Leading Coefficient:

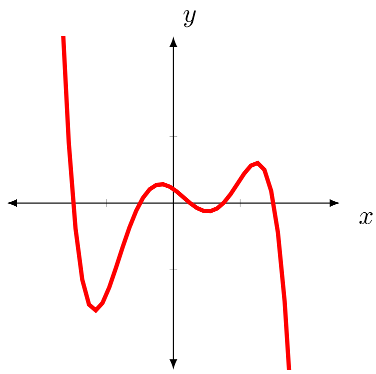
(b)



Degree:

Leading Coefficient:

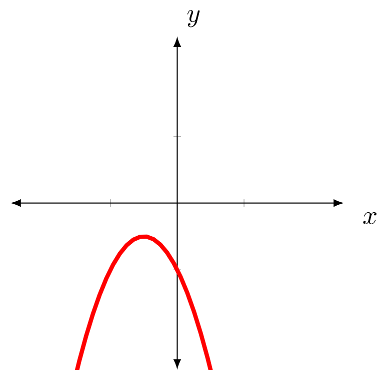
(c)



Degree:

Leading Coefficient:

(d)

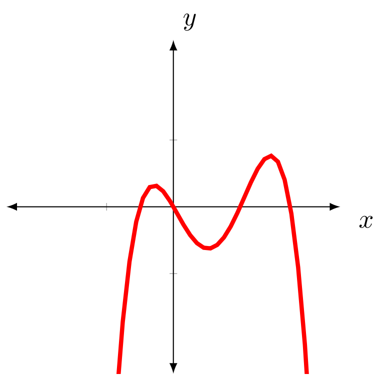


Degree:

Leading Coefficient:

You Try It! 7. Identify whether the function graphed has an odd or even degree and a positive or negative leading coefficient.

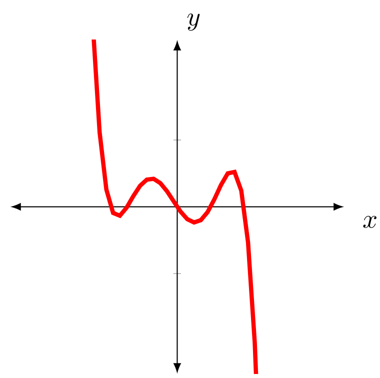
(a)



Degree:

Leading Coefficient:

(b)



Degree:

Leading Coefficient:

6.7 Homework: page 457   2-9, 15-21 odd

6.8

Objective:

Definition 6.8.1. Definition

Example 1. Example

$\boxed{\textbf{6.8 Homework}\text{: page}}$

6.9

Objective:

Definition 6.9.1. Definition

Example 1. Example

6.9 Homework: page