Chapter 14 Test

Multivariable Calculus Name: ______________________

Calculator and 3×5 note card permitted Date: ______________________

Instructions: Show all of your work and justify all steps. Correct answers without supported work receive reduced credit. Write legibly. (90 points)

True/False Questions [2 points each]

(1) ____ The Jacobian for spherical coordinates is $J(\rho, \theta, \phi) = \rho \sin \phi$.

(2) ____ The integral $\int_0^{2\pi} \int_0^2 \int_r^2 dz\, dr\, d\theta$ represents the volume enclosed by the cone $z = \sqrt{x^2 + y^2}$ and the plane $z = 2$.

(3) ____ If G is the rectangular box defined by $a \le x \le b, c \le y \le d, k \le z \le l$, then

$$\iiint_R f(x)g(y)h(z)\, dV = \left[\int_a^b f(x)\, dx\right]\left[\int_c^d g(y)\, dy\right]\left[\int_k^l h(z)\, dz\right]$$

(4) ____ The following parametric equations represent a sphere.

$$x(u,v) = u, \quad y(u,v) = \sqrt{4-u^2}\cos(v), \quad z(u,v) = \sqrt{4-u^2}\sin(v)$$

(5) ____ The center of gravity of a homogeneous lamina in a plane is located at the lamina's centriod.

Multiple Choice Questions [4 points each]

(6) ____ Find the surface area of the portion of the plane $x + y + z = 3$ in the first octant.

A. $9\sqrt{3}$ B. $\frac{9}{2}\sqrt{3}$ C. $\frac{9}{2}\sqrt{2}$ D. $9\sqrt{2}$

(7) ____ Find the Jacobian of the following change of variables,

$$x(\alpha, \beta) = 5\alpha \sin\beta, \qquad y(\alpha, \beta) = 4\alpha\cos\beta$$

A. $\frac{\partial(x,y)}{\partial(\alpha,\beta)} = 9\alpha$ B. $\frac{\partial(x,y)}{\partial(\alpha,\beta)} = -20\alpha\sin\beta\cos\beta$

C. $\frac{\partial(x,y)}{\partial(\alpha,\beta)} = -20\alpha$ D. $\frac{\partial(x,y)}{\partial(\alpha,\beta)} = -\alpha$

(8) ______ Evaluate $\displaystyle\iiint_E \sqrt{x^2+y^2}\,dV$ where E is the region that lies inside the cylinder $x^2+y^2=4$ and between the planes $z=-4$ and $z=7$.

A. 252.61 B. 184.31 C. 168.58 D. 126.99

(9) ______ Given $\displaystyle\int_0^2 \int_0^{\sqrt{4-x^2}} \sin(x^2+y^2)\,dy\,dx$, when changing to polar coordinates what are the limits for θ

A. $\left[0, \frac{\pi}{2}\right]$ B. $[0, \pi]$ C. $[0, 2\pi]$ D. $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

(10) ______ What is the principal unit normal vector to the parametric surface,

$$x(u,v)=u, \quad y(u,v)=v, \quad z(u,v)=u^2+v^2, \qquad \textit{Recall}\text{: } \mathbf{n} = \frac{\frac{\partial \mathbf{r}}{\partial u} \times \frac{\partial \mathbf{r}}{\partial v}}{\left\| \frac{\partial \mathbf{r}}{\partial u} \times \frac{\partial \mathbf{r}}{\partial v} \right\|}$$

A. $\dfrac{\langle -2u, 2v, 1\rangle}{\sqrt{4u^2+4v^2+1}}$ B. $\langle -2u, -2v, 1\rangle$

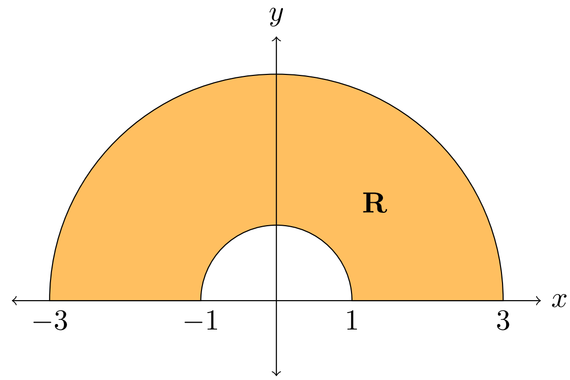
C. $\dfrac{\langle -2u, -2v, 1\rangle}{\sqrt{4u^2+4v^2+1}}$ D. $\langle -2u, 2v, 1\rangle$

Short Answer Questions

(11) Find the tangent plane to the parametric surface in #10 at the point $(1,2,5)$.

[4 points]

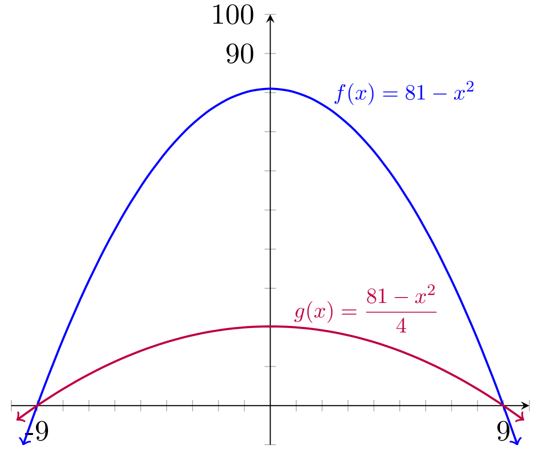
(12) Find the value of $\displaystyle\iint_R f(x,y)\,dA$ for the function $f(x,y) = \sqrt{x^2+y^2}$, given the semi-annulus region R shown below. [6 points]



(13) Use spherical coordinates to find the volume of the solid that lies within the sphere $x^2+y^2+z^2=9$ above the xy-plane and below the cone $z=\sqrt{x^2+y^2}$ [10 points]

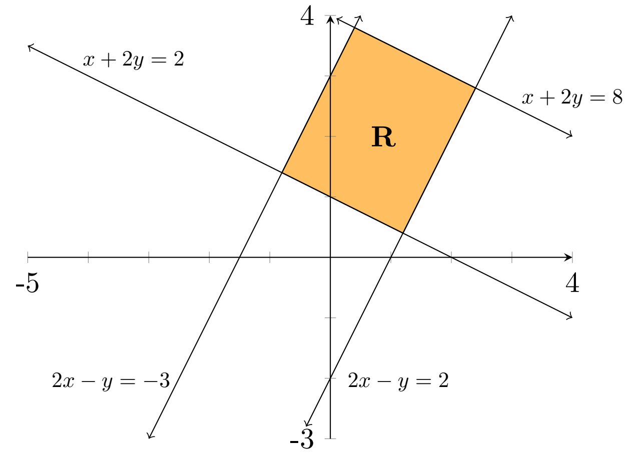
(14) Find the centroid for the region bounded by the graphs of $f(x) = 81 - x^2$ and $g(x) = \dfrac{81 - x^2}{4}$. Integral set-up must be shown, you may use a calculator for integral evaluation.

[10 points]



(15) Use the transformation $u(x,y) = 2x - y$ and $v(x,y) = x + 2y$ to find $\displaystyle\iint_R \frac{2x-y}{x+2y}\,dA$. Where R is the region shown in the graph below. [10 points]

Hint: $\quad x = \dfrac{2}{5}u + \dfrac{1}{5}v, \qquad y = -\dfrac{1}{5}u + \dfrac{2}{5}v$



(16) Find the average value of $f(x,y) = 8y^2$ over the triangle with vertices $(0,0)$, $(2,0)$, $(2,3)$. [5 points]

(17) Set up, **do not evaluate**, the triple integral $\iiint\limits_R y\,dV$ where R is the "ice-cream cone" region enclosed by a sphere $x^2 + y^2 + z^2 = 4$ centered at the origin and the cone $z = \sqrt{3x^2 + 3y^2}$:

a. Use rectangular coordinates. [5 points]

$$\iiint\limits_R y\,dV =$$

b. Use cylindrical coordinates. [5 points]

$$\iiint\limits_R y\,dV =$$

c. Use spherical coordinates. [5 points]

$$\iiint\limits_R y\,dV =$$

(18) **BONUS: Points are for correct answer only. You must show all work to recieve credit.**

The function $f(x, y) = ax^2 + 4y$ has an average value of 2 on the triangle with vertices $(0, 0), (0, 1)$, and $(1, 0)$. What can you say about the constant a? [2 points]