Chapter 6 Systems of Equations and Inequalities

6.1 Solving Systems by Graphing

Definition 6.1.1. A **system of linear equations** is a set of two or more linear equations containing two or more variables. A **solution of a system of linear equations** with two variables is an ordered pair that satisfies each equation in the system.

Example 1. Tell whether the ordered pair is a solution of the given system.

(a) (4,1); $\begin{cases} x + 2y = 6 \\ x - y = 3 \end{cases}$

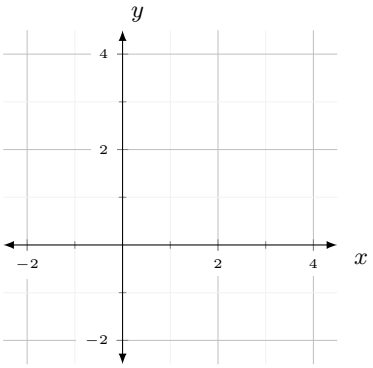
(b) (-1,2); $\begin{cases} 2x + 5y = 8 \\ 3x - 2y = 5 \end{cases}$

(c) (1,3); $\begin{cases} 2x + y = 5 \\ -2x + y = 1 \end{cases}$

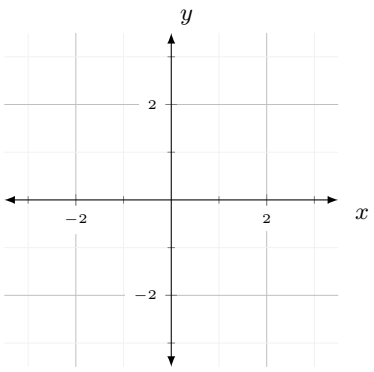
(d) (2,-1); $\begin{cases} x - 2y = 4 \\ 3x + y = 6 \end{cases}$

Example 2. Solve each system by graphing. Check your answer.

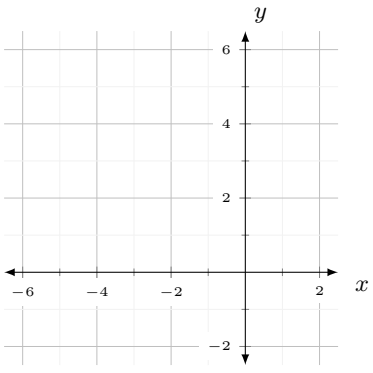
(a) $\begin{cases} y = x - 3 \\ y = -x - 1 \end{cases}$



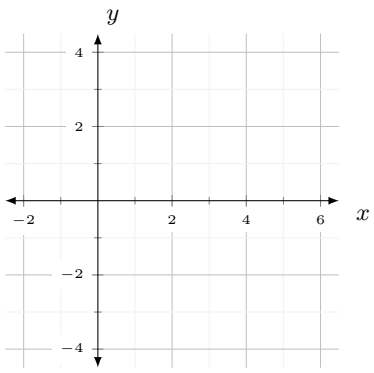
(c) $\begin{cases} x + y = 0 \\ y = -\frac{1}{2}x + 1 \end{cases}$



(b) $\begin{cases} y = -2x - 1 \\ y = x + 5 \end{cases}$



(d) $\begin{cases} y = \frac{1}{3}x - 3 \\ 2x + y = 4 \end{cases}$



6.1 Homework : page 386, 1-15 odds

6.2 Solving Systems by Substitution

Steps for Solving Systems of Equations by Substitution

Step 1 Solve for one variable in at least one of the equations.

Step 2 Substitute the resulting expression into the other equation.

Step 3 Solve that equation to get the value of the first variable.

Step 4 Substitute that value into one of the original equations and solve.

Step 5 Write the values from **Step 3** and **Step 4** as an ordered pair, (x, y), and check.

Example 1. Solve each system by stubstitution.

(a) $\begin{cases} y = 2x \\ y = x + 5 \end{cases}$

(c) $\begin{cases} 2x + y = 5 \\ y = x - 4 \end{cases}$

(b) $\begin{cases} x + 4y = 6 \\ x + y = 3 \end{cases}$

(d) $\begin{cases} y = x + 3 \\ y = 2x + 5 \end{cases}$

Example 2. Solve each system using substitution and distribution.

(a) $\begin{cases} 4y - 5x = 9 \\ x - 4y = 11 \end{cases}$

(c) $\begin{cases} -2x + y = 8 \\ 3x + 2y = 9 \end{cases}$

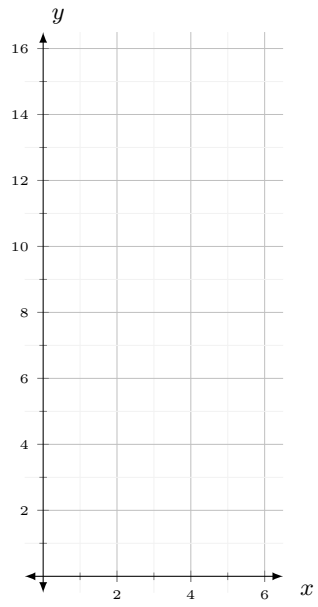
(b) $\begin{cases} 2x + y = 14 \\ -3x + 4y = -10 \end{cases}$

(d) $\begin{cases} 4x = y - 1 \\ 6x - 2y = -3 \end{cases}$

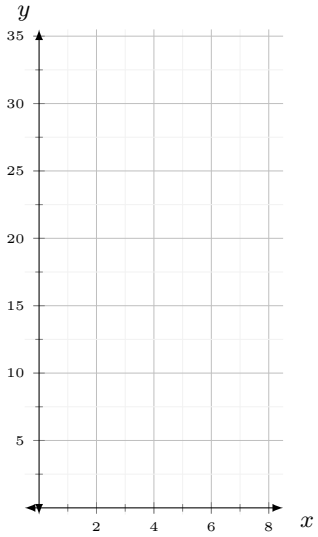
6.2 Homework: page 394 1-5 odd, 9-15 odd

6.1 and 6.2 Solving Systems Word Problems

Example 1. Bowl-o-Rama charges \$2.50 per game plus \$2 for shoe rental, and Bowling Pinz charges \$2 per game plus \$4 for shoe rental. For how many games will the cost to bowl be the same at both places? What is that cost?



Example 2. Streaming Service A charges \$10 for subscription and \$3 per movie rental. Streaming Service B charges \$15 for subscription and \$2 per movie rental. For how many movie rentals will the cost be the same for both streaming services? What is the cost?



Example 3. One high-speed Internet provider has \$50 setup fee and costs \$30 per month. Another provider has no setup fee and costs \$40 per month. In how many months with both providers cost the same? What will the cost be?

6.1 and 6.2 Word Problems Homework: page 386 8, 25, 26; page 394 7, 17 25

6.3 Solving Systems by Elimination

Definition 6.3.1. Elimination is a method used to solve a system of equations in which one variable is eliminated by adding or subtracting two equations of the system.

Steps for Solving Systems of Equations by Elimination

Step 1 Write the system so that the terms are aligned.

Step 2 Eliminate one of the variables and solve for the other variable.

Step 3 Substitute the value of the variable into one of the original equations and solve for the other variable.

Step 4 Write the values from **Step 2** and **Step 3** as an ordered pair, (x, y), and check.

Example 1. Solve each of the following systems by elimination with addition.

(a) $\begin{cases} x - 2y = -19 \\ 5x + 2y = 1 \end{cases}$

(b) $\begin{cases} y + 3x = -2 \\ 2y - 3x = 14 \end{cases}$

Example 2. Solve each of the following systems by elimination with subtraction.

(a) $\begin{cases} 3x + 4y = 18 \\ -2x + 4y = 8 \end{cases}$

(b) $\begin{cases} 3x + 3y = 15 \\ -2x + 3y = -5 \end{cases}$

Example 3. Solve each of the following systems by elimination.

(a) $\begin{cases} 2x + y = 3 \\ -x + 3y = -12 \end{cases}$

(b) $\begin{cases} 7x - 12y = -22 \\ 5x - 8y = -14 \end{cases}$

(c) $\begin{cases} 3x + 2y = 6 \\ -x + y = -2 \end{cases}$

(d) $\begin{cases} 2x + 5y = 26 \\ -3x - 4y = -25 \end{cases}$

6.3 (day 1) Homework: page 401 1-9 all

Elimination Day 2 Practice

Example 4. Solve each of the following systems by elimination.

(a) $\begin{cases} 4x + y = --1 \\ 2x - y = -5 \end{cases}$

(b) $\begin{cases} x + 2y = -1 \\ x + y = 2 \end{cases}$

(c) $\begin{cases} x + y = 12 \\ 2x + 5y = 27 \end{cases}$

(d) $\begin{cases} 3x - 2y = -6 \\ \frac{1}{3}x + 3y = 9 \end{cases}$

Example 5. Solve each system using **any** method you choose.

(a) $\begin{cases} 3x + y = 2 \\ y = -4x \end{cases}$

(b) $\begin{cases} 2y = -3x \\ y = -2x + 2 \end{cases}$

(c) $\begin{cases} y = \frac{1}{3}x - 6 \\ y = -2x + 1 \end{cases}$

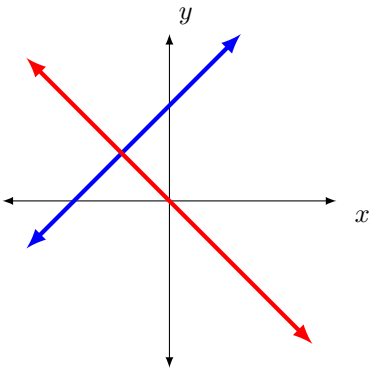
(d) $\begin{cases} x - y = 0 \\ 3x + y = 8 \end{cases}$

6.3 (day 2) Homework: page 401 11-19 all

6.4 Solving Special Systems

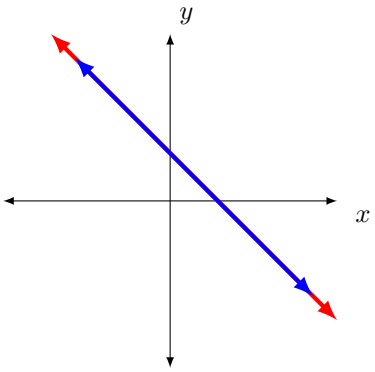
Definition 6.4.1. Systems with at least one solution are called **consistent**. Systems with no solutions are called **inconsistent**.

Exactly One Solution



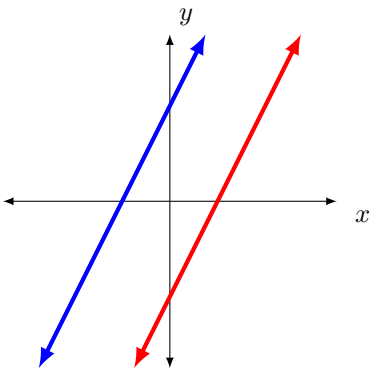
Consistent system. The graphs are intersecting lines with different slopes.

Infinitely Many Solutions



Consistent system. The graphs are coinciding lines; same slope and y-intercept.

No Solution



Inconsistent system. Parallel lines; same slope different y-intercept.

Example 1. Solve and classify each system and determine the number of solutions.

(a) $\begin{cases} y = x - 1 \\ -x + y = 2 \end{cases}$

(b) $\begin{cases} y = -2x + 5 \\ 2x + y = 1 \end{cases}$

(c) $\begin{cases} y = 2x + 1 \\ 2x - y + 1 = 0 \end{cases}$

(d) $\begin{cases} y = x - 3 \\ x - y - 3 = 0 \end{cases}$

(e) $\begin{cases} 2y = x + 2 \\ -\frac{1}{2}x + y = 1 \end{cases}$

(f) $\begin{cases} y = 2(x - 1) \\ y = x + 1 \end{cases}$

(g) $\begin{cases} x + 2y = -4 \\ -2(y + 2) = x \end{cases}$

(h) $\begin{cases} y = -2(x - 1) \\ y = -x + 3 \end{cases}$

6.4 Homework: page 409 1-10 all

6.5 Solving Linear Inequalities

Definition 6.5.1. A **Linear Inequality** is similar to a linear equation, but the equal sign is replaced with an inequality symbol (i.e. $>$, $<$, $\geq$, $\leq$, $\neq$). A **solution of a linear inequality** is any ordered pair that makes the inequality true (usually a shaded graph of coordinates that satisfy the inequality).

Example 1. Tell whether the ordered pair is a solution of the inequality.

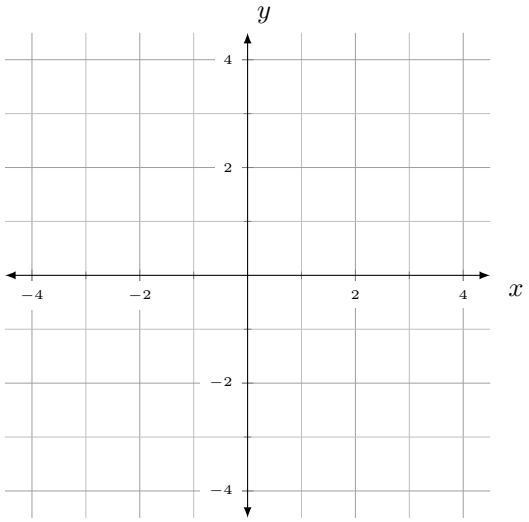
(a) $(7, 3); y < x - 1$

(b) $(4, 5); y > 3x + 2$

Graphing Linear Inequalities	
Step 1	Solve the inequality for y (slope-intercept form)
Step 2	Graph the boundry line. Use a solid line for $\leq$ or $\geq$. Use a dashed line for $>$ or $<$
Step 3	Shade the half-plane above the line for $y >$ or $y \geq$. Shade the half-plane below the line for $y <$ or $y \leq$. Check your answer.

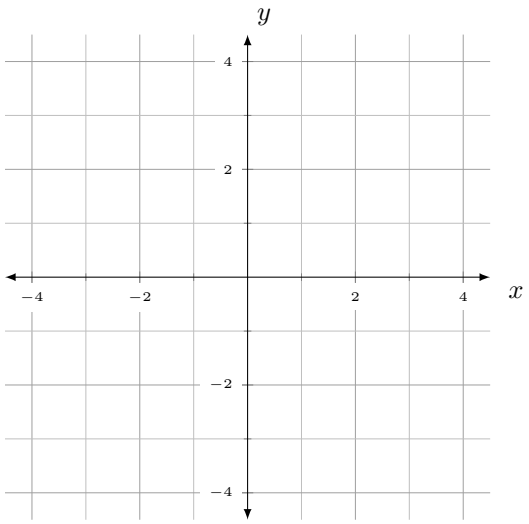
Example 2. Graph the solutions of each linear inequality.

$y < 3x + 4$



Example 3. Graph the solutions of each linear inequality.

$3x + 2y \geq 6$



6.5 (day 1) Homework: page 418 2-8

Solving Linear Inequalities Day 2 Practice

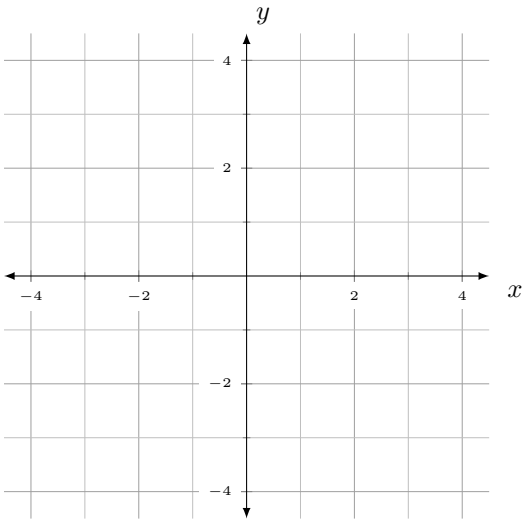
Example 4. Tell whether the ordered pair is a solution of the inequality.

(a) $(4, 5); y < x + 1$

(b) $(1, 1); y > x - 7$

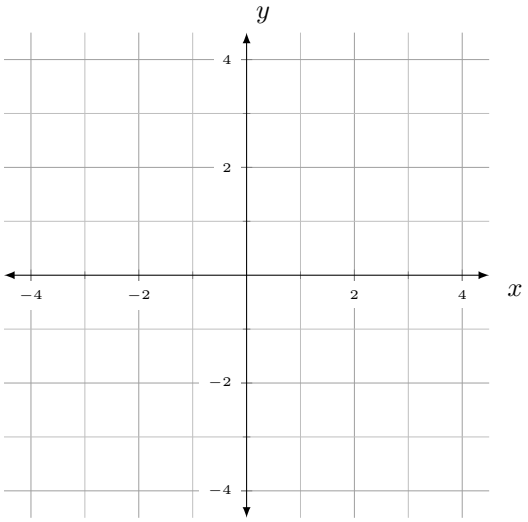
Example 5. Graph the solutions of each linear inequality.

$4x - 3y > 12$



Example 6. Graph the solutions of each linear inequality.

$2x - y - 4 > 0$



6.5 (day 2) Homework: page 418 12-18

6.6 Solving Systems of Linear Inequalities

Definition 6.6.1. A **system of linear inequalities** is a set of two or more linear inequalities containing two or more variables. The **solutions of a system of linear inequalities** consists of all the ordered pairs that satisfy all the linear inequalities in the system.

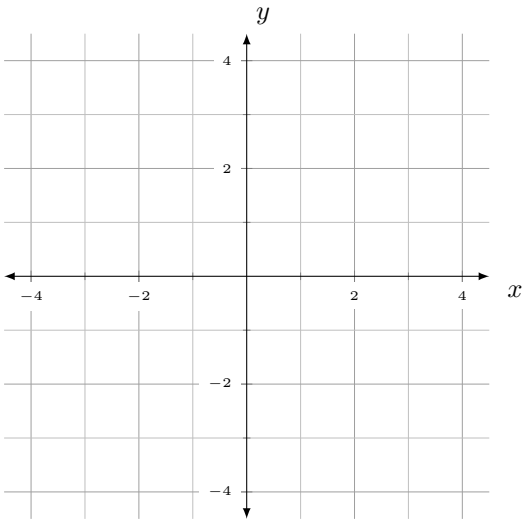
Example 1. Tell whether the ordered pair is a solution of the given system.

(a) $(2,1)$; $\begin{cases} y < -x + 4 \\ y \leq x + 1 \end{cases}$

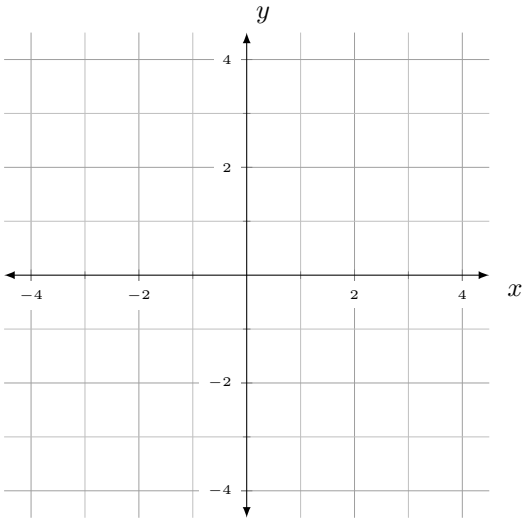
(b) $(2,0)$; $\begin{cases} y \geq 2x \\ y < x + 1 \end{cases}$

Example 2. Graph the system of linear inequalities. Give two ordered pairs that are solutions and two that are not solutions.

(a) $\begin{cases} 8x + 4y \leq 12 \\ y > \frac{1}{2}x - 2 \end{cases}$



(b) $\begin{cases} y > x - 4 \\ 3x + 6y \leq 12 \end{cases}$



6.6 (day 1) Homework: page 424 2-8 all, 19, 21

Solving Systems of Linear Inequalities Day 2 Practice

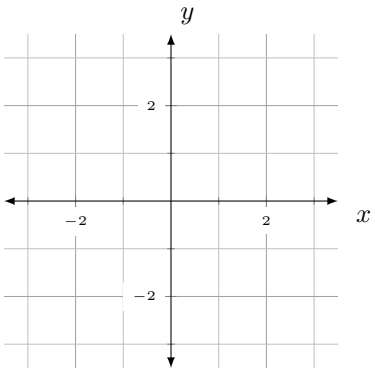
Example 3. Tell whether the ordered pair is a solution of the given system.

(a) $(0, 1)$; $\begin{cases} y < -3x + 2 \\ y \geq x - 1 \end{cases}$

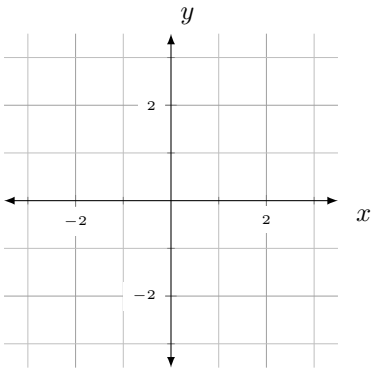
(b) $(0, 0)$; $\begin{cases} y > -x + 1 \\ y > x - 1 \end{cases}$

Example 4. Graph the system of linear inequalities. Give two ordered pairs that are solutions and two that are not solutions.

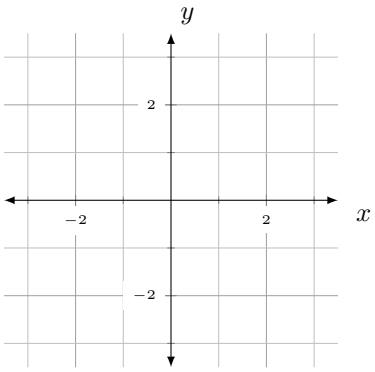
(a) $\begin{cases} y < 2x - 3 \\ y > 2x + 2 \end{cases}$



(b) $\begin{cases} y > x - 3 \\ y \leq x + 1 \end{cases}$



(c) $\begin{cases} y \leq x + 1 \\ y > 2 \end{cases}$



6.6 (day 2) Homework: page 424 9, 11, 13, 16, 17, 20, 22, 23

Chapter 6 Review (day 1)

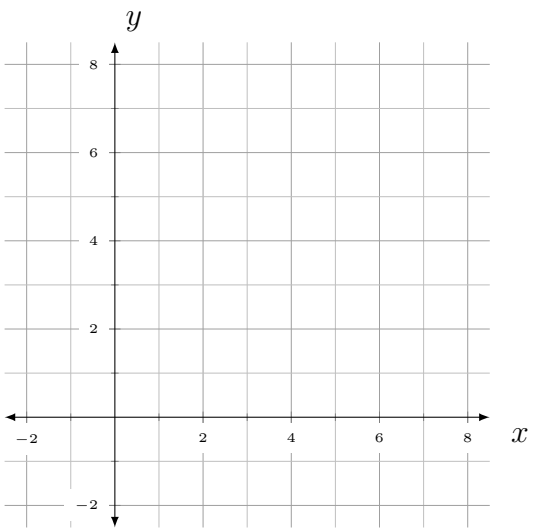
Tell whether the ordered pair is a solution of the given system of equations.

1. $(1, -4)$; $\begin{cases} x - 2y = 8 \\ 4x - y = 8 \end{cases}$

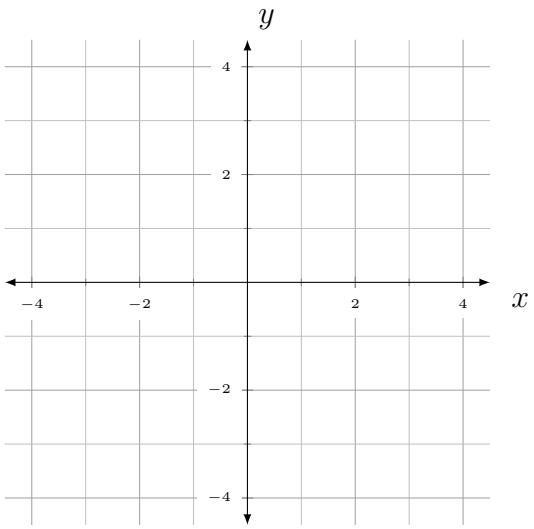
2. $(5, 2)$; $\begin{cases} 2x + y = 12 \\ -3y - x = -11 \end{cases}$

Solve the following systems by graphing

3. $\begin{cases} y = 2x - 2 \\ y = -\frac{1}{2}x + 8 \end{cases}$



4. $\begin{cases} x + y = 2 \\ y = x - 4 \end{cases}$



Solve the following systems by substitution or elimination

5. $\begin{cases} 2x - 3y = -2 \\ -3x + y = 10 \end{cases}$

6. $\begin{cases} 2x - 3y = -8 \\ -x + 4y = 7 \end{cases}$

Ch 6 Review (day 1) Homework: page 430 6, 9, 10, 14, 15, 20, 21, 27

Chapter 6 Review (day 2)

Classify each system. Give the number of solutions.

7. $\begin{cases} y = 3x + 4 \\ 6x - 2y = -8 \end{cases}$

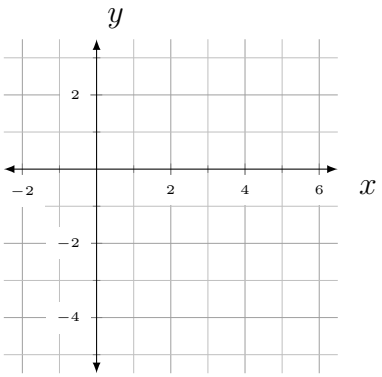
8. $\begin{cases} y = 2x - 1 \\ 6x - 2y = -8 \end{cases}$

9. $\begin{cases} x - y = 6 \\ y = x - 1 \end{cases}$

Is the given ordered pair a solution? Graph the solutions of the following inequalities.

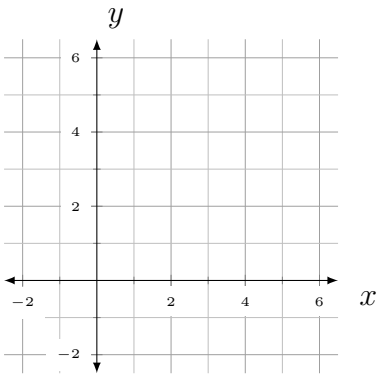
10. $x - 2y < 6$

$(3, -3)$



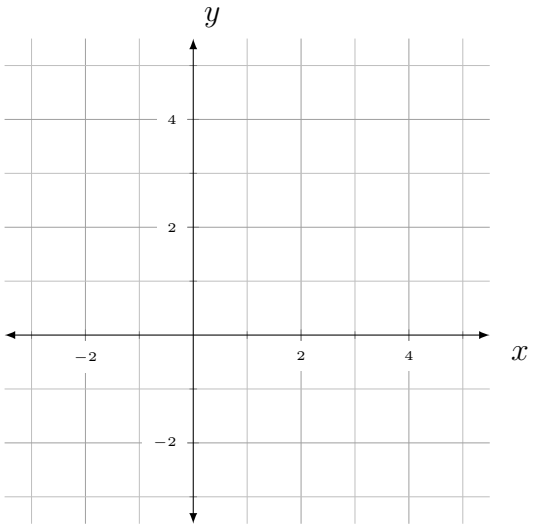
11. $5 - y \geq 2x$

$(2, 2)$

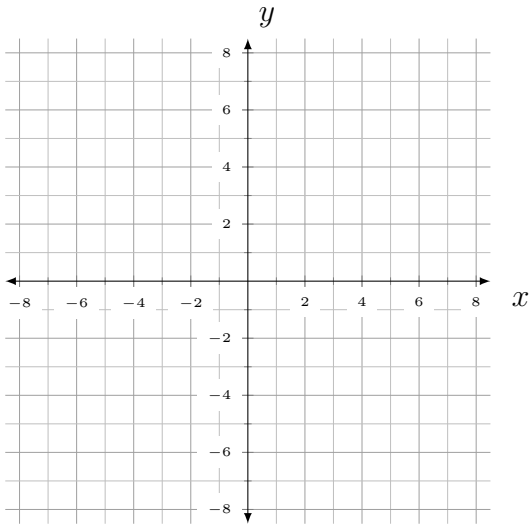


Graph each system of linear inequalities. Give **two ordered pairs** that are solutions and two that are not solutions.

12. $\begin{cases} y < -x + 5 \\ y \geq 2x - 3 \end{cases}$



13. $\begin{cases} y \leq -2x + 8 \\ y > 3x - 5 \end{cases}$



Ch 6 Review (day 2) Homework: page 432 28, 29, 30, 42, 43, 46, 50, 53, 54, 55, 59