



Paper



Code



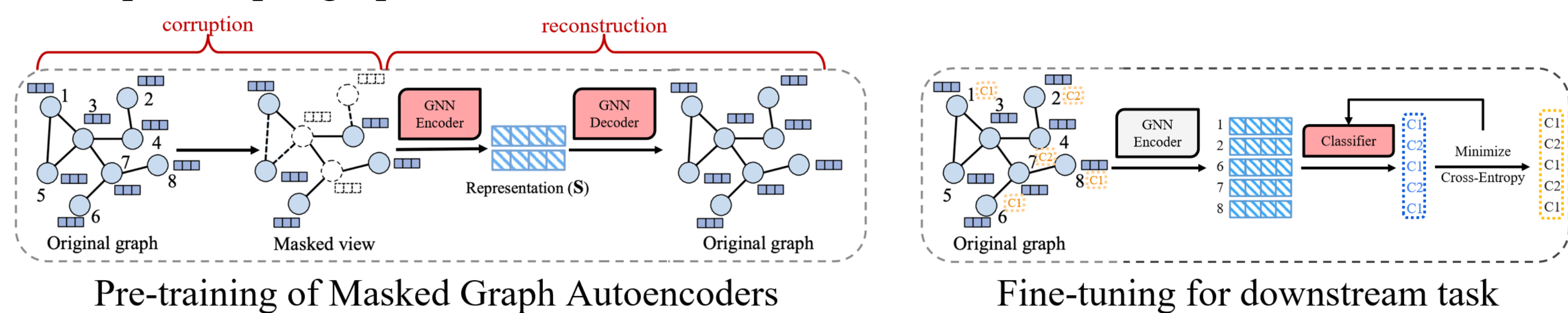
Graph Positional Autoencoders as Self-supervised Learners

Yang Liu*, Deyu Bo*, Wenxuan Cao, Yuan Fang, Yawen Li, Chuan Shi†

❖ Motivation

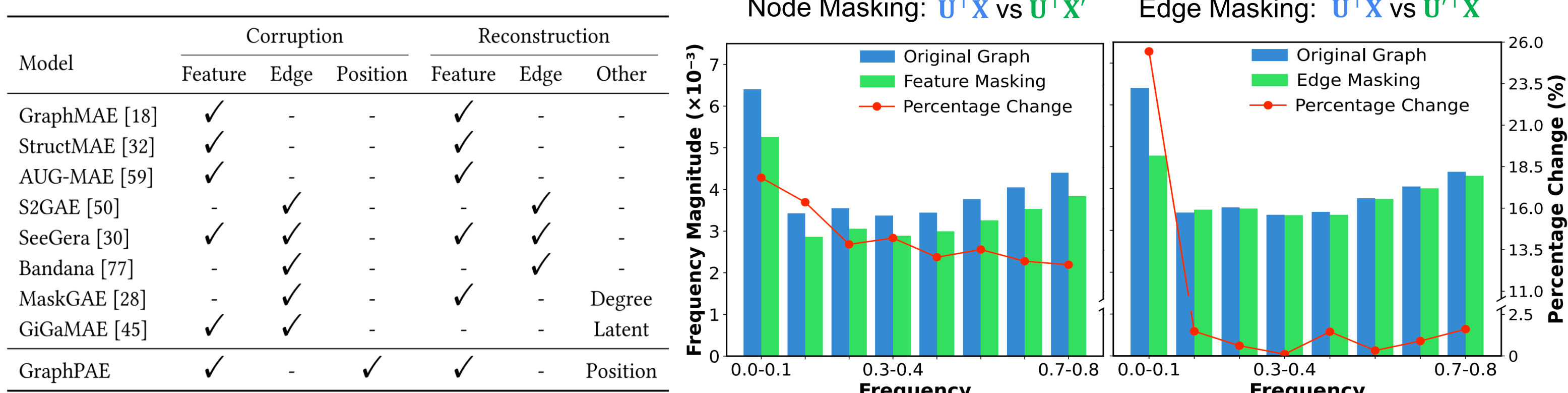
➤ Background

Masked Graph Autoencoders (GAEs) follow a corruption-reconstruction framework, which learns graph representations by recovering the missing information of the incomplete input graphs.



➤ Limitations of existing methods

Existing masked GAEs tend to focus on reconstructing low-frequency information of graphs while overlooking high-frequency information.



Comparison between different masked graph autoencoders.

Comparison of the frequency magnitudes between original and corrupted graphs.

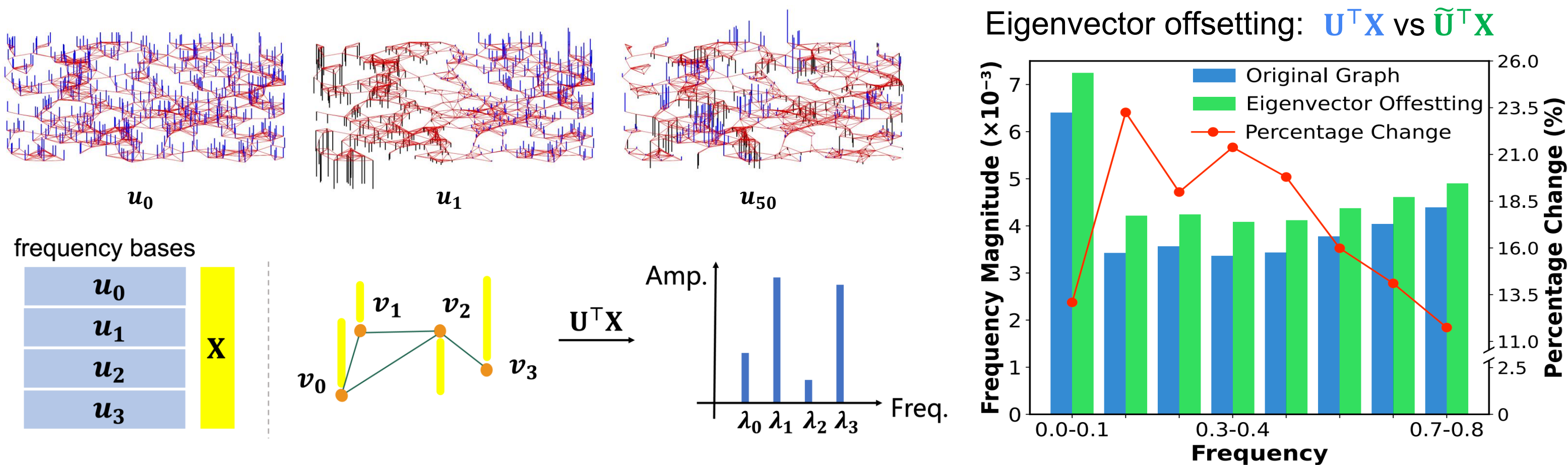
➤ Our Goal

To equip GAEs with the ability to exploit the diverse frequency information.

❖ Inspired by Spectral Theory

➤ A Spectral Perspective on the Corruption Strategy

Eigenvectors of the graph Laplacian represent different frequencies, acting as frequency bases in the spectral domain.

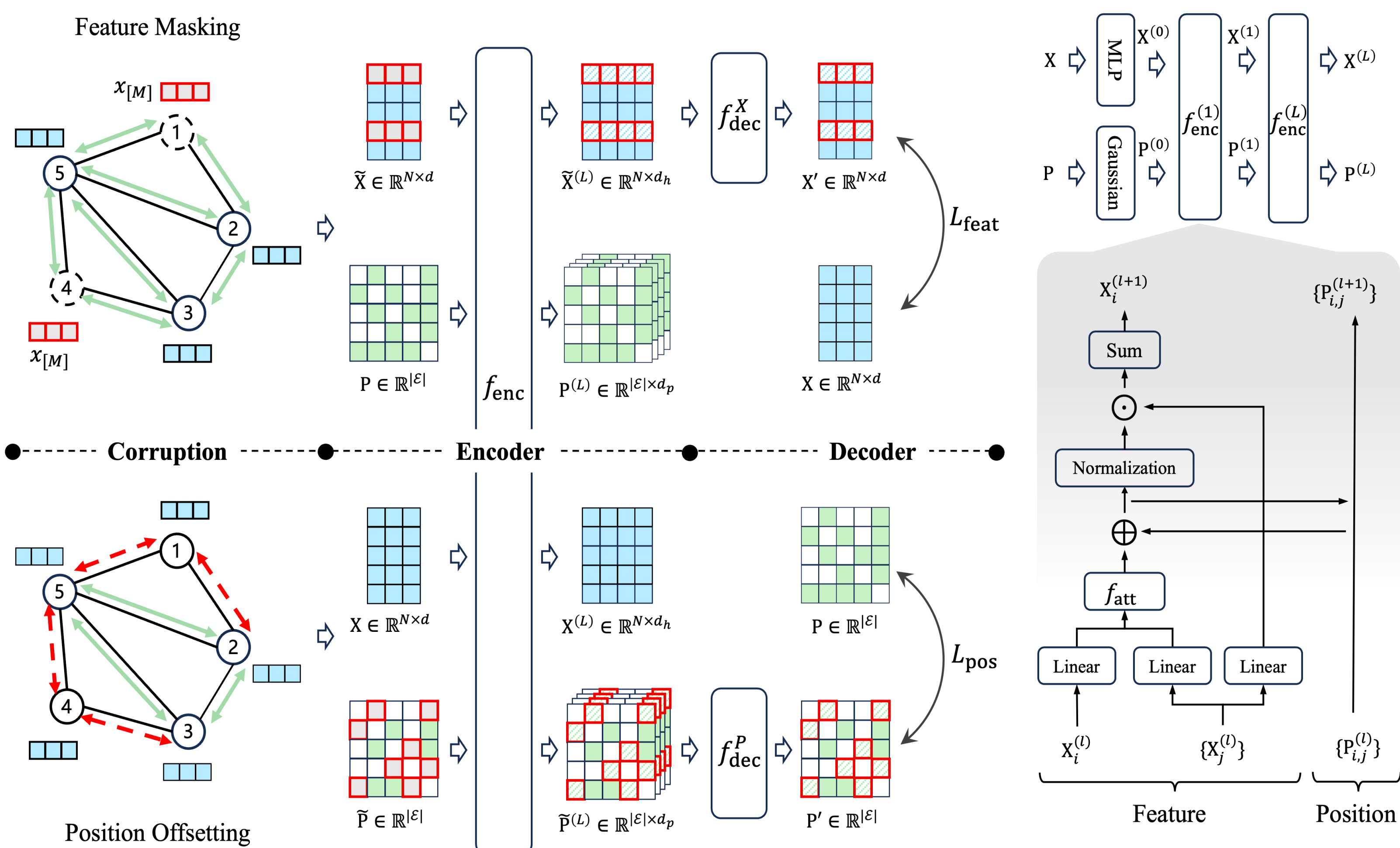


➤ Eigenvectors as Positional Encoding

Eigenvectors are utilized as enhanced features (positional encodings) to boost expressivity of MPNN.

🔧 **Incorporating eigenvector, i.e., position, corruption-reconstruction into masked GAEs.**

❖ Proposed Framework: GraphPAE



➤ **GraphPAE** uses a dual-path architecture to separately reconstruct node features and positions, overcoming the expressivity and ambiguity challenges.

- **Feature Path:** Integrates positional encodings to enhance message-passing expressivity for improved feature reconstruction.
- **Position Path:** Leverages node representations to refine positional encodings, enabling the model to learn diverse frequency information.
- **Reconstruction Strategy:** Reconstructs relative node distances as a surrogate objective to avoid the ambiguity of eigenvectors.

➤ Encoder

$$\mathbf{X}_i^{(l+1)}, \mathbf{P}_i^{(l+1)} = f_{\text{enc}}^{(l+1)} \left(\mathbf{X}_i^{(l)}, \left\{ \mathbf{X}_j^{(l)} \right\}_{j \in \mathcal{N}_i}, \mathbf{P}_i^{(l)} \right)$$

$$\alpha_{i,j}^{(l)} = f_{\text{att}} \left(\mathbf{X}_i^{(l)}, \mathbf{X}_j^{(l)} \right), \quad \alpha_{i,j}^{(l)} \in \mathbb{R}^d,$$

$$\mathbf{X}_i^{(l+1)} = \sum_{j \in \mathcal{N}_i} \left(\alpha_{i,j}^{(l)} + \mathbf{P}_{i,j}^{(l)} \right) \odot \text{MLP} \left(\mathbf{X}_j^{(l)} \right)$$

$$\mathbf{P}_{i,j}^{(l+1)} = \alpha_{i,j}^{(l)} + \mathbf{P}_{i,j}^{(l)}.$$

➤ Decoder

$$\mathcal{L}_{\text{feat}} = \frac{1}{|\tilde{\mathcal{V}}|} \sum_{v_i \in \tilde{\mathcal{V}}} \left(1 - \frac{\mathbf{X}_i^T \mathbf{X}'_i}{\|\mathbf{X}_i\| \cdot \|\mathbf{X}'_i\|} \right)^{\gamma}$$

$$\mathcal{L}_{\text{pos}}^{i,j} = \begin{cases} \frac{(\mathbf{P}'_{i,j} - \mathbf{P}_{i,j})^2}{2}, & \text{if } |\mathbf{P}'_{i,j} - \mathbf{P}_{i,j}| < 1 \\ |\mathbf{P}'_{i,j} - \mathbf{P}_{i,j}| - \frac{1}{2}, & \text{otherwise} \end{cases}$$

$$\mathcal{L}_{\text{pos}} = \frac{1}{\sum_{v_i \in \tilde{\mathcal{V}}} |\mathcal{N}_i|} \sum_{v_i \in \tilde{\mathcal{V}}, j \in \mathcal{N}_i} \mathcal{L}_{\text{pos}}^{i,j}$$

❖ Experiments

➤ Performance of Node Classification

Dataset	Small Graphs				Large Graphs	
	BlogCatalog	Chameleon	Squirrel	Actor	arXiv-year	Penn94
Supervised	80.52±2.10	80.02±0.87	71.91±1.03	33.93±2.47	46.02±0.26	81.53±0.55
DGI	72.07±0.16	43.83±0.14	34.56±0.10	27.98±0.09	-	-
BGRL	79.74±0.46	61.24±1.07	43.24±0.52	26.61±0.57	<u>41.43±0.04</u>	63.31±0.49
MVGRL	63.24±0.94	73.19±0.42	60.09±0.44	34.64±0.20	-	-
CCA-SSG	74.00±0.28	75.00±0.75	61.58±1.98	27.79±0.58	40.78±0.01	62.63±0.20
Sp ² GCL	72.73±0.46	78.88±1.04	62.61±0.87	<u>34.70±0.92</u>	39.09±0.02	68.80±0.45
VGAE	60.47±1.84	62.32±1.90	42.50±1.35	31.57±0.75	36.39±0.21	55.31±0.28
GraphMAE	79.90±1.13	<u>79.50±0.57</u>	61.13±0.60	32.15±1.33	40.30±0.04	67.97±0.21
GraphMAE2	77.34±0.12	79.13±0.19	<u>70.27±0.88</u>	34.48±0.26	38.97±0.03	67.86±0.42
MaskGAE	73.10±0.08	74.50±0.87	68.53±0.44	33.44±0.34	40.59±0.04	63.84±0.03
S2GAE	75.76±0.43	60.24±0.37	68.60±0.56	26.17±0.38	40.32±0.12	<u>70.24±0.09</u>
AUG-MAE	<u>82.03±0.69</u>	70.10±1.88	62.57±0.67	33.42±0.38	37.10±0.13	69.90±0.43
GraphPAE	85.76±1.22	80.51±1.25	72.05±1.40	38.55±1.35	41.85±0.04	71.79±0.37

➤ Performance of Graph Prediction

Task	Regression (Metric: RMSE ↓)			Classification (Metric: ROC-AUC% ↑)			
	molosol	molipo	molreesolv	molbase	molbbbp	molclintox	moltoxc21
Supervised	1.173±0.057	0.757±0.018	2.755±0.349	80.42±0.96	68.17±1.48	88.14±2.51	74.91±0.51
InfoGraph	1.344±0.178	1.005±0.023	10.005±8.147	73.64±3.64	66.33±2.79	64.50±5.32	69.74±0.57
GraphCL	1.272±0.089	0.910±0.016	7.679±2.748	73.32±2.70	68.22±2.19	74.92±4.42	72.40±1.07
MVGRL	1.433±0.145	0.962±0.036	9.024±1.982	74.88±1.43	67.24±3.19	73.84±2.75	70.48±0.83
JOAO	1.285±0.121	0.865±0.032	5.131±0.782	74.43±1.94	67.62±1.29	71.28±4.12	71.38±0.92
Sp ² GCL	1.235±0.119	<u>0.835±0.026</u>	4.144±0.573	78.76±1.43	68.72±1.53	80.88±3.86	73.06±0.75
GraphMAE	<u>1.050±0.034</u>	0.850±0.022	2.740±0.233	79.14±1.31	66.55±1.78	80.56±5.55	73.84±0.58
GraphMAE2	1.225±0.081	0.885±0.019	2.913±0.293	<u>80.74±1.53</u>	67.67±1.44	75.75±3.65	72.93±0.69
StructMAE	1.499±0.043	1.089±0.002	2.568±0.262	77.75±0.42	65.66±1.16	79.42±4.56	71.13±0.61
AUG-MAE	1.248±0.026	0.917±0.013	<u>2.395±0.158</u>	78.54±2.49	67.05±0.63	<u>82.66±1.98</u>	<u>74.33±0.07</u>
GraphPAE	1.015±0.045	0.810±0.018	2.058±0.188	81.11±1.24	<u>68.56±0.71</u>	82.69±3.39	74.46±0.54

➤ Performance of Transfer Learning on QM9

Target	μ	α	ϵ_{homo}	ϵ_{lumo}	$\Delta\epsilon$	R^2	ZPVE	U_0	U	H	G	C_v
Unit	D	a_0^3	10^{-2}meV	10^{-2}meV	10^{-2}meV	a_0^2	10^{-2}meV	meV	meV	meV	meV	cal/mol/K
GraphCL	1.035	2.321	2.030	<u>3.667</u>	4.523	40.725	<u>2.063</u>	2.461	<u>1.745</u>	<u>1.734</u>	<u>1.751</u>	<u>1.747</u>
GraphMAE	1.030	2.924	2.407	6.373	4.813	41.955	4.623	<u>1.411</u>	2.207	2.208	2.207	2.200
Mole-BERT	1.031	<u>1.918</u>	<u>1.477</u>	4.127	4.240	44.374	2.190	2.532	2.509	2.511	2.516	2.508
SimSGT	1.064	2.413	2.837	4.227	<u>4.107</u>	<u>40.504</u>	2.127	1.948	2.420	2.416	2.416	2.410
GraphPAE	0.703	0.879	1.199	2.141	2.289	36.480	0.502	0.510	0.639	0.639	0.641	0.643

➤ Ablation Studies

