Shared Variables and Interference-Freedom

CS 536: Science of Programming, Spring 2022

A. *Why*

- Parallel programs can coordinate their work using shared variables, but it's important for threads to not interfere (to not invalidate conditions that the other thread relies on).

B. *Objectives*

At the end of these practice questions you should be able to

- Check for interference between the correctness proofs of the sequential threads of a shared memory parallel program.

C. *Problems*

1. Say we have a parallel program that fails one of its interference checks.
   a. What does that tell us about how the program will behave at runtime? (Assume we start it in a state satisfying the precondition.)
   b. Is it impossible to get a proof of correctness for the program?

2. List the interference freedom checks for the following two standard proof outlines. Remember, you only need to see if atomic statements cause interference.[1]
   - $\{p_1\}$ *if* B_1 *then* $\{\{p_2\} < S_1 >\}$ *else* $\{\{p_3\}$ *skip*$\}\{p_4\}$
   - $\{q_1\} < T_1 >$; $\{inv\ q_2\}$ *while* C_1 $\{\{q_3\} < T_2 > \}$ $\{q_4\}$

For each of the following sets of threads, (a) List the triples to check for interference freedom, (b) Say whether each of the interference triples is true or false, and (c) Say whether the threads interfere or not.

3. $\{even(x)\}\ x := x+1\ \{odd(x)\}$ and $\{x \geq 0\}\ x := x+2\ \{x \geq 0\}$
4. $\{T\}\ x := 0\ \{x = 0\}$ and $\{x = 0\}\ x := x+2\ \{x = 2\}$
5. $\{T\}\ x := 0\ \{x = 0\}$ and $\{x = 0\}\ x := x+2\ \{x = 0 \vee x = 2\}$
6. $\{x \geq 0\}\ x := x+1\ \{x \geq 1\}$; $x := x*2\ \{x \geq 2\}$ and $\{x \geq 0\}\ x := x+2\ \{x \geq 2\}$; $x := x*3\ \{x \geq 6\}$

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1 Interference-freedom checks will appear on the Final Exam. Question 1 is important too.

Solution to Practice 25 (Shared Variables and Interference-Freedom)

1. (Parallel program fails an interference check.)
   a. Failing an interference check only tells us that if we execute the failed triple, interference is not impossible. It doesn't guarantee that interference will occur at runtime; it just says its possible.
   b. It may or may not be possible to prove correctness. It's possible that if the proof outline's conditions are modified, we'd be able to prove interference freedom. It's also possible that proof outline can't be modified to prove interference freedom without modifying the given final postcondition. E.g., in Example 1, if we start with $x = 10$ and insist on showing that $x = 25$ when the program finishes, then no proof of correctness is possible because we might end with $x = 28$.

2. (Interference checks)

   For thread 1 vs thread 2:

   - $\{p_2 \wedge q\}\ S_1\ \{q\}$ where $q = q_1, q_3, q_4$. (Since q_2 is not in front of an atomic statement, including $q = q_2$ is not necessary, but including it is relatively harmless.)
   - Technically, we also need $\{p_3 \wedge q\}\ skip\ \{q\}$ for the same q, but they're trivially valid.

   For thread 2 vs thread 1:

   - $\{q_1 \wedge p\}\ T_1\ \{p\}$ where $p = p_1, p_2, p_3, p_4$.
   - $\{q_3 \wedge p\}\ T_2\ \{p\}$ for the same p.

3. All the triples to check are valid, so there's no interference:

   - $\{even(x) \wedge x \geq 0\}\ x := x+1\ \{x \geq 0\}$ *// the precondition of x := x+2*
   - $\{even(x) \wedge x \geq 0\}\ x := x+1\ \{x \geq 0\}$ *// the postcondition of x := x+2*
   - $\{x \geq 0 \wedge even(x)\}\ x := x+2\ \{even(x)\}$
   - $\{x \geq 0 \wedge odd(x)\}\ x := x+2\ \{odd(x)\}$

4. The 2nd and 4th triples are invalid, so the threads interfere

   - $\{T \wedge x = 0\}\ x := 0\ \{x = 0\}$
   - $\{T \wedge x = 2\}\ x := 0\ \{x = 2\}$ *// fails*
   - $\{x = 0 \wedge T\}\ x := x+2\ \{T\}$
   - $\{x = 0 \wedge x = 0\}\ x := x+2\ \{x = 0\}$ *// fails*

5. The last triple is invalid, so the threads interfere

   - $\{T \wedge x = 0\}\ x := 0\ \{x = 0\}$
   - $\{T \wedge (x = 0 \vee x = 2)\ 0\}\ x := 0\ \{x = 0 \vee x = 2\}$

- $\{x = 0 \wedge T\}\ x := x+2\ \{T\}$
- $\{x = 0 \wedge x = 0\}\ x := x+2\ \{x = 0\}$ *// fails*

6. There are 9 triples to check, and all of them are valid, so there's no interference
   - $\{x \geq 0 \wedge x \geq 0\}\ x := x+1\ \{x \geq 0\}$
   - $\{x \geq 0 \wedge x \geq 2\}\ x := x+1\ \{x \geq 2\}$
   - $\{x \geq 0 \wedge x \geq 6\}\ x := x+1\ \{x \geq 6\}$

   - $\{x \geq 0 \wedge x \geq 0\}\ x := x*2\ \{x \geq 0\}$
   - $\{x \geq 0 \wedge x \geq 2\}\ x := x*2\ \{x \geq 2\}$
   - $\{x \geq 0 \wedge x \geq 6\}\ x := x*2\ \{x \geq 6\}$

   - $\{x \geq 0 \wedge x \geq 0\}\ x := x+2\ \{x \geq 0\}$
   - $\{x \geq 0 \wedge x \geq 1\}\ x := x+2\ \{x \geq 1\}$
   - $\{x \geq 0 \wedge x \geq 2\}\ x := x+2\ \{x \geq 2\}$

   - $\{x \geq 2 \wedge x \geq 0\}\ x := x*3\ \{x \geq 0\}$
   - $\{x \geq 2 \wedge x \geq 1\}\ x := x*3\ \{x \geq 1\}$
   - $\{x \geq 2 \wedge x \geq 2\}\ x := x*3\ \{x \geq 2\}$