

Weakest Preconditions

Part 2: Calculating wp, wlp; Domain Predicates

CS 536: Science of Programming, Spring 2022

For Problems 2 – 4, just syntactically calculate the *wlp*; don't also logically simplify the result.

2. Calculate the *wlp* in each of the following cases.
 - a. $wlp(k := k - s, n = 3 \wedge k = 4 \wedge s = -7)$.
 - b. $wlp(n := n^*(n-k), n = 3 \wedge k = 4 \wedge s = -7)$.
 - c. $wlp(n := n^*(n-k) ; k := k - s, n > k + s)$
3. Let $Q(k, s) \equiv 0 \leq k \leq n \wedge s = \text{sum}(0, k)$ where $\text{sum}(u, v)$ is the sum of $u, u+1, \dots, v$ (when $u \leq v$) or 0 (when $u > v$).
 - a. Calculate $wp(k := k+1; s := s+k, Q(k, s))$.
 - b. Calculate $wp(s := s+k+1; k := k+1, Q(k, s))$.
 - c. Calculate $wp(s := s+k; k := k+1, Q(k, s))$. (This one isn't compatible with $s = \text{sum}(0, k)$.)
4. Calculate the *wp* below.
 - a. $wp(\text{if } e \text{ then } \{x := x/2\} \text{ else } \{\text{skip}\}; y := x, x = 5 \wedge y = z)$.
 - b. $wp(\text{if } x \geq 0 \text{ then } \{x := x*2\} \text{ else } \{x := y\}; x := c*x, a \leq x < y)$

For Problems 5 and 6, don't forget the domain predicates. You can logically simplify as you go.

5. Calculate p to be the *wp* in $\{p\} x := y/b[k] \{x > 0\}$.
6. Calculate p_1 and p_2 to be the *wp*'s in $\{p_1\} y := \text{sqrt}(b[k]) \{z < y\}$ and $\{p_2\} k := x/k \{p_1\}$.

*Solution to Practice 11 (Weakest Preconditions, pt. 2)*2. (Calculate wlp)

- a. $wlp(k := k - s, n = 3 \wedge k = 4 \wedge s = -7) \equiv n = 3 \wedge k - s = 4 \wedge s = -7$
- b. $wlp(n := n*(n-k), n = 3 \wedge k = 4 \wedge s = -7) \equiv n*(n-k) = 3 \wedge k = 4 \wedge s = -7$
- c. $wlp(n := n*(n-k); k := k-s, n > k+s)$
 $\equiv wlp(n := n*(n-k), wlp(k := k-s, n > k+s))$
 $\equiv wlp(n := n*(n-k), n > k-s+s)$
 $\equiv n*(n-k) > k-s+s$

3. (wp involving sums) We have $Q(k, s) \equiv 0 \leq k \leq n \wedge s = \text{sum}(0, k)$.

- a. $wp(k := k+1; s := s+k, Q(k, s))$
 $\equiv wp(k := k+1, wp(s := s+k, Q(k, s)))$
 $\equiv wp(k := k+1, Q(k, s+k))$
 $\equiv wp(k := k+1, 0 \leq k \leq n \wedge s+k = \text{sum}(0, k))$
 $\equiv 0 \leq k+1 \leq n \wedge s+k+1 = \text{sum}(0, k+1)$
- b. $wp(s := s+k+1; k := k+1, Q(k, s))$
 $\equiv wp(s := s+k+1, wp(k := k+1, Q(k, s)))$
 $\equiv wp(s := s+k+1, Q(k+1, s))$
 $\equiv wp(s := s+k+1, 0 \leq k+1 \leq n \wedge s = \text{sum}(0, k+1))$
 $\equiv 0 \leq k+1 \leq n \wedge s+k+1 = \text{sum}(0, k+1)$
- c. $wp(s := s+k; k := k+1, Q(k, s))$
 $\equiv wp(s := s+k, wp(k := k+1, Q(k, s)))$
 $\equiv wp(s := s+k, Q(k+1, s))$
 $\equiv wp(s := s+k, 0 \leq k+1 \leq n \wedge s = \text{sum}(0, k+1))$
 $\equiv 0 \leq k+1 \leq n \wedge s+k = \text{sum}(0, k+1)$. Note this isn't compatible with $s = \text{sum}(0, k)$.

4. (wp of if-then)

- a. $wp(\text{if } e \text{ then } \{x := x/2\} \text{ else } \{\text{skip}\}; y := x, x = 5 \wedge y = z)$
 $\equiv wp(\text{if } e \text{ then } \{x := x/2\} \text{ else } \{\text{skip}\}, wp(y := x, x = 5 \wedge y = z))$
 $\equiv wp(\text{if } e \text{ then } \{x := x/2\} \text{ else } \{\text{skip}\}, x = 5 \wedge x = z)$
 $\equiv (e \rightarrow wp(x := x/2, x = 5 \wedge x = z)) \wedge (\neg e \rightarrow wp(\text{skip}, x = 5 \wedge x = z))$
 $\equiv (e \rightarrow x/2 = 5 \wedge x/2 = z) \wedge (\neg e \rightarrow x = 5 \wedge x = z)$

$$\begin{aligned}
 b. \quad & wp(if \ x \geq 0 \ then \ \{x := x*2\} \ else \ \{x := y\}; \ x := c*x, \ a \leq x < y). \\
 & \equiv wp(S, wp(x := c*x, a \leq x < y)) \quad \text{where } S \text{ is the if statement} \\
 & \equiv wp(S, a \leq c*x < y) \\
 & \equiv wp(if \ x \geq 0 \ then \ \{x := x*2\} \ else \ \{x := y\}, a \leq c*x < y) \\
 & \equiv (x \geq 0 \rightarrow wp(x := x*2, a \leq c*x < y)) \wedge (x < 0 \rightarrow wp(x := y, a \leq c*x < y)) \\
 & \equiv (x \geq 0 \rightarrow a \leq c*(x*2) < y) \wedge (x < 0 \rightarrow a \leq c*y < y)
 \end{aligned}$$

5. For $\{p\}$ $x := y/b[k]$ $\{x > 0\}$,
let $p \Leftrightarrow wp(x := y/b[k], x > 0)$
 $\equiv wlp(x := y/b[k], x > 0) \wedge D(x := y/b[k])$
 $\equiv y/b[k] > 0 \wedge b[k] \neq 0 \wedge D(b[k])$
 $\equiv y/b[k] > 0 \wedge b[k] \neq 0 \wedge 0 \leq k < \text{size}(b)$

6. For $\{p_1\}$ $y := \sqrt{b[k]}$ $\{z < y\}$
let $p_1 \Leftrightarrow wp(y := \sqrt{b[k]}, z < y)$
 $\equiv wlp(y := \sqrt{b[k]}, z < y) \wedge D(y := \sqrt{b[k]})$
 $\equiv z < \sqrt{b[k]} \wedge b[k] \geq 0 \wedge D(b[k])$
 $\equiv z < \sqrt{b[k]} \wedge b[k] \geq 0 \wedge 0 \leq D(b[k] < \text{size}(b))$

For $\{p_2\}$ $k := x/k$; $\{p_1\}$, let
 $p_2 \Leftrightarrow wp(k := x/k, p_1)$
 $\equiv wlp(k := x/k, p_1) \wedge D(k := x/k)$
 $\equiv p_1[x/k / k] \wedge k \neq 0$
 $\equiv (z < \sqrt{b[k]} \wedge b[k] \geq 0 \wedge 0 \leq D(b[k] < \text{size}(b)) [x/k / k] \wedge k \neq 0)$
 $\equiv z < \sqrt{b[x/k]} \wedge b[x/k] \geq 0 \wedge 0 \leq D(b[x/k] < \text{size}(b)) \wedge k \neq 0$