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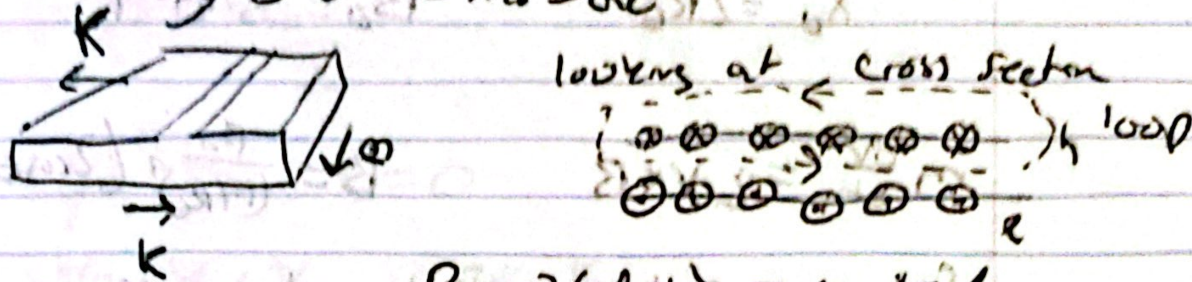
HW 8

1) Ampere's Law

$$\oint \vec{B}\cdot dl = \mu_0 I_{enc}$$



$$B\cdot -2(l+h) = \mu_0 K l$$

if $h \ll l$

$$B = \frac{-\mu_0 K}{2}$$

$$\mathcal{E} = -\frac{\partial \Phi}{\partial t} \qquad \Phi_m = \int \vec{B}\cdot d\vec{a}$$

$$A = wh$$

$$\Phi_m = BA = -\frac{\mu_0 K}{2}(A)$$

$$\mathcal{E} = -\frac{\partial \Phi}{\partial t} = +\frac{\mu_0 A}{2}\frac{\partial K}{\partial t} \qquad \cancel{\mathcal{L} = \frac{\mu_0 A}{2}}$$

$$\mathcal{L} = \frac{\Phi_m}{Kl} \qquad Kl\mathcal{L} = \Phi_m \qquad \mathcal{L} = \frac{-\mu_0 A}{2l}$$

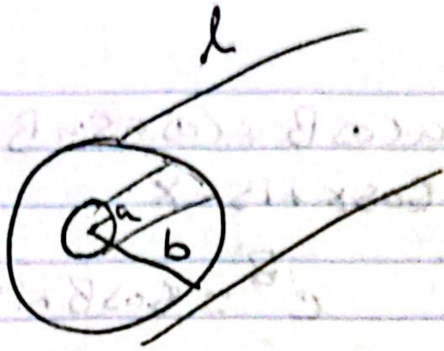
$$\mathcal{E} = -l\mathcal{L}\frac{\partial K}{\partial t}$$

3) $\mu_0 \mathcal{L} I^2 = \int B^2 dV \qquad W = \frac{1}{2}LI^2$

$$\mu_0 \mathcal{L} I^2 = \left(-\frac{\mu_0 K}{2}\right)^2 lwh \qquad W = \frac{1}{2\mu_0}\int_{\text{all space}} B^2 d\tau$$

$$\mu_0 \mathcal{L}(K^2 l^2) = \frac{K^2 l A \mu_0}{4} \qquad \mathcal{L} = \frac{\mu_0 A}{4l} \qquad \text{Why different by } (-1/2)$$

2)



$\oint E\cdot dl = -\frac{d\Phi}{dt}$

$\mathcal{L} = \Phi/I$

$\mu_0 \mathcal{L} I^2 = \int B^2\, dV \quad = \int_0^l \int_a^b \int_0^{2\pi} B^2\, r\, d\theta\, dr\, dz$

$\mu_0 \mathcal{L} I^2 = B^2\, 2\pi\left(\frac{b^2-a^2}{2}\right) l$

B is the same regardless of path

Bc Symmetry

$\oint B\cdot dl = \mu_0 I_{enc} \qquad B = \frac{\mu_0 I_{enc}}{2\pi r}$

$\mathcal{L} = \left(\frac{\mu_0 I}{2\pi r}\right)^2 2\pi \cancel{\left(\frac{b^2-a^2}{2}\right)} l \,/\, \mu_0 I^2$

$\mathcal{L} = \frac{\mu_0 2\pi \cancel{(b^2-a^2)} l}{4\pi^2 r^2}$

$\cancel{\mathcal{L} = \frac{\mu_0 b^2 l}{4\pi a^2} - \frac{\mu_0 a^2 l}{4\pi a^2}}$

$\cancel{\mathcal{L} = \frac{\mu_0 b^2 l}{4\pi a^2} - \frac{\mu_0 l}{4\pi}}$

$\mathcal{L} = \frac{\mu_0 l}{2\pi}$

8.3

1) $\sin(a+B) = \sin a\cos B + \cos a\sin B$

using $e^{ix} = \cos x + i\sin x$

$e^{ia} = \cos a + i\sin a \qquad e^{iB} = \cos B + i\sin B$

$e^{ia}\cdot e^{iB} = e^{i(a+B)} = (\cos a + i\sin a)(\cos B + i\sin B)$

$= \cos a\cos B + i\sin a\cos B + i\sin B\cos a - \sin a\sin B$

$e^{i(a+B)} = \cos a\cos B - \sin a\sin B + i(\sin a\cos B + \sin B\cos a)$

$e^{i(a+B)} = \cos(a+B) + i\sin(a+B)$

$\mathrm{Im}[e^{i(a+B)}] = \sin(a+B) = \sin a\cos B + \sin B\cos a$

2) $V(z,t) = \cos(\omega t - Bz) + a\cos(\omega t + Bz)$

Show $V = A\cos\omega t\cos Bz + B\sin\omega t\sin Bz$

using Re of above

$\mathrm{Re}[e^{i(a+B)}] = \cos a\cos B - \sin a\sin B = \cos(a+B)$

$V = \cos(\omega t - Bz) + a(\cos\omega t\cos Bz - \sin\omega t\sin Bz)$

$= \cos\omega t\cos(-Bz) - \sin\omega t\sin(-Bz)$
$+ a(\cos\omega t\cos Bz - \sin\omega t\sin(Bz))$

$\sin(-x) = -\sin(x) \qquad \cos(-x) = \cos(x)$

$$V = (1+a)\cos\omega t\cos\beta z + (1-a)\sin\omega t\sin\beta z$$

$$A = 1+a \qquad B = 1-a$$

3) $A\cos(\theta+\delta_1) + B\cos(\theta+\delta_2)$ in the form $A\cos(\theta+\delta)$

$Ae^{i\theta+\delta_1} + Be^{i\theta+\delta_2}$ using $\cos(x+y) = \cos x\cos y - \sin x\sin y$

$$A\cos\theta\cos\delta_1 - A\sin\theta\sin\delta_1 + B\cos\theta\cos\delta_2 - B\sin\theta\sin\delta_2$$

$$\cos\theta(A\cos\delta_1 + B\cos\delta_2) - \sin\theta(A\sin\delta_1 + B\sin\delta_2)$$

to go back to $A\cos(\theta+\delta)$

$$\cos\theta(A\cos\delta_1 + B\cos\delta_2) = A\cos\theta\cos\delta$$

$$\sin\theta(A\sin\delta_1 + B\sin\delta_2) = C\sin\theta\sin\delta$$

$$A\cos\delta_1 + B\cos\delta_2 = C\cos\delta \quad \text{*same*}$$
$$A\sin\delta_1 + B\sin\delta_2 = C\sin\delta$$

$$C = A\cos\delta_1 + B\cos\delta_2 / \cos\delta$$

$$A\sin\delta_1 + B\sin\delta_2 = \frac{A\cos\delta_1 + B\cos\delta_2}{\cos\delta}\sin\delta$$

$$\frac{A\sin\delta_1 + B\sin\delta_2}{A\cos\delta_1 + B\cos\delta_2} = \tan\delta$$

$$\delta = \tan^{-1}\left(\frac{A\sin\delta_1 + B\sin\delta_2}{A\cos\delta_1 + B\cos\delta_2}\right)$$ plug into C

$A e^{ix} = A\cos x + iA\sin x$

$$A\cos(\omega t+\theta) = Re[Ae^{i(\omega t+\theta)}]$$
$$= Re[Ae^{i\theta}e^{i\omega t}]$$
$$= Re[Ae^{i\omega t}]$$

$A_1\cos(\theta+\delta_1) + B\cos(\theta+\delta_2)$

$A\,Re[e^{i(\theta+\delta_1)}] + B\,Re[e^{i(\theta+\delta_2)}]$

$A\,Re[e^{i\theta}e^{i\delta_1}] + B\,Re[e^{i\theta}e^{i\delta_2}]$

$Re[Ae^{i\theta}e^{i\delta_1} + Be^{i\theta}e^{i\delta_2}]$

$Re[(Ae^{i\delta_1} + Be^{i\delta_2})e^{i\theta}]$

$Re[(A(\cos\delta_1 + i\sin\delta_1) + B(\cos\delta_2 + i\sin\delta_2))e^{i\theta}]$

$Re[(A\cos\delta_1 + B\cos\delta_2 + Ai\sin\delta_1 + Bi\sin\delta_2)e^{i\theta}]$

~~Re~~ need $A\cos\delta_1 + B\cos\delta_2 = C\cos\delta$ *same*
$A\sin\delta_1 + B\sin\delta_2 = C\sin\delta$

b/c I need to get back to $e^{i\delta} = \cos\delta + i\sin\delta$

This will give same result