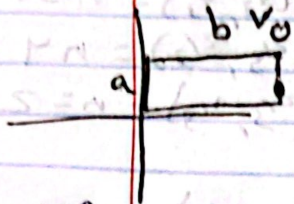
Sam Murphy

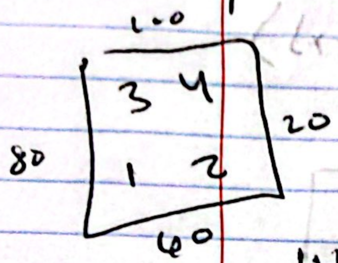
HW5

1) $\psi(x,y) = \frac{-2V_0}{b} \sum_{n=1}^{n} \frac{(-1)^n}{a_n} \frac{\sinh(a_n y)}{\sinh(a_n a)} \cos(a_n x)$



$a_n = (2n-1)\pi/2b$

$a=1 \quad 2b=1$



$\psi(x,y) = -4V_0 \sum_{n=1} \frac{(-1)^n}{a_n} \cdot \frac{\sinh(a_n y)}{\sinh(a_n)} \cos(a_n x)$

$a_n = (2n-1)\pi$

$\psi(x,y) = \frac{4V}{\pi} \sum_{n=1}^{n} \frac{\sin(n\pi y/y_0)}{n \sinh(n\pi y_0/x_0)} \cdot \sinh(n\pi(x_0-x)/y_0)$

$n=1 \quad (1/3, 1/3)$

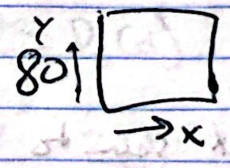
$n=2 \quad (2/3, 1/3)$

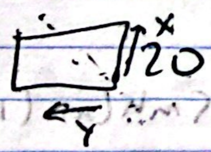
$n=3 \quad (1/3, 2/3)$

$n=4 \quad (2/3, 2/3)$

@ $y_0=1 \quad x_0=1$

$\psi(x,y) = \frac{4V}{\pi} \sum_{n=1}^{\infty} \frac{\sin(n\pi y)}{n \sinh(n\pi)} \sinh\left(n\pi\left(\frac{1-x}{1}\right)\right)$





$\psi(x,y) = -4V \sum_{n=1} \frac{(-1)^n}{a_n} \frac{\sinh(a_n y)}{\sinh(a_n)} \cos(a_n x)$

$a_n = (2n-1)\pi$

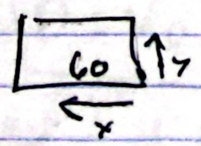
$n=1 \quad (1/3, 2/3)$

$n=2 \quad (1/3, 1/3)$

$n=3 \quad (2/3, 2/3)$

$n=4 \quad (2/3, 1/3)$

Invert $x=-y$

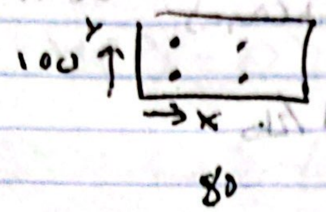


$\psi(x,y) = -4V \sum \frac{(-1)^n}{a_n} \frac{\sinh(a_n(-x))}{\sinh(a_n)} \cos(a_n(-y))$

$(1/3, 1/3) = n=2 = (2/3, 1/3) \quad (1/3, 2/3) = n=4$

$(2/3, 1/3) = n=1 = 1/3, 1/3 \quad (2/3, 2/3) = n=3$

Rotate inside for final step



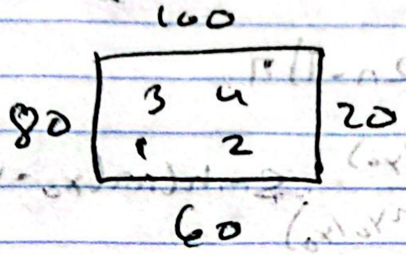
$i=1 = (1/3, 1/3) = (1/3, 2/3) = n=3$

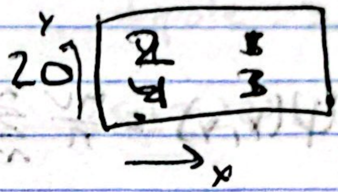
$i=2 = (2/3, 1/3) = (1/3, 1/3) = n=1$

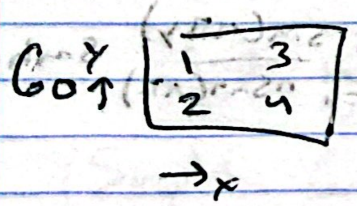
$i=3 = (1/3, 2/3) = (2/3, 2/3) = n4$

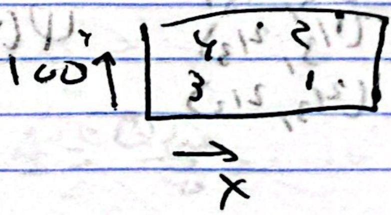
$i=4 = (2/3, 2/3) = (2/3, 1/3) = n=2$

$$\psi(x,y) = \frac{4V}{\pi} \sum \frac{\sin(n\pi y)}{n \sinh(n\pi)} \sinh(n\pi(1-x))$$







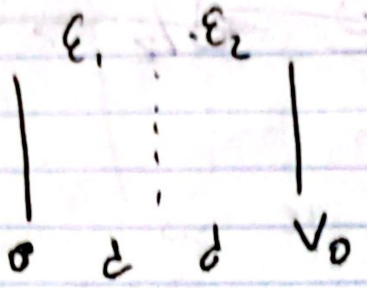


Solutions in text seem to have ~240 error

used $\psi(x,y) = \frac{4V}{\pi} \sum \frac{\sin(n\pi y)}{n \sin(n\pi)} \sinh(n\pi(1-x))$

for all, rotated points as above

5.2)



$\vec{P}\cdot\hat{n} = \sigma_b = \varepsilon_0 \chi_e E_{tot}$

$E_{tot} = \sigma_b / \varepsilon_0 \chi_e$

$\Psi_1 = Ax + B$ $\qquad$ $\Psi_1(0) = 0 = B + A(0)$ $\quad \boxed{B=0}$

$\Psi_2 = Cx + D$ $\qquad$ $\Psi_2(2d) = Cd(2) + D = V_0$

$V_0 - 2Cd = D$

$\Psi_1(d) = \Psi_2(d)$ $\qquad$ $D_1(d) = D_2(d)$

$Ad = Cd + D$

$D = \varepsilon \frac{\partial \varphi}{\partial x}\Big|_d \Rightarrow D_1 = \varepsilon_1 A$

$D_2 = \varepsilon_2 C$

$D_1(d) = \varepsilon_1 A = \varepsilon_2 C = D_2(d)$ $\qquad$ $C = \frac{\varepsilon_1}{\varepsilon_2} A$

$Ad = \frac{\varepsilon_1}{\varepsilon_2} Ad + V_0 - 2\left(\frac{\varepsilon_1}{\varepsilon_2} A\right) d$

$Ad = -\frac{\varepsilon_1}{\varepsilon_2} Ad + V_0$ $\qquad$ $Ad + \frac{\varepsilon_1}{\varepsilon_2} Ad = V_0$

$\boxed{A = V_0 / d(1 + \varepsilon_1/\varepsilon_2)}$ $\qquad$ $C = \varepsilon_1/\varepsilon_2 \frac{V_0}{d(1 + \varepsilon_1/\varepsilon_2)}$

$\boxed{C = V_0 / d\left(\frac{\varepsilon_2}{\varepsilon_1} + 1\right)}$

$D = V_0 - 2d\left(\frac{V_0}{d(\varepsilon_2/\varepsilon_1 + 1)}\right)$

$\boxed{D = V_0\left(1 - \frac{2}{\left(\frac{\varepsilon_2}{\varepsilon_1} + 1\right)}\right)}$

$\Psi_1 = \frac{V_0}{d(1 + \varepsilon_1/\varepsilon_2)} x$

$\Psi_2 = \frac{V_0}{d\left(\frac{\varepsilon_2}{\varepsilon_1} + 1\right)} x + V_0\left(1 - \frac{2}{\left(\frac{\varepsilon_2}{\varepsilon_1} + 1\right)}\right)$

$\cancel{\varphi_1 (d/2)} \quad \cancel{\varphi = 2 \text{ @ } d/2 \;\; \vdots \;\; 3d/2}$

$\cancel{\varphi_1 (d/2) = 0}$

if $\varepsilon_1 = \varepsilon_2 = \varepsilon_0$

$\varphi_1 = \frac{V_0}{2d} x$ ✓

$\varphi_2 = \frac{V_0}{2d} x + V_0 (1 - \frac{2}{2}) = \frac{V_0}{2d} x$ ✓

$\varepsilon_1 = \varepsilon_0 (1 + \chi_1) \qquad \varepsilon_2 = \varepsilon_0 (1 + \chi_2)$

$P = D - \varepsilon_0 E \qquad D = \varepsilon E$

$E = \frac{\partial \varphi}{\partial x} \qquad E_1 = \frac{V_0}{d(1 + \varepsilon_1/\varepsilon_2)}$

$E_2 = V_0 / d(\varepsilon_2/\varepsilon_1 + 1)$

$P_1 = \varepsilon_1 E_1 - \varepsilon_0 E_1 = E_1 (\varepsilon_1 - \varepsilon_0) = V_0 / d(1 + \varepsilon_1/\varepsilon_2) \, (\varepsilon_1 - \varepsilon_0)$

$P_2 = (\varepsilon_2 - \varepsilon_0) V_0 / d(\varepsilon_2/\varepsilon_1 + 1)$

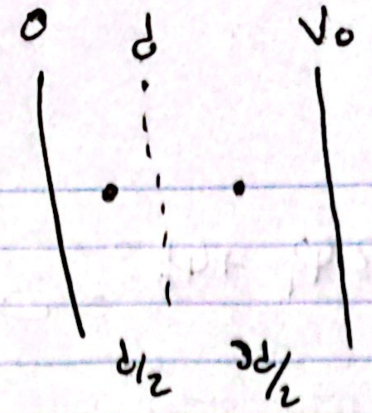
$\sigma_b = P \cdot \hat{n} \qquad \sigma_b = \frac{V_0 (\varepsilon_1 - \varepsilon_0)}{d(1 + \varepsilon_1/\varepsilon_2)} (1)$ @ $x = 0$ $\qquad \varphi_1$

$\sigma_b = \frac{V_0 (\varepsilon_1 - \varepsilon_0)}{d(1 + \varepsilon_1/\varepsilon_2)} (-1)$ @ $x = d^-$

$\sigma_b = \frac{V_0 (\varepsilon_2 - \varepsilon_0)}{d(\varepsilon_2/\varepsilon_1 + 1)} (1)$ @ $x = d^+$

$\sigma_b = \frac{V_0 (\varepsilon_2 - \varepsilon_0)}{d(\varepsilon_2/\varepsilon_1 + 1)} (-1)$ @ $x = 2d$

5.3)



Simple averaging
Start w/ values in
Space $\psi = V_0/2$

$\psi_0\ \psi_1\ \psi_2\ \psi_3\ \psi_4$

$$\frac{\psi_2 - 2\psi_1 + \psi_0}{(d/2)^2} = \nabla^2\psi = 0$$

$$\frac{\psi_4 - 2\psi_3 + \psi_2}{(d/2)^2} = \nabla^2\psi = 0$$

$D_1(d) = D_2(d)$ $\qquad \varepsilon_1\left(\frac{V_0}{d(1+\varepsilon_1/\varepsilon_2)}\right) = \varepsilon_2\left(\frac{V_0}{d(\varepsilon_2/\varepsilon_1 + 1)}\right)$

$D = \varepsilon\frac{d\psi}{dx} \approx$ $\quad D_1 = \varepsilon_1 \frac{\psi_2 - \psi_1}{d/2}$ backward Euler

$D_2 = \varepsilon_2 \frac{\psi_3 - \psi_2}{d/2}$ forward Euler

$$\frac{\psi_4 - 2\psi_3 + \psi_2}{(d/2)^2} = 0 \qquad -2\psi_3 = -V_0 - \frac{V_0}{2}$$

$$\boxed{\psi_3 = 3V_0/4}$$

$$\frac{\varepsilon_2}{d/2}\left(\frac{3V_0}{4} - \psi_2\right) = \frac{\varepsilon_1}{(d/2)}\left(\psi_2 - \frac{V_0}{2}\right) \qquad \frac{3\varepsilon_2 V_0}{4} + \frac{\varepsilon_1 V_0}{2} = \psi_2(\varepsilon_1 + \varepsilon_2)$$

$$\frac{3\varepsilon_2 V_0}{4} - \varepsilon_2\psi_2 = \varepsilon_1\psi_2 - \frac{\varepsilon_1 V_0}{2} \qquad \boxed{\frac{V_0}{2(\varepsilon_1+\varepsilon_2)}\left(\frac{3}{2}\varepsilon_2 + \varepsilon_1\right) = \psi_2}$$

$$\frac{\frac{V_0}{2(\varepsilon_1+\varepsilon_2)}\left(\frac{3\varepsilon_2}{2}+\varepsilon_1\right) - 2\varphi_1 + \varphi_0}{\varepsilon/2} = 0$$

$$\boxed{\varphi_1 = \frac{V_0}{4(\varepsilon_1+\varepsilon_2)}\left(\tfrac{3}{2}\varepsilon_2+\varepsilon_1\right)}$$