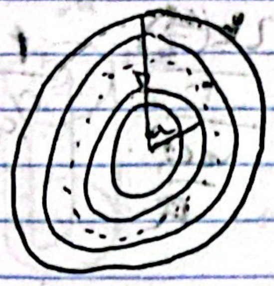
Sam Murphy

HW3

(3.1)



New shell on dotted line w/ charge $Q/2$

- Same charge until $\frac{a+b}{2}$ b/c Q_{enc} does not change

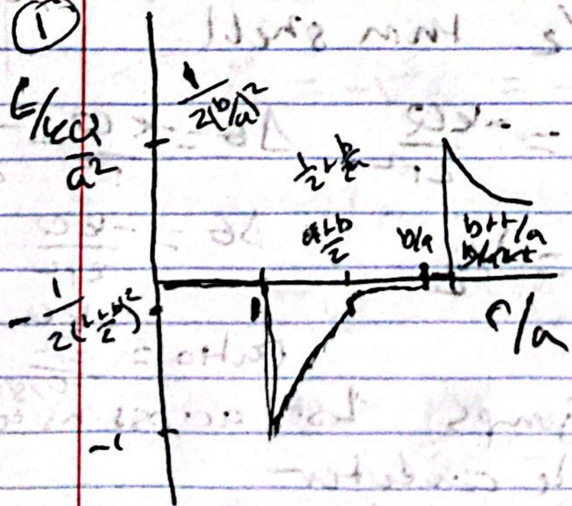
Uniformly charged spherical shell => have to assume "thin" otherwise would imply some E inside thickness w $-Q + Q/2$

To balance $E=0$ inside outer thickness only half of charge is inside shell, => other half moves out.

$E = \frac{kQ}{r^2}\hat{r}$

@ $r/a = 1$ $\quad E = \frac{\frac{k(-Q)}{r^2}}{\frac{kQ}{a^2}} = \frac{-1}{(r/a)^2} = -1$

(1)

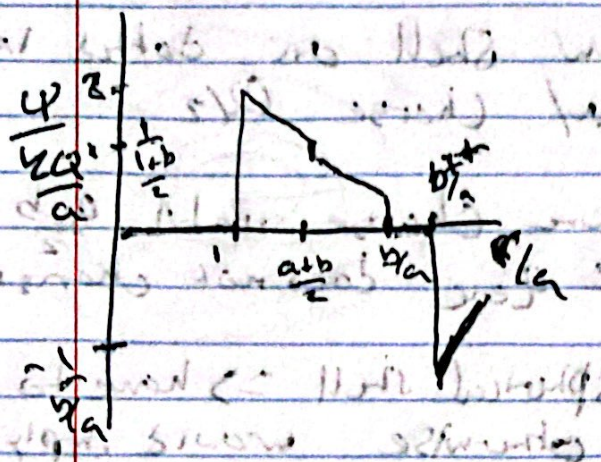


@ $r/a = \frac{\left(\frac{a+b}{2}\right)}{a}$ $\quad E = \frac{\frac{-Q/2}{r^2}}{\frac{kQ}{a^2}}$

$E = \frac{-1}{2(r/a)^2}$

(2) $\varphi(r) - \varphi(0) = -\int_{r_0}^{r} E\,dl \qquad \varphi(0) = 0 \qquad \varphi(1) = -\int_0^r \frac{kQ}{r^2}\,dr$

$\varphi(r) = -\frac{2kQ}{r}\Big]_0^r = \frac{-2kQ}{r}$

② 

$\frac{a+b}{2}$ $\qquad \frac{-2k(\frac{Q}{2})/r}{kQ/a}$ $\qquad \frac{1}{r/a}$ $\qquad \frac{1}{1+\frac{b}{2}}$

b^+/a $\qquad \frac{-2k(\frac{Q}{2})/r}{kQ/a}$ $\qquad -\frac{1}{b/a}$

③ Jump 1 outer surface of inner shell

$G_1 = 0 \qquad G_2 = k\frac{-Q}{r^2} \qquad \Delta G = -\frac{kQ}{r^2} - 0$

$\text{ratio}_{12} = G_2/G_1 = \frac{-kQ}{r^2}/0 = -\infty \qquad \text{ratio} = \frac{\Delta G}{\frac{-Q}{4\pi r^2}} = \frac{\Delta G}{\sigma_0}$

$\text{ratio} = 1/\varepsilon_0$

Jump 2 near $a+b/2$ thin shell

$G_2 = \frac{-kQ}{r^2} \qquad G_3 = \frac{-kQ}{2r^2} \qquad \Delta G = \frac{-kQ}{r^2} - \frac{-kQ}{2r^2}$

$\text{ratio}_{23} = \frac{-kQ}{2r^2}/\frac{-kQ}{r^2} = 1/2 \qquad \Delta G = \frac{-kQ}{2r^2} \qquad \sigma_0 = \frac{+\frac{Q}{2}}{4\pi r^2}$

$\text{ratio} = \frac{\frac{-kQ}{2r^2}}{-Q/8\pi r^2} = \frac{1}{\varepsilon_0}$

Jump 3 => two jumps 1st. across inside of shell to $G = 0$ inside conductor
Second w/ fully Q enclosed

$\Delta G = \frac{-kQ}{2r^2} - \frac{kQ}{2r^2} = \frac{-kQ}{r^2}$

$G_3 = \frac{-kQ}{2r^2} \qquad G_4 = \frac{kQ}{2r^2}$

$\sigma = \frac{Q}{4\pi r^2}$

$\text{ratio}_{34} = \frac{G_4}{G_3} = \frac{kQ}{2r^2}/\frac{-kQ}{2r^2} = -1 \qquad \frac{\Delta G}{\sigma} = \frac{-\frac{kQ}{r^2}}{\dots} = \frac{1}{\varepsilon_0}$

(3.2)

$\psi(b)-\psi(a) = -\int_a^b E\,dl$

$\Delta\psi = \psi(b)-\psi(a)$

$\Delta\psi = -\int_a^b \frac{k(-Q)}{r^2}\,dr$

$\Delta\psi = -2kQ/r$

$C = \frac{Q}{|\Delta\psi|}$ $\quad Q = -Q$

$C = \frac{-Q}{\left|-\frac{2kQ}{r}\right|} = \frac{-r}{2k}$

$\psi(a) = V_0 \quad \psi(b) = 0 \quad \Delta\psi = 0$

$\nabla^2\psi = 0$

$E = -\nabla\psi = -\hat{x}\frac{\partial\psi}{\partial x} - \hat{y}\frac{\partial\psi}{\partial y} = -\hat{r}\frac{\partial\psi}{\partial r} + \hat{z}\frac{\partial\psi}{\partial z} + \hat{\theta}\frac{\partial\psi}{\partial\theta}$

"by symmetry" ? infinite reduce to

$E = -\nabla\psi = -\hat{r}\frac{\partial\psi}{\partial r}$

$\cancel{\frac{\partial^2\psi}{\partial r^2} = 0}$ $\quad \frac{\partial\psi}{\partial r} = A \quad \psi = Ar + B$

$\cancel{\psi(a) = 0}$ $\quad V_0 = A(a) + B$

$0 = A(b) + B$

$2\pi r h = A \quad \sigma = Q/A$

$$\frac{1}{r}\frac{d}{dr}\left(r\frac{d\psi}{dr}\right)=0$$

$$\frac{d}{dr}\left(r\frac{d\psi}{dr}\right)=0 \qquad r\frac{d\psi}{dr}=A$$

$$\frac{d\psi}{dr}=\frac{A}{r} \qquad \psi=A\ln(r)+B$$

$$\psi(a)=V_0=A\ln(a)+B$$
$$\psi(b)=0=A\ln(b)+B$$

$$V_0=A(\ln(a)-\ln(b))$$
$$V_0=A\left(\ln\left(\frac{a}{b}\right)\right) \qquad A=\frac{V_0}{\ln(a/b)}$$

$$0=\frac{V_0}{\ln(a/b)}\ln(b)+B \qquad B=\frac{-V_0\ln(b)}{\ln(a/b)}$$

$$\psi=\frac{V_0}{\ln(a/b)}\ln(r)-\frac{V_0\ln(b)}{\ln(a/b)}$$

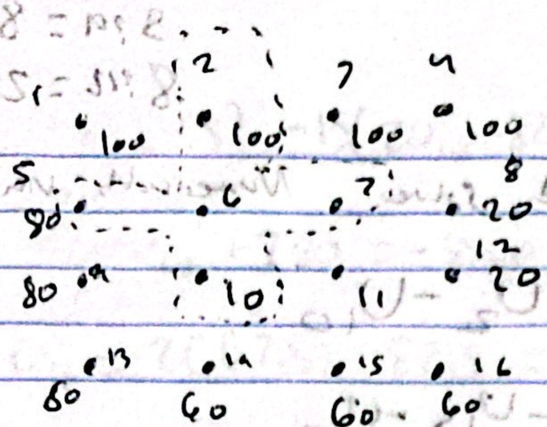
$$E=-\nabla\psi=-\frac{\partial}{\partial r}\left(\frac{V_0\ln(r)}{\ln(a/b)}-\frac{V_0\ln(b)}{\ln(a/b)}\right)$$

$$E=\frac{V_0}{\ln(a/b)}\left(\frac{-1}{r}\right)=\frac{-\sigma}{\varepsilon_0} \qquad \sigma=\frac{V_0\varepsilon_0}{\ln(a/b)r}$$

$$Q/A=\sigma \qquad Q=\sigma A \qquad A=2\pi rh$$

$$C=\frac{\frac{V_0\varepsilon_0}{\ln(a/b)r}(2\pi rh)}{V_0}=\frac{\varepsilon_0 2\pi h}{\ln(a/b)}$$

(3.3)



$\nabla^2 \phi = 0$

$$\frac{U_{i+1} + 2U_i + U_{i-1}}{h^2}$$

Central difference

$$\frac{U_{i+1j} + U_{ij+1} + U_{i-1j} + U_{i-1j} - 4U_{ij}}{h^2}$$

5 point laplaceanstencil

$\cancel{U_7 = 2U_6}$

$\Phi_1 = U_6$ $\Phi_2 = U_7$

$\Phi_3 = U_{80}$ $\Phi_4 = U_{11}$

$U_7 + U_5 + U_{10} + U_2 - 4U_6 = 0$ ∴ $U_7 = -80 - U_{10} - 100 + 4U_6$

$65.63 + 80 + 70.63 + 100 - 4(77.5) = 0$ $U_7 = -180 - U_{10} + 4U_6$

$U_{11} + U_3 + U_8 + U_6 - 4U_7 = 0$ $U_7 = \frac{U_{11} + 100 + 20 + U_6}{4}$

$U_{14} + U_6 + U_{11} + U_9 - 4U_{10} = 0$

$U_{12} + U_{10} + U_{15} + U_7 - 4U_{11} = 0$ $U_7 = 4U_{11} - 60 - U_{10} - 20$

$U_7 = 4U_{11} - 80 - U_{10}$

$U_7 = -180 - U_{10} + 4U_6$

$- \quad U_7 = 4U_{11} - U_{10} - 80$

$0 = 4U_6 - 4U_{11} - 100$ $U_6 = U_{11} + 25$

$$\begin{bmatrix} 1 & 0 & \dots & 1 & -4 & 1 & \dots & 0 & 1 \\ & & & & & & & & \end{bmatrix} \begin{bmatrix} U_1 \\ \\ U_n \end{bmatrix} = \begin{bmatrix} 0 \\ \\ 0 \end{bmatrix} h^2$$

$1 \to 4 = 100$
$13 \to 16 = 60$
$5 \,\&\, 9 = 80$
$8 \,\&\, 12 = 20$

Thats how I would solve. Numerically via python

$U_7 = 4U_6 - U_5 - U_2 - U_{10}$

$- \quad U_7 = 4U_{11} - U_{12} - U_{15} - U_{10}$

$0 = 4U_6 - 4U_{11} - U_5 - U_2 + U_{12} + U_{15}$

$4U_{11} = 4U_6 - 80 - 100 + 20 + 60$

$U_{11} = U_6 - 25 \qquad U_6 = U_{11} + 25$

$(U_6 - 25) + 100 + 20 + U_6 = 4U_7$

$2U_6 + 95 = 4U_7 \qquad U_6 = 2U_7 - \frac{95}{2}$

$20 + U_{10} + 60 + U_7 - 4U_{11} = 0$

$20 + U_{10} + 60 + \left(\frac{U_6}{2} + \frac{95}{4}\right) - 4(U_6 - 25) = 0$

$\frac{7}{2}U_6 = 180 + \frac{95}{4} + U_{10} \qquad U_{10} = \frac{7}{2}U_2 - 180 - \frac{95}{4}$

$\cancel{\left(\frac{U_6}{2} + \frac{95}{4}\right) + 80 + \left(\frac{7}{2}U_6 - 180 - \frac{95}{4}\right)}$

$\cancel{(U_6 - 25) + 100 + 20 + U_6 - 4\left(\frac{U_6}{2} + \frac{95}{4}\right)}$

$U_{11} + 25 + U_{11} - 4U_{10} = -U_{14} - U_9 = -140 - 25$

$2U_{11} - 4U_{10} = -165 \qquad U_{11} = 2U_{10} - 82.5$

$U_6 = 2U_{10} - 82.5 + 25 = 2U_{10} - 57.5$

$U_7 = 4U_6 - U_{10} - 180$

$U_{11} + U_6 - 4(4U_6 - U_{10} - 180) = -120$

$U_{11} - 15U_6 + 4U_{10} = -840$

$(2U_{10}-82.5) - 15(2U_{10}-82.5+25) + 44U_{10} = -840$

$-24U_{10} + 780 = -840 \quad U_{10} = 67.5 \quad \Phi_3$

$U_4 = 2(67.5) - 82.5 = 52.5 \quad \Phi_4$

$U_6 = 2(67.5) - 57.5 = 77.5 \quad \Phi_1$

$U_7 = 4(77.5) - 67.5 - 180 = 62.5 \quad \Phi_2$

~~Corrected~~

Using method

$$\Phi_1 = \frac{65+65+100+80}{4} = 77.5$$

$$\Phi_2 = \frac{100+20+65+65}{4} = 62.5$$

$$\Phi_3 = \frac{80+60+65+65}{4} = 67.5$$

$$\Phi_4 = \frac{65+65+20+60}{4} = 52.5$$

But use new #

$$\Phi_2 = \frac{65+77.5+100+20}{4} = 65.625$$

$$\Phi_3 = \frac{77.5+65+80+60}{4} = 70.625$$

$$\Phi_4 = \frac{70.625+65.625+20+60}{4} = 54.0625$$

2)

	0	0	0	0	
	1	2	3	4	
80	5	6	7	8	0
80	9	10	11	12	0
	13	14	15	16	
	0	0	0	0	

Solve like previous

average = 20