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HW 9

1) $\frac{dI}{dt} + \frac{I}{\tau} = \frac{V_0}{\mathcal{L}} \cos(\omega t + \phi)$

Find Steady state $I(t)$

assume soln in form $I(t) = Re[\tilde{I} e^{i\omega t}]$

$$\frac{V_0}{\mathcal{L}} \cos(\omega t + \phi) = Re[I_0 i\omega e^{i\omega t}] + \frac{1}{\tau} Re[I_0 e^{i\omega t}]$$

$$= Re[I_0 i\omega(\cos\omega t + i\sin\omega t)]$$

$$+ \left[\frac{I_0}{\tau}(\cos\omega t + i\sin\omega t)\right]$$

$$= -I_0 \omega \sin\omega t + \frac{I_0}{\tau}\cos\omega t$$

$$I_0 = \frac{V_0}{\mathcal{L}} \frac{\cos(\omega t + \phi)}{\cos(\omega t) - \frac{\omega}{\tau}\sin(\omega t)}$$

b) $\mathcal{L}\frac{dI}{dt} + IR + Q/C = V_0 \cos(\omega t)$

$I = dQ/dt$

$$\mathcal{L}\frac{\partial^2 Q}{\partial t^2} + \frac{\partial Q}{\partial t}R + Q/C = V_0 \cos(\omega t)$$

Assume soln in form $Q(t) = Re[\tilde{Q} e^{i\omega t}]$

$Q'' = Re[-Q_0 \omega^2 e^{i\omega t}]$ $\qquad \tilde{Q} = Q_0$

$Q' = Re[Q_0 i\omega e^{i\omega t}]$

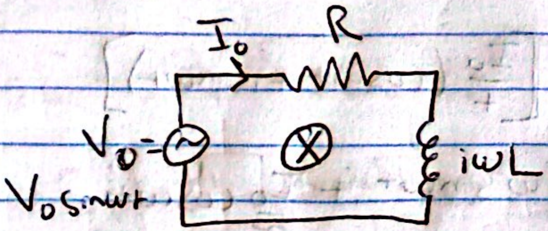
$$\mathcal{L}\,Re[-Q_0\omega^2 e^{i\omega t}] + R\,Re[Q_0 i\omega e^{i\omega t}] + Q_0/C = V_0\cos(\omega t)$$

$$e^{i\omega t} = \cos\omega t + i\sin\omega t$$

$$-\mathcal{L}Q_0\omega^2\cos\omega t + RQ_0(-\omega\sin\omega t) + Q_0/C = V_0\cos(\omega t)$$

$$Q_0 = \frac{V_0\cos\omega t}{\frac{1}{C} - \mathcal{L}\omega^2\cos\omega t - R\omega\sin\omega t}$$

2) ~~$I_0 = V_0 - R - i\omega L$~~



$\underbrace{V_0\sin\omega t - IR - L\frac{dI}{dt}}_{} = 0$

homogenous part + particular part

① $\frac{R}{\mathcal{L}}dt = \frac{-dI}{I}$ integrate

② $I_p(t) = A\cos\omega t + B\sin\omega t$

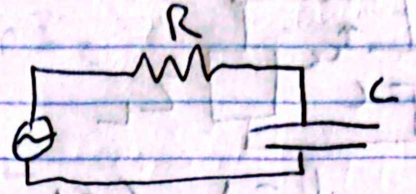
③ $V_0\sin\omega t = L\,dI/dt + IR$

assume soln in form $I(t) = Re[\tilde{I}e^{i\omega t}]$

$$V_0\sin\omega t = L\,Re[\tilde{I}i\omega e^{i\omega t}] + R.Re[\tilde{I}e^{i\omega t}]$$

$$V_0\sin\omega t = L(-I(t)\omega\sin\omega t) + R(I(t)\cos\omega t)$$

Kirchoff $\sum V_i = 0$ $\oint \vec{E}\cdot dl = -d\Phi/dt$



$V_0 = V_0 \sin\omega t$ $C = Q/V$

$I = dQ/dt$ $Q = VC$

$V_0 \sin\omega t = I_0 R_0 + I_0 X_C$ $\frac{1}{C} Q = V$

$V_0 \sin\omega t = I_0 R_0 + Q/C$

$V_0 \sin\omega t = \frac{dQ}{dt} R_0 + Q/C$ assume

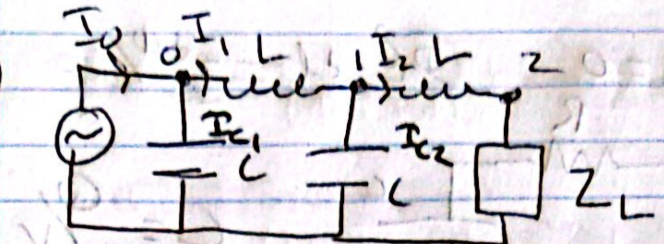
$\int dQ = \int I\, dt$ $I = Re(I_0 e^{i\omega t})$

$Q = \frac{I}{i\omega} + \cancel{\text{const}}$ $V = Q/C$ $V = \frac{I}{i\omega C}$

$V_0 \sin\omega t = I_0 R_0 + \frac{I_0}{i\omega C} = I_0 R_0 + I X_C$

$X_C = 1/i\omega C$

9.3)



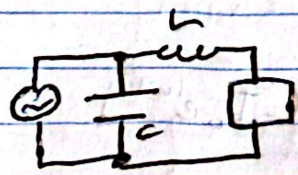
find complex currents

$V_0 = \cos(\omega t)$ $\quad L=1 \quad C=1$

$I_0 = I_1 + I_{C_1} \qquad I_1 = I_2 + I_{C_2}$

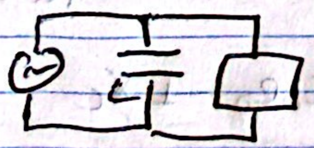
Resistors in ||

$$Z = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2}} = \frac{1}{\frac{1}{C} + \frac{1}{Z_L + L}}$$



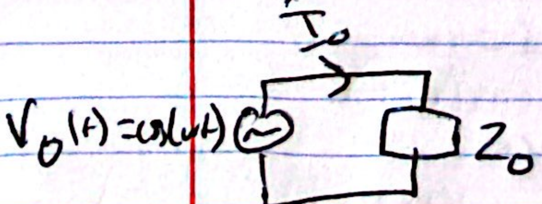
$$Z = \frac{1}{\frac{1}{C} + \frac{1}{Z_L + L}}$$

$$Z = L + \frac{1}{\frac{1}{C} + \frac{1}{Z_L + L}}$$



$$Z = L + \frac{1}{\frac{1}{C} + \frac{1}{Z_L + L}}$$

$$Z_0 = \frac{1}{\frac{1}{C} + \frac{1}{L + \frac{1}{\frac{1}{C} + \frac{1}{Z_L + L}}}}$$



$$V_0(t) - \bar{I}_0 Z_0 = 0$$

$$\bar{I}_0 = \frac{\cos(\omega t)}{Z_0}$$

$I_1 = I_0(1 - i\omega C Z_0)$

$I_2 = I_1(1 - i\omega C Z_1)$

$$V_0 = (I_0 - I_1)\left(\frac{1}{j\omega C}\right)$$

$$V_1 = (I_1 - I_2)\left(\frac{1}{j\omega C}\right)$$

$$V_2 = I_2 Z_L$$

$$\frac{V_0}{I_0} = \frac{I_0 - I_1}{i\omega C} \Big/ \frac{\cos(\omega t)}{Z_0}$$

$$= \frac{\cancel{\frac{\cos(\omega t)}{Z_0}} - \cancel{\frac{\cos(\omega t)}{Z_0}}(1 - i\omega C Z_0)}{i\omega C} \Big/ \cancel{\frac{\cos \omega t}{Z_0}}$$

$$\frac{V_0}{I_0} = \frac{1 - (1 - i\omega C Z_0)}{i\omega C} = Z_0$$

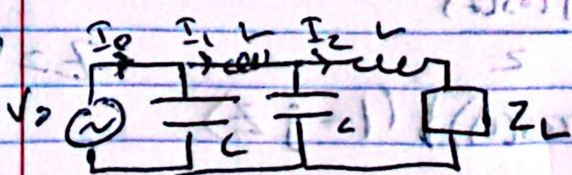
$Z_L = 1$ $\quad \frac{1}{1+\frac{1}{1+1+\frac{1}{1+\frac{1}{2}}}} = \frac{1}{1+\frac{1}{1+\frac{1}{3/2}}} = \frac{1}{1+\frac{1}{1+2/3}} = \frac{1}{1+\frac{1}{5/3}}$

$= \frac{1}{8/5} = 5/8$

$$Z_0 = \frac{1}{\frac{1}{j\omega C}+\frac{1}{\frac{1}{j\omega L}+\frac{1}{\frac{1}{j\omega C}+\frac{1}{Z_L+j\omega L}}}}$$

$$\frac{V_0}{I_0} = \frac{1 - i\omega C}{i\omega L} \qquad -i + \frac{C}{L} \qquad \omega = 1 \quad C = 1 \quad L = 1 \quad Z_L = 1$$

$$Z_0 = \frac{1}{-j+\frac{1}{-j+\frac{1}{-j+\frac{1}{1+j}\frac{(1-j)}{(1-j)}}}}$$



$$\frac{1}{-j+\frac{1-j}{1+1+j-j}}$$

$$\frac{1}{\frac{-10j}{10}+\frac{2+6j}{10}} \quad \frac{1}{-j+\frac{2+6j}{1+9}} \quad \frac{1}{-j+\frac{2}{1-3j}\frac{(1+3j)}{(1+3j)}} \quad \frac{1}{\frac{1-3j}{2}} = \frac{1}{\frac{2j}{2}+\frac{1-j}{2}} \quad \frac{1}{-j+\frac{1-j}{2}}$$

$$\frac{1}{\frac{2+4j}{10}} \quad \frac{1}{\frac{1-2j}{5}} \quad , -j+\frac{1}{\frac{5}{1-2j}\frac{(1+2j)}{(1+2j)}} \quad \frac{1}{-j+\frac{5+10j}{1+4}}$$

$$\frac{1}{-j+1+2j} \quad \frac{1}{1+j} = Z_0 = \frac{1}{1+j}\frac{(1-j)}{(1-j)} = \frac{1-j}{1+1}$$

$$Z_0 = 1-j/2 \checkmark$$

assume ssln in form $I = Re[I_0 e^{j\omega t}]$

$V_0 = \cos(\omega t) = I_0 Z_0$; $R\omega = 1$

$$I_0 = \frac{2\cos(t)}{1-j}\frac{(1+j)}{1+j} = \frac{2\cos(t) + 2j\cos(t)}{2}$$

$I_0 = \cos(t) + j\cos(t)$

$I_1 = I_0(1-j(1-j/2)) = (\cos(t)+j\cos(t))(1-j-1/2)$

$I_1 = (\cos(t)+j\cos(t))(1-3j/2)$

$I_1 = \cos(t) + j\cos(t) - \frac{3j}{2}\cos(t) + 3/2\cos(t)$

$I_1 = 5/2\cos(t) - \frac{j\cos(t)}{2}$

$Z = 1+2j$

$I_2 = 1/2(5\cos(t) - j\cos(t))(1-jZ)$

$I_2 = 1/2(5\cos(t) - j\cos(t))(1-j+2)$

$= \cancel{5\cos(t)} \; 1/2(15\cos(t) - 3j\cos(t) - 5j\cos(t) - \cos(t))$

$I_2 = \cancel{\tfrac{12}{2}\cos} \tfrac{1}{2}(14\cos(t) - 8j\cos(t))$

$I_2 = 7\cos(t) - 4j\cos(t)$

$V_1 = I_1 Z \qquad V_1 = \tfrac{1}{2}(5\cos(t) - j\cos(t))(1+2j)$

$V_1 = \tfrac{1}{2}(5\cos(t) - j\cos(t) + 10j\cos(t) + 2\cos(t))$

$V_1 = \tfrac{1}{2}(7\cos(t) + 9j\cos(t))$

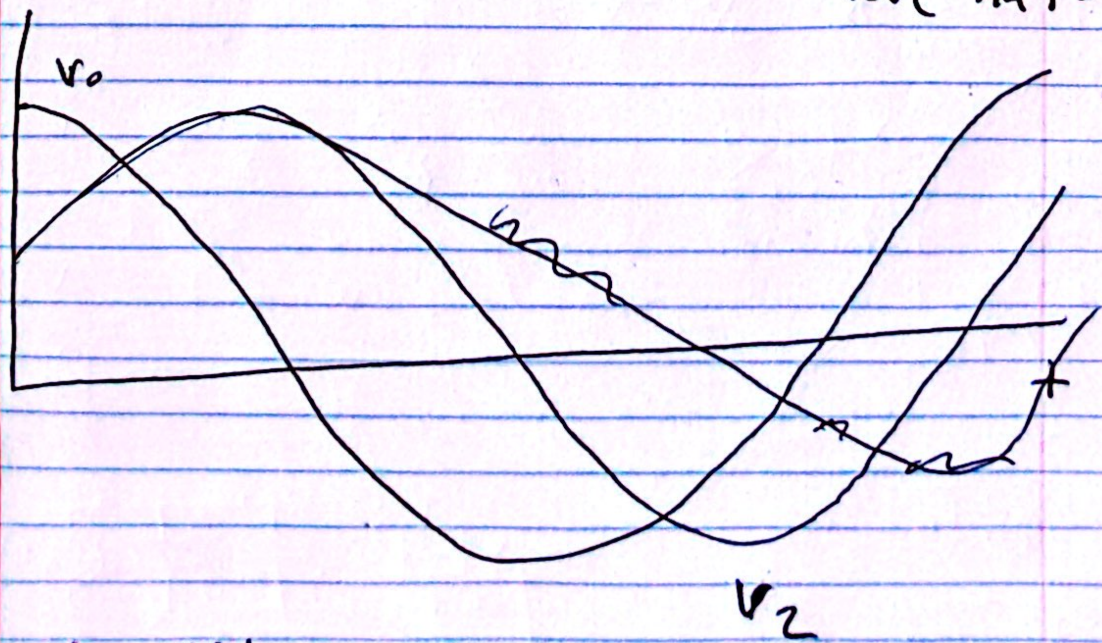
$V_2 = I_2 Z \quad = (7\cos(t) - 4j\cos(t))\left(\frac{1+3j}{5}\right)$

$V_2 = \tfrac{1}{5}(7\cos(t) - 4j\cos(t) + 21j\cos(t) + 12\cos(t))$

$V_2 = \tfrac{1}{5}(19\cos(t) + 17j\cos(t)) \qquad V_2 = \cos(t)\left(\frac{19}{5} + \frac{17}{5}j\right)$

$\tan^{-1}(17/19) = \phi$

V_1



$\phi = 41.8°$ phase shift