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# HW 2

2.1 from HW 1

$$G_z = 2\pi k\sigma R\left(\frac{1}{\sqrt{R^2+(z-h/2)^2}} - \frac{1}{\sqrt{R^2+(z+h/2)^2}}\right)$$

$$R\sqrt{1+\left(\frac{z-h/2}{R}\right)^2}$$

$$R\sqrt{1+\left(\frac{z}{R}-\frac{h}{2R}\right)^2}$$

$$G_z = 2\pi k\sigma\left(\frac{1}{\sqrt{1+(z/R-\frac{h}{2R})^2}} - \frac{1}{\sqrt{1+(\frac{z}{R}+h/2R)^2}}\right)$$

$$G_0 = kQ/R^2 \qquad Q = 2\pi Rh\sigma$$

$$G_0 = k2\pi Rh\sigma/R^2 = k2\pi h\sigma/R$$

$$G_z/G_0 = \frac{\frac{1}{\sqrt{1+(z/R-h/2R)^2}} - \frac{1}{\sqrt{1+(\frac{z}{R}+\frac{h}{2R})^2}}}{h/R}$$

$x = z/R$ $\quad \frac{G_z}{G_0}=R/h \quad \frac{1}{\sqrt{x^2-\frac{2\phi x}{2}+\phi^2+1}} - \frac{1}{\sqrt{x^2+\frac{2\phi x}{2}+\phi^2+1}}$

$\phi = h/R$

peaks around $z/R$

larger $h/R$ means smaller field (based on $R/h$)

point charge $\quad E_z = \frac{kQ}{R^2}$

$E_z/E_0 = 1 \quad E_0 = \frac{kQ}{R^2}$

no plot
no change

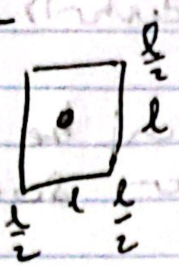
Ring of charge
from class $\perp$

~~$E_z = k \frac{R}{\sqrt{h^2+R^2}}$ $\quad E_z/E_0 = \frac{1}{\sqrt{h^2+R^2}}$~~

$E = E_0 \frac{R}{\sqrt{z^2+R^2}} \quad E_z/E_0 = \frac{R}{\sqrt{z^2+R^2}} = \frac{1}{\sqrt{(\frac{z}{R})^2+1}}$

Falls off a $z/R \to \infty$

2.2




$\oint G\, da = \frac{q_{enc}}{\epsilon_0}$

point chose @ center

w/ cube even faces

$GA = \frac{q_{enc}}{6\epsilon_0}$

$E = \frac{kQ}{r^2}\hat{r}$

assume upper face $z = \ell/2$

$r = \sqrt{x^2+y^2+z^2} = \sqrt{x^2+y^2+\frac{\ell^2}{4}}$

$E = \frac{kQ}{r^2}\frac{\hat{r}}{\sqrt{r}} = \frac{kQ}{(x^2+y^2+\frac{\ell^2}{4})^{1/2}}$

$dA = \hat{z}\, dx\, dy$

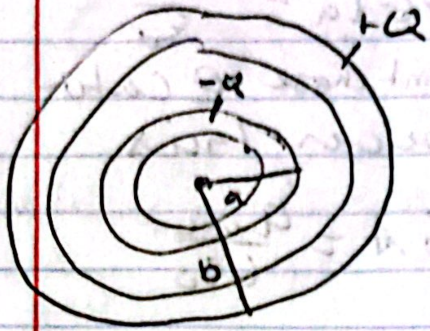
$\oint G\, dA = kQ \oint \frac{\ell/2}{(x^2+y^2+\frac{\ell^2}{4})^{3/2}}\, dx\, dy$

$= kQ \int_{-\frac{\ell}{2}}^{\frac{\ell}{2}} \int_{-\frac{\ell}{2}}^{\frac{\ell}{2}} \frac{\ell/2}{(x^2+y^2+\frac{\ell^2}{4})^{3/2}}\, dx\, dy$

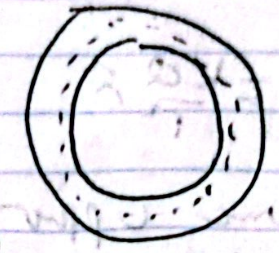
Wolfram alpha $= kq \int_{-\frac{\ell}{2}}^{\frac{\ell}{2}} \frac{\ell}{2}\left(\frac{1}{y^2+\frac{\ell^2}{4}}\right) dy$

Wolfram alpha $= kq\frac{\ell}{2}\left(\frac{4}{3\ell}\pi\right) = \frac{q}{4\pi\epsilon_0}\frac{2}{3}\pi = \frac{q}{6\epsilon_0}$

## 2.3 Electric field inside conductors $0$:




There can be no charge on inner surface

① 

As we move to center of spheres entire $-Q$ must be enclosed

$$E = -\frac{kQ}{r^2}$$

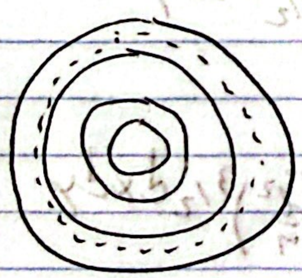
Gaussian sphere here

$E = 0$ $\quad \oint E\,dA = 0 = \frac{Q_{enc}}{\varepsilon_0}$

$Q_{enc} = 0$

$\therefore$ thus all $-Q$ on outside of sphere

② Outer conductor also $E = 0$ inside

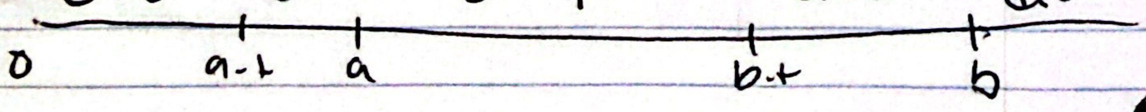


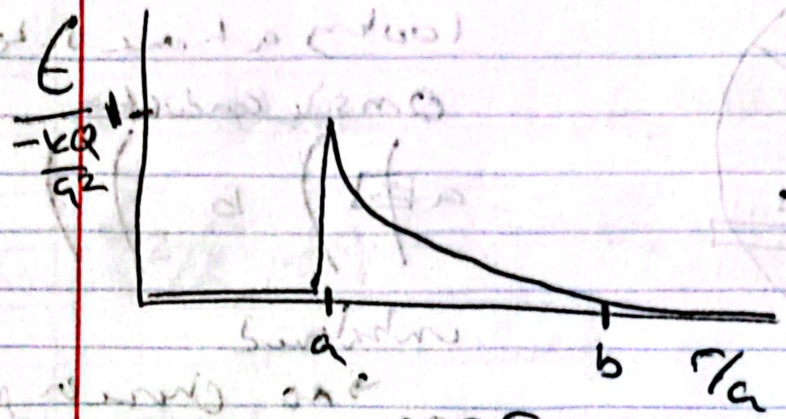
$E$ still must $= 0$

$Q+$ must be inside this sphere $\Rightarrow$ all $Q+$ on inside

If all $Q+$ is on inside of outer, outside of outer $\Rightarrow 0$

| | | | | |
|---|---|---|---|---|
| $Q_{enc} = 0$, $E = 0$ | $Q_{enc} = 0$, $E = 0$ | $E = \frac{kQ}{r^2}$ | $E = 0$, $Q = 0$ | $E = 0$, $Q = 0$ |

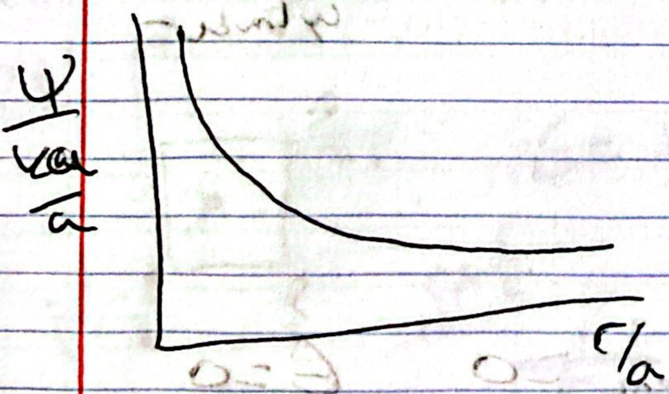





2.4 $\Psi(r) = -\int_0^r E_r(r)\,dr$ $\qquad E_r(r) = \frac{kQ}{r^2}$

$\Psi(r) = \frac{-kQ}{-2r}\Big]_0^r = \frac{kQ}{2r}$

plot $= \frac{\Psi(r)}{kQ/a} = \frac{a}{2r} = \frac{1}{2r/a}$ $\qquad r/a = x$ $\qquad \frac{1}{2x}$




Same by symmetry

surface doesnt matter