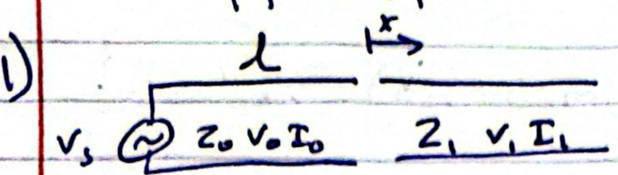
Sam Murphy

HW 11

1)



$$V_0(x,t) = Re[e^{i\omega t}(V_0^+ e^{-iBx} + V_0^- e^{iBx})]$$

$$I_0(x,t) = Re[e^{i\omega t}(I_0^+ e^{-iBx} + I_0^- e^{iBx})]$$

$$I_0(x,t) = Re[e^{i\omega t}(\frac{V_0^+}{Z_0} e^{-iBx} + \frac{V_0^-}{Z_0} e^{iBx})]$$

Continuity $V_0(0,t) = V_1(0,t)$ $\quad I_0(0,t) = I_1(0,t)$

$V_1^- = 0 \qquad V_1^+ = V_s$

$\phi @ x=0$

$$Re[e^{j\omega t}(V_0^+ \cancel{e^{-iBx}} + V_0^-)] = Re[e^{j\omega t}(V_1^+ + V_1^-)]$$

$$Re[e^{i\omega t}(V_0^+ + V_0^-)] = Re[e^{i\omega t}(V_1^+)]$$

$1 @ t=0$ $\quad V_1^+/Z_1)]$

$$Re[\cancel{e^{i\omega t}}(\frac{V_0^+}{Z_0} - \frac{V_0^-}{Z_0})] = Re[e^{i\omega t}(\cancel{V_0^+ e^{-iB(-l)}} + \cancel{V_0^- e^{iB(-l)}})]$$

$$Re[V_{s_0} e^{i\omega t}] = Re[e^{i\omega t}(V_0^+ e^{-iB(-l)} + V_0^- e^{iB(-l)})]$$

$$V_{s_0} = V_0^+(e^{iBl} + \frac{V_0^-}{V_0^+} e^{-iBl}) \qquad V_0^-/V_0^+ = \tilde{\rho}$$

$V_0^+ + V_0^- = V_1^+ \quad V_0^+(1+\rho) = V_1^+ \quad V_{s_0} = V_0^+ \cos(Bl)(1+\rho)$

$V_0^+(1 - \frac{V_0^-}{V_0^+}) = \frac{Z_0}{Z_1} V_1^+ \qquad \frac{Z_0}{Z_1} V_1^+ = V_0^+(1 - \rho)$

$\cancel{V_0^+(1-\rho)} \quad 1 - \rho = \frac{Z_0}{Z_1}(1+\rho) \qquad \cancel{V_{s_0} = \frac{Z_0}{Z_1} V_1^+ \cos(Bl)}$

$1 - \rho = \frac{Z_0}{Z_1} + \frac{Z_0}{Z_1}\rho \quad 1 - \frac{Z_0}{Z_1} = \rho(1 + \frac{Z_0}{Z_1}) \qquad \cancel{V_1^+ = \frac{Z_1}{Z_0} \cos(Bl) V_{s_0}}$

$\rho = \frac{1 - \frac{Z_0}{Z_1}}{} \quad \boxed{\rho = Z_1 - Z_0 / Z_1 + Z_0}$

$$V_0^- = \frac{Z_1 - Z_0}{Z_1 + Z_0} V_0^+$$

$$\cancel{V_1^+ = \frac{Z_1}{Z_0}\left(V_{s0}\frac{Z_1-Z_0}{Z_1+Z_0}\right)/\cos(\beta l)\left(1+\frac{Z_1-Z_0}{Z_1+Z_0}\right)}$$

$$V_0^+ = V_{s0} / \cos(\beta l)(1+\rho)$$

$$\cancel{V_1^+ = 3V_{s0}(1/2)(1-1/2) / (1)(1+1/2)}$$

$$V_1^+ = \frac{3}{4}\frac{V_{s0}}{3/2}$$

$$\boxed{V_0^+ = V_{s0} / \cos(\beta l)\left(1+\frac{Z_1-Z_0}{Z_1+Z_0}\right)}$$

$$\boxed{V_0^- = \left(V_{s0}\frac{Z_1-Z_0}{Z_1+Z_0}\right) / \cos(\beta l)\left(1+\frac{Z_1-Z_0}{Z_1+Z_0}\right)}$$

$$\boxed{\cancel{V_1^+ = \frac{Z_1}{Z_0} V_{s0}\frac{Z_1-Z_0}{Z_1+Z_0} / \cos(\beta l)\left(1+\frac{Z_1-Z_0}{Z_1+Z_0}\right)}}$$

2) x/l plot $\quad \beta l = 4\pi \quad Z_1 = 3Z_0$

$$\rho = \frac{3Z_0 - Z_0}{3Z_0 + Z_0} = \frac{2Z_0}{4Z_0} = 1/2 \checkmark$$

$$V_0^+ = V_{s0} / (1)(1+1/2) = 2/3\, V_{s0}$$

$$V_0^- = V_{s0}(1/2) / (1)(3/2) \qquad V_0^- = 1/3\, V_{s0}$$

$$\cancel{V_1^+ = 3V_{s0}(3/2)/(1)(1+1/2) \quad V_1^+ = 3V_{s0}}$$

$$\boxed{V_1^+ = \frac{Z_1}{Z_0} V_0^+ (1-\rho)} \qquad V_1^+ = (3)(2/3\, V_{s0})(1-1/2)$$

$$V_1^+ = (2V_{s0})(1/2) = V_{s0}$$

$$V_0(x,t) = \mathrm{Re}\left[e^{i\omega t}\left(V_0^+ e^{-i\beta x} + V_0^- e^{i\beta x}\right)\right]$$

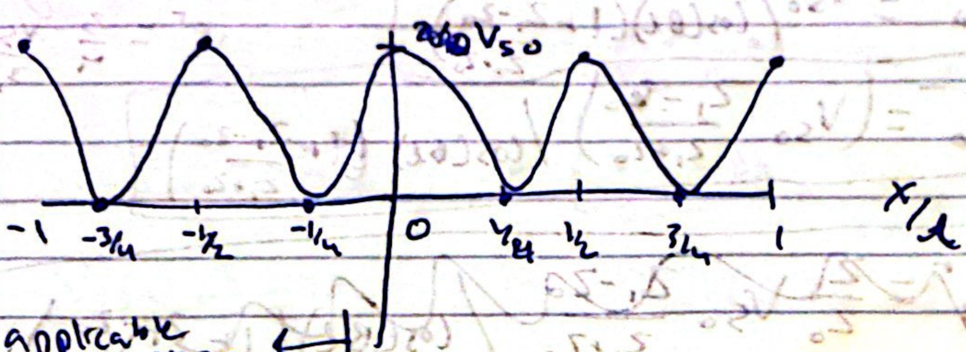
$$V_0(x,t) = V_0^+ e^{i(\omega t - \beta x)} + V_0^- e^{i(\omega t + \beta x)}$$

$$V_0(x,t) = 2/3\, V_{s0}\, e^{i\omega t} e^{i(\omega t - \beta x)} + 1/3\, V_{s0}\, e^{i\omega t} e^{i(\omega t + \beta x)}$$

$V_0(x,t) = \frac{2}{3} V_{s0} \cos(2\omega t - Bx)$

$V_0(x,0) = \frac{2}{3} V_{s0} \cos(-Bx) = \frac{2}{3} V_{s0} \cos(Bx)$

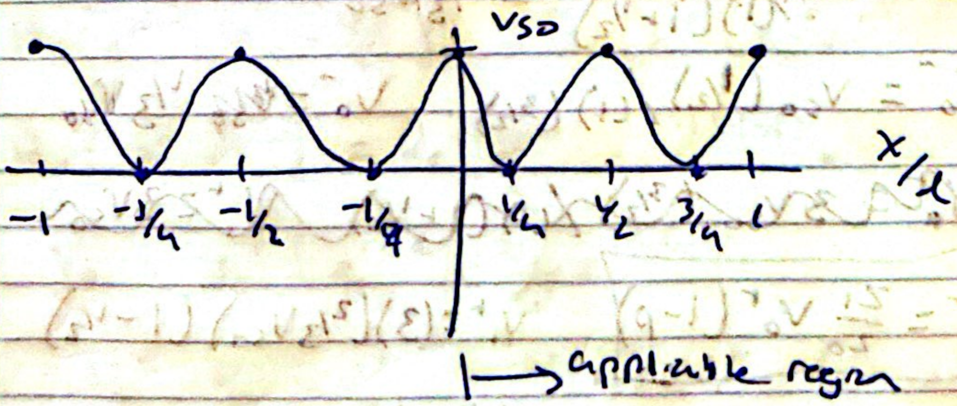
→ not sure how to get ℓ out from $\cos(x)$ and*



$V_1(x,t) = e^{i\omega t} V_1^+ e^{-iBx}$ $\qquad V_1^+ = V_{s0}$

$V_1(x,t) = V_{s0} e^{i\omega t} e^{i\omega t} e^{-iBx} = V_{s0} \cos(2\omega t - Bx)$

$V_1(x,0) = V_{s0} \cos(-Bx) = V_{s0} \cos(Bx)$



$\tilde{V}_0(x) = \tilde{V}_0^+ + V_0^- = \frac{2}{3} V_{s0} \cos(-Bx) + \frac{1}{3} V_{s0} \cos(Bx)$

$= V_{s0} (\cos(Bx))$

3) $\tilde{V}_0(x) = \tilde{V}_0^+(x) + \tilde{V}_0^-(x)$

$$|\tilde{V}_0(x)|e^{i\phi} = \frac{V_{s0}e^{i\omega t}}{e^{iBx}(1+p)} + p\left(\frac{V_{s0}e^{i\omega t}}{e^{iBx}(1+p)}\right)$$

$$|\tilde{V}_0(x)|e^{i\phi} = \frac{2V_{s0}e^{i(\omega t - Bx)}}{3} + \frac{1}{2}\left(\frac{2V_{s0}e^{i(\omega t - Bx)}}{3}\right.$$

$$|\tilde{V}_0(x)|e^{i\phi} = \tilde{V}_{s0}e^{i(\omega t - Bx)}$$

$$|\tilde{V}_0(x)| = (V_0^+ + V_0^-)(V_0^+ - V_0^-)$$

$$\tilde{V}_0(x) = \tilde{V}_0^+(e^{-iBx} + pe^{iBx})$$

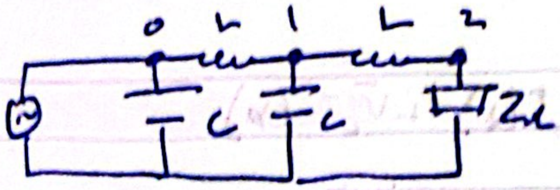
$$= (V_0^+)^2 \cos(Bx) - i\sin(Bx) + p(\cos(Bx) + i\sin(Bx))$$

$$= p\cos^2(Bx) + \sin^2(Bx)$$

$$= (\cos(Bx) - i\sin(Bx))(\cos(Bx) + i\sin(Bx)) + p(\cos(Bx) + i\sin(Bx))(\cos$$

$$\cos^2(Bx^2) + \sin^2(Bx) + p(1 + 2\cos$$

$$|\tilde{V}_0(x)| = V_{s0} \qquad \phi = \omega t - Bx$$

2) $V_0(t)$ 

$$\frac{\partial I}{\partial x} = -C\frac{\partial V}{\partial t} \qquad \frac{\partial V}{\partial x} = -L\frac{\partial I}{\partial t}$$

$$\frac{\partial^2 V}{\partial x^2} = LC\frac{\partial^2 V}{\partial t^2}$$

$$V_1 - V_0 = -L\frac{\partial I_1}{\partial t}$$

$$V_2 - V_1 = -L\frac{\partial I_2}{\partial t}$$

$$\frac{\partial I_1}{\partial t} = a_{11}I_1 + a_{12}I_2 + a_{13}V_1 + b_1U_1$$

$$\frac{\partial I_2}{\partial t} = a_{21}I_1 + a_{22}I_2 + a_{23}V_1 + b_2U_2$$

$$\frac{\partial V_1}{\partial t} = a_{31}I_1 + a_{32}I_2 + a_{33}V_1 + b_3U_2$$

$$\frac{\partial I_1}{\partial t} = \overset{a_{11}}{0} + \overset{a_{12}}{0} + \overset{a_{13}}{(-1)}V_1 + \overset{b_3\ V_1}{(1)\cos(t)} \qquad \frac{\partial I_1}{\partial t} = \frac{V_0 - V_1}{L}$$

$$V_2 = I_2 Z_L = I_2 R$$

$$\frac{\partial I_2}{\partial t} = \frac{-RI_2}{L} + \frac{V_1}{L} = \overset{a_{21}}{0} + \overset{a_{22}}{-1} + \overset{a_{33}}{1} + \overset{b_2U_2}{0}$$

$$\frac{\partial V_1}{\partial t} = \frac{I_1 - I_2}{-C}$$

$$\begin{array}{ccccc} & a_{31} & a_{32} & a_{33} & b_3 u_3 \\ \frac{\partial V_1}{\partial t} = & +1 & + -1 & +0 & +0 \end{array}$$

$$\begin{array}{cccc} & & & \text{RHS} \\ & & & u \end{array}$$

$$\left[\begin{array}{ccc|c} 0 & 0 & -1 & 1 \\ 0 & -1 & 1 & 0 \\ 1 & -1 & 0 & 0 \end{array}\right] \begin{bmatrix} I_1(t) \\ I_2(t) \\ V_1(t) \end{bmatrix} = \begin{bmatrix} \frac{\partial I_1}{\partial t} \\ \frac{\partial I_2}{\partial t} \\ \frac{\partial V_1}{\partial t} \end{bmatrix}$$

Steady State $\partial/\partial t = 0$

$$\begin{bmatrix} 0 & 0 & -1 \\ 0 & -1 & 1 \\ 1 & -1 & 0 \end{bmatrix} \begin{bmatrix} I_1(t) \\ I_2(t) \\ V_1(t) \end{bmatrix} = \begin{bmatrix} -\cos(t) \\ 0 \\ 0 \end{bmatrix}$$

$$V_1(t) = \cos(t)$$

$$-I_2(t) + V_1(t) = 0$$

$$I_2(t) = V_1(t) = \cos(t)$$

$$I_1(t) - I_2(t) = 0$$

$$I_1(t) = I_2(t) = \cos(t)$$