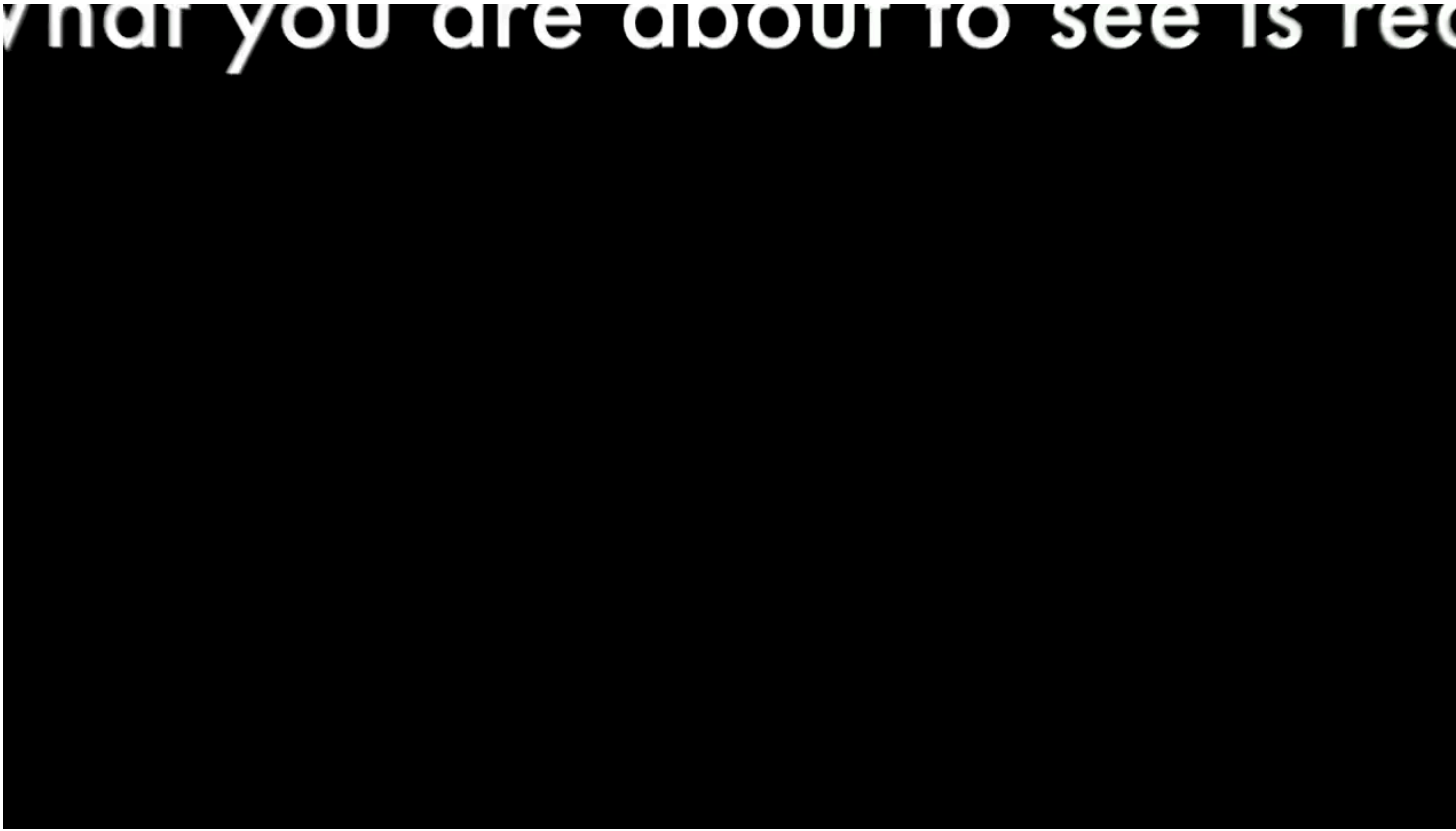
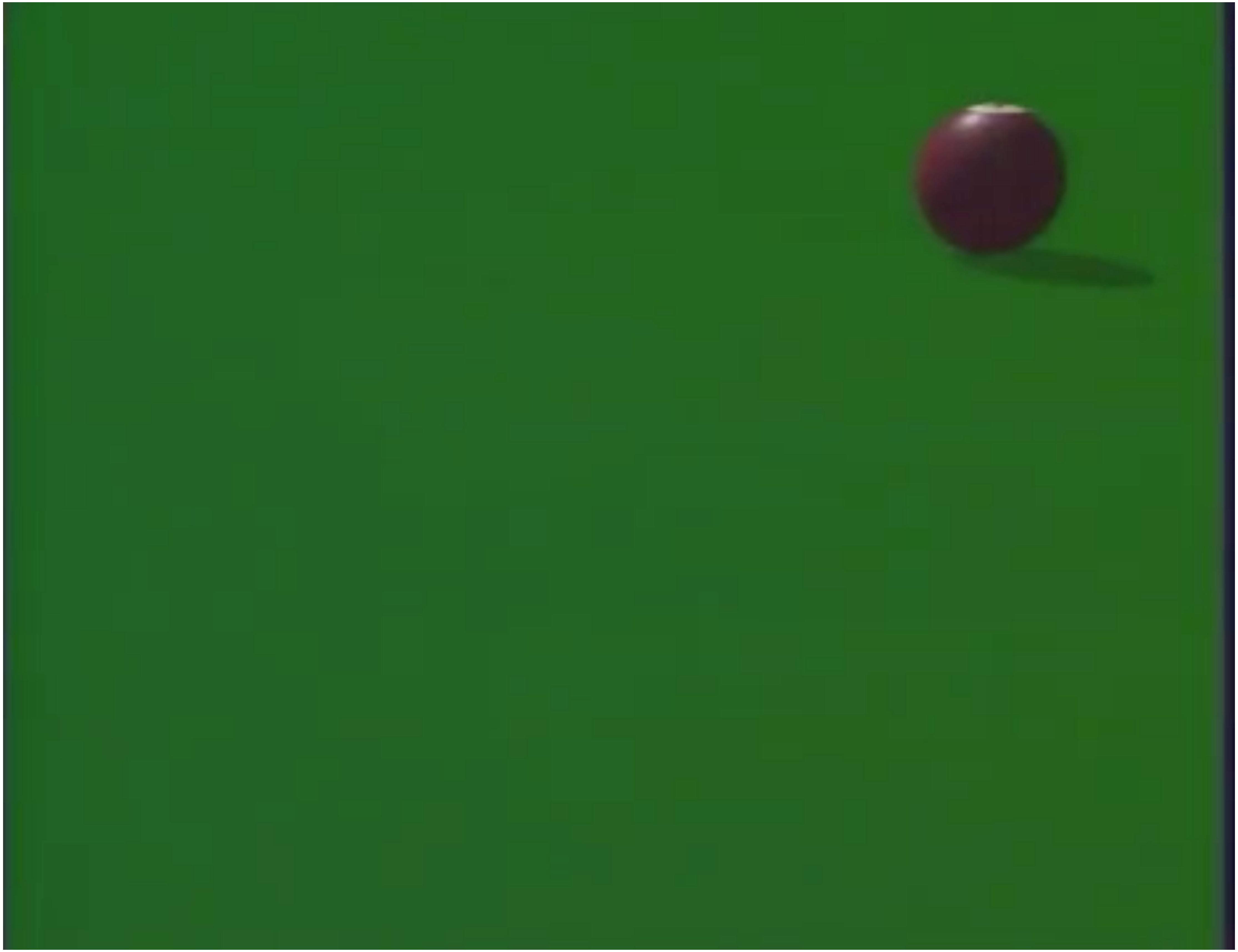
Modern Physics Lecture #34

Chapter 2: Special Relativity

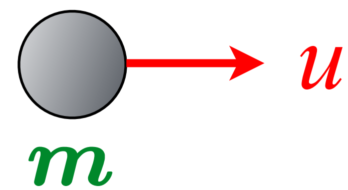


A collision of equal-mass billiard balls at slow velocities



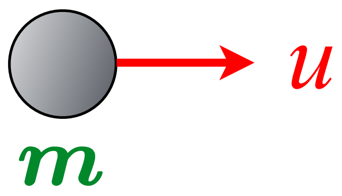
A collision of equal-mass billiard balls at high velocities

Velocity and Momentum



[Newtonian] velocity $u = dx/dt$ between 0 and c	[Newtonian] momentum $p = (m)(u)$ between 0 and mc
Proper velocity $\eta = dx/d\tau = \gamma_u\, u$ between 0 and ∞	Proper momentum $p = (m)(\eta) = \gamma_u\, mu$ between 0 and ∞

Total Energy, Rest Energy and Kinetic Energy



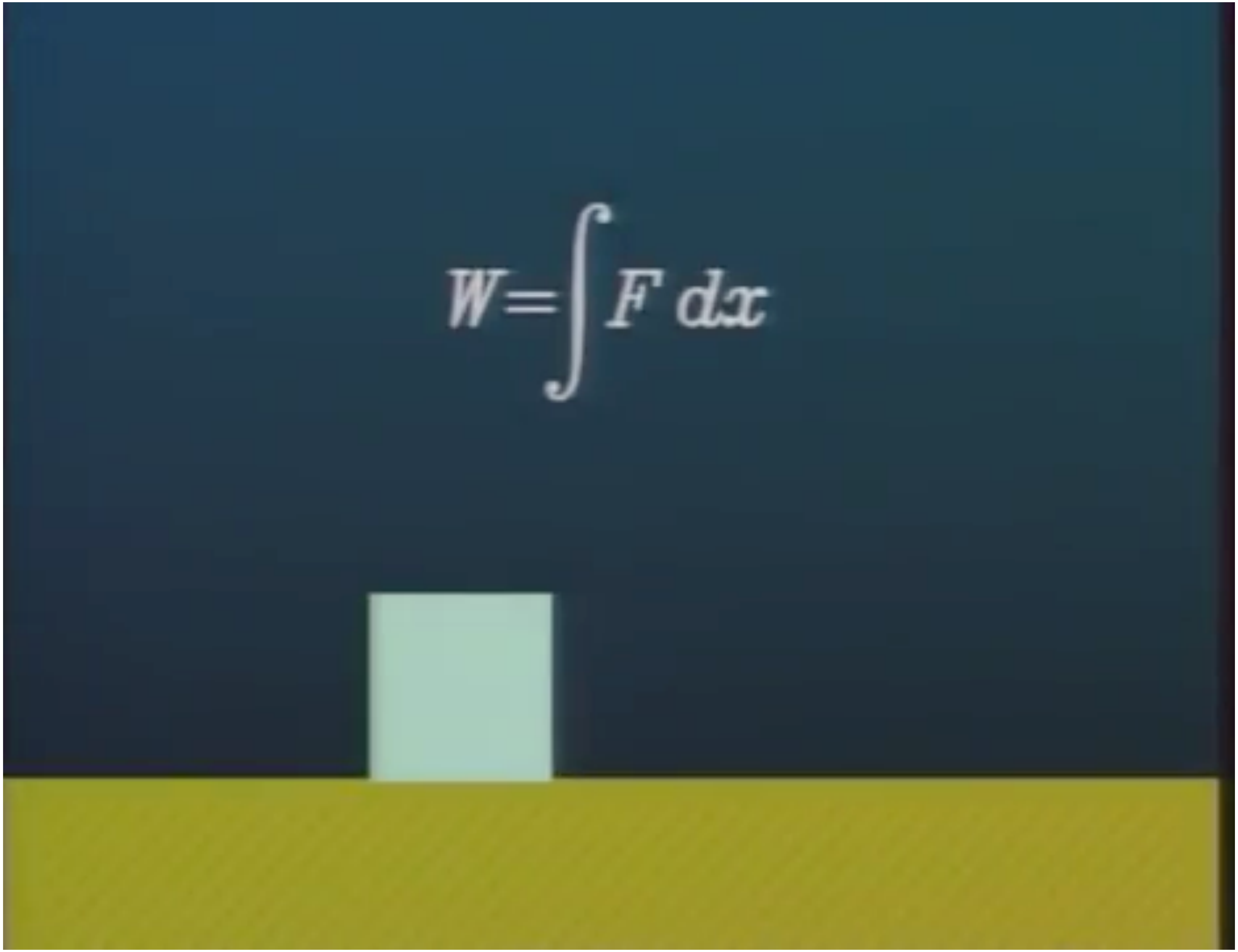
From Lecture 33 slide 19, $E^2 = (pc)^2 + (mc^2)^2$

so the total energy is
$$E = \sqrt{(pc)^2 + (mc^2)^2}$$
$$= \sqrt{[(\gamma_u\, m\, u)\, c]^2 + (mc^2)^2}$$
$$= \sqrt{[\gamma_u^2\, u^2 + c^2]\, m^2\, c^2}$$
$$= \sqrt{[\gamma_u^2\, c^2]\, m^2\, c^2}$$
$$= \gamma_u\, m\, c^2$$

while the rest energy is $E_0 = m\, c^2$

so the kinetic energy is $K \equiv E - E_0 = (\gamma_u - 1)\, m\, c^2$

Energy at high velocities



Invariant: same value in all inertial frames

Conserved: same value as time progresses

Quantity	Conserved?	Invariant?
Mass		
Energy		
Momentum		
Electric charge		
Velocity		

Invariant: same value in all inertial frames

Conserved: same value as time progresses

Quantity	Conserved?	Invariant?
Mass	NO	YES
Energy	YES	NO
Momentum	YES	NO
Electric charge	YES	YES
Velocity	NO	NO

Electric charge is a *Lorentz invariant.*

How do we know this?

- ✓ 2 protons and 2 electrons in hydrogen molecule are neutral overall to a very high precision. The protons are ~10^{-10} m apart (~0.1 nanometer) and moving at a non-relativistic speed of ~$0.001c$.
- ✓ 2 protons and 2 electrons in helium atom are neutral overall to a very high precision. The protons are ~10^{-15} m apart (~1 femtometer) and moving at a much faster speed of ~$0.2c$.
- ✓ The cancellation of proton and electron charge is unaffected by the vast range in proton speeds.

– from Purcell's *Electricity and Magnetism*, 2e

If you observe a baseball at rest, what is its four-momentum?

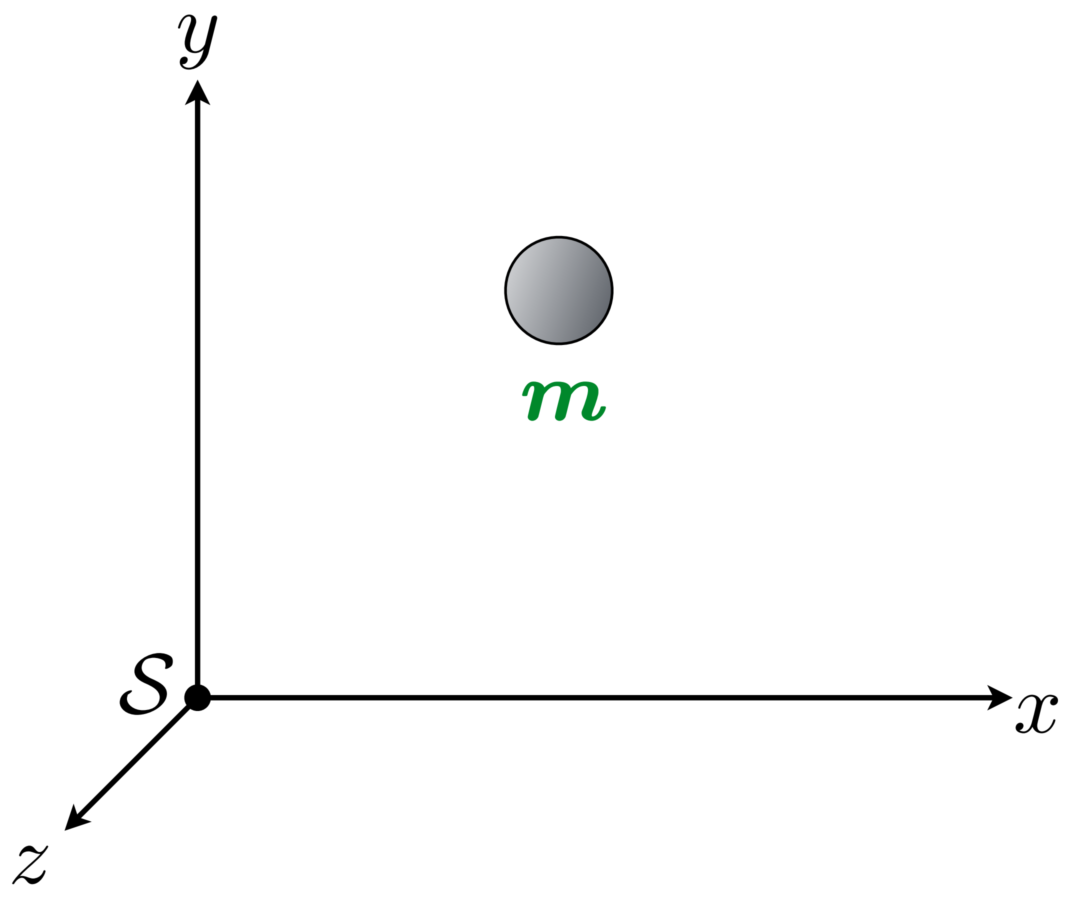
A) zero

B) $(-mc, 0, 0, 0)$

C) (mc, mc, mc, mc)

✓ D) $(mc, 0, 0, 0)$

E) $(0, 0, 0, 0)$



If you observe a baseball moving along the x axis at 0.6 times the speed of light, what is its four-momentum?

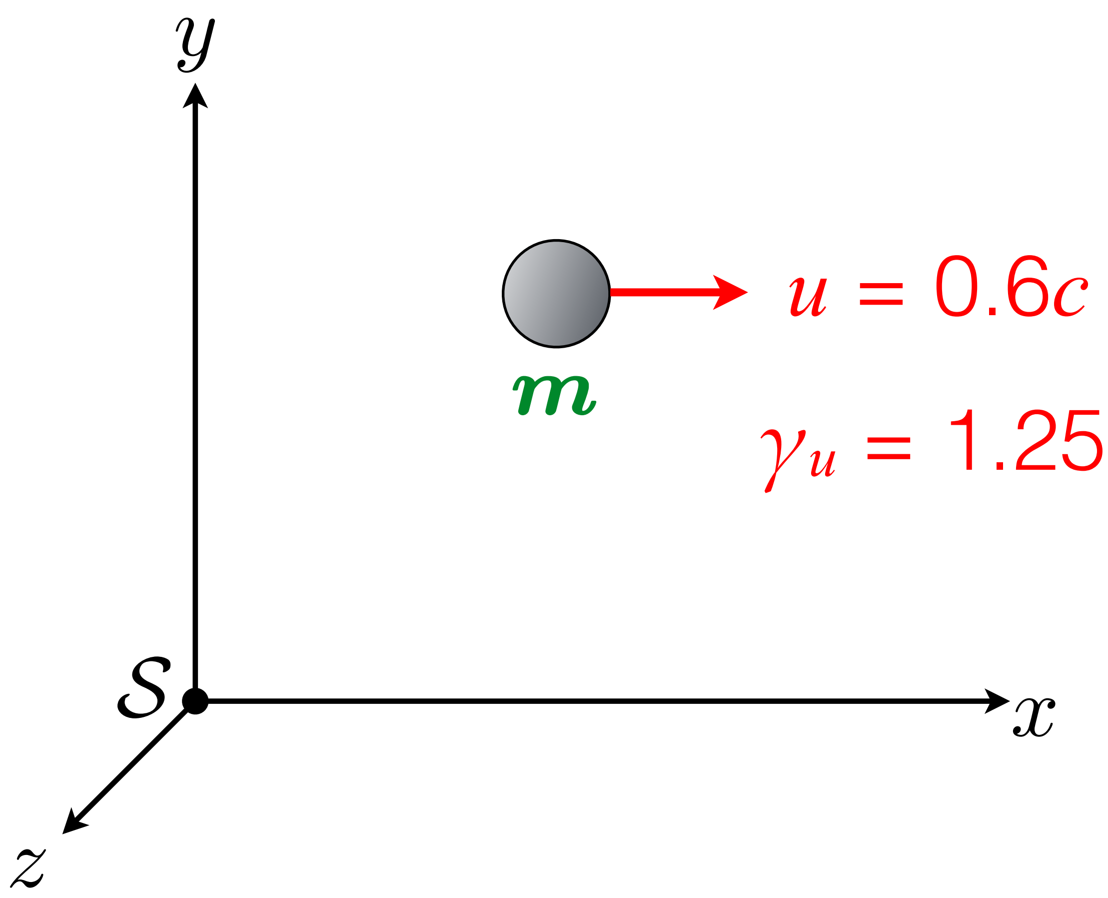
A) $(mc, 0.6mc, 0, 0)$

B) $(1.25mc, 0.6mc, 0, 0)$

✓ C) $(1.25mc, 0.75mc, 0, 0)$

D) $(0.8mc, 0.48mc, 0, 0)$

E) $(0.8mc, 0.6mc, 0, 0)$



What is the “momentum invariant” for a baseball at rest?

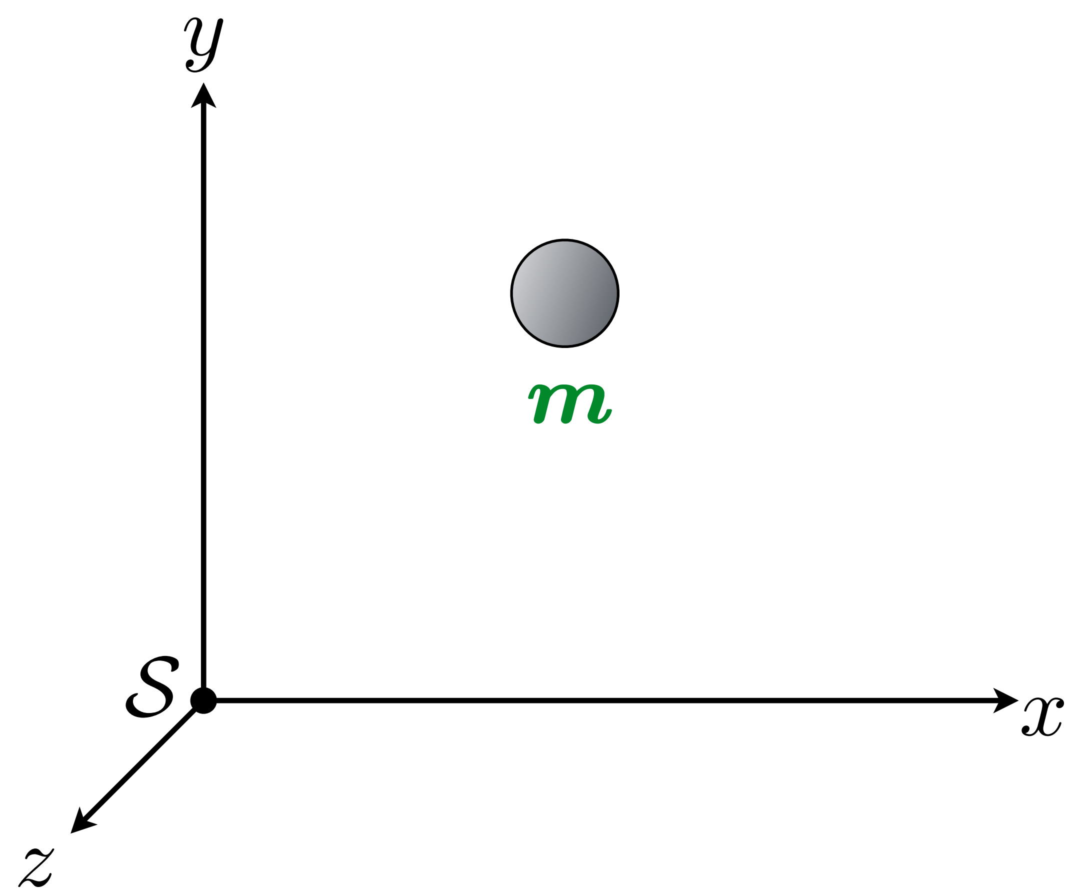
A) zero

B) m^2c^2

✓ C) $-m^2c^2$

D) $2m^2c^2$

E) $-2m^2c^2$



$$\boldsymbol{P} = -\frac{E^2}{c^2} + p_x^2 + p_y^2 + p_x^2$$

What is the "momentum invariant" for a baseball moving at a speed of 0.6 times the speed of light?

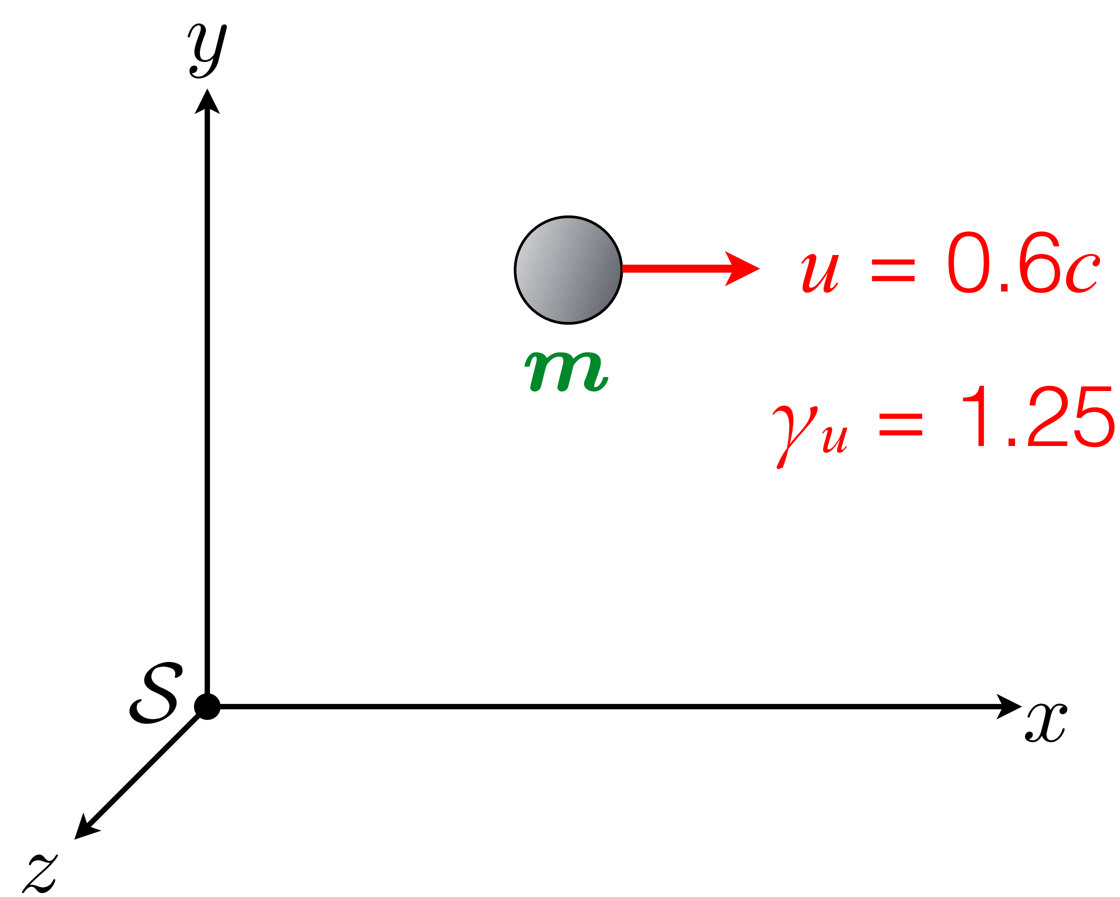
✓ A) $-m^2c^2$

B) $-1.5625m^2c^2$

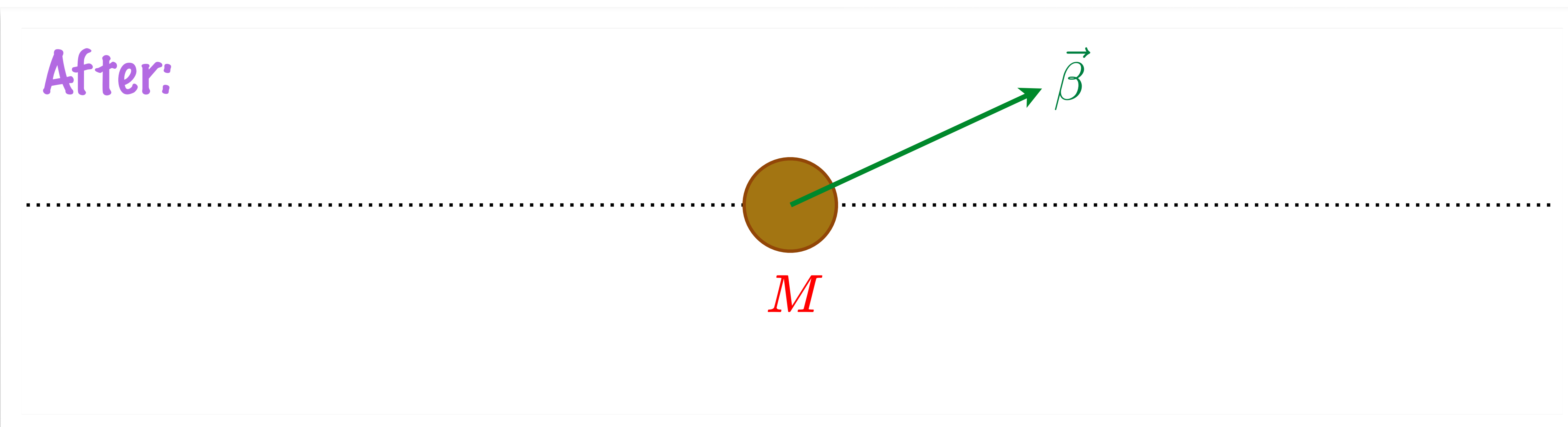
C) $-0.36m^2c^2$

D) $-0.64m^2c^2$

E) $-0.5625m^2c^2$



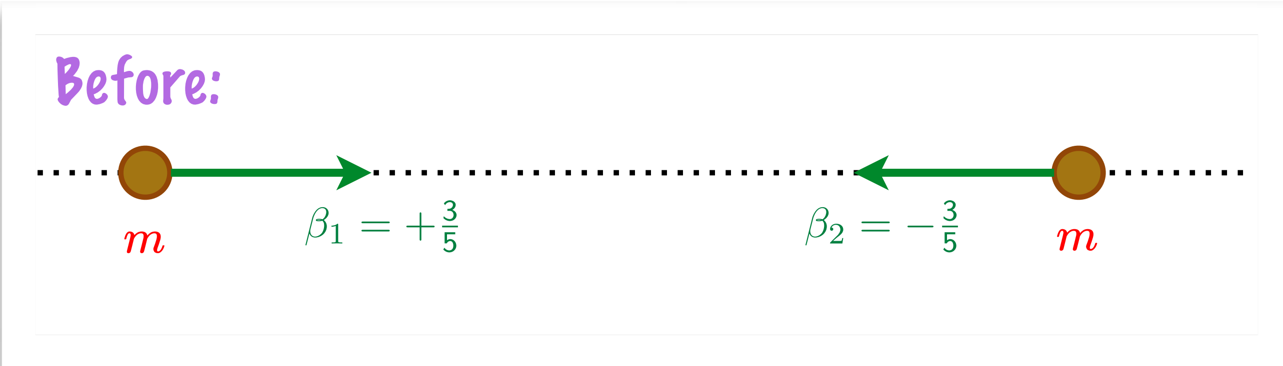
Relativistic 4-momentum conservation in a collision of billiard balls

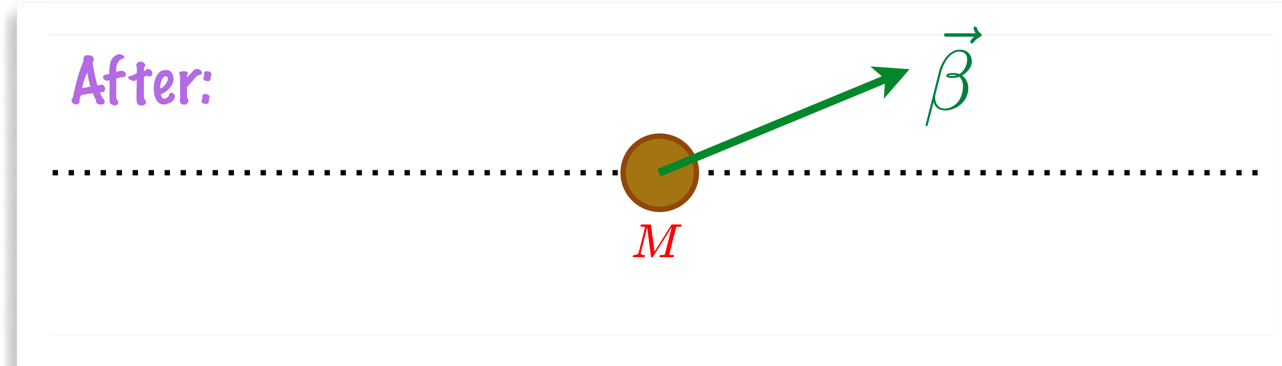


What are $\vec{\beta}$ and M, given m?

$c = 1$

Relativistic 4-momentum conservation in a collision of billiard balls





Before:

$$p_1^\mu = \frac{5}{4} m (1,\ +\frac{3}{5},\ 0,\ 0)$$

$$p_2^\mu = \frac{5}{4} m (1,\ -\frac{3}{5},\ 0,\ 0)$$

$$\gamma_1 = \gamma_2 = \frac{1}{\sqrt{1 - (\frac{3}{5})^2}} = \frac{5}{4}$$

After:

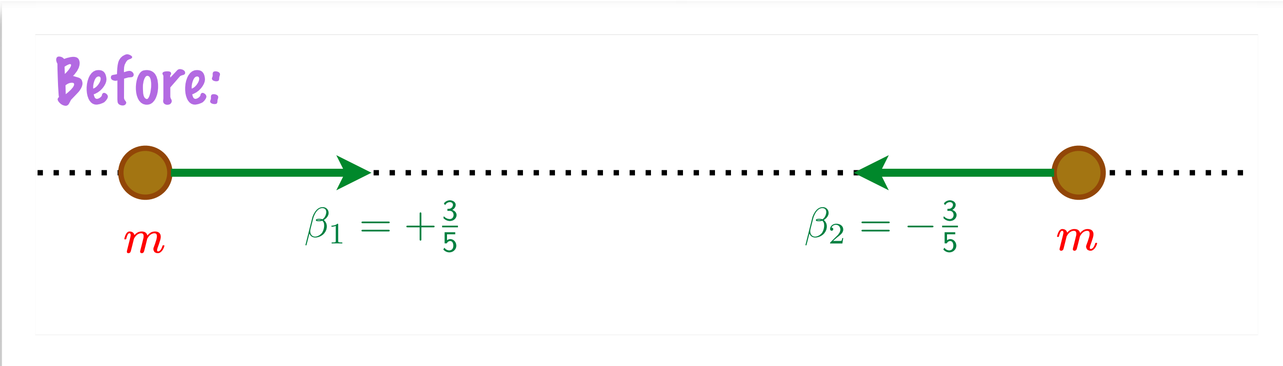
$$p^\mu = \gamma M (1,\ \beta_x,\ \beta_y,\ \beta_z)$$

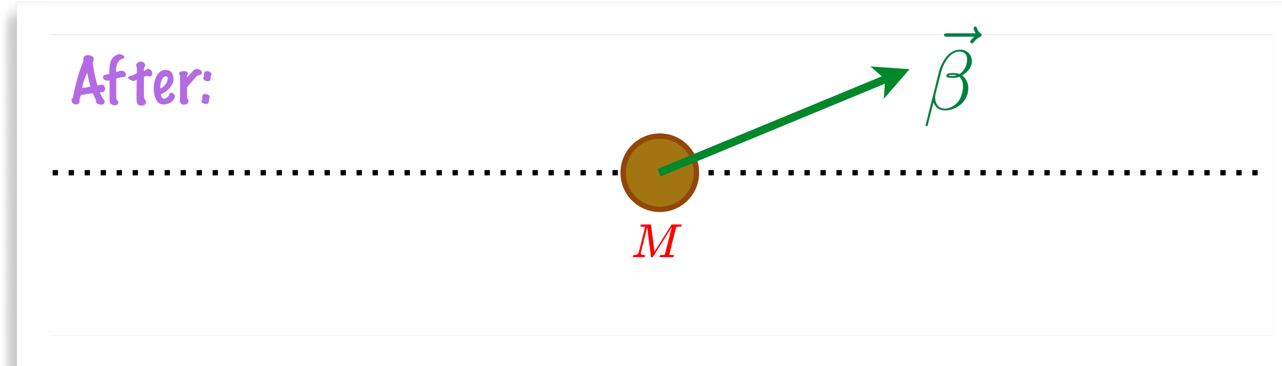
not the same as γ_1 – to be determined

What are $\vec{\beta}$ and M, given m?

Use conservation of 4-momentum!

Relativistic 4-momentum conservation in a collision of billiard balls





Before: $p_1^\mu = \frac{5}{4}m(1,\, +\frac{3}{5},\, 0,\, 0)$

$p_2^\mu = \frac{5}{4}m(1,\, -\frac{3}{5},\, 0,\, 0)$

After: $p^\mu = \gamma M(1,\, \beta_x,\, \beta_y,\, \beta_z)$

⓪ $\boxed{\gamma M = \frac{5}{4}m + \frac{5}{4}m = \frac{5}{2}m}$

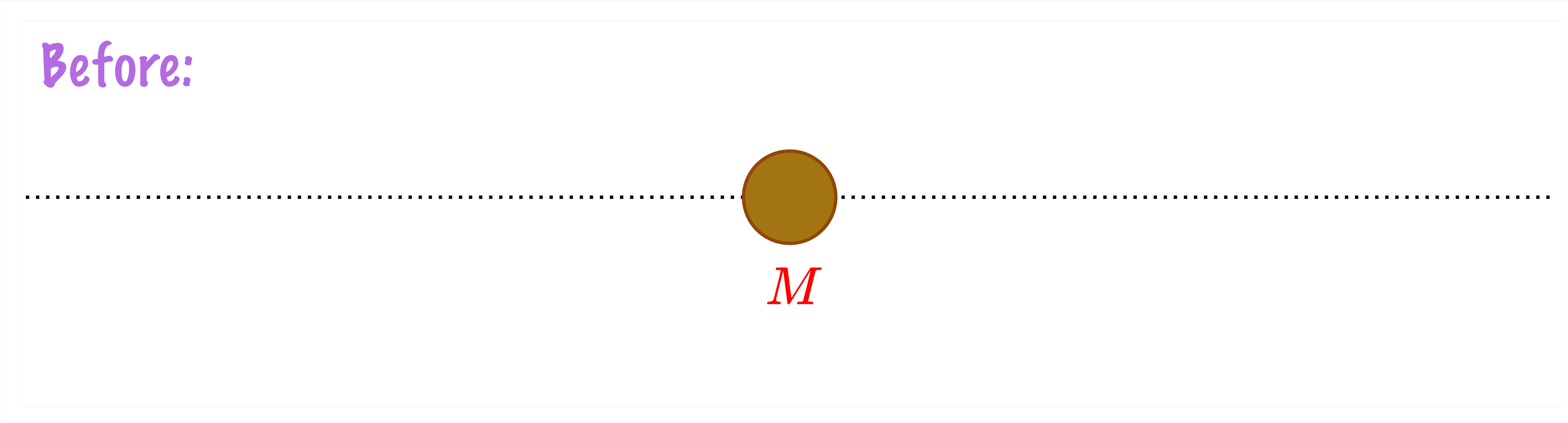
① $\boxed{\beta_x = +\frac{3}{5} - \frac{3}{5} = 0}$

② $\boxed{\beta_y = 0 + 0 = 0}$

③ $\boxed{\beta_z = 0 + 0 = 0}$

Then $\beta = \sqrt{\beta_x^2 + \beta_y^2 + \beta_z^2} = 0 \quad \Rightarrow \quad \gamma = 1 \quad \Rightarrow \quad M = \frac{5}{2}m$

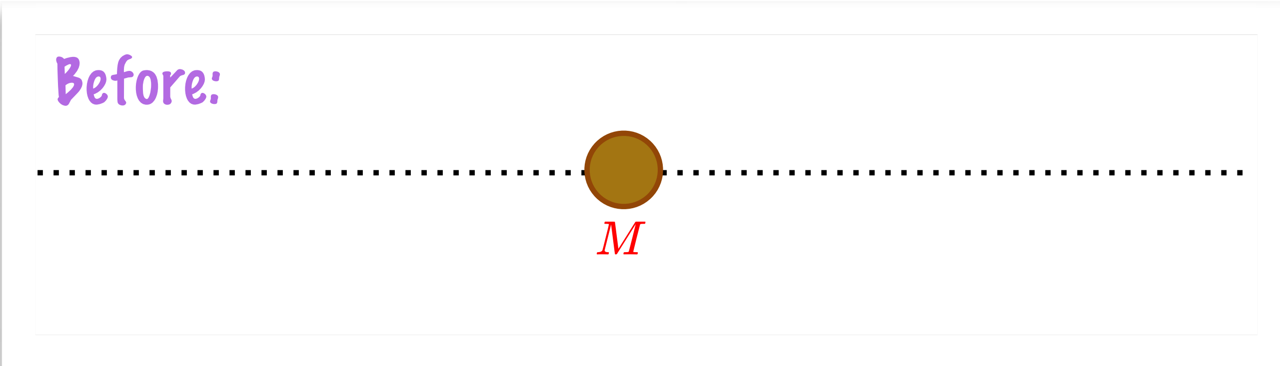
Relativistic 4-momentum conservation in a disintegration into two equal-mass daughters

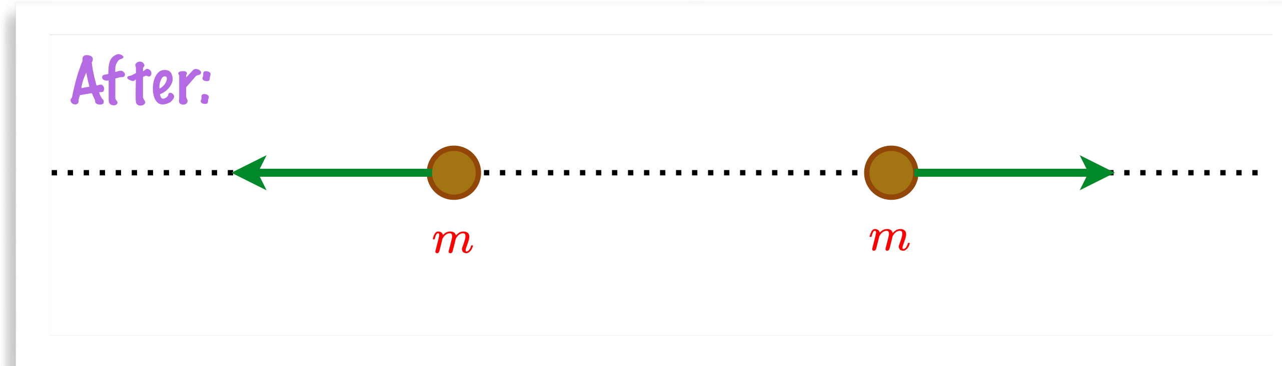


What is β, given M and m?

$\boxed{c = 1}$

Relativistic 4-momentum conservation in a disintegration into two equal-mass daughters





Before: $p^\mu = M(1,\,0,\,0,\,0)$

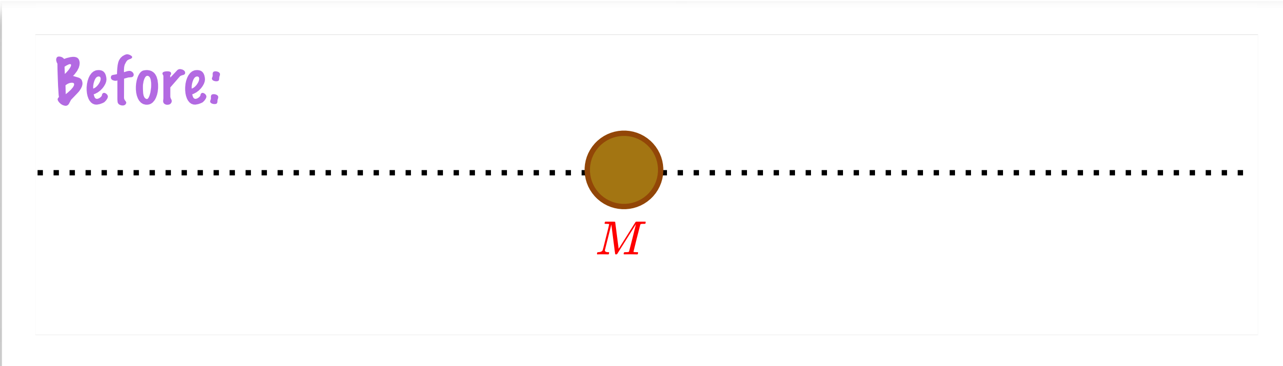
After: $p_1^\mu = \gamma m(1,\,-\beta,\,0,\,0)$

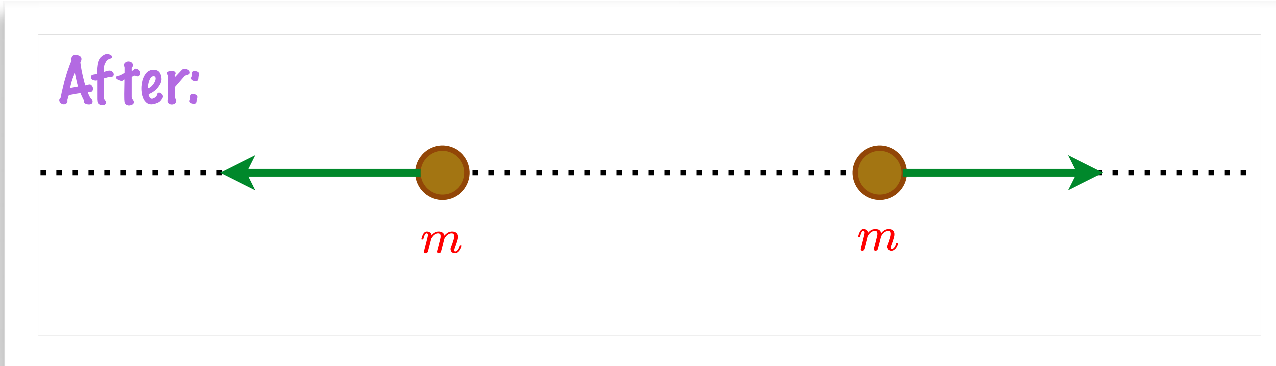
$$p_2^\mu = \gamma m(1,\,+\beta,\,0,\,0)$$

What is β, given M and m?

Use conservation of 4-momentum!

Relativistic 4-momentum conservation in a disintegration into two equal-mass daughters





Before: $p^\mu = M(1,\ 0,\ 0,\ 0)$

After: $p_1^\mu = \gamma m(1,\ -\beta,\ 0,\ 0)$

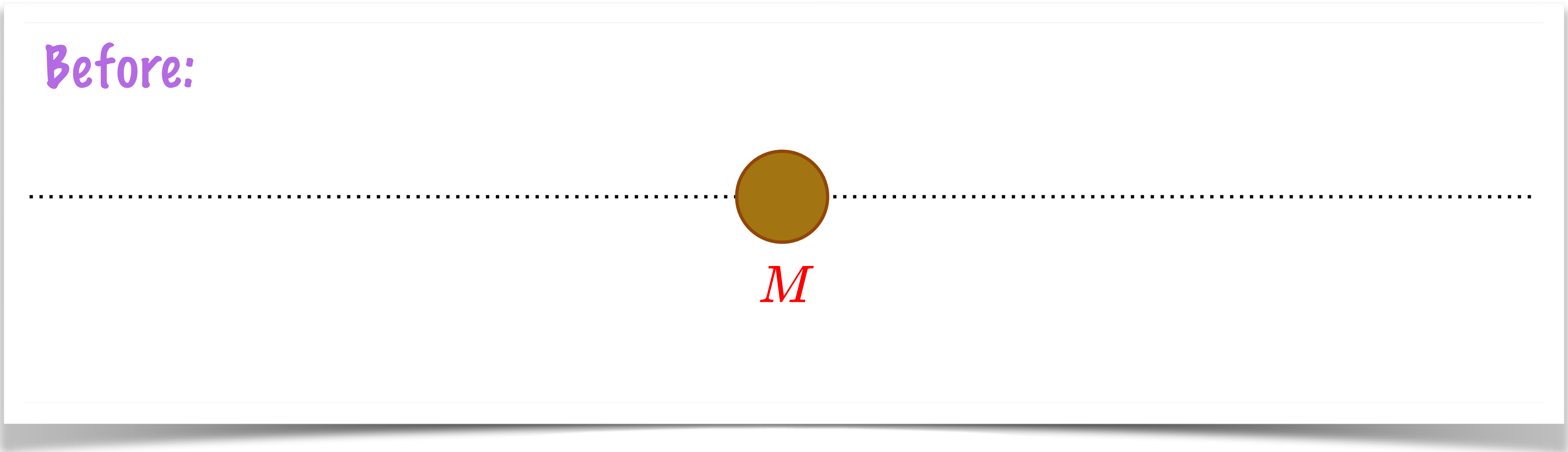
$p_2^\mu = \gamma m(1,\ +\beta,\ 0,\ 0)$

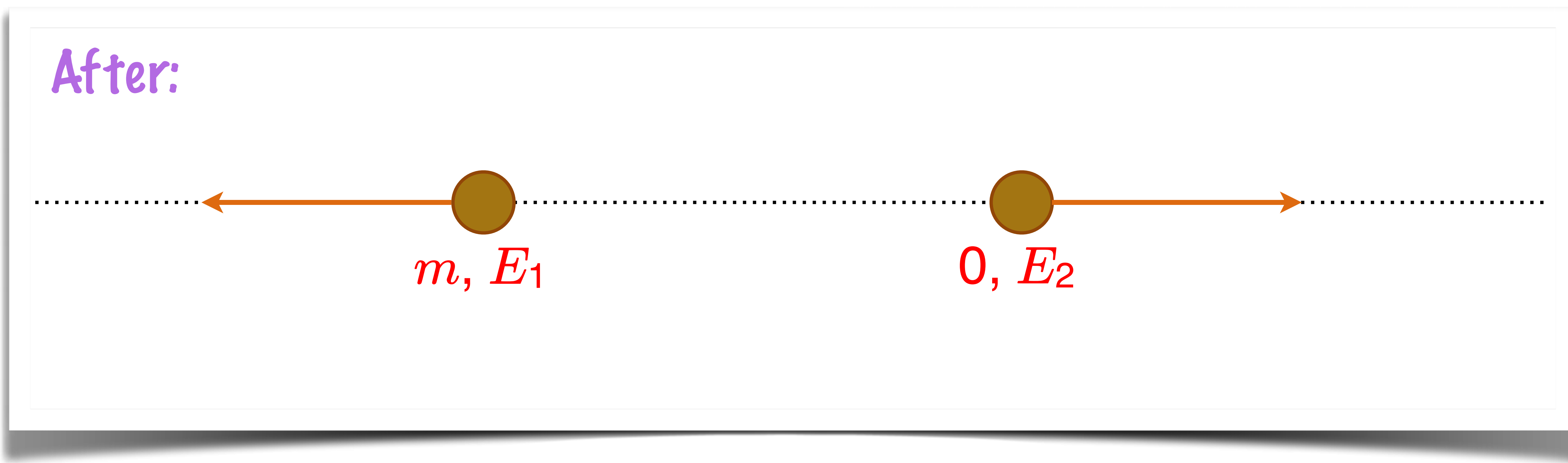
⓪

$$\boxed{M = \gamma m + \gamma m} \quad \Rightarrow \quad \gamma = \frac{M}{2m} = \frac{1}{\sqrt{1-\beta^2}} \quad \Rightarrow \quad \beta = \sqrt{1 - \left(\frac{2m}{M}\right)^2}$$

E.g., $\Upsilon(4S) \to B^0 + \bar{B}^0$: $\beta = \sqrt{1 - \left(\frac{2 \times 5279.6}{10579.4}\right)^2} = 0.0618$

Relativistic 4-momentum conservation in a disintegration into two unequal-mass daughters

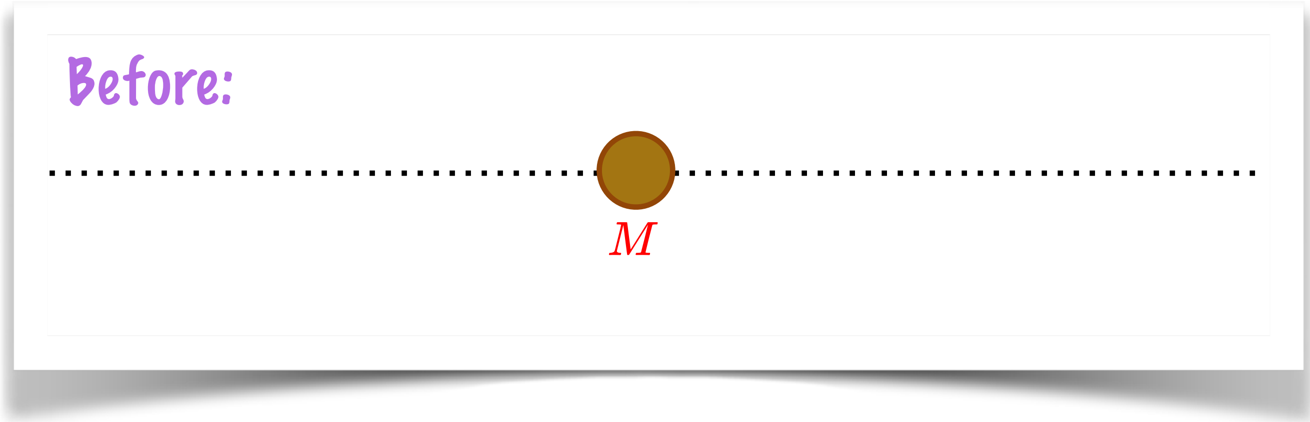


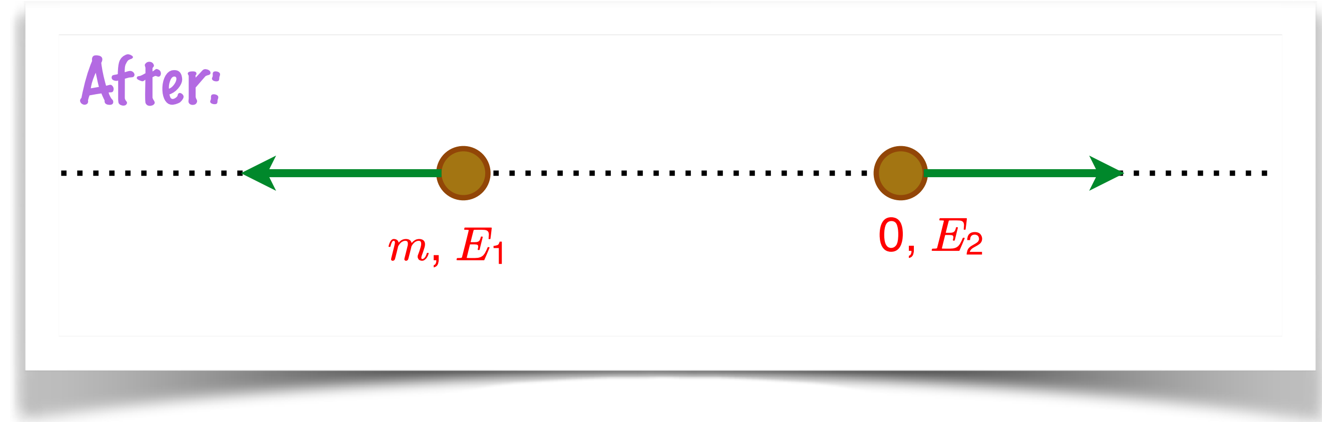


What are E_1 and E_2, given M and m?

$c = 1$

Relativistic 4-momentum conservation in a disintegration into two unequal-mass daughters





Before: $p^\mu = M(1,\,0,\,0,\,0)$

After: $p_1^\mu = \left(E_1,\,-\sqrt{E_1^2 - m^2},\,0,\,0\right)$

$p_2^\mu = (E_2,\,+E_2,\,0,\,0)$

using $E^2 = p^2 + m^2$ for p_1
and $E = p$ for p_2 $(m = 0)$

What are E_1 and E_2, given M and m?

Use conservation of 4-momentum!

Relativistic 4-momentum conservation in a disintegration into two unequal-mass daughters

Before: $p^\mu = M(1,\, 0,\, 0,\, 0)$

After: $p_1^\mu = (E_1,\, -\sqrt{E_1^2 - m^2},\, 0,\, 0)$

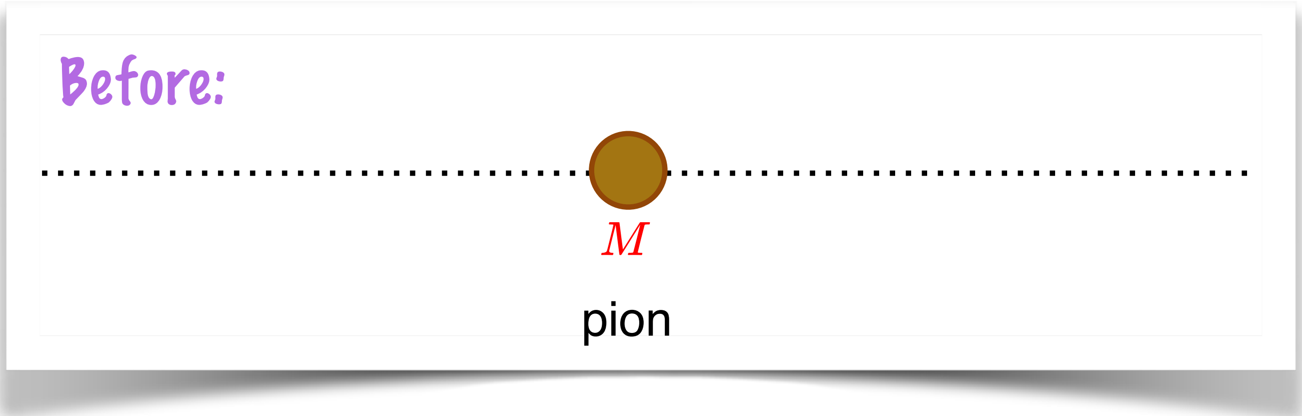
$$p_2^\mu = (E_2,\, +E_2,\, 0,\, 0)$$

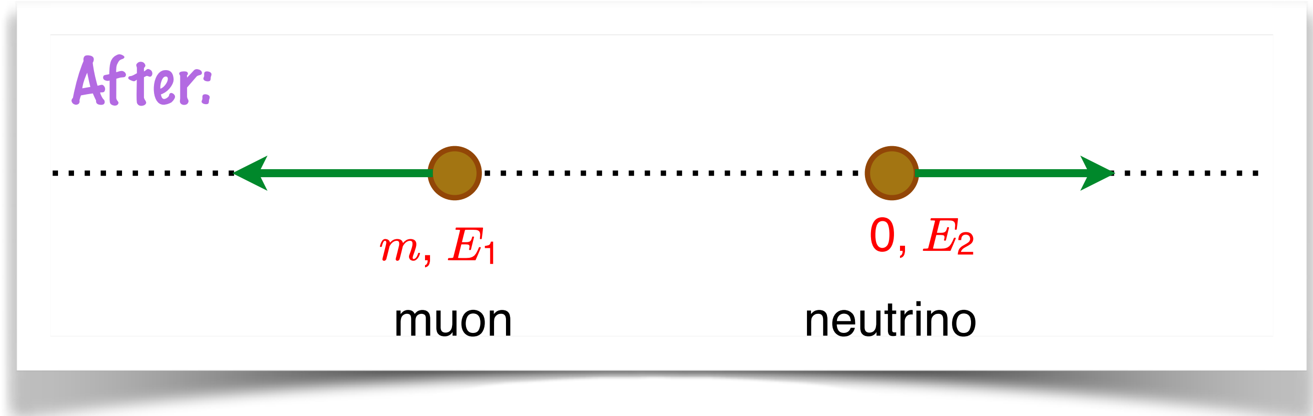
⓪ $\boxed{M = E_1 + E_2}$

① $\boxed{0 = -\sqrt{E_1^2 - m^2} + E_2}$

$$\Rightarrow \quad E_1 = \frac{M^2 + m^2}{2M} \qquad E_2 = \frac{M^2 - m^2}{2M}$$

Relativistic 4-momentum conservation in a disintegration into two unequal-mass daughters





Example:

$$\pi^+ \to \mu^+ + \bar{\nu}_\mu$$

Energy and mass measured in MeV.

$$E_1 = \frac{M^2 + m^2}{2M} = \frac{139.57^2 + 105.66^2}{2 \cdot 139.57} = 109.78$$

$$E_2 = \frac{M^2 - m^2}{2M} = \frac{139.57^2 - 105.66^2}{2 \cdot 139.57} = 29.8$$

Then, muon kinetic energy is

$$K_1 = E_1 - m = 109.78 - 105.66 = 4.12$$

and muon's relativistic factors are

$$\gamma = \frac{E_1}{m} = \frac{109.78}{105.66} = 1.04 \qquad \beta = 0.27$$