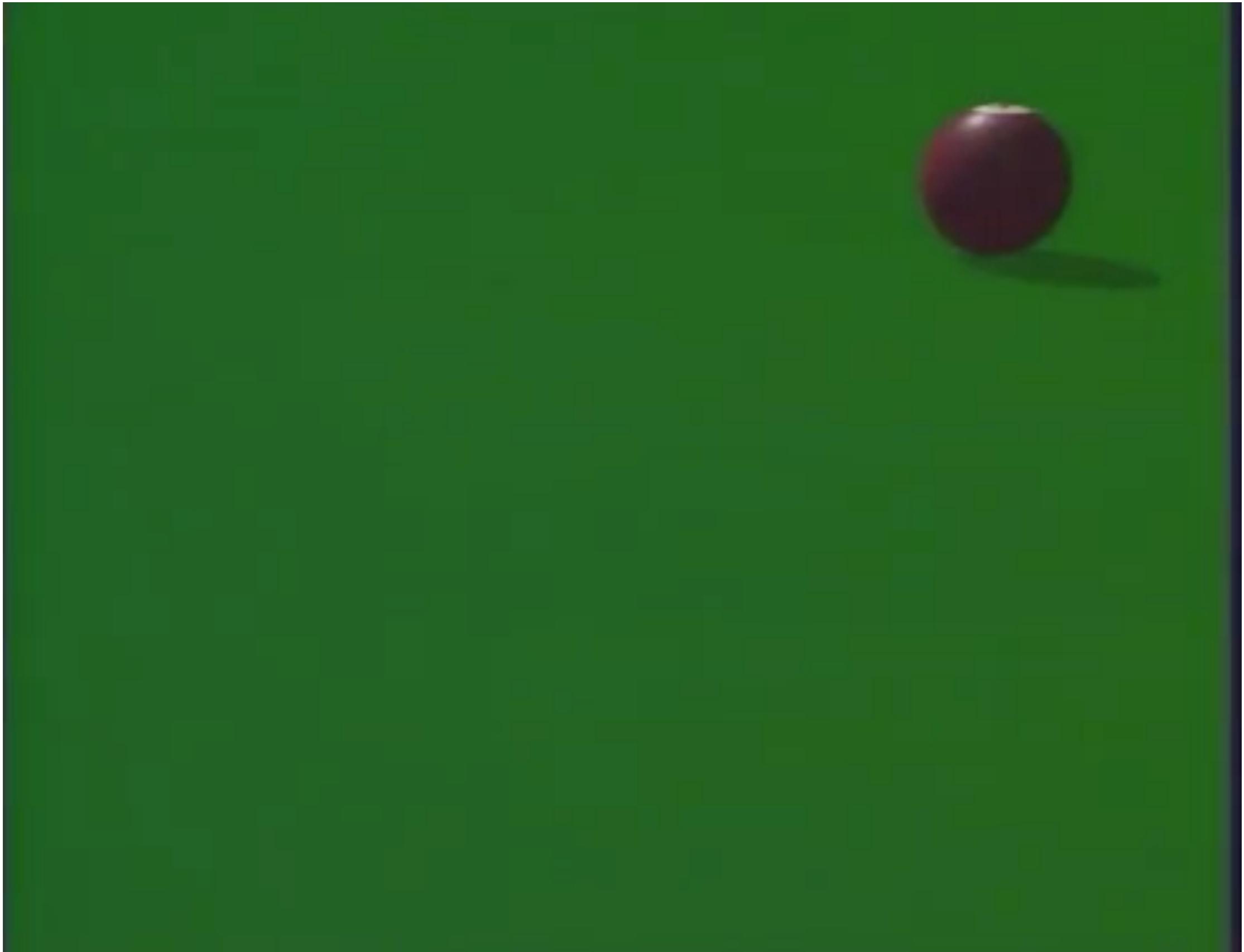


Modern Physics Lecture #34

Chapter 2: Special Relativity

What you are about to see is redacted

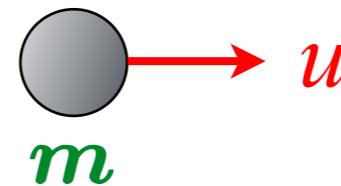
A collision of equal-mass billiard balls at slow velocities



A collision of equal-mass billiard balls at high velocities

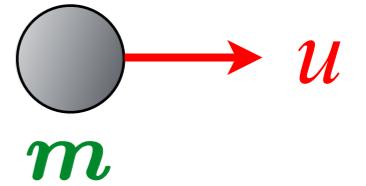


Velocity and Momentum



[Newtonian] velocity	[Newtonian] momentum
$u = dx/dt$	$p = (m)(u)$
between 0 and c	between 0 and mc
Proper velocity	Proper momentum
$\eta = dx/d\tau = \gamma_u u$	$p = (m)(\eta) = \gamma_u mu$
between 0 and ∞	between 0 and ∞

Total Energy, Rest Energy and Kinetic Energy



From Lecture 33 slide 19, $E^2 = (pc)^2 + (mc^2)^2$

so the total energy is

$$\begin{aligned} E &= \sqrt{(pc)^2 + (mc^2)^2} \\ &= \sqrt{[(\gamma_u m u) c]^2 + (mc^2)^2} \\ &= \sqrt{[\gamma_u^2 u^2 + c^2] m^2 c^2} \\ &= \sqrt{[\gamma_u^2 c^2] m^2 c^2} \\ &= \gamma_u m c^2 \end{aligned}$$

while the rest energy is

$$E_0 = m c^2$$

so the kinetic energy is

$$K \equiv E - E_0 = (\gamma_u - 1) m c^2$$

Energy at high velocities

$$W = \int F \, dx$$



Invariant: same value in all inertial frames

Conserved: same value as time progresses

Quantity	Conserved?	Invariant?
Mass		
Energy		
Momentum		
Electric charge		
Velocity		

Invariant: same value in all inertial frames

Conserved: same value as time progresses

Quantity	Conserved?	Invariant?
Mass	NO	YES
Energy	YES	NO
Momentum	YES	NO
Electric charge	YES	YES
Velocity	NO	NO

Electric charge is a *Lorentz invariant*.

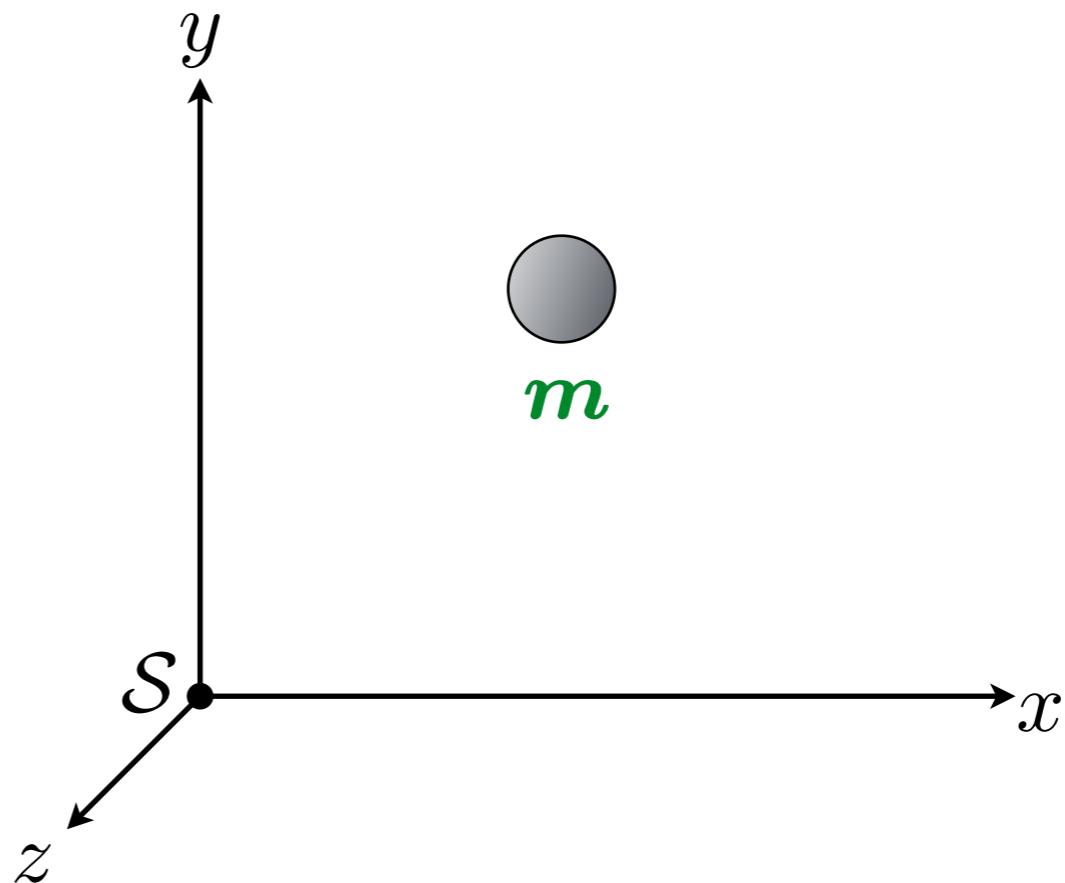
How do we know this?

- ✓ 2 protons and 2 electrons in hydrogen molecule are neutral overall to a very high precision. The protons are $\sim 10^{-10}$ m apart (~ 0.1 nanometer) and moving at a non-relativistic speed of $\sim 0.001c$.
- ✓ 2 protons and 2 electrons in helium atom are neutral overall to a very high precision. The protons are $\sim 10^{-15}$ m apart (~ 1 femtometer) and moving at a much faster speed of $\sim 0.2c$.
- ✓ The cancellation of proton and electron charge is unaffected by the vast range in proton speeds.

– from Purcell's *Electricity and Magnetism*, 2e

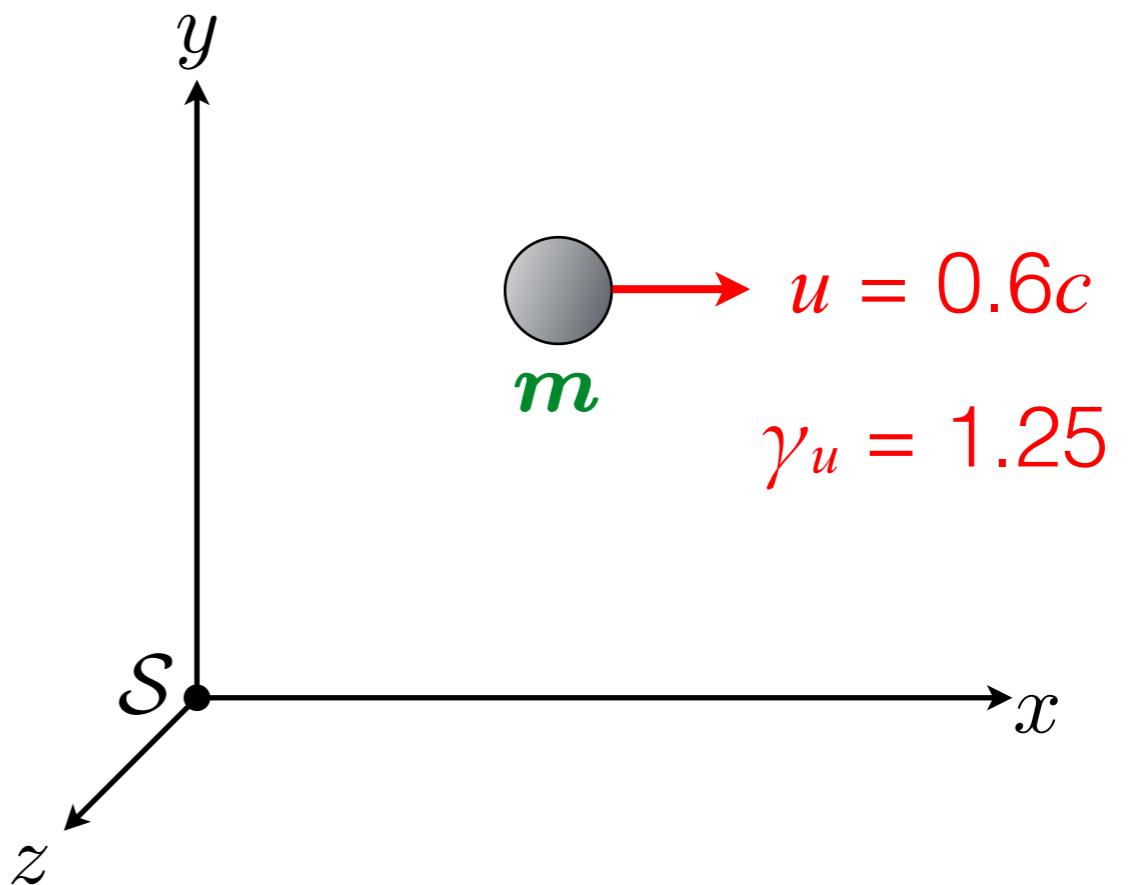
If you observe a baseball at rest, what is its four-momentum?

- A) zero
- B) $(-mc, 0, 0, 0)$
- C) (mc, mc, mc, mc)
- D) $(mc, 0, 0, 0)$
- E) $(0, 0, 0, 0)$



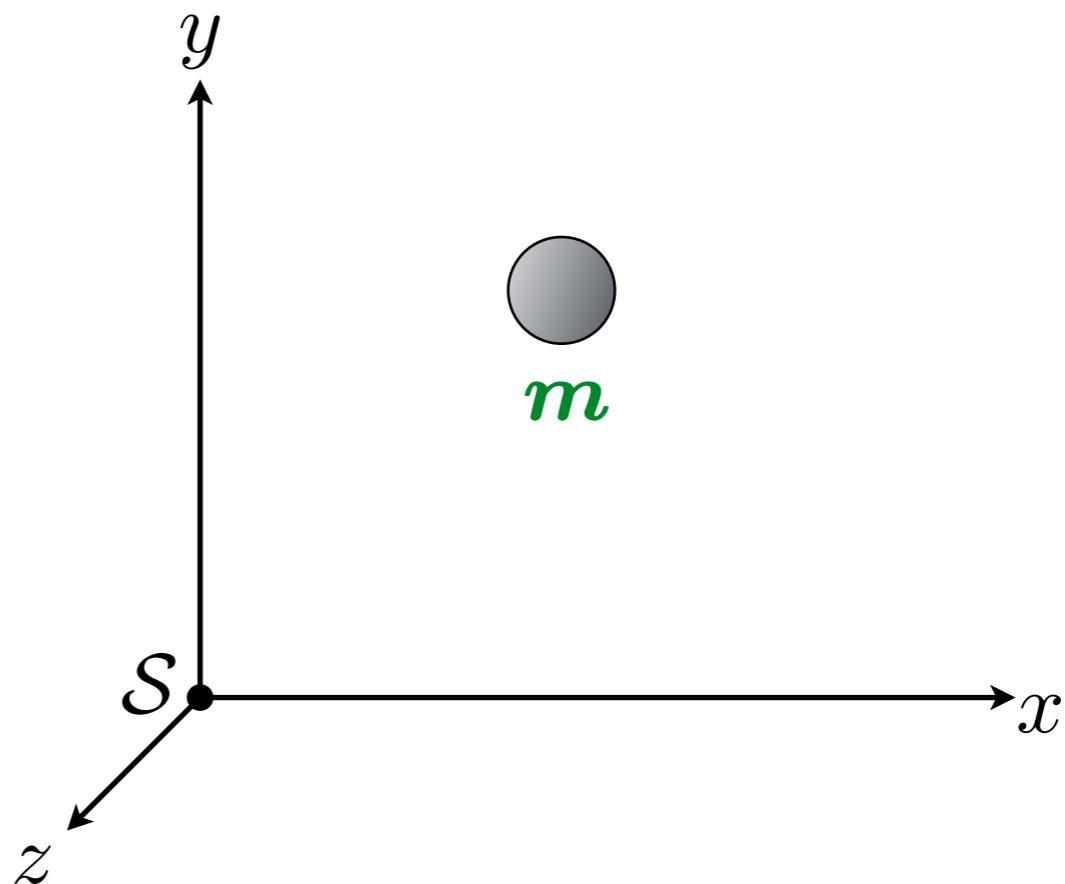
If you observe a baseball moving along the x axis at 0.6 times the speed of light, what is its four-momentum?

- A) $(mc, 0.6mc, 0, 0)$
- B) $(1.25mc, 0.6mc, 0, 0)$
- C) $(1.25mc, 0.75mc, 0, 0)$
- D) $(0.8mc, 0.48mc, 0, 0)$
- E) $(0.8mc, 0.6mc, 0, 0)$



What is the “momentum invariant” for a baseball at rest?

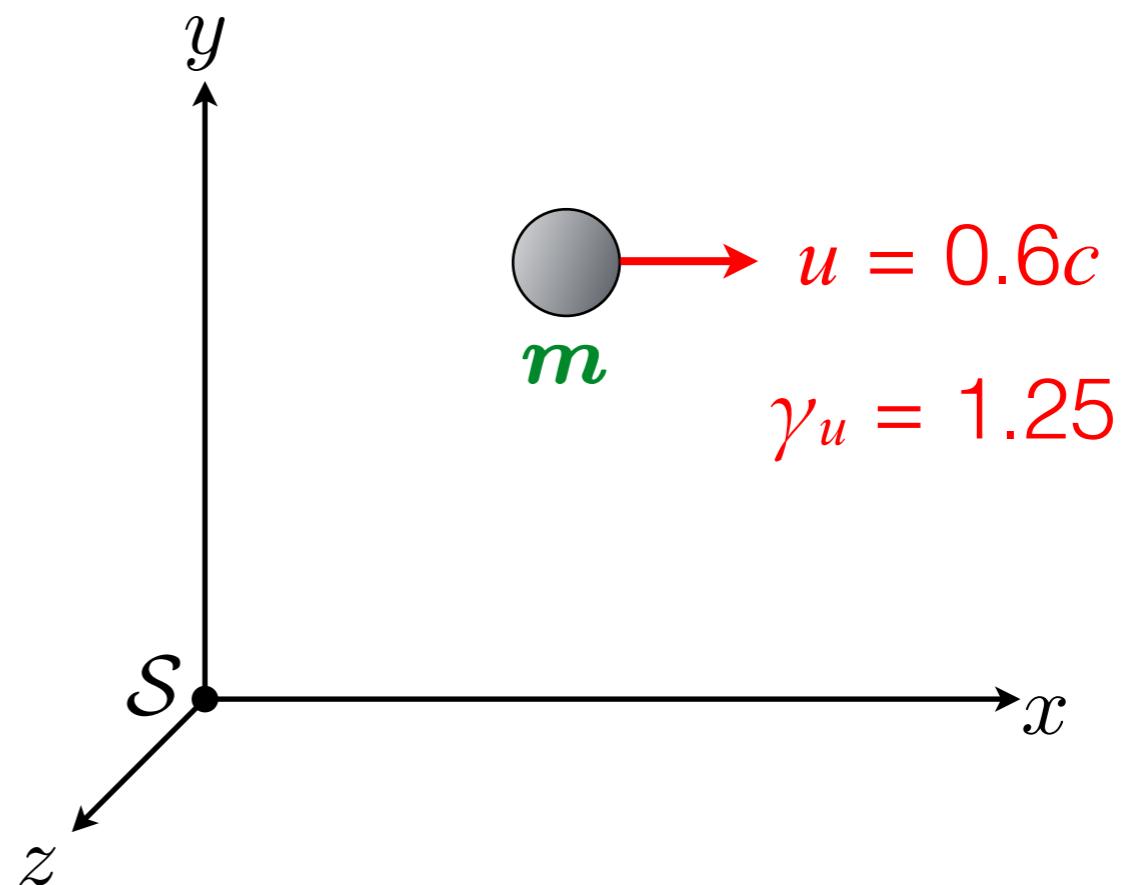
- A) zero
- B) m^2c^2
- ✓ C) $-m^2c^2$
- D) $2m^2c^2$
- E) $-2m^2c^2$



$$\mathbf{P} = -\frac{E^2}{c^2} + p_x^2 + p_y^2 + p_z^2$$

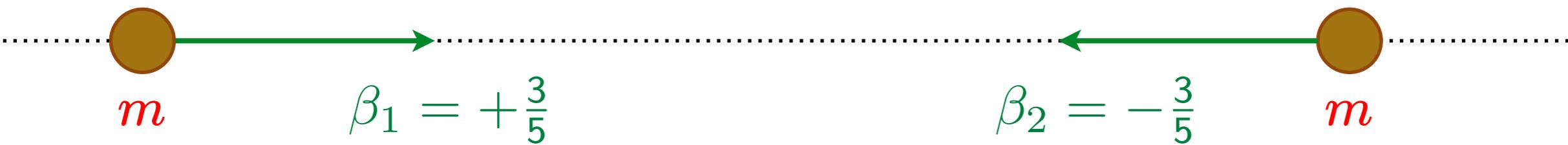
What is the “momentum invariant” for a baseball moving at a speed of 0.6 times the speed of light?

- A) $-m^2c^2$
- B) $-1.5625m^2c^2$
- C) $-0.36m^2c^2$
- D) $-0.64m^2c^2$
- E) $-0.5625m^2c^2$

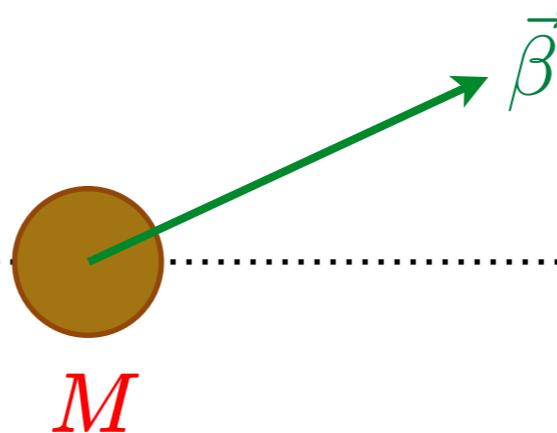


Relativistic 4-momentum conservation in a collision of billiard balls

Before:



After:

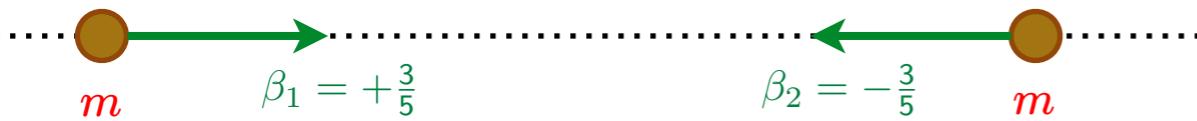


What are $\vec{\beta}$ and M , given m ?

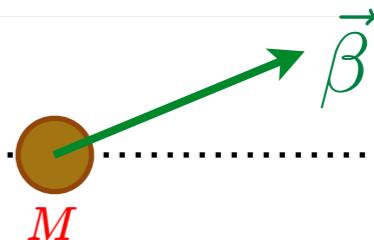
$$c = 1$$

Relativistic 4-momentum conservation in a collision of billiard balls

Before:



After:



Before: $p_1^\mu = \frac{5}{4}m(1, +\frac{3}{5}, 0, 0)$

$$p_2^\mu = \frac{5}{4}m(1, -\frac{3}{5}, 0, 0)$$

$$\gamma_1 = \gamma_2 = \frac{1}{\sqrt{1 - (\frac{3}{5})^2}} = \frac{5}{4}$$

After: $p^\mu = \gamma M(1, \beta_x, \beta_y, \beta_z)$

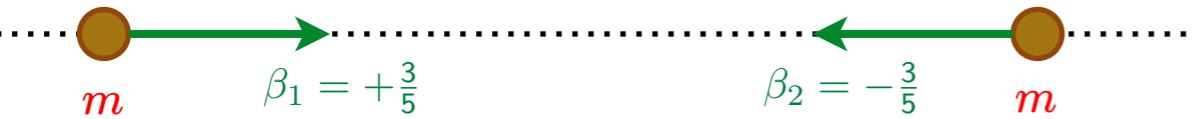
not the same as γ_1
– to be determined

What are $\vec{\beta}$ and M , given m ?

Use conservation of 4-momentum!

Relativistic 4-momentum conservation in a collision of billiard balls

Before:



After:



Before: $p_1^\mu = \frac{5}{4}m(1, +\frac{3}{5}, 0, 0)$

$$p_2^\mu = \frac{5}{4}m(1, -\frac{3}{5}, 0, 0)$$

After: $p^\mu = \gamma M(1, \beta_x, \beta_y, \beta_z)$

①

$$\gamma M = \frac{5}{4}m + \frac{5}{4}m = \frac{5}{2}m$$

②

$$\beta_x = +\frac{3}{5} - \frac{3}{5} = 0$$

③

$$\beta_y = 0 + 0 = 0$$

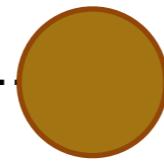
④

$$\beta_z = 0 + 0 = 0$$

Then $\beta = \sqrt{\beta_x^2 + \beta_y^2 + \beta_z^2} = 0 \Rightarrow \gamma = 1 \Rightarrow M = \frac{5}{2}m$

Relativistic 4-momentum conservation in a disintegration into two equal-mass daughters

Before:



M

After:

$-\beta$

m

$+\beta$

m

What is β , given M and m ?

$c = 1$

Relativistic 4-momentum conservation in a disintegration into two equal-mass daughters

Before:



M

After:



m



m

Before: $p^\mu = \cancel{M}(1, 0, 0, 0)$

After: $p_1^\mu = \gamma \cancel{m}(1, -\beta, 0, 0)$

$$p_2^\mu = \gamma \cancel{m}(1, +\beta, 0, 0)$$

What is β , given \cancel{M} and \cancel{m} ?

Use conservation of 4-momentum!

Relativistic 4-momentum conservation in a disintegration into two equal-mass daughters

Before:



M

After:



m



m

Before: $p^\mu = M(1, 0, 0, 0)$

After: $p_1^\mu = \gamma m(1, -\beta, 0, 0)$

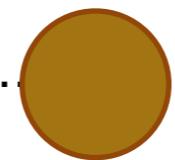
$$p_2^\mu = \gamma m(1, +\beta, 0, 0)$$

$$\boxed{\textcircled{0} \quad M = \gamma m + \gamma m} \Rightarrow \gamma = \frac{M}{2m} = \frac{1}{\sqrt{1 - \beta^2}} \Rightarrow \beta = \sqrt{1 - \left(\frac{2m}{M}\right)^2}$$

E.g., $\Upsilon(4S) \rightarrow B^0 + \bar{B}^0$: $\beta = \sqrt{1 - \left(\frac{2 \times 5279.6}{10579.4}\right)^2} = 0.0618$

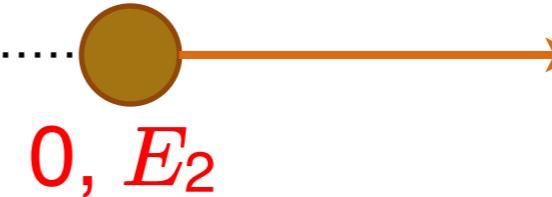
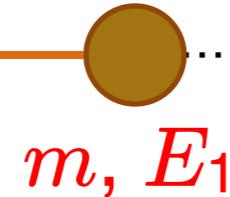
Relativistic 4-momentum conservation in a disintegration into two unequal-mass daughters

Before:



M

After:



What are E_1 and E_2 , given M and m ?

$c = 1$

Relativistic 4-momentum conservation in a disintegration into two unequal-mass daughters

Before:



M

After:



m, E_1



$0, E_2$

Before: $p^\mu = \textcolor{red}{M}(1, 0, 0, 0)$

After: $p_1^\mu = (\textcolor{teal}{E}_1, -\sqrt{\textcolor{teal}{E}_1^2 - \textcolor{red}{m}^2}, 0, 0)$

$$p_2^\mu = (\textcolor{teal}{E}_2, +\textcolor{teal}{E}_2, 0, 0)$$

using $E^2 = p^2 + m^2$ for p_1
and $E = p$ for p_2 ($m = 0$)

What are E_1 and E_2 , given M and m ?

Use conservation of 4-momentum!

Relativistic 4-momentum conservation in a disintegration into two unequal-mass daughters

Before:



M

After:



m, E_1



$0, E_2$

$$\mathbf{Before:} \quad p^\mu = M(1, 0, 0, 0)$$

$$\mathbf{After:} \quad p_1^\mu = (E_1, -\sqrt{E_1^2 - m^2}, 0, 0)$$

$$p_2^\mu = (E_2, +E_2, 0, 0)$$

①

$$M = E_1 + E_2$$

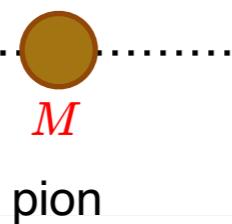
②

$$0 = -\sqrt{E_1^2 - m^2} + E_2$$

$$\Rightarrow \quad E_1 = \frac{M^2 + m^2}{2M} \quad E_2 = \frac{M^2 - m^2}{2M}$$

Relativistic 4-momentum conservation in a disintegration into two unequal-mass daughters

Before:



After:



Example:

$$\pi^+ \rightarrow \mu^+ + \bar{\nu}_\mu$$

Energy and mass measured in MeV.

$$E_1 = \frac{M^2 + m^2}{2M} = \frac{139.57^2 + 105.66^2}{2 \cdot 139.57} = 109.78$$

$$E_2 = \frac{M^2 - m^2}{2M} = \frac{139.57^2 - 105.66^2}{2 \cdot 139.57} = 29.8$$

Then, muon kinetic energy is

$$K_1 = E_1 - m = 109.78 - 105.66 = 4.12$$

and muon's relativistic factors are

$$\gamma = \frac{E_1}{m} = \frac{109.78}{105.66} = 1.04 \quad \beta = 0.27$$