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Assignment 3#

Problem 1

Problem 1

$$X = \begin{bmatrix} 7 & 13 \\ 8 & 9 \\ 8 & 10 \end{bmatrix} \rightarrow \bar{X} = [6 \quad 10]$$

$$n = 4$$

$$p = 2$$

$$\mu = [7, 11]$$

$$\begin{aligned} s_{11} &= \text{Cov}(X_1, X_1) = 8 \\ s_{12} &= \text{Cov}(X_1, X_2) = -10/3 \\ s_{22} &= \text{Cov}(X_2, X_2) = 2 \end{aligned}$$

$$T^2 = n(\bar{X} - \mu)^T E^{-1} (\bar{X} - \mu) = 4 \begin{bmatrix} -1 \\ -1 \end{bmatrix}^T \begin{bmatrix} 8 & -10/3 \\ -10/3 & 2 \end{bmatrix} \begin{bmatrix} -1 \\ -1 \end{bmatrix} = 4 \times 3.411$$

$$= 13.64$$

$$T^2 \sim \frac{\frac{(n-1)p}{2} \cdot \frac{19}{2}}{\frac{n-p}{2}} F_{\frac{p(n-p)}{2}, \frac{n-p}{2}}(\alpha) = T^2 \sim F_{\frac{p(n-p)}{2}, \frac{n-p}{2}}(\alpha)$$

$$T^2 = 13.64 < 3 \times 19 = 57 \Rightarrow \text{We cannot reject the Null hypothesis}$$

which is $\mu_0 = \mu$

Problem 2

Problem 2

for Column 2

$$\bar{X} = 2626.875$$

$$\sigma^2 = 1458543$$

$$UCL = \bar{X} + 3\sigma = 6299.98$$

$$LCL = \bar{X} - 3\sigma = -946.60 \Rightarrow LCL = 0$$

Both of them are I.C.

100

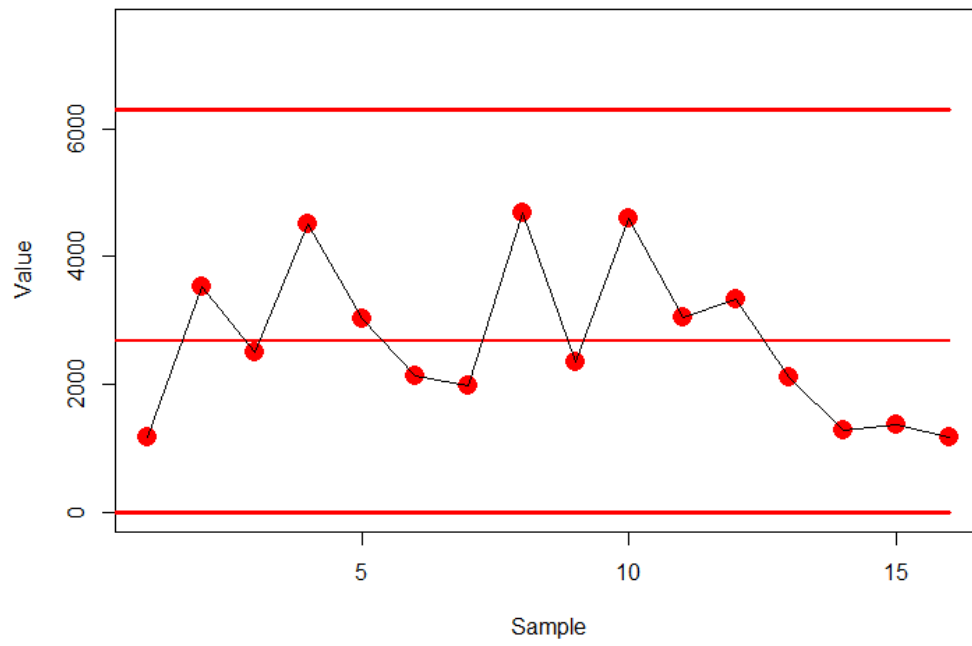
for Column 3

$$\bar{X} = 13563.625$$

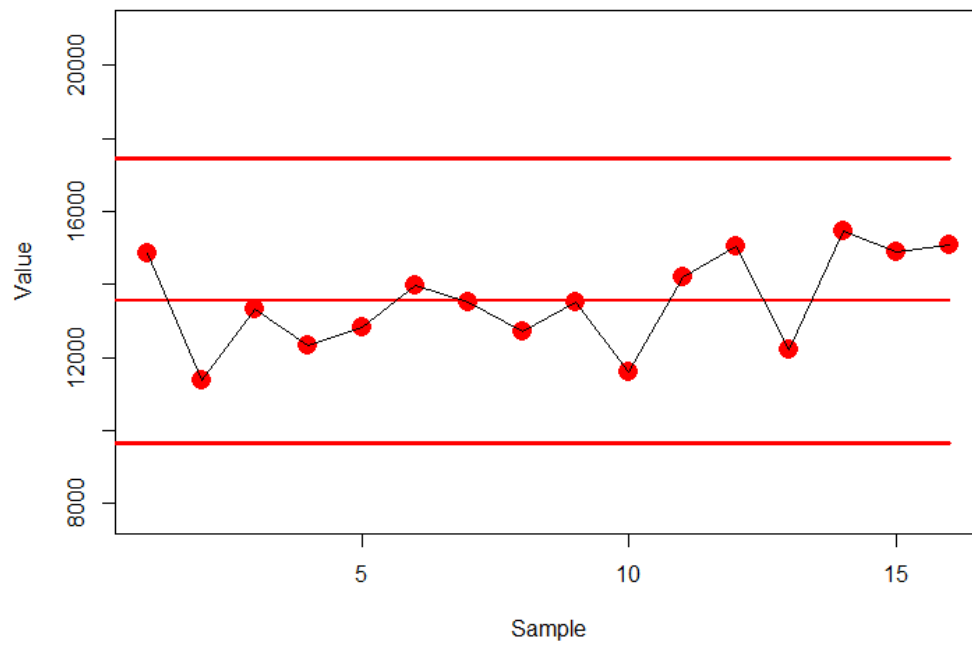
$$\sigma^2 = 1698324$$

$$UCL = 17473$$

$$LCL = 9654$$



For x4



Problem 3

Problem 3

Because sample size is 1 : not
we consider phase 1

$$\begin{aligned} m &= 16 \\ p &= 2 \\ n &= 1 \end{aligned}$$

$$UCL = \frac{(m-1)^2}{m} p_{\frac{1}{2}}, p_{\frac{1}{2}}, (m-p-1)/2$$

$$\frac{(m-1)^2}{m} = 14.0625$$

$$p_{\frac{5}{2}, 1, \frac{16-2-1}{2}} = p_{\frac{5}{2}, 1, 6.5} = 0.4330703$$

$$p_{1-\frac{5}{2}, 1, \frac{16-2-1}{2}} = 0.003887$$

$$UCL = 6.09$$

$$LCL = 0.55$$

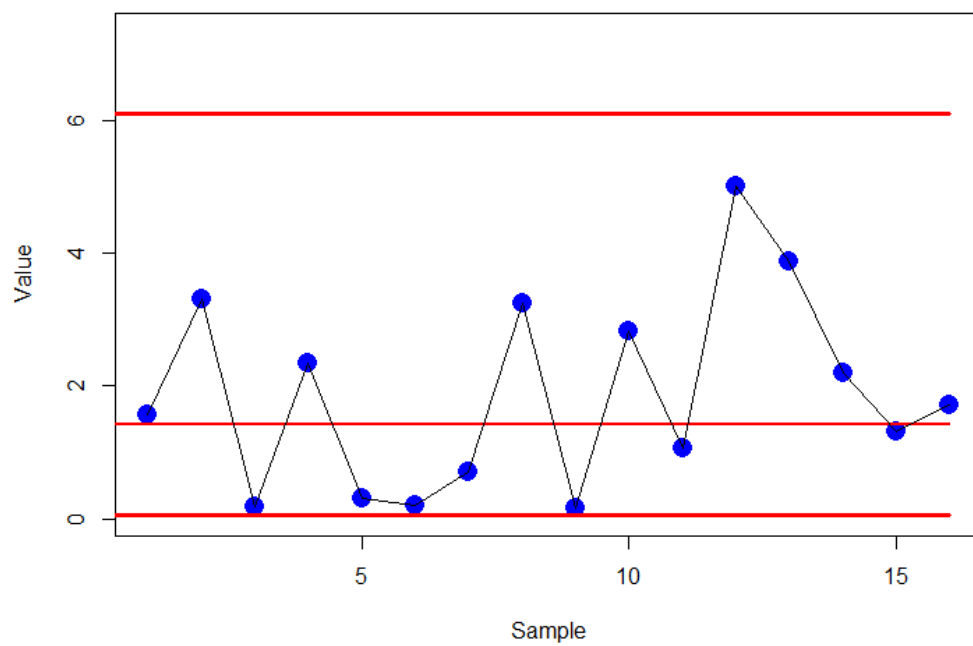
$$T^2 = n(X - \bar{X})^T R^{-1} (X - \bar{X})$$

$$T^2 = \begin{bmatrix} 1.56 \\ 2.54 \\ 0.06 \\ 2.38 \\ 0.72 \\ 0.42 \\ 0.66 \\ 2.20 \end{bmatrix} \begin{bmatrix} 0.16 \\ 2.96 \\ 1.54 \\ 6.13 \\ 2.39 \\ 2.15 \\ 1.18 \\ 2.34 \end{bmatrix}$$

$$\bar{X} = [2676.875 \quad \quad \quad]$$

$$\Sigma = \begin{bmatrix} 14585413 & -113810 \\ -1113810 & 698324 \end{bmatrix}$$

Although the stage change but the process is still I.C.
However, sometimes ~~they~~ using this method may change the decision.



Problem 4

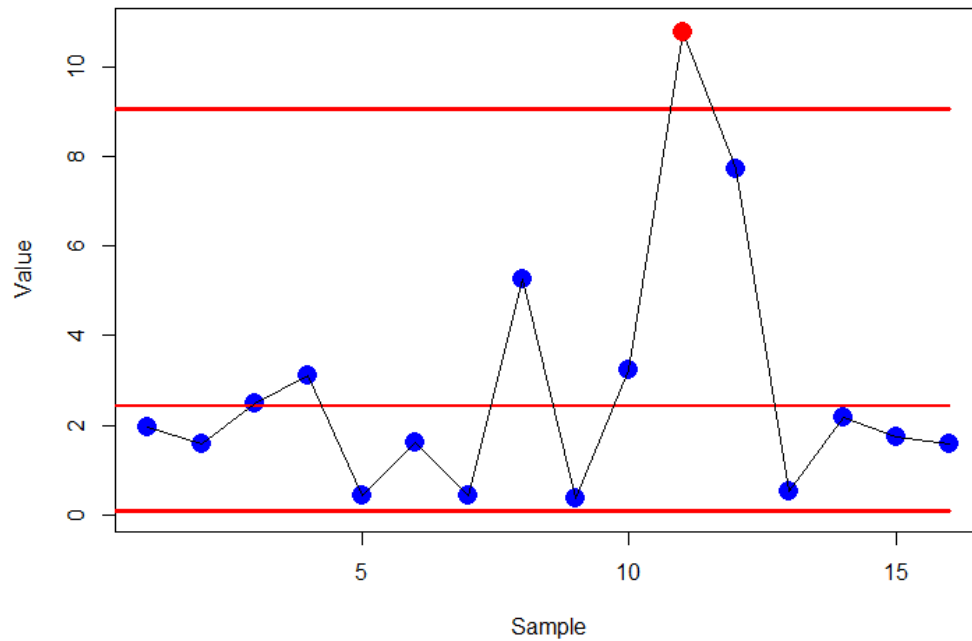
Although adding X_3 results in changes in the figure; however, Sample 11 is still OC. This shows the problem should be either in X_1 or X_2 . It should be mentioned $\alpha = 1\%$.

$$\bar{X} = [3557.75 \quad 1478.438 \quad 2676.875]$$

$$S_{\text{pooled}} = \begin{bmatrix} 367684 & -72093 & 857118 \\ -72093 & 1399053 & 483999 \\ 857118 & 483999 & 14456543 \end{bmatrix}$$

$$UCL = 9.052$$

$$LCL = 0.081$$

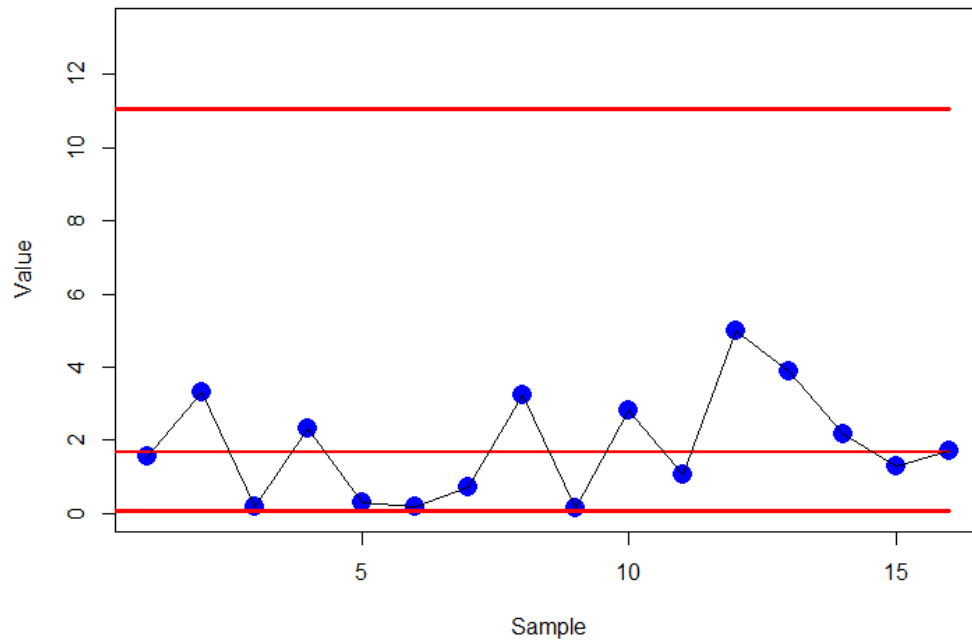


Problem 5

Problem 5

$$UCL = \frac{p(m+1)(m-1)}{m(m-p)} F_{\alpha/2, p, m-p} = 2.277 \times 4.86 = 11.058$$

$$LCL = \sim F_{1-\alpha/2, p, m-p} = 2.277 \times 0.025 = 0.056$$



Problem 6

The is the script results in R. The script its own is also attached.

```
Call:
lm(formula = Data$Literacy.Rate ~ Data$Newspapers + Data$Radios +
    Data$TV)

Residuals:
    Min       1Q   Median       3Q      Max
-0.233963 -0.069603 -0.007276  0.127095  0.188900

Coefficients:
              Estimate Std. Error t value Pr(>|t|)
(Intercept)   0.5148602   0.0936762    5.496  0.00152 **
Data$Newspapers 0.0005421   0.0008653    0.626  0.55410
Data$Radios    -0.0003535   0.0003285   -1.076  0.32330
Data$TV        0.0019882   0.0015503    1.282  0.24699
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 0.1865 on 6 degrees of freedom
Multiple R-squared:  0.6988, Adjusted R-squared:  0.5482
F-statistic: 4.64 on 3 and 6 DF, p-value: 0.05255

> confint(Regression,level = 1-alpha)
              2.5 %      97.5 %
(Intercept)  0.285642871 0.7440774959
Data$Newspapers -0.001575298 0.0026594242
Data$Radios    -0.001157286 0.0004503745
Data$TV        -0.001805185 0.0057815707
```

The answer from R and MATLAB are exactly the same; MATLAB script is also attached.

Based on the model, the most important parameter is Intercept and after that TV. The p-value of the whole model is around 5% rather low and the R-Squared is around 0.6 rather high; therefore, we can claim the model is working good.

Linear regression model:

LiteracyRate ~ 1 + Newspapers + Radios + TV

Estimated Coefficients:

| | Estimate | SE | tStat | pValue |
|-------------|-------------|------------|---------|-----------|
| (Intercept) | 0.51486 | 0.093676 | 5.4962 | 0.0015203 |
| Newspapers | 0.00054206 | 0.00086532 | 0.62643 | 0.5541 |
| Radios | -0.00035346 | 0.00032851 | -1.0759 | 0.3233 |
| TV | 0.0019882 | 0.0015503 | 1.2825 | 0.24699 |

Number of observations: 10, Error degrees of freedom: 6

Root Mean Squared Error: 0.186

R-squared: 0.699, Adjusted R-Squared 0.548

F-statistic vs. constant model: 4.64, p-value = 0.0526

ans =

| | |
|---------|--------|
| 0.2856 | 0.7441 |
| -0.0016 | 0.0027 |
| -0.0012 | 0.0005 |
| -0.0018 | 0.0058 |