1.

(a). Evaluate T^2, for testing $H_0 : \mathbf{\mu}' = [7,11]$ using the data

$$\mathbf{X} = \begin{bmatrix} 2 & 12 \\ 8 & 9 \\ 6 & 9 \\ 8 & 10 \end{bmatrix}$$

(b). Specify the distribution of T^2 for the situation in (a).

(c). Using (a) and (b), test H_0 at the $\alpha = .05$ level. What conclusion do you reach?

2. Using the Madison, Wisconsin, Police Department data in (**Appendix**) Table 1, construct individual $\bar{\mathbf{X}}$ charts for $x_3 =$ holdover hours and $x_4 =$ COA hours. Do these individual process characteristics seem to be in control? (That is, are they stable?) Comment.

3. Refer to problem 2. Using the data on the holdover and COA overtime hours, construct a T^2 chart. Does the process represented by the bivariate observations appear to be in control? (That is, is it stable?) Comment. Do you learn something from the multivariate control charts that was not apparent in the individual $\bar{\mathbf{X}}$ charts?

4. Construct a T^2 chart using the data on $x_1 =$ legal appearances overtime hours, $x_2 =$ extraordinary event overtime hours, and $x_3 =$ holdover hours from table 1. Compare this chart with the chart in Figure 5.8 of Example 5.10. Does plotting T^2 with an additional characteristic change your conclusion about process stability? Explain.

5. Using the data on $x_3 =$ holdover hours and $x_4 =$ COA hours from Table 1, construct a T^2 control chart for **future observation** $\mathbf{x}^T = (x_3, x_4)$.

6. Literacy rate is a reflection of the educational facilities and quality of education available in a country, and mass communication plays a large part in the educational process. In an effort to relate the literacy rate of a country to various mass communication outlets, a demographer has proposed to relate literacy rate to the following variables: number of daily newspaper copies (per 1000 population), number of radios (per 1000 population), and number of TV sets (per 1000 population). Table 2 (**Appendix**) shows the data for a sample of 10 countries.

(a) Construct a multiple linear regression model and calculate the least square estimators for $\boldsymbol{\beta}$ and σ^2.

(b) Calculate 95% confidence intervals for $\boldsymbol{\beta}$ and σ^2.

(c) Identify the important predictors and conclude your model.

(d) Calculate the R^2 to evaluate how well the model fit the data.

Appendix

Table 1. Five Types of Overtime Hours for the Madison, Wisconsin, Police Department

x_1 Legal Appearances Hours	x_2 Extraordinary Event Hours	x_3 Holdover Hours	x_4 COA[1] Hours	x_5 Meeting Hours
3387	2200	1181	14,861	236
3109	875	3532	11,367	310
2670	957	2502	13,329	1182
3125	1758	4510	12,328	1208
3469	868	3032	12,847	1385
3120	398	2130	13,979	1053
3671	1603	1982	13,528	1046
4531	523	4675	12,699	1100
3678	2034	2354	13,534	1349
3238	1136	4606	11,609	1150
3135	5326	3044	14,189	1216
5217	1658	3340	15,052	660
3728	1945	2111	12,236	299
3506	344	1291	15,482	206
3824	807	1365	14,900	239
3516	1223	1175	15,078	161

[1] Compensatory overtime allowed.

Example 5.10 (A T^2 chart for overtime hours)

Using the police department data in Table 1, we construct a T^2 plot based on the two variables x_1 = legal appearances overtime hours and x_2 = extraordinary event overtime hours. α level is set to be .01.

T^2 chart in figure 5.8 reveals that the pair (legal appearances, extraordinary event) hours for period 11 is out of control. Further investigation confirms that this is due to the large value of extraordinary event overtime during that period.
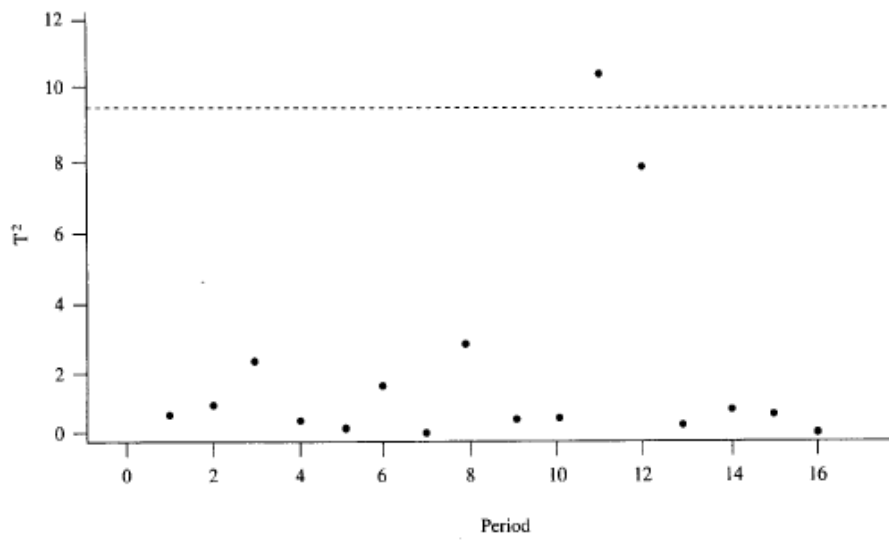
Figure 5.8 The T^2 chart for legal appearances hours and extraordinary event hours, $\alpha = .01$.

Table 2. Literacy rate data

Country	newspapers	radios	tv sets	literacy rate
Czech Republic / Slovakia	280	266	228	0.98
Italy	142	230	201	0.93
Kenya	10	114	2	0.25
Norway	391	313	227	0.99
Panama	86	329	82	0.79
Philippines	17	42	11	0.72
Tunisia	21	49	16	0.32
USA	314	1695	472	0.99
Russia	333	430	185	0.99
Venezuela	91	182	89	0.82