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AN EXPERIMENTALLY DERIVED MODEL FOR ACTUAL EVAPOTRANSPIRATION

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ABSTRACT

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Experimental data from several different climatic regions were used to develop a statistical model for actual water loss rates from land surfaces. The actual evapotranspiration rate was considered to be influenced by the amount of available water in the soil and by meteorological and plant conditions which determine the potential evapotranspiration rate. Experimental measurements of these three variables from various environmental conditions were combined into a single model expressing the composite relationship. Actual evapotranspiration rates were calculated from this relationship using the variables soil moisture and potential evapotranspiration. Initial testing showed that the model gave satisfactory results when used for estimating moisture changes in the soil.

INTRODUCTION

The amount of water which escapes from land surfaces to the atmosphere is a very important element of the hydrologic cycle which is also very difficult to quantify. In addition to the environmental variables which determine the potential evapotranspiration rate, the actual water loss rate is strongly influenced by the amount of available water in the soil. In order to obtain estimates of the actual evapotranspiration rate, a simple straight line relationship between the relative evapotranspiration rate, ratio of actual to potential, and the soil moisture content was used by THORNTHWAITE and MATHER (1948) and VAN BAVEL (1953). A different linear relationship was proposed by VIEHMEYER and HENDRICKSON (1955). Other more recent investigations, such as PIERCE (1958), DENMEAD and SHAW (1962), EAGLEMAN and DECKER (1965), and VAN BAVEL (1967), have revealed curvilinear decreases in the relative evapotranspiration as the amount of moisture in the soil decreased. This article describes an investigation of the relationship of such experimental results obtained under various climatic conditions. It was found that they could be combined into a single regression model which may be useful for calculating actual evapotranspiration rates for specified amounts of soil moisture and atmospheric demands.

DEVELOPMENT OF THE MODEL

The experimental results which were used as the basic data for this study were obtained under various climatic conditions and with different vegetative covers. The three variables measured or calculated in each case were soil moisture, actual and potential evapotranspiration rates. In the experiment by DENMEAD and SHAW (1962), corn was grown in 20-gallon containers placed in a corn field to eliminate boundary effects. The actual water loss rate was determined by the decreased weight of water in the containers and the potential loss rate from other containers maintained at field capacity. This experiment differed slightly from the others since the soil surface was covered with plastic to prevent evaporation from the soil. This component of evapotranspiration decreases in importance for complete vegetative cover and actively growing crops. Since these two conditions are necessary in order to obtain the potential evapotranspiration this experiment was included with the others which did not have this restriction. In the investigation by EAGLEMAN and DECKER (1965), the actual evapotranspiration rate was determined by water balance measurements for several moisture regimes within a soybean field. The potential evapotranspiration was calculated from PENMAN's (1956) equation. In the experiment by VAN BAVEL (1967), actual evapotranspiration from alfalfa in Arizona was measured by a lysimeter and the potential evapotranspiration rate was calculated from a modified form of Penman's equation. PIERCE (1958) investigated the actual water loss rates from meadow grasses in Ohio as measured by a lysimeter. The potential evapotranspiration rate was calculated by a modified form of THORNTHWAITE's (1948) equation. Various other investigations have been conducted in which soil moisture, actual and potential evapotranspiration have been determined under less natural environments such as in greenhouses or from plants growing in small containers. These were not included in the analysis because of the unknown influence of the individual components of the environment which differed from natural conditions. Therefore, the results of this analysis should be applicable to actual water loss rates from land surfaces.

Each of the described investigations presented results showing a curvilinear relationship between some measure of moisture content and the ratio obtained by dividing the actual evapotranspiration (AE) by the potential evapotranspiration (PE). A uniform expression for water content was required in order to compare these results. The moisture ratio (MR) was used and is defined as:

$$MR = (SM - WP)/(FC - WP) \qquad (1)$$

where: SM is the measured soil moisture content; WP is the moisture content at the wilting point for which the moisture content corresponding to 15 atm suction was used; and FC is the moisture content at field capacity for which 1/3 atm of suction was used.

These constants are commonly used for expressing the available water for plants. Even if they were entirely accurate for plants, some additional moisture is available for evaporation from the soil after the plants have wilted and decayed. Therefore, there is no reason for assuming that some water will not be lost when the value of MR is zero or less. The various experiments indicate that this frequently occurs. A definition such as the moisture ratio is essential for evaluating the influence of moisture on the actual evapotranspiration rate since water loss is more related to the degree of attraction between the water and soil particles than to the quantity of water in the soil.

Some of the investigations, e.g., DENMEAD and SHAW (1962), show results of the relative evapotranspiration (AE/PE) as a function of soil moisture which indicate a cubic relationship. Results of all the experiments showed that the relative evapotranspiration remained high as initial decreases of moisture content occurred with a sharper drop as the moisture content decreased further. Since the results of each experiment were presented showing the relative evapotranspiration as a function of soil moisture, regression coefficients were obtained for each set of data for the cubic equation:

$$AE/PE = A + B(MR) + C(MR)^2 + D(MR)^3 \tag{2}$$

The regression coefficients A, B, C, and D determined for each set of data are given in Table I together with the data source. These coefficients produce curves so similar to the figures in each of the references that the cubic relationships might still be considered to be experimentally derived. This step was necessary in order to make the results from the separate studies comparable.

The degree of effect of soil moisture content and potential evapotranspiration on the actual evapotranspiration rate was investigated in terms of these equations. The additional influence of the potential rate was determined by a regression analysis of each of the coefficients in Table I on potential evapotranspiration. The

TABLE I

COMPARABLE REGRESSION COEFFICIENTS FOR EQ. 2 BASED ON THE RESULTS FROM SEVERAL INVESTIGATIONS

A	*B*	*C*	*D*	*PE* (*mm/day*)	*Data source*
0.3440	3.36	−5.34	2.68	2.0	DENMEAD and SHAW (1962)
0.0870	3.40	−4.11	1.62	3.3	DENMEAD and SHAW (1962)
0.2180	2.78	−3.28	1.29	3.6	PIERCE (1958)
0.0334	2.58	−1.83	0.185	4.1	DENMEAD and SHAW (1962)
0.2419	1.46	−1.23	0.553	5.1	EAGLEMAN (1963)
0.0186	1.05	1.58	−1.69	5.6	DENMEAD and SHAW (1962)
0.0361	−0.412	1.93	−0.465	6.4	DENMEAD and SHAW (1962)
0.0599	−0.359	4.87	−3.62	9.0	VAN BAVEL (1967)

results, Fig.1–4, show that a very good relationship exists for all except the *A* coefficients in Fig.1. The correlation coefficient for Fig.1 was 0.73. For the other three coefficients, *B*, *C*, and *D*, it varied from 0.93 to 0.97. Various types of equations were considered for the data in Fig.1. A computer program which performed correlations using various forms of the independent variable was used.

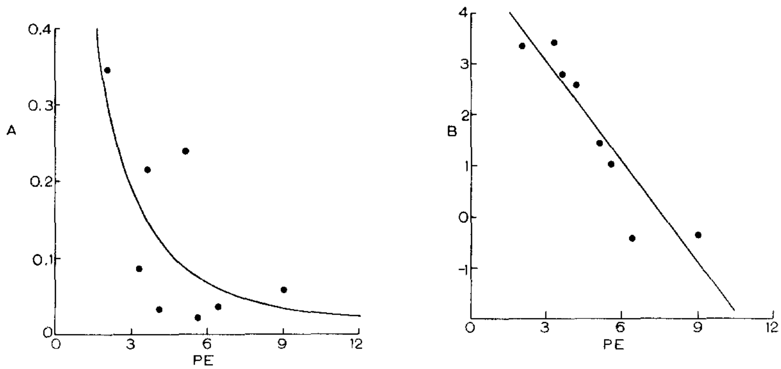

Fig.1. The *A* coefficient of eq.2 as a function of potential evapotranspiration (mm/day).
Fig.2. The *B* coefficient of eq.2 as a function of potential evapotranspiration (mm/day).

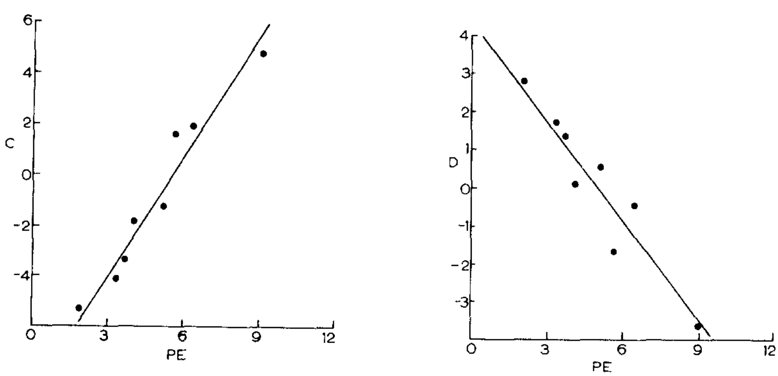

Fig.3. The *C* coefficient of eq.2 as a function of potential evapotranspiration (mm/day).
Fig.4. The *D* coefficient of eq.2 as a function of potential evapotranspiration (mm/day).

The equilateral hyperbola using logarithms of the independent variable gave the best correlation coefficient of 0.74 with the potential evapotranspiration rate in millimeters and a coefficient of only 0.56 with the rate expressed in inches. The equilateral hyperbola without logarithms gave a correlation coefficient of 0.73 in either case. Therefore this relationship was used. There may be physical reasons why this type of equation gave the best fit.

From eq.2 it can be seen that A is the value for the relative evapotranspiration when the available water is equal to zero. Therefore, A should never become negative but should approach zero as higher potential evapotranspiration rates are encountered. The equilateral hyperbola is a type of equation which satisfies this requirement. There are several reasons why the A coefficients show more scatter than the others. One probable reason is that the experimental data are much more limited at the moisture content corresponding to the wilting point. In most of the experiments water was supplied to the plants before the permanent wilting point was reached so that data were much more limited at this point or were obtained by extrapolation. Since the four points below the line in Fig.1 are all from the experiment by DENMEAD and SHAW (1962), much of the scatter may arise because of the plastic cover over the soil in this experiment. Resulting differences because of this treatment would be most apparent at the wilting point for plants since the small amount of evaporation from the soil is the only loss of moisture after the plants are no longer active. Another contributing cause of the scatter may be the rapid change in the relative evapotranspiration near the wilting point for low or medium potential evapotranspiration rates. This causes additional difficulty with the accuracy of experimental data obtained near the wilting point.

The coefficients A, B, C, and D in eq. 2 expressed as functions of potential evapotranspiration are:

$$A = -0.050 + 0.732/(PE) \tag{3}$$
$$B = +4.97 - 0.661\,(PE) \tag{4}$$
$$C = -8.57 + 1.56\,(PE) \tag{5}$$
$$D = +4.35 - 0.880\,(PE) \tag{6}$$

The actual evapotranspiration rate in terms of the moisture ratio and potential evapotranspiration then becomes:

$$\begin{aligned} AE = {} & 0.732 - 0.050\,(PE) + [4.97\,(PE) - 0.661\,(PE)^2]\,MR \\ & - [8.57\,(PE) - 1.56\,(PE)^2]\,MR^2 + [4.35\,(PE) - 0.880\,(PE)^2]\,MR^3 \end{aligned} \tag{7}$$

Actual evapotranspiration rates from eq.7 calculated for various soil moisture conditions and potential evapotranspiration rates are shown in Fig.5. This model indicates that for higher potential rates of evapotranspiration, the actual rate decreases as the potential rate increases when over one-half of the available moisture for plants has been depleted from the soil. The physical condition in support of this model is the development of a dry surface layer which interferes with diffusion of soil moisture under high evaporative demands of the environment when the soil temperature gradient would probably favor a downward movement of moisture. The line corresponding to a moisture ratio of 1.0 in Fig.5 varies slightly from an idealistic forty-five degree straight line. However, the maximum

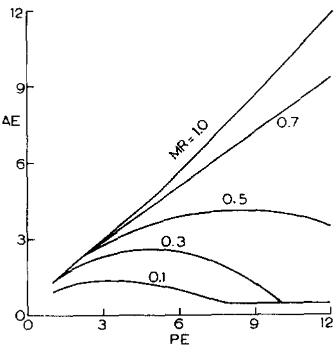

Fig.5. The relationship between actual evapotranspiration (*AE*), various values of moisture ratio (*MR*), and potential evapotranspiration (*PE*) (mm/day).

difference between the actual and potential evapotranspiration is only 0.49 mm which occurs at a potential evapotranspiration rate of 7.0 mm and a calculated actual rate of 6.51. Since the range of the experimental data was from 2.0 to 9.0 mm of potential evapotranspiration the curves outside this range are extrapolated, but appear to be reasonable except for extreme values of moisture ratio. On the low end of the scale at a potential evapotranspiration rate of 2.0 mm the calculated rate is 2.20 mm. This difference is not very significant in terms of amount but is greater in terms of percent error. On the high end of the scale for *PE* values greater than 9.0 mm the calculated actual evapotranspiration rate becomes slightly negative for very low moisture ratios. There is probably in reality some small continued actual evaporation rate from vapor diffusion from the soil after the plants have wilted and the moisture ratio has dropped below zero.

TESTING OF THE MODEL

The statistical model shown in Fig.5 is undoubtedly still a simplification. A test of its applicability was made by comparing the calculated actual evapotranspiration rate with the measured water loss rate in the field. In order to make this comparison, a computer program (see Appendix) was developed which depleted moisture from the soil according to the daily actual evapotranspiration rate. The potential evapotranspiration was calculated by a method previously described (EAGLEMAN, 1967). The data used have been described in detail by EAGLEMAN (1963). Briefly, they consisted of weekly measurements of soil moisture together with environmental data, precipitation and runoff from a soybean field. A comparison of the soil moisture estimated by eq.7 and the measured soil moisture is shown in Table II for twenty-six days in August for which continuous measurements were available. The estimated soil moisture was set equal to the measured soil moisture at the start of this period on August 2. The moisture was then depleted by twenty-six calculations of the following equation:

TABLE II

CALCULATIONS AND COMPARISONS FROM THE COMPUTER PROGRAM FOR ESTIMATING SOIL MOISTURE

Day (1962)	*PE (mm)*	*AE (mm)*	*Precip. (mm)*	*Runoff (mm)*	*Measured soil moisture (mm)*	*Estimated soil moisture (mm)*
Aug. 2	6.2	4.4	0.	0.	260.1	260.1
Aug. 3	6.0	3.9	11.4	0.		255.8
Aug. 4	6.5	4.8	0.	0.		263.3
Aug. 5	6.8	4.4	0.	0.		258.5
Aug. 6	8.7	4.0	0.	0.		254.1
Aug. 7	6.6	3.5	0.	0.	262.9	250.3
Aug. 8	6.0	3.1	0.	0.		246.8
Aug. 9	6.1	2.9	0.	0.		243.6
Aug. 10	5.7	2.6	0.	0.		240.8
Aug. 11	8.1	1.6	0.	0.		238.1
Aug. 12	7.3	1.8	0.	0.		236.6
Aug. 13	5.0	2.2	0.	0.		234.8
Aug. 14	4.9	2.0	0.	0.	247.9	232.6
Aug. 15	5.8	1.7	0.	0.		230.5
Aug. 16	5.9	1.5	0.	0.		228.8
Aug. 17	5.2	1.5	0.	0.		227.3
Aug. 18	7.5	0.7	4.3	0.		225.7
Aug. 19	7.3	1.1	0.	0.		229.4
Aug. 20	10.0	0.5	0.	0.		228.3
Aug. 21	10.6	0.5	0.	0.	226.1	227.8
Aug. 22	8.0	0.6	0.	0.		227.3
Aug. 23	7.9	0.6	0.	0.		226.7
Aug. 24	6.8	1.0	0.	0.		226.2
Aug. 25	3.9	1.4	0.	0.		225.2
Aug. 26	5.0	1.2	0.	0.		223.7
Aug. 27	6.6	0.8	0.	0.	222.5	222.4

$$SME = SMS - AE + P - RO \tag{8}$$

SME is the soil moisture content estimated for the end of the day; SMS is the soil moisture content at the start of the day; AE is the calculated actual evapotranspiration using climatic data for the day and the moisture ratio from the end of the previous day; P is the amount of precipitation during the day; and RO is the runoff. Impervious conditions in this soil allowed the assumption of no percolation below the depth of measurements.

After the first calculation no adjustments were made on days when soil moisture measurements were available and did not agree with the predicted amount so that the estimated amount of moisture after twenty-six days was influenced only by the amount measured at the start of the period. Table II shows that for these data the model underestimated the soil moisture content on August 7 and 14 by 5 and 6% respectively, but was very close to the other measured values.

CONCLUSIONS

The fact that such a good relationship exists for the *B*, *C*, and *D* coefficients is quite interesting in view of the several distinctly different types of vegetation and climatic conditions included in the analysis. This implies that the general response of these different types of vegetation to their environment was quite similar. In each of the experiments the moisture changes corresponded to different depths because of differences in rooting characteristics of the plants. Therefore, if this variable is properly considered, a somewhat orderly relationship exists between actual evapotranspiration, potential evapotranspiration and soil moisture. This relationship can be used to obtain the estimated moisture content of the root zone by utilizing the calculated potential evapotranspiration based on climatic data, or it can be used for a general estimate of the actual evapotranspiration from land surfaces.

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APPENDIX

FORTRAN IV PROGRAM

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C CALCULATION OF POTENTIAL AND ACTUAL EVAPOTRANSPIRATION AND COMPARISON
C OF MEASURED AND ESTIMATED SOIL MOISTURE CONTENT.
C REQUIRED VARIABLES ARE NUMBER OF OBSERVATIONS (NOB), SOIL MOISTURE AT
C FIELD CAPACITY (SMFC), SOIL MOISTURE AT WILTING POINT (SMWP), TEMPERATURE
C IN DEGREES F (TT), RELATIVE HUMIDITY IN PERCENT (RH), PRECIPITATION IN
C MM (P), RUNOFF IN MM (RO), AND MEASURED SOIL MOISTURE (SMM).
      DIMENSION TT (200), RH( 200), P(200), SMM(200), SMP(200), RO(200),
      9PE(200), AE(200), MT(13)
      REAL MR
      NST = 1
      READ (5,1) NOB, SMFC, SMWP
    1 FORMAT (I10, 2F10.2)
    2 FORMAT (13A6)
      READ (5,2) MT
      READ (5,MT) (TT(I) , I = 1,NOB)
      READ (5,MT) (RH(I) , I = 1,NOB)
      READ (5,MT) (P(I) , I = 1,NOB)
      READ (5,MT) (RO(I) , I = 1,NOB)
      READ (5,MT) (SMM(I) , I = 1,NOB)
C CALCULATION OF POTENTIAL EVAPOTRANSPIRATION (EAGLEMAN, 1967)
      SMP (NST) = SMM(NST)
      DO 16 I = NST,NOB
      TEMPA = 0.5555555556 * (TT(I) - 32.) + 273.16
      IF (TT(I) .GT. 31.) GO TO 11
C  SATURATION VAPOR PRESSURE COMPUTED BY TETONS METHOD.
      EXP=(21.8745584*(TEMPA-273.16))/(TEMPA-7.66)
      ES = 6.1078*(2.71828**EXP)
      PE(I) = 0.0175*ES*((100. - RH(I))**.5)
      GO TO 12
   11 EXP=(17.2693882*(TEMPA-273.16))/(TEMPA-35.86)
      ES = 6.1078*(2.71828**EXP)
      CR = 0.200 + 0.0133 * TT(I)
      IF (TT(I) .GE. 70.) CR = 1.130
      PE(I) = 0.0292*CR * ES * ((100. -RH(I)) ** .5)
   12 CONTINUE
```

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C  CALCULATION OF ACTUAL EVAPOTRANSPIRATION
      A = -0.050 + 0.732 / PE(I)
      B = +4.97 - 0.661 * PE(I)
      C - -8.57 + 1.56 * PE(I)
      D = +4.35 - 0.880 * PE(I)
      MR = ((SMP(I) -SMWP)/(SMFC - SMWP))
      IF (MR .LT. 0.) MR = 0.
   15 RET = A + B * MR + C * MR ** 2 + D * MR ** 3
      IF (RET .GT. 1.0) RET = 1.0
      IF (RET .LT. 0.05) RET = 0.05
      AE(I) = RET * PE(I)
      IPO = I + 1
   16 SMP(IPO) = SMP(I) - AE(I) + (P(I) - RO(I))
      WRITE(6,17)
   17 FORMAT (1H1, 1X,3HDAY,13X,2HPE,14X,2HAE,10X,6HPRECIP,10X
     96HRUNOFF,7X, 9HSM, MEAS.,7X,9HSM, PRED. //)
      WRITE(6,19) (I,PE(I),AE(I),P(I),RO(I),SMM(I),SMP(I),I=NST,NOB)
   19 FORMAT (I4, 6F16.2)
      STOP
      END
```