

Mathematical Modelling (APM1514)

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August 23, 2016

Question 1

$$\begin{aligned}\frac{dy}{dt} &= 2(4 - y) \\ \Rightarrow \frac{dy}{4-y} &= 2dt \\ \Rightarrow \int \frac{dy}{4-y} &= \int 2dt \\ \Rightarrow -\ln|4-y| &= 2t + C \\ \Rightarrow \ln|4-y| &= -2t + C \\ \Rightarrow 4-y &= e^{-2t+C} = Ae^{-2t} \\ \Rightarrow -y &= -4 + Ae^{-2t} \\ \therefore y &= 4 + Ae^{-2t}\end{aligned}$$

Question 4

$$\begin{aligned}\frac{dy}{dt} &= 1 - 4y, y(1) = 5 \\ \frac{dy}{1-4y} &= dt, \text{ separation of variables} \\ \int \frac{dy}{1-4y} &= \frac{1}{-4} \int \frac{1}{y-\frac{1}{4}} dt \\ \int \frac{dy}{y-\frac{1}{4}} &= -4 \int dt, \text{ divide LHS \& RHS denominators by -4} \\ \ln|y-\frac{1}{4}| &= -4t + C \\ y-\frac{1}{4} &= e^{-4t+C} \\ \Rightarrow y &= \frac{1}{4} + Ae^{-4t}, A = e^C \\ \text{From initial conditions, } y(1) &= 5 \\ \Rightarrow 5 &= \frac{1}{4} + Ae^{-4 \times 1} \\ \Rightarrow \frac{19}{4} &= \frac{A}{e^4} \\ \Rightarrow A &= \frac{19e^4}{4}\end{aligned}$$
