Lab 3: Fourier Series Analysis Using MATLAB

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A:

A.1:

A.1: $x_1(t) = \cos\frac{3\pi}{10}t + \frac{1}{2}\cos\frac{\pi}{10}t$

$$= \frac{1}{2}e^{j3\pi/10t} + \frac{1}{2}e^{-j3\pi/10t} + \frac{1}{2}\left(\frac{1}{2}e^{j\pi/10t} + \frac{1}{2}e^{-j\pi/10t}\right)$$

$$= \frac{1}{2}e^{j3\pi/10t} + \frac{1}{2}e^{-j3\pi/10t} + \frac{1}{4}e^{j\pi/10t} + \frac{1}{4}e^{-j\pi/10t}$$

Fundamental frequency:

$\frac{3\pi}{10} \cdot \frac{10}{\pi} = 3$

$\omega_{01} = \frac{3\pi}{10} \quad \omega_{02} = \frac{\pi}{10} \qquad T_0 = \frac{2\pi}{\left(\frac{\pi}{10}\right)} = 2\pi \cdot \frac{10}{\pi} = 20$

$\frac{GCF}{LCM} = \frac{\pi}{10} = \pi/10$

$jn\frac{\pi}{10}t = j\frac{3\pi}{10}t \qquad jn\,\pi/10\,t = j\pi/10\,t$

$n = 3 \quad n = -3 \qquad n = 1 \quad n = -1$

$$\boxed{D_3 = \frac{1}{2} \quad D_{-3} = \frac{1}{2} \quad D_1 = \frac{1}{4} \quad D_{-1} = \frac{1}{4}}$$

$$D_n = \frac{1}{20}\int_{-10}^{10}\left[\frac{1}{2}e^{j3\pi/10t} + \frac{1}{2}e^{-j3\pi/10t} + \frac{1}{4}e^{j\pi/10t} + \frac{1}{4}e^{-j\pi/10t}\right]e^{-j\pi/10nt}\,dt$$

$$D_n = \frac{1}{20}\left[\frac{e^{j(3-n)\pi} - e^{-j(3-n)\pi}}{2j(3-n)\pi/10} + \frac{e^{j(3+n)\pi} - e^{-j(3+n)\pi}}{2j(3+n)\pi/10} + \frac{e^{j(1+n)\pi} - e^{-j(1+n)\pi}}{4j(1+n)\pi/10} + \frac{e^{j(1-n)\pi} - e^{-j(1-n)\pi}}{4j(1-n)\pi/10}\right]$$

$$\therefore D_n = \frac{1}{2}\left[\mathrm{sinc}[(3-n)\pi] + \mathrm{sinc}[(3+n)\pi] + \frac{1}{2}\mathrm{sinc}[(1+n)\pi] + \frac{1}{2}\mathrm{sinc}[(1-n)\pi]\right]$$

A.2:

A.2: $x_2(t)$: $T_0 = 20$ $\omega_0 = \frac{2\pi}{20} = \frac{\pi}{10}$

$$D_n = \frac{1}{20}\left[\int_{-5}^{5} (1)\, e^{-jn\pi/10\, t}\, dt\right] = \frac{1}{20}\left[\frac{1}{-jn\frac{\pi}{10}} e^{-jn\pi/10\, t}\right]_{-5}^{5}$$

$$D_n = \frac{1}{20}\left[\frac{-10}{jn\pi} e^{-jn\pi/2} + \frac{10}{jn\pi} e^{jn\pi/2}\right] = \frac{1}{n\pi}\sin\left(\frac{\pi n}{2}\right)$$

$$\therefore D_n = \frac{1}{n\pi}\sin\left(\frac{n\pi}{2}\right)$$

$x_3(t)$: $T_0 = 40$ $\omega_0 = \frac{2\pi}{40} = \frac{\pi}{20}$ | $D_n = \frac{1}{40}\int_{-5}^{5} (1)\, e^{-jn\pi/20\, t}\, dt$

$$D_n = \frac{1}{40}\left[\frac{1}{-jn\pi/10} e^{-jn\pi/10\, t}\right]_{-5}^{5}$$

$$D_n = \frac{1}{40}\left[\frac{-20}{jn\pi} e^{-jn\pi/4} + \frac{20}{jn\pi} e^{jn\pi/4}\right]$$

$$\therefore D_n = \frac{1}{n\pi}\sin\left(\frac{n\pi}{4}\right)$$

A.3:

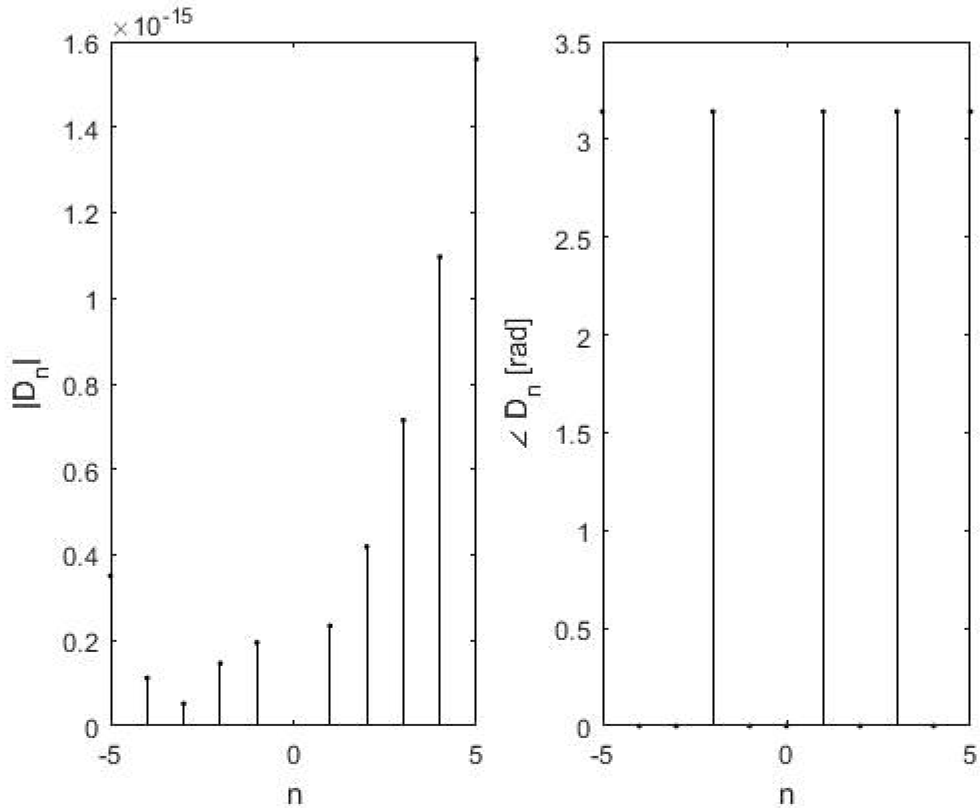
Matlab code:

```
---------------------------------
function [D]=Dn(d,n)
D1 = [0.5,0,-0.5*1i,0,0.5*1i,0,0.5];
D2 = (1/(n.*pi)*sin((n*pi)/2));
D3 = (1/(n.*pi)*sin((n*pi)/4));
if (d==1)
        D=D1;end
if (d==2)
        D=D2;end
if (d==3)
        D=D3;end
end
------------------------------------
```

A.4:

a)

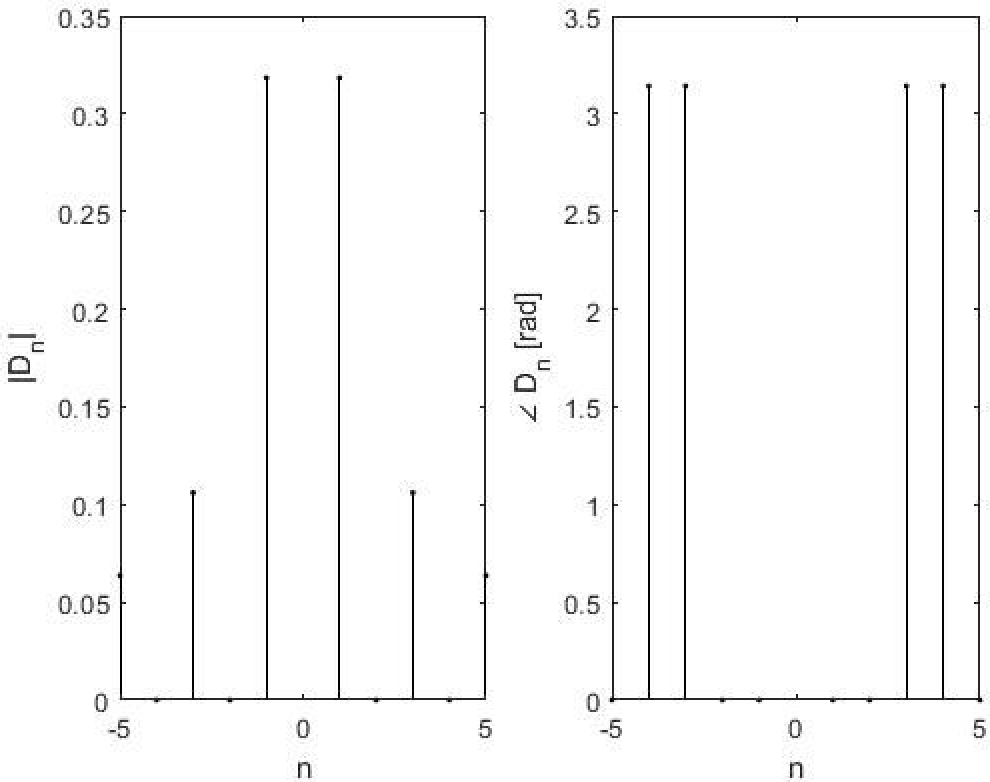
x1(t):



Matlab:

```
----------
clf;
n = (-5:5);
D_n = 1./2.*((1./(pi.*n)).*sin((3-n).*pi )) + (1./pi.*n).*sin((3+n).*pi) + (1./(2.*n.*pi).*sin((1+n).*pi)) +
(1./(2.*n.*pi).*sin((1-n).*pi)) ;
subplot(1,2,1); stem(n,abs(D_n),'.k');
xlabel('n'); ylabel('|D_n|');
subplot(1,2,2); stem(n,angle(D_n),'.k');
xlabel('n'); ylabel('\angle D_n [rad]');
------------
```

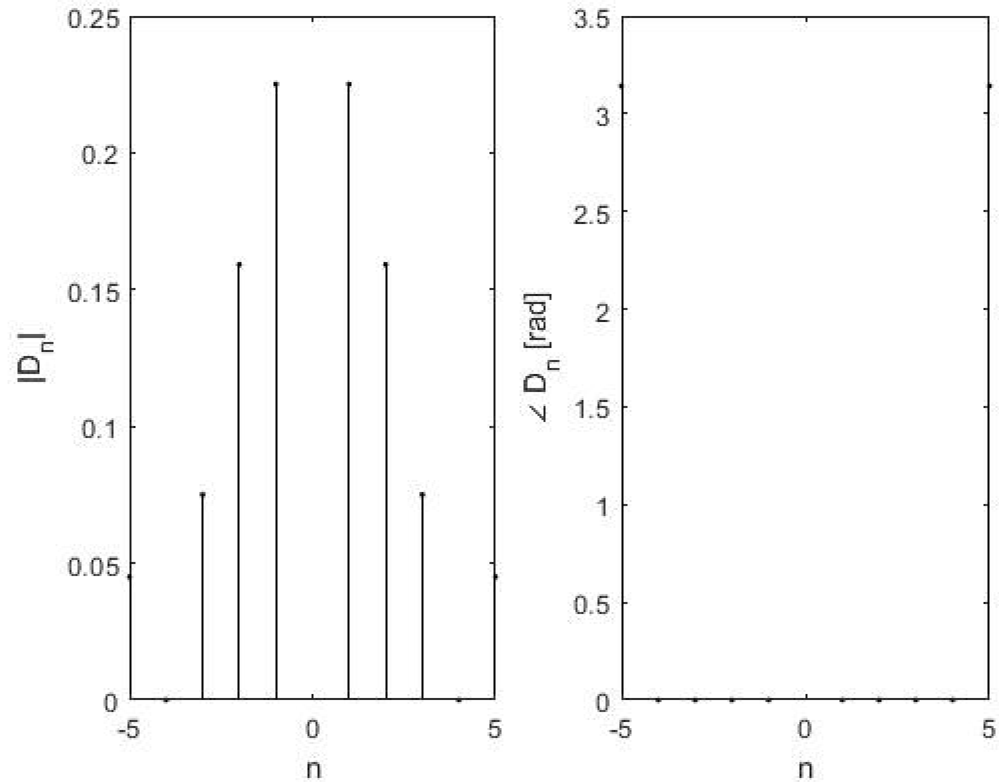
x2(t):



Matlab

```
----------------------------
clf;
n = (-5:5);
D_n = (1./(n.*pi).*sin((n.*pi)./2));
subplot(1,2,1); stem(n,abs(D_n),'.k');
xlabel('n'); ylabel('|D_n|');
subplot(1,2,2); stem(n,angle(D_n),'.k');
xlabel('n'); ylabel('\angle D_n [rad]');
----------------------------
```

x3(t):



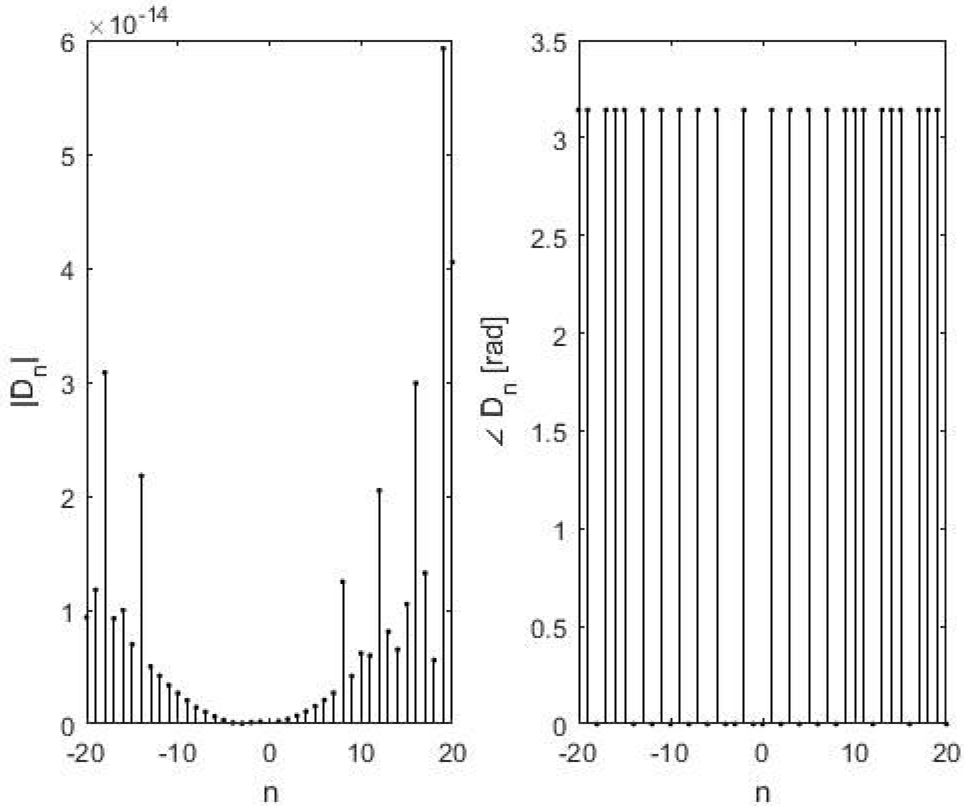
Matlab:

```
clf;
n = (-5:5);
D_n = (1./(n.*pi).*sin((n.*pi)./4));
subplot(1,2,1); stem(n,abs(D_n),'.k');
xlabel('n'); ylabel('|D_n|');
subplot(1,2,2); stem(n,angle(D_n),'.k');
xlabel('n'); ylabel('\angle D_n [rad]');
```

----

b)

x1(t):

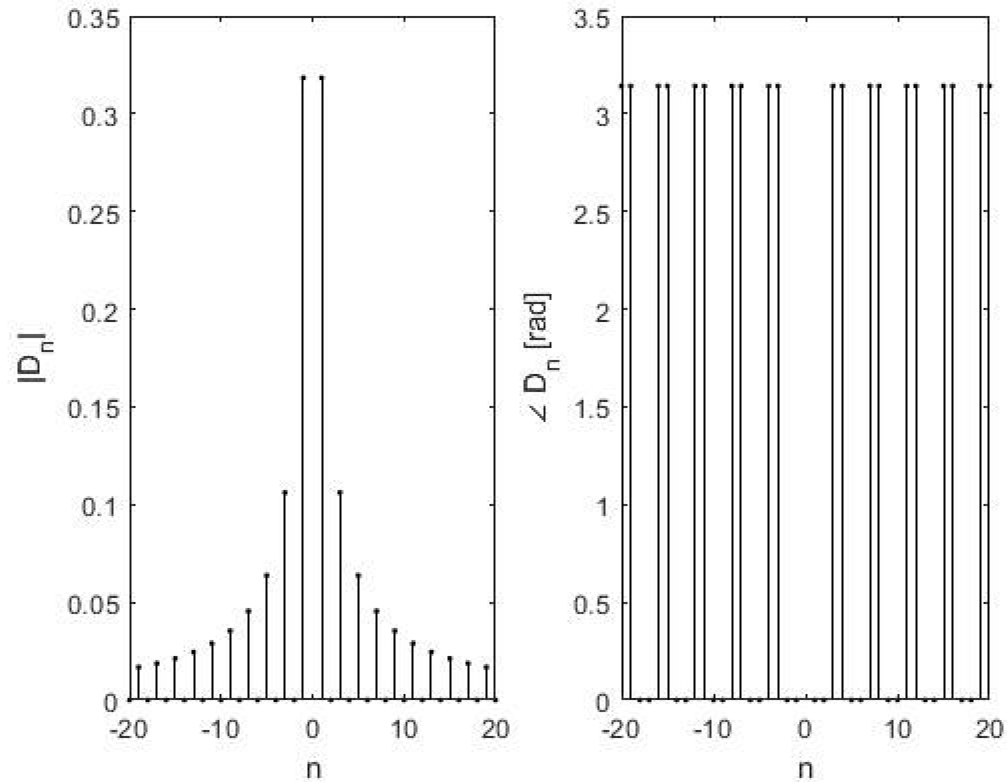


Matlab:

```
clf;
n = (-20:20);
D_n = 1./2.*((1./(pi.*n)).*sin((3-n).*pi )) + (1./pi.*n).*sin((3+n).*pi) + (1./(2.*n.*pi).*sin((1+n).*pi)) +
(1./(2.*n.*pi).*sin((1-n).*pi)) ;

subplot(1,2,1); stem(n,abs(D_n),'.k');
xlabel('n'); ylabel('|D_n|');
subplot(1,2,2); stem(n,angle(D_n),'.k');
xlabel('n'); ylabel('\angle D_n [rad]');
```

----

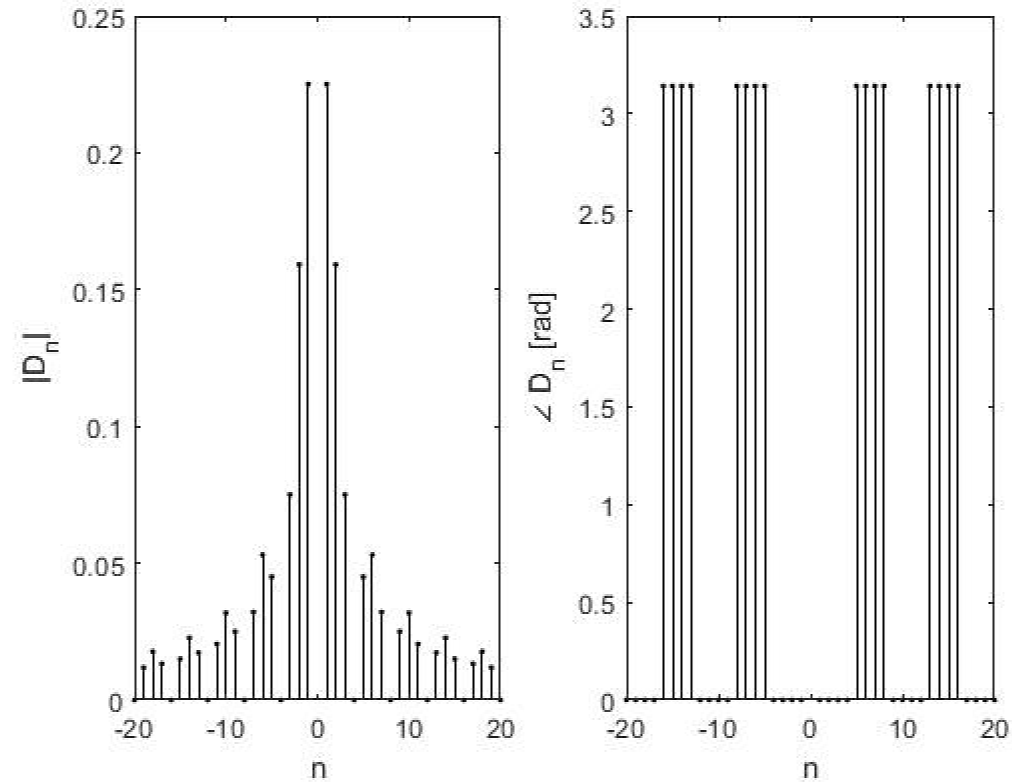
x2(t):



Matlab:

```
-----------------
clf;
n = (-20:20);
D_n = (1./(n.*pi).*sin((n.*pi)./2));

subplot(1,2,1); stem(n,abs(D_n),'.k');
xlabel('n'); ylabel('|D_n|');
subplot(1,2,2); stem(n,angle(D_n),'.k');
xlabel('n'); ylabel('\angle D_n [rad]');
-----------------
```

x3(t):

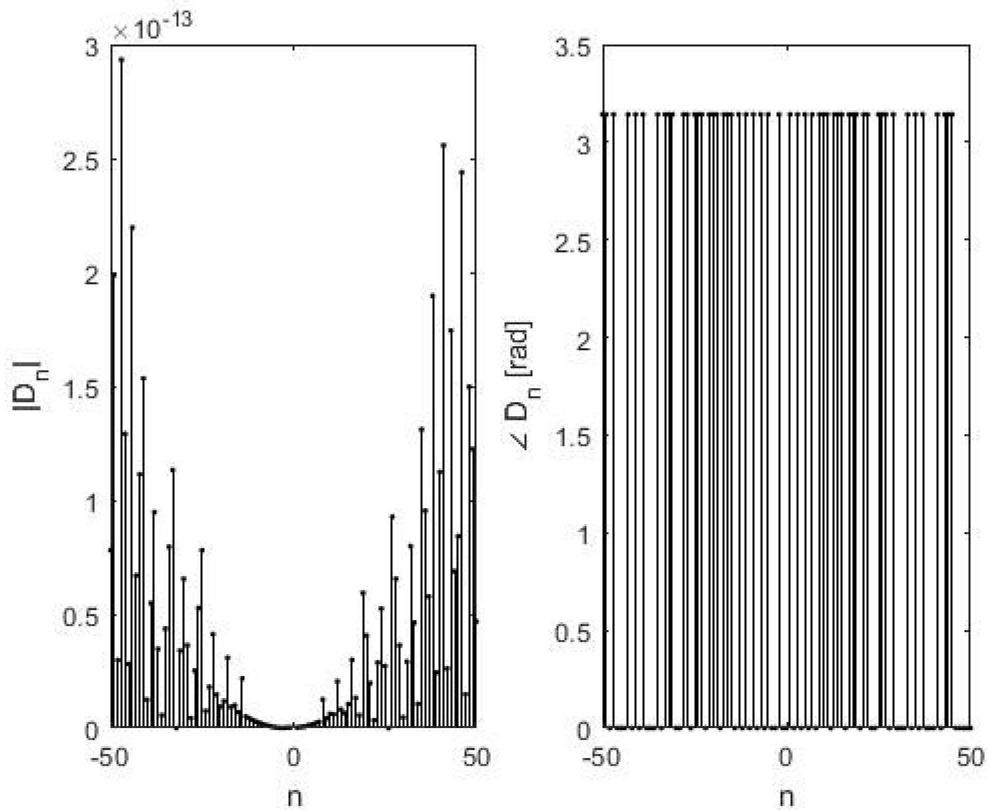


Matlab

```
------------
clf;
n = (-20:20);
D_n =  (1./(n.*pi).*sin((n.*pi)./4));

subplot(1,2,1); stem(n,abs(D_n),'.k');
xlabel('n'); ylabel('|D_n|');
subplot(1,2,2); stem(n,angle(D_n),'.k');
xlabel('n'); ylabel('\angle D_n [rad]');
----------
```

c)

x1(t):

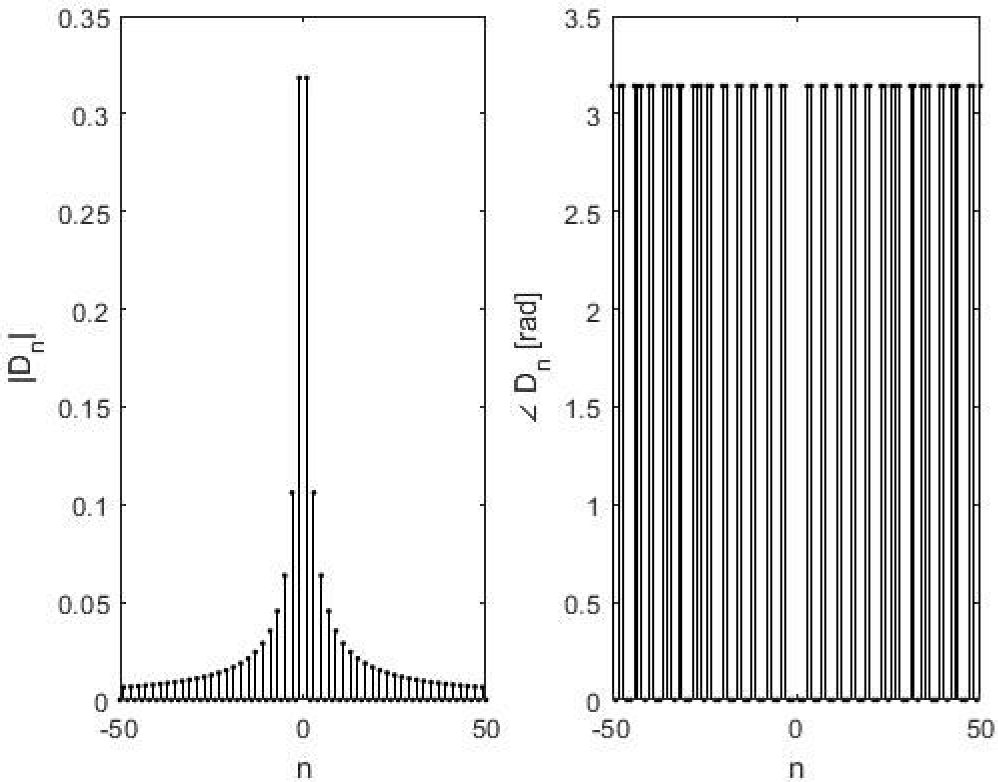


Matlab

```
clf;
n = (-50:50);
D_n = 1./2.*((1./(pi.*n)).*sin((3-n).*pi )) + (1./pi.*n).*sin((3+n).*pi) + (1./(2.*n.*pi).*sin((1+n).*pi)) +
(1./(2.*n.*pi).*sin((1-n).*pi)) ;

subplot(1,2,1); stem(n,abs(D_n),'.k');
xlabel('n'); ylabel('|D_n|');
subplot(1,2,2); stem(n,angle(D_n),'.k');
xlabel('n'); ylabel('\angle D_n [rad]');
```

----

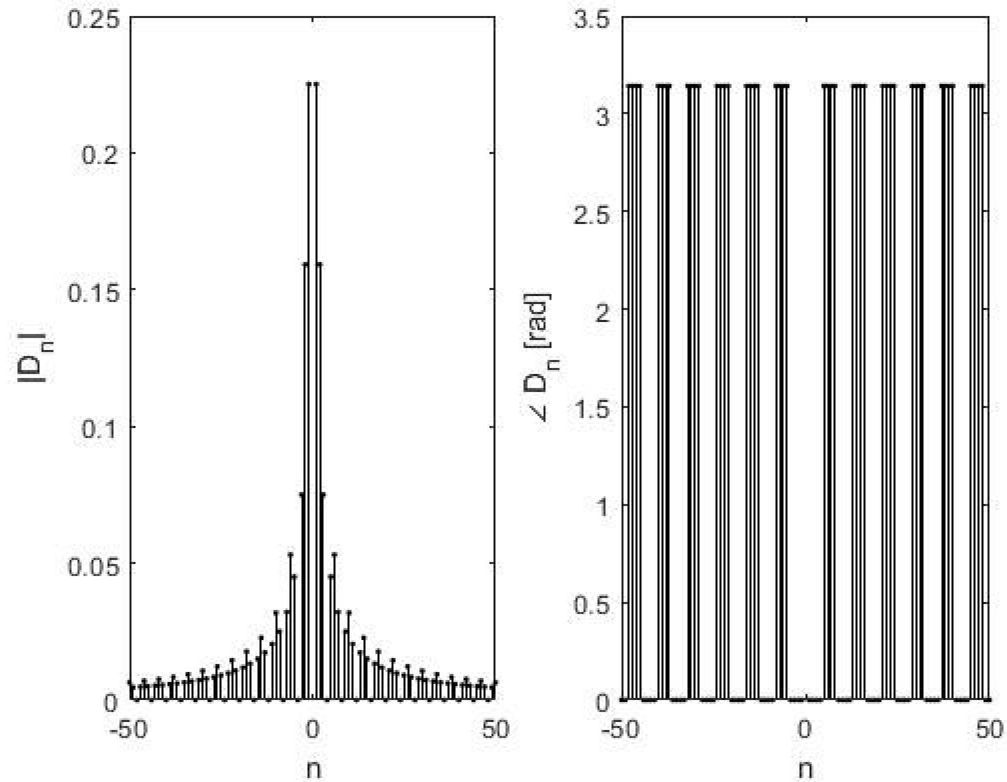
x2(t):



Matlab

```
---
clf;
n = (-50:50);
D_n = (1./(n.*pi).*sin((n.*pi)./2));

subplot(1,2,1); stem(n,abs(D_n),'.k');
xlabel('n'); ylabel('|D_n|');
subplot(1,2,2); stem(n,angle(D_n),'.k');
xlabel('n'); ylabel('\angle D_n [rad]');
---
```

x3(t):

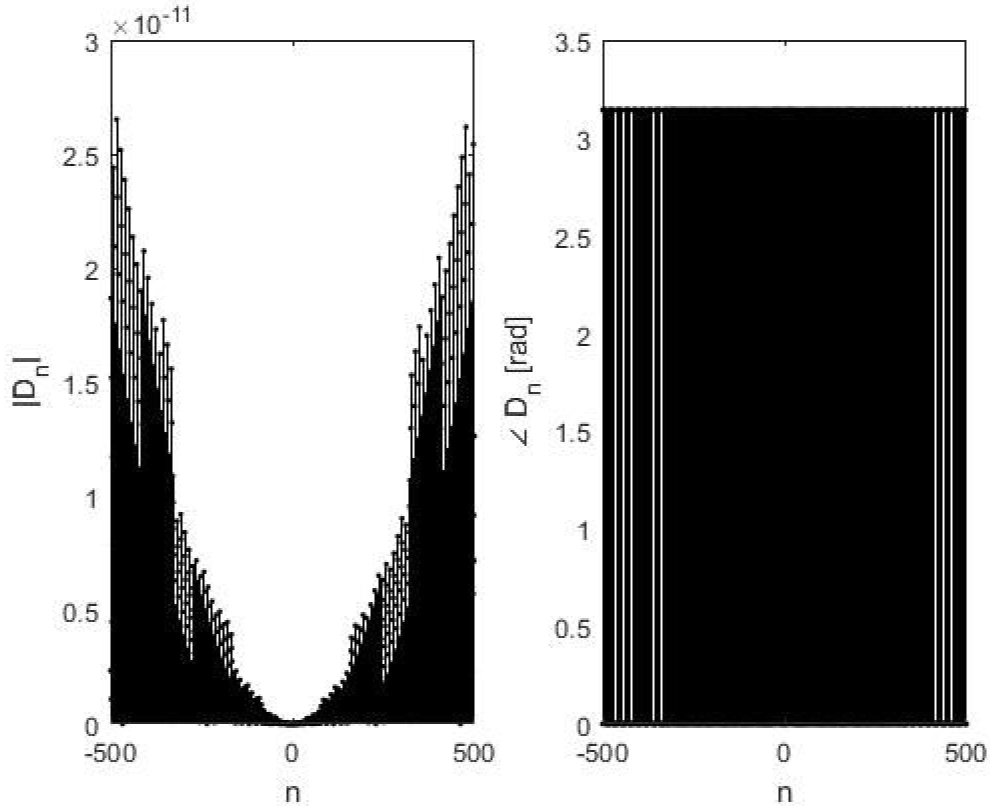


Matlab:

```
----
clf;
n = (-50:50);
D_n = (1./(n.*pi).*sin((n.*pi)./4));

subplot(1,2,1); stem(n,abs(D_n),'.k');
xlabel('n'); ylabel('|D_n|');
subplot(1,2,2); stem(n,angle(D_n),'.k');
xlabel('n'); ylabel('\angle D_n [rad]');
----
```

d)

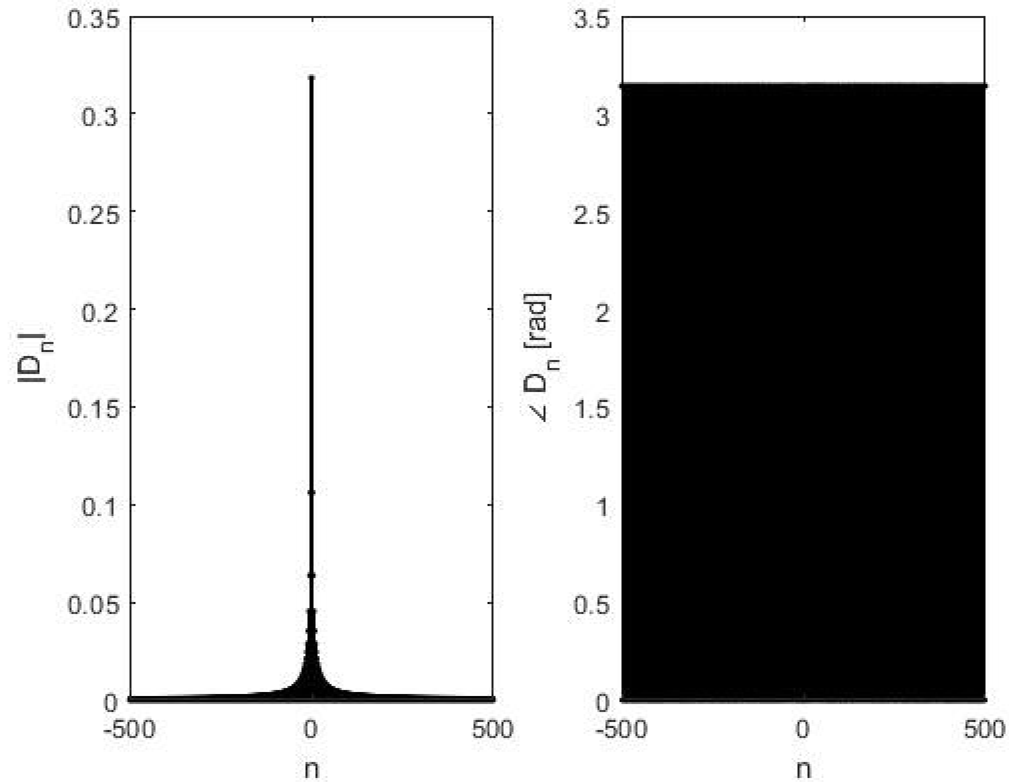
x1(t)



Matlab:

```
---
clf;
n = (-500:500);
D_n = 1./2.*((1./(pi.*n)).*sin((3-n).*pi )) + (1./pi.*n).*sin((3+n).*pi) + (1./(2.*n.*pi).*sin((1+n).*pi)) +
(1./(2.*n.*pi).*sin((1-n).*pi)) ;


subplot(1,2,1); stem(n,abs(D_n),'.k');
xlabel('n'); ylabel('|D_n|');
subplot(1,2,2); stem(n,angle(D_n),'.k');
xlabel('n'); ylabel('\angle D_n [rad]');
---
```

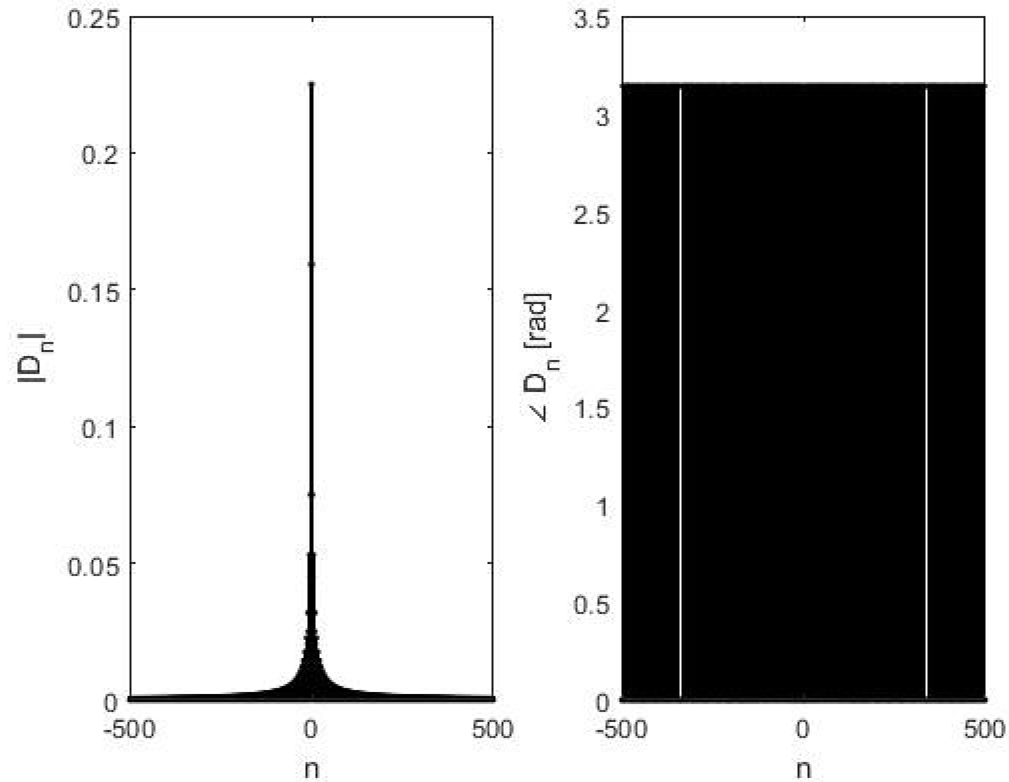
x2(t):



Matlab:

```
--
clf;
n = (-500:500);
D_n = (1./(n.*pi).*sin((n.*pi)./2));


subplot(1,2,1); stem(n,abs(D_n),'.k');
xlabel('n'); ylabel('|D_n|');
subplot(1,2,2); stem(n,angle(D_n),'.k');
xlabel('n'); ylabel('\angle D_n [rad]');
---
```

x3(t):



Matlab:

```
---
clf;
n = (-500:500);
D_n = (1./(n.*pi).*sin((n.*pi)./4));


subplot(1,2,1); stem(n,abs(D_n),'.k');
xlabel('n'); ylabel('|D_n|');
subplot(1,2,2); stem(n,angle(D_n),'.k');
xlabel('n'); ylabel('\angle D_n [rad]');
----
```

A.5:

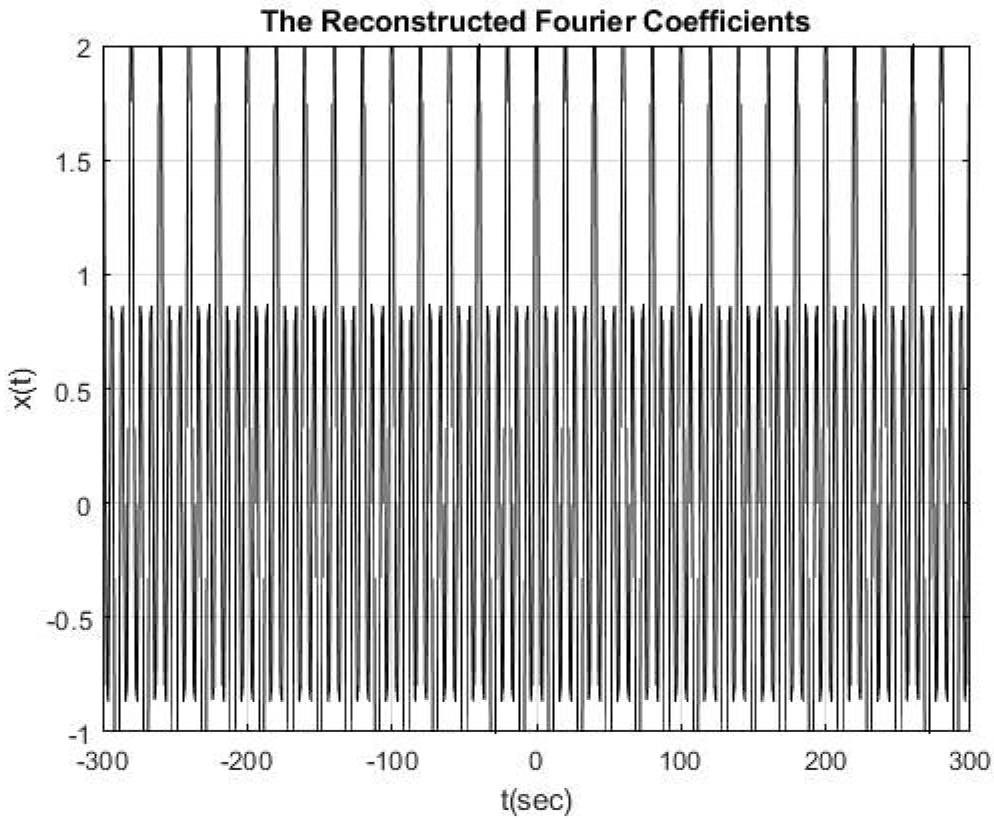
Matlab:

```
-----
function [D] = a5(Dn)
n=-500:500;
D=Dn;
```

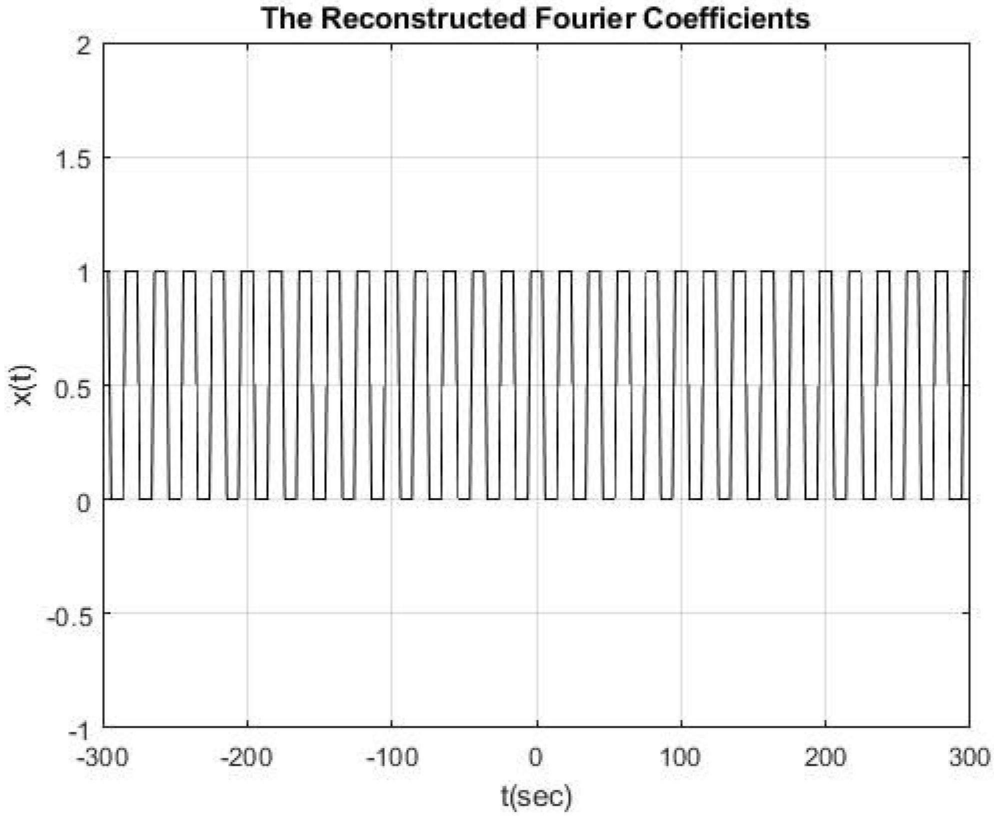
```
t=[-300:1:300];
w=pi*0.1;
x=zeros(size(t));
for i = 1:length(n)
 x=x+D(i)*exp(j*n(i)*w*t);
 't'
end

figure(5);
plot(t,x,'k')
xlabel('t(sec)');
ylabel('x(t)');
axis([-300 300 -1 2]);
title('The Reconstructed Fourier Coefficients');
grid;
-----
```

A.6:

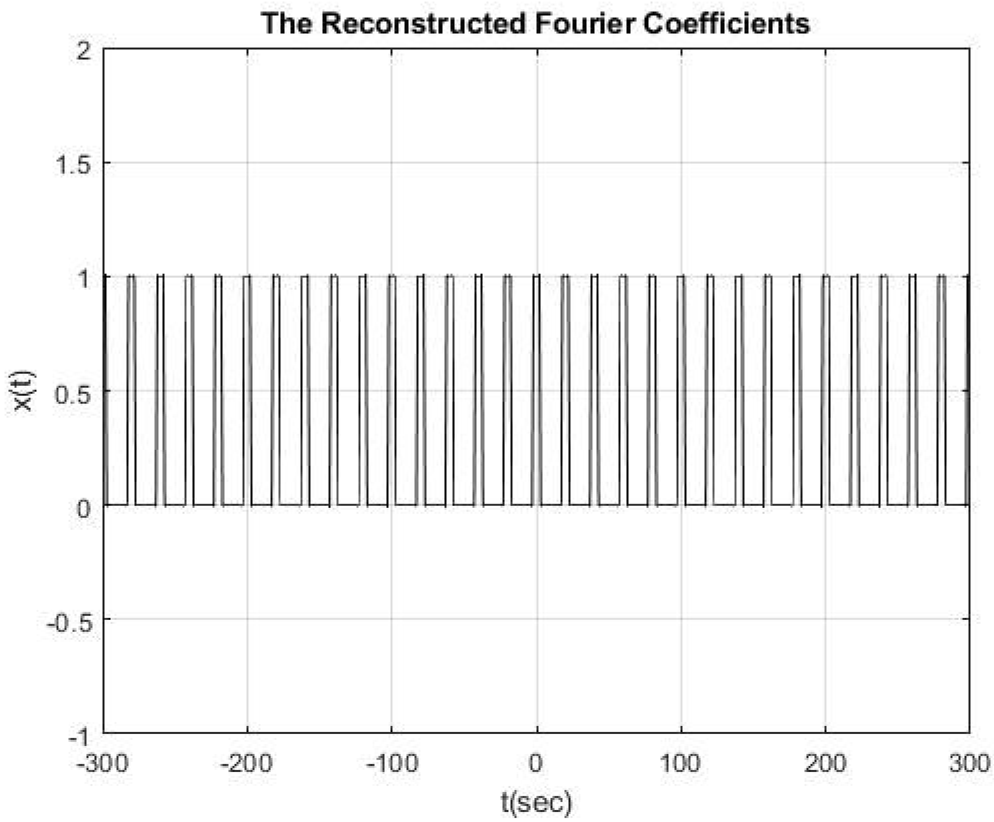
x1(t):


The Reconstructed Fourier Coefficients
2
1.5
1
0.5
0
-0.5
-1
x(t)
-300
-200
-100
0
100
200
300
t(sec)

x2(t):


The Reconstructed Fourier Coefficients
2
1.5
1
0.5
0
-0.5
-1
x(t)
-300
-200
-100
0
100
200
300
t(sec)

x3(t):



The Reconstructed Fourier Coefficients
x(t)
t(sec)
2
1.5
1
0.5
0
-0.5
-1
-300
-200
-100
0
100
200
300

B:

B.1)

$x_1(t) = \cos\left(\frac{3\pi}{10}\right)t + \frac{1}{2}\cos\left(\frac{\pi}{10}\right)t,$

$$\omega_{o1} = \frac{3\pi}{10},\ \omega_{o2} = \frac{\pi}{10}$$

$$\omega_o = \frac{G.C.F\ of\ numerator}{L.C.M\ of\ denominator} = \frac{\pi}{10} = 0.314\ rad/s$$

For $x_2(t) \rightarrow T_o = 20\ s$

$$\omega_o = \frac{\pi}{10} = 0.314\ rad/s$$

For $x_3(t) \rightarrow T_o = 40\ s$

$$\omega_o = \frac{\pi}{20} = 0.157\ rad/s$$

B.2) The main differences between the fourier coefficients of x1(t) and x2(t) is that one consists of sinc and the other consists of sin functions respectively. Furthermore, x1(t) has four distinct fourier series coefficients, while x2(t) has infinite fourier coefficients for Dn.

B.3)Signal x3(t) has a smaller fundamental frequency value compared to signal x2(t) for it's Fourier coefficients.

B.4)Do = 0.5 for signal x4(t), derived from x2(t) .

B.5) Since x1(t) has a finite number of Dn values, nothing will change if the Fourier coefficients are increased. However, for x2(t) and x3(t), increasing values of Dn results in higher accuracy.

B.6) Again, since x1(t) has a finite number of Dn values, we would only need four Fourier series coefficients, in this case, to perfectly reconstruct. However, for x2(t) and x3(t), we would need an infinite number of Dn for perfect reconstruction.

B.7) Since a periodic signal has an infinite number of Dn values, it is not viable. However, if it is finite like x1(t), then the values of Dn can be stored. However, this is not recommended for signals which have a large amount of finite Dn values as they would tend to waste space.