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Instructor:	Dimitri Androutsos
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<i>Assignment/Lab Number:</i>	3
<i>Assignment/Lab Title:</i>	Fourier Series Analysis Using MATLAB

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Lab 3: Fourier Series Analysis Using MATLAB

By Syed Yousuf and Khaled Hashem

A:

A.1:

$$\begin{aligned} \text{A.1: } x_1(t) &= \cos \frac{3\pi}{10} t + \frac{1}{2} \cos \frac{\pi}{10} t \\ &= \frac{1}{2} e^{j\frac{3\pi}{10} t} + \frac{1}{2} e^{-j\frac{3\pi}{10} t} + \frac{1}{2} \left(\frac{1}{2} e^{j\frac{\pi}{10} t} + \frac{1}{2} e^{-j\frac{\pi}{10} t} \right) \\ &= \frac{1}{2} e^{j\frac{3\pi}{10} t} + \frac{1}{2} e^{-j\frac{3\pi}{10} t} + \frac{1}{4} e^{j\frac{\pi}{10} t} + \frac{1}{4} e^{-j\frac{\pi}{10} t} \end{aligned}$$

Fundamental frequency:

$$\frac{3\pi}{10} \cdot \frac{10}{\pi} = 3$$

$$\omega_1 = \frac{3\pi}{10} \quad \omega_2 = \frac{\pi}{10}$$

$$T_0 = \frac{2\pi}{(\frac{\pi}{10})} = 2\pi \cdot \frac{10}{\pi} = 20$$

$$\frac{\text{GCF}}{\text{LCM}} = \frac{\pi}{10} = \pi/10$$

$$jn \frac{\pi}{10} t = j \frac{3\pi}{10} t \quad n=3$$

$$n=-3$$

$$jn \frac{\pi}{10} t = j \frac{\pi}{10} t \quad n=1$$

$$n=-1$$

$$\left\{ D_3 = \frac{1}{2} \quad D_{-3} = \frac{1}{2} \quad D_1 = \frac{1}{4} \quad D_{-1} = \frac{1}{4} \right\}$$

$$D_n = \frac{1}{20} \int_{-10}^{10} \left[\frac{1}{2} e^{j\frac{3\pi}{10} t} + \frac{1}{2} e^{-j\frac{3\pi}{10} t} + \frac{1}{4} e^{j\frac{\pi}{10} t} + \frac{1}{4} e^{-j\frac{\pi}{10} t} \right] e^{-jn\frac{\pi}{10} t} dt$$

$$D_n = \frac{1}{20} \left[\frac{e^{j(3-n)\pi} - e^{-j(3-n)\pi}}{2j(3-n)\frac{\pi}{10}} + \frac{e^{j(3+n)\pi} - e^{-j(3+n)\pi}}{2j(3+n)\frac{\pi}{10}} + \frac{e^{j(1+n)\pi} - e^{-j(1+n)\pi}}{4j(1+n)\frac{\pi}{10}} + \frac{e^{j(1-n)\pi} - e^{-j(1-n)\pi}}{4j(1-n)\frac{\pi}{10}} \right]$$

$$\therefore D_n = \frac{1}{2} [\text{sinc}[(3-n)\pi] + \text{sinc}[(3+n)\pi] + \frac{1}{2} \text{sinc}[(1+n)\pi] + \frac{1}{2} \text{sinc}[(1-n)\pi]]$$

A.2:

A.2: $x_2(t) : T_0 = 20 \quad \omega_0 = \frac{2\pi}{20} = \frac{\pi}{10}$

$$D_n = \frac{1}{20} \left[\int_{-5}^5 (1) e^{-jn\pi/10 t} dt \right] = \frac{1}{20} \left[\frac{1}{-jn\pi/10} e^{-jn\pi/10 t} \right]_{-5}^5$$

$$D_n = \frac{1}{20} \left[\frac{-10}{jn\pi} e^{-jn\pi/2} + \frac{10}{jn\pi} e^{jn\pi/2} \right] = \frac{1}{n\pi} \sin\left(\frac{n\pi}{2}\right)$$

$$\therefore D_n = \frac{1}{n\pi} \sin\left(\frac{n\pi}{2}\right)$$

$x_3(t) : T_0 = 40 \quad \omega_0 = \frac{2\pi}{40} = \frac{\pi}{20} \quad D_n = \frac{1}{40} \int_{-5}^5 (1) e^{-jn\pi/20 t} dt$

$$D_n = \frac{1}{40} \left[\frac{1}{-jn\pi/20} e^{-jn\pi/20 t} \right]_{-5}^5 \quad \leftarrow D_n = \frac{1}{40} \left[\frac{-20}{jn\pi} e^{-jn\pi/4} + \frac{20}{jn\pi} e^{jn\pi/4} \right]$$

$$\therefore D_n = \frac{1}{n\pi} \sin\left(\frac{n\pi}{4}\right)$$

A.3:

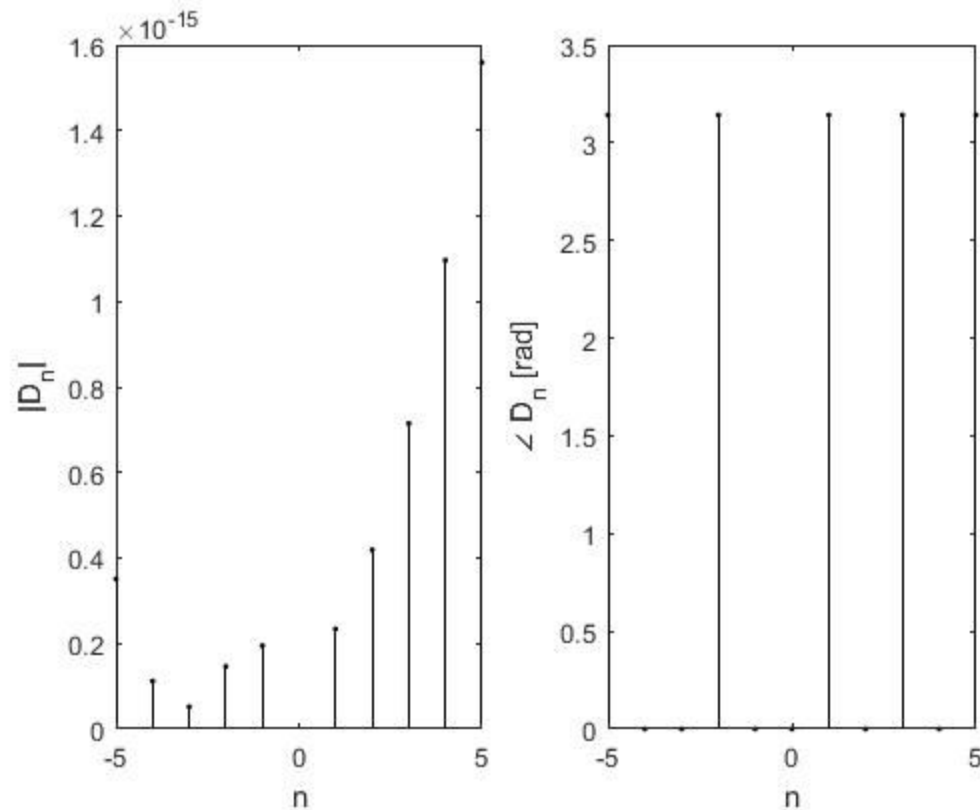
Matlab code:

```
function [D]=Dn(d,n)
D1 = [0.5,0,-0.5*1i,0,0.5*1i,0,0.5];
D2 = (1/(n.*pi)*sin((n*pi)/2));
D3 = (1/(n.*pi)*sin((n*pi)/4));
if (d==1)
    D=D1;end
if (d==2)
    D=D2;end
if (d==3)
    D=D3;end
end
```

A.4:

a)

$x_1(t)$:



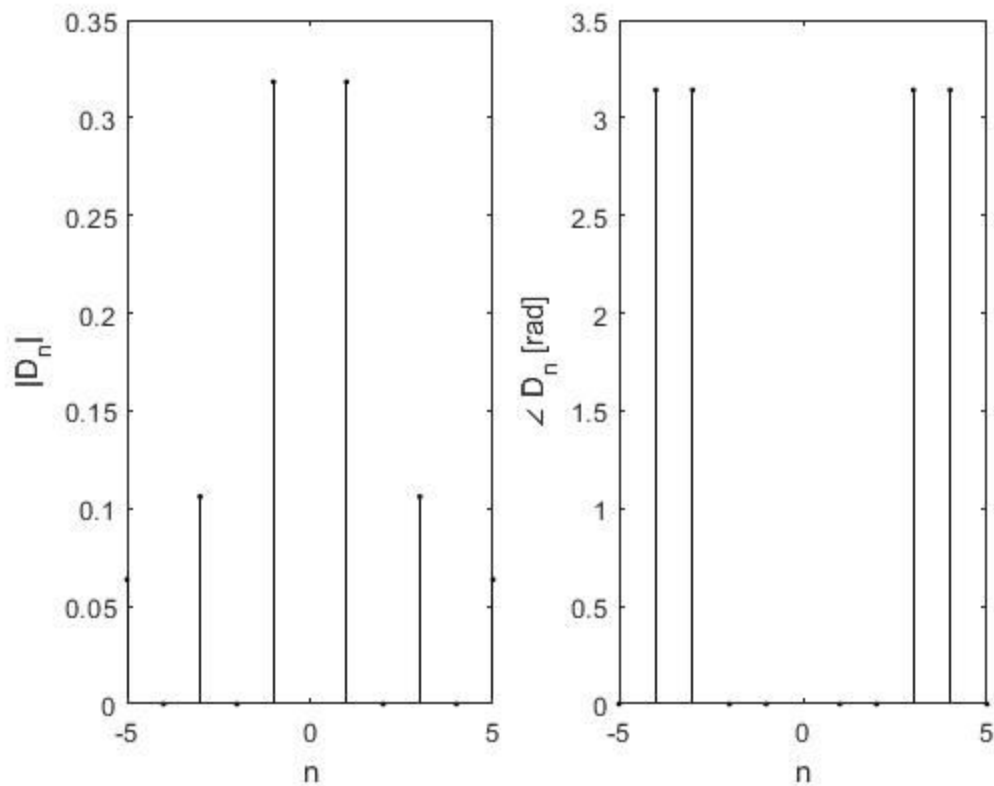
Matlab:

```

-----
clf;
n = (-5:5);
D_n = 1./2.*((1./(pi.*n)).*sin((3-n).*pi)) + (1./pi.*n).*sin((3+n).*pi) + (1./(2.*n.*pi)).*sin((1+n).*pi) +
(1./(2.*n.*pi)).*sin((1-n).*pi);
subplot(1,2,1); stem(n,abs(D_n),'k');
xlabel('n'); ylabel('|D_n|');
subplot(1,2,2); stem(n,angle(D_n),'k');
xlabel('n'); ylabel('\angle D_n [rad]');
-----

```

$x_2(t)$:



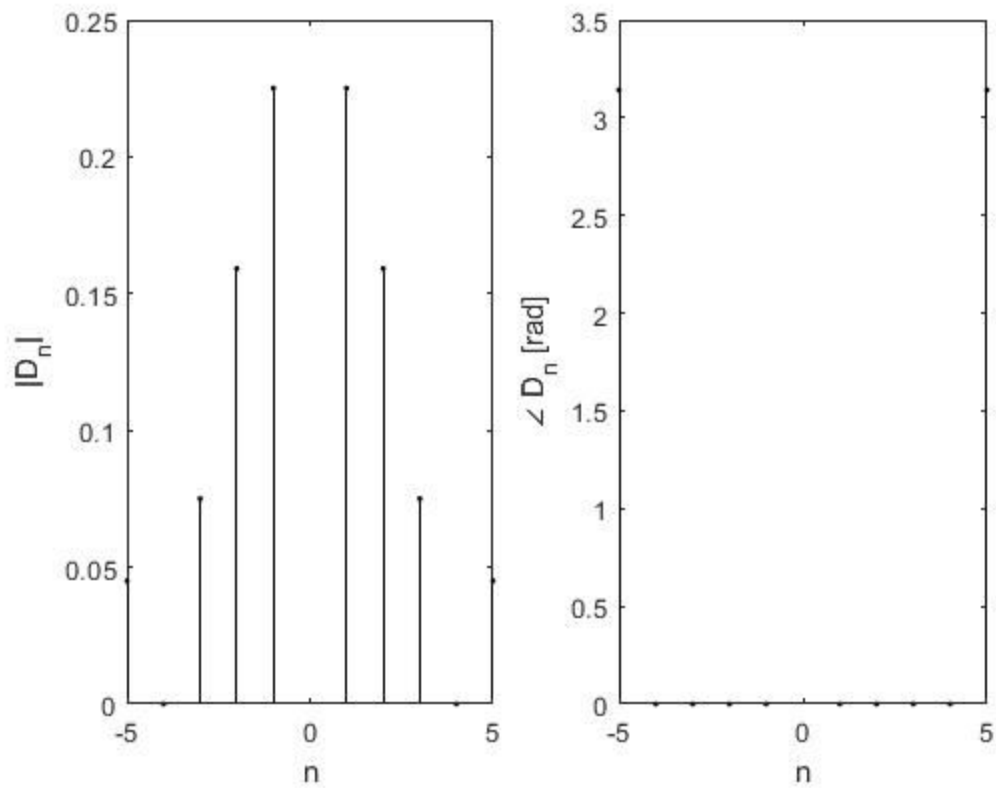
Matlab

```

-----
clf;
n = (-5:5);
D_n = (1./(n.*pi)).*sin((n.*pi)./2);
subplot(1,2,1); stem(n,abs(D_n),'.k');
xlabel('n'); ylabel('|D_n|');
subplot(1,2,2); stem(n,angle(D_n),'.k');
xlabel('n'); ylabel('\angle D_n [rad]');
-----

```

$x_3(t)$:

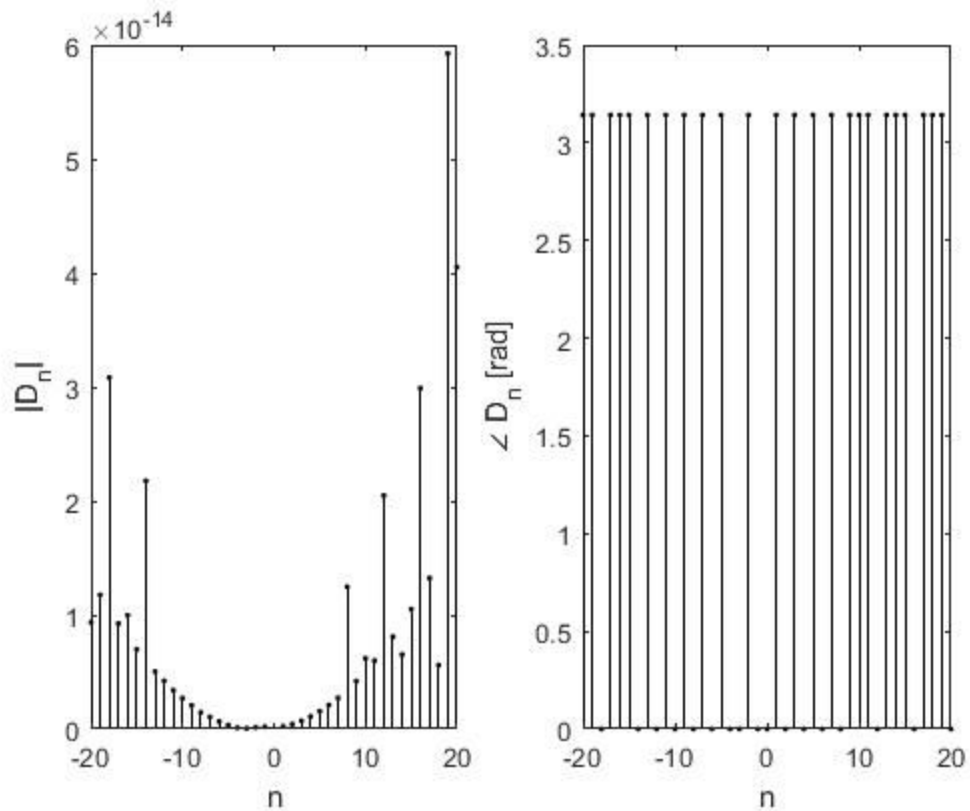


Matlab:

```
clf;
n = (-5:5);
D_n = (1./(n.*pi)).*sin((n.*pi)./4);
subplot(1,2,1); stem(n,abs(D_n),'k');
xlabel('n'); ylabel('|D_n|');
subplot(1,2,2); stem(n,angle(D_n),'k');
xlabel('n'); ylabel('\angle D_n [rad]');
```

b)

$x_1(t)$:



Matlab:

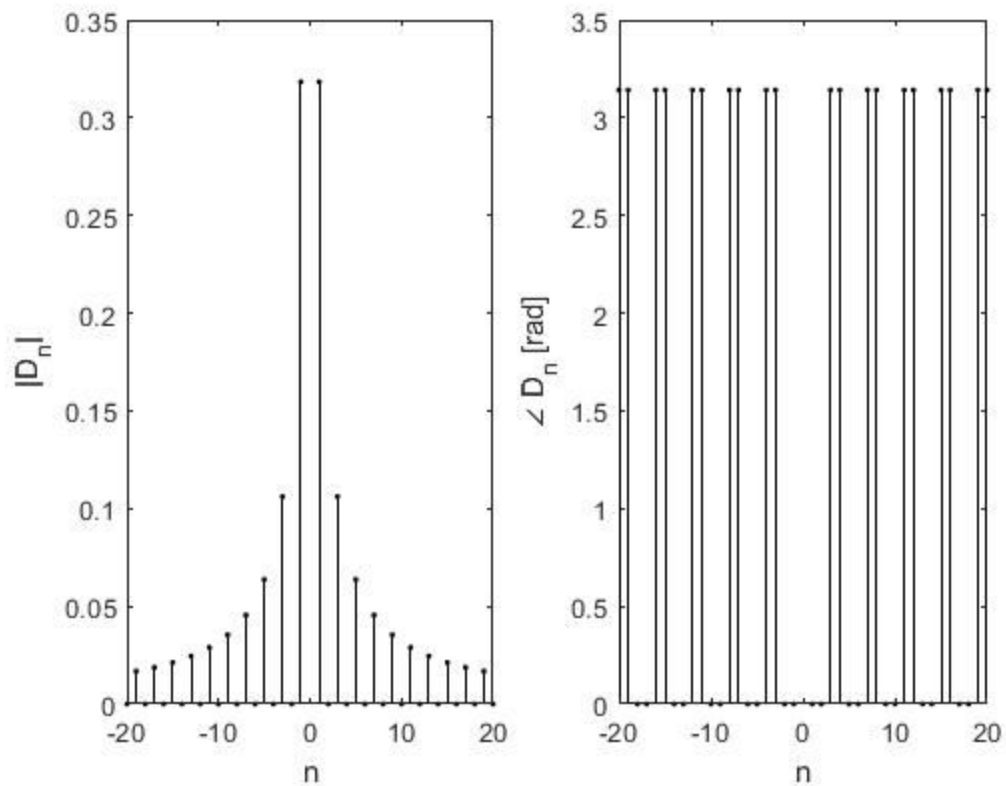
```

-----
clf;
n = (-20:20);
D_n = 1./2.*((1./(pi.*n)).*sin((3-n).*pi )) + (1./pi.*n).*sin((3+n).*pi) + (1./(2.*n.*pi)).*sin((1+n).*pi)) +
(1./(2.*n.*pi)).*sin((1-n).*pi)) ;

subplot(1,2,1); stem(n,abs(D_n),'.k');
xlabel('n'); ylabel('|D_n|');
subplot(1,2,2); stem(n,angle(D_n),'.k');
xlabel('n'); ylabel('\angle D_n [rad]');
-----

```

x2(t):



Matlab:

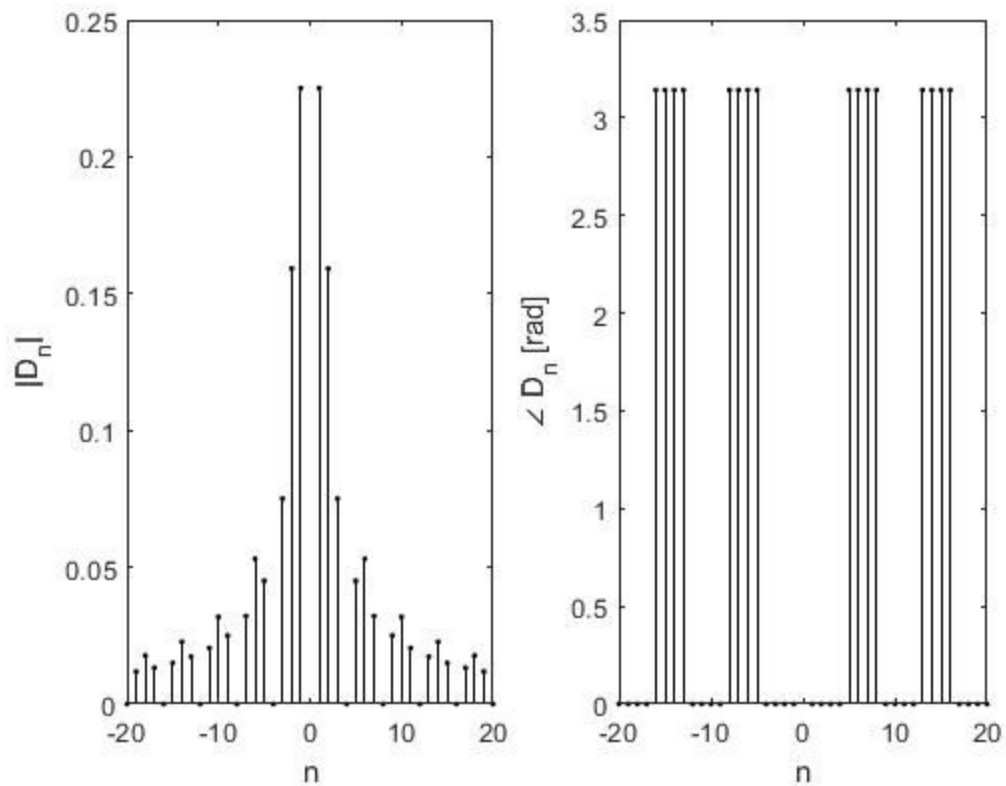
```

-----
clf;
n = (-20:20);
D_n = (1./(n.*pi).*sin((n.*pi)./2));

subplot(1,2,1); stem(n,abs(D_n),'.k');
xlabel('n'); ylabel('|D_n|');
subplot(1,2,2); stem(n,angle(D_n),'.k');
xlabel('n'); ylabel('\angle D_n [rad]');
-----

```

$x_3(t)$:



Matlab

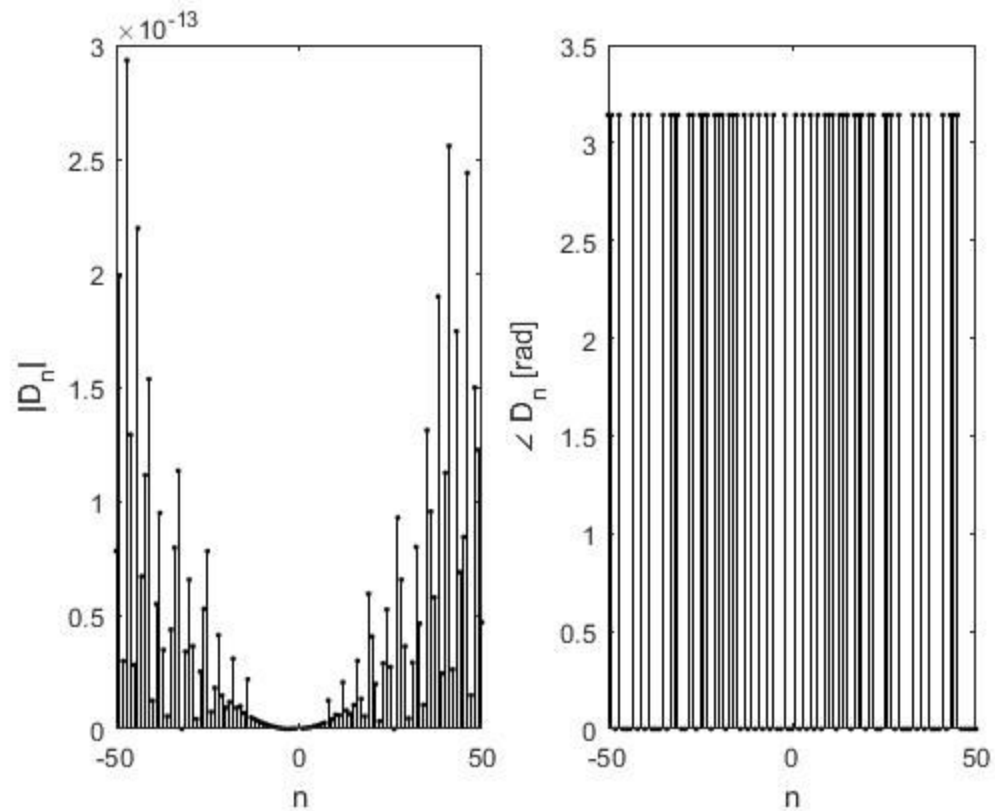
```

-----
clf;
n = (-20:20);
D_n = (1./(n.*pi)).*sin((n.*pi)./4);

subplot(1,2,1); stem(n,abs(D_n),'.k');
xlabel('n'); ylabel('|D_n|');
subplot(1,2,2); stem(n,angle(D_n),'.k');
xlabel('n'); ylabel('\angle D_n [rad]');
-----

```

c)
x1(t):

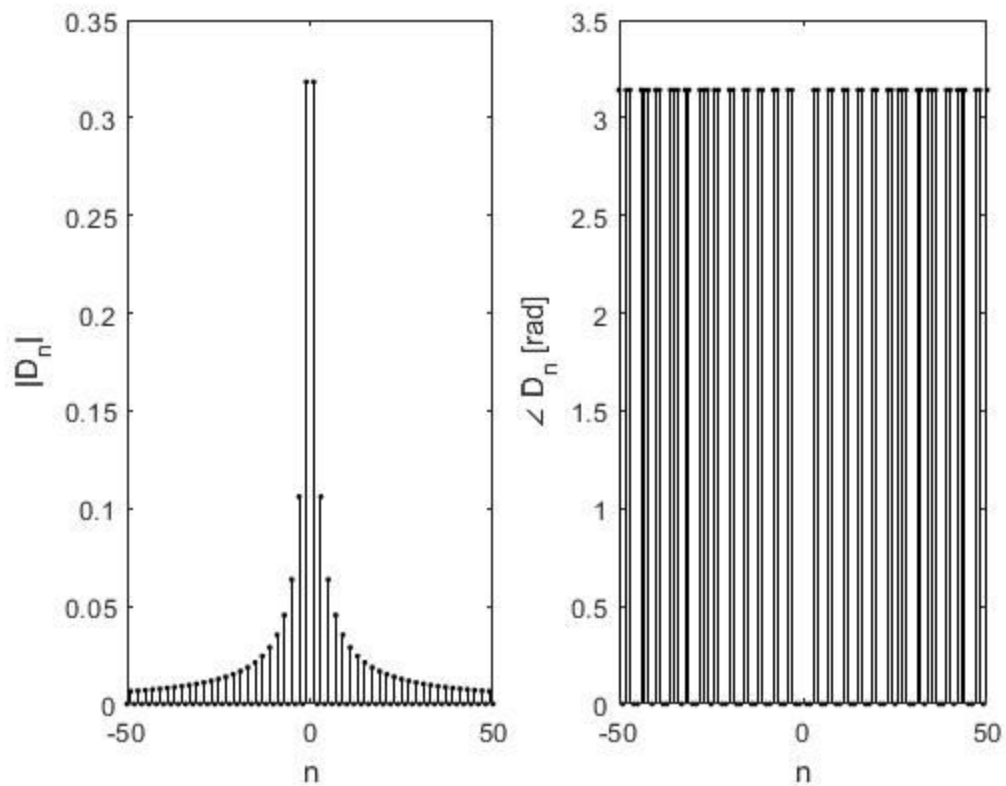


Matlab

clf;
n = (-50:50);
 $D_n = 1./2.*((1./(pi.*n)).*sin((3-n).*pi)) + (1./pi.*n).*sin((3+n).*pi) + (1./(2.*n.*pi)).*sin((1+n).*pi)) + (1./(2.*n.*pi)).*sin((1-n).*pi))$;

subplot(1,2,1); stem(n,abs(D_n),'k');
xlabel('n'); ylabel('|D_n|');
subplot(1,2,2); stem(n,angle(D_n),'k');
xlabel('n'); ylabel('\angle D_n [rad]');

x2(t):



Matlab

clf;

n = (-50:50);

D_n = (1./(n.*pi)).*sin((n.*pi)./2));

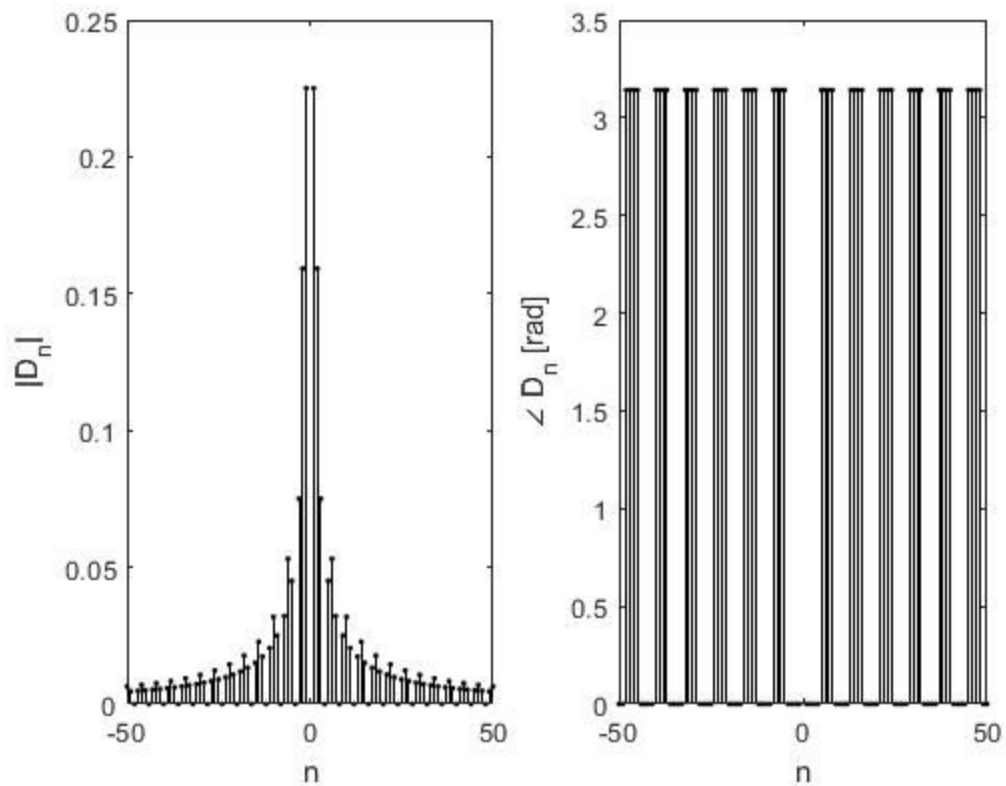
subplot(1,2,1); stem(n,abs(D_n),'.k');

xlabel('n'); ylabel('|D_n|');

subplot(1,2,2); stem(n,angle(D_n),'.k');

xlabel('n'); ylabel('\angle D_n [rad]');

x3(t):

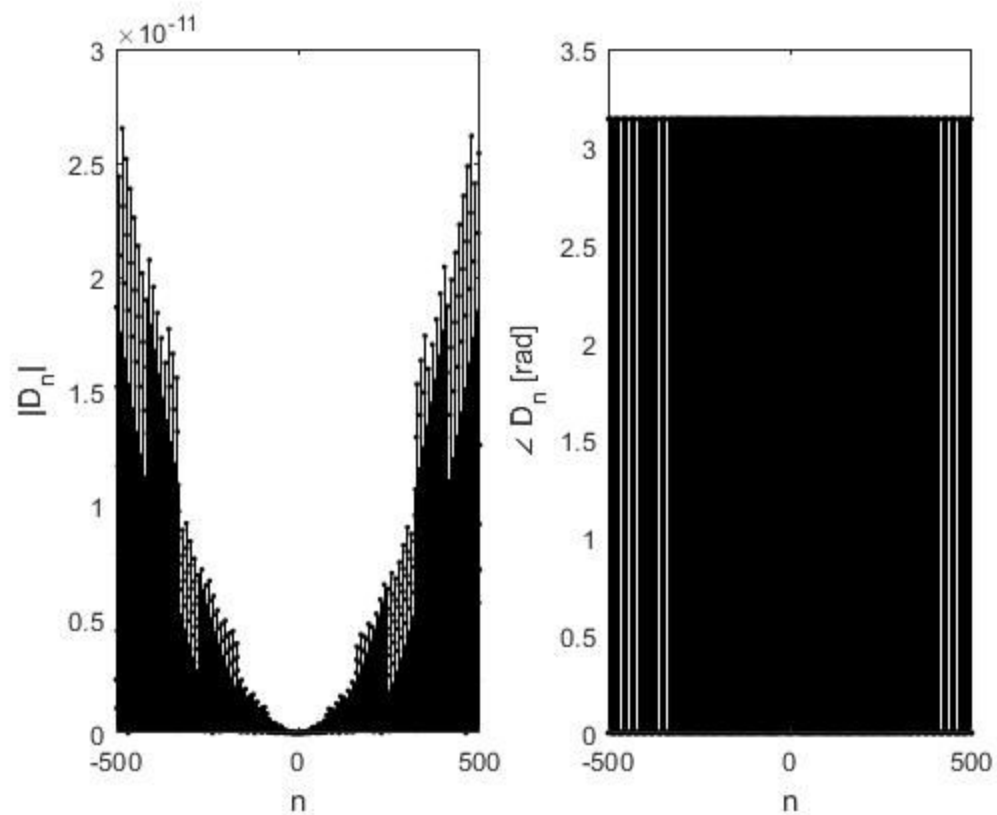


Matlab:

```
clf;
n = (-50:50);
D_n = (1./(n.*pi)).*sin((n.*pi)./4);
```

```
subplot(1,2,1); stem(n,abs(D_n),'.k');
xlabel('n'); ylabel('|D_n|');
subplot(1,2,2); stem(n,angle(D_n),'.k');
xlabel('n'); ylabel('\angle D_n [rad]');
```

d)
x1(t)



Matlab:

clf;

n = (-500:500);

D_n = 1./2.*((1./(pi.*n)).*sin((3-n).*pi)) + (1./pi.*n).*sin((3+n).*pi) + (1./(2.*n.*pi)).*sin((1+n).*pi)) + (1./(2.*n.*pi)).*sin((1-n).*pi)) ;

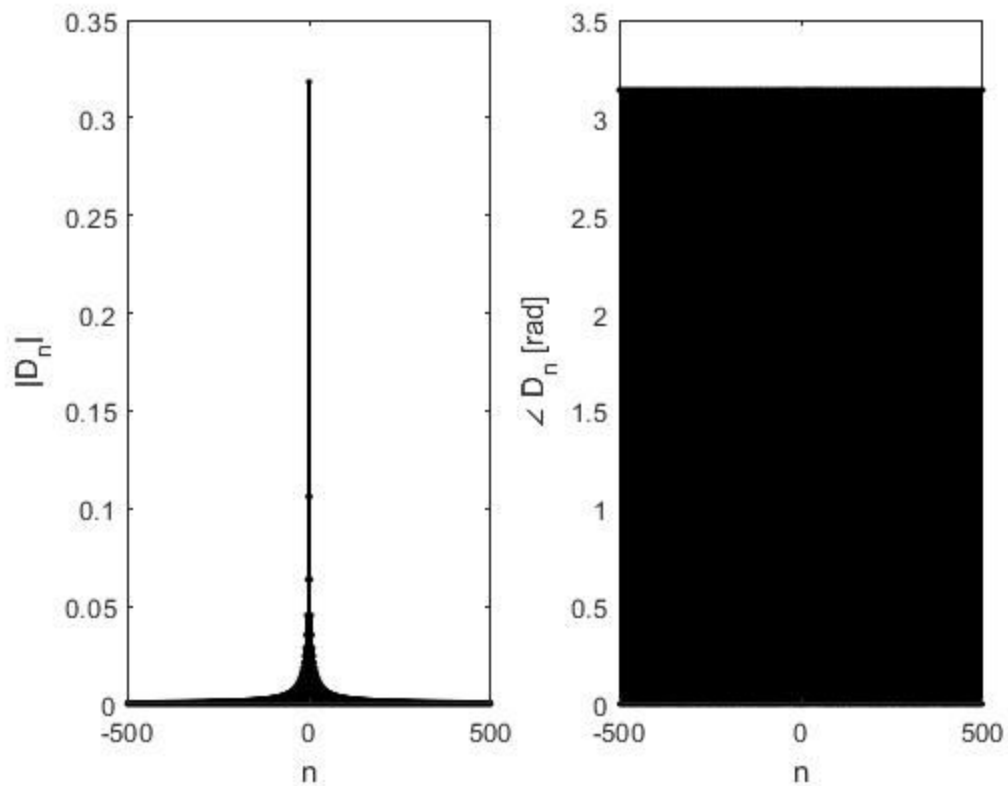
subplot(1,2,1); stem(n,abs(D_n),'.k');

xlabel('n'); ylabel('|D_n|');

subplot(1,2,2); stem(n,angle(D_n),'.k');

xlabel('n'); ylabel('\angle D_n [rad]');

x2(t):



Matlab:

--

clf;

$n = (-500:500);$

$D_n = (1./(n.*pi).*\sin((n.*pi)./2));$

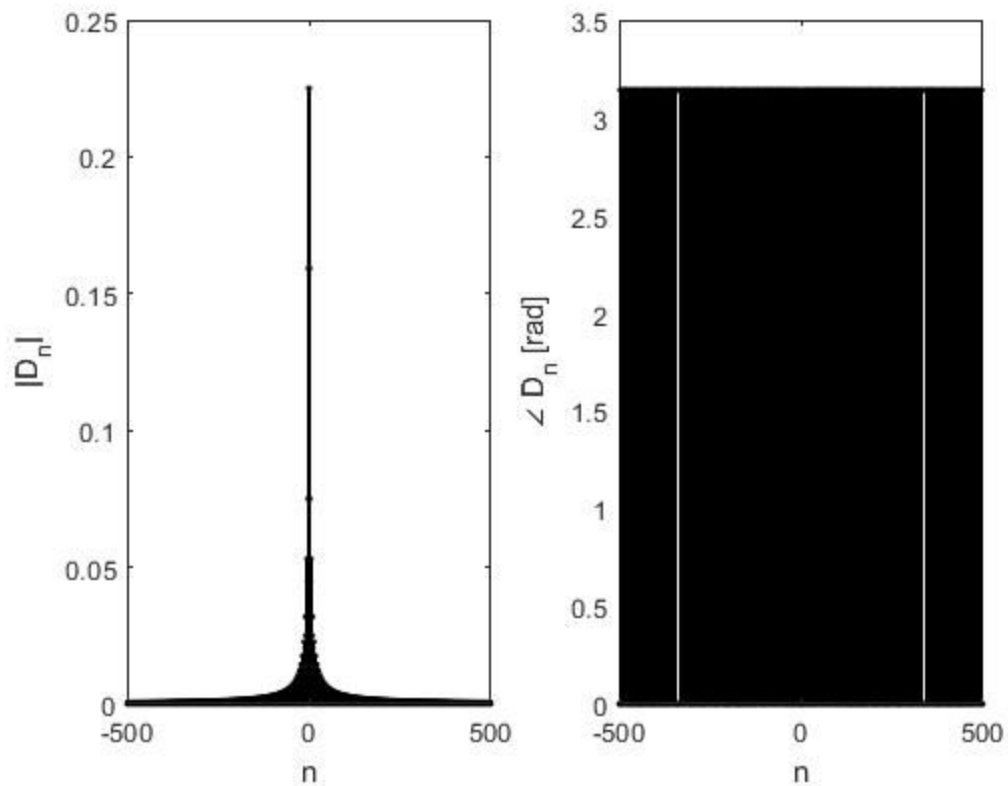
subplot(1,2,1); stem(n,abs(D_n),'k');

xlabel('n'); ylabel('|D_n|');

subplot(1,2,2); stem(n,angle(D_n),'k');

xlabel('n'); ylabel('\angle D_n [rad]');

$x_3(t):$



Matlab:

```
clf;
n = (-500:500);
D_n = (1./(n.*pi)).*sin((n.*pi)./4);
```

```
subplot(1,2,1); stem(n,abs(D_n),'k');
xlabel('n'); ylabel('|D_n|');
subplot(1,2,2); stem(n,angle(D_n),'k');
xlabel('n'); ylabel('\angle D_n [rad]');
```

A.5:

Matlab:

```
function [D] = a5(Dn)
n=-500:500;
D=Dn;
```



```

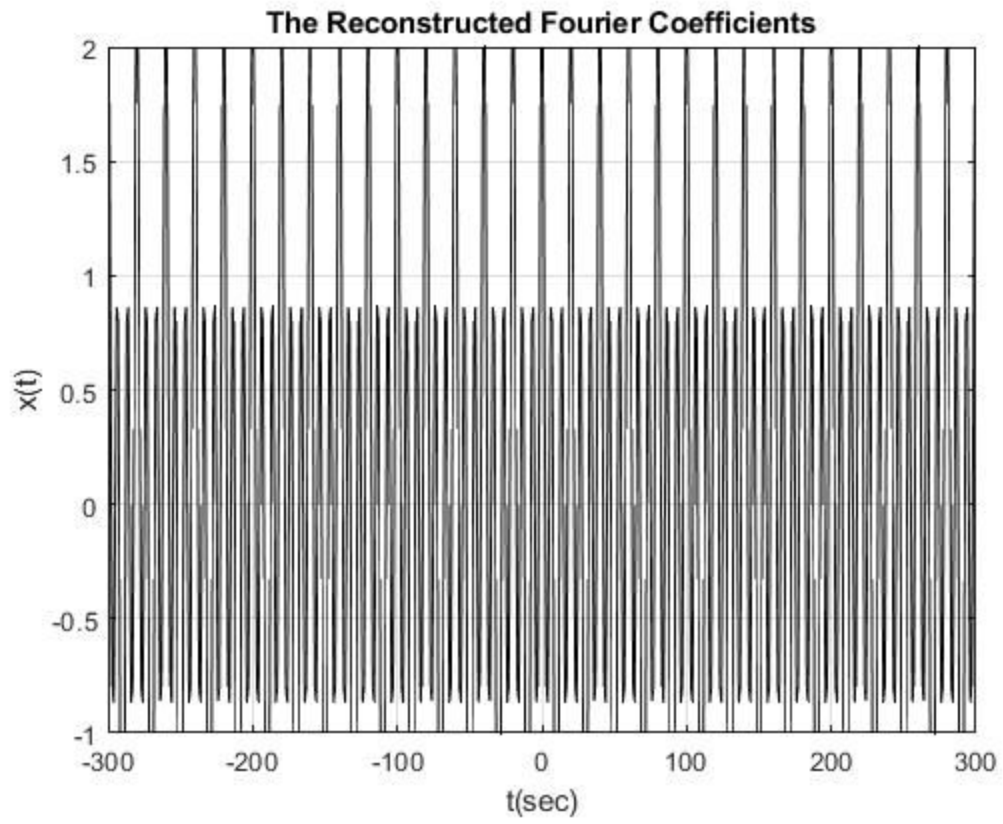
t=[-300:1:300];
w=pi*0.1;
x=zeros(size(t));
for i = 1:length(n)
    x=x+D(i)*exp(j*n(i)*w*t);
    't'
end

figure(5);
plot(t,x,'k')
xlabel('t(sec)');
ylabel('x(t)');
axis([-300 300 -1 2]);
title('The Reconstructed Fourier Coefficients');
grid;
-----

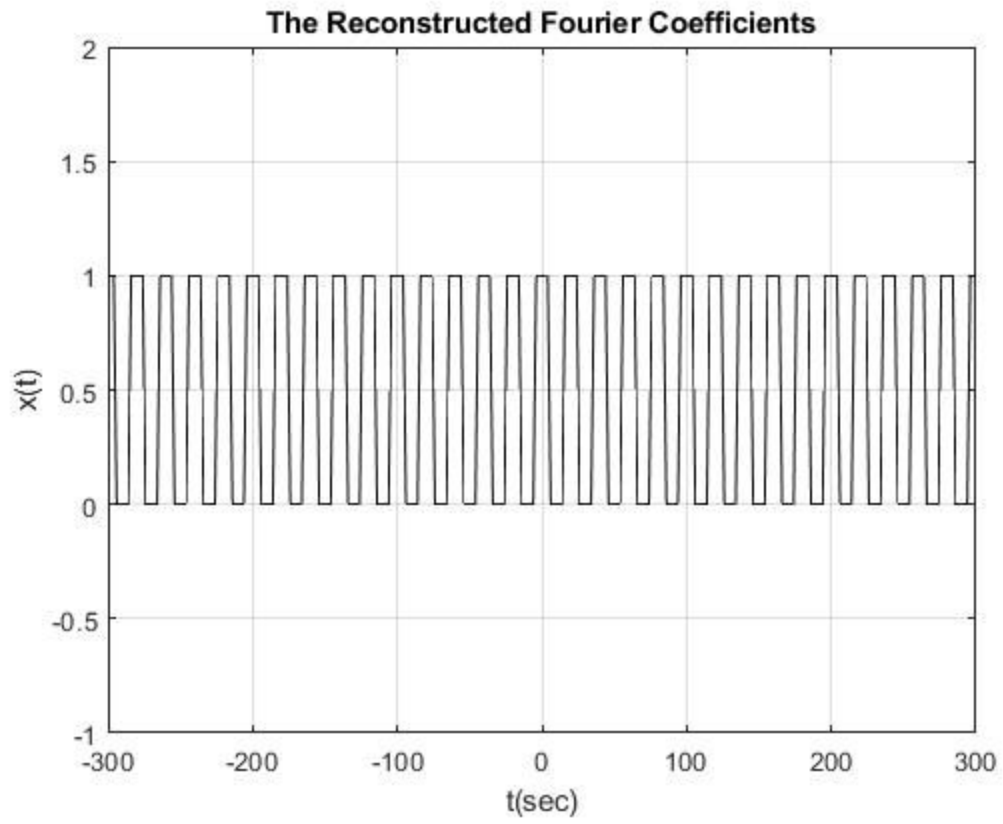
```

A.6:

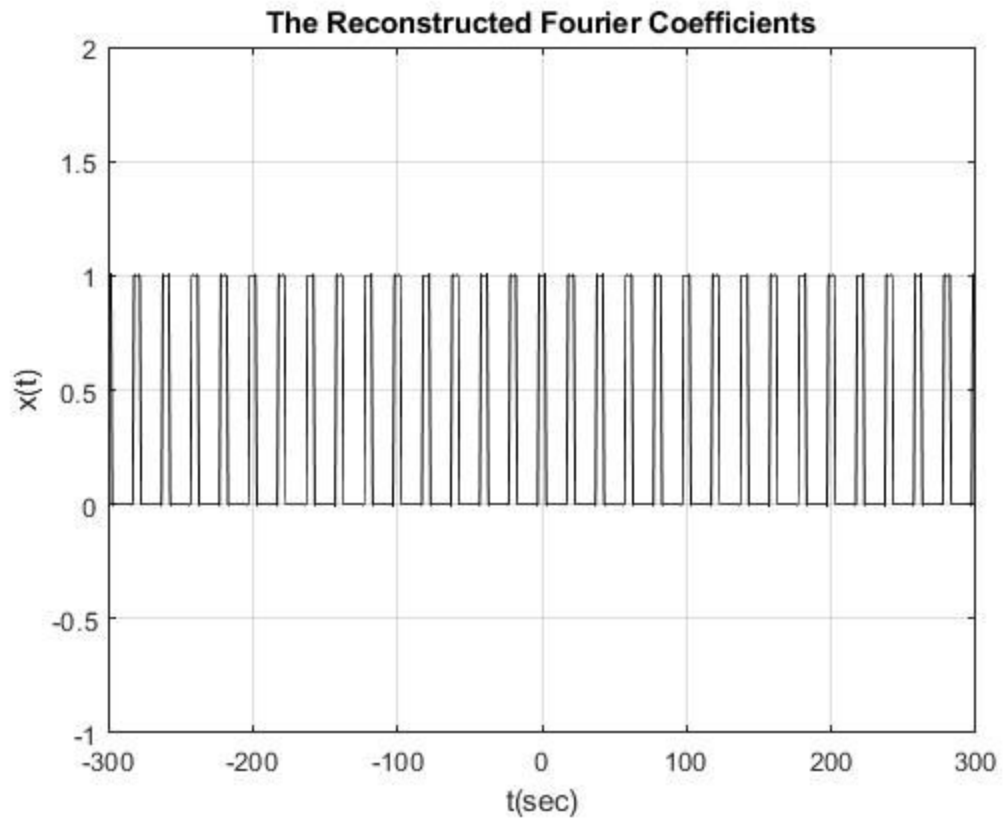
$x_1(t)$:



$x_2(t)$:



$x_3(t)$:



B:

B.1)

$$x_1(t) = \cos\left(\frac{3\pi}{10}t\right) + \frac{1}{2}\cos\left(\frac{\pi}{10}t\right)$$

$$\omega_{o1} = \frac{3\pi}{10}, \quad \omega_{o2} = \frac{\pi}{10}$$

$$\omega_o = \frac{G.C.F \text{ of numerator}}{L.C.M \text{ of denominator}} = \frac{\pi}{10} = 0.314 \text{ rad/s}$$

For $x_2(t) \rightarrow T_o = 20 \text{ s}$

$$\omega_o = \frac{\pi}{10} = 0.314 \text{ rad/s}$$

For $x_3(t) \rightarrow T_o = 40 \text{ s}$

$$\omega_o = \frac{\pi}{20} = 0.157 \text{ rad/s}$$

B.2) The main differences between the fourier coefficients of $x_1(t)$ and $x_2(t)$ is that one consists of sinc and the other consists of sin functions respectively. Furthermore, $x_1(t)$ has four distinct fourier series coefficients, while $x_2(t)$ has infinite fourier coefficients for D_n .

B.3) Signal $x_3(t)$ has a smaller fundamental frequency value compared to signal $x_2(t)$ for it's Fourier coefficients.

B.4) $D_o = 0.5$ for signal $x_4(t)$, derived from $x_2(t)$.

B.5) Since $x_1(t)$ has a finite number of D_n values, nothing will change if the Fourier coefficients are increased. However, for $x_2(t)$ and $x_3(t)$, increasing values of D_n results in higher accuracy.

B.6) Again, since $x_1(t)$ has a finite number of D_n values, we would only need four Fourier series coefficients, in this case, to perfectly reconstruct. However, for $x_2(t)$ and $x_3(t)$, we would need an infinite number of D_n for perfect reconstruction.

B.7) Since a periodic signal has an infinite number of D_n values, it is not viable. However, if it is finite like $x_1(t)$, then the values of D_n can be stored. However, this is not recommended for signals which have a large amount of finite D_n values as they would tend to waste space.