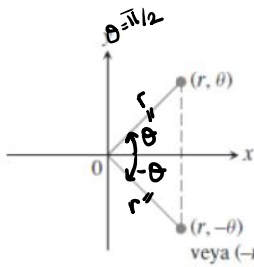
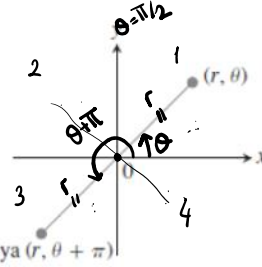


SİMETRİ (Bir eğride birden fazla simetri olabilir)

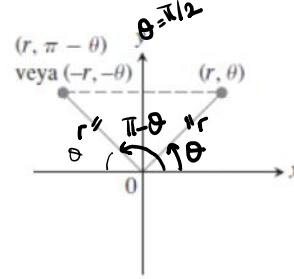
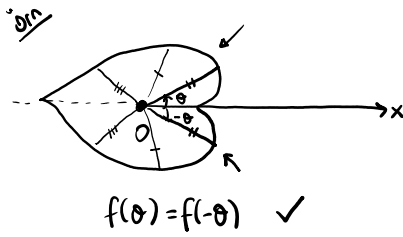
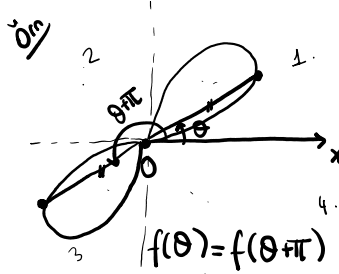
- ! $f(\theta) = f(-\theta)$ ise kutup eksenine göre simetri vardır.
- ! r yerine $-r$ değişmez veya $f(\theta) = f(\theta + \pi)$ ise orjine göre simetri vardır.
- ! $f(\theta) = f(\pi - \theta)$ ise $\theta = \pi/2$ doğrusuna göre simetri vardır.



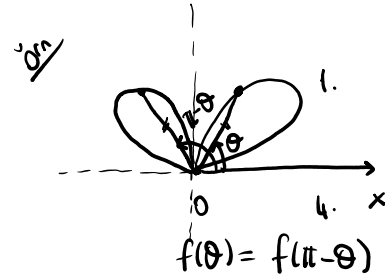
Kutup eksenine göre



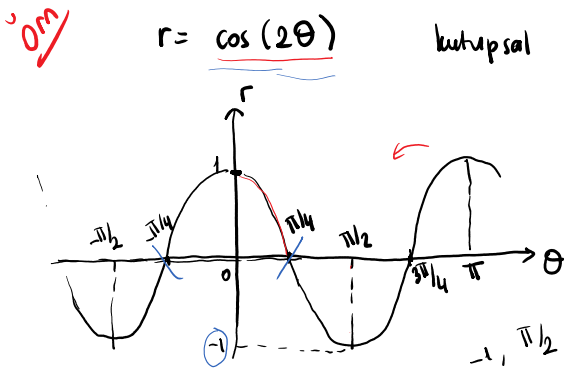
Kutup Noktasına göre

 $\theta = \pi/2$ 'ye göre* $(0, \pi)$ arası çizmek yeterli.

* $(0, \pi/2)$ → 1. bölge = 3. bölge
 $(\pi/2, \pi)$ → 2. bölge = 4. bölge

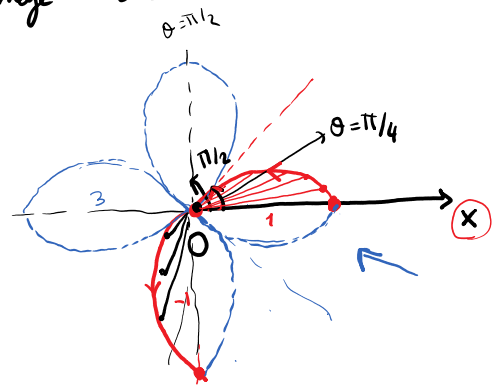


* $(-\pi/2, \pi/2)$ arası çizmek yeterli.



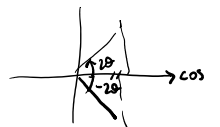
kutupsal eğrisini çizmeye çalışalım.

$$\begin{aligned} \theta = 0 & \quad r = 1 \\ \theta = \pi/4 & \quad r = 0 \\ \theta = \pi/2 & \quad r = -1 \end{aligned}$$



✓ Kutup eksenine göre simetri? $f(\theta) \stackrel{?}{=} f(-\theta)$

$$f(-\theta) = \cos(-2\theta) = \cos 2\theta = f(\theta) \quad \checkmark$$



✓ Orjine göre simetri? $f(\theta) \stackrel{?}{=} f(\theta + \pi)$

$$f(\theta + \pi) = \cos(2(\theta + \pi)) = \cos(2\theta + 2\pi) = \cos 2\theta = f(\theta) \quad \checkmark$$

1. bölge yeterli
 $\rightarrow 1$

1. bölge yeterli
3
(0, π/2)

$$f(\theta + \pi) = \cos(2(\theta + \pi)) = \cos(2\theta + 2\pi) = \cos 2\theta = f(\theta) \checkmark$$

(✓) θ = π/2'ye göre simetri?

$$f(\theta) \stackrel{?}{=} f(\pi - \theta)$$

$$f(\pi - \theta) = \cos(2(\pi - \theta)) = \cos(2\pi - 2\theta) = \cos 2\theta = f(\theta) \checkmark$$

TÜREV VE EĞİM

$$r = f(\theta)$$

$$x = r \cos \theta = f(\theta) \cdot \cos \theta$$

$$y = r \sin \theta = f(\theta) \cdot \sin \theta$$

$$\frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta}$$

$$= \frac{\frac{d}{d\theta}(f(\theta) \cdot \sin \theta)}{\frac{d}{d\theta}(f(\theta) \cdot \cos \theta)}$$

$$= \frac{\frac{df}{d\theta} \sin \theta + f(\theta) \cos \theta}{\frac{df}{d\theta} \cos \theta - f(\theta) \sin \theta}$$

$$= \frac{\frac{df}{d\theta} \sin \theta + f(\theta) \cos \theta}{\frac{df}{d\theta} \cos \theta - f(\theta) \sin \theta}$$

$$\frac{dx}{d\theta} = f'(\theta) \cos \theta - f(\theta) \sin \theta$$

$$\frac{dy}{d\theta} = f'(\theta) \sin \theta + f(\theta) \cos \theta$$

$r = f(\theta)$ Eğrisinin Eğimi

(r, θ) noktasında $dx/d\theta \neq 0$ olması şartıyla,

$$\frac{dy}{dx} \Big|_{(r, \theta)} = \frac{f'(\theta) \sin \theta + f(\theta) \cos \theta}{f'(\theta) \cos \theta - f(\theta) \sin \theta}$$

→ Türev

* $\frac{dx}{d\theta} = 0 \Rightarrow$ türev yoktur (dikiz teğet durumu)

* $\frac{dy}{d\theta} = 0 \left(\frac{dx}{d\theta} \neq 0 \right) \Rightarrow$ türev = 0 (yatay teğet durumu)

Ör/ a) $r = 1 + \sin \theta$ kardioidinin $\theta = \pi/3$ noktasındaki teğetinin eğimi nedir?

b) Yatay / dikiz teğet olan noktaları bulalım.

$$y = r \sin \theta$$

$$\frac{dy}{dx} = \frac{f'(\theta) \sin \theta + f(\theta) \cos \theta}{f'(\theta) \cos \theta - f(\theta) \sin \theta} = \frac{\cos \theta \sin \theta + (1 + \sin \theta) \cos \theta}{\cos \theta \cos \theta - (1 + \sin \theta) \sin \theta}$$

$$= \frac{\cos \theta (1 + 2 \sin \theta)}{(1 + \sin \theta) (1 - 2 \sin \theta)}$$

$$\frac{dy}{dx} \Big|_{\theta = \pi/3} = \frac{1/2 (1 + \sqrt{3})}{(1 + \sqrt{3}) (1 - \sqrt{3})} = \frac{(1 + \sqrt{3})}{(2 + \sqrt{3})(1 - \sqrt{3})} = \frac{1 + \sqrt{3}}{-1 - \sqrt{3}} = -\frac{1}{7}$$

2 - 3

Dikiz teğet

dx

$$(1 + \sin \theta)(1 - 2 \sin \theta) = 0$$

/ 1. \

~~Diğer~~ ~~te~~

$$\frac{dx}{d\theta} = 0$$

$$(1 + \sin \theta)(1 - 2 \sin \theta) = 0$$
$$\sin \theta = -1 \quad \sin \theta = 1/2 \quad \boxed{\pi/6, 5\pi/6}$$

↓ $\boxed{3\pi/2}$

$$\left(\frac{dy}{d\theta} \neq 0 \right)$$

~~Yatay~~ ~~te~~

$$\frac{dy}{d\theta} = 0$$

$$\cos \theta (1 + 2 \sin \theta) = 0$$

$$\cos \theta = 0 \rightarrow \boxed{\pi/2}, \boxed{3\pi/2}$$

$$1 + 2 \sin \theta = 0 \quad \sin \theta = -1/2 \quad \boxed{7\pi/6, 11\pi/6}$$

lim
 $\theta \rightarrow 3\pi/2$