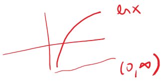
1st Week

21 Şubat 2023 Salı 13:44

Kayıt Anahtarı : mat116esp23

1.) Anti- Derivatives



$\int f(x)\,dx = F(x) + C \qquad F'(x) = f(x)$

$x^{-1} = \frac{1}{x}$ $\quad$ ln x

$\int x^n\,dx = \frac{x^{n+1}}{n+1} + C \quad (n \neq -1)$ $\qquad$ $\int \frac{1}{x}\,dx = \ln|x| + C$

$\int e^x\,dx = e^x + C$ $\qquad$ $\int a^x\,dx = \frac{a^x}{\ln a} + C$

$\int \sin x\,dx = -\cos x + C$ $\qquad$ $\int \cos x\,dx = \sin x + C$

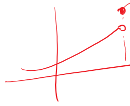
$-\sin(x)$

$\int \sec^2 x\,dx = \tan x + C$ $\qquad$ $\int \csc^2 x\,dx = -\cot x + C$

$\int \sec x \tan x\,dx = \sec x + C$ $\qquad$ $\int \csc x \cot x\,dx = -\csc x + C$

$a^x \to a^x \ln a$

$\int a^x\,dx = \frac{a^x}{\ln(a)}$



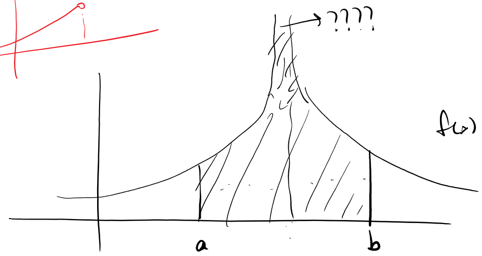
Indefinite Integrals } Definite Integrals

$\int f(x)\,dx = F(x)+C$

$\int_a^b f(x)\,dx = ?$ $\quad F(b) - F(a)$

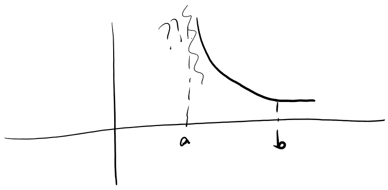
should be continuous on $[a,b]$!

(integrable)

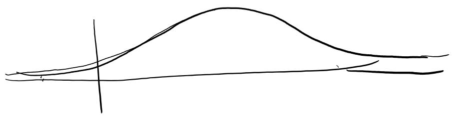


$\int_a^b f(x)\,dx \Rightarrow$ Not a definite integral $\rightarrow$ (Improper Integrals)

[Genelleştirilmiş]



$\int_a^b f(x)\,dx \Rightarrow$ Not a definite integral



$\int_{-\infty}^{\infty} \qquad \int_{-\infty}^{a} \qquad \int_{a}^{\infty} \rightarrow$ Not a definite integral

$*\quad \int f(x) \pm g(x)\,dx = \int f(x)\,dx \pm \int g(x)\,dx$

$*\quad c \in \mathbb{R}, \quad \int c\,f(x)\,dx = c\int f(x)\,dx$

2) Substitution Rule (u-substitution)

Örn $\int \tan(x)\,dx = \int \frac{\sin(x)\,dx}{\cos(x)}$ $\quad -du$

$\boxed{u = \cos(x)}$

$du = -\sin(x)\,dx$

~~$u = \sin(x)$~~

~~$du = \cos(x)\,dx$~~

$= \int -\frac{1}{u}\,du = -\ln|u| + C$

$= -\ln|\cos(x)| + C$

$$= \ln \left| \cos(x)^{-1} \right| + C \qquad = \ln \left| \frac{1}{\cos(x)} \right| + C$$

$$= \ln \left| \sec(x) \right| + C$$

$\int \tan x \, dx = \ln|\sec x| + C$ $\qquad$ $\int \cot x \, dx = \ln|\sin x| + C$

$\int \frac{1}{x^2 + a^2} dx = \frac{1}{a} \cancel{\tan^{-1}} \overset{\text{arctan}}{}\left(\frac{x}{a}\right) + C$ $\qquad$ $\int \frac{1}{\sqrt{a^2 - x^2}} dx = \cancel{\sin^{-1}} \overset{\text{arcsin}}{}\left(\frac{x}{a}\right) + C, \quad a > 0$

$$[\arctan(x)]' = \frac{1}{1+x^2} \qquad \rightarrow \int \frac{1}{1+x^2} dx = \arctan(x) + C$$

Ex

$$\int \frac{1}{x^2+9} dx \qquad \frac{1}{x^2+9} = \frac{1}{9\left(1+\frac{x^2}{9}\right)} = \frac{1}{9\left(1+\left(\frac{x}{3}\right)^2\right)}$$

$$= \frac{1}{3} \arctan\left(\frac{x}{3}\right) + C \qquad \left[\arctan\left(\frac{x}{3}\right)\right]' = \frac{1}{3} \cdot \frac{1}{1+\left(\frac{x}{3}\right)^2}$$

$$[\arcsin(x)]' = \frac{1}{\sqrt{1-x^2}} \qquad \int \frac{1}{\sqrt{1-x^2}} dx = \arcsin(x) + C$$

Ex

$$\int \frac{1}{\sqrt{25-x^2}} dx \qquad \frac{1}{\sqrt{25-x^2}} = \frac{1}{\sqrt{25\left(1-\frac{x^2}{25}\right)}} = \frac{1}{5\sqrt{1-\left(\frac{x}{5}\right)^2}}$$

$$\left[\arcsin\left(\frac{x}{5}\right)\right]' = \frac{1}{5} \cdot \frac{1}{\sqrt{1-\left(\frac{x}{5}\right)^2}}$$

$$= \arcsin\left(\frac{x}{5}\right) + C$$

$$! \quad \int f(x)\, g(x)\, dx \neq \int f(x) dx \cdot \int g(x) dx$$

3) Integration By Parts (Kısmi İntegrasyon)

Product Rule for derivative $\rightarrow$ $[f(x) \cdot g(x)]' = f'(x)\, g(x) + f(x)\, g'(x)$

Product Rule for derivative $\rightarrow$ $\left[f(x) \cdot g(x) \right]' = f'(x)\,g(x) + f(x)\,g'(x)$

$$\cancel{\int} \left[f(x) \cdot g(x) \right]' = \int f'(x)\,g(x)\,dx + \int f(x)\,g'(x)$$

$u = f(x)$

$du = f'(x)\,dx$

$$f(x) \cdot g(x) = \int \underbrace{g(x)}_{v}\,\underbrace{f'(x)\,dx}_{du} + \int \underbrace{f(x)}_{u}\,\underbrace{g'(x)\,dx}_{dv}$$

$v = g(x)$

$dv = g'(x)\,dx$

$$u \cdot v = \int v \cdot du + \int u \cdot dv$$

Integration by parts

$$\boxed{\int u \cdot dv = u \cdot v - \int v\,du}$$

priority for being u $\rightarrow$ LAPTE

L $\rightarrow$ log, A $\rightarrow$ arctan, P $\rightarrow$ poly, T $\rightarrow$ trig, E $\rightarrow$ exponential

Ex

$$\int 1 \cdot \ln x \; dx$$

$u = \ln x$

$u = \ln x \xrightarrow{\text{derivative}} du = \frac{1}{x}\,dx$

$\int dv = \int 1 \cdot dx \xrightarrow{\text{integrate}} v = x$

$$\int 1 \cdot \ln x\,dx = \ln x \cdot x - \underbrace{\int \cancel{x} \cdot \frac{1}{\cancel{x}}\,dx}_{x} = \ln x \cdot x - x + C$$