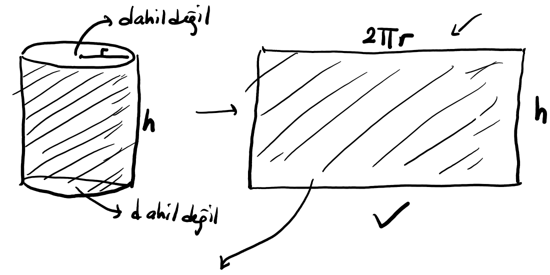
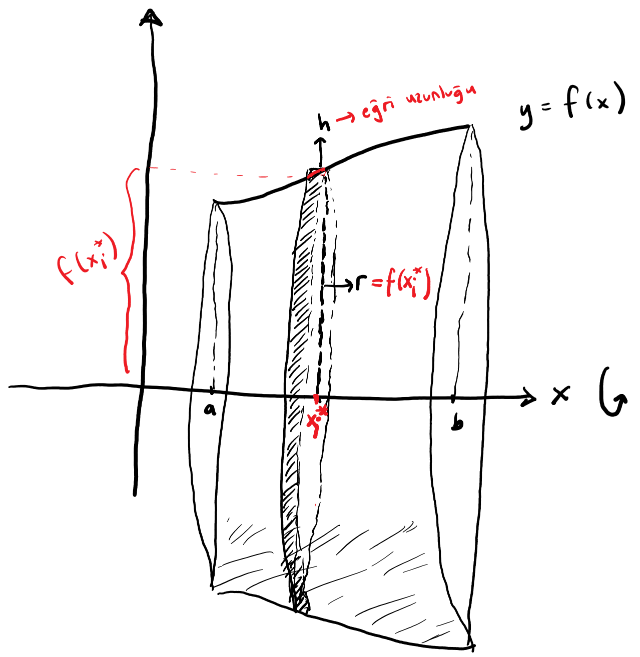
4. Hafta Perşembe Dersi

18 Mart 2021 Perşembe 11:39

Dönel Yüzeylerin Alanları



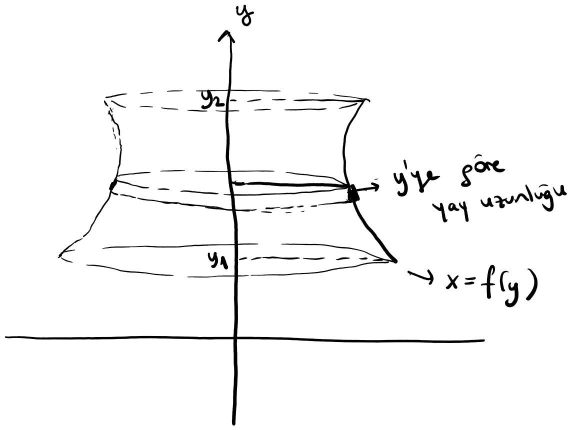
Silindir Yüzey Alanı : $\boxed{A = 2\pi r \cdot h}$



Tek Silindir için $A = 2\pi \cdot f(x_i^*) \cdot h$ → yay uzunluğu

$$\lim_{n\to\infty}\sum_{i=1}^{n} 2\pi f(x_i^*)\, h_i$$

$$\boxed{A = \int_a^b 2\pi f(x)\sqrt{1+f'(x)^2}\,dx}$$



$$\boxed{A = \int_{y_1}^{y_2} 2\pi f(y)\sqrt{1+f'(y)^2}\,dy}$$

Örn/ $\boxed{y = x^2}$ parabolünün $(1,1)$ ve $(2,4)$ noktaları arasındaki parçasının y ekseni etrafında döndürülmesiyle oluşan yüzeyin alanı nedir?

$x = f(y)$

$y = x^2 \Rightarrow x = \sqrt{y}$

$f(y) = \sqrt{y}$

$$A = \int^{4} 2\pi \sqrt{y}\sqrt{1+\frac{1}{4y}}\,dy$$

$f(y) = \sqrt{y}$

$f'(y) = \dfrac{1}{2\sqrt{y}}$

$f'(y)^2 = \dfrac{1}{4y}$

$$A = \int_1^4 2\pi \sqrt{y} \sqrt{1 + \frac{1}{4y}} \, dy$$

$$= \int_1^4 \cancel{2}\pi \cancel{\sqrt{y}} \, \frac{\sqrt{4y+1}}{\cancel{2\sqrt{y}}} \, dy$$

$u = 4y + 1$

$du = 4\,dy$

$$\int \frac{\sqrt{u}}{4} \, du \qquad \frac{2}{3} \, \frac{u^{3/2}}{4 \; 2}$$

$$= \frac{\pi}{6} (4y+1)^{3/2} \Big]_1^4$$

$$= \frac{\pi}{6} \left(17\sqrt{17} - 5\sqrt{5} \right) \; br^2$$

Örn $y = \sqrt{4 - x^2}$ $\quad -1 \leq x \leq 1 \quad$ x - ekseni etrafında dya?

$f(x) = \sqrt{4 - x^2}$

$f'(x) = -\cancel{2}x \cdot \dfrac{1}{\cancel{2}\sqrt{4-x^2}}$

$= \dfrac{-x}{\sqrt{4 - x^2}}$

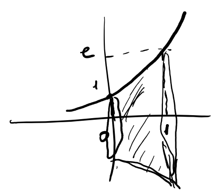
$f'(x)^2 = \dfrac{x^2}{4 - x^2}$

$$A = \int_{-1}^{1} 2\pi \sqrt{4 - x^2} \sqrt{1 + \frac{x^2}{4 - x^2}} \, dx$$

$$= \int_{-1}^{1} 2\pi \, \cancel{\sqrt{4 - x^2}} \, \frac{\sqrt{4 - \cancel{x^2} + \cancel{x^2}}}{\cancel{\sqrt{4 - x^2}}} \, dx$$

$$= 4\pi x \Big]_{-1}^{1} = 4\pi - (-4\pi) = 8\pi$$

Örn $y = e^x$, $\quad 0 \leq x \leq 1 \quad$ x - ekseni etrafında



$f(x) = e^x$

$f'(x) = e^x$

$f'(x)^2 = e^{2x}$

$$A = \int_0^1 2\pi f(x) \sqrt{1 + f'(x)^2} \, dx = \int_0^1 2\pi \, e^x \sqrt{1 + (e^x)^2} \, dx$$

$1 + (e^x)$.

$\sqrt{1 + u^2}$

$\boxed{u = e^x}$

$du = e^x dx$

$f'(x) = e^x$

$f'(x)^2 = e^{2x}$

$\int_0 \ldots f'(x)\, \sqrt{1+f'(x)^2}\, dx = \int_0 2\pi (e^x) \sqrt{1+(e^x)^2}$ $\sqrt{1+u^2}$ $\boxed{u = e^x}$ $du = e^x dx$

$\int \sqrt{1+u^2}\, du$ $\frac{u = \tan\theta}{du = \sec^2\theta\, d\theta}$

$x_1 = 0 \longrightarrow e^x = \tan\theta$, $\theta = \arctan(e^0)$, $\theta = \pi/4$

$x_2 = 1 \rightarrow e^1 = \tan\theta$, $\theta = \arctan(e)$

$$= \int \sec\theta \sec^2\theta\, d\theta = \int \sec^3\theta\, d\theta = \frac{1}{2}\left(\sec\theta\tan\theta + \ln|\sec\theta + \tan\theta|\right)$$

$$= \pi \left(\underbrace{\sec\theta}_{\sqrt{e^2+1}} \underbrace{\tan\theta}_{e} + \ln|\sec\theta + \underbrace{\tan\theta}_{e}| \right) \Big|_{\pi/4 = \theta_1}^{\arctan(e) = \theta_1}$$

e, $\sqrt{e^2+1}$, θ, 1 $\frac{1}{\cos\theta}$

$$\pi\left(e\sqrt{e^2+1} + \ln\left|\sqrt{e^2+1}+e\right| - \sqrt{2}.1 - \ln\left|\sqrt{2}+1\right|\right)$$

Örn $y = \sqrt[3]{x}$ $1 \leq y \leq 2$ y-ekseni etrafında d.y.a ?

$x = f(y)$

$x = y^3$

$f(y) = y^3$

$f'(y) = 3y^2$

$f'(y)^2 = 9y^4$

$$A = \int_1^2 2\pi f(y) \sqrt{1+f'(y)^2}\, dy$$

$$= \int_1^2 2\pi y^3 \sqrt{\underbrace{1+9y^4}_{u}}\, dy$$

$u = 1 + 9y^4$

$du = 36y^3\, dy$

$2\pi \frac{du}{36} \sqrt{u}$

$$\Rightarrow \frac{\pi}{\cancel{18}_9} \frac{\cancel{2}}{3} \cdot u^{3/2} = \frac{\pi}{27}\left(1+9y^4\right)^{3/2} \Big]_{y=1}^{y=2}$$