11. Hafta Salı Dersi

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EXAMPLE 1 $\sum_{n=1}^{\infty} \frac{n-1}{2n+1} \to \neq 0 \to$ ıraksaklık için n. terim testi $\Rightarrow$ ıraksaktır.

Since $a_n \to \frac{1}{2} \neq 0$ as $n \to \infty$, we should use the Test for Divergence.

EXAMPLE 2 $\sum_{n=1}^{\infty} \frac{\sqrt{n^3}+1}{3n^3+4n^2+2}$ $\cdot \frac{3n^3}{\sqrt{n^3}} \to \sqrt{\frac{n^3+1}{n^3}} \to \sqrt{1+\frac{1}{n^3}} = 1 > 0$ $b_n = \frac{\sqrt{n^3}}{3n^3} =$

Since a_n is an algebraic function of n, we compare the given series with a p-series. The comparison series for the Limit Comparison Test is $\Sigma\, b_n$, where

$\Rightarrow \sum a_n$ yakınsaktır.

$$b_n = \frac{\sqrt{n^3}}{3n^3} = \frac{n^{3/2}}{3n^3} = \frac{1}{3n^{3/2}} \to \text{yakınsak}$$

EXAMPLE 3 $\sum_{n=1}^{\infty} ne^{-n^2}$

Since the integral $\int_1^{\infty} xe^{-x^2}\,dx$ is easily evaluated, we use the Integral Test. The Ratio Test also works.

EXAMPLE 4 $\sum_{n=1}^{\infty} (-1)^n \frac{n^3}{n^4+1}$ yakınsak

Since the series is alternating, we use the Alternating Series Test.

EXAMPLE 5 $\sum_{k=1}^{\infty} \frac{2^k}{k!}$ $\lim_{n\to\infty} \frac{a_{n+1}}{a_n}$

Since the series involves $k!$, we use the Ratio Test.

11.7 Exercises

$\frac{2n+1}{n^2}$ $\frac{1}{2}$

1–38 Test the series for convergence or divergence.

1. $\sum_{n=1}^{\infty} \frac{1}{n+3^n} < \frac{1}{3^n}$ $r=\frac{1}{3}<1$ yak.
2. $\sum_{n=1}^{\infty} \frac{(2n+1)^n}{n^{2n}} \to$ yak.
3. $\sum_{n=1}^{\infty} (-1)^n \frac{n}{n+2} \to$ ıraksak
4. $\sum_{n=1}^{\infty} (-1)^n \frac{n}{n^2+2} \to$ yak.
5. $\sum_{n=1}^{\infty} \frac{n^2 2^{n-1}}{(-5)^n}$ oran, kök
6. $\sum_{n=1}^{\infty} \frac{1}{2n+1} \to$ ıraksak.
7. $\sum_{n=2}^{\infty} \frac{1}{n\sqrt{\ln n}}$ integ. ıraksak
8. $\sum_{k=1}^{\infty} \frac{2^k k!}{(k+2)!} \to$ oran
9. $\sum_{k=1}^{\infty} k^2 e^{-k}$
10. $\sum_{n=1}^{\infty} n^2 e^{-n^3} \to$ integral
11. $\sum_{n=1}^{\infty} \left(\frac{1}{n^3}+\frac{1}{3^n}\right) \to$ yakınsak (yak, yak)
12. $\sum_{n=1}^{\infty} \frac{1}{k\sqrt{k^2+1}} < \frac{1}{k\sqrt{k^2}} = \frac{1}{k^2}$ $p>1$ yakınsak
13. $\sum_{n=1}^{\infty} \frac{3^n n^2}{n!}$ oran $\quad \frac{3^{n+1}(n+1)^2}{(n+1)!}\cdot\frac{n!}{3^n n^2} = 0 < 1$
14. $\sum_{n=1}^{\infty} \frac{\sin 2n}{1+2^n} < \frac{1}{1+2^n} < \frac{1}{2^n} \to r=1/2$ geo. yak.
15. $\sum_{k=1}^{\infty} \frac{2^{k-1}3^{k+1}}{k^k}$ kök $\frac{1}{2}\cdot 3 \quad \frac{6}{n}$
16. $\sum_{n=1}^{\infty} \frac{n^2+1}{n^3+1}$ $b_n = \frac{n^2}{n^3} = \frac{1}{n} \to$ ıraksak. limit k. $\Rightarrow$ ıraksak.
17. $\sum_{n=1}^{\infty} \frac{1\cdot 3\cdot 5\cdot\cdots\cdot(2n-1)}{2\cdot 5\cdot 8\cdot\cdots\cdot(3n-1)}$ oran
18. $\sum_{n=2}^{\infty} \frac{(-1)^{n-1}}{\sqrt{n}-1} \to$ alterne seri testi

13) $\frac{3(n+1)^2}{n^2(n+1)}$

$2n-1+2$

(n+1). terim:

17) $\lim_{n\to\infty} \frac{\cancel{1.3.5\ldots}(2n-1)(2n+1)}{\cancel{2.5.8\ldots}(3n-1)(3n+2)} \cdot \frac{\cancel{2.5\ldots(3n-1)}}{\cancel{1.3\ldots(2n-1)}}$

$= \frac{2}{3} < 1 \Rightarrow$ yak.

5) $\lim_{n\to\infty} \sqrt[n]{\left|\frac{n^2\cdot 2^n}{2\cdot(-5)^n}\right|}$ $\quad \frac{2}{5}\sqrt[n]{\frac{n^2}{2}} \to 1$ $= \frac{2}{5} < 1$

16) $\lim_{n\to\infty} \frac{\frac{n^2+1}{n^3+1}}{\frac{n^2}{n^3}} = \lim_{n\to\infty} \frac{n^2+1}{n^3+1}\cdot\frac{n^3}{n^2} = 1 > 0$

6) $\lim_{n\to\infty} \frac{\frac{1}{2n+1}}{\frac{1}{n}} = \frac{1}{2n+1}\cdot\frac{n}{1} = \frac{1}{2} > 0$ $\quad \frac{1}{n} \to$ ıraksak

7) $\int_1^{\infty} \frac{1}{x\sqrt{\ln x}}\,dx = \lim_{t\to\infty}\int_1^t \cdots = \lim_{t\to\infty} 2\sqrt{\ln x}\,\Big]_{x=1}^{x=t} = \frac{2\sqrt{\ln t}}{\infty} - 0 = \infty$

$u = \ln x$

ıraksak

$\int \frac{1}{\sqrt{u}}\,du \quad u^{-1/2}\,du \to \frac{u^{1/2}}{1/2} \to 2\sqrt{\ln x}$

9) $\sum_{n=1}^{\infty} n^2 e^{-n} \to$ yakınsak

$\rightarrow \frac{n^2}{e^n}$ azalan $\frac{2n e^n - n^2 e^n}{e^{2n}} < 0$

$$\int_1^{\infty} x^2 e^{-x}\,dx \qquad u = x^2 \quad dv = e^{-x}dx \quad du = 2x\,dx \quad v =$$

$5/e \to$ yakınsak

Örn

$$\sum_{n=1}^{\infty} \frac{1}{n(1+\ln^2 n)}\,dx \Rightarrow \text{yakınsak.}$$

sürekli, pozitif, azalan $\to$ integral testi

$$\int_1^{\infty} \frac{1}{x(1+\ln^2 x)}\,dx \qquad u = \ln x \qquad \int \frac{1}{1+u^2}\,du$$

$$= \lim_{t\to\infty} \arctan(\ln x)\Big|_{x=1}^{x=t} =$$

$$\underbrace{\arctan(\ln t)}_{\pi/2} - \underbrace{\arctan 0}_{0} = \pi/2 \to \text{yakınsak.}$$

Örn

$$\sum_{n=2}^{\infty} \frac{1+n\ln n}{n^2+5} \to \text{ıraksaktır} \qquad b_n = \frac{n}{n^2} = \frac{1}{n} \to \text{harmonik seri ıraksak}$$

Limit Karşılaştırma Testi.

$$\lim_{n\to\infty} \frac{a_n}{b_n} = \lim_{n\to\infty} \frac{1+n\ln n}{n^2+5} \cdot \frac{n}{1} = \lim_{n\to\infty} \frac{n+n^2\ln n}{n^2+5} \quad \frac{\infty}{\infty}.$$

$$\lim_{n\to\infty} \frac{1+2n\ln n+n}{2n} \quad \frac{\infty}{\infty}$$

$$\lim_{n\to\infty} \frac{2\ln n+2+1}{2} = \infty$$