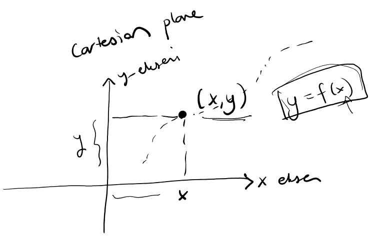
8th Week

11 Nisan 2023 Salı 13:31

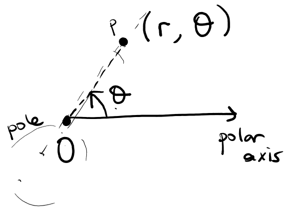
~~integration techniques~~ ~~parametric eqns & curve~~ polar coord curves

kartezyen $y = f(x)$

$x = f(t)$
$y = f(t)$



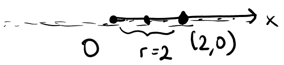
Polar Coordinates



$r =$ distance of the point to the pole

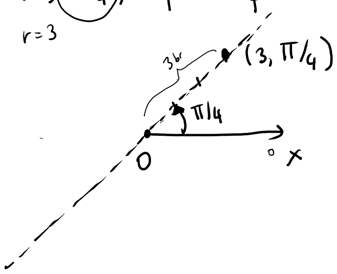
✓ $\theta =$ counter clock wise angle from polar axis

Ex $(2, 0)$ polar point

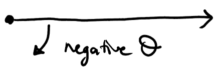


Ex $(3, \pi/4)$ polar point

$r = 3$



negative θ:



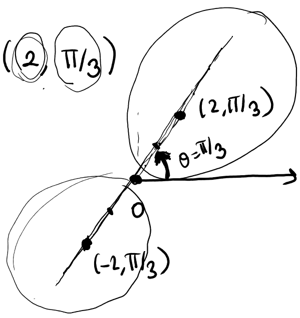
clock-wise rotation

negative r:

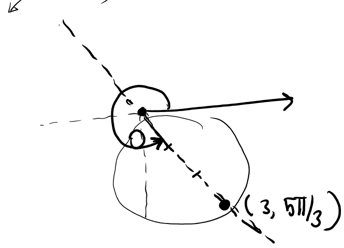
! positive r ⇒ point ⟵ same quadrant with θ

negative r ⇒ point ⟵ opposite quadrant with θ

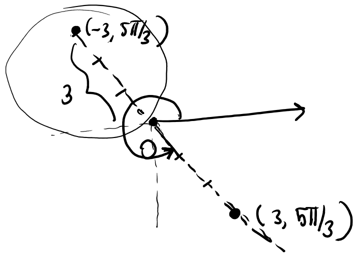
Ex $(2, \pi/3)$ $(-2, \pi/3)$



Ex $(3, \frac{5\pi}{3})$ polar point $\qquad \frac{5\pi}{3} = \pi + \frac{2\pi}{3}$

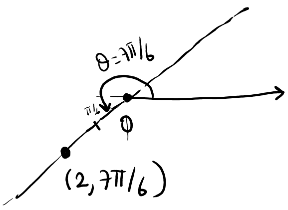


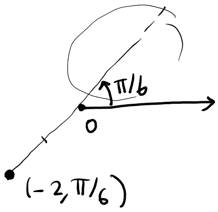
Ex $(-3, 5\pi/3)$

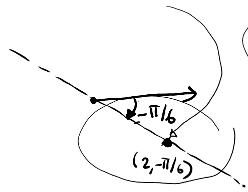


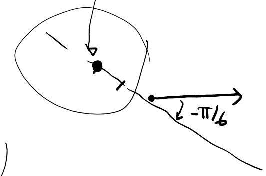
Ex/ $(2, 7\pi/6)$ polar point $=$ Ex/ $(-2, \pi/6)$ Ex/ $(2, -\pi/6)$ Ex/ $(-2, -\pi/6)$

$\pi + \frac{\pi}{6}$





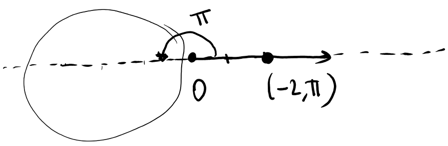


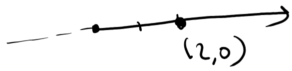


$$(-r, \theta + \pi) = (r, \theta)$$

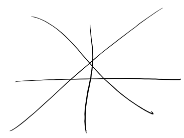
$$(r, \theta + \pi) = (-r, \theta)$$

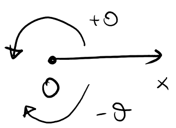
Ex/ $(-2, \pi) = (2, 0)$



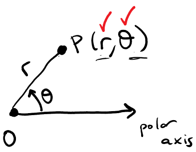


! There are infinitely many polar coordinates representing the same point.

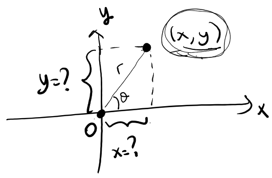




Polar Coordinates vs. Cartesian Coordinates



$\Rightarrow$



$x = ?$
$y = ?$

(r, θ) $(-r, \theta + \pi) = (-r, \theta - \pi)$

$(r, \theta + 2\pi) = (r, \theta - 2\pi)$

$\Leftarrow$?

$x = r\cos\theta$ ✓

$y = r\sin\theta$ ✓

(x, y) 9889

$(r, \theta + 2\pi) = (r, \theta - 2\pi)$ $\leftarrow$

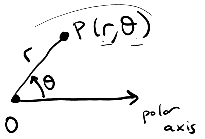
$y = r\sin\theta$ ✓

9889

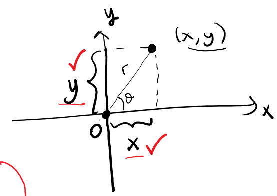
Ex/ $\left(2, \frac{\pi}{3}\right)$ polar point $\Rightarrow$ cartesian ? $(1, \sqrt{3})$

$r = 2 \quad \theta = \pi/3$

$$\left.\begin{aligned} x &= r\cos\theta = 2 \cdot \underbrace{\cos\pi/3}_{1/2} = 1 \\ y &= r\sin\theta = 2 \cdot \underbrace{\sin\pi/3}_{\sqrt{3}/2} = \sqrt{3} \end{aligned}\right\} (1, \sqrt{3})$$



$\Leftarrow$



$r = ?$
$\theta = ?$

$$\boxed{\begin{aligned} x &= r\cos\theta \\ y &= r\sin\theta \end{aligned}}$$

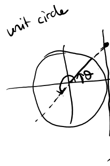
arccos $\left(\frac{x}{r}\right)$

$\frac{x}{y} = \frac{\cancel{r}\cos\theta}{\cancel{r}\sin\theta}$

$\frac{y}{x} = \frac{\cancel{r}\sin\theta}{\cancel{r}\cos\theta} = \tan\theta$

$\tan\theta = \frac{y}{x}$

unit circle



$\rightarrow \Rightarrow \boxed{\theta = \arctan\left(\frac{y}{x}\right)} \rightarrow$

$(r, \theta) \quad (r, \pi+\theta)$

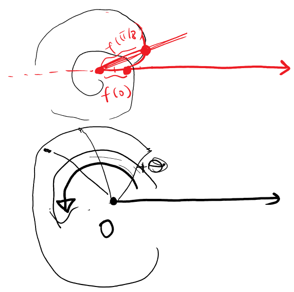
$$x^2 + y^2 = r^2\cos^2\theta + r^2\sin^2\theta = r^2(\underbrace{\cos^2\theta + \sin^2\theta}_{1}) = r^2$$

$$\boxed{r = \sqrt{x^2+y^2}}$$

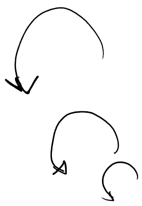
$(-r)^2 = r^2$
$(r)^2 = r^2$

Polar Curves

$\boxed{r = f(\theta)} = 3 + \sin\theta - \cos\theta$

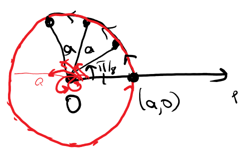


θ	r
0	f(0)
$\frac{\pi}{8}$	$f(\pi/8)$



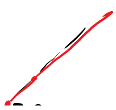
Ex/ $r = a$: $a \in \mathbb{R}$ $f(\theta)$

$(a,$



represents a circle centered at the pole (O) with radius a.

θ	r
0	a
π/8	a
π/6	a
π/2	



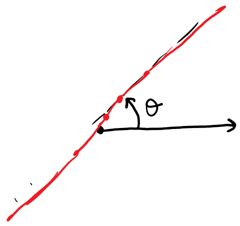
represents a line passing through the pole.

$\pi/2$

Ex $\theta =$ an angle here

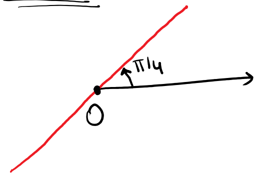
(r, θ) fixed

all possible r's

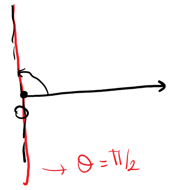


represents a line passing through the pole.

Ex $\theta = \pi/4$ is a line as a polar curve

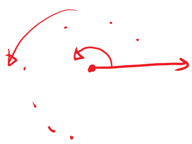


Ex $\theta = \pi/2$ line



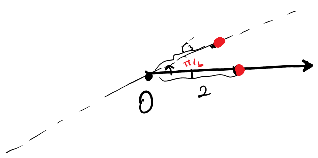
$r = f(\theta)$

θ	r
0	

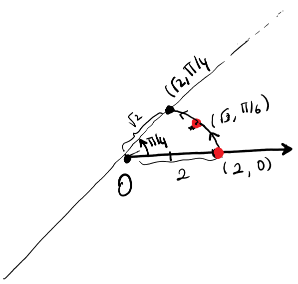


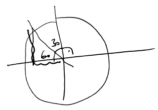
Ex $\boxed{r = 2\cos\theta}$

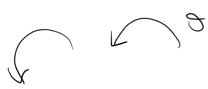
$r = f(\theta)$

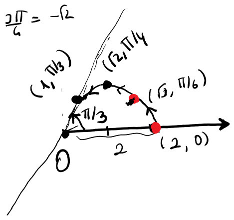


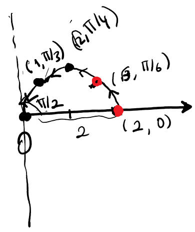
θ	r
0	$2\underbrace{\cos 0}_{1} = 2$
$\pi/6$	$2\underbrace{\cos\frac{\pi}{6}}_{\sqrt{3}/2} = \sqrt{3}$
$\pi/4$	$2\underbrace{\cos\pi/4}_{\sqrt{2}/2} = \sqrt{2}$
$\pi/3$	$2\underbrace{\cos\pi/3}_{1/2} = 1$
$\pi/2$	$2\underbrace{\cos\pi/2}_{0} = 0$
$\frac{\pi}{2}+\frac{\pi}{6} = \frac{2\pi}{3}$	$2\underbrace{\cos\frac{2\pi}{3}}_{-1/2} = -1$
$\frac{\pi}{2}+\frac{\pi}{4} = \frac{3\pi}{4}$	$2\cos\frac{3\pi}{4} = -\sqrt{2}$

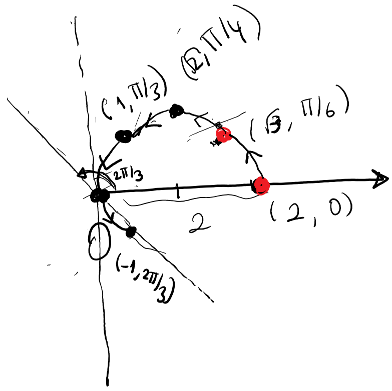






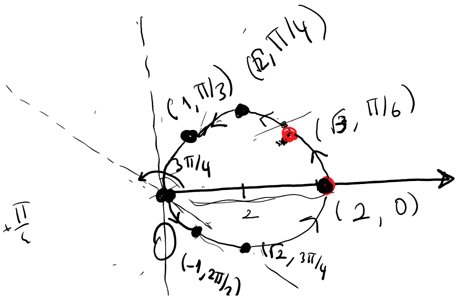




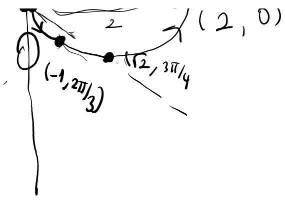


$\frac{\pi}{2}+\frac{\pi}{3}$

$\frac{\pi}{2}+\frac{\pi}{4}$



$\frac{\pi}{2}+\frac{\pi}{6}$



a>0

* $r = a\cos\theta$, $r = a\sin\theta$, $r = -a\cos\theta$, $r = -a\sin\theta$ → circle

(max r) diameter = a

passes through pole

! max r (→ diameter) (min r) at which θ = ?

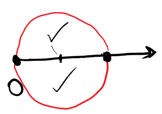
a>0

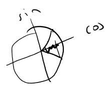
* $r = a\cos\theta = a \cdot 1$

$-1 \leqslant \cos\theta \leqslant 1$

for max r, $\cos\theta = 1$ ⇒ when? $\boxed{\theta = 0}$

max r = a





sym

$r = a\cos\theta$

$f(\theta) \overset{?}{=} f(-\theta)$

$a\cos\theta \overset{?}{=} a\cos(-\theta)$ ✓

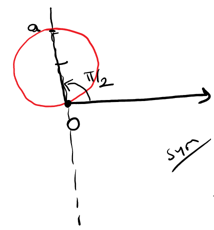
$r = a\sin\theta$

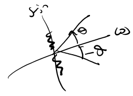
$-1 \leqslant \sin\theta \leqslant 1$

for max r, $\sin\theta = 1$ ⇒ when? $\boxed{\theta = \pi/2}$

max r = a

diameter





sym

$r = a\sin\theta$

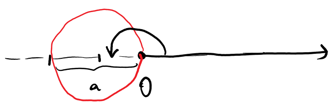
$f(\theta) \overset{?}{=} f(-\theta)$

$a\sin\theta \underset{?}{\neq} a\sin(-\theta)$

$r = -a\cos\theta = a$

$\cos\theta = -1$

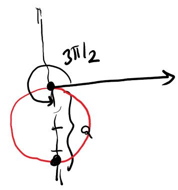
for max r, $\cos\theta = -1$, ⇒ when? $\theta = \pi$



$r = -a\sin\theta = a$

$\sin\theta = -1$

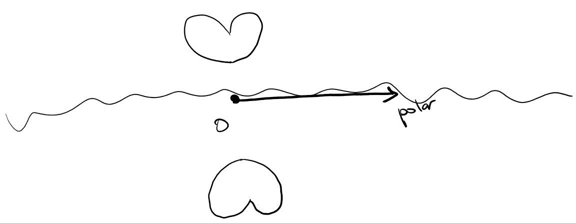
for max r, $\sin\theta = -1$ ⇒ $\theta = 3\pi/2$

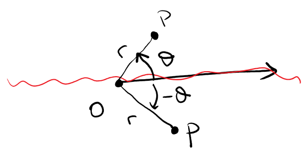


Symmetry

$r = f(\theta)$

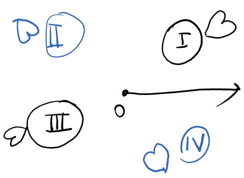
1) wrt the polar axis

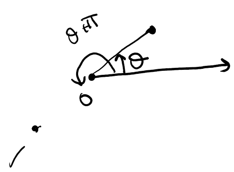




$\boxed{f(\theta) \overset{?}{=} f(-\theta)}$

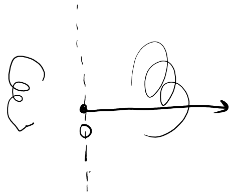
2) wrt the pole

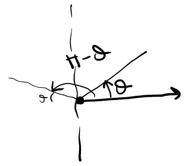




$\boxed{f(\theta) = f(\theta + \pi)}$

3) wrt the line $\theta = \pi/2$





$\boxed{f(\theta) = f(\pi - \theta)}$