2nd Week Tuesday

28 Şubat 2023 Salı 13:33

Ex

$$\int e^{\sqrt{x}} \cdot 1\, dx = ?$$

exp. poly.

LAPTE

$u = 1$ $\quad \cancel{dv = e^{\sqrt{x}} dx}$ $\quad \int \quad v = ?$

→ This is not available directly for integration by parts. → use u-substitution first.

$$\int e^{\sqrt{x}} dx = \int e^{u} \underset{!?}{dx} = \int e^{u} \underset{?!}{2\sqrt{x}}\, du = \int e^{u} \cdot 2u\, du$$

$\boxed{u = \sqrt{x}}$

$du = \frac{1}{2\sqrt{x}} dx \rightarrow dx = 2\sqrt{x}\, du$

we can apply int. by parts now!

$$\left(\int e^{x} \cdot x\, dx = x.e^{x} - \int e^{x} \cdot 1 dx = x.e^{x} - e^{x} + C \right)$$

LAPTE

$u = x$, $du = 1dx$ $\quad \int dv = \int e^{x} dx$, $v = e^{x}$

$$= 2\left(\sqrt{x}.e^{\sqrt{x}} - e^{\sqrt{x}}\right) + C$$

Trigonometric Integrals

m, n → integers

$$\int \sin^{m}(x) \cos^{n}(x)\, dx = ?$$

u-substitution:

choose the one with even power to be u

if m is even; $\boxed{u = \sin(x)}$ or

if n is even; $\boxed{u = \cos(x)}$

$$\boxed{\sin^2(x) + \cos^2(x) = 1}$$

✓ if both are odd ⇒ any of them may be u.

✓ if both are even ⇒ use half-angle formulas

$\cos^2(x) = \frac{1 + \cos(2x)}{2}$

$\sin^2(x) = \frac{1 - \cos(2x)}{2}$

$u = \cos(2x)$

$du = -2\sin(2x) \} -4\sin(x)\cos(x)\,du$

$$\int \sec^{m}(x) \tan^{n}(x)\, dx = ?$$

if m is even ⇒ $\boxed{u = \tan(x)}$ ✓ $\quad u = \tan(x)$, $du = \sec^2(x)\,dx$

if n is odd ⇒ $\boxed{u = \sec(x)}$ ✓ $\quad u = \sec(x)$, $du = \sec(x)\tan(x)\,dx$

$$\boxed{1 + \tan^2(x) = \sec^2(x)} \qquad \boxed{\tan^2(x) = \sec^2(x) - 1}$$

otherwise, we should try integration by parts

$\int u\, dv \rightarrow$ $u =$ $\quad \int dv =$

$du =$ $\quad v$

$= uv - \int v\, du$

$$\int \tan(x)\,dx = \ln|\sec(x)| + C$$

$$\int \sec(x)\,dx = \ln|\sec(x) + \tan(x)| + C$$

Ex

$$\int \cos^3(x)\, dx = ? \rightarrow \int \underbrace{\cos^2(x)}_{1-u^2} \underbrace{\cos(x)\,dx}_{\text{reserved for } du} = \int (1 - u^2)\, du$$

$\sin^{0}(x)$ → even

$\boxed{u = \sin(x)}$ → $\sin^2(x) + \cos^2(x) = 1$

$\boxed{du = \cos(x)\,dx}$ $\quad u^2 + \cos^2(x) = 1$

$$= \int 1\, du - \int u^2\, du$$

$$= u - \frac{u^3}{3} + C$$

$$= \sin(x) - \frac{\sin^3(x)}{3} + C$$

Ex

$$\int \sin^5(x)\, dx = ? \rightarrow = -\int \underbrace{\sin^4(x)}_{(1-u^2)^2} \cdot \underbrace{\sin(x)\,dx}_{du} = -\int (1-u^2)^2 du$$

$\cos^{0}(x)$ → even

Ex

$$\int \sin^5(x)\,dx = ? \rightarrow = \ominus \int \underbrace{\sin^4(x)}_{(1-u^2)^2} \cdot \underbrace{\sin(x)\,dx}_{du} = -\int (1-u^2)^2\,du$$

$\cos^0(x)$ even

$\boxed{u=\cos(x)} \longrightarrow \sin^2+\cos^2=1$, $\sin^2+u^2=1$, $\sin^2=(1-u^2)^2$, $\sin^4=$

$du=-\sin(x)\,dx$

$$= -\int (1-2u^2+u^4)\,du$$

$$= -u + \frac{2u^3}{3} - \frac{u^5}{5} + C$$

Ex

$$\int \sin^6(x)\cos^3(x)\,dx = \int \underbrace{\sin^6(x)}_{u^6}\,\underbrace{\cos^2(x)}_{(1-u^2)}\,\underbrace{\cos(x)\,dx}_{du} = \int u^6(1-u^2)\,du$$

even

$\boxed{u=\sin(x)} \rightarrow \sin^2(x)+\cos^2(x)=1$

$du=\cos(x)\,dx$

$$= \int u^6 - u^8\,du$$

$$= \frac{u^7}{7} - \frac{u^9}{9} + C$$

$$= \frac{\sin^7(x)}{7} - \frac{\sin^9(x)}{9} + C$$

Ex

$$\int \sin^3(x)\cos^5(x)\,dx = \int \underbrace{\sin^3(x)}_{u^3}\,\underbrace{\cos^4(x)}_{(1-u^2)^2}\,\underbrace{\cos(x)\,dx}_{du}$$

$\boxed{u=\sin(x)} \longrightarrow \sin^2+\cos^2=1$, $u^2+\cos^2=1$, $\cos^2=1-u^2$

$du=\cos(x)\,dx$

$$= \int u^3(1-u^2)^2\,du$$

$$= \int u^3(1-2u^2+u^4)\,du$$

$$= \frac{u^4}{4} - 2\frac{u^6}{6} + \frac{u^8}{8} + C$$

2nd way

$\boxed{u=\cos(x)}$

$du=-\sin(x)\,dx$

$$= -\int \underbrace{\cos^5(x)}_{u^5}\,\underbrace{\sin^2(x)}_{1-u^2}\,\underbrace{\sin(x)\,dx}_{du}$$

$$= -\int u^5(1-u^2)\,du = -\frac{u^6}{6} + \frac{u^8}{8} + C$$

Ex

$$\int \sin^2(x)\cos^4(x)\,dx = \int \left(\frac{1-\cos 2x}{2}\right)\left(\frac{1+\cos 2x}{2}\right)^2 dx$$

use half-angle formulas

$$\sin^2(x) = \frac{1-\cos(2x)}{2}$$

$$\cos^2(x) = \frac{1+\cos(2x)}{2}$$

$1-\cos^2 2x = \sin^2 2x$

$$= \int \frac{(1-\cos 2x)(1+\cos 2x)(1+\cos 2x)}{8}\,dx$$

$$= \frac{1}{8}\int \sin^2 2x\,(1+\cos 2x)\,dx = \frac{1}{8}\left[\int \sin^2 2x\,dx + \int \underbrace{\sin^2 2x}_{u^2}\,\underbrace{\cos 2x\,dx}_{\frac{1}{2}du}\right]$$

$\sin^2 2x = \frac{1-\cos 4x}{2}$

$u=\sin 2x$

$du = 2\cos 2x\,dx$

$$\int \frac{1}{2}dx - \int \frac{\cos 4x}{2}dx$$

$$= \frac{1}{8}\left[\left(\frac{x}{2} - \frac{\sin 4x}{8}\right) + \frac{\sin^3(2x)}{3}\right] + C$$

Ex

$$\int \sin^4(x)\,dx = \int \left(\frac{1-\cos 2x}{2}\right)^2 dx = \frac{1}{4}\int \left(1 - 2\cos 2x + \cos^2 2x\right) dx$$

$$\sin^2(x) = \frac{1-\cos 2x}{2}$$

$(1-\cos 2x)^2$

$$= \frac{1}{4}\left[\int 1\,dx - \int 2\cos 2x\,dx + \int \cos^2 2x\,dx\right]$$

$\cos^2 2x = \frac{1+\cos 4x}{2}$

$$= \frac{1}{4}\left[x - \sin 2x + \frac{x}{2} + \frac{\sin 4x}{8}\right] + C$$

Ex

$$\int \sec^4(x)\tan^6(x)\,dx = \int \underbrace{\tan^6(x)}_{u^6}\cdot\underbrace{\sec^2(x)}_{(1+u^2)}\cdot\underbrace{\sec^2(x)\,dx}_{du} = \int u^6(1+u^2)\,du$$

$$= \frac{u^7}{7} + \frac{u^9}{9} + C$$

$u = \tan(x)$ → $1+\tan^2 x = \sec^2 x$

$du = \sec^2(x)\,dx$

Ex

$$\int \sec^6(x)\tan^3(x)\,dx = \int \underbrace{\sec^5(x)}_{u^5}\cdot\underbrace{\tan^2(x)}_{(u^2-1)}\cdot\underbrace{\sec(x)\tan(x)\,dx}_{du} = \int u^5(u^2-1)\,du$$

$$= \frac{u^8}{8} - \frac{u^6}{6} + C$$

$u = \sec(x)$

$du = \sec(x)\tan(x)\,dx$

$1+\tan^2 x = \sec^2 x$

$1+\tan^2 x = u^2$

Ex

$$I: \int \sec(x)\overset{\sec^2(x)-1}{\tan^2(x)}\,dx = \int u\,dv = uv - \int v\,du.$$

use integration by parts.

$$= \int \left(\sec^3(x) - \sec(x)\right) dx = \int \sec^3(x)\,dx - \int \sec(x)\,dx$$

$\ln(\sec(x)+\tan(x))$

$$I = \int \cancel{\sec^3(x)}\,dx - \ln|\sec(x)+\tan(x)|$$

$$I = \sec(x)\tan(x) - \int \cancel{\sec^3(x)}\,dx$$

$$\Rightarrow 2I = \sec(x)\tan(x) - \ln|\sec(x)+\tan(x)|$$

$$I = \frac{\sec(x)\tan(x) - \ln|\sec(x)+\tan(x)|}{2}$$

$$\int \sec^3(x)\,dx = uv - \int v\,du = \sec(x)\tan(x) - \int \tan(x)\sec(x)\tan(x)\,dx$$

$u = \sec(x)$ $\quad dv = \sec^2(x)\,dx$

$du = \sec(x)\tan(x)\,dx$ $\quad v = \tan(x)$

$$\int \sec(x)\tan^2(x)\,dx \qquad \int \sec^3(x)\,dx$$