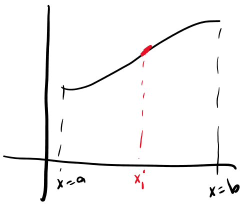
5th Week Thursday

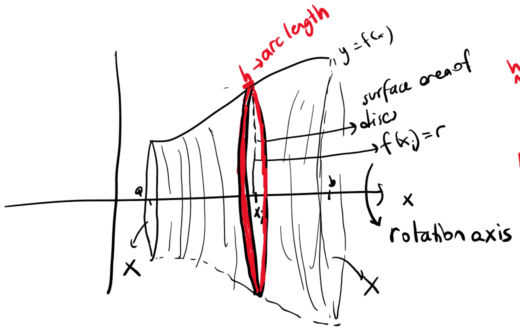
23 Mart 2023 Perşembe 11:43

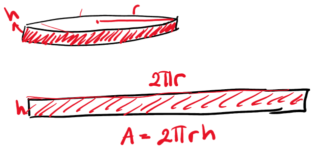
Arc Length



$$L = \int \sqrt{1 + f'(x)^2}\, dx$$

Area of a Surface of Revolution

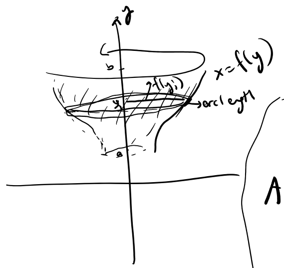




$$\lim_{n\to\infty} \sum_{i=1}^{n} 2\pi \underset{\text{radius}}{f(x_i)} \cdot \underset{\text{height}}{\text{arc length}} = \lim_{n\to\infty} \sum_{i=1}^{n} 2\pi f(x_i) \sqrt{1 + f'(x_i)^2}\, \Delta x_i$$

x-axis

$$\boxed{A = \int_{x=a}^{x=b} 2\pi \underset{\text{radius}}{f(x)} \underbrace{\sqrt{1 + f'(x)^2}}_{\text{arc length}}\, dx}$$



$$\boxed{A = \int_{y=a}^{y=b} 2\pi \underset{\text{radius}}{f(y)} \sqrt{1 + f'(y)^2}\, dy}$$

Ex/ Find the area of the surface generated by rotating the curve $y = x^2$ between the points $(1,1)$ and $(2,4)$ about the y-axis.

$1 \leq x \leq 2$

$y = f(x)$

$x = f(y)$ ←

$$A = \int_{y=1}^{y=4} 2\pi f(y) \sqrt{1 + f'(y)^2}\, dy = \int_{y=1}^{y=4} 2\pi \sqrt{y} \sqrt{1 + \frac{1}{4y}}\, dy =$$

$$\to \frac{\sqrt{4y+1}}{\sqrt{4y}} = \frac{\sqrt{4y+1}}{2\sqrt{y}}$$

$y = x^2$ $\quad x = \sqrt{y}$ $\quad f(y) = \sqrt{y}$ $\quad f'(y) = \dfrac{1}{2\sqrt{y}}$ $\quad f'(y)^2 = \dfrac{1}{4y}$

$$= \int_{y=1}^{y=4} \cancel{2}\pi \cancel{\sqrt{y}} \frac{\sqrt{4y+1}}{\cancel{2\sqrt{y}}}\, dy = \int_{y=1}^{y=4} \pi \sqrt{4y+1}\, dy = \frac{\pi}{6} (4y+1)^{3/2} \Big]_{y=1}^{y=4} = \frac{\pi}{6}\left[17^{3/2} - 5^{3/2}\right] br^2$$

$u = 4y+1 \quad du = 4dy$

$$\int \frac{1}{4} \sqrt{u}\, du = \frac{\cancel{2}}{3} \cdot \frac{1}{\cancel{4}_2} u^{3/2}$$

Ex.

Find the area of the surface generated by rotating the curve $y=\sqrt{4-x^2}$ between $-1 \le x \le 1$ about the x-axis.

$$A=\int_{x=-1}^{x=1} 2\pi f(x)\sqrt{1+f'(x)^2}\,dx = \int_{x=-1}^{x=1} 2\pi\sqrt{4-x^2}\sqrt{1+\frac{x^2}{4-x^2}}\,dx = \int_{x=-1}^{x=1} 2\pi \cancel{\sqrt{4-x^2}}\,\frac{2}{\cancel{\sqrt{4-x^2}}}\,dx$$

$$\frac{\sqrt{4-x^2+x^2}}{\sqrt{4-x^2}} \rightarrow 2$$

$$f(x)=\sqrt{4-x^2} \qquad f'(x) = -\cancel{2}x\cdot\frac{1}{\cancel{2}\sqrt{4-x^2}} \qquad f'(x)^2=\frac{x^2}{4-x^2}$$

$$=4\pi x\Big]_{x=-1}^{x=1} = 4\pi-(-4\pi) = 8\pi$$

Ex. Find the area of the surface generated by rotating the curve $y=e^x$ between $0 \le x \le 1$ about the x-axis.

$$A=\int_{x=0}^{x=1} 2\pi f(x)\sqrt{1+f'(x)^2}\,dx = \boxed{\int_{x=0}^{x=1} 2\pi e^x\sqrt{1+e^{2x}}\,dx}$$

$\cancel{u=1+e^{2x}}$ $\cancel{du=2e^{2x}dx}$

$\cancel{u=\sqrt{1+e^{2x}}}$ $\cancel{u^2=1+e^{2x}}$ $\cancel{2u\,du=2e^{2x}dx}$

$\boxed{u=e^x \quad du=e^x dx}$

$$f(x)=e^x \qquad f'(x)=e^x \qquad f'(x)^2=e^{2x}$$

$$\int 2\pi\sqrt{1+u^2}\,du = \int 2\pi \sec\theta\sec^2\theta\,d\theta = \int \cancel{2}\pi\sec^3\theta\,d\theta = \pi\left(\sqrt{1+e^{2x}}\cdot e^x + \ln\left|\sqrt{1+e^x}+e^x\right|\right)\Bigg]_{x=0}^{x=1}$$

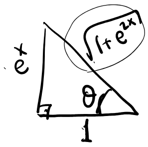
$u=e^x$

$\boxed{u=\tan\theta}$

$du=\sec^2\theta\,d\theta$

$\sec\theta$

$$\frac{1}{\cancel{2}}\left(\sec\theta\tan\theta+\ln|\sec\theta+\tan\theta|\right)$$



$$=\pi\left[\sqrt{1+e^2}\cdot e+\ln\left|\sqrt{1+e^2}+e\right| - \sqrt{2}-\ln\left|\sqrt{2}+1\right|\right]$$

Ex. Find the area of the surface generated by rotating the curve $y=\sqrt[3]{x}$ between $1 \le y \le 2$ about the y-axis.

$y=f(x)$ $\quad x=f(y)$

$y=\sqrt[3]{x}$ $\quad x=y^3$ $\quad f(y)=y^3$

$$A=\int_{y=1}^{y=2} 2\pi f(y)\sqrt{1+f'(y)^2}\,dy = \int_{y=1}^{y=2} 2\pi y^3\sqrt{1+9y^4}\,dy = \frac{\pi}{27}(1+9y^4)^{3/2}\Big]_{y=1}^{y=2} = \frac{\pi}{27}\left[145^{3/2}-10^{3/2}\right]$$

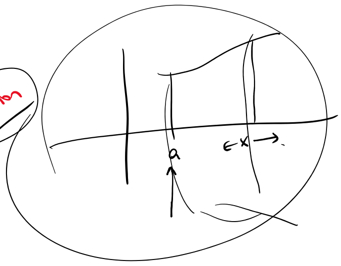
$$\int_{y=1} 2\pi f(y)\sqrt{1+f'(y)^2}\,dy = \int_{y=1} 2\pi y^3\sqrt{1+9y^4}\,dy = \frac{\pi}{27}(1+9y^4)\Big]_{y=1} = \frac{\pi}{27}\left[145 - \ldots\right]$$

$$u = 1+9y^4$$
$$du = 36y^3\,dy$$

$$\int \frac{2\pi}{36}u^{1/2}\,dy \rightarrow \frac{2\pi}{36}\cdot\frac{2}{3}u^{3/2}$$

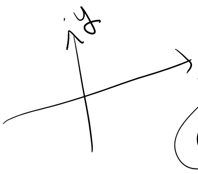
$f(y) = y^3$ $\quad f'(y) = 3y^2$ $\quad f'(y)^2 = 9y^4$

Surface Area Function



$$A(x) = \int_{a}^{\boxed{x}} 2\pi f(t)\sqrt{1+f'(t)^2}\,dt$$

X cinsinden



Cartesian

Calculus $\longrightarrow$ Parametric

Satır

Tür
Integr
Yay İnal

Polar

Jüpit