11. Hafta Perşembe Dersi

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$\sum_{n=1}^{\infty} ar^{n-1} \to$ geometrik serisi

$|r|<1 \Rightarrow = \frac{a}{1-r}$

$|r|\geq 1 \Rightarrow$ ıraksaktır.

$\frac{a_{n+1}}{a_n} = \frac{\cancel{a}.r^{n+1-1}}{\cancel{a}\, r^{n-1}} = r$

Örn $\sum_{n=1}^{\infty} \frac{3^{n-1}-1}{6^{n-1}} = ?$

$$\sum_{n=1}^{\infty}\left(\frac{3^{n-1}}{6^{n-1}} - \frac{1}{6^{n-1}}\right) \overset{?\checkmark}{=} \sum_{n=1}^{\infty}\left(\frac{1}{2}\right)^{n-1} - \sum_{n=1}^{\infty}\frac{1}{6^{n-1}}$$

$a=1$, $r=\frac{1}{2}$; $a=1$, $r=\frac{1}{6}$

$$= \frac{1}{1-\frac{1}{2}} - \frac{1}{1-\frac{1}{6}} = 2 - \frac{6}{5} = \frac{4}{5}$$

Örn $\sum_{n=1}^{\infty} \frac{4}{n^2+4n+3} = ?$

$$\frac{4}{(n+3)(n+1)} = \frac{A}{(n+3)} + \frac{B}{(n+1)} = \frac{-2}{n+3} + \frac{2}{n+1}$$

$An + A + Bn + 3B = 4$

$A+B=0$

$A+3B=4$

$2B=4$, $B=2$ $\Rightarrow A=-2$

$= \lim_{n\to\infty} s_n$

$$s_n = \underbrace{\left(\frac{-2}{4}+\frac{2}{2}\right)}_{a_1} + \underbrace{\left(\frac{-2}{5}+\frac{2}{3}\right)}_{a_2} + \underbrace{\left(\frac{-2}{6}+\frac{2}{4}\right)}_{a_3} + \underbrace{\left(\frac{-2}{7}+\frac{2}{5}\right)}_{a_4} + \underbrace{\left(\frac{-2}{8}+\frac{2}{6}\right)}_{a_5} + \dots$$

$$+ \left(\frac{-2}{n-1+3} + \right) + \left(\frac{-2}{n+3} + \right)$$

$n-1+3$; $n-1$; n

$$s_n = 1 + \frac{2}{3} - \frac{2}{n+2} - \frac{2}{n+3}$$

$$\lim_{n\to\infty} s_n = \lim_{n\to\infty} 1 + \frac{2}{3} - \frac{2}{n+2} - \frac{2}{n+3} = 1 + \frac{2}{3} = \frac{5}{3}$$

Örn $\sum_{n=1}^{\infty} \frac{2n+1}{n^2.2^n}$ yakınsaklık durumu?

$b_n = \sum \frac{1}{n}$ harmonik seri $\to$ ıraksak

Limit Karşılaştırma $\lim_{n\to\infty} \frac{2n+1}{\cancel{n^2}.2^n} \cdot \frac{\cancel{n}}{1} \overset{L'}{\Rightarrow} \lim_{n\to\infty} \frac{2}{1.2^n + n.2^n.\ln 2} = 0 \Rightarrow$ çalışmadı

a_n

Karşılaştırma $\quad \lim_{n\to\infty} \frac{\cdots}{n^2 \cdot 2^n} \quad \lim_{n\to\infty} (n \cdot 2^n + n \cdot 2^n \cdots)$

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$\lim_{n\to\infty} \frac{a_n}{b_n}$

$b_n = \sum \frac{1}{2^n}$ geometrik seri $a = \frac{1}{2}$ $r = \frac{1}{2} < 1 \rightarrow$ yakınsak $\Big\} \Rightarrow \sum a_n$ yakınsak

Limit karşılaştırma $\lim_{n\to\infty} \frac{2n+1}{n^2 \cdot \cancel{2^n}} \cdot \frac{\cancel{2^n}}{1} = \lim_{n\to\infty} \frac{2n+1}{n^2} = 0$

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Oran testi $\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = \lim_{n\to\infty} \frac{2(n+1)+1}{(n+1)^2 \cdot \cancel{2^{n+1}}_{2}} \cdot \frac{n^2 \cdot \cancel{2^n}}{2n+1} = \lim_{n\to\infty} \frac{(2n+3)n^2}{(n+1)^2 \, 2 \, (2n+1)} \rightarrow \frac{2n^3}{4n^3} = \frac{1}{2} < 1$

$\Rightarrow$ yakınsaktır.

Örn $\sum_{n=1}^{\infty} \frac{1+\ln n}{\sqrt[3]{n}}$

$\underbrace{\frac{1+\ln n}{\sqrt[3]{n}}}_{\text{ıraksaktır.}} > \frac{1}{\sqrt[3]{n}}$ $\quad (\ln n > 1)$ $\quad p = \frac{1}{3} < 1 \Rightarrow$ ıraksak

$\lim_{n\to\infty} \frac{1+\ln n}{\sqrt[3]{n}} \cdot \frac{\sqrt[3]{n}}{1} \quad \infty \quad b_n \rightarrow$ ıraksak $\Big\} \Rightarrow \sum a_n$ ıraksaktır

Örn $\sum_{n=1}^{\infty} \left(\sqrt{n^3} - \sqrt{n^3 - 1} \right)$ yakınsaklık durumu.

$b_n = \frac{1}{\sqrt{n^3}}$ p-serisi $p = \frac{3}{2} > 1 \Rightarrow$ yakınsaktır.

Limit karşılaştırma

$\lim_{n\to\infty} \frac{a_n}{b_n} = \lim_{n\to\infty} \left(\sqrt{n^3} - \sqrt{n^3-1} \right) \sqrt{n^3}$

$n^3 - (n^3 - 1) = 1$

$= \lim_{n\to\infty} \frac{\left(\sqrt{n^3} - \sqrt{n^3-1}\right)\left(\sqrt{n^3} + \sqrt{n^3-1}\right)\sqrt{n^3}}{\left(\sqrt{n^3} + \sqrt{n^3-1}\right)}$

$n\to\infty \quad \frac{1}{n} \to 0$

$= \lim_{n\to\infty} \frac{\sqrt{n^3}}{\left(\sqrt{n^3} + \sqrt{n^3-1}\right)} \xrightarrow{\sqrt{n^3} \text{ 'e bölelim}} = \lim_{n\to\infty} \frac{1}{\left(1 + \sqrt{1 - \frac{1}{n^3}}\right)} = \frac{1}{2} > 0$

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$\Rightarrow$ iki seri aynı durumdadır.

$\Rightarrow$ Seri yakınsaktır.

$b_n = \sum \sqrt{n^3} \Rightarrow$ ıraksak alıp deneseydik;

$\lim_{n\to\infty} \frac{a_n}{b_n} = \lim_{n\to\infty} \frac{\sqrt{n^3} - \sqrt{n^3-1}}{\sqrt{n^3}} = \frac{1 - \sqrt{1 - \frac{1}{n^3}}}{1} = \frac{1-1}{1} = 0$

$\nexists$ limit karşılaştırma çalışmaz.