3. Hafta 1.Tip Genelleştirilmiş İntegraller

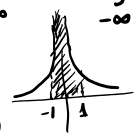
09 Mart 2021 Salı 12:46

Genelleştirilmiş İntegraller ⇝ Yakınsak, Iraksak → hesaplanamaz.

1. Tip : Sonsuz Aralıklar (Yatay asimptotlar); $\int_a^\infty f(x)\,dx$, $\int_{-\infty}^a f(x)\,dx$, $\int_{-\infty}^\infty f(x)\,dx$

2. Tip : Sonsuz Süreksizlik (Dikey asimptotlar) → $[a,b]$; $\int_a^b f(x)\,dx$

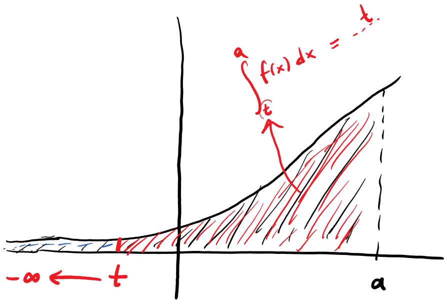
$0\in[-1,1] \leftarrow \int_{-1}^{1} \frac{1}{x^2}$



$-\frac{1}{2x}\Big]_{-1}^{1}$

$-\frac{1}{2}-\left(\frac{1}{2}\right)=-\frac{1}{4}$

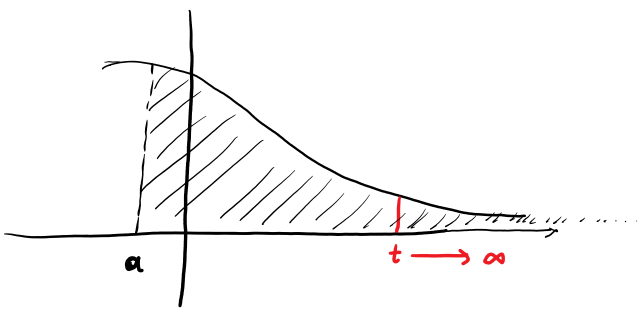
1. Tip Sonsuz Aralıklar



$$\int_{-\infty}^{a} f(x)\,dx = \lim_{t\to-\infty}\left(\int_{t}^{a} f(x)\,dx\right) \to$$

Limit varsa yakınsaktır. $=L$

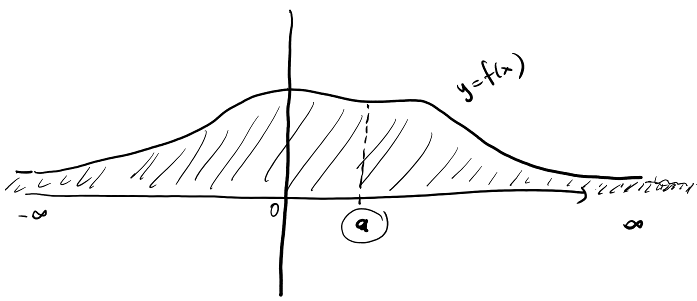
Limit yoksa, sonsuzsa ıraksaktır.



$$\int_{a}^{\infty} f(x)\,dx = \lim_{t\to\infty}\left(\int_{a}^{t} f(x)\,dx\right) \to$$

Limit varsa yakınsaktır $=L$

Limit yoksa, sonsuzsa ıraksaktır.



$$\int_{-\infty}^{\infty} f(x)\,dx = \underbrace{\int_{-\infty}^{a} f(x)\,dx}\ + \underbrace{\int_{a}^{\infty} f(x)\,dx}$$

yakınsaktır. ← ikisi de yakınsak

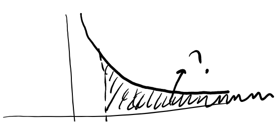
$\boxed{\text{Iraksaktır.}}$ ← herhangi biri ıraksak

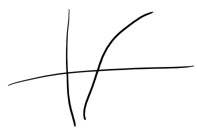
$$\lim_{t\to-\infty}\int_t^a f(x)\,dx + \lim_{t\to\infty}\int_a^t f(x)\,dx$$

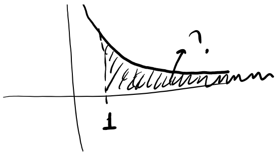
Örn

$$\int_1^\infty \frac{1}{x}\,dx = \lim_{t\to\infty}\left(\int_1^t \frac{1}{x}\,dx\right) = \lim_{t\to\infty}\left(\ln|x|\Big]_1^t\right)$$

$$= \lim_{t\to\infty}\left(\underset{\infty}{\ln|t|} - \underset{0}{\ln 1}\right)$$







$= \infty \longrightarrow$ ıraksaktır.

Örn

$$\int_{-\infty}^{0} x e^{x}\,dx = \lim_{t\to-\infty}\left(\int_{t}^{0} x e^{x}\,dx\right)$$

$u = x \qquad dv = e^{x}dx$

$u.v - \int v\,du = x.e^{x} - e^{x}$

$$= \lim_{t\to-\infty}\left(\left(xe^{x}-e^{x}\right)\Big]_{t}^{0}\right) = \lim_{t\to-\infty}\left(-1-\left(te^{t}-e^{t}\right)\right)$$

$0.e^{0} - e^{0}$

$\lim_{t\to-\infty} e^{t} = e^{-\infty} = \frac{1}{e^{\infty}} = \frac{1}{\infty} = 0$

$\lim_{t\to-\infty} \frac{t}{e^{-t}} \overset{L'}{=}$ $e^{-t} \to e^{-(-\infty)} = e^{\infty} = \infty$

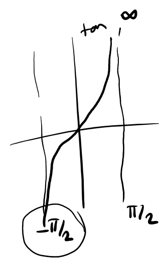
$L' \to \lim_{t\to-\infty} \frac{1}{-e^{-t}} = -\frac{1}{\infty} = 0$

$$= -1-0+0 = -1 \qquad \to \text{Yakınsaktır.}$$

Örn

$$\int_{-\infty}^{\infty} \frac{1}{1+x^{2}}\,dx = \int_{-\infty}^{0} \frac{1}{1+x^{2}}\,dx + \int_{0}^{\infty} \frac{1}{1+x^{2}}\,dx$$

$$= \lim_{t\to-\infty}\left(\arctan x\right)_{t}^{0} + \lim_{t\to\infty}\left(\arctan x\right)_{0}^{t}$$



$$= \lim_{t\to-\infty}\left(\arctan 0 - \arctan t\right) + \underbrace{\lim_{t\to\infty}\left(\arctan t - \arctan 0\right)}_{\frac{\pi}{2}}$$

$$= 0 - \left(-\frac{\pi}{2}\right) + \frac{\pi}{2} - 0$$

$$= \frac{\pi}{2} + \frac{\pi}{2} = \pi \qquad \to \text{yakınsak}$$