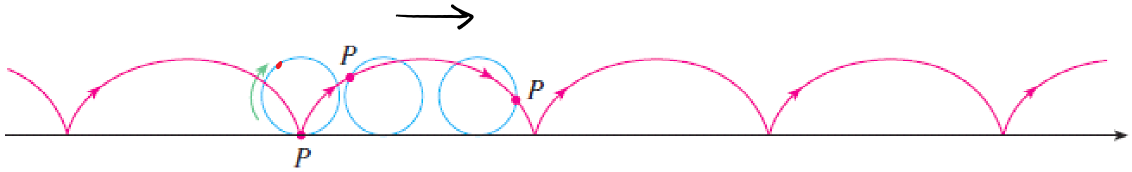
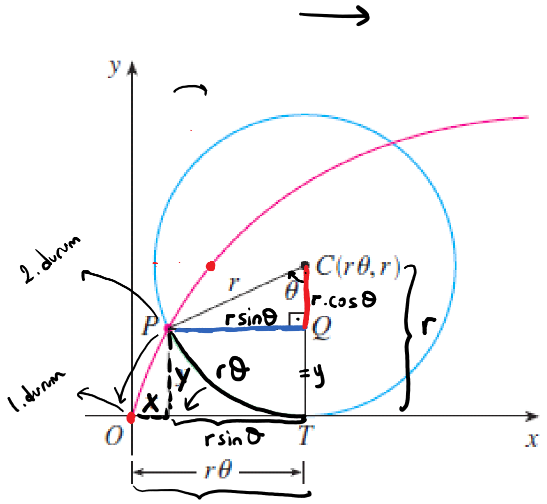
5. Hafta Perşembe Dersi

25 Mart 2021 Perşembe 11:36

Yuvarlanma Eğrisi (Cycloid)





Tekerlek θ açısı kadar döndüğünde P noktası O → P 'ye geliyor.

P → (0,0) P → (x,y) $\boxed{x=?\ y=?}$

$$\overset{\frown}{PT} = \cancel{2\pi} r \cdot \frac{\theta}{\cancel{2\pi}} = r\theta$$

$$\boxed{\begin{aligned} y &= r - r\cos\theta \\ x &= r\theta - r\sin\theta \end{aligned}}$$

$r \to y$

$\pi/2$ 1 $r\left(\frac{\pi}{2}-1\right) \to x$

Örn $x = \tan t + \sec t$, $y = \tan t - \sec t$, $t \in I$. Kartezyen denklemini bulalım.

$$\boxed{1+\tan^2 t = \sec^2 t}$$

$\frac{\sin^2 t}{\cos^2 t}$ $\frac{1}{\cos^2 t}$

$$xy = (\tan t + \sec t)(\tan t - \sec t) = \tan^2 t - \sec^2 t = -1$$

$$\Rightarrow xy = -1 \quad \Rightarrow \boxed{y = -\frac{1}{x}}$$

Örn $x = \cos t$

$$y = \cos 2t = 2\cos^2 t - 1 = 2x^2 - 1$$

$$\boxed{y = 2x^2 - 1}$$

$\cos(2x) = 2\cos^2(x) - 1$

$\sin(2x) = 2\sin(x)\cos(x)$

Parametrik Eğrilerde Yay Uzunluğu

$$\lim_{n\to\infty} \sum_{i=1}^{n} \underbrace{\sqrt{(\Delta x_i)^2+(\Delta y_i)^2}}_{|P_{i-1}P_i|}$$

$$\boxed{\begin{aligned} x &= f(t) \\ y &= g(t) \end{aligned}}$$

$$\boxed{L=\int_{t=a}^{t=b} \sqrt{f'(t)^2+g'(t)^2}\, dt}$$

Örn

$x = r\cos t$

$y = r\sin t$

$0 < t < 2\pi$

Eğrinin uzunluğu nedir?

$$L = \int_0^{2\pi} \sqrt{\underbrace{r^2\sin^2 t + r^2\cos^2 t}_{r^2(\underbrace{\sin^2 t+\cos^2 t}_{1})}}\, dt$$

$\dfrac{dx}{dt} = -r\sin t$

$\dfrac{dy}{dt} = r\cos t$

$$= \int_0^{2\pi} r\cdot dt = r.t\Big]_{t=0}^{t=2\pi} = 2\pi r - 0 = 2\pi r$$

Örn

$x = t - \sin t = f(t)$

$y = 1 - \cos t = g(t)$

$0 \leq t \leq 2\pi$

Eğrinin uzunluğu nedir?

$\dfrac{dx}{dt} = 1-\cos t$

$\dfrac{dy}{dt} = \sin t$

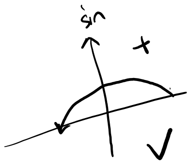
$$L = \int_{t=0}^{t=2\pi} \sqrt{f'(t)^2+g'(t)^2}\, dt$$

$$L = \int_0^{2\pi} \sqrt{(1-\cos t)^2+\sin^2 t}\, dt$$

$$L = \int_0^{2\pi} \sqrt{1-2\cos t+\underbrace{\cos^2 t+\sin^2 t}_{1}}\, dt = \int_0^{2\pi} \sqrt{2(1-\cos t)}\, dt$$

$$L = \int_0 \sqrt{1 - 2\cos t + \underbrace{\cos^2 t + \sin^2 t}_{1}}\, dt = \int_0 \sqrt{2(\underbrace{1-\cos t}_{})}\, dt$$

$2\sin^2 x = 1 - \cos 2x$



$$L = \int_0^{2\pi} \sqrt{2 \cdot 2 \sin^2 \frac{t}{2}}\, dt$$

$$L = 2 \int_0^{2\pi} \left| \sin \frac{t}{2} \right| = -4\cos \frac{t}{2} \Big]_{t=0}^{t=2\pi} = 4 - (-4) = \underline{\underline{8}}$$

$0 < \frac{t}{2} < \pi$

Örn/ $x = 1 + e^t = f(t)$ eğrinin x-ekseni ile arasında kalan alan nedir?

$y = t - t^2 = g(t)$

$0 \leq t \leq 1$

$$A = \int_0^1 g(t)\, f'(t)\, dt$$

$$\frac{dx}{dt} = 0 + e^t$$

$$A = \int_{t=0}^{t=1} (t - t^2) e^t\, dt = \ldots\ldots$$

Parametrik Eğrilerde Dönel Yüzey Alanı $= \boxed{\int_{t=a}^{t=b} 2\pi\, g(t) \cdot \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt}$

$x = f(t)$
$y = g(t)$

Consider a circle of radius $a = 4$ centered at $(0, a)$, as in the figure. Let a line from the origin O to a point A on the circle intersect the line $y = 2a$ at B. Finally, let C be the point of intersection of a horizontal line through A and a vertical line through B. As t, the angle OA makes with the positive x-axis varies, point C traces out a curve called the **witch of Agnesi**.

(a) Find a vector-parametric equation for the point A in terms of the parameter t. Your answer should be of the form $\langle x(t), y(t) \rangle$ and include the angle brackets.

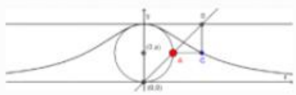
$\vec{r}_A(t) =$

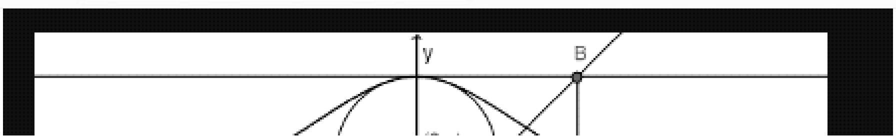
(b) Find a vector-parametric equation for the point B in terms of the parameter t.

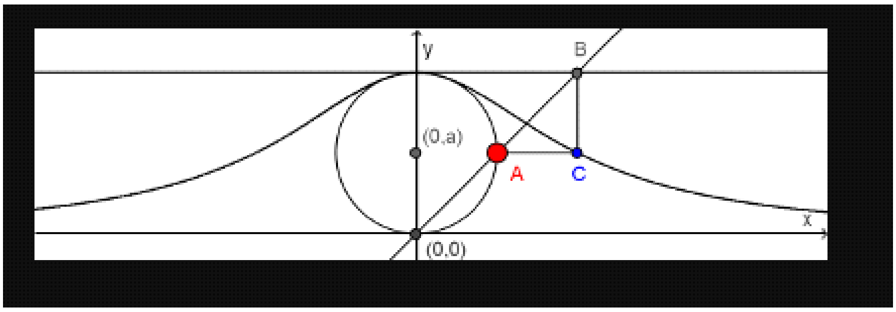
$\vec{r}_B(t) =$

(c) Find a vector-parametric equation for the point C in terms of the parameter t.

$\vec{r}_C(t) =$






y
B
(0,a)
A
C
x
(0,0)