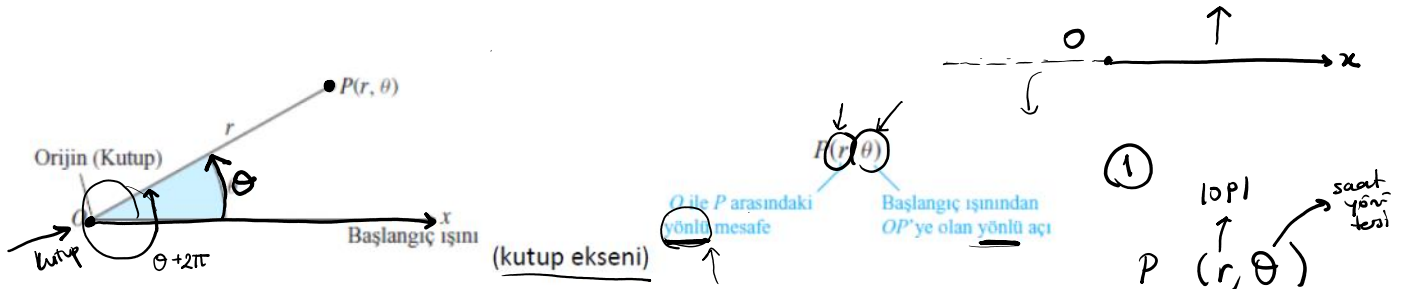


## (POLAR)

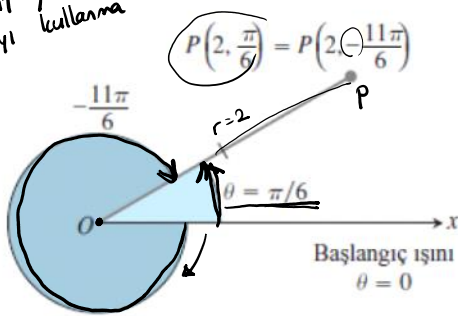
## #10.3 KUTUPSAL KOORDİNATLAR



! Bir noktayı temsil eden sonsuz sayıda koordinat vardır;

negatif yönlü kullanma  
ağırlı

②

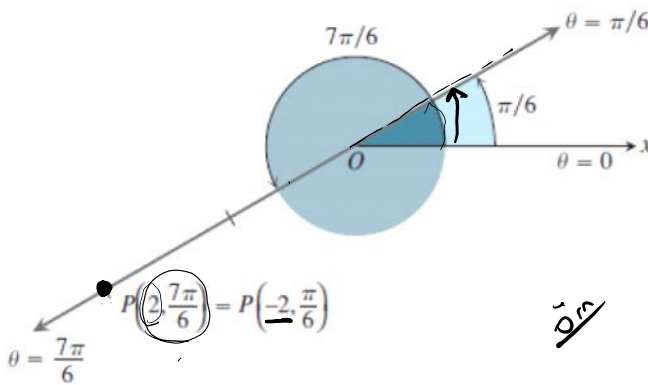


$$2i - \frac{11}{6}$$

$$P(2, \frac{\pi}{6}) = P(2, -\frac{11\pi}{6})$$

①  $|OP|$   
 $P(r, \theta)$   
 $= P(r, \theta \pm 2\pi)$   
 $\neq (r, \theta \pm 2n\pi)$

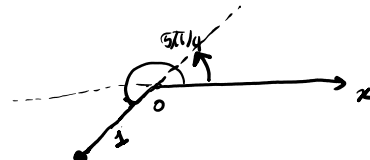
! Negatif r değeri durumu;



$\pm 2\pi k$

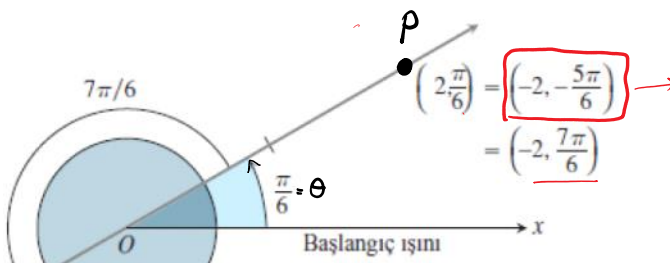
$(r, \theta)$   
 $(-r, \theta - \pi)$   
 $(-r, \theta)$   
 $(r, \theta + \pi)$

$$P(1, \frac{5\pi}{4}) = (-1, \frac{\pi}{4})$$

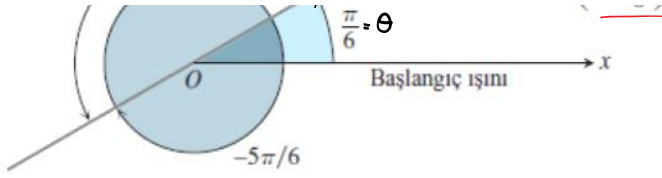


Örnek.

$P(2, \pi/6)$  noktasının tüm kutupsal koordinatları;



$r \rightarrow -r$   
 $(-2, -\frac{5\pi}{6})$



$$\boxed{r \rightarrow -r} \quad \left(-2, -\frac{5\pi}{6}\right) \quad +2\pi$$

$$\frac{\pi}{6} - \pi = \left(-\frac{5\pi}{6}\right)$$

$r = 2$  için, açılarının tam bir listesi

$$\left(\frac{\pi}{6}, \frac{\pi}{6} \pm 2\pi, \frac{\pi}{6} \pm 4\pi, \frac{\pi}{6} \pm 6\pi, \dots\right)$$

$r = -2$  içinse açılar

$$\left(-\frac{5\pi}{6}, -\frac{5\pi}{6} \pm 2\pi, -\frac{5\pi}{6} \pm 4\pi, -\frac{5\pi}{6} \pm 6\pi, \dots\right)$$

Bir noktanın tüm kutupsal koordinatları

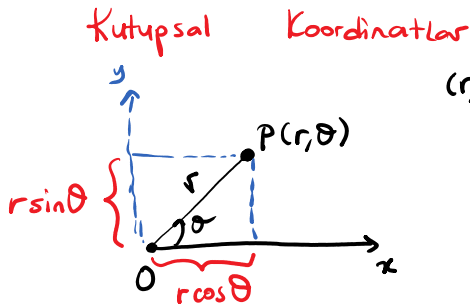
Yani;

$$\left(2, \frac{\pi}{6} + 2n\pi\right), \quad n = 0, \pm 1, \pm 2, \dots$$

$$\left(-2, -\frac{5\pi}{6} + 2n\pi\right), \quad n = 0, \pm 1, \pm 2, \dots$$

$$P(r, \theta) = (r, \theta + 2\pi k)$$

$$= (-r, (\theta - \pi) + 2\pi k)$$



$(r, \theta)$

VS.



Kartzyen Koordinatlar  $(x, y)$

$$\begin{cases} x = r \cos \theta \\ y = r \sin \theta \end{cases}$$

$$\begin{aligned} x^2 + y^2 &= r^2 \cos^2 \theta + r^2 \sin^2 \theta \\ &= r^2 \Rightarrow \boxed{r = \sqrt{x^2 + y^2}} \end{aligned}$$

$$\frac{y}{x} = \frac{r \sin \theta}{r \cos \theta} = \tan \theta \Rightarrow \boxed{\theta = \arctan\left(\frac{y}{x}\right)}$$

\*  $(r, \theta)$  biliniyorsa

$$x = r \cos \theta \quad y = r \sin \theta \rightarrow (x, y)$$



$(x, y)$  biliniyorsa

$$(r, \theta) = ? \quad \left( \sqrt{x^2 + y^2}, \arctan\left(\frac{y}{x}\right) \right)$$

ör

$$P(2, \pi/3)$$

kartzyen ?

$$\begin{cases} x = r \cos \theta = 2 \cdot \cos \pi/3 = 1 \\ y = r \sin \theta = 2 \cdot \sin \pi/3 = \sqrt{3} \end{cases} \Rightarrow (1, \sqrt{3})$$

ör

$$(1, -1)$$

hangisi bölgede? kutupsal (polar) ?

$$x = 1$$

$$y = -1$$

$$r^2 = x^2 + y^2 = 1^2 + (-1)^2 = 2$$

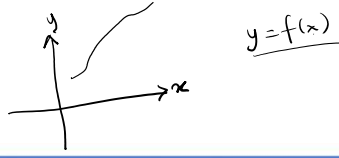
$$r = \sqrt{2}$$

$$\theta = -\frac{\pi}{4}$$

$$\theta = \arctan\left(\frac{y}{x}\right) = \arctan(-1)$$

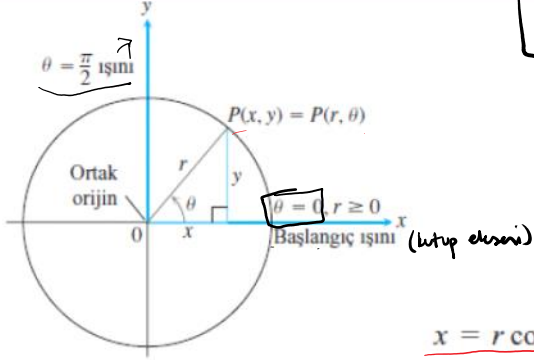
$$\left(\sqrt{2}, -\frac{\pi}{4}\right) \rightarrow \text{sadece bir tane!}$$





## KUTUPSAL EĞRİLER

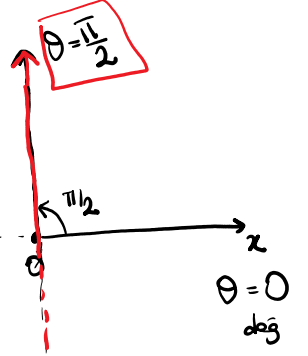
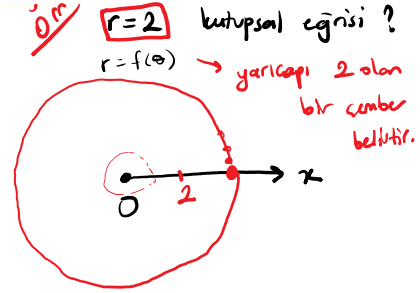
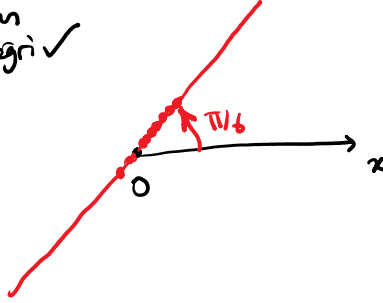
$$r = f(\theta)$$



$$x = r \cos \theta, \quad y = r \sin \theta, \quad x^2 + y^2 = r^2$$

$r = f(\theta)$  kutupsal eğri denklemini sağlayan tüm  $(r, \theta)$  noktalarının oluşturduğu eğri ✓

Ör  $\theta = \frac{\pi}{6} \rightarrow$  bir doğru belirtir.



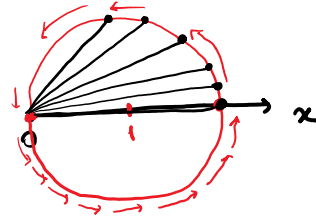
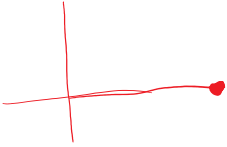
Ör  $r = 2 \cos \theta$

$$x = r \cos \theta$$

$$\frac{r}{2} = \frac{x}{r} \Rightarrow r^2 = 2x$$

$$x^2 + y^2 = r^2$$

$$x^2 + y^2 = 2x$$



$$x^2 - 2x + 1 + y^2 - 1 = 0$$

$$(x-1)^2 + y^2 = 1$$

$$(x-a)^2 + (y-b)^2 = r^2$$

$r=1$  yarıçaplı  $(1,0)$  merkezli çember.

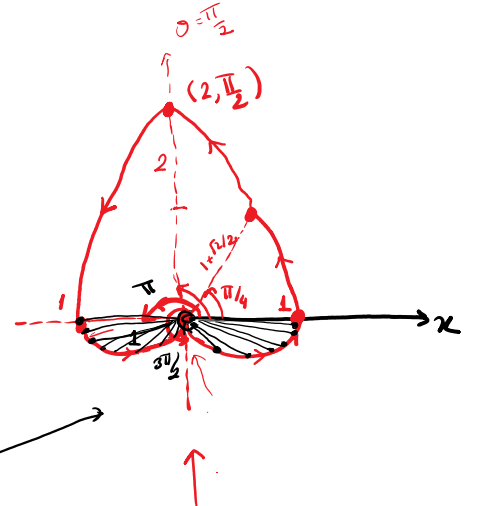
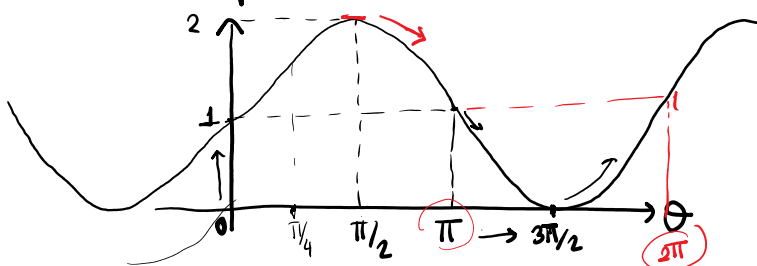
Ör

$$r = 1 + \sin \theta$$

kutupsal eğrisi çizelim.

$$1 + \frac{r}{2}$$

$$r = f(\theta)$$





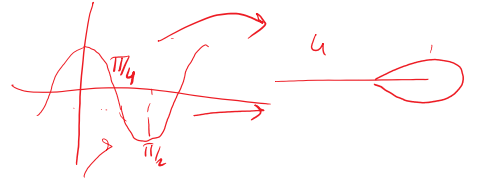
$$\begin{aligned} \theta = 0 & \quad r = 1 \\ \theta = \pi/2 & \quad r = 2 \\ \theta = \pi & \quad r = 1 \\ \theta = 3\pi/2 & \quad r = 0 \end{aligned}$$

$$(1, 0)$$

$$(2, \pi/2)$$

$$\frac{-11}{2}, \frac{11}{2}$$

$$r = \cos 2\theta$$



!

## SİMETRİ $r = f(\theta)$

- !  $f(\theta) = f(-\theta)$  ise kutup eksenine göre simetri vardır. (x eksen)  $(0, \pi)$  yarıyolu
- !  $r$  yerine  $-r$  değişmez veya  $f(\theta) = f(\theta + \pi)$  ise orjine göre simetri vardır. (aynı iş)
- !  $f(\theta) = f(\pi - \theta)$  ise  $\theta = \pi/2$  doğrusuna göre simetri vardır.  $(-\pi/2, \pi/2)$  yarıyolu

