# 3. Hafta Salı Dersi

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Örn

$$\int \frac{x+1}{(x-2)^2(x^2+3)(x^2+1)}\,dx = \int \left( \frac{A_1}{(x-2)} + \frac{A_2}{(x-2)^2} + \frac{Bx+C}{(x^2+3)} + \frac{Dx+E}{(x^2+1)} \right) dx$$

$\rightarrow (x-2)(x^2+3)(x^2+1)$ $\quad (x^2+3)(x^2+1)$ $\quad (x-2)^2(x^2+1)$ $\quad (x-2)^2(x^2+3)$

Tekrarlı lineer — tekrarsız kuadratik — tekrarsız kuadratik

Örn

$$\int \frac{2x^2-x+4}{x^3+4x}\,dx = \int \frac{2x^2-x+4}{x(x^2+4)}\,dx = \int \left( \frac{A}{x} + \frac{Bx+C}{x^2+4} \right) dx$$

lineer — indirgenemez kuadratik

$(x^2+4)$ $\quad (x)$

$$2x^2-x+4 = A(x^2+4)+(Bx+C)x$$

$x^2$ : $2 = A+B$ $\quad B=1$

$x$ : $-1 = C$ $\quad C=-1$

sabit : $4 = 4A$ $\quad A=1$

$$= \int \frac{1}{x}\,dx + \int \frac{x-1}{x^2+4}\,dx$$

$$= \ln|x| + \int \frac{x}{x^2+4}\,dx - \int \frac{1}{x^2+4}\,dx$$

$u=x^2+4$, $du=2x\,dx$, $\int \frac{du}{2u}$ $\quad \frac{1}{a}\arctan\frac{x}{a}$

$$= \ln|x| + \frac{1}{2}\ln|x^2+4| + \frac{1}{2}\arctan\left(\frac{x}{2}\right)+C$$

Örn

$$\int \frac{1-x+2x^2-x^3}{x(x^2+1)^2}\,dx = \int \left( \frac{A}{x} + \frac{B_1x+C_1}{x^2+1} + \frac{B_2x+C_2}{(x^2+1)^2} \right) dx$$

lineer — 2 tanesi tekrarlı kuad.

$x(x^2+1)^2$

$(x^2+1)^2$ $\quad x(x^2+1)$ $\quad (x)$

$$1-x+2x^2-x^3 = A(x^4+2x^2+1)+(B_1x+C_1)(x^3+x)+(B_2x+C_2)x$$

$x^4$ : $0 = A+B_1$ $\quad B_1=-1$

$x^3$ : $-1 = C_1$ $\quad C_1=-1$

$x^2$ : $2 = 2A+B_1+B_2$ $\quad B_2=1$

$x$ : $-1 = C_1+C_2$ $\quad C_2=0$

sabit : $1 = A$ $\quad A=1$

$$= \int \frac{1}{x}\,dx + \int \frac{-x-1}{x^2+1}\,dx + \int \frac{x}{(x^2+1)^2}\,dx$$

$u=x^2+1$, $du=2x\,dx$

$$= \ln|x| - \int \frac{x}{x^2+1}\,dx - \int \frac{1}{x^2+1}\,dx + \int \frac{du}{2u^2} = \ln|x| - \frac{1}{2}\ln|x^2+1| - \arctan(x) + -\frac{1}{2(x^2+1)} + C$$

$u=x^2+1$, $du=2x\,dx$, $\frac{du}{2u}$

Örn

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$$\int \frac{4x^2-3x+2}{4x^2-4x+3}\,dx = \int\left(1+\frac{x-1}{4x^2-4x+3}\right)dx$$

$(2x-1)^2 = 4x^2-4x+1$

$$\begin{array}{r|l} 4x^2-3x+2 & 4x^2-4x+3 \\ -\,4x^2-4x+3 & \hline 1 \\ \hline 0 \quad x-1 & \end{array}$$

$$= \int\left(1+\frac{x-1}{(2x-1)^2+2}\right)dx = x + \int \frac{\frac{u+1}{2}-1}{u^2+2}\,\frac{du}{2}$$

$\frac{u-1}{2}$

$u = 2x-1 \rightarrow x = \frac{u+1}{2}$

$du = 2\,dx$

$$= x + \frac{1}{4}\int \frac{u-1}{u^2+2}\,du = \frac{1}{4}\left(\int \frac{u}{u^2+2}\,du - \int \frac{1}{u^2+2}\right)$$

$t = u^2+2$, $dt = 2u\,du$, $\frac{dt}{2t}$

$\frac{1}{\sqrt{2}}\arctan\left(\frac{u}{\sqrt{2}}\right)$

$$= x + \frac{1}{4}\ln\left|(2x-1)^2+2\right| - \frac{1}{4\sqrt{2}}\arctan\left(\frac{2x-1}{\sqrt{2}}\right)+C$$

$\frac{1}{4}\ln|u^2+2| - \frac{1}{4\sqrt{2}}\arctan\frac{u}{\sqrt{2}}$