

1. Evaluate the following integrals:

(a)  $\int (3+2x-x^2)^{3/2} dx.$

(b)  $\int_1^2 \frac{2^{\ln x}}{x} dx.$

(c)  $\int \frac{1}{x^3+x} dx.$

(d)  $\int x \arctan(x^2) dx.$

(e)  $\int \frac{x^3+4x^2}{x^2+4x+3} dx.$

(f)  $\int_1^2 \frac{\sqrt{x^2-1}}{x} dx.$

(g)  $\int \frac{dt}{t^2-2t+5}.$

(h)  $\int \frac{x^3}{(x+1)^{10}} dx.$

(i)  $\int \frac{\sin^3 x}{\cos^6 x} dx.$

(j)  $\int \sec^6 x \sqrt{\tan x} dx.$

(k)  $\int \frac{\sin 3z}{\cos 7z} dz.$

(l)  $\int \frac{\sqrt{1-x^2}}{x^4} dx.$

(m)  $\int \frac{x+1}{x^4+6x^3+9x^2} dx.$

(n)  $\int \sec^2 x \tan x dx.$

2. Find the area between the curve  $y = 2(\ln x)/x$  and the  $x$ -axis from  $x = 1$  to  $x = e$ .

3. Evaluate the following improper integrals:

(a)  $\int_0^\infty \frac{2}{x^2-2x+2} dx.$

(d)  $\int_{-\infty}^0 \frac{1}{x^2+2x+5} dx.$

(b)  $\int_0^3 \frac{1}{\sqrt[3]{x-1}} dx.$

(e)  $\int_0^1 \frac{x+1}{\sqrt{x^2+2x}} dx.$

(c)  $\int_{-\infty}^\infty \frac{1}{(x^2+1)(1+\arctan(x))} dx.$

(f)  $\int_1^3 \frac{dt}{t^2-6t+8}.$

4. Check for convergence or divergence:

(a)  $\int_1^\infty \frac{e^x}{\sqrt{x}} dx.$

(d)  $\int_1^\infty \frac{1+\sin^2(2x)}{x^2+\cos^2 x} dx.$

(b)  $\int_\pi^\infty \frac{1+\sin x}{x^2} dx.$

(e)  $\int_1^\infty \frac{x}{\sqrt{x^6-x+2}} dx.$

(c)  $\int_0^\infty \frac{1}{\sqrt{x^6+x^3+1}} dx.$

(f)  $\int_1^\infty \frac{dt}{t+\cos^2 t}.$

5. Determine the arc length of the path  $x(t) = e^t + e^{-t}$ ;  $y(t) = 5 - 2t$ ,  $0 \leq t \leq 4$ .

6. A ball rolls along a marked table and its position at any time can be determined by the parametric equations:  $x(t) = t^3 - t^2$  and  $y(t) = t^3 - 3t$ . Determine  $dy/dx$  when  $t = 3$ .

7. Consider the curve of the function  $f(x) = \ln(\cos x)$  from  $x = 0$  to  $x = \pi/3$ .
- (a) Find the arc length of the curve.
  - (b) **Set up** an integration representing the area of the surface generated by rotating the curve about the  $x$ -axis.
8. i.) Sketch the curve by using the parametric equations to plot points. Indicate with an arrow the direction in which the curve is traced as  $t$  increases.
- ii.) Eliminate the parameter to find a Cartesian equation of the curve.
- (a)  $x = 1 - 2t$ ,  $y = t/2 - 1$ ,  $-2 \leq t \leq 4$ .
  - (b)  $x = t - 1$ ,  $y = t^3 + 1$ ,  $-2 \leq t \leq 2$ .
9. A curve  $C$  is defined by the parametric equations

$$x = t^2 \text{ and } y = t^3 - 3t, \quad -2.5 \leq t \leq 2.5.$$

- (a) Find the horizontal and vertical velocities and describe the motion at  $t = 2$ .
- (b) Find the points on  $C$  where the tangent is horizontal or vertical.
- (c) Determine where the curve is concave upward or downward.
- (d) Sketch the curve.
- (e) Find the slope of the tangent to the curve at  $t = 2$ .
- (f) Find the area inside the loop.
- (g) Set up an integral (Do not evaluate) that represents the arc length of the loop.
- (h) Set up an integral (Do not evaluate) that represents the area of the surface generated by rotating the loop about the  $y$ -axis.