1. Hafta Perşembe Dersi

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13. $\int t\sec^2 2t\,dt$
14. $\int s2^s\,ds$
15. $\int (\ln x)^2\,dx$
16. $\int t\sin \pi t\,dt$
17. $\int e^{2\theta}\sin 3\theta\,d\theta$
18. $\int e^{-\theta}\cos 2\theta\,d\theta$
19. $\int z^3e^z\,dz$
20. $\int x\tan^2 x\,dx$
21. $\int \frac{xe^{2x}}{(1+2x)^2}\,dx$
22. $\int (\arcsin x)^2\,dx$
23. $\int_0^{1/2} x\cos \pi x\,dx$
24. $\int_0^1 (x^2+1)e^{-x}\,dx$
25. $\int_0^1 t\cosh t\,dt$
26. $\int_4^9 \frac{\ln y}{\sqrt{y}}\,dy$
27. $\int_1^3 r^3\ln r\,dr$
28. $\int_0^{2\pi} t^2\sin 2t\,dt$

14) $u=s$, $du=ds$, $dv=2^s\,ds$, $v=\int 2^s\,ds=\frac{2^s}{\ln 2}$

$uv-\int v\,du = s.\frac{2^s}{\ln 2}-\int \frac{2^s}{\ln 2}\,ds$

20) $u=x$, $du=dx$, $dv=\tan^2 x\,dx$, $1+\tan^2x=\sec^2x$

$v=\int \tan^2x\,dx=\int(\sec^2x-1)\,dx=\int \sec^2x\,dx-\int 1.dx=\tan x-x$

$u.v-\int v\,du = x(\tan x - x)-\int(\tan x - x)\,dx$

$= x\tan x - x^2 - \ln|\sec x| + \frac{x^2}{2}$

13.) $\int x\sec^2 2x\,dx = ?$ $u=x$, $du=dx$, $dv=\sec^2 2x\,dx$, $v=\int \sec^2 2x\,dx=\frac{\tan 2x}{2}$

$= u.v-\int v\,du = x.\frac{\tan 2x}{2}-\int\frac{\tan 2x}{2}\,dx$ $\quad \frac{1}{2}\int\frac{\sin 2x}{\cos 2x}\,dx$ $\quad u=\cos 2x$, $du=-2\sin 2x\,dx$

19.) $\int x^3e^x\,dx = ?$ $u_1=x^3$, $du_1=3x^2\,dx$, $dv=e^x\,dx$, $v=\int e^x\,dx=e^x$

$= u_1v-\int v\,du_1 = x^3.e^x-\boxed{\int e^x.3x^2\,dx}$ $\quad u_2=3x^2$, $du_2=6x\,dx$, $dv=e^x\,dx$, $v=e^x$

$u_2v-\int v\,du_2 = 3x^2.e^x-\boxed{\int e^x.6x\,dx}$ $\quad u_3=6x$, $du_3=6dx$, $dv=e^x\,dx$, $v=e^x$

$u_3v-\int v\,du_3 = 6xe^x-\underbrace{\int e^x.6\,dx}_{6e^x}$

$= x^3e^x-\left(3x^2e^x-\left(6xe^x-6e^x\right)\right)$

21. $\int \frac{xe^{2x}}{(1+2x)^2}\,dx$

$u=xe^{2x}$, $du=\underbrace{e^{2x}+2xe^{2x}}_{e^{2x}(1+2x)}\,dx$ $\quad dv=\frac{1}{(1+2x)^2}\,dx$, $v=\int\frac{1}{(1+2x)^2}\,dx=-\frac{1}{2(1+2x)}$

$(1+2x)^{-1}\quad -2(1+2x)^{-2}$

$\frac{1}{1+2x}\xrightarrow{\text{türev}}\frac{-2}{(1+2x)^2}$

$u.v-\int v\,du = \frac{-xe^{2x}}{2(1+2x)}+\int\frac{e^{2x}\cancel{(1+2x)}}{2\cancel{(1+2x)}}\,dx = \frac{-xe^{2x}}{2(1+2x)}+\frac{e^{2x}}{4}+c$

$$u.v - \int v\,du = \frac{-xe^{2x}}{2(1+2x)} + \int \frac{e^{2x}\cancel{(1+2x)}}{2\cancel{(1+2x)}}dx = \frac{-xe^{2x}}{2(1+2x)} + \frac{e^{2x}}{4} + c$$

22. $\int (\arcsin x)^2 dx$ $\quad u^2$

$u = \arcsin x \longrightarrow \boxed{\sin u = x}$

$du = \frac{1}{\sqrt{1-x^2}}dx \rightarrow du \sqrt{1-x^2} = dx \rightarrow du \cos u = dx$

$\sqrt{1-\sin^2 u}$

$\sqrt{\cos^2 u} = \cos u$

u – dönüşümü

$\int u^2 \cos u\, du = ?$ Kısmî integrasyon

$t = u^2$ $\quad dv = \cos u\,du$

$tv - \int v\,dt$ $\quad dt = 2u\,du$ $\quad v = \int \cos u\,du = \sin u$

$= u^2 \sin u - \int \sin u\, 2u\, du$ $\quad t = 2u$, $dt = 2\,du$ $\quad dv = \sin u\,du$, $v = \int \sin u\,du = -\cos u$

$= u^2 \sin u - \left(-2u\cos u - \int -\cos u . 2\,du\right)$

$= u^2 \sin u + 2u\cos u - 2\sin u + c = (\arcsin x)^2 . x + 2\arcsin x \sqrt{1-x^2} - 2x + c$

$u = \arcsin x \rightarrow \sin u = x$

$\cos u = \sqrt{1-x^2}$

26. $\int_4^9 \frac{\ln y}{\sqrt{y}}dy$

$u = \ln y$ $\quad dv = \frac{1}{\sqrt{y}}dy$ $\quad -\frac{1}{2\sqrt{y}} \leftarrow \sqrt{y}$

$du = \frac{1}{y}dy$ $\quad y^{-1/2}$ $\quad \frac{y^{1/2}}{-1/2}$

$v = \int \frac{1}{\sqrt{y}}dy = -2\sqrt{y}$

$\int \frac{1}{\sqrt{y}}dy = 2\sqrt{y}$

$= \left(uv - \int v\,du\right)\Big]_4^9 = ?$

$uv - \int v\,du = \ln y . 2\sqrt{y} + \int \frac{2\sqrt{y}}{y}dy$ $\quad \frac{y^{1/2}}{y} \rightarrow y^{-1/2}$

$= \ln y . 2\sqrt{y} - 4\sqrt{y}$

$= \ln y 2\sqrt{y} - 4\sqrt{y}\Big]_4^9$

$= (6\ln 9 - 12) - (4\ln 4 - 8) = 6\ln 9 - 4\ln 4 - 4$

69. Suppose that $f(1) = 2$, $f(4) = 7$, $f'(1) = 5$, $f'(4) = 3$, and f'' is continuous. Find the value of $\int_1^4 xf''(x)\,dx$.

$$\int_1^4 \underbrace{x}_{u} \cdot \underbrace{f''(x)\,dx}_{dv} \rightarrow \quad u = x \quad du = dx \qquad dv = f''(x)\,dx \qquad v = \int f''(x)\,dx = f'(x)$$

$$\int u\,dv = u.v - \int v\,du = x.f'(x) - \underbrace{\int f'(x)\,dx}_{f(x)}$$

$$= \Big(x\,f'(x) - f(x)\Big)\Big]_1^4 = 4.f'(4) - f(4) - \big(1.f'(1) - f(1)\big)$$

$$= 4.3 - 7 - (5 - 2) = 2$$

Trigonometrik İntegraller

$$\int \sin^{m} x \cos^{n} x\,dx = ?$$

Çift olana u de! → $\sin^2 x + \cos^2 x = 1$

söz konusu değilse →

$$\left(\sin^2 x = \frac{1-\cos 2x}{2} \qquad \cos^2 x = \frac{1+\cos 2x}{2} \right)$$

$$\int \sec^m x \tan^n x\,dx = ?$$

m çift ise $u = \tan x$ → $1 + \tan^2 x = \sec^2 x$

n tek ise $u = \sec x$

söz konusu değilse

(kısmi integrasyon)

$(\tan x)' = \sec^2 x$

$(\sec x)' = \sec x \tan x$

$\int \tan x = \ln|\sec x| + c$

$\int \sec x = \ln|\sec x + \tan x| + c$

Örn

$$\int \sin^5 x\,dx = ? \qquad \cos^0 x$$

$u = \cos x$

$\boxed{du = -\sin x\,dx}$

$\sin^2 x + \cos^2 x = 1$

$$\int \underbrace{\sin^4 x}_{(1-\cos^2 x)^2 \;\to\; (1-u^2)^2} \underbrace{\sin x\,dx}_{-du} = -\int (1-u^2)^2\,du = -\int (1 - 2u^2 + u^4)\,du$$

$$= -\frac{u^5}{5} + \frac{2u^3}{3} - u + c$$

$$= -\frac{\cos^5 x}{5} + \frac{2\cos^3 x}{3} - \cos x + c$$

Örn/ $\int \sin^2 x \cos^3 x \, dx = ?$ $\quad u = \sin x$, $\boxed{du = \cos x \, dx}$ $\quad \boxed{\sin^2 x + \cos^2 x = 1}$

$$= \int \underbrace{\sin^2 x}_{u^2} \underbrace{\cos^2 x}_{1-u^2} \underbrace{\cos x \, dx}_{du} = \int u^2(1-u^2)\,du = \frac{u^3}{3} - \frac{u^5}{5} + c$$

$$= \frac{\sin^3 x}{3} + \frac{\sin^5 x}{5} + c$$

Örn/ $\int \sin^3 x \cos^5 x \, dx = ?$ $\quad \underbrace{\sin^2 x = \frac{1-\cos 2x}{2}} \quad \underbrace{\cos^2 x = \frac{1+\cos 2x}{2}}$

$\boxed{u = \cos 2x}$

$du = -2 \sin 2x \, dx$

$du = -2 . 2 \sin x \cos x \, dx$

$$= \int \underbrace{\sin^2 x}_{\frac{1-u}{2}} \underbrace{\cos^4 x}_{\left(\frac{1+u}{2}\right)^2} \underbrace{\sin x \cos x \, dx}_{-\frac{du}{4}} = ?$$

$u^3 + u^2 - u - 1$

$$= \int \frac{(u-1)(1+2u+u^2)}{32}\,du = \int \frac{\cancel{u} + 2u^2 + u^3 - 1 - \cancel{2u} - \cancel{u^2}}{32}\,du = \frac{1}{32}\left(\frac{\cos^4 2x}{4} + \frac{\cos^3 2x}{3} - \frac{\cos^2 2x}{2} - \cos 2x\right) + c$$

Örn/ $\int \sec^4 x \, dx = ?$ $\quad u = \tan x$, $du = \sec^2 x \, dx$ $\quad 1 + \tan^2 x = \sec^2 x$ ($1+u^2$)

$$= \int \underbrace{\sec^2 x}_{1+u^2} \underbrace{\sec^2 x \, dx}_{du} = \int (1+u^2)\,du = u + \frac{u^3}{3} + c = \tan x + \frac{\tan^3 x}{3} + c$$

Örn/ $\int \sec x \tan^3 x \, dx = ?$ $\quad u = \sec x$, $du = \sec x \tan x \, dx$ $\quad 1 + \tan^2 x = \sec^2 x$ (u)

$$= \int \underbrace{\tan^2 x}_{u^2-1} \underbrace{\sec x \tan x \, dx}_{du} = \int (u^2-1)\,du = \frac{u^3}{3} - u + c = \frac{\sec^3 x}{3} - \sec x + c$$

Örn/ $\int \sec^3 x \, dx = \int \underbrace{\sec x}_{u} \underbrace{\sec^2 x \, dx}_{dv} = uv - \int v\,du = \sec x . \tan x - \int \sec x \tan^2 x \, dx$

ör/ $\underbrace{\int \sec^3 x\, dx}_{I} = \int \underbrace{\sec x}_{u} \cdot \underbrace{\sec^2 x\, dx}_{dv} = uv - \int v\,du = \sec x . \tan x - \int \sec x \underbrace{\tan^2 x}_{(\sec^2 x - 1)} dx$

$u = \sec x$ $\quad dv = \sec^2 x\, dx$

$du = \sec x \tan x\, dx$ $\quad v = \tan x$

$= \sec x \tan x - \int \sec x (\sec^2 x - 1)\, dx$

$= \sec x \tan x - \underbrace{\int \sec^3 x\, dx}_{I} + \underbrace{\int \sec x\, dx}_{\ln|\sec x + \tan x|}$

$\Rightarrow 2I = \sec x \tan x + \ln|\sec x + \tan x|$

$\Rightarrow I = (\sec x \tan x + \ln|\sec x + \tan x|)/2 + c$

ör/ $\int \sec x \tan^2 x\, dx = \int \sec^3 x\, dx - \int \sec x\, dx = \dfrac{\sec x \tan x - \ln|\sec x + \tan x|}{2} + c$

ör/ $\int \cos^5(3x)\, dx = ?$ $\quad u = \sin(3x)$, $du = 3\underbrace{\cos(3x)dx}$ $\longrightarrow$ $\cos^2(3x) = 1 - \sin^2(3x) = 1 - u^2$

$= \int \underbrace{\cos^4(3x)}_{(1-u^2)^2} . \underbrace{\cos(3x)dx}_{\frac{du}{3}} = \int \frac{1}{3} (1-u^2)^2\, du = \frac{1}{3}\left(u - \frac{2u^3}{3} + \frac{u^5}{5}\right) + c$

$1 - 2u^2 + u^4$

$= \dfrac{\sin(3x)}{3} - \dfrac{2\sin^3(3x)}{9} + \dfrac{\sin^5(3x)}{15} + c$