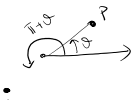
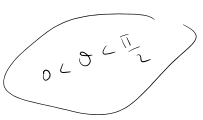
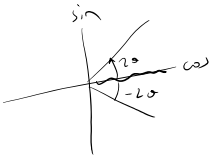
8th Week Thursday

13 Nisan 2023 Perşembe 11:30







Ex / $r = \cos(2\theta)$

symmetry ?

polar axis: $f(\theta) \stackrel{?}{=} f(-\theta)$

pole: $f(\theta) \stackrel{?}{=} f(\pi+\theta)$

$\theta = \pi/2$: $f(\theta) \stackrel{?}{=} f(\pi-\theta)$

$\cos(2\theta) \stackrel{?}{=} \cos(2(-\theta))$ ✓

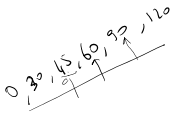
$=$ ✓ $\quad -2\theta$

$\cos(2\theta) \stackrel{?}{=} \cos(2(\pi+\theta))$ ✓

✓ $\quad 2\pi + 2\theta = 2\theta$

$\cos(2\theta) \stackrel{?}{=} \cos(2(\pi-\theta))$ ✓

✓ $\quad 2\pi - 2\theta = -2\theta$

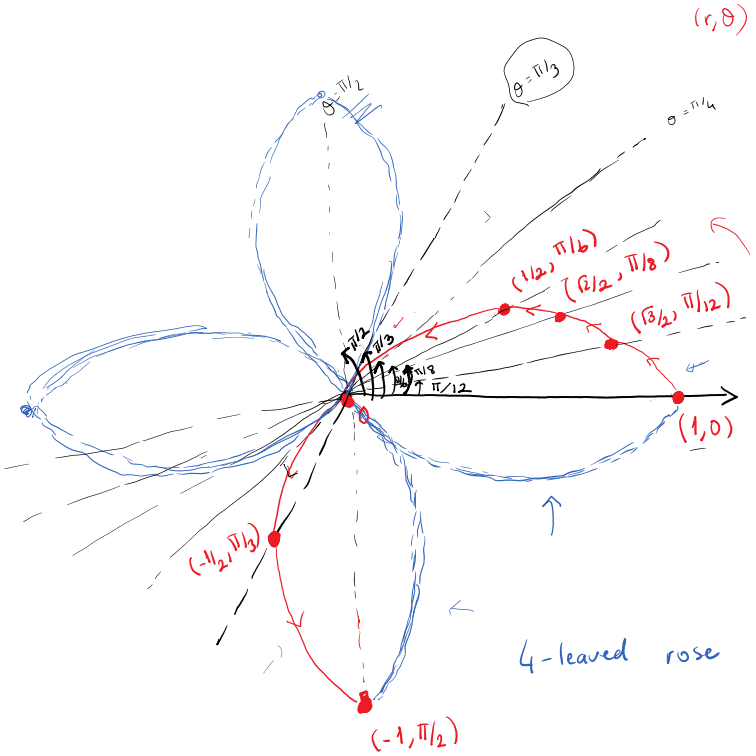


$\cos(2\theta)$ $\quad \frac{2\pi}{3}$

$0 < \theta < \pi/2$

$$0 < \frac{\pi}{12} < \frac{\pi}{8} < \frac{\pi}{6} < \frac{\pi}{4} < \frac{\pi}{3} < \frac{\pi}{2}$$

θ	$r = \cos(2\theta)$
0	$\cos(2\theta) = 1$
$\frac{\pi}{12}$	$\cos(2\theta) = \frac{\sqrt{2}}{2}$ $\quad 2\theta = \pi/6$
$\pi/8$	$\cos(2\theta) = \frac{\sqrt{2}}{2}$ $\quad 2\theta = \pi/4$
$\pi/6$	$\cos(2\theta) = 1/2$ $\quad 2\theta = \pi/3$
$\pi/4$	$\cos(2\theta) = 0$ $\quad 2\theta = \pi/2$
$\pi/3$	$\cos(2\theta) = -1/2$ $\quad 2\theta = 2\pi/3 \Rightarrow 120°$
$\pi/2$	$\cos(2\theta) = -1$ $\quad 2\theta = \pi$



! $r = \cos(a\theta)$, $r = \sin(a\theta)$ ⇝ leaved roses $\quad a \neq \pm 1 \quad a \in \mathbb{Z}$

a = even ⇒ $2a$ leaves

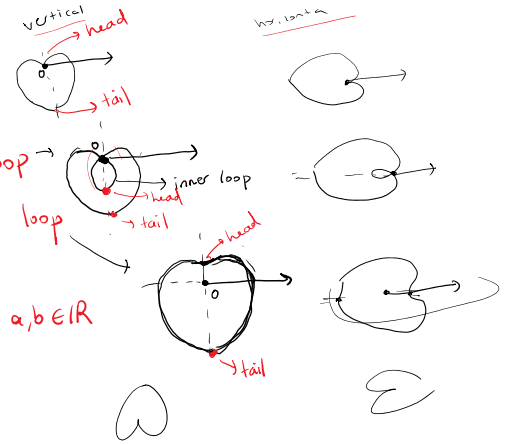
a = odd ⇒ a leaves

Cardioids and Limaçons

Limaçons → Cardioid

Limaçons → with inner loop

Limaçons → without inner loop



! $r = a + b\cos\theta$, $r = a + b\sin\theta$ $\quad a, b \neq 0 \quad a, b \in \mathbb{R}$

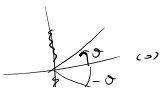
$r = a + b\cos\theta$: head, tail on the horizontal axis

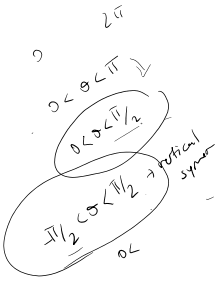
$r = a + b\sin\theta$: head, tail on vertical axis

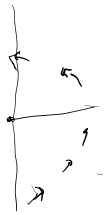
Ex / $r = 1 - \sin\theta$

symmetry ?

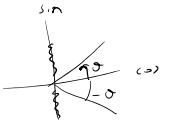
polar axis / $f(\theta) \stackrel{?}{=} f(-\theta)$? $\quad 1 - \sin\theta \stackrel{?}{=} 1 - \sin(-\theta)$ ✗



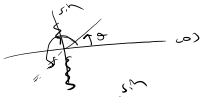




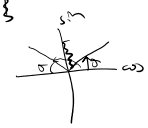
polar axis: $f(\theta) \overset{?}{=} f(-\theta)$? $\quad 1-\sin\theta \overset{?}{=} 1-\sin(-\theta)$ ✗



pole: $f(\theta) \overset{?}{=} f(\pi+\theta)$? $\quad 1-\sin\theta \overset{?}{=} 1-\sin(\pi+\theta)$ ✗



$f(\theta) = f(\pi-\theta)$? $\quad 1-\sin\theta \overset{?}{=} 1-\sin(\pi-\theta)$ ✓

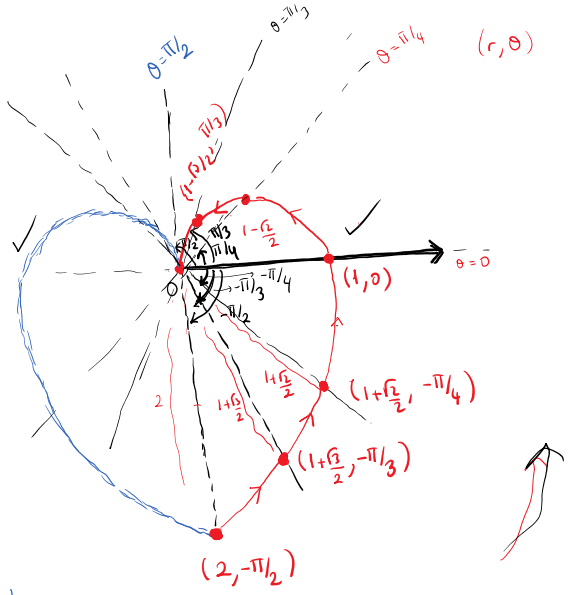


$-\pi/2 < \theta < \pi/2$ →

$-\pi/2 < -\pi/3 < -\pi/4 < -\pi/6 < 0 < \pi/6 < \pi/4 < \pi/3 < \pi/2$ →

$r = 1-\sin\theta$ →

θ	r
$-\pi/2$	$1-\sin(\theta) = 1-(-1) = 2$
$-\pi/3$	$1-\sin(\theta) = 1-(-\frac{\sqrt{3}}{2}) = 1+\frac{\sqrt{3}}{2}$
$-\pi/4$	$1-\sin(\theta) = 1-(-\frac{\sqrt{2}}{2}) = 1+\frac{\sqrt{2}}{2}$
0	$1-\sin(\theta) = 1$
$\pi/4$	$1-\sin(\theta) = 1-\frac{\sqrt{2}}{2}$
$\pi/3$	$1-\sin(\theta) = 1-\frac{\sqrt{3}}{2}$
$\pi/2$	$1-\sin(\theta) = 1-1 = 0$



Kardioid

$r = a + b\sin\theta$ → vertical

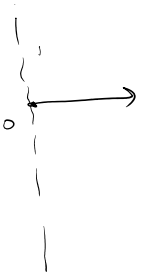
$r = a + b\cos\theta$ → horizontal

✓ This is a limaçon! (vertical, horizontal)

✓ Tail ⇝ max r =? at which θ =?

✓ Head ⇝ min r =? at which θ =?

$-1 \le \sin\theta \le 1$

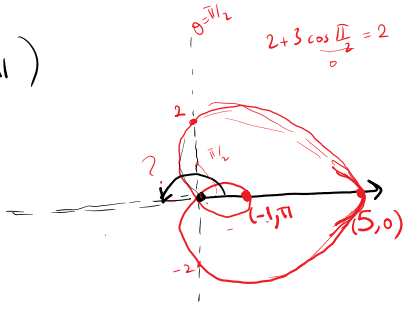


Ex: $r = 2 + 3\cos\theta$ This is a limaçon! (Horizontal)

$-1 < \cos\theta < 1$

Tail ? $\max r = 5 \Rightarrow \cos\theta = 1 \Rightarrow \theta = 0$

Head ? $\min r = 2+3(-1) = -1 \Rightarrow \cos\theta = -1 \Rightarrow \theta = \pi$

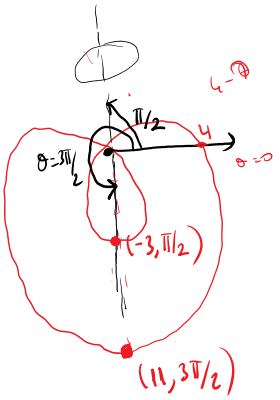


Ex: $r = 4 - 7\sin\theta$ This is a limaçon (vertical)

Ex $r = 4 - 7\sin\theta$ This is a limaçon (vertical)

Tail? max $r = 4+7 = 11$ $\sin\theta = -1$ $\theta = 3\pi/2$

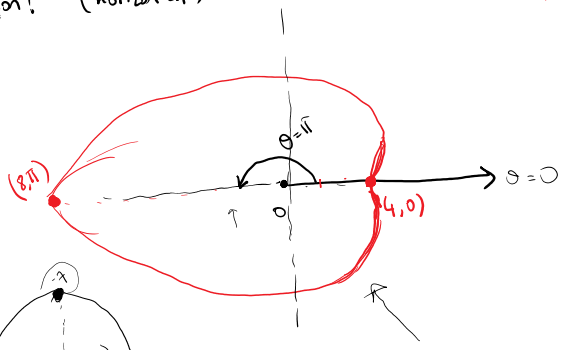
Head? min $r = 4-7 = -3$ $\sin\theta = 1$ $\theta = \pi/2$



Ex $r = 6 - 2\cos\theta$ This is a limaçon! (horizontal)

✓ Tail? max $r = 8$, $\cos\theta = -1$, $\theta = \pi$

✓ Head? min $r = 4$, $\cos\theta = 1$, $\theta = 0$



$r = -3 + 4\sin\theta$

tail → max $r = 1$ $\theta = \pi/2$

head → min $r = -7$ $\theta = 3\pi/2$