1. Hafta Salı Dersi

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$\int x^n dx = \frac{x^{n+1}}{n+1} + c \quad (n \neq -1)$

$\int e^x dx = e^x + c$

$c \in \mathbb{R}$

$\int c\, f(x)\, dx = c \int f(x)\, dx$

$\rightarrow \int (f(x) + g(x))\, dx = \int f(x)\, dx + \int g(x)\, dx$

$\lim_{n \to \infty} \sum \Delta x \rightsquigarrow \int dx$

Toplam

1 Table of Indefinite Integrals

$\int cf(x)\,dx = c\int f(x)\,dx$ ✓	$\int [f(x)+g(x)]\,dx = \int f(x)\,dx + \int g(x)\,dx$ ✓
$\int k\,dx = kx + C$	* π, e → sabit sayılar
$\int x^n\,dx = \frac{x^{n+1}}{n+1} + C \quad (n \neq -1)$ ✓	$\int \frac{1}{x}\,dx = \ln\lvert x\rvert + C$ ✓
$\int e^x\,dx = e^x + C$ ✓	$\int a^x\,dx = \frac{a^x}{\ln a} + C$ ✓
$\int \sin x\,dx = -\cos x + C$	$\int \cos x\,dx = \sin x + C$
$\int \sec^2 x\,dx = \tan x + C$	$\int \csc^2 x\,dx = -\cot x + C$
$\int \sec x \tan x\,dx = \sec x + C$	$\int \csc x \cot x\,dx = -\csc x + C$
→ $\int \frac{1}{x^2+1}\,dx = \tan^{-1} x + C$	$\int \frac{1}{\sqrt{1-x^2}}\,dx = \sin^{-1} x + C$

$(a^x)' = a^x \cdot \ln a$

$\int \tan x\,dx = \ln\lvert \sec x\rvert + c$

$\int \sec x\,dx = \ln\lvert \sec x + \tan x\rvert + c$

$\int \frac{1}{a^2+u^2}\,du = \frac{1}{a}\arctan\left(\frac{u}{a}\right) + c$

$\int \frac{1}{\sqrt{a^2-u^2}}\,du = \arcsin\left(\frac{u}{a}\right) + c$

* Belirli integralde verilen aralıkta süreklilik/tanımlılık bozulmadığı müddetçe çalışır.

$\int_{-1}^{1} \frac{1}{x}\,dx = \times \quad (x=0)$

$\int_a^b f(x)\,dx = F(x)\Big]_a^b$ $[a,b]$ sürekli ✓

Kısmî İntegrasyon

u = ? → LAPTÜ → Hangi fonksiyona u demeliyiz?

1. Logaritmik ($\log_a x$, $\ln x$) 2. Ters (Ters-trig) 3. Polinom fonk. ($a_0+a_1x+a_2x^2+\dots+a_nx^n$) 4. Trig. (sin, cos, ..) 5. Üstel fonk. (e^x, 3^x, .. a^x)

$\int [f(x)\,g(x)]' = \int (f'(x)\,g(x) + f(x)\,g'(x))$

$f(x)\,g(x) = \int f'(x)\,g(x)\,dx + \int f(x)\,g'(x)\,dx$

($f'(x)\,dx = du$, $g(x) = v$; $f(x) = u$, $g'(x)\,dx = dv$)

$u.v - \int v.\,du = \int u\,dv$

$\int u\,dv = uv - \int v\,du$

Örn $\int e^x \sin x\,dx$ — üstel, trig. → $u = \sin x$ — 2 T →

$\int u\,dv = uv - \int v\,du$

LAPTÜ = ?

Örn

$\int e^x \sin x\,dx = ?\ I \rightarrow$ (üstel, trig. → $u = \sin x$)

$\int u\,dv = uv - \int v\,du$ ($u = \sin x$, $dv = e^x dx$)

LAPTÜ = ?

(1) $u = \sin x \xrightarrow{\text{türev}} du = \cos x\,dx$

$\int dv = \int e^x dx \xrightarrow{\text{integral}} v = e^x$

$\rightarrow I = \int u\,dv = uv - \int v\,du = \sin x \cdot e^x - \int e^x \cos x\,dx$

(2) $u = \cos x \xrightarrow{\text{türev}} du = -\sin x\,dx$

$dv = e^x dx \xrightarrow{\text{integral}} v = e^x$

$\int u\,dv = uv - \int v\,du = \cos x \cdot e^x - \int e^x (-\sin x)\,dx$

$I = \sin x e^x - \left(\cos x \cdot e^x - \int e^x \cdot (-\sin x)\,dx\right)$

$\Rightarrow I = \sin x e^x - \left(\cos x \cdot e^x + \int e^x \sin x\,dx\right)$, $\int e^x \sin x\,dx = I$

$\Rightarrow I = \sin x e^x - \cos x e^x - I$

$\Rightarrow 2I = \sin x e^x - \cos x e^x$

$\rightarrow I = \dfrac{\sin x e^x - \cos x e^x}{2}$

Örn

LAPTÜ

$\int \ln x\,dx = ? = uv - \int v\,du$

$u = \ln x \xrightarrow{\text{türev}} du = \frac{1}{x}\,dx$

$dv = 1\,dx \xrightarrow{\text{integral}} v = x$

$\int u\,dv = uv - \int v\,du = \ln x \cdot x - \int x \cdot \frac{1}{x}\,dx = \ln x \cdot x - x + c$

$\sin(x^2)$ $\quad 2x \cdot \cos(x^2)$

Örn

$\int \ln(\sqrt[3]{x})\,dx = ? = u.v - \int v\,du$

$u = \ln(\sqrt[3]{x})$ ($f(g(x))$) $\xrightarrow{\text{türev}} du = \frac{1}{\sqrt[3]{x^2}} \cdot \frac{1}{\sqrt[3]{x}}\,dx = \frac{1}{3x}\,dx$

$x^{1/3} \xrightarrow{\text{türev}} \frac{1}{3} x^{-2/3}$

$\frac{1}{3\sqrt[3]{x^2}}$

$dv = dx \xrightarrow{\text{integral}} v = x$

$= u.v - \int v.du = \ln \sqrt[3]{x} \cdot x - \int x \cdot \frac{1}{3x}\,dx$

$= \ln \sqrt[3]{x} \cdot x - \frac{x}{3} + c$

Örn

LAPTU

$\int e^{\sqrt{x}}\,1\,dx = \int \sqrt{x}\cdot\frac{e^{\sqrt{x}}}{\sqrt{x}}\,dx = u.v - \int v\,du$

$u = \sqrt{x}$ (u=1, $dx = e^{\sqrt{x}}\,dx$)

$u = \sqrt{x} \xrightarrow{\text{türev}} du = \frac{1}{2\sqrt{x}}\,dx$

$dv = \frac{e^{\sqrt{x}}}{\sqrt{x}}\,dx \xrightarrow{\text{integral}} v = 2e^{\sqrt{x}}$

$= \sqrt{x}.2e^{\sqrt{x}} - \int 2.e^{\sqrt{x}}.\frac{1}{2\sqrt{x}}\,dx$

$= \sqrt{x}.2e^{\sqrt{x}} - 2e^{\sqrt{x}} + c$

Örnek Ödev Soruları

3. $\int x\cos 5x\,dx$
4. $\int ye^{0.2y}\,dy$
5. $\int te^{-3t}\,dt$
6. $\int (x-1)\sin \pi x\,dx$
7. $\int (x^2+2x)\cos x\,dx$
8. $\int t^2\sin \beta t\,dt$
9. $\int \ln \sqrt[3]{x}\,dx$ ✓
10. $\int \sin^{-1}x\,dx$
11. $\int \arctan 4t\,dt$
12. $\int p^5 \ln p\,dp$
13. $\int t\sec^2 2t\,dt$
14. $\int s2^s\,ds$
15. $\int (\ln x)^2\,dx$
16. $\int t\sinh mt\,dt$
17. $\int e^{2\theta}\sin 3\theta\,d\theta$
18. $\int e^{-\theta}\cos 2\theta\,d\theta$
19. $\int z^3e^z\,dz$
20. $\int x\tan^2 x\,dx$
21. $\int \frac{xe^{2x}}{(1+2x)^2}\,dx$
22. $\int (\arcsin x)^2\,dx$
23. $\int_0^{1/2} x\cos \pi x\,dx$
24. $\int_0^1 (x^2+1)e^{-x}\,dx$
25. $\int_0^1 t\cosh t\,dt$
26. $\int_4^9 \frac{\ln y}{\sqrt{y}}\,dy$
27. $\int_1^3 r^3\ln r\,dr$
28. $\int_0^{2\pi} t^2\sin 2t\,dt$