Classical Questions

1. Evaluate the following integrals:

   (a) $\int \left(2x+3x^2\right)\ln(x)\,dx$. (b) $\int \sqrt{36-25x^2}\,dx$. (c) $\int \dfrac{\cos x}{\sin^2(x)-25}dx$.

2. Evaluate the following improper integrals:

   (a) $\displaystyle\int_0^\infty \frac{3x-1}{4x^4-x^2}\,dx$. (b) $\displaystyle\int_0^\infty x^2e^{-x}\,dx$. (c) $\displaystyle\int_0^3 \frac{dx}{\sqrt{9-x^2}}\,dx$.

3. Check for convergence:

   (a) $\displaystyle\int_1^\infty \frac{e^{-x}}{x^{3/2}}\,dx$. (b) $\displaystyle\int_3^\infty \frac{dx}{x+e^x}\,dx$. (c) $\displaystyle\int_3^\infty \frac{x^2+1}{(x-2)(x^2+2)}\,dx$.

4. Determine whether the following series converge or not:

   (a) $\displaystyle\sum_{n=1}^{\infty} n^2e^{-n^3}$ (c) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n\ (\ln\ n)^2}{n+1}$ (e) $\displaystyle\sum_{n=1}^{\infty} \frac{n^n}{n^2 6^{n^2}}$

   (b) $\displaystyle\sum_{n=1}^{\infty} \frac{5^n\ n!}{(2n)!}$ (d) $\displaystyle\sum_{n=1}^{\infty} \frac{(\ln\ n)^3}{n^{3/2}}$ (f) $\displaystyle\sum_{n=1}^{\infty} \frac{\sqrt{n^2+1}}{(n^5-n^2+n+1)^{1/3}}$

5. Decide whether the following series is absolutely convergent, conditionally convergent or divergent.

   (a) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n\sin(n)}{n^2+1}$ (b) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n n\ln(n)}{n^2+n+1}$

6. Find the radius and interval of convergence for the following power series:

   (a) $\displaystyle\sum_{n=1}^{\infty} \frac{n(x+2)^n}{(n+1)\ 3^n}$ (b) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n n}{4^n}(x+3)^n$

7. Find a power series representation of the following functions:

(a) $\dfrac{4x}{1-4x^2}$ (b) $\ln(1-x^2)$.

8. Find the Taylor series for the function $f(x) = \cos(x)$ at $\pi/3$.
9. Consider the function $f(x) = (1+2x)^{1/3}$.
   (a) Find the first four terms (up to x^3) of the Taylor expansion about $x = 0$.
   (b) Use the Taylor polynomial $P_2(x)$ of $f(x)$ near $x = 0$, to estimate the value of $\sqrt[3]{2}$.
   (c) Estimate the error using the remainder formula and compare with the exact error.
10. A curve C is defined by the parametric equations $x = t^2, y = t^3 - 3t, t^2 \leq 3$.
    (a) Show that C has two tangents at the point $(3, 0)$ and find their equations.
    (b) Find the points on C where the tangent is horizontal or vertical
    (c) Determine where the curve is concave upward or downward
    (d) Sketch the curve
    (e) Find the area of the region inside the loop
    (f) Find the area of the surface generated by rotating the curve about the $x-$axis.
11. Find parametric equations describing the given curves
    (a) The line segment from $(1, 2)$ to $(-1, 5)$
    (b) The circle of radius 5 centered at $(2, 1)$, drawn counterclockwise.
12. Consider the two polar curves $r = 6\cos\theta$ and $r = 2 + 2\cos\theta$.
    (a) Identify the symmetries, if any , for each curve
    (b) Sketch the graph of the two curves on the same coordinates
    (c) Find the intersection points
    (d) Find the area inside the curve the first curve and outside the second curve.
13. Find the area of the region shared by the graph of $r = 2 + 2\cos\theta$ and $r = 3$.

Multiple Choice Questions

(Circle the correct answer.)

M.1 Which of the following sequences converge?

I. $\left\{n \sin \frac{1}{n}\right\}$ II. $\left\{n^{1/n}\right\}$ III. $\left\{\frac{\sqrt{n^6+n}}{n^2+1}\right\}$ IV. $\left\{n \cos n\pi\right\}$

a.) Only I. and II.

b.) Only II. and III.

c.) Only II. and IV.

d.) Only I. and IV.

M.2 If $a_n = \frac{1}{\sqrt{n^{1.1}}}$, then

a.) The series $\sum_{n=1}^{\infty} a_n$ converges.

b.) The series $\sum_{n=1}^{\infty} a_n$ diverges.

c.) There is not enough information to determine whether the series converges or diverges.

M.3 If $a_n > \frac{1}{n^2}$, then

a.) The series $\sum_{n=1}^{\infty} a_n$ converges.

b.) The series $\sum_{n=1}^{\infty} a_n$ diverges.

c.) There is not enough information to determine whether the series converges or diverges.

M.4 If $\sum_{n=1}^{\infty} \sqrt{n}\ a_n$ converges and $a_n > 0$ then

a.) The series $\sum_{n=1}^{\infty} a_n$ converges.

b.) The series $\sum_{n=1}^{\infty} a_n$ diverges.

c.) There is not enough information to determine whether the series converges or diverges.

M.5 The series $\sum_{n=1}^{\infty}(-1)^n \frac{n^2+1}{2-n^2}$

a.) converges conditionally.

b.) diverges by the $n-th$ term test.

c.) converges absolutely.

d.) converges by the limit comparison test.

M.6 The sum of the series $\sum_{n=1}^{\infty} 2^{-2n}(-3)^n$ is equal to

a.) $3/7$ b.) $-3/7$ c.) ∞ d.) $4/7$ e.) None of these.

M.7 Which of the following statements is most correct regarding $\sum_{n=2}^{\infty} \frac{1}{n\sqrt{1+\ln n}}$

a.) It diverges according to the $n-th$ term test.

b.) It diverges according to the integral test.

c.) It diverges according to the ratio test.

d.) It converges according to the $n-th$ term test.

e.) It converges according to the ratio test.

M.8 The equation $r = \tan\theta \sec\theta$ can be expressed in a rectangular coordinates by the equation

a.) $x = y^2$ b.) $y = x^2$ c.) $x = \frac{1}{y^2}$ d.) $y^2 = x^2$

M.9 Which of the following statements is FALSE?

a.) $(-3, \pi/6)$ and $(3, \frac{7\pi}{6})$ represent the same point.

b.) The graph of $r\sec\theta = 2$ is a circle.

c.) The curve $r = 4 + 3\sin\theta$ passes through the origin.

d.) The circles $r = 2\sin\theta$ and $r = 2\cos\theta$ intersect at only two points.