1st Week Thursday

23 Şubat 2023 Perşembe 11:22

Integration By Parts

LAPTE → expo. ($e^x, 2^x, \ldots$); trig; poly; arc; log.

$$\int u \cdot dv = ?$$

$u = \ldots \xrightarrow{\text{differentiate}} du = \ldots$ ✓

$dv = \ldots dx \xrightarrow{\text{integrate}} v =$ ✓

$$= u.v - \int v.du$$

$$\int u\,dv = uv - \int v\,du$$

Örn

$$\int 1.\ln(x)\,dx = uv - \int v\,du = \ln(x).x - \int \cancel{x}\,\frac{1}{\cancel{x}}\,dx = \ln(x).x - x + C$$

$\int u\,du$ (poly., log.)

$u = \ln(x)$ $\int dv = \int 1.dx$

diff. $\quad du = \frac{1}{x}dx$ int. $\quad v = x$

Örn

$$\int x.\cos(5x)\,dx = u.v - \int v.du = x.\frac{\sin(5x)}{5} - \int \frac{\sin(5x)}{5}.1.dx$$

(poly, trig.) $\int u\,dv$

LAPTE

$u = x$ $\int dv = \int \cos(5x)\,dx$

diff. $\quad du = 1.dx$ int. $\quad v = \frac{\sin(5x)}{5}$

$$= x.\frac{\sin(5x)}{5} - \frac{1}{5}.\frac{-\cos(5x)}{5} + C$$

Örn

$$\int x^2 e^x\,dx = u.v - \int v\,du = x^2.e^x - \int e^x.2x\,dx$$

$\int u\,dv$ (poly., exp.)

LAPTE

$u = x^2$ $\int dv = \int e^x\,dx$

diff. $\quad du = 2x\,dx$ int. $\quad v = e^x$

it need int. by parts once more!

$uv - \int v\,du$

$$\int e^x.x\,dx = x.e^x - \int e^x.1\,dx$$

$u = x$ $dv = e^x\,dx$

dif. $\quad du = 1.dx$ int. $\quad v = e^x$

e^x

$$= x^2.e^x - 2\left(x.e^x - e^x\right) + C$$

Ex

$uv - \int v\,du$

$$\int x^3 e^x\,dx = x^3.e^x - \int e^x.3x^2\,dx$$

$u = x^3$ $\int dv = \int e^x\,dx$

$uv - \int v\,du$

$$\int e^x.x^2\,dx = x^2.e^x - \int e^x.2x\,dx$$

$u = x^3 \quad \int dv = \int e^x\,dx$

$du = 3x^2\,dx \quad v = e^x$

$uv - \int v\,du$

$\int e^x \cdot x^2\,dx = x^2 \cdot e^x - \int e^x \cdot 2x\,dx$

$u = x^2 \quad dv = e^x dx$

$du = 2x\,dx \quad v = e^x$

$= uv - \int v\,du$

$\int e^x \cdot x\,dx = x \cdot e^x - \int e^x \cdot 1\,dx$

e^x

$u = x \quad dv = e^x dx$

$du = 1.dx \quad v = e^x$

$$\int x^3 e^x\,dx = x^3 \cdot e^x - 3\left(x^2 \cdot e^x - 2\left(x \cdot e^x - e^x\right)\right) + C$$

$$\int x^n e^x\,dx = x^n \cdot e^x - n\left(x^{n-1} e^x - (n-1)\left(x^{n-2} \cdot e^x - \ldots\right)\right)$$

Ex

$$\int 1 \cdot \ln\left(\sqrt[3]{x}\right) dx = \int \ln\left(x^{1/3}\right) dx = \int \frac{1}{3} \ln(x)\,dx = \frac{1}{3}\left[\ln(x) \cdot x - x\right] + C$$

reminder

!

$$\ln(x^2) = 2 \cdot \ln(x) \checkmark \quad \neq \quad (\ln(x))^2 = \ln(x) \cdot \ln(x)$$

Ex

$$\int (\ln(x))^2 \cdot 1\,dx = uv - \int v\,du = (\ln(x))^2 \cdot x - \int \cancel{x} \cdot \frac{1}{\cancel{x}} \cdot 2\ln(x)\,dx$$

u dv

$u = (\ln(x))^2 \quad \int dv = \int 1.dx$

diff. int.

$du = \frac{1}{x} \cdot 2 \cdot \ln(x)\,dx \quad v = x$

$$= (\ln(x))^2 \cdot x - 2\int \ln(x)\,dx$$

$\ln(x) \cdot x - x$

$$= (\ln(x))^2 \cdot x - 2\left(\ln(x) \cdot x - x\right) + C$$

Ex

$\int u\,dv$

$$\int \arctan(4x) \cdot 1\,dx = uv - \int v\,du = \arctan(4x) \cdot x - \int x \cdot 4 \cdot \frac{1}{1+(4x)^2}\,dx$$

arc poly.

$\frac{1}{8}du$

u

LIAP

$u = \arctan(4x) \quad \int dv = \int 1.dx$

diff. int.

$du = 4 \cdot \frac{1}{1+(4x)^2}\,dx \quad v = x$

$u = 1 + 16x^2$

$du = 32x\,dx$

$\int \frac{1}{8u}du = \frac{1}{8}\ln|u|$

$\frac{1}{1+(x)^2}$

$$= \arctan(4x) \cdot x - \frac{1}{8}\ln\left(1+16x^2\right) + C$$

Ex

$\int u\,dv$

$$\int x^3 \cdot \ln(x)\,dx = uv - \int v\,du = \ln(x) \cdot \frac{x^4}{4} - \int \frac{x^4}{4} \cdot \frac{1}{x}\,dx$$

x^3

Ex

$$\int x^3 . \ln(x)\, dx = uv - \int v\, du = \ln(x).\frac{x^4}{4} - \int \frac{x^4}{4}.\frac{1}{x}\, dx$$

poly. log.

LAPTE

$u = \ln(x)$ (diff.) $du = \frac{1}{x}dx$

$\int dv = \int x^3 dx$ (integ.) $v = \frac{x^4}{4}$ ($n+1$)

$$= \ln(x).\frac{x^4}{4} - \frac{1}{4}\int x^3 dx$$

$\frac{x^4}{4}$

$$\int x^3 \ln(x)\, dx = \ln(x).\frac{x^4}{4} - \frac{1}{4}.\frac{x^4}{4} + C$$

$$\int x^n \ln(x)\, dx = \ln(x).\frac{x^{n+1}}{n+1} - \frac{1}{n+1}.\frac{x^{n+1}}{n+1} + C$$

Ex

$$\int e^x . \sin(x)\, dx = u.v - \int v.du = \sin(x).e^x - \int e^x . \cos(x)\, dx$$

exp. trig. $I = ?$

LAPTE

$u = \sin(x)$ (dif.) $du = \cos(x)dx$

$\int dv = \int e^x dx$ (int.) $v = e^x$

$$\int e^x \cos(x)\, dx = u.v - \int v\, du = \cos(x).e^x - \int e^x . -\sin(x)\, dx$$

exp. - trig.

$u = \cos(x)$, $du = -\sin(x)dx$

$\int dv = \int e^x dx$, $v = e^x$

$$= \cos(x).e^x + \int e^x . \sin(x)\, dx \rightarrow I$$

$$\Rightarrow I = \sin(x).e^x - \left(\cos(x).e^x + I\right)$$

$$I = \sin(x).e^x - \cos(x).e^x - I \Rightarrow 2I = \sin(x).e^x - \cos(x).e^x$$

$$\Rightarrow I = \frac{1}{2}\left(\sin(x).e^x - \cos(x).e^x\right) + C$$

Ex

$$\int \sin(\ln(x))\, dx = \int \sin(u).(x.)du = \int \sin(u).e^u.du$$

$u = \ln(x) \rightarrow x = e^u$

LAPTE

$du = \frac{1}{x} dx$

LAPTÜ

$u = \ln(x) \rightarrow x = e^u$

$du = \frac{1}{x} dx$

$$= \frac{1}{2}\left(\sin(u).e^u - \cos(u).e^u\right)$$

$$= \frac{1}{2}\left[\sin(\ln(x)).x - \cos(\ln(x)).x\right] + C$$

EX/

$$\int \cos(\sqrt{x})\,dx = \int \cos(u)\,2\sqrt{x}\,du = \int \cos(u).2u\,du$$

$u = \sqrt{x}$

$du = \frac{1}{2\sqrt{x}} dx \longrightarrow dx = 2\sqrt{x}.du$

$$= 2.\,u\sin(u) + \cos(u)$$

$$= 2.\sqrt{x}.\sin(\sqrt{x}) + \cos(\sqrt{x}) + C$$

LAPTE

$$\int \cos(x).x\,dx = uv - \int v\,du = x.\sin(x) - \int \sin(x).1\,dx = x.\sin(x) + \cos(x) + C$$

$-\cos(x)$

diff. $u = x$, $du = 1dx$

$\int dv = \int \cos(x)dx$, $v = \sin(x)$

69. Suppose that $f(1) = 2$, $f(4) = 7$, $f'(1) = 5$, $f'(4) = 3$, and f'' is continuous. Find the value of $\int_1^4 xf''(x)\,dx$.

$$\int_1^4 \underbrace{x}_{u}\, \underbrace{f''(x)\,dx}_{dv} = uv - \int v\,du = x.\,f'(x) - \underbrace{\int f'(x).1.dx}_{f(x)}$$

diff. $u = x$, $du = 1.dx$

integ. $\int dv = \int f''(x)\,dx$, $v = f'(x)$

$$= x.f'(x) - f(x) \quad + C$$

$$= \left(x.f'(x) - f(x)\right)\Big]_1^4$$

$$= 4.\overset{3}{f'(4)} - \overset{7}{f(4)} - 1.\overset{5}{f'(1)} - \overset{2}{f(1)}$$

$$= 12 - 7 - 3 = 2$$