2. Hafta Perşembe Dersi

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- Kısmî integrasyon: $\int u\,dv = uv - \int v\,du$
- Trigonometrik integraller: $\int \sin^m \cos^n \dots \tan^m \sec^n$
- Trigonometrik Dönüşümler: $\sqrt{a^2-x^2}$ (b^2) $\sqrt{a^2+x^2}$ (b^2) $\sqrt{x^2-a^2}$ (b^2)

$bx = a\sin\theta$, $a\tan\theta$, $a\sec\theta$

Rasyonel Fonksiyonların Basit Kesirlere Ayrılması

Rasyonel fonk. → $\frac{p(x)}{q(x)}$ → $p(x)$ polinom, $q(x)$ polinom, $q(x)\neq 0$

$a_0 + a_1x + a_2x^2 + \dots + a_nx^n$ → polinom (kuvvetler $\in \mathbb{N}$, $a_i \in \mathbb{R}$)

$\int \left(\frac{p(x)}{q(x)}\right) dx$

1) derece$(p(x)) >$ derece$(q(x)) \Rightarrow p(x) \,|\, q(x)$ polinom bölmesi ile basit kesirli hale getir.

2) $\int \frac{2x+1}{x^2+x-2}\,dx = \int \frac{A=?}{x+2}\,dx + \int \frac{B=?}{x-1}\,dx = A\ln|x+2| + B\ln|x-1| + C$

$x^2+x-2 = (x+2)(x-1)$ → çarpanlarına ayır.

* Her polinom, lineer ve/veya indirgenemez kuadratik çarpanlara ayrılabilir.

lineer → 1. dereceden polinom

indirgenemez kuadratik → 2. dereceden polinom

$(x^2+1)^3(x-1)(x+1)$

indirgeniyor → $x^2-2 = (x-\sqrt{2})(x+\sqrt{2})$

x^2+1, x^2+x+1

Tekrarsız Lineer Faktör	Tekrarlı Lineer Faktör	Tekrarsız İndirgenemez Kuadratik Faktör	Tekrarlı İndirgenemez Kuadratik Faktör
$(ax+b)$	$(ax+b)^m$, $m>1$	(ax^2+bx+c)	$(ax^2+bx+c)^m$
$\to A \in \mathbb{R}$	$\frac{A_1}{ax+b} + \frac{A_2}{(ax+b)^2} + \dots + \frac{A_m}{(ax+b)^m}$	$\to \frac{Ax+B}{ax^2+bx+c}$	$\frac{A_1x+B_1}{ax^2+bx+c} + \frac{A_2x+B_2}{(ax^2+bx+c)^2} + \dots + \frac{A_mx+B_m}{(ax^2+bx+c)^m}$
$\frac{A}{(ax+b)}$			

Örn

$$\int \frac{x^2+2x-1}{2x^3+3x^2-2x}\,dx = \int \left(\frac{A=?}{2x-1} + \frac{B=?}{x+2} + \frac{C=?}{x}\right) dx$$

$2x^3+3x^2-2x = x(2x^2+3x-2) = (2x-1)(x+2)x$

$(x+2)x$, $(2x-1)x$, $(2x^2+3x-2)$

$2x \cdot 2 - x \cdot 1 = 4x - x = 3x$

$$x^2+2x-1 = A(x^2+2x) + B\cdot(2x^2-x) + C(2x^2+3x-2)$$

	soldaki katsayı		sağdaki katsayı
x^2	1	=	$A+2B+2C$
x	2	=	$2A-B+3C$
sabit	-1	=	$-2C$ → $C = 1/2$

$A+2B+1 = 1$ → $A+2B=0$ → $A=-2B$

$2A-B+\frac{3}{2} = 2$ → $2A-B = \frac{1}{2}$

$-4B-B = \frac{1}{2}$

$-5B = \frac{1}{2}$

| sabit | -1 | $-2C$ | $\rightarrow C = 1/2$ |

$-4B - B = \frac{1}{2}$

$-5B = \frac{1}{2}$

$A = 1/5$ ve $B = -1/10$

$$\int \left(\frac{A=?}{2x-1} + \frac{B=?}{x+2} + \frac{C=?}{x} \right) dx = \int \frac{1}{5(2x-1)}\,dx + \int \frac{-1}{10(x+2)}\,dx + \int \frac{1}{2x}\,dx$$

$((x+2)x)$ $((2x-1)x)$ $(2x^2+3x-2)$

$$= \frac{1}{5}\ln|2x-1| - \frac{1}{10}\ln|x+2| + \frac{1}{2}\ln|x| + c$$

Örn

$$\int \frac{x^4 - 2x^2 + 4x + 1}{x^3 - x^2 - x + 1}\,dx = ?$$

$$\begin{array}{r|l} x^4 - 2x^2 + 4x + 1 & x^3 - x^2 - x + 1 \\ -x^4 + x^3 + x^2 - x & x + 1 \\ \hline x^3 - x^2 + 3x + 1 & \\ -x^3 + x^2 + x - 1 & \\ \hline 4x & \end{array}$$

$$= \int (x+1)\,dx + \int \frac{4x}{x^3 - x^2 - x + 1}\,dx$$

tam kısım

$$\int \frac{4x}{x^3 - x^2 - x + 1}\,dx = \int \frac{A=?}{x+1} + \frac{B_1=?}{(x-1)} + \frac{B_2=?}{(x-1)^2}\,dx$$

$(x+1)(x^2-2x+1)$, $(x-1)^2$

tekrarsız lineer faktör — tekrarlı lineer faktör

$$\begin{array}{r|l} x^3 - x^2 - x + 1 & x + 1 \\ -x^3 - x^2 & x^2 - 2x + 1 \\ \hline -2x^2 - x + 1 & \\ +2x^2 + 2x & \\ \hline x + 1 & \\ -x - 1 & \\ \hline 0 & \end{array}$$

$(x+1)(x-1)^2$

$\frac{7}{2} = 3 + \frac{1}{2}$ ✓

$$\frac{4x}{x^3 - x^2 - x + 1} = \frac{-1}{x+1} + \frac{1}{(x-1)} + \frac{2}{(x-1)^2}$$

$(x-1)^2$ $(x+1)(x-1)$ x^2-1 $x+1$

$$4x = A(x^2 - 2x + 1) + B_1(x^2 - 1) + B_2(x+1)$$

	solda	sağda
x^2	0	$A + B_1$
x	4	$-2A + B_2$
sabit	0	$A - B_1 + B_2$

-1 · $A + B_1 = 0$ → $B_1 = 1$

$-2A + B_2 = 4$

$+\quad A - B_1 + B_2 = 0$

$2B_2 = 4 \Rightarrow B_2 = 2$

$A = -1$

$$\int \frac{x^4 - 2x^2 + 4x + 1}{x^3 - x^2 - x + 1}\,dx = \int (x+1)\,dx + \int \frac{-1}{x+1}\,dx + \int \frac{1}{x-1}\,dx + \int \frac{2}{(x-1)^2}\,dx$$

$u = x - 1$, $du = dx$

$2\int \frac{du}{u^2}$

$-\frac{2}{u} + c$

$$= \frac{x^2}{2} + x - \ln|x+1| + \ln|x-1| - \frac{2}{x-1} + c$$

$$\ln\left|\frac{x-1}{x+1}\right|$$

Ödev

$$\int \frac{x^2 + 4}{(x+1)(x-5)^3(x^2+1)^2(x^2+x+1)}\,dx = \frac{A_0}{x+1} + \frac{A_1}{(x-5)} + \frac{A_2}{(x-5)^2} + \frac{A_3}{(x-5)^3} + \frac{Bx + C_1}{x^2+1} + \frac{B_2x + C_2}{(x^2+1)^2} + \frac{D_1x + E_1}{x^2+x+1}$$

$$\int \frac{x^2+4}{(x+1)(x-5)^3(x^2+1)^2(x^2+x+1)}\,dx = \frac{A_0}{x+1} + \frac{A_1}{(x-5)} + \frac{A_2}{(x-5)^2} + \frac{A_3}{(x-5)^3} + \frac{B_1x+C_1}{x^2+1} + \frac{B_2x+C_2}{(x^2+1)^2} + \frac{B_3x+C_3}{x^2+x+1}$$

lineer

kuadratik