3rd Week Friday

10 Mart 2023 Cuma 10:15

Transpose of a Matrix

$$A_{m\times n} = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{12} & \cdots & a_{2n} \\ \vdots & \vdots & & \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix} \qquad A^T = \begin{bmatrix} a_{11} & a_{21} & a_{m1} \\ a_{12} & a_{22} & a_{m2} \\ \vdots & \vdots & \vdots \\ a_{1n} & a_{2n} & a_{mn} \end{bmatrix}_{n\times m}$$

Ex

$$A = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 5 & 6 & 7 & 8 \\ -1 & 2 & 3 & 4 \end{bmatrix}_{3\times 4} \Rightarrow A^t = \begin{bmatrix} 1 & 5 & -1 \\ 2 & 6 & 2 \\ 3 & 7 & 3 \\ 4 & 8 & 4 \end{bmatrix}_{4\times 3}$$

Square Matrix $\rightarrow A_{n\times n} \rightarrow$ # of rows = # of columns

! Diagonal of a square matrix $\rightarrow$

$$A_{n\times n} = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & & a_{2n} \\ \vdots & & a_{33} & \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{bmatrix}_{n\times n}$$

reverse diagonal

(If the matrix is not square, we can not talk about a diagonal)

$n\times n$!

Triangular Matrices (n×n)

1) Upper Triangular Matrices

$$\begin{bmatrix} d_1 & \# & \# & \# \\ & d_2 & & \# \\ 0 & & \ddots & \\ & & & d_n \end{bmatrix}_{n\times n}$$

$\rightarrow$ non-zero entries may exist only on the diagonal and the upper part of the diagonal

Ex

$$A = \begin{bmatrix} 1 & 3 & 0 & 5 \\ 0 & 2 & -4 & 1 \\ 0 & 0 & 0 & 3 \\ 0 & 0 & 0 & 4 \end{bmatrix}_{4\times 4}$$

2) Lower Triangular Matrices

$$\begin{bmatrix} d_1 & & & 0 \\ & d_2 & & \\ \# & & \ddots & \\ \# & \# & & d_n \end{bmatrix}_{n\times n}$$

$\rightarrow$ non zero entries may exist only - - - and the lower part

Ex

$$A = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ -1 & 0 & 3 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

3) Diagonal Matrices

$$\begin{bmatrix} d_1 & & 0 \\ & d_2 & \\ & & \ddots \\ 0 & & & d_n \end{bmatrix}_{n\times n}$$

Identity Matrix (n×n)

diagonal $d_1 = d_2 = \cdots = d_n = 1$

$I_3 \rightarrow$ 3x3 identity matrix

$$I_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$I_n \Rightarrow$ n×n identity matrix

$$I_n = \begin{bmatrix} 1 & & 0 \\ & 1 & \\ & & \ddots \\ 0 & & & 1 \end{bmatrix}_{n\times n}$$

$a \in \mathbb{R}$

$a \cdot 1 = a$

$1 \cdot a = a$

$A_{m\times n} \cdot I_{n\times n} = A_{m\times n}$ ✓

$I_{n\times n} \rightarrow$ is the identity element of the

$1 \cdot a = a$ ✓

$A_{m\times n} \cdot I_{n\times n} = A_{m\times n}$ ✓

$I_{n\times n} \rightarrow$ is the identity element of the matrix multiplication!

Ex: $A = \begin{bmatrix} 2 & 3 & 4 & 5 \\ -2 & -1 & 0 & -3 \end{bmatrix}_{2\times 4} \cdot I = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{4\times 4} = \begin{bmatrix} 2 & 3 & 4 & 5 \\ -2 & -1 & 0 & -3 \end{bmatrix}_{2\times 4}$

$$\boxed{A_{n\times n} \cdot I_n = I_n \cdot A_{n\times n} = A_{n\times n}} \ ✓$$

The Multiplicative Inverse of a Square Matrix

$1 \rightarrow$ identity element of multiplication of real number

$3 \in \mathbb{R} \rightarrow$ Multiplicative inverse of $3 = \frac{1}{3} = 3^{-1}$ $\qquad 3 \cdot 3^{-1} = 3^{-1} \cdot 3 = 1$

$$\boxed{A \cdot A^{-1} = A^{-1} A = I_n} \qquad (A \rightarrow n\times n \quad A^{-1} \Rightarrow n\times n)$$

A^{-1}: the multiplicative inverse of the matrix A. $\rightarrow$ unique.

! Not all matrices, not all square matrices have mult. inverses

! If A^{-1} exists, we call A as an "invertible matrix". $\rightarrow$ non-singular.

! If A^{-1} does not exist, we call A as a "singular matrix". $\rightarrow$ non-invertible

Ex: $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}_{2\times 2}$ $\qquad \underbrace{\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}}_{A} \cdot \underbrace{\begin{bmatrix} ? \end{bmatrix}}_{A^{-1}} = \underbrace{\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}}_{I_2}$

$\underbrace{\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}}_{A} \cdot \underbrace{\begin{bmatrix} a & b \\ c & d \end{bmatrix}}_{A^{-1}} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$1 \cdot a + 2 \cdot c = 1$
$1 \cdot b + 2 \cdot d = 0$
$3 \cdot a + 4 \cdot c = 0$
$3 \cdot b + 4 \cdot d = 1$

a, b, c, d.

4x4 system of l.e.

not a good way of finding the inverse

! What if the matrix has a bigger size? You need to solve an $n^2 \times n^2$ system of LE.

$$\begin{array}{cccc} a & b & c & d \end{array}$$
$$\left[\begin{array}{cccc|c} 1 & 0 & 2 & 0 & 1 \\ 0 & 1 & 0 & 2 & 0 \\ 3 & 0 & 4 & 0 & 0 \\ 0 & 3 & 0 & 4 & 1 \end{array}\right] \xrightarrow{-3r_1 + r_3 \rightarrow r_3} \left[\begin{array}{cccc|c} 1 & 0 & 2 & 0 & 1 \\ 0 & 1 & 0 & 2 & 0 \\ 0 & 0 & -2 & 0 & -3 \\ 0 & 3 & 0 & 4 & 1 \end{array}\right]$$

$4 - 6 = -2$

$$\xrightarrow{-3r_2+r_4\to r_4}\left[\begin{array}{cccc|c}1&0&2&0&1\\0&1&0&2&0\\0&0&-2&0&-3\\0&0&0&-2&1\end{array}\right]\xrightarrow{-\frac{1}{2}r_3\to r_3}\left[\begin{array}{cccc|c}1&0&2&0&1\\0&1&0&2&0\\0&0&1&0&3/2\\0&0&0&-2&1\end{array}\right]\xrightarrow{-\frac{1}{2}r_4\to r_4}\left[\begin{array}{cccc|c}1&0&2&0&1\\0&1&0&2&0\\0&0&1&0&3/2\\0&0&0&1&-1/2\end{array}\right]$$

REF

$a+2c=1$ $\quad$ $b+2d=0$ $\quad$ $c=3/2$ $\quad$ $d=-1/2$

$a+3=1 \Rightarrow a=-2$ $\quad$ $b-1=0 \Rightarrow b=1$

$$\Rightarrow A^{-1}=\begin{bmatrix}-2&1\\3/2&-1/2\end{bmatrix} \checkmark$$

We will continue → on Wed.

Symmetric Matrices (nxn) $\quad A^T = A$

Ex/

$$A=\begin{bmatrix}1&2&3\\2&5&7\\3&7&9\end{bmatrix} = A^T=\begin{bmatrix}1&2&3\\2&5&7\\3&7&9\end{bmatrix} \rightarrow A \text{ is symmetric.} \checkmark$$

Anti-symmetric Matrices $\quad (-1).A^T = A$

Ex/

scalar multiplication

$$A=\begin{bmatrix}0&1&2&-3\\-1&0&4&-5\\-2&-4&0&6\\3&5&-6&0\end{bmatrix} \qquad (-1).A^T = (-1).\underbrace{\begin{bmatrix}0&-1&-2&3\\1&0&-4&5\\2&4&0&-6\\-3&-5&6&0\end{bmatrix}}_{A^T} = \begin{bmatrix}0&1&2&-3\\-1&0&4&-5\\-2&-4&0&6\\3&5&-6&0\end{bmatrix}$$

=

$\Rightarrow A$ is an anti-symmetric matrix