## 3rd Week Friday's Lecture

18 Ekim 2023 Çarşamba 15:15

Add/multiply/subtract two integers, the result is an integer.

If 
$$b \neq 0$$
 is  $a \neq 0$   $\Rightarrow ab \neq 0$ 

If  $ab = 0$   $\Rightarrow contain = 0$  intime:  $contain = 0$  in  $contain = 0$   $contain = 0$  intime:  $contain = 0$   $contain =$ 

Proof: assumption case! Let 
$$p(x)$$
 hold.

assumption case? Let  $q(x)$  hold.

3 - - - -  $\Rightarrow r(x)$ .

19. Prove that for all integers  $n$   $n^2 - n + 3$  is odd.

 $n = even$ 
 $n =$ 

proof: ( proof by cases)

case 1: Let 
$$n \in \mathbb{Z}$$
 be even.  

$$\Rightarrow n = 2k, \exists k \in \mathbb{Z}.$$

$$\Rightarrow n^2 - n + 3 = (2k)^2 - 2k + 3 = 4k^2 - 2k + 3$$

$$= 2(2k^2 - k + 1) + 1 \Rightarrow n^2 - n + 3 \text{ is odd.}$$

$$\Rightarrow n = 2k + 1, \exists k \in \mathbb{Z}$$

$$\Rightarrow n^2 - n + 3 = (2k + 1)^2 - (2k + 1) + 3$$

$$= 4k^2 + 4k + 1 - 2k + 3 = 4k^2 + 2k + 3$$

$$= 2(2k^2 + k + 1) + 1 \Rightarrow n^2 - n + 3 \text{ is odd.}$$

Theorem 4.4.1 The Quotient-Remainder Theorem

Given any integer n and positive integer d, there exist unique integers q and r such that

$$n = dq + r$$
 and  $0 \le r < d$ .

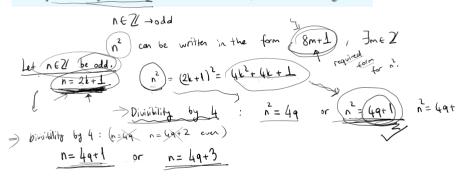
$$= \frac{10 \mid 3}{1}$$

$$0,1,2 \rightarrow \text{cenododes}$$

$$1 \uparrow \uparrow$$

n may be written: 3k, 3k+1, 3k+2 in one of these

The square of any odd integer has the form 8m + 1 for some integer m. Prose it.



$$\Rightarrow \frac{\text{casel}:}{\Rightarrow n^2} : \text{ let } \frac{n = 4q + 1}{1} \quad \exists q \in \mathbb{Z}$$

$$\Rightarrow n^2 = (4q + 1)^2 = 16q^2 + 8q + 1 = 8(2q^2 + q) + 1 \quad \checkmark$$