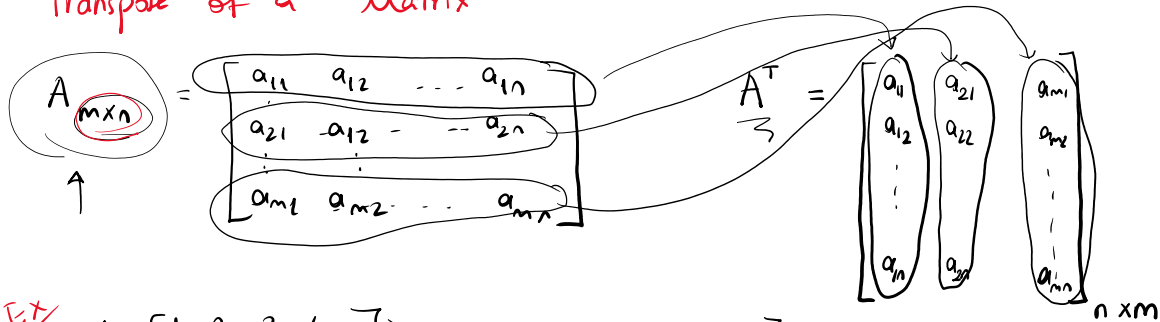


Transpose of a Matrix

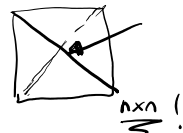
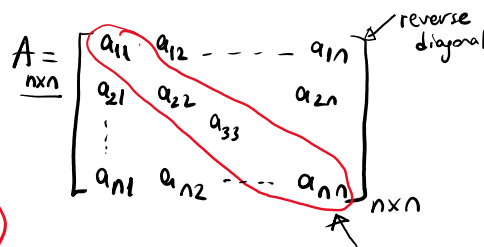


Ex $A = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 5 & 6 & 7 & 8 \\ -1 & 2 & 3 & 4 \end{bmatrix}_{3 \times 4} \Rightarrow A^T = \begin{bmatrix} 1 & 5 & -1 \\ 2 & 6 & 2 \\ 3 & 7 & 3 \\ 4 & 8 & 4 \end{bmatrix}_{4 \times 3}$

Square Matrix $\rightarrow A_{n \times n} \rightarrow \# \text{ of rows} = \# \text{ of columns}$

! Diagonal
of a square
matrix

(If the matrix is not square,
we can not talk about a diagonal)



Triangular Matrices ($n \times n$)

1) Upper Triangular
Matrices

$\begin{bmatrix} d_1 & \# & \# & \# \\ & d_2 & & \\ & & \ddots & \\ 0 & & & d_n \end{bmatrix}_{n \times n}$ \rightarrow non-zero entries
may exist
only on the diagonal
and the upper part of the
diagonal

Ex $A = \begin{bmatrix} 1 & 3 & 0 & 5 \\ 0 & 2 & 4 & 1 \\ 0 & 0 & 0 & 3 \\ 0 & 0 & 0 & 4 \end{bmatrix}_{4 \times 4}$

2) Lower Triangular
Matrices

$\begin{bmatrix} d_1 & & 0 \\ \# & d_2 & \\ \# & \# & \ddots \\ \# & \# & & d_n \end{bmatrix}_{n \times n}$ \rightarrow non zero entries
may exist
only
and the lower part

Ex $A = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ -1 & 0 & 3 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

3) Diagonal Matrices

$\begin{bmatrix} d_1 & & 0 \\ & d_2 & \\ & & \ddots \\ 0 & & & d_n \end{bmatrix}_{n \times n}$

$d_1 = d_2 = \dots = d_n = 1$

Identity Matrix ($n \times n$) diagonal

$I_3 \rightarrow 3 \times 3$ identity
matrix

$I_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$I_n \Rightarrow n \times n$
identity
matrix

$I_n = \begin{bmatrix} 1 & & 0 \\ & \ddots & \\ 0 & & 1 \end{bmatrix}_{n \times n}$

$a \cdot 1 = a$
 $1 \cdot a = a$

$A_{m \times n} \cdot I_{n \times n} = A_{m \times n}$

$I_{n \times n} \rightarrow$ is the identity
element of
the

$$\frac{a \cdot 1 = a}{1 \cdot a = a}$$

$$I_{n \times n} \cdot A_{n \times n} = A_{n \times n}$$

$I_{n \times n} \rightarrow$ the identity element of the matrix multiplication!

$$A = \begin{bmatrix} 2 & 3 & 4 & 5 \\ -2 & -1 & 0 & -3 \end{bmatrix}_{2 \times 4} \cdot I = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{4 \times 4} = \begin{bmatrix} 2 & 3 & 4 & 5 \\ -2 & -1 & 0 & -3 \end{bmatrix}_{2 \times 4}$$

$$A_{n \times n} \cdot I_n = I_n \cdot A_{n \times n} = A_{n \times n}$$

The Multiplicative Inverse of a Square Matrix

$1 \rightarrow$ identity element of multiplication of real number

$$3 \in \mathbb{R} \rightarrow \text{multiplicative inverse of } 3 = \frac{1}{3} = 3^{-1} \quad 3 \cdot 3^{-1} = 3^{-1} \cdot 3 = 1$$

$$A \cdot A^{-1} = A^{-1} \cdot A = I_n \quad (A \rightarrow n \times n \quad A^{-1} \rightarrow n \times n)$$

the multiplicative inverse of the matrix A . \rightarrow unique.

! Not all matrices, not all square matrices have mult. inverses

! If A^{-1} exists, we call A as an "invertible matrix". \rightarrow non-singular.

! If A^{-1} does not exist, we call A as a "singular matrix". \rightarrow non-invertible.

$$A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}_{2 \times 2}$$

$$\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \cdot \begin{bmatrix} ? \\ ? \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$A \quad \uparrow \quad A^{-1} \quad I_2$

$$\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \cdot \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\begin{aligned} 1 \cdot a + 2 \cdot c &= 1 \\ 1 \cdot b + 2 \cdot d &= 0 \\ 3 \cdot a + 4 \cdot c &= 0 \\ 3 \cdot b + 4 \cdot d &= 1 \end{aligned}$$

a, b, c, d .

4x4 system of le.

not a good way of finding the inverse

! What if the matrix has a bigger size?

You need to solve an $n^2 \times n^2$ system of LE.

$$4 - b = 2$$

$$\begin{bmatrix} 1 & 0 & 2 & 0 & 1 \\ 0 & 1 & 0 & 2 & 0 \\ 3 & 0 & 4 & 0 & 0 \\ 0 & 3 & 0 & 4 & 1 \end{bmatrix} \xrightarrow{-3r_1 + r_3} \begin{bmatrix} 1 & 0 & 2 & 0 & 1 \\ 0 & 1 & 0 & 2 & 0 \\ 0 & 0 & -2 & 0 & -3 \\ 0 & 3 & 0 & 4 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 2 & 0 & 1 \\ 0 & 1 & 0 & 2 & 0 \\ 0 & 0 & -2 & 0 & -3 \\ 0 & 3 & 0 & 4 & 1 \end{bmatrix} \xrightarrow{-4r_2 + r_4} \begin{bmatrix} 1 & 0 & 2 & 0 & 1 \\ 0 & 1 & 0 & 2 & 0 \\ 0 & 0 & -2 & 0 & -3 \\ 0 & 0 & 0 & -4 & 1 \end{bmatrix}$$

$$\begin{array}{c}
 \text{LU } 5 \ 0 \ 4 \ 1 \ 1 \\
 \text{LU } 10 \ 13 \ 0 \ 4 \ 1 \ 1 \\
 -3r_2 + r_4 \rightarrow r_4 \rightarrow \left[\begin{array}{cccc|c} 1 & 0 & 2 & 0 & 1 \\ 0 & 1 & 0 & 2 & 0 \\ 0 & 0 & -2 & 0 & -3 \\ 0 & 0 & 0 & -2 & 1 \end{array} \right] \xrightarrow{-\frac{1}{2}r_3 \rightarrow r_3} \left[\begin{array}{cccc|c} 1 & 0 & 2 & 0 & 1 \\ 0 & 1 & 0 & 2 & 0 \\ 0 & 0 & 1 & 0 & 3/2 \\ 0 & 0 & 0 & -2 & 1 \end{array} \right] \xrightarrow{-\frac{1}{2}r_4 \rightarrow r_4} \left[\begin{array}{cccc|c} 1 & 0 & 2 & 0 & 1 \\ 0 & 1 & 0 & 2 & 0 \\ 0 & 0 & 1 & 0 & 3/2 \\ 0 & 0 & 0 & 1 & -1/2 \end{array} \right] \leftarrow \text{RREF}
 \end{array}$$

$$a + 2c = 1$$

$$a + 3 = 1 \Rightarrow a = -2$$

$$b + 2d = 0$$

$$b - 1 = 0 \Rightarrow b = 1$$

$$c = 3/2$$

$$d = -1/2$$

$$\Rightarrow A^{-1} = \begin{bmatrix} -2 & 1 \\ 3/2 & -1/2 \end{bmatrix} \checkmark$$

We will continue \rightarrow on Wed.

Symmetric Matrices ($n \times n$) $A^T = A$

$$\text{Ex/ } A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 5 & 7 \\ 3 & 7 & 9 \end{bmatrix} \quad A^T = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 5 & 7 \\ 3 & 7 & 9 \end{bmatrix} \rightarrow A \text{ is symmetric. } \checkmark$$

Anti-symmetric Matrices $(-1) \cdot A^T = A$

$$\text{Ex/ } A = \begin{bmatrix} 0 & 1 & 2 & -3 \\ -1 & 0 & 4 & -5 \\ -2 & -4 & 0 & 6 \\ 3 & 5 & -6 & 0 \end{bmatrix} \quad (-1) \cdot A^T = (-1) \cdot \begin{bmatrix} 0 & -1 & -2 & 3 \\ 1 & 0 & -4 & 5 \\ 2 & 4 & 0 & -6 \\ -3 & -5 & 6 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 2 & -3 \\ -1 & 0 & 4 & -5 \\ -2 & -4 & 0 & 6 \\ 3 & 5 & -6 & 0 \end{bmatrix}$$

$\underbrace{\hspace{10em}}_{A^T}$

$\Rightarrow A$ is an anti-symmetric matrix