

$$|R^{s} = \{(a,b,c) : a,b,c \in |R\}$$

$$|R^{s} = \{(x_{1},x_{2,2-1},x_{n}) : x_{i} \in |R\}$$

$$V = |R^{2} \rightarrow S = \{ (x,y) : x < y \}$$
 Is S a subspace of $|R^{3}|$?
$$S \neq |R^{3}| \Rightarrow S \neq |R^{3}|$$



$$\frac{2}{(x_{1},y_{1},\overline{z_{1}}) \in S} \Rightarrow \frac{(x_{1},y_{1},\overline{D})}{(x_{2},y_{2},\overline{D})} > (x_{1},y_{1},\overline{D}) \oplus (x_{2},y_{2},\overline{D}) \oplus (x_{1},x_{2},y_{1}+y_{2},\overline{D}) \oplus S$$

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$$\frac{2}{(x_{1},y_{2},\overline{D})} \oplus S$$

$$\frac{3}{\sqrt{r \in \mathbb{R}}} \frac{(x,y,z) \in S}{\sqrt{r \in \mathbb{R}}}, z=0$$

$$\frac{r_0(x,y,0)}{\sqrt{r_0(x,y,0)}} = (rx,ry,\underline{r,0}) \in S$$

$$\Rightarrow S \leq V$$

$$V = IR^{2}$$

$$\Rightarrow S = \{ (x,y) : (x,y) \}$$

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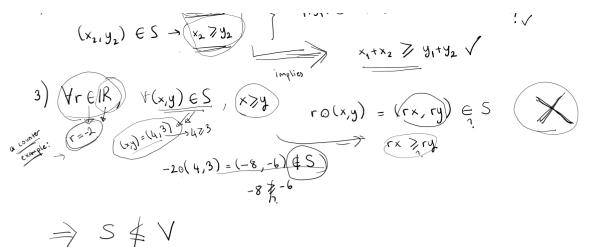
2)
$$(x_1, y_1) \in S \rightarrow (x_1 \nearrow y_1)$$

 $(x_2, y_2) \in S \rightarrow (x_2 \nearrow y_2)$

$$(x_1, y_1) \oplus (x_2, y_2) = (x_1 + x_2, y_1 + y_2) \in S \qquad ? \checkmark$$

$$(x_1, y_2) \in S \rightarrow (x_2 \nearrow y_2)$$

$$x_1 + x_2 \nearrow y_1 + y_2 \checkmark$$



1. Determine whether the following sets form subspaces of (\mathbb{R}^2)

(a)
$$\{(x_1, x_2)^T \mid x_1 + x_2 = 0\}$$

(b)
$$\{(x_1, x_2)^T \mid x_1 x_2 = 0\}$$

(c)
$$\{(x_1, x_2)^T \mid x_1 = 3x_2\}$$

$$\rightarrow$$
 (d) $\{(x_1, x_2)^T \mid |x_1| = |x_2|\}$

(e)
$$\{(x_1, x_2)^T | x_1^2 = x_2^2 \}$$

s form sub-
$$(2,2) \in S$$

$$(-2,2) \in S$$

$$(3,5) \notin S$$

$$d) S = \left\{ (x_1, x_2) : |x_1| = |x_2| \right\} \quad S \subseteq \mathbb{R}^2 \quad \text{is } S$$

$$(x_{1},x_{2}) \in S \Rightarrow |x_{1}| = |x_{2}|$$

$$(y_{1},y_{2}) \in S \Rightarrow |y_{1}| = |y_{2}|$$

$$(x_{1},x_{2}) + (y_{1},y_{2}) = (x_{1}+y_{1},x_{1}+y_{2})$$

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2. Determine whether the following sets form sub-

spaces of
$$[x_1, x_2, x_3]^T \mid x_1 + x_3 = 1]$$
 $0, 45$

(b)
$$\{(x_1, x_2, x_3)^T \mid x_1 = x_2 = x_3\}$$

(c)
$$\{(x_1, x_2, x_3)^T \mid x_3 = x_1 + x_2\}$$

$$\rightarrow$$
 (d) $\{(x_1, x_2, x_3)^T \mid x_3 = x_1 \text{ or } x_3 = x_2\}$

sub-

$$S = \begin{cases} (x_1, x_2, x_3) : x_3 = x_1 \text{ or } x_3 = x_2 \end{cases}$$

$$(x_{1}, x_{2}, x_{3}) \in S$$

$$(y_{1}, y_{2}, y_{3}) \in S$$

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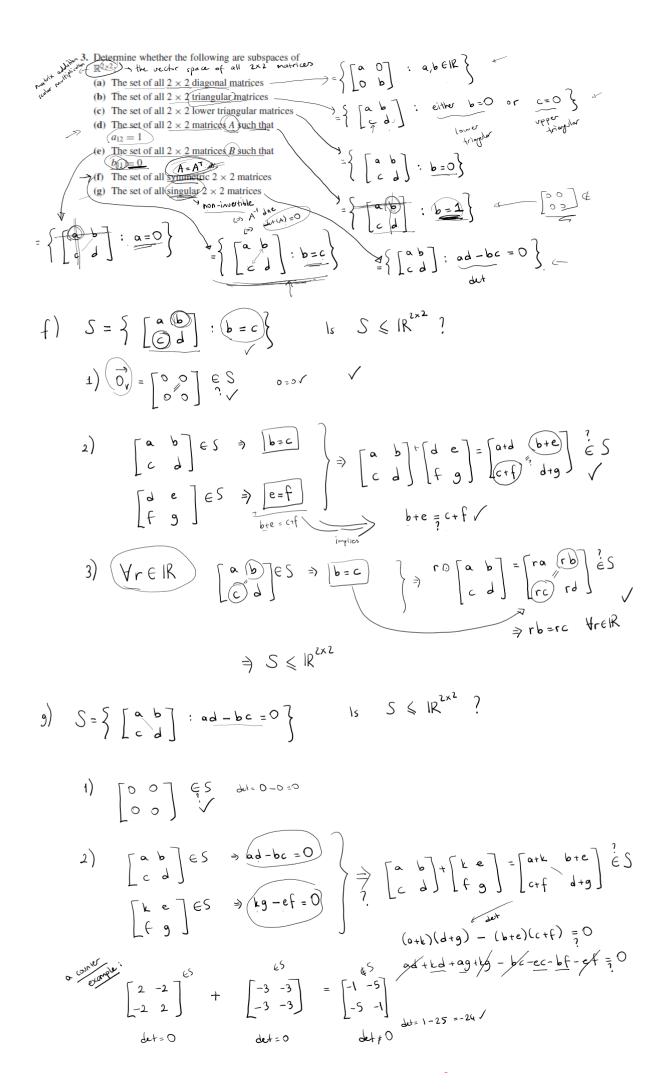
$$(1,2,1)+(4,3,3)=(5,5,4) \notin S$$

$$(y_{1}, y_{2}, y_{3}) \in S \Rightarrow y_{3} = y_{1} \text{ or } y_{3} = y_{2}$$

$$(x_{1}, x_{2}, x_{3}) \oplus (y_{1}, y_{2}, y_{3}) = (x_{1} + y_{1}, x_{2} + y_{2}, x_{3} + y_{3})$$

$$(x_{1}, x_{2}, x_{3}) \oplus (y_{1}, y_{2}, y_{3}) = (x_{1} + y_{1}, x_{2} + y_{2}, x_{3} + y_{3})$$

$$\Rightarrow 5 \notin \mathbb{R}^3$$



Some Special Subspaces (related to matrices)



$$N(A) =$$
 the set of all solutions of $A \times = 0$

1)
$$(0,0,...0) = 0_{\mathbb{R}^n} \in N(A) \checkmark \rightarrow \text{trivial solution}$$

2)
$$(x_1,x_2,...,x_n) \in N(A)$$
 $(y_1,y_2,...,y_n) \in N(A)$ $A(\underbrace{x+y}) = Ax + Ay = 0$

$$Ay = 0$$

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$$E(A)$$

$$= N(A) \leq \mathbb{R}^{n}$$

$$= (rx_1, rx_2, ..., rx_n) = (rx_1, rx_2, ..., rx_n) \qquad A(rx_1) = 0$$

$$= N(A) \leq \mathbb{R}^{n}$$

$$A = \begin{bmatrix} 1 & 2 & -3 & -1 \\ -2 & -4 & 6 & 3 \end{bmatrix}$$

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$$\begin{bmatrix} 1 & 2 & -3 & -1 & 0 \\ -2 & -4 & 6 & 3 & 0 \end{bmatrix} \xrightarrow{2r_1+r_2\to r_2} \begin{bmatrix} 1 & 2 & -3 & -1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \xrightarrow{0} x_4=0$$

$$x_2=relR$$

$$\Rightarrow x_1 = \underbrace{3s - 2r}_{s}$$