6th Week Friday

31 Mart 2023 Cuma 10:26

closed
1) $\vec{v}_1 \oplus \vec{v}_2 \in V$ → closed property of $\oplus$ on V ✓ ?
2) $r \odot \vec{v} \in V$ → " " " $\odot$ on V ✓ ?

$\oplus$
→ 1) $\vec{v}_1 \oplus \vec{v}_2 = \vec{v}_2 \oplus \vec{v}_1$ → Commutativity of $\oplus$ ✓
2) $\vec{v}_1 \oplus (\vec{v}_2 \oplus \vec{v}_3) = (\vec{v}_1 \oplus \vec{v}_2) \oplus \vec{v}_3$ → associativity of $\oplus$ ✓
identity 3) $\vec{0}_v \in V$ $\quad \vec{v} \oplus \vec{0}_v = \vec{v}$ identity element of $\oplus \in V$ ✗
inverse 4) $-\vec{v} \in V$ $\quad \vec{v} \oplus \underset{\in V}{-\vec{v}} = \vec{0}_v$ inverse element of $\oplus$ for any vector $\in V$ ✗

$\vec{0}, \vec{0}$ $\forall r,s \in \mathbb{R}$
$\odot$ 1) $r \odot (\vec{v}_1 \oplus \vec{v}_2) = (r \odot \vec{v}_1) \oplus (r \odot \vec{v}_2)$ → Distributivity [D-1] ✓
2) $(r+s) \odot \vec{v} = (r \odot \vec{v}) \oplus (s \odot \vec{v})$ → " [D-2] ✗
3) $(rs) \odot \vec{v} = r \odot (s \odot \vec{v})$ → [D-3]
4) $1 \in \mathbb{R}$ $\quad 1 \odot \vec{v} = \vec{v}$ → identity element of $\odot$

$\oplus$

! $(V, \oplus, \odot)$ → Is a vector space?

a set of vectors
→ n-tuples
→ matrices
→ polynom

Ex Is $V = \left\{ \begin{bmatrix} x \\ y \end{bmatrix} : x, y \in \mathbb{R}, \ x \leq 0 \right\}$,

with usual vector addition and scalar multiplication a vector space? **NO!**

$\forall r \in \mathbb{R}$ $\left\{ \begin{array}{l} (x_1, y_1) \oplus (x_2, y_2) = (x_1 + x_2, y_1 + y_2) \\ r(x, y) = (rx, ry) \end{array} \right\}$ usual

$(\underset{\leq 0}{x_1}, y_1) \oplus (\underset{\leq 0}{x_2}, y_2) = (\underset{\leq 0}{x_1 + x_2}, y_1 + y_2) \in V$ ✓ closed under $\oplus$

$r \in \mathbb{R}$ $\quad r(\underset{\leq 0}{x}, y) = (\underset{? \leq 0}{rx}, ry) \overset{?}{\in} V$ not closed under $\odot$

counter example → $r = -2$ $\quad (-3, 5) \in V$ ✗

$-2(-3, 5) = (\underset{\geq 0}{6}, -10) \notin V$

Ex Is $\mathbb{R}^2$ with operations;

→ $(x_1, y_1) \oplus (x_2, y_2) = (x_1 + x_2, y_1 + y_2) \in \mathbb{R}^2$

$\forall r \in \mathbb{R}$, $\quad r \odot (x, y) = (\underset{\in \mathbb{R}}{rx}, \underset{\in \mathbb{R}}{y}) \in \mathbb{R}^2$

1) ✓ closed $\oplus$
2) ✓ closed $\odot$

a vector space?

1) LHS: $r \odot \left(\underbrace{(x_1, y_1) \oplus (x_2, y_2)}_{(x_1 + x_2, \ y_1 + y_2)} \right) = (r(x_1 + x_2), y_1 + y_2)$ ✓

RHS: $\underbrace{r \odot (x_1, y_1)}_{(rx_1, y_1)} \oplus \underbrace{r \odot (x_2, y_2)}_{(rx_2, y_2)} = (rx_1 + rx_2, y_1 + y_2)$ ✓

$\forall r, s \in \mathbb{R}$

2) LHS: $(r+s) \odot (x, y) = ((r+s)x, y) \neq$ RHS: $\underbrace{r \odot (x, y)}_{(rx, y)} \oplus \underbrace{s \odot (x, y)}_{(sx, y)} = (rx + sx, y + y)$

Counter example $\quad r = 3 \quad s = -1 \quad (1, 2)$

LHS $\quad \underbrace{[3 + (-1)]}_{} \odot (1, 2) = (2, 2)$ $\quad$ RHS $\neq$ $\quad 3 \odot (1, 2) \oplus (-1) \odot (1, 2) = (3, 2) \oplus (-1, 2) = (2, 4)$

$2 \odot (1, 2)$

12. Let R^+ denote the set of positive real numbers. Define the operation of scalar multiplication, denoted $\circ$, by

$$\alpha \circ x = x^\alpha$$

for each $x \in R^+$ and for any real number α. Define the operation of addition, denoted $\oplus$, by

$$x \oplus y = x \cdot y \qquad \text{for all} \qquad x, y \in R^+$$

Thus, for this system, the scalar product of -3 times $\frac{1}{2}$ is given by

$$\underset{\in \mathbb{R}}{-3} \circ \underset{\in \mathbb{R}^+}{\frac{1}{2}} = \left(\frac{1}{2}\right)^{-3} = 8 \in \mathbb{R}^+ \ ??$$

and the sum of 2 and 5 is given by

$$\underset{\in \mathbb{R}^+}{2} \oplus \underset{\in \mathbb{R}^+}{5} = 2 \cdot 5 = 10$$

Is R^+ a vector space with these operations? Prove your answer.

2) closed
1) $\underset{\in V}{\vec{v}_1} \oplus \underset{\in V}{\vec{v}_2} \in V$ → closed property of $\oplus$ on V → ✓
$r \odot \vec{v} \in V$ → " " " $\odot$ on V ✓

$\oplus$ $\quad \vec{0}_v \in V$
→ 1) $\vec{v}_1 \oplus \vec{v}_2 = \vec{v}_2 \oplus \vec{v}_1$ → Commutativity of $\oplus$ ✓
2) $\vec{v}_1 \oplus (\vec{v}_2 \oplus \vec{v}_3) = (\vec{v}_1 \oplus \vec{v}_2) \oplus \vec{v}_3$ → associativity of $\oplus$ ✓ $x + (y + z) = x(yz) = xyz$
$(xy)z = xyz$
identity 3) $\vec{0}_v \in V$ $\quad \vec{v} \oplus \vec{0}_v = \vec{v}$ identity element of $\oplus \in V$ ✓
inverse 4) $-\vec{v} \in V$ $\quad \vec{v} \oplus \underset{\in V}{-\vec{v}} = \vec{0}_v$ inverse element of $\oplus$ for any vector $\in V$ ✓

$\forall r, s \in \mathbb{R}$
→ ✓ 1) $r \odot (\vec{v}_1 \oplus \vec{v}_2) = (r \odot \vec{v}_1) \oplus (r \odot \vec{v}_2)$ → Distributivity [D-1] $\quad (v_1 v_2)^r \overset{\checkmark}{=} v_1^r \oplus v_2^r = (v_1 v_2)^r$
→ ✓ 2) $(r+s) \odot \vec{v} = (r \odot \vec{v}) \oplus (s \odot \vec{v})$ → " [D-2] $\quad v^{r+s} \overset{\checkmark}{=} v^r \oplus v^s = v^{r+s}$
→ ✓ 3) $(rs) \odot \vec{v} = r \odot (s \odot \vec{v})$ → [D-3] $\quad v^{rs} \overset{\checkmark}{=} (v^s)^r$
→ ✓ 4) $1 \in \mathbb{R}$ $\quad 1 \odot \vec{v} = \vec{v}$ → identity element of $\odot$

$1 \in \mathbb{R}^+$ → identity $\oplus$

citizens $\quad \forall \mathbb{R}^+$ $\quad \underset{\in \mathbb{R}^+}{x} \oplus \underset{\in \mathbb{R}^+}{y} = x \cdot y \in \mathbb{R}^+$ ✓

Is $\mathbb{R}^+, \oplus, \odot$ vector space?

$\rightarrow 2 \oplus 5 = 2 \cdot 5 = 10$

Is R^+ a vector space with these operations? Prove your answer.

$\mathbb{R}^+$?

$x \oplus y = x \cdot y \in \mathbb{R}^+$

$\forall r \in \mathbb{R}$

$r \odot x = x^r \in \mathbb{R}^+$

$x^1 = x$

Is $\mathbb{R}^+, \oplus, \odot$ vector space?

$x \in \mathbb{R}^+$ inverse $\oplus \Rightarrow \frac{1}{x} \in \mathbb{R}^+$ ✓