2nd Week Friday

03 Mart 2023 Cuma 10:29

Elementary Row Operations

1) $r_i \leftrightarrow r_j$
2) $cr_i \to r_i$ $\quad c \neq 0,\ c \in \mathbb{R}$ — while trying to obtain a "leading 1"
3) $cr_i + r_j \to r_j$ → make zeroes

Gauss Elimination Method

system → Augmented Matrix $\left[\begin{array}{c|c} \text{Coeff} & \text{results} \end{array}\right]$ → → → ... REF

elementary row operations

Gauss-Jordan Elimination Method

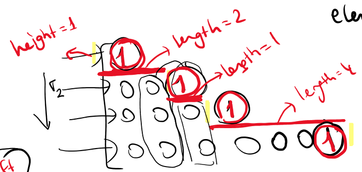
→ → ... RREF

elementary row operation

Ex

$$\begin{array}{c} r_1 \to \\ \\ r_3 \to \end{array}\left[\begin{array}{ccccc|c} 3 & \# & \# & \# & \# & \\ & & & & & \\ 1 & \# & \# & \# & \# & \end{array}\right] \xrightarrow{r_1 \leftrightarrow r_3}$$

Ex



REF

the height of each step = 1
the length of a step may change ≥ 1

top-left ① → bottom right

! obtain a "leading 1" for each row

Ex

$$\left[\begin{array}{cccc|c} 0 & & & & \\ 0 & & & & \\ 0 & & & & \\ 0 & & & & \end{array}\right]$$

move to the next column to search for the leading 1 of this row!

Ex obtain a leading 1 for r_1:

$$\begin{array}{c} r_1 \to \\ \\ r_3 \to \end{array}\left[\begin{array}{ccccc|c} 3 & \# & \# & \# & \# & \\ & & & & & \\ 1 & \# & \# & \# & \# & \end{array}\right] \xrightarrow{r_1 \leftrightarrow r_3} \begin{array}{c} r_1 \\ r_2 \to \\ r_3 \to \\ r_4 \to \end{array}\left[\begin{array}{ccccc|c} 1 & \# & \# & \# & \# & \\ -2 & & & & & \\ 3 & \# & \# & \# & \# & \\ 4 & & & & & \end{array}\right] \xrightarrow[\substack{-3r_1 + r_3 \to r_3 \\ -4r_1 + r_4 \to r_4}]{2r_1 + r_2 \to r_2} \left[\begin{array}{cccc|c} 1 & \# & \# & \# & \\ 0 & \# & \# & \# & \\ 0 & \# & \# & \# & \\ 0 & \# & \# & \# & \end{array}\right]$$

instead $\frac{1}{3}r_1 \to r_1$

leading 1 for the 1st row

make all these = zero!

use multiples of the row where your leading 1 is.

! these will never change until the end of the process

→ obtain a leading 1 for r_2

$$\begin{array}{c} \\ r_2 \to \\ \\ \\ \end{array}\left[\begin{array}{cccc|c} 1 & \# & \# & \# & \\ 0 & 4 & \# & \# & \\ 0 & 3 & \# & \# & \\ 0 & 5 & \# & \# & \end{array}\right] \xrightarrow{\frac{1}{4}r_2 \to r_2} \begin{array}{c} \\ r_2 \\ \\ \\ \end{array}\left[\begin{array}{cccc|c} 1 & \# & \# & \# & \\ 0 & 1 & \# & \# & \\ 0 & 3 & \# & \# & \\ 0 & 5 & \# & \# & \end{array}\right] \xrightarrow[-5r_2 + r_4 \to r_4]{-3r_2 + r_3 \to r_3} \left[\begin{array}{cccc|c} 1 & \# & \# & \# & \\ 0 & 1 & \# & \# & \\ 0 & 0 & \# & \# & \\ 0 & 0 & \# & \# & \end{array}\right]$$

what if all these were zero?

bottom-right

! make all these = zero!

use multiples of r_2

bottom

... continue ... until you reach bottom-right.

Ex

6- (b)
$$\begin{aligned} x_1 + 3x_2 + x_3 + x_4 &= 3 \\ 2x_1 - 2x_2 + x_3 + 2x_4 &= 8 \\ 3x_1 + x_2 + 2x_3 - x_4 &= -1 \end{aligned}$$

augmented matrix

leading 1 of r_1

$$\begin{array}{c} r_1 \to \\ \\ \\ \end{array}\left[\begin{array}{cccc|c} 1 & 3 & 1 & 1 & 3 \\ 2 & -2 & 1 & 2 & 8 \\ 3 & 1 & 2 & -1 & -1 \end{array}\right] \xrightarrow[-3r_1 + r_3 \to r_3]{-2r_1 + r_2 \to r_2} \begin{array}{c} \\ r_2 \to \\ \\ \end{array}\left[\begin{array}{cccc|c} 1 & 3 & 1 & 1 & 3 \\ 0 & -8 & -1 & 0 & 2 \\ 0 & -8 & -1 & -4 & -10 \end{array}\right]$$

make all these = zero

! use multiples of r_1

$$\begin{array}{c} \\ r_2 \to \\ \\ \end{array}\left[\begin{array}{cccc|c} 1 & 3 & 1 & 1 & 3 \\ 0 & -8 & -1 & 0 & 2 \\ 0 & -8 & -1 & -4 & -10 \end{array}\right] \xrightarrow{-\frac{1}{8}r_2 \to r_2} \left[\begin{array}{cccc|c} 1 & 3 & 1 & 1 & 3 \\ 0 & 1 & 1/8 & 0 & -1/4 \\ 0 & -8 & -1 & -4 & -10 \end{array}\right] \xrightarrow{8r_2 + r_3 \to r_3} \begin{array}{c} \\ r_2 \to \\ r_3 \to \end{array}\left[\begin{array}{cccc|c} 1 & 3 & 1 & 1 & 3 \\ 0 & 1 & 1/8 & 0 & -1/4 \\ 0 & 0 & 0 & -4 & -12 \end{array}\right]$$

make this = zero
use multiples of r_2!

?

$$-\frac{1}{4}r_3 \to r_3 \quad \begin{array}{c} r_1 \to \\ r_2 \to \\ r_3 \to \end{array} \left[\begin{array}{cccc|c} 1 & 3 & 1 & 1 & 3 \\ 0 & 1 & 1/8 & 0 & -1/4 \\ 0 & 0 & 0 & 1 & 3 \end{array}\right]$$

REF ✓✓

top left → bottom-right

REF

To go through RREF, this time your job is to make zeroes above the "leading 1"s.

from bottom-right → top-left.

$$\begin{array}{c} r_1 \to \\ r_2 \to \\ r_3 \to \end{array} \left[\begin{array}{cccc|c} 1 & 3 & 1 & 1 & 3 \\ 0 & 1 & 1/8 & 0 & -1/4 \\ 0 & 0 & 0 & 1 & 3 \end{array}\right] \xrightarrow[-1r_3 + r_1 \to r_1]{} \left[\begin{array}{cccc|c} 1 & 3 & 1 & 0 & 0 \\ 0 & 1 & 1/8 & 0 & -1/4 \\ 0 & 0 & 0 & 1 & 3 \end{array}\right]$$

use multiples of r_3

use multiples of r_2

$-\frac{3}{8}+1$

$$\xrightarrow[-3r_2 + r_1 \to r_1]{} \left[\begin{array}{cccc|c} 1 & 0 & 5/8 & 0 & 3/4 \\ 0 & 1 & 1/8 & 0 & -1/4 \\ 0 & 0 & 0 & 1 & 3 \end{array}\right] \to \text{RREF ✓✓}$$