Assignment 4

ELEC 442 - Introduction to Robotics

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I. Two-Link Manipulator Open Loop Simulation

Considering the two link manipulator described on page 87 in chapter 6 of the notes, the equations of motion are given by equation (209) to (233). In general we have the Euler-Lagrange equations of motion given as

$$\frac{\mathrm{d}}{\mathrm{d}t}\left[\frac{\partial L}{\partial \dot{q}_i}(q,\dot{q})\right] - \frac{\partial L}{\partial q_i}(q,\dot{q}) = \tau_i$$

where

$$L(\boldsymbol{q},\dot{\boldsymbol{q}}) = T(\boldsymbol{q},\dot{\boldsymbol{q}}) - V(\boldsymbol{q})$$

and τ_i is the generalized force or torque associated with coordinate i. This can be written on standard form as

$$D(\boldsymbol{q})\ddot{\boldsymbol{q}} + C(\boldsymbol{q},\dot{\boldsymbol{q}})\dot{\boldsymbol{q}} + G(\boldsymbol{q}) = \boldsymbol{u} + \underline{J}_n^\top \begin{bmatrix} \underline{f}_e \\ \underline{\tau}_e \end{bmatrix}$$

where the last term can be ignored as we do not interact with the environment. This gives us the secod order system

$$\ddot{\boldsymbol{q}} = D^{-1}(\boldsymbol{q})\left[\boldsymbol{u} - C(\boldsymbol{q},\dot{\boldsymbol{q}})\dot{\boldsymbol{q}} - G(\boldsymbol{q})\right]$$

with $D(\boldsymbol{q})$ given by equation (222), $C(\boldsymbol{q},\dot{\boldsymbol{q}})$ given by (232) and $G(\boldsymbol{q})$ by (233), and $\boldsymbol{q} = \begin{bmatrix}\theta_1 \\ \theta_2\end{bmatrix}$. To generate these matrices in Simulink, we implement block functions shown in Figure 5, Figure 6 and Figure 7 in Appendix A. The complete mainpulator dynamics are implemented in Figure 8.

i).

With $\boldsymbol{x}(0) = \begin{bmatrix}0 & 0 & 0 & 0\end{bmatrix}^\top$ and $\tau_1 = \tau_2 = 0$ we get the response shown in Figure 1.

ii).

With $\boldsymbol{x}(0) = \begin{bmatrix}0 & \frac{\pi}{2} & 0 & 0\end{bmatrix}^\top$ and $\tau_2 = 5\,\mathrm{Nm}$ we get the response shown in Figure 2. Here the total energy of the system is increasing, which is seen in the plots for the angular velocities.

iii).

With $\boldsymbol{x}(0) = \begin{bmatrix}0 & 0 & 0 & 0\end{bmatrix}^\top$ and friction modeled as $\tau_1 = -0.5\dot{\theta}_1$ and $\tau_2 = -0.5\dot{\theta}_2$ we get the response shown in Figure 3. Here we see that the amplitude is decreasing, wich makes sense because the total energy of the system is decreasing when friction is added.

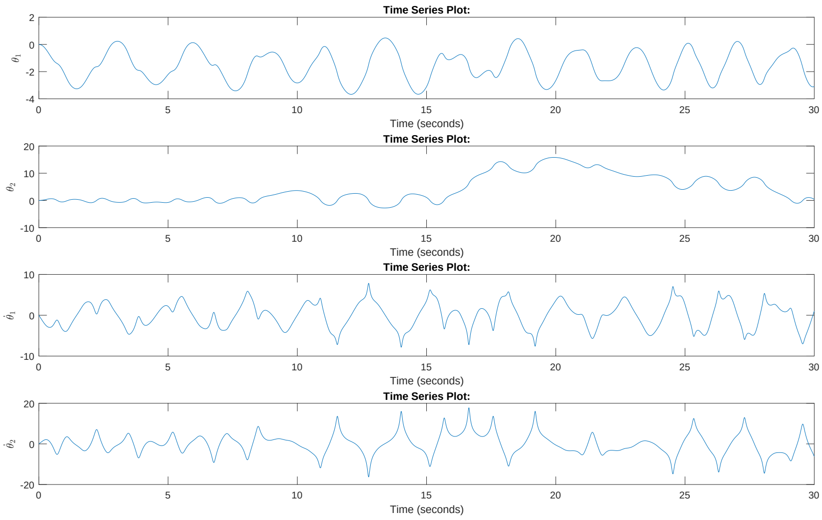

Figure 1: Response of all states with all states initialized to zero

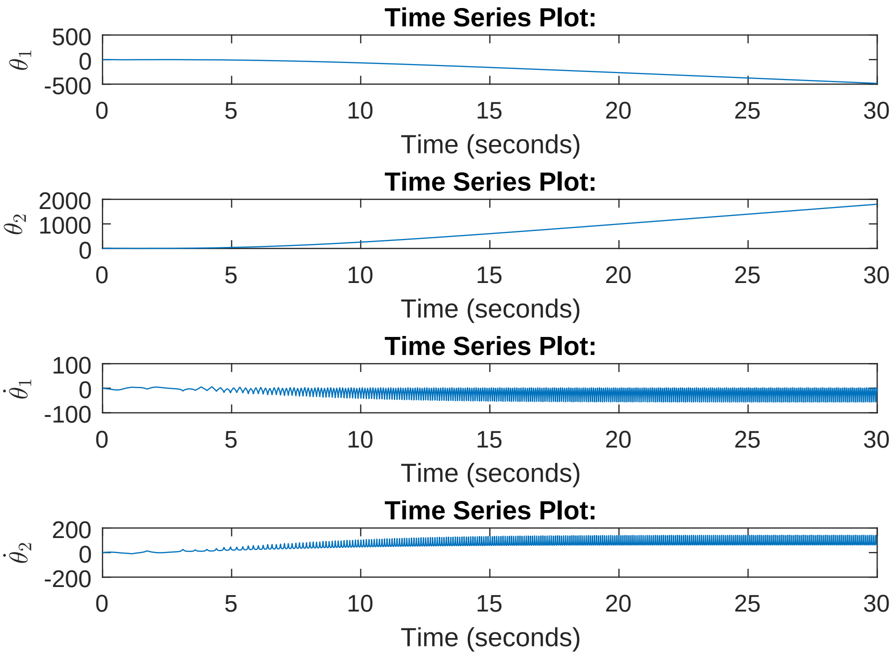

Figure 2: Response of all states with $\boldsymbol{x}(0) = \left[0\ \frac{\pi}{2}\ 0\ 0\right]^\top$ and $\tau_2 = 5\,\mathrm{Nm}$

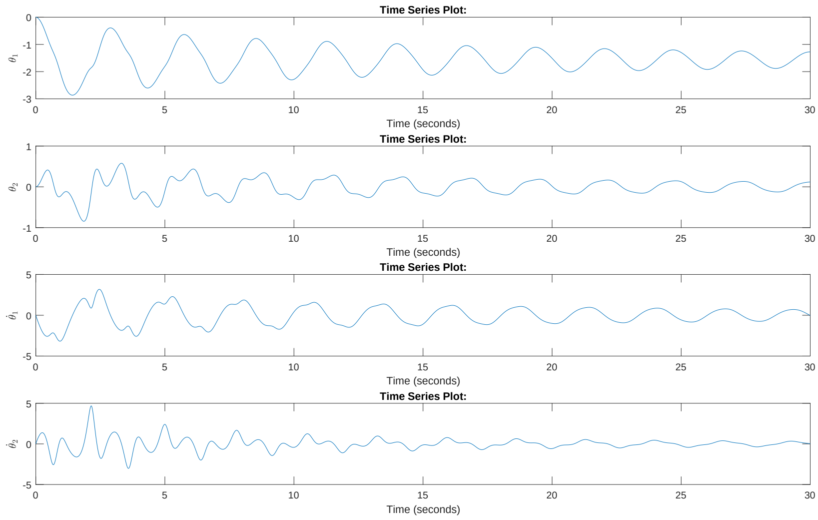

Figure 3: Response of all states with all states initialized to zero and friction modeled as $\tau_1 = -0.5\dot{\theta}_1$ and $\tau_2 = -0.5\dot{\theta}_2$

II. Controller Implementation

i). Closed loop joint-space control

With the PD + gravity controller we get the input vector

$$\boldsymbol{u} = \underbrace{G(\boldsymbol{q})}_{\text{Gravity terms}} + \underbrace{K_p(\boldsymbol{q}_d - \boldsymbol{q}) - K_v\dot{\boldsymbol{q}}}_{\text{PD-controller}}$$

where we require

$$K_p, K_v \succ 0$$

which is implemented as the Simulink diagram shown in Figure 9. With this implementation and the initial conditions

$$\begin{aligned} \boldsymbol{x}(0) &= \begin{bmatrix} -\frac{\pi}{2} & 0 & 0 & 0 \end{bmatrix}^\top \\ \boldsymbol{q}_d &= \begin{bmatrix} 0 & \frac{\pi}{2} \end{bmatrix}^\top \\ K_p &= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \\ K_v &= \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} \end{aligned}$$

we get the output shown in Figure 4.

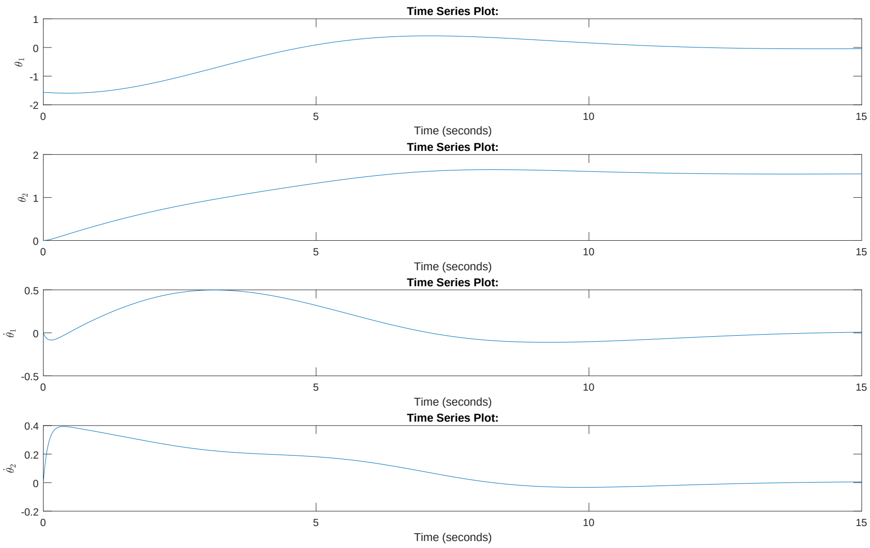

Figure 4: Output states with the PD + gravity controller

Appendices

A. Simulink diagrams

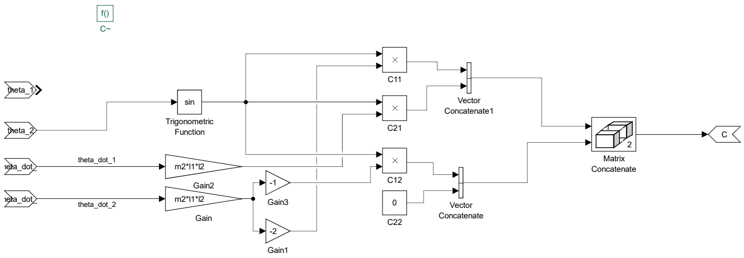

Figure 5: Simulink function block to generate $C(\boldsymbol{q}, \dot{\boldsymbol{q}})$

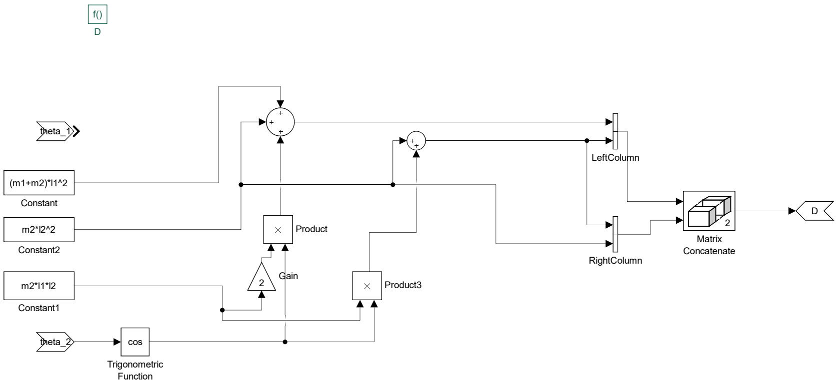

Figure 6: Simulink function block to generate $D(\boldsymbol{q})$

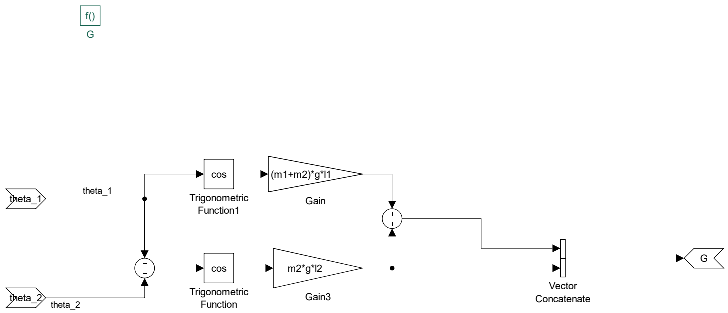

Figure 7: Simulink function block to generate $G(\boldsymbol{q})$

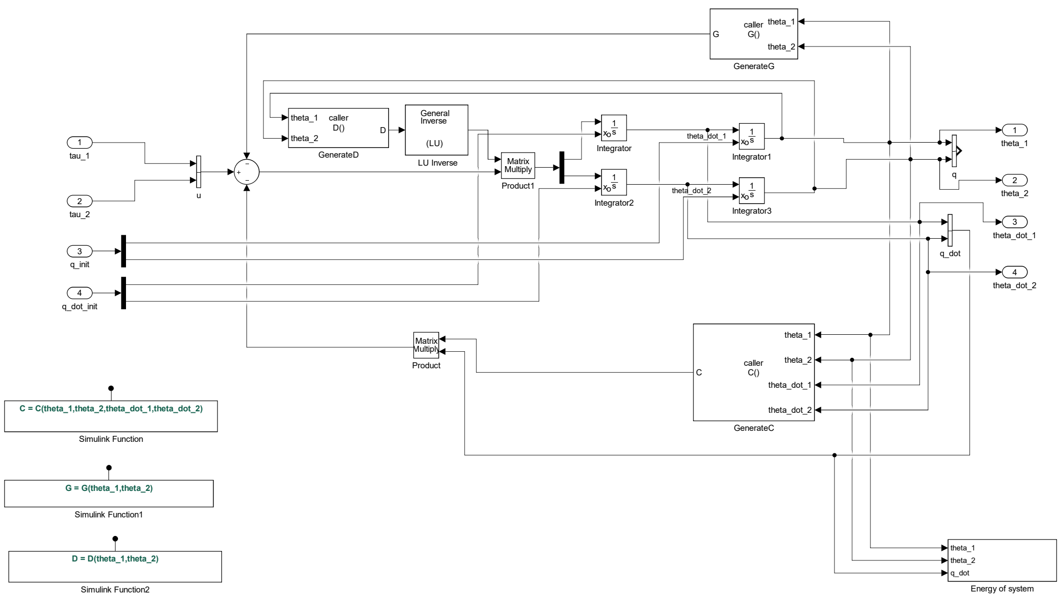

Figure 8: Simulink block to the complete manipulator dynamics

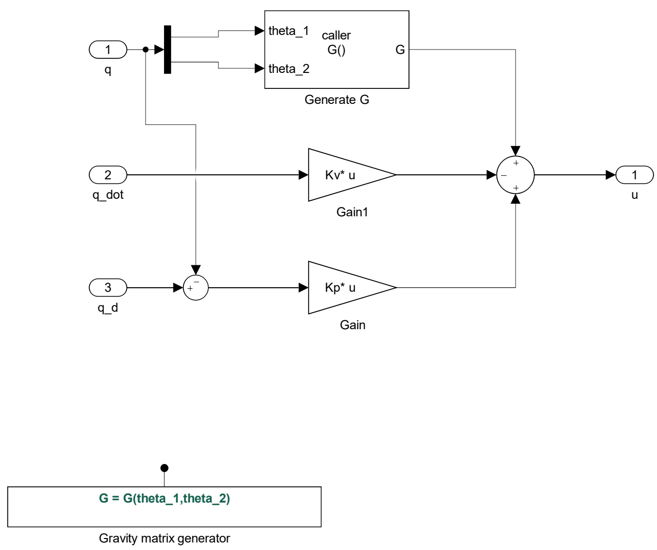

Figure 9: Simulink block for the PD + gravity controller