

ELEC 442 - Introduction to Robotics

Assignment 1

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1

Given the homogenous transformation

$$\begin{bmatrix} \mathbf{y} \\ 1 \end{bmatrix} = \underbrace{\begin{bmatrix} Q & \mathbf{d} \\ \mathbf{0}^\top & 1 \end{bmatrix}}_T \begin{bmatrix} \mathbf{x} \\ 1 \end{bmatrix}$$

where Q and \mathbf{d} accounts for rotation and translation, respectively. We have that the inverse is given by

$$T^{-1} = \begin{bmatrix} \tilde{Q} & \tilde{\mathbf{d}} \\ \mathbf{0}^\top & 1 \end{bmatrix}$$

where we know that $T^{-1}T$ is equal to the 4×4 identity matrix. This yields

$$\begin{aligned} T^{-1}T &= \begin{bmatrix} \tilde{Q} & \tilde{\mathbf{d}} \\ \mathbf{0}^\top & 1 \end{bmatrix} \begin{bmatrix} Q & \mathbf{d} \\ \mathbf{0}^\top & 1 \end{bmatrix} \\ &= \begin{bmatrix} \tilde{Q}Q & \tilde{Q}\mathbf{d} + \tilde{\mathbf{d}} \\ \mathbf{0}^\top & 1 \end{bmatrix} = \mathbf{I}_{4 \times 4} \\ \Rightarrow &\begin{cases} \tilde{Q}Q &= \mathbf{I}_{3 \times 3} \\ \tilde{Q}\mathbf{d} + \tilde{\mathbf{d}} &= \mathbf{0} \end{cases} \\ \Rightarrow &\begin{cases} \tilde{Q} &= Q^{-1} = Q^\top \\ \tilde{\mathbf{d}} &= -\tilde{Q}\mathbf{d} = -Q^\top \mathbf{d} \end{cases} \\ \Rightarrow &T^{-1} = \underline{\begin{bmatrix} Q^\top & -Q^\top \mathbf{d} \\ \mathbf{0} & 1 \end{bmatrix}} \end{aligned}$$