Assignment 2

ELEC 442 - Introduction to Robotics

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Solving the inverse kinematics problem for the PUMA 560 robot

For the first angle, θ_1 we use that the centre of the spherical wrist, $\underset{\sim}{o_4}$, compared to the xy-plane is only dependent on θ_1. We also know that for $\theta_1 = 0$ $\underset{\sim}{o_4}$ must be in the plane defined by $y - 149.09\text{mm} = 0$. This gives us the relation between u and w, where w is the vector between the origin and $\underset{\sim}{o_4}$ and u is the vector to where $\underset{\sim}{o_4}$ would be if $\theta_1 = 0$, such that we kan use the inplemented function (KahanP2). The relation is given as

$$
\begin{aligned}
\|w\|^2 = o_{4,x}^2 + o_{4,y}^2 + o_{4,z}^2 = u_x^2 + u_y^2 + u_z^2 = \|u\|^2 & \\
\implies o_{4,x}^2 + o_{4,y}^2 + \cancel{o_{4,z}^2} = u_x^2 + 149.09^2 + \cancel{o_{4,z}^2} & \quad \text{as desired } z\text{-value doesn't} \\
\implies u_x = \sqrt{o_{4,x}^2 + o_{4,y}^2 - 149.09^2} & \quad \text{change with } \theta_1
\end{aligned}
$$

Given this and that $\underset{\sim}{o_0} = \begin{pmatrix} 0 & 0 & 0 \end{pmatrix}^\top$ we can use the implemented function `KahanP2` with $s = \boldsymbol{k}_0$, $\hat{\boldsymbol{u}} = \begin{bmatrix} u_x & 149.09 & o_{4,z} \end{bmatrix}$ and $\boldsymbol{w} = \underset{\sim}{o_4} - \underset{\sim}{o_0}$. The function is implemented as shown in Listing 1. In this implementation the function itself normalizes the vectors such that we do not have to think about feeding this into the function.

```
function theta = KahanP2(s,u,w)
    s_hat = s/norm(s);
    u_hat = u/norm(u);
    w_hat = w/norm(w);
    if s_hat'*cross(s_hat,u_hat) == s_hat'*cross(s_hat,w_hat)
        theta = 2*atan(norm(cross(s_hat,(u_hat-w_hat)))/norm(cross(s_hat,(u_hat+w_hat))));
        if w_hat'*cross(s_hat,(u_hat-w_hat)) < 0
            theta = -theta;
        end
    else
        theta = 'The solution does not exist';
    end
end
```

Listing 1: MATLAB implementation of the Kahan P2 problem