ELEC 442 - Introduction to Robotics

Assignment 1

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Given the homogenous transformation

$$\begin{bmatrix} \boldsymbol{y} \\ 1 \end{bmatrix} = \underbrace{\begin{bmatrix} Q & \boldsymbol{d} \\ \boldsymbol{0}^\top & 1 \end{bmatrix}}_{T} \begin{bmatrix} \boldsymbol{x} \\ 1 \end{bmatrix}$$

where Q and $\boldsymbol{d}$ accounts for rotation and translation, respectively. We have that the inverse is on the form

$$T^{-1} = \begin{bmatrix} \tilde{Q} & \tilde{\boldsymbol{d}} \\ \boldsymbol{0}^\top & 1 \end{bmatrix}$$

where we know that $T^{-1}T$ is equal to the 4×4 identity matrix. This yields

$$\begin{aligned}
T^{-1}T &= \begin{bmatrix} \tilde{Q} & \tilde{\boldsymbol{d}} \\ \boldsymbol{0}^\top & 1 \end{bmatrix} \begin{bmatrix} Q & \boldsymbol{d} \\ \boldsymbol{0}^\top & 1 \end{bmatrix} \\
&= \begin{bmatrix} \tilde{Q}Q & \tilde{Q}\boldsymbol{d} + \tilde{\boldsymbol{d}} \\ \boldsymbol{0}^\top & 1 \end{bmatrix} = \mathbf{I}_{4\times 4} \\
\implies & \begin{cases} \tilde{Q}Q & = \mathbf{I}_{3\times 3} \\ \tilde{Q}\boldsymbol{d} + \tilde{\boldsymbol{d}} & = \boldsymbol{0} \end{cases} \\
\implies & \begin{cases} \tilde{Q} & = Q^{-1} = Q^\top \\ \tilde{\boldsymbol{d}} & = -\tilde{Q}\boldsymbol{d} = -Q^\top \boldsymbol{d} \end{cases} \\
\implies T^{-1} &= \underline{\begin{bmatrix} Q^\top & -Q^\top \boldsymbol{d} \\ \boldsymbol{0}^\top & 1 \end{bmatrix}}
\end{aligned}$$

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Considering the homogenous transformation matrix

$${}^0T_1 = \begin{bmatrix} Q & \boldsymbol{d} \\ \boldsymbol{0}^\top & 1 \end{bmatrix}$$

with

$$Q = \underbrace{\begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ 0 & 0 & 1 \end{bmatrix}}_{Q_1} \underbrace{\begin{bmatrix} 1 & 0 & 0 \\ 0 & -\frac{1}{2} & -\frac{\sqrt{3}}{2} \\ 0 & \frac{\sqrt{3}}{2} & -\frac{1}{2} \end{bmatrix}}_{Q_2}$$

and

$$\boldsymbol{d} = \begin{bmatrix} -\frac{5}{\sqrt{2}} \\ \frac{5}{\sqrt{2}} \\ 4 \end{bmatrix} \text{cm}$$

2.a

2.b

For the point represented in coordinate system 1 by $\underset{\sim}{\boldsymbol{x}} = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix}^\top$ cm we get the representation in system 0 given by

$$\begin{bmatrix} {}^0\boldsymbol{x} \\ 1 \end{bmatrix} = {}^0T_1 \begin{bmatrix} {}^1\boldsymbol{x} \\ 1 \end{bmatrix}$$

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A MATLAB fuction named `DH_homog` is implemented with the code shown in Listing 1.

Listing 1: MATLAB code to generate homogenous transformation matrix based on the Denavit-Hartenberg convention

```matlab
function T = DH_homog(theta, d, a, alpha)
    i=[1;0;0];
    k=[0;0;1];
    angle = [expm(theta*skew(k)) zeros(3,1); zeros
       (1,3) 1];
    offset = [eye(3) d*k; zeros(1,3) 1];
    length = [eye(3) a*i; zeros(1,3) 1];
    twist = [expm(alpha*skew(i)) zeros(3,1); zeros
       (1,3) 1];
    T = angle*offset*length*twist;
end
```