Assignment 1

ELEC 442 - Introduction to Robotics

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1.

Given the homogenous transformation

$$\begin{bmatrix} \boldsymbol{y} \\ 1 \end{bmatrix} = \underbrace{\begin{bmatrix} Q & \boldsymbol{d} \\ \boldsymbol{0}^\top & 1 \end{bmatrix}}_{T} \begin{bmatrix} \boldsymbol{x} \\ 1 \end{bmatrix}$$

where Q and $\boldsymbol{d}$ accounts for rotation and translation, respectively. We have that the inverse is on the form

$$T^{-1} = \begin{bmatrix} \tilde{Q} & \tilde{\boldsymbol{d}} \\ \boldsymbol{0}^\top & 1 \end{bmatrix}$$

where we know that $T^{-1}T$ is equal to the 4×4 identity matrix. This yields

$$\begin{aligned} T^{-1}T &= \begin{bmatrix} \tilde{Q} & \tilde{\boldsymbol{d}} \\ \boldsymbol{0}^\top & 1 \end{bmatrix} \begin{bmatrix} Q & \boldsymbol{d} \\ \boldsymbol{0}^\top & 1 \end{bmatrix} \\ &= \begin{bmatrix} \tilde{Q}Q & \tilde{Q}\boldsymbol{d} + \tilde{\boldsymbol{d}} \\ \boldsymbol{0}^\top & 1 \end{bmatrix} = \mathbf{I}_{4\times 4} \\ &\implies \begin{cases} \tilde{Q}Q & = \mathbf{I}_{3\times 3} \\ \tilde{Q}\boldsymbol{d} + \tilde{\boldsymbol{d}} & = \boldsymbol{0} \end{cases} \\ &\implies \begin{cases} \tilde{Q} & = Q^{-1} = Q^\top \\ \tilde{\boldsymbol{d}} & = -\tilde{Q}\boldsymbol{d} = -Q^\top \boldsymbol{d} \end{cases} \\ \implies T^{-1} &= \underline{\begin{bmatrix} Q^\top & -Q^\top \boldsymbol{d} \\ \boldsymbol{0}^\top & 1 \end{bmatrix}} \end{aligned} \tag{1}$$

For T^{-1} to exist obvously T must be invertible, and for this to be fulfilled we require full rank. In this case $\text{rank}(T) = 4$ since $\text{rank}(Q) = 3 \ \forall Q$ as Q is a rotation matrix, and the $T_{4,4} = 1 \ \forall T$. Thus $\forall \{Q, \boldsymbol{d}\}$ we have $\text{rank}(T) = 4$ and T^{-1} exists.

2.

Considering the homogenous transformation matrix

$${}^0T_1 = \begin{bmatrix} Q & \boldsymbol{d} \\ \boldsymbol{0}^\top & 1 \end{bmatrix}$$

with

$$Q = \underbrace{\begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ 0 & 0 & 1 \end{bmatrix}}_{Q_1} \underbrace{\begin{bmatrix} 1 & 0 & 0 \\ 0 & -\frac{1}{2} & -\frac{\sqrt{3}}{2} \\ 0 & \frac{\sqrt{3}}{2} & -\frac{1}{2} \end{bmatrix}}_{Q_2}$$

and

$$\boldsymbol{d} = \begin{bmatrix} -\frac{5}{\sqrt{2}} \\ \frac{5}{\sqrt{2}} \\ 4 \end{bmatrix} \text{ cm}$$

2a).

By obserwing the rotation matrices Q_1 and Q_2 we see that Q_1 is a simple rotation around the $\boldsymbol{k}$-axis. The angle of this rotation is given by $\theta = \arccos\left(\frac{1}{\sqrt{2}}\right) = \pm\frac{\pi}{4}$. As $\sin\left(-\frac{\pi}{4}\right) = -\frac{1}{\sqrt{2}}$ we get $\theta = -\frac{\pi}{4}$. Q_2 is a simple rotation arond the $\boldsymbol{i}$-axis and the rotation angle is given by $\alpha = \arccos\left(-\frac{1}{2}\right) = \frac{2\pi}{3}$. To determine d_1 and a_1 we recognize that a homogenous transformation matrix can be written ass the product of four transformation matrices; angle, offset, length and twist. This gives us

$$\begin{aligned}
{}^0T_1 = \begin{bmatrix} Q & \boldsymbol{d} \\ 0^\top & 1 \end{bmatrix} &= \underbrace{\begin{bmatrix} \exp(\theta\boldsymbol{k}\times) & \boldsymbol{0} \\ \boldsymbol{0}^\top & 1 \end{bmatrix}}_{\text{angle}} \underbrace{\begin{bmatrix} \boldsymbol{I} & d\boldsymbol{k} \\ \boldsymbol{0}^\top & 1 \end{bmatrix}}_{\text{offset}} \underbrace{\begin{bmatrix} \boldsymbol{I} & a\boldsymbol{i} \\ \boldsymbol{0}^\top & 1 \end{bmatrix}}_{\text{length}} \underbrace{\begin{bmatrix} \exp(\alpha\boldsymbol{i}\times) & \boldsymbol{0} \\ \boldsymbol{0}^\top & 1 \end{bmatrix}}_{\text{twist}} \\
&= \begin{bmatrix} \exp(\theta\boldsymbol{k}\times + \alpha\boldsymbol{i}\times) & \exp(\theta\boldsymbol{k}\times)(a\boldsymbol{i} + d\boldsymbol{k}) \\ \boldsymbol{0}^\top & 1 \end{bmatrix} = \begin{bmatrix} Q & \boldsymbol{d} \\ \boldsymbol{0}^\top & 1 \end{bmatrix} \\
\implies \exp(\theta\boldsymbol{k}\times)(a\boldsymbol{i} + d\boldsymbol{k}) &= \begin{bmatrix} -\frac{5}{\sqrt{2}} \\ \frac{5}{\sqrt{2}} \\ 4 \end{bmatrix} \\
\implies \exp(\theta\boldsymbol{k}\times) \begin{bmatrix} a \\ 0 \\ d \end{bmatrix} &= \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} a \\ 0 \\ d \end{bmatrix} = \begin{bmatrix} \frac{a}{\sqrt{2}} \\ -\frac{a}{\sqrt{2}} \\ d \end{bmatrix} = \begin{bmatrix} -\frac{5}{\sqrt{2}} \\ \frac{5}{\sqrt{2}} \\ 4 \end{bmatrix} \\
\implies &\begin{cases} a = -5 \\ d = 4 \\ \theta = -\frac{\pi}{4} \\ \alpha = \frac{2\pi}{3} \end{cases}
\end{aligned}$$

And we have numeric values for all our four DH parameters.

2b).

For the point represented in coordinate system 1 by $\underset{\sim}{\boldsymbol{x}} = \begin{bmatrix}1 & 0 & 0\end{bmatrix}^\top$ cm we get the representation in system 0 given by

$$
\begin{aligned}
\begin{bmatrix} {}^0\boldsymbol{x} \\ 1 \end{bmatrix} &= {}^0T_1 \begin{bmatrix} {}^1\boldsymbol{x} \\ 1 \end{bmatrix} \\
&= \begin{bmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{2\sqrt{2}} & -\frac{\sqrt{3}}{2\sqrt{2}} & -\frac{5}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} & -\frac{1}{2\sqrt{2}} & -\frac{\sqrt{3}}{2\sqrt{2}} & \frac{5}{\sqrt{2}} \\ 0 & \frac{\sqrt{3}}{2} & -\frac{1}{2} & 4 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 0 \\ 1 \end{bmatrix} \\
&= \begin{bmatrix} \frac{4}{\sqrt{2}} \\ -\frac{4}{\sqrt{2}} \\ 4 \\ 1 \end{bmatrix} \\
\implies {}^0\underset{\sim}{\boldsymbol{x}} &= \begin{bmatrix} \frac{4}{\sqrt{2}} \\ -\frac{4}{\sqrt{2}} \\ 4 \end{bmatrix} \text{ cm}
\end{aligned}
$$

2c).

For the opposite case, that we have a point represented in cooridinate system 0 by $\underset{\sim}{\boldsymbol{x}} = \begin{bmatrix}1 & 0 & 0\end{bmatrix}^\top$ cm we apply the inverse transformation matrix that is on the form we found in (1). This gives

$$
\begin{aligned}
\begin{bmatrix} {}^1\boldsymbol{x} \\ 1 \end{bmatrix} &= {}^0T_1^{-1} \begin{bmatrix} {}^0\boldsymbol{x} \\ 1 \end{bmatrix} \\
&= \begin{bmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 & 5 \\ -\frac{1}{2\sqrt{2}} & -\frac{1}{2\sqrt{2}} & \frac{\sqrt{3}}{2} & -2\sqrt{3} \\ -\frac{\sqrt{3}}{2\sqrt{2}} & -\frac{\sqrt{3}}{2\sqrt{2}} & -\frac{1}{2} & 4 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 0 \\ 1 \end{bmatrix} \\
&= \begin{bmatrix} \frac{10+\sqrt{2}}{2} \\ -\frac{8\sqrt{3}+\sqrt{2}}{4} \\ \frac{16-\sqrt{6}}{4} \\ 1 \end{bmatrix} \\
\implies {}^1\underset{\sim}{\boldsymbol{x}} &= \begin{bmatrix} \frac{10+\sqrt{2}}{2} \\ -\frac{8\sqrt{3}+\sqrt{2}}{4} \\ \frac{16-\sqrt{6}}{4} \end{bmatrix} \text{ cm}
\end{aligned}
$$

2d).

The angular velocity vector represented in coordinate system 0 by $\begin{bmatrix}1 & 0 & 0\end{bmatrix}^\top$ is given by

$$\begin{aligned}
\begin{bmatrix}\boldsymbol{\omega}_{1,1}\\ 0\end{bmatrix} &= {}^0T_1^{-1}\begin{bmatrix}\boldsymbol{\omega}_{1,0}\\ 0\end{bmatrix}\\
&= \begin{bmatrix}\frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 & 5\\ -\frac{1}{2\sqrt{2}} & -\frac{1}{2\sqrt{2}} & \frac{\sqrt{3}}{2} & -2\sqrt{3}\\ -\frac{\sqrt{3}}{2\sqrt{2}} & -\frac{\sqrt{3}}{2\sqrt{2}} & -\frac{1}{2} & 4\\ 0 & 0 & 0 & 1\end{bmatrix}\begin{bmatrix}1\\0\\0\\0\end{bmatrix}\\
&= \begin{bmatrix}\frac{1}{\sqrt{2}}\\ -\frac{1}{2\sqrt{2}}\\ -\frac{\sqrt{3}}{2\sqrt{2}}\\ 0\end{bmatrix}\\
\implies \boldsymbol{\omega}_{1,1} &= \begin{bmatrix}\frac{1}{\sqrt{2}}\\ -\frac{1}{2\sqrt{2}}\\ -\frac{\sqrt{3}}{2\sqrt{2}}\end{bmatrix}
\end{aligned}$$

3.

A MATLAB fuction named `DH_homog` is implemented with the code shown in Listing 1. This function has two return values; the homogenous transformation matrix and the rotation matrix.

```
function [T, C] = DH_homog(theta, d, a, alpha)
    i=[1;0;0];
    k=[0;0;1];
    angle = [expm(theta*skew(k)) zeros(3,1); zeros(1,3) 1];
    offset = [eye(3) d*k; zeros(1,3) 1];
    length = [eye(3) a*i; zeros(1,3) 1];
    twist = [expm(alpha*skew(i)) zeros(3,1); zeros(1,3) 1];
    C = expm(theta*skew(k)) * expm(alpha*skew(i));
    T = angle*offset*length*twist;
end
```

Listing 1: MATLAB code to generate homogenous transformation matrix based on the Denavit-Hartenberg convention

4.

A sketch of the "home" position can be found in Figure 1.
The Jacobian for a manipulator like this can generally be expressed as

$$\underline{J}_{i-1} = \begin{cases} \begin{bmatrix} \underline{\boldsymbol{k}}_{i-1} \times (\underset{\sim}{\boldsymbol{o}_n} - \underset{\sim}{\boldsymbol{o}_{i-1}}) \\ \underline{\boldsymbol{k}}_{i-1} \end{bmatrix} & \text{when joint } i \text{ is revolute} \\ \begin{bmatrix} \underline{\boldsymbol{k}}_{i-1} \\ \mathbf{0} \end{bmatrix} & \text{when joint } i \text{ is prismatic} \end{cases} \tag{2}$$

as given in (10) in Salcudean's notes. This gives us the Jacobian

$$\underline{J} = \begin{bmatrix} \underline{J}_0 & \underline{J}_1 & \underline{J}_2 \end{bmatrix} = \begin{bmatrix} \underline{\boldsymbol{k}}_0 \times (\underset{\sim}{\boldsymbol{o}_3} - \underset{\sim}{\boldsymbol{o}_0}) & \underline{\boldsymbol{k}}_1 \times (\underset{\sim}{\boldsymbol{o}_3} - \underset{\sim}{\boldsymbol{o}_1}) & \underline{\boldsymbol{k}}_2 \\ \underline{\boldsymbol{k}}_0 & \underline{\boldsymbol{k}}_1 & 0 \end{bmatrix}$$

We know that sigularities occur every time $\underline{J}$ loses rank. As this happens when $\underline{\boldsymbol{k}}_0 \times (\underset{\sim}{\boldsymbol{o}_3} - \underset{\sim}{\boldsymbol{o}_0})$, $\underline{\boldsymbol{k}}_1 \times (\underset{\sim}{\boldsymbol{o}_3} - \underset{\sim}{\boldsymbol{o}_1})$ and $\underline{\boldsymbol{k}}_2$ are coplanar we firstly see that $\underline{\boldsymbol{k}}_0 \times (\underset{\sim}{\boldsymbol{o}_3} - \underset{\sim}{\boldsymbol{o}_0})$ and $\underline{\boldsymbol{k}}_1 \times (\underset{\sim}{\boldsymbol{o}_3} - \underset{\sim}{\boldsymbol{o}_1})$ are never coplanar because of the geometrical position of the two origins. Secondly we see that $\boldsymbol{k}_2$ is parallell with the plane spanned by $\underset{\sim}{\boldsymbol{o}_3} - \underset{\sim}{\boldsymbol{o}_1}$ and $\boldsymbol{k}_1$ and therefore its never coplanar with the cross product of these vectors, hence we never have any singularities with this configuration.

5.

5a).

The different coordinate frames are sketched and the completed table is found in Figure 2. I see now that it should have been in degrees, but I have filled the table with the radian values.

5b).

We know that the relationship between base $\{\underset{\sim}{\boldsymbol{o}_0}, \underline{C}_0\}$ and the end effector $\{\underset{\sim}{\boldsymbol{o}_6}, \underline{C}_6\}$ is given by

$$\begin{aligned} \begin{bmatrix} \underline{C}_n & \underset{\sim}{\boldsymbol{o}_n} \\ \mathbf{0}^\top & 1 \end{bmatrix} &= \begin{bmatrix} \underline{C}_0 & \underset{\sim}{\boldsymbol{o}_0} \\ \mathbf{0}^\top & 1 \end{bmatrix} {}^0T_1(q_1)\, {}^1T_2(q_2) \dots {}^{n-1}T_n(q_n) \\ \implies \begin{bmatrix} \underline{C}_6 & \underset{\sim}{\boldsymbol{o}_6} \\ \mathbf{0}^\top & 1 \end{bmatrix} &= \begin{bmatrix} \underline{C}_0 & \underset{\sim}{\boldsymbol{o}_0} \\ \mathbf{0}^\top & 1 \end{bmatrix} {}^0T_6 \end{aligned} \tag{3}$$

where the matrices ${}^{i-1}T_i$ is given by

$$
\begin{aligned}
{}^{0}T_1 &= \begin{bmatrix} \exp\left(\theta_1 \boldsymbol{k}\times\right) \exp\left(-\frac{\pi}{2}\boldsymbol{i}\times\right) & \mathbf{0} \\ \mathbf{0}^\top & 1 \end{bmatrix} \\
{}^{1}T_2 &= \begin{bmatrix} \exp\left(\theta_2 \boldsymbol{k}\times\right) \exp\left(\pi\boldsymbol{i}\times\right) & \exp\left(\theta_2 \boldsymbol{k}\times\right)\left(431.80\boldsymbol{i}\right)\mathrm{mm} \\ \mathbf{0}^\top & 1 \end{bmatrix} \\
{}^{2}T_3 &= \begin{bmatrix} \exp\left(\theta_3 \boldsymbol{k}\times\right) \exp\left(\frac{\pi}{2}\boldsymbol{k}\times\right) \exp\left(\frac{\pi}{2}\boldsymbol{i}\times\right) & \exp\left(\theta_3 \boldsymbol{k}\times\right)\left(-149.09\boldsymbol{k} + 20.32\boldsymbol{j}\right)\mathrm{mm} \\ \mathbf{0}^\top & 1 \end{bmatrix} \\
{}^{3}T_4 &= \begin{bmatrix} \exp\left(\theta_4 \boldsymbol{k}\times\right) \exp\left(\frac{\pi}{2}\boldsymbol{i}\times\right) & \exp\left(\theta_4 \boldsymbol{k}\times\right)\left(433.07\boldsymbol{k}\right)\mathrm{mm} \\ \mathbf{0}^\top & 1 \end{bmatrix} \\
{}^{4}T_5 &= \begin{bmatrix} \exp\left(\theta_5 \boldsymbol{k}\times\right) \exp\left(-\frac{\pi}{2}\boldsymbol{i}\times\right) & \mathbf{0} \\ \mathbf{0}^\top & 1 \end{bmatrix} \\
{}^{5}T_6 &= \begin{bmatrix} \exp\left(\theta_6 \boldsymbol{k}\times\right) & \exp\left(\theta_6 \boldsymbol{k}\times\right)\left(60.00\boldsymbol{k}\right)\mathrm{mm} \\ \mathbf{0}^\top & 1 \end{bmatrix}
\end{aligned}
\tag{4}
$$

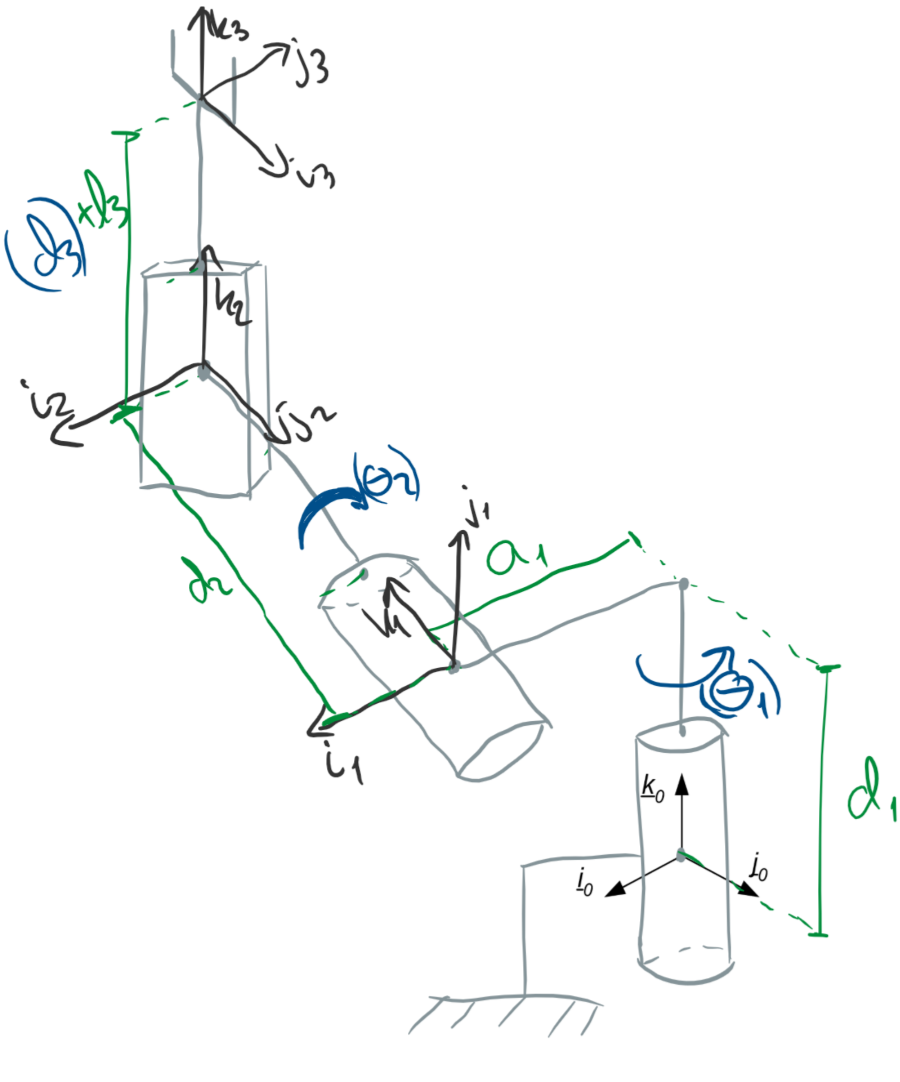


Figure 1: Sketch of “home” position based on the DH-table provided

Based on (3) and (4) we have the relationship between base $\{\underset{\sim}{\boldsymbol{o}}_0, \underline{C}_0\}$ and the end effector $\{\underset{\sim}{\boldsymbol{o}}_6, \underline{C}_6\}$ on the form presented by Salcudean's example 2.5.

5c).

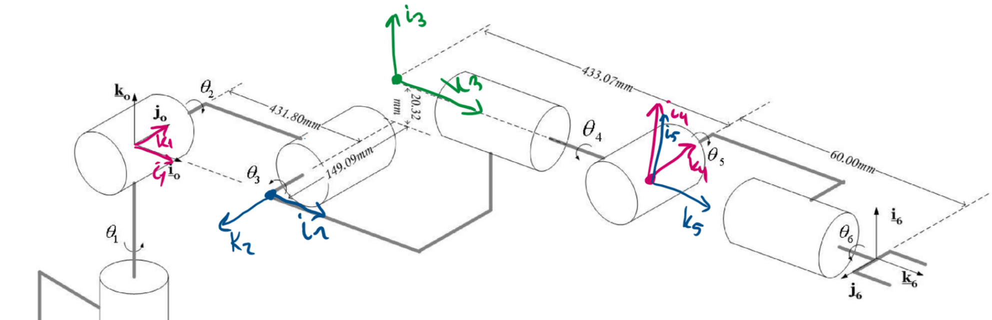
Using the same recipe as in (2) we can derive the Jacobian for the Puma 560 can be found by

$$\underline{J} = \begin{bmatrix} \underline{\boldsymbol{k}}_0 \times (\underset{\sim}{\boldsymbol{o}}_6 - \underset{\sim}{\boldsymbol{o}}_0) & \underline{\boldsymbol{k}}_1 \times (\underset{\sim}{\boldsymbol{o}}_6 - \underset{\sim}{\boldsymbol{o}}_0) & \underline{\boldsymbol{k}}_2 \times (\underset{\sim}{\boldsymbol{o}}_6 - \underset{\sim}{\boldsymbol{o}}_2) & \underline{\boldsymbol{k}}_3 \times (\underset{\sim}{\boldsymbol{o}}_6 - \underset{\sim}{\boldsymbol{o}}_3) & \underline{\boldsymbol{k}}_4 \times (\underset{\sim}{\boldsymbol{o}}_6 - \underset{\sim}{\boldsymbol{o}}_4) & \underline{\boldsymbol{k}}_5 \times (\underset{\sim}{\boldsymbol{o}}_6 - \underset{\sim}{\boldsymbol{o}}_4) \\ \underline{\boldsymbol{k}}_0 & \underline{\boldsymbol{k}}_1 & \underline{\boldsymbol{k}}_2 & \underline{\boldsymbol{k}}_3 & \underline{\boldsymbol{k}}_4 & \underline{\boldsymbol{k}}_5 \end{bmatrix}$$

Doing the row operation row1 $\leftarrow$ row1 + $(\underset{\sim}{\boldsymbol{o}}_6 - \underset{\sim}{\boldsymbol{o}}_4) \times$ row2 implies

$$\underline{J} \sim \left[\begin{array}{cccc:cc} \underline{\boldsymbol{k}}_0 \times (\underset{\sim}{\boldsymbol{o}}_4 - \underset{\sim}{\boldsymbol{o}}_0) & \underline{\boldsymbol{k}}_1 \times (\underset{\sim}{\boldsymbol{o}}_4 - \underset{\sim}{\boldsymbol{o}}_0) & \underline{\boldsymbol{k}}_2 \times (\underset{\sim}{\boldsymbol{o}}_4 - \underset{\sim}{\boldsymbol{o}}_2) & \underline{\boldsymbol{k}}_3 \times (\underset{\sim}{\boldsymbol{o}}_4 - \underset{\sim}{\boldsymbol{o}}_3) & 0 & 0 \\ \hdashline \underline{\boldsymbol{k}}_0 & \underline{\boldsymbol{k}}_1 & \underline{\boldsymbol{k}}_2 & \underline{\boldsymbol{k}}_3 & \underline{\boldsymbol{k}}_4 & \underline{\boldsymbol{k}}_5 \end{array}\right]$$

$$\sim \left[\begin{array}{c:c} J_{11} & \mathbf{0} \\ \hdashline J_{21} & J_{22} \end{array}\right]$$



DH Parameter	θ_i (deg)	d_i (mm)	a_i (mm)	α_i (deg)
Link 1	θ_1	0	0	$-\pi/2$
Link 2	θ_2	0	431.80	π
Link 3	$\theta_3+\frac{\pi}{2}$	-149.09	20.32	$\pi/2$
Link 4	θ_4	433.07	0	$\pi/2$
Link 5	θ_5	0	0	$-\pi/2$
Link 6	θ_6	60.06	0	0

Figure 2: Sketch of the coordinate frames according to the DH-convention

Based on this Jacobian we see that we will not get any wrist singularities with this setup, based on that $\underline{\boldsymbol{k}}_4$ will never be parallell to $\underline{\boldsymbol{k}}_5$ in J_{22}, but when we look at the J_{11} part we immediately see that singularities occur. As singularities occur when the elements in J_{11} are coplanar, and since we have 4 3×1 vectors they will be linearly dependent, hence we get singularities. These singularities will occur when $\boldsymbol{o}_4$ align with the $\boldsymbol{k}_0$ or $\boldsymbol{i}_0$ axes. These are respectively called shoulder and elbow singularities. When hitting the shoulder singularity we lose the ability to move the point $\boldsymbol{o}_4$ parallell to the $\boldsymbol{j}_0$ axis, and when the elbow singularity occur we lose the ability to move positively along the $\boldsymbol{i}_0$ axis.

5d).

The MATLAB code used to in subsection 5d) is listed in Listing 2. The user is prompted for six joint avariables, the Jacobian and the total homogenous transformation matrix is calculated aswell as the link origins is plotted by using the `plot3` command. As Figure 3 shows there are 5 different link origins, as expected by (4) since origin 0 and 1, and 4 and 5 is the same point. These points are he origins when $\theta_i = 0\ \forall i$.

```
close all
clear all
clc

%% 5d
i = [1;0;0];
j = [0;1;0];
k = [0;0;1];

inputangle = ['1 ';'2 ';'3 ';'4 ';'5 ';'6 '];
theta = [0 0 0 0 0 0];
for n = 1:6
    theta(n) = degtorad(input(inputangle(n)));
end

[T1,C01] = DH_homog(theta(1), 0, 0, -pi/2);
[T2,C12] = DH_homog(theta(2), 0, 431.8, pi);
[T3,C23] = DH_homog(theta(3) + pi/2, -149.09, 20.32, pi/2);
[T4,C34] = DH_homog(theta(4), 433.07, 0, pi/2);
[T5,C45] = DH_homog(theta(5), 0, 0, -pi/2);
[T6,C56] = DH_homog(theta(6), 60, 0, 0);

%Computation of homogenous transformation matrix
T= T1*T2*T3*T4*T5*T6;

```

```
C0 = eye(3); C1 = C0*C01; C2 = C1*C12;
C3 = C2*C23; C4 = C3*C34; C5 = C4*C45;

%Computing origins
o0 = [0;0;0];
o1 = o0;
o2 = o1 + C1*expm(theta(2)*skew(k))*431.8*i;
o3 = o2 + C2*expm(theta(3)*skew(k))*(-149.09*k + 20.32*j);
o4 = o3 + C3*expm(theta(4)*skew(k))*433.07*k;
o5 = o4;
o6 = o5 + C5*expm(theta(6)*skew(k))*60*k;

%Computing k-vectors
k0 = [C0(3,1); C0(3,2); C0(3,3)]; k1 = [C1(3,1); C1(3,2); C1
    (3,3)];
k2 = [C2(3,1); C2(3,2); C2(3,3)]; k3 = [C3(3,1); C3(3,2); C3
    (3,3)];
k4 = [C4(3,1); C4(3,2); C4(3,3)]; k5 = [C5(3,1); C5(3,2); C5
    (3,3)];

%Computing Jacobian
J0 =[skew(k0)*(o6-o0); k0]; J1 = [skew(k1)*(o6-o1);k1];
J2 =[skew(k2)*(o6-o2); k2]; J3 = [skew(k3)*(o6-o3);k3];
J4 =[skew(k4)*(o6-o4); k4]; J5 = [skew(k5)*(o6-o5);k5];
J = [J0 J1 J2 J3 J4 J5];

%% Plotting
x = [o0(1) o1(1) o2(1) o3(1) o4(1) o5(1) o6(1)];
y = [o0(2) o1(2) o2(2) o3(2) o4(2) o5(2) o6(2)];
z = [o0(3) o1(3) o2(3) o3(3) o4(3) o5(3) o6(3)];

figure
    hold on; view(3); grid on;
    plot3(x,y,z, '*');
    xlabel('$x$-coordinate', 'interpreter', 'latex');
    ylabel('$y$-coordinate', 'interpreter', 'latex');
    zlabel('$z$-coordinate', 'interpreter', 'latex');
```

Listing 2: MATLAB code used to generate homogenous transformation matrix for angles chosen by the user

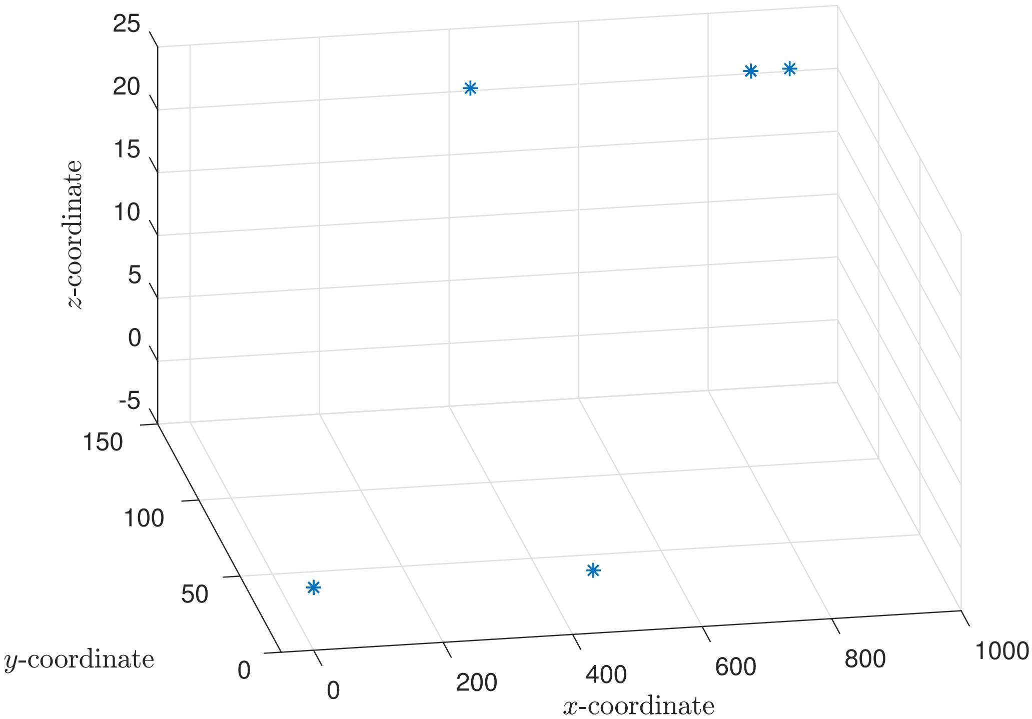
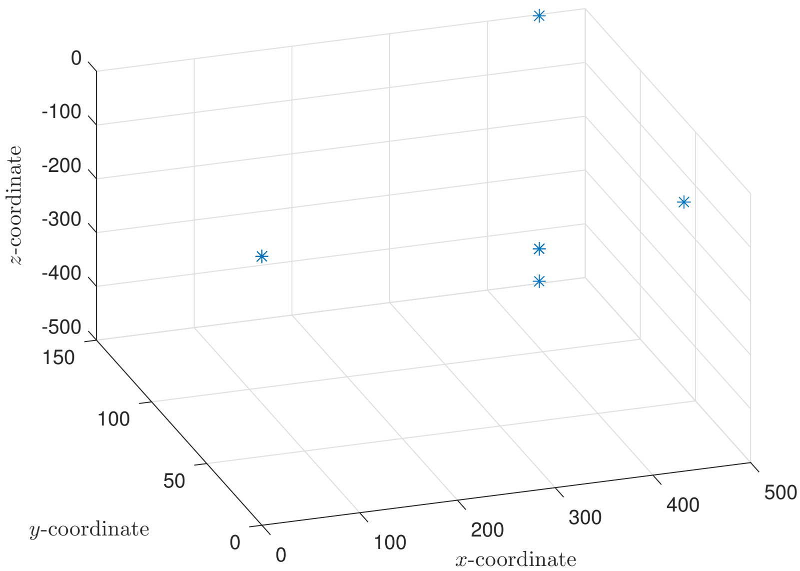


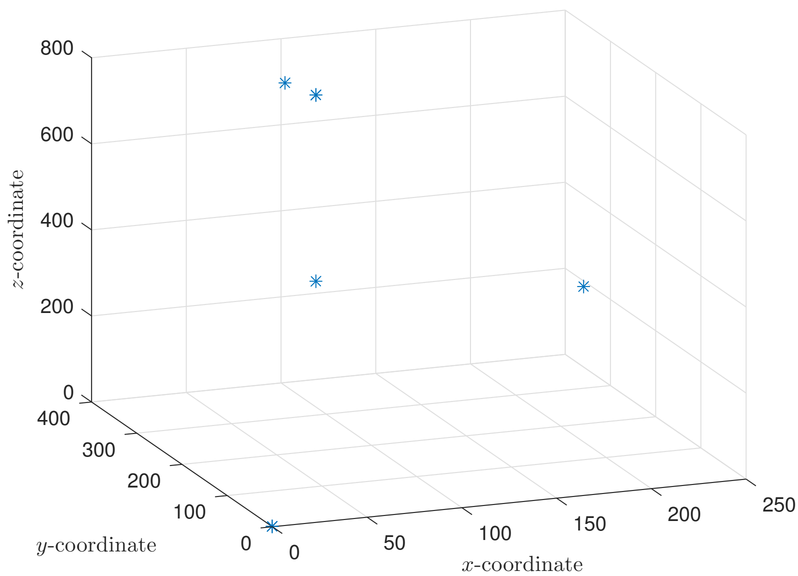
Figure 3: Location of the link origins with $\theta_i = 0 \ \forall i$

5e).

Link origins for $\boldsymbol{q}_A = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}^\top$ is shown in Figure 3. For $\boldsymbol{q}_B = \begin{bmatrix} 0 & 0 & -90 & 0 & 0 & 180 \end{bmatrix}^\top$ we have the link origins shown in Figure 4a, and for the angle set of $\boldsymbol{q}_C = \begin{bmatrix} 45 & -45 & 45 & 0 & -30 & 90 \end{bmatrix}^\top$ we see the origins in Figure 4b.



(a) Position of origins with angles given by $\boldsymbol{q}_B$



(b) Position of link origins with angles given by $\boldsymbol{q}_C$

Figure 4: Position of link origins with different link angle vectors