ELEC442
Assgt #1
Due Oct 1, 2017 (11:59pm)

- You may submit your work through Connect in teams of up to 3 individuals (include, as appropriate, a weight allocation table). Only one member needs to submit but be sure that names and student numbers for all team members are clearly indicated.
- Submit (a) scans of hand-written/typeset work with page numbers and (b) all necessary Matlab m-files. **NB: Document your code. Even if the output is correct, marks may be deducted for poorly documented and/or sloppy code.** Documentation referring to submitted pages is acceptable (e.g., "% see submitted homework, page 3, eqn (C) for equation derivation.").

1. A rigid motion transforms the coordinates $\boldsymbol{x}$ (of a point $\underset{\sim}{\boldsymbol{x}}$) to $\boldsymbol{y}$ according to the homogenous transformation $\begin{bmatrix} \boldsymbol{y} \\ 1 \end{bmatrix} = \begin{bmatrix} Q & \boldsymbol{d} \\ \boldsymbol{0}^{\boldsymbol{T}} & 1 \end{bmatrix} \begin{bmatrix} \boldsymbol{x} \\ 1 \end{bmatrix}$ where Q and $\boldsymbol{d}$ account for the rotation and translation, respectively. Determine the inverse of this transformation (find this as a matrix, expressed in terms of Q and $\boldsymbol{d}$, and does not require matrix inversion; e.g., instead of Q^{-1}, use Q^T). Explain briefly whether this inverse always exists (hint: think about whether, $\forall Q$ and $\forall \boldsymbol{d}$, the rank of the homogenous transformation matrix is less than 4).

2. Consider the homogenous transformation matrix ${}^{0}T_{1} = \begin{bmatrix} Q & \boldsymbol{d} \\ 0 & 1 \end{bmatrix}$ relating coordinate systems 0 & 1 in the usual way with $Q = \underbrace{\begin{bmatrix} 1/\sqrt{2} & 1/\sqrt{2} & 0 \\ -1/\sqrt{2} & 1/\sqrt{2} & 0 \\ 0 & 0 & 1 \end{bmatrix}}_{Q_1} \underbrace{\begin{bmatrix} 1 & 0 & 0 \\ 0 & -1/2 & -\sqrt{3}/2 \\ 0 & \sqrt{3}/2 & -1/2 \end{bmatrix}}_{Q_2}$ and $\boldsymbol{d} = \begin{bmatrix} -5/\sqrt{2} \\ 5/\sqrt{2} \\ 4 \end{bmatrix}$ centimeters.

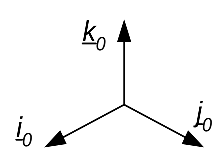
   NB: Q_1 and Q_2 are each rotation matrices.

   a) Determine the numeric values of the DH parameters (θ_1, d_1, a_1 and α_1) for this link.
   b) For the point represented in coordinate system 1 by $[1\ 0\ 0]^T$ centimeters, determine its representation in coordinate system 0.
   c) For the point represented in coordinate system 0 by $[1\ 0\ 0]^T$ centimeters, determine its representation in coordinate system 1.
   d) For the angular velocity vector represented in coordinate system system 0 by $[1\ 0\ 0]^T$ rad/s, determine its representation in coordinate system 1.

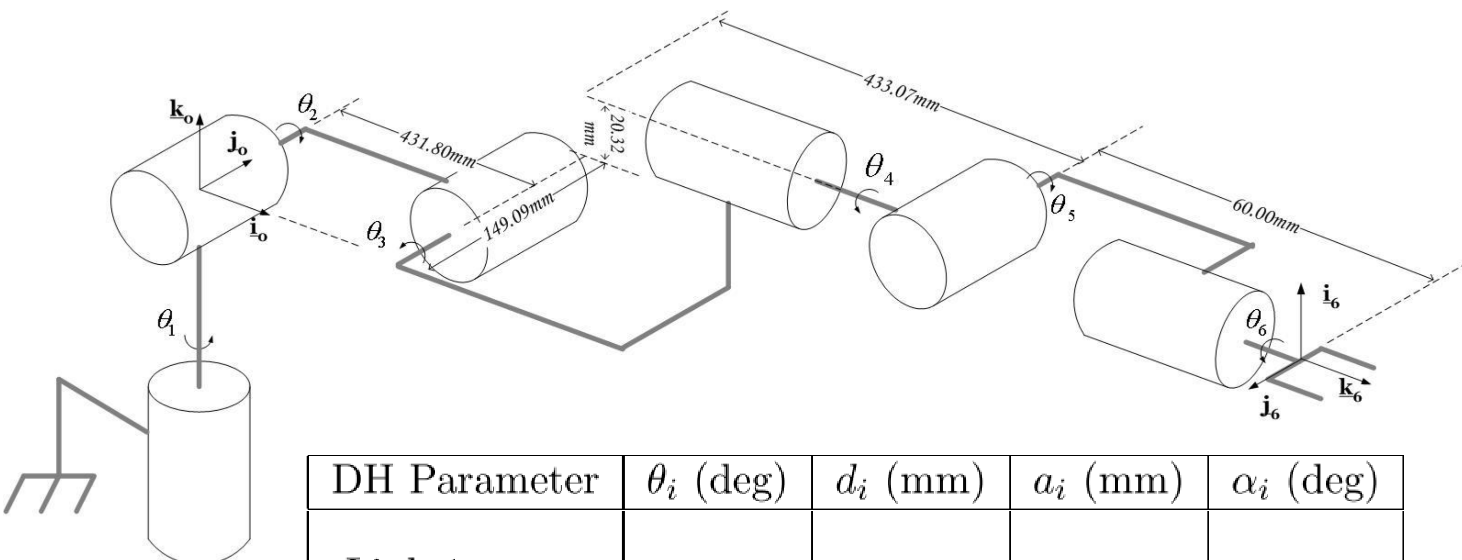
3. Create a Matlab function "DH_homog.m" that accepts the four D-H parameters between two links and outputs the homogenous transformation matrix that relates the two coordinate systems. Your function is to be called in your code for Question 5(d).

4. Sketch the "home" position of the manipulator described by the table of D-H parameters below, starting from the base coordinate system shown. Label all coordinate systems (only need to label **i** and **k** vectors in frames), dimensions, and joint displacements (show polarity). In the table, joint variables are enclosed in parentheses. Find the abstract expression for the geometric Jacobian and discuss the existence of singularities.

	θ_i	d_i	a_i	α_i
Link 1	(θ_1)	d_1	a_1	$\pi/2$
Link 2	(θ_2)	d_2	0	$-\pi/2$
Link 3	$\pi/2$	$(d_3)+l_3$	0	0



5. The Puma 560 has a reach of 0.92m and a payload capacity of 2.3kg, making it ideal for medium-to-lightweight assembly, welding, materials handling, packaging and inspection applications. Using the schematic on the next page, do the following:
   a. Directly on the schematic, assign coordinate frames according to the D-H convention (only need to label **i** and **k** vectors for each frame). Assume $\underline{C}_0$ and $\underline{C}_6$ as illustrated are in the ``home" position. Fill in Table 1 the values of the DH-parameters. For each joint, consider the positive rotation to be in the *right-handed sense.* (NB: This was not always the case in the notes).
   b. Compose a chain of transformations that give the relationship between the base ({$\boldsymbol{o}_0$, $\underline{C}_0$}) and end-effector ({$\boldsymbol{o}_6$, $\underline{C}_6$}) coordinate systems (use notation from Salcudean notes as was done for example 2.5 on p 31).
   c. Determine the manipulator Jacobian symbolically and discuss when singularities occur.
   d. Write a Matlab m-file which prompts the user for the sequence of 6 joint angles in degrees (e.g., "45,-45,45,0,-30,90""), then outputs the resulting homogenous transformation matrix (relating the base and end effector) and the manipulator Jacobian (relating joint velocities to end-effector velocities). As well, graphically plot the location of each link origin (e.g., using Matlab's ``plot3" function, indicate each origin with an "*") for the given joint angles.
   e. Use your code to compute end-effector transformations and manipulator Jacobians for the joint vector sets: $\boldsymbol{q}_A$=[0°;0°;0°;0°;0°;0°]^T, $\boldsymbol{q}_B$=[0°;0°;-90°;0°;0°;180°]^T, $\boldsymbol{q}_C$=[45°;-45°;45°;0°;-30°;90°]^T.



DH Parameter	θ_i (deg)	d_i (mm)	a_i (mm)	α_i (deg)
Link 1				
Link 2				
Link 3				
Link 4				
Link 5				
Link 6				