ELEC 442 - Introduction to Robotics

Assignment 1

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1

Given the homogenous transformation

$$\begin{bmatrix} \boldsymbol{y} \\ 1 \end{bmatrix} = \underbrace{\begin{bmatrix} Q & \boldsymbol{d} \\ \boldsymbol{0}^\top & 1 \end{bmatrix}}_T \begin{bmatrix} \boldsymbol{x} \\ 1 \end{bmatrix}$$

where Q and \boldsymbol{d} accounts for rotation and translation, respectively. We have that the inverse is given by

$$T^{-1} = \begin{bmatrix} \tilde{Q} & \tilde{d} \\ \mathbf{0}^\top & 1 \end{bmatrix}$$

where we know that $T^{-1}T$ is equal to the 4×4 identity matrix. This yields

$$T^{-1}T = \begin{bmatrix} \tilde{Q} & \tilde{d} \\ \mathbf{0}^{\top} & 1 \end{bmatrix} \begin{bmatrix} Q & d \\ \mathbf{0}^{\top} & 1 \end{bmatrix}$$
$$= \begin{bmatrix} \tilde{Q}Q & \tilde{Q}\mathbf{d} + \tilde{d} \\ \mathbf{0}^{\top} & 1 \end{bmatrix} = \mathbf{I}_{4\times 4}$$
$$\implies \begin{cases} \tilde{Q}Q & = \mathbf{I}_{3\times 3} \\ \tilde{Q}\mathbf{d} + \tilde{\mathbf{d}} & = \mathbf{0} \end{cases}$$
$$\implies \begin{cases} \tilde{Q} & = Q^{-1} = Q^{\top} \\ \tilde{\mathbf{d}} & = -\tilde{Q}\mathbf{d} = -Q^{\top}\mathbf{d} \end{cases}$$
$$\implies T^{-1} = \begin{bmatrix} Q^{\top} & -Q^{\top}\mathbf{d} \\ \mathbf{0} & 1 \end{bmatrix}$$

2

Considering the homogenous transformation matrix

$${}^{0}T_{1} = \begin{bmatrix} Q & \boldsymbol{d} \\ \boldsymbol{0}^{\top} & 1 \end{bmatrix}$$

with

$$Q = \underbrace{\begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0\\ -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0\\ 0 & 0 & 1 \end{bmatrix}}_{Q_1} \underbrace{\begin{bmatrix} 1 & 0 & 0\\ 0 & -\frac{1}{2} & -\frac{\sqrt{3}}{2}\\ 0 & \frac{\sqrt{3}}{2} & -\frac{1}{2} \end{bmatrix}}_{Q_2}$$

and

$$\boldsymbol{d} = \begin{bmatrix} -\frac{5}{\sqrt{2}} \\ \frac{5}{\sqrt{2}} \\ 4 \end{bmatrix} \text{cm}$$