Assignment 1

ELEC 442 - Introduction to Robotics

Sondre Myrberg

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THE UNIVERSITY OF BRITISH COLUMBIA

1.

Given the homogenous transformation

$$\begin{bmatrix} \boldsymbol{y} \\ 1 \end{bmatrix} = \underbrace{\begin{bmatrix} Q & \boldsymbol{d} \\ \mathbf{0}^\top & 1 \end{bmatrix}}_{T} \begin{bmatrix} \boldsymbol{x} \\ 1 \end{bmatrix}$$

where Q and $\boldsymbol{d}$ accounts for rotation and translation, respectively. We have that the inverse is on the form

$$T^{-1} = \begin{bmatrix} \tilde{Q} & \tilde{\boldsymbol{d}} \\ \mathbf{0}^\top & 1 \end{bmatrix}$$

where we know that $T^{-1}T$ is equal to the 4×4 identity matrix. This yields

$$\begin{aligned}
T^{-1}T &= \begin{bmatrix} \tilde{Q} & \tilde{\boldsymbol{d}} \\ \mathbf{0}^\top & 1 \end{bmatrix} \begin{bmatrix} Q & \boldsymbol{d} \\ \mathbf{0}^\top & 1 \end{bmatrix} \\
&= \begin{bmatrix} \tilde{Q}Q & \tilde{Q}\boldsymbol{d} + \tilde{\boldsymbol{d}} \\ \mathbf{0}^\top & 1 \end{bmatrix} = \mathbf{I}_{4\times 4} \\
\implies & \begin{cases} \tilde{Q}Q & = \mathbf{I}_{3\times 3} \\ \tilde{Q}\boldsymbol{d} + \tilde{\boldsymbol{d}} & = \mathbf{0} \end{cases} \\
\implies & \begin{cases} \tilde{Q} & = Q^{-1} = Q^\top \\ \tilde{\boldsymbol{d}} & = -\tilde{Q}\boldsymbol{d} = -Q^\top \boldsymbol{d} \end{cases} \\
\implies T^{-1} &= \underline{\begin{bmatrix} Q^\top & -Q^\top \boldsymbol{d} \\ \mathbf{0}^\top & 1 \end{bmatrix}}
\end{aligned}$$

2.

Considering the homogenous transformation matrix

$${}^{0}T_1 = \begin{bmatrix} Q & \boldsymbol{d} \\ \mathbf{0}^\top & 1 \end{bmatrix}$$

with

$$Q = \underbrace{\begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ 0 & 0 & 1 \end{bmatrix}}_{Q_1} \underbrace{\begin{bmatrix} 1 & 0 & 0 \\ 0 & -\frac{1}{2} & -\frac{\sqrt{3}}{2} \\ 0 & \frac{\sqrt{3}}{2} & -\frac{1}{2} \end{bmatrix}}_{Q_2}$$

and

$$\boldsymbol{d} = \begin{bmatrix} -\frac{5}{\sqrt{2}} \\ \frac{5}{\sqrt{2}} \\ 4 \end{bmatrix} \text{ cm}$$

2.a).

By obserwing the rotation matrices Q_1 and Q_2 we see that Q_1 is a simple rotation around the $\boldsymbol{k}$-axis. The angle of this rotation is given by $\theta = \arccos\left(\frac{1}{\sqrt{2}}\right) = \frac{\pi}{4}$. Q_2 is a simple rotation arond the $\boldsymbol{i}$-axis and the rotation angle is given by $\alpha = \arccos\left(-\frac{1}{2}\right) = \frac{2\pi}{3}$. To determine d_1 and a_1 we recognize that a homogenous transformation matrix can be written ass the product of four transformation matrices; angle, offset, length and twist. This gives us

$$
\begin{aligned}
{}^0T_1 = \begin{bmatrix} Q & \boldsymbol{d} \\ 0^\top & 1 \end{bmatrix} &= \underbrace{\begin{bmatrix} \exp(\theta\boldsymbol{k}\times) & \mathbf{0} \\ \mathbf{0}^\top & 1 \end{bmatrix}}_{\text{angle}} \underbrace{\begin{bmatrix} \boldsymbol{I} & d\boldsymbol{k} \\ \mathbf{0}^\top & 1 \end{bmatrix}}_{\text{offset}} \underbrace{\begin{bmatrix} \boldsymbol{I} & a\boldsymbol{i} \\ \mathbf{0}^\top & 1 \end{bmatrix}}_{\text{length}} \underbrace{\begin{bmatrix} \exp(\alpha\boldsymbol{i}\times) & \mathbf{0} \\ \mathbf{0}^\top & 1 \end{bmatrix}}_{\text{offset}} \\
&= \begin{bmatrix} \exp(\theta\boldsymbol{k}\times + \alpha\boldsymbol{i}\times) & \exp(\theta\boldsymbol{k}\times)(a\boldsymbol{i} + d\boldsymbol{k}) \\ \mathbf{0}^\top & 1 \end{bmatrix} = \begin{bmatrix} Q & \boldsymbol{d} \\ \mathbf{0}^\top & 1 \end{bmatrix} \\
\implies \exp(\theta\boldsymbol{k}\times)(a\boldsymbol{i} + d\boldsymbol{k}) &= \begin{bmatrix} -\frac{2}{\sqrt{2}} \\ \frac{5}{\sqrt{2}} \\ 4 \end{bmatrix} \\
\implies \exp(\theta\boldsymbol{k}\times) \begin{bmatrix} a \\ 0 \\ d \end{bmatrix} &= \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} a \\ 0 \\ d \end{bmatrix} = \begin{bmatrix} \frac{a}{\sqrt{2}} \\ -\frac{a}{\sqrt{2}} \\ d \end{bmatrix} = \begin{bmatrix} -\frac{5}{\sqrt{2}} \\ \frac{5}{\sqrt{2}} \\ 4 \end{bmatrix} \\
\implies \begin{bmatrix} a \\ d \end{bmatrix} &= \begin{bmatrix} -5 \\ 4 \end{bmatrix}
\end{aligned}
$$

And we have numeric values for all our four DH parameters.

2.b).

For the point represented in coordinate system 1 by $\underset{\sim}{\boldsymbol{x}} = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix}^\top$ cm we get the representation in system 0 given by

$$
\begin{aligned}
\begin{bmatrix} {}^0\boldsymbol{x} \\ 1 \end{bmatrix} &= {}^0T_1 \begin{bmatrix} {}^1\boldsymbol{x} \\ 1 \end{bmatrix} \\
&=
\end{aligned}
$$

3.

A MATLAB fuction named `DH_homog` is implemented with the code shown in Listing 1.

Listing 1: MATLAB code to generate homogenous transformation matrix based on the Denavit-Hartenberg convention

```
function T = DH_homog(theta, d, a, alpha)
```

```
    i=[1;0;0];
    k=[0;0;1];
    angle = [expm(theta*skew(k)) zeros(3,1); zeros(1,3) 1];
    offset = [eye(3) d*k; zeros(1,3) 1];
    length = [eye(3) a*i; zeros(1,3) 1];
    twist = [expm(alpha*skew(i)) zeros(3,1); zeros(1,3) 1];
    T = angle*offset*length*twist;
end
```