TTK4210 Advanced Control of Industrial Processes
Department of Engineering Cybernetics
Norwegian University of Science and Technology
Spring 2018 - Assignment 4
Due date: Friday 23 February at 16:00

1. Consider the system

$$G(s) = \begin{bmatrix} \frac{3(s+2)}{(s+3)(s+1)} & \frac{2}{s+1} \\ \frac{1}{s+2} & \frac{1}{s+3} \end{bmatrix} \tag{1}$$

   (a) Calculate the poles and zeros for the system (1). How many poles does the system have? Are there any right halfplane (RHP) zeros?

   (b) Find the input pole vectors and output pole vectors. Also find the pole directions. What can we say about the system's observability and controllability based on the pole vectors?

   (c) Find the relative gain array (RGA) for the system (1) and suggest a pairing. Design a decentralized controller based on two PI-controllers and simulate with a unit step in reference $y_{1,\,\mathrm{ref}}$ at $t = 5$ s and $y_{2,\,\mathrm{ref}}$ at $t = 10$ s.

   (d) Try to simulate with the other set of pairings (i.e. if you have chosen a diagonal pairing, try now to simulate with an off-diagonal pairing).

Appendix

Hermitian transpose

G^H (The *Hermitian transpose*, or *conjugate transpose*), represents the transpose of $\bar{G}$ (the matrix with complex conjugated entries), i.e.

$$G^H = (\bar{G})^T$$

In Matlab, G^H is calculated as `G'` and G^T is calculated as `G.'`.
Naturally, for a real matrix the relationship is $G^H = G^T$.

Use of RGA

The RGA matrix can be used for choosing a pairing of manipulated (input) and controlled (output) variables.
Using a pairing corresponding to a negative steady state RGA for loop k (for stable systems, with integral action in all loops), we know that *at least one* of the following will be true:

1. The overall system will be unstable
2. The system will be unstable if loop k is out of operation
3. Loop k is unstable on its own.

Therefore - since instability of the overall system is clearly unacceptable - for a 2x2 system with a pairing corresponding to a negative RGA, we must actually design one of the loops to be unstable on its own. Remember that both loops have the same RGA for 2x2 systems (only!). For a pairing corresponding to a negative RGA we therefore use the 'wrong' sign of the gain for one of the loops - hoping this will give a stable overall system.
For tuning of $|K_p|$ and T_i, other methods are needed.