Assignment 4

TTK4210 - Advanced Control of Industrial Processes

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1.

Considering the system

$$G(s) = \begin{bmatrix} \frac{3(s+2)}{(s+3)(s+1)} & \frac{2}{s+1} \\ \frac{1}{s+2} & \frac{1}{s+3} \end{bmatrix} \tag{1}$$

a.

The poles of the system can by Theorem 4.4 in the book be found by observing $\phi(s)$. This corresponds to the least common denominator of all non-identically zero minors of all orders of $G(s)$. By looking at (1) we have four minors of order 1, the elements of $G(s)$, and one minor of order 2, which is $\det\{G(s)\}$. This yields

$$\begin{aligned} \det\{G(s)\} &= \frac{3(s+2)}{(s+3)^2(s+1)} - \frac{2}{(s+2)(s+1)} = \frac{3(s+2)^2 - 2(s+3)^2}{(s+3)^2(s+2)(s+1)} \\ &= \frac{s^2-6}{(s+3)^2(s+2)(s+1)} \end{aligned}$$

We then easily see that

$$\phi(s) = (s+3)^2(s+2)(s+1)$$

and we have two poles at $s=-3$, one pole at $s=-2$ and one pole at $s=-1$.
The zeros of the system can by Theorem 4.5 in the book be found by looking at the greatest common divisor of all the numerators of all order-r minors of $G(s)$, where r is the normal rank of $G(s)$. In this case we have that r is 2, hence we must look at the second order minor $\det\{G(s)\}$. This gives us that the zeros of $\det\{G(s)\} = \frac{\prod_i (s-z_i)}{\phi(s)}$ are also the zeros of the system. In our case this gives us zeros at $s = \pm\sqrt{6}$, and we have one RHP-zero at $s=\sqrt{6}$.

b.

The pole vectors of a system is given by

$$\begin{aligned} u_{p_i} &\triangleq B^H q_i \\ y_{p_i} &\triangleq C t_i \end{aligned}$$

for respectively input pole vecor and output pole vector corresponding to the i'th pole and corresponding eigenvector. By using the code shown in Listing 1

Listing 1: Matlab code to generate pole vectors

```
g11 = tf([3 6],[1 4 3]); g12 = tf(2,[1 1]);
g21 = tf(1, [1 2]);       g22 = tf(1, [1 3]);
G = [g11 g12 ; g21 g22];

sys_ss = ss(G);
```

```
A = sys_ss.A; B = sys_ss.B; C = sys_ss.C; D = sys_ss.D;

[T, Po] = eig(A);
yp = C*T;
[Q, Pi] = eig(A');
up = B'*Q;
```

we get the output

```
up =

  -1.7889   -1.1094    1.0000         0         0
        0         0         0    2.0000    1.0000

   yp =

  -0.4160   -0.6708         0    1.0000         0
        0         0    1.0000         0    1.0000
```

which is the input and output pole vectors of the system. The pole directions are given as the normalized pole vectors, which in this case is trivial to compute as all pole vectors have only one non-zero entry.
By Theorem 4.1 we have that a system is controllable if and only if every pole p_i is controllable, and the pole p_i is controllable if and only if

$$u_{p_i} = B^H q_i \neq 0$$

From above we see that all columns of `up` are non-zero, hence the system is controllable. Similarly for observability we have by Theorem 4.2 that a system is observable if and only if every mode p_i is observable, and the mode p_i is observable if and only if

$$y_{p_i} = Ct_i \neq 0$$

As above we see that all columns of `yp` are non-zero, and our system is observable.

c.

The relative gain array of a system is given as

$$\mathrm{RGA}(G) = \Lambda(G) \triangleq G \times (G^{-1})^\top$$

where $\times$ denotes the element wise multiplication of the matrices G and $(G^{-1})^\top$.