TTK4210 Advanced Control of Industrial Processes
Department of Engineering Cybernetics
Norwegian University of Science and Technology
Spring 2018 - Assignment 5
Due date: Friday 2 March at 16:00.

Before doing this assignment, we advice you to read Example 10.7 in S&P.

Selecting controlled outputs

Given the cost function

$$J(u,d) = (u-2d)^2$$

and the possible measurements of the process

$$\begin{aligned} y_1 &= 0.2(u-d) \\ y_2 &= u \\ y_3 &= 8u-5d \end{aligned}$$

The magnitude of the expected disturbances W_d and expected implementation errors associated with individual controlled variables W_e are

$$\begin{aligned} W_d &= 1 \\ W_e &= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \end{aligned}$$

Note that W_e above gives the expected implementation errors for the three candidate measurements. When calculating the loss corresponding to using a single measurement, we select the corresponding diagonal element of W_e. When using a linear combination of measurements $z = Hy$, we use $\tilde{W}_e = HW_e$ when calculating the loss.

1. From (10.12) in S&P the worst-case loss for the selected measurement is given by

$$\max_{\left\|\begin{bmatrix} d' \\ e' \end{bmatrix}\right\|_2 \leq 1} L = \frac{1}{2}\bar{\sigma}\left(\begin{bmatrix} M_d & M_e \end{bmatrix}\right)^2 \tag{1}$$

   where

$$\begin{aligned} M_d &= J_{uu}^{1/2}(J_{uu}^{-1}J_{ud} - G^{-1}G_d)W_d \\ M_e &= J_{uu}^{1/2}G^{-1}W_e \end{aligned}$$

   We can write

$$\begin{bmatrix} M_d & M_e \end{bmatrix} = J_{uu}^{1/2}(HG^y)^{-1}H\begin{bmatrix} FW_d & W_e \end{bmatrix}$$

   where

$$F = -G^y J_{uu}^{-1}J_{ud} + G_d^y \quad , \quad G = HG^y$$

   and $z = Hy$ denotes the selected measurements used for control.

   With the *exact local method* the measurement with the lowest loss L is selected. Calculate L for the three possible measurements y_1, y_2 and y_3 and suggest which of the measurements that should be used ($z = y_i$). Do not use MATLAB or any other computer software.

2. Find a linear combination H of measurements, $z = Hy$, which minimizes the loss L. Instead of the numerical search proposed in S&P to find the optimal measurement combination H, you may use the analytical method proposed in section 3 of [1], obtainable from
   `http://www.nt.ntnu.no/users/skoge/publications/2009/alstad_extended_nullspace_jpc/JJPC852.pdf`.
   In short, the method in [1] - when cancelling a factor that does not affect the loss - simplifies to
$$H^T = (YY^T)^{-1}G^y$$
   where $Y = \begin{bmatrix} FW_d & W_e \end{bmatrix}$. What is the loss now? Compare to a).

3. (a) Use the *nullspace method* (S&P, p. 397 or Lecture notes, pp. 117–118) to find the optimal combination of measurements, H, and the corresponding loss. Compare your result with the other methods.
   (b) Use the *extended nullspace method* (Lecture notes, p. 118) to find the optimal combination of measurements, H, and the corresponding loss, when accounting for implementation error. Compare your result with the other methods.

References

[1] V. Alstad, S. Skogestad, and E. S. Hori. Optimal measurement combinations as controlled variables. *Journal of Process Control*, 19:138–148, 2009.

Appendix

G^H (The *Hermitian transpose*, or *conjugate transpose*), represents the transpose of $\bar{G}$ (the matrix with complex conjugated entries), i.e.

$$G^H = (\bar{G})^T$$

In Matlab, G^H is calculated as `G'` and G^T is calculated as `G.'`.
Naturally, for a real matrix the relationship is $G^H = G^T$.